Coherence, progression, and pathways in NCEA Mathematics and Statistics: Participation and achievement data analysed by school deciles, 2013-2019

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Abstract

Mathematics education plays an important role in social justice and equity. Access for all students to a coherent mathematics education that supports progression and pathways to future learning is crucial for reducing inequities in education. The National Certificate of Education Achievement [NCEA] was introduced to recognise a wider range of achievement and decrease inequity in educational outcomes. However, concerns have been raised that it has not achieved these goals. The overall goals of this study were, first, to develop an understanding of which NCEA achievement standards support progression and future pathways in Mathematics and Statistics; second, to analyse by school decile (which acts as a proxy for the socio-economic status of the communities that schools draw from) the patterns of participation and achievement in these NCEA standards between the implementation of these New Zealand Curriculum [NZC] aligned standards in 2013 until 2019. This is predominantly a quantitative research methods study with an initial qualitative component which established the evaluative framework of key standards for progression and future pathways used for the quantitative analysis. The research finds that there has been a decline in participation in many of the key standards in Mathematics and Statistics from 2013-2019, raising concerns about equity of opportunities and whether students are best prepared for progression and future pathways. Analysis by school decile confirms concerns about inequities for low socio-economic students. This study’s findings have implications for students’ progression and future pathways and for policy and professional development for teachers as the NCEA Change Package is implemented over the next few years.

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Glossary of key words and abbreviations

Achievement standards	In NCEA, achievement standards each have a credit value and have four grades associated with them: Not Achieved (a failing grade), Achieved, Merit, and Excellence.
Coherence (or coherent)	Course coherence is a term used to describe programmes or courses of learning which, through deliberate design, provide connections between conceptual areas to build disciplinary knowledge, and best provide for progression within a subject (Hipkins, Johnston, & Sheehan, 2016).
Decile (or School decile)	School deciles indicate the extent to which schools draw on students from low socio-economic communities (Ministry of Education, 2020c). Decile 1 schools are the 10% of schools which have the highest proportion of students from low socio-economic communities. The factors used to determine this are: the percentage of households with income in the lowest 20% nationally; percentage of employed parents in the lowest skill level occupational groups; household crowding; percentage of parents with no educational qualifications; and percentage of parents receiving income support benefits. These are determined using the latest New Zealand Census data. School decile can act as a proxy for the socio-economic status of the communities that the schools draw from (Haque, 2014).
Internally assessed standards	Internally assessed (unit or achievement) standards in NCEA are assessed through tasks set and marked by students' teachers.

Externally assessed standards	Externally assessed achievement standards in NCEA are set and marked by an external team. In Mathematics and Statistics these are time-bound examinations at the end of the year, except for AS1.2 (Algebra), which is assessed in September.
Key standards	I have coined the term 'key standards' for this research to represent the achievement standards in each of Mathematics and Statistics, which best support progression through the levels of NCEA and onto first year university study in these disciplines. The list of these standards can be found in Chapter 3.
Ministry of Education	Formed in 1989, the Ministry of Education is the government department which oversees the New Zealand education system.
National Certificate of Education Achievement [NCEA]	The national qualification for senior secondary school students in New Zealand. There are three levels and these are usually done in Years 11-13 (from 15-18 years of age).
New Zealand Curriculum [NZC]	The national curriculum for students in Years 1-13 in New Zealand, which was introduced in 2007 (Ministry of Education, 2007).
New Zealand Qualifications Authority [NZQA]	Established in 1989, NZQA is a Crown entity in New Zealand which overseas assessment and qualifications. It administers NCEA and provides quality assurance for non-university tertiary training providers.
Pathways	Pathways will be used in this research to convey the ability to transition to further study (Ministry of Education, 2007). For this research it will primarily be

	used for pathways onto the core first year university papers in Mathematics and Statistics.
Progression	Progression of learning is the increased levels of sophistication in students' thinking over time in a subject (Fonger, Stephens, Blanton, Isler, & Knuth, & Gardener, 2018). For this research, I will primarily use the term progression to describe the move from one NCEA level to another.
Unit standards	Unit standards in NCEA each have a credit value and generally have Not Achieved and Achieved as their two grades.
University entrance [UE]	UE is the minimum requirement to gain entrance to a New Zealand university. To gain UE students must have: NCEA Level 3, 14 credits in each of three approved subjects, 10 literacy credits at Level 2 or above (5 credits in reading, 5 credits in writing), and 10 Numeracy credits at Level 1 or above (either in achievement standards from an approved list or in a package of three numeracy unit standards) (NZQA, 2020d).
Vertical knowledge structure	Bernstein (1999) theorised that disciplines can be categorised into those with horizontal and vertical knowledge structures. Vertical knowledge structures are hierarchical in nature and require the careful sequencing and pacing of learning to build blocks in layers will lead to abstraction and generalisation. Mathematics and Statistics are examples of disciplines with vertical knowledge structures (Bernstein, 1999).

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Chapter 1: Introduction and Literature Review

1.1 Introduction

There are concerns in New Zealand about both the declining levels of achievement in Mathematics and the persisting inequity in achievement experienced by Māori, Pacific, and low socio-economic students (Collins, 2017; Haque, 2014; Long & Te, 2019). A New Zealand report on the 2018 PISA findings stated that since 2003, the average PISA test scores in Mathematics have declined from 523 to 494 points and New Zealand's educational ranking has dropped internationally (May, Jang-Jones, & McGregor, 2019). As well, the gaps between the average overall achievement in Mathematics and the averages for Māori, Pacific, and low socio-economic students have not decreased over time (May, Jang-Jones, & McGregor, 2019).

While there have been many changes in policy and educational practice in this time, one area receiving attention but surprisingly little research is the National Certificate of Educational Achievement [NCEA], New Zealand's senior secondary school qualification. Introduced in 2002, NCEA intended to recognise a greater range of achievement and improve inequities for Māori, Pacific, and low socio-economic students (Hipkins, Johnston, & Sheehan, 2016; Openshaw, 2009; Yoon & Rata, 2018). However, previous literature over the years has identified problems with how NCEA is implemented, such as undermining subject coherence, a loss of pathways due to credit collecting, the effect of assessment on teaching, and persisting inequities (for example: Black, 2001; Education Review Office, 2013; Elley, Hall, & Marsh, 2004; Locke, 2001; Locke, 2018; Munro, 2018; Rata & Taylor, 2015, Rawlins, 2010; Wilson, Madjar, & McNaughton, 2016; Yoon & Rata, 2018). The current NCEA Change Programme, introduced in 2019, intends to address many of these issues through the reform of NCEA to be implemented from 2025 (Ministry of Education, 2019c).

In this dissertation I explore the patterns of participation and achievement in the National Certificate of Education Achievement [NCEA] in Mathematics and Statistics between 2013 and 2019 as a way to understand the access to Mathematics learning in senior high schools and how these patterns might equip students for future pathways. My interest here is to understand how the concerns about inequities for low socio-economic students in Mathematics have manifested in the patterns of participation and achievement in NCEA

Mathematics and Statistics. This research analyses the data by school decile groups, which act a proxy for the socio-economic status of the communities that the schools draw from (Haque, 2014), to provide insight into the inequities in participation and achievement in Mathematics and Statistics in senior high school that exist for low socio-economic students. The goals of my research are: First, to develop ideas about what aspects of NCEA Mathematics and Statistics learning enhance progression and future pathways; and second, to examine how patterns of NCEA Mathematics and Statistics participation and achievement between 2013-2019 may shape students' potential progression and future pathways.

This dissertation consists of five chapters. Following this introduction, I present an overview of the literature on Mathematics education and examine why access to a rich Mathematics programme is an issue of social justice. I also present an overview of the history of NCEA and its critiques, and prior research into NCEA Mathematics and Statistics. In the second chapter, I describe the underpinning theoretical framework for this research and the analytical approach used. In Chapter 3, I propose an evaluative framework of the achievement standards which incorporate the knowledge required for successful progression within NCEA and pathways on to university.[1] Chapter 4 reports on the exploration of the NCEA participation and achievement in the standards proposed in the evaluative framework, analysed by decile groups, from 2013-2019. Chapter 5 concludes the dissertation by answering the research questions, proposes explanations for the findings, and considers the implications for both students and education policy implementation. Limitations and further research are also identified.

Key terms and abbreviations used in this research can be found in the *Glossary of key words and abbreviations* at the start of this dissertation for easy reference.

1.2 Literature review

There are three sections to this literature review. In the first section a brief description of Mathematics and Mathematics education is provided and an explanation as to why Mathematics education is an equity and social justice issue. The second section presents a

[1] I have intentionally used the word *university* rather than *higher education* because the focus of this research is on the transition to university courses in Mathematics and Statistics. Broadly, I recognise that rich mathematics and statistics programmes for progression also applies to other higher education institutions but narrowed the focus to universities to be more precise.

description of NCEA and its history and critiques to provide an understanding of the context for this dissertation and some of the issues this research hopes to shed light on. The final section of this literature review focuses on research that has been carried out on NCEA Mathematics and Statistics.

1.2.1 Mathematics education

The discipline of Mathematics[2] is described by Golding (2018) as the "exploration of, and connections between, patterns that often arise from the external world, their abstraction and their relationships as established through reasoning" (p. 461). This definition resonates strongly with the description provided in the New Zealand Curriculum [NZC] of the Mathematics and Statistics learning area (Ministry of Education, 2007). What is experienced in a Mathematics classroom by students should be a near-authentic experience with students having the opportunity to be novice mathematicians who are being inducted into the mathematics subculture (Golding, 2017). This is consistent with Bernstein's (2000) conception that school subjects are a recontextualisation of the parent subject, based on the epistemic knowledge of the discipline. Exposure to deep and robust conceptual understandings and experiencing mathematically valued ways of working are central to the development of disciplinary knowledge in Mathematics (Golding, 2018). This means that the learning experiences of students must go beyond facts and procedural skills and include modelling of real-world situations, problem solving, developing creativity, and building an understanding of how knowledge within the discipline is both discovered and invented (Golding, 2018).

Access to a mathematics education which supports this mathematical participation and progression for all students is a social justice and equity issue (Golding, 2018). The term 'critical filter' was coined to describe the social function of Mathematics, which has a role in the provision or obstruction of social justice within schools, due to the sorting and preparation role it has for further learning (or the restriction of further learning) in Mathematics and other subjects and therefore affecting future pathways (Ernest, 2007). Mathematics plays a significant role in ranking students (Bourdieu, 1998, as cited in Gates, 2018) and preparing and sorting them into different social stations (Ernest 2007). Research

[2] In the education literature the overarching subject is Mathematics, with Statistics as a component within this. The literature this section draws on refers to the subject as Mathematics.

shows there are inequitable outcome for students based on socio-economic and ethnic minority groups (Gates, 2020). Gates (2020) asserts that one of the ways that students are held back is through the restriction of curriculum when students are organised into groups or classes based on their ‘ability’. ‘Streaming’, as it is commonly referred to in New Zealand, is applied in Mathematics in many countries (Gates, 2020), including New Zealand (O’Callaghan, 2020), despite evidence showing it increases the gap in achievement outcomes between students (Gates, 2020; Pomeroy, Jones, Azarmandi, & Tolbert, 2020). There are also concerns that students are put into groups based on factors other than their prior achievement, such teachers’ perceptions of students’ behaviour and ability (Gates, 2020, Hood, 2020). For social justice and equity to take place, all students must be entitled to access a full and rich mathematics curriculum, without the downgrading of knowledge to match assessment or restriction of their curriculum through streaming.

1.2.2 The history of NCEA and critiques

Assessment has a powerful influence over curriculum, teaching, and learning (Black, 2001; Hipkins et al., 2016; Priestley & Sinnema, 2014). Johnston, Hipkins, and Sheehan (2017) assert that since its implementation in 2002, NCEA has dominated curriculum decisions in senior secondary schools.

The intended outcome for assessment reform in New Zealand which resulted in the introduction of NCEA was to provide “the enhanced flexibility for schools to offer broader and deeper learning for all students” (Mallard, 2001). It was introduced in response to an increasing demand for students to gain qualifications at a time when students were staying at school longer due to an increase in youth unemployment (Hipkins et al., 2016). Under the previous qualification system, which was norm-referenced, around 50% of students would gain the qualification at each level (Hipkins et al., 2016). There were concerns over the suitability of the previous system and inequities for Māori, Pacific, and low socio-economic students (Hipkins et al., 2016; Openshaw, 2009; Yoon & Rata, 2018). The government’s desire to recognise the achievement of a wider range of students and in a wider range of skills lead to the assessment reform (Hipkins et al., 2016).

NCEA is attained by students accumulating credits from achievement standards or unit standards, in both academic and vocational subjects and courses (Haque, 2014). Achievement standards have four grades associated with them: Not Achieved (a failing grade), Achieved, Merit, and Excellence. Unit standards generally have Not Achieved and Achieved as their two grades. When NCEA was introduced for school subjects, phased in from 2002-2004 for Year 11-13 students, curriculum subjects were assessed with achievement standards. Each subject typically had four to seven achievement standards each worth credits, for a total of 24 credits (Hipkins et al., 2016). Achievement standards were either internally or externally assessed. Internally assessed standards were assessed through tasks set and marked by students' teachers, whereas externally assessed standards were generally examinations that were set and marked by an external team and schools could use a combination of both types of standards when designing courses (Hipkins et al., 2016). Additional to gaining a certain number of credits for each level of qualification, students needed to meet the literacy and numeracy requirements (Hipkins et al., 2016).

Following a review of NCEA and the implementation of the New Zealand Curriculum [NZC] in 2010 (Ministry of Education, 2007), changes were made to NCEA which included the alignment of the achievement standards in curriculum subjects to the NZC (Haque, 2014; Hipkins et al., 2016). Some achievement standards underwent significant changes and unit standards in curriculum subjects were mostly removed as they allowed double-dipping, replaced if necessary with internally assessed achievement standards in the subject (Hipkins et al., 2016). Subjects were no longer limited to 24 credits and in some subjects, such as Mathematics and Statistics, the number of credits at Levels 1 and 2 almost doubled. Changes to the numeracy and literacy requirements occurred, with a requirement of 10 credits from either achievement standards or a bundle of three unit standards for each. Achievement standards which provided evidence of literacy and numeracy could come from a range of learning areas, not just from English and Mathematics, respectively (NZQA, 2019).

There have been concerns raised over NCEA and its implementation over the years. These started before its implementation (for example, Black, 2001; Elley, Hall, & Marsh, 2004; Locke, 2001). Haque (2014) argues that there was insufficient policy analysis, development, and research for NCEA given that it was such an ambitious and all-encompassing process of assessment reform and that issues in the early years could have been avoided if the processes had been carried out properly. Risk to course coherence and compromises in the development of disciplinary knowledge caused by fragmentation of the discipline, as well as concerns about students leaving schools without the development of clear pathways due to a focus on credit collection have been raised over the years by various authors (for example: Elley et al., 2004; Hipkins et al., 2016; Education Review Office, 2013; Locke, 2018; Munro, 2018; Rata & Taylor, 2015). Ongoing concerns about assessment reliability and validity, teacher workload, and teacher pedagogy have also been well-documented (for example: Alison, 2005; Elley, Hall, & Marsh, 2004; Locke, 2001; Rawlins, 2010).

Despite increased student achievement across all groups over the years, there remains inequity in achievement outcomes (Haque, 2014). For example, an analysis of University Entrance [UE][3] results from 2005-2017 found that despite increasing Level 2 NCEA achievement of students and a decreasing achievement gap between Māori and all students, the UE achievement rate was fairly consistent over this period (Yoon & Rata, 2018). 30% of Māori students were gaining UE, compared to 50% of all students and compromises in the quality of learning with an increasing quantity of credits gained was offered as the reason behind the increased gap between Level 2 and UE achievement for Māori and all students (Yoon & Rata, 2018). The preference for internally assessed standards, which are perceived to be easier, over externally assessed standards, and the avoidance of high-literacy standards are all strategies employed by schools to improve student achievement rates in a range of subject areas (Hipkins et al., 2016; McPhail, 2019; Munro, 2018; Wilson, Madjar, & McNaughton, 2016). This strategy of entering students into a greater proportion of internally assessed standards would appear to be successful for improving results in low decile schools with analysis showing that the achievement gap between internally assessed and externally assessed standards is greater at lower decile schools than higher decile schools (Hipkins et al., 2016; Riley, 2014). This approach risks course coherence (McPhail, 2019; Munro, 2018), and impacts on both students' further pathways and success at university (Jensen, Madjar, & McKinley, 2010; Wilson et al., 2016; Wilson, McNaughton, & Zhu, 2017). Māori, Pacific, and low socio-economic students are found to be disproportionately impacted by these strategies as they are more likely to end up in courses which limit their academic pathways (Jensen et al., 2010).

These critiques have led to growing concerns about NCEA and, following the election of the Labour-led government in 2017, an extensive review of NCEA was announced which intended to address these concerns. A taskforce was formed which released their first report

[3] For the definition and requirements of UE see the Glossary of key terms and abbreviations at the start of this dissertation.

with proposed changes in 2018 (NCEA Review Ministerial Advisory Group, 2018). In the most significant reform of assessment in New Zealand since the introduction of NCEA, all existing achievement standards are being replaced by the start of 2025 with the intention of improving well-being, equity, coherence, pathways, and credibility (Ministry of Education, 2020e). At the time this study was conducted, the process of development of the new standards had started, albeit slower than intended due to the impact of COVID-19. A number of subjects are having achievement standards and supporting material developed for feedback and four subjects (English, Religious Studies, Science, and Visual Arts) are ready for mini-pilots in 2021 (Ministry of Education, 2020e).

1.2.3 Mathematics and Statistics – The New Zealand Curriculum [NZC] and NCEA

The past two decades have been characterised by considerable change in mathematics curriculum policy. In 2007, the NZC (Ministry of Education, 2007) introduced significant changes to the learning area of Mathematics. One shift was the increased prominence of Statistics within the Mathematics curriculum. Whereas previously, Statistics was a strand of the national curriculum (Ministry of Education, 1992), the learning area was renamed Mathematics and Statistics in the NZC (Ministry of Education, 2007). Changes to Statistics itself were also significant with an increased emphasis on real investigations and real data, moving away from the algorithms and procedures which were the focus in the past (Pfannkuch, Regan, Wild, & Horton, 2010).

The alignment of NCEA with the NZC (Ministry of Education, 2007) resulted in the introduction of many new achievement standards in Mathematics and Statistics (Table 1.1) (NZQA, 2020c). At Level 3 there are three approved UE subjects in this learning area: Calculus, Statistics, and Mathematics, each requiring 14 achievement standard credits (NZQA, 2020b). For UE Calculus, the credits must come from the Mathematics sub-field at Level 3. UE Statistics requires the 14 Level 3 credits to come from the Statistics and Probability sub-field. Finally, UE Mathematics is a hybrid subject which can draw the required credits from both sub-fields at Level 3. The flexible approach in this learning area compounds some of the difficulties to ensure the participation in a coherent course for progression and pathways and is in part why more research into this area is needed.

Table 1.1: The number of standards and credits available in Mathematics and Statistics at Levels 1-3

	Level 1		Level 2		Level 3	
Sub-field	Number of standards	Number of credits	Number of standards	Number of credits	Number of standards	Number of credits
Mathematics	9	30	8	25	8[a, c]	32
Statistics and Probability	4	14	6	18	7[b, c]	28
Total	13	44	14	43	15	60

Source: NZQA, 2020c.

Notes: a - *standards that can contribute to UE Calculus.* b - *standards that can contribute to UE Statistics.* c - *standards that can contribute to UE Mathematics.*

Similar to patterns noted earlier across NCEA, there have been concerns about Mathematics and Statistics and the impacts on teaching, learning, and assessment practices due to the implementation of NCEA (Black, 2001; Hipkins & Neill, 2006; Rawlins, 2010). Early studies of NCEA implementation raised concerns about NCEA and standards-based assessment for Mathematics learning. In a report on NCEA implementation, Mathematics teachers reported doing fewer open-ended and higher order thinking tasks compared to before implementation (Hipkins & Neill, 2006). Black (2001) described the issue of fragmentation in standards-based assessment, using the example of Mathematics: "a set of good scores in every separate technique in mathematics does not inform the user about capacity to appraise a real problem and so select the relevant techniques and co-ordinate their application" (p. 7-8). He suggests that without the ability to bring the fragments together to make the discipline whole, the individual components are without value (Black, 2001). In a study of Level 2 Mathematics teachers, Rawlins (2010) questioned whether the intended outcome of enhanced learning was being delivered when considering the practice of teachers around feedback.

Studies have also raised equity concerns in NCEA Mathematics and Statistics. For example, Wilson, McNaughton, and Zhu (2017), found that students in 22 low-to-mid decile schools in Auckland and Northland had lower participation rates compared to national data in the externally assessed high-literacy standards in English, Biology, and Mathematics and there had been no improvement over time. Through classroom observations they found that these students also had very few opportunities to develop their subject-area literacy in class.

As my interest in this study was to consider what makes for coherent and future-enhancing pathways in Mathematics, I examined the limited studies which have focused on the NCEA standards which best prepare students for university study in Mathematics. As this greatly informed my answer to Research Question 1 (see Section 1.3 below), I have placed this in Chapter 3 (Section 3.3.1).

1.3 Research questions

While previous studies have pointed to growing concerns in the practices associated with NCEA, limited studies to date have examined what is happening within specific assessment practices in Mathematics over time. The research questions for this dissertation were developed based on gaps in the research literature. While the literature has identified issues associated with NCEA, in general or within a number of subjects, very little research has been done on the participation and achievement of students in NCEA Mathematics and Statistics over time, especially since alignment with the NZC in 2013. Therefore, the objective of this research project was to answer the following research questions:

> *RQ1: Which of the NCEA achievement standards in Mathematics and Statistics, herein known as the 'key standards', best provide for coherent programmes of learning and future pathways within these disciplines?*
>
> *RQ2: For the key standards in Mathematics and Statistics, what are the patterns of student participation and achievement from 2013 to 2019 in relation to school deciles?*

In the following chapter I will propose an evaluative framework for progression and pathways in NCEA Mathematics and Statistics to answer RQ1. In Chapter 4 I will analyse the NCEA participation and achievement data from 2013-2019 for the key standards, from the framework proposed in Chapter 3, to answer RQ2.

Chapter 2: Theoretical framing and methodology

This chapter begins by describing the theoretical framework that underpinned my approach in this study. I then outline the research methods I used to access the statistical data from NZQA and the analysis I employed in this in this three-phase mixed methods research.

2.1 Theoretical framework

One of the concerns which led me to this research was that the way NCEA appeared to be implemented was reducing the disciplinary value and coherence of mathematical knowledge. This drew me to a body of work on the significance of knowledge and disciplines in an era of constructivist educational philosophies (Priestley & Sinnema, 2014; Sinnema, 2014). In particular, my study was underpinned theoretically by a social realist position of powerful knowledge (Young, 2008), which is built upon Bernstein's (1999) theorisation on knowledge discourses and the structures of knowledge. These theoretical ideas will be briefly described and contextualised to Mathematics education.

2.2.1 Bernstein's knowledge discourses and structures

Bernstein's (1999) theorisation distinguishes between two types of knowledge discourses, or forms of knowledge: vertical and horizontal discourses. Horizontal discourses are the everyday or common-sense knowledge, which have features including being localised, context dependent, tacit, and multi-layered. It is subjective, that is it can be different across culture groups. Vertical discourses are forms of knowledge that we think of as the disciplines.

Vertical discourses are categorised into *vertical* and *horizontal* knowledge structures. These knowledge structures distinguish between two types of disciplines. Horizontal knowledge structures "consist of a series of specialised languages with specialised modes of interrogation and criteria for the construction and circulation of texts" (Bernstein, 1999, p. 163). The humanities and social sciences are examples of disciplines with horizontal knowledge structures. Development of horizontal knowledge structures involves the introduction of a new language, providing new perspectives, questions, and connections. Vertical knowledge structures are hierarchically organised. They are defined to have a "coherent, explicit, and systematically principled structure" (Bernstein, 1999, p. 161). Mathematics and Sciences are examples of disciplines with vertical knowledge structures.

Vertical knowledge structures develop by integrating knowledge at lower levels, building on these, and tending towards abstraction and generalisation. The implications of this are that to support learning in Mathematics, the concepts must be correctly sequenced and paced to build up the layers of learning (Brabrand & Dahl, 2009; Fonger, Stephens, Blanton, Isler, Knuth, & Gardiner, 2018; Hudson, 2018). These ideas suggest that a disciplinary approach to Mathematics can build a more coherent body of knowledge, which Young (2008) builds upon in his idea of 'powerful knowledge'.

2.1.2 Powerful knowledge

'Powerful knowledge' was a term coined by Young (2008) to describe the knowledge that allows students to make sense of the natural and social worlds, through various disciplinary ways of thinking, and participate fully in society through debate. It is seen as being powerful as it allows students to go beyond their individual experiences to better understand the world (Young, 2008).

Drawing on Bernstein's earlier work, Young (2014) suggests three criteria for defining powerful knowledge: First, it is distinct from the knowledge gained through everyday experiences, or 'common-sense' knowledge that is generally learned outside of school. Second, it is systematic. Concepts relate to each other in a systematic way in groups, which are the school subjects or disciplines. This allows for generalisability beyond specific contexts, making the knowledge context-independent. Finally, the knowledge is specialised. It has been developed by specialists in the area and has a clearly defined focus or field of enquiry, known as the discipline or subject.

Powerful knowledge is positioned as a curriculum principle to promote equity and social justice (Muller, 2016). A curriculum must be carefully designed to ensure that all students, regardless of their background, have the potential to acquire powerful knowledge. The selection, sequencing, and pacing of knowledge are important, as seen in Bernstein's model of knowledge (1999). Giving students too much choice in their curriculum, for example by choosing from a series of modules in a subject, risks undermining this and further disadvantages students from disadvantaged backgrounds (Young, 2008).

2.1.3 Powerful knowledge in Mathematics education

What Bernstein's (1999) theorisation means in a subject like Mathematics is that the less abstract, foundational layers must be learned sufficiently for the increasingly abstract ideas to be built upon them. This means that to progress, they need to reach a level of automaticity with the concepts in Mathematics (Johnston, Wood, Cherrington, Boniface, & Mortlock, 2020, under review), and students missing foundation blocks in the layers will struggle to learn ideas later on. Reiterating Bernstein's structure of knowledge ideas, curriculum coherence is important in vertical subjects: pace, sequencing and prerequisite knowledge are important to progress learning (Muller 2016).

Mathematics is positioned as powerful knowledge when it is of high epistemic quality, which is involves both the 'know that' (conceptual knowledge) and 'know how' (procedural knowledge, in this case mathematical thinking and the associated processes of creative reasoning) (Hudson, 2018). This contrasts with a fragmented, rule following, low epistemic quality approach with an over-emphasis on practice. Golding (2018) explains that a deep and robust conceptual understanding and an experience of mathematically valued ways of working are central to the development of powerful knowledge. This means that the learning experiences of students must go beyond facts and procedural skills and include modelling of real-world situations, problem solving, developing creativity, and building an understanding of how knowledge within the discipline is both discovered and invented (Golding, 2018).

This theoretical approach was important to this study as it helped me construct a framing of the type of mathematics learning that needed to be aspired to, and I use this for my first research question (see Chapter 3). In addition, the ideas of powerful knowledge shaped my approach to the data which I outline in the section below.

2.2 Research and analysis methods

This section describes the mixed methods approach employed, describing the three phases of research, and linking these to the Research questions introduced in section 1.3.

2.2.1 Mixed methods research

This project primarily uses a quantitative methods approach with a smaller qualitative component. Mixed methods research has been chosen as a pragmatic approach which allows the strategic choice of qualitative and quantitative data to serve multiple purposes (Johnson & Christensen, 2012). The rationale for using a mixed methods design in this research is primarily development, as it allowed me to develop a framing construct and then apply it empirically in sequential phases (Greene, Caracelli, & Graham, 1989). The research will take place in three sequential phases which are described section 2.2.2 below.

2.2.2 Phases of research

There were three phases to this research. The first involved the development of an evaluative framework. Second, I undertook an analysis of NCEA Mathematics and Statistics participation and achievement data for individual standards and combinations of standards between 2013-2019. Finally, I conducted an analysis of differences in the participation and achievement of students in standards and programmes supporting the development of powerful knowledge by decile groups. These three phases are discussed in detail below.

Phase 1: Development of an evaluative framework: The first phase of this research addressed Research Question 1 (section 1.3) and involved the development of an evaluative framework that sought to outline a suite of achievement standards in NCEA that could provide coherence, and support progression and pathways in Mathematics and Statistics. Phase 1 had three sources of data that were sought to inform the analysis of the NCEA standards in Mathematics and Statistics. The first source of data for this qualitative phase was the research literature on developing powerful knowledge in Mathematics education and on the key concepts for progression in Mathematics and Statistics to identify the important concepts for building disciplinary knowledge (see for examples, Golding, 2017; Golding, 2018; Hudson, 2018). The second source of data was the research literature on progression within NCEA and on to university. This included the identification of which Level 3 Mathematics achievement standards may best predict success in first year university study in core Mathematics papers in previous research based in New Zealand (James, Clemency, & Williams, 2008; Johnston, Wood, Cherrington, Boniface, & Mortlock, 2020, under review; The University of Auckland Planning Unit, 2017), literature about the development of mathematical and statistical literacy through NCEA standards (Wilson & McNaughton, 2014;

Wilson, McNaughton, & Zhu, 2017), and the types of standards predicting success in the progression through NCEA in other hierarchical knowledge structure disciplines (Johnston, Hipkins, & Sheehan, 2017). The third source of data in Phase 1 came from investigating entry criteria on New Zealand university websites. This was important as the research into progression in Mathematics and Statistics within NCEA is limited. The required or recommended NCEA standards (and grades if applicable) for the core Mathematics and Statistics first year papers at New Zealand universities were collected from universities' websites. The core papers being the ones defined as being the minimum papers required for a major in Statistics or Mathematics. These helped to confirm the challenge universities were finding in relation to entry criteria relating to NCEA and university courses.

These three sources of data were combined with knowledge from my experiences working across a variety of roles in Mathematics and Statistics education over the past 11 years as a secondary school teacher, resource developer, curriculum leader, and senior teaching fellow in Statistics. The NCEA achievement standards at Levels 1-3 were examined and the key achievement standards for progression from Level 1 to Level 3 Calculus or Statistics and then pathways onto first year university study in these disciplines were proposed. A variety of data sources allowed for convergence within this evaluative framework for the New Zealand assessment context to be reached. The proposed evaluative framework was used for this study to examine student uptake of those standards and is intended to be open to feedback and critique by other educators and researchers.

Phase 2: Analysis of NCEA Mathematics and Statistics participation and achievement data from 2013-2019: The second phase involved gaining access to the large databases on NCEA participation held by the New Zealand Qualifications Authority [NZQA] and involved quantitative analysis of these data. An analysis of student participation and achievement data in the key standards in Mathematics and Statistics proposed in the evaluative framework in Phase 1 above from 2013-2019 was carried out to answer RQ2. This analysis used existing data for all students participating in NCEA from 2013-2019, sourced from NZQA. The year 2013 was used as the starting point for analysis as it was the first year that the NZC-aligned standards were implemented across all three levels so allowed a common starting point. The latest complete data I could source was for 2019.

All analysis was done using Excel after SPSS was used to extract from the original file the data on all the Mathematics and Statistics standards organised into decile groups 1-10. The numbers attempting each of the key standards, and their combinations at each level, count only the students who were entered for the standard in the typical year level of study. For example, Year 11 students doing Level 1 standards. This was done so that the numbers could be converted into roll-based participation rates using the data available from the Education Counts website (Ministry of Education, 2020d), as the numbers of students from each school decile are different and change from year to year. The roll-based numbers used in this analysis can be found in the Appendix A. Each result was indexed to a student using an arbitrary number unique to that student, allowing each student's results to be identified as belonging to the same person. This allowed for the combinations of standards being done by students to be determined.

Phase 3: Analysing for differences in the participation and achievement of students in programmes supporting the development of powerful knowledge by school decile: This phase looked in more detail at the effect that school decile, as a proxy for socio-economic status, had on the participation of students in the key standards. It focused on comparing the participation and achievement of students from decile 1 and decile 10 schools to illustrate the socio-economic extremes that exist in New Zealand education. To simplify the presentation of findings the comparisons focused on the start and end of the analysis period, rather than all the points in between, in order to simplify the key messages and communication of these. This project had initially intended to disaggregate the data by gender and ethnicity as well but this was deemed to be outside of the scope of this dissertation.

2.2.3 Ethical considerations

Ethical approval was not required from the Human Ethics Committee from Victoria University of Wellington as existing data was sourced from NZQA and public websites. Discussion about ethics were held with NZQA before receiving the data to ensure that the data provided by NZQA did not allow individual students to be directly identified. Unique identifiers other than through their National Student Number were used. However, as the data could indirectly lead to results and achievements of individual students to be identified, certain precautions were required. These included: storing data privately and confidentially with password protection, limiting access to the data, destroying data by 20 December 2020, and ensuring reporting does not compromise the privacy of any individual (for example by not publishing data with less than five individuals in a cell). These conditions were presented to me by NZQA as a condition of using the data which I readily agreed with in consultation with my supervisor.

2.2.4 Limitations

This was a relatively small-scale study that was conducted for a 60 point dissertation and therefore had a number of limitations in timeframes, scope, and scale. First, the proposal of the evaluative framework which suggest a set of key standards for progression and pathways to university is necessarily a tentative one and open to debate and critique by other educators and researchers. This study's application of this framework can be seen as a pilot study and further testing using New Zealand data and the NCEA assessment system is recommended. Its application to different assessment contexts would also enhance its credibility and generalisability.

Second, my study assumes that the achievement standards act as a proxy for the curriculum and learning of students. Whilst one would hope that assessment was not sum total of curriculum, we know that assessment is one of the strongest drivers of what ends up being taught, and NCEA in particular, has had a significant influence on what is taught, often at the expense of a broad and rich curriculum (Black, 2001; Hipkins, Johnston, & Sheehan, 2016; Johnston et al., 2017; Priestley & Sinnema, 2014). It is not uncommon for students to opt out of the assessment of standards (Alison, 2005; Cherrington, Johnston, Wood, Mortlock, & Boniface, 2018), which may lessen their learning in that subject, and therefore a study of which achievement standards are undertaken tells us a fair bit (but not everything) about their mathematics experiences in senior high school.

This analysis left out a number of groups, such as students participating in New Zealand secondary school courses that solely use alternative qualification systems such as Cambridge examinations and International Baccalaureate as the focus was on NCEA. Also, excluded from the analysis are students who attend a school without a decile rating (Te Aho o Te Kura Pounamu, formerly The Correspondence School), new schools, and independent (private) schools) or who are participating in standards a level higher or lower than their year level, for example Year 12 students doing Level 1 standards. However, the numbers in these categories are small. It is not uncommon for Year 10 students to sit one or two NCEA assessments in

Mathematics and Statistics. However, these are usually entered in Year 11 and will count still towards their programme at Level 1.

Chapter 3: Identification of the key standards in Mathematics and Statistics

3.1 Introduction

The purpose of this chapter is to answer RQ1: *Which of the NCEA achievement standards in Mathematics and Statistics, herein known as the 'key standards', best provide for coherent programmes of learning and future pathways within these disciplines?*

This section is concerned with forming an evaluative framework which consists of the NCEA achievement standards which best support course coherence, student progression, and future pathways. These standards will be called the *key standards*. This does not suggest that other standards available from the suite of achievement standards at each level do not provide valuable learning, but the key standards have been identified as a possible strong option amongst a plethora of possible standards to choose from. There are a number of terms used in this section: *progression*, *pathways*, and *coherence*, which are defined in the *Glossary of key terms and abbreviations* at the front of this dissertation.

This chapter will examine New Zealand universities' prerequisites for core papers in Mathematics and Statistics, the literature on key concepts in Mathematics and Statistics required for progression, and the NCEA progression literature to propose an evaluative framework of key standards.

3.2 International literature on Mathematics education

As discussed in Chapter 2, mathematics students need to develop conceptual understanding (the 'know that') and experience mathematically valued ways of thinking to develop their 'know how', such as problem solving, modelling real world situations, developing creativity (Golding, 2018; Hudson, 2017). Given its vertical knowledge structure (Bernstein, 1999), effective mathematics education requires the careful design of sequential blocks of learning and appropriate pacing, under the guidance of specialist teachers (Young & Muller, 2013).

Mathematics education: I have positioned access to a broad and rich Mathematics education as a social equity issue due to its role of sorting and ranking students both within school and in the community (Bourdieu, 1998, as cited in Gates, 2018; Ernest, 2007). Further, Algebra acts as a 'gatekeeper' within the subject (Fonger, Stephens, Blanton, Isler, Knuth, & Gardiner, 2017; Knuth, 2016), stopping students from progressing if they do not participate or achieve in Algebra at early stages. In America, Algebra plays an important role in determining who attends university and in students becoming mathematically literate which contributes to providing economic access and full citizenship (Occupy Math, 2018). Learning Calculus requires prior understanding of variables and functions (Kidron, 2020), which are fundamental to Algebra. There are many key concepts in Calculus which are repeatedly revisited, such as tangents, limits, derivatives, and integrals, which cause students to have cognitive difficulty both at high school and at university (Kidron, 2020). Algebra has a strong presence in the NZC at all levels and Calculus is introduced into the NZC at Curriculum Level 7 (Year 12) (Ministry of Education, 2007).

Statistics education: Ben-Zhi (2020) stated that there is widespread agreement in the statistics education community about what needs to be included in an effective statistics programme at high school level: exploring patterns in data, planning and conducting a study (including sampling), exploring random phenomena using models, probability and simulation, and statistical inference. Further, research identifying threshold concepts in university statistics courses found regression, chi-square testing, hypothesis testing, confidence intervals, p values, analysis of variance all presented challenges to students (Bulmer, O'Brien, & Price, 2007; Khan, 2014). Some concepts, such as regression and confidence intervals, are introduced in the NZC. Other concepts are beyond what is done in high school Statistics but the required prior knowledge is taught in the NZC.

3.3 Progression and pathways in Mathematics and Statistics, the New Zealand context.

This section considers both the research literature on progression and pathways in NCEA Mathematics and Statistics, and the prerequisites for first year university study in New Zealand in core papers in Mathematics and Statistics, to understand what is required during senior high school for successful study at university.

3.3.1 Literature on NCEA progression and pathways in Mathematics and Statistics

A study carried out pre-alignment of NCEA to the NZC investigated which achievement standards were the most important for predicting success in first year Mathematics at the University of Canterbury (James, Montelle, & Williams, 2008). They found the Differentiation and Integration standards were the most important. Further, the quality of the results was investigated and it was found students needed Merit or Excellence in these standards to maximise their chances of success in the first-year courses. Although this work concerns the old version of the standards, it is still suggestive of the importance of acquiring conceptual knowledge in differentiation and integration to increase success in the core papers at university. James et al.'s (2008) findings around grade quality were consistent with those of Shulruf, Hattie, & Tumen (2008) who investigated several alternative models for predicting first-year university results across a range of subject areas and found that the grades obtained in NCEA at Level 3 were an important predictor of success at university.

The importance of coherent mathematics programmes has also been the focus of a number of other studies in New Zealand. Since alignment of NCEA with the NZC, analysis was carried out at The University of Auckland to gain insight into the preparation required for success in first year courses in Mathematics (The University of Auckland Planning Unit, 2017). Two Level 3 standards were identified as key indicators of success in the core Mathematics courses: AS3.6 (Differentiation) and AS3.7 (Integration). Further, they found that a Merit or Excellence grade was important as students with this level of achievement had at least 80% chance of success in the university courses. Internally assessed standards were not found to be predictive of success in first year Mathematics courses, although the authors state that this does not mean the content is irrelevant (The University of Auckland Planning Unit, 2017).

Confirming this finding, a further recent study of first-year students at Victoria University of Wellington found that the better predictors of success in first year core Mathematics courses were the performance in the NCEA external Calculus standards at Level 3 rather than the internally assessed standards (Johnston, Wood, Cherrington, Boniface, & Mortlock, 2020, under review). The explanation offered was that the time-limited examinations, which are the method of assessment in externally assessed standards for this subject, play an important role in developing automaticity in Mathematics (Johnston et al., 2020). This automaticity is essential for knowledge building in a discipline such as Mathematics which, employing Bernstein's (1999) theory, has a hierarchical knowledge structure, where knowledge is built in layers and requires the less abstract knowledge to be well-understood before more abstract knowledge can be acquired. The paper does not distinguish if some of the Calculus externals were more strongly associated with success in first year Mathematics than others. This study also examined a number of other NCEA subjects, which helped to examine findings through Bernstein's (1999) notion of structures of knowledge. First year university performance in core Chemistry courses, a discipline which also has a hierarchical knowledge structure, was also found to be more strongly associated with external assessment than with internal assessment. This contrasted with first year courses in three social sciences and humanities disciplines which Bernstein (1999) characterised as having horizontal knowledge structures: English, Geography and History, where university performance was more strongly associated with NCEA Level 3 internal assessment performance than external assessment performance. These findings were consistent with an earlier study on NCEA achievement in History and Biology (Johnston, Hipkins, & Sheehan, 2017). For Biology, performance in the externally assessed standards at NCEA Levels 1 and 2 was a stronger predictor of success in subsequent levels than the internally assessed standards. The opposite was true for performance in the History NCEA achievement standards, with the internally assessed standards more strongly predicting success. This study affirms the role of the externally assessed standards for predicting future success in subjects with hierarchical knowledge structures and points to the importance of these distinctions in knowledge structures.

Wilson and McNaughton (2014) have argued for the inclusion of high subject-area (disciplinary) literacy standards in students' courses as these standards are important to support them to develop their literacy and disciplinary knowledge and to be successful in further study. They identified several externally assessed high-literacy standards in Mathematics and Statistics: AS1.6 (Geometric Reasoning), AS1.12 (Probability), AS2.6 (Algebra), AS2.12 (Probability), AS3.12 (Statistical Reports), and AS3.13 (Probability),[4] which can be used to profile senior students' subject-area literacy.

[4] For the full titles of the standards and their details see NZQA (2020c).

3.3.2 New Zealand university courses and their prerequisites

As the studies described above have shown, students who have completed the externally assessed standards in Level 3 Calculus, particularly AS3.6 (Differentiation) and AS3.7 (Integration) are better prepared for the core papers in first year university Mathematics, especially if they have gained Merit or Excellence grades.

To add to these findings, I have sourced the prerequisites for each university in New Zealand which offers a major in Mathematics and Statistics to understand what is perceived to be important prior learning for successful study at university. Many universities have multiple pathways to majoring in these subjects so I have chosen to select the core papers with the lowest entry requirements which satisfy the major requirements. The findings can be found in Appendix B.

Mathematics: The prerequisites and recommended NCEA standards at Level 3 for entry into the core first year papers in Mathematics differ between universities. However, the externally assessed AS3.6 (Differentiation) and AS3.7 (Integration) feature strongly in the list of required standards, with three of the universities (The University of Auckland, Victoria University of Wellington, and University of Otago) requiring both these standards and a further two universities (University of Waikato and Massey University) requiring two out of three of AS3.5 (Algebra), AS3.6 (Differentiation), and AS3.7 (Integration). In response to students not having these prerequisites, a reflection on the fragmentation caused by the flexibility of NCEA, most universities offer several bridging papers in Mathematics. The University of Auckland is an example of this and their flow chart of options can be viewed in Appendix C. This shows that universities are struggling to compensate for patchy NCEA courses and that they recognise the need for foundational learning.

Statistics: With the core papers in Statistics there is very little by way of Level 3 Statistics achievement standards prerequisites. The University of Canterbury is the only university to have Statistics to Year 13 down as a prerequisite but specific standards are not given. The University of Auckland has no prerequisites given on its website but the

CensusAtSchool New Zealand website,[5] run by the Department of Statistics at The University of Auckland, includes a recommendation that AS3.9 (Bivariate), AS3.10 (Inference), and 3.10 (Probability) are included in a Year 13 course to prepare students for studying Statistics at university (The University of Auckland, Department of Statistics, 2020). Many universities have a mathematics component to their Statistics degree, which requires varying levels of prior knowledge in Mathematics. The University of Otago has set the highest minimum standard for entry with AS3.6 (Differentiation) and AS3.7 (Integration) required.

3.4 The key standards which best provide for coherency, progression, and pathways in Mathematics and Statistics

As a result of my analysis of previous research and university entry requirements I have proposed a framework for the key standards for progression in NCEA Mathematics and Statistics and to support pathways to university (Table 3.1 and Figure 3.1). This framework aims to propose the minimum number of standards per level in each of Mathematics and Statistics, not a full programme of learning, recognising that the flexibility of NCEA means that there are many ways to design courses to build knowledge in these subjects. It might be that at Level 1, all four key standards (both Mathematics and Statistics) could be included in a programme to prepare students for specialisation at higher levels. This framework is open to feedback and critique by other educators and researchers.

[5] The CensusAtSchool NZ website is a joint collaboration between the Department of Statistics, StatsNZ, and the Ministry of Education and curates statistics resources at all curriculum levels from Statistics education researchers and teachers.

Table 3.1: The key standards in Mathematics and Statistics.

Level	Mathematics	Statistics
1	AS1.2 (Algebra) (E) AS1.3 (Tables, Equations and Graphs) (E)	AS 1.10 (Inference) (I) AS1.12 (Chance and Data) (E)
2	AS2.6 (Algebra) (E) AS2.7 (Calculus) (E)	AS2.9 (Inference) (I) AS2.12 (Probability) (E)
3	AS3.6 (Differentiation) (E) AS3.7 (Integration) (E)	AS3.9 (Bivariate) (I) AS3.10 (Inference) (I) AS3.13 (Probability) (E)

Notes: (E) denotes externally assessed standards. (I) denotes internally assessed standard. Full details of the standards can be found in Appendix D.

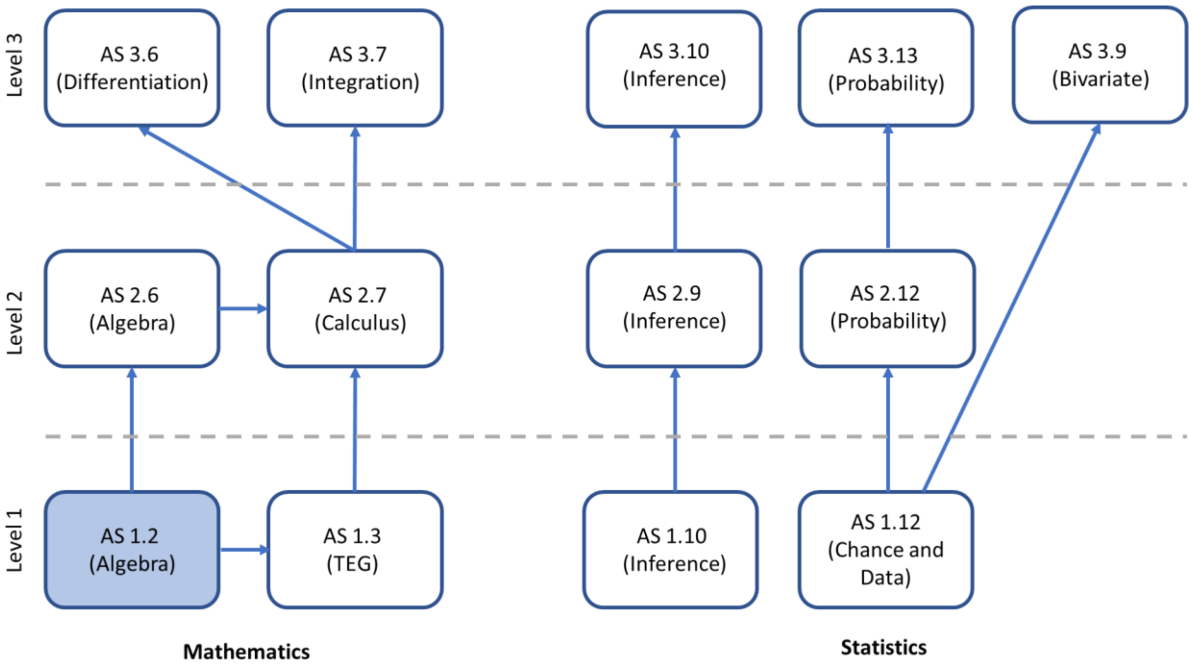

Figure 3.1: The key standards for progression and pathways in Mathematics and Statistics.

Notes: The arrows represent the contributing prior learning for each standard, not alternative pathways to reach the standard. Full details of the standards can be found in Appendix D. AS1.2 (Algebra) has been shaded as it acts as 'gatekeeper' standard for progression in Mathematics.

Mathematics: In Mathematics, both the literature and an analysis of the prerequisites for first year core university papers in Mathematics at universities in New Zealand strongly suggest that the two most important standards to be included in a Level 3 Calculus course are the externally assessed standards AS3.6 (Differentiation) and AS3.7 (Integration). Using Bernstein's (1999) theorisation of vertical knowledge structures, the learning needs to be carefully sequenced to build to these standards and the concepts they represent. Learning Calculus requires prior understanding of variables and functions (Kidron, 2020), which are fundamental to Algebra. For students to be successful in AS3.6 (Differentiation) and AS3.7 (Integration) they must have built a solid foundation of conceptual knowledge in the Algebra and Calculus standards at the lower level. Therefore, at Level 2 the framework proposes the key two standards to build prior knowledge for Level 3 are two of the externally assessed standards: AS2.6 (Algebra) and AS2.7 (Calculus) (Figure 2.1). At Level 1, the proposed standards are again external standards: AS1.2 (Algebra) and AS1.3 (Tables, Equations, and Graphs). Based on this hierarchical structure, depicted in Figure 3.1, I am suggesting that the standard of AS1.2 (Algebra) acts as an important 'gatekeeper' in NCEA Mathematics progression. If students do not participate in this standard or perform well in the assessment, they will be highly unlikely to progress through an Algebra and Calculus pathway. This is also affirmed through my experiences working in secondary schools. This framework has also included one of the high-literacy standards identified by Wilson and McNaughton (2014), namely AS2.6 (Algebra).

While all these key Mathematics standards are externally assessed, this framework is not intending to suggest that the internally assessed standards at each of the three levels are not an important component of a coherent course in Mathematics. Given the time-limited nature of the examinations, which are the assessment method for externally assessed standards in the subject, they have an important role in assessing students' conceptual knowledge (the 'know that'), and developing automaticity in these (Johnston et al., 2020). The internally assessed standards, which allow for greater flexibility in their means of assessment encourage enquiry, creativity, and complex problem solving, the 'know how' alongside the 'know that'. There is greater flexibility in the selection of internally assessed standards to achieve the development of both types of knowledge. Both components are required for Mathematics to be positioned as powerful knowledge so ideally courses would be designed to include both types of standards.

The key standards for Mathematics are also not an exhaustive set of the knowledge required for progression through to a full Level 3 course in Calculus. For example, Trigonometry would be likely to feature in the progression to Level 3 Calculus but has not been included in this list of key standards. It could be developed at Levels 1 and 2 through a selection of one or more of these standards: AS1.6 (Geometry), AS1.7 (Right angled triangles), or AS2.4 (Trigonometry).

Statistics: In Statistics, there is much less evidence to be considered in the identification of the key standards, arguably this may be because it is not as hierarchical as Mathematics. The recommendations from The University of Auckland’s Department of Statistics on the CensusAtSchool NZ website are that students should be taking AS3.9 (Bivariate), AS3.10 (Inference), and AS3.13 (Probability) to prepare for university study in Statistics (The University of Auckland, Department of Statistics, 2020). The first two of these standards are internally assessed, the last one is an externally assessed through an examination. Statistics in the NZC has an increased emphasis on real investigations and real data compared to previous curricula (Pfannkuch, Regan, Wild, & Horton, 2010). The internally assessed standards provide opportunities to investigate data sets and carry out statistical enquiries. Ben-Zvi (2020) identified the importance of enquiry to promote students’ statistical reasoning and their construction of meaningful knowledge. The standards identified also cover many of the things that Ben-Zvi (2020) identified need to be covered in an effective programme at high school level and the concepts identified as being threshold concepts (Bulmer, O’Brien, & Price, 2007; Khan, 2014). Therefore, these three standards: AS3.9 (Bivariate), AS3.10 (Inference), and AS3.13 (Probability) are the three key Statistics standards at Level 3.

Again, using Bernstein’s (1999) structures of vertical knowledge approach we must consider the sequencing of standards at Levels 1 and 2. For students to be successful in the three key standards identified in the framework at Level 3, the standards at Levels 1 and 2 must build a strong knowledge foundation for students to be able to build on. Both AS2.9 (Inference) and AS2.12 (Probability) are identified as containing the key conceptual knowledge for progression to the key Level 3 standards. At NCEA Level 1, the conceptual knowledge and statistical reasoning from AS1.10 (Inference) and 1.12 (Chance and Data) are important prior learning for the key Level 2 standards. Bivariate data analysis at Level 3 does not immediately appear to have a precursor standard in this framework. At Level 2 it is not included in any of the NCEA standards but at Level 1 it is a component of AS1.12 (Chance and Data). Although AS1.11 (Bivariate) fully focuses on bivariate analysis, it has not been included since this framework aims to identify the minimum number of standards at each level rather than define a full programme. The focus in the NZC on Inference means the Inference standard (AS1.10) at Level 1 has been prioritised instead of AS1.11 (Bivariate) in this framework. I suggest, that in comparison to the key Mathematics standards, which has a 'gatekeeper' standard: AS1.2 (Algebra), there is a greater flexibility in the selection of standards to achieve progression within the subjects. For example, I have known schools that have chosen to forego either AS1.10 (Inference) or AS2.9 (Inference) in their preparation for AS3.10 (Inference) as it was perceived that there was too much repetition of concepts and a more coherent programme was served by offering alternative standards. There are externally assessed high-literacy standards, as identified by Wilson and McNaughton (2014), included in this framework: AS1.12 (Chance and Data), AS2.12 (Probability), and AS3.13 (Probability).

3.5 Summary

This chapter has drawn on a variety of sources of literature and on New Zealand universities' prerequisite recommendations to propose an evaluative framework of key standards in Mathematics and Statistics consistent with Bernstein's (1999) vertical structures of knowledge in Mathematics and Statistics and Hudson's (2017) and Golding's (2018) conceptualisation of Mathematics as a form of powerful knowledge. The evaluative framework is open to critique by other educators and researchers and is proposed to be the minimum number of standards required as part of a coherent NCEA course in Mathematics and Statistics, which provides the optimum preparation for progression and pathways onto university. It places no judgement on the standards which were not selected for this framework and I argue that both externally and internally assessed standards must both be included in a coherent course to support the development of the conceptual and procedural knowledge. This evaluative framework of key standards will be used to focus the analysis of the NCEA participation and achievement data from 2013-2019 in the following chapter (Chapter 4).

Chapter 4: Analysis of the NCEA participation and achievement data in Mathematics and Statistics, 2013-2019

4.1 Introduction

In this chapter, the NCEA data provided by NZQA will be explored looking at the patterns of participation and achievement from 2013-2019 in the key standards and combinations of the key standards as identified for Mathematics and Statistics progression in Chapter 3 (see Section 3.4 and definitions in the *Glossary of key terms and abbreviations* at the start of this dissertation).

Participation rates and achievement for each decile group have been calculated using the number of students at the corresponding year level in each decile group entered for the standard (regardless of result) and the roll count for that year level in each decile group, for each year from 2013-2019.

My analysis through school deciles was to explore aspects of equity as deciles can be used to indicate the extent to which schools draw from low socio-economic communities (Ministry of Education, 2020c). The decile groupings of decile 1, 2-3, 4-6, 7-9, and 10 are used as they shared similarities in patterns of participation in these standards over time and reduced the complexity of visual graphs and analysis somewhat. The relationships between participation rates in the individual, and combinations of, key standards with school deciles were also examined to understand if the opportunities for students to participate in these standards were based on the decile of the school they attend, with a focus on the start and end of the analysis period. The participation and achievement of students from decile 1 and decile 10 will be compared in further detail to provide insight into the equity of opportunity based on socio-economic status.

In Chapter 3, I answered the first research question: *Which of the NCEA achievement standards in Mathematics and Statistics, herein known as the 'key standards', best provide for coherent programmes of learning and future pathways within these disciplines?* Having identified the key standards, the purpose of this chapter is to answer the following research question:

RQ2: For the key standards in Mathematics and Statistics, what are the patterns of student participation and achievement from 2013 to 2019 in relation to school deciles?

To keep data representation and analysis manageable, selected key standards will be used to illustrate the patterns of participation and achievement in each section. These have been chosen both to illustrate what is typical but also to show some novel patterns. Additional graphs referred to in this chapter can be found in the appendices.

4.2 Decreasing rates of participation in the key standards (2013-2019), analysed through school deciles

This section analyses the participation data from 2013-2019 in the key Mathematics and Statistics standards identified in Chapter 3 and shown in Table 3.1.

4.2.1 Mathematics

In the key Mathematics standards, which are all externally assessed, there has been an overall decline in the participation rates at Levels 1 and 2 since 2013 for all decile groups. The overall decrease is largest for the Level 1 standards. AS1.2 (Algebra) exhibits the most dramatic of the decreases. It is an important standard for discussion as Algebra is known to be a 'gatekeeper' for progression in Mathematics (Knuth, 2016), and AS1.2 has been identified as a 'gatekeeper' standard for progression in NCEA Mathematics (see Figure 3.1).

Figure 4.1 illustrates the rates of participation of Year 11 students in AS1.2 (Algebra) by decile grouping from 2013-2019. Across all decile groupings there has been a decrease in participation rates in 2019 compared to 2013, although the patterns of decline are different. While demonstrating an overall decrease, decile 10 schools' participation rates initially increased from 73.1% in 2013 to a high of 83.0% in 2015. Since then, there has been a decrease in participation rates each year, dropping to 49.4% in 2019. This means that at its peak in 2015, Year 11 students at decile 10 schools were almost 1.7 times more likely to be participating in AS1.2 (Algebra) than in 2019.

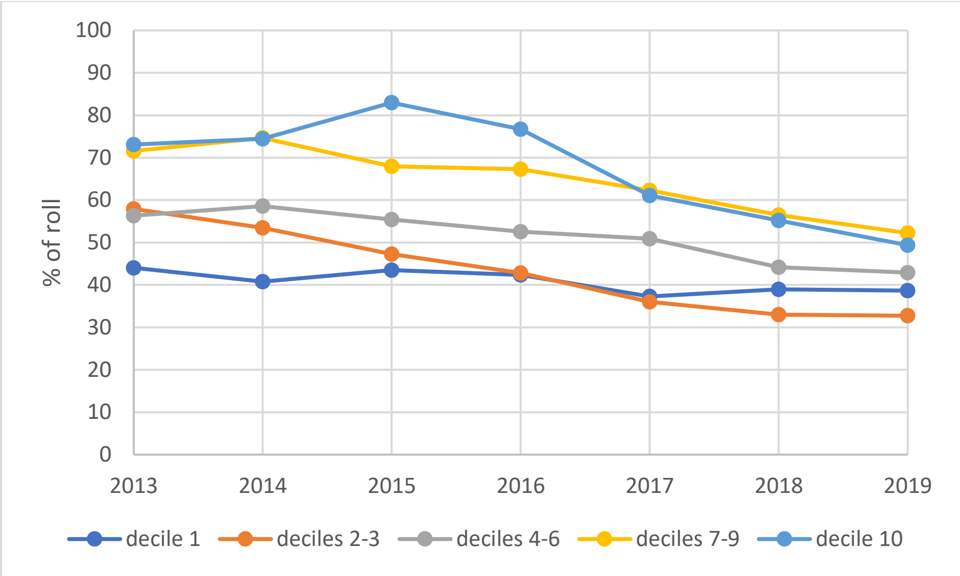

Figure 4.1: Rates of participation of Year 11 students in AS1.2 (Algebra), 2013-2019.

Rates of participation of Year 11 students at decile 7-9 schools start and finish at a similar rate to decile 10 schools over 2013-2019, but without the same degree of increase seen from 2013-2015. Decile 7-9 participation rates show a small increase from 71.6% in 2013 to 74.6% in 2014, before decreasing each year until 52.2% in 2019, slightly higher than that for decile 10 (49.4%). Year 11 students at decile 7-9 schools in 2014 were approximately 1.4 times more likely to be participating in AS1.2 than in 2019. A similar overall decrease is seen for Year 11 students in decile 4-6 schools, with students being almost 1.4 times more likely to have been participating in AS1.2 (Algebra) at its peak in 2014 (58.6%), compared to in 2019 (42.9%).

Participation rates for Year 11 students at decile 2-3 schools have dropped each year since 2013. In 2013, the participation rate of 57.9% was similar to that of students at decile 4-6 schools, dropping to 32.7% in 2019. Therefore, Year 11 students at a decile 2-3 school were almost 1.8 times more likely to be participating in AS1.2 (Algebra) in 2013 than in 2019. The changes in Year 11 participation in AS1.2 (Algebra) at decile 1 schools are much smaller than that shown for the other decile groups. There have been modest increases and decreases over the years, overall decreasing from 44.0% in 2013 to 38.6% in 2019.

A similar pattern of decline was seen for AS1.3 (Tables, Equations and Graphs) (Figure 4.2). Year 11 students were 1.2-1.4 times more likely to be participating in the standard at the peak in 2014/2015, for all decile groups except decile 1, which was higher at 1.6 times. Participation rates for Year 11 students at decile 10 schools increased from 64.5% in 2013 to

73.5% in 2015, and in 2019 was down to 55.1%. For Year 11 students at decile 1 schools, the highest participation rate was in 2015 at 22.6%, which fell to 14.1 % in 2019.

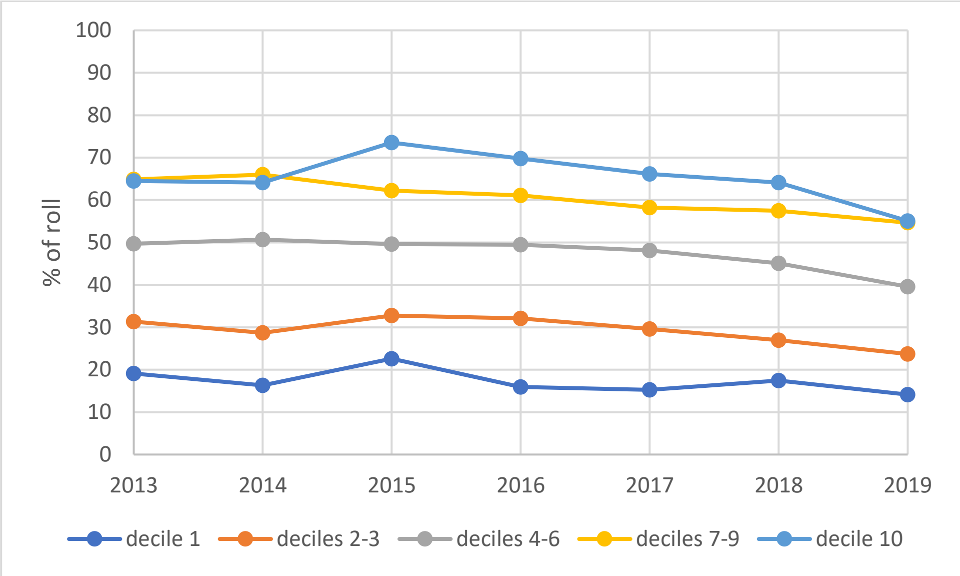

Figure 4.2: Rates of participation of Year 11 students in AS1.3 (Tables, Equations and Graphs), 2013-2019.

For the Level 2 standards, AS2.6 (Algebra) and AS2.7 (Calculus), similar overall decreases from the peak in 2013-2015 until 2019 are observed (Appendices E and F). At the peak, Year 12 students were 1.2-1.6 times more likely to be participating in these standards compared to in 2019, depending on the decile group. For example, decile 10 schools showed similar patterns of increase in participation rates from 2013-2015 as at Level 1, with peaks at 51% for both AS2.6 (Algebra) and AS2.7 (Calculus) in 2015, decreasing to 41.0% and 40.4%, respectively, in 2019. The decile group exhibiting the biggest drops was for decile 2-3 for both standards. For AS2.6, the participation rate was around 29% in 2013-2014, dropping each year until 2019 (18.8%). A slight increase to 27.7% was seen in 2014 from 25.8% in 2013 for AS2.7 (Calculus) before it began to fall, reaching 20.1% in 2019.

At Level 3, the participation rates of Year 13 students in AS3.6 (Differentiation) and AS3.7 (Integration) have remained fairly consistent (Appendices G and H). Participation rates show small increases and decreases from 2013-2019. The maximum changes were within five percentage points and these larger fluctuations may be attributable in part to small participation rates at this level. For example, for Year 13 students at decile 4-6 schools, the participation rates in the AS3.6 (Differentiation) varied from 15-17%, and for AS3.7 (Integration) the rate was 15-16%. This consistency over time is not surprising as the smaller numbers of students who opt into the Level 3 Calculus courses (compared to Levels 1 and 2), which would likely include both these standards, are those who have experienced success so far in Mathematics. They are unlikely to be the ones who were offered courses designed with a focus on maximising credits, such as internal only courses.

The differences seen in the analysis over time of the key standards suggest there is a relationship between the rates of participation and school deciles. Data for the years of 2013 and 2019 were analysed to examine what has happened over the period of analysis and to simplify the graphs and analysis. For each of the key standards, as the deciles of schools increase there is generally an increase in participation rates. An example of the relationship between participation rates and school deciles is shown in Figure 4.3. This shows the rates of participation of Year 11 students in AS1.2 (Algebra) against school decile for 2013 and 2019. Each year, the positive relationship is apparent, although there are examples of school deciles performing better or worse than the trend would suggest. For example, in 2013 decile 9 schools had higher participation rates than the decile 10 schools. The trendline for 2019 is slightly less steep than that of 2013, suggesting the difference in participation rates between deciles have decreased slightly. This is consistent with what was seen in Figure 4.1, where the differences in participation between decile groups in 2013 narrowed by 2019, primarily due to the decrease in participation rates for all decile groups except for decile 1, which remained fairly consistent. Perhaps surprisingly, the deciles 2 and 3 participation rates dropped below decile 1 in 2018 and remained lower in 2019.

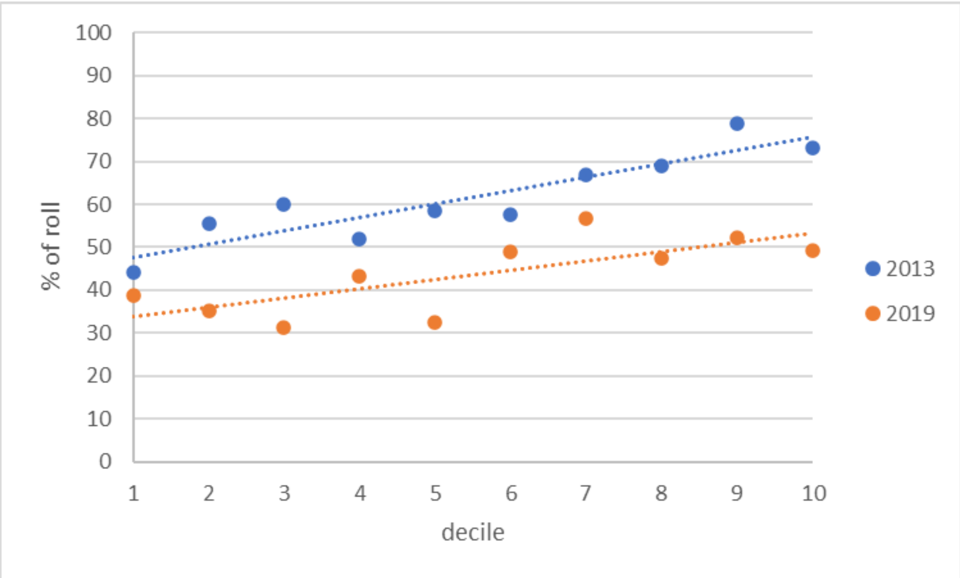

Figure 4.3: Rates of participation of Year 11 students in AS1.2 (Algebra) against school decile.

To illustrate the inequalities that exist to access the key standards in Mathematics based on the socio-economic status of the school communities, the implications of these findings are considered for students from decile 1 schools in comparison to those from decile 10 schools. In 2013, students from decile 10 schools were between 1.7 and 3.4 times more likely than students from decile 1 schools to be participating in the key Mathematics standards, depending on the standard. Across the levels, for all standards but one, the rates either stayed the same or increased in 2019. AS3.7 (Integration) showed the largest increase in differences between decile 10 and 1 schools, going from a rate of 3.4 times in 2013 to 4.2 times in 2019, representing an increased gap in participation over time. AS1.2 (Algebra) was the only standard to show a reduction in the participation gap, as discussed above. In 2013, Year 11 students from decile 10 schools were 1.7 times more likely to be participating in AS1.2 than Year 11 students from decile 1 schools. This ratio decreased to 1.3 times in 2019.

4.2.2. Statistics

In the key Statistics standards, the patterns of change in participation rates over time are more complex than is seen for the key Mathematics standards. This may in part be due to the inclusion of both internally and externally assessed standards in the key Statistics standards, whereas Mathematics only has externally assessed key standards. The internally and externally assessed standards will be grouped separately to analyse these trends. As seen in Table 3.1, the externally assessed key standards are externally assessed are: AS1.12 (Chance and Data), AS2.12 (Probability), and AS3.13 (Probability) The internally assessed key standards are: AS1.10 (Inference), AS2.9 (Inference), AS3.9 (Bivariate), and AS3.10 (Inference). The graphs will be shown in this section for selected standards and additional graphs can be found in the appendices.

The participation rates in the externally assessed key Statistics standards at all levels generally show decreases for each of the decile groups from 2013-2019, although often to a lesser extent than for the key Mathematics standards. The exception to this is an increase seen for decile 1 and 2-3 schools in the Level 1 external, AS1.12 (Chance and Data) (Figure 4.4). The decile 7-9 and decile 10 groups show the most consistent decreases in the three externally assessed key Statistics standards so are the focus of this analysis on decreasing participation in the externals. At Level 1, decreases are seen for AS1.12 (Chance and Data) for the decile 7-9 and decile 10 groups, with participation rates for Year 11 students from decile 7-9 schools decreasing from 57.4% at the peak in 2014 to 48.9% in 2019 (Figure 4.4).

For decile 10 schools, participation rates dropped from a peak of 54.1% in 2015 to 43.8% in 2019. This means that at the peak, Year 11 students at decile 7-9 and decile 10 schools were 1.2 and 1.3 times, respectively, more likely to be participating in AS1.12 at their peak than in 2019. The Levels 2 and 3 externally assessed standards showed similar rates of decreases since their peaks for these decile groups.

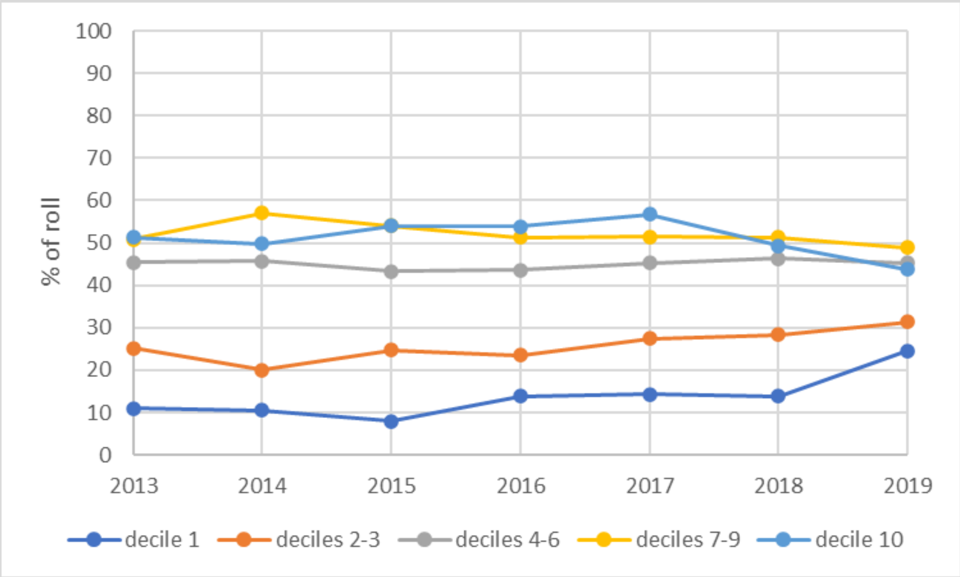

Figure 4.4: Rates of participation of Year 11 students in AS1.12 (Chance and Data), 2013-2019.

Going against this trend, the participation rates in AS1.12 (Chance and Data) for Year 11 students at decile 1 and 2-3 schools increased over time from 2013-2019 (Figure 4.4). For Year 11 students at decile 1 schools, the participation rate had small changes from 2013-2018, decreasing slightly from 11.0% in 2013 to a low of 8.0% in 2015 before increasing to 14% for 2016-2018. However, in 2019 there was a large increase to 24.6%. This means that in 2019 Year 11 students at decile 1 schools were over twice as likely to be participating in this standard compared to in 2013, and over three times as likely compared to its lowest point in 2013. Participation rates at decile 2-3 schools also increased, although the increase was more modest than for the decile 1 schools. In 2019 the participation rates were at its highest at 31.3%, up from 20.0% at its lowest point in 2014. This means that Year 11 students in 2019 were almost 1.6 times more likely to be doing this standard in 2019 than in 2014. This bucks the trend for decreasing participation rates in externals by students in low decile schools. A reason for this may be that it is perceived as being an ‘easy’ external to pass for students who might not have the knowledge required for the Mathematics externals at this level, such as AS1.2 (Algebra), which had declined over the same time (Figure 4.1).

For most of the internally assessed key Statistics standards, declines in the participation rates are generally seen for higher decile groups and increases for lower decile groups. For AS1.10 (Inference) and AS3.9 (Bivariate), decile 7-9 and decile 10 schools both show notable decreases in participation rates. For example, Year 11 students at decile 10 schools had participation rates in AS1.10 (Inference) which fell from 65.8% at its peak in 2014 to 52.1% in 2019 (Appendix I). Decile 7-9 schools has similar rates (64.2% in 2014 and 52.3% in 2019). For AS3.9 (Bivariate), the participation rates are similar for these two decile groups with the peak participation rates being 42% for both (Figure 4.5). For decile 10 schools, there was an increase that occurred in 2017 and 2018 before falling to its lowest level in 2019. In 2019, Year 13 participation rates were at 36.4% and 34.6% for decile 7-9 and decile 10, respectively. This means that at the peak, students were 1.2-1.3 times more likely to be taking these standards than in 2019. The biggest decline in any of the standards was seen for AS3.10 (Inference) for the decile 10 group (Appendix J). At its peak in 2015, the participation rate for decile 10 Year 13 students in this standard was 38.9% dropping to 27.0% in 2019, meaning that they were 1.4 times more likely to be participating in this standard in 2015 than in 2019.

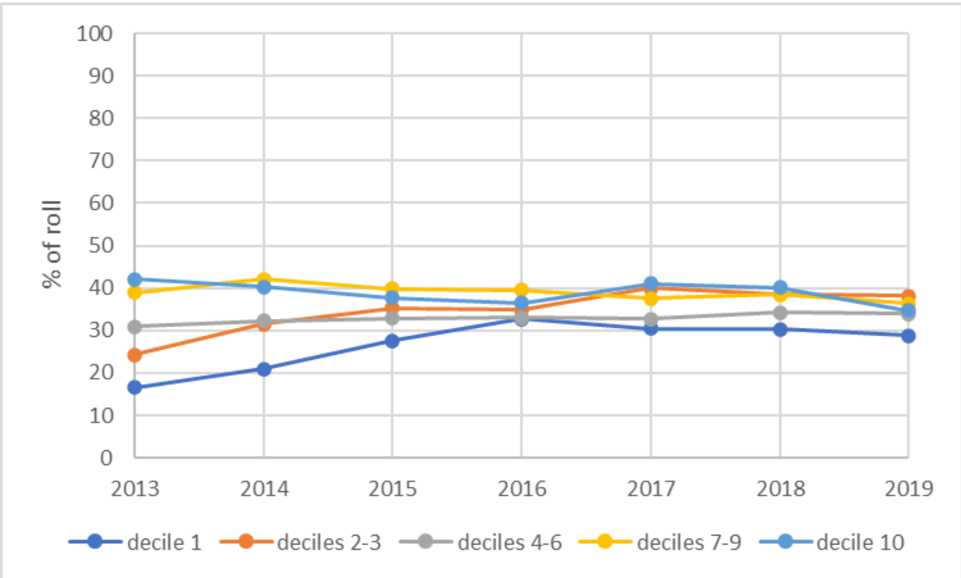

Figure 4.5: Rates of participation of Year 13 students in AS3.9 (Bivariate), 2013-2019.

Increases in participation rates were seen in the Level 3 key Statistics standards AS3.9 (Bivariate) (Figure 4.5) and AS3.10 (Inference) (Appendix J) for Year 13 students from decile 1 and 2-3 schools as well as for Year 12 students from decile 1 schools in AS2.9 (Inference) (Appendix K). For example, Year 13 participation rates for AS3.9 (Bivariate), the most popular of the four internally assessed standards in Statistics, increased fairly steadily for decile 1 schools from 16.6% in 2013 to 32.8% in 2016, before levelling off. A similar pattern of participation was seen for students from decile 1 schools in AS3.10, although the participation rates were much lower (Appendix J).

The participation data for the key Statistics standards for 2013 and 2019 against school decile was analysed and there is a clear relationship between participation rates and school decile for the externally assessed standards and for some of the internally assessed standards. For the externally assessed key Statistics standards: AS1.12 (Chance and Data), AS2.12 (Probability), and AS3.13 (Probability), as the deciles of schools increase the participation rates for each of these standards generally increases (Appendices L and M and Figure 4.6). Caution does need to be applied when interpreting the patterns for AS3.13 as the numbers enrolled in the standard are very small. The participation rates of the internally assessed AS3.10 (Inference) also show a positive relationship with decile (Appendix N). For the internally assessed standards AS1.10 (Inference) and AS2.9 (Inference) there does not appear to be a relationship between participation rates and school deciles (Appendices O and P).

The external AS2.12 (Probability) exhibits some of the strongest decile relationships in 2013 and 2019 (Figure 4.6) and is an important standard to consider as it one of the high-literacy standards identified by Wilson and McNaughton (2014). Wilson, McNaughton, and Zhu (2017) found there was lower participation and achievement in these high literacy standards in low decile schools than the national average. Figure 4.6 shows that the situation has not improved since their study, with low decile schools experiencing much lower participation rates than high decile schools in 2019, with no notable improvement since 2013. Some decile groups in 2019 are performing better than the trend might otherwise predict, such as decile 3 schools having comparable participation rates to decile 4 schools. However, all participation rates are the same or lower for all deciles in 2019 compared to 2013, showing a similar pattern of decline as Mathematics.

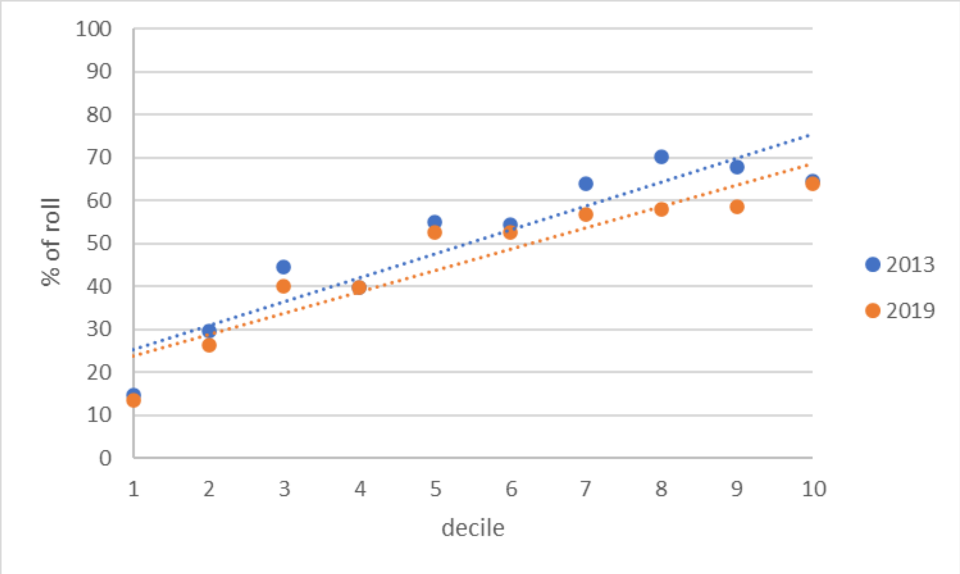

Figure 4.6: Rates of participation of Year 12 students in AS2.12 (Probability) against school decile.

Some of the key Statistics standards have shown a decrease in the participation gap since 2013. One of these was an externally assessed standard AS1.12 (Chance and Data) and the others were the internally assessed standards: AS3.9 (Bivariate) and AS3.10 (Inference) (Appendix N), although the numbers participating in AS3.10 from low deciles schools are very small, especially in 2013. Figure 4.7 shows the participation rates in AS3.9 (Bivariate) for Year 13 students in 2013 and 2019 and is the only key Statistics standard to have undergone the apparent removal of the participation gap which existed when the standards were first introduced. In 2013, there was a positive relationship between the Year 13 participation rates and school decile. In 2019, the participation rates of deciles 1-5 had increased and the rates for decile 6-10 schools had either decreased or remained the same.

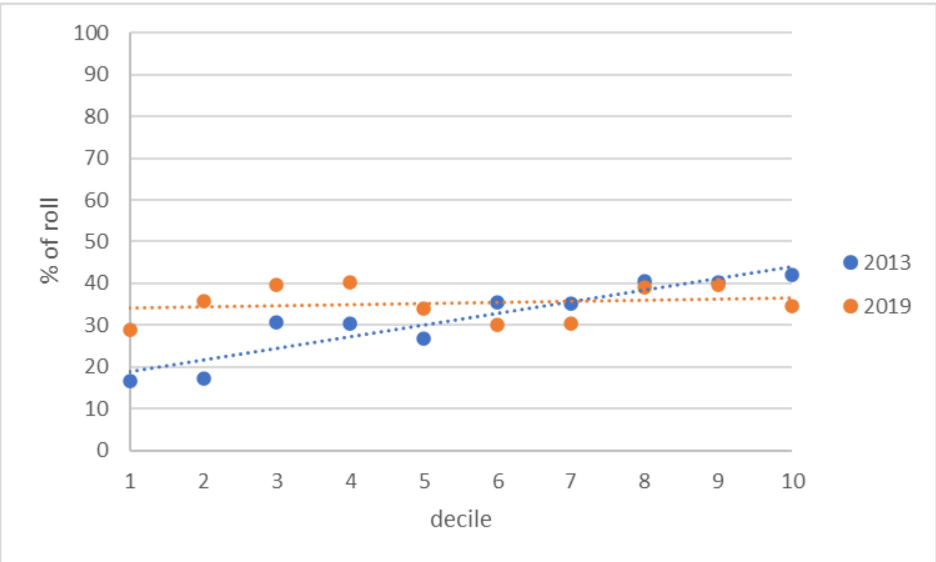

Figure 4.7: Rates of participation of Year 13 students in AS3.9 (Bivariate) against school decile.

To illustrate the inequalities that exist to access the key standards in Statistics based on the socio-economic status of the school communities, the implications of these findings are considered for students from decile 1 schools in comparison to those from decile 10 schools.

In 2013, students from decile 10 schools were between 4.4 and 5.6 times more likely than students from decile 1 schools to be participating in the externally assessed key Statistics standards. For the internally assessed key standards, the rates were 1.1-6.4 times in 2013. Many of the standards saw improvements in 2019, although the discrepancy clearly remained. Notable improvements include the external AS1.12 which went from Year 11 students at decile 10 schools being 4.6 times more likely to be participating in the standard than decile 1 students in 2013, to 1.8 times more likely in 2019. The internally assessed standards AS1.10 (Inference) and AS2.9 (Inference) had slightly greater rates of participation for students from decile 10 schools than from decile 1 schools in 2013, 1.1 times for AS1.10 and 1.4 times for AS2.9. In 2019, greater rates of students were doing AS1.10 from decile 1 schools than decile 10 schools and the rate for AS2.9 was the same for the two decile groups.

4.2.3 Summary of findings

There has been an overall decline during 2013-2019 in the participation rates for the key Mathematics standards at Levels 1 and 2 for all decile groups. The findings for the key Statistics standards have been a little more mixed. The participation rates in the externally assessed key Statistics standards generally showed declines for all decile groups at all levels over the period 2013-2019, the exception being an increase for decile 1 and 2-3 schools in AS1.12 (Chance and Data). The participation rates in most of the internally assessed key Statistics standards at all levels increased for the lower decile groups and decreased for the higher decile groups during 2013-2019. When analysing the relationship between participation rates and decile, it was found that for most of the key standards in Mathematics and Statistics, as the deciles of schools increases there is generally an increase in the participation rates. Students from decile 10 schools in 2013 were more likely than students from decile 1 schools to be participating in the key standards in Mathematics and Statistics. Most of the differences between these decile groups stayed the same or worsened for Mathematics by 2019. For Statistics, many of the differences improved over time with a decrease in the participation gaps.

4.3 Achievement in the key standards (2013-2019), analysed through school deciles

Participation rates tell us about the enrolment in the standard but that only tells part of the story of inequality. The quality of the participation also needs to be considered. The percentage of students not doing the external examinations either due to absence or not attempting the paper has been calculated for each standard. Achievement is considered in terms of both pass rates for those that attempt the examination (after those who are absent and those who did not attempt the paper are removed) and the percentage of students getting Merit or Excellence. The grade level is important as it both indicates the depth of understanding and it is often used as selection or progression criteria. As was shown in section 3.3.2, many of the universities have grade criteria in their prerequisites for the recommended Level 3 standards (see for example University of Auckland's selection criteria in Appendix C) and schools often set required grades for progression, such as a minimum of a Merit in AS1.2 (Algebra) to continue to an Algebra-based Level 2 Mathematics course. This analysis will provide further insight into the quality of learning of the key standards for students from schools with different deciles.

In this section, selected data will be provided which illustrates changes over time in the proportion of students not awarded a grade in the external examinations (due to absences or papers not attempted), the pass rates, and the percentage of Merit and Excellence grades. The data presented will be of the start (2013) and end (2019) of the analysis period and will consider the implications for students from decile 1 and decile 10 schools. Focusing on the 2019 data to provide an up-to-date snapshot of the situation in New Zealand, the relationships between achievement and school deciles will also be examined.

4.3.1 Mathematics

The achievement data for the key Mathematics standards for 2013 and 2019 for students from decile 1 and 10 schools can be found in Table 4.1. The percentage of students not being awarded a grade from Not Achieved to Excellence due to absence or not attempting the examination paper, is higher for students from decile 1 schools than for decile 10 schools and generally higher in 2019 than in 2013.

Table 1.1: Achievement data in the key Mathematics standards for students from decile 1 and decile 10 schools in 2013 and 2019.

		Decile 1			Decile 10		
Standard	**Year**	% no grade	% pass	% Merit and Excellence	% no grade	% pass	% Merit and Excellence
AS1.2	2013	16.9	62.5	15.7	3.1	89.9	59.3
	2019	11.5	58.3	21.9	2.0	90.4	62.0
AS1.3	2013	15.6	44.4	10.6	3.8	87.5	54.4
	2019	19.0	55.5	14.1	12.3	89.7	59.2
AS2.6	2013	25.0	35.7	6.0	7.4	82.9	45.5
	2019	35.9	34.0	10.3	14.7	84.3	62.7
AS2.7	2013	24.7	37.7	5.8	9.6	81.4	49.1
	2019	31.7	30.4	7.6	9.4	80.3	52.6
AS3.6	2013	12.1	44.8	13.1	4.7	77.7	50.8
	2019	24.2	42.3	16.0	8.6	83.7	53.3
AS3.7	2013	10.8	34.3	7.1	10.2	79.1	49.4
	2019	34.2	47.1	17.6	16.9	82.7	52.1

Notes: '% no grade' denotes the percentage of students who were absent or did not attempt the paper in external examinations. '% pass' denotes the percentage pass rates for those who attempted the assessment. '% Merit and Excellence' denotes the percentage of students who gained Merit or Excellence grades out of those who attempted the assessment.

AS1.2 (Algebra) is an important standard to examine due to its 'gatekeeper' role, although it is not typical of the trends seen for most standards when comparing 2013 and 2019 data. The general trend of the proportion of absences and papers not attempted being higher for students from decile 1 schools than for decile 10 schools in both years was seen for AS1.2. In 2019, only 2% of students from decile 10 schools were absent or did not attempt the paper compared to 11% of students from decile 1 schools. These figures are down on the 2013 figures, which were 3% and 17% for decile 1 and 10 schools, respectively, which goes against the general trend of an increase in this proportion in 2019.

The other key standards also show much higher rates of standards with no marks due to absences and papers not attempted. The rates for decile 1 schools in 2019 vary from 19% for

AS1.3 (Tables, Equations and Graphs) to 36% for AS3.6 (Algebra) and are all increased from the 2013 rates. For decile 10 schools, the rates are lower than for decile 1, ranging from 9% for AS2.7 (Calculus) and AS3.6 (Differentiation) to 17% for AS3.7 (Integration). It is logical that AS1.2 (Algebra) has lower proportions of absences and papers not attempted compared to the other external exams as it is an externally set examination held outside the end-of-year examination period (Common Assessment Task) during regular term time in September each year. This means that students have only the one examination to focus on at that time and, being earlier in the year, students will have fewer of their standards completed so are less likely to opt out due to already having completed what they view to be sufficient standards.

For students attempting the examinations, there is very little change from 2013 to 2019 in the percentage pass rates of the key standards at each level, for both deciles. Being entirely externally assessed standards, this is perhaps unsurprising as externally assessed standards have Profiles of Expected Performance (PEPs) for each standard which are a monitoring tool for the markers which provide consistency from year to year (NZQA, 2018). PEPs are ranges for each of the grades that indicate the distribution of results that are expected. Interestingly, it appears there may be slight increases over time in the percentages of Merit and Excellence grades. Looking at the intervening years, there have been fluctuations in the percentages for each standard and it is not the case that these percentages have been increasing steadily over this period.

The percentage pass rates for students from decile 10 schools are higher than those for students from decile 1 schools for all of the key standards, as is the proportion of students getting the higher grades of Merit and Excellence. This is true across the years. The 2019 data are used here to exemplify the achievement gaps and provide the most up-to-date snapshot of the current situation for students. For AS1.2, the pass rate for Year 11 students at decile 1 schools doing AS1.2 (Algebra) is 58%, similar to that for AS1.3 (Table, Equations and Graphs) at 55%. The pass rate for Year 11 students from decile 10 schools is 90% for both AS1.2 and AS1.3. Rates of Merit and Excellence are also higher for students from decile 10 schools, compared to decile 1 schools, for both standards. For AS1.2, 62% of Year 11 students from decile 10 schools who sat the examination obtained Merit or Excellence and only 22% of those from decile 1 schools obtaining those grades.

The achievement gap widens going into Level 2. The pass rates for Year 12 students from decile 1 schools in the key Mathematics standards were 34% and 30% for AS2.6

(Algebra) and AS2.7, respectively, a large drop from the Level 1 pass rates. The percentage of the students from decile 1 schools sitting the examination who got Merit and Excellence were 10% for AS2.6 and 8% for AS2.7. The figures for Year 12 students from decile 10 schools do not show the same drop from Level 1. For AS2.6, the pass rate was 84% and the percentage of Merit and Excellence was 63%. For AS2.7, the pass rate was 80%, and the percentage of Merit and Excellence was 53%.

The situation does improve slightly at Level 3 with pass rates for Year 13 students from decile 1 schools up over 40% and the proportion of Merit and Excellence rates for this group up slightly too at 16% and 18% for AS3.6 (Differentiation) and AS3.7 (Integration), respectively. For Year 13 students from decile 10 schools, the pass rates and percentages of Merit and Excellence are comparable to that at Level 2.

The differences in achievement were further investigated by analysing the relationships between pass rates and school decile and the rates of Merit and Excellence grades and school decile using the 2019 data. For these key Mathematics standards, there is a clear positive relationship in both cases, as the decile of schools increases both achievement measures tend to increase also. An example of this can be seen in Figure 4.8 for AS1.2 (Algebra), which is fairly representative of all of the key Mathematics standards.

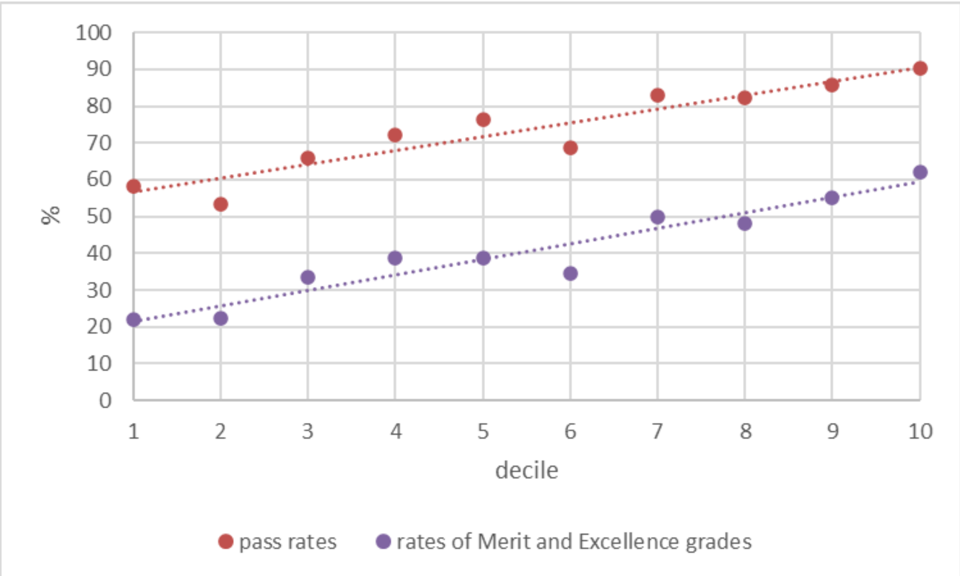

Figure 4.8: Pass rates and rates of Merit and Excellence grades for Year 11 students against school deciles for AS1.2 (Algebra) in 2019.

These findings for the key Mathematics standards are consistent with what has happened generally in externally assessed standards across all subjects (NZQA, 2020a). The pass rates for all externally assessed standards at all three levels are similar over the years 2015-2019, with perhaps a slight drop visible in the data. The distributions of grades by decile groups show that students from lower decile schools (decile 1-3) have lower pass rates and percentages of the higher grades than those from the middle decile schools (decile 4-6), which in turn are lower than those students from higher decile schools (decile 8-10). This is the case across all three levels of NCEA.

Combining participation rates, the proportion of students who attempted the examination, and the pass rates paints a very sobering picture of the unequal opportunity for achievement for students from decile 1 schools compared to their counterparts at decile 10 schools. To illustrate these cumulative effects, imagine a random sample of 1000 Year 11 students selected from each of the decile 1 and decile 10 schools in 2019. For those 1000 students from the decile 10 schools, it would be expected that 436 students would pass AS1.2 (Algebra), with 300 of those students getting Merit or Excellence. In contrast, for 1000 students from decile 1 schools, only 199 would be expected to pass the standard, with 76 of those getting Merit or Excellence.

While AS1.2 (Algebra) is important as many schools use it as a gatekeeper for further Algebra, it is not one of the most unequal in its participation and achievement. Any of the other key Mathematics standards exhibit greater inequality. The follow-on standard to AS1.2, AS2.6 (Algebra), will be used to illustrate this. Again, taking a sample of 1000 students, this time from Year 12 in 2019 from each of the decile groups, we see the number of students passing AS2.6 is much higher for decile 10 schools (293 students) compared to decile 1 schools (35 students). 220 of the students from decile 10 schools passing the standard received a Merit or Excellence grade, compared to the 10 students from a decile 1 school.

4.3.2 Statistics

The achievement data for the key Statistics standards for 2013 and 2019 for students from decile 1 and 10 schools can be found in Table 4.2. As was generally seen for the key Mathematics standards, the percentage of students not being awarded a grade from Not Achieved to Excellence due to absence or not attempting the examination paper, is higher for students from decile 1 schools than for decile 10 schools and has generally increased in 2019 compared to 2013 for both decile groups. For example, 7.3% of Year 12 students from decile 10 schools enrolled in AS2.12 (Probability) received no grade for the standard, which increased to 13.8% in 2019. For students from decile 1 schools the proportions were much higher with 23.3% of Year 12 students from decile 1 schools in 2013 enrolled in AS2.12 (Probability) received no grade in the examination, which increased to 34.8% in 2019.

Table 4.2: Achievement data in the key Statistics standards for students from decile 1 and decile 10 schools in 2013 and 2019.

		Decile 1			Decile 10		
Standard	**Year**	% no grade	% pass	% Merit and Excellence	% no grade	% pass	% Merit and Excellence
AS1.10	2013	-	76.1	14.2	-	86.8	50.9
	2019	-	81.8	22.4	-	87.0	53.8
AS1.12	2013	28.0	43.8	11.1	3.5	86.4	54.9
	2019	28.7	40.3	11.7	8.1	87.1	53.5
AS2.9	2013	-	65.5	13.7	-	75.7	35.5
	2019	-	71.3	30.9	-	79.5	53.3
AS2.12	2013	23.3	30.8	6.1	7.3	88.2	59.9
	2019	34.8	32.8	10.2	13.8	84.7	57.7
AS3.9	2013	-	76.0	19.1	-	90.2	48.5
	2019	-	81.6	28.3	-	91.0	58.9
AS3.10	2013	-	74.8	15.7	-	84.5	45.1
	2019	-	85.1	27.2	-	84.1	55.3
AS3.13	2013	25.2	38.6	13.9	14.3	83.6	50.8
	2019	36.8	14.9	s	22.9	84.2	49.8

Notes: '% no grade' denotes the percentage of students who were absent or did not attempt the paper in external examinations. Internally assessed standards do not have a '% no grade' as this only applies to externally assessed standards. '% pass' denotes the percentage pass rates for those who attempted the assessment. '% Merit and Excellence' denotes the percentage of students who gained Merit or Excellence grades out of those who attempted the assessment. 's' denotes small numbers which cannot be reported due to privacy considerations.

For students attempting the examinations for the externally assessed standards, there is very little change from 2013 to 2013 in the percentage pass rates of the key standards at each level, for both deciles, except for AS3.13 (Probability). For this standard, there appears to be a large decrease in the percentage pass rate but this may be in part explained by the small numbers doing this standard from decile 1 schools. The percentage pass rates for the internally assessed key Statistics standards appear to have increased from 2013-2019 for students from decile 1 schools and stayed similar for students from decile 10 schools. For example, for AS1.10, the pass rate increased from 76.1% in 2013 for decile 1 students to 81.8% in 2019. For decile 10 students the pass rates were similar between the years, 86.8% in 2013 and 87.0% in 2019.

The pattern of lower achievement which was seen in the Mathematics key standards for students from decile 1 schools compared to those from decile 10 schools is seen in the external key Statistics standards. In 2019, the pass rate for AS1.12 (Chance and Data) for Year 11 students at decile 1 schools was 40%, with 12% of those attempting the examination getting Merit or Excellence. For students from decile 10 schools the pass rate was 87%, with 53% of students getting Merit or Excellence. For AS2.12 (Probability) this gap was slightly higher with 33% of Year 12 students who attempted the examination from decile 1 schools obtaining the standard (10% getting Merit or Excellence), compared to 85% from decile 10 schools (58% getting Merit or Excellence).

For the internally assessed key Statistics standards, the difference in pass rates is much smaller between the decile groups, but the percentage of students getting the higher grades is still noticeably different. In 2019, the pass rates for these internally assessed key Statistics standards for students from decile 1 schools varied from 71-85%. For students from decile 10 schools, the pass rates varied from 79-91%. The proportion of Merit and Excellence for decile 10 students were much higher than for students from decile 1 schools. For example, in AS2.9 (Inference) the percentage of Merit and Excellence was 31% for Year 12 students from decile 1 schools and 53% for students from decile 10 schools. It is worth noting that over the period 2013-2019, the pass rates for students from decile 1 schools increased for all internally assessed key Statistics standards as have the proportions of Merit and Excellence grades. For example, for AS1.12 (Chance and Data), the pass rate increased from 76% in 2013 to 82% in 2019, accompanied by an increase in Merit and Excellence grades from 14% (2013) to 22% (2019). Decile 10 schools have not seen the same changes in pass rates between these two years, but the percentages of students gaining the higher grades has also increased, the most notable increase being for AS2.9, which went from 36% Merit and Excellence in 2013 to 53% in 2019. These patterns in achievement in the internals are unsurprising as previous NCEA studies have shown that the achievement differences between decile groups in internally assessed standards are less than in externally assessed standards (Jones & Singh, 2014; NZQA, 2020a).

The differences in achievement were further investigated by analysing the relationships between pass rates and school decile and the rates of Merit and Excellence grades and school decile using the 2019 data. For the externally assessed key Statistics standards, there is a clear positive relationship in both cases, as was seen for Mathematics. As the decile of schools increases both achievement measures tend to increase also. An example of this can be seen in Figure 4.9 for AS1.12 (Chance and Data), which is fairly representative of all of the externally assessed key Statistics standards. For the internally assessed key Statistics standards, there does not appear to be a relationship between the pass rates and the school decile, the pass rates are similar across all deciles, for example in Figure 4.10 for AS1.10 (Inference), but the positive relationship is there between the rate of Merit and Excellence grades and school deciles, with the students from the higher decile schools tending to have higher rates of these grades. These findings for the key Statistics standards are consistent with what has happening generally in internal assessments across all subjects (NZQA, 2020a). The pass rates for internally assessed standards at each level across the deciles groups are closer than for all externally assessed standards.

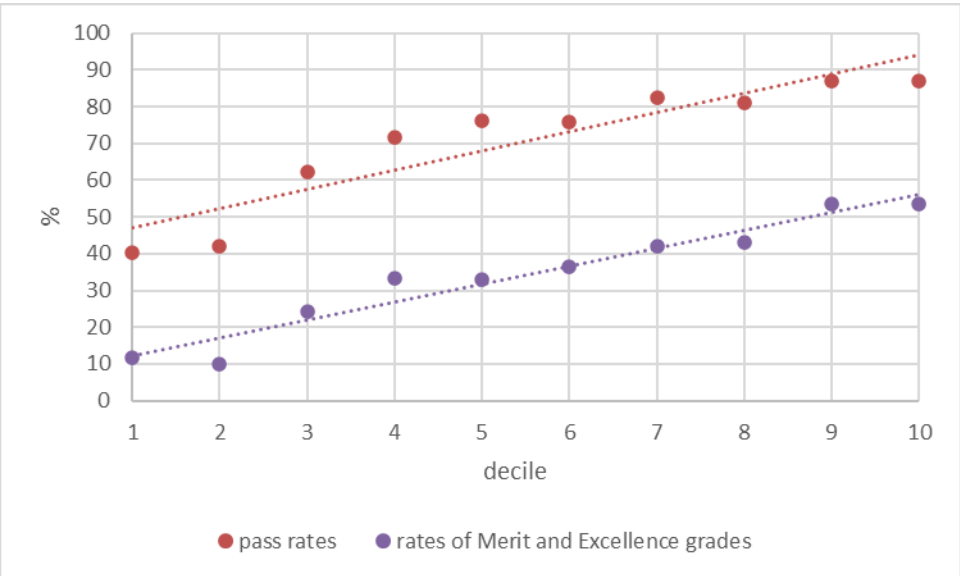

Figure 4.9: Pass rates and rates of Merit and Excellence grades for Year 11 students against school deciles for AS1.12 (Chance and Data) in 2019.

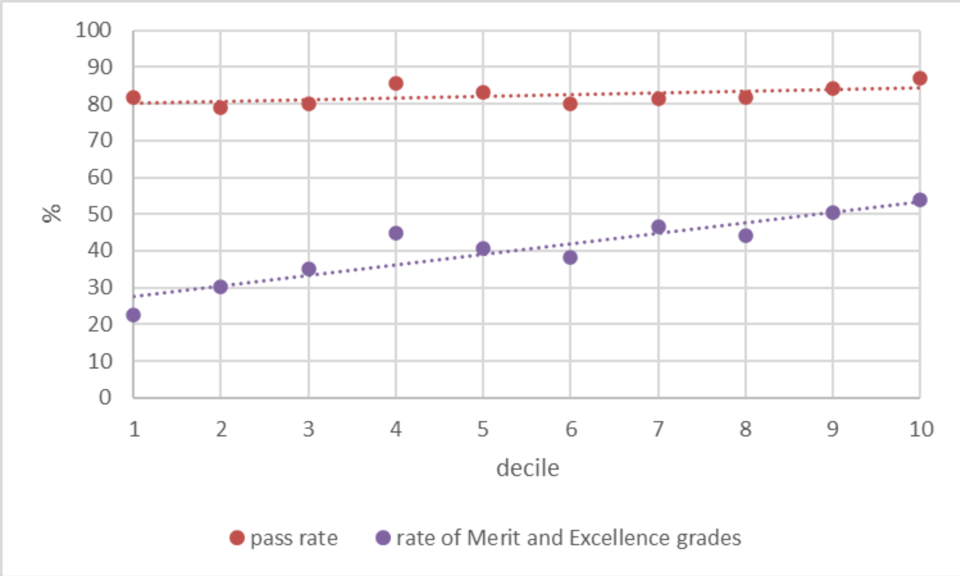

Figure 4.10: Pass rates and rates of Merit and Excellence grades for Year 11 students against school deciles for AS1.10 (Inference) in 2019.

As was done for Mathematics, participation rates, the proportion of students attempting the examination, and the pass rates are combined for the externals to illustrate the cumulative effects these have which result in unequal opportunity for achievement for students from decile 1 and 10 schools. Again, we can imagine taking a random sample of 1000 Year 12 students from each of the decile 1 and decile 10 groups of schools in 2019. Out of the 1000 Year 12 students from the decile 1 schools, only 29 are expected to gain AS2.12 (Probability), with 9 of those gaining Merit or Excellence. For the 1000 Year 12 students from decile 10 schools, the likelihood of gaining the standard is 16 times higher with 470 students being expected to gain the standard, 320 of these with Merit or Excellence grades.

The results are not so extreme for the internally assessed standards due to factors such as the smaller differences in pass rates between deciles and some internals not showing a strong decile relationship. For example, for 1000 Year 12 students in 2019 from decile 1 schools, 280 students would be expected to gain the AS2.9 (Inference) standard (with 122 gaining Merit or Excellence) compared to from decile 10 schools, where 325 would expect to gain the standard (with 218 gaining Merit or Excellence).

4.3.3 Summary of findings

Over the period 2013-2019 there has been no discernible change in the pass rates of the externally assessed key standards in Mathematics and Statistics at all levels, except for AS3.13 (Probability) but that is likely due to small numbers. Decile 1 students have experienced increased pass rates over this period for the internally assessed key Statistics standards. At all levels, students at decile 10 schools had lower rates of absences and not attempted papers for the external examinations in Mathematics and Statistics than students from decile 1 schools and had higher pass rates and percentages of Merit and Excellence grades in both the internally and externally assessed standards. The differences in achievement for these two decile groups were larger for the externally assessed key standards than the internally assessed key standards. The cumulative effects of these findings are that students from decile 1 schools are far less likely to be awarded the key standards in Mathematics and Statistics than students from decile 10 schools.

4.4 Decreasing rates of participation in the combinations of the key standards 2013-2019, analysed through school deciles

In this section the rates of participation in combinations of the key standards at each level are analysed from 2013-2019. The combinations of key standards at each level for each of Mathematics and Statistics are given in Table 3.1. The purpose of this analysis is to understand how prepared students are for the subsequent level of study based on the premise that the best preparation is to have completed all the key standards at each level (see section 3.4).

Participation rates in the combinations of key standards at each level will be affected by both the participation rates in the individual standards (analysed in section 4.2) and the composition of the courses that students are taking. To gain insight into course composition, the percentage of students who are doing both (or all) key standards at a level out of all students entered for either standard has been calculated for each decile group. This indicates the percentage of students (out of students doing either standard) who are in a course which best prepares them for progression, as I argued earlier in Chapter 3. The NCEA data cannot definitively tell us about the composition of the courses that students take. However, I propose in this research that the rates of participation in the combination of both key standards at a level (out of the students doing at least one key standard) can be a proxy for the course expectations set by schools. It provides a tentative view into the courses that students experience, how they have changed over time, and how they compare between decile groups. The primary limitation of this calculation is that it will not include students who take courses where both standards are included but are later withdrawn from one of the standards by their teacher (in consultation with the student). It does include students who ‘self-withdraw’ by either not turning up or not attempting the examination paper for the standard. Additionally, it does not tell the full picture of the various courses that schools might offer given the flexibility of NCEA. Further qualitative research would be required to fully understand course composition in schools.

4.4.1 Mathematics

For Levels 1 and 2 there has been a decrease in the percentage of the roll doing both key Mathematics standards for all decile groups from 2013-2019 (refer Table 3.1 for the key standards). For Level 3, it has been fairly steady for all decile groups over this time.

At Level 1, the percentages of Year 11 students doing both AS1.2 (Algebra) and AS1.3 (Tables, Equations and Graphs) have declined overall from 2013-2019, although there was a clear increase initially for decile 10 schools (Figure 4.11). In 2013, 61.6% of Year 11 students in decile 10 schools were doing both standards. This increased to 71.9% in 2015 and decreased down to 43.4% in 2019. This means that at the peak in 2015, Year 11 students at decile 10 schools were 1.7 times more likely to be doing both key Mathematics standards than in 2019. Decile 7-9 schools match closely for most years except they did not have the increase in 2015, instead showing a steadier decline. Changes at decile 1 schools has been the most modest, with a drop from the highest percentage of 18.8% in 2015 to 13.0% in 2019. But even with this more modest drop, Year 11 students from decile 1 schools were 1.4 times more likely to be doing both standards in 2015 than in 2019. These changes in participation in the combination of AS1.2 and AS1.3 over time mirror the changes seen in the rates of participation in the individual standards over time (Figures 4.1 and 4.2).

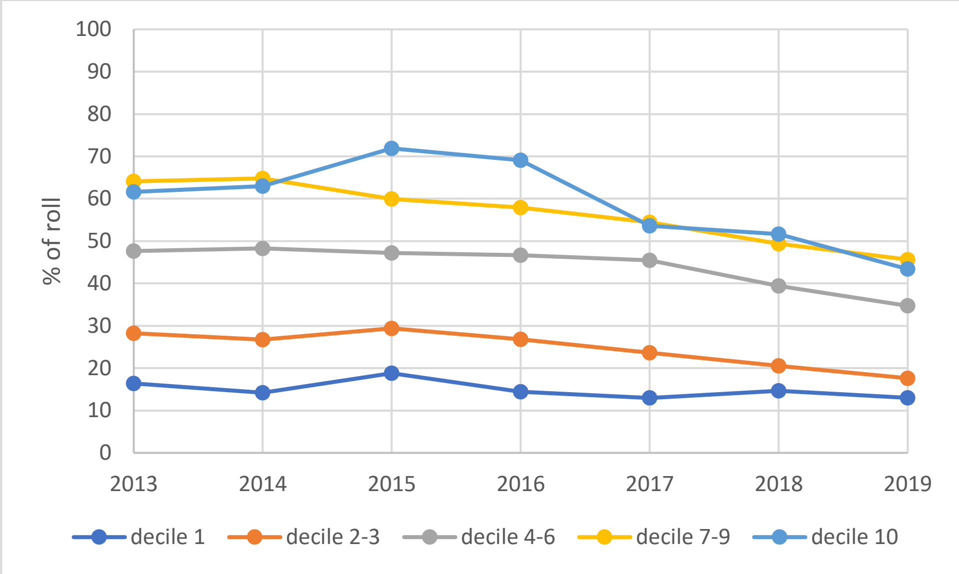

Figure 4.11: Rates of participation of Year 11 students in the combination of AS1.2 (Algebra) and AS1.3 (Tables, Equations and Graphs), 2013-2019.

Another contributing factor to consider in the overall decile of the participation in both key standards at each level is the composition of the courses that the students doing these standards experience, that is whether they have the opportunity to do both standards within their courses. For the Level 1 key Mathematics standards this is shown in Figure 4.12. For all decile groups there has been a decline in the percentage of Year 11 students doing at least one of these standards who are doing courses which result in them participating in both standards. This suggests that over time, some courses which initially included both standards may have dropped one of the standards, despite some initial increases. Decile 10, 7-9, and 4-6 have seen quite large drops. For example, in 2013 decile 7-9 schools had 88.6% of courses with either AS1.2 (Algebra) or AS1.3 (Tables, Equations and Graphs) included both standards but this dropped to 74.5% in 2019. The large drops seen at this level may be due to strategic course choices being made at the level where many students do Mathematics in order to meet the Numeracy requirement of their NCEA qualification.

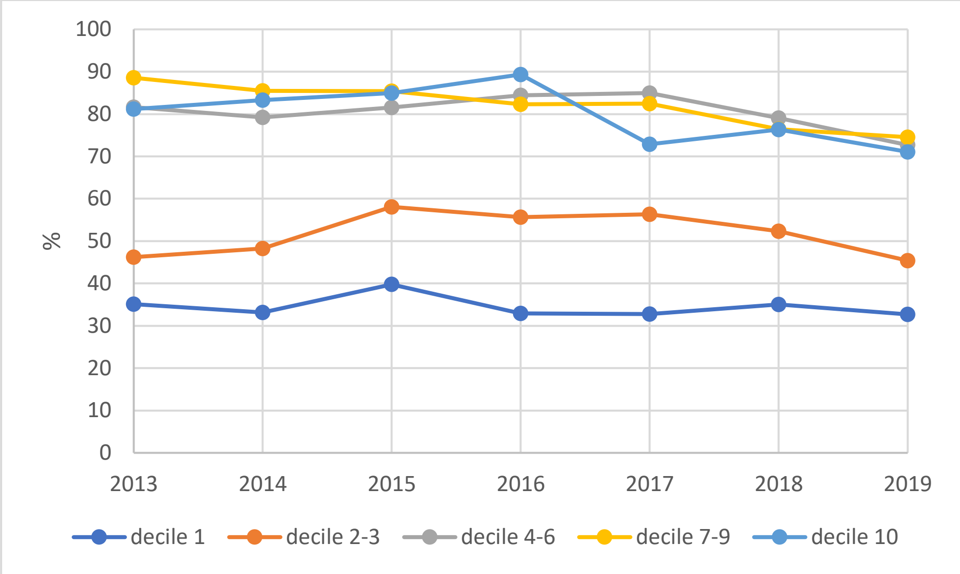

Figure 4.12: Percentage of Year 11 students who participated in AS1.2 (Algebra) or AS1.3 (Tables, Equations and Graphs) who are doing both, 2013-2019.

The composition of courses at Levels 2 and 3 do not show such large changes for the higher decile groups, consistent with schools providing more specialised courses which students have opted into now that the subject is no longer likely to be compulsory. For example, as seen in Figure 4.13, for decile 10 and 7-9 most Year 12 students doing at least one of AS2.6 (Algebra) or AS2.7 (Calculus) were doing courses with both standards included, with percentages over the years being between 87.4 and 99.3%. This high percentage makes sense as these two standards are crucial for continuation on to Calculus in Year 13 and both of these standards are in courses selected by students who are dedicated to the subject at a stage where Mathematics is no longer compulsory. Decile 1 and 2-3 had increased from the 2013 figures of 54.6% and 80.1%, respectively, to 76.3 % (decile 1) and 88.1% (decile 2-3) in 2015. However, these figures dropped to 61.3% (decile 1) and 71.7% (decile 2-3) in 2019. This suggests the courses they are experiencing are less likely to be providing the optimum preparation for progression to Level 3 Calculus than those experienced by the higher decile schools. This may either be due to schools not designing courses with the same expectations of progression or because students are being selectively withdrawn from standards by their teachers, which also impacts on their preparation for subsequent levels of study.

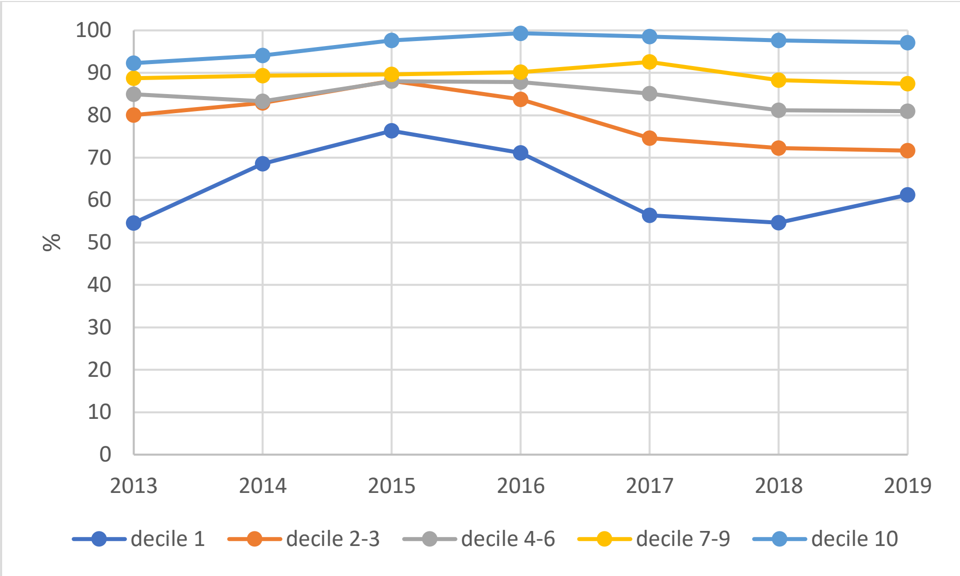

Figure 4.13: Percentage of Year 12 students who participated in AS2.6 (Algebra) or AS2.7 (Calculus) who are doing both, 2013-2019.

The relationships between the rates of participation in the combinations of the key Mathematics standards at each level and the school decile for 2013 and 2019 are suggested in the analysis above and closely mirror those seen for the individual standards in section 4.2.1. There is a positive relationship between participation rates in the combinations of key standards and the school decile, with students from high decile schools being more likely than those in low decile schools to have participated in both key standards at each level.

Analysis of the percentage of students doing both standards out of all students enrolled in at least one standard against school decile in 2013 and 2019 also shows positive relationships. This means that students from high decile schools are more likely to be participating in courses that support them to do both key Mathematics standards, rather than just one, than students from low decile schools. Using the example of Level 2 to illustrate this, the positive relationship between the rates of participation in the combination of both AS2.6 (Algebra) and AS2.7 (Calculus) and school decile can be seen in Figure 4.14. The rate of participation in the combination of standards is higher in 2013 than 2019 for all deciles. The relationship between the percentage of students doing both standards (out of those participating in either) and school decile is also positive, as seen in Figure 4.15. Year 12 students doing at least one of these standards are more likely at a high decile school to be doing both, than those at a low decile school.

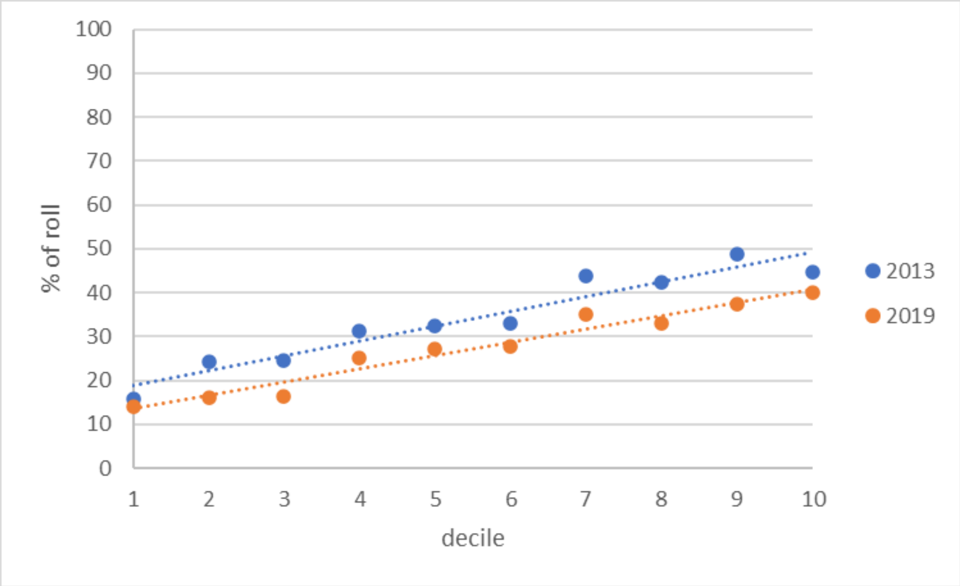

Figure 4.14: Rates of participation of Year 12 students in both AS2.6 (Algebra) and AS2.7 (Calculus) against school decile.

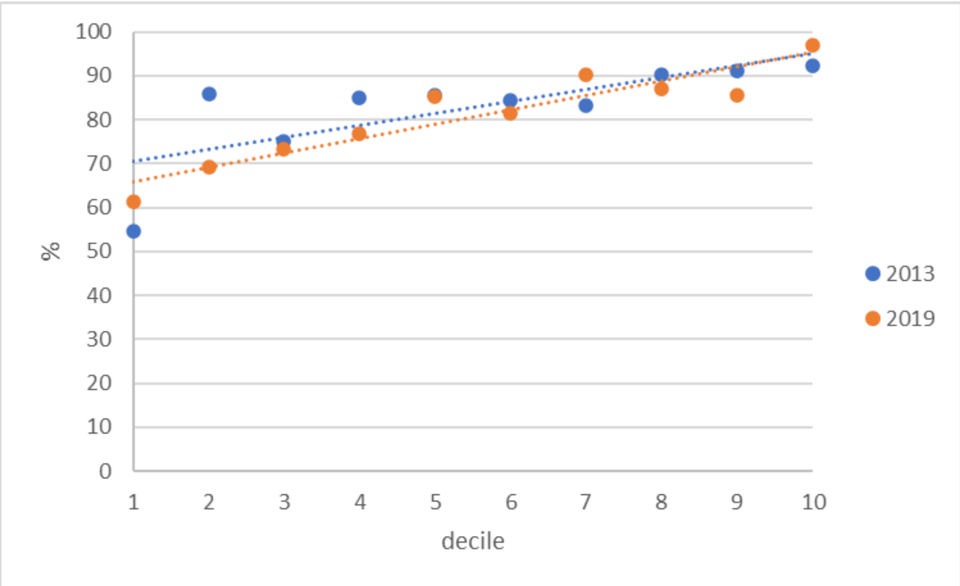

Figure 4.15: Percentage of Year 12 students doing AS2.6 (Algebra) or AS2.7 (Calculus) who are doing both against school decile.

To understand the implications on students from schools with different decile ratings, the opportunities for students from decile 1 and decile 10 schools are compared, examining the start and end of the analysis period (2013 and 2019). Students from decile 10 schools were 2.8-3.8 times more likely than students from decile 1 schools to be doing both key Mathematics standards at each level in 2013. The disparity at each level remained in 2019 and while Level 1 was slightly better (from 3.8 to 3.3 times), the difference at Level 3 was worse having increased from 3.4 times to 4.3 times, although the small numbers for decile 1 schools with only 149 Year 13 students (6% of the roll of decile 1 schools) doing both standards in 2019 means findings must be interpreted tentatively (Appendix Q).

The composition of the courses was also different for these two decile groups for students doing the key standards. The key Mathematics standards are positioned as being the optimum ones for progression within the subject and ideally students would be offered the opportunity through their courses to do both (refer section 3.4). Students from decile 1 schools doing the key Mathematics standards were less likely to be doing courses with both key standards offered at each level, compared to students from decile 10 schools doing the key Mathematics standards. For example, at Level 1 in 2019, 32.7% of Year 11 students from decile 1 schools doing AS1.2 (Algebra) or AS1.3 (Tables, Equations and Graphs) were doing both. 64.5% of the students were doing AS1.2 only and 2.8% were doing AS1.3 only. It is heartening to see that most students doing only one of key Mathematics standards at this level are doing AS1.2, given its role as gatekeeper for further Algebra. In comparison, 71.1% of Year 11 students from decile 10 schools doing the key Mathematics standards at Level 1 are doing both, with 9.8% and 19.1% doing only AS1.2 or AS1.3, respectively. What this means is that Year 11 students doing one or more of these key Level 1 standards at decile 10 schools, in 2019, are 2.2 times more likely to be participating in a course which offers both standards than students from decile 1 schools.

In 2019, Year 12 students from decile 10 schools who did one or more of the key Level 2 Mathematics standards are 1.6 times more likely, than the corresponding students from decile 1 schools, to be participating in both AS2.6 (Algebra) and AS2.7 (Calculus). At Level 2 in 2019, 97.1% of Year 12 students in decile 10 schools who were doing at least one of AS2.6 (Algebra) or AS2.7 (Calculus) were doing both standards. This suggests a perception that these two standards provide an important pairing in Mathematics learning for progression to Level 3 Calculus. It has been over 90% each year from 2013. Despite the evidence for the importance of this pairing, only 61.3% of Year 12 students from decile 1 schools who participated in at least one of these standards in 2019 did both. 9.7% did AS2.6 only and 29.1% did AS2.7 only. While it may be that students were withdrawn by their teachers from one of the externals part way through the year, rather than the standard not being on offer during the course, it does suggest that over a third of these students in 2019 were not sufficiently prepared for progression to a Level 3 Calculus courses. The percentage doing both is up on the 2013 figure of 54.6% but is down on the 2015 peak of 76.3% in 2015.

Similarly at Level 3, in 2019 Year 13 students from decile 10 schools who did one or more of AS3.6 (Differentiation) or AS3.7 (Integration) were 1.4 time more likely, than students from decile 1 schools, to be doing both of these key Mathematics standards (Appendix R). In 2019, 97% of Year 13 students from decile 10 schools doing at least one of these standards were participating in both in 2019, a proportion that has been over 95% each year since 2013, suggesting the perceived importance of this pairing in decile 10 schools. In contrast, in 2019 approximately two-thirds of Year 13 students from decile 1 school doing one of these standards were doing both. While the findings need to be applied tentatively given the small numbers doing both from decile 1 schools each year, what can be seen is that the percentage from decile 1 schools have been between 57.6% to 80.3% each year since 2013, always lower than the decile 10 schools' percentage.

4.4.2 Statistics

Analysing the data for the percentage of the roll doing both (or all three) key Statistics standards from 2013-2019 shows a more complex picture than for Mathematics, consistent with what was seen for the individual standards in section 4.2.2.

At Level 1, the percentage of Year 11 students doing both AS1.10 (Inference) and AS1.12 (Chance and Data) had increased for the lower decile schools and decreased for the higher decile schools over the period 2013-2019 (Figure 4.16). The greatest increase was seen for decile 1 schools, which went from 4.3% in 2013 to 15.7% in 2019. The overall decreases for decile 7-9 and decile 10 schools were similar. In 2013, 34.2% of Year 11 students from decile 7-9 schools participated in both standards, increasing to 40.5% in 2014 before dropping to 26.7% in 2019. This means that at the peak in 2014, Year 11 students from decile 7-9 schools were 1.3 times more likely to be doing both key Statistics standards than in 2019.

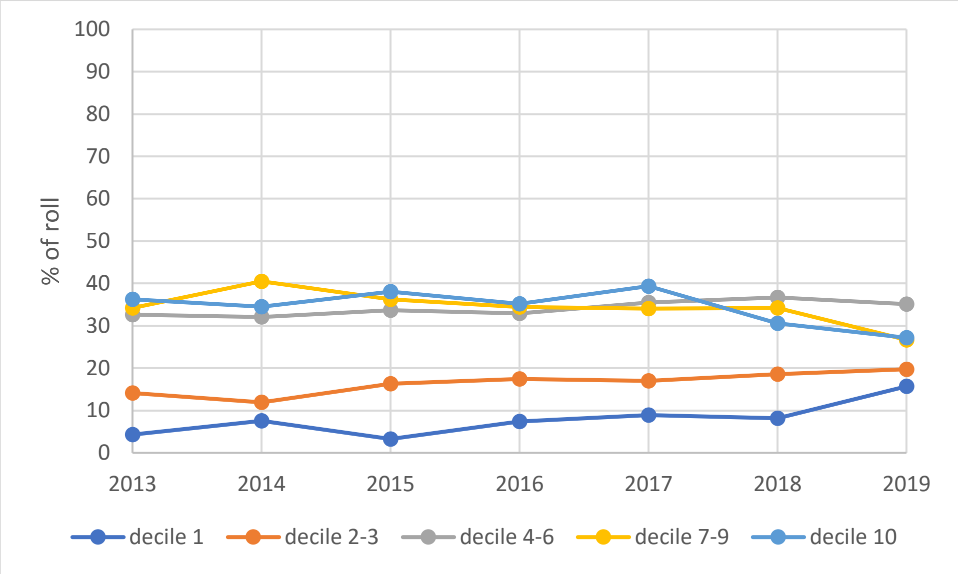

Figure 4.16: Rates of participation of Year 11 students in the combination of AS1.10 (Inference) and AS1.12 (Chance and Data), 2013-2019.

At Level 2 there were only slight decreases in the percentages of Year 12 students doing both the key Statistics standards AS2.9 (Inference) and AS2.12 (Probability) from 2013-2019 for each of the decile groups (Appendix S). The biggest change occurred for decile 4-6 schools. In 2013, the percentage of Year 12 students doing both key Statistics standards initially increased from 33.5% in 2013 to 38.4% in 2015 before decreasing and in 2019 was 29.8%. So, Year 12 students from decile 4-6 schools in 2015 were 1.3 times more likely to be doing both key Statistics standards at Level 2 than the corresponding students in 2019.

The higher decile schools (decile 7-9 and 10) experienced decreases in the percentage of Year 13 students doing all three of the key Statistics standards: AS3.9 (Bivariate), AS3.10 (Inference), and AS3.13 from 2013-2019, consistent with the decreases seen for these groups in the participation rates for AS3.10 and AS3.13 (Appendix T). The biggest decline was for decile 10 schools. In 2013, 30.2% of Year 13 students from decile 10 schools were doing all three of the key Statistics standards and this had dropped to 12.7% in 2019. This means that in 2013 Year 13 students from decile 10 schools were 2.4 times more likely be doing all three of the key Statistics standards than students in 2019. This is despite recommendations that all three standards are important preparation for undergraduate study in Statistics at universities in New Zealand (University of Auckland, Department of Statistics, 2020). The lower decile schools (decile 1, 2-3 and 4-6) had low participation rates in the combination of these three standards which did not show much change over time.

The changes in the composition of the courses may also contribute to the changes in the percentage of students doing the combinations of key standards at each level. At Levels 1 and 2 there are no discernible patterns to the changes over time (Appendices U and V). However, at Level 3 there is a clear pattern of change suggested in the composition of courses (Figure 4.17). There is a decrease across all decile groups in the percentage of those students who are doing all three out of those who are doing at least one of the key standards. Notably decile 10 schools have decreased sharply from 65.0% in 2013 to 25.4% in 2019, making it comparable to the percentage for decile 2-3 schools in 2019 (25.0%). This decline across all decile groups suggests that the courses being offered to Level 3 students have over time become less than optimal to prepare them for progression to Statistics at university. It is interesting to note how this compares to Mathematics, which did not see a decline for high decile schools although there was a decline for decile 1 and 2-3 schools. A possible reason for this change in Statistics is that the subject is widely viewed as something that can be started at any stage, including at university, in comparison to Mathematics which the hierarchical knowledge structure is firmly ascribed to by schools, for example, students would not be allowed into a Level 3 Calculus course without having completed a Level 2 Mathematics course. The Statistics standards, especially the internally assessed standards, also feature in the hybrid Mathematics courses at Level 3, less so than AS3.6 (Differentiation) and AS3.7 (Integration) do.

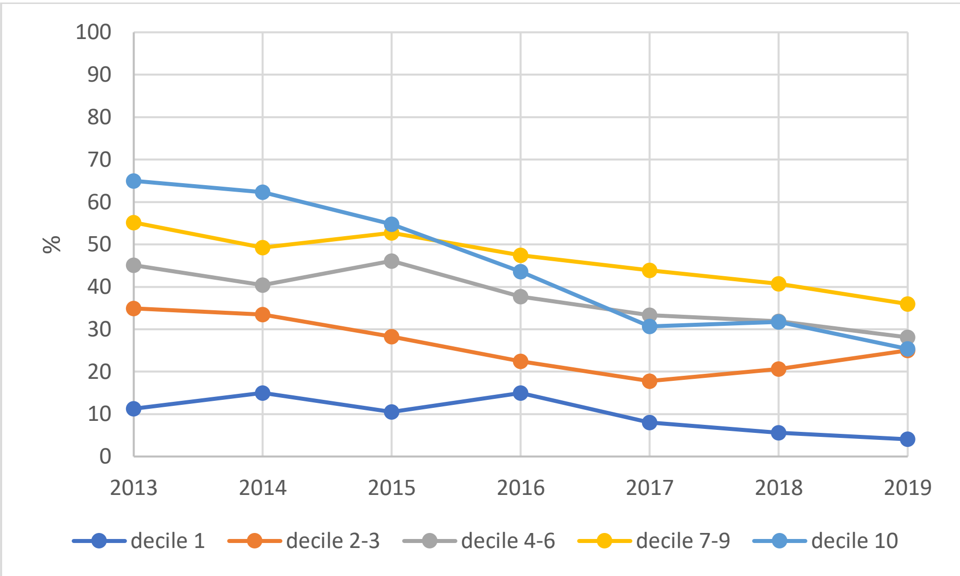

Figure 4.17: Percentage of Year 13 students who participated in AS3.9 (Bivariate), AS3.10 (Inference), or AS3.13 (Probability) who are doing all three, 2013-2019.

There appears to be a positive relationship between the rate of participation of students in the combination of both (or all three) key Statistics standards and school decile at each level. The higher decile schools tend to have higher rates of participation in the combination of standards than the lower decile schools. The percentage of students doing both standards out of all students enrolled in at least one standard against school decile (2013-2019) also shows a positive relationship at each level, meaning that students from high decile schools are more likely than students from low decile schools to be participating in courses that support them to do both key Statistics standards.

The combination of the Level 2 key Statistics standards, AS2.9 (Inference) and AS2.12 (Probability), is used to exemplify the relationship seen at each level. The overall positive relationship between the participation rate of Year 12 students in both AS2.9 and AS2.12 and school decile can be seen in Figure 4.18. For 2013, there is a very clear linear relationship between participation and school decile for decile 1-7 schools, before levelling off at around 40% for the higher decile schools. In 2019, decile 1-5 schools show this positive, linear relationship and the participation rates for decile 7-10 schools are quite similar at around 38%. Very similar relationships can be seen can be seen in Figure 4.19, which show the positive relationship between the percentage of students doing both standards (out of those participating in either) and school decile for both 2013 and 2019.

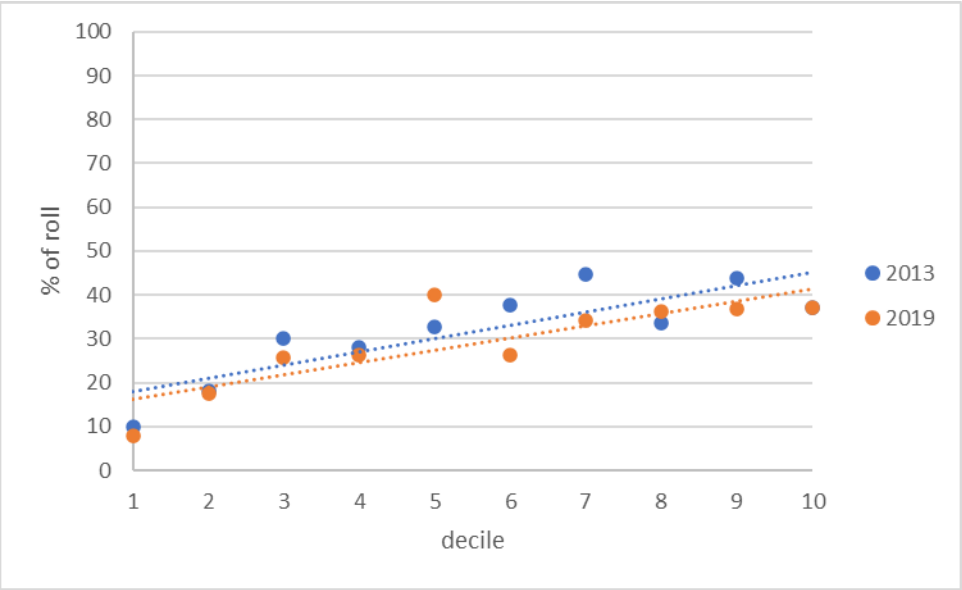

Figure 4.18: Rates of participation of Year 12 students in both AS2.9 (Inference) and AS2.12 (Probability) against school decile.

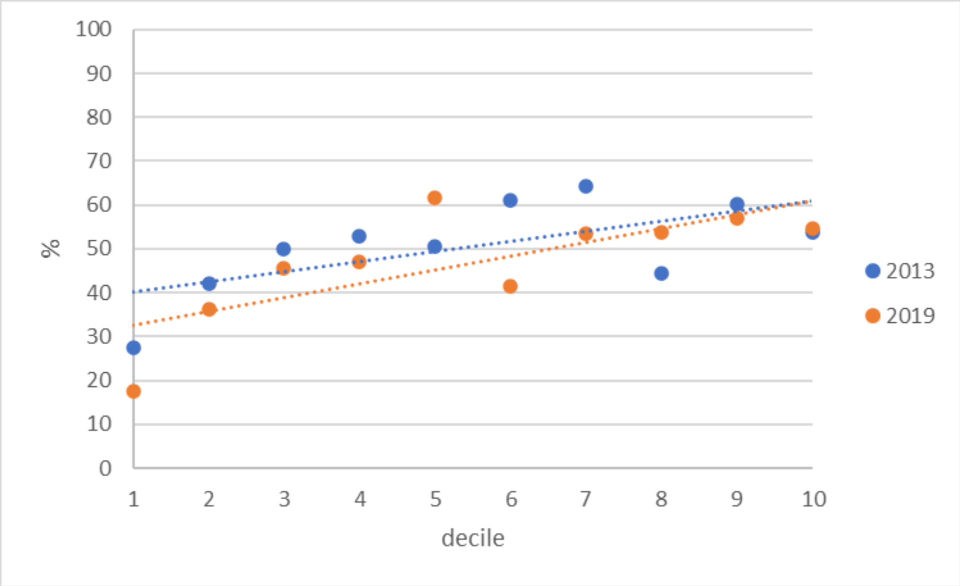

Figure 4.19: Percentage of Year 12 students doing AS2.9 (Inference) or 2.12 (Probability) who are doing both against school decile.

The opportunities for students from decile 1 and decile 10 schools are compared, examining the start and end of the analysis period (2013 and 2019) to understand the implications for students from either end of the decile range. In 2013, students from decile 10 schools were more likely than students from decile 1 schools to be doing all of the key Statistics standards at each level and this gap remained in 2019, although at Level 1 it did lessen. In 2013, Year 11 students at decile 10 schools were 8.4 times more likely to be doing both of AS1.10 (Inference) and AS1.12 (Chance and Data) than Year 11 students at decile 1 schools (Figure 4.16). In 2019 this ratio decreased to 1.7 times due to both an increase in the proportion from decile 1 schools and a decrease in the proportion from decile 10 schools who are doing both. In 2013, Year 12 students from decile 10 schools were 3.8 times more likely to be doing both AS2.9 (Inference) and AS2.12 (Probability) than Year 12 students from decile 1 schools (Figure 4.18). This gap widened slightly in 2019, to 4.7 times. The gap at Level 3 is the largest but there very small percentages of students participating in the combination of all three of AS3.9 (Bivariate), AS3.10 (Inference), and AS3.13 (Probability) from decile 1 schools so it is difficult to compare changes over time with ratios (Appendix V). In 2013, only 40 Year 13 students (2.0%) at decile 1 schools were doing all three standards, with this decreasing to 32 students (1.3%) in 2013. 2015 was the only year that the numbers went higher than 100. The low participation in all three standards for decile 1 schools is not surprising given the low rates of participation in the individual standards AS3.10 and AS3.13 for this group. Students at decile 10 schools have been doing the combination of all three key Statistics standards at a higher rate, although it has dropped over the years. 30.2% of Year 13 students from decile 10 schools participated in all three standards in 2013, dropping to 12.7% in 2019.

For students doing the key standards in Statistics, the composition of the courses was different for students from decile 1 schools compared to students from decile 10 schools. Students from decile 1 schools are less likely than students from decile 10 schools to be getting the optimum preparation for progression in Statistics in part due to the composition of the courses they take. The key Statistics standards that this research suggests have been selected as the standards required to best prepare students for progression to the next level of NCEA and onto first year university courses in Statistics. To exemplify these differences, the key Statistics standards at Level 2 in 2019 are considered: AS2.9 (Inference) and AS2.12 (Probability). In 2019, the percentage of students who were participating in at least one of these standards who were doing both was 54.6%. 39.5% were doing AS2.12 only in their courses. At decile 1 schools the figures were lower for the combination. In 2019, the proportion of students doing both standards in their courses (of those doing at least one) had decreased to 17.7% and 70.2% were doing AS2.9 only. What this suggests is that, in 2019, Year 12 students doing one or more of these key Level 2 standards at decile 10 schools are 3.1 times more likely to be participating in a course which offers both standards.

4.4.3 Summary of findings

There have been decreases in the rates of participation in the combination of both key Mathematics standards for all decile groups from 2013-2019 at Levels 1 and 2. For Level 3, there has been little change over this period. At Levels 2 and 3, the percentages of students at decile 10 schools doing at least one of the key Mathematics standards who are doing both at each level, have been consistently high, suggesting the pairings of these standards in courses at Levels 2 and 3 are perceived as being important for progression. Decile 1 students have not experienced these high proportions of pairings of key standards in their courses. For lower decile schools (decile 1, 2-3 and 4-6), there has been a decrease in the offerings of the pairs of key Mathematics standards at Levels 2 and 3. At Level 1 a decrease is seen for all decile groups.

For the combinations of the key Statistics standards, the changes in participation over time have been more mixed than for Mathematics but the high decile groups (decile 7-9 and

10) have decreased across all three levels. At Level 3 there have been decreases across all decile groups in the percentage of those students doing all three key Statistics standards, out of those who are doing at least one. There are no discernible changes in the Statistics composition of Level 1 and 2 courses over time.

There are clear relationships between participation rates in the combinations of standards and school deciles. It was found that students from high decile schools are more likely than those in low decile schools to have participated in both (or all) key standards in Mathematics or Statistics in both 2013 and 2019. Analysis of the course compositions also found that students from high decile schools are more likely than students from low decile schools to be participating in courses that support them to do both (or all) of the key standards in Mathematics or Statistics, rather than just one of the key standards.

4.5 Summary

Through the analysis of NCEA participation and achievement data from 2013-2019 of the key standards (see Table 3.1), I have shown that there has been a decline in many of the key standards across all decile groups. Additionally, there has been decreases in the rates of participation in in many of the combinations of the key standards at each level in Mathematics and Statistics over this period. There are equity concerns around both participation and achievement revealed through the analysis. There is a clear relationship between the rates of participation in many of the individual, and combinations of, key standards in Mathematics and Statistics and school decile at each level, with students from high decile schools being more likely than those from low decile schools to be participating in these standards. Additionally, there is a clear relationship between achievement (pass rates and rates of Merit and Excellence grades) for the externally assessed key standards, with students from higher decile schools more likely to achieve higher. By combining the cumulative effects of participation rates, proportions of those not attempting the examination, and pass rates, students at decile 1 schools are less likely to be awarded the key standards in Mathematics and Statistics than students from decile 10 schools. This has significant implications for future progression in Mathematics and Statistics. In Chapter 5, I discuss these implications in greater depth.

Chapter 5: Discussion and Conclusion

5.1 Introduction

Previous literature on NCEA has exposed many issues such as risks to both course coherence and the development of disciplinary knowledge, and the negative implications for students of a focus on credit collection without the development of opportunities for pathways (for example: Elley, Hall, & Marsh, 2004; Hipkins, Johnston, & Sheehan, 2016; Education Review Office, 2013; Locke, 2018; Munro, 2018; Rata & Taylor, 2015). Despite the intended NCEA policy goal of enhancing equitable learning opportunities and outcomes (Mallard, 2001), many researchers have raised concerns about inequalities in learning opportunities, achievement, and pathways seen through ethnicity and school decile (for example: Haque, 2014; Hipkins et al., 2016; Jensen, Madjar, & McKinley, 2010; Jensen, McKinley, & Madjar, 2010; Riley, 2014; Wilson, McNaughton, & Zhu, 2017; Yoon & Rata, 2018). This literature prompted my research to understand how these concerns have manifested in NCEA Mathematics and Statistics participation and achievement over time and the potential implications of these patterns for students from low decile schools.

My research was underpinned by Bernstein's (1999) theorisation of structures of knowledge and Young's (2008) notion of powerful knowledge. The implications of these for Mathematics, which has a vertical knowledge structure, is that for equitable outcomes all students must have access to coherent courses which support progression. This means the building blocks of layers of learning at the fundamental layers must be carefully designed to be correctly sequenced and paced to allow for increasing sophistication in thinking through higher order abstraction and generalisation.

A coherent course in Mathematics and Statistics in NCEA is designed to develop disciplinary knowledge and prepare students for progression within NCEA and potentially onto university. In Chapter 3, I proposed an evaluative framework of a set of the key standards in each of Mathematics and Statistics, based on Bernstein's (1999) theory of vertical knowledge (see Table 3.1). With a large numbers of achievement standards at each level of NCEA, careful course design is required to provide the optimal learning opportunities for progression. As I argued in Chapter 3, there is also a social equity issue at play here, with Algebra acting as a gatekeeper for progression in Mathematics (Knuth, 2016), and the standard AS1.2 (Algebra) acting as a gatekeeper at Level 1 in NCEA (see Figure 3.1). It is important that participation and achievement in Mathematics supports all learners to succeed and progress.

The findings in Chapter 4 showed declines in participation rates of many of the key standards and their combinations over the years. In addition, the chapter revealed differences in both participation and achievement for students from different school deciles. This section discusses some possible reasons for these findings and their implications for students' pathways and for policy.

5.2 Possible explanations for findings

National trends in NCEA (2009-2019): The first major finding of this research was that there was declining participation in the key standards in Mathematics and Statistics and that declines were more pronounced in those standards that were externally assessed (for example: Figures 4.1 and 4.2). This decline in external assessment is consistent with the overall national data from 2009-2019, which showed a clear decrease over time in the percentage of results gained through externally assessed standards (Table 5.1), although not to the same extent as was seen in some of the Mathematics and Statistics externals. The national data shows a clear increase over this period in internally assessed standards, partly due to the phasing out of curriculum-based unit standards from 2011, but also due to the decrease in participation in externally assessed standards. The increase in internally assessed standards was seen for the lower decile groups in some of the key standards in Statistics (for example see Chapter 4, Figure 4.4). Externally assessed standards are generally perceived to be harder to achieve in than internally assessed standards, which may be contributing to the decline in the participation of the externally assessed key standards in Mathematics and Statistics. The overall national data for 2019 would support this belief with the pass rate for externally assessed standards at 78.4% being lower than the pass rate for internally assessed standards at 84.7% (NZQA, 2020a). Further, the higher grades of Merit and Excellence were more likely to be gained in internally assessed standards (50.5% of results) than in externally assessed standards (40.6%).

Table 5.1: Percentage of results by standard type (2009-2019).

Year	Externally assessed achievement standards (%)	Internally assessed achievement standards (%)	Unit standards (%)
2009	30.4	31.5	38.1
2010	31.0	32.9	36.1
2011	28.4	40.4	31.2
2012	27.8	48.7	23.5
2013	26.5	53.2	20.5
2014	26.1	54.8	19.1
2015	25.3	55.6	19.1
2016	24.6	56.2	19.2
2017	23.9	57.3	18.8
2018	23.2	57.5	19.2
2019	22.5	58.2	19.3

Source: NZQA, 2010-2020.

Achievement differences between internally and externally assessed standards can be seen in the Mathematics and Statistics key standards, especially for the students from low decile schools (Tables 4 and 5). While critics of internal assessment have attributed these higher pass rates in internally assessed standards to being due to less robust and reliable assessment of students' academic achievements, advocates of internal assessment offer a number of explanations for higher achievement in internally assessed standards (Hipkins et al., 2016). These include an assessment environment which is lower pressure and more flexible (not a limited time, closed book examination which most externally assessed standards are), resubmission and reassessment opportunities offered by schools, and teachers choosing contexts for assessment relevant and accessible to students (Hipkins et al., 2016). Nonetheless, I have argued that selecting standards which do not form a coherent programme hinders progression and reduces pathways in these subjects, and therefore while the tendency to focus on internal assessment has some merits, it also is reducing pathways in Algebra and Calculus, where externally assessed standards hold the potential for richer future pathways.

Policy context in New Zealand (2012-2019): There are a number of drivers for teachers selecting what are perceived to be the 'easier' standards, usually the internally assessed standards and those with lower literacy demands, which may be contributing to a decline in participation in the externally assessed key standards in Mathematics and Statistics over time. Some of these are discussed in this section. The first of these is the Better Public Service (BPS) target, introduced in 2012 by the National government, for at least 85% of 18-year olds to achieve NCEA Level 2 or higher by 2017 (New Zealand Government, 2018). The intended outcome of the goal was to have more students keeping their learning pathways open as Level 2 was considered the minimum standard required for further study and to increase employment prospects (Hipkins et al., 2016). The government's Māori education strategy, Ka Hikitia – Accelerating Success 2013-2017, also set an 85% target for Māori students to gain NCEA Level 2, intended to close the gap in achievement in NCEA (Ministry of Education, 2013). These targets resulted in many schools setting their own targets for increasing achievement overall and for Māori students, incentivising teachers to look for ways to recognise the achievements of students, which may include selecting standards perceived to be easier (Hipkins at al, 2016). This target was revoked by the Labour government early-2018 (New Zealand Government, 2018).

The BPS goal was arguably successful in increasing the overall achievement of NCEA Level 2 for 18-year olds which increased from 77.2% in 2012 to 84.9% in 2017 and 85.1% in its final year of monitoring in 2018 (Ministry of Education, 2019a). 18-year old Māori students over this same period increase their attainment of NCEA Level 2 from 60.9% to 76.1%, thus narrowing the gap between Māori and non-Māori students. Data from NZQA on the percentage attainment rates of Year 11 students gaining Level 1, Year 12 students gaining Level 2, and Year 13 students gaining Level 3 from 2011-2019 showed an increase over the years up to around 2017 (NZQA, 2020a). After that it continued to increase for Level 3, levelled off at Level 2, and declined for Level 1. Despite these increases at all levels, the attainment rate of UE has stayed fairly steady. Yoon and Rata (2018) examined this issue with a focus on ethnicity groups. They found that approximately 50% of all students gained UE each year from 2005-2017. The percentage of Māori students gaining UE also stayed fairly consistent at 30%, which given the increase in Level 2 attainment meant that there was an increasing gap between Level 2 achievement and UE achievement for all students, but more so for Māori students. Yoon and Rata (2018) suggested this was due to compromises in the *quality* of what students were learning despite an increase in the *quantity* of credits being gained. This provides a potential explanation for what is happening in Mathematics and Statistics. We saw a decline in the externally assessed standards in Mathematics and

Statistics, as they are likely perceived to be harder therefore do not serve the purpose of readily increasing the achievement of students. Further research is needed on the Mathematics and Statistics standards outside of those defined as the ‘key standards’ in this study to look at their pattern of participation over time. It may be that given the range of internally assessed standards on offer, some others of these have had increased participation rates.

Published school league tables: Published school league tables, which encourage comparison and competition between schools, may also incentivise a focus on achieving credits rather than coherent courses and pathways. Concerns have been raised that league tables incentivise schools to withdraw students from standards that they do not think they will achieve in or encourage lower achieving students into ‘alternative courses’, designed to enhance student achievement through the selection of ‘easier’ standards (Hipkins et al., 2016). School league tables may provide a contributing explanation in the decline of the participation in the key standards, especially if we take into account the competition built into Tomorrow’s Schools (Haque, 2014). This comes at the cost of coherent courses in Mathematics and Statistics which support progression and pathways.

Student choice: Students and their families do not always understand the implications of their choices at course selection time, and that they may be limiting pathways for future studies within a subject, especially if they select ‘alternative courses’, such as internally assessed versions of courses (Jensen, McKinley, & Madjar, 2010). Mathematics and Statistics tends to be offered as multiple (typically streamed) courses in the senior school, with different standards on offer in the various courses. This could be a potential reason for students not participating in the key standards, especially at lower decile schools where the rates of participation are lower (see for example Chapter 4, Figures 4.1 and 4.2).

Student choice within courses also plays a part in the coherence of subjects. Students opting out of standards is an issue in NCEA, which has contributed to the decline in coherent courses which may be affecting the participation and achievement in the key standards in Mathematics and Statistics. Courses can be well-designed to support disciplinary development, but gaps are created if students then opt out of standards (Yoon & Rata, 2018). In a study of first year students at Victoria University, it was found that over 70% of students were at least ‘at times’ choosing to drop standards to reduce their workload, with 14% reporting that they did this ‘a great deal’ (Cherrington, Johnston, Wood, Mortlock, &

Boniface, 2018). They found that for externally assessed standards, students were often dropping standards to give themselves longer in the examination to focus on fewer standards or because they already had sufficient credits for their qualification. The data for the externally assessed key standards for Mathematics and Statistics suggest that this strategy is being increasingly employed by students in this subject with the rates of absences and papers not attempted increasing since 2013 for most of the externally assessed key standards (see Chapter 4, Tables 4 and 5).

School programme design: 'Personalised' or 'individualised' NCEA courses where students, supported by a teacher, design their course from a suite of standards, within and across subjects, are also increasing in popularity (for example: Milmine, 2019) and such approaches may lead to a further decline in course coherence in Mathematics and Statistics. These approaches are situated within the policy context described above which incentivises achievement. There are examples of this style of flexible NCEA course being implemented in schools for Mathematics and Statistics and promoted by the Ministry of Education (2020a). Young (2008) stated that when students are entrusted with the responsibility for constructing their curriculum from a number of modules, it undermines the conditions required for the learning of a discipline. Knowledge also risks losing its authority when student choice directs learning (Hirshman & Wood, 2018). While the intended course can be guided by the teacher, if the learning is self-directed and students control the pace, and therefore the amount of content or standards covered, this risks student learning (Riley, 2014). These personalised courses provide a challenge to ensure every student has access to a coherent course which builds disciplinary knowledge, supports progression and pathways within Mathematics and Statistics, and does not increase the achievement gaps which I have demonstrated are evident in the NCEA data.

5.3 Implications

My study appears to show that we have reached the stage in New Zealand where students are less likely now to be receiving courses which include one, or both, of the key standards in Mathematics and Statistics at each level than they were approximately five years earlier (see sections 4.2 and 4.4). This decline is further compounded by the differences in both participation and achievement experienced by students from low decile schools (see Section

4.3). So, what are the implications of these findings for students and their pathways to university?

Equity: First, we have an equity issue. As the results of this dissertation show, Mathematics and Statistics in general and Algebra in particular, which acts as a gateway to further learning in Mathematics (Knuth, 2016), are not equally participated or achieved in by students from different school deciles. The differences in participation rates and achievement in the key standards in Mathematics and Statistics between students from high and low decile schools that my study identified are consistent with the NCEA equity issues previously documented, for example by Wilson, McNaughton, and Zhu (2017). The findings in this dissertation support their findings of lower participation and achievement of students from low decile schools in the high-literacy standards of AS1.12 (Chance and Data), AS2.6 (Algebra), AS2.12 (Probability), and AS3.13 (Probability) (see Section 4.2). The only externally assessed standard in my study to show improvement in participation rates over the period 2013-2019 for students from low decile schools was AS1.12, arguably because it is perceived to be the easiest of the externally assessed standards at this level. The achievement rates for Levels 1-3 NCEA by decile groups 1-3, 4-7, and 8-10 from 2010-2019 also show that the higher decile groups have a higher attainment rate than the low decile groups (NZQA, 2020a). The differences in achievement rates for these groups have decreased over time but still persist (NZQA, 2020a). The case is the same when looking at the achievement standards, with students from higher decile schools consistently having higher pass rates and are more likely to gain higher grades, than students from low decile schools in 2019 (NZQA, 2020a). This achievement gap is consistent with what was seen for the key standards in Mathematics and Statistics (section 4.3).

My analysis in Chapter 4 showed that there was a clear difference in the achievement of the externally assessed key standards in Mathematics and Statistics for students from decile 1 and decile 10 schools, but the differences in pass rates for the internally assessed key standards was much less (see Chapter 4, Tables 4 and 5). This is consistent with other analyses, both overall and for specific subjects for high and low decile groups (NZQA, 2020a; Riley, 2014), although the differences in Mathematics and Statistics are much higher for decile 1 groups. With these differences in mind, it would appear that schools' strategies of selecting internally assessed standards over externally assessed standards are successful for raising student achievement in lower decile schools, which do not historically experience the same level of success in externally assessed standards as students from higher decile schools. However, this approach risks undermining course coherence and future pathways (McPhail, 2019; Munro, 2018). Students from low decile schools are less likely to be successful in the externally assessed key standards in Mathematics and are therefore less prepared for progression and pathways in these subjects.

Pathways to the future: Given the hierarchical structure of Mathematics knowledge, students who are do not participate in optimal NCEA courses in high school are likely to be inadequately prepared for university study in this area. Universities are responding to this by offering first year courses to bridge the gaps, an example being the University of Auckland, which has a number of pathways depending on the standards done during NCEA Level 3 and the level of achievement (Appendix C). While the attempts to accommodate students who have not attained the ideal prerequisites provide an opportunity to continue with pathways previously cut off to them, it must be remembered that these bridging papers have a cost associated with them, either in terms of the fees for the paper, or the lost opportunity to do another paper to add depth or diversity to a qualification. For Statistics, many universities have no specific prerequisites. However, the Department of Statistics at University of Auckland has recommended standards (The University of Auckland, Department of Statistics, 2020), which are the three key Statistics standards at Level 3 also proposed in this study (see Chapter 3, Table 3.1). Failing to study these standards means that students are not best prepared to succeed. Further, NCEA Mathematics standards are recommended or required for some of the Statistics papers at Universities in New Zealand and the best preparation for Statistics study at university would be the key standards in both Mathematics and Statistics. Mathematics and Statistics papers are also required as components of degrees with majors in subjects such as Medicine, Commerce, Psychology, Physics, Computer Science, and Engineering. Therefore, the decline and inequities in participation in the key standards impacts on pathways to many other fields outside of Mathematics and Statistics. The decline in the externally assessed standards, which are assessed by examinations in Mathematics and Statistics, also means that students are poorly set up for university study due to the automaticity they promote (Johnston, Wood, Cherrington, Boniface, & Morlock, 2020, under review) and the skills in examination-taking they develop (Jensen, Madjar, & McKinley, 2010). The inequity in achievement seen in Mathematics and Statistics in the key standards for students from low decile schools means that these students' chances of success at university are lowered (James, Montelle, & Williams, 2008; Johnston et al., 2020; The University of Auckland Planning Unit, 2017).

5.4 Future implications

NCEA Change Package: This study is timely given the changes to NCEA that are underway due to the identified problems with NCEA that need addressing. Announcing the NCEA review in 2018, Minister Hipkins reported that the intended policy goals of NCEA had not yet been met, which he attributed in part to the accountability targets driving implementation (Hipkins, 2018). There are several changes to be introduced by 2025 which are supported by and relevant to the findings of this study. The NCEA Change Package describing the intended changes to NCEA has a strong message about increasing course coherence in NCEA to simplify course design for teachers and develop clear pathways for students through the reduction in the numbers of standards and credits available in subjects (Ministry of Education, 2019c). Many of the changes identified to NCEA mention increased equitable outcomes and the recognition of te ao Māori and mātauranga Māori (the Māori World and knowledge), and Pacific knowledge (Ministry of Education, 2019c), as one of the guiding principles of the review was *Equity and inclusion* (NCEA Review Ministerial Advisory Group, 2018). There are also changes to the literacy and numeracy requirement to strengthen these.

To achieve coherence for each subject, the knowledge structure of the subjects must be understood by the panels who are clarifying the 'essence' of learning areas and how, in subjects with hierarchical knowledge structures, the learning must be carefully sequenced and paced. A challenge exists in the learning area of Mathematics and Statistics when reducing the current standards down to four standards for a total of 20 credits, where currently there are 13-15 standards, with a total of 43-60 credits, available at each level. While it may be anticipated that Statistics and Calculus will remain as separate subjects at Level 3 given the distinct pathways they provide to university, it is unclear if this will happen at Level 2, and at Level 1 it has been decided that there will be a single subject. This reduction in standards will require the panels to have a deep understanding of the knowledge to prioritise for progression, which this research has proposed, although this study does not claim to provide the exhaustive set of concepts required for a full and rich course. There are always unintended consequences for policy that will not be understood until after they are implemented (Rizvi & Lingard, 2010), but the decision to have a single or two subjects at Level 2 for Mathematics and Statistics will need to be carefully weighed up as there is the potential for negative consequences for either decision.

The changes to numeracy (and literacy) are intended to address concerns that some students are leaving school without the numeracy (and literacy) levels to be able to engage in the community and in work or further study (Ministry of Education, 2019c). It is suggested that this is because it is being assessed in different ways, as numeracy can be gained through either unit standards or achievement standards, and these achievement standards can be from a range of subjects, not just Mathematics and Statistics. It is planned that these will be externally assessed standards that sit outside of the Mathematics and Statistics achievement standards and will be a co-requisite to NCEA qualifications. Currently, most schools require Year 11 students to take a Mathematics and Statistics course at NCEA Level 1 to gain their Numeracy credits, with some doing this as the unit standards Numeracy course, but most doing achievement standards. A separate co-requisite, which it is suggested that students could sit earlier in Years 9 or 10, will be set at Curriculum Levels 4-5 (Ministry of Education, 2019b). This means that there will no longer be a need for students to do NCEA Level 1 Mathematics and Statistics, risking the decline in participation in this subject area if schools no longer make this subject compulsory, with a particular risk to students from low decile schools who have experienced less success on average than those from higher decile schools. This would undermine the equity and inclusion principle of the review. Setting the numeracy co-requisite at Curriculum Levels 4-5 is also unlikely to raise numeracy levels as it sits lower than the NCEA Level 1 achievement standards, which are at Curriculum Level 6.

Teacher professional development: Educators play an important role in shaping and enacting education policy (Bell & Stevenson, 2006) and there needs to be sufficient support for teachers for this round of NCEA assessment reform for the goals of course coherence, equity and inclusion, and improved numeracy levels, to be enacted. It will be important to critically analyse the NCEA reform process over the years, both at the initial implementation of NCEA and the NCEA-NZC alignment (2011-2013) when planning for the implementation of this NCEA Change Package. Insufficient support was provided for teachers for the development of coherent curriculum planning and to implement the improvements to teaching and learning that were assumed would eventuate (Haque, 2014; Locke, 2010; Locke, 2018). Professional learning opportunities were focused on assessment only: the tasks, exemplars of student responses, and how to make judgements (Locke, 2010). For course coherence in Mathematics and Statistics to be realised through these new standards, course planning must start with a deep understanding of disciplinary knowledge and the sequencing and pacing of learning required to build knowledge in subjects with vertical knowledge structures. From there, opportunities for assessment should be identified. This differs to the typical way in which courses are defined in Mathematics and Statistics, which is typically by listing the achievement standards. Mathematics and Statistics teachers will require professional learning in both coherent course design and the new achievement standards and sufficient resourcing on the new assessments, as well as the time to be able to implement these changes.

These changes to course coherence and numeracy requirements, even if successfully implemented as intended, are unlikely in themselves to result in equitable outcomes for all learners. Considering the participation rates in the Mathematics and Statistics key standards over 2013-2019 for students from different school deciles, having fewer standards as part of a more coherent course will reduce the differences in the standards that students participate in, reducing the inequities in participation experienced by students from lower decile schools. However, the picture is made more complex when the rates of students not attempting externally assessed standards and the achievement of students are considered for students from different decile schools. This shows that the solution is not solely to have students do the same standards as it is unlikely that the inequitable achievement will be fixed by a standardised course. Instead, a wider view of the education system and educational achievement of students is required. Achievement differences in Mathematics for students from different school deciles are seen for students at primary school (for example: Ministry of Education, 2017). Changes would need to occur across the entire educational system, from Year 1 to Year 13, to identify students progressing more slowly in Mathematics and to accelerate their learning to be at the expected level. It also requires high teacher expectations of all students. Without this, students who are not achieving as highly in Mathematics will continue to opt out when they get the opportunity or be streamed into lower courses, a practice which is prevalent in New Zealand (Hood, 2020; O’Callaghan, 2020).

5.5 Limitations and future research

This is a small-scale study which proposed a framework of key standards for progression and future pathways in Mathematics and Statistics in a single qualification system (NCEA) in New Zealand and then tracked participation and achievement of students in the past seven years. This framework is open to critique from researchers and educators as it is unlikely to capture all possible options for ensuring a rich programme in Mathematics and Statistics. However, it does open up the conversation about how well the system of NCEA is performing, and what may be needed to ensure that all students receive equitable opportunities.

The key standards proposed in this research were selected based on the scope of the standards and the nature of their assessments. This research has not established whether achievement in these standards predicts success at subsequent levels of NCEA. This research would benefit from a longitudinal study of a cohort of students progressing through NCEA and onto university study in both Mathematics and Statistics. Further statistical analysis, similar to that carried out by Johnston, Hipkins, & Sheehan (2017) on predicting success in Biology and History NCEA achievement standards, would be recommended to interrogate the proposed key standards and their role in progression within NCEA and onto university, especially in Statistics where there is a gap in the research.

This study was limited by time and size and other demographics, such as ethnicity and gender, were outside of scope of this study. Recognising these limitations, future research could include deeper investigation into how patterns of participation may differ for boys and girls, and for different ethnicities to gain a fuller picture of the equity issues within NCEA Mathematics and Statistics. My study did not consider the transferability of this model to other assessment systems in New Zealand or internationally. However, this also would make for an interesting exploration, as at the same time as this study in NZ, other countries such as England and Australia were introducing a much more prescribed curriculum model (Sinnema, 2015).

In my analysis, I suggested that one could assume the composition of courses that were available to students from schools with different deciles. This was a tentative suggestion which had limitations given what is known about students self-selecting standards and the range of courses on offer due to the common practice of streaming in schools. Further qualitative research is needed to further understand the composition of Mathematics and Statistics courses on offer to students, the selection criteria for courses, and the curriculum decision making that takes place in schools. It would be of interest to use case studies from a variety of schools, including exploring how streaming may or may not affect curriculum choices and outcomes. For example, a low decile school which had abolished streaming and experienced high success with their students in AS1.2 (Algebra) and improved retention rates was recently profiled in the media (O'Callaghan, 2020) and this would be very interesting to compare with schools which have introduced 'individualised' Mathematics programmes (for example: Ministry of Education, 2020a). This would provide insight into the curriculum design across a range of settings, teacher decision making, and the outcomes for student equity in achievement and participation.

This research has focused on pathways to university study in Mathematics and Statistics and would also benefit from a wider university perspective. University courses such as Engineering, Computer Science, Psychology, and Medicine, to name a few, all require high levels of Mathematics or Statistics. It would be of interest to understand the role that participation and achievement in these proposed key standards and other NCEA standards in Mathematics and Statistics, play in predicting success in these university courses.

5.6 Conclusion

This research sought to answer the following research questions:

> *RQ1: Which of the NCEA achievement standards in Mathematics and Statistics, herein known as the 'key standards', best provide for coherent programmes of learning and future pathways within these disciplines?*
>
> *RQ2: For the key standards in Mathematics and Statistics, what are the patterns of student participation and achievement from 2013 to 2019 in relation to school deciles?*

In answer to Research Question 1, the key standards that I proposed are fundamental to a coherent programme of learning with pathways for many future options. These were theoretically based on Bernstein's (1999) vertical knowledge structure for disciplines such as Mathematics and the research literature on progression within Mathematics and Statistics. These proposed key standards are not intended to constitute a full course of study at each level but instead, prioritise the standards with concepts, skills, and knowledge which are intended to optimise progression. I have argued that these standards are necessary for both equity and future pathways for students. These key standards proposed are given in Table 3.1.

To answer Research Question 2, the NCEA participation and achievement data for the key standards was analysed by decile groups from 2013-2019. There are clear patterns of declining participation over this time in many of the key standards, and especially in the externally assessed standards. This decrease in participation in individual standards, combined with a decrease in the courses which appear to offer combinations of the key standards, has resulted in declines in the participation rates in combinations of the key standards at each level.

My analysis of these patterns of participation rates revealed that there was a clear association between the likelihood of participating in courses offering the combination of key standards in each of Mathematics and Statistics and the decile of the school that students attend. This pattern was also confirmed in my analysis of student achievement against school deciles. The implications of this are concerning due to both inequitable learning opportunities and outcomes in Mathematics and Statistics. I demonstrated in my analysis that students at decile 1 schools were far less likely than those at decile 10 schools to be awarded the key standards, especially for the externally assessed standards. This has consequences for their preparedness and future success at university in Mathematics and Statistics, as well as in a range of subjects which require mathematical and statistical knowledge, such as engineering and psychology.

Further research in this area will be important to elucidate the predictive properties of the key standards for future academic success and to understand curriculum design and teacher decision making within schools. While the NCEA Change Package (Ministry of Education, 2019c) aims to improve subject coherence and equitable outcomes such as those identified in this study the jury is still out about whether these changes will result in greater equity and coherence or if schools will find another way to ‘soften’ their expectations. Many of the details are still unknown and heavy investment in teacher professional development will be crucial to realise the policy intentions.

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Appendices

Appendix A

Year 11 roll numbers (Ministry of Education, 2020d)

Decile	2013	2014	2015	2016	2017	2018	2019
1	2618	2647	3213	3269	3296	3273	3276
2	3634	3618	3272	3095	2924	2819	2820
3	3866	3712	4626	4647	4418	4263	4534
4	5337	5113	5745	5806	5660	5592	5584
5	5920	5853	4837	4698	4653	4443	4705
6	7437	7233	8168	8202	7921	7946	7991
7	6054	5961	7419	7604	7491	7586	7633
8	8403	8124	7118	7282	7064	7019	7014
9	6900	6967	8152	8250	8285	8139	8402
10	8598	8534	6287	6633	6647	6862	7264

Year 12 roll numbers (Ministry of Education, 2020d)

Decile	2013	2014	2015	2016	2017	2018	2019
1	2355	2253	2800	2786	2904	2731	2793
2	3134	2870	2635	2682	2624	2440	2380
3	3219	3324	4109	4031	3976	3829	3693
4	4653	4727	5072	5209	5323	5042	4997
5	5316	5259	4139	4365	4241	4105	3986
6	6610	6684	7205	7408	7457	7254	7102
7	5484	5582	6905	7010	7187	6996	7201
8	7505	7850	6444	6911	7019	6699	6597
9	6908	6851	7756	8026	8082	8044	7943
10	8391	8787	6091	6590	6726	6695	6817

Year 13 roll numbers (Ministry of Education, 2020c)

Decile	2013	2014	2015	2016	2017	2018	2019
1	1981	2050	2478	2511	2548	2494	2435
2	2582	2555	2169	2130	2120	2094	1949
3	2764	2698	3402	3498	3352	3306	3090
4	3946	3805	4267	4070	4106	4102	4015
5	5746	5506	3271	3228	3336	3215	3241
6	5270	5098	7069	6937	7125	7022	6692
7	4656	4489	5811	5605	5709	5878	6383
8	6219	6237	5518	5444	5818	5802	5539
9	5866	5926	6821	6718	7015	6965	6909
10	7516	7450	5497	5659	5971	5947	6021

Appendix B

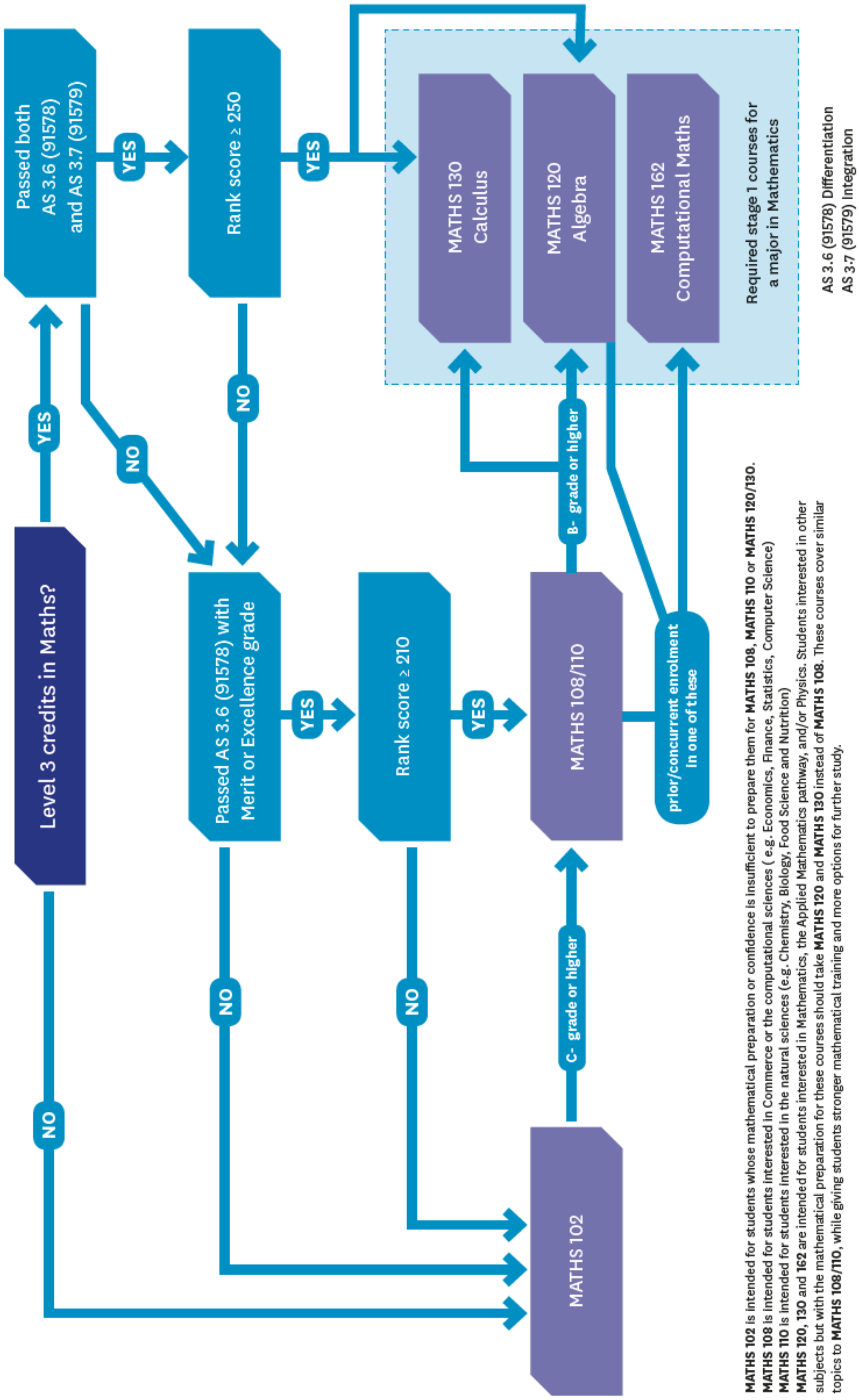
Minimum first year course prerequisites for the core papers in Mathematics and Statistics for New Zealand universities.

University	Subject	Minimum recommended/required NCEA papers and grades for a major in Mathematics/Statistics
The University of Auckland Source: The University of Auckland (2020a); The University of Auckland (2020b); The University of Auckland Planning Unit (2017); The University of Auckland, Department of Statistics (2020)	Mathematics	Passed both AS 3.6 (Differentiation) and AS 3.7 (Integration) and rank score of at least 250. At least 18 credits in Mathematics at NCEA Level 3 including at least 9 credits at Merit or Excellence. It is strongly recommended that NCEA students have a Merit or Excellence AS3.6 (Differentiation) and AS3.7 (Integration).
	Statistics	None stated on the university's website. Through the Census at School website (The University of Auckland, Department of Statistics, 2020), the Department of Statistics, recommends AS3.10 (Inference), AS3.9 (Bivariate), and AS3.13 (Probability) as a statistics core course for the approved subject Statistics or for a Mathematics course for students hoping to proceed to study Statistics at University.
Auckland University of Technology Source: Auckland University of Technology (2020a); Auckland University of Technology (2020b)	Applied Mathematics	None stated. For enrolment, preference will be given to applicants with one or more NCEA Level 3 subjects from Calculus, Mathematics or Statistics. Passes in Physics and Computing at NCEA Level 3 are an advantage.
	Analytics	None stated. For enrolment, preference will be given to applicants with one or more Level 3 subjects from Calculus, Mathematics or Statistics. Students who do not have the above background may be directed to take certain papers in their first year of study.
The University of Waikato Source: The University of Waikato (2019)	Mathematics	16 credits in NCEA Level 3 Calculus including at least 11 credits from: AS3.5 (Algebra), AS3.6 (Differentiation), and AS3.7 (Integration)
	Statistics	Minimum prerequisites: 18 credits at Level 2 in NCEA Mathematics, or 10 credits at Level 3 in NCEA Calculus, or 14 credits at Level 3 in NCEA Mathematics.

Massey University Source: Massey University (2020a); Massey University (2020b)	Mathematics	At least 16 credits in the NCEA Level 3 Mathematics sub-field, including at least two of the following standards: AS3.5 (Algebra), AS3.6 (Differentiation), AS3.7 (Integration)
	Statistics	Required: At least 16 credits in NCEA Level 2 from the Mathematics sub-field. Recommended: At least 16 credits in the NCEA Level 3 Mathematics sub-field.
Victoria University of Wellington Source: Victoria University of Wellington (2020a); Victoria University of Wellington (2020b)	Mathematics	AS3.6 (Differentiation) achieved with Excellence, AS3.7 (Integration), and one of: AS3.1 (Conics), AS3.3 (Trigonometry), or AS3.5 (Algebra). Students need a Merit or Excellence grade in at least one of AS3.1, AS3.3, AS3.5, or AS3.7.
	Statistics	AS2.6 (Algebra) is recommended
University of Canterbury Source: University of Canterbury (2020a); University of Canterbury (2020b)	Mathematics	NCEA 14 credits at Level 3 in Mathematics
	Statistics	Statistics to Year 13 level
The University of Lincoln	n/a	
University of Otago Source: University of Otago (2020a); University of Otago (2020b)	Mathematics	AS3.6 (Differentiation) AS3.7 (Integration)
	Statistics	AS3.6 (Differentiation) AS3.7 (Integration)

Appendix C

First year Mathematics course details and prerequisites for The University of Auckland.



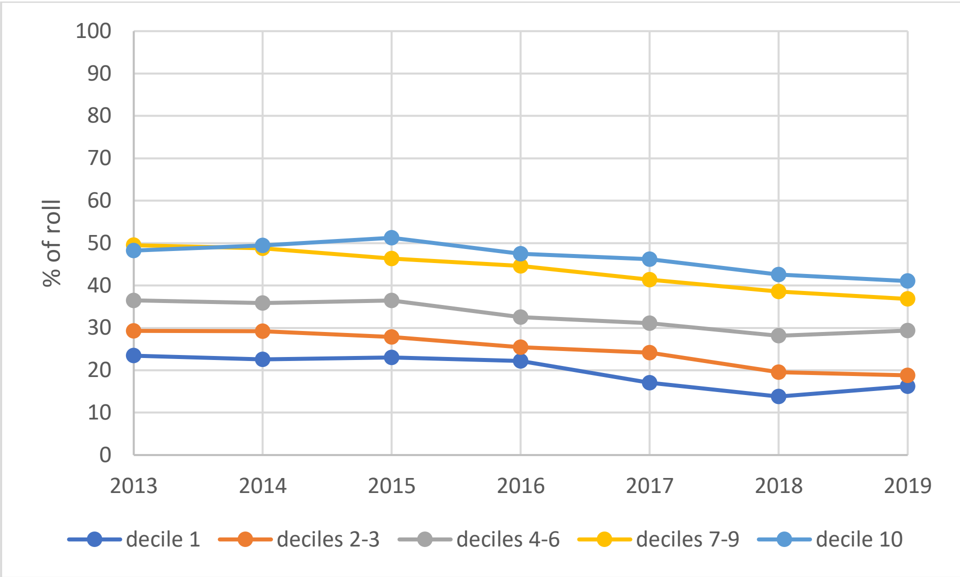
Source: The University of Auckland Planning Unit (2017).

Appendix D

Full details of the key standards

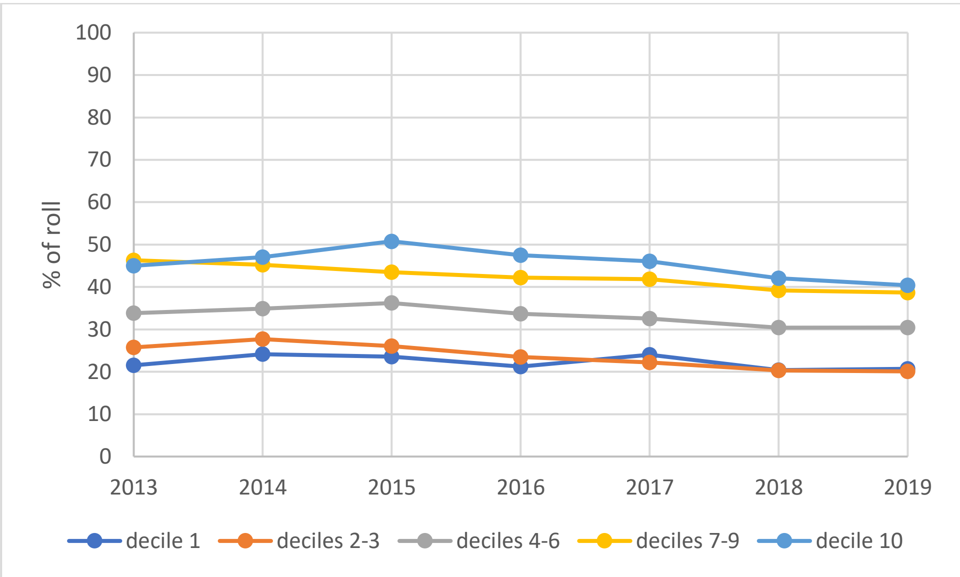
Level 1	Level 2	Level 3
AS91027 1.2 Apply algebraic procedures in solving problems 4 credits External Sub-field: Mathematics Short name: AS1.2 (Algebra)	AS91261 2.6 Apply algebraic methods in solving problems 4 credits External Sub-field: Mathematics Short name: AS2.6 (Algebra)	AS91578 3.6 Apply differentiation methods in solving problems 6 credits External Sub-field: Mathematics Short name: AS3.6 (Differentiation)
AS90128 1.3 Investigate relationships between tables, equations and graphs 4 credits External Sub-field: Mathematics Short name: AS1.3 (Table, Equations and Graphs)	AS91262 2.6 Apply calculus methods in solving problems 5 credits External Sub-field: Mathematics Short name: AS2.7 (Calculus)	AS91579 3.6 Apply integration methods in solving problems 6 credits External Sub-field: Mathematics Short name: AS3.7 (Integration)
AS91035 1.10 Investigate a given multivariate data set using the statistical enquiry cycle 4 credits Internal Sub-field: Statistics and Probability Short name: AS1.10 (Inference)	AS91264 2.9 Use statistical methods to make an inference 4 credits Internal Sub-field: Statistics and Probability Short name: AS2.9 (Inference)	AS91581 3.9 Investigate bivariate measurement data 4 credits Internal Sub-field: Statistics and Probability Short name: AS3.9 (Bivariate)
AS91037 1.12 Demonstrate understanding of chance and data 4 credits External Sub-field: Statistics and Probability Short name: AS1.12 (Chance and Data)	AS91267 2.12 Apply probability methods in solving problems 4 credits External Sub-field: Statistics and Probability Short name: AS2.12 (Probability)	AS91582 3.10 Use statistical methods to make a formal inference 4 credits Internal Sub-field: Statistics and Probability Short name: AS3.10 (Inference)
		AS91585 3.13 Apply probability concepts in solving problems 4 credits External Sub-field: Statistics and Probability Short name: AS3.13 (Probability)

Appendix E



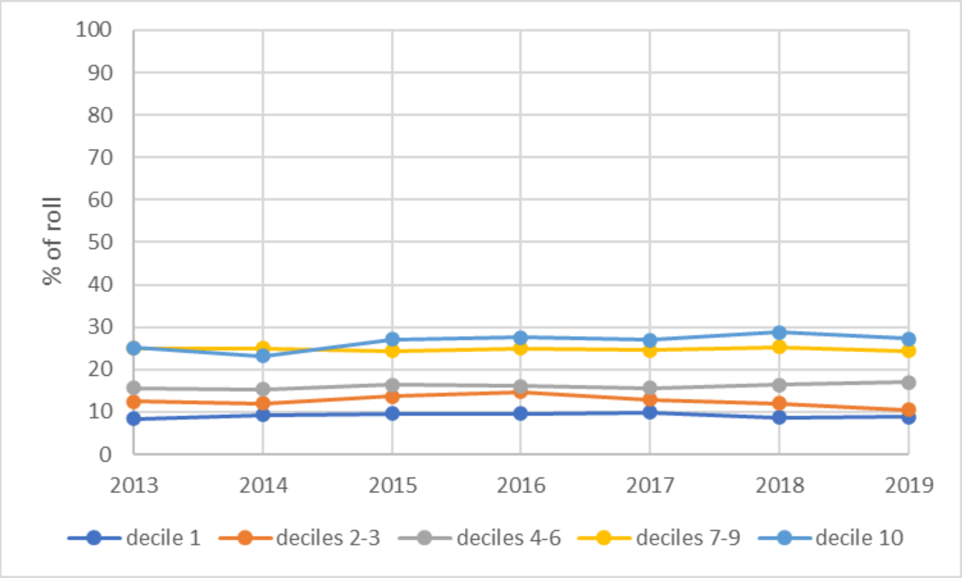
Rates of participation of Year 12 students in AS2.6 (Algebra), 2013-2019.

Appendix F



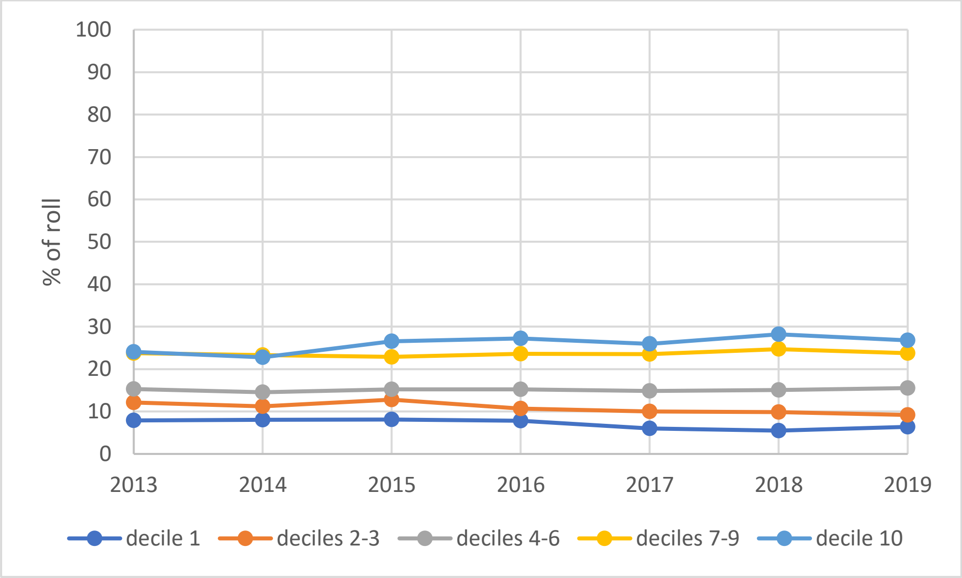
Rates of participation of Year 12 students in AS2.7 (Calculus), 2013-2019.

Appendix G



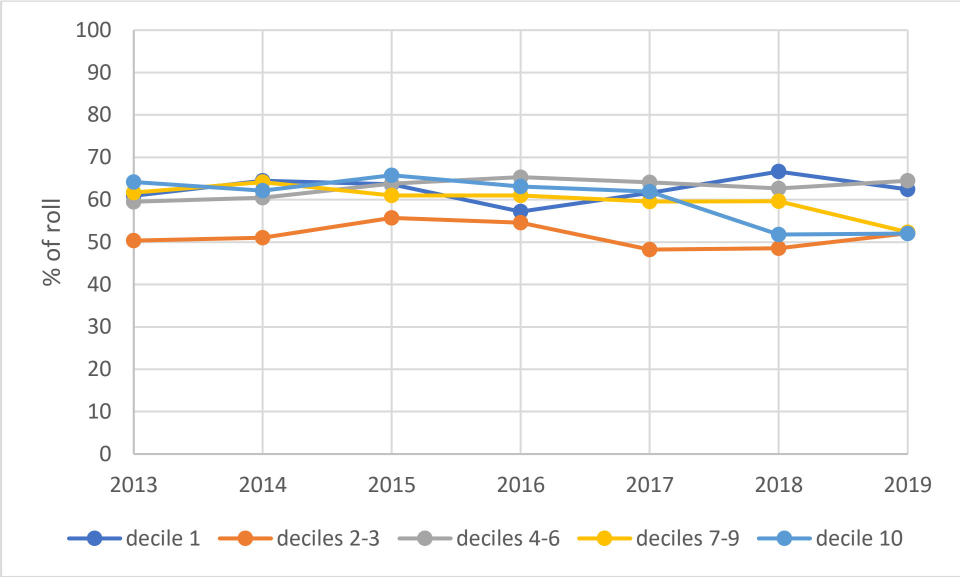
Rates of participation of Year 13 students in AS3.6 (Differentiation), 2013-2019.

Appendix H



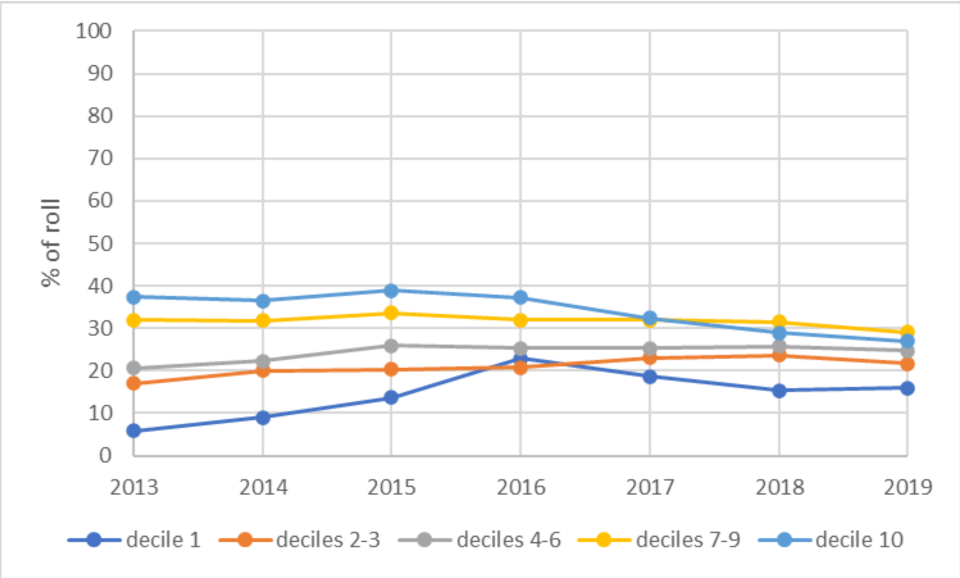
Rates of participation of Year 13 students in AS3.7 (Integration), 2013-2019.

Appendix I



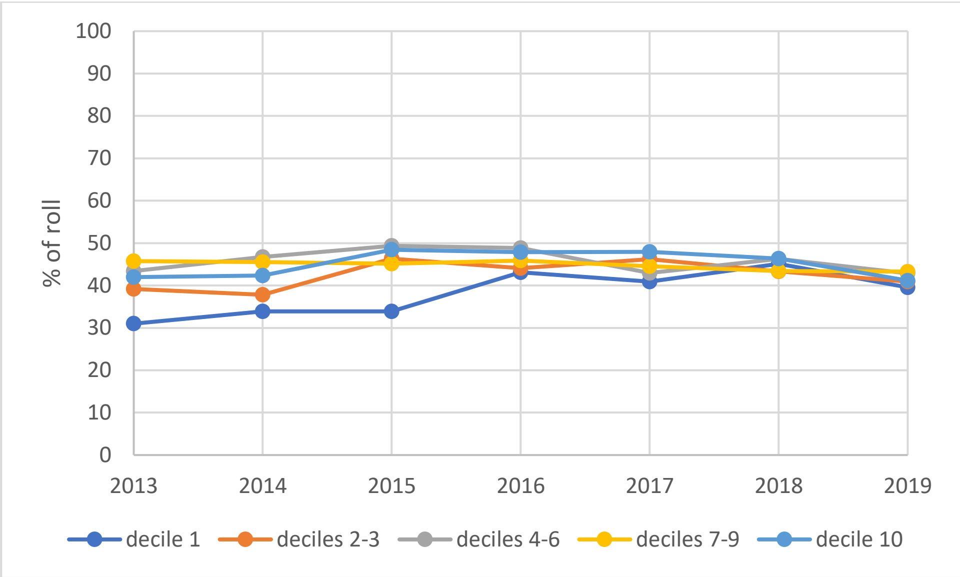
Rates of participation of Year 11 students in AS1.10 (Chance and Data), 2013-2019.

Appendix J



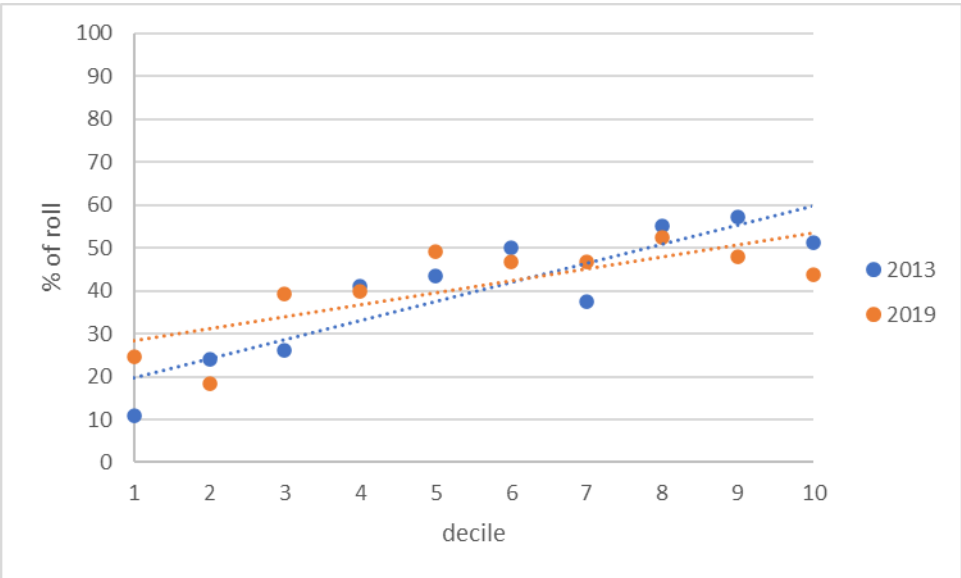
Rates of participation of Year 13 students in AS3.10 (Inference), 2013-2019.

Appendix K



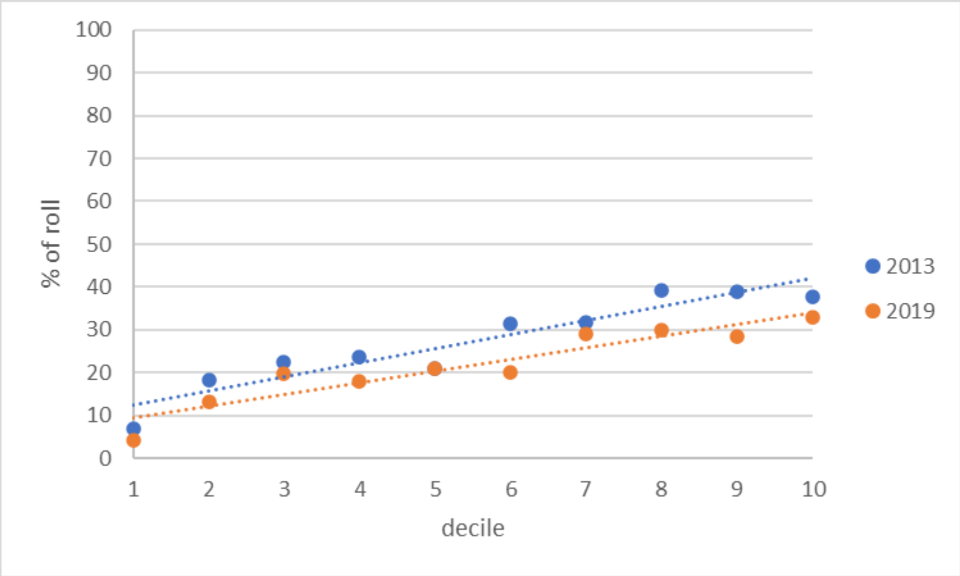
Rates of participation of Year 12 students in AS2.9 (Inference), 2013-2019.

Appendix L



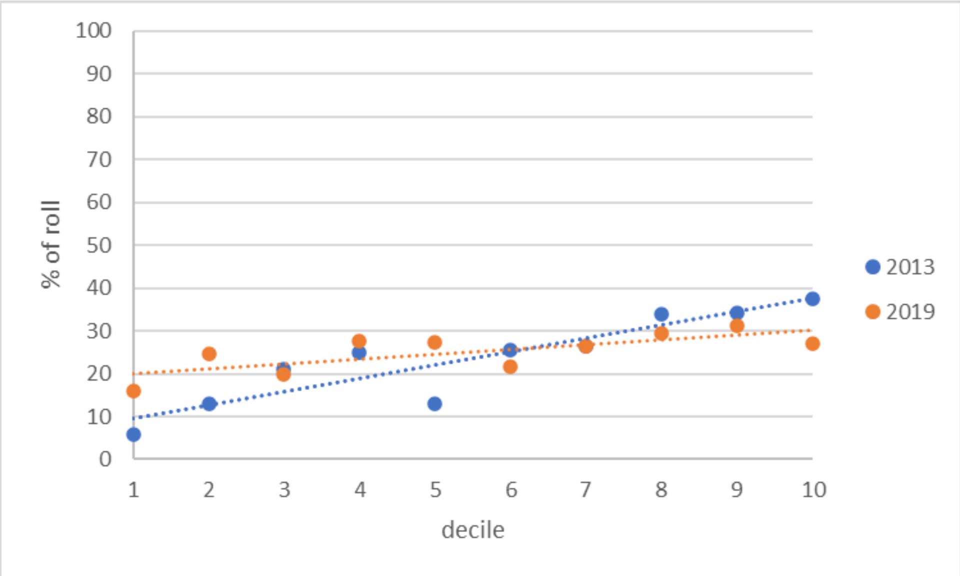
Rates of participation of Year 11 students in AS1.12 (Chance and Data) against school decile.

Appendix M



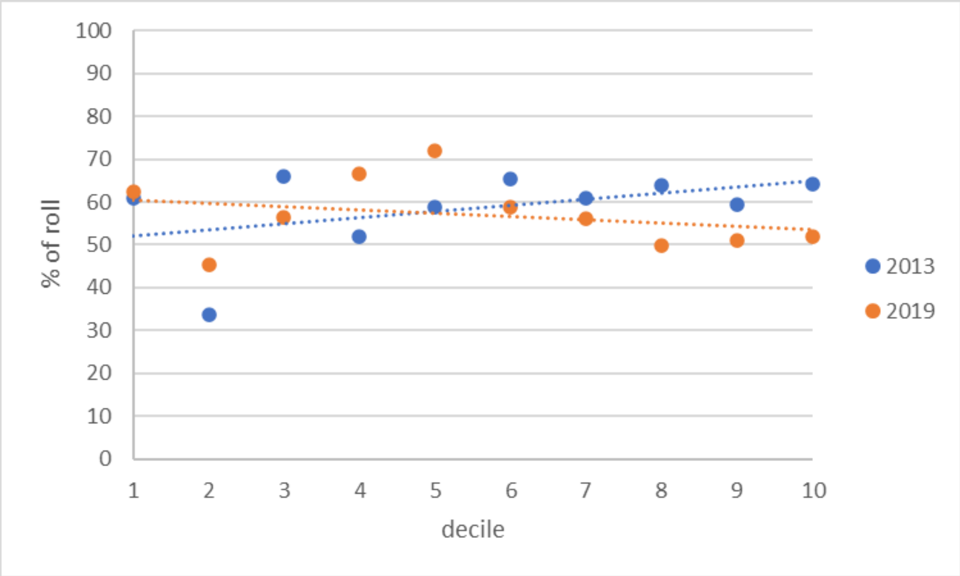
Rates of participation of Year 13 students in AS3.13 (Probability) against school decile.

Appendix N



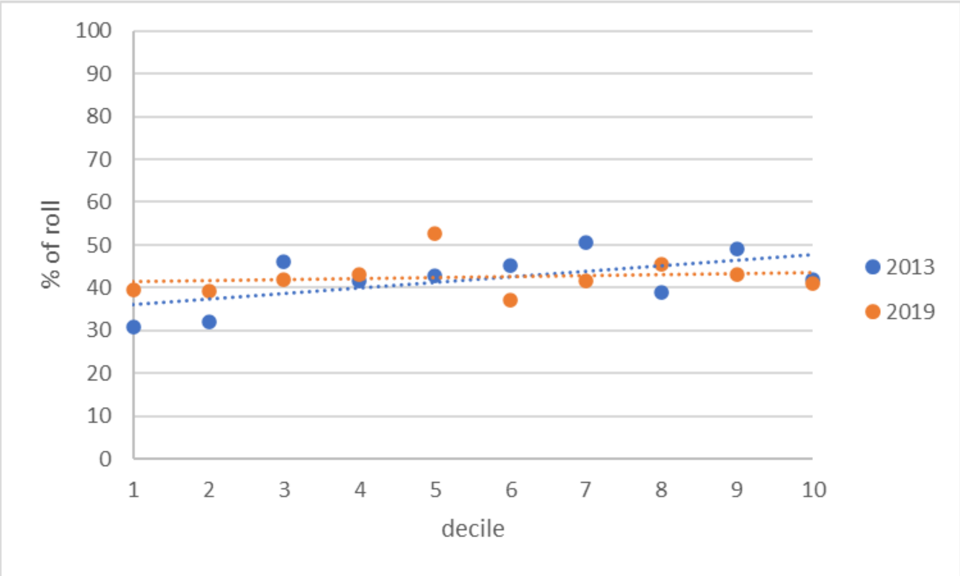
Rates of participation of Year 13 students in AS3.10 (Inference) against school decile.

Appendix O



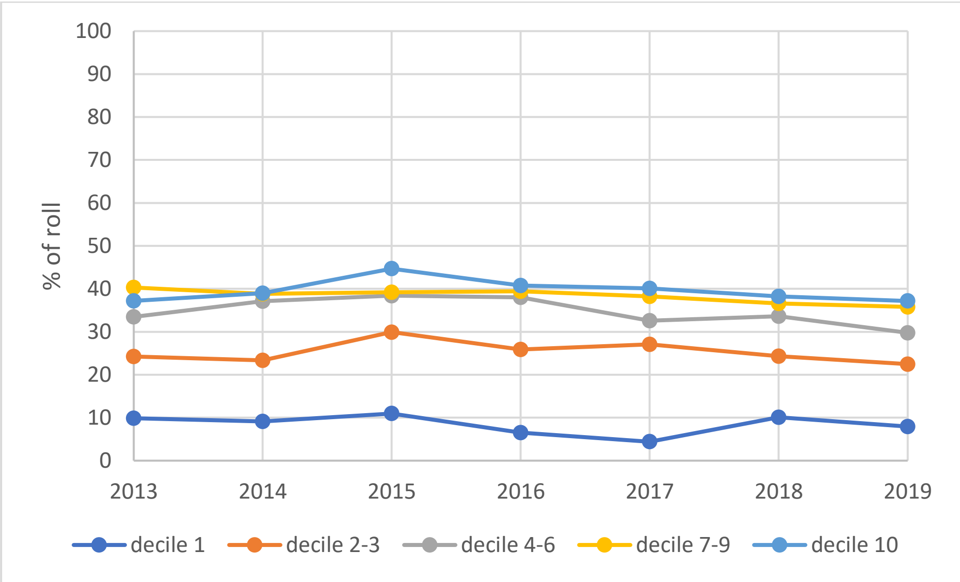
Rates of participation of Year 11 students in AS1.10 (Inference) against school decile.

Appendix P



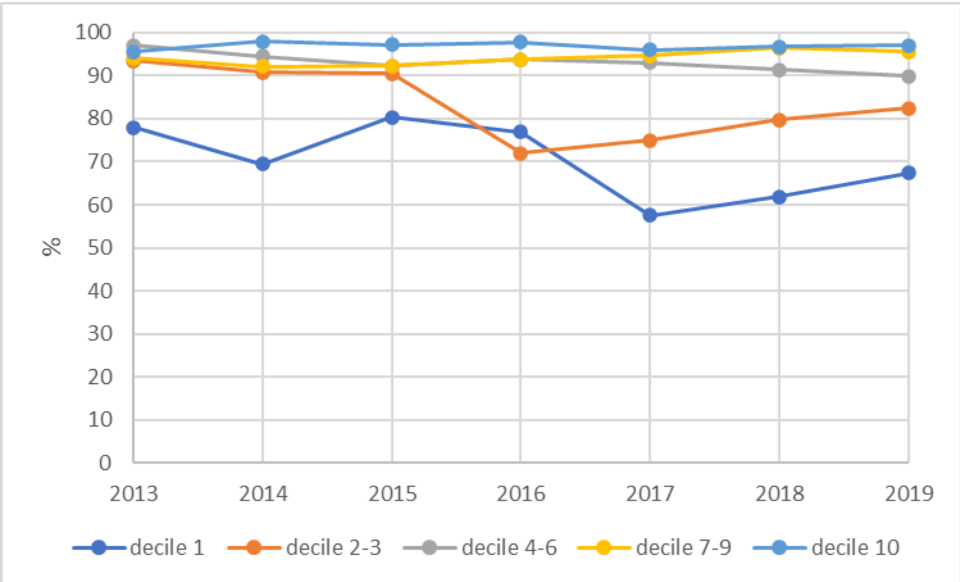
Rates of participation of Year 12 students in AS2.9 (Inference) against school decile.

Appendix Q



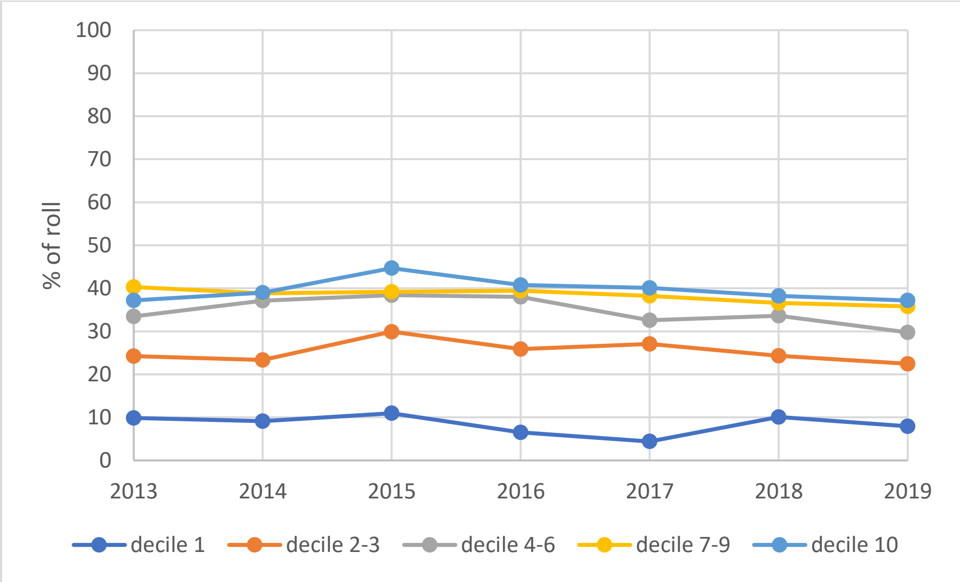
Rates of participation of Year 13 students in the combination of AS3.6 (Differentiation) and AS3.7 (Integration), 2013-2019.

Appendix R



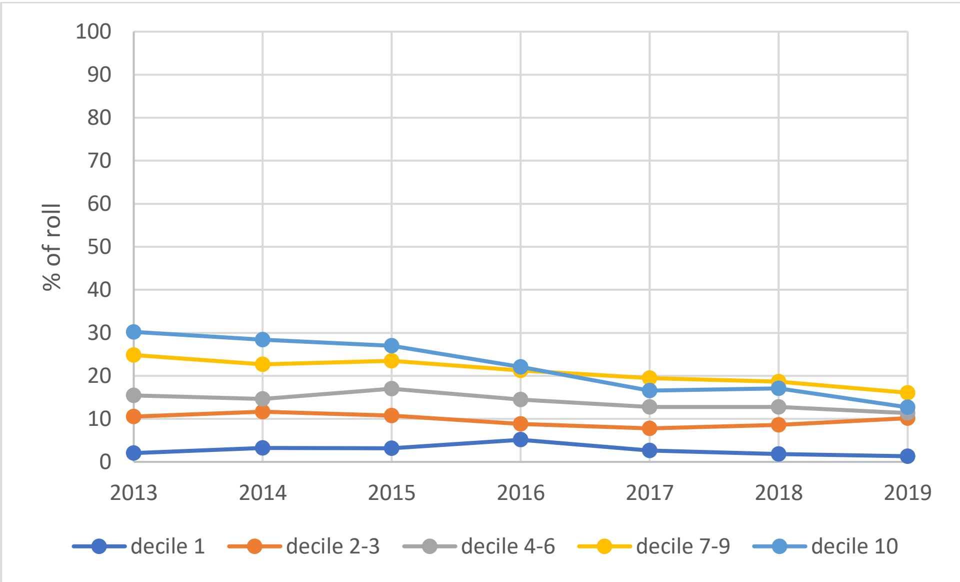
Percentage of Year 13 students who participated in AS3.6 (Differentiation) or AS3.7 (Integration) who are doing both, 2013-2019.

Appendix S



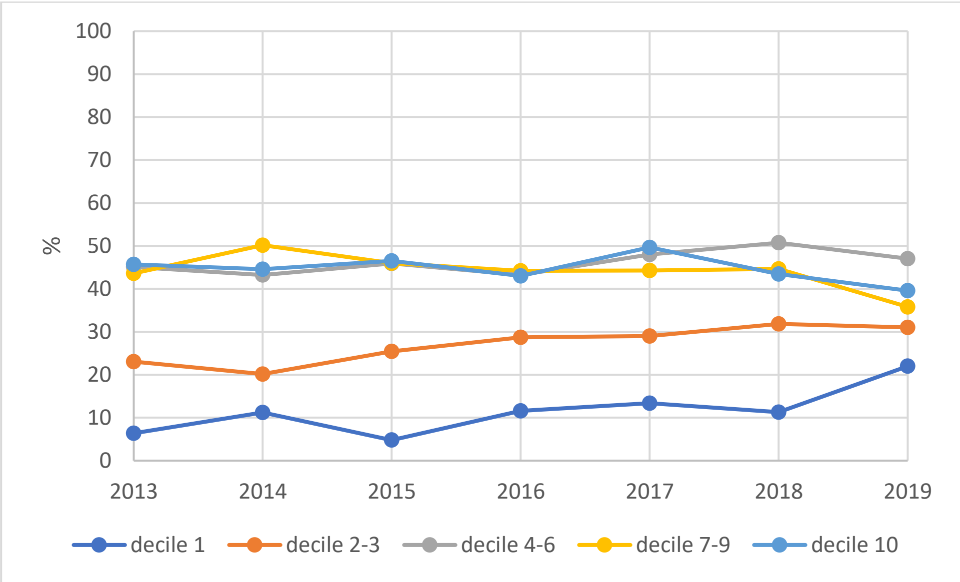
Rates of participation of Year 12 students in the combination of AS2.9 (Inference) and AS2.12 (Probability), 2013-2019.

Appendix T



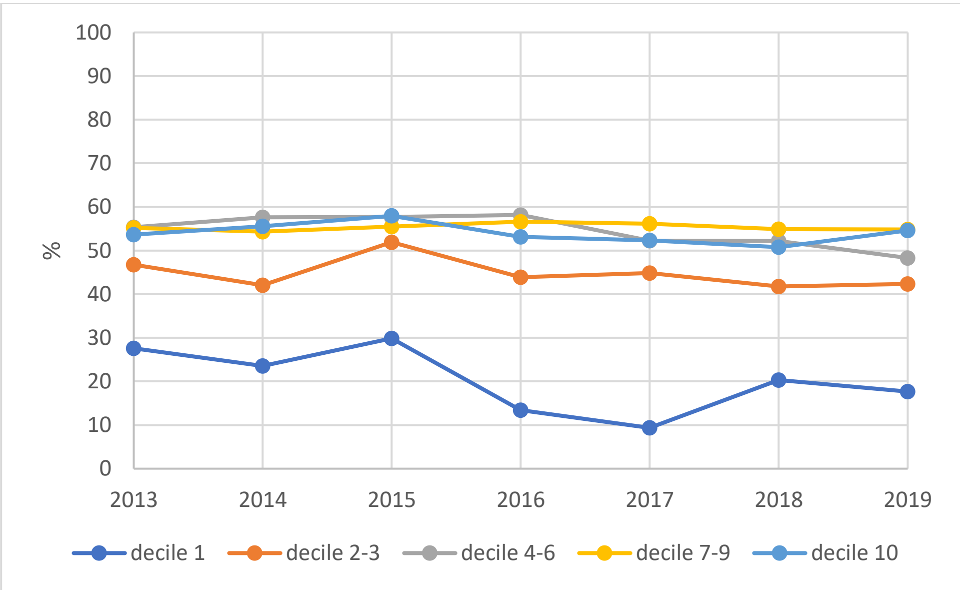
Rates of participation of Year 13 students in the combination of AS3.9 (Bivariate), AS3.10 (Inference), and AS3.13 (Probability), 2013-2019.

Appendix U



Percentage of Year 11 students who participated in AS1.10 (Inference) or AS1.12 (Chance and Data) who are doing both, 2013-2019.

Appendix V



Percentage of Year 12 students who participated in AS2.9 (Inference) or AS2.12 (Probability) who are doing both, 2013-2019.