**Appendix A:**

***A1 Growth***

Molting and growth are assumed to happen after the fishery in this assessment model. In reality, krill molt almost every two weeks, which is more frequent than most other crustaceans (Tarling et al., 2006). Krill may also shrink after molting, and its growth varies seasonally (Ikeda et al., 1982). Krill’s small size and the inhospitable physical environment of the Southern Ocean make it difficult to obtain tagging data. All of these factors make the growth per molt increment hard to characterize at the temporal scale (biweekly) that it occurs. So, instead of trying to capture these dynamics, we estimated a yearly size transition matrix. The length increment per molt within this size transition matrix was based on a linear relationship between pre-molt and post-molt length. The distribution of the growth increment was given by a discretized and renormalized gamma distribution that used the growth increment as input:

 (A1)

 (A2)

 (A3)

Where *n* are the size classes;

 is the midpoint of lengths in size class n;

 is the expected size after molting in size class n;

 is the parameter which can define the variability in growth increment, the “2.5” is the half one bin’s size;

 (A5)

 (A6)

Where *a* and *b* are the parameters of relationship between length and growth increment；

 is the difference in length between midpoints of size class n and n+1；

We use the empirical formula to estimate the weight at length (Hewitt et al., 2004).

 (A7)

Where n is the midpoint of size class n.

***A2 Fishing mortality and selectivity***

Fishery selectivity was assumed to be a logistic function of size and constant over time, all fishery selectivity parameters were estimated.

 (A8)

Where is the length at which 50% of krill are selected, is the length at which 95% of krill are selected, is the length in size class n.

 Survey selectivity was assumed to be a logistic function of size and constant over time, survey selectivity parameters were estimated;

 (A9)

Where, A is the survey catch-ability coefficient, is the length at which 50% of krill are selected, is the length at which 95% of krill are selected,

 Fishing mortality was assumed to be applied regardless of krill maturity state. Fishing mortality is estimated by:

 (A10)

Where, is the fishery selectivity in size class *n*;

 is the natural logarithm of the average fully-selected fishing mortality;

 is the yearly deviation from average fishing mortality.

 and were estimated within the model.

***A3 Recruitment***

A mean recruitment () and annual deviations from this recruitment (di) were estimated within the model. Annual recruitment in a given year was:

 (A11)

The proportion of recruitment allocated to a given size class was calculated in a manner similar to the growth increment. The fraction of the annual recruitment which recruits to size-class *n* is based on a gamma function:

 (A12)

 (A13)

 (A14)

 (A15)

Where and are parameters in the gamma function that defined the shape of the curve that allocates recruitment to a given size bin. They were specified such that recruitment entered the model in the first three size bins.

 (A16)

Proxies for biomass and fishing mortality reference points were calculated using spawner-per-recruit methods (e. g. Clarke, 1991), including F35% (the fishing mortality that reduces the spawners per recruit to 35% of unfished levels) and B35% (the mature biomass at the time of mating resulting from fishing at F35%).

***A4 Model predictions***

 The model is fit to several data sources, for which model predictions are calculated as follows. The model prediction of the number of krill at length at the time of survey:

 (A17)

Where is the survey selectivity at size class *n*;

 is the number of krill in size class *n* at year *i*.

 (A18)

Where is the prediction of survey number in year *I*;

The model prediction of the catch number of krill at length:

 (A19)

 (A20)

Where is the total catch number of krill in size class *n* in year *I,* and it was estimated within the model;

 is the number of krill in size class *n* before fishery occurs during year *I*;

 is survival to the fishing;

M is natural mortality;

1. is the proportion of krill caught by fishery at size class *n* during year *i；*

 is the fishing mortality in size-class *n* during year i.

***A5 Likelihood components***

The model predictions are compared to the observed quantities and the related log likelihoods are minimized. The likelihood for survey number at length and catch at length is multinomial:

 (A21)

Where, is the contribution to the objective function of the fit to survey or fishery length composition; is the sample size of survey and fishery in year I; is the observed survey or fishery length frequency in size class n during year I; is the estimated data of survey or fishery length frequency in size class n during year I through model; is a small number (i.e. 0.00001) to avoid taking the log of zero.

The likelihood for total survey number by year was lognormal and calculated as:

 (A22)

Where, is the contribution to the objective function of the fit to survey number;

 is the observed survey number in year I;

 is the estimated data of survey number in year I through model;

 is the observed coefficient of variation for ;

The likelihood for catch biomass by year was also lognormal and calculated as:

 (A24)

Where is the contribution to the objective function of the fit to catch biomass;

 is the observed catch biomass in year I;

 is the estimated data of catch biomass in year I through model;

 is the coefficient of observed catch biomass.

***A6 Penalties component***

Small first-difference penalties were used in the assessment model to improve model stability when deviations were estimated from a mean (e. g. recruitment and fishing mortality). The penalties (λ) were calculated as:

 (A25)

Where, the , is the quantity in question and is the weighting factor.

Tab. A-1 Parameter setting in the model

|  |  |  |
| --- | --- | --- |
| **Fixed parameters** | **Variable name** | **Range** |
| Weight at length | a, b | 0.00000503; 3.283 |
| **Estimated parameters** |  |  |
| CatchabilityNatural Mortality | AM | (0,1)(0.1,0.8) |
| Fishing Selectivity (50%) | L50, fishery | (10,70) |
| Fishing Selectivity (95%) | L95, fishery | (10,70) |
| Survey Selectivity (50%) | L50, survey | (9,70) |
| Survey Selectivity (95%) | L95, survey | (10,70) |
| Average Fishing Mortality | Log (F mean) | 1 |
| Fishing Mortality Deviation | F di | (styr,endyr,-100,100) |
| Average Recruitment | Log (R mean) | 1 |
| Recruitment Deviation | R di | (styr,endyr,-200,200) |
| Parameters of relationship between length and growth increment | am, bm | (0,100)；(0.6,1) |
| Length Increment pre. molt | G alphal | (0.1,3) |

Tab. A-2 Fixed parameter in the model

|  |  |
| --- | --- |
| **Parameter name** | **Value** |
| Small number added to log calcs | 0.01 |
| Initial smoothness weighting | 20 |
| Time-varying catch-ability smoothness | 20 |
| Shrink factor for CV in survey | 1 |
| Smoothness for recruitment | 5 |
| Smoothness for F | 5 |
| Smoothness for fishing selectivity | 2 |