

**Numerical assessments of excess ice impacts on permafrost and greenhouse gases  
in a Siberian tundra site under a warming climate**

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## Appendix

Soil water content,  $\theta$ , is calculated using a finite difference form of the Richardson equation. The water content of each soil layer,  $i$ , is represented as:

$$\frac{d\theta}{dt} = \frac{\partial W}{\partial z} - R \quad (\text{A1})$$

where  $z$  is the vertical coordinate,  $W$  is a flux and  $R$  is a sink term that represents the extraction of water by plant roots in the layer, including evaporation in the top layer.

The vertical flux is calculated by Darcy's law:

$$W = K_h \left( \frac{\partial \psi}{\partial z} + 1 \right) \quad (\text{A2})$$

where  $K_h$  is the hydraulic conductivity and  $\psi$  is the water suction at a given thickness ( $z$ ) of soil layer. The top boundary condition for Eq. (A1) is the infiltration of precipitation water at the soil surface, and the lower boundary condition is drainage, which contributes to subsurface flow. As permafrost exists, the drainage occurs at just above the permafrost table because of water blocking by the permafrost.

To close the model, it is necessary to assume forms of the hydraulic conductivity and the soil water suction as a function of the soil moisture content. We use the hydraulic relationship between soil water content, suction, and hydraulic conductivity, given by van Genuchten (1980), which is a more complex formulae but more scientifically robust:

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = S = \frac{1}{[1 + (\alpha\psi)^n]^m} \quad (\text{A3})$$

$$K_h = K_{hs} S^{0.5} [1 - (1 - S^{1/m})^m]^2 \quad (\text{A4})$$

where  $\theta_s$  and  $\theta_r$  are the saturated and residual soil moisture content, respectively,  $K_{hs}$  is the saturated hydraulic conductivity, and  $\alpha$ ,  $n$ , and  $m$  are soil parameters. As excess ice exists,  $\theta_s$  of the soil layer increases as much as the excess ice content.

The excess ice affects soil thermal properties. The soil thermal conductivity ( $\lambda$ ) is represented including the impact of the excess ice:

$$\lambda = (\lambda_s - \lambda_{dry})K_e + \lambda_{dry} \quad (A5)$$

where  $\lambda_s$  and  $\lambda_{dry}$  are the saturated and dry thermal conductivity, respectively, and  $K_e$  is the Kersten number.  $\lambda_{dry}$  was already parameterized considering the impact of soil organic carbon (Park et al., 2014). The saturated thermal conductivity is:

$$\lambda_s = \frac{\lambda_w^{f_w \theta_s} \lambda_i^{f_i \theta_s}}{\lambda_w^{\theta_s}} \lambda_{s0} \quad (A6)$$

where

$$f_w = \frac{\theta_w}{(\theta_w + \theta_i + \theta_{exice})}; \quad f_i = \frac{\theta_i + \theta_{exice}}{\theta_w + \theta_i + \theta_{exice}}$$

where  $\lambda_w$  and  $\lambda_i$  are the water and ice thermal conductivity, respectively, and  $\lambda_{s0}$  is the saturated thermal conductivity when the soil is entirely unfrozen (Park et al., 2014). Soil heat capacity ( $C$ ) is as follows:

$$C = C_s(1 - \theta_s)\Delta z + w_i c_i + w_l c_l + w_{exice} c_{exice} \quad (A7)$$

where  $C_s$  is the saturated soil heat capacity,  $\Delta z$  is the soil layer thickness,  $c_i$ ,  $c_l$ , and  $c_{exice}$  are the specific heat of ice, soil, and excess ice, respectively, and  $w_i$ ,  $w_l$ , and  $w_{exice}$  are ice, liquid, and excess ice content, respectively. The phase change energy ( $E_p$ ) is modified to:

$$E_p = L_f \left( \frac{w_{i0} - w_i}{\Delta t} \right) + L_f \left( \frac{w_{exice0} - w_{exice}}{\Delta t} \right) \quad (A8)$$

where  $L_f$  is the latent heat of fusion for ice,  $\Delta t$  is the time step, and  $w_{i0}$  and  $w_{exice0}$  are the initial mass of ice and excess ice, respectively. The phase change energy equation allows excess ice to melt.

The treatment of excess ice increases soil layer thickness and the saturated soil water content. As excess ice melts, subsidence occurs, which decreases the thickness of

the soil layer and the water storage capacity. The melting reduces the saturated soil water content, consequently causing changes in the hydraulic properties in equations (A1) and (A4). The meltwater is also added to the liquid water of the soil layer or the upper layer, and it participates in soil water movement and runoff generation mechanisms in CHANGE. Furthermore, the changed soil water condition affects the soil thermal properties (equations A5 to A7). It is important to note here the melting of excess ice results in changes in soil structure as well as soil water and heat fluxes.