Supplementary Material

1 DETERMINING WATER CONTENT: NUMERICAL SOLUTION OF THE STEFAN PROBLEM

The calorimetric method used to determine water content follows closely that of Cohen (1999) who modeled a one-dimensional, moving phase boundary through temperate ice and is based conceptually on the method used by Duval (1976, 1977). The initial condition for the model is the ice ring at its pressure-melting temperature θ_m. Temperature at the lateral edges of ice ring is decreased as abruptly as possible, creating a freezing phase boundary. The temperature decrease with time at the edges of the ice ring is measured by thermistors mounted flush with the surfaces of the walls in contact with the ice. The phase boundary moves away from the heat sink and into the ice (Figure S1), thus separating a zone of cold ice near the lateral walls of the ice ring from a zone of temperate ice away from the walls.

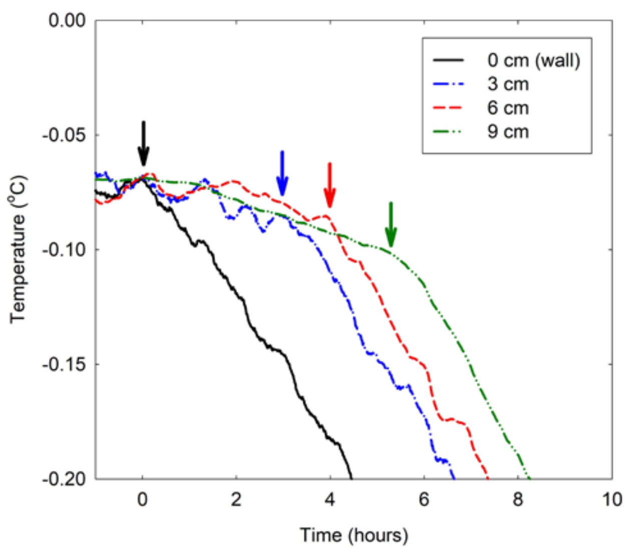


Figure S1. Temperatures recorded by four thermistors during the passage of a freezing front. One thermistor was flush with the inner surface of the wall of the ice chamber, and three thermistors were in the ice along a radial transect at different distances from the wall. Arrows point to inferred arrival times of the freezing front.

The heat equation describes the temperature distribution in the ice with time for a given half-width of the ice ring. The following assumptions are made in the analysis: (1) The ice is a homogenous ice-water mixture; (2) Thermal conductivity, specific heat capacity, and density of ice are fixed values; (3) The ice ring is perfectly insulated at its top and bottom surfaces, so the heat flux is strictly horizontal across the width of the ice ring. Let

θ : temperature

θ_c : heat sink temperature

θ_m : phase boundary temperature

t : time

ζ : radial position

ζ_c : heat sink radius

λ : geometric constant

ρ : density of ice

K : thermal conductivty of ice

c : specific heat capcity of ice

$\alpha := \frac{K}{\rho c}$: thermal diffusivity

L : latent heat of fusion of water

σ : freezing front radial position w : water content.

The Stefan problem can be described as

$$\frac{\partial\theta}{\partial t} = \alpha\frac{\partial^2\theta}{\partial\zeta^2} + \alpha\frac{(\lambda-1)}{\zeta}\frac{\partial\theta}{\partial\zeta}, \qquad \text{(A.1a)}$$

with boundary conditions:

$$\theta = \theta_c(t), \qquad \zeta = \zeta_c, \qquad \text{(A.1b)}$$

at the ice-chamber wall and

$$\frac{\partial\theta}{\partial\zeta} = \frac{Lw\rho\dot{\sigma}}{K}, \qquad \zeta = \sigma(t), \qquad \text{(A.1c)}$$

$$\theta = \theta_m \qquad \text{(A.1d)}$$

at the moving freezing front. Equation A.1b describes the temperature, θ_c, that decreases with time, t, at the ice-wall interface, ζ_c (wall thermistor temperature record), and equation A.1c indicates how the temperature gradient depends on water content, w, and the speed of the freezing front, $\dot{\sigma}$. The initial conditions are

$$\theta(\zeta_c, t=0) = \theta_c(0), \qquad \text{(A.1e)}$$

$$\sigma(0) = \zeta_c, \qquad \text{(A.1f)}$$

The geometric constant λ has a value of either 1, 2, or 3 for Cartesian, cylindrical, and spherical geometries. The goal is to find $\theta(\zeta, t)$ and $\sigma(t)$, for $\zeta_c \le \zeta \le \sigma(t), t > 0$.

The problem is simplified by dimensionally reducing parameter values. Dimensionless temperature, position, and time become

$$u = \frac{(\theta-\theta_m)}{\theta_o},$$

$$x = \frac{\zeta}{\zeta_o},$$

$$t = \frac{t}{t_o},$$

where t is retained for dimensionless time and where

$$\theta_o = \frac{Lw}{c},$$

$$\zeta_o = \sqrt{\alpha},$$

$$t_o = 1 \text{ second}.$$

The heat equation is then

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{(\lambda - 1)}{x}\frac{\partial u}{\partial x}, \tag{A.2a}$$

with boundary conditions

$$u = u_c(t), \qquad x = x_c, \tag{A.2b}$$

$$\frac{\partial u}{\partial x} = \dot{s}, \qquad x = s(t), \tag{A.2c}$$

$$u = 0, \tag{A.2d}$$

where s is the dimensionless moving phase boundary radius. The initial conditions are

$$u(x_c, 0) = u_c(0), \tag{A.2e}$$

$$s(0) = x_c, \qquad . \tag{A.2f}$$

where

$$x_c = \frac{\zeta_c}{\zeta_o},$$

$$u_c(t) = \frac{\theta_c}{\theta_o}$$

$$s(t) = \frac{\sigma(t)}{\zeta_o}.$$

The goal is to find $u(x,t)$ and $s(t)$, for $x_c \leq x \leq s(t), t > 0$.

With the variables dimensionally reduced, the moving phase boundary is then eliminated to make the problem tractable. The following transformation was proposed by Crank (1984):

$$\xi = \frac{x - x_c}{s - x_c}, \qquad 0 \leq \xi \leq 1. \tag{A3.3}$$

The heat equation is then

$$\frac{\partial u}{\partial t} = (s - x_c)^{-2}\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial u}{\partial \xi}\left(\frac{(\lambda - 1)}{(s - x_c)(\xi(s - x_c) + x_c)} + \frac{\xi \dot{s}}{s - x_c}\right), \tag{A.4a}$$

with boundary conditions

$$u = u_c(t), \qquad \xi = 0, \tag{A.4b}$$

$$\frac{\partial u}{\partial \xi} = \dot{s}(s - x_c), \qquad \xi = 1, \tag{A.4c}$$

$$u = 0, \tag{A.4d}$$

and initial conditions

$$u(x_c, 0) = u_c(0), \tag{A.4e}$$

$$s(0) = x_c. \tag{A.4f}$$

The problem is then to find $u(\xi, t)$ and $s(t)$, for $0 \leq \xi \leq 1, t > 0$.

Equations A.4 are solved numerically using the finite difference method. To solve the moving-boundary problem a variable time-step method proposed by Asaithambi (1988) is used, in which the time for the phase boundary to move a specified increment is calculated iteratively.

The equations are discretized using a central difference stencil in space with forward/backward differences at boundaries. The dimensionless spatial mesh is divided into N_ξ intervals of equal length $\Delta\xi$ such that the coordinate points are $\xi_i = i\Delta\xi$ for i = 0; 1, . . . N_ξ. To find the incremental time value necessary to move the boundary the specified distance, the heat equation with boundary conditions A.4b-d is first solved numerically using an initial guess $\Delta t^{(k)}$. The discretization of the Stefan boundary condition at $\xi = 1$ is then used to iterate for the next guess for a time step $\Delta t^{(k+1)}$ as

$$\Delta t^{(k+1)} = \frac{2(s_{j+1} - x_c)(s_{j+1} - s_j)\Delta\xi}{3u^{(k)}_{N_\xi, j+1} - 4u^{(k)}_{N_\xi - 1, j+1} + u^{(k)}_{N_\xi - 2, j+1}}, \tag{A.5}$$

where the indices k and j denote the iteration number and time step, respectively. The values of the discrete temperature solution are taken at the three nodes in the mesh closest to the boundary $\xi = 1$ $(\text{indices } N_\xi - 2, N_\xi - 1, N_\xi)$. By resolving the heat equation, the time step is iterated on until a desired convergence ϵ is achieved.

The method is summarized as follows:

1. For a chosen initial guess for the time step ($\Delta t^{(k)}$) necessary to move the phase boundary to the next known position (s_{j+1}), solve the heat equation to obtain a temperature profile $u^{(k)}$ at time j+1.
2. Compute a refined value of the time step, $\Delta t^{(k+1)}$, using equation A.5.
3. Repeat steps 1 and 2 until $\left|\Delta t^{(k+1)} - \Delta t^{(k)}\right| < \epsilon$.

After desired convergence is achieved, time and position are dimensionally restored by reversing the earlier listed reductions to construct a position-time plot for the model freezing front.

With the Stefan model, various water contents are used to compute position-time plots for freezing fronts to find the water content that best fits the data by minimizing the standard error (Figure S2). Results for the four transects are averaged.

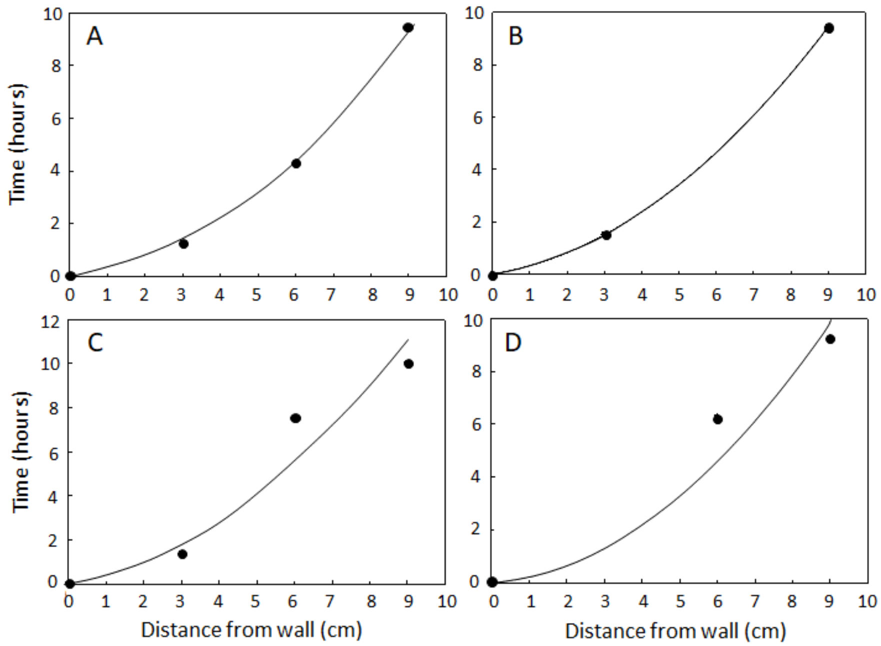
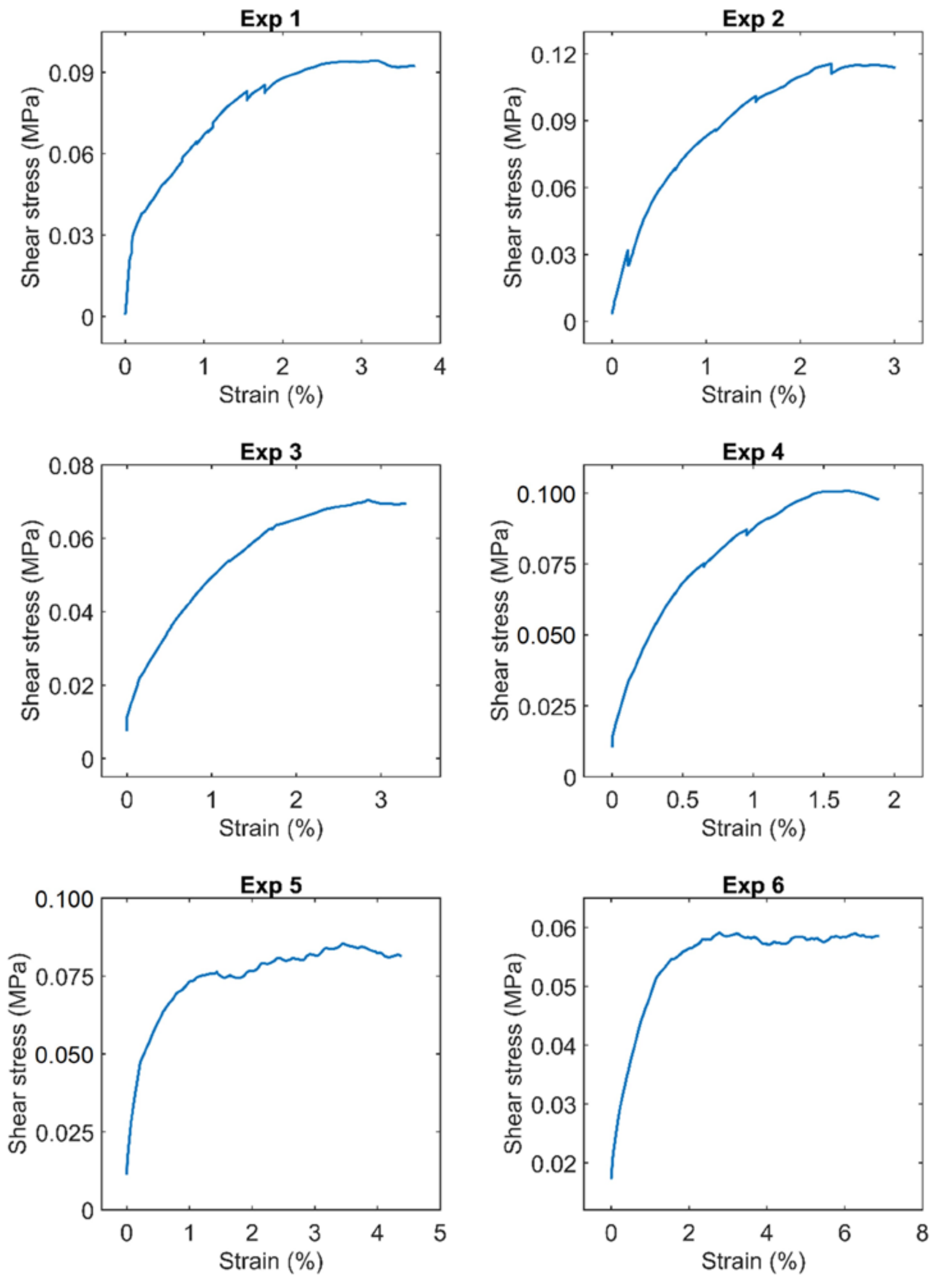


Figure S2. Model fits to freezing-front arrival-time data from Experiment 9. Radial thermistor transects contained three or four thermistors (including those in the walls). Transects were at diametrically opposed locations around the ice ring and adjacent to either the (A and B) outer wall or (C and D) inner wall. These data indicate a water content of 1.33 ± 0.07%.

2 STRESS-STRAIN DATA



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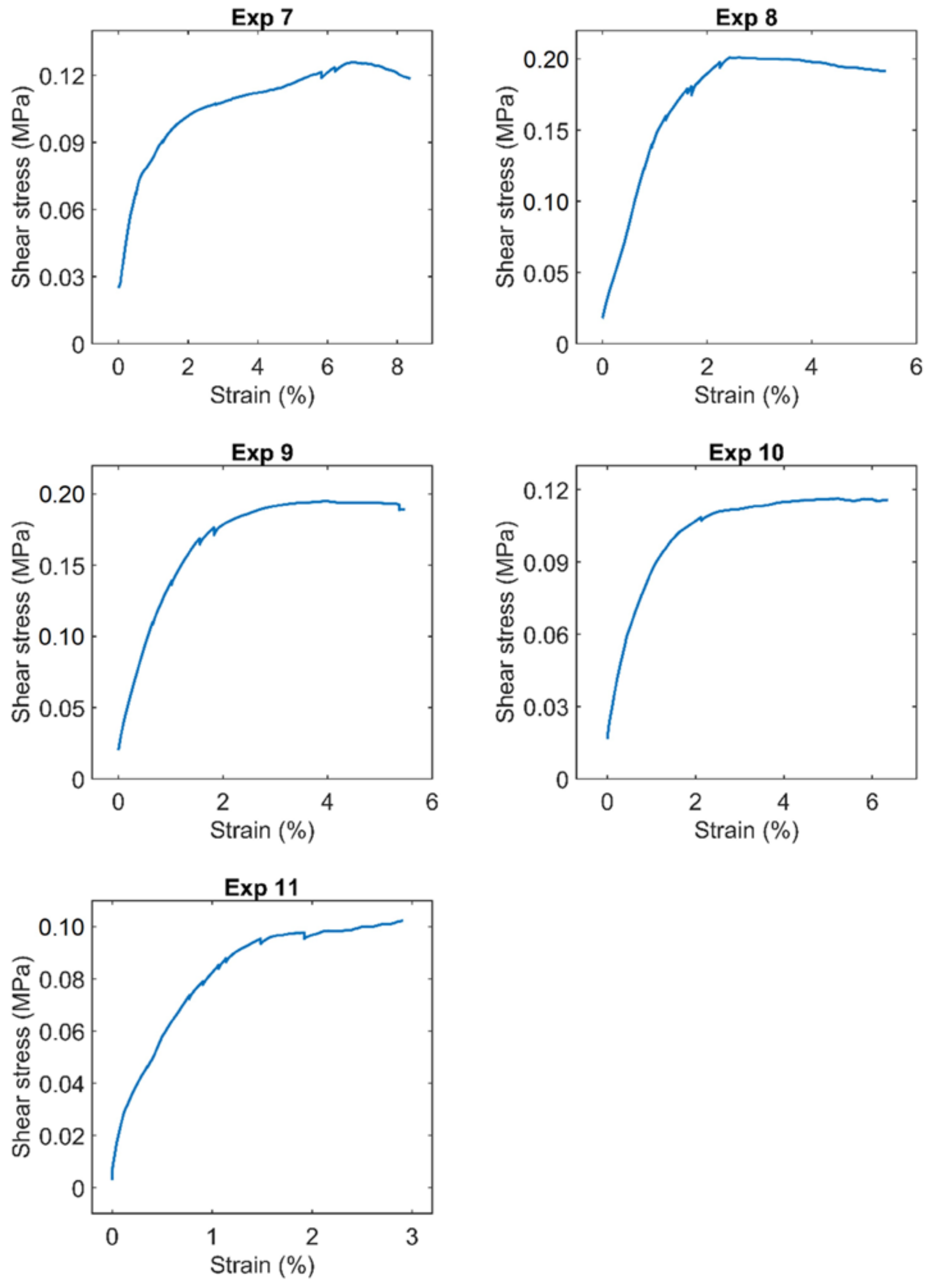

Figure S3. Stress as a function of strain for 11 experiments conducted to secondary creep. In experiments 1-5 shearing speed was reduced by a factor of 3.0 at a strain less than 0.25%. The stress drop in Experiment 2 at a strain of ~0.2% resulted from a power outage that transiently shut off the motors that drive the rotation of the upper platen. See Table 1 of the article for experimental parameter values.

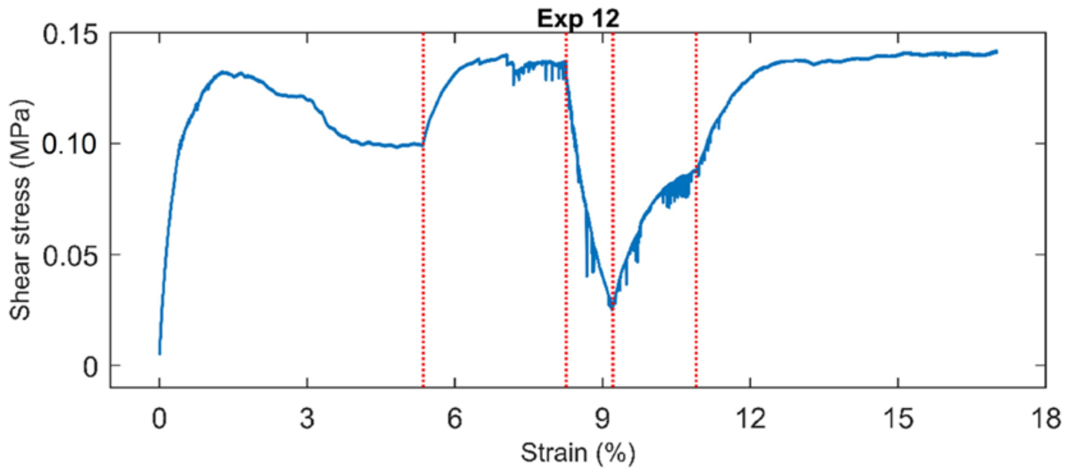

Figure S4. Stress as a function strain for Experiment 12 conducted to tertiary creep. Red dotted lines indicate when shearing speed was increased or decreased. The flow stress in tertiary creep was taken to be the average value over a strain of 15-17%. Water content was measured only at the end of the experiment and likely evolved as strain accrued at various rates, so stresses measured at strains of 0-15% are not considered. See Table 1 of the article for experimental parameter values.

REFERENCES

Asaithambi, N.S. (1988). On a variable time-step method for the one-dimensional Stefan problem. Comput. Methods Appl. Mech. Engrg. 71, 1–13.

Cohen, D. (1999). Rheology of basal ice at Engabreen, Norway. [Dissertation, Minneapolis, MN]: University of Minnesota

Crank, J. (1984). Free and Moving Boundary Problems: Oxford, UK, Clarendon Press.

Duval, P. (1976). Fluage et recristallisation des glaces polycristallines. [Dissertation, Grenoble, France]: Université Scientifique et Médicale de Grenoble.

Duval, P. (1977). The role of the water content on the creep rate of polycrystalline ice. Proc Grenoble Symposium, 1975, IAHS 118, 29-33.