

An Empirical Method for Evaluating Robustness of Network Centrality Measures

The Case of “Dark Networks”

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The Origins of “Dark Networks”

- **First instance** of “Dark Networks”—Raab & Milward (2003) –Dark Networks as Problems
- Milward and Raab, 2006; Milward, 2006—Dark Networks as Organizational Problems
- Xu & Chen, 2008—The Topology of Dark Networks
- Bohannon, 2009—Investigating Networks: The Dark Side [**Science Editorial**]
- Keller; Atkinson; Roberts; Keegan (all 2010)
- 2010+ Deluge of “Dark Network” claims

DARK NETWORKS

by

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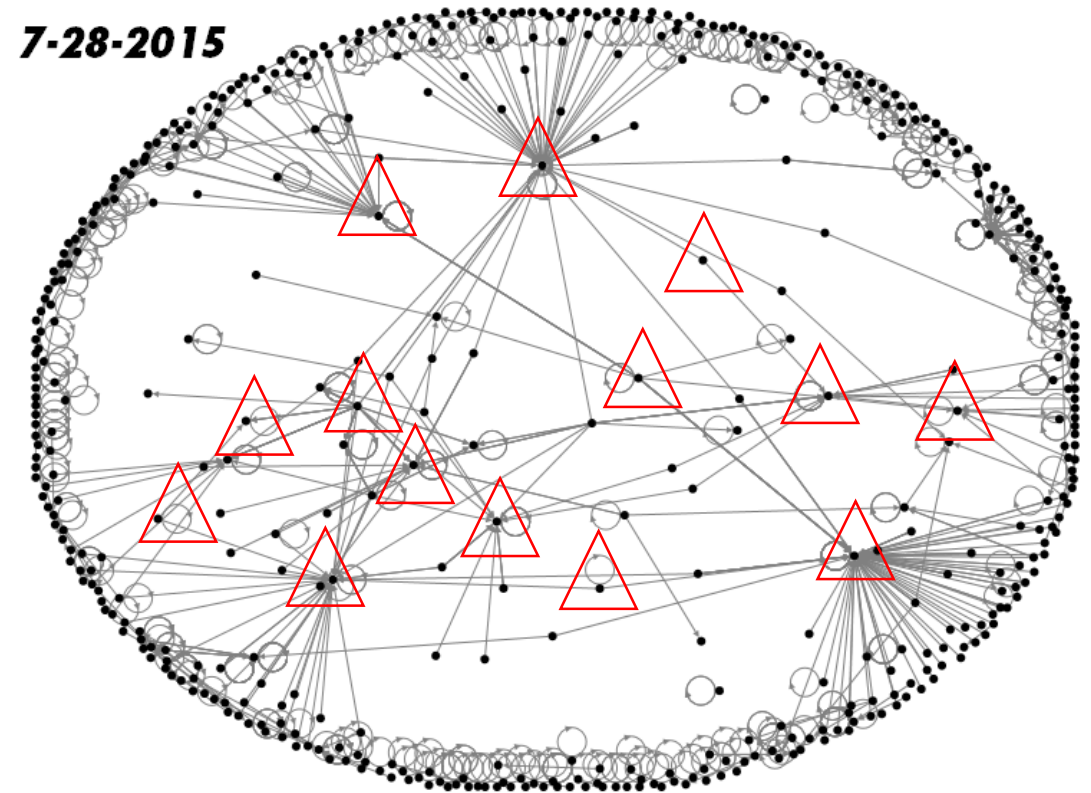
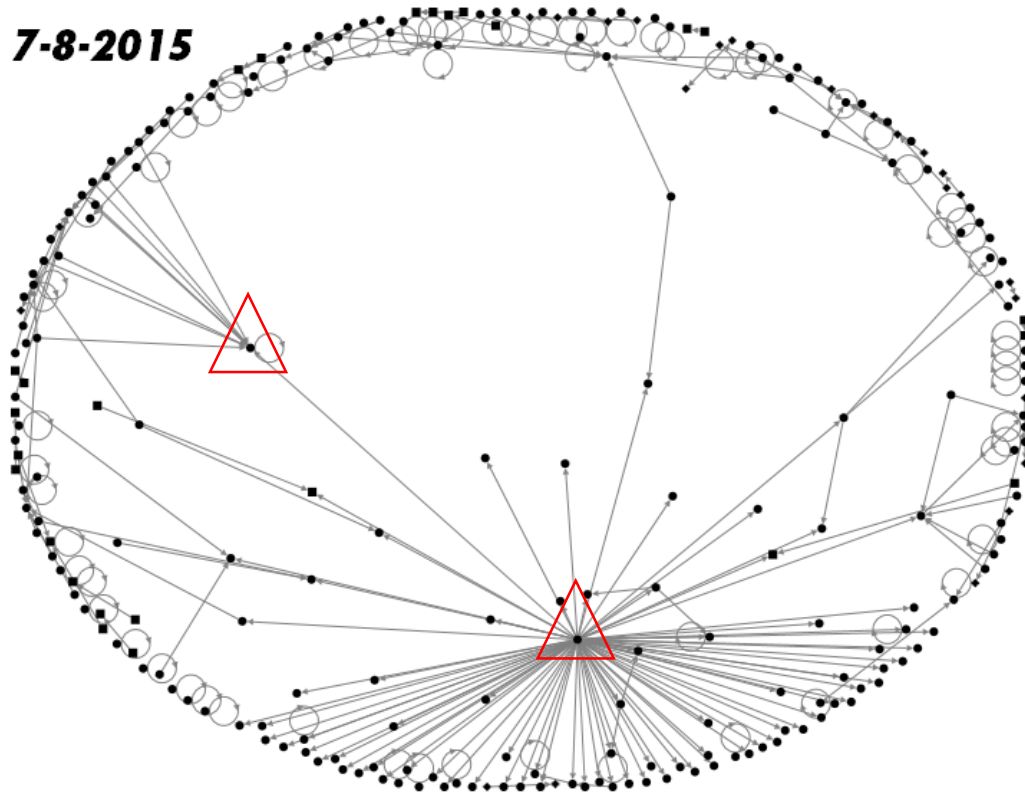
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Peters, Douglas; Dark Networks: Screenplay, 2006

Salient Themes

- Dark Networks are *different*. (Raab & Milward)
- *Centrality* is important (Xu and Chen)
- Remove the most central actor and someone else “moves in” (Tsevatat & Carley)
- *SNA inappropriate* for this domain (Valente)
- There are *key players* that can be identified; their removal would “break” the network (Borgatti)
- **Main Theme:** Here are a *collection of methods drawn from classical SNA*...
Now, let’s talk about bad people. Occasionally, here is a method or two from the network resilience line of inquiry clothed in Network Science sprinkled with a few agent-based models.

Example of current Framework

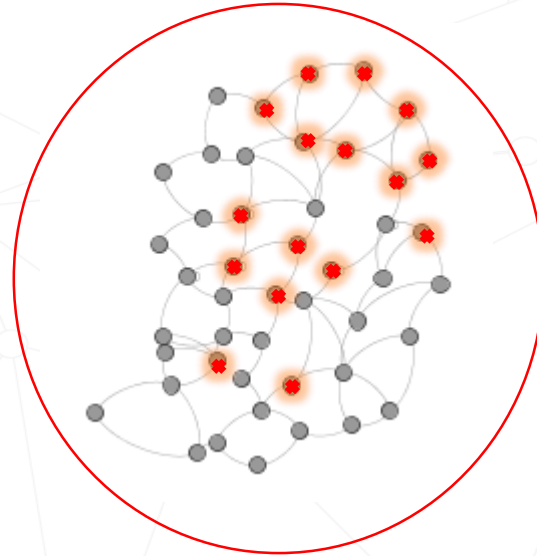


Source: **Shaheen, J. A. E.** (2015). *Network of Terror: How Daesh Uses Adaptive Social Networks To Spread its Message*. NATO Stratcom Centre of Excellence (Vol. 1). Retrieved from <https://www.stratcomcoe.org/network-terror-how-daesh-uses-adaptive-social-networks-spread-its-message>

What is a successful framework?

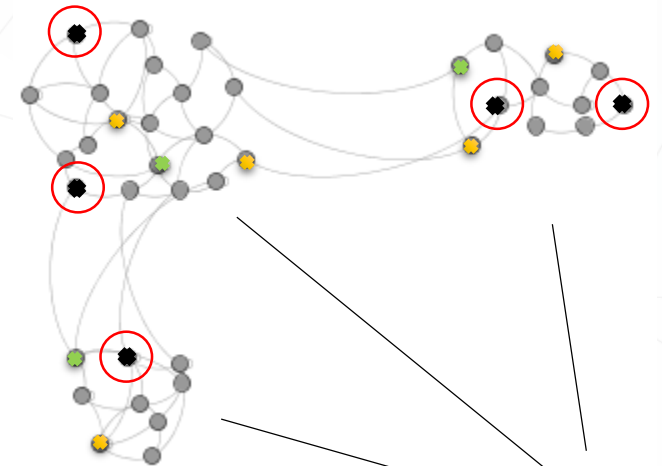
- Should it explain?
Predict?
- Predictive Power !=
Explanatory Power
- A collection of **effective**
ideas, theories, tools
(methodological) and
principles.

**Dark
Networks**



*"Dark Network"
(the whole thing)*

Dark Actor



Communities

Reality

Claim: “Surprise” as Risk Management

- Analytical tools for security policy-making **should enrich**...security policy-making.
- In the absence of additional network information that can plainly identify dark actors (such as behavioral), one must, through an analytical tool **identify the risk** (e.g. structural, processual) of central actors becoming “dark”.
- **Analytical conclusions** can then be made, and interventions, such as “Hardening” can then be employed.
- There is a branch of applied mathematics that offers a measure of surprise:
Information Entropy

Differential network entropy reveals cancer system hallmarks

James West^{1,2,3}, Ginestra Bianconi⁴, Simone Severini^{2,3} & Andrew E. Teschendorff^{1,3}

Hardening

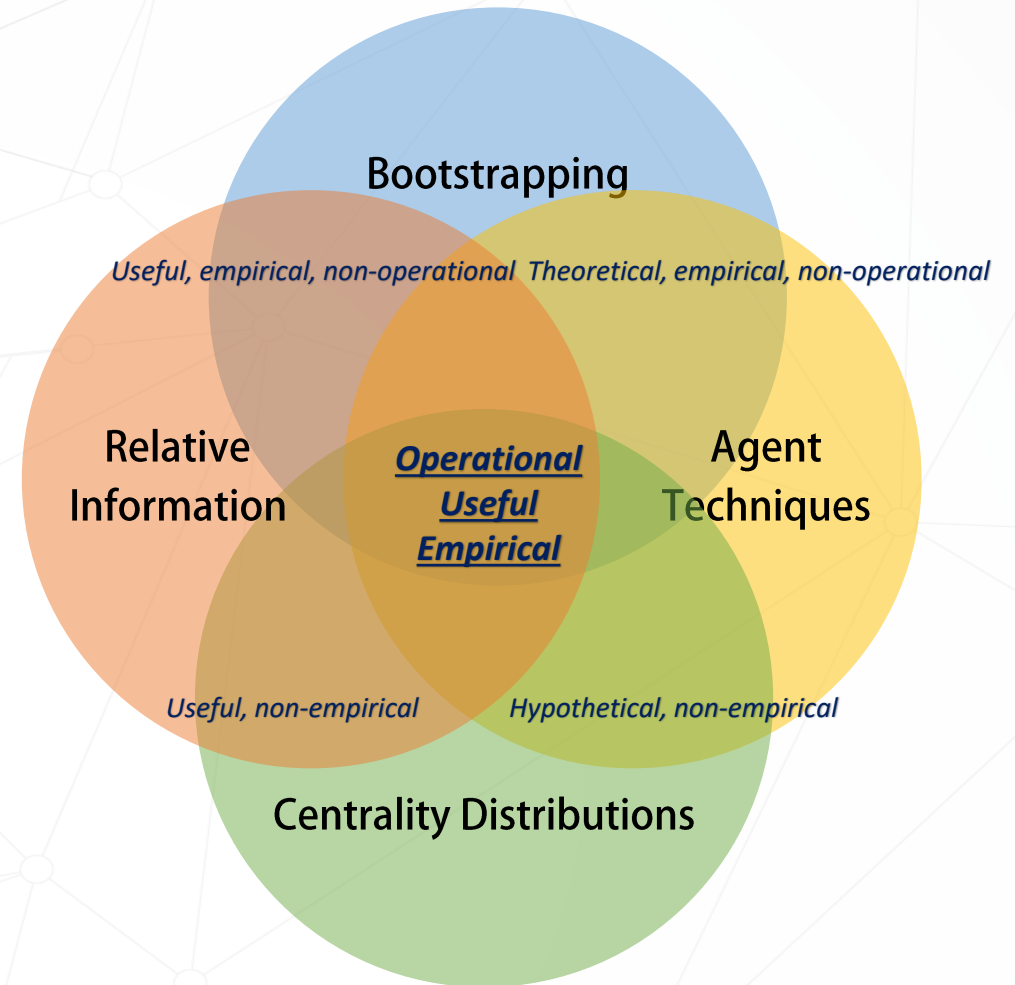


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Simply Put: Instead/addition of/to who should we target for node deletion, the framework should answer who causes maximum “surprise” if they were to be a “Dark Actor”

Methodology

- Information Entropy is a **representation the Information contained within** a closed system. For the majority of parametric distributions a closed form solution exists
- Relative Entropy or the **Kullback-Leibler Divergence** is a measure of **information loss or gain**, or the divergence of one distribution's information content from another
- The **Configuration Model** is a generalized random graph model that relies on fixing the degree distribution of a random network –so called **degree sequence**. Computationally, it uses a **bootstrapping** technique along with a vast amount of theoretical and mathematical tools, allowing for the permutation/perturbation of degree sequences (and distributions) while measuring a variable or effect of interest.
- Agent-based modeling is a set of techniques that connect behaviors at the micro-states to properties of the system (we won't need that today)
- → **Network ensemble comparison**



Methodology

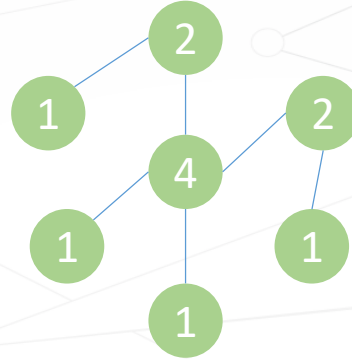
1. Given some graph \mathbf{G} , with n nodes and m edges, and some fixed degree sequence $k = \{k_1, k_2, \dots, k_n\}$ and some normalized centrality measure $c = \{c_1, c_2, \dots, c_n\}$ let us calculate the **change in entropy D** (KL Divergence) of \mathbf{G} given some new degree-preserving configuration (matching) \mathbf{G}' .
2. Furthermore, we must find a **re-wiring method** that can approximate the contribution of some node i to the $D_{KL}(\mathbf{G} || \mathbf{G}')$ for each configuration

Methodology

$$C(x) = \frac{N}{\sum_y d(y, x)}$$

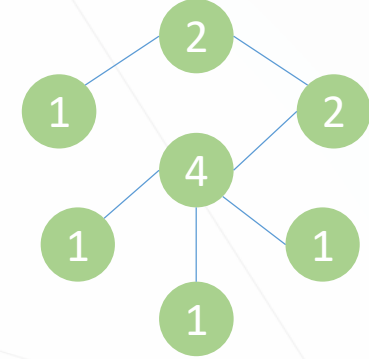
$$D_{\text{KL}}(P \parallel Q) = \sum_{x \in \mathcal{X}} P(x) \log \left(\frac{P(x)}{Q(x)} \right).$$

Original Network



Degree preserving

Configuration/Match



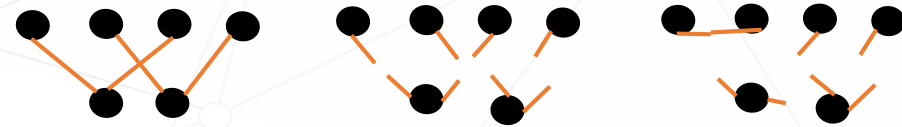
- Assume simple and undirected graphs
- MCMC Sampling
- We will use degree-preserving rewiring (sometimes known as **double-edge-swap vertex labeled rewiring**. Rewiring will allow for self-loops
- Applied to configuring the whole network 2 stubs at a time.
- Rely on theoretical derivations from Zichao, L. I., Mucha, P. J., & Taylor, D. (2018). Network-ensemble comparisons with stochastic rewiring and von neumann entropy*. *SIAM Journal on Applied Mathematics*, 78(2), 897–920. <https://doi.org/10.1137/17M1124218>

Methodology

- We run into an issue if we want to know the **specific contribution** of $D_{KL,i}(c)$ —or the entropy loss/gain given the re-wiring of a specific node.
- Remember: We'd like to know the given amount of topological **"surprise"** given a fixed degree sequence as we re-wire the network
- Heuristic Solution: Since the minimum number of rewires needed to sample a single configuration/match is $\sum_i k_i = \frac{2m}{2} = m$ (reconfigured 2 at a time), and that option is not available to us, because we need the node-level entropy change, **we'll need to reconfigure 4 at a time**, or a double dyad and so at a rate of $\sum_i k_i = 2 \frac{2m}{2} = 2m$. But that still doesn't solve the attribution problem
- We must modify the double-edge-swap vertex labeled rewiring method and introduce the **double-dyad-swap vertex labeled rewiring** method.

double-edge-swap vertex labeled rewiring

1. Original 2. Stub network 3. Rewiring 1 dyad



Ok try to calculate D_{kl} now please. **Dude, I can't. I'm not done yet.**

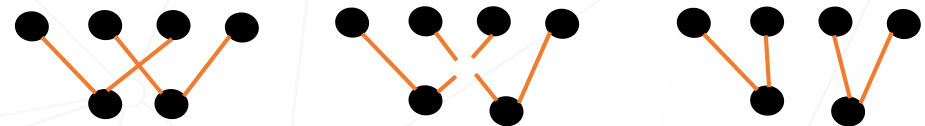
$$N(\{k_i\}) = \prod_i k_i!$$

$$N(\{k_i\}) = \frac{\prod_i k_i!}{\prod_{i < j} A_{ij}! \prod_i A_{ii}!!}$$

Departing from the Configuration Model

double-dyad-swap vertex labeled rewiring

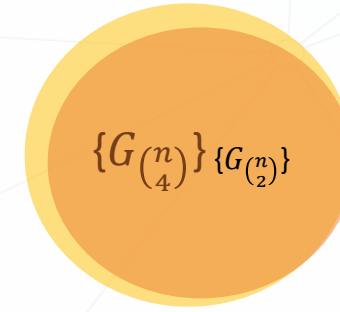
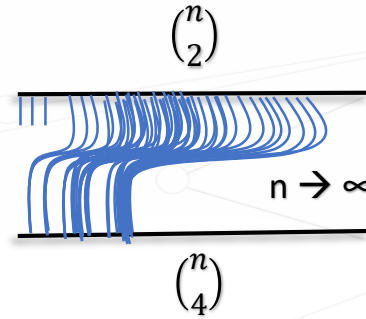
1. Original 2. Stub network 3. Rewiring 2 dyads



$$N(\{k_i\}) = \prod_i k_i!$$

Methodology

- We have **departed from the configuration model**: Our rewiring method fundamentally contrasts the network generation mechanism that the configuration model relies on → Now it's just bootstrapping.
- Based on **cardinality principles** of both sequence iterations, $\binom{n}{2}$ (CM) will differ from $\binom{n}{4}$ so we should estimate the relative sampling rate and adjust by simply sampling more
- Note that the graph set $\{G_{(2)}^{(n)}\} \subseteq \{G_{(4)}^{(n)}\}$ but what we care about is the **rate of sampling**. We want to cover enough of the former while implementing the later.

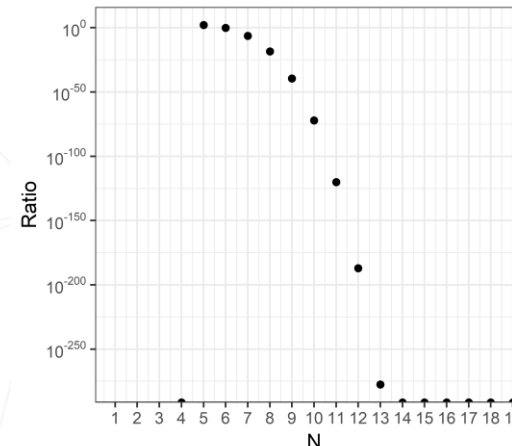


$$P\left(G_{(2)}^{(n)}\right) = p^m(1-p)^{\binom{n}{2}-m} \quad \text{Also...} \quad P\left(G_{(4)}^{(n)}\right) = p^m(1-p)^{\binom{n}{4}-m}$$

Via Newman (2010)

$$\text{ratio} = \frac{P(G_{(2)}^{(n)} \cap G_{(4)}^{(n)})}{P(G_{(4)}^{(n)})} = \frac{P(G_{(2)}^{(n)})}{P(G_{(4)}^{(n)})} = \frac{p^m(1-p)^{\binom{n}{2}-m}}{p^m(1-p)^{\binom{n}{4}-m}} = \text{how much more sampling}$$

some algebra + making use of stirling's approximation
+ solving numerically



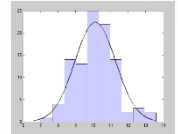
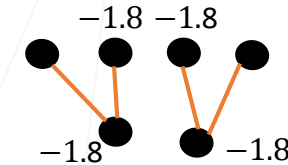
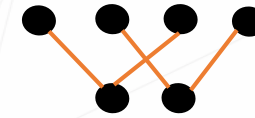
Note that what we're actually measuring is:

$$P\left(G_{(2)}^{(n)}|G_{(4)}^{(n)}\right) = \frac{P(G_{(2)}^{(n)} \cap G_{(4)}^{(n)})}{P(G_{(4)}^{(n)})}$$

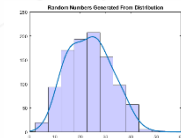
Methodology

- Now, we can bootstrap appropriately and **attribute KL divergence to a set of 4 nodes/2 dyads/2 edges**.
- We can calculate statistics on the aggregate and **attribute entropy loss to a small number of nodes and dyads**.
- On average, we will be able to measure the information loss, or “surprise” on the whole, given the **permutation of any centrality measure** (or property of any node for that matter) on a fixed degree sequence—this later part is critical.
- We can identify nodes and dyads that are surprisingly important (by any statistic we choose) to the whole distribution (say centrality) **while holding their degree constant**.
- *Comment: not sure if we should normalize by node degree or not.*

$$C(x) = \frac{N}{\sum_y d(y, x)}$$



rewire

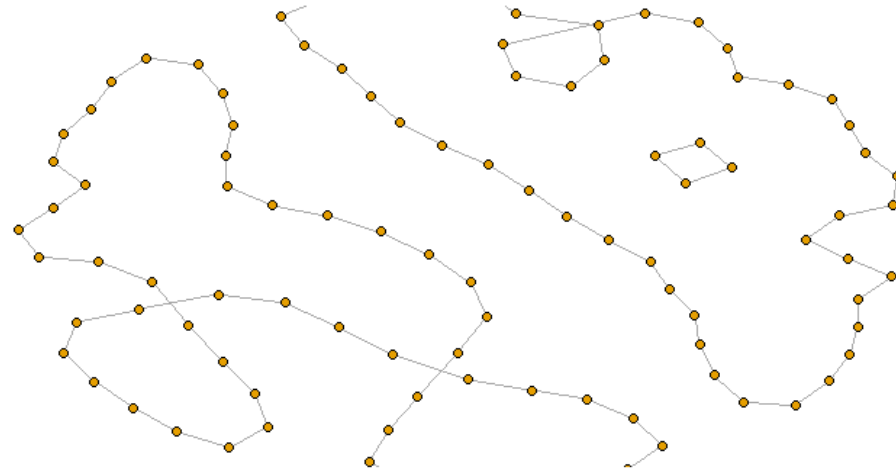


$$D_{KL}(P||Q) = \sum_{x \in X} P(x) \log \left(\frac{P(x)}{Q(x)} \right) = -7.2 \text{ bits}$$

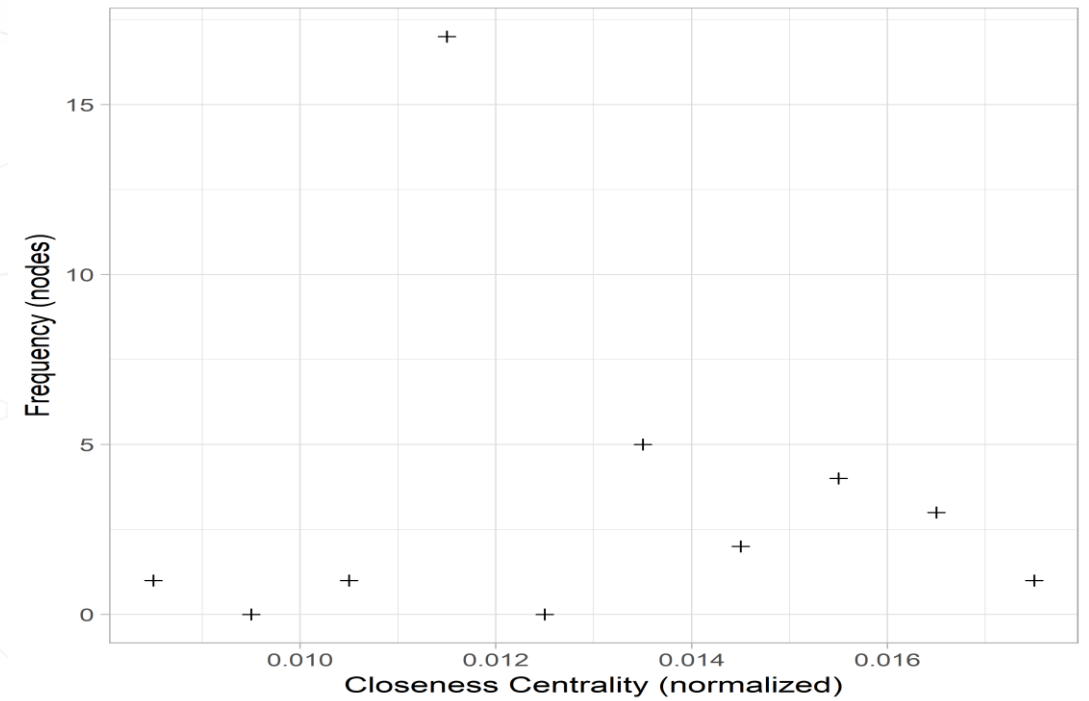
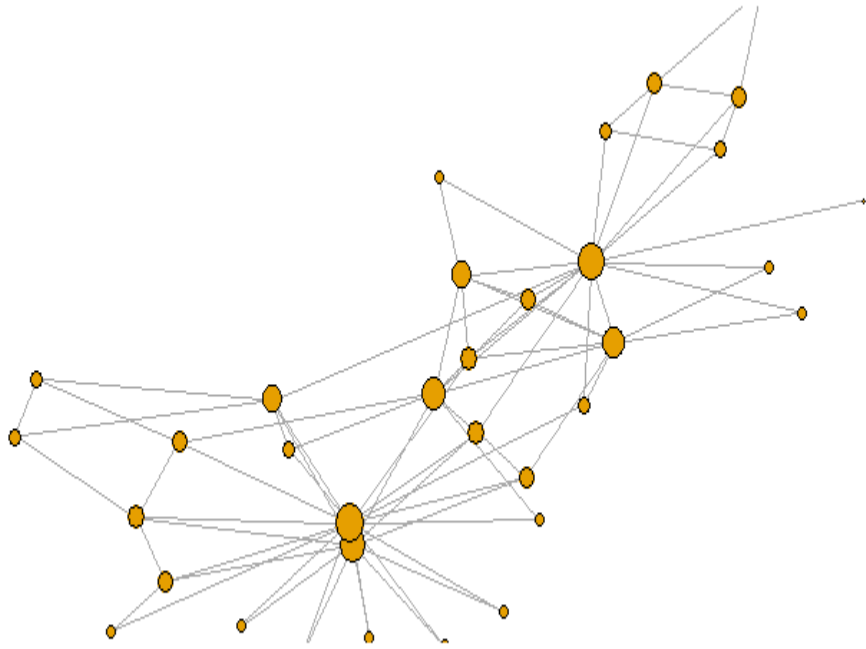
All the Pieces

- A framework that allows for **diverse interventions**—Targeting, hardening etc. instead of only **node deletion**
- A meaningful scalar statistic—**Entropy**—that fits right into security and target policy-making
- An empirical framework to borrow from—the **Configuration Model**
- A **permutation/bootstrapping** method that covers enough sample space, and allows attribution of aggregate level properties to individual nodes
- An expandable **Dark Networks 2.0** framework that could easily integrate several other toolsets, including temporal, longitudinal, and agentized methods (consider permuting according to an activation rate, simulating time, and consider permuting according to some “rule”).

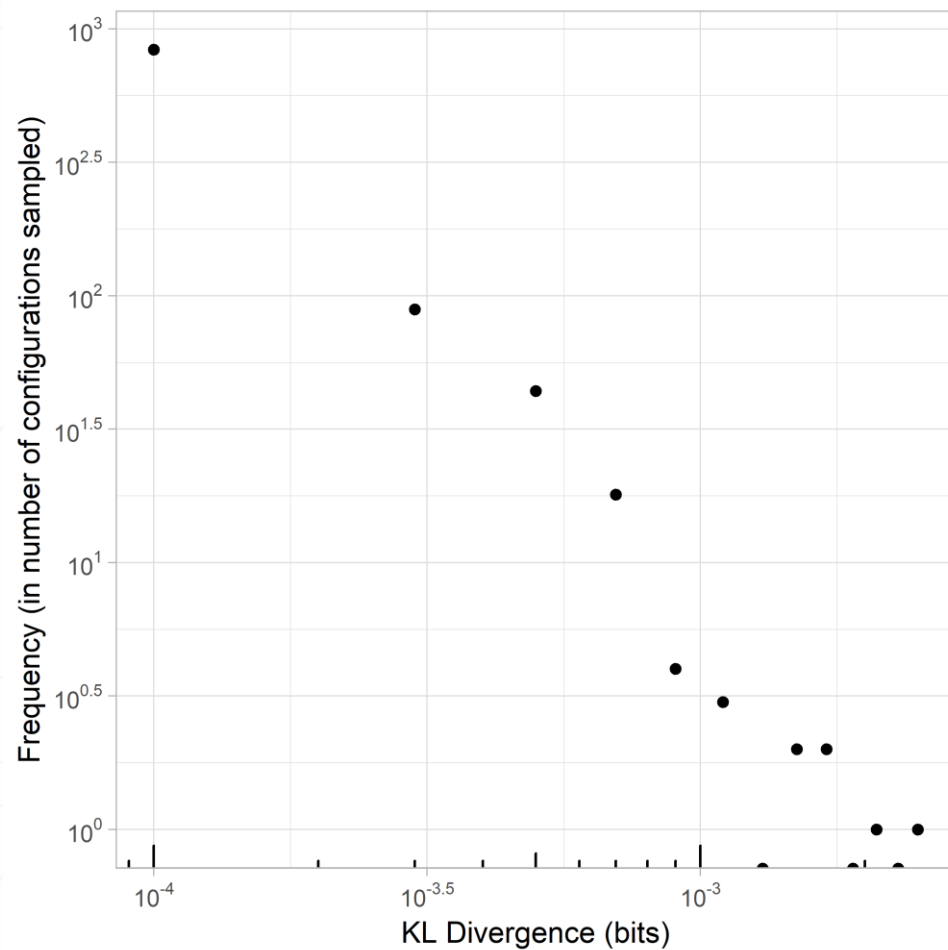
Theoretical Test: Ring Networks



Zachary's Karate Club



Zachary's Karate Club



Is “Surprise” Power Law?

Asking for a friend!

Future Work—So much to do

- Expanding into joint entropy distributions (multinomial) allowing for multiple statistics to be considered
- Testing on a variety of real-world networks
- Validating predictive performance
- Incorporating temporal rules and behaviors into the permutation scheme (think empirical agent-based modeling!!!)

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Thank you to my mentors and sponsors.