Errors in Infinite Computations

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When the error rate is not absolutely zero, infinite computations are necessarily erroneous, rendering them uninformative. This restricts the practical value of those hypercomputers that perform infinite computations.

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§1 Introduction

¶1 · For Shagrir & Pitowsky (2003): "A hypercomputer is a physical or an abstract system that computes functions that cannot be computed by a universal Turing machine." Then they present "the only known physical digital hypercomputer. This device [HM] performs a supertask, that is, it can carry out an infinite number of steps in a finite time-span." And finally, they "consider three objections to [their] contention that HM is a physical system that computes the function h", where "h characterizes the self-halting states of Turing machines." This note presents another objection.

§2 Memory

¶1 · The hyper-machine HM is "made up of a pair of communicating standard computers, T_A and T_B." By using a relativistic spacetime somewhat resembling that of the "twin paradox" fame, T_B can compute an infinite number of steps and can communicate with T_A, for which the elapsed time is finite, in such a way that T_B works as an oracle for T_A on infinite computations; see Turing (1938) on oracles.

¶2 · Turing (1936) machines are mathematical idealizations that abstract away some practical limitations of real computing: Turing machines are unbounded in time and tape, and they are error-free, see Casares (T). The hyper-machine HM resolves the limitation of time, but it is silent on the other two limitations: memory and errors.

¶3 · Though not all non-halting computations require unbounded memories, some will expand beyond any finite memory, and then, in resolving the halting problem, T_B will sometimes meet memory limitations. This is, in fact, objection #1 to which Shagrir & Pitowsky (2003) reply starting in page 88. In any case, regarding memory limitations, HM could not compute function h in every case, because the computation would be aborted whenever T_B's memory were exhausted.

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§3 Errors

¶1 · In practice, the error rate P_e cannot be absolutely zero. Except when the temperature of a system is absolute zero, we cannot control the system completely. And then, as 0 K is impossible to achieve, the error rate is always positive: $0 < P_e \leq 1$. Then, assuming a constant error rate, the probability of an error-free computation of n steps is $(1 - P_e)^n$, and then it is zero when the number of steps grows to the infinite:

$$0 < P_e \leq 1; \quad \lim_{n \to \infty} (1 - P_e)^n = 0.$$

¶2 · Assuming a constant error rate is assuming thermal equilibrium, which is the most sensible assumption. Other assumptions are possible but, as long as the error rate would not decay to zero (asymptotically, since it cannot be zero), the conclusion will be the same, see §6: the probability of an error-free infinite computation is zero. Button (2009; §2.3) argues similarly and he also reaches this conclusion.

¶3 · This means that T_B's answers as an oracle for T_A on infinite computations are as bad as those provided by a random source. In other words, regarding infinite computations, T_B's answers are completely uninformative. And, since finite computations do not need any oracle, the conclusion is that, when taking into account errors, the hyper-machine HM is not better than a universal Turing machine.

¶4 · This is not only applicable to the hyper-machine HM, because this conclusion can be extended to any non-error-free hyper-machine performing infinite computations.

§4 Discussion

¶1 · Regarding errors, finite and infinite computations are very different. While in finite computations errors can be reduced to as close to zero as desired, either by redundancy or by repetition, in the case of infinite computations, as seen, errors will always happen for sure so they cannot be eliminated. This explains why errors can be tolerated in computing, but are disqualifying in hypercomputing when an infinite computation is performed. This difference is easily neglected when we use Turing machines as models, because they are error-free idealizations.

§5 Conclusion

¶1 · Contrary to Shagrir & Pitowsky's "contention that HM is a physical system that computes the function h" (page 88), our analysis shows that, either HM is not a physical system, but an idealization that ignores its errors, or, if we take into account its errors, then HM does not compute function h.

§6 Appendix: General case

¶1 · In the general case, if $P_e(n)$ is the probability that a machine makes an error in step n, so $0 \leq P_e(n) \leq 1$ for every n, then the probability of an error-free infinite computation is:

$$P_\infty = \prod_{n=1}^{\infty}(1 - P_e(n)).$$

If for every n, $P_e(n) \geq P_\epsilon > 0$, that is, if the error rate of the machine cannot be less than P_ϵ, meaning that P_ϵ is the absolute error rate limit of the machine, then:

$$P_\infty = \prod_{n=1}^{\infty}(1 - P_e(n)) \leq \prod_{n=1}^{\infty}(1 - P_\epsilon) = \lim_{n\to\infty}(1 - P_\epsilon)^n = 0 \quad\Longrightarrow\quad P_\infty = 0.$$

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