Appendices

A Lemmas

**Lemma 1**  is a strictly increasing and strictly concave function of  on  where  and  Moreover:



**Proof.** The definition of  given in equation (14) directly implies . To establish the second limit, we rewrite equation (14) as



(32)

As the second ratio on the right-hand side of equation (32) approaches 1 if , we conclude that  Moreover, to establish that  is a strictly increasing and strictly concave function of , we observe that



(33)

and



(34)

Finally, note that  is strictly increasing in both  and  (equation 8). As a result, we conclude that  and  

**Lemma 2**  is a strictly concave function of  on . At   while



*So there exists a unique*  *such that* ,  *for all*   *and*  *for all*  .

**Proof.**  can be written as  where



(35)

and



(36)

If  Lemma 1 and equations (8) and (21) imply  and . Moreover, it follows from equation (8) that the latter expression is equivalent to  Therefore, for the case considered, 

The derivatives of  and  with respect to  are given by, respectively,



(37)

and



(38)

To establish the strict concavity of  on , observe that Lemma 1 and equation (34) imply that  is a strictly decreasing function of  for all . As a result, the strict concavity of  follows if  is a strictly decreasing function of , too. Because  is a strictly decreasing function of , a sufficient condition for this is



(39)

for all  When we use in equation (39) the explicit expressions for the two partial derivatives (given in equations 33 and 34, respectively), we obtain



(40)

Since  for all  considered,  holds, and we conclude that  is strictly concave on .

If  equations (37) and (38) simplify considerably, and we obtain



(41)

where the sign follows from  and  (assumptions R1 and R2, respectively). If , it follows from equations (14), (33), and (37) that  Similarly, we can conclude that —given by equation (38)—approaches  as  Therefore,



(42)

The existence of threshold  now directly follows from the established results. 

B Propositions

**Proof of Proposition 1**. The Lagrangian associated with optimization problem (7) reads



In this Lagrangian,  is the multiplier associated with constraint . The first-order condition with respect to  is given by



(43)

The set of conditions that determine the solution to optimization problem (7) consists of the first-order condition (43), the constraint , the complementary slackness condition , and the non-negativity constraint on the multiplier. We now have to distinguish between two different parameter constellations.

(i) . For , first-order condition (43) leads to , implying that  holds. Since  guarantees that the complementary slackness condition holds, too, it follows that  is a candidate solution to optimization problem (7). Moreover, it is the only one: For , the complementary slackness condition implies . However, if  and , first-order condition (43) is violated.

(ii) . For , first-order condition (43) again leads to , implying that  is violated. For , the complementary slackness condition implies that , and this is the only candidate solution to optimization problem (7).

Since the objective function is a strictly concave function of  on , the candidate solutions characterized under (i) and (ii) must be maxima. Thus, taken together, (i) and (ii) imply that  if  and that  if . 

**Proof of Proposition 3.** The Lagrangian associated with optimization problem (20) reads



where  and  are stated in equations (19) and (18), respectively, and  are the two Lagrange multipliers. The first-order conditions with respect to  and  are given by



(44)

and



(45)

respectively. Now assume that there exists a candidate solution to optimization problem (20) that involves  (strictly positive transfers). Then, this candidate solution necessarily involves . With  first-order condition (45) implies



(46)

Observing that  is strictly positive, we conclude that the candidate solution under consideration necessarily involves . To find the explicit expression for  stated in equation (21), use equation (46) to eliminate  from first-order condition (44), remember that  and then rearrange terms. To derive the explicit expression for  stated in equation (22), substitute  for  in equation (18) and then solve for 

The next step is to verify that the expressions for  and  stated in the proposition are consistent with the assumption  and thus indeed part of a candidate solution to the optimization problem (together with  and the expression for  that is stated in equation 46). To do so, we first establish two interim results: (i) it follows from Lemma 1 and equations (8) and (21) that  (ii) because of (i) and equation (19),  is a sufficient condition for .

On the basis of (ii), we now complete the verification by establishing that



(47)

In what follows, the arguments of  and  are dropped for notational convenience. Taking into account equation (23), condition (47) is equivalent to



(48)

To prove that condition (48) holds, first observe that  if Therefore, if , the left-hand side (LHS) and right-hand side (RHS) of condition (48) are exactly equal. To establish that condition (48) holds for , too, it is sufficient to show that for all  the derivative of the LHS with respect to , LHS, is strictly greater than RHS:



(49)

where  is stated in equation (33). Rearranging terms yields the equivalent condition



(50)

Condition (50) in fact holds for all , as can be immediately concluded from the following two observations: first, if  condition (50) simplifies to  an inequality that holds because  and  (assumptions R1 and R2, respectively); second, the right-hand side of condition (50) is a decreasing function of  since  is a decreasing function of  and  is an increasing function of  (equation 33 and Lemma 1, respectively). To summarize: for all  condition (49) holds, implying that condition (47) does so, too; since the latter is sufficient for , the verification is completed.

We now proceed by showing that the considered candidate solution is in fact the global maximizer of  In this regard, note that there are no candidate solutions involving : both possible constellations— and  —violate first-order condition (45). Thus,  and  As a result, we can find the solution to optimization problem (20) by solving the following single-variable optimization problem:



(51)

where



(52)

The objective function of problem (51) is a strictly concave function of  on , and one can show that the expression for  stated in the proposition is the global maximizer of this function. At the same time, in the preceding paragraph, we have established that this expression satisfies . Thus, the proposition follows. 

**Proof of Proposition 4.** In this proof, the arguments of  and  are dropped for notational convenience. Using in condition (25) the corresponding explicit expressions for consumption given in equations (9), (10), (23), and (24), we obtain



(53)



Condition (53) can be rewritten as



(54)

where



(55)

To prove that condition (54) holds, first observe that  implies  and  as a result, the two sides of condition (54) are exactly equal if  To establish that condition (54) holds for all , it is sufficient to show that for all  the derivative of the LHS with respect to , LHS, is strictly greater than RHS. The two derivatives read



and



where  is given by equation (33). It follows that LHSRHS is equivalent to



(56)

To see that (56) in fact holds for all , consider the following two results: first, if  condition (56) simplifies to  an inequality that holds because  and  (assumptions R1 and R2, respectively); second, the left-hand side of condition (56) is an increasing function of  while the right-hand side is a decreasing function of  To summarize: for all  condition (56) holds, implying that condition (54) holds for all ; since the latter is equivalent to condition (25), the proposition follows. 

**Proof of Proposition 9.** Parts *a.* and *b.* of the proposition, describing the behavior of the group in power if , are just a restatement of results established prior to the proposition and hence do not need any proof. So we focus on showing that the group in power always relies on repression if  We have to distinguish between two cases:  and . The former interval is non-empty if , i.e., if  becomes negative before  reaches  So suppose that  belongs to the non-empty interval  For this case, Lemma 2 establishes that there is a loss in individual overall utility in group  if the group switches from repression (as described in Proposition 5) to institutional reform (as described in Proposition 8). As a result, group  prefers the repression approach to institutional reform. Considering this, and the fact that it is never optimal to abstain from repression while keeping the degree of institutional cohesiveness unchanged (see Footnote 12), it follows that group  will adopt the repression strategy.

Now suppose that  In this case,  is strictly positive even if  (Lemma 1), implying that institutional reform alone cannot prevent group  from an attempt to take power in period . We now complete the proof by showing that—irrespective of the choice of —group  always prefers to retain political power. We start by re-emphasizing Footnote 4.2, which says that in period  group  will never abstain from institutional reform () while, at the same time, abstaining from repression. Therefore, what remains to be established is that in period  group  will never opt for institutional reform () while, at the same time, abstaining from repression. To do so, suppose  and assume that in period  group  opts for the “ad-hoc” repression approach  and , where  is given by equation (22). Then, equation (11) can be rewritten as



(57)

Now suppose that group  is contemplating whether it should abstain from repression and opt for  instead (while, at the same time, sticking to  as this is the optimal level of spending on public services if —see the discussion in Subsection 4.2). Group  will prefer the above ad-hoc repression approach if



(58)



By rearranging terms in condition (58), and by taking into account that **, we obtain



(59)

Now observe that equations (9) and (10) imply that the right-hand side of equation (57) and the right-hand side of condition (59) are identical. Therefore, condition (59) holds if



(60)

Since  and  this is indeed the case. As a result, we conclude that group  prefers the ad-hoc repression approach introduced above to abstaining from repression and thus losing power (which would happen if  and 

Finally, observe that group  has an even stronger preference for repression if it relies on the optimal approach—described in Proposition 3, with the value of  evaluated at  —instead of the ad-hoc variant: the optimal approach maximizes first-period consumption, subject to the constraint that the choice of  discourage group  from taking power. As a result, we conclude that in period  group  will never combine institutional reform () with absence of repression, and so the proposition follows. 

**Proof of Proposition 10.** Equation (41) implies that  approaches  if . Since  is strictly concave on  (Lemma 2), we conclude that  approaches  if . Considering this result, the definition of  in equation (31), and the fact that , the first statement in the proposition follows.

As for the second statement, first observe that equation (21) implies that  is a strictly decreasing function of . Therefore, it follows from equation (30) that, at any given , an increase in  raises . As a result, an increase in  raises  Finally, since we focus on —and hence —the second statement follows. 

**Proof of Proposition 11.** Group  strictly prefers repression without institutional reform over a combination of repression and institutional reform if



(61)



where



(62)

and



(63)

Moreover,



(64)

where  is given by equation (8). Because , it follows that , , and . Rearranging terms, condition (61) can be rewritten as



(65)

Now consider the limit of  if :



(66)

So, in the limit, group  strictly prefers repression without institutional reform if



(67)

By rearranging terms, we obtain



(68)

a condition which holds for any . This is the case because  is strictly less than  and because . Given that  is continuous in  for any , it follows from  that there must exist a value of public revenues  such that  for any . 