Uniform Top Hat Illumination for Extended Sources Using Only Spherical Lenses

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ABSTRACT
A technique is presented for achieving flat top irradiance distributions for planar extended Lambertian sources (such as high-powered array LEDs). It is also briefly discussed how to generalize the technique for sources with arbitrary radiant intensity profiles, provided that such profiles are rotationally symmetric.

1. INTRODUCTION

Gao *et al.* [1] had shown that the spherical aberration of a spherical singlet can be tailored to improve the uniformity of illumination from a small LED. By applying flux conservation principles, I showed how to produce a top hat illumination (i.e., irradiance) distribution from a small disk Lambertian source using an air-spaced spherical doublet [2, 3]. The design was performed completely by sequential ray tracing on a commercial optical design program (Zemax® OpticStudio®). In this paper, I extend the technique to account for extended Lambertian sources, as well as sources possessing any radiant intensity (flux per unit solid angle) profile, provided that the intensity profile is rotationally symmetric.

2. THEORY

Applying the geometry of rays as before [2, 3], let us now include an off-axis ray given by the ray segment $\overline{O'YH'}$ as shown in Fig. 1:

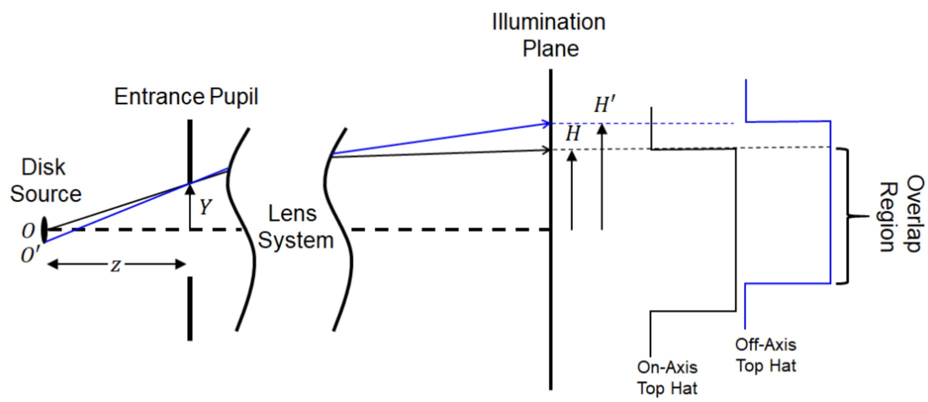

Fig. 1 Geometry for axial and off-axis rays traversing the edge of the entrance pupil at full height Y, and arriving at the edges of their respective top hat distributions.

In Fig.1, the off-axis ray from point O' at the source strikes at height H' at the top edge of the illumination plane, while ray segment $\overline{OYH}$ strikes at height H. From this illustration, we can see that there is a region of overlap between the central top hat distribution (arising from an on-axis point at the source) and the upper top hat distribution (arising from an off-axis point at the source). In this overlap region, there is a superposition of "top hat point spread functions (PSFs)" arising from all points across the source, provided that all these PSFs are the same across the illumination plane (i.e., if this optical system is shift-invariant). This is somewhat analogous to the convolution of a PSF with an ideal geometric projection of the disk source. In either case (i.e., regardless of whether or not we have a shift-invariant system), it is only in the overlap region that a top hat distribution can be generated, due to the superposition of all PSFs in that region. Outside of this region, the irradiance would "taper down" with a slope — perhaps monotonically — until the irradiance reaches zero beyond where rays strike the illumination plane. It may then be concluded that in order to have a uniform top hat distribution at the illumination plane for an extended source, we must have either one or both of the following two conditions:

(1) The PSF area must be made larger so that the extent of the overlap region is larger.
(2) The angular field of view (FOV) given by the ratio $\overline{OO'}/z$ is reduced. For a fixed source size, this may be achieved by increasing the distance z between the source and the entrance pupil.

It is unlikely for the doublet example from the previous study [2, 3] to satisfy the two conditions above — at least in a practical manner — because the element surface curvatures would probably end up being too high. Hence, a different luminaire design form is needed. One alternative is to consider splitting the design into two element groups: a "condenser" (first group) and a "shaper" (second group) that act together to form the final irradiance distribution (Fig. 2).

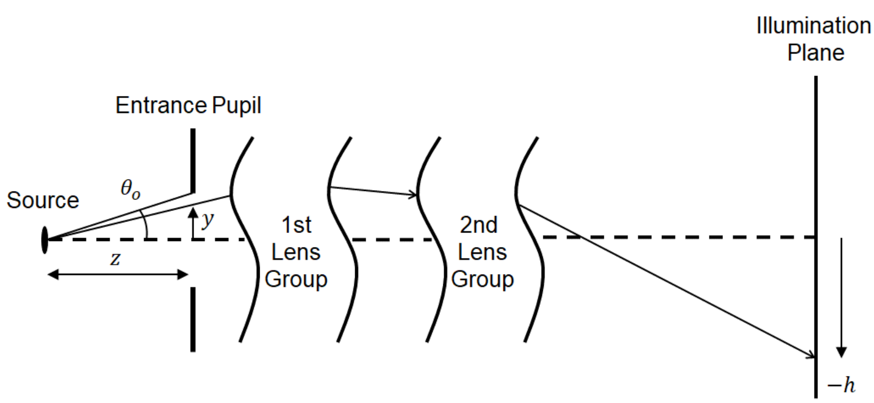

Fig. 2 Concept of a "condenser" (1st Lens Group) and "shaper" (2nd Lens Group) design form for spherical elements to shape an incoherent extended source into a top hat irradiance distribution.

Applying the geometry from Fig. 2 and flux conservation between the entrance pupil and illumination plane as before [2, 3], negative-valued marginal ray heights h at the illumination plane may be expressed in terms of entrance pupil heights y as follows:

$$h = -\frac{H}{\sin\theta_o}\frac{1}{\sqrt{1+(z^2/y^2)}}\,. \tag{1}$$

3. OPTICAL DESIGN EXAMPLE

The reader is encouraged to apply Eq. (1) to arrive at a solution similar to that shown in Fig. 3, whose prescription is provided in Table 1. As an exercise, for example, one may simply modify the prescription and re-optimize using the following values for the variables: $H = 90$ mm, $z = 35$ mm, and entrance pupil semi-diameter $Y = 31.5$ mm. Thus, $\sin\theta_o = Y/\sqrt{z^2 + Y^2} = 0.69$. The design wavelength is 550 nm.

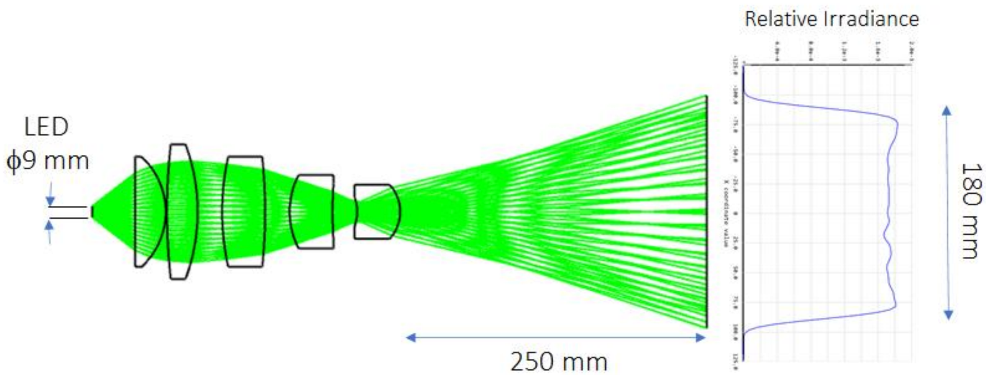

Fig. 3 Five-element "condenser-shaper" design using spherical elements producing a top hat irradiance distribution at a plane at 250 mm from the right vertex of the last element. The source is assumed to be a light emitting diode (LED) with 9 mm diameter having radiance that is independent of angle (i.e., it is a Lambertian emitter).

Table 1. Prescription for the lens system shown in Fig. 3.

Surface	Comment	Radius	Thickness	Glass	Semi-Diameter
0	Source	Infinity	35.00		4.50
1	STOP	Infinity	0.00		31.50
2	-	Infinity	25.00	N-SF6	45.00
3	-	-60.893	1.00		45.00
4	-	485.855	25.00	N-SF6	55.00
5	-	-157.402	20.00		55.00
6	-	208.330	35.00	N-SF6	45.00
7	-	-422.661	20.00		45.00
8	-	65.099	35.00	N-SF6	30.00
9	-	370.783	20.00		25.00
10	-	-34.018	35.00	N-SF6	12.00
11	-	-33.045	250.00		22.00
12	Illum. Plane	Infinity	0.00		95.00

The design possesses certain interesting optical characteristics. First, one would note that the shape of the perimeter surrounding the illumination distribution is independent of the shape of the source, so no source structures would be visible at the illumination plane, which is useful when a high-powered array LED is used as a source. Although this is similar to Kohler projection, it is not a Kohler projection system in that the shaper elements are not imaging the entrance pupil onto the illumination plane. However, the shape of the illumination is indeed dependent on the shape of the entrance pupil. For instance, if the aperture stop (surface 1 in Table 1) is rectangular, then the illumination is rectangular — but it is not a projected image of the entrance pupil (it is more like a "shadow"). This system is therefore a non-imaging luminaire.

Another useful characteristic is that certain elements may be used as "compensators" for the irradiance profile. For instance, the air space between surfaces 5 and 6, and 7 and 8, may be adjusted to vary the irradiance profile between being a "valley" profile to being a "hill" profile (while the top hat profile is maintained if no adjustment is made). This may be useful for compensating any manufacturing errors that result in a non-flat irradiance distribution. Or perhaps this adjustment may be used for varying the irradiance to match some specific application. Further, if a "tunable lens" (or a tunable-focusing element, such as a liquid lens) is mounted between surfaces 9 and 10, such an element may also be used to vary the irradiance profile. This is depicted in Fig. 4, where a thin paraxial lens model is mounted half-way between the fourth and fifth elements.

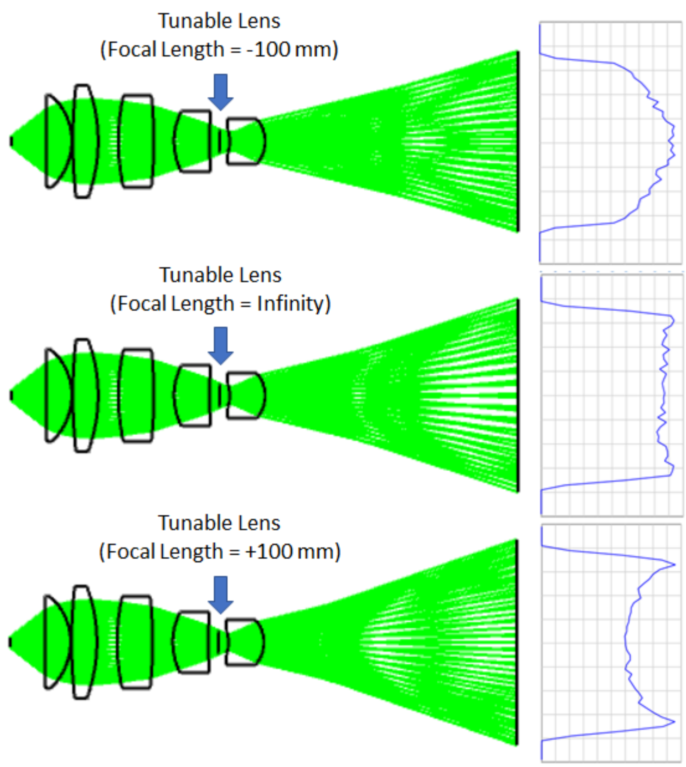

Fig. 4 Varying the irradiance profile by mounting a tunable lens between the last two elements.

The optical characteristics of such an all-spherical luminaire design form may not necessarily be limited to those described above. If more elements are added, one may be able to modify and apply the design to several applications involving illumination, such as fluorescence detection, microscopy, and machine vision. If, for example, a sufficiently large positive powered lens is mounted at the illumination plane, one may collimate the rays to produce uniform telecentric illumination for industrial imaging and inspection systems. If, on the other hand, the source conjugate is designed to fit into a microscope objective, then it may be possible to design the all-spherical lens elements such that the illumination is flat across a bio-sample at the objective's focal plane. However, it is important to note that such all-spherical luminaire designs are based on having a sort of "deterministic" ray density profile at the entrance pupil, such as that given by a Lambertian source whose radiant intensity is well-known in closed-form, or even a LED whose radiant intensity profile is known through measurement (see Sec. 4). If, for example, source rays are coupled into a mixing rod, the output rays become rather — well, mixed up — yielding a ray density profile with direction cosines that may be difficult to characterize.

4. GENERALIZED THEORY FOR NON-LAMBERTIAN SOURCES

Let us assume that we have a circular entrance pupil. If it is desired to have a top hat irradiance distribution at the illumination plane, then, as before [2, 3], the irradiance E' at the illumination plane is constant for any marginal ray height h. Thus, the flux ϕ contained in a circular area of radius h is

$$\phi(h) = E'\pi h^2. \tag{2}$$

The total flux ϕ_o at the full radial height H (see, e.g., Fig. 1) is therefore

$$\phi_o = E'\pi H^2. \tag{3}$$

Substituting $E'\pi$ from Eq. (3) into Eq. (2) we have

$$\phi(h) = \phi_o \frac{h^2}{H^2}. \tag{4}$$

If $\phi(y)$ is the flux in a half-cone subtended by a ray emitting from the center of the source towards height y at the entrance pupil, then by conservation of flux between the entrance pupil and the illumination plane, we have

$$\phi(y) = \phi(h) = \phi_o \frac{h^2}{H^2}. \tag{5}$$

Solving for h in Eq. (5) we have

$$h = \pm H \sqrt{\frac{\phi(y)}{\phi_o}}, \tag{6}$$

where the $\pm$ symbol denotes that h could be either above or below the optic axis, depending on which luminaire design form is applied (e.g., as in either Fig. 1 or in Fig. 2). Eq. (6) is general, for any flat source with a radiant intensity (flux per unit solid angle) profile that is rotationally symmetric about the optic axis. If the radiant intensity as a function of half-cone angle θ is $I(\theta)$, then the formulary of radiometry will show that the flux contained in θ is

$$\phi(\theta) = 2\pi \int_0^\theta I(\theta) \sin\theta \, d\theta \,. \tag{7}$$

For any radiant intensity profile, Eq. (7) may be numerically integrated, and an association must be made between θ and y at the entrance pupil. That is all there is to it.

It is also a straightforward matter if one wishes to consider point sources (i.e., sources whose radiation is isotropic such that the radiant intensity is not a function of angle). In this case, Eq. (7) would end up being $\phi(\theta) = 2\pi I(1 - \cos\theta)$, and $\cos\theta$ may be expressed in terms of y and z. But how realistic are point sources? Perhaps a fluorophore (or a quantum dot) is a likely candidate, but a small flat source whose radiance is not a function of angle is not a point source — it is still a Lambertian source (it's just a very small one). In any case, if a source is sufficiently small and has sufficiently isotropic radiation, the technique will work to produce uniform top hat illumination.

5. CONCLUSION

It is possible to have only spherical lens elements to generate top hat irradiance distributions at a plane of illumination. At least two design forms have been presented to achieve this. One form is to have a pair of spherical meniscus doublets [2, 3], while the other form is described in this paper. In this paper, the latter design form is called a "condenser-shaper" luminaire system. While aspheres and freeform surfaces have been shown to be effective and efficient (with perhaps higher flux transfer efficiency and reduced number of lens elements), lens suppliers often find it easier to manufacture lenses with spherical surfaces. Thus, the method and design form presented in this paper should be quite useful for practical applications.

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AUTHOR

Ronian Siew is a consultant in optical engineering with more than 20 years of professional experience. His optical designs have involved fluorescence detection systems for DNA analysis (such as qPCR and digital PCR instruments), and various imaging and illumination systems. He is also the author of several journal papers and books on optical design. For more information, please visit the website below.

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