Parallel Roads to the Traveling Salesman 3/2-Approximation

Metric Traveling Salesman Problem (TSP)

Find: A shortest tour visiting each vertex of an edge-weighted graph at least once.
NP-hard: No polynomial-time algorithm is known.

In 1976, a polynomial-time algorithm for computing a tour of length no more than 3/2 of the optimum was discovered simultaneously in the English and Russian literature.

Actually, it is the end of a series of *three* parallel discoveries.

1973/74: The Chinese Postman Problem (CPP)

Find: A shortest tour visiting all edges of an edge-weighted graph. Solvable by computing a maximum-weight matching in an auxiliary graph.

1973: Christofides finds the connection between CPP and matching, solves the resulting matching problem in polynomial time using Edmonds' 1965 algorithm.

1974: Serdyukov finds the connection between CPP and matching, solves the resulting matching problem in exponential time, being unaware of Edmonds' algorithm.

1975: Faster Matching Algorithms

Lawler: $O(n^3)$-time implementation of Edmonds' matching algorithm.

Karzanov: $O(n^3 \log n)$-time implementation of Edmonds' matching algorithm.

Likely, Serdyukov learned about the polynomial-time solvability of the maximum-weight matching problem from a preliminary version of Karzanov's article.

Both results are published only in 1976.

1976: 3/2-Approximation Algorithm for TSP

Algorithm: First compute a minimum-weight spanning tree and then a shortest tour traversing it, via matching, like for CPP.

February: Christofides describes the algorithm in a technical report at Carnegie Mellon University.

January: Serdyukov submits it to the journal *Upravlyayemye Sistemy* of the Institute of Mathematics in Novosibirsk.

Full article: R. van Bevern and V. A. Slugina: A historical note on the 3/2-approximation algorithm for the traveling salesman problem. *Historia Mathematica*, 2020. doi:10.1016/j.hm.2020.04.003, arXiv:2004.02437