A Systematic Assessment of National, and Regional Under-Five Mortality Rate by Wealth Quintiles and Identification of Countries with Outlying Levels Using a Bayesian Hierarchical Time Series Model

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Abstract

National-level estimates of the under-five mortality rate (U5MR) may mask disparities at the subnational level. We developed a Bayesian hierarchical time series model to assess disparities in the U5MR by household-level socioeconomic status (measured through wealth quintiles). In the model, country-specific ratios of wealth-quintile-specific U5MRs are modelled as the product of an expected ratio and a country-specific deviation from that expected ratio. We obtained quintile-specific estimates for 96 countries from 1990 (or the earliest year of data collection) to 2015 and pinpointed countries with unusually large or small disparities. We find that important differences exist between children from the poorest and richest households with respect to survival up to age five.

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1 Introduction

Under-five mortality rate (U5MR), the probability of a child dying before the age of five, is one of the most important indicators of health infrastructure and living standard of countries. Since 1990, the progress of achieving Millennium Development Goals (MDGs) 4, which is to reduce child mortality, has been remarkable, with a reduction of more than 50% of U5MR by 2015; the reduction in rate has significantly accelerated in recent years (UNICEF et al. [2015]). Despite the many achievements on reducing child mortality, progress has been uneven within and across countries (Unicef et al. [2015]). In particular, millions of children have left behind, especially those from the poorest households. The disparity of childrenâs survival between the wealthiest and the poorest household still remain (Hosseinpoor et al. [2005], Organization [2000], Wagstaff [2000]) or could even worsen, especially in developing countries (Kraft et al. [2013], Östlin et al. [2011], Wang [2003]).

Monitoring U5MR by household socioeconomic status is challenging. This is because the household wealth level is difficult to quantify. Few national data sources provide information on household wealth status. While estimates of U5MR by socioeconomic strata have been reported previously for either one country (Axelson et al. [2012], Jain et al. [2013], Rarani et al. [2016], Vapattanawong et al. [2007]) or multiple countries (Minujin and Delamonica [2003]), they did not publish uncertainty intervals, nor did they provide enough evidence to validate their estimate. To date, no studies have provided a clear overview on how U5MR inequality on wealth status changes over time across countries. Identifying countries with unusually large or small disparities on U5MR between the poor and rich is challenging as well, because the size of the mortality gap between the poorest and the richest or the way it varies in relation to the total level of U5MR is not as well known.

To better understand who and where the most disadvantaged and vulnerable children are and how the socioeconomic disparities changes over time, we aim to measure levels of and trends in U5MR disparities by wealth quintiles, which are based on household-level socioeconomic status, for 96 countries from 1990 (or earlier, depending on data availability) to 2015. The proposed estimation method takes into account various data quality issues, includes an uncertainty assessment and is shown to perform well in validation exercises. The model estimation of ratios of quintile-specific U5MR and total U5MR were combined with estimates of total U5MR from the United Nations Inter-agency Group for Child Mortality Estimation (UN IGME) to obtain quintile-specific U5MR estimates, which were published in UN IGMEâs 2015 publication of mortality estimates (UNICEF et al. [2015]). Moreover, in the model, the overall relation between ratios among quintile-specific U5MR and total U5MR levels is assessed using all available survey data and modelled with a flexible splines regression model. The resulting overall relation is subsequently used to pinpoint countries with outlying U5MR disparity on wealth status.

2 Methods

2.1 Data

The quintile-specific U5MR refers to the probability for those children born in a certain wealth quintile group to die before age five. The data used in this study are observed U5MR by wealth quintiles from Demographic Health Surveys (DHS) and Multiple Indicator Cluster Surveys (MICS), where all the surveys were conducted between 1990 and 2014. DHS and MICS are retrospective sampling surveys. They employ an asset index (also known as wealth index) composed of a set of variables asked in household questionnaires that describes household assets and utility services (Shea Oscar Rutstein [2004]). The wealth index is then used as a proxy for household income.

The variables constituting the wealth index are a standard set, although it has developed somewhat from early surveys to later ones. The wealth index is constructed based on a principal components analysis, using the set of wealth indices for a certain survey. The wealth quintiles computed in our study used the product of the sampling weight and the number of births to ensure that there are the same number of births in each quintile. This approach differs from the conventional way of deriving quintiles, which weigh the households by the product of the sampling weight and the household size, so that the numbers of individuals recorded in the survey for each quintile are the same. Apart from the fact that our focus is on the distribution of births, our procedure has the benefit that the estimate of the 5th (richest) quintile becomes more stable since more births fall inside this quintile.

For data from full birth histories collected, five-year estimates were used. For surveys in which only summary birth histories were collected, the indirect method of time since first birth (TSFB) was used to get indirect estimates of child mortality data and only those estimates that are 5 to 9 years since the first birth are used (Hill and Figueroa [1999]). We included livebirths born in the 10 years previous to the survey.

2.2 Statistical analysis

We developed a statistical model to estimate levels and trends in U5MR by wealth quintiles over time for 96 countries. In the model, the U5MRs from the 1st, 2nd, 4th, and 5th quintiles are modelled relative to the U5MR from the 3rd quintile, these ratios are referred to as “S ratios”. This approach was used (i) to exchange information within countries over time and across countries on expected and actual U5MR ratios, and (ii) to incorporate the constraint that the sum of quintile-specific under-five deaths is equal to the total number of under-five deaths.

A flexible regression model (penalized B-splines regression) was used to represent the overall expected relation between the S-ratios of the U5MR from the 1st, 2nd, 4th, and 5th quintile to that from the 3rd quintile and total U5MR. Country-specific S-ratios were modelled by the product of the expected ratio (based on the flexible regression model and the total U5MR in the country-year) and a country-specific multiplier, which represents the relative difference between the country-specific ratio amongst quintiles to the overall expected ratio amongst quintiles. These multipliers were modelled with a flexible time series model in which country-specific levels are assumed to fluctuate around one. By simultaneously estimating the overall regression model fit and the country-specific ratios, country-specific information informs the overall relation and vice verse, the overall relation informs the country-specific estimates.

The model was fitted to available DHS and MICS data. We used the ratio of quintile-specific U5MR to total U5MR (called “ratio R”) in the data quality model, instead of using quintile-specific U5MR directly, to reduce the effect of level biases in U5MR data series (Alkema et al. [2014]). The data quality model incorporated sampling variance which takes into account survey structures.

Estimates of the ratios were combined with estimates of total U5MR to obtain quintile-specific U5MR, accounting for the uncertainty in total U5MR (UNICEF et al. [2015]). Deaths were obtained through a standard life table approach adopted by the UN IGME (Fund et al. [2007]), using information on population numbers from the 2015 Revision of World Population Prospects and life table entries set by WHO (United Nations, Department of Economic and Social Affairs, Population Division [2015]). We did not impute estimates for countries without data. Aggregated estimates for all the 96 countries and UNICEF regions were based on the totals for the number of deaths and population numbers by countries from each region.

To pinpoint countries with unusually large or small wealth quintile disparities, we defined and calculated the expected quintile-specific U5MR that is associated with an expected ratio S and the UN IGME estimate of total U5MR for a given country-year, and defined and calculated excess quintile-specific U5MR as the difference between the expected quintile-specific U5MR and the estimated rate for the country-year (where negative outcomes refer to lower-than-expected quintile-specific U5MR).

We used a Markov Chain Monte Carlo (MCMC) algorithm to generate samples of the posterior distributions of the parameters (Gelfand and Smith [1990]). This approach produced a set of trajectories of ratios of quintile-specific U5MR to total U5MR (ratio R) for each country, and associated measures of quintile-specific U5MR, excess quintile-specific under-five mortality and deaths. We computed 90% uncertainty intervals (UIs) for all indicators of interest using the 5th and 95th percentiles of the posterior distributions (90% UIs are the standard choice in UN IGME reporting as opposed to the more standard 95% intervals given the inherent uncertainty in child mortality related outcomes). The uncertainty in estimates follows from the limitations of the available data at the country-level, especially for recent years. We defined country-years to have outlying disparities on quintile-specific U5MR if the absolute value of the point estimate for excess quintile-specific U5MR is greater than one per 1,000 and if the posterior probability that the excess quintile-specific mortality is either negative or positive is more than 90%, corresponding to a chance of one in ten of incorrectly flagging an outlying country.

The MCMC sampling algorithm was implemented using JAGS 4.0.1 Open Source software (Plummer [2003]), and the analysis was carried out in R version 3.2.2 (R Core Team [2016]).

Model performance was assessed through an out-of-sample validation. Model validation exercises suggest that our model was well calibrated.

3 Results

3.1 Overall relation between ratio-S and total U5MR

Figure 1 shows the model results for the overall relation between total U5MR levels and expected ratios of the U5MR from the 1st, 2nd, 4th, and 5th quintile to that from the 3rd quintile (ratio S) for a given level of total U5MR. Comparing to the Loess curves in green, within the 90% bounds of total U5MR, the model estimates and Loess curves agree with each other.

The expected ratio of the 1st to the 3rd quintile-specific U5MR is estimated to be slightly above one when mortality is high (around 200 deaths per 1,000 live births). The ratio have a steady increase of more than 30% as total U5MR decreases to around 20 deaths per 1,000 live births and remains stable as the total U5MR decreases. For the ratio between the 5th and 3rd quintile-specific U5MR, the expected ratios are significantly below one and remain stable for the entire range of total U5MR at around 0.7.

3.2 Country-specific results

Figure 2 shows the model results from Albania for the ratio of quintile-specific to total U5MR (top row), and quintile-specific U5MR (bottom row) for all quintile groups over time. For Albania, the estimated and expected ratio (red and green curves respectively) have very similar level throughout the whole time period. Hence, the estimated quintile-specific U5MR (in blue) for all the quintile groups are not much differ from their corresponding total U5MR (in black) as shown in the bottom row.

Figure 3 shows the model results from Peru. For Peru, the estimated and expected ratio between the 1st and the 5th quintile groups are significantly different from each other for the whole observation period since their credible bounds do not overlap until in the recent period (which is due to lack of data). More specifically, the U5MR for the 1st/poorest quintile is higher than expected, and to the contrary, the U5MR for the 5th/richest quintile is much lower than expected. The estimated quintile-specific U5MR for the 1st and 5th quintiles are significantly higher and lower than their corresponding total U5MR for the entire time period.

Figure 4 shows the model results from Chad. The estimated and expected ratio between the 1st and the 5th quintile groups are significantly different from each other for the whole observation period. However, the situation is the opposite to Peru. In particular, the U5MR for the 1st/poorest quintile is significantly lower than expected, and the U5MR for the 5th/richest quintile is significantly higher than expected. The estimated quintile-specific U5MR for the 1st and 5th quintiles show the same story.

Figure 5 gives an overview of the countries with outlying U5MR for the 1st and 5th quintiles in either 1990 or 2015. "Outlying" means that the 90% credibal bounds of the excess U5MR is entirely above or below zero and the 90% credibal bounds of the ratio of estimated to expected quintile-specific U5MR is completely above or below one. The top row lists countries with higher-than-expected U5MR for the 1st/poorest quintile in 1990 and/or 2015. There are 11 countries fall into this category. The excess 1st quintile U5MR decreases for all the 11 countries from the year 1990 to 2015. Only 4 of them still have the excess 1st quintile U5MR significantly higher than zero in 2015: Bolivia, Nigeria, Peru, and Philippines. However, when looking at the ratio of estimated to expected 1st quintile U5MR, the improvement on the relative scale is less remarkeable than that on the absolute scale. The decrease of the ratio over time is not significant for all the 11 countries. Furthermore, 5 countries still have their ratio significantly higher than 1 in 2015: Peru, Philippines, Bolivia, Indonesia, and Nigeria. For Nigeria, although the point estimate for excess 1st quintile U5MR decreased over time, its ratio of estimated to expected 1st quintile U5MR increased during the period. This suggests that on absolute scale, the disparity of the 1st quintile is diminishing over time, but given that the total U5MR is also decreasing, on relative scale, the disparity of the 1st quintile is almost the same from 1990 to 2015.

The bottom row lists countries with lower-than-expected U5MR for the 5th/richest quintile in 1990 and/or 2015. 12 countries are identified here. In general, the excess 5th quintile U5MR become closer to zero from the year 1990 to 2015 for all the 12 countries (implies less disparity), the ratio of the estiamted to expected 5th quintile U5MR are quite similar for 1990 and 2015. Among the 12 countries, 8 of them also have higher-than-expected 1st quintile U5MR as shown in the top row. They are: Philippines, Indonesia, Turkey, Egypt, Peru, Bolivia, Nigeria, and India.

3.3 Aggregated results

Figure 6 shows the summary of aggregated results based on weighted average of all country-level results. Overall, the estimated and expected quintile-specific U5MR have decreased since 1990 for all the quintiles. The 1st quintile-specific U5MR is estimated to be above its expected level, and the 5th quintile-specific U5MR is estimated to be below the expected level throughout the whole period. The overlapping during recent period is mainly due to lack of data and hence wider uncertainty intervals. The estimated ratio of 1st quintile U5MR to the total U5MR is significanly above the expected ratio expect for recent period. Both the estimated and expected ratio is becoming smaller from the 1st/poorest quintile to the 5th/richest quintile. For the 5th quintile, the estimated ratio is significantly lower than its expected level (despite slight overlap in recent). For the quintile-specific under-5 deaths, the number decreased over time for all the quintiles. However, the number of under-5 deaths from the 1st quintile is the highest comparing to all the rest quintiles in the same year. Furthermore, the median estimate of the number of under-5 death for the 1st quintile in 2015 is still 16% higher than that for the 5th quintle in 1990. Note that the aggregated results only summarize the weighted average of the 96 countries with data, but not all countries around the world.

4 Discussion

To our best knowledge, our study is the first systematic analysis to provide an assessment of levels and trends of socioeconomic inequalities in U5MR with a wide coverage of developing countries. Our findings confirmed evidence from previous studies (Jain et al. [2013], Mohammad and Tabassum [2016], Rarani et al. [2016]) that the gaps of survival and accessing healthcare resources between the under-five children from the richest and poorest households have remained or even worsen for some countries since 1990 even though the overall healthcare situation has improved. In addition, we identified countries with unexpectedly high mortality in the poorest group and low mortality in the richest group. Further research should focus on explaining the outlying disparities across countries. More efforts from international organizations should be drawn to the children aged under-five from those groups with outlying disparities.

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5 Tables and Figures

UNICEF Region	**Country with data**	**Country w/o data**
South Asia	Afghanistan; Bangladesh; Bhutan; India; Maldives; Nepal; Pakistan	Sri Lanka
CEE/CIS	Albania; Armenia; Azerbaijan; Belarus; Georgia; Kazakhstan; Kyrgyz Republic; Macedonia; Republic of Moldova; Serbia; Tajikistan; Turkey; Ukraine; Uzbekistan	Bosnia and Herzegovina; Bulgaria; Croatia; Montenegro; Romania; Russian Federation; Turkmenistan
Sub-Saharan Africa	Angola; Benin; Burkina Faso; Burundi; Cameroon; Central African Republic; Chad; Comoros; Congo; Democratic Republic of the Congo; Cote d'Ivoire; Equatorial Guinea; Eritrea; Ethiopia; Gabon; The Gambia; Ghana; Guinea; Guinea-Bissau; Kenya; Lesotho; Liberia; Madagascar; Malawi; Mali; Mauritania; Mozambique; Namibia; Niger; Nigeria; Rwanda; Sao Tome and Principe; Senegal; Sierra Leone; Somalia; South Africa; South Sudan; Sudan; Swaziland; Tanzania; Togo; Uganda; Zambia; Zimbabwe	Botswana; Cape Verde; Djibouti; Mauritius; Seychelles
Latin America and the Caribbean	Belize; Bolivia (Plurinational State of); Brazil; Colombia; Dominican Republic; Guatemala; Guyana; Haiti; Honduras; Nicaragua; Paraguay; Peru; Suriname; Trinidad and Tobago	Antigua and Barbuda; Argentina; Bahamas; Barbados; Chile; Costa Rica; Cuba; Dominica; Ecuador; El Salvador; Grenada; Jamaica; Mexico; Panama; Saint Kitts and Nevis; Saint Lucia; Saint Vincent and the Grenadines; Uruguay; Venezuela (Bolivarian Republic of)
East Asia and Pacific	Cambodia; Indonesia; Lao People's Democratic Republic; Mongolia; Philippines; Thailand; Timor Leste; Vanuatu; Vietnam	Brunei; China; Cook Islands; Federated States of Micronesia; Fiji; Kiribati; Democratic People's Republic of Korea; Republic of Korea; Malaysia; Marshall Islands; Myanmar; Nauru; Niue; Palau; Papua New Guinea; Samoa; Singapore; Solomon Islands; Tonga; Tuvalu
Middle East and North Africa	Egypt; Iraq; Jordan; Morocco; State of Palestine; Syria; Tunisia; Yemen	Algeria; Bahrain; Iran (Islamic Republic of); Kuwait; Lebanon; Libya; Oman; Qatar; Saudi Arabia; United Arab Emirates

Table 1: **Classification of countries by UNICEF regions.** The 96 countries with DHS and/or MICS data are listed in column "Country with data". Countries with no data are listed in column "Country w/o data". Countries are categorized by UNICEF regions.

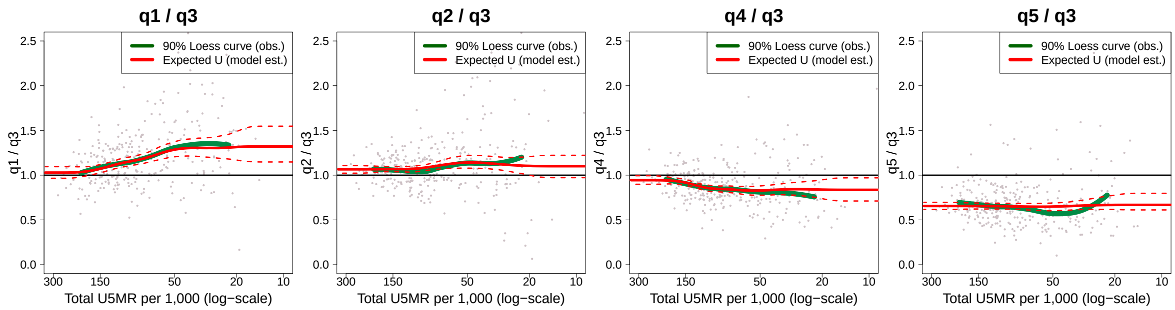

Figure 1: **Overall relation between ratio-S and total U5MR.** Ratio of the 1st, 2nd, 4th, 5th to the 3rd quintile-specific U5MR (ratio-S) is plotted against the total U5MR per 1000 live births on the log-scale in the descending order (since in general the U5MR decrease over time). The red solid lines are the median estimates of ratio-S, the red dashed lines are the 90% credible intervals. The green curves are Loess curves between the 5th and 95th percentiles of the total U5MR.

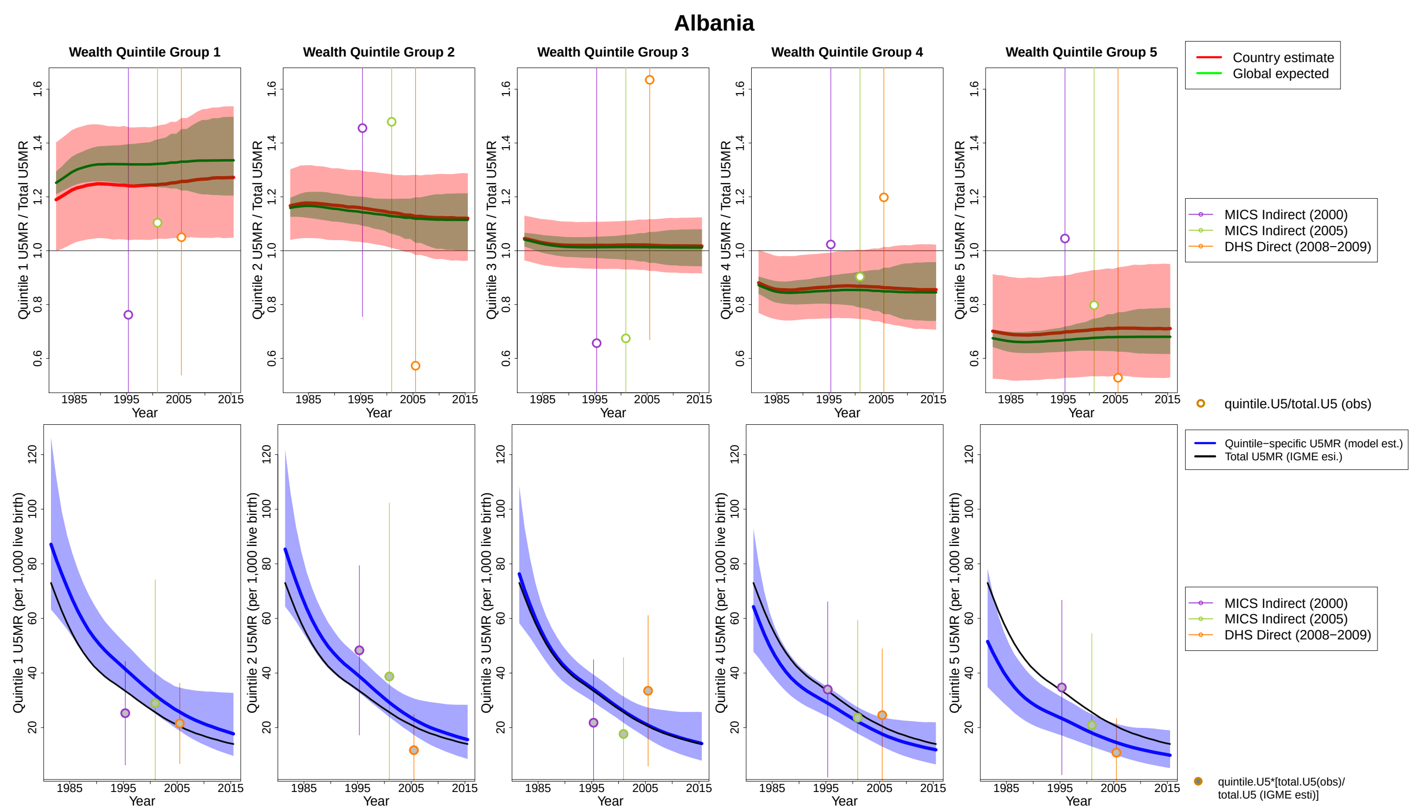

Figure 2: **Ratio of quintile-specific U5MR to total U5MR, and quintile-specific U5MR for Albania.** Top row: ratio of quintile-specific U5MR to total U5MR over time. Observations from different data series are differentiated by colored dots. Dots are observed quintile-specific U5MR over observed total U5MR from survey. The vertical line segments around the dots are $+/-2$ sampling errors. The red lines and shades are the country-specific estimates and their 90% credible bounds for ratio. The green lines and shades are the global expected level and their 90% credible bounds for ratio. Bottom row: quintile-specific U5MR. Dots are observed quintile-specific U5MR adjusted by the ratio of observed total U5MR to IGME median estimates of total U5MR. Blue lines and shades are the country-specific estimates and their 90% credible bounds for quintile-specific U5MR. The black curves are the total country-specific U5MR (IGME 2015 median estimates). The 5 columns refer to quintile groups 1 (poorest) to 5 (richest).

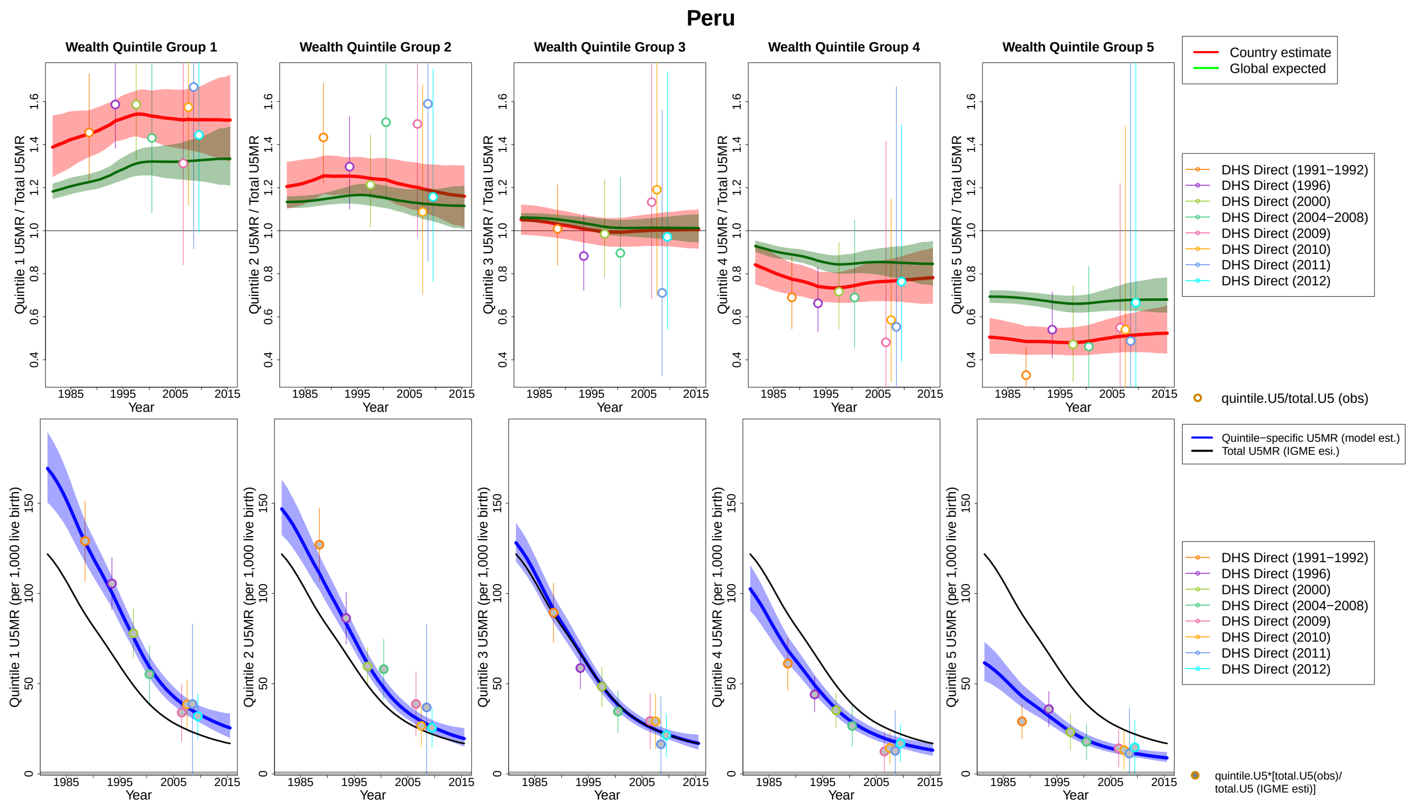

Figure 3: **Ratio of quintile-specific U5MR to total U5MR, and quintile-specific U5MR for Peru.** Top row: ratio of quintile-specific U5MR to total U5MR over time. Observations from different data series are differentiated by colored dots. Dots are observed quintile-specific U5MR over observed total U5MR from survey. The vertical line segments around the dots are $+/-2$ sampling errors. The red lines and shades are the country-specific estimates and their 90% credible bounds for ratio. The green lines and shades are the global expected level and their 90% credible bounds for ratio. Bottom row: quintile-specific U5MR. Dots are observed quintile-specific U5MR adjusted by the ratio of observed total U5MR to IGME median estimates of total U5MR. Blue lines and shades are the country-specific estimates and their 90% credible bounds for quintile-specific U5MR. The black curves are the total country-specific U5MR (IGME 2015 median estimates). The 5 columns refer to quintile groups 1 (poorest) to 5 (richest).

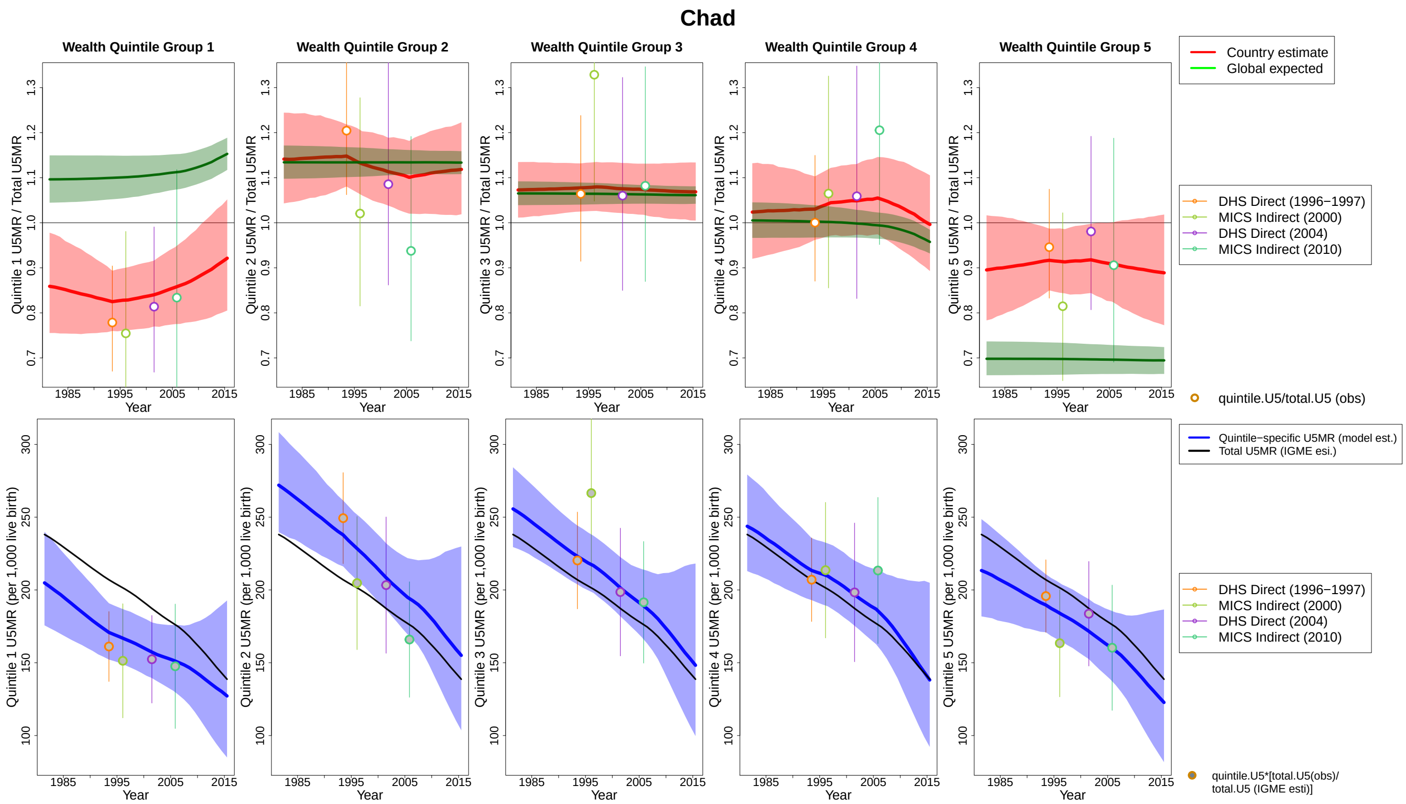

Figure 4: **Ratio of quintile-specific U5MR to total U5MR, and quintile-specific U5MR for Chad.** Top row: ratio of quintile-specific U5MR to total U5MR over time. Observations from different data series are differentiated by colored dots. Dots are observed quintile-specific U5MR over observed total U5MR from survey. The vertical line segments around the dots are $+/-2$ sampling errors. The red lines and shades are the country-specific estimates and their 90% credible bounds for ratio. The green lines and shades are the global expected level and their 90% credible bounds for ratio. Bottom row: quintile-specific U5MR. Dots are observed quintile-specific U5MR adjusted by the ratio of observed total U5MR to IGME median estimates of total U5MR. Blue lines and shades are the country-specific estimates and their 90% credible bounds for quintile-specific U5MR. The black curves are the total country-specific U5MR (IGME 2015 median estimates). The 5 columns refer to quintile groups 1 (poorest) to 5 (richest).

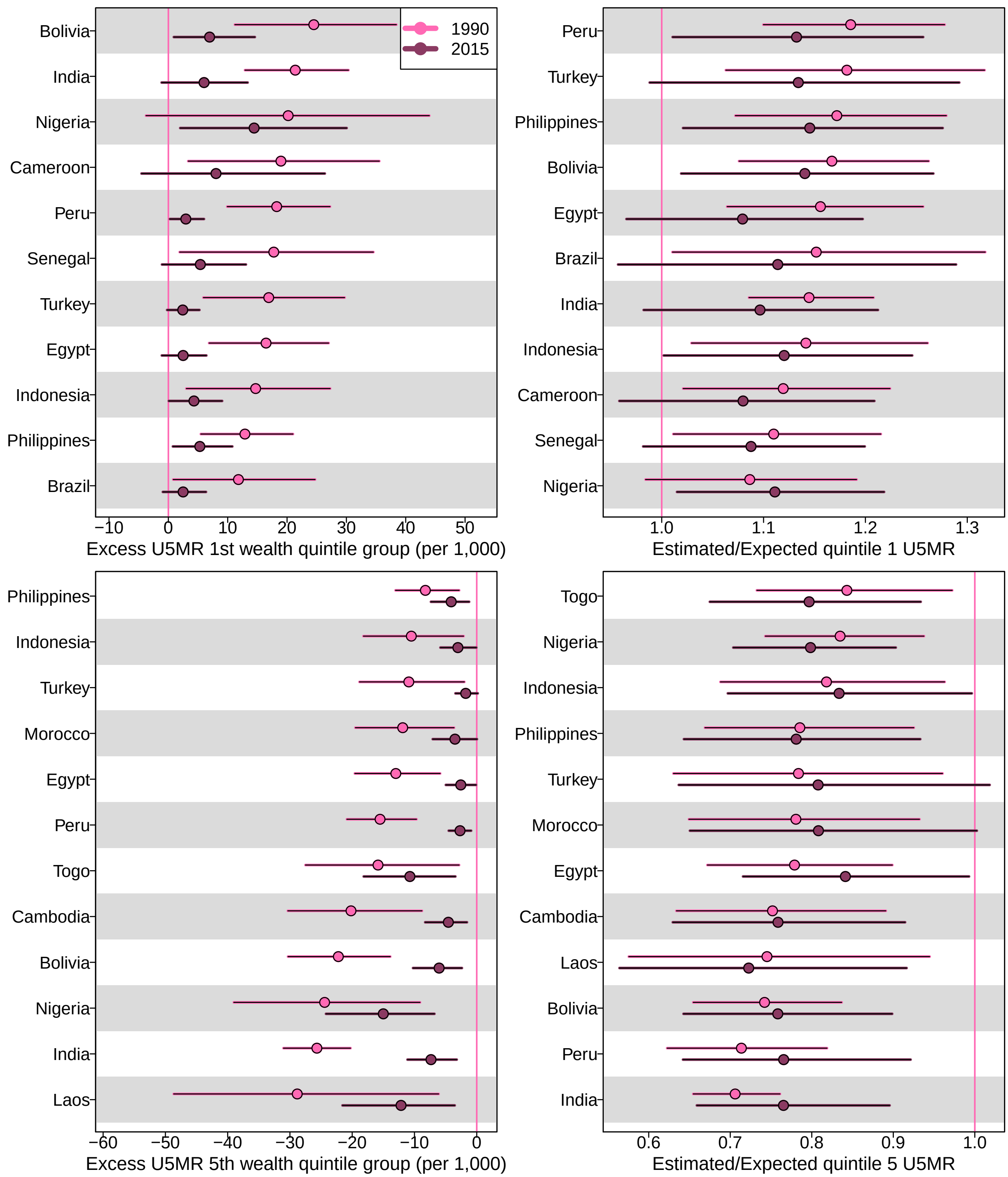

Figure 5: **Overview of countries with outlying quintile-specific U5MR in 1990 and/or 2015.** Left column: excess quintile-specific U5MR. Right column: the ratio of estimated to expected quintile-specific U5MR. Top row: countries with higher-than-expected quintile-specific U5MR in 1990 and/or 2015 for the 1st/poorest quintile. Bottom row: countries with lower-than-expected quintile-specific U5MR in 1990 and/or 2015 for the 5th/richest quintile. Countries are ordered by decreasing point estimates for the year 1990. Dots indicate median estimates, and horizontal lines refer to 90% uncertainty intervals. Light pink refers to results in 1990, dark pink refer to results in 2015.

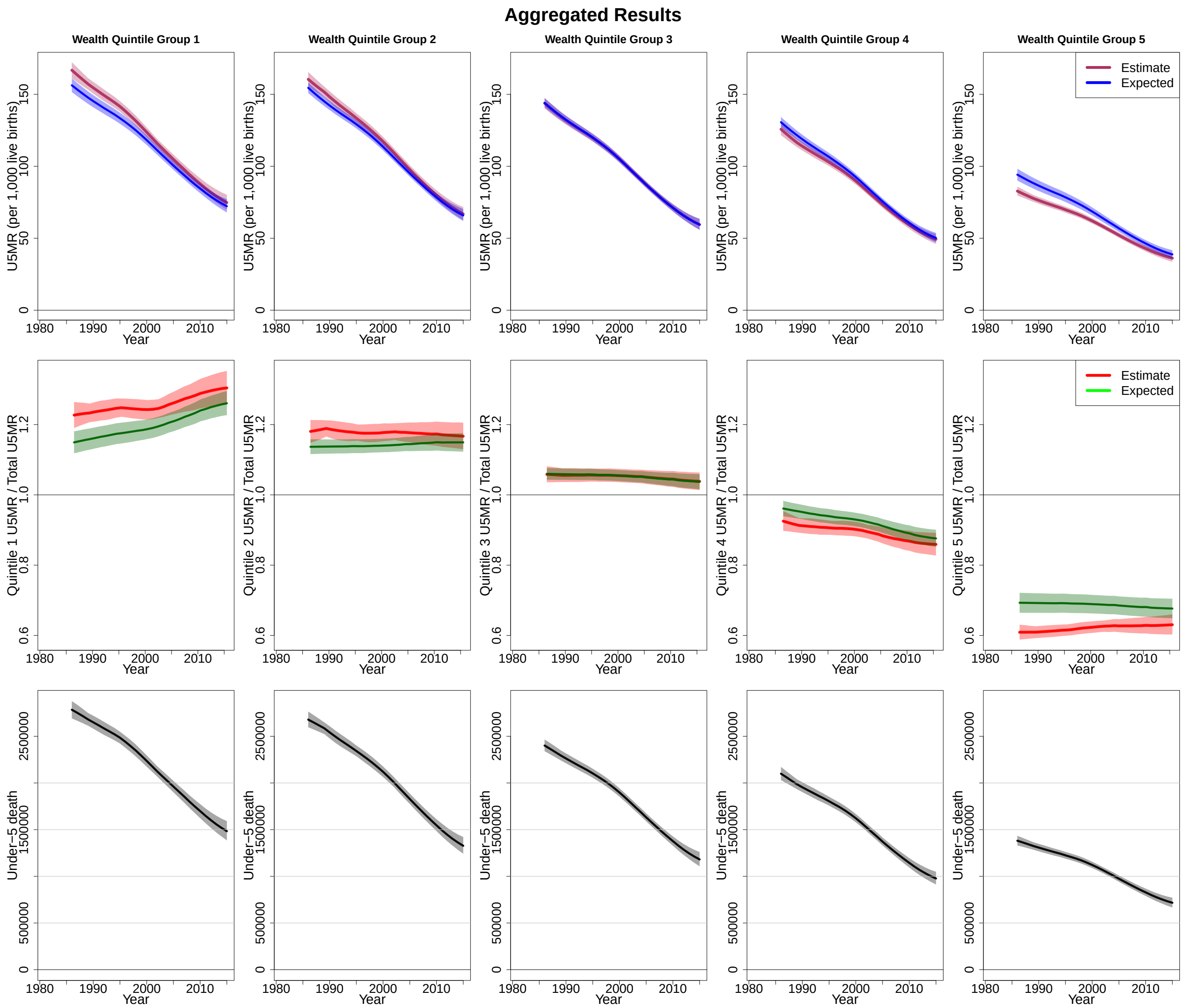

Figure 6: **Aggregated results by quintiles.** Curves are median estimates. Shades are 90% credible bounds. 1st row: Maroon refers to estimated quintile-specific U5MR. Blue refers to expected quintile-specific U5MR. 2nd row: Red refers to estimated ratio of quintile-specific U5MR to the total U5MR. Green refers to expected ratio of quintile-specific U5MR to the total U5MR. 3rd row: estimated number of quintile-specific under-5 deaths. The 5 columns refer to 5 quintiles respectively.

Appendix A Technical details for methods

We want to estimate the quintile-specific U5MR for country c year t, and they are constraint to the total U5MR in the relation as below:

$$
\begin{aligned}
Q_{c,t}^{total} &= D_{c,t}^{total}/B_{c,t}^{total}, \\
Q_{c,t}^{a} &= D_{c,t}^{a}/(B_{c,t}^{total}/5), \\
D_{c,t}^{total}/B_{c,t}^{total} &= \sum_{a=1}^{5} D_{c,t}^{a}/(B_{c,t}^{total}/5), \\
Q_{c,t}^{total} &= \sum_{a=1}^{5} Q_{c,t}^{a}/5.
\end{aligned}
$$

where $Q_{c,t}^{total}$ is the total U5MR, $Q_{c,t}^{a}$ is the quintile-specific U5MR from the a-th quintile group, $D_{c,t}^{total}$ is the total number of under-5 deaths, $B_{c,t}^{total}$ is the total number of live birth, and $D_{c,t}^{a}$ is the number of under-5 deaths from the a-th quintile group. All notations are referring to country c in year t. In order to incorporate this constraint, we estimate the ratio of $Q_{c,t}^{a}$ for $a = 1, 2, 4, 5$ to $Q_{c,t}^{a=3}$.

A.1 Wealth quintile U5MR model

For country c, year t, the ratio (denoting as $S_{u,c,t}$ for $u = 1, 2, 3, 4$) of $Q_{c,t}^{a}$ for $a = 1, 2, 4, 5$ to $Q_{c,t}^{a=3}$ is modeled as follows:

$$
\begin{aligned}
S_{u=1,c,t} &= Q_{c,t}^{1}/Q_{c,t}^{3}, \\
S_{u=2,c,t} &= Q_{c,t}^{2}/Q_{c,t}^{3}, \\
S_{u=3,c,t} &= Q_{c,t}^{4}/Q_{c,t}^{3}, \\
S_{u=4,c,t} &= Q_{c,t}^{5}/Q_{c,t}^{3}, \\
Q_{c,t}^{3} &= 5 \cdot Q_{c,t}^{total}/(S_{u=1,c,t} + S_{u=2,c,t} + S_{u=3,c,t} + S_{u=4,c,t} + 1),
\end{aligned}
$$

We use the 3rd wealth quintile group as the reference group because that is the group where we expect the proportion of deaths to be closest to 20%.

A.2 Model of ratio-S

Figure 7 shows the expected relationship between the $S_{u,c,t}$ to the total U5MR $Q_{c,t}^{total}$ for $u = 1, 2, 3, 4$ in the four plots respectively. Given the loess curve in the 1st plot goes up while the total U5MR decreases, it implies that when the total U5MR is decreasing, there are more children under-5 from the 1st (poorest) quintile die w.r.t. the 3rd (medium) quintile. The decreasing trend of the loess curve in the last plot indicates that while the total U5MR is recuding over time, the decrease of the U5MR from the 5-th quintile (richest) is even faster comparing to the 3rd quintile.
Hence, by using the information of the total U5MR $Q_{c,t}^{total}$, $S_{u,c,t}$ is modeled as the product of two components:

$$
S_{u,c,t} = U_{u,c,t} \cdot P_{u,c,t}, \text{ for } u = 1, 2, 3, 4.
$$

where $U_{u,c,t}$ is the expected relative difference.
For the j-th unique value of the total U5MR Q_{j}^{total}, the splines function for the u-th ratio group is:

$$
\begin{aligned}
f_u(Q_j^{(total)}) &= b_u + \boldsymbol{Z_{j,1:H}} \times \boldsymbol{\delta_u}, \text{ where } H = K - 1, \text{ and } K = 9, \\
\delta_{u,h} &\sim N(0, \sigma_{\delta_u}^2), \text{ for } u = 1, \ldots, 4, \text{ and } h = 1, \ldots, H.
\end{aligned}
$$

We used an AR(1) process to model the $P_{u,c,t}$ on the log-scale:

$$
\log(P_{u,c,t}) \sim N(\rho_u \cdot \log(P_{u,c,t-1}), \sigma_{\epsilon}^2).
$$

We assume that for each group of u, the rate of convergence back to 0 on log-scale is different for each group of u. Hence, we use ρ_u instead of a global parameter ρ.

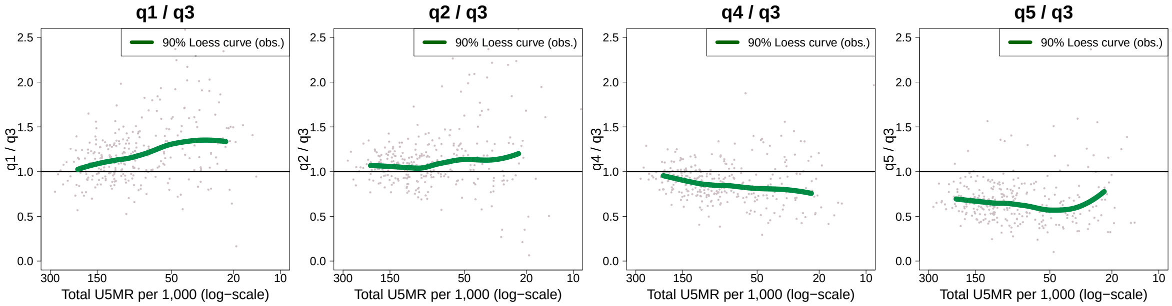

Figure 7: **90% Loess curves for expected ratio against total U5MR based on observations.** The grey dots are observed ratio of $Q^a_{c,t}$ for $a = 1, 2, 4, 5$ to $Q^{a=3}_{c,t}$ respectively for the four plots. The green curves are Loess curves between the 5th and 90th percentiles of the total U5MR.

Specification of overall relation between $U_{u,c,t}$ and total U5MR levels We used flexible penalized B-spline regression models Eilers and Marx [1996, 2010] to estimate the overall relation between total U5MR and expected relative difference, denoted by function $f^{(u)}(\cdot)$, for $u = 1, 2, 3, 4$. The function $f^{(u)}(q)$ for some value q for total U5MR was specified as follows:

$$\log(f^{(u)}(q)) = \sum_{k=1}^{K} B_k(q)\alpha_k^{(u)}, \quad (1)$$

where $B_k(q)$ refers to the k-th B-spline evaluated at q and $\alpha_k^{(u)}$ to the k-th spline coefficients. The expected relative difference for country c, year t with the total U5MR for that country-year $Q^{total}_{c,t}$ is given by $U_{u,c,t} = f^{(u)}(Q^{total}_{c,t})$ for $u = 1, 2, 3, 4$ (where $Q^{total}_{c,t}$'s are rounded to three decimal places to reduce the number of splines evaluations).

The B-splines used in the regression models are illustrated in Figure 7. We used symmetric third-order polynomials, equally spaced on the log-transformed total U5MR scale (knots are set to be 0.3 apart). The resulting splines add up to unity at any level of total U5MR. To avoid extreme extrapolations, splines are combined for total U5MR less than 20 per 1,000, and for total U5MR greater than the 95-th percentile of $Q^{total}_{c,t}$.

When fitting the splines model to observations, first-order differences in adjacent splines coefficients were penalized to guarantee smoothness of the global relation between total mortality and expected relative difference. The remainder of this subsection discusses the implementation details.

The splines regression model is specified as follows:

$$\log(f^{(u)}(\tilde{\boldsymbol{q}})) = \tilde{\boldsymbol{B}}\boldsymbol{\alpha}^{(u)}, \quad (2)$$

where $\tilde{\boldsymbol{q}}$ represents the vector of unique values $Q^{total}_{c,t}$ (rounded to three digits), $\tilde{\boldsymbol{B}} = \boldsymbol{B}(\tilde{\boldsymbol{q}})$ the matrix of splines evaluated at each entry of $\tilde{\boldsymbol{q}}$, and $\boldsymbol{\alpha}^{(u)}$ the vector of splines coefficients of length K. The splines equation can be written as follows Eilers2010,Currie2002,Eilers1999:

$$\begin{aligned} \tilde{\boldsymbol{B}}\boldsymbol{\alpha}^{(u)} &= b^{(u)} + \boldsymbol{Z}\boldsymbol{e}^{(u)}, \quad (3)\\ \boldsymbol{G} &= (\boldsymbol{1}_K\ \boldsymbol{g}_K), \text{ where } \boldsymbol{g}_K = (1 - K/2, \ldots, K - K/2)', \\ \boldsymbol{Z} &= \tilde{\boldsymbol{B}}\boldsymbol{D}'_K(\boldsymbol{D}_K\boldsymbol{D}'_K)^{-1}, \end{aligned}$$

where the elements of difference matrix $\boldsymbol{D}_K$ are given by $D_{K,i,i} = -1$, $D_{K,i,i+1} = 1$ and $D_{K,i,j} = 0$ otherwise. The first part in Eq.(3), $b^{(u)}$, describes the constant level in the expected relative difference, and the second part $\boldsymbol{Z}\boldsymbol{e}^{(u)}$ describes the fluctuations around the linear trend. $\boldsymbol{e}^{(u)} = (e_1^{(u)}, \ldots, e_J^{(u)})'$, with $J = K - 1$. First-order differences are penalized by imposing

$$e_j^{(u)} \sim N(0, \sigma_u^2), \text{ for } j = 1, \ldots, J, \text{ and } u = 1, \ldots, 4,$$

where variance σ_u^2 determines the extent of smoothing. Spread out prior distributions were used for the splines model parameters.

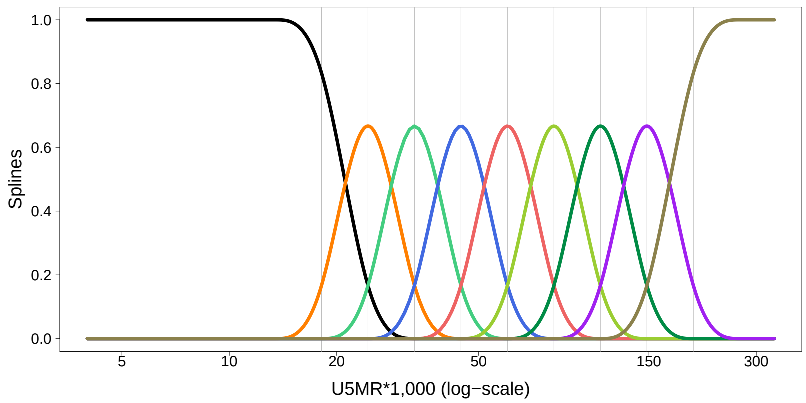

Figure 8: **Illustration plot for splines.** The splines over the U5MR per 1000 live births on log-scale. The grey vertical lines indicate that knots.

Notations Table 2 summarizes the notation and indices for this project:

Symbol	Description
t	Indicator for year, $t = 1981, \ldots, 2015$.
c	Indicator for country, $c = 1, \ldots, 96$.
a	Indicator for wealth quintile groups, $a = 1, \ldots, 5$. $a = 1$ refers to the poorest wealth quintile group, and $a = 5$ refers to the richest wealth quintile group.
u	Indicator for ratio between quintile groups, $u = 1, \ldots, 4$. $u = 1$ refers to the ratio between the 1st wealth quintile group to the 3rd wealth quintile group; $u = 2$ refers to the ratio of the 2nd to the 3rd quintile group; $u = 3$ refers to the ratio of the 4th to the 3rd quintile group; $u = 4$ refers to the ratio of the 5th to the 3rd quintile group.
i	Indicator for the i-th observation within a certain quintile group.
$r_{a,i}$	The i-th observed ratio of the a-th quintile-specific U5MR to the total U5MR.
$\gamma_{a,i}$	The i-th sampling error for $r_{a,i}$.
$q_{a,i}$	The i-th observed quintile-specific U5MR from the a-th quintile group.
q_i	The i-th observed total U5MR.
$R^a_{c,t}$	The true ratio of the ratio between the a-th quintile-specific U5MR to the total U5MR for country c in year t.
$S_{u,c,t}$	The ratio between the 1st, 2nd, 4th, and 5th quintile-specific U5MR to the 3rd quintile-specific U5MR, for $u = 1, 2, 3, 4$ respectively.
ρ_u	Autoregressive parameter for AR(1) time series model for $\log(P_{u,c,t})$, for $u = 1, \ldots, 4$.
σ^2_ϵ	Variance of distortion terms in AR(1) time series model for $\log(P_{u,c,t})$.

Table 2: **Notation summary.**

In summary:

$$\begin{aligned}
S_{u=1,c,t} &= Q^1_{c,t}/Q^3_{c,t} = R^1_{c,t}/R^3_{c,t}, \\
S_{u=2,c,t} &= Q^2_{c,t}/Q^3_{c,t} = R^2_{c,t}/R^3_{c,t}, \\
S_{u=3,c,t} &= Q^4_{c,t}/Q^3_{c,t} = R^4_{c,t}/R^3_{c,t}, \\
S_{u=4,c,t} &= Q^5_{c,t}/Q^3_{c,t} = R^5_{c,t}/R^3_{c,t}, \\
Q^3_{c,t} &= 5 \cdot Q^{total}_{c,t}/(S_{u=1,c,t} + S_{u=2,c,t} + S_{u=3,c,t} + S_{u=4,c,t} + 1), \\
S_{u,c,t} &= U_{u,c,t} \cdot P_{u,c,t}, \text{ for } u = 1, 2, 3, 4, \\
\log(U_{u,c,t}) &= f_u(Q^{(total)}_j), \\
\log(P_{u,c,t}) &\sim N(\rho_u \cdot \log(P_{u,c,t-1}), \sigma^2_\epsilon).
\end{aligned}$$

Non-informative priors are assigned to hyper-parameters.

A.3 Data model

Instead of using the observed quintile-specific U5MR in the data model, we used the observed ratio of quintile-specific to the total U5MR. In this way, we are able to diminish the survey biases by taking into account both the observed quintile-specific U5MR and the observed total U5MR based on survey data. For the i-th observed ratio of the a-th quintile-specific U5MR to the total U5MR $r_{a,i}$, which is from country $c[i]$, in year $t[i]$, the data model is:

$$\log(r_{a,i}) \sim N(\log(R^a_{c[i],t[i]}), \gamma^2_{a,i}),$$

where $r_{a,i}$ is the ratio of the i-th observed U5MR from the a-th quintile $q_{a,i}$ to the i-th observed total U5MR q_i. $\gamma^2_{a,i}$ is the sampling variance for the i-th observation for the a-th quintile group. The $R^a_{c,t}$ can be recovered as below:

$$\begin{aligned} R^3_{c,t} &= 5/(S_{u=1,c,t} + S_{u=2,c,t} + S_{u=3,c,t} + S_{u=4,c,t} + 1), \\ R^1_{c,t} &= S_{u=1,c,t} \cdot R^3_{c,t}, \\ R^2_{c,t} &= S_{u=2,c,t} \cdot R^3_{c,t}, \\ R^4_{c,t} &= S_{u=3,c,t} \cdot R^3_{c,t}, \\ R^5_{c,t} &= S_{u=4,c,t} \cdot R^3_{c,t}. \end{aligned}$$

Appendix B Model validation

B.1 Leaving out data based on survey year

The model performance was assessed by out-of-sample validation. Instead of leaving out observations randomly, we left out observations that are obtained after a certain year so that around 20% observations were left out (Alkema et al. [2012]). Based upon the current database, all data that were collected in the year 2011 onwards were left out. We fitted the model to the training data set, and got point estimates and uncertainty intervals that would have been constructed based on available data set in the year 2011. We computed mean and median errors, and coverage based on left-out observations and based on estimates obtained from the full data set and estimated obtained from training data set.

For the left-out observations, errors are defined as $e_{a,i} = r_{a,i} - \widetilde{r}_{a,i}$, where $\widetilde{r}_{a,i}$ refers to the posterior median of the predictive distribution based on training data set for the left-out observation $r_{a,i}$. Coverage is given by $1/n \cdot \sum 1[r_{a,i} \geq l_{a,i}] \cdot 1[r_{a,i} \leq u_{a,i}]$, where n refers to the number of left-out observations, and $l_{a,i}$ and $u_{a,i}$ correspond to the lower and upper bounds of the 90% or 80% prediction interval for the left-out observation $r_{a,i}$. The validation measures were calculated for 100,000 sets of left-out observations, where each set consisted only one randomly selected left-out observation from each country. The reported validation results were based on the mean of the outcomes from the 100,000 sets of left-out observations.

For the point estimates based on full data set and training data set, errors are defined as $e^a_{c,t} = \widehat{R}^a_{c,t} - \widetilde{R}^a_{c,t}$, where $\widehat{R}^a_{c,t}$ is the posterior median for country c in year t for the a-th quintile based on full data set, and $\widetilde{R}^a_{c,t}$ is the posterior median for the same country-year and quintile based on the training data set. Coverage was computed in a similar manner as for the left-out observations, based on the lower and upper bounds of the 95% uncertainty interval of $\widetilde{R}^a_{c,t}$ from the training data set.

B.2 Leaving out data randomly

We also checked the model performance using the traditional way of leaving out data, i.e. leaving out 20% data randomly. We then compute the median and mean errors and coverages using the same method as described in Section B.1. We repeat the process for 30 times and report the average median errors, median absolute errors, and coverage of 80% and 90% PIs for all the quintiles.

Appendix C Validation results

C.1 Leaving out data based on survey year

We left out all observations that were collected in or after the year 2011: 264 observations were left out, corresponding to 17.9% of all observations. Table 3 summarizes the results related to the left-out observations for the validation exercise based on 90% and 80% prediction intervals (PIs). Median errors were very close to zero for left-out observations in all the wealth quintile groups. Coverage of 90% PIs were higher than expected: 100%, 96.1%, 100%, 98.2%, 92.6% for the 5 wealth quintile groups respectively. Coverage of 80% PIs were higher than expected at 93.6%, 95%, 97.3%, 84.7%, and 90.5% for each wealth quintile group.

Table 4 shows the results for the comparison between estimates obtained based on the full data set, and estimates based on the training set. Median errors and the median absolute errors were close to zero. The proportion of updated estimates that fell outside the uncertainty intervals constructed based on the training set was well below 5%.

Wealth Quintile Group	1st	2nd	3rd	4th	5th
Median of Error	-0.06	-0.08	0.03	0.04	0.05
Median of absolute Error	0.13	0.11	0.09	0.11	0.09
Left-out observations fall below 90% PI (%)	0.0	3.9	0.0	0.0	1.1
Left-out observations fall above 90% PI (%)	0.0	0.0	0.0	1.8	6.3
Expected proportions (%)	**5**	**5**	**5**	**5**	**5**
Left-out observations fall below 80% PI (%)	3.9	5.0	1.6	7.2	3.2
Left-out observations fall above 80% PI (%)	2.5	0.0	1.1	8.1	6.3
Expected proportions (%)	**10**	**10**	**10**	**10**	**10**

Table 3: **Validation results for left-out observations by wealth quintile groups.** Errors are defined as the difference between a left-out observation and the posterior median of its predictive distribution.

Wealth Quintile Group	1st		2nd		3rd		4th		5th	
Year	**2000**	**2005**	**2000**	**2005**	**2000**	**2005**	**2000**	**2005**	**2000**	**2005**
Median of Error	-0.00	-0.00	-0.01	-0.01	-0.00	-0.00	0.01	0.01	0.01	0.01
Median of absolute Error	0.01	0.01	0.01	0.02	0.00	0.00	0.01	0.01	0.01	0.01
Below 90% CI of validation run (%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0
Above 90% CI of validation run (%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0
Expected proportions (%)	**≤5**	**≤5**	**≤5**	**≤5**	**≤5**	**≤5**	**≤5**	**≤5**	**≤5**	**≤5**

Table 4: **Summary of differences in ratio estimates in observation years 2000 and 2005 based on training set and full data set.** Errors are defined as the differences between estimates based on the full dataset and the training set. The proportions refer to the proportions (%) of countries in which the median ratio estimates based on the full data set fall below or above their corresponding 90% uncertainty intervals based on the training dataset. The results are broken down by wealth quintile groups and observation years.

C.2 Leaving out data randomly

We also did validation exercises by randomly leaving out 20% of data. We repeat this validation for 30 times. Table 5 shows the average of these 30 validation exercises. Median of error and median of absolute error are close to zero for all the wealth quintile groups. The proportions of left-out data falling outside the 90% and 80% PIs are lower than expected for all the quintile groups. This means that the PIs the model are more conservative than expected. No systematic bias are observed for PIs.

Wealth Quintile Group	1st	2nd	3rd	4th	5th
Median of Error	-0.01	-0.01	-0.01	0.01	-0.00
Median of absolute Error	0.11	0.09	0.08	0.09	0.09
Left-out observations fall below 90% PI (%)	2.6	1.1	1.0	3.0	3.3
Left-out observations fall above 90% PI (%)	2.9	0.7	2.5	1.2	2.6
Expected proportions (%)	**5**	**5**	**5**	**5**	**5**
Left-out observations fall below 80% PI (%)	5.9	4.8	4.2	8.8	7.1
Left-out observations fall above 80% PI (%)	4.5	3.9	4.9	5.0	5.0
Expected proportions (%)	**10**	**10**	**10**	**10**	**10**

Table 5: **Average of 30 validation results by randomly leaving out 20% data.** Errors are defined as the difference between a left-out observation and the posterior median of its predictive distribution.