ADDENDUM: Robust Inference in Generalized Linear Models. Poisson Model.

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1 Monte Carlo Experiments

We run three different Monte Carlo experiments. The first experiment is based on the Poisson family, the second one on the Gamma family and the last one on the Inverse Gaussian family. Details and results are reported in the next subsections.

1.1 Poisson Model.

The Monte Carlo experiment has the following factors:

- Family: $y \sim \text{Poisson}(\lambda(x^t\beta))$;
- Link: log link;
- Predictors: $X_i \sim \text{Discrete Uniform on } (-3, -2, -1, 0, 1, 2, 3), i = 1, 2, 3, 4,$ independent each others;

- Linear predictor vector parameter: $\beta = (-0.25, -0.25, 0, 0)$ plus intercept equal to 2;
- Dispersion parameter known and fixed to 1;
- Sample size: 70, 140, 280, 560;
- Level of contamination: 0%, 5%, 10%;
- Type of contamination: linear predictor parameter (10, 10, 10, 10) and intercept equal to 0. The contaminated mean value is limited to 200;
- Monte Carlo replications: 5000;
- Methods:
 - Maximum Likelihood (MLE) as implemented in function `glm` of **R**,
 - Robust approach by Cantoni and Ronchetti (ROB) as implemented in function `glmrob` in the package `robustbase` of **R**,
 - Weighted Likelihood, as implemented in function `wle.glm` in the package `wle` of **R**, for it we consider the following factors
 - * Residual Adjustment Function: Generalized Kullback–Leibler with $\tau = 0.05$;
 - * Classical weights (WLE) and weights based on the Anscombe residuals (WLE Asy).

In figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 we report a comparison between a non parametric density estimates of the Anscombe residuals, evaluated at the true parameters, for a given combination of sample size and level of contamination and the standard normal density. Crosses are the position of the observations from the model, while circles are the position of the outliers.

Figure 1: Poisson, Anscombe Residuals, $\varepsilon = 0\%$, $n = 70$

Figure 2: Poisson, Anscombe Residuals, $\varepsilon = 5\%$, $n = 70$

Figure 3: Poisson, Anscombe Residuals, $\varepsilon = 10\%$, $n = 70$

Figure 4: Poisson, Anscombe Residuals, $\varepsilon = 0\%$, $n = 140$

Figure 5: Poisson, Anscombe Residuals, $\varepsilon = 5\%$, $n = 140$

Figure 6: Poisson, Anscombe Residuals, $\varepsilon = 10\%$, $n = 140$

Figure 7: Poisson, Anscombe Residuals, $\varepsilon = 0\%$, $n = 280$

Figure 8: Poisson, Anscombe Residuals, $\varepsilon = 5\%$, $n = 280$

Figure 9: Poisson, Anscombe Residuals, $\varepsilon = 10\%$, $n = 280$

Figure 10: Poisson, Anscombe Residuals, $\varepsilon = 0\%$, $n = 560$

Figure 11: Poisson, Anscombe Residuals, $\varepsilon = 5\%$, $n = 560$

Figure 12: Poisson, Anscombe Residuals, $\varepsilon = 10\%$, $n = 560$

The results are summarized as follows:

- In Tables (1,2,3,4) we report the actual level of significance for different nominal level (0.01, 0.025, 0.05, 0.10), sample size and contamination level for the following hypothesis test

$$\begin{cases} H_0 : \beta_3 = 0 \\ H_1 : \beta_3 \neq 0 \end{cases}$$

- Figures (13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24) report the bias of the parameters (Intercept and β s).
- Figures (25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36) report the values of the test statistic for each parameter in the hypothesis system $H_0 : \beta_i = 0$ against $H_1 : \beta_i \neq 0$ for (i=0,1,2,3,4). Asymptotic bands $(-1.96, 1.96)$ are reported for the critical region at $\alpha = 0.05$.
- Figures (37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48) report the distribution of the p-values for the following Analysis of Variance:

$$\begin{cases} H_0 : \beta_3 = \beta_4 = 0 \\ H_1 : H_0^c \end{cases}$$

Since the null hypothesis is true we expect a uniform distribution of the p-values.

Size	Lev.	MLE	WLE	WLE Asy	ROB
70	0%	0.010	0.006	0.010	0.009
	5%	0.747	0.009	0.012	0.025
	10%	0.846	0.016	0.026	0.057
140	0%	0.010	0.007	0.011	0.011
	5%	0.811	0.008	0.011	0.025
	10%	0.895	0.008	0.013	0.071
280	0%	0.011	0.007	0.011	0.011
	5%	0.867	0.007	0.014	0.035
	10%	0.947	0.010	0.015	0.104
560	0%	0.011	0.006	0.010	0.010
	5%	0.934	0.011	0.015	0.052
	10%	0.989	0.011	0.018	0.190

Table 1: Poisson model. Actual level of significance for the test on β_3 . Nominal level: $\alpha = 0.01$.

Size	Lev.	MLE	WLE	WLE Asy	ROB
70	0%	0.025	0.017	0.024	0.024
	5%	0.775	0.020	0.025	0.051
	10%	0.865	0.032	0.045	0.099
140	0%	0.029	0.021	0.028	0.027
	5%	0.836	0.020	0.027	0.051
	10%	0.907	0.023	0.032	0.119
280	0%	0.025	0.018	0.026	0.026
	5%	0.884	0.023	0.032	0.072
	10%	0.954	0.024	0.032	0.164
560	0%	0.027	0.022	0.027	0.027
	5%	0.943	0.023	0.034	0.090
	10%	0.991	0.029	0.041	0.277

Table 2: Poisson model. Actual level of significance for the test on β_3 . Nominal level: $\alpha = 0.025$.

Size	Lev.	MLE	WLE	WLE Asy	ROB
70	0%	0.051	0.042	0.050	0.050
	5%	0.796	0.039	0.052	0.086
	10%	0.880	0.056	0.073	0.154
140	0%	0.052	0.040	0.052	0.053
	5%	0.856	0.040	0.051	0.085
	10%	0.920	0.048	0.055	0.176
280	0%	0.051	0.038	0.049	0.048
	5%	0.901	0.047	0.060	0.112
	10%	0.959	0.045	0.061	0.235
560	0%	0.052	0.044	0.051	0.053
	5%	0.948	0.048	0.065	0.137
	10%	0.992	0.054	0.072	0.364

Table 3: Poisson model. Actual level of significance for the test on β_3 . Nominal level: $\alpha = 0.05$.

Size	Lev.	MLE	WLE	WLE Asy	ROB
70	0%	0.101	0.088	0.101	0.100
	5%	0.824	0.088	0.105	0.146
	10%	0.899	0.104	0.124	0.229
140	0%	0.103	0.085	0.107	0.103
	5%	0.878	0.085	0.104	0.150
	10%	0.934	0.094	0.112	0.265
280	0%	0.096	0.085	0.096	0.098
	5%	0.920	0.095	0.115	0.182
	10%	0.965	0.095	0.117	0.331
560	0%	0.094	0.086	0.097	0.096
	5%	0.956	0.094	0.110	0.222
	10%	0.993	0.107	0.127	0.462

Table 4: Poisson model. Actual level of significance for the test on β_3 . Nominal level: $\alpha = 0.1$.

Figure 13: Poisson, bias, $\varepsilon = 0\%$, $n = 70$

Figure 14: Poisson, bias, $\varepsilon = 5\%$, $n = 70$

Figure 15: Poisson, bias, $\varepsilon = 10\%$, $n = 70$

Figure 16: Poisson, bias, $\varepsilon = 0\%$, $n = 140$

Figure 17: Poisson, bias, $\varepsilon = 5\%$, $n = 140$

Figure 18: Poisson, bias, $\varepsilon = 10\%$, $n = 140$

Figure 19: Poisson, bias, $\varepsilon = 0\%$, $n = 280$

Figure 20: Poisson, bias, $\varepsilon = 5\%$, $n = 280$

Figure 21: Poisson, bias, $\varepsilon = 10\%$, $n = 280$

Figure 22: Poisson, bias, $\varepsilon = 0\%$, $n = 560$

Figure 23: Poisson, bias, $\varepsilon = 5\%$, $n = 560$

Figure 24: Poisson, bias, $\varepsilon = 10\%$, $n = 560$

Figure 25: Poisson, $\varepsilon = 0\%$, $n = 70$

Figure 26: Poisson test, $\varepsilon = 5\%$, $n = 70$

Figure 27: Poisson, test, $\varepsilon = 10\%$, $n = 70$

Figure 28: Poisson, test, $\varepsilon = 0\%$, $n = 140$

Figure 29: Poisson, test, $\varepsilon = 5\%$, $n = 140$

Figure 30: Poisson, test, $\varepsilon = 10\%$, $n = 140$

Figure 31: Poisson, test, $\varepsilon = 0\%$, $n = 280$

Figure 32: Poisson test, $\varepsilon = 5\%$, $n = 280$

Figure 33: Poisson, test, $\varepsilon = 10\%$, $n = 280$

Figure 34: Poisson test, $\varepsilon = 0\%$, $n = 560$

Figure 35: Poisson, test, $\varepsilon = 5\%$, $n = 560$

Figure 36: Poisson, test, $\varepsilon = 10\%$, $n = 560$

Figure 37: Poisson, anova, $\varepsilon = 0\%$, $n = 70$

Figure 38: Poisson, anova, $\varepsilon = 5\%$, $n = 70$

Figure 39: Poisson, anova, $\varepsilon = 10\%$, $n = 70$

Figure 40: Poisson, anova, $\varepsilon = 0\%$, $n = 140$

Figure 41: Poisson, anova, $\varepsilon = 5\%$, $n = 140$

Figure 42: Poisson, anova, $\varepsilon = 10\%$, $n = 140$

Figure 43: Poisson, anova, $\varepsilon = 0\%$, $n = 280$

Figure 44: Poisson, anova, $\varepsilon = 5\%$, $n = 280$

Figure 45: Poisson, anova, $\varepsilon = 10\%$, $n = 280$

Figure 46: Poisson, anova, $\varepsilon = 0\%$, $n = 560$

Figure 47: Poisson, anova, $\varepsilon = 5\%$, $n = 560$

Figure 48: Poisson, anova, $\varepsilon = 10\%$, $n = 560$