Effective dynamic properties of random complex media

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Introduction

When an acoustic wave, having wavelength larger than the dimension of inhomogeneities (e.g. particles, cracks, voids), propagates through a complex medium, the wave is unaffected by the substructure (e.g. the arrangement of the scatterers) of the medium. Instead the wave "sees" the composite as an entirely different *homogeneous* medium with well-defined "effective" properties, different from its constituent materials' properties.

Modelling of wave propagation in a random medium is almost impossible to solve analytically, being a multi-body interaction problem with continuity requirements at each particle surface and infinite multiple scattering events. It is, however, often possible to solve and simplify the problem by replacing the complex medium with a homogeneous "effective medium" which can be characterized by its effective properties. These properties are dynamic, relating to a harmonic frequency-dependent process, and may be different from the static effective properties. Here, we consider, in particular, the effects of a *viscous* suspending fluid on the dynamic effective properties of solid particle suspensions.

Model

In recent years it has been theoretically and experimentally demonstrated that, as particle concentration in a suspension increases, the viscous nature of the host fluid becomes more and more significant and thereby needs to be taken into consideration when calculating effective properties for acoustic propagation [1]. One approach to the problem is to incorporate wave conversion phenomena, primarily between compressional and shear wave modes, into the models for effective properties.

We employ an effective medium model to determine the effective dynamic properties of a random suspension of spherical solid particles in a viscous fluid (Figure 1). Analysis proceeds by use of the *self-consistency* condition [2] (i.e. no scattering from the core-shell system), using Rayleigh partial wave analysis for monopole and dipole terms, and using long compressional wavelength (small $k_c a$). Here, we include shear wave motion in the viscous fluid, differentiating from the simpler inviscid case.

Figure 1(a): an incident plane compressional wave with wavenumber k_c is propagating though an viscous suspension. The wave gets scattered due the particles and hence experiences attenuation.

Results

The effective bulk modulus B_e obtained from the monopole mode is

$$\frac{B_c - B}{3B_c - 4\mu} = \frac{i}{\left(k_c b\right)^3} T_0^{cc}$$

where B and μ are bulk and shear moduli of the host fluid respectively.

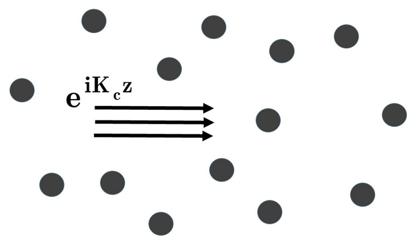
The static mass density of a composite is the volume average of its components' densities.

$$\rho_e = \left(1-\phi\right)\rho + \phi\,\rho'$$

If, however, the wave mode conversion is taken into account, the effective mass density becomes dynamic (frequency-dependent) at higher concentration and/or frequency which can be calculated from the dipole mode in the following form :

$$\rho_e = \left(1-\phi\right)\rho + \phi\rho' + i\omega\chi\left(\phi\right) + \omega^2\psi\left(\phi\right)$$

where $i=\sqrt{-1}$, ϕ and ω are, imaginary unit, volume concentration and angular frequency, respectively. $\chi\left(\phi\right)$ and $\psi\left(\phi\right)$ are analytical functions of concentration.



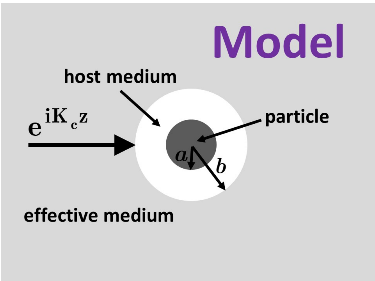


Figure 1(b): the effective medium model treats this as equivalent to a particle ("core") embedded in the viscous host liquid ("shell"), surrounded by the effective medium. Scattering from the core-shell particle should vanish (self-consistency condition). Concentration in the shell is equivalent to that in the suspension as a whole. Here ρ , μ and λ are density and two Lamé parameters of the host fluid respectively.

Numerical Calculation

Figure 2(a) shows the variation of real part of the normalized effective density with the real part of the normalized shear wavenumber.

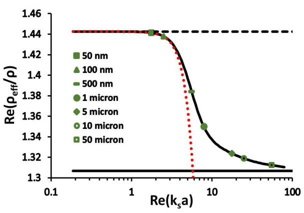
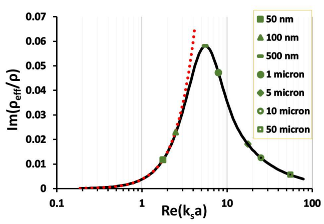


Figure 2(b) shows the variation of imaginary part of the normalized effective density with the real part of the normalized shear wavenumber. The imaginary part is related to the translational viscosity of the suspension. When the thickness of the viscous boundary layer becomes comparable to the dimension of the core-shell particle, the system undergoes maximum attenuation due to viscosity, which being reflected in the pronounced peak.



The real part of the effective density is related to the effective inertia of the suspension. At very low $k_s a$ values, the steady Stokesian drag dominates over the inertial force, the effective density being the static volume-averaged density. As the frequency increases, the viscous boundary layer starts to become thinner. At sufficiently elevated frequencies, the viscous regime gives way to the inertial regime: the fluid behaves an inviscid one, and the effective mass density is given by the Ament's inviscid formula.

Conclusion

- A self-consistent effective medium method is used to calculate the effective dynamic properties of a random composite.
- The model is applied to solid particles in a viscous liquid
- The model takes into account the wave *mode conversion* by including shear waves in the viscous liquid.
- Effective bulk modulus and effective dynamic density have been derived analytically.
- Bulk modulus is equivalent to static case, effective density is frequency-dependent and reaches the static or inviscid limit at different conditions.

References

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2. Mei J, Liu Z, Wen W, Sheng P. Effective dynamic mass density of composites. Physical Review B. 2007 Oct 16;76(13):134205.