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Ways of Being Strands: Exploration of Textile Craft Knots by Hand and Mathematics

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Abstract: The paper presents an ongoing collaborative project between a textile practitioner-researcher and a textile practitioner-mathematician that investigates the relationship between mathematical knot theory and knotted textiles. It examines how multiple monochrome textile knots may be characterised using mathematical analysis and how this in turn may facilitate the conceptualisation, design and production of knotted textiles.

Mathematical investigation of Nimkulrat's knotted textile practice through the use of mathematical knot diagrams by Matthews revealed knot properties such as strand start/end positions and strand active/passive roles which were indiscernible from the work alone. This approach led to a way of visualising knot designs using more than one colour prior to making. Further iterative design experimentation and material properties led to the creation of a new striped pattern and a three-dimensional artefact which will be exhibited.

The result of this current research phase illuminates the role of mathematics in making the knotting process explicit. It demonstrates the influence of mathematical analysis on craft practice and the significance of cross-disciplinary collaboration on the development of knotted pattern design.

Keywords: Craft; Knot; Knot Diagram; Knot Theory; Mathematics; Textiles.

Introduction: Mathematics, Craft and Textiles

The intersection of mathematics and craft[1] can be seen throughout the last decades. Mathematicians have noticed the potential for the expression of mathematical concepts within craft media. Textile craft is used here as a means of making mathematics explicit. It transforms a mathematical idea into a material object in order to demonstrate proof of concept and to facilitate the understanding of the textile craft, and simultaneously gives insight into the underlying mathematics for textile practitioners. Conversely, mathematicians, with their mathematics expertise, analyse art and design works made by textile craft techniques and sometimes offer an inspiring perspective on the creation or even new designs of textiles. Crocheting, knitting and weaving are textile craft techniques that mathematicians often use to explore and concretely communicate wide-ranging mathematics including topology, geometry etc. In crocheting, Osinga and Krauskopf (2004; 2014) transform computer-generated images of the Lorenz manifold into a crocheted piece that explicitly conveys its properties, while Taimina (2009) crochets models to illustrate the concept and historical account of hyperbolic geometry. In knitting, Isaksen and Petrofsky (1999) study topological principles of knitting and invent two new homogeneous methods to knit mobius strips. In weaving, Feijs (2012) applies tessellation theory, computational tools and laser cutting to examine the houndstooth pattern, resulting in a new weaving pattern. Other studies include Harris (1988; 1997) and Belcastro and Yackel (2008; 2011), which examine the mathematical content of various textile activities and illustrate how mathematics skills may be acquired in the learning of textile crafts.

These examples present mathematics research exploring connections between mathematics and textiles conducted by mathematicians who, within their research, may act as non-professional designers or crafters and perform their research through craft. As the connections between the two disciplines are studied from mathematicians' or scientists' viewpoints, the results of their research that may be useful for the design of textiles are communicated in the language of mathematics that designers may not understand. More recently research performed collaboratively between mathematicians and professional designers has begun to emerge. Feijs in collaboration with Toeters, a fashion designer, uses fractals to examine basic fabric constructions and develops new pattern designs: one pattern for laser cutting derived from the houndstooth weave (Feijs and Toeters 2013) and one for digital printing derived from warp knitting (Feijs et al. 2014). The 132 5 ISSEY MIYAKE series is also the result of a designer/mathematician collaboration to explore the mathematics of folding.[2]

Collaboration between the mathematician and the designer seems to be a key approach to successfully make research into this area understandable by designers, who may generally not have had a formal mathematics training. The ongoing research presented in this paper contributes to this approach. It is a collaborative project investigating the relationship between mathematical knot theory and knotted textiles. The project started when Nimkulrat, a designer-researcher who has used knotting as a technique for creating textiles for nearly a decade, came across diagrams of knot theory that appeared as if they were visualisations of her knotted structure. Although Nimkulrat studied mathematics at school level, this did not include topology. To investigate possible links between the observed knot diagrams, supporting theory and her physical textiles, collaboration was necessary. Research questions include: (1) whether craft and mathematical knots share comparable characteristics, (2) whether knot theory can examine the mathematical properties of knotted textile structures and (3) how knot theory can facilitate the design and production of knotted textiles. This research contributes to existing work investigating connections between mathematics and textiles, by utilising knot theory, which is a mathematical concept not having been studied in relation to textiles, to examine knotted textiles from both the viewpoint of a designer and a mathematician. Along with this paper, an artefact created within the inquiring process will be included in the exhibition (Figure 1). The artefact illustrates a how knot theory may inform and promote the design of knotted textiles – the lesson learnt from the collaborative process.

Figure 1a. Black & White Striped Armchair (2014).

Figure 1a and 1b. Black & White Striped Armchair (2014) details, dimensions: 62cm x 50cm x 63cm, weight 1.5kg, material: paper string. Photograph: Nithikul Nimkulrat.

Mathematical Knot Theory and Craft Knots

To answer the above questions is both to conduct interdisciplinary research through design practice (Koskinen et al. 2011) and to understand the basics of mathematical knot theory. Knot theory is a part of mathematical topology that studies properties of one-dimensional idealised objects, including knots, links, braids and tangles[3] that are made up of infinite thread and can be continuously deformed without breaking (Sossinski 2002). A mathematical knot is a "closed loop in three-dimensional space" which has "no free ends" (Delvin 1998, p. 248-249). Physical properties of the objects are not considered in knot theory, only properties relating to the positions of threads in space (Adams 1994).

The study of knots in mathematics concerns the patterns of knots, and ignores physical properties, such as tightness, size and the shape of individual loops (Delvin 1998). A fundamental problem in knot theory is determining whether two knots are equivalent. Solving this problem involves seeking a property of each knot that does not change when the knot is subject to manipulations, or a knot invariant (ibid.). A knot diagram may be used to illuminate knot properties and equivalence if it exists. A knot diagram is a representation of a mathematical knot that does not depict a physical knot

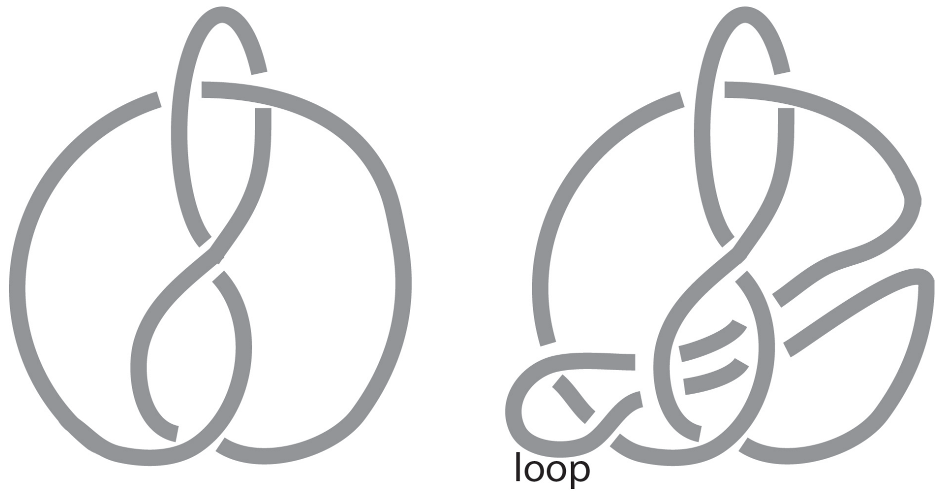


Figure 2. Knot diagrams illustrate crossings. Both are representations of the same knot. The loop in the right diagram may be removed to restore the original figure-eight knot in left diagram. Diagram: Janette Matthews.

Nimkulrat, Matthews

made, but uses simple line drawings to indicate the knot pattern. Lines in the diagram are broken to show where the knot crosses itself (Figure 2). The use of diagrams makes problem solving highly visual.

Drawing may also be used to depict a physical craft knot made from string. However, unlike a mathematical knot diagram, the illustration of a craft knot attempts to provide realistic representation, using continuous bordered/textured lines to depict loosened or tightened knots and additional arrows to demonstrate the knotting process (e.g. Ashley 1993). While a craft knot illustration aims to reveal the process, a knot diagram focuses on the knot pattern and the position of strands. This differentiation can be seen elsewhere in textiles. Weaving illustrations also use simple lines, similarly to mathematical knot diagrams, to show the position of warps and wefts. This can be seen in weaving software for computer-controlled looms, e.g. WeavePoint.

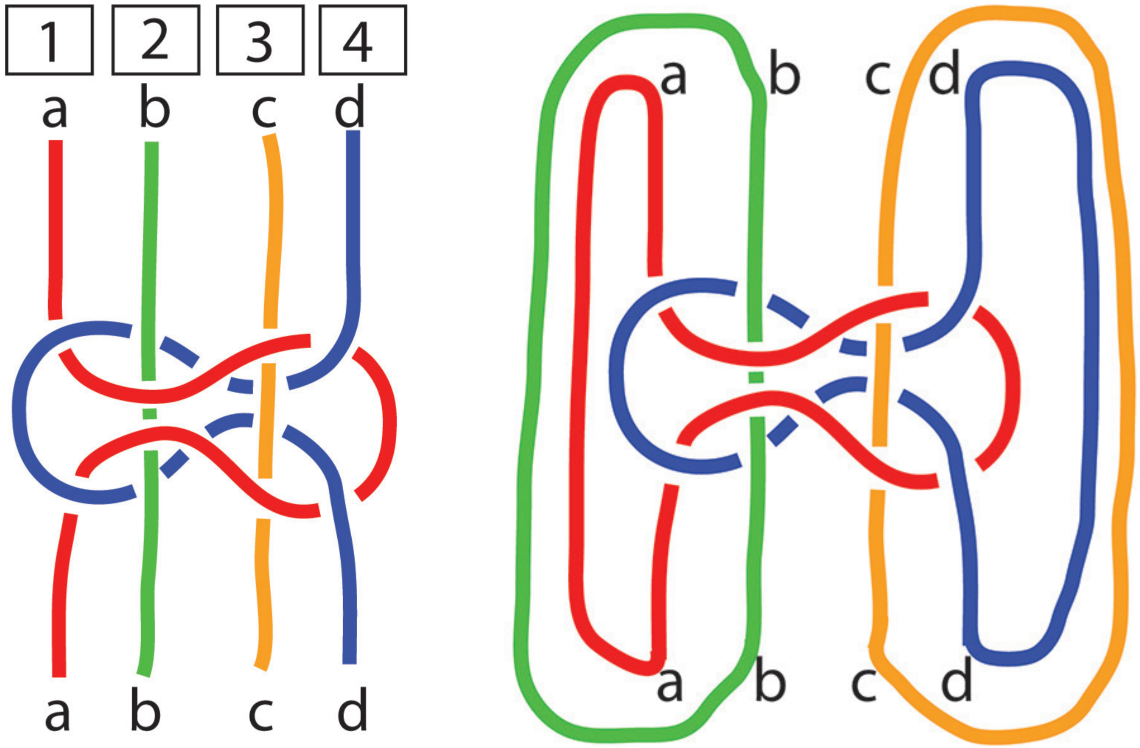
Research through Design: Collaborative Approach of Textile Knotting and Mathematical Knot Theory

While Nimkulrat is a textile practitioner, Matthews is a textile practitioner with a first degree in mathematics. Through mathematical characterisation, Nimkulrat and Matthews (2013) revealed differences between textile knot practice and mathematical knot theory through an examination of a single craft knot used in Nimkulrat's work (Table 1). The difference in ends was explored further using a knot diagram and its colourable property (Adam 1994). The colouring of the diagram of Nimkulrat's knot after all ends were joined showed that the knot was composed of four trivial knots (by definition a knot with no crossings) tangled together (Figure 3b). This aspect stimulated new ideas, and supported the design and production of a new knot structure (Figure 4) that contains no loose ends (Nimkulrat and Matthews 2014). It also suggested a different material (neoprene) which Nimkulrat would not have normally selected.

Property	Textile knot practice	Mathematical knot theory
Ends	May have loose ends	Continuous curve with no loose ends.
Material	Material dependent	Not concerned with materiality. Cross section of strand deemed to be a point.
Tension	Tension dependent	A tight knot has the same representation as a loose knot so are considered equivalent.
Form	The addition of extra loops changes the appearance of a knot	If a knot may be simplified to the same representation as another knot, they are considered equivalent.

Table 1. Differences between textile knot practice and mathematical knot theory (Nimkulrat 2013, p. 7).

Figure 3. Diagram of a craft knot showing the position of strands (left). Four tangled trivial knots formed by joining all loose four ends of a craft knot (right). Diagram: Janette Matthews.



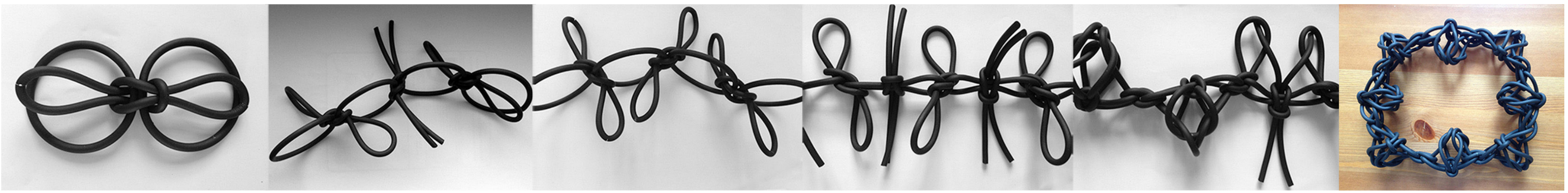


Figure 4. A new knot structure with no loose ends that have the starting point from Figure 3. It is made of neoprene cord. Photograph: Nithikul Nimkulrat.

Knotted Pattern Designs Influenced by Mathematics

In Nimkulrat and Matthews (2013), the colouring of the diagram reveals the positions of strands finish in the same place that they start, i.e. strand a starts and ends in position 1, likewise strand b remains in position 2, c in 3 and d in 4 (Figure 3a). This property is not obvious in Nimkulrat's craft practice. The positions of strands in the diagram imply that various knot patterns may be created by changing positions of strands.

This paper therefore examines multiple knots in Nimkulrat's work (Figures 5), in order to understand the positions of strands that compose these knots and to illustrate how the craft is influenced by mathematics. It investigates whether a group of knots may be characterised using mathematical analysis and diagrams in the same way as a single knot, and how this in turn may facilitate the pattern design and production of knotted textiles.

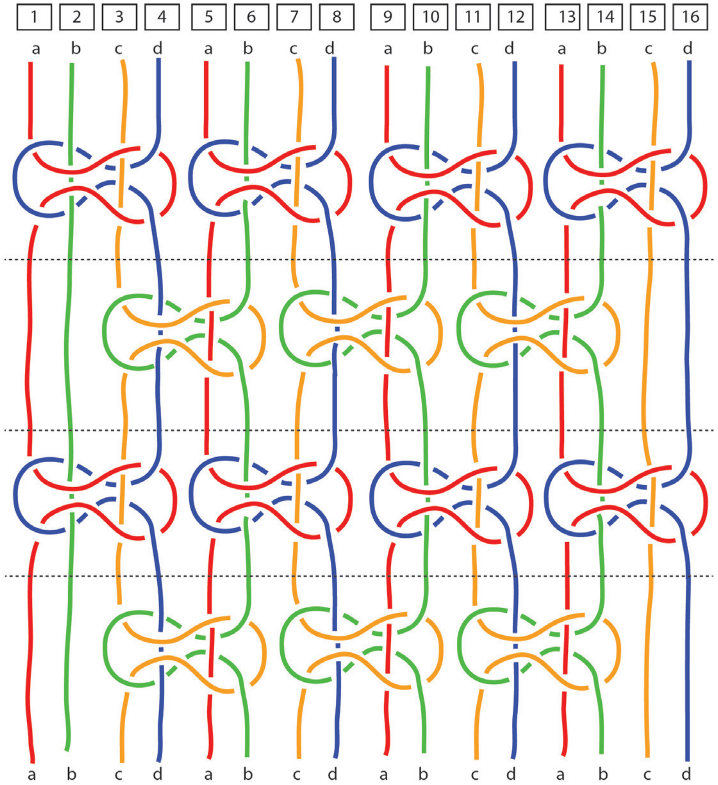
Figure 5a. The White Forest (2012), dimensions: 150cm x 100cm x 200cm, material: paper string. Photograph: Nithikul Nimkulrat.

Figure 5b. The White Forest (2012), the structure of an individual piece from the installation made from the same knot used in repeats (b). Photograph: Nithikul Nimkulrat.

Figure 5b.

Matthews utilised the diagrammatic method used in knot theory to characterise a group of knots starting with four knots, each requiring four strands. In colouring the diagram, four colours were used alternately as in the characterisation of the single knot (Figure 3a). Figure 6 shows that there are potentially four knots on each row and the remaining strands may be used to tie an additional knot which would result in a tubular structure as shown in Figure 5b. All knots are tied identically and each knot contains two active and two passive strands. The first row of knots (Figure 6, from top) uses strands a (red) and d (blue) for knotting, while strands b (green) and c (yellow) are not touched. The knots on the second row have as active strands b (green) and c (yellow). The active strands from the previous row, a (red) and d (blue), are now passive. The third row uses the same strands as the knots on the first row and the fourth the same as the second. Again, all strands finish in the same positions in which they started.

Characteristics (e.g. start/end positions, active/passive roles) indiscernible through observing the work alone become evident in Figure 6. As textile practitioners, both authors immediately recognised in the diagram design repeats – groups of four strands creating a knot along a row and every alternate row utilising the same strands for tying. This act of recognising suggests two potentials for generating repeating knot pattern designs: first, through the use of colours, and second, through changing which strands are active and which are passive.



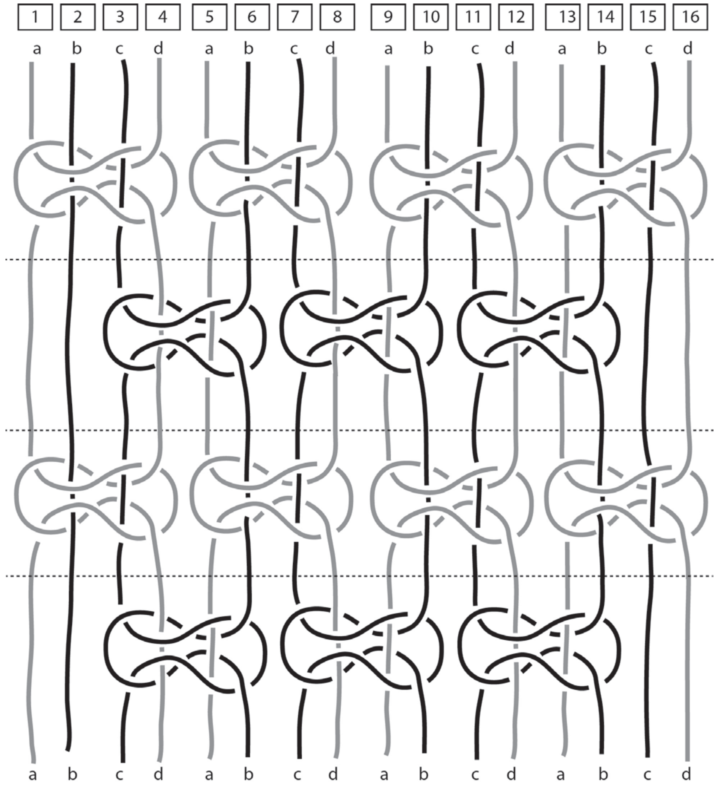


Figure 6 (far left). Diagram of four rows of knots showing rows one and two (from top) are repeated. Diagram: Janette Matthews.

Figure 7 (left). Diagram of four knots using two colours. Diagram: Janette Matthews.

In mathematics, a knot diagram's colourable property is used for studying the positions of strands in space (Adam 1994). This was adopted to further explore the craft knots. The use of colour was considered. To generate a clear repeating knot pattern, the authors decided to reduce the number of colours to two. Matthews then re-coloured Figure 6 with black and grey. The selected colours were neutral because the authors aimed to focus on the pattern generated from two different colours with little influence of colours themselves. While grey replaced red and blue, black replaced green and yellow. Re-colouring the diagram showed that a variety of design outcomes could be achieved by altering the strands' colours (Figure 7). This approach provides, prior to making, a way of visualising two-tone knot patterns. Incorporating colours in textile knot practice is an aspect Nimkulrat wanted to explore, but had not done so. Although she is a skilful practitioner, experimenting with colours in the making of knotted textiles without the diagrammatic approach seemed difficult to handle, so was not attempted. On examination of the diagram, the authors observed that a black knot is created when the strands tying it are black and a grey knot results when the active strands are grey (Figure 7). A further insight is that the strands linking the knots would appear to form either black or grey rounded rectangles – not a mixture of black and grey.

To confirm this design possibility, Nimkulrat followed Figure 6 in knotting paper string. The knotting started with using white string for the strands that actively tie the knot on the first row (grey) and black string for the middle strands (black). The order of the colours of the strands from left to right is white-black-black-white. An individual knot created with the active white strands and the passive black ones appears white (Figure 8a). Knots tied with the active black strands and the middle white strands are black (Figure 8b). White and black strands alternately played an active role in the tying of knots, and eventually black and white circles became apparent (Figures 8c and 8d). The material properties of paper string transformed rounded rectangles in the diagram into circles (Figure 8d).

Nimkulrat then experimented with different positions of black/white strands. Coloured strands were put in the following positions from left to right: black-black-white-white (Figure 9a). Both black and white strings simultaneously played a direct role in tying the knot. To link individual knots, one knot was flipped before the tying took place (Figure 9b), allowing all four strings of the same colour (two strings from one individual knot and two from another) to form a pure colour knot, black or white (Figure 9c). The process continued alternately between a row of mixed colour knots and that of pure black and white ones. This iterative process

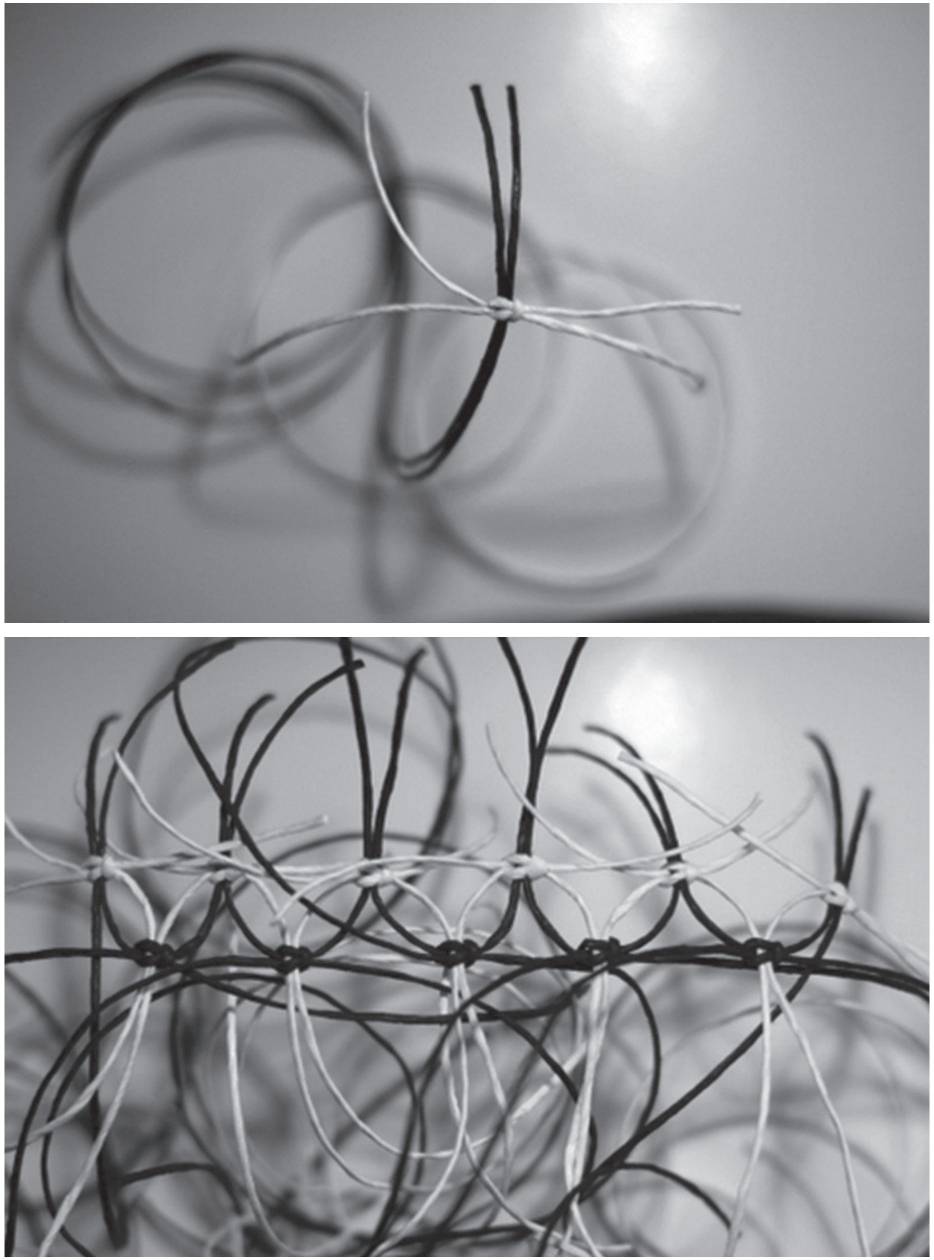
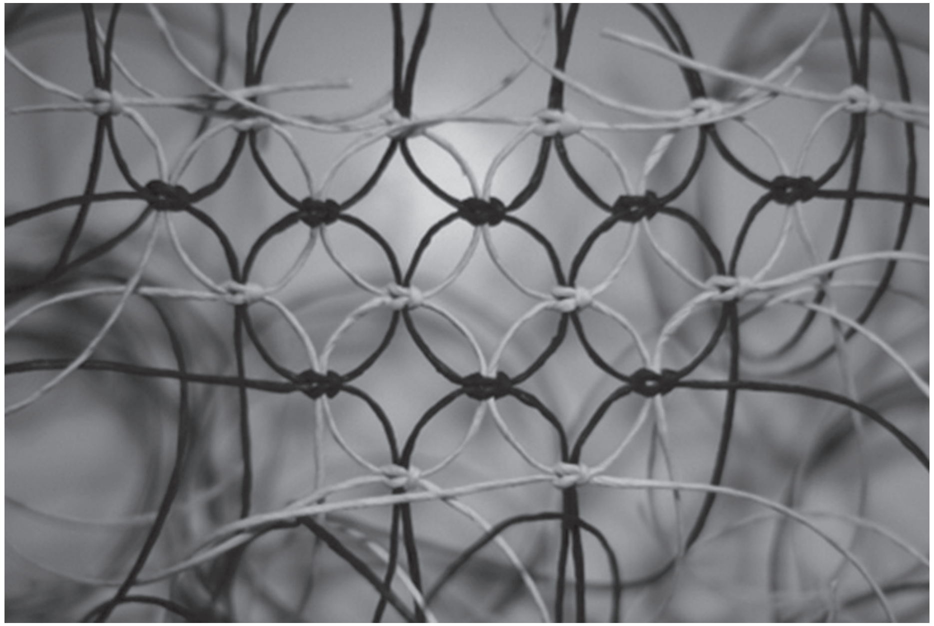


Figure 8a (top) and 8b (bottom): The knotting process using black and white paper string to create a circle pattern. Photograph: Nithikul Nimkulrat.



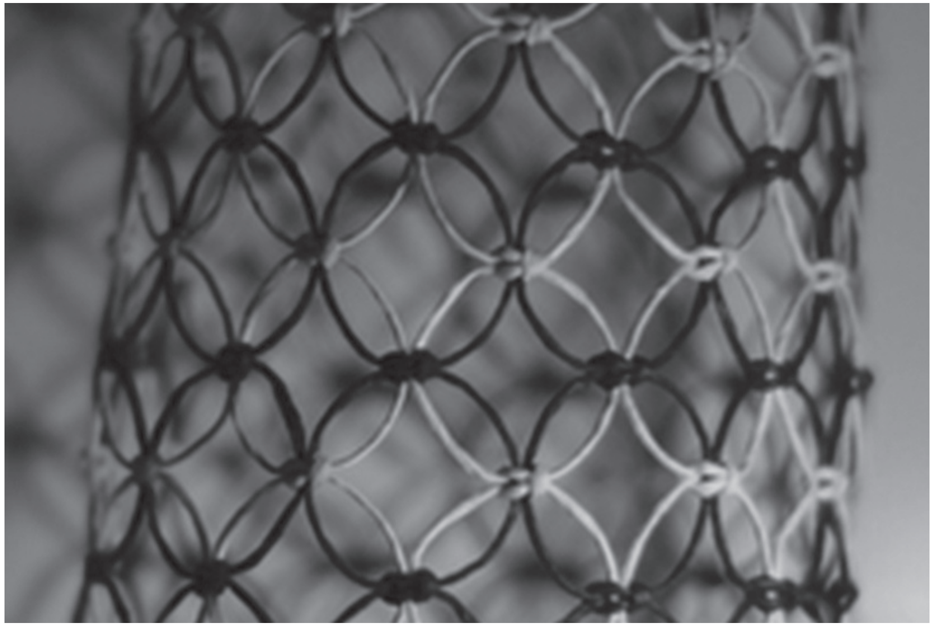


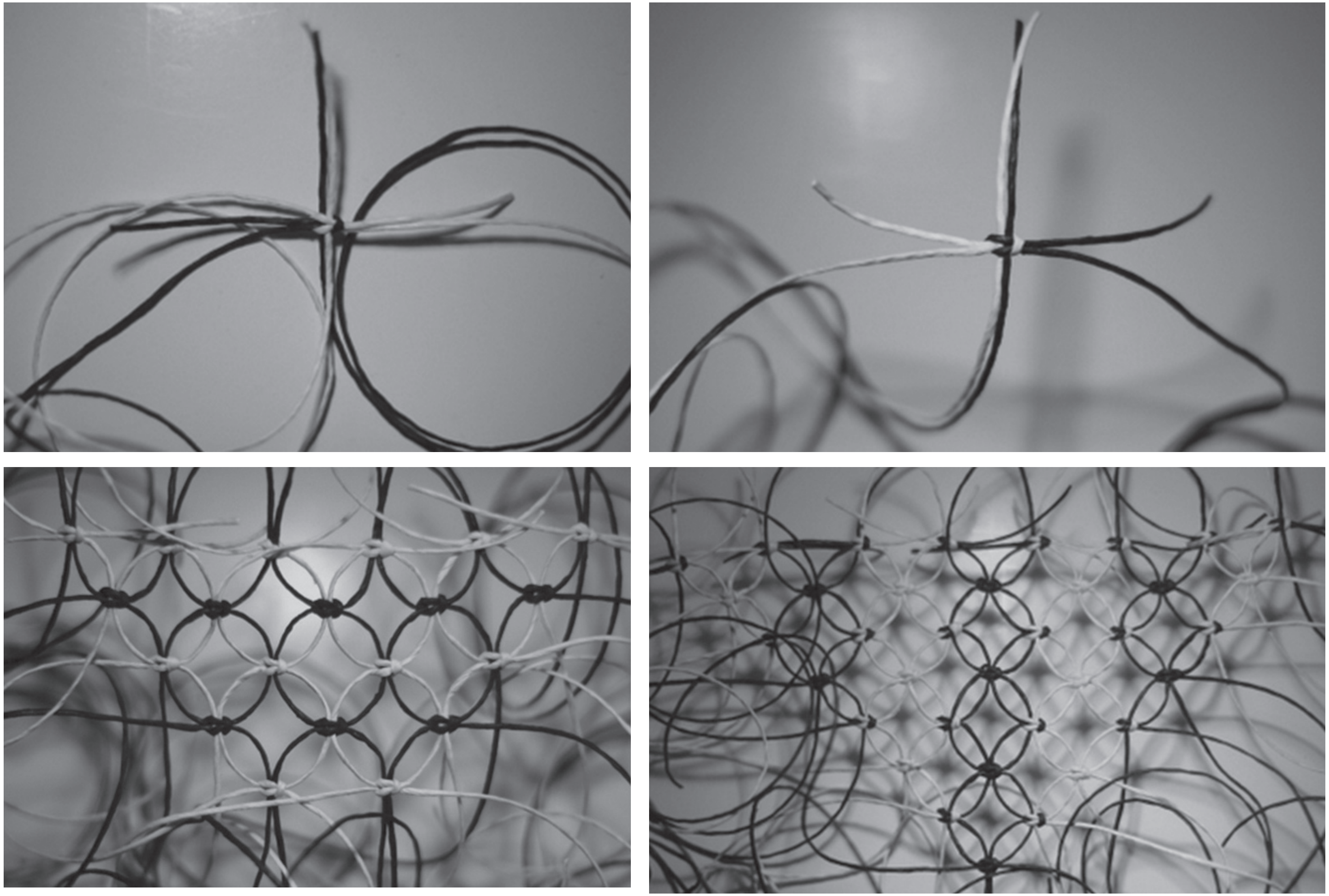
Figure 8c (top) and 8d (bottom): The knotting process using black and white paper string to create a circle pattern. Photograph: Nithikul Nimkulrat.

led to a striped knot pattern (Figure 9d). Samples in Figures 8d and 9d will be included in the exhibition alongside the completed artefact (Figure 1) to provide an opportunity to visually explain the technicalities set out above.

Following Nimkulrat's knotting, Matthews examined the knotted sample from Figure 9d to determine whether this pattern could have been predicted from a coloured diagram. Figure 10 shows the diagram coloured on the basis of the starting position described above, black(a)-black(b)-white(c)-white(d), and then in the opposite order, white(d)-white(c)-black(b)-black(a), with white replaced with grey. It can be clearly seen that the pattern shown in Figure 9d emerges. There are alternate vertical columns – mixed-coloured knots, solid-grey knots, mixed-coloured knots and solid-black knots. The solid-colour knots are attached to rings of the same colour. Furthermore, it can be seen that there is a row of mixed-colour knots and a row of solid alternating colour knots.

Nimkulrat's new understanding of the positions of colour strands in relation to the pattern inspired new creation. Being a basic pattern for interior textiles, the stripe pattern inspired the utilisation of this knotted structure for a functional form connected to interiors (Figures 1). Made of paper string, this artefact entitled Black & White Striped Armchair is intended for inclusion in the exhibition. The artefact consists of three parts: the seat, the back- and armrest and four legs. The striped knot pattern was used in the seat and the back- and armrest. The artefact can be displayed by

From top left clockwise - Figure 9a, 9b, 9c and 9d. The knotting process using black and white paper strings in a different position of strands. Photograph: Nithikul Nimkulrat.



itself without any assisting structure. It is worth noting here that while hand knotting the seat of the three-dimensional artefact, Nimkulrat observed that the stripe pattern was continuously created in all sides of the seat (Figures 1a and c). Although the knot diagrams did not facilitate the design of the three-dimensional form of the artefact, the stripe pattern had its origin in the diagrams. The positions of strands designated in the diagrams created the pattern on the three-dimensional form.

The collaborative process between the authors presented above can be summarised visually as Figure 11.

Conclusion

This paper provides evidence of mathematics influencing craft practice. The pattern development for knotted structures can be explored, predicted and modelled through the mathematical characterisation process and knot diagrams. The use of diagramming textile techniques in colour to show the process is not new. What is new here is the application of the characteristics and philosophy of mathematical knot diagrams to describe craft knots, i.e. simple lines (sometimes broken) depicting positions of strands and the knot pattern. The diagrams used are not graphical illustrations of textile knots but mathematical representations of a textile process. Embedded in these are knotting characteristics. The use of colour in the diagram makes explicit the positions and roles of strands in

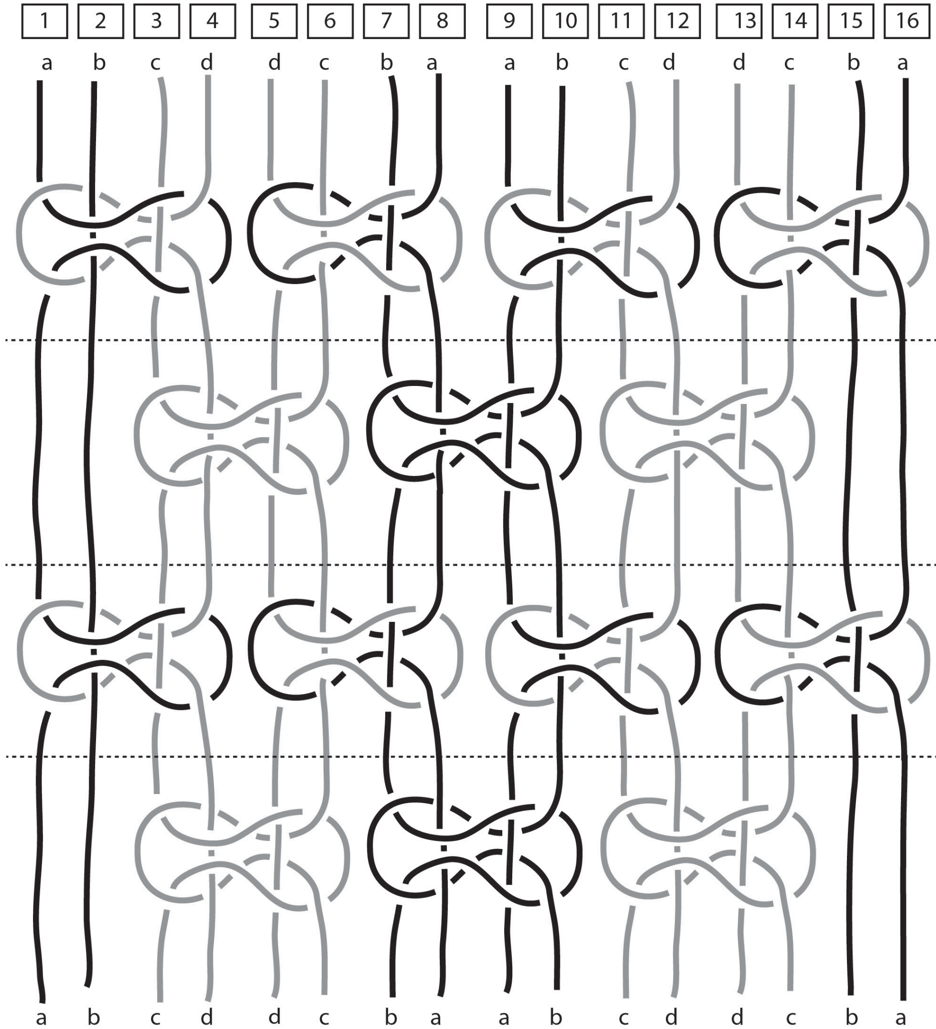
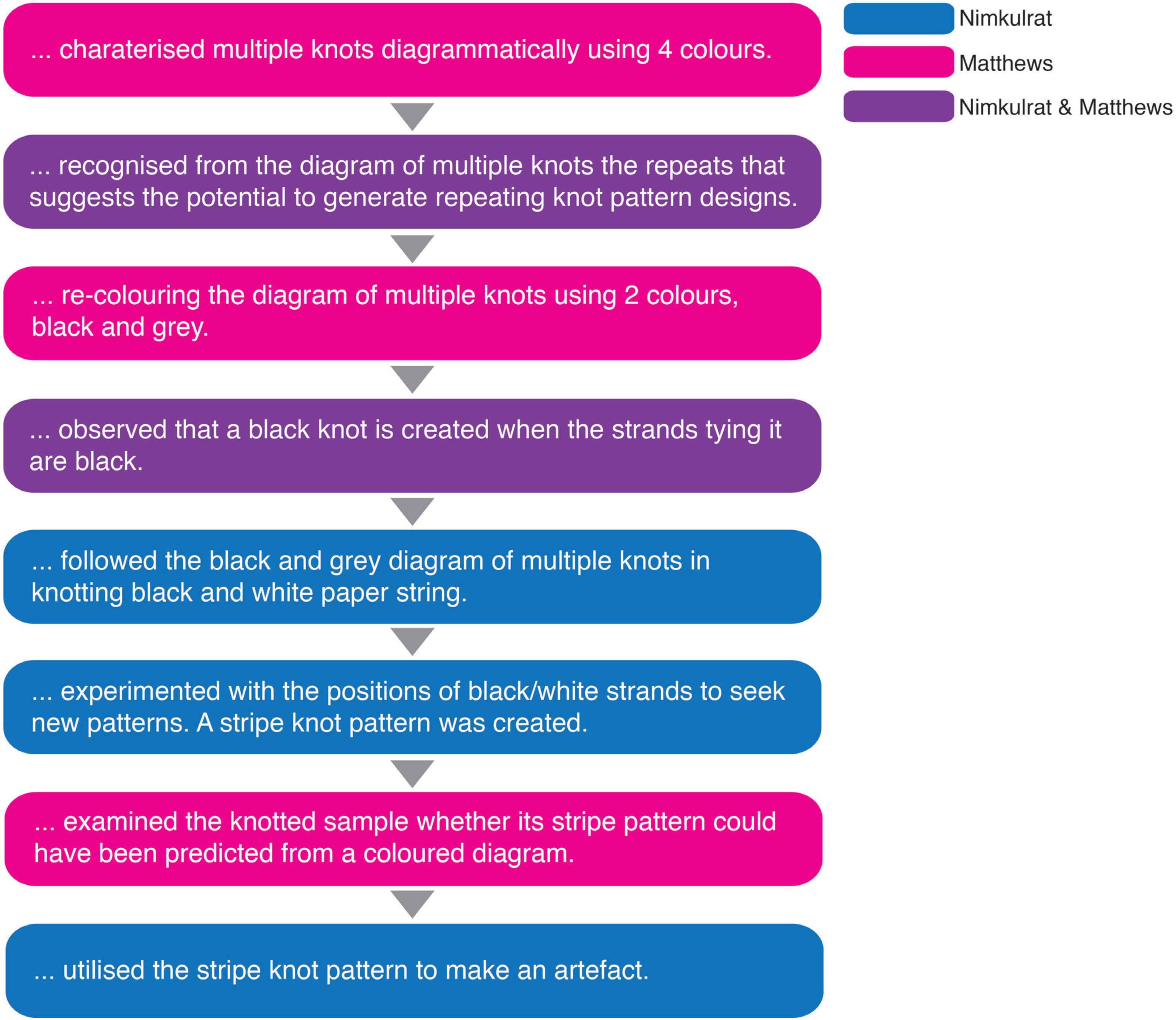


Figure 10. Diagram with two colours and alternate flipped knots on row one. Diagram: Janette Matthews.

Figure 11. Collaborative process between the authors presented in this paper. Diagram: Nithikul Nimkulra



a knotted structure, and leads to an exploration of knotted pattern designs that may not have occurred otherwise.

The colouring of knot diagrams may be developed further into software that could assist the design of patterns, using a variety of different types of knots and positions of coloured strands. This requires collaboration with a software engineer, and could be an area for future research. Such software may benefit textile practitioners who utilise knotting as a technique and could be used not only to design pattern (e.g. coloured stripes) but also to model structures for artefacts (e.g. the number of strands and knot combinations needed to create a chair back in Figure 1a).

The result of this phase illuminates the role of mathematics in making craft knot practice explicit and demonstrates the significance of cross-disciplinary collaboration on the development of practice. The research contributes to the research through design community by elucidating collaboration between a mathematician and a designer as a design research approach.

Mathematicians (e.g. Taimina 2009; Harris 1997) illuminate the role textiles may play in illustrating mathematics to general audiences, who may or may not have received formal mathematical education. Initiatives (e.g. the Museum of Mathematics[4]) demonstrate an appetite to explore the creativity of mathematics through hands-on activities. Through the artefact and samples to be exhibited and associated knot diagrams, this research has the potential to make the mathematics of knot theory explicit to a general audience. Further research will centre on the role of knotted textile practice in mathematics education.

Notes

[1] In this paper, craft is used to identify neither a discipline in its own right nor a sub-discipline of art or design, but a method of logically and intellectually thinking through the hand manipulating a material that is utilised in disciplines such as textiles, ceramics, etc. For further discussion, see Nimkulrat 2010; 2012; Gray and Burnett 2009; Sennett 2008.

[2] http://www.isseymiyake.com/en/brands/132_5.html.

[3] A link is a combination of several knots inter-chained together; a braid is a set of ascending threads with end points fixed on two parallel lines, one under the other; a tangle is an arbitrary set of threads with fixed end points (Adam 1994).

[4] http://momath.org.

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