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Essays on Financial Markets Theory

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A thesis submitted for the degree of
Doctor of Philosophy at the University of Leicester

April 2019

To Kostas
To my family

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by

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Abstract

This thesis examines three different, self-contained topics in Financial Markets Theory.

In the first chapter, we present a model of financial intermediation, in which misperception of small probability events determines the allocation of systematic risk in an economy. Particularly, we analyse an economy where households can invest directly in projects or indirectly via bank deposits. We show that, when households overweight the probability of an unlikely bad state, they prefer to invest through deposits, thus allocating systematic risk to the financial sector. Although this creates financial fragility in the market, it insures households as they are less exposed to risk. Furthermore, we examine any demand externalities that arise due to the households' investment decision and how they affect the real economy.

In the second chapter, we develop a model of entrepreneurial finance in which financiers search entrepreneurs in two financial markets. The key assumption of the model is that markets are heterogeneous with respect to the number of entrepreneurs located in each one. From a financier's perspective, the market with the higher number of entrepreneurs gives a higher chance of finding an entrepreneur, and thus it is perceived to be larger compared to the other. We identify the conditions such that financiers tend to overcrowd the larger market leaving the other one with potential undermatched entrepreneurs. We show that over-concentration of financiers in one market may lead to excessive systematic risk in the economy and to higher financial fragility. Thus, asymmetry in the size of financial markets may accentuate systematic risk and it is one systemic variable that policy makers need to take into consideration.

In the final chapter, we study Bayesian persuasion in a strategic environment, where a seller wishes to influence a buyer to buy a security. When the two agents share a common prior belief, we characterise the optimal signals. The novel feature of our model is that we also allow the buyer to strategically choose her own prior ex-ante. We find that a pessimistic prior belief is optimal and that as the buyer becomes more sceptical, the seller is more prone to truthful communication. Both evolutionary and psychological interpretations are discussed.

Acknowledgements

First and foremost, I would like to express my sincerest gratitude to my supervisors Dr. Fabrizio Adriani and Dr. Aristotelis Boukouras for their continuous support, advice and patience. Thank you for all your guidance and encouragement; I could not have imagined having two better mentors during these remarkable years.

I would like to thank the Division of Economics, School of Business for accepting me into the PhD program and sponsoring my studies through the GTAsip grant. Thank you for providing me with the opportunity to meet all these wonderful people who made my dream a reality. I would particularly like to thank Dr. Subir Bose and Ms Samantha Hill for their help and support. I especially wish to extend my thanks to Dr. David Rojo-Arjona for his comments and feedback. Finally, I would also like to thank my examiners Dr. Piercarlo Zanchettin and Dr. Christopher Wilson for their invaluable suggestions.

Special thanks go to my friends and fellow PhD students for making these years so much enjoyable. Wish you guys all the best!

Kostas, meeting you thirteen years ago changed my life. Thank you for always being there and for all your support.

Finally, I would like to thank my family; my parents Maria, Panos and my brother Chris. Thank you for encouraging me to pursue my dream no matter how difficult and challenging it seemed at the beginning. You never stopped believing in me, and for that I am grateful.

Declaration

Chapter 2 entitled “Investors’ Misperception and the Allocation of Systematic Risk” is joint work with Dr. Fabrizio Adriani and Dr. Aristotelis Boukouras. Chapter 3 entitled “Searching for Borrowers: The Allocation of Financiers across Markets and Systematic Risk” is joint work with Dr. Aristotelis Boukouras. Chapter 4 entitled “Bayesian Persuasion and Bayesian Scepticism” is joint work with Dr. Fabrizio Adriani.

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Chapter 1

Introduction

This thesis explores three different topics in the area of Financial Markets Theory. The first chapter focuses on the relationship between the misperception of low probability events and the allocation of systematic risk in an economy. In the past years, there has been a continuous interest regarding how individuals perceive rare, high-impact events. Both theoretical and empirical studies in Behavioural literature reveal that people tend to overweight the probability of an unlikely bad event (e.g. a financial crisis, a terrorist attack, a large-scale nuclear accident, etc.) in their minds than is normatively appropriate (e.g. Fox and Tversky (1998); Burns, Chiu and Wu, (2010); Snowberg and Wolfers (2010)). So, why do we associate systematic risk?

The latter represents the risk of an external aggregate event which hits the financial system and may prevent it from functioning properly (e.g. the Great Recession of 2008 provides an example of systematic risk). In essence, it constitutes a potential source of fragility for the financial system. Motivated by the aforementioned, this study contributes to the literature by examining how fragile the financial sector can become when people overweight the probability of a rare, bad event. The two-period model developed in this chapter consists of a simple economy with entrepreneurs who want to finance their projects and households who contract directly or indirectly with the former and can fund these projects. Entrepreneurs may be endowed either with correlated projects which carry systematic risk and uncorrelated projects which carry idiosyncratic risk (project-specific risk). The key assumption of our model is that households, on average, tend to overweight systematic risk.

Furthermore, there is a continuum of small, risk-neutral banks which represent

the financial sector. The banks compete in deposits, can invest in either type of project and can issue deposits to the households. Finally, there is the real sector which is represented by a monopolistic producer. The latter only trades with the households for his consumption good. In the case of demand fluctuations from the part of the households, the producer bears a capacity adjustment cost in terms of production (as in Plantin, 2015). This allows us to investigate how the households' investment decision affects the real economy in the presence of demand externalities.

The main results of our analysis can be summarised as follows: We find that misperception of systematic risk induces households to prefer intermediated investment, and thus leading systematic risk to being concentrated in the financial sector. This creates a trade-off: On the one hand, as more systematic risk is allocated to the financial sector, it means that households are better insured and there is potentially less risk for the real economy in the presence of demand externalities. The intuition is that in the presence of misperception, the effect on the real sector decreases due to the safety net that is provided by banks. On the other hand, the financial sector bears excessive risk, which makes bank bailouts more likely and costly for taxpayers. Finally, by focusing on welfare, we find that, when bailout costs are small compared to the demand externality, the socially optimal amount invested in deposits is greater than in the decentralised economy. Intuitively, in terms of welfare, it means that higher average misperception may be beneficial as households tend to invest more in deposits, and thus increasing the amount of risk taken by the financial sector.

In the second chapter, we ask why is it that the largest markets, who also have the most experienced and sophisticated financiers, concentrated so much exposure to systematic risk instead of the smaller, less-sophisticated markets (e.g. the 2008 financial crisis)? We address this question through the lens of search frictions. Search and matching theory has been extensively studied within the contexts of random search (e.g. Burdett and Judd (1983); Trejos and Wright (1995); Shi (1995)) and of directed search (for a recent survey on directed search literature, see Wright et al., (2017)). This study is motivated by the work of Lagos (2000a) who constructs a directed search and matching model in a decentralised dynamic market for taxicab rides.

In this paper, we model the search and matching process between financiers and

entrepreneurs across two financial markets. In particular, we develop a model in which only financiers who are looking for new investment opportunities to finance, search for entrepreneurs. We assume that search is random. Once the agents form the match, they bargain over the distribution of returns generated by the investment.

In this environment, entrepreneurs are exogenously located across the two markets without the ability to move across them due to geographical and financing restrictions, while financiers actively decide which market to enter. The key assumption of the model is that markets are heterogeneous with respect to the number of entrepreneurs located in each one. In particular, we assume that one market has a higher number of entrepreneurs compared to the other. Thus, from a financier’s perspective, this market is perceived to be “larger” than the other.

We show that overcrowding of financiers in the larger (or faster-growing) market is generated via two main channels. The first channel corresponds to the relative strength of the “stepping-on-toes” effect across the two markets, while the second channel is the relative ratio of the social value of matches in comparison to the financiers’ private value of the matches. By introducing a source of aggregate risk in the model, we are able to examine the implications of the allocation of financiers across the two financial markets on systematic risk. We show that, in the case where markets are asymmetrically affected by an external shock so as the ratios of private and social values of a successful match are unequal, overcrowding will occur only in the market which is relatively less affected by the shock. Furthermore, we show that if overcrowding takes place in the market, this market is exposed to excessive systematic risk.

Chapter 4 focuses on the topic of Bayesian persuasion. Based on the motivational work by Kamenica and Gentzkow (2011), we study a standard problem of Bayesian persuasion where a seller (the sender) wishes to persuade a buyer (the receiver) to buy his financial security. This chapter is related to several papers in the rapidly growing Bayesian persuasion literature which study similar settings, e.g. Alonso and Câmara (2016), Kolotilin et al., (2017) just to name a few. It is also closely related to models of optimal beliefs such as Brunnermeier and Parker (2005). In our paper, we assume that the buyer holds pessimistic beliefs about the value of the security. The seller knows that the buyer is pessimistic and that the latter, based solely on

the prior information, is not inclined to buy the security. By examining the strategic communication between the two agents, we ask: Can the seller persuade the buyer to take an action which benefits the former?

In our model, the seller commits to an informative, costless signal. The buyer observes the outcome of the signal, she updates her prior according to Baye’s rule and then chooses whether to buy or not the security. We start our analysis with the assumption that both the seller and the buyer share a common prior belief over the value of the security. Under this assumption, we find the optimal signals for which the seller gains from persuasion, i.e. the seller through his choice of signals, maximize his expected utility.

The innovative part of our work, and thus our contribution to the existing literature, is that later on we depart from the assumption of the common prior. In particular, we allow the buyer to “choose” her own prior strategically. We show that by optimally choosing an extremely pessimistic prior, the buyer induces the seller to “almost” truthfully communicate the signal realisation. The result suggests a natural reaction to persuasion, i.e. people become more sceptical in the face of manipulative behavior. We interpret this finding through both psychological and evolutionary channels.

Finally, Chapter 5 concludes the thesis.

Chapter 2

Investors' Misperception and the Allocation of Systematic Risk

Abstract

We present a model of financial intermediation, in which misperception of small probability events determines the allocation of systematic risk in an economy. We find that, when households overweight the probability of the bad state, they prefer to invest via deposits, thus allocating systematic risk to the financial sector. Although this creates financial fragility in the market, it insures households as they are less exposed to risk. Within the framework of misperception, we also examine the demand externalities that arise due to the households' investment decision and how these demand fluctuations affect the real economy.

2.1 Introduction

There is significant evidence on how people filter information and how cognitive forces determine their decision-making process. No matter how much we try to discipline our minds, we are still influenced by sentiment and emotion. This is of great importance, especially when it comes to understanding the way investors perceive rare, high-impact events and how this perception affects the functioning of financial markets.

In this paper, we develop a theory of financial intermediation regarding the relationship between the perception of low probability events and the level of risk-taking and allocation. Specifically, we examine how systematic risk is allocated between households who overweight the probability of an unlikely bad state and a sophisticated financial sector. Furthermore, we analyse how the households' investment decision affects the real economy.

The model consists of a simple economy with entrepreneurs who need to raise capital for their projects and households who contract directly or indirectly with the entrepreneurs and can fund these projects. In particular, the entrepreneurs may be endowed either with correlated or uncorrelated projects. The uncorrelated projects carry idiosyncratic (project-specific and diversifiable) risk, but require active monitoring. The correlated projects carry systematic (aggregate and undiversifiable) risk, but can be funded without monitoring. This reflects the fact that, since all correlated projects pay off in the same states, their returns are publicly known. There is also a continuum of households with heterogeneous expectations about the return of projects. Particularly, a given household may over or underestimate the likelihood that correlated projects will default, but in the aggregate, they tend to overweight systematic risk. Households can fund the projects directly or acquire deposits from the financial sector.

The financial sector is modelled as a continuum of small, risk-neutral banks who compete in deposits. The banks can invest in either type of project and can issue deposits to the households. Following Diamond's (1984) seminal work, the main role of the financial sector in our setting is to reduce monitoring costs by acting as a delegated monitor for uncorrelated projects. Finally, based on Plantin (2015), representing the real economy in the model is a producer who trades with the households

for his consumption good.

Our main result shows that misperception of systematic risk induces households to prefer intermediated over direct investment, thus, leading systematic risk to being concentrated in the financial sector. In this environment, a trade-off is created. On the one hand, more systematic risk allocated to the financial sector means that households are better insured, and hence there is potentially less risk for the real economy in the presence of demand externalities. On the other hand, the financial sector bears excessive risk, thus making bank bailouts more likely and costly for taxpayers.

Furthermore, we examine how the households' investment decision affects the producer. In particular, we find that, in the presence of misperception, the effect on the producer decreases due to the safety net that is provided by the financial intermediaries. The intuition is the following: By observing the proceeds that households receive from investing, the producer can infer the demand for his product and can adjust his production. However, this adjustment comes at a cost for the producer. As misperception increases and households are more inclined to invest through the financial sector, the adjustment cost decreases, thus, decreasing the effect on the real economy.

Finally, within our framework, we investigate the role of a social planner. We show that when bailout costs are small compared to the demand externality, the socially optimal amount invested in deposits is greater than in the decentralised economy. In terms of welfare, it means that higher average misperception may be beneficial as it increases the amount of risk taken by the financial sector. The opposite applies when bailout costs are large compared to the demand externality.

The behavioural literature, both theoretical and empirical, has sparked much interest on how people perceive rare, high-impact events. According to Fox and Tversky (1998), the psychological process of an individual when considering rare events follows two steps. Initially, the individual assesses the probability of a tail event and then, proceeds to make a decision. The first step is about beliefs, while the second is about preferences. On the beliefs' side, evidence reveal that people overestimate the probability of a tail event, while on the preferences' side, if the individual is aware of the likelihood of a rare event, he will overweight the possible

outcome in his mind¹. This view is the basic component of Tversky and Kahneman's (1992) Prospect Theory, where the weighting function transforms subjective probabilities into decision weights and consequently the individual overweights the tails of the distribution (Barberis, 2013).

Our paper is closely related to studies in which low probability events are overweighted. According to Hertwig and Erev (2009), overestimation and overweighting operate to increase the impact of low probability, negative events on people's choices. In particular, decision-makers tend to overestimate the chance that rare events will occur and small probabilities are overweighted in terms of their impact on decisions. Combined, these two reasons suggest that rare events, i.e. a nuclear accident or a terrorist attack are given greater psychological weight in our minds than is normatively appropriate (Burns, Chiu and Wu, 2010). Snowberg and Wolfers (2010) deserve a special mention. The authors study the long-standing betting irregularity that is known as the "favorite-long shot bias". Particularly, they find empirical evidence in favor of the view that misperception of probabilities and not risk-loving drive the favorite-long shot bias in betting markets where long-shots (unlikely odds) are over-bet, while favorites are under-bet.

Note here that there is also the view that people may underweight the likelihood of rare events². The work of Camerer and Kunreuther (1989) reveal a dichotomy in perceptions, where some individuals downplay or reject low probabilities and others might tend to overweight them when given additional information (Richter, Schiller and Schlesinger, 2014). McClelland, Schulze and Hurd (1990), by conducting a study near a landfill site, show that dichotomy in perception drive some people to dismiss risk and conclude there is no hazard, while others place a relatively high value on risk.

Our paper also relates to the strand of literature that examines the potential sources of fragility in the financial system linked to systematic risk. The latter represents the risk of an external aggregate event; it captures the exogenous risk that hits the financial system and may prevent it from functioning properly. This in turn can lead to its failure, i.e. bank failures (Zigrand, 2014). Gennaioli, Shleifer and

¹Comparative to the weight it would receive under the expected utility paradigm.

²See Hertwig et al., (2004); Coval, Jurek and Stafford (2009); Foote, Gerardi and Willen (2012); Gennaioli, Shleifer and Vishny, (2015) for further reading.

Vishny (2013) deserve a particular mention. The authors construct a model where intermediaries originate or acquire safe and risky loans. The latter are subject to both idiosyncratic and systematic risk. Due to diversification (by buying and selling the risky loans with each other), the systematic risk of their portfolios increases, and thus the intermediaries become exposed to tail aggregate risks. This in turn may lead to extreme financial fragility, when, in quiet times, investors and intermediaries neglect these risks. Our work is in contrast to the latter, although the results may go in the same direction, i.e. excess fragility of the financial sector.

Another potential source of financial instability is the maturity mismatch between a bank's assets and liabilities. In particular, financial intermediaries fund long-term, illiquid loans with short-term liabilities, often payable on demand. If a substantial fraction of creditors simultaneously call in their funds, an intermediary may find itself unable to pay its debts. Diamond and Dybvig (1983) present a model where bank runs are caused by maturity mismatch. Adrian and Shin (2008) as well as Diamond and Rajan (2009) link the 2008 financial crisis to illiquidity issues due to maturity mismatch. Farhi and Tirole (2012) have called the recent financial crisis "one of wide-scale maturity mismatch. It is also one of substantial systematic risk exposure..."

To our best knowledge, this work represents a first attempt to theoretically establish how fragile the financial sector can become when people overweight the probability of rare events. With this in mind, we address the allocation of systematic risk within an economy. The structure of the paper is as follows: In Section 2.2, we introduce the basic setting of the model. We continue to the analysis and the existence of equilibrium in Section 2.3. Section 2.4 focuses on the trade-off between financial and demand stability and Section 2.5 on welfare analysis. In Section 2.6, we discuss the main implications of our work, we propose possible future directions and we conclude. Most of the proofs are found in Appendix A.

2.2 The Model

We develop a two-period model ($T = 1, 2$) of financial intermediation with the following types of agents: Entrepreneurs, Households, Banks and a Producer. All agents

are risk-neutral. Furthermore, there are two goods in the economy: a numéraire good which acts as a measure of value for all trading and a consumption good which is produced by the producer and only consumed by the households. In this simple economy, households can invest directly in projects or indirectly through bank deposits.

2.2.1 Projects

In this economy, there are two types of projects: correlated (or type k) and uncorrelated (or type u) projects. All projects are funded in period 1 and all pay off in period 2. Let p_k and p_u denote the price of correlated and uncorrelated projects, respectively. Let $\Phi_k(p_k)$ denote the supply function of type k projects with the following properties: $\Phi_k : \mathbb{R}^+ \rightarrow [0, 1]$, $\Phi_k^1 > 0$ (non-decreasing) and $\lim_{p_k \rightarrow \infty} \Phi_k(p_k) = 1$. Similarly, the supply function of type u projects is $\Phi_u(p_u)$, where the same properties apply. Maximum supply of projects is normalized to 1 (as the relevant price goes to infinity). We need to mention here that increasing supply functions implicitly assume an increasing marginal cost of running additional projects for the entrepreneurs (which are not modelled explicitly).

There are two states of nature: the good state (H) and the bad state (L). All correlated projects pay 1 unit of the numéraire good in period 2 under the good state (H), which occurs with probability $\theta \in [0, 1]$ and 0 under the bad state (L), which occurs with probability $1 - \theta$. We interpret the bad state as a small probability “catastrophic event” and accordingly consider the case where θ is close to 1. Uncorrelated projects pay 1 unit of the numéraire good in period 2 with probability $\mu \in [0, 1]$ and 0 with probability $1 - \mu$. For simplicity, we assume that their mean μ is equal to θ . The realized return of a type u project is independent of the return of its own class of projects as well as with the class of correlated projects, and hence with the aggregate shock. Therefore, type k projects carry systematic risk which implies that they are either all profitable or loss-making, whereas type u projects carry idiosyncratic risk which is a project-specific risk.

We assume that the return of an individual project is known to the entrepreneur, but verification is costly for the other agents in the economy. In particular, verifying the return of a project costs $c_b > 0$ to the bank, that is the present value of the

verification cost in period 1 and ∞ to the households. In our model, the financial intermediary is thus delegated (on behalf of the depositors) to monitor the loan contract, as it has a substantial cost advantage in acquiring information³. We also assume that the state of nature can be verified at no cost and most importantly after the verification of the projects' return. In this perspective, type k projects can be funded without monitoring since knowing the state of one project, effectively makes the state of the rest of the projects known, while type u projects require monitoring from the bank.

2.2.2 Households

There is a continuum of risk-neutral households endowed with wealth $W_1 > 0$ in period 1. The households can invest directly in type k projects or indirectly in both types of projects through risk-free deposits (note here that a household's investment cannot exceed the available funds W_1). Households can also store units of the first-period endowment and use them to buy a second-period consumption good without incurring any depreciation, i.e. endowment is not perishable. Households only consume in period 2 after the trade with the producer takes place. Their discount factor is normalised to one.

A crucial feature of our setting is heterogeneity across households' beliefs, implying that some households overweight the unlikely possibility of the bad state occurring and others underweight it. However, we assume that on average, households overweight the probability of the bad state. In particular, household i 's perception of the good state θ is given by $w_i \sim U[\theta - \bar{w} - \varepsilon, \theta - \bar{w} + \varepsilon]$, where $\bar{w} \in [0, \varepsilon]$ corresponds to the average misperception bias and $\varepsilon \leq \min\{\theta - \bar{w}, 1 - (\theta - \bar{w})\}$ constitutes an idiosyncratic shock. When $\bar{w} = 0$, households have on average accurate beliefs. However, when $\bar{w} > 0$, households on average underestimate θ ; this essentially means that households underweight the probability of the good state and overweight the probability of the bad state of nature.

³Alternatively, either households would face a large expenditure on monitoring costs or nobody would monitor as the share of benefit would be small (a free rider problem), see Diamond (1984).

2.2.3 Financial Sector and Deposit Insurance

There is a continuum of small, risk-neutral and sophisticated bankers whose size is normalized to one, each endowed with $E_1 > 0$ units of finance. Bankers can invest in bank's equity or in deposits. The representative bank invests a share of $\alpha \in (0, 1)$ of its funds in type k projects and a share of $1 - \alpha$ in type u projects. Also, the bank issues risk-free deposits to households. A deposit contract has a price p_d ⁴ in period 1 and pays 1 unit of the numéraire good to the household in period 2.

In our model, deposits are insured by a deposit insurance scheme provided by the government with a shadow cost of funds $\xi \geq 0$, that is, when bank's liabilities (deposits) are larger than its assets, society bears a cost of ξ times the shortfall. This cost reflects the fact that the government may need to raise funds through distortionary taxation. Essentially, it captures the social costs of banks' bailouts. If the bank defaults in the bad state and its period-2 assets are lower than its liabilities, then any shortfall in capital will be covered by the deposit insurance scheme.

2.2.4 Real Sector

In our model, the financial system affects the real economy in two ways. First, through the quantity and type of projects being funded and second, as in Plantin (2015), through demand externalities borne by a monopolistic producer. The latter produces a consumption good in period 2, which only households consume. As households gain utility only from the second-period consumption good, the producer faces an inelastic demand up to each household's second period wealth W_2 , as arising from the realized returns of their first-period endowment investments. As a monopolist, the producer sets a maximal price of one per unit (as households value one unit of output as much as one unit of the numéraire good) to extract all consumer surplus, i.e. the equilibrium utility of each household will be zero.

In period 1, the producer chooses an initial capacity (costlessly) $N_1 \geq 0$. This happens before the producer can observe the investment proceeds of the households. In period 2, the producer chooses output $N_2 \geq 0$, where output has a marginal cost

⁴ p_d is endogenously given.

of $c \in (0, 1)$ and a symmetric quadratic adjustment cost

$$\frac{l}{2}(N_2 - N_1)^2$$

where $l > 0$ is the adjustment cost parameter, if output differs from the capacity level.

2.2.5 Timing

The timing of the model can be summarized as follows: In period 1, households invest in projects directly or indirectly via bank deposits and the producer makes an initial capacity choice in terms of production. In period 2, the producer observes the households' proceeds from investment and makes an adjustment choice to his capacity. Finally, the households and the producer trade the numéraire good for the producer's output.

2.3 Equilibrium Analysis

2.3.1 Demand for Projects and Deposits

Households allocate their available funds W_1 directly to type k projects and indirectly to both types of projects by depositing their income to the bank. Since households are unable to monitor type u projects, they invest in these projects only through the bank. If households invest in deposits, their utility in period 2 is

$$u(W_2) = \frac{W_1}{p_d} \tag{2.1}$$

and if households invest in type k projects, their period-2 utility is

$$u(W_2) = \begin{cases} \frac{W_1}{p_k}, & \text{if good state (H);} \\ 0, & \text{if bad state (L).} \end{cases} \tag{2.2}$$

A crucial feature of the model is that households misperceive the systematic risk realizing from demanding type k projects. This will affect their expected utility in period 2:

$$E_i[u(W_2)] = \begin{cases} \frac{W_1}{p_d}, & \text{if households invest in deposits;} \\ w_i \frac{W_1}{p_k}, & \text{if households invest directly in type } k \text{ projects.} \end{cases} \quad (2.3)$$

Since households have heterogeneous beliefs in regard to their investment decision, the marginal household w^* is indifferent between investing in type k projects and deposits. Hence, its expected utility in period 2 must be:

$$w^* \frac{W_1}{p_k} = \frac{W_1}{p_d} \quad (2.4)$$

The above equation implicitly assumes that households invest in type k projects in equilibrium. Solving for w^* will give us the cut-off rate, the threshold that will allow us to determine households' demand for deposits and type k projects. However, we first solve for the prices of type k and u projects, p_k and p_u respectively. The following Lemma establishes the prices of both types of projects. It tells us that in the long-run equilibrium, in a perfectly competitive market, the net present value (NPV) of projects must be zero. In particular, this result is derived by imposing that in equilibrium new banks have no incentive to enter the market and that profits must be the same across projects' types. For the representative bank, this gives an opportunity cost of capital of $1/p_d$.

Lemma 2.3.1. *In any equilibrium, the prices of type u and type k projects are $p_u = \theta p_d - c_b$ and $p_k = \theta p_d$, respectively.*

Proof. see Appendix □

Substituting for $p_k = \theta p_d$ in expression (2.4), we get that :

$$w^* \frac{W_1}{\theta p_d} = \frac{W_1}{p_d} \Rightarrow \quad (2.5)$$

$$w^* = \frac{\theta p_d}{p_d} \Rightarrow \quad (2.6)$$

$$w^* = \theta \quad (2.7)$$

Inspection of equation (2.7) informs us that the marginal household's belief of the good state w^* is equal to the objective probability θ . This is an interior solution where for households to invest in type k projects in equilibrium it must be assumed that $\varepsilon > \bar{w}$, otherwise there cannot be any household with $w_i > w^* = \theta$. This requires the presence of over-optimistic households in the economy. Above this threshold, households want to invest in correlated projects. It follows that the households' demand for type k projects is:

$$\begin{aligned} D_k &= W_1 \frac{(\theta - \bar{w} + \varepsilon) - \theta}{2\varepsilon} \\ &= W_1 \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \end{aligned} \quad (2.8)$$

Below the threshold, hence, households invest in deposits:

$$\begin{aligned} D_d &= W_1 \frac{\theta - (\theta - \bar{w} - \varepsilon)}{2\varepsilon} \\ &= W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \end{aligned} \quad (2.9)$$

Having established households' demand for deposits and type k projects, we are now able to turn the analysis to the conditions that are required for the projects' market to clear.

2.3.2 Market Clearing for Projects

In the presence of financial intermediation, the clearing conditions for projects' market require the total supply of funds to be equal to the total quantity of projects being funded. Recall that type u projects can only be funded by the bank via deposits and equity. Thus, the market for type u projects clears only if the following condition is satisfied:

$$(1 - \alpha) \left[E_1 + W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right] = \Phi_u(p_u)(p_u + c_b) \Rightarrow \quad (2.10)$$

$$(1 - \alpha) \left[E_1 + W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right] = \Phi_u(\theta p_d^* - c_b)\theta p_d^* \Rightarrow \quad (2.11)$$

The left-hand side of equation (2.10) gives us the total funds which are intended to fund type u projects, where $\alpha \in [0, 1]$ is the share of bank's funds invested in type k projects. The right-hand side of equation (2.10) corresponds to the total quantity demanded for investment. Equation (2.12) gives us the equilibrium share of bank's funds $1 - \alpha^*$ intended for type u projects as a function of the equilibrium price of a deposit contract, p_d^* :

$$1 - \alpha^* = \frac{\Phi_u(\theta p_d^* - c_b)\theta p_d^*}{E_1 + W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right)} \quad (2.12)$$

Similarly, the market for type k projects clears when the total supply of funds, that is the direct investment by households, the bank's period-1 equity and deposits is equal to the total quantity of type k projects demanded for investment, as shown in equation (2.13). The equilibrium share of bank's funds α^* intended for type k projects is obtained by equation (2.15).

$$W_1 \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) + \alpha \left[E_1 + W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right] = \Phi_k(p_k)p_k \Rightarrow \quad (2.13)$$

$$W_1 \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) + \alpha \left[E_1 + W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right] = \Phi_k(\theta p_d^*)\theta p_d^* \Rightarrow \quad (2.14)$$

$$\alpha^* = \frac{\Phi_k(\theta p_d^*)\theta p_d^* - W_1 \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right)}{E_1 + W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right)} \quad (2.15)$$

Inspection of market clearing conditions (2.10) and (2.13), informs us that an increase in misperception ($\uparrow \bar{w}$) will increase the amount of deposits used to fund type u and type k projects and will decrease the direct investment of households in type k projects. Intuitively, this means that an increase in average misperception induces households to prefer intermediated over direct investment, leading systematic risk to being concentrated in the financial sector. Hence, any change in misperception has a direct effect on the allocation of systematic risk between the financial sector and the households.

We can now derive the total market clearing condition by summing up equations (2.10) and (2.13) and substituting for the prices of both types of projects:

$$W_1 + E_1 = \theta p_d^* [\Phi_u(\theta p_d^* - c_b) + \Phi_k(\theta p_d^*)] \quad (2.16)$$

The left-hand side of condition (2.16) shows that the total financing for both types of projects is fixed and independent of misperception and the right-hand side is increasing in p_d^* , the equilibrium price on deposits. The following Lemma determines p_d^* , as Figure 2.1 shows:

Lemma 2.3.2. *Under the condition $W_1 + E_1 < \theta[\Phi_u(\theta - c_b) + \Phi_k(\theta)]$, there exists an equilibrium price on deposits p_d^* solving equation (2.16).*

Proof. see Appendix □

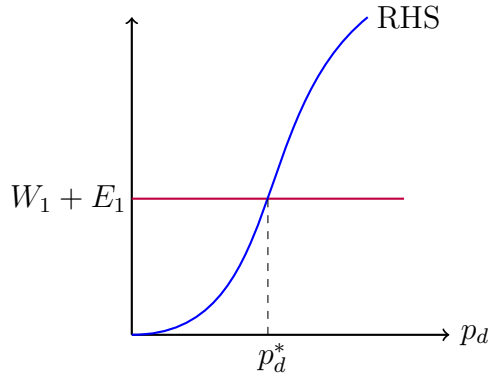


Figure 2.1: Total Market Clearing

The following Lemma shows that the amount invested in type k projects is independent of misperception:

Lemma 2.3.3. *The aggregate amount invested in type k projects does not depend on misperception, but only on the equilibrium price on deposits (p_d^*).*

Proof. We know that households' demand for deposits is $W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right)$ and households' demand for type k projects is $W_1 \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right)$. The total amount invested in type k is $\Phi_k(\theta p_d^*)$, but from equation (2.16) we see that p_d^* is independent of $\bar{w}$. This implies that $\Phi_k(\theta p_d^*)$ is constant with respect to $\bar{w}$. □

Furthermore, from equation (2.16), we can see that if the bank becomes better capitalised (there is an increase in period-1 equity, E_1), it will increase the total supply of funds, which in turn will increase the equilibrium price on deposits p_d^* , as Figure 2.2 shows. Intuitively, an increase in E_1 will first, reduce the return on

deposits, i.e. increase the price of a deposit contract and second, it will affect the composition of projects in the economy.

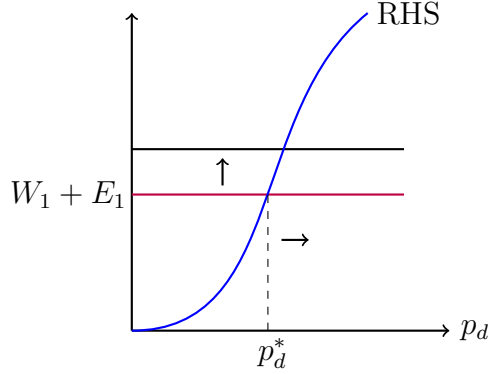


Figure 2.2: Increase in the Supply of Funds

The latter depends on the steepness of $\Phi_u(p_u)$ and $\Phi_k(p_k)$. If $\Phi_u(p_u)$ is steeper relatively to $\Phi_k(p_k)$, then an increase in E_1 will increase type k projects in the economy. If the opposite occurs, then an increase in E_1 will increase type u projects in the economy. Finally, it is important to mention that an increase in the expected return of projects θ , will induce the bank to invest more in both types of projects, while an increase in the cost of monitoring c_b will decrease the price of type u projects, p_u .

2.3.3 Demand Externality

In this section, which is based on Plantin's (2015) framework, the analysis focuses on the demand externality that arises from the households' investment choice. In particular, when households decide whether to invest in deposits or type k projects, they do not take into account the effect on the real economy. Recall that the real sector in our model is represented by a producer, who decides on an initial production scale N_1 . By observing the returns of the households' investment, the producer has the ability to adjust his initial production in period 2. However, this adjustment comes at a cost of $\frac{l}{2}(N_2 - N_1)^2$ for the producer.

An important assumption is that the producer does not commit himself to producing output in period 2. He finds it optimal to wait until the uncertainty from the households' investment decision clears up (in period 2). As a monopolist, he then charges a price of one per unit and adjusts his production scale equal to the

period-2 realized wealth of the households. To understand this, we revisit Lemma 1 and 2 of Plantin (2015). Solving backwards from period 2, suppose that the wealth of households is $W_2 \geq N_1$. After observing W_2 , the producer chooses $N_2 \in [N_1, W_2]$ in order to maximize his profits:

$$\begin{aligned} \max_{N_2} \quad & (1-c)N_2 - \frac{l}{2}(N_2 - N_1)^2 \\ \text{subject to} \quad & N_2 \leq W_2 \end{aligned}$$

For l small enough, profit is maximised at $N_2 = W_2$ since the producer produces the maximum output in order to match the period-2 wealth of the households. Suppose now that $W_2 < N_1$, that is the realized wealth of the households in period 2 is smaller than the initial production scale of the producer. In this case, the producer chooses $N_2 \in [W_2, N_1]$ in order to maximize his profits:

$$\begin{aligned} \max_{N_2} \quad & W_2 - cN_2 - \frac{l}{2}(N_2 - N_1)^2 \\ \text{subject to} \quad & W_2 \leq N_2 \end{aligned}$$

Note here that the producer can get at most W_2 from the households and when l is small enough, we have that $N_2 = W_2$. The following Lemma formalizes the above insights. If the producer believes that the period-2 wealth of the household $\tilde{W}_2$ is stochastic, then:

Lemma 2.3.4. *If $l \leq \min\left(\frac{1-c}{(1-\theta)B}, \frac{c}{\theta B}\right)$ where $B \equiv \frac{W_1}{W_1+E_1} \left(1 - \frac{\bar{w}}{\varepsilon}\right)$, the producer chooses an initial capacity $N_1 = E(\tilde{W}_2) = \frac{W_1}{p_d}$ and always adjusts his capacity from N_1 to $N_2 = W_2$.*

Proof. see Appendix □

Lemma 2.3.4 tells us that, under the stated condition, the producer chooses an initial capacity equal to the expected wealth of households in period 2. However, he will always adjust his initial production choice to match the households' resources in period-2. The intuition is as follows: Since only the households value and consume the producer's good, the latter would not want a production level greater than the period-2 realized wealth of the households as he would not be able to sell the excess product. Similarly, it would not be sensible and profitable for the producer to set a production level smaller than the period-2 realized wealth of households as he would

sell less. If adjustment costs are not too large, the producer always wants to match the realized demand. Hence, the total expected profit of the producer is:

$$\begin{aligned} E(\Pi_p) &= (1 - c)E(\tilde{W}_2) - \frac{l}{2}E[(W_2 - E[\tilde{W}_2])^2] \\ &= (1 - c)E(\tilde{W}_2) - \frac{l}{2}Var(\tilde{W}_2) \end{aligned}$$

where the term $\frac{l}{2}Var(\tilde{W}_2)$ characterizes the negative demand externality that arises in the economy. Intuitively, higher variance of households' wealth implies higher costs to the producer (when investing, households do not internalize these costs). The following Proposition characterizes the relationship between investors' misperception and demand externality.

Proposition 2.3.1. *An increase in misperception will decrease the period-2 variance of wealth of households. Thus, decreasing the demand externality.*

Proof. see Appendix □

The above Proposition states that an increase in misperception will effectively decrease the demand externality. Intuitively, as households prefer intermediated investment, this will result in the concentration of systematic risk to the financial sector. Households are better insured and there may be less risk for the real economy in the presence of demand externalities. If misperception was not present, households would have the tendency to overinvest in type k projects. This would result to systematic risk being allocated to the households. Hence, in the presence of misperception, any investment decision plays a crucial part on the economy's entirety.

2.3.4 Equilibrium

We now introduce the equilibrium definition of the model:

1. No entry in the financial sector.
2. Market for type k and type u projects clears.
3. Deposit market clears.
4. Market for producer's good clears and producer maximizes profits.

5. Equity market clears.

Proposition 2.3.2. *Given $\bar{w} \in [0, \varepsilon]$, the adjustment condition $l \leq \min\left(\frac{1-c}{(1-\theta)B}, \frac{c}{\theta B}\right)$ where $B \equiv \frac{W_1}{W_1+E_1} \left(1 - \frac{\bar{w}}{\varepsilon}\right)$ and the condition $W_1 + E_1 < \theta[\Phi_u(\theta - c_b) + \Phi_k(\theta)]$, there exists a unique interior equilibrium. In this unique interior equilibrium:*

1. *The price of type u projects is $p_u = \theta p_d^* - c_b$.*
2. *The price of type k projects is $p_k = \theta p_d^*$.*
3. *Households invest $W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon}\right)$ in deposits and $W_1 \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon}\right)$ in type k projects.*
4. *The equilibrium price on deposits p_d^* satisfies equation (2.16).*
5. *The equilibrium value α^* satisfies equations (2.12) and (2.15).*
6. *The producer chooses initial capacity $N_1 = \frac{W_1}{p_d}$ and always adjusts his production scale to $N_2 = W_2$.*
7. *The return on bank's equity r_e equals $1/p_d^*$.*

Proof. see Appendix for the proof of Equilibrium Statement 7. All other Equilibrium Statements have been previously proved. $\square$

Let us discuss the above Proposition in detail. Equilibrium Statements 1 and 2 establish the prices of the projects in the economy as in equilibrium banks have no incentive to enter the market and the existing ones must price these projects the same. Equilibrium Statements 3, 4 and 5 are derived by analysing the total and individual market clearing conditions for projects. Equilibrium Statement 6 provides us with the producer's choice of production scale for both periods. Finally, Equilibrium Statement 7 requires that bank shareholders must obtain their opportunity cost for supplying the bank with their funds; it is essential for the equity market to clear.

2.4 The Trade-off between Financial and Demand Stability

In this section, we analyse how misperception affects the link between the financial and the real sector through the channel of systematic risk. In particular, we determine the trade-off between potential financial disruption and real sector activi-

ties. The following Propositions establish that an increase in misperception allocates more systematic risk to the financial sector. This in turn leads to higher expected bank losses and bailout costs:

Proposition 2.4.1. *Given $\bar{w} \in [0, \varepsilon]$, an increase in misperception will decrease the direct investment of households in type k projects and will increase the investment in deposits. Thus, more risk will be allocated to the financial sector.*

Proof. Follows from market clearing conditions (2.10) and (2.13). $\square$

Proposition 2.4.2. *Given $\bar{w} \in [0, \varepsilon]$, by allocating more risk to the financial sector, an increase in misperception will increase the expected bank losses, thus increasing expected bailout costs.*

Proof. We begin by examining the assets and liabilities of the representative bank under the bad state of nature in period 2. On the assets' side of the bank, we only have the cash flows from type u projects (as mentioned before, all type k projects pay zero in the bad state), which is $\Phi_u(p_u)\theta$. On the liabilities' side, the bank is funded by deposits $\frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right)$ so that its period-2 equity is

$$E_2 = \Phi_u(p_u)\theta - \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \quad (2.17)$$

The above expression is derived by subtracting total liabilities from total assets. If the bank defaults in the bad state and is unable to repay its depositors (as all equity is wiped out), then the government finances a bailout and society pays a cost of

$$\xi \min(0, E_2) \quad (2.18)$$

i.e., a bailout is necessary only if $E_2 < 0$. Since the shortfall E_2 is decreasing in misperception, the higher the misperception, the more exposed to systematic risk the bank is and hence, the higher is the social cost of a bailout by the government. $\square$

Note here that both the above Propositions are true as long as the equilibrium is “interior” ($\varepsilon > \bar{w}$) in that also households invest in type k projects. By comparing Proposition 2.3.1 with Propositions 2.4.1 and 2.4.2, we see that when misperception increases, a trade-off emerges in terms of financial and real stability. On the one hand, more risk taken by the financial sector leads to higher expected bank losses and

bailout costs. On the other hand, less risk is borne by households, thus decreasing demand externality.

It is important to discuss here how the composition of projects in the economy affects demand stability. Recall that any increase of period-1 equity (banks become better capitalised) affects the composition of projects in the economy. This depends on the steepness of $\Phi_u(p_u)$ and $\Phi_k(p_k)$. If $\Phi_u(p_u)$ is steeper relatively to $\Phi_k(p_k)$, then there would be more type k projects in the economy. Effectively, this would increase the variance of period-2 wealth of the households, and thus increase the demand externality. If the opposite occurs, i.e. $\Phi_k(p_k)$ is steeper relative to $\Phi_u(p_u)$, then the composition of projects would move towards type u projects, which would decrease the variance of period-2 wealth, and thus decrease demand externality.

2.5 Optimal Demand for Deposits

In the presence of misperception, the demand for deposits may vary from the welfare-maximizing one. In this section, we analyse the optimal demand for deposits when a social planner allocates funds being only interested in the objective probability θ . He thus determines the share β of wealth to be invested in deposits and the share $1 - \beta$ to be invested in type k projects. We want to compare β with the solution for the decentralised case found in Proposition 2.3.2, i.e $\beta^{dec} = (\frac{1}{2} + \frac{\bar{w}}{2\varepsilon})$.

In the planner's case, welfare consists of the following: The households' period-2 expected utility, the expected shortfall of bank capital⁵ and the producer's period-2 expected utility. We do not include the surplus of projects' owners and the rate of return of bank's shareholders (which equals $1/p_d^*$) as they are constant with respect to β :

- The households' expected utility in period 2 is

$$E(U_H) = \beta \frac{W_1}{p_d^*} + (1 - \beta) \frac{W_1}{p_k} \theta \quad (2.19)$$

where the first term corresponds to the share invested in deposits and the second term to the share invested in type k projects.

⁵The expected shortfall is considered a cost in our model, hence it should have a negative sign. However, for simplicity we use it in absolute value.

- The expected shortfall of bank capital is derived by subtracting the share invested in deposits (liabilities' side of the bank) from the total assets:

$$|ES| = \left| \theta \Phi_u(\theta p_d^* - c_b) - \beta \frac{W_1}{p_d^*} \right| \quad (2.20)$$

- Finally, the producer's expected utility⁶ which coincides with his expected profits in period 2 is:

$$\begin{aligned} E(\Pi_p) &= (1 - c)E(\tilde{W}_2) - \frac{l}{2}Var(\tilde{W}_2) \\ &= (1 - c) \left[\beta \frac{W_1}{p_d^*} + (1 - \beta) \frac{W_1}{p_k} \theta \right] - \frac{l}{2} \theta (1 - \theta) (1 - \beta)^2 \left(\frac{W_1}{p_k} \right)^2 \end{aligned} \quad (2.21)$$

By summing up equations (2.19), (2.20) and (2.21) and substituting for $p_k = \theta p_d^*$, we obtain the total welfare in the economy:

$$TW = (2 - c) \frac{W_1}{p_d^*} + \xi \left[\theta \Phi_u(\theta p_d^* - c_b) - \beta \frac{W_1}{p_d^*} \right] - \frac{l}{2} \frac{1 - \theta}{\theta} (1 - \beta)^2 \left(\frac{W_1}{p_d^*} \right)^2 \quad (2.22)$$

where p_d^* is determined by equation (2.16), which is unchanged. The planner determines the optimal share β by solving the following maximization problem:

$$\max_{\beta} \quad (2 - c) \frac{W_1}{p_d^*} + \xi \left[\theta \Phi_u(\theta p_d^* - c_b) - \beta \frac{W_1}{p_d^*} \right] - \frac{l}{2} \frac{1 - \theta}{\theta} (1 - \beta)^2 \left(\frac{W_1}{p_d^*} \right)^2$$

Welfare is maximized by taking the first order condition with respect to β :

$$\beta^{opt} = 1 - \frac{\xi}{l} \frac{p_d^*}{W_1} \frac{\theta}{1 - \theta} \quad (2.23)$$

Equation (2.23) gives us the optimal amount invested in deposits by households. As ξ increases, the optimal β becomes smaller. Intuitively, this tells us that as the cost of a bank bailout becomes higher for the society, households should invest less in deposits. On the other hand, an increase in l affects the adjustment cost of the producer. Effectively, it increases the demand externality that arises from investing in correlated projects, which in turn increases the optimal β . Hence, households

⁶see Appendix A.6 for proof.

should invest more in deposits. It follows that the optimal choice largely depends on these cost parameters.

Having solved for the welfare-maximizing β , we are now able to solve for the optimal demand for deposits:

$$\begin{aligned}
D_d^{opt} &= W_1 \beta^{opt} \\
&= W_1 \left(1 - \frac{\xi}{l} \frac{p_d^*}{W_1} \frac{\theta}{1 - \theta} \right) \\
&= W_1 - \frac{\xi}{l} \frac{\theta}{1 - \theta} p_d^*
\end{aligned} \tag{2.24}$$

Hence, the difference between optimal and decentralised demand for deposits is

$$\begin{aligned}
D_d^{opt} - D_d^{dec} &= W_1 - \frac{\xi}{l} \frac{\theta}{1 - \theta} p_d^* - W_1 \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \\
&= W_1 \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) - \frac{\xi}{l} \frac{\theta}{1 - \theta} p_d^*
\end{aligned} \tag{2.25}$$

The following Proposition summarizes the main result from comparing the optimal and the decentralized demand for deposits:

Proposition 2.5.1. *Given $\bar{w} \in [0, \varepsilon]$, if ξ/l is large, then there is excessive intermediation in the decentralized economy. If ξ/l is small, then there is too little intermediation in the decentralised economy.*

Proof. If ξ/l is large, this means that $\beta^{opt} < \beta^{dec}$, hence households will invest more in deposits in the decentralised economy. Note that β^{dec} does not depend on neither ξ nor l . Hence, for ξ/l sufficiently large we have $\beta^{opt} < \beta^{dec}$. The opposite occurs when ξ/l is small. $\square$

An implication of this result is that, when ξ/l is small (so that $\beta^{opt} > \beta^{dec}$), higher average misperception may be beneficial in terms of welfare, as it increases the amount of risk taken by the financial sector. Intuitively, this is the case when bailout costs are small compared to the demand externality. The opposite applies when ξ/l is large.

2.6 Conclusion

How is systematic risk allocated when households overweight the probability of an unlikely bad event? We answered this question by analysing an economy where households either invest in projects directly or indirectly through issued deposits by financial intermediaries. Our main results showed that within an environment where investors misperceive systematic risk, a trade-off arises. In particular, our model indicates that households prefer intermediated investment, thus allocating systematic risk into the financial sector. On the one hand, this may create financial instability as it increases the potential bank losses and makes bank bailouts more likely. On the other hand, in the presence of demand externalities, households and the real economy are more protected against this type of risk.

Although our prime findings highlight the main implications emerging from our environment of financial intermediation and misperception, it would be interesting to explore different directions. A possible route would be to endogenize the shadow cost of funding ξ in our analysis, e.g. by introducing labour supply and distortionary taxation. It is clear that the choice of distortional taxes by the government plays a crucial role as it affects the labor supply decision of tax payers as well as their consumption decision, when considering contracting with financial intermediaries or a producer (Cardia, Kozhaya and Ruge-Murcia, 2003).

Another possible extension would be to introduce competition in the production of the consumption good. Competition would benefit households as the latter would buy in a lower equilibrium price. Thus, it would be interesting to see what type of demand externalities would arise and how these would be affected by misperception. Finally, another intriguing route would be to consider the case where the social planner chooses a partial bailout. How this notion of optimal deposit insurance would affect the perception of small probability events by the households?

Chapter 3

Searching for Borrowers: The Allocation of Financiers across Markets and Systematic Risk

Abstract

In this paper, we develop a model of entrepreneurial finance in which financiers search for entrepreneurs in two financial markets. The key assumption of the model is that markets are heterogeneous with respect to the number of entrepreneurs located in each one. From a financier's perspective, the market with the higher number of entrepreneurs gives a higher chance of finding a customer and so, given everything else, it is perceived to be "larger" compared to the other. We identify the conditions such that financiers tend to overcrowd the larger market leaving the other one with potential undermatched entrepreneurs. We show that over-concentration of financiers in one market may lead to excessive systematic risk in the economy and to higher financial fragility. Thus, asymmetry in the size of financial markets may accentuate systematic risk and it is one systemic variable that policy makers need to take into consideration.

3.1 Introduction

The 2008 financial crisis has been described as “one of substantial systematic risk” (Farhi and Tirole, 2012) as it was the largest financial markets in terms of both geographical locations and financial products that concentrated most financiers and led the developed economies to the over-exposure of systematic risk, e.g. AAA-ratings were given to billions of dollars of structured finance products of companies listed in London and New York, whose yields failed to account for declines in aggregate economic conditions (Coval, Jurek and Stafford, 2009). Why is it that the largest markets, who also have the most experienced and sophisticated financiers, concentrated so much exposure to systematic risk instead of the smaller, less-sophisticated markets? This paper tries to address this question through the lens of search frictions.

In this paper, we construct a model of entrepreneurial finance in which financiers and entrepreneurs search and match with each other in two financial markets (e.g. London and Hong-Kong). In the words of Phelps (2009), “the capital market is a sort of matching process that matches a financier to an entrepreneur, whom the former sees as having a model compatible with his own model”. Entrepreneurs in each market have a project at their disposal but no funds, while financiers have the funds and seek out projects to invest in. Importantly, entrepreneurs are assumed to be already located in one market or the other and unable to move, while financiers choose freely and without cost in which market to search. Once a financier locates himself in a market and he finds an entrepreneur, they bargain over the distribution of returns associated with the investment. We assume that the bargaining power between the two which determines the final terms of the financial contract, is exogenous.

As noted above, the meeting process between the two agents explicitly relies on the following key features. First, this is an environment where financiers actively decide which market to enter in order to reach an entrepreneur. Second, only the financiers search for the entrepreneurs. In particular, we assume that the latter face restrictions, either geographical or in regard to the type of financing they require, making them immobile between the markets. For this reason, the entrepreneurs are modelled to be exogenously located across the two markets. Most importantly for

the purposes of this model, we assume that markets are heterogeneous with respect to the number of entrepreneurs located in each one. Specifically, we assume that there is a higher number of entrepreneurs in one market compared to the other, implying that from a financier’s perspective, this market is perceived to be “larger”, i.e. given everything else, the probability of being matched to an entrepreneur is greater. One possible interpretation is that the number of entrepreneurs in each market represents the new and unexploited investment options, and hence the larger market is the one with the faster growth of investment opportunities. To the extent that this interpretation is relevant, the model generates predictions about the growth of investment opportunities and systematic risk. Given the number of entrepreneurs and financiers allocated across the two markets, the total number of successful deals in each market is determined via the use of a *matching function*, which gives the number of contracts between searchers, on both sides of the market, at any moment in time (Lagos, 2000a).

Our model identifies and interprets two main channels that generate overcrowding in the larger (or faster-growing) market. In particular, the first channel corresponds to the relative strength of the “stepping-on-toes” effect which arises due to financiers not internalising the negative externality of their individual entry on the other financiers’ probability to be matched. By examining the decentralised private equilibrium relative to the social planner’s optimal allocation of financiers, we find that there is overcrowding in the market where the relative “stepping-on-toes” effect is stronger. The intuition is the following.

In the decentralised equilibrium, any financier cares only of his own probability to be matched and hence of the average probabilities of matches across the markets, while the social planner cares of the marginal effect of additional financiers in the two markets. Under certain conditions that relate to the elasticity of the matching function with respect to the entry of financiers in each market, the average probability to be matched is higher than the marginal one, and more so in the larger market than in the smaller one. We show that the more inelastic is the matching function in the larger market to financiers’ entry, the larger is the discrepancy between the average and the marginal probabilities of matching, and thus the higher is the overcrowding effect in the larger market.

The second channel stems from the different valuation of a successful match, i.e. the relative ratio of the social value of matches across the two markets in comparison to the financiers' private value of the matches. If the ratio of the private values is smaller than the ratio of the social values, then financiers find the “larger” market relative more profitable than the other market in comparison to the planner and so they over-allocate themselves in this market. This effect depends on the bargaining power of financiers and on the probability of a shock hitting each market.

In addition, we introduce a source of aggregate risk in the model in order to examine the implications on systematic risk. In particular, the two markets are susceptible to an external shock which can affect either one market or both markets simultaneously. In the markets affected by the shock, all projects yield no returns. We show that, in the case where markets are asymmetrically affected by the shock so as the ratios of private and social values of a successful match are unequal, overcrowding will occur only in the market which is relatively less affected by the shock. Hence, private and social allocations of financiers do not coincide.

Note that, in line with other studies in the literature, we use the *Value at Risk* (VaR) as a measure of systematic risk. In essence, VaR is equal to the total expected losses that arise from the projects in the event of the external shock. Besides its widespread use in the literature, we use this measure of systematic risk because of its relevance to the latest financial crisis, i.e. it was the measure used by many financial institutions in order to assess their exposure to systematic risks. By comparing the decentralised to the planner's solution, we find a set of necessary and efficient conditions under which the expected losses are higher in the decentralised equilibrium. Thus, if overcrowding takes place in the market, this market is exposed to excessive systematic risk and this excess more than makes up for the reduction of systematic risk in the smaller market.

Our paper is broadly related to the vast literature regarding random and directed search models. The former typically assumes that market participants possess limited information and therefore meet at random¹ (the “nobody knows where anything is” assumption) (Lagos, 2000a). However, the resulting equilibrium of these models

¹See Burdett and Judd (1983); Shi (1995); Trejos and Wright (1995) on goods market. See Mortensen (1988) or Shimer and Smith (2000) on marriage. Papers by Bowden (1980); Blanchard et al., (1989); Smith (1994); Coles and Smith (1996, 1998); Lagos (2000b); Pissarides (2000); Burdett and Smith (2002) are some papers that contribute to labour market applications.

in which agents take the matching frequency as given, is inefficient as it is not able to internalize the externalities in the search process. As a new mechanism, directed search allows agents to target their search and “enables the market to produce the efficient allocation under the constraint of the matching technology” (Shi, 2008).

Lagos (2000a) deserves a special mention. The author constructs a directed² search and matching model in a decentralised dynamic market for taxicab rides. The author allows both searching agents, i.e. taxicabs and customers to direct their search by choosing their locations. By letting this, the author shows that some equilibria may exhibit frictions as long as not all locations are identical from the searching agents’ perspective. He finds that if at least one location is better than another, then taxicabs may overcrowd this location. For all possible contacts that take place in each meeting point, cabs may distribute themselves in such a way where some of them will not find customers and some customers will not be able to find cabs. A key difference from Lagos (2000a) is that in our analysis we specify the matching function exogenously³. From a theoretical point of view, assuming that search is, at least partially, random allows us to examine the effects of search externalities on the levels of systematic risk. It is also empirically relevant for our topic of investigation, as participants in financial markets may fail to find a suitable counter-party, even after a considerable effort or time, due to the idiosyncratic nature of various projects financing conditions.

This paper is closely connected to the literature which depicts entrepreneurial finance (e.g. in venture capital markets) as a search and matching process between financiers and entrepreneurs. Few papers study bilateral random matching and bargaining such as Silveira and Wright (2010, 2016). Other papers focus on the link between entrepreneurial finance, innovation and economic growth, e.g. Giordani (2015) incorporates the process of entrepreneurial finance into an endogenous growth model. Relevant contributions have also been made by Inderst and Muller (2004), Michelacci and Suarez (2004), Wasmer and Weil (2004), Sorensen (2007) and Cipollone and Giordani (2016a, 2016b). Our paper contributes to this literature by

²Introduced by Moen (1997), directed search models have been analysed through a variety of directions. To name a few, see Butters (1977); Peters (1984, 1991); Acemoglu and Shimer (1999); Burdett, Shi, and Wright (2001). For a recent survey on directed search literature, see Wright et al., (2017).

³In Lagos (2000a), the matching function is derived endogenously as any changes in parameters affect the agents’ search strategies, and hence the shape of the matching function.

linking market heterogeneity to overcrowding and systematic risk, something that none of these papers examines.

Our paper is also related to the finance literature that links the fragility of financial markets to systematic risk. Proposed by the seminal work of Markowitz (1952) on portfolio choice as well as by Sharpe (1964), Lintner (1965a, 1965b) and Mossin (1966), systematic or “aggregate” risk represents the risk of an exogenous aggregate event to the market (Zigrand, 2014). This type of risk is undiversifiable and unavoidable. To name a few examples, systematic risk can take the form of an aggregate technology shock, monetary shock or that of a bank failure⁴. These types of systematic events may correspond to extreme shocks, such as in Gabaix (2009) and Barro and Ursua (2011) (Zigrand, 2014). It is important to mention as well that some papers focus their analysis on the determinants of systematic risk in particular industries (e.g. airlines industry) such as Lee and Jang (2007) and Park and Kim (2016). We contribute to this literature by studying the effect of market-size or market-growth asymmetries on aggregate systematic risk.

The structure of the rest of the paper is as follows. In Section 3.2, we present the basic setting and analysis of the model. We continue by examining relevant numerical examples in Section 3.3. In Section 3.4, we propose possible future directions and we conclude the paper.

3.2 The Model

3.2.1 Setting

Consider an economy which consists of $m = \{1, 2\}$ markets across which there is a population of financiers and entrepreneurs. In particular, there is a continuum ϕ of financiers who can enter freely in any market and whose size is normalised to one. There is also a continuum of k entrepreneurs who want to finance their projects and whose mass is also normalised to 1. Entrepreneurs are exogenously split across the two markets. Specifically, a mass of k_1 is allocated in market 1 and a mass of k_2 is allocated in market 2 with $k_1 + k_2 = k$. We consider this exogenous allocation to represent geographical or financial-product related restrictions which

⁴See Sun, Wu and Zhao (2018) for further reading.

entrepreneurs can not relax. For example, entrepreneurs in market 1 may own a firm located in London, while entrepreneurs in market 2 may own a firm in Hong-Kong. Hence, they look for financing from each location's financial centre. Therefore, we assume that entrepreneurs are immobile across the two markets, while financiers are perfectly mobile and they actively decide which market to enter in order to search for entrepreneurs.

Each entrepreneur has an idea for a project which requires an initial investment of I funds. If the project is carried out, then it yields a net return equal to r_p , out of which r_f accrues to the financier and the remainder $r_e = r_p - r_f$ accrues to the entrepreneur. Note that, for simplicity, r_p and r_f are the same for all projects in both markets. Therefore, when the project yields its returns, the financier receives back the amount $(1 + r_f)I$ of funds and the entrepreneur the amount $(1 + r_e)I$. Generically, $r_p \geq r_f$, so that the return to each one of the two investment parties depends implicitly on the split of the bargaining power between the two. When $r_p = r_f$, then financiers have all the bargaining power, while when $r_f = 0$, then entrepreneurs have all the bargaining power. In this model, we consider r_f to be exogenously given.

In each market, financiers enter and search for entrepreneurs. Search is random and risky. With some probability, each financier succeeds in finding an entrepreneur and agrees to finance his project. With the remainder probability, the financier does not find an entrepreneur and no project is financed. For simplicity, we assume that each financier can match with only one entrepreneur at most. Note here that searching/matching, and hence financiers' entry and bargaining is "one-shot" as there is no further searching and re-matching upon disagreement at a first match. The probability of finding an entrepreneur in a particular market depends on the total number of matches between financiers and entrepreneurs in this market, which in turn depends on the mass of the two types in it.

Let ϕ_m and k_m denote the mass of financiers and entrepreneurs in market m , respectively. Then the total mass of successful matches in market m is given by the function

$$T = T(\phi_m, k_m) \tag{3.1}$$

As it is standard in the search and matching literature, this function is called the

matching function and it is assumed to be continuous and twice differentiable. In addition, it is increasing in both its inputs and concave, that is to say

$$\textbf{(i)} \quad \frac{\partial T}{\partial \phi_m} > 0, \frac{\partial T}{\partial k_m} > 0 \text{ and } \textbf{(ii)} \quad \frac{\partial^2 T}{\partial \phi_m^2} < 0, \frac{\partial^2 T}{\partial k_m^2} < 0$$

implying that the number of matches is increasing in both financiers and entrepreneurs, but at a decreasing rate. Furthermore, we assume that there exists $\underline{\phi}$, with $0 \leq \underline{\phi} < 1/2$, such that:

$$\textbf{(iii)} \quad T(\underline{\phi}, k) = 0 \text{ for all } k > 0 \text{ and } \textbf{(iv)} \quad \lim_{\phi_m \rightarrow \underline{\phi}^+} \frac{\partial T}{\partial \phi_m} \rightarrow +\infty$$

Assumption (iii) means that if the mass of financiers in a market reaches the critical minimal threshold $\underline{\phi}$, then the number of successful matches becomes zero. The definition of the critical value $\underline{\phi}$ is general enough to allow it to take the value zero, in which case assumption (iii) reduces to the assumption $T(0, k) = 0$, i.e. when there are no financiers in a market there can be no successful matches. This is the typical assumption used in the literature. Assumption (iv) is a generalised version of the usual Inada condition, i.e. when the critical minimum level of financiers is reached, the marginal change in the probability of creating a successful match becomes infinite.

Because all financiers are homogeneous in their ability to find entrepreneurs, and conditional on the aggregate variables ϕ_m and k_m per market, they all have the same probability of being successfully matched in each market. This probability is equal to the average matches per financier and it is given by

$$q(\phi_m, k_m) = \frac{T(\phi_m, k_m)}{\phi_m} \tag{3.2}$$

with $q : \mathbb{R}^+ \rightarrow [0, 1]$. Note that the earlier assumptions on $T(\cdot)$ imply that the probability of a financier⁵ to be matched is decreasing in the number of competing financiers and increasing in the number of entrepreneurs, respectively. The intuition is straightforward. Essentially, a financier's entry into a market creates two externalities. The first is a negative externality as a sort of a "stepping-on-toes" effect where an additional financier decreases the matching frequency of the other financiers. The

⁵Similarly, the individual probability of an entrepreneur being funded is decreasing in the number of competing entrepreneurs and increasing in the number of financiers.

other externality is positive, where the entry of an additional financier increases the matching frequency of the entrepreneurs (Shi, 2008).

As we mentioned earlier, one of the key features of the model is that markets are heterogeneous with respect to the number of entrepreneurs located in each one, and in particular, market 1 has more entrepreneurs than market 2: $k_1 > k_2$. Since the average probability of being matched for a financier is increasing in the mass of entrepreneurs, then, *ceteris paribus*, a financier has a higher probability of being matched in market 1 than in market 2. In this respect, market 1 can be thought of as “larger” compared to market 2, namely it offers more investment opportunities. One interpretation is that market 1 is growing faster than market 2. Intuitively, k_m represents the mass of new entrepreneurs in the respective market, and hence market 1 can be interpreted as growing faster than market 2. Thus, the model can be seen as a way to link financial market growth to the incentives of financiers to serve certain market segments.

In order to expand this interpretation further and to create theoretical implications for systematic risk, the model needs a source of aggregate risk. This is done by assuming that the economy is susceptible to an external shock which is strong enough to affect a single market or even both markets. If the shock hits, then the affected projects become worthless and yield no returns as a result. To be more precise, there is a probability $s \in [0, 1]$ of a financial shock hitting the economy. This shock can be either local, i.e. hitting only one market or global, hitting both markets at the same time.

Conditional on the financial shock materialising, $p_1 \in [0, 1]$ is the probability that it hits only market 1, $p_2 \in [0, 1]$ is the probability that it hits only market 2 and $p_{1,2} \in [0, 1]$ is the probability that it hits both markets, with $p_1 + p_2 + p_{1,2} = 1$. These three events are taken to be independent from each other. Any market affected by the shock yields zero gross returns from all its projects, namely $r_p = 0$ for the affected markets. Overall, the probability that the shock hits market 1 is $s(p_1 + p_{1,2})$, the probability that the shock hits market 2 is $s(p_2 + p_{1,2})$, and the probability that it does not hit the economy is equal to $1 - s(p_1 + p_2 + p_{1,2}) = 1 - s$. Table 3.1 summarizes the above assumptions. The main intuition is that the allocation of financiers across the two markets is important for determining the systematic risk

in the economy through the implications of the shock for the “Value of investment at Risk”, namely the total investment that will become valueless if the shock realises.

	Shock hits market 2	Shock does not hit market 2
Shock hits market 1	$sp_{1,2}$	sp_1
Shock does not hit market 1	sp_2	$1 - s$

Table 3.1: External shock and ex-ante probabilities

3.2.2 The Planner’s Problem and Solution

Before we proceed to analyse the decentralised equilibrium of the economy, it is useful to present the benchmark solution of a benevolent social planner who decides how to allocate the population of financiers across the two markets in order to maximize the total welfare in the economy. In particular, the total ex-ante economic value V_m^s of a project located in market m is given by:

$$V_m^s = [1 - s(p_m + p_{m,n})](1 + r_p)I - I = [r_p - s(p_m + p_{m,n})(1 + r_p)]I \quad (3.3)$$

where p_m is the conditional probability that the shock hits only market m and $p_{m,n}$ is the conditional probability that the shock hits both markets. Note that V_m^s is obtained by summing the cash inflows when the projects are successful in the respective markets, reduced by the initial capital investment I . In addition, because of the assumption that the net returns are the same within and across markets, V_m^s is independent of the allocation of types, i.e. both financiers and entrepreneurs. Thus, the social planner’s problem becomes one of finding the optimal allocation of financiers $\{\phi_1, \phi_2\}$ that maximizes the ex-ante economic value of both markets:

$$\begin{aligned} & \max_{\{\phi_1, \phi_2\}} \{T(\phi_1, k_1)V_1^s + T(\phi_2, k_2)V_2^s\} \\ & \text{subject to } \sum_m \phi_m = 1 \end{aligned} \quad (3.4)$$

In order to solve the planning problem, we use the Lagrange multipliers method. Let λ denote the Lagrange multiplier on the constraint with regards to the aggregate mass of financiers. Given this constraint, we have:

$$\mathcal{L} = T(\phi_1, k_1)V_1^s + T(\phi_2, k_2)V_2^s - \lambda(\phi_1 + \phi_2 - 1) \quad (3.5)$$

The first order conditions for this problem are:

- $[\phi_1]: \frac{\partial T(\phi_1, k_1)}{\partial \phi_1} V_1^s - \lambda = 0$
- $[\phi_2]: \frac{\partial T(\phi_2, k_2)}{\partial \phi_2} V_2^s - \lambda = 0$
- $[\lambda]: \phi_1 + \phi_2 = 1$

At the optimum, the planner equates the marginal matches between the two markets:

$$\begin{aligned} \{\phi_1^*, \phi_2^*\} : \frac{\partial T(\phi_1, k_1)}{\partial \phi_1} V_1^s &= \frac{\partial T(\phi_2, k_2)}{\partial \phi_2} V_2^s \Rightarrow \\ \frac{\partial T(\phi_1^*, k_1)/\partial \phi_1}{\partial T(\phi_2^*, k_2)/\partial \phi_2} &= \frac{V_2^s}{V_1^s} \end{aligned} \quad (3.6)$$

The above condition characterizes the optimal allocation of financiers in the planner's problem. In particular, it tells us that in equilibrium, the optimal allocation of financiers is achieved by equating the ratio of marginal matches to the inverse of the ratio of the social benefits resulting from these matches. From this point of view, the social planner wants to balance the social benefit across the markets from adding an extra financier. Intuitively, we can interpret this as a solution of optimal allocation where the planner thinks on the marginal. Moreover, the combination of the Inada conditions with the monotonicity of the ratio $(\partial T/\partial \phi_1)/(\partial T/\partial \phi_2)$ with respect to ϕ_1 implies that the planner's solution always exists, it is interior and unique.

Proposition 3.2.1. *The solution to the Planner's Problem is characterized by equation (3.6) and it is always interior and unique.*

Proof. By the Inada conditions, $\lim_{\phi \rightarrow \underline{\phi}} \partial T/\partial \phi_1 = +\infty$ for some $\underline{\phi}$ with $0 \leq \underline{\phi} < 1/2$. Therefore

$$\lim_{\phi_1 \rightarrow \underline{\phi}} \{(\partial T/\partial \phi_1)/(\partial T/\partial \phi_2)\} = +\infty \quad \text{and}$$

$$\lim_{\phi_1 \rightarrow 1-\underline{\phi}} \{(\partial T/\partial \phi_1)/(\partial T/\partial \phi_2)\} = 0$$

By continuity, there exists ϕ_1^* and $\phi_2^* = 1 - \phi_1^*$, with $0 \leq \underline{\phi} < \phi_1^* < 1 - \underline{\phi} \leq 1$ such that $(\partial T/\partial \phi_1)/(\partial T/\partial \phi_2) = V_2^s/V_1^s > 0$. Moreover

$$\frac{\partial [(\partial T/\partial \phi_1)/(\partial T/\partial (1 - \phi_1))]}{\partial \phi_1} = \frac{\frac{\partial^2 T}{(\partial \phi_1)^2}(\partial T/\partial (1 - \phi_1)) + (\partial T/\partial \phi_1)\frac{\partial^2 T}{[\partial (1 - \phi_1)]^2}}{[\partial T/\partial (1 - \phi_1)]^2} < 0$$

Since the ratio $(\partial T/\partial \phi_1)/(\partial T/\partial \phi_2) = V_2^s/V_1^s > 0$ is monotonic in ϕ_1 , the solution $\{\phi_1^*, \phi_2^*\}$ is unique. $\square$

3.2.3 The Decentralised Equilibrium

In the decentralised economy, meetings between financiers and entrepreneurs occur bilaterally. Since markets are not identical from a financier's perspective, the financier's expected payoff in market m is given by:

$$\begin{aligned} U_{\phi,m} &= q(\phi_m, k_m)[(1 - sp_m - sp_{m,n})I(1 + r_f) + (sp_m + sp_{m,n})0 - I] \\ &= q(\phi_m, k_m)I[r_f - s(p_m + p_{m,n})(1 + r_f)] \end{aligned} \quad (3.7)$$

where the term $q(\phi_m, k_m)$ corresponds to the probability of meeting an entrepreneur and the term $I[r_f - s(p_m + p_{m,n})(1 + r_f)]$ corresponds to the financier's private benefit generated by investing in the entrepreneur's project. Particularly, the latter term is obtained by summing up the positive cash inflows when the project is successful, reduced by the initial capital investment I (this is essentially the project's Net Present Value). Let $V_m^d \equiv I[r_f - s(p_m + p_{m,n})(1 + r_f)]$, then equation (3.7) becomes

$$U_{\phi,m} = q(\phi_m, k_m)V_m^d \quad (3.8)$$

The above equation gives us the financier's expected payoff in each market. Financiers care about two key features of the markets when deciding where to locate themselves. First, the number of potential financiers and entrepreneurs in each market and second, the profitability of the match. This implies that the higher the probability of being matched within a market, the greater is the payoff for the financier. Following Lagos (2000a), in equilibrium, there is a distribution of financiers and

entrepreneurs across the two markets, such that, given this distribution, financiers maximize their payoff by optimally choosing where to locate themselves.

In equilibrium, financiers must be indifferent between the two markets as ex-ante they are identical. This is achieved as long as financiers' expected payoff is equal across the two markets. The following Lemma provides us with the necessary equilibrium condition, under which, financiers are indifferent between locating themselves in markets 1 or 2:

Proposition 3.2.2. *There exists a pair $\{\tilde{\phi}_1, \tilde{\phi}_2\}$, with $\tilde{\phi}_2 = 1 - \tilde{\phi}_1$ and $0 \leq \underline{\phi} < \tilde{\phi}_1 < 1 - \underline{\phi} \leq 1$, such that the decentralised equilibrium condition $q(\tilde{\phi}_1, k_1)V_1^d = q(\tilde{\phi}_2, k_2)V_2^d$ holds. Moreover, the solution $\{\tilde{\phi}_1, \tilde{\phi}_2\}$ always exists, it is interior and unique.*

Proof. In the decentralised equilibrium all financiers are indifferent between selecting one market or the other. If this were not true, then there is an incentive for some financiers to reallocate across markets, which would invalidate the original conjecture that the original allocation is an equilibrium. Thus, the condition $q(\phi_1, k_1)V_1^d = q(\phi_2, k_2)V_2^d$ holds in any equilibrium. By substituting equation (3.2) into this condition, we get the following expression:

$$\frac{T(\phi_1, k_1)/\phi_1}{T(\phi_2, k_2)/\phi_2} = \frac{V_2^d}{V_1^d} \quad (3.9)$$

By the assumption that $T(\cdot)$ is a concave function in ϕ and the Inada conditions, we have that $T(\phi_1, k_1)/\phi_1 \rightarrow +\infty$ as $\phi_1 \rightarrow \underline{\phi}$, and similarly, $T(\phi_2, k_2)/\phi_2 = T(1 - \phi_1, k_2)/(1 - \phi_1) \rightarrow +\infty$ as $\phi_1 \rightarrow 1 - \underline{\phi}$. Therefore the ratio on the left-hand side of (3.9) approaches $+\infty$ as ϕ_1 approaches $\underline{\phi}$ and approaches zero as ϕ_1 approaches $1 - \underline{\phi}$. By continuity, there exists $\tilde{\phi}_1$ and $\tilde{\phi}_2 = 1 - \tilde{\phi}_1$, with $\underline{\phi} < \tilde{\phi}_1 < 1 - \underline{\phi}$, such that (3.9) holds exactly.

Moreover, we have that the left-hand side of (3.9) is monotonic in ϕ_1

$$\frac{\partial [(T_1/\phi_1)/(T_2/(1 - \phi_1))]}{\partial \phi_1} = \frac{\left(\frac{T_1' \phi_1 - T_1}{(\phi_1)^2} \right) \frac{T_2}{1 - \phi_1} + \frac{T_1}{\phi_1} \frac{T_2' (1 - \phi_1) - T_2}{(1 - \phi_1)^2}}{\left[\frac{T_2}{1 - \phi_1} \right]^2} < 0$$

Hence, the solution to (3.9) is unique. □

In terms of economic intuition, the decentralised equilibrium condition described by equation (3.9) has a clear interpretation. The ratio of the matches per financier in each market is equal to the inverse of the ratio of the private benefits to financiers resulting from these matches. Essentially, it tells us that in equilibrium, financiers' decision regarding their choice of markets depends on the average matches per market rather than the marginal number of matches, which is the preferred criterion of the social planner. As expected, and as we will demonstrate subsequently, this divergence in choice criteria between the decentralised economy and the planner's solution leads to potentially different allocations and the inefficiency of the decentralised solution.

3.2.4 Systematic Risk

As discussed earlier, the purpose of this paper is to explore the implications of the allocation of financiers across financial markets on systematic risk. In this section, we define the measure of systematic risk that will be used in later analysis. Our preferred measure is the total *Value at Risk* (VaR) in the economy, in the event of the shock. Formally, this is equal to the total expected losses from the projects in the economy, i.e. the loss of the initial funds invested in them:

$$E(VaR) = s(p_1 + p_{1,2})T(\phi_1, k_1) + s(p_2 + p_{1,2})T(\phi_2, k_2) \quad (3.10)$$

where the first and second term correspond to the expected losses resulting from matches in markets 1 and 2, respectively. Clearly, systematic risk depends on the allocation of financiers across the two markets as this determines the total number of projects funded. Thus, systematic risk under the planner's solution may not be equal to the systematic risk in the decentralised equilibrium. We denote the former by $E(VaR^p)$ and the latter by $E(VaR^d)$.

Also note that VaR is one possible measure of systematic risk. Another measure that one could adopt is the aggregate variance of the economy. We use the VaR measure because of its relevance to the latest financial crisis as it was used by many financial institutions in order to assess their exposure to systematic risks as well as its wide-spread use in the financial literature.

3.2.5 Comparison of Allocations and Implications for Efficiency

In this section, we examine the conditions under which the planner's solution differs from the decentralised equilibrium and their implications for economic efficiency. In particular, we seek to identify necessary and sufficient conditions so that the decentralised equilibrium generates a greater concentration of financiers than the planner's solution in the largest of the two markets, namely market 1. As mentioned in the introduction, the motivation for this is the stylised fact that in the financial crisis of 2008 it was the largest financial markets (in terms of both geographical location and financial products) that concentrated most financiers and (over-)exposed western economies to systematic risk.

As we will show shortly, there are two main channels that may create this divergence depending on their direction. One stems from the different way that the matching function enters the welfare calculations of the planner and individual financiers ("marginal" versus "average" calculus, respectively) and one stems from the different valuation of a successful match (total valuation versus financiers' valuation). The following Proposition demonstrates these two channels in a formal manner:

Proposition 3.2.3. *If*

$$\frac{q_1}{q_2} \Big|_{\{\phi_1^*, \phi_2^*\}} \geq \frac{\partial T / \partial \phi_1}{\partial T / \partial \phi_2} \Big|_{\{\phi_1^*, \phi_2^*\}} = \frac{V_2^s}{V_1^s} \geq \frac{V_2^d}{V_1^d} \quad (3.11)$$

then $\tilde{\phi}_1 \geq \phi_1^$, with $\tilde{\phi}_1 > \phi_1^*$ if any of the above inequalities is strict.*

Proof. The two inequalities of (3.11) in tandem imply that $\frac{q_1}{q_2} \Big|_{\{\phi_1^*, \phi_2^*\}} \geq \frac{V_2^d}{V_1^d}$. Therefore, $q(\phi_1^*, k_1)V_1^d \geq q(\phi_2^*, k_2)V_2^d$. Thus, if financiers allocate themselves across the two markets according to the planner's solution, the private benefit from market 1 is at least as large as the private benefit of market 2, and no financier wants to move from market 1 to market 2. Hence, a decentralised allocation with $\tilde{\phi}_2 > \phi_2^*$ can not exist, and so at the decentralised condition where $q(\tilde{\phi}_1, k_1)V_1^d \geq q(\tilde{\phi}_2, k_2)V_2^d$, it must hold that $\tilde{\phi}_1 \geq \phi_1^*$. The last result of the Proposition follows immediately in the above analysis if one of the inequalities in (3.11) is strict. $\square$

Proposition 3.2.3 provides a set of sufficient conditions so that the decentralised allocation generates at least as much concentration of financiers in market 1 as the planner's solution. The reason that it is presented this way is because it disentangles the two channels that may generate over-concentration in market 1. Specifically, the first inequality, namely $\frac{q_1}{q_2}|_{\{\phi_1^*, \phi_2^*\}} > \frac{\partial T / \partial \phi_1}{\partial T / \partial \phi_2}|_{\{\phi_1^*, \phi_2^*\}}$ corresponds to channel one, while the second inequality $\frac{V_2^s}{V_1^s} > \frac{V_2^d}{V_1^d}$ corresponds to channel two.

Channel one is the relative “stepping-on-toes” effect across the two markets. It captures the degree with which the ratio of the average probability of being matched in each market diverges from the ratio of the marginal matching probabilities. The higher this discrepancy is, the higher is the relative matching externality of market 1 in comparison to market 2, and so the greater the over-allocation of financiers in that market. The significance of this channel depends on the curvature of the matching function $T(\cdot)$. For some specific categories of functions, this effect may disappear altogether. For example, if the matching function is a Cobb-Douglas function, then the average value at a point ϕ is equal with the marginal value and the “stepping-on-toes” channel is not operational.

It is worth pointing out that the condition of channel one can be written more compactly in terms of elasticities of the matching function with respect to the mass of financiers:

$$\begin{aligned} \frac{\partial T / \partial \phi_1}{\partial T / \partial \phi_2}|_{\{\phi_1^*, \phi_2^*\}} < \frac{q_1}{q_2}|_{\{\phi_1^*, \phi_2^*\}} &\Leftrightarrow \frac{\partial T / \partial \phi_1}{q_1}|_{\{\phi_1^*, \phi_2^*\}} < \frac{\partial T / \partial \phi_2}{q_2}|_{\{\phi_1^*, \phi_2^*\}} \Leftrightarrow \\ \frac{\partial T(\phi_1^*, k_1)}{\partial \phi_1} \frac{\phi_1^*}{T(\phi_1^*, k_1)} < \frac{\partial T(\phi_2^*, k_2)}{\partial \phi_2} \frac{\phi_2^*}{T(\phi_2^*, k_2)} &\Leftrightarrow \epsilon_{T|\phi}(\phi_1^*) < \epsilon_{T|\phi}(\phi_2^*) \Leftrightarrow \\ \frac{\epsilon_{T|\phi}(\phi_1^*)}{\epsilon_{T|\phi}(\phi_2^*)} < 1 \end{aligned} \tag{3.12}$$

where $\epsilon_{T|\phi}$ denotes the elasticity of the matching function with respect to ϕ . Thus, equation (3.12) states that if the mass-elasticity of market 1 is lower than the mass-elasticity of market 2 at the planner's solution, then the “stepping-on-toes” effect tends to generate over-allocation of financiers in market 1.

Channel two is the relative ratio of the social value of matches across the two markets in comparison to the financiers' private value of the matches. If the ratio

of the private values is smaller than the ratio of the social values, then financiers find market 1 relative more profitable than market 2 in comparison to the planner and so they over-allocate themselves in this market. Clearly, this effect depends on the bargaining power of financiers and on the probability of the shock hitting each market. In the limit cases, where either financiers have all the bargaining power and they accrue all the profits ($r_p = r_f$) or where the two markets are equally likely to be hit by the shock ($p_1 = p_2$), the ratio of private returns equates the ratio of social returns and channel two is not operational.

Proposition 3.2.3 is useful in presenting the general intuition of why there may be over-allocation in market 1, however it requires conditions on endogenous variables that make it hard to discern when exactly it might hold or not. The following Propositions shed more light in this respect by identifying sufficient conditions on exogenous variables such that each channel is operational and induces over-allocation when the other channel is inactive.

Proposition 3.2.4. *Suppose that the matching function is Cobb-Douglas, $T(\phi_m, k_m) = \tau \phi_m^\alpha k_m^{1-\alpha}$, so that the “stepping-on-toes” effect is not operational. Then, $\tilde{\phi}_1 > \phi_1^*$ if and only if:*

$$(r_p - r_f)(p_2 - p_1) > 0 \quad (3.13)$$

Proof. Since the “stepping-on-toes” effect is not operational, then there can be over-allocation of financiers in market 1 if and only if the ratio of social and private values of the two markets diverge:

$$\frac{V_2^s}{V_1^s} > \frac{V_2^d}{V_1^d} \Leftrightarrow \frac{r_p - s(p_1 + p_{1,2})(1 + r_p)}{r_p - s(p_2 + p_{1,2})(1 + r_p)} < \frac{r_f - s(p_1 + p_{1,2})(1 + r_f)}{r_f - s(p_2 + p_{1,2})(1 + r_f)}$$

By doing the algebra, the above expression is equivalent to the one below:

$$\begin{aligned} & r_p[(p_2 + p_{1,2})(1 - s(p_1 + p_{1,2})) - (p_1 + p_{1,2})(1 - s(p_2 + p_{1,2}))] > \\ & r_f[(p_2 + p_{1,2})(1 - s(p_1 + p_{1,2})) - (p_1 + p_{1,2})(1 - s(p_2 + p_{1,2}))] \Leftrightarrow \end{aligned}$$

$$(r_p - r_f)(p_2 - p_1) > 0$$

□

Proposition 3.2.4 gives us the necessary and sufficient condition for over-allocation in market 1 when the “stepping-on-toes” effect is not operational. The intuition of equation (3.13) is clear. There is over-allocation of financiers in market 1 if they do not have full bargaining power and so some surplus goes to entrepreneurs, and if market 2 has a higher probability of being affected by the shock than market 1. When these two conditions are satisfied, market 1 is more attractive than market 2 from a financier’s perspective because it faces less systematic risk, so the expected profit from a successful match in market 1 is greater.

Proposition 3.2.5. *Suppose that $r_p = r_f$ so that channel two is not operational, and suppose that $\frac{\partial \epsilon_{T|\phi}}{\partial \phi} < 0$, $\forall k$. Then, $\tilde{\phi}_1 > \phi_1^*$ if $\phi_1^* > \hat{\phi}$, where the cut-off $\hat{\phi}$ is determined by:*

$$\epsilon_{T|\phi}(\hat{\phi}, k_1) = \epsilon_{T|\phi}(1 - \hat{\phi}, k_2) \quad (3.14)$$

Proof. Since channel two is not operational, $\tilde{\phi}_1 > \phi_1^*$ if and only if $\frac{q_1}{q_2}|_{\{\phi_1^*, \phi_2^*\}} > \frac{\partial T / \partial \phi_1}{\partial T / \partial \phi_2}|_{\{\phi_1^*, \phi_2^*\}}$, or equivalently $\epsilon_{T|\phi}(\phi_1^*, k_1) < \epsilon_{T|\phi}(1 - \phi_1^*, k_2)$. Because of the assumption that $\frac{\partial \epsilon_{T|\phi}}{\partial \phi} < 0$, we have that

$$\frac{\partial \{\epsilon_{T|\phi}(\phi_1, k_1) - \epsilon_{T|\phi}(1 - \phi_1, k_2)\}}{\partial \phi_1} < 0$$

with $\epsilon_{T|\phi}(\phi_1, k_1) - \epsilon_{T|\phi}(1 - \phi_1, k_2) > 0$ as $\phi_1 \rightarrow \underline{\phi}$ and $\epsilon_{T|\phi}(\phi_1, k_1) - \epsilon_{T|\phi}(1 - \phi_1, k_2) < 0$ as $\phi_1 \rightarrow 1 - \underline{\phi}$. Therefore, by continuity and monotonicity, there exists a unique cut-off value $\hat{\phi}$ such that $\epsilon_{T|\phi}(\hat{\phi}, k_1) = \epsilon_{T|\phi}(1 - \hat{\phi}, k_2)$. Moreover, for any $\phi > \hat{\phi}$ one obtains that $\epsilon_{T|\phi}(\phi_1, k_1) - \epsilon_{T|\phi}(1 - \phi_1, k_2) < 0$ and, thus, if $\phi_1^* > \hat{\phi}$ then $\epsilon_{T|\phi}(\phi_1^*, k_1) < \epsilon_{T|\phi}(1 - \phi_1^*, k_2)$ as required. □

Proposition 3.2.5 states that, as long as the elasticity of the matching function with respect to the mass of financiers is decreasing, then there will be an internal cut-off mass of financiers such that the elasticities of the two markets are equalised. In this case, if the planner’s solution lies to the right of the cut-off point, then the elasticity of market 1 will be lower than the elasticity of market 2 and so the decentralised solution will generate more concentration of financiers than the planner’s solution.

The intuition of the result lies with the fact that the discrepancy between marginal and average thinking increases as the difference of elasticities across the two markets increases, with market 1 having lower elasticity than market 2. If the cut-off that equates elasticities across the two markets is below the planner's solution, then in the planner's solution, marginal and average matches differ and so market 1 must have even more concentration of financiers in the decentralized equilibrium.

Thus, Proposition 3.2.5 gives a sufficient condition in terms of the two critical values $\hat{\phi}$ and ϕ_1^* such that the over-crowding of the market with the most entrepreneurs takes place. While the cut-off point $\hat{\phi}$ depends on the curvature of the matching function $T(\cdot)$, the planner's solution ϕ_1^* is still endogenously determined, and so it is still not easy to discern exactly when this result applies or not. However, in section 3.3, we conduct several numerical examples. One of them is devoted to showing that the conditions of Propositions 3.2.3 and 3.2.5 are not vacuous. On the contrary, one can find reasonable parametric values such that channel one generates over-concentration of financiers in market 1, even if channel two is inactive. The next section examines the impact of over-concentration on systematic risk.

3.2.6 Systematic Risk and Over-Concentration of Financiers

As mentioned earlier, in the event of a shock, VaR under the planner's solution is equal to:

$$E(VaR^p) = s(p_1 + p_{1,2})T(\phi_1^*, k_1) + s(p_2 + p_{1,2})T(\phi_2^*, k_2) \quad (3.15)$$

where the first and second term corresponds to the expected losses resulting from matches in markets 1 and 2, respectively. Similarly, VaR for the decentralised economy is equal to:

$$E(VaR^d) = s(p_1 + p_{1,2})T(\tilde{\phi}_1, k_1) + s(p_2 + p_{1,2})T(\tilde{\phi}_2, k_2) \quad (3.16)$$

By comparing the two, one can find a set of necessary and sufficient conditions under which VaR is higher in the decentralised economy, as characterised by Proposition 3.2.6. In particular, we show that in the event of an external shock hitting the markets, the latter will be faced with potential losses. However, the more financiers

overcrowd the larger market, the higher will be the resulting expected losses, thus leading the market to become concentrated with excessive systematic risk.

Proposition 3.2.6.

(a) (Sufficient): If $\tilde{\phi}_1 > \phi_1^*$ and if $\frac{dVaR}{d\phi_1} > 0$ for any $\phi \in [\phi_1^*, \tilde{\phi}_1]$, then there is excessive systematic risk in the decentralised economy. **(b) (Necessary):** Systematic risk is excessive in the decentralised economy only if $\frac{T(\tilde{\phi}_1, k_1) - T(\phi_1^*, k_1)}{T(\tilde{\phi}_2, k_2) - T(\phi_2^*, k_2)} < -\frac{p_2 + p_{1,2}}{p_1 + p_{1,2}}$.

Proof. First, we look at the necessary condition under which the *Value at Risk* in the decentralised economy exceeds the *Value at Risk* in the planner's economy. Hence, from equations (3.15) and (3.16) we have that:

$$\begin{aligned}
E(VaR^d) &> E(VaR^p) \Rightarrow \\
s(p_1 + p_{1,2})T(\tilde{\phi}_1, k_1) + s(p_2 + p_{1,2})T(\tilde{\phi}_2, k_2) &> s(p_1 + p_{1,2})T(\phi_1^*, k_1) + s(p_2 + p_{1,2})T(\phi_2^*, k_2) \Rightarrow \\
(p_1 + p_{1,2})[T(\tilde{\phi}_1, k_1) - T(\phi_1^*, k_1)] + (p_2 + p_{1,2})[T(\tilde{\phi}_2, k_2) - T(\phi_2^*, k_2)] &> 0 \Rightarrow \\
\frac{T(\tilde{\phi}_1, k_1) - T(\phi_1^*, k_1)}{T(\tilde{\phi}_2, k_2) - T(\phi_2^*, k_2)} &< -\frac{p_2 + p_{1,2}}{p_1 + p_{1,2}} \tag{3.17}
\end{aligned}$$

The sufficient part follows directly: if $\tilde{\phi}_1 > \phi_1^*$ and if $\frac{dVaR}{d\phi_1} > 0$ in the interval $[\phi_1^*, \tilde{\phi}_1]$, then $VaR(\tilde{\phi}_1) > VaR(\phi_1^*)$ and the result is obtained. $\square$

Intuitively, Proposition 3.2.6 tells us that if there is an over-allocation of financiers in the decentralised economy and VaR is increasing in the number of financiers, then this economy is faced with excessive systematic risk. We also derive the necessary condition under which the systematic risk becomes excessive in the decentralised economy. Thus, we show that the allocation of financiers across markets may have substantial implications on systematic risk. This is in line with the 2008 financial crisis where the world saw major financial markets to become over-exposed to systematic risk.

3.3 Examples

In this section, we present numerical examples which demonstrate that Proposition 3.2.4, i.e. when the matching function is a Cobb-douglas and the “stepping-on-toes”

effect is not operational, there is an over-allocation of financiers in the larger market and that Proposition 3.2.5, i.e. over-allocation of financiers in the larger market, are not vacuous.

In particular, under the Cobb-Douglas matching function, both decentralised and planner's equilibrium outcomes are equal in the case where financiers hold all the bargaining power, a result that follows when both channels are not in operation (see Proposition 3.2.3). However, if financiers hold some bargaining power, we show that there is an over-allocation of financiers in the decentralised market with the lowest systematic risk (market 1 in our example). This is in accordance with Proposition 3.2.4. However, when we use the Gorman matching function, which makes the “stepping-on-toes” channel operational for some parameters, we find that there is always an over-allocation of financiers in the larger market regardless of the distribution of bargaining power between financiers and entrepreneurs. This validates that Proposition 3.2.5 is not vacuous.

The following table presents the numerical values that we assign to the primitive parameters of the model for the demonstration of these numerical examples:

$\alpha = 0.4$	$k_1 = 10$ and $k_2 = 2$	$\tau = 1.3$
$s = 0.01$	$r_p = r_f = 0.0014$ or $r_p = 0.0017 > r_f = 0.0014$	$\underline{\phi} = 0.1$
$p_1 = 0.25$	$p_2 = 0.6$	$p_{1,2} = 0.15$

Table 3.2: Parameters' numerical values

3.3.1 Cobb-Douglas Matching Function

Consider the Cobb-Douglas matching function of the form

$$T(\phi_m, k_m) = \tau \phi_m^\alpha k_m^{1-\alpha}$$

with $\tau > 0$ and $0 < \alpha < 1$. In the decentralised equilibrium, we have that:

$$\begin{aligned} \frac{T(\tilde{\phi}_1, k_1)}{T(\tilde{\phi}_2, k_2)} &= \frac{V_2^d \tilde{\phi}_1}{V_1^d \tilde{\phi}_2} \Rightarrow \frac{\tau \tilde{\phi}_1^\alpha k_1^{1-\alpha}}{\tau \tilde{\phi}_2^\alpha k_2^{1-\alpha}} = \frac{V_2^d \tilde{\phi}_1}{V_1^d \tilde{\phi}_2} \Rightarrow \\ \tilde{\phi}_1^{\alpha-1} \tilde{\phi}_2^{1-\alpha} \left(\frac{k_1}{k_2} \right)^{1-\alpha} &= \frac{V_2^d}{V_1^d} \Rightarrow \tilde{\phi}_1^{\alpha-1} \tilde{\phi}_2^{1-\alpha} = \frac{V_2^d}{V_1^d} \left(\frac{k_2}{k_1} \right)^{1-\alpha} \Rightarrow \end{aligned}$$

$$\tilde{\phi}_1^{\alpha-1} \tilde{\phi}_2^{1-\alpha} = V_d \quad (3.18)$$

where $V_d \equiv \frac{V_2^d}{V_1^d} \left(\frac{k_2}{k_1} \right)^{1-\alpha}$ for simplicity. In the planner's equilibrium, we have:

$$\begin{aligned} \frac{\partial T(\phi_1^*, k_1)/\partial \phi_1^*}{\partial T(\phi_2^*, k_2)/\partial \phi_2^*} &= \frac{V_2^s}{V_1^s} \Rightarrow \frac{\alpha \tau \phi_1^{*\alpha-1} k_1^{1-\alpha}}{\alpha \tau \phi_2^{*\alpha-1} k_2^{1-\alpha}} = \frac{V_2^s \phi_1^*}{V_1^s \phi_2^*} \Rightarrow \\ \phi_1^{*\alpha-1} \phi_2^{*1-\alpha} \left(\frac{k_1}{k_2} \right)^{1-\alpha} &= \frac{V_2^s}{V_1^s} \Rightarrow \phi_1^{*\alpha-1} \phi_2^{*1-\alpha} = \frac{V_2^s}{V_1^s} \left(\frac{k_2}{k_1} \right)^{1-\alpha} \Rightarrow \\ \phi_1^{*\alpha-1} \phi_2^{*1-\alpha} &= V_s \end{aligned} \quad (3.19)$$

where $V_s \equiv \frac{V_2^s}{V_1^s} \left(\frac{k_2}{k_1} \right)^{1-\alpha}$ for simplicity.

As mentioned above, under the Cobb-Douglas matching function and in the case where financiers hold all the bargaining power ($r_p = r_f$), both decentralised and planner's equilibrium outcomes are equal. By using the parameters' values (as presented in the above table) and solving for the decentralised and the planner's equilibrium solutions (equations (3.18) and (3.19)), we obtain the following: $\tilde{\phi}_1 = 0.543$ and $\tilde{\phi}_2 = 0.457$, whereas $\phi_1^* = 0.543$ and $\phi_2^* = 0.457$. It follows that $\tilde{\phi}_1 = \phi_1^*$ and $\tilde{\phi}_2 = \phi_2^*$, that is the allocation of financiers across markets is equal when both channels are not in operation.

We next show that in the case where financiers hold some of the bargaining power ($r_p > r_f$), there is an over-allocation of financiers in the decentralised market with the lowest systematic risk, that is in market 1 ($p_1 < p_2$). By using the parameters' values from the above table and solving for the decentralised and the planner's equilibrium solutions (equations (3.18) and (3.19)), we obtain the following: $\tilde{\phi}_1 = 0.543$ and $\tilde{\phi}_2 = 0.457$, whereas $\phi_1^* = 0.508$ and $\phi_2^* = 0.492$. It follows that $\tilde{\phi}_1 > \phi_1^*$ and $\tilde{\phi}_2 < \phi_2^*$, that is there an over-allocation of financiers in the decentralised larger market when the “stepping-on-toes” channel is not operational.

3.3.2 Gorman Matching Function

Consider the Gorman matching function of the form

$$T(\phi_m, k_m) = \tau(\phi_m - \underline{\phi})^\alpha k_m^{1-\alpha}$$

with $0 < \alpha < 1$, $\tau > 0$ and $0 \leq \underline{\phi} < \frac{1}{2}$. In the decentralised equilibrium, we have that:

$$\begin{aligned} \frac{T(\tilde{\phi}_1, k_1)}{T(\tilde{\phi}_2, k_2)} &= \frac{V_2^d \tilde{\phi}_1}{V_1^d \tilde{\phi}_2} \Rightarrow \frac{\tau(\tilde{\phi}_1 - \underline{\phi})^\alpha k_1^{1-\alpha} \tilde{\phi}_2}{\tau(\tilde{\phi}_2 - \underline{\phi})^\alpha k_2^{1-\alpha} \tilde{\phi}_1} = \frac{V_2^d}{V_1^d} \Rightarrow \\ \frac{\tilde{\phi}_2}{\tilde{\phi}_1} \left(\frac{\tilde{\phi}_1 - \underline{\phi}}{\tilde{\phi}_2 - \underline{\phi}} \right)^\alpha \left(\frac{k_1}{k_2} \right)^{1-\alpha} &= \frac{V_2^d}{V_1^d} \Rightarrow \frac{\tilde{\phi}_2}{\tilde{\phi}_1} \left(\frac{\tilde{\phi}_1 - \underline{\phi}}{\tilde{\phi}_2 - \underline{\phi}} \right)^\alpha = \frac{V_2^d}{V_1^d} \left(\frac{k_2}{k_1} \right)^{1-\alpha} \Rightarrow \\ \frac{\tilde{\phi}_2}{\tilde{\phi}_1} \left(\frac{\tilde{\phi}_1 - \underline{\phi}}{\tilde{\phi}_2 - \underline{\phi}} \right)^\alpha &= V_d \end{aligned} \quad (3.20)$$

where $V_d \equiv \frac{V_2^d}{V_1^d} \left(\frac{k_2}{k_1} \right)^{1-\alpha}$ for simplicity. In the planner's equilibrium, we have:

$$\begin{aligned} \frac{\partial T(\phi_1^*, k_1)/\partial \phi_1}{\partial T(\phi_2^*, k_2)/\partial \phi_2} &= \frac{V_2^s}{V_1^s} \Rightarrow \frac{\alpha \tau(\phi_1^* - \underline{\phi})^{\alpha-1} (1 - \underline{\phi}) k_1^{1-\alpha}}{\alpha \tau(\phi_2^* - \underline{\phi})^{\alpha-1} (1 - \underline{\phi}) k_2^{1-\alpha}} = \frac{V_2^s}{V_1^s} \Rightarrow \\ \left(\frac{\phi_1^* - \underline{\phi}}{\phi_2^* - \underline{\phi}} \right)^{\alpha-1} \left(\frac{k_1}{k_2} \right)^{1-\alpha} &= \frac{V_2^s}{V_1^s} \Rightarrow \left(\frac{\phi_1^* - \underline{\phi}}{\phi_2^* - \underline{\phi}} \right)^{\alpha-1} = \frac{V_2^s}{V_1^s} \left(\frac{k_2}{k_1} \right)^{1-\alpha} \Rightarrow \\ \left(\frac{\phi_1^* - \underline{\phi}}{\phi_2^* - \underline{\phi}} \right)^{\alpha-1} &= V_s \end{aligned} \quad (3.21)$$

where $V_s \equiv \frac{V_2^s}{V_1^s} \left(\frac{k_2}{k_1} \right)^{1-\alpha}$ for simplicity.

First inspection of equations (3.20) and (3.21), tells us that the decentralised and the planner's equilibrium solutions are not equal. In the case where financiers hold all the bargaining power and solving for equations (3.20) and (3.21) by using the parameters' values in Table 3.2, we obtain the following: $\tilde{\phi}_1 = 0.895$ and $\tilde{\phi}_2 = 0.105$, whereas $\phi_1^* = 0.535$ and $\phi_2^* = 0.465$. It follows that $\tilde{\phi}_1 > \phi_1^*$ and $\tilde{\phi}_2 < \phi_2^*$, that is there is an over-allocation of financiers in the decentralised larger market. However, in the case where financiers hold some of the bargaining power, we obtain that: $\tilde{\phi}_1 = 0.895$ and $\tilde{\phi}_2 = 0.105$, whereas $\phi_1^* = 0.506$ and $\phi_2^* = 0.494$. It follows that $\tilde{\phi}_1 > \phi_1^*$ and $\tilde{\phi}_2 < \phi_2^*$, that is there is an over-allocation of financiers again in the larger market.

Hence, when we use the Gorman matching function which makes the “stepping-on-toes” channel operational for some parameters, we find that there is always an over-allocation of financiers in the larger market regardless of the distribution of bargaining power between financiers and entrepreneurs. Furthermore, in what fol-

lows, we provide a numerical example that demonstrates that market 1 has a lower elasticity than market 2. This is in accordance with Proposition 3.2.5:

Solving for

$$\epsilon_{T|\phi}(\phi_1^*, k_1) = \frac{\partial T(\phi_1^*, k_1)}{\partial \phi_1} \frac{\phi_1^*}{T(\phi_1^*, k_1)}$$

where $\phi_1^* = 0.506$, we obtain that

$$\epsilon_{T|\phi}(\phi_1^*, k_1) = 0.671$$

whereas for

$$\epsilon_{T|\phi}(\phi_2^*, k_2) = \frac{\partial T(\phi_2^*, k_2)}{\partial \phi_2} \frac{\phi_2^*}{T(\phi_2^*, k_2)}$$

and $\phi_2^* = 0.494$, we obtain that:

$$\epsilon_{T|\phi}(\phi_2^*, k_2) = 0.680$$

It follows that $\epsilon_{T|\phi}(\phi_1^*, k_1) < \epsilon_{T|\phi}(\phi_2^*, k_2)$.

3.4 Conclusion

In this paper, we presented a model of entrepreneurial finance in which financiers search and match with entrepreneurs who are exogenously located across two financial markets. Motivated and based on the work by Lagos (2000a), we showed how asymmetries in size and growth across markets (in terms of successful matches) can impact the systematic risk when financiers over-allocate themselves in the market with the highest individual benefit.

In particular, we showed that financiers overcrowd the larger market leaving the other market with potential undermatched entrepreneurs. We did this by comparing the decentralised and the planner's equilibrium outcome. In addition, we showed that, under certain conditions, there is excessive systematic risk in the decentralised compared to the planner's economy. We demonstrated our main results via the use of numerical examples, in which we employed the Cobb-Douglas and the Gorman matching function.

A potential direction for future research is to consider a generalised search and

matching environment with $m \geq 2$ markets across which population of financiers and entrepreneurs may locate themselves. In addition, it would be interesting to examine the case where both agents search and match with costly effort in order to locate themselves in the market with the better matches.

Chapter 4

Bayesian Persuasion and Bayesian Scepticism

Abstract

We study Bayesian persuasion in a strategic environment, where a seller wishes to influence a buyer to buy a security. When the two agents share a common prior belief, we characterise the optimal signals. The innovative feature of our model is that we also allow the buyer to strategically choose her own prior ex-ante. We find that a pessimistic prior belief is optimal and that as the buyer becomes more pessimistic, the seller is more prone to truthful communication. Both evolutionary and psychological interpretations are discussed.

4.1 Introduction

Persuasion plays a crucial role in many real-life situations, i.e. in advertising, in political campaigns, in lobbying, in financial information disclosure and many others. Following the seminal work of Kamenica and Gentzkow (2011) in “Bayesian Persuasion” (henceforward KG), many studies have focused their interest on strategic settings with this type of communication. In particular, one agent, call him sender, seeks to influence another agent, call her receiver, to take a specific action. The receiver is a rational Bayesian individual who understands that the sender (who has full control over information disclosure about the state of the world) will tailor what information to communicate with the sole intent to influence the receiver’s action. This begs the question of which are the optimal signals that the sender must choose in order to gain from persuasion.

Motivated and based on the main framework by KG, we first study a standard problem of Bayesian persuasion, where a seller wishes to persuade a pessimistic buyer to buy a security. Specifically, we ask what are the optimal signals that maximize the seller’s expected profits when the buyer holds pessimistic beliefs regarding the value of the security, i.e. the buyer tends to overestimate the likelihood that the security’s value is lower than its corresponding pricing. In our model, the seller commits to an informative, costless signal which constitutes a mapping from the true state to a distribution over some finite signal realization space (Gentzkow and Kamenica, 2014). The buyer observes the outcome of the signal, updates her prior according to Baye’s rule and then chooses whether to buy or not the security. Both agents are expected utility maximizers.

We start our analysis with the assumption that the seller and the buyer share a common prior belief¹ over the state of the world; that is the value of the security. The seller knows that the buyer is pessimistic and that the buyer, based solely on the prior information, is not inclined to buy the security. In this informational environment, we find the optimal signals under which the seller gains from persuasion. In particular, we establish the necessary conditions under which the seller, through his choice of signals, maximize his expected utility (by making the buyer to buy the

¹Papers such as Kamenica and Gentzkow (2011), Gentzkow and Kamenica (2017a, 2017b) to name a few, study Bayesian persuasion under the assumption of the common prior belief.

security).

We next depart from the common prior assumption and we allow the buyer to strategically “choose” her own prior ex-ante. In this case, the buyer holds a subjective belief over the state, which again is pessimistic. We make the following two assumptions regarding of how the two agents view and process the information in this environment. First, the fact that the players now hold different priors about the state is common knowledge, that is they “agree to disagree”. Second, the new, subjective belief of the buyer works as a commitment device, i.e. she is committed to use it. We show that as the buyer becomes more pessimistic, the seller is induced to “almost” truthfully communicate the true state to the buyer. We find that the buyer optimally becomes extremely sceptical, indicating a natural response when faced with situations of manipulative behaviour.

According to Aytekin (2015), a sceptical mind may doubt almost everything. For example, a sceptical consumer is more likely to question an ad claim critically and not accept it at face value or even disbelieve it entirely. In the field of marketing, researchers argue that consumer scepticism towards advertising claims is a necessary and beneficial skill that protects consumers from the persuasive and potentially misleading efforts of advertisers. In other words, a consumer may “choose” to be sceptical and thus, his or her scepticism may act as a defensive coping mechanism towards persuasive messages (Koslow, 2000). Our work is motivated by this view by showing that a buyer’s pessimistic belief towards the persuasive intent of the seller may become highly sceptical to achieve protective benefit from any potential manipulative claim. To understand how defense motivated consumers respond to such claims, we consider two interpretations.

One interpretation is a psychological one, where we consider that the process from objective to subjective probabilities is a “hard-wired subconscious” one as in Brunnermeier and Parker (2005). In particular, the strategic setting may directly cause an unconscious processing of data in the mind of the buyer, thus creating behavioural tendencies. For any degree of pessimism, this automatic processing may affect the buyer to naturally respond with scepticism in order to resist or cope with situations that are perceived to be deceptive. Another interpretation considers behavior which confers evolutionary advantage. In particular, the buyer’s choice of

a sceptical prior belief turns out to be optimal since its effects are advantageous to the buyer, i.e. it confers a higher fitness. Nevertheless, each approach signifies how defense motivated consumers may challenge a system of manipulative tactics and persuasive claims.

Several papers in the rapidly growing Bayesian persuasion literature study Bayesian communication settings, but with a variation of assumptions and features. Kolotilin et al., (2017) study persuasion mechanisms, where an uninformed sender wishes to influence a receiver who privately knows his preference type. This approach differs from ours, as in our model the sender knows that the receiver is pessimistic. Other papers such as Perez-Richet (2014) and Alonso and Câmara (2018) study Bayesian persuasion with a privately informed sender. An environment with multiple senders is analysed in Li and Norman (2015), Board and Lu (2016) and Gentzkow and Kamenica (2017a, 2017b). Gentzkow and Kamenica (2014) introduce cost functions into their Bayesian persuasion analysis, while Gentzkow and Kamenica (2017b) study competition in persuasion.

Our paper also relates to the strand of literature that analyses persuasion and heterogeneous priors. In particular, Alonso and Câmara (2016) study the gain that a sender receives from controlling the information available to a decision maker when both individuals hold different prior beliefs; they openly disagree. Our work differs, as we focus on a situation where heterogeneity in prior beliefs stems from the receiver who strategically chooses her own prior. Other papers such as Yildiz (2004), Van den Steen (2004, 2009, 2011), Che and Kartik (2009) and Hirsch (2016) show that heterogeneous priors incentivize more the agents to acquire information, as each one believes that new evidence will support his or hers view and hence “persuade” others (Alonso and Câmara, 2016).

Finally, this paper is closely related to models of optimal beliefs which focus on anticipatory utilities of forward-looking agents and assume that individuals choose subjective biased beliefs departing from the real probabilities (Chen, 2013). Brunnermeier and Parker (2005) deserve a special mention as they show that forward-looking agents who care about their expected future utility, obtain higher current felicity if they have optimistic biased beliefs. Our main difference is that in a strategic setting, the bias may take another direction; it may be towards pessimism rather

than optimism.

In Chen’s (2013) model, agents choose the optimal subjective beliefs and optimal expectations as in Brunnermeier and Parker (2005), but they are reference dependent and loss averse. The author shows that higher anticipation produces current felicity but also greater future loss. Agents manipulate their beliefs in order to trade off between the gain from an optimistic future and the cost from a loss. Spiegel (2008) proved that in Brunnermeier and Parker (2005) the decision maker is not averse to information as the support of subjective beliefs does not update to signals that entail uncertainty (Chen, 2013). Mayraz (2011) by providing experimental evidence shows that all subjective judgments are subject to wishful thinking bias.

The structure of the paper is as follows. In the following section we present the basic setting and analysis of the model. Specifically, we characterise the equilibrium in the case where both agents share the same prior as well as in the case where the buyer chooses her own prior. In Section 4.3, we discuss the main implications of our work and we interpret our findings. Section 4.4 proposes directions for further research and concludes the paper.

4.2 The Model

4.2.1 The Setting

Consider the seller of a security, who is trying to persuade a buyer to buy it at a fixed price $p \in (0, 1)$. Let v denote the value of the security to the buyer. The security can take two values: 1 with prior probability $\alpha \in [0, 1]$ and 0 with probability $1 - \alpha$. For the moment, we assume that both seller and buyer share the same prior belief $Pr(v = 1) = \alpha$, which is the true, “objective” prior. Later on, we will allow the buyer to choose her own prior. A key assumption of the model is that the buyer is sufficiently pessimistic about the value of the security, i.e. $\alpha < p$, so that, based solely on the prior information, she would not buy.

The game between the seller and the buyer is as follows. The seller chooses a signal π , which consists of a finite realization space S and a family of probability distributions $\{Pr(\cdot|\omega)\}_{\omega \in \Omega}$ over S , where ω is the state of the world. We assume that signals are costless to the seller and we restrict attention to binary signals with

possible realizations H and L. The buyer observes the seller's choice of signal and each signal realization $s \in S$ induces the buyer to form a posterior belief over the states. The buyer updates her beliefs according to Bayes's rule and chooses whether to buy or not. Lastly, payoffs are realized and the game ends.

In our model, the seller is not committed to truthfully communicate the signal realization to the buyer, hence the choice of signal is a complete set of conditional probabilities denoted as **a)** $Pr(H|v = 1) = \mu$ **b)** $Pr(L|v = 1) = 1 - \mu$ **c)** $Pr(H|v = 0) = q$ and **d)** $Pr(L|v = 0) = 1 - q$. In the case where the seller truthfully communicates to the buyer, then $\mu = 1$ and $q = 0$. Regardless of the state, the seller's payoff is

$$V_s = \begin{cases} p, & \text{if buyer buys;} \\ 0, & \text{otherwise.} \end{cases} \quad (4.1)$$

and the buyer's payoff is

$$V_b = \begin{cases} v - p, & \text{if buyer buys;} \\ 0, & \text{otherwise.} \end{cases} \quad (4.2)$$

4.2.2 Optimal Signals

In this section, we examine the optimal signals that maximize seller's expected profits. The following Lemma provides the seller with the necessary conditions in terms of choice of signals, which induce the buyer to always buy when receiving the signal H:

Lemma 4.2.1. *It is always optimal for the seller to set $\mu = 1$ and q such that $Pr(v = 1|H) = p$, i.e. buyer is indifferent between buying and not buying when observing $s = H$.*

Proof. We begin by showing that it is optimal for the seller to choose $\mu = 1$. In particular, we show that there is no equilibrium where the buyer buys under both signal realizations H and L. Suppose that he does buy:

- Buyer buys when $s = H$:

$$Pr(v = 1|H) \geq p \Rightarrow \quad (4.3)$$

$$\frac{\alpha\mu}{\alpha\mu + (1-\alpha)q} \geq p \quad (4.4)$$

- Buyer buys when $s = L$:

$$Pr(v = 1|L) \geq p \Rightarrow \quad (4.5)$$

$$\frac{(1-\mu)\alpha}{(1-\mu)\alpha + (1-q)(1-\alpha)} \geq p \quad (4.6)$$

We show that equations (4.4) and (4.6) do not hold simultaneously. From equation (4.4), we solve for μ and we find that

$$\mu \geq \frac{(1-\alpha)pq}{\alpha(1-p)} \quad (4.7)$$

Simplifying inequality (4.6) gives us:

$$\begin{aligned} (1-\mu)\alpha &\geq p(1-\mu)\alpha + p(1-\alpha)(1-q) \Rightarrow \\ (1-\mu)\alpha - p(1-\mu)\alpha &\geq p(1-\alpha)(1-q) \Rightarrow \\ (1-\mu)\alpha(1-p) &\geq p(1-\alpha)(1-q) \end{aligned} \quad (4.8)$$

Substituting the lower bound of μ from (4.7), we obtain the following contradiction

$$\begin{aligned} \alpha(1-p) - (1-\alpha)pq &\geq (1-\alpha)(1-q)p \Rightarrow \\ \alpha(1-p) &\geq (1-\alpha)(1-q)p + (1-\alpha)pq \Rightarrow \\ \alpha(1-p) &\geq (1-\alpha)p[(1-q) + q] \Rightarrow \\ \alpha(1-p) &\geq (1-\alpha)p \Rightarrow \\ \alpha - \alpha p &\geq p - \alpha p \Rightarrow \\ \alpha &\geq p \end{aligned} \quad (4.9)$$

which does not hold by assumption. Hence, the buyer cannot buy for both signal realizations. Without loss of generality, we thus restrict attention to the buyer not buying when $s = L$. The expected utility of the seller is:

$$\begin{aligned}
E(U_s) &= Pr(v = 1)[Pr(H|v = 1)(p) + Pr(L|v = 1)(0)] \\
&\quad + Pr(v = 0)[Pr(H|v = 0)(p) + Pr(L|v = 0)(0)]
\end{aligned} \tag{4.10}$$

and by substituting we get that

$$\begin{aligned}
E(U_s) &= \alpha(\mu p + 0) + (1 - \alpha)(qp + 0) \\
&= \alpha\mu p + (1 - \alpha)qp \\
&= p[\alpha\mu + (1 - \alpha)q]
\end{aligned} \tag{4.11}$$

The above equation is increasing in both μ and q . This informs us that equation (4.4) must be binding. If it was slack, then an increase in q would increase the expected utility of the seller. Hence, since it is binding, we have that

$$\begin{aligned}
\alpha\mu &= p\alpha\mu + p(1 - \alpha)q \Rightarrow \\
q &= \frac{\alpha(1 - p)\mu}{p(1 - \alpha)}
\end{aligned} \tag{4.12}$$

Substituting equation (4.12) in equation (4.11), we find that the expected utility of the seller is

$$E(U_s) = \alpha\mu \tag{4.13}$$

which is maximized for $\mu = 1$. Hence, substituting $\mu = 1$ in equation (4.12), we obtain

$$q = \frac{\alpha(1 - p)}{p(1 - \alpha)} \tag{4.14}$$

□

The above Lemma establishes that the seller will send the H signal when the security has no value with some positive probability $q > 0$. Intuitively, the seller could lie a bit while still not stopping the buyer from finding it optimal to buy under the H signal, i.e. the buyer would still gain surplus. The propensity to lie, q , can then be increased to the point where the buyer becomes indifferent between buying or not buying upon observing the H signal. Note here that when the buyer is indifferent, his buying probability is equal to 1. If the buyer was mixing, the seller

would have an incentive to reduce q . We also need to mention that there exists an equivalent situation where the seller sets μ and q such that $Pr(v = 1|L) = p$, i.e. the buyer only buys if signal is L.

Proposition 4.2.1. *There exists an equilibrium where for $\mu = 1$ and $q = \frac{\alpha(1-p)}{p(1-\alpha)}$, the buyer only buys when the signal realization is $s = H$.*

The above is not a unique equilibrium as there also exists a “mirror image” equilibrium, where for $q = 1$ and $\mu = \frac{p(1-\alpha)}{\alpha(1-p)}$, the buyer buys when observes $s = L$ and never buys when $s = H$. Also, note that truthtelling is generally not an equilibrium. The seller prefers to send the “good” signal with some probability even when the security has no value ($v = 0$).

4.2.3 Buyer’s Optimal Prior

If the buyer could credibly *commit* ex-ante to a degree of scepticism, i.e. a different prior, what would she strategically choose? In this section, we consider whether the buyer would benefit from adopting a prior other than the true, “objective” prior belief α . Let $\hat{\alpha} < \alpha$ denote the subjective, sceptical prior of the buyer. We assume, for technical reasons, that $\hat{\alpha}$ is bounded away from 0, i.e. $\hat{\alpha} \in [\varepsilon, 1]$ where $\varepsilon > 0$ is arbitrarily small. Otherwise, there would exist no solution to the buyer’s optimal prior problem as there would be no room for persuasion.

We also assume that when evaluating her own welfare ex-ante, the buyer uses the objective prior (see Brunnermeier and Parker, 2005). In this perspective, we can think of the objective prior as an anchor, which helps us to assess the effectiveness of adjusting to the subjective prior. The following Lemma shows that by strategically choosing her own prior, the buyer can induce the seller to (almost) truthfully reveal the value of the security.

Definition 1. *Let $q(\hat{\alpha}) \equiv \frac{\hat{\alpha}(1-p)}{p(1-\hat{\alpha})}$ denote the probability of signal H with $v = 0$ chosen by the seller, when the buyer’s prior is $\hat{\alpha}$.*

Lemma 4.2.2. *The more pessimistic the buyer becomes, the more is the seller induced to truthfully communicate the signal realization s to the buyer, i.e. $q(\hat{\alpha})$ is increasing in $\hat{\alpha}$.*

Proof. The proof follows from Definition 1. □

Proposition 4.2.2. *The optimal prior for the buyer involves extreme scepticism: $\hat{\alpha} = \varepsilon$. In equilibrium, the seller misleads the buyer with vanishingly small probability, i.e. $q = \varepsilon$.*

Proof. The problem of the buyer is to maximize her expected utility:

$$\begin{aligned} E(U_b) = & Pr(v = 1)[Pr(H|v = 1)(1 - p) + Pr(L|v = 1)(0)] \\ & + Pr(v = 0)[Pr(H|v = 0)(-p) + Pr(L|v = 0)(0)] \end{aligned} \quad (4.15)$$

Since seller and buyer do not share the same prior and with $\hat{\alpha}$ being the subjective prior of the buyer, we have that $Pr(H|v = 0) = q(\hat{\alpha})$, where $q(\hat{\alpha}) = \frac{\hat{\alpha}(1-p)}{p(1-\hat{\alpha})}$. By substituting, we get that:

$$\begin{aligned} E(U_b) &= \alpha(1 - p) + (1 - \alpha)[q(\hat{\alpha})(-p) + 0] \\ &= \alpha(1 - p) - (1 - \alpha)q(\hat{\alpha})p \\ &= \alpha(1 - p) - (1 - \alpha) \left[\left(\frac{\hat{\alpha}(1 - p)}{p(1 - \hat{\alpha})} \right) p \right] \end{aligned} \quad (4.16)$$

Inspection of equation (4.16) tells us that the expected utility of the buyer is maximized for $\hat{\alpha} = \varepsilon$. This means that the buyer becomes even more sceptic about the value of the security, since her subjective prior $\hat{\alpha}$ gets the lowest value. Effectively, as the buyer becomes even more pessimistic about buying, the seller is induced to choose $q = \varepsilon$. □

Hence, $\hat{\alpha}$ constitutes the optimal subjective prior of the buyer, which drives the seller to “almost” truthfully communicate s .

4.3 Discussion and Interpretation

In this paper, we consider a standard Bayesian persuasion model, where the seller (sender) of a security attempts to persuade a buyer (receiver) to buy it. Following the framework of KG, the seller chooses an informative signal regarding the state

of the world, the buyer observes the realization of the signal, updates her beliefs according to Baye’s rule and then takes an action.

In the standard model, both seller and buyer share a common prior belief. The seller chooses his signals in order to achieve the desired action from the buyer. Proposition 4.2.1 shows that the optimal signals which maximize the expected utility of the seller, involve misleading the buyer with positive probability when the security value is low. We see that the seller benefits from sending the “good” signal, even if the security has no value, as the buyer will always buy when she observes an H signal realization (unless the seller sends the H signal too often).

The main innovation of this paper is that we also allow the buyer to choose her prior strategically. In this respect, the standard Bayesian persuasion model has a partial equilibrium flavour and if the buyer’s prior is allowed to change, it may change in a way that adversely affects the seller. The key assumption is commitment from the part of the buyer, in that the prior the buyer “chooses” must be known to the seller and that the buyer is committed to use it². This commitment assumption is strong, as it ensures the seller’s knowledge of the buyer’s subjective prior. A standard justification regarding seller’s knowledge comes for “learning”; the seller either through his own experiences or through communication with others learns about the buyer’s prior before choosing a signal (Acemoglu, Chernozhukov and Yildiz, 2006).

In Proposition 4.2.2, we show that by choosing an extremely pessimistic prior, the buyer induces the seller to “almost” truthfully communicate the signal realization. The result suggests a natural reaction to persuasion, i.e. people become more sceptical. There are possible interpretations regarding the meaning and significance of this result. First, we consider an evolutionary approach. The motivation behind this is that evolution shapes individual beliefs, and thus behaviour. Similar in spirit to the approach taken by several studies on preference evolution, e.g. Bester and Güth (1998) and subsequent literature, here, we consider the evolution of beliefs in a strategic environment.

According to Robson and Samuelson (2011), one view is that evolution “hard-wires” individuals with behaviour, equipping them with a rule on how to respond

²In this case, both agents in the model “agree to disagree” since the buyer is now endowed with a subjective belief regarding the value of the security.

in each possible situation. Alternatively, we could think of evolution as a process which shapes our preferences rather than simply programming us with behaviour. Nevertheless, the focus lies primarily “on behaviour which confers evolutionary advantage”. In our model, the buyer chooses a prior belief which turns out to be optimal. The fact that her perceptual ability allows her to select an action whose effects carry on well with her beliefs, may suggest a behaviour which is evolutionary advantageous. To clarify this point, consider the following. In the case where the buyer uses her true prior, her expected utility becomes zero:

$$\begin{aligned}
E(U_b)_{true} &= \alpha(1-p) + (1-\alpha)[q(\alpha)(-p) + (1-q(\alpha))(0)] \\
&= \alpha(1-p) - (1-\alpha) \left[\frac{\alpha(1-p)}{p(1-\alpha)} p \right] \\
&= \alpha(1-p) - \alpha(1-p) \\
&= 0
\end{aligned} \tag{4.17}$$

However, when the buyer uses her optimal prior, her expected utility is:

$$\begin{aligned}
E(U_b)_{opt} &= \alpha(1-p) + (1-\alpha)[q(\hat{\alpha})(-p) + (1-q(\hat{\alpha}))(0)] \\
&= \alpha(1-p) + (1-\alpha)[q(\hat{\alpha})(-p)]
\end{aligned} \tag{4.18}$$

Note here that the objective prior applies in the buyer’s ex-ante calculation in order to assess whether the “choice” of a sceptical prior confers higher fitness. The above equation is maximized when her optimal prior gets the lowest possible value, i.e. $\hat{\alpha} = \varepsilon$ and $\varepsilon \approx 0$. By substituting $q(\hat{\alpha}) = \frac{\hat{\alpha}(1-p)}{p(1-\hat{\alpha})} = \frac{\varepsilon(1-p)}{p(1-\varepsilon)}$ in equation (4.18), we have that

$$\begin{aligned}
E(U_b)_{opt} &= \alpha(1-p) + (1-\alpha) \left[\frac{\varepsilon(1-p)}{p(1-\varepsilon)} (-p) \right] \\
&= \alpha(1-p) - (1-\alpha) \frac{\varepsilon}{1-\varepsilon} (1-p) > 0
\end{aligned} \tag{4.19}$$

Inspection of equations (4.17) and (4.19), tells us that the expected utility of the buyer is lower under the true prior rather than under the optimal prior ($E(U_b)_{true} <$

$E(U_b)_{opt}$) showing that “rational” individuals have a lower fitness than sceptical individuals, thus indicating an evolutionary advantage for the latter.

Another interpretation is a psychological one *à la* Brunnermeier and Parker (2005). The authors, view the process from objective to subjective probabilities as a “hard-wired subconscious” one. This framework allows us to interpret our result based rather on the automatic and unconscious process of the limbic system³ than that of the conscious control of perception (Bargh and Chartrand, 1999). In particular, we may consider that the resultant behaviour of the buyer constitutes an automatic route in perception which is activated by the external environment. The strategic setting may directly cause an unconscious processing of data in the mind of the buyer, thus creating behavioural tendencies. If this automatic processing is pessimistic to start with, the agent may then tend to naturally respond to situations with pessimistic biases (Brunnermeier and Parker, 2005).

4.4 Conclusion

In this paper, we studied Bayesian persuasion of a pessimistic receiver. Based on the influential work by Kamenica and Gentzkow (2011), we obtained the optimal signals that maximize the seller’s expected utility. Later on, we departed from the assumption of the common prior and we allowed the buyer to commit to a subjective prior *ex-ante*. We found that a pessimistic prior is optimal. This suggests that when people are faced with manipulative behaviour, they may respond to situations with greater scepticism. We interpreted this result through psychological channels as well as through the evolutionary route of shaping beliefs.

It would be interesting to analyse the case of an endogenous rather than a fixed price. Specifically, how the subjective prior of the buyer would be affected if the seller were to choose a particular pair of signal structure and price? Would it still remain optimal? Another possible route would be to examine the case of multiple senders. In particular, examine how each seller’s choice of signal will affect the prior of the buyer when it tends towards pessimism. What would be the impact to the buyer’s prior if the sellers collude as in Gentzkow and Kamenica (2017a)?

³The limbic system is the part of the brain which supports a variety of functions such as behavior, emotion and others.

Chapter 5

Conclusion

This thesis has studied three topics in Financial Markets Theory. Each chapter constitutes a self-contained paper which explores different, theoretical in nature, financial frameworks.

In Chapter 2, we examined how systematic risk is allocated when households overweight the probability of an unlikely bad event. In particular, we analysed an economy where households invest in projects directly or indirectly through bank deposits. Our main findings showed that in an environment where investors misperceive systematic risk, they prefer intermediated over direct investment, resulting in the allocation of systematic risk to the financial sector. In this perspective, a trade-off arises. On the one hand, this may create financial fragility as it increases the potential bank losses and makes bank bailouts more likely. On the other hand, in the presence of demand externalities, households and the real economy are more protected against this type of risk.

In Chapter 3, we investigated a model of entrepreneurial finance in which financiers search and match with entrepreneurs across two financial markets. The key assumption of the model is that markets are heterogeneous with respect to the number of entrepreneurs located in each one, and hence the market with the higher number of entrepreneurs is perceived to be larger from a financier's perspective. We identified the conditions such that financiers tend to overcrowd the larger market leaving the other one with potential undermatched entrepreneurs. We showed that over-concentration of financiers in one market may lead to excessive systematic risk in the economy and to higher financial fragility. Thus, asymmetry in the size of

financial markets may accentuate systematic risk. We demonstrated these results with the use of numerical examples.

In Chapter 4, we studied Bayesian persuasion of a pessimistic receiver. Motivated and based on the influential work by Kamenica and Gentzkow (2011), we obtained the optimal signals that maximize the seller's expected utility in the case where both seller and buyer share a common prior belief. Later on, we departed from the assumption of the common prior and we allowed the buyer to commit to a subjective prior ex-ante. We found that a pessimistic prior is optimal. This finding suggests that when people are faced with manipulative behaviour, they tend to respond to these situations with greater scepticism. We interpreted this result through psychological and evolutionary channels.

Appendices

Appendix A

Appendix to Chapter 2

A.1 Proof of Lemma 2.3.1

Proof. We start by showing that the Net Present Value of type k projects (NPV_k) and the Net Present Value of type u projects (NPV_u) must be zero.

The NPV of type k projects is

$$\frac{\theta}{R_B} - p_k \tag{A.1}$$

and the NPV of type u projects is

$$\frac{\theta}{R_B} - p_u - c_b \tag{A.2}$$

where R_B is the cost of capital.

- If $NPV_k < 0$ and $NPV_u < 0$, then the bank would invest less in both projects (the market for projects would not clear).
- If $NPV_k < 0$ and $NPV_u > 0$ or $NPV_k > 0$ and $NPV_u < 0$, then the bank would transfer funds to the projects with the higher NPV (the market for projects would not clear).
- If $NPV_k > 0$ and $NPV_u > 0$, then there would be incentive for another bank to enter the market and offer higher return in a smaller price (the market for projects would not clear).

Hence, we have that $NPV_k = 0$ and $NPV_u = 0$. Finally, we have to prove that $R_B = \frac{1}{p_d}$. We know that the cost of capital R_B is the weighted average of the cost

of equity (r_e) and cost of debt ($\frac{1}{p_d}$):

$$R_B = r_e \frac{E}{D + E} + \frac{1}{p_d} \frac{D}{D + E} \quad (\text{A.3})$$

where for simplicity, D and E denote deposits and equity, respectively.

- If $R_B > \frac{1}{p_d}$, then $r_e > \frac{1}{p_d}$. This means that equity is more expensive and the bank would offer more debt (the market for equity does not clear).
- If $R_B < \frac{1}{p_d}$, then $r_e < \frac{1}{p_d}$. This means that equity is cheaper, i.e. more supply and less demand for equity (the market for equity does not clear).

Hence, from the above we have that $R_B = \frac{1}{p_d}$. Therefore, we have that $p_k = \theta p_d$ and $p_u = \theta p_d - c_b$. $\square$

A.2 Proof of Lemma 2.3.2

Proof. First, we show uniqueness. Note that the left-hand side (LHS) of equation (2.16) is a constant. Hence, it is sufficient for uniqueness that the right-hand side (RHS) of equation (2.16) is monotonic. Differentiating the right-hand side with respect to p_d yields

$$\theta[\Phi_u(\theta p_d - c_b) + \Phi_k(\theta p_d)] + \theta^2 p_d [\phi_u(\theta p_d - c_b) + \phi_k(\theta p_d)] > 0$$

since $\Phi_u > 0$, $\Phi_k > 0$, $\phi_u > 0$ and $\phi_k > 0$. Second, we show existence. Note that for $p_d = 0$, the right-hand side of equation (2.16) is zero. Since the LHS > 0 , we just need to check that LHS $<$ RHS when p_d is at its upper bound, $p_d = 1$ (If $p_d > 1$, households strictly prefer holding their cash to investing it). This requires

$$E_1 + W_1 < \theta[\Phi_u(\theta - c_b) + \Phi_k(\theta)] \quad (\text{A.4})$$

$\square$

A.3 Proof of Lemma 2.3.4

Proof. We start our proof by showing that $N_1 = E(W_2) = \frac{W_1}{p_d}$. The profit of the producer is

$$\Pi_p = (1 - c)W_2 - \frac{l(W_2 - N_1)^2}{2} \quad (\text{A.5})$$

He finds it optimal to adjust capacity so as to match the household's resources at the price of one per unit. The producer chooses his initial capacity N_1 in order to maximize his profits:

$$\Pi_p = (1 - c)E(W_2) - \frac{lE[(W_2 - N_1)^2]}{2} \quad (\text{A.6})$$

We take the FOC with respect to N_1 and we impose that :

$$\frac{\partial \Pi_p}{\partial N_1} = 0 \quad (\text{A.7})$$

$$-2\frac{l}{2}N_1 + 2\frac{l}{2}E(W_2) = 0 \Rightarrow \quad (\text{A.8})$$

$$lN_1 = lE(W_2) \Rightarrow \quad (\text{A.9})$$

$$N_1 = E(W_2) \quad (\text{A.10})$$

where

$$E(W_2) = \theta W_1 \left[\frac{1}{\theta p_d} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) + \frac{1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right] + (1 - \theta) \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \quad (\text{A.11})$$

In the above equation, the first term corresponds to the households' period-2 wealth in the good state and the second term corresponds to the households' period-2 wealth in the bad state (recall that in the bad state, we only have deposits since type k projects pay zero). Thus, solving equation (A.11) we obtain

$$N_1 = E(W_2) = \frac{W_1}{p_d} \quad (\text{A.12})$$

We next show that $N_2 = W_2$. Suppose that $W_2 \geq N_1$, then the producer chooses

$N_2 \in (N_1, W_2)$ in order to maximize his profits ($N_1 < N_2 \leq W_2$):

$$\begin{aligned} \max_{N_2} \quad & (1-c)N_2 - \frac{l(N_2 - N_1)^2}{2} \\ \text{subject to} \quad & N_2 \leq W_2 \end{aligned}$$

By using the Lagrange multipliers method, we have that:

$$\mathcal{L}(N_2) = (1-c)N_2 - \frac{l(N_2 - N_1)^2}{2} - \lambda(N_2 - W_2) \quad (\text{A.13})$$

Provided the constraint is binding, the solution must satisfy the following conditions:

$$\frac{\partial \mathcal{L}}{\partial N_2} = 0 \Rightarrow \quad (\text{A.14})$$

$$(1-c) - lN_2 + lN_1 - \lambda = 0 \Rightarrow \quad (\text{A.15})$$

$$\lambda = (1-c) - l(N_2 - N_1) \quad (\text{A.16})$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \quad (\text{A.17})$$

$$N_2 = W_2 \quad (\text{A.18})$$

For $\lambda \geq 0$, we have that:

$$(1-c) - l(N_2 - N_1) \geq 0 \Rightarrow \quad (\text{A.19})$$

$$l \leq \frac{1-c}{N_2 - N_1} \quad (\text{A.20})$$

Suppose now that $W_2 < N_1$, then the producer chooses $N_2 \in (W_2, N_1)$ so as to maximize his profits ($W_2 \leq N_2 < N_1$):

$$\begin{aligned} \max_{N_2} \quad & W_2 - cN_2 - \frac{l(N_2 - N_1)^2}{2} \\ \text{subject to} \quad & W_2 \leq N_2 \end{aligned}$$

By using the Lagrange multipliers method, we have that:

$$\mathcal{L}(N_2) = W_2 - cN_2 - \frac{l(N_2 - N_1)^2}{2} - \lambda(W_2 - N_2) \quad (\text{A.21})$$

Provided the constraint is binding, the solution must satisfy the following conditions:

$$\frac{\partial L}{\partial N_2} = 0 \Rightarrow \quad (\text{A.22})$$

$$-c - lN_2 + lN_1 + \lambda = 0 \Rightarrow \quad (\text{A.23})$$

$$\lambda = c + lN_2 - lN_1 \quad (\text{A.24})$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \quad (\text{A.25})$$

$$N_2 = W_2 \quad (\text{A.26})$$

For $\lambda \geq 0$, we have that:

$$c + l(N_2 - N_1) \geq 0 \Rightarrow \quad (\text{A.27})$$

$$l \geq \frac{-c}{N_2 - N_1} \quad (\text{A.28})$$

but $(N_2 - N_1) < 0$, hence

$$l \leq \frac{c}{N_1 - N_2} \quad (\text{A.29})$$

In the case where $N_2 \geq N_1$, we have that:

$$N_2 - N_1 = N_2^g - N_1 = W_1 \left[\frac{1}{\theta p_d} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) + \frac{1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right] - \frac{W_1}{p_d} \Rightarrow \quad (\text{A.30})$$

$$N_2 - N_1 = \frac{W_1}{\theta p_d} \left[\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} + \theta \frac{1}{2} + \theta \frac{\bar{w}}{2\varepsilon} - \theta \right] \Rightarrow \quad (\text{A.31})$$

$$N_2 - N_1 = \frac{W_1}{\theta p_d} \left[\frac{1}{2}(1 - \theta) - \frac{\bar{w}}{2\varepsilon}(1 - \theta) \right] \Rightarrow \quad (\text{A.32})$$

$$N_2 - N_1 = \frac{1 - \theta}{\theta} \frac{W_1}{p_d} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \quad (\text{A.33})$$

In the case where $N_2 < N_1$, we have that:

$$N_1 - N_2 = N_1 - N_2^b = \frac{W_1}{p_d} - \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \Rightarrow \quad (\text{A.34})$$

$$N_1 - N_2 = \frac{\theta W_1}{\theta p_d} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \quad (\text{A.35})$$

Consider equation (2.16):

$$W_1 + E_1 = \theta p_d [\Phi_u(\theta p_d - c_b) + \Phi_k(\theta p_d)] \quad (\text{A.36})$$

Since $\Phi_k(p_k) \leq 1$ and $\Phi_u(p_u) \leq 1$, then it must be that $p_d \geq \frac{W_1 + E_1}{2\theta}$. This provides a lower bound for p_d . The condition for $N_2 = W_2$ requires that

$$l \leq \min \left\{ \frac{1 - c}{(1 - \theta)A(p_d)}, \frac{c}{\theta A(p_d)} \right\}, \quad (\text{A.37})$$

where

$$A(p_d) \equiv \frac{W_1}{\theta p_d} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \quad (\text{A.38})$$

Since $A(p_d)$ is inversely related to p_d , a sufficient condition for (A.37) to hold for all p_d is that it holds when p_d is at the lower bound, i.e.

$$l \leq \min \left\{ \frac{1 - c}{(1 - \theta)B}, \frac{c}{\theta B} \right\}, \quad (\text{A.39})$$

where, replacing p_d with its lower bound,

$$B \equiv \frac{W_1}{W_1 + E_1} \left(1 - \frac{\bar{w}}{\varepsilon} \right). \quad (\text{A.40})$$

□

A.4 Proof of Proposition 2.3.1

Proof. The producer's expected profits are

$$\begin{aligned} E(\Pi_p) &= (1 - c)E(\tilde{W}_2) - \frac{l}{2}E[(W_2 - E[\tilde{W}_2])^2] \\ &= (1 - c)E(\tilde{W}_2) - \frac{l}{2}Var(\tilde{W}_2) \end{aligned} \quad (\text{A.41})$$

We start by looking at the aggregate wealth of the households in period 2, in both states of nature. If households in period 1 have wealth W_1 and invest in deposits and type k projects, their period-2 wealth in the good state is

$$W_2^g = \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) + \frac{W_1}{p_k} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \quad (\text{A.42})$$

and in the bad state¹

$$W_2^b = \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) + 0 \quad (\text{A.43})$$

Therefore, the variance of households' wealth in period 2 will be:

$$Var(W_2) = E[W_2]^2 - (E[W_2])^2 \Rightarrow \quad (\text{A.44})$$

$$\begin{aligned} Var(W_2) &= \theta \left[\frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) + \frac{W_1}{p_k} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \right]^2 + (1 - \theta) \left[\frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right]^2 \\ &\quad - \left[\theta \left(\frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) + \frac{W_1}{p_k} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \right) + (1 - \theta) \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right]^2 \end{aligned} \quad (\text{A.45})$$

For simplicity, we denote

$$x \equiv \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \quad (\text{A.46})$$

and

$$y \equiv \frac{W_1}{p_k} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \quad (\text{A.47})$$

Hence, equation (A.45) becomes:

¹In the bad state, type k projects default, hence they pay zero.

$$Var(W_2) = \theta[x+y]^2 + (1-\theta)x^2 - [\theta(x+y) + (1-\theta)x]^2 \Rightarrow \quad (A.48)$$

$$Var(W_2) = \theta[x+y]^2 + x^2 - \theta x^2 - \theta^2[x+y]^2 - (1-\theta)^2 x^2 - 2x\theta[x+y](1-\theta) \Rightarrow \quad (A.49)$$

$$Var(W_2) = \theta(1-\theta)[x+y]^2 + \theta(1-\theta)x^2 - 2x\theta(1-\theta)[x+y] \quad (A.50)$$

and taking as common factor $\theta(1-\theta)$, we have

$$Var(W_2) = \theta(1-\theta)[x+y]^2 + x^2 - 2x[x+y] \Rightarrow \quad (A.51)$$

$$Var(W_2) = \theta(1-\theta)[x^2 + y^2 + 2xy + x^2 - 2x^2 - 2xy] \Rightarrow \quad (A.52)$$

$$Var(W_2) = \theta(1-\theta)y^2 \quad (A.53)$$

Substituting for y , we get that the variance of period-2 wealth is:

$$Var(W_2) = \theta(1-\theta) \left[\frac{W_1}{p_k} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) \right]^2 \quad (A.54)$$

For all $\bar{w} \in [0, \varepsilon]$, the above expression is decreasing in $\bar{w}$. Thus, equation (A.54) shows that an increase in misperception, will reduce the variance of period-2 wealth. This in turn will reduce the adjustment cost $\frac{l}{2}(N_2 - N_1)^2$, thus decreasing the demand externality. $\square$

A.5 Proof of Proposition 2.3.2

Here, we prove Equilibrium Statement 7 where the return on bank's equity r_e equals $1/p_d^*$:

Proof. In the case where there is no shortfall, the return on equity in the good state is:

$$r_e^g = \frac{1}{E_1} \left[\Phi_u(p_u)\theta + \Phi_k(p_k) - \frac{W_1}{p_k} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) - \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right] \quad (A.55)$$

and in the bad state is:

$$r_e^b = \frac{1}{E_1} \left[\Phi_u(p_u)\theta - \frac{W_1}{p_d} \left(\frac{1}{2} + \frac{\bar{w}}{2\varepsilon} \right) \right] \quad (\text{A.56})$$

From equilibrium, we have that (for simplicity, we denote deposits as D_1):

$$\Phi_k(p_k)\theta p_d^* = W_1 \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) + \alpha(E_1 + D_1) \Rightarrow \quad (\text{A.57})$$

$$\Phi_k(p_k) = \frac{W_1}{\theta p_d^*} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) + \alpha \frac{E_1 + D_1}{\theta p_d^*} \quad (\text{A.58})$$

and

$$\Phi_u(p_u)\theta = \frac{(1 - \alpha)(E_1 + D_1)}{p_d^*} \quad (\text{A.59})$$

Substituting for $p_k = \theta p_d^*$ and from equilibrium in the good state, we have:

$$r_e^g = \frac{1}{E_1} \left[\frac{(1 - \alpha)(E_1 + D_1)}{p_d^*} + \frac{W_1}{\theta p_d^*} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) + \alpha \frac{E_1 + D_1}{\theta p_d^*} - \frac{W_1}{\theta p_d^*} \left(\frac{1}{2} - \frac{\bar{w}}{2\varepsilon} \right) - \frac{1}{p_d^*} D_1 \right] \quad (\text{A.60})$$

Simplifying and multiplying by θ , we get that:

$$r_e^g = \frac{1}{E_1} \frac{1}{p_d^*} [E_1(\theta - \alpha\theta + \alpha) + D_1(\alpha - \alpha\theta)] \quad (\text{A.61})$$

Substituting from equilibrium in the bad state, we have:

$$r_e^b = \frac{1}{E_1} \left[\frac{(1 - \alpha)(E_1 + D_1)}{p_d^*} - \frac{1}{p_d^*} D_1 \right] \quad (\text{A.62})$$

Simplifying and multiplying by $(1 - \theta)$, we get that

$$r_e^b = \frac{1}{E_1} \frac{1}{p_d^*} [E_1(1 - \alpha - \theta + \alpha\theta) + D_1(\alpha\theta - \alpha)] \quad (\text{A.63})$$

Summing up equations (A.61) and (A.63), the return on equity is:

$$r_e = \frac{1}{p_d^*} \quad (\text{A.64})$$

□

A.6 Proof of Producer's Expected Profits

Proof. The producer's expected profit in period 2 is

$$E(\Pi_p) = (1 - c)E(\tilde{W}_2) - \frac{l}{2}Var(\tilde{W}_2)$$

The expected wealth of the households in period 2 comes from investing in deposits and type k projects:

$$E(\tilde{W}_2) = \beta \frac{W_1}{p_d} + (1 - \beta) \frac{W_1}{p_k} \theta \quad (\text{A.65})$$

From the aggregate wealth of the households in period 2, in both states of nature, we can solve for the period-2 variance of wealth:

$$\begin{aligned} VarW_2 &= \theta \left[(1 - \beta) \frac{W_1}{p_k} - \theta \frac{W_1}{p_k} (1 - \beta) \right]^2 + (1 - \theta) \left[-\theta \frac{W_1}{p_k} (1 - \beta) \right]^2 \\ &= \theta \left[(1 - \beta) \frac{W_1}{p_k} (1 - \theta) \right]^2 + (1 - \theta) \left[-\theta \frac{W_1}{p_k} (1 - \beta) \right]^2 \\ &= \theta(1 - \beta)^2 \left(\frac{W_1}{p_k} \right)^2 (1 - \theta)^2 + (1 - \theta) \theta^2 \left(\frac{W_1}{p_k} \right)^2 (1 - \beta)^2 \quad (\text{A.66}) \\ &= (1 - \beta)^2 \left(\frac{W_1}{p_k} \right)^2 \theta(1 - \theta)[1 - \theta + \theta] \\ &= \theta(1 - \theta)(1 - \beta)^2 \left(\frac{W_1}{p_k} \right)^2 \end{aligned}$$

Substituting equations (A.65) and (A.66) into the producer's expected profit function, we obtain

$$E(\Pi_p) = (1 - c) \left[\beta \frac{W_1}{p_d^*} + (1 - \beta) \frac{W_1}{p_k} \theta \right] - \frac{l}{2} \theta(1 - \theta)(1 - \beta)^2 \left(\frac{W_1}{p_k} \right)^2 \quad (\text{A.67})$$

□

Bibliography

- [1] Acemoglu, D. and Shimer, R., (1999), *Holdups and Efficiency with Search Frictions*, International Economic Review, Vol. 40, No. 4, pp. 827-849.
- [2] Acemoglu, D., Chernozhukov, V. and Yildiz, M., (2006), *Learning and disagreement in an uncertain world*, NBER Working Paper, No. 12648.
- [3] Adrian, T. and Shin, H., (2008), *Financial Intermediaries, Financial Stability and Monetary Policy*, Federal Reserve Bank of New York Staff Reports, No. 346, pp. 1-37.
- [4] Alonso, R. and Câmara, O., (2018), *On the Value of Persuasion by Experts*, Journal of Economic Theory, Vol. 174, pp. 103-123.
- [5] Alonso, R. and Câmara, O., (2016), *Bayesian Persuasion with Heterogeneous Priors*, Journal of Economic Theory, Vol. 165, pp. 672-706.
- [6] Aytekin, P., (2015), *Persuasion Knowledge and Ad Scepticism*, International Journal of Research in Social Sciences, Vol. 4, No. 10, pp. 8-14.
- [7] Barberis, N., (2013), *The Psychology of Tail Events: Progress and Challenges*, American Economic Review, Vol. 103, pp. 611-616.
- [8] Bargh, J. A. and Chartrand, T. L., (1999), *The Unbearable Automaticity of Being*, American Psychologist, Vol. 54, No. 7, pp. 462-479.
- [9] Barro, R. J. and Ursua, J. F., (2011), *Rare Macroeconomic Disasters*, Annual Review of Economics, Vol. 4, No. 1, pp. 83-109.
- [10] Bartlett, J., (2001), *Sea change*, The VC Buzz, <http://vcexperts.com>.
- [11] Bester, H. and Güth, W., (1998), *Is Altruism Evolutionarily Stable?*, Journal of Economic Behavior and Organization, Vol. 34, pp. 193-209.

- [12] Blanchard, O., Diamond, P., Hall, R. and Yellen, J., (1989), *The Beveridge Curve*, Brookings Papers on Economic Activity, Vol. 1989, No. 1, pp. 1-76.
- [13] Board, S. and Lu, J., (2016), *Competitive Information Disclosure in Search Markets*, mimeo.
- [14] Bowden, R., (1980), *On the Existence and Secular Stability of u - v Loci*, *Economica*, Vol. 47, No. 185, pp. 35-50.
- [15] Brunnermeier, M. K., and Parker, J. A., (2005), *Optimal Expectations*, *American Economic Review*, Vol. 95, No. 4, pp. 1092-1118.
- [16] Burdett, K. and Judd, K., (1983), *Equilibrium Price Dispersion*, *Econometrica*, Vol. 51, No. 4, pp. 955-969.
- [17] Burdett, K., Shi, S. and Wright, R., (2001), *Pricing and Matching with Frictions*, *Journal of Political Economy*, Vol. 109, No. 5, pp. 1060-1085.
- [18] Burdett, K. and Smith, E., (2002), *The Low Skill Trap*, *European Economic Review*, Vol. 46, pp. 1439-1451.
- [19] Burns, Z., Chiu, A. and Wu, G., (2010), *Overweighting of Small Probabilities*, *Encyclopedia of Operations Research and Management*, John Wiley and Sons.
- [20] Butters, G. R., (1977), *Equilibrium Distributions of Sales and Advertising Prices*, *Review of Economic Studies*, Vol. 44, No. 3, pp. 465-491.
- [21] Camerer, C. F. and Kunreuther, H., (1989), *Decision Processes for Low Probability Events: Policy Implications*, *Journal of Policy Analysis and Management*, Vol. 8, No. 4, pp. 565-592.
- [22] Cardia, E., Kozhaya, N. and Ruge-Murcia, F. J., (2003), *Distortionary Taxation and Labor Supply*, *Journal of Money, Credit and Banking*, Vol. 35, No. 3, pp. 351-373.
- [23] Che, Y. and Kartik, N., (2009), *Opinions as Incentives*, *Journal of Political Economy*, Vol. 117, No. 5, pp. 815-860.
- [24] Chen, S., (2013), *Optimistic versus Pessimistic-Optimal Judgmental Bias with*

- Reference Point*, MPRA Paper No. 50693, University of Oxford, Available online at: <http://mpra.ub.uni-muenchen.de/50693/>.
- [25] Cipollone, A. and Giordani, P. E., (2016a), *Market Frictions in Entrepreneurial Innovation: Theory and Evidence*, LUISS, mimeo.
 - [26] Cipollone, A. and Giordani, P. E., (2016b), *When Entrepreneurs Meet Financiers: Evidence from the Business Angel Market*, University Library of Munich in its series MPRA Paper with number 69545.
 - [27] Coles, M., and Smith, E., (1996), *Cross Section Estimation of the Matching Function: Evidence from England and Wales*, *Economica*, New Series, Vol. 63, No. 252, pp. 589-597.
 - [28] Coles, M., and Smith, E., (1998), *Marketplaces and Matching*, *International Economic Review*, Vol. 39, No. 1, pp. 239-254.
 - [29] Coval, J., Jurek, J. and Stafford, E., (2009), *The Economics of Structured Finance*, *Journal of Economic Perspectives*, Vol. 23, No. 1, pp. 3-25.
 - [30] Diamond, D. W. and Dybvig, P. H., (1983), *Bank Runs, Deposit Insurance and Liquidity*, *Journal of Political Economy*, Vol. 91, pp. 401-419.
 - [31] Diamond, D. W., (1984), *Financial Intermediation and Delegated Monitoring*, *The Review of Economic Studies*, Vol. 51, Issue 3, pp. 393-414.
 - [32] Diamond, D. W. and Rajan, R. G., (2009), *The Credit Crisis: Conjectures about Causes and Remedies*, *American Economic Review*, Vol. 99, pp. 606-10.
 - [33] Diamond, P., (1982), *Aggregate Demand Management in Search Equilibrium*, *Journal of Political Economy*, Vol. 90, No. 5, pp. 881-894.
 - [34] Diamond, P., (1984), *Money in Search Equilibrium*, *Econometrica*, Vol. 52, No. 1, pp. 1-20.
 - [35] Farhi, E. and Tirole, J., (2012), *Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts*, *American Economic Review*, Vol. 102, pp. 60-93.
 - [36] Foote, C. L., Gerardi, K. S. and Willen P. S., (2012), *Why Did So Many People*

- Make So Many Ex Post Bad Decisions? The Causes of the Foreclosure Crisis*, The National Bureau of Economic Research, Working Paper No. 18082.
- [37] Fox, C. and Tversky, A., (1998), *A Belief-based Account of Decision Under Uncertainty*, Management Science, Vol. 44, pp. 879-895.
 - [38] Gabaix, X., (2009), *Power Laws in Economics and Finance*, Annual Review of Economics, Vol. 1, pp. 255-293.
 - [39] Gennaioli, N., Shleifer, A. and Vishny, R., (2013), *A Model of Shadow Banking*, Journal of Finance, Vol. 68, pp. 1331-1363.
 - [40] Gennaioli, N., Shleifer, A. and Vishny, R., (2015), *Neglected Risks: The Psychology of Financial Crises*, American Economic Review: Papers and Proceedings 2015, Vol. 105, No. 5, pp. 310-314.
 - [41] Gentzkow, M. and Kamenica, E., (2014), *Costly Persuasion*, American Economic Review: Papers & Proceedings, Vol. 104, No. 5, pp. 457-462.
 - [42] Gentzkow, M. and Kamenica, E., (2017a), *Bayesian Persuasion with Multiple Senders and Rich Signal Spaces*, Games and Economic Behavior, Vol. 104, pp. 411-429.
 - [43] Gentzkow, M. and Kamenica, E., (2017b), *Competition in Persuasion*, The Review of Economic Studies, Vol. 84, No. 1, pp. 300-322.
 - [44] Giordani, P., (2015), *Entrepreneurial Finance and Economic Growth*, Journal of Economics, Vol. 115, No. 2, pp. 153-174.
 - [45] Gompers, P. A. and Lerner J., (1999), *The Venture Capital Cycle*, Cambridge: MIT Press, Available at: eBook Collection (EBSCOhost).
 - [46] Gompers, P. A. and Lerner J., (2000), *Money Chasing Deals? The Impact of Fund Inflows on Private Equity Valuation*, Journal of Financial Economics, Vol. 55, pp. 281-325.
 - [47] Hertwig, R., Barron, G., Weber, E. U. and Erev, I., (2004), *Decisions from experience and the effect of rare events in risky choice*, Psychological Science, Vol. 15, pp. 534-539.

- [48] Hertwig, R. and Erev, I., (2009), *The Description-Experience Gap in Risky choice*, Trends in Cognitive Science, Elsevier Ltd, Vol. 13, Issue 12, pp. 517-523.
- [49] Hirsch, A. V., (2016), *Experimentation and Persuasion in Political Organizations*, American Political Science Review, Vol. 110, No. 1, pp. 68-84.
- [50] Hosios, A., (1990), *On the Efficiency of Matching and Related Models of Search and Unemployment*, The Review of Economic Studies, Vol. 57, No. 2, pp. 279-298.
- [51] Howitt, P. and McAfee, R., (1987), *Costly Search and Recruiting*, International Economic Review, Vol. 28, No. 1, pp. 89-107.
- [52] Inderst, R. and Muller, H., (2004), *The Effect of Capital Market Characteristics on the Value of Start-up Firms*, Journal of Financial Economics, Vol. 72, pp. 319-356.
- [53] Kamenica, E. and Gentzkow, M., (2011), *Bayesian Persuasion*, American Economic Review, Vol. 101, pp. 2590-2615.
- [54] Kolotilin, A., Mylovannov, T., Zapechelnyuk, A. and Li, M., (2017), *Persuasion of a Privately Informed Receiver*, Econometrica, Vol. 85, pp. 1949-1964.
- [55] Koslow, S., (2000), *Can the Truth Hurt? How Honesty and Persuasive Advertising can Unintentionally Lead to Increased Consumer Skepticism*, Journal of Consumer Affairs, Vol. 34, No. 2, pp. 245-268.
- [56] Lagos, R., (2000a), *An Alternative Approach to Search Frictions*, Journal of Political Economy, Vol. 108, No. 5, pp. 851-873.
- [57] Lagos, R., (2000b), *Unemployment and the Rental Rate of Capital*, Bulletin of Economic Research, Vol. 52, No. 4, pp. 297-322.
- [58] Lee, J. S. and Jang, S. C., (2007), *The Systematic-Risk Determinants of the US Airline Industry*, Tourism Management, Vol. 28 No. 2, pp. 434-442.
- [59] Li, F. and Norman, P., (2015), *On Bayesian Persuasion with Multiple Senders*, University of North Carolina, mimeo.
- [60] Lintner, J., (1965a), *The Valuation of Risk Assets and the Selection of Risky*

- Investments in Stock Portfolios and Capital Budgets*, Review of Economics and Statistics, Vol. 47, pp. 13-37.
- [61] Lintner, J., (1965b), *Security Prices, Risk and Maximal Gains from Diversification*, Journal of Finance, Vol. 20, pp. 587-615.
 - [62] Markowitz, H., (1952), *Portfolio Selection*, Journal of Finance, Vol. 7, pp. 77-91.
 - [63] Mayraz, G., (2011), *Wishful Thinking*, Discussion Paper No. 1092, Centre for Economic Performance, London School of Economics.
 - [64] McClelland, G. H., Schulze, W. D. and Hurd, B., (1990), *The Effects of Risk Beliefs on Property Values: A Case Study of a Hazardous Waste Site*, Risk Analysis, Vol. 10, pp. 485-497.
 - [65] Michelacci, C. and Suarez, J., (2004), *Business Creation and the Stock Market*, Review of Economic Studies, Vol. 71, pp. 459-481.
 - [66] Moen, E. R., (1997), *Competitive Search Equilibrium*, Journal of Political Economy, Vol. 105, No. 2, pp. 385-411.
 - [67] Mortensen, D. T., (1988), *Matching: Finding a Partner for Life or Otherwise*, American Journal Sociology, Vol. 94, pp. 215-240.
 - [68] Mossin, J., (1966), *Equilibrium in a Capital Asset Market*, Econometrica, Vol. 35, pp. 768-783.
 - [69] Park, S. Y. and Kim, S. H., (2016), *Determinants of Systematic Risk in the US Restaurant Industry: A Technical Perspective*, Tourism Economics, Vol. 22, No. 3, pp. 621-628.
 - [70] Perez-Richet, E., (2014), *Interim Bayesian Persuasion: First Steps*, The American Economic Review, Vol. 104, No. 5, pp. 469-474.
 - [71] Peters, M., (1984), *Bertrand Equilibrium with Capacity Constraints and Restricted Mobility*, Econometrica, Vol. 52, No. 5, pp. 1117-1127.
 - [72] Peters, M., (1991), *Ex-Ante Price Offers in Matching Games: Non-Steady State*, Econometrica, Vol. 59, No. 5, pp. 1425-1454.
 - [73] Phelps, E., (2009), *Toward a Model of Innovation and Performance Along the*

- Lines of Knight, Keynes, Hayek and M. Polanyi*, In Z. Acs, D. Audretsch and R. Strom (Eds), *Entrepreneurship, Growth and Public Policy*, Cambridge University Press, Cambridge.
- [74] Pissarides, C., (2000), *Equilibrium Unemployment Theory*, Cambridge: MIT Press.
 - [75] Plantin, G., (2015), *Shadow Banking and Bank Capital Regulation*, Review of Financial Studies, Vol. 28, pp. 146-175.
 - [76] Richter, A., Schiller, J. and Schlesinger, H., (2014), *Behavioral Insurance: Theory and Experiments*, Journal of Risk and Uncertainty, Vol. 48, No. 2, pp. 85-96.
 - [77] Robson, A. J. and Samuelson, L., (2011), *The Evolutionary Foundations of Preferences*, Handbook of Social Economics, North-Holland, Vol. 1, pp. 221-310.
 - [78] Sharpe, W. F., (1964), *Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*, Journal of Finance, Vol. 19, pp. 425-442.
 - [79] Shi, S., (1995), *Money and Prices: A Model of Search and Bargaining*, Journal of Economic Theory, Vol. 67, pp. 467-496.
 - [80] Shi, S., (2008), *Search Theory (New Perspectives)*, in S.N. Durlauf and L.E. Blume eds., *The New Palgrave Dictionary of Economics*, 2nd ed. Macmillan.
 - [81] Shimer, R. and Smith, L., (2000), *Assortative Matching and Search*, Econometrica, Vol. 68, No. 2, pp. 343-369.
 - [82] Silveira, R. and Wright, R., (2010), *Search and the Market for Ideas*, Journal of Economic Theory, Vol. 145, Issue 4, pp. 1550-1573.
 - [83] Silveira, R. and Wright, R., (2016), *Venture Capital: A Model of Search and Bargaining*, Review of Economic Dynamics, Vol. 19, pp. 232-246.
 - [84] Smith, E., (1994), *Taxation in a Two Sided Search Model*, Journal of Public Economics, Vol. 54, pp. 423-435.
 - [85] Snowberg, E. and Wolfers, J., (2010), *Explaining the Favorite-Long Shot Bias*:

- Is It Risk-Love or Misperceptions?*, Journal of Political Economy, Vol. 118, Issue 4, pp. 723-746.
- [86] Sorensen, M., (2007), *How Smart is Smart Money? A Two-Sided Matching Model of Venture Capital*, Journal of Finance, Vol. 62, pp. 2725-2762.
 - [87] Spiegel, R., (2008), *On Two Points of View Regarding Revealed Preference and Behavioral Economics*, In The Foundations of Positive and Normative Economics, Oxford University Press, pp. 95-115.
 - [88] Sun, J., Wu, D. and Zhao, X., (2018), *Systematic Risk Factors and Bank Failures*, Journal of Economics and Business, Available at: <https://www.sciencedirect.com/science/article/pii/S014861951730190X>.
 - [89] Trejos, A. and Wright, R., (1995), *Search, Bargaining, Money and Prices*, Journal of Political Economy, Vol. 103, No. 1, pp. 118-141.
 - [90] Tversky, A. and Kahneman, D., (1992), *Advances in Prospect Theory: Cumulative Representation of Uncertainty*, Journal of Risk and uncertainty, Vol. 5, pp. 297-323.
 - [91] Van den Steen, E., (2004), *Rational Overoptimism (and Other Biases)*, American Economic Review, Vol. 94, No. 4, pp. 1141-1151.
 - [92] Van den Steen, E., (2009), *Authority Versus Persuasion*, American Economic Review, Vol. 99, No. 2, pp. 448-453.
 - [93] Van den Steen, E., (2011), *Overconfidence by Bayesian-Rational Agents*, Management Science, Vol. 57, No. 5, pp. 884-896.
 - [94] Wasmer, E. and Weil, P., (2004), *The Macroeconomics of Labor and Credit Market Imperfections*, American Economic Review, Vol. 94, No. 4, pp. 944-963.
 - [95] Wright, R., Kircher, P., Julien, B. and Guerrieri, V., (2017), *Directed Search: A Guided Tour*, NBER Working Paper No. 23884. Available at SSRN: <https://ssrn.com/abstract=3046353>.
 - [96] Yildiz, M., (2004), *Waiting to Persuade*, The Quarterly Journal of Economics, Vol. 119, No. 1, pp. 223-248.

- [97] Zigrand, J. P., (2014), *Systems and Systemic Risk in Finance and Economics*, Systemic Risk Centre Special Papers, No. 1, London School of Economics and Political Science.