<u>Investment, Valuation, and the</u>
<u>Managerial Theory of the Firm</u>

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Section 1 : Managerial Theorising and the Concept of Strategy
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Modelling the structure, thinking and behaviour of business
management is at once among the oldest, most important and least
settled areas of the theory of the market economy. Following the
recognition many years ago that management is typically divorced
from ownership and enjoys considerable independence from both owners
and the capital market, theorists have been searching for new
certainties to replace the old.[1] The questioning of the concept
of profit maximisation as the businessman's objective in the real
world provided an additional incentive for those seeking to develop
a new theory of the firm.

This search has for the most part taken as its model a
particular specification of the modern business: management and
ownership are effectively distinct groups; the firm is sufficiently
well established to anticipate meeting most of its need for new
equity capital from retained earnings; it has achieved a stock
exchange quotation for its equity shares; lack of information
available to the capital market and institutional factors working
in favour of established management make the latter secure against
most threats to its position; the competitive situation in product
markets does not exclude managerial discretion or compel profit

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(1)   The outstanding early work on the ownership-management
        distinction was A.A. Berle and G.C. Means, The Modern
        Corporation and Private Property, 1932, which may fairly
        claim to have provided much of the evidence and ideas
        from which managerial theorising has grown. For a
        valuable review of current thought on the original Berle
        and Means propositions, see J.R. Wildsmith, Managerial
        Theories of the Firm, 1973, Chapter 1.

maximising behaviour. Such characteristics have been widely accepted as typical of much of modern business, and the kind of enterprise meeting this description is the subject matter of this study.

Agreement on the outwardly observable situation of the typical enterprise has not brought unanimity to the theory of its objectives and decision making, and for reasons explained in the following section this is neither surprising nor discouraging. The present study is intended as a contribution to the managerial theory of the firm, an area of theory in which the traditional conception of the businessman or management as a unified and optimising personality is translated into conditions where considerable freedom exists to define the objectives and dictate the operation of the business.

Managerial theories therefore represent a minimum of departure from the simple concept of the firm as a profit (or value) maximiser,[2] and the basic reason for preferring a managerial approach to the latter in this study is that it provides a framework in which the existence of a strategic level of decision making is recognised and emphasised. The managerial approach remains closer to the traditional than to other modern departures: the behavioural theory of Cyert and March, for example, shows the business organisation responding in predictable ways to external developments and their internal consequences without attempting to estimate the range of possible performance outcomes over time implied by the

(2)   See F. Machlup, "Theories of the Firm: Marginalist, Behavioural, Managerial," American Economic Review (March 1967), pp.5-6

interactions of events and responses.[3] This view is practically a negation of strategic decision making as described in this study, where high confidence is required on the part of the decision maker that all possible developments up to a limited horizon can be identified. However, the paradox in Cyert and March's model lies in their demonstration that the firm's decision making according to its own rules and procedures can be simulated: essentially the present study goes beyond the behavioural model only in assuming a willingness on top management's part to compare (ranges of) prospects for different internal decision procedures and choose between procedures in the light of that comparison.

Another denial of the strategic aspects of managerial decision making comes from Galbraith, who argues in these terms:

> "It follows from both the tendency for decision making to pass down into the organisation and the need to protect the autonomy of the group that those who hold high formal rank in an organisation ....exercise only modest powers of substantive decision. .... Nor is it a valid explanation that the boss, though impotent on specific questions, acts on broad issues of policy. Such issues of policy, if genuine, are pre-eminently the ones that require the specialised information of the group. Leadership does cast the membership of the groups that make the decisions ...... This is its most important function." (4)

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(3) R.M. Cyert and J.G. March, _A Behavioural Theory of the Firm_, 1963.

(4) J.K. Galbraith, _The New Industrial State_, 1967, pp.78-9 (Pelican edition, 1968). Galbraith's definition and analysis of "groups" can be found earlier in the same chapter of the book, _op.cit._, pp.69-77.

This small sampling of anti-managerial opinion is offered here because the aspect of managerial theory with which this study is largely concerned is that of strategy choice. Without attempting at this stage to describe the strategic decision process, the reason for accepting its existence may be stated briefly: management's widely recognised concern for its own security and for the survival and growth of the enterprise it controls all suggest that it does indeed assess in a distinct decision context the long-term implications of those policies which bear most directly on those objectives.

Methodological preliminaries too can be summarised briefly. In theorising about decision making there is an important stage at which the terms and complexity of the decision maker's model of reality — the important relationships, as he sees them, between his actions and their results — and the nature of his related utility function have to be discussed. This stage precedes any attempt to define his underlying motivation and predict actual choices in different situations. In the profit-maximising theory of the firm this preliminary stage was easily overlooked, with the firm's objective predetermined, the businessman's model of reality (cost and revenue functions) dictated by his objective, and his policy instruments (output level, advertising, etc.) dictated by the objective and the model. Beyond traditional maximising theory, however, it becomes impossible to postulate a unique arrangement of the decision maker's model, objective function and choice of instruments — or even to be certain of the order in which he settles these matters.

Fortunately this latter problem can be disregarded here: in common with other managerial theories of strategy choice the model developed in Part II from the orthodox theory of Part I allows for a variety of managerial motivations in conjunction with a standard managerial model of reality and choice of strategic instruments. However, as a qualification to this independence, the theory in Part II makes the recognition of the risks involved in growth the central feature of management's model of reality, with the result that certain managerial motivations are unavoidably excluded: even if management wished simply to maximise equity market value, its own risk model would oblige it to adopt some other, perhaps related, objective more suited to the realities of the risky prospects for market value.

Finally, this study attempts to match other managerial theories in deriving a "timeless" theory of strategy choice. If one strategic option is expected to remain superior to all others presently available as time unfolds, the assumption of its permanence is logically sound. In Part I this condition is easily met as management's objective of maximising the market value of investors' wealth is achieved by a continuing process of investment decision making. In a very different way, in Marris's steady-state growth context the once-for-all aspect of strategic choice is also easy to appreciate: given the terms in which managerial utility is expressed, a strategic choice maintains in perpetuity the initially preferred combination of growth rate and valuation ratio.[5] However, with both growth

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(5) See R.L. Marris, The Economic Theory of Managerial Capitalism, 1964, especially Chapter 6; and "Theories of Corporate Growth" in R.L. Marris and A.J.B. Wood (eds), The Corporate Economy, 1971. Marris's steady-state growth framework is discussed at length in Chapter 14.

and equity market value subject to risks, the possibility that management will feel able to reach a once-for-all strategy decision is more difficult to demonstrate - even though the alternative, a management expectation that strategy will require alteration as time passes, is itself logically unsatisfactory.

Section 2 :   Background and Main Features of the Study

1 : 2 (i)   The development and diversity of managerial theories.

There is now a long and fruitful tradition in the managerial theory of the firm. The main prerequisites for "managerial" behaviour appear to be : (i) the separation of ownership from management functions; (ii) a capital market imperfectly aware of a firm's full earnings potential; (iii) the existence of a coherent managerial "personality" in an enterprise, a personality possessing a consistent preference system or utility function; (iv) the exercise of managerial discretion in many areas of decision making, with a unique objective (maximum profit) probably replaced by a small number of conflicting objectives (growth versus market valuation,

profit _versus_ sales volume, etc.). [6] Of these, (i) and (ii)
are conditions necessary for the emergence of "managerial" or
discretionary behaviour, while (iii) and (iv) effectively link the
new theories to the traditional - a unified managerial personality
pursuing a comparatively narrow range of objectives. The
distinction between traditional and managerial theories is blurred
even more by the realisation that even in the former the profit
maximising motivation is not absolute, that discretion and compromise
are implicit and unavoidable. [7] Even if managerial theory has
not succeeded in detaching itself fully from the traditional, so
that its parentage on one side is easy to trace, its contribution

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(6)   Some comments on these prerequisites:
(i) Attempts to link management's motivation to its system of
remuneration or to the size of its shareholdings have not
produced a conclusive pecuniary model of either a profit-
maximising or a value-maximising motivation. On the other
hand, Marris argues that sociological, professional and
organisational factors combine with the system of rewards
for managers to produce a distinctive "managerial" motivation.
See Marris, _op.cit._, 1964 Chapter 2; Wildsmith, _op.cit._,
1973, pp.3-5 and 115-6.
(ii) It is usual to refer separately to management's
ability to carry out investment programmes through internal
financing; however, in a fully informed capital market
management's position would be seriously threatened if it
undertook to reinvest profit at less than investors' required
rate of return. The absence of perfect information contributes
to management's independence, as do inadequate institutional
arrangements under which shareholders can mount a challenge:
see Wildsmith, _op.cit._, 1973, pp.6-12.
(iii) It will be seen that the managerial "personality"
is strictly necessary only at the top level of decision making,
where outcomes of lower-level decision processes are taken
into account in determining strategy. Thus, organisational
factors and even satisficing behaviour can, if necessary, be
accommodated within a managerial model.

(7)   See C.J. Hawkins, _Theory of the Firm_, 1973, pp.60-1.

has been most impressive. Discussion of managerial objectives has gone much farther than mere recognition that businessmen are not out-and-out profit maximisers.

In early contributions to managerial theory the understandable reaction against the concept of profit maximising produced a tendency to downgrade the importance to management of financial performance – in terms of profit or equity market value – to the level of a constraint or to parity with other, non-financial, aspects of performance. Recognition of management's typical financial independence contributed to this tendency. Thus Scitovsky saw businessmen as maximising utility in terms of both profit and leisure, so that a rather exceptional motivation was required to generate a maximum level of profit.[8] Baumol originally saw management as maximising sales revenue subject to a minimum profit constraint.[9] O.E. Williamson defined managerial utility to include the various uses to which discretionary profit might be put.[10]

However, the development of theories which place growth at, or near, the centre of management's concern has had an effect on the tendency to downgrade the importance to management of financial performance. The importance of profit for financing growth outlays – either directly or through favourable effects on equity market value –

(8)   T. Scitovsky, "A Note on Profit Maximisation and its Implications," *Review of Economic Studies* (1943).

(9)   W.J. Baumol, *Business Behaviour, Value and Growth*, 1959, Chapters 6 – 8.

(10)   O.E. Williamson, *The Economics of Discretionary Behaviour*, 1967.

has given it a new prominence among managerial objectives.[11]
In addition, it has all along been recognised that financial
performance may be a perfectly understandable objective, either
because of arrangements under which management benefits financially
from the performance of the enterprise, or because "stock market
approval", to use Marris's term, brings its own satisfaction and
security. The advent of managerial theory never seemed likely to
eliminate the profit motive, but recognition of management's
desire for growth brought with it a timely reappraisal of the
relative importance of financial performance.

There would be widespread agreement to the proposition that
managerial theory remains at a formative stage, with theorists
still exploring new ways of expressing the complexities of management's
structure, modes of thought, preferences and decision making. A
continuing state of experimentation and exploration is not necessarily
to be deplored. For one thing, contributions have dealt with
different aspects and areas of decision making: the most important
division being that between static and dynamic approaches, but with
considerable differentiation within each. Second, there ought to be
room within managerial theory for differing accounts of the process
by which management structures its decision making so that the
performance aspects most important to it, and the instruments by
which those aspects are most effectively influenced, are isolated

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(11)   A good example of this is Baumol's recognition ("On the
         theory of Expansion of the Firm," American Economic Review
         (December 1962), p.1085) that the profit constraint in his
         static model (Baumol, op.cit.,1959) "may have seemed to be
         arbitrarily imposed from the outside," whereas "From the
         point of view of a long-run growth (or sales) maximiser,
         profit no longer acts as a constraint. Rather it is an
         instrumental variable - a means whereby management works
         towards its goals."

and the final or top-level decision process reduced to manageable
size. Either explicitly or implicitly, a feature of managerial
theories is the assumption of some such division between the bulk
of decisions and those that are taken in accordance with an
objective function relating to a small number of important aspects
of performance. But the recognition of management's wide
discretionary power would scarcely be consistent with the imposition
at the theoretical level of a uniform framework of thought, preferences
and decision structure: for this reason alone a diversity of
managerial models is welcome.[12]

1 : 2 (ii)   Main features of this study.

Having established a strong case for diversity in managerial
theory, the general intentions of this study can now be indicated.[13]
Central to the whole study is a particular conception of a partitioned
structure of decision making, though interest largely focusses on

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(12)   Hawkins (op.cit., 1973, p.62) emphasises that managerial
utility theory remains in its infancy, and that attempts to
provide a "simplified general approximation of firms' behaviour"
should take precedence over the construction of models
reflecting individual situations. Machlup makes much the
same point in arguing that "the firm is only a theoretical
link.... helping to explain how one gets from the cause to
the effect. This is altogether different from explaining
the behaviour of a firm." : F. Machlup, op.cit., 1967, p.9
Neither writer believes that managerial theorising is pointless,
and Machlup in particular appears to be making a macro-
micro type distinction in the field of price theory.

(13)   The following section describes in greater detail, and with
chapter references, the components and structures of the
models developed in Parts I and II.

what is called the _strategic_ or top-level decision. As indicated
above, managerial theories generally assume that by some means
management isolates a final stage of decision making involving
manageable numbers of policies and performance aspects. However,
theories may differ in their assumptions about the processes by
which this necessary reduction is achieved: here it is assumed
that in comparing alternative strategies management assumes a
common background of other policies which, in a partitioned
decision making structure, have all been determined in various ways
prior to the strategic (top-level) choice.[14]

In keeping with recent trends in managerial theorising the
strategic decision is assumed to relate to the firm's policy in
respect of investment and growth. However, both in Part I and in
Part II, the firm's equity market value is assigned greater importance
in management's preference system and in its necessarily selective
model of reality than is customary among managerial theories – even
those oriented towards growth. So, while the general presumption
stands against disqualifying any internally consistent managerial
model, there remains a need for some explanation of this particular
feature of the proposed theory.

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(14)   A striking contrast exists between the partitioned decision
       structure which results in _all_ subordinate policies
       being determined prior to the strategic decision, and the
       implication in Marris's steady-state growth model that the
       choice of growth rate (the strategic decision) also determines
       what may be called the product and market policy of the firm –
       a policy without an obvious counterpart in the managerial
       utility function : see Chapters 14 :2 (ii) and 15 : 3 (iii).

The basic explanation for the market value emphasis in
Part II's managerial theory, and incidentally for its concentration
on investment decision rules as the appropriate strategic instrument,
is that it represents a logical development of a traditional view of
management's financial objective and method: that management's
objective is to maximize the market value of investors' wealth,
and that in relation to this objective investment decision making
is an instrument of major importance.[15] While no attempt is
made to establish, or re-establish, market value maximisation as a
unique managerial objective, it is assumed that a strong residue
of an original, basic, instinct to think in terms of market value
survives into the managerial age. However, it is essential to
distinguish between the terms in which management's preference
system is expressed and the motivation underlying the choice of
strategy: while it is natural that management should compare strategic
options in terms of their market value prospects, and should find
the comparison important, its underlying motivation may be "managerial"
or traditional (in the general sense of consciously attempting to
reflect investors' preferences where possible in its decisions).
There ought to be a clear distinction between the terms in which
management envisages the outcomes of its strategic options and the

(15)   For a statement of the case in favour of wealth maximisation
        as opposed to profit maximisation as management's objective,
        see E. Solomon, The Theory of Financial Management, 1963,
        pp.19-20. In discussing possible conflicts between managerial
        and investor viewpoints Solomon is perhaps more sanguine
        than managerial theorists that the wealth – maximising motivation
        will prevail (op.cit., p.24), but he appears to leave some
        room for management to operate as a value or wealth maximiser
        subject to self-imposed constraints on its behaviour as a
        profit maximiser. Behaviour of this kind is possible given
        the partitioned decision structure assumed throughout the
        present study: see Section 3 (ii) below.

nature of the motivation underlying its choice.

One further consideration relevant to the market value – investment decision making orientation of managerial theory in this study is that management's interest and expertise in financial aspects of decision making has probably been increasing in recent years. The growth of managerial theory has been paralleled by that of what may be called the financial theory of the firm, comprising, for example, capital budgeting, dividend policy and debt financing, with valuation theory as background; and by the application of modern computer techniques to the theoretical problems defined in these areas., Widening awareness of developments in financial analysis may have little impact on underlying managerial motivations (nor is such analysis generally intended to replace the managerial function), but may produce changes in the importance management assigns to different aspects of performance and instruments of policy.[16]

Turning to the role of investment, and to the suggestion that investment decision making stands at the centre of the process of strategic choice, there is a very important distinction to be drawn between Parts I and II. In the certainty context considered in Part I a simple and objective decision rule is shown to be consistent with the simple objective of maximising investors' wealth; while in Part II where investment returns are subject to _ex ante_ risk, the rules governing investment choice seem likely to be dictated by managerial rather than market criteria.[17] Nonetheless, given

(16)   For references to some of the important developments in
        quantitative financial analysis, see footnotes (22) and (23) below.

(17)   The return referred to is the mean value of the probability
        distribution of the investment's periodic earnings.

that both growth and valuation are inextricably linked with the success
of investments, it seems reasonable to describe management's rules
for investment choice as the main variable component of strategy.
In thus singling out investment decision making the model assigns
different roles to other equally discretionary areas of decision.
The basic choice of an area of business activity in which to operate,
structures of operating and competitive rules of behaviour; dividend
and financing policies – all are important areas in which management
exercises discretion, and in some of these the choice might arguably
be described as "strategic". However, for reasons that differ
according to the area in question, such decisions are seen as
providing the essential common background against which manage-
ment compares alternative sets of rules governing investment choice.

The distinction between investment decision rules and other
areas of decision ensures a simple and neat strategy choice situation
of the kind favoured in managerial theories, with investment policy
the instrument and market value prospects the determinant of
managerial utility. The following section contains some discussion
of the model of decision structure underlying this particular
simplification of management's top-level choice, and the subject
is developed at appropriate points in later chapters. It should
perhaps be emphasised that a partitioned decision structure simply
implies that decisions are taken in some order, with higher decisions
based upon lower-level ones: it is not intended to suggest that
management derives utility solely from the outcome of the top-level
decision in the sequence.

The managerial theory developed in this study has two levels, general and particular. At the general level the intention is as described above: to suggest a managerial approach to strategic decision making - the performance aspects determining (strategic) utility and the instruments employed. The particular aspects of the theory are the attempts to fill the various empty boxes of the general argument with appropriate components. But the general argument is intended to have a validity that does not depend on the particular ways in which the various boxes are filled: the latter are to be seen in each case as illustrating the kind of managerial thinking or decision making that could constitute a component part of the model.

The following section enumerates the different area or boxes of the overall structure and briefly indicates how, in this study, appropriate components have been defined.

Section 3   Main Components of a Market Value-Based Managerial Theory.

1 : 3 (i)   Management's financial decision making and valuation model.

The managerial theory developed in Part II of this study should be seen as a logical progression from the models of dividend and financing policy and valuation to be explained in Part I. Both components are modified as necessary for the very different conditions described in Part II, but the lines of descent are obvious and deliberate.

Management's assumed dividend and financing policies are inter-related and can be described briefly. Aiming at both a high level of internal financing and a constant target level of ex ante dividend risk for each period, management plans its debt financing (and occasional external equity financing) accordingly, taking into account the known risk properties of all other components of cash flow. Planning of this kind is described in relation to the simple case of a certain growth-path of investment (Chapter 7) and the multiple growth-path context (Chapter 8). An appropriate definition of ex ante dividend risk is given in Chapter 2 : 2 (iv) and is retained throughout.

As far as Part I is concerned, where a unique growth-path of investment and expected earnings is assumed, this managerial approach to dividend and financing policy confines optimising behaviour within narrower limits than are often suggested. As shown in Chapters 3,5, and 7, dividend risk policy and its associated debt financing requirements effectively determine a firm's cost of capital, leaving management free to optimise only by selecting acceptable investments. The importance attached to achieving an optimal capital structure is assumed to be small (Chapters 3 : 4 (i) and 5 : 4), and in any case a single instrument - dividend risk policy - cannot simultaneously satisfy targets for dividend risk, capital structure and internal financing of investment. When perfect foresight obtains in the context of Part I, making possible the forward planning of expected dividend and financing (Chapter 7 : 2 (ii)), awkward choices between retained earnings and external equity finance are avoided; and only for a completely unforeseen investment opportunity need a departure from

established dividend policy be considered, along with the valuation
issues it raises (Chapter 5 : 5). Even when future investment
demand is known only in probabilistic terms - the Part II context -
the conditional forward planning of constant-risk dividend and
financing can proceed quite mechanically (Chapter 8 : 2 (ii)).

It would be wrong to conclude that the suggested managerial
approach to financial decision making entirely ignores or replaces
the customary tripartite view of the subject: investment decision
making, capital structure and dividend policy.[18] Given the firm's
underlying business risk (Chapter 4) a decision on dividend risk
effectively commits management in all three areas as far as conditions
assumed in Part I are concerned. (In Part II investment policy remains
a separate area of decision.) But a prior decision on dividend
risk is unlikely to be taken in vacuo, and undoubtedly its feasibility
of implementation and other implications are considered at that stage.[19]

Closely related to management's dividend and financing policy
is its interpretation of valuation. As is obvious from the position-
ing of dividend risk policy at the centre of the suggested model of
financial decision making, the valuation model attributed to manage-
ment is one in which expected dividends are discounted by the market
at a rate determined by the level of dividend risk. Such a model
may be incomplete in several ways. First, management may have only

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(18)   See J.C. Van Horne, _Financial Management and Policy_,
        second edition 1971, pp.9-11; L.P. Anderson, V.V. Miller and
        D.L. Thompson, _The Finance Function_, 1971, Chapters 4-7.

(19)   See Chapter 8, footnote 10.

an imprecise notion of the complete functional relationship between
discount rate and dividend risk (Chapter 3 : 4 (i)). More
important, management may have little interest in, or capacity to
resolve, the issues relating to capital structure and investment
financing raised by Modigliani and Miller.[20] As indicated above,
such issues arise only on the rather unlikely occasion of a completely
unforeseen investment opportunity: for all other occasions they are
effectively bypassed by management's conditional planning of
dividend and financing (Chapters 2 : 4 and 3 : 4 (ii)).

The interpretation of valuation imputed to management is
justified by its correspondence with the latter's financial decision
making. Constant dividend risk is seen by management as an end in
itself and as essential to its ability to comprehend the market
valuation process, while more ambitious objectives for financial
decision making run into serious complexities on the side of valuation
theory. To summarise: in the light of the purpose for which manage-
ment requires a valuation model the shortcomings of a simple
dividend valuation model are excusable; and a limited model has
the advantages of being relatively easy to identify (Chapters 9,10)
and, once accepted, relatively difficult to dismiss (Chapter 13).

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(20)   The two articles in which Modigliani and Miller stated
        their central propositions are: F. Modigliani and M.H.Miller,
        "The Cost of Capital, Corporation Finance and the Theory
        of Investment," American Economic Review (June 1958);
        F. Modigliani and M.H. Miller, "Dividend Policy,
        Growth and the Valuation of Shares," Journal of Business
        (October 1961). Relevant aspects of the "M & M debate"
        are discussed in Chapters 2 : 4 and 3 : 4 (ii). For
        confirmation of management's inclination to adopt a simple
        view of valuation see J.C.T. Mao, "Survey of Capital
        Budgeting: Theory and Practice," Journal of Finance (May 1970),
        p.352.

As will become obvious, the applicability of computer techniques - especially simulation - to financial analysis is essential to the feasibility of the strategic decision frameworks suggested in both parts of this study; and a brief indication of similarities and differences between the present study and some of the well-known work in the fields of simulation and capital budgeting may be appropriate here. First, the use of simulation techniques in the identification of probability distributions of individual and multiple investment returns suggests that the characteristics of business risk can be identified with reasonable confidence by management (Chapters 2 : 3 (i) and 4).[21] The precise treatment of risky investments in Part II (Chapter 8 : 1), chosen mainly for expository purposes and as an extension of the Part I treatment of "safe" investments, may differ from those followed elsewhere; but (i) it lends itself equally well to simulation (Chapter 15 : 2), and (ii) it can with little difficulty be modified for greater generality.

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(21)   The variety of possible assumptions and the simulation approaches employed in this field are illustrated in the following selection of articles:
J.F. Magee, "Decision Trees for Decision Making," Harvard Business Review (July-August 1964); J.F. Magee, "How to Use Decision Trees in Capital Investment," Harvard Business Review (September-October 1964); D.B. Hertz, "Risk Analysis in Capital Investment," Harvard Business Review (January-February 1964); R. Hespos and P. Strassman, "Stochastic Decision-Trees for the Analysis of Investment Decisions," Management Science (August 1965); F.S. Hillier, "The Derivation of Probabilistic Information for the Analysis of Risky Investments," Management Science (April 1963); J.C. Van Horne, "Capital Budgeting Decisions Involving Combinations of Risky Investments," Management Science (October 1966). Probability distribution approaches to investment appraisal are reviewed in J.C. Van Horne, Financial Management and Policy, second edition 1971, pp.131-52, 166-93.

An important outgrowth of the use of simulation and other
computer techniques has been the interest shown in the expression
of management's financial objectives and in the development of
related capital budgeting techniques.[22] The significance of
such models lies in their recognition (common to managerial theories
in general) that managerial discretion is both typical and necessary
to the formulation of an overall financial objective. Where the
present study stands in relation to such studies is that it accepts
with little discussion the existence of investment decision
procedures (Chapter 11: 3) and concentrates on the ways in which
management is likely to envisage their results over time. Further,
where numerous models consider the selection of investments from a
known and unchanging set of opportunities, the emphasis in both Parts
I and II — especially Part II — is on management's interest in the
implications of different selection procedures and criteria, rather
than in the implications of particular investment portfolios.
Management's strategic choice is assumed to involve the criteria and
procedures it will apply in investment decision making, not the
precise projects that will be chosen. The latter type of future
choice situation is not realistically foreseeable in unique terms.[23]

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(22)   Weingartner's work on mathematical programming and capital
       budgeting (H.M. Weingartner, Mathematical Programming and
       the Analysis of Capital Budgeting Problems, 1963 ; "Capital
       Budgeting of Interrelated Projects: Survey and Synthesis,"
       Management Science (March 1966).) has been developed and
       extended in a number of ways to allow for risks in invest-
       ment performance. Examples are : B. Naslund, "A model of
       Capital Budgeting Under Risk," Journal of Business (April
       1966) (chance — constrained programming), and R.C. Salazar
       and S.K. Sen, "A Simulation Model of Capital Budgeting Under
       Uncertainty," Management Science (December 1968) (stochastic
       linear programming).

(23)   See Van Horne, op.cit., 1971, p. 84.

Following a brief treatment in Chapter 2 of certain central concepts in orthodox valuation theory, the theory itself – as it applies to the static firm – is briefly described in Chapter 3. Cost of capital and related dividend and financing rules for various comparative-static and simple growth situations are derived in Chapters 5 – 7. In Chapter 10 a modified version of the basic valuation model is suggested, taking into account management's recognition that the firm's future growth is not uniquely pre-determined. This later version, a central component of the managerial theory developed in Part II, builds on the foundation of valuation theory and managerial dividend and financing policies established in Part I. However, management's motivation in Part II is necessarily different from that assumed in Part I, for the same reason that a modified valuation model is required – the presence of risks affecting both growth and market value.

1 : 3 (ii)   The structure of decision making.

Another box in managerial theory is reserved for a model of the structure of decision making. As stated earlier, a theory involving a top level of decision making must contain or imply some account of the processes by which the final choice is reduced to manageable size. As will be shown (especially in Chapter 14) managerial models do not all meet this requirement in the same way, and the implications of different approaches are themselves instructive. The decision structure assumed throughout this study is one in which management operates at two levels, determining (or accepting) a large and varied mass of subordinate policy, against which background a top-level choice of strategy (the rules governing

investment selection) can be made. The processes of subordinate policy making and interactions between subordinate decisions are not examined in detail, and it is important to stress that the model is perfectly compatible with the many different processes by which firms can determine subordinate policy matters. Thus, utility maximising, target setting, constraint observation and satisficing behaviour may all figure at some stages of subordinate policy making - either in relation to isolated decisions or in relation to forms of simultaneous decision making.

Clearly, given the acceptance of diversity between firms in subordinate decision processes as well as the diversity of the decision situations within any one firm, there is little real prospect that a standardised or general account can emerge. However, if the concept of managerial utility is believed relevant to the subordinate as well as the top level of decision making, a simple condition ensures that the top-level utility maximising exercise remains quite independent. (It is not strictly necessary to extend the concept of managerial utility into all levels of decision making, and it may actually be incorrect to do so: nevertheless the possibility is worth considering.) The simple condition is that a distinction or partition exists between subordinate utility and strategic or top-level utility, so that the two are independent of each other: management's subordinate policy decisions give utility of one kind, while its strategic decision gives utility of a different kind. Because the different utilities are strictly non-comparable, subordinate decision making can proceed regardless of the fact that the strategic decision remains outside that process; and at the strategic decision stage utility derives only from the prospects

of the policy to be determined at that stage. This condition merely restates the original account of the decision structure to accommodate the concept of managerial utility explicitly in both stages of the model of decision making.

Different interpretations of management's structure and "personality" are capable of yielding essentially the same picture of a rigidly partitioned decision structure. In the model suggested above management remains a single psychological entity, but with different areas of consciousness and interests. The same result would occur if management itself was recognised as divided into a hirerarchy, with subordinate decision making largely the preserve of levels below top management. The conclusion is that a decision structure such as that described is consistent with the possibility of utility-maximising behaviour at all levels if either a structurally unified management is psychologically divided, so that different utilities register in different compartments of its consciousness, or if management itself is structured in such a way that higher levels must largely accept decisions, customs and attitudes prevailing at lower levels and shaping subordinate policy. It is advisable to repeat the earlier cautionary note that nothing that has been said indicates that utility maximising is an accurate description of what motivates decision making at subordinate levels. The point of the digression has merely been to show that such a possibility is perfectly compatible with the assumed structure of decision making, given a quite reasonable assumption about the non-comparability of utilities derived from very different sources.

One further point of clarification is required. In Part I subordinate policies, and in Part II both subordinate and top-level policies, must be understood as taking account of risk factors. Thus it is assumed that subordinate decisions, where necessary, take the form of conditional rules of behaviour for the various situations that can arise (Chapter 12 : 4). Thus if utility is a meaningful conception in relation to subordinate decision processes it derives from the prospective pattern of a policy's results rather than from a unique outcome. This point makes no difference to the central assumptions, that subordinate decisions provide a common background against which alternative strategies can be compared, and that management's strategic utility is a quite separate concept from whatever measures of satisfaction apply at subordinate levels.

The partitioned decision structure implies very little about the relative importance to management of the prospective results of different levels of decision making. (This follows of course from the assumption of non-comparability of different utilities, where the concept of utility is believed relevant thoughout the decision structure.) Certain decisions, while necessarily subordinate - for example, dividend risk policy and conditional rules for competitive behaviour - are obviously very important to management. This importance is perhaps best expressed not through explicit incorporation in management's top-level utility function, but in the role of such policies as constraints or targets which influence the possibilities that may arise under any specified top-level strategy. Management in such cases operates as a constrained

maximiser of (strategic) utility — a perfectly acceptable view of
its behaviour.

1 : 3 (iii)   The growth model and its implications.

A third box is reserved for a model of growth. In Part
I this is perfectly simple, with management accepting the concept of
a unique predetermined programme of future investment, expected
profit, expected dividend, equity market value and debt financing.
(Conditions linking the different stages of traditional valuation,
investment and financing theory in Part I are defined in Chapter
3 : 1 (i).) In Part II the picture is very different, with each
set of rules governing investment choice associated with a probabilistic
pattern of interactions between the firm's profit and its invest-
ment (Chapter 8). This pattern, together with the firm's predetermined
conditional financing policy, traces out all possible growth-paths
of constant-risk dividend that are inherent in the rules under
consideration. Different investment rules have different associated
probability distributions of growth-paths.

This Part II growth model calls for modifications to both
of the components previously discussed. As indicated earlier,
management's interpretation of valuation, its dividend planning approach
and the role of investment decisions in implementing strategy are
all modified by the change to a context of multiple growth-paths.

The effect of recognising that management sees growth in
terms of a probability distribution of outcomes for any specified
strategy is extremely important. It means, for one thing, that a

(managerial) utility function based upon equity market value does not automatically require management to be labelled "profit maximising" or "market-oriented" : given the risks affecting growth and market value, the latter can provide management with a suitable medium in which to express its preference system while its underlying motivation is in no way constrained or influenced by that medium of expression (Chapters 11 : 4 (ii) and 12 : 5). More fundamentally, a probability distribution of growth-paths necessitates a complete removal of the distinction between "managerial" and "market-oriented" motivations : all strategic motivations are inevitably "managerial" when there exists no course of action that can be described as obviously and unambiguously market or investor-oriented. This conclusion holds irrespective of the terms in which management's preference system is expressed and the instruments employed in its strategic decision (Chapter 12 : 5 (iii) - (iv)).

1 : 3 (iv)   Sufficient confidence in beliefs.

Central to the entire study is a view of management's attitude towards its beliefs, including its probability distribution estimates, and a brief introduction to this subject is required. Management is assumed to foresee the outcomes of its decisions in terms of _risk_ rather than _uncertainty_. [24] However, the theoretical

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(24)   The distinction is the standard one established by F.H. Knight in _Risk, Uncertainty and Profit,_ 1921. In risk situations the probabilities of the various possible outcomes are known, while in uncertainty situations they are not.

distinction between objectivity and subjectivity in relation to probability is quite inappropriate in the present context, and the assumption adopted is that a decision maker accepts probability estimates (and other beliefs) if, within the relevant time horizon, he does not anticipate having to revise them. A brief indication can be given of how, in the differing situations of Parts I and II, this assumption is applied.

In Part I management confronts a limited range of risk aspects (Chapters 2 : 3 (i) and 4) and the pursuit of its top-level objective - the maximisation of the market value of shareholders' wealth - involves frequent or continuous decision making. This process is not impeded either by ignorance or risk in relation to long-term growth prospects (Chapter 7 : 6) or by awareness that assumptions about valuation and business risk may be reappraised in the future. For the Part I context - and, for a different reason, for a simple version of the Part II managerial utility function - management's horizon can be defined as immediate, in the sense that continuous decision making must at all times be based on the best probability information available. Thus the question of anticipation of errors in estimated probabilities is not considered in Part I, though its later treatment (Chapter 13) is applicable. Further justification for this approach lies in the fact that the relationships and risks that are relevant to the strategic decision (on the cost of capital) are those most likely to be well understood by management, whose confidence in the durability of assumptions and objectivity of probability estimates can be assumed.

In Part II the relationship between the managerial horizon, the nature of the strategic decision and the attitude towards beliefs is quite different. The strategic objective is very likely to be differently formulated (Chapters 11 : 4 (ii) and 12, Sections 1 (i), 5 (iii), 5 (iv)), particularly in respect of management's horizon; the range of relevant risk aspects and other beliefs is wider than in Part I and does include risks relating to future growth performance; and the strategic decision itself is of the once-only type. The assumption stated above is now taken to apply to an extended but still finite horizon, the exact position of which is one aspect of management's own discretionary thinking.

Part I

Cost of Capital Theory, Managerial Dividend and
Financing Policies

Chapter 2

Concepts and Methodology of Traditional Valuation Theory.

Plan of Chapter

Section 1 : Introduction.

Section 2 : The Valuation of Financial Assets.

    (i)   Long-term debt.

    (ii)   Equity Shares.

    (iii) A digression on variable equity prices under "ideal" conditions.

    (iv) Dividend risk defined.

Section 3 : Representing Business Earnings.

    (i)   The earnings probability distribution.

    (ii)   Representing the earnings of investments.

Section 4 : Modigliani and Miller and Valuation.

Section 1 :   Introduction.

This chapter introduces several of the concepts and conventions
that figure prominently in traditional capital structure and cost
of capital theories. For the most part the treatment is entirely
customary and intentionally economical, given the familiarity of
the subject matter and the abstract level of discussion that is to
follow in the chapters based upon these preliminaries.

Section 2 introduces traditional concepts of financial
asset valuation and, typical of the intended level of abstraction,
divides financial assets cleanly into those promising risk-free
periodic incomes and redemption values and those offering residual
equity income to the holder. (This distinction is relaxed in a
limited way in Chapter 3 : 3 (i), but is retained implicitly
throughout most of this study.) A further simplification is the
concentration on long-term sources of business finance; so that in
addition to equity capital, which fits well enough into this picture,
debt capital is treated as long-term. The entire theory of
valuation relates to a condition in which capital markets are in an
unchanging equilibrium, with the price of each particular type of
asset tending towards its equilibrium level.

Section 3 prepares the way for the capital structure analysis
of Chapter 3 and for the latter's subsequent developments. The
problem it deals with is the conversion of the earnings probability
distribution of the static firm to a basis on which capital structure
theory can usefully build.

Section 4 returns to the theme of the traditional valuation model and, without joining the mainstream of the Modigliani and Miller (M & M) debate, rejects one of the supporting arguments employed by M & M which directly contradicts both a basic assumption of the traditional model and a typical managerial attitude.[1] However, the broader question raised by M & M - the nature of investors' valuation in conditions of uncertainty - is reserved for discussion in Chapter 13.

Section 2 :   The Valuation of Financial Assets.

2 : 2 (i)   Long-term debt.

There are two purposes in this brief sub-section: to show the sense in which the value of the long-term debt of a static firm with unchanging capital structure is constant; and to demonstrate that if the capital structure of such a firm is altered debt-holders need expect neither a capital gain nor a loss on their holdings.

It is assumed that a company's debt is risk-free, as regards both interest payments and redemption value. The market value of a debt instrument at any time is the sum of the appropriately discounted future receipts due to the holder - interest and redemption value. The discount rate in this situation - un-

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(1)   The particular M & M contribution discussed in Section 4 is F. Modigliani and M.H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review, (1958).

changing equilibrium prices ruling throughout the capital market -
gives rise to no difficulty, being the straightforward risk-
free interest rate. Although there are many bond-type financial
instruments it is neither necessary nor interesting here to
discuss their various features: under the market equilibrium
conditions assumed the precise debt instrument adopted is
largely irrelevant. However, two very different types of
instrument can be described and used to illustrate the main
points mentioned above.[2]

A conventional bond offers the holder a regular interest
payment expressed as a proportion of the bond's redemption value.
Let i be the risk-free market rate of interest per annum, and
M the bond's redemption value: then annual interest is always
equal to iM. The market value of such a bond at any time is the
discounted sum of all of its future payments to the bond-holder,
so that one year before its maturity at the end of year n it
is worth

$$V_{n-1} = \frac{M + iM}{1+i} = M \qquad\qquad (i).$$

Similarly, two years before its maturity it is worth the sum of
the discounted value of one year's interest and market value at
the end of $n - 1$:

(2)   On the questions of forms of debt and considerations
       important in choosing the debt instruments to be issued,
       a helpful descent from the present level of abstraction
       is provided in S. Friedland, The Economics of Corporate
       Finance, 1966, Chapters 8,9.

$$V_{n-2} = \frac{M + iM}{1 + i} = M \qquad\qquad \text{(ii)},$$

and so on. It follows that the issue price of the bond must also be equal to M, since the market would not pay more and the issuer should not accept less. An undated bond is just as simply handled: let the issue price of such a bond be I and annual interest to be paid to the holder be iI. Then at any date (say $t = o$) the bond's market value equals

$$V_o = \sum_{t=1}^{\infty} \frac{iI}{(1+i)^t} = I \qquad\qquad \text{(iii)}.$$

If the borrower's financial prospects remain constant, dated bonds can be replaced on exactly the same terms, with the new bonds again holding their constant capital value to maturity.

The other type of instrument considered is one by which the long-term borrower raises the sum T on the basis of no periodic interest payment and final redemption value $T(1+i)^n$ after n years, where i is again the risk-free interest rate. The loan is renewed every n years as it matures. The initial market value of the debt is given by

$$\frac{T(1+i)^n}{(1+i)^n} = T \qquad\qquad \text{(iv)},$$

but it is clear that the market value of the debt rises as maturity approaches. However, as far as the shareholders of a static firm are concerned, the firm's debt has a constant value equal to the amount of new equity they would have to subscribe to

re-purchase on the market the whole of the title to the final payment $T(1+i)^n$. This is because the firm is assumed regularly to withhold from shareholders an interest charge at rate i on the sum borrowed, T, allowing it to accumulate through time at the risk-free interest rate ; so that at the redemption date (or any other time) shareholders need only raise an additional T to pay off the entire obligation.

The other aspect of capital structure theory requiring elaboration at this point concerns the possibility that a change in the firm's capital structure (a change in the equity-debt balance in its financing) may alter the terms on which outside lenders would be prepared to renew their lending to the firm. If capital structure is to be altered it is easy in principle to ensure that any overall capital gain or loss accrues solely to equity holders: if necessary the firm can repay its entire structure of all types of long-term debt and, immediately its new intentions become clear to the market, borrow the new planned level (and composition) of debt at rate(s) that are appropriate to the new situation.[3]

2 : 2 (ii)   Equity shares [4]

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(3)   Any change in the interest rate on a firm's debt consequent upon a change in capital structure implies the presence of at least subjective risk in the minds of lenders. This need not affect the constancy of the value of debt in the new capital structure nor the need for the firm to make a constant periodic appropriation for debt interest. The consequences of objective risk affecting debt are considered briefly in Chapter 3 : 3 (i).

(4)   The traditional view of equity valuation, to be suggested here as the kind of model management would prefer to work with, is explained and analysed in innumerable books and articles. See, for example H. Bierman and S. Smidt, The Capital Budgeting Decision, 3rd ed. 1971, pp.155-9; W.H. Jean, The Analytical Theory of Finance, 1970, pp.126-9; J.C. Van Horne, Financial Management and Policy, 2nd edition, 1971, p.94

At this point the emphasis in the exposition changes, and the concern now is to develop the kind of limited but internally consistent interpretation of equity market valuation that is likely to reflect management's preconceptions. It is assumed throughout that management's inclination is towards a self-centred valuation model rather than towards a full analysis of the forces determining equilibrium price relationships throughout the capital market. Management accepts the role of price-taker as far as the capital market is concerned: the financial instruments it issues are expected to settle at the equilibrium prices for instruments of those types.[5] Further, it is assumed that management, along with many theorists, accepts that the ultimate basis of equilibrium equity valuation is dividend expectations.[6]

The fundamental equity valuation model employed in Part I (and extended as appropriate in Part II) is

$$S_0 = \sum_{t=1}^{\infty} \frac{\overline{D}_t}{(1+k)^t} \qquad \text{(v)},$$

(5)   In addition, the growth of the firm _per se_ is not expected to alter the equilibrium market value of a unit of its expected dividend.

(6)   The case for accepting the presence of "dividend effect" is well reviewed in J.E. Walter, "Dividend Policy: Its Influence on the Value of the Enterprise," _Journal of Finance_ (May 1963). Rival theories of valuation are developed and compared in J.C.T. Mao, _Quantitative Analysis of Financial Decisions_, 1969, Chapter 12. Confirming the chosen approach is the fact that management does _not_ regard its future investment programme as fixed irrespective of dividend policy.

where S_o is the present-day ex-dividend market value of the firm's
equity capital; $\overline{D}_t$ $(t = 1 \ldots \ldots \infty)$ is the expected value of
dividend in period t ; k is the market rate of discount
appropriate to the risk of the dividend payment in each period.
The special case of (v) for a constant periodic dividend expectation
is

$$S_o = \frac{\overline{D}}{k} \qquad\qquad (va),$$

the dividend expectation capitalised at the appropriate discount
rate. At this stage it is assumed that both management and investors
agree on the objective risk properties of all D-distributions, so
that an ideal situation exists as far as the availability of
information is concerned. It is assumed in (v) but demonstrated
in later chapters that objective dividend risk in a sense to be
defined can indeed be held constant through time, justifying
the use of a single discount rate in (v). Points for discussion,
if not for final resolution in a preamble as brief as this, are
the concept of k itself and the implication of assuming a horizon
in an investor's valuation of equity.

There exists a strong line of criticism against the concept
of a discount rate performing at once the separate functions of
discounting cash flows over time and allowing for risk. But it
should be made clear that the discussion here is not about whether
investors discount future expectations; rather about the manner
in which they do so. A particular objection to k is that it
implies that the investor's adjustment for risk changes

geometrically with remoteness in time, effectively excluding any other pattern of risk adjustment. In defence of k several points can be made, the most important being that it does possess the algebraic property of reducing to equality with present-day price the stream of future dividend expectations, and therefore carries the authenticity of a "true" rate of return. Second, accepting for the moment that the concept of the investor discounting certainty equivalents is theoretically preferable to the model expressed by (v), a constant value of k is exactly equivalent to a certainty-equivalent factor declining geometrically with remoteness in time. Thus if

$$S_0 = \sum_{t=1}^{\infty} \frac{\overline{D}_t}{(1+k)^t} = \sum_{t=1}^{\infty} \frac{\lambda(t)\, \overline{D}_t}{(1+i)^t} \qquad (vi)$$

are two ways of explaining the same present-day equity market value, with $\lambda(t)$ the certainty-equivalent factor for period t and i the (constant) risk-free interest rate, the corresponding individual terms in the two versions must be equal if $\lambda(t)$ takes the form

$$\lambda(t) = \lambda_0^{\,t} \quad (0 < \lambda_0 < 1) \qquad (vii).$$

Given this form of certainty equivalent dependence on t , any pair of corresponding discount factors, $\left(\frac{1}{1+k}\right)^t$ and $\left(\frac{\lambda_0}{1+i}\right)^t$, must be equal, since the $\overline{D}_t$ series and S_0 are common to both versions. Obviously this demonstration exposes the limitation that the constant k is equivalent to only one of many possible $\lambda(t)$ functions: but on the other hand, if management actually considers the matter, the function shown in (vii) is likely to

seem both reasonable and not easily capable of disproof.[7]

There remains a difficulty associated with the simple equity valuation model (v) which arises when investors are recognised as having finite horizons. The difficulty is not that the continuing (indeed infinite) life of an equity share cannot be maintained when the owner's finite horizon is recognised, but with the valuation implications that arise in the process of linking the two ideas. In principle,

$$S_o = \sum_{t=1}^{\infty} \frac{\bar{D}_t}{(1+k)^t} \tag{v},$$

$$\text{or} \quad S_o = \sum_{t=1}^{n} \frac{\bar{D}_t}{(1+k)^t} + \sum_{t=n+1}^{\infty} \frac{\bar{D}_t}{(1+k)^t} \tag{viii},$$

$$\text{or} \quad S_o = \sum_{t=1}^{n} \frac{\bar{D}_t}{(1+k)^t} + \frac{S_n}{(1+k)^n} \tag{ix},$$

where S_n is the equilibrium ex-dividend market value of the firm's equity at $t = n$, the date at which a hypothetical investor intends to sell his holding. In (ix) S_n is necessarily discounted back

(7) The case for basing valuation theory on the discounting of certainty equivalents has been most forcefully argued by A.A. Robichek and S.C. Myers in Optimal Financing Decisions, 1965, pp.79-86, and in "Conceptual Problems in the Use of Risk-Adjusted Discount Rates," Journal of Finance (December 1966). These authors do admit the possible importance of geometrically declining $\alpha(t)$ (op.cit., 1965, pp.83-4). Also, contrary to the impression they give, it is likely to be extremely difficult in practice for management to discover that its use of constant k is leading to distorted decision making: in general, the longer a project's life the less obvious is any error likely to appear. In addition, the enormous problems of inferring the true $\alpha(t)$ series and the interest rate time series expected by investors seem likely to create their own distortions. On balance it does appear likely that management will prefer the k concept on practical if not intellectual grounds, and will find it hard to fault in practice.

to the present at the same rate as are the individual dividend expectations in (viii), otherwise S_o in (ix) could not equal S_o in (viii) . Yet the difficulty in accepting this necessity is that equilibrium equity values, present and future, have so far been regarded as not subject to risk and therefore ineligible for discounting at the rate appropriate to the individual dividend expectations.[8]

The only resolution of the paradox that does not make nonsense of the whole concept of an equilibrium market value independent of the investor's horizon is to accept that the future equilibrium value of equity _is_ recognised by the market as subject to risk in the sense that the actual value ruling may differ from the equilibrium calculated according to (v). This resolution has its good and bad aspects, but on the whole it can be expected to sustain and strengthen the belief of a not-too-critical supporter of the simple valuation model.

In favour of the suggested resolution of the paradox is its obvious appeal to the casual observer of movements in market values of equity shares: the model appears to correspond admirably

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(8)   Many expositions of the traditional dividend valuation model fail to recognise or deal with this difficulty. For example, Van Horne, op.cit., 1971, p.17, and E.M. Lerner and W.T. Carleton, _A Theory of Financial Analysis_, 1966, pp.123-5, both use expressions comparable to (ix) without acknowledging the logical difficulty. Robichek and Myers recognise the importance of the investor's horizon, but employ certainty equivalent analysis to handle the problem: see Robichek and Myers, op.cit., 1965, p.149.

to real-world events. Indeed, management may accept this
correspondence cheerfully, as offering justification for its
concentration on the long-term, or equilibrium, effects of
its policies on share value; and for its general disinclination
to become involved in or take responsibility for day to day
fluctuations in its own equity price.

There remains a particularly awkward question for the
intellectually curious management: why should S_n, admittedly
subject to the risk that actual market value at n may differ
from it, qualify for exactly the same discount rate in (ix)
as does $\bar{\bar{D}}_n$? The factors determining the discount rate on $\bar{\bar{D}}_n$
are, in the model in question, the risk properties of the D_n
probability distribution: but the resolution of the paradox
certainly does not show or imply that a future equity value has
a probability distribution as risky as that of one of the individual
dividends in the stream to which it relates. There is no reason
why the degree of price variability anticipated, given the
vagaries of capital market supply and demand conditions affecting
a particular share, should exactly equal the objective risk inherent
in that share's periodic dividend - though it is easy for the
casual observer to accept that the two kinds of risk are likely
to be positively related, with high dispersion of the typical
D-probability distribution contributing to a wide margin of possible
divergence of share price from equilibrium.

This tendency towards similar degrees of risk, together
with the fact that management can, if it wishes, envisage period
n as extremely remote in time, thus diminishing the importance of the

problem in its own thinking, means that the simple valuation model becomes an accepted and important part of management's intellectual apparatus.

2 : 2 (iii)   <u>A digression on variable equity prices under "ideal" conditions.</u>

The main point - managements acceptance of the traditional valuation model - may be taken as demonstrated. But it seems important to pursue briefly an outstanding logical difficulty in the "casual observer's" resolution of the paradox. It is difficult to reconcile a condition in which, while the capital market is aware of the objective dividend probability distributions for all future periods, there remains an expectation that the future price of equity at any date may diverge from its equilibrium value. The question of the basis of valuation is to be thoroughly discussed in Chapter 13, by which stage the nature of the difficulties confronting a (modified) traditional view ought to be very clear. For the moment an explanation is sought in a minimum departure from ideal capital market conditions rather than in some premature major overhaul of the valuation concept itself. A minimum departure in this sense would be that justifiable anticipations of a variable level of, and fluctuations in, dividend create a wide-spread expectation of unco-ordinated changes in financial plans of investors. Thus a low or reduced dividend is expected to lead to sales of shares by some investors without the kind of increased demand from others that would preserve the ideal valuation of each share. Similarly, high or increased dividend is expected, because investors find themselves with an above-average income, to lead to a higher demand for shares not matched by sufficient sales to hold the price down to its ideal level.

To this basic explanation for a widespread expectation
of non-ideal prices two points should be added. First, the
majority of companies' profits and dividends move roughly in
line with macroeconomic performance: total dividend income cannot
be expected to be stable, with individual movements largely
cancelling each other. An investor whose income from one share
is disappointingly low will be fortunate if his shortfall (against
his planned spending) is entirely made good by his other holdings,
and some general pressure to sell assets is likely. Thus it is
not suggested that it is the dividend performance of a particular
firm that may cause its share price to diverge from the ideal.

Second, there might appear to be an asymmetry in this
explanation, between falls and rises in price. A fall is easy to
understand, with sellers anxious to replenish their liquidity
and the fortunate minority of still-liquid investors willing to
see prices slide somewhat before offering support. A rise is
different: why should even a majority of liquid investors pay
more for an asset than its known ideal value? The answer is
actually no different from that explaining price falls: short-
term speculation explains the developments of divergences that do
occur, in either direction. A general shortage of shares at
ideal prices may not immediately raise prices of shares already
at those levels; but if surplus funds spill over and begin to
raise bond and other asset prices, as well as prices of other
forms of property, then even ideally priced equities become suitable

speculative prospects.[9]

To summarise, the expectation that equity prices will diverge from their known ideal values is understandable in terms of a commonsense view of capital market conditions. No change in the concept of ideal valuation itself is required, and that concept as expressed in (ix) can be retained – subject only to the doubt about using the same discount rate for $\bar{D}_n$ and S_n. Generalised conditions of high or low liquidity, in relation to investors' plans, create conditions in which prices can diverge from ideal levels: short-term speculation shapes the actual developments that occur.

2 : 2 (iv)   Dividend risk defined.

The actual measure of dividend risk assumed throughout the study is the coefficient of variation – the ratio of the standard deviation to the mean of the periodic dividend probability

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(9)   This account makes no pretence at fully explaining the origins and workings of speculative activity: indeed, by retaining the assumption of known ideal values it deliberately excludes a major basis for such activity. The point established is limited but methodologically important: a person considering the suggested valuation model need not reject it as an ideal merely because under the best possible conditions it appears to imply an inconsistency (the possibility of non-equilibrium price levels). And having accepted the model for the ideal conditions defined, it is a much smaller step to accept it as predicting the equilibrium values towards which market prices would tend if ideal information conditions did exist. It is of course in this second sense that management will be supposed to employ any valuation model, but the importance of the model's acceptability as an ideal cannot be overlooked.

distribution.[10] This measure is not without its critics, but its comparative simplicity means that it will serve admirably in its intended purpose of reflecting management's interpretation of valuation and its concept of risk. Discount rates on equity income streams are assumed to vary with risk, as defined, with the risk-free rate as the base of the rate structure. As will emerge in Chapter 3, management's valuation model may be incomplete as well as partial or self-centred, in that its knowledge of discount rates for risks differing markedly from that on its own equity may be extremely sketchy.

Section 3 :   Representing Business Earnings.

2 : 3 (i)   The earnings probability distribution.

The concept of a probability distribution of the periodic earnings accruing to the long-term capital of a business is central to the theory developed in subsequent chapters; and some preliminary description of the basic idea with clarification on certain aspects is advisable before the concept itself is put to work. In Chapter 3 the distribution is assumed to be stationary through time, but in later chapters its position is assumed to shift as investments decisions take effect. Whether static or shifting, the distribution shows the probability associated with each

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(10)   The same concept has been referred to as _financial risk_ : see J.C. Van Horne, _op.cit._, 1971, pp.198-201. The coefficient of variation applied to the firm's earnings probability distribution (see Section 2 : 3 (i) which follows) is used in Chapter 4 to define overall business risk.

possible level of earnings in the period to which it relates, thus embodying the combined effects of all external and internal determinants of performance. External factors include such variables as the general economic situation, the state of the particular markets in which the firm sells its products and obtains its inputs, the competitive structure of its industry and the behaviour of rival firms in any given situation. Internal factors include the firm's ability to vary output to meet the prevailing level of demand, more generally its control over both output and costs at different demand levels, its management of inventory and working capital, its willingness and ability to compete in various ways with rivals, and the extent to which its competitive behaviour affects earnings in different situations.[11] Evidently the earnings probability distribution is a catch-all concept which assigns a probability weighting to every possible level of periodic earnings of long-term capital.[12]

There are two definitional problems to consider before the concept of an earnings probability distribution is ready for use in capital structure analysis. The first, in the present context, is a matter of definition only, and may be disposed of briefly. The values used in the distribution are those of a firm's true earnings, E, after the deduction of an amount

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(11)   Thus the distribution reflects management's conditional decisions in relation to many aspects of operations, as well as the underlying and unalterable risks of the business.

(12)   In Chapter 8 : 4 (ii) it is argued that a "period" in management's financial planning is of considerably longer duration than a standard accounting year.

necessary to maintain the earnings potential of its physical
capital and other resources. While obtaining the correct value
for such a periodic deduction may represent a headache for
accountants, it is a comparatively easy procedure to accept in
principle. [13]

The second problem has practical as well as definitional
aspects. It concerns the interpretation of the whole E-distribution
if there is some possibility of negative periodic earnings. Even
without long-term debt in the capital structure a problem would
exist, because while firms may adopt various means of financing
an occasional loss they can rarely resort to the expedient of
raising additional equity for that purpose. Theory must recognise
that the lowest periodic return anticipated by shareholders is
zero. Nevertheless, to avoid both bankruptcy and take over
management must plan to finance its occasional losses in some way.

A correct perspective on this problem is that E will
be positive on most occasions. A loss occurs when in the face of
adverse external conditions the firm is unable to maintain its
revenue and/or reduce its costs sufficiently within the defined

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(13)   Under the static conditions envisaged it should not be
       difficult to compute a constant periodic amount which,
       on chosen assumptions, will maintain the E-distribution
       or earnings potential of the business : see J.W. Bennet,
       J. McB Grant and R.H. Parker, Topics in Business Finance
       and Accounting, 1964, pp.83-91. In the dynamic and
       risky conditions studied in Part II, an ex ante risk in
       relation to an investment's depreciation requirement may
       be understood as one of the factors creating the general
       condition of ex ante risk in respect of the earnings
       distributions of investments: see Chapters 8 : 1 and
       11 : 2 (ii).

period: to the extent that management is aware of the possibility
of adverse conditions it is more likely to be prepared for
appropriate cost-saving measures. The main reason, however, for
expecting a loss to be a comparatively infrequent and unimportant
occurence is the probable avoidance by management of activities
and markets which involve large foreseeable losses on a significant
number of occasions. All business risks are assumed foreseeable
and quantifiable, an assumption which yields an L-distribution of
known shape and position.

To a great extent, but not completely, the provision of
finance to meet occasional losses can be treated as a simple
insurance problem, provided only that the expected value of the
L-distribution is positive – a condition which scarcely requires
emphasis. The basic principle involved is the allocation of a
predetermined amount from positive L-values, when they occur,
so that the expected value of such allocations offsets the expected
value of losses. The simplest system of allocation to a reserve
fund would be the setting aside of a constant fraction, s, of all
positive L-values. To calculate the required value of s is
straight-forward, as is shown by Diagram 1 and the related proof.

probability

$E(-)$ A   O   B $E(+)$
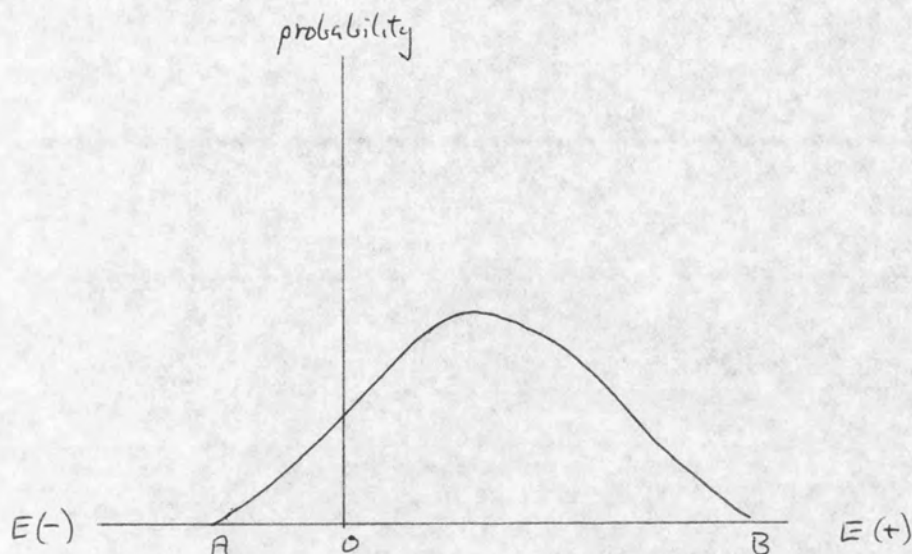
Diagram 1 : An earnings probability distribution.

Let a constant fraction of all positive E-values be allocated
to a reserve fund, and let the expected value of payments into
the fund be E V (R). Then

$$E\,V\,(R) = \sum_{E=0}^{E=B} s\,E\,p(E) = s\sum_{E=0}^{E=B} E\,p(E) \qquad (x).$$

Similarly, let the expected value of the firms losses, when they
occur, be EV(L). Then

$$E\,V\,(L) = \sum_{E=A}^{E=0} E p(E). \qquad (xi).$$

Given that E V (R) is intended to offset E V (L), s is obtained:

$$s = \quad - \quad \frac{\displaystyle\sum_{E=A}^{E=0} E\,p\,(E)}{\displaystyle\sum_{E=0}^{E=B} E\,p\,(E)} \qquad (xii)$$

This simple procedure for providing in good times for a foreseeable quota of bad times is approximately the one that prudent management will adopt. More complicated allocation formulae could be suggested, but the essential principle would be unaltered. Complications in the procedure are caused by either an accumulating surplus in the reserve-against-loss fund or an emerging deficit due to an exceptionally long and/or deep trough in the firm's fortunes. Before considering these the main effect of the reserve allocation principle must be made clear.

When the E-distribution embodies a probability of occasional loss a modification is necessary in order to derive a distribution that can be employed in the analysis of capital structure. This modification is simply the reduction in positive E-values in the original distribution and the elimination of negative E-values: the resulting distribution is the important one for this study and is referred to from now on as the Q-distribution. The probability given to the zero Q-value equals the sum of probabilities of zero and negative E-values in the original distribution :

$$p \, (Q = 0) \; = \; p \, (E \leq 0) \qquad\qquad \text{(xiii)}.$$

An example using a discrete E-distribution illustrates both the computation of s and the derivation of a Q-distribution. For the E-distribution in Table 1 the expected value of positive E - values is 27.0 and that of negative E-values is - 0.5.

E	-10	0	10	20	30	40	50
p(E)	.05	.05	.10	.15	.40	.15	.10

Table 1 : An example E-distribution

Applying (xii) yields s = 0.01852 : a fraction of all positive earnings, rather less than two percent, is an adequate provision for the expected value of losses. The Q-distribution derived from the original E-distribution is shown in Table 2.

Q	0	9.815	19.629	29.444	39.259	49.074
p(Q)	.10	.10	.15	.40	.15	.10

It is obvious, given the shape of the E-distribution, that the Q-values depend on the formula chosen for the allocation of earnings to reserve: the example illustrates only the simplest procedure — a uniform percentage allocation of all positive E-values — but other possibilities are obvious.

The complications mentioned earlier, an accumulating surplus or exhaustion of reserves, can be disposed of briefly. If E V (R) equals —E V (L) there should not in the long run be any upward trend in the level of the reserve fund, so the important problem is the possible exhaustion of reserves. To appreciate the nature of a solution it is helpful to imagine a very simple model of a firm's arrangements for financing its operations: any loss on current operations in excess of accumulated reserves is automatically and exactly met by bank lending to the firm as the latter is unable to make complete repayment of working capital borrowed during the period. This automatic loss financing has to be repaid, with interest, when earnings recover; but provided the firm's bankers remain satisfied as to the basic viability of the enterprise there

should be little difficulty in following such a course. Any
difference between the rate of interest payable on such
borrowings and that obtainable on surplus reserves has to be
taken into account in determining the value of s.

The Q-distribution derived in the way described here
is one of the central concepts in the discussion of orthodox
capital structure theory in Chapter 3. The reason for this is
that Q is defined as a non-negative level of earnings entirely
available for payments to the suppliers of the firm's long-term
capital. It was necessary to explain in a general way the derivation
of Q from the closely related concept E, because long-term
investors are not expected to directly perform the function of
financing occasional losses. The way is now clear for the new
concept to be used in its intended role in Chapter 3.

2 : 3 (ii)   Representing the earnings of investments.

This is a suitable point at which to indicate briefly
the basic way in which the prospects of investments will be
treated throughout this study. The approach chosen lacks
realism but lends itself to the straightforward symbolic
development of the main arguments — on cost of capital theory and,
later, multiple growth possibilities — without prejudice to the
generality of the results obtained.

An investment's earnings prospects are treated in much
the same way as the Q-distribution introduced in this section.

A proposal is assumed to have an identified probability distribution of periodic earnings, its ΔQ-distribution, which remains constant through time. For a proposal costing an initial amount I, the mean of the ΔQ-distribution, $\overline{\Delta Q}$, can be expressed as

$$\overline{\Delta Q} = r\ I \qquad\qquad\qquad (ixv),$$

so that r is an obvious measure of the proposal's rate of return. Throughout Part I investments are assumed to be "safe" — that is, their ΔQ-distributions are assumed identified ex ante so that no risk attaches to their r-values. In Part II the ΔQ-distribution for a given outlay is seen as subject to ex ante risk regarding its position.

Section 4 : Modigliani and Miller and Valuation Theory.

Returning to the subject of valuation after the apparently unconnected explanation of the Q-distribution may seem a disordered procedure, but the valuation interpretation attributed to management in Section 2 depends so heavily on the Q-distribution that any challenge to that interpretation should be seen against the background it provides.

The context assumed in Part I of this study, and indeed as far as Chapter 12 in Part II, is a straightforward one in which investors value objective probability distributions of dividend (by appropriately discounting expected dividends) and in which management, recognising this to be the nature of

valuation, is able to make capital structure and investment decisions with confidence in their long-term market value effects. The importance of stressing the long term aspect is obvious: in new or changing conditions objective knowledge of risks such as those of the Q-distribution does take time to become widespread, but this kind of departure from an ideal is not regarded by management as very damaging to the prospect of long-term equilibrium valuation as suggested by traditional theory. Indeed, the solution to the "finite investor horizon/infinite share life" paradox, implying some risk in relation to future equity prices, seems to recognise that fully ideal conditions are acknowledged to be unlikely.

One of the most powerful critiques of this traditional conception of valuation - and by implication much of the managerial decision making based upon it - was begun by Modigliani and Miller (M & M) in their celebrated 1958 paper.[14] The relationship of M & M's "irrelevance" propositions on capital structure and investment financing to management's actual financial decision making is discussed in Chapter 3:4(iii); but it is appropriate here to consider briefly their underlying conception of valuation which appears in complete contrast to the traditional interpretation and which threatens a breakdown in the belief in limited but sufficient communication between management and capital market on which much of this study is based.

(14)   Modigliani and Miller, op.cit., (1958)

According to M & M, investors do not, indeed cannot, value objective probability distributions, with risk measured objectively as the dispersion of the typical distribution. Instead, they form an admittedly subjective view of the probability distribution of the firm's average earnings over a period of time, and their conception of risk is (some measure of) the dispersion of this subjective distribution.[15] This view of the essence of risk is supported by the argument that "real" risk is a comparatively unimportant feature, in the sense that if a true earnings probability distribution were known the variability of earnings about the mean would be of minor importance in determining the present market value of all future periodic earnings randomly generated from the distribution.[16]

The present stage is not the appropriate place to deal at length with M & M's main suggestion - that investors value a subjective probability distribution of average earnings. For the moment both management and investors are assumed to recognise the objective risks embodied in the firm's Q-distribution: the M & M suggestion presumably becomes operative, if at all, in some context other than the static. In much of the development that is to follow, through comparative-static into growth contexts, the assumption that management recognises objective risks and relies on capital market valuation to converge towards the ideal is felt to be sufficient support for the main arguments on managerial objectives and decision making procedures. Nevertheless a point is reached,

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(15)   Ibid., pp.265-6.

(16)   Ibid., p.266.

in Chapter 13, at which the underlying nature of investors' valuation must again be examined, in the light of the complexities in objective risk situations and implied valuation of risks that will by then have arisen.

For the moment it only remains to take issue on behalf of traditional theory with M & M's supporting point, that "the effect of variability _per se_ on the valuation of the stream is at best a second-order one which can safely be neglected for our purposes (and indeed most others too)." [17] It is certainly true that in the long run an asset holder receives approximately the expected total value of earnings, regardless of the risk associated with individual receipts: but it does not follow automatically that the present amount he would be prepared to pay for the asset depends very little on the known risk properties of the periodic earnings distribution. The traditional dividend valuation model expresses the summation of _separately_ discounted expectations, and that model sees the investor as looking at each expectation and its risk separately, rather than valuing the long-term total expectation in a single mental process. [18] M & M's position in this supporting argument

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(17)   _Ibid._, p.266

(18)   Gordon in particular stressed this point in his dividend valuation model which implies that investors discount objectively equal dividend prospects at a rate that increases with futurity: see M.J. Gordon, _The Investment, Financing and Valuation of the Corporation_, 1962, pp.63-6. Gordon's approach to valuation is rejected here in favour of a version (Chapter 10:3) which plausibly represents investors' attitudes towards the greater risk they associate with more remote dividend expectations, and which lends itself to management's strategic decision making in the Part II context.

appears to derive from their well-known and much criticised image of the investor as engaged in a considerable borrowing and lending activity.[19] Their argument in response to criticism has been that the arbitrage results they describe (their "capital structure irrelevance" proposition) would be achieved if a small number of investors were so inclined, and this aspect of their general theory is still in dispute.[20] But in the present context their supporting argument (on the near-irrelevance of variability) appears to imply that all investors willingly engage in lending and borrowing to offset unexpected surpluses or deficits of income in relation to planned spending levels.

When investors are not credited with this kind of indifference towards frequent financial adjustment and are instead recognized as preferring stability and predictability, the traditional view of the separate importance of single results is confirmed. And from what is known of management's typical attitudes toward dividend stability it seems reasonable to attribute to it that traditional view.[21] The strength of that

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(19)   Modigliani and Miller, op.cit., (1958), pp.269-71.

(20)   For summaries of the various positions on M & M's
        leverage proposition see Robinchek and Myers, op.cit.,
        1965, Chapters II, IV; Van Horne, op.cit., 1971, pp.211-22.

(21)   Management's own concern for dividend stability was
        impressively demonstrated in J. Lintner, "Distribution
        of Incomes of Corporations among Dividends, Retained
        Earnings and Taxes," American Economic Review (Papers and
        Proceedings), (1956).

view in the present context lies in this correspondence between its assumptions and known managerial attitudes and preconceptions, and, as will become clear, in its usefulness and versatility in different areas of financial decision making.

The following chapter deals with the first of these areas, the capital structure decision of the static business firm.

Chapter 3

The Traditional Theory of Capital Structure

Plan of Chapter

Section 1 :   Introduction.

(i)   The fundamental relationship between capital structure theory and cost of capital theory.

(ii)   Simplifying capital structure theory.

(iii)   Dividend policy.

Section 2 :   Traditional Capital Structure Theory.

(i)   Shareholders' wealth and the market value of the firm.

(ii)   Dividing Q into separate income streams.

(iii)   The market value cost of capital concept.

Section 3 :   The Inconclusiveness of Capital Structure Theory.

(i)   Interest payments and risk.

(ii)   Falling V as B increases.

(iii)   Increasing V as B increases.

(iv)   Constant V at all levels of B.

(v)   Summary.

Section 4 :   Conclusions.

(i)   OCS in perspective.

(ii)   The relevance of the Modigliani and Miller propositions.

Section 1 : Introduction

3 : 1 (i) The fundamental relationship between capital structure theory and cost of capital theory.

This chapter describes the capital structure decision of the static business firm in terms of what might be described as an "orthodox" or "traditional" view of the subject. As is frequently the case with such labels, the user has an obligation to emphasise the diversity of views to be found among individuals placed for convenience within a single category, and in showing the wide range of possible solutions to the problem of optimum capital structure this chapter adequately fulfils the obligation. As explained in Chapter 2, the choice of theoretical model is intended to represent a possible managerial approach to understanding and applying market valuation concepts. Management's interests are entirely practical: in the present static context its concern is to improve the value of share-holders' wealth in the only way open to it — the choice of capital structure. In later chapters it will be seen that the same theoretical view of market valuation provides management with practical guidance on questions of invest-ment and financing. Finally, in the complex expectations model developed in Part II management can refine and adopt essentially the same valuation model, though the questions on which it will be expected to provide guidance are very different.

There is a precise expression of the connection between the capital structure theory of this chapter and the cost of capital models to be developed in Chapters 5 - 7. The former is simply the most elementary member of a "family" of situations, all of which satisfy certain conditions. These are:

(i) the firm's future investment programme is uniquely determined ex-ante, but not necessarily known to management or investors;

(ii)   the firm's Q-distribution (See chapter 2 : 3) in each future period
is also uniquely determined _ex ante_, but not necessarily known at
the present, even when the distribution shifts through time;

(iii) the risk class of the Q-distribution is known to be constant through time.

These conditions are satisfied in a trivial sense by a firm which
has no net investment plans and which expects its Q-distribution to remain
unchanged. But the connection between the static case – the subject matter
of this chapter – and dynamic situations which also satisfy the conditions
is not a trivial one. The cost of capital concept that can be derived for
the simplest case will be shown in later chapters to apply to investment
decision making in various dynamic contexts satisfying conditions (i) – (iii).
This is not an appropriate point to subject these standardising conditions
to critical scrutiny: their implications and practical relevance will
become clear at each stage of the analysis in later chapters.

3 : 1 (ii)   Simplifying capital structure theory.

The level of abstraction in this and later chapters is comparable
with the approach of many accounts of capital structure theory.[1] The
most obvious simplification is that the wide variety of financial assets
is condensed into just two categories : long-term debt and equity capital.
The exclusion of short-term debt is justified on the grounds that its
normal function is to finance current operations, and as such its level
fluctuates with the firm's activity: in these circumstances it is appropriate
to treat the interest cost of short-term debt like any other cost attributable

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(1)   See for example, E. Solomon, _The Theory of Financial Management_, 1963,
esp. Chapters VIII, IX ; D. Durand, "Costs of Debt and Equity Funds
for Business: Trends and Problems of Measurement," in E. Solomon (ed.)
The Management of Corporate Capital, 1959; S. Friedland, _The Economics
of Corporate Finance_, 1966, Chapters 8,10.

to current production - as a deduction from gross cash receipts. [2]
As far as debt that is undeniably long-term in nature is concerned, it is
convenient but not essential to assume homogeneity in all aspects, including
the rate of interest. The main requirement for capital structure analysis
is that no action by management should affect the market value of any part
of the firm's outstanding long-term debt: the nominal and market values of
long-term debt remain equal and constant. This is a requirement manage-
ment will be anxious to satisfy: its continuing ability to attract long-term
debt capital depends on the security of the capital value of its borrowings.

In many accounts of capital structure theory the terms "gearing"
and "leverage" are employed, as of course they are in comment on companies'
accounts in the financial press. These terms have come to possess a number
of closely related meanings, all relating to the importance of debt in a
firm's capital structure. Suggested ratios include $\frac{B}{K}$, $\frac{B}{E}$, $\frac{B}{V}$ and $\frac{B}{S}$
; where B is the market (and nominal) value of debt, K is the book
value of long-term investment in the firm, E is the book value of equity
investment, S is the market value of equity capital, and V is the sum
of S and B. While these ratios all appear to be reasonably good indicators
of a firm's reliance on debt, in this chapter B alone is used as the measure
of leverage. This approach avoids a commitment to any one of the suggested
measures, and is made possible by conducting the discussion in relation to

(2)   Short-term borrowing which effectively contributes to the financing
      of long-term investment should be treated as one element in long-term
      debt. Thus if the minimum short-term credit enjoyed by the firm
      exceeds its minimum requirement for financing current operations, the
      difference should be recognised as "disguised" long-term debt. For a
      discussion of alternative treatments of working capital see A.J.
      Merret and A. Sykes, The Finance and Analysis of Capital Projects,
      1963, pp.49-50 .

the earnings probability distribution - the Q-distribution - of a static firm. In this context the absolute value of B is a quite unambiguous measure of the importance of debt in the firm's capital structure.[3]

3 : 1 (iii) Dividend Policy.

It is a convenient feature of the static situation considered in this chapter that the dividend policy of the firm need not be specified or considered separately. With a static Q-distribution, if the firm succeeds in maximising the market value of shareholders' wealth it will by definition have chosen the optimum dividend policy, although the exercise is usually described as choosing the optimum capital structure. In later chapters it will no longer be possible to see dividend policy as a kind of by-product of capital structure policy : in fact the position will be that financing decisions (and hence capital structure) are directed towards a predetermined level of dividend risk.

Section 2 : Traditional Capital Structure Theory.[4]

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(3) The ratios $\frac{B}{V}$ and $\frac{B}{S}$ are open to the objection that a change in the numerator is _intended_ to produce a change in the denominator. In using either ratio as the measure of leverage to explain what happens as leverage changes, the analyst finds that his explanatory variable is itself dependent on the result he is trying to explain. See H Bierman, Financial Policy Decisions, 1970, pp.87-91, and Solomon, op.cit., 1963, p.81, for the various definitions of leverage.

(4) The context assumed in this section is a once-only choice of capital structure. Shifts from one structure to another can be carried out, either by raising new debt and making the proceeds available to shareholders, or by raising extra equity capital to allow the level of debt to be reduced. Whether the intention is to increase or reduce the debt level, the objective is the same: to increase the market value of investors' wealth. This being so, there is no difficulty in principle in planning a change in capital structure. The main technical problem in such a change is to ensure that debt holders experience neither a capital gain or a capital loss in the process, and this can ultimately be accomplished by repaying all debt at its nominal value and re-borrowing the desired amount from lenders at the interest rate appropriate to the new debt level. For an account of methods by which capital structure can be changed, see J.C. Van Horne, Financial Management and Policy, 2nd ed. 1971, Chapter 7.

3 : 2 (i) <u>Shareholders' wealth and the market value of the firm.</u>

It is easy to symbolise the problem facing the financial management of a business. Let the capital outlays required to establish the business be K ; and let that part of the total raised from equity sources be E , the remainder being raised as long-term debt, B . Then

$$K = E + B \qquad (i).$$

It is assumed that capital market equilibrium prevails, so that all securities remain at their true or correct market values.

The market value of all securities issued by the firm equals the sum of the market value of its issued equity, S , and the market value of its long-term debt. Since no capital gain or loss accrues to the holders of long-term debt, the market value of the latter equals its nominal value, B. Thus the market value of all issued securities, V, is the sum of S and B:

$$V = S + B \qquad (ii).$$

However, since $B = K - E$ in (i), it follows that

$$V = (S - E) + K \qquad (iii),$$

which expresses the firm's market value as the sum of the outlays required to establish it plus whatever capital gain/loss accrues to suppliers of equity capital. This is obvious, since the possibility of capital gain/ loss to the only other type of investor is ruled out. From (iii) it follows that V is maximised when (S-E) is maximised: the gain to equity investors is at a maximum. If management wishes to maximise the market value of equity investors' wealth, its immediate objective should be to maximise V.

Before proceeding to the theory relating the firm's market value to its capital structure, one further issue deserves consideration. An element of doubt in connection with V-maximising as a means of maximising investors' wealth is that in investing in one firm investors may inadvertently forego

more attractive opportunities elsewhere, either through lack of knowledge or because of the timing of different opportunities. This objection may be dismissed for two reasons. First, share issuing and valuation processes are assumed to work so rapidly that an investor's capital is involuntarily tied up for a very brief period, after which he is free at any time to capitalise his share of S for re-investment elsewhere. A second and more significant reason for upholding the simple view of management's correct decision rule lies in the limited extent of the latter's responsibilities: there is the possibility that investors may make decisions which with hindsight they will regret, but in the division of responsibility between management and investors the latter are seen as willing risk-bearers as far as "opportunity loss" mistakes are concerned.

3 : 2 (ii) Dividing Q into separate income streams.

The theory of optimum capital structure (OCS) is concerned with the possibility of maximising V when the only instrument available to management is the proportion of debt included in the firm's capital structure. The original investment decision and its earnings probability distribution are not in question at this stage. A static probability distribution for periodic earnings, Q, is shown in Diagram 1.
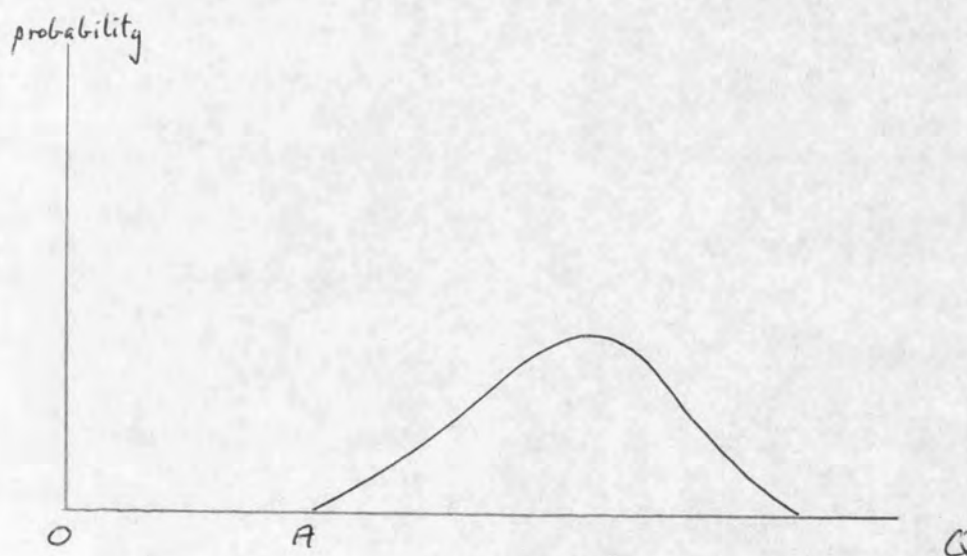

Diagram 1 : distribution with definite minimum value.

In Diagram 1 Q may be divided into "certain" and "risky" components, with A the exact dividing point between the two. [5] The level of periodic fixed interest charges incurred by management in its choice of capital structure may be shown as a horizontal distance measured along the Q-axis from the origin, leaving the residual amount up to the actual value of Q as equity income for the period. The advantage of assuming a definite minimum Q-value is that it provides a maximum level of risk-free periodic interest charge which the firm and its long-term debt holders will accept. In the absence of this simplification there would exist for any level of interest charge a possibility, however remote, that in a particular period the firm would be compelled to default on at least part of the charge. The theoretical consequence of this would be a supply schedule of debt capital less attractive than that facing a firm with the type of Q-distribution shown in Diagram 1, though the practical importance of the difference may be doubted as far as the lower end of the Q-distribution is concerned. It seems reasonable for both management and lenders to think in terms of a definite limit on the firm's capacity to meet interest charges reliably, and for the former to ensure that charges fall comfortably within the limit; so that interest payments may be regarded as virtually risk-free. [6]

Let the interest rate at which the firm borrows be i, and the nominal (and market) value of its debt be B. As long as i times B lies below OA in Diagram 1, changes in the total interest charge affect the mean or expected value of residual equity income but do not affect the absolute

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(5) It is assumed throughout that management, investors and lenders are in agreement about the shape of the Q-distribution: this is in keeping with the timeless approach adopted, the assumption that long-term static eqilibrium prevails.

(6) The simplifying assumption of constant i is dropped in Section 3, and in the meantime its presence has little effect on the variety of capital structure effects that can occur within the framework of traditional theory.

measures of its variability: a change in iB produces an equal change in the expected value of equity income but leaves the latter's standard deviation unaltered. An increase in iB increases the risk of equity income in relation to its expected value, and the effect is to reduce the equilibrium market value per unit of expected equity income.

Let the expected value of Q be symbolised by $\overline{Q}$, so that the expected value of equity income is $(\overline{Q} - iB)$. Instead of S the expression $\frac{(\overline{Q} - iB)}{k}$ can be used to denote the equilibrium market value of the firm's equity, where k is the market rate of discount applying to an income stream having the risk of the $(\overline{Q} - iB)$ distribution. Equation (ii) for V becomes

$$V = \frac{(\overline{Q} - iB)}{k} + B \qquad\qquad\text{(iv)}$$

The problem of OCS is that of balancing an increase in B against the reduction in $\frac{(\overline{Q} - iB)}{k}$ caused by (a) the resulting fall in the numerator as B increases, and (b) the rise in the denominator, k, as the risk per unit of expected equity income increases.[7]

3 : 2 (iii) The market value cost of capital concept.

In the situation considered no investment decision is in question: given the previously determined investment plan with its Q-distribution the primary interest is in the possibility of maximising V. Nevertheless it is appropriate here to derive the cost of capital concept which will later prove relevant in investment decision making, though at present the concept must be seen as simply another way of looking at the V-maximising problem.

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(7)   For completeness, if additional debt does give rise to an increase in i, the effects (a) and (b) are reinforced.

Let $c = \dfrac{\overline{Q}}{V}$ \hfill (v).

Then, by expansion,

$$c = \frac{\dfrac{(\overline{Q} - iB)}{k}\, k \quad + iB}{V}$$ \hfill (vi)

or $\quad c = \dfrac{Sk + iB}{V}$ \hfill (vii).

Thus $c = \dfrac{Sk}{V} + \dfrac{Bi}{V} = \dfrac{\overline{Q}}{V}$ \hfill (viii).

For any Q-distribution, whatever value is assigned to B — and hence whatever values emerge for k, S, V and i — c can be interpreted as a weighted-average cost concept since, by definition, $\dfrac{S}{V} + \dfrac{B}{V} = 1$. It is clear from (v) that the debt policy which minimises c is the one which maximises V, so the OCS problem may be presented in either way.

The view of the OCS problem as one of balancing using B against falling S to maximise $V = S + B$ (and minimise c) may be illustrated for any Q-distribution by a diagram showing schedules of i, k and c in relation to levels of B. This method illustrates the fact that c is a weighted average of its basic components i and k, and that these in turn depend upon the level of B. In Diagram 2, i is assumed constant within the range of debt levels considered; while k originates at a higher level than the i-schedule, reflecting the risk of the "unleveraged" Q-distribution, and rises as increasing debt raises the risk per unit of expected equity income.
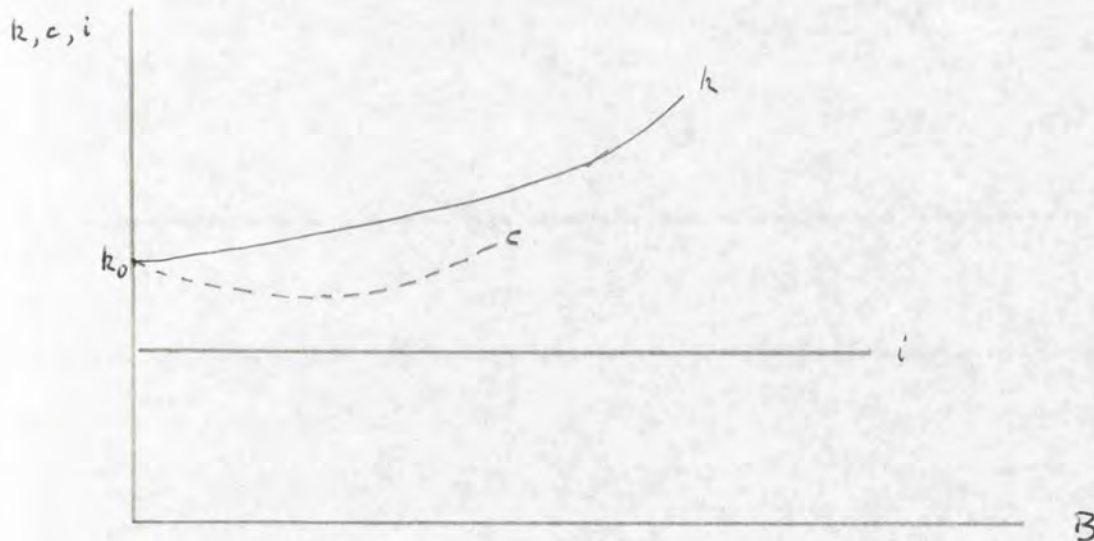
Diagram 2 : Relationship between i, k and c in traditional theory.

The weighted average of k and i is shown in Diagram 2 as the dotted schedule c: it originates at k_o where the capital structure is 100 per cent equity so that c equals k. Initially the weighted average c is drawn downwards by the increasing weight, $\frac{B}{V}$, attached to i which lies below k . $\frac{B}{V}$ is increasing because the addition of debt to the capital structure at this stage raises V by less than the value of debt added. This effect is assumed strong enough to offset the inevitable rise in k, and of course the matching decline in $\frac{S}{V}$ helps to lower the $\frac{S}{V}$ k component of c. Ultimately, however, the weighted average can be forced no lower through additional borrowing: if more debt is added the dominating influence becomes the increasing value of k, reinforced of course by the steadily rising $\frac{B}{V}$ i component (V having passed its maximum).

The tautological element in this demonstration of an optimal debt level is undeniable. If c reaches a minimum, V must reach a maximum. A rising c beyond B= OA requires a decline in V so that $\frac{Bi}{V}$ may rise. Without going beyond the framework of orthodox capital structure theory it might as easily be argued that V decreases or remains constant as debt

is added to the capital structure: there is nothing inevitable in a U-shaped c-schedule, just as there is no inescapable logic pointing to either a saucer-shaped or horseshoe-shaped schedule once a lower turning point is admitted. All that can be offered in the orthodox context are alternative sets of assumptions together with their expected results, and this is the task of the following section.

Section 3 The Inconclusiveness of Capital Structure Theory.

3 : 3 (i) Interest payments and risk.

Before considering the variety of possible c-schedule shapes, it is helpful to extend the treatment of debt beyond the level reached in Section 2. A completely risk-free type of debt (in respect of capital value and interest receipts) may not be inconsistent with a rising i-schedule, [8] but a complete explanation of increasing i should refer to objective risk factors of one kind or another; and here the treatment of debt is generalised by introducing the risk of an occasional default on the payment of (some or all) regular interest. As before, the redemption value of dated debt is assumed to be completely secure: a firm with a static Q-distribution operating in unchanging capital markets is able to renew its loans, as they mature, out of new borrowing. Neither the firm nor its lenders, in theory, will undertake loans which will not prove to be renewable on the same terms.

For a Q-distribution such as that shown in Diagram 3, there is no level of fixed interest obligations than can be regarded as completely risk-free.

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(8) If lenders identify risk in terms of the ratio of long-term debt to the balance sheet measure of net worth, the interest rate on long-term debt may increase with debt even before total interest payments approach the safe limit of minimum earnings. See Merret and Sykes, op.cit., 1963, pp.107 and 111.
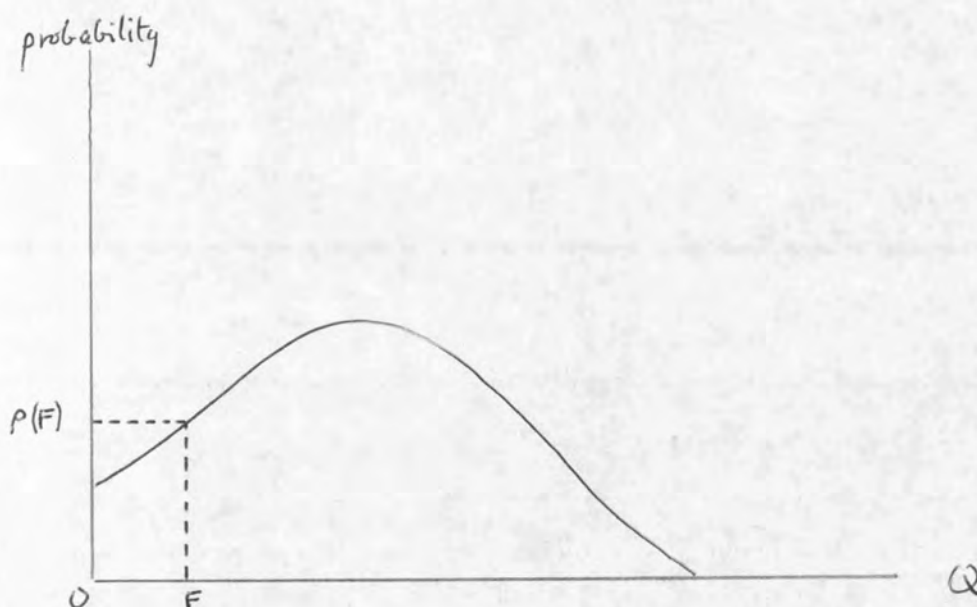
Diagram 3 : Q-distribution skewed towards lower end.

On a small proportion of occasions the firm will be unable to meet all of its fixed obligation, OF , out of current earnings; and although there are complicated ways in which it could plan to circumvent this difficulty, the simplest solution is for the firm and its potential lenders to accept the slight element of risk associated with the payment of annual interest and consequently a lower market value for the firm's debt. In such cases the market value of debt is expressed as

$$B = \frac{EV(I)}{i_1} \qquad\qquad (ix),$$

where $EV(I)$ is the expected value of interest receipts and i_1 is the market discount rate applying to income streams with the risk properties of the I-distribution. $EV(I)$ is of course lower than OF, the "face value" of the firm's obligations. As before, B is the nominal and market value of the firm's debt.[9]

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(9)   See A.J. Boness, "A Pedagogic Note on the Cost of Capital", Journal of Finance XIX (1964), for a full discussion of the treatment of debt in these circumstances.

The complete similarity between the valuation of equity income and interest income streams is obvious. Since both k and i have comparable roles in the expression (viii) for c , it is obviously correct that they should have a common theoretical origin - each expressing the market valuation of the objective risks of an income probability distribution. In addition, the sum of the expected values of interest payments, $EV(I)$, and of dividend receipts, $EV(D)$, must equal the expected value of the Q-distribution, $\overline{Q}$. No part of $\overline{Q}$ remains uncounted and no part is double-counted in the valuation model which generates V and c.

Having derived a concept to stand in place of the risk-free interest rate employed in Section 2, two obvious points may be noted briefly. First, the original concept, i , like the version now adopted, i_1 , applies to the expected value of interest payments: the only difference is that originally iB was a risk-free expected value. Second, it is easy to explain a rising i_1 - schedule as debt is increased with a given Q-distribution: the increasing risk in relation to $EV(I)$ causes i_1 to rise for exactly the same reason that k rises in this situation.

3 : 3 (ii) Falling V as B increases.[10]

Turning now to situations for which a traditional OCS solution may not exist, the first possibility is that V falls (c rises) as soon as any debt is introduced into the capital structure. This may be due to:

(a)   a sharply rising k-schedule;

(b)   a sharply rising i_1-schedule;

(c)   (a) and (b) together.

These will be considered briefly in turn.

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(10) Merret and Sykes (op.cit., 1963, pp.397-8) deal with the practical reasons for doubting the occurrence of such situations: this sub-section deals only with the theory.

The explanation for a rapid rise in k as B increases must lie with the arrangements made for the security of interest payments, or with the shape of the Q-distribution, or both. One way in which a borrowing firm can circumvent the problem of an occasional excess of iB over Q is to operate a reserve into which a sufficient amount is paid in good years to cover the known probability of a poor result.[11] The effect would be to maintain the near-absolute security of fixed interest payments at the expense of the average return to equity in periods when a dividend is paid. As long as the level of debt does not make such an "insurance policy" impossibly expensive the firm may continue to borrow at a near-constant interest rate, while the equity income distribution bears the full burden in the form of lower average return and higher risk per unit of return. The result might be a rise in c from the very outset of a debt increase policy.[12]

Turning to cause (b), a sharply rising i_1-schedule, it is easy to suggest a Q-distribution shape to which it might apply.

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(11) Such a scheme would have to take into account the chances of a run of bad years, but no doubt a policy could be worked out to provide for almost any level of security for the firm's commitments.

(12) While a rise in k has been explained exclusively in terms of increasing risk per unit of expected equity income, it is true that as debt increases there is a rise in the proportion of occasions on which no dividend at all is paid. This may be an additional factor contributing to the rise in k.

probability
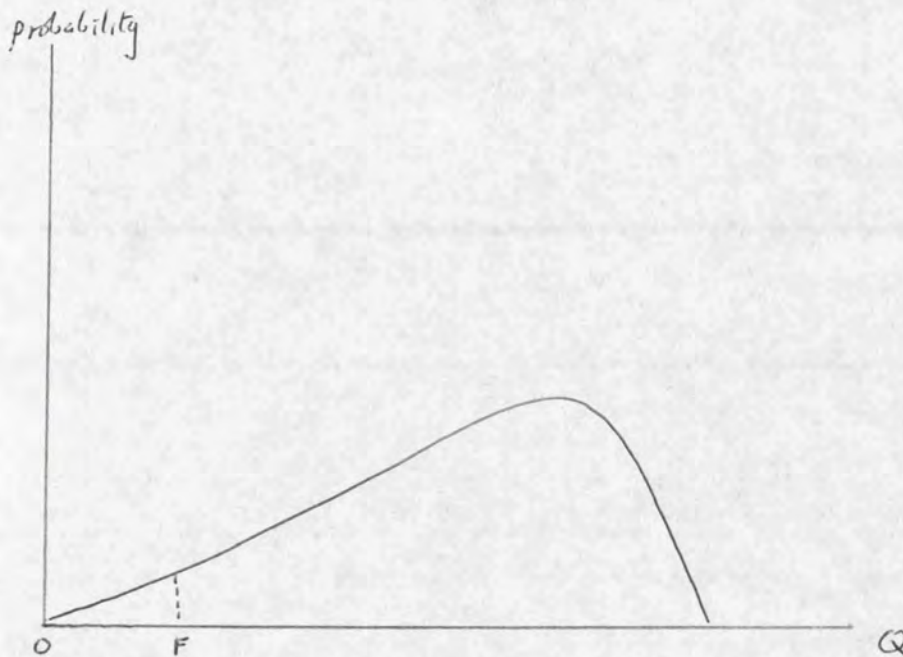
Diagram 4 : Q-distribution skewed towards upper end.

Suppose a Q-distribution with the general shape shown in Diagram 4 and suppose also that debt holders bear the risk of default on the fixed interest charge, OF . In this case the firm can only raise more debt on a rising i_1-schedule. Each time the point F moves a given horizontal distance to the right there is smaller rise in the expected value of interest payments, EV(I). Each increment to OF brings a smaller increment to B as the firm has to offer a higher face value of interest for a given amount of extra debt. The effect of this on equity return and risk is fairly insignificant and is not likely at first to imply a rapid increase in k: only a small area of the Q-distribution is involved and the risk of equity income in relation to expected value is not greatly affected by a change in the position of F when OF is low in relation to $\bar{Q}$.

As for cause (c), a certain shape of Q-distribution may cause (a) and (b) to reinforce one another. A rapid rise in i_1 with debt may mean a rapid fall in $(\bar{Q} - iB)$ and a marked rise in its risk and k-value. In a distribution like that shown in Diagram 3 a rapid rise in i_1 is likely if interest

payments are subject to the risk of default. The counterpart of this is a rapid fall in $(\overline{Q} - iB)$ as debt increases and a marked increase in its risk per unit of expected dividend, as well as an increasing probability of a zero dividend on occasion.

3 : 3 (iii) Increasing V as B increases.

The possibility that V may rise (and c fall) without limit as debt increases can safely be dismissed. In terms of its weighted-average interpretation, c could only fall to the limit set by i_1 , which in turn could at the very best remain constant. The one practical possibility is an abrupt limit on debt beyond a certain level, bringing a rise in V and fall in c to a premature halt. In other words, debt expansion would stop at some point before a traditional OCS was reached: lenders, management or equity investors would impose an absolute limit on the proportion of debt in the firm's capital structure.

3 : 3 (iv) Constant V at all levels of B.

The lower turning point of a traditional c-schedule reflects an exact mathematical balance between the effects of the increases in k and i_1, the decline in $\frac{S}{V}$ and the rise in $\frac{B}{V}$. In terms of the traditional theory of capital structure, the existence of such a precise balance at every level of debt is a state of affairs not to be taken seriously. It is only within the theoretical framework suggested by Modigliani and Miller that a rigourous argument for constant V and c can be made out.[13] Management is assumed not to desire or require a valuation model intended to answer the kind of questions that prompted Modigliani and Miller's contributions, and

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[13] The relationship of the present study to the Modigliani and Miller propositions is discussed in Section 4 (ii) below.

the likelihood that it accepts the theoretical constancy of V in terms of
a traditional valuation model can be dismissed.

3 : 3 (v) Summary

The position now is that management is assumed to employ a traditional
valuation model for its own limited purposes. This section has shown that
such a model provides no general expectation of the effect of debt upon
market value under equilibrium conditions, so that at its present capital
structure management can have no precise idea of the change in V that will
follow a change in structure - or even of the direction that the change
will take. This general unhelpfulness of traditional valuation theory
is one of two main factors leading to a realistic redefinition of manage-
ment's financial objective in the following section.

Section 4 : Conclusions.

3 : 4 (i) OCS in perspective

The theory outlined in this chapter presupposes the existence of
capital market equilibrium in which management has no difficulty in
identifying its optimum debt level. It remains to add to the picture the
elements of the real-world situation which are likely to interfere with the
process of achieving OCS and divert management's interest towards a different
objective.

First, only at the beginning of its existence does the firm have
maximum freedom to choose between capital structures. An orderly change
of structure is always possible at a later date, but is likely to involve
costs which have to be set against whatever gain in V is achieved.

Second, the pattern of equilibrium $prices$ of financial assets is unlikely to be fully obvious to the real-world observer: in practice management can be expected to have a clearer idea of the equilibrium value of its equity at the present risk level than for alternative levels. Third, the uniqueness of each firm's Q-distribution means that even if equilibrium valuation relationships were observable management would expect its own OCS to differ from those of other firms - even when the latter are engaged in activities similar to its own. Fourth, what is observable in the real world is a considerable variety of debt policies; so that a particular management may conclude that if other firms reach widely differing decisions on the quantity of debt, the sensitivity of V and c to capital structure is probably not very great.

These factors suggest managerial uncertainty or indifference about the effects of change in capital structure: given the certainty that a change is costly it is reasonable to expect that even in the present static context inertia tends to be a more powerful factor than a desire to achieve OCS. This in turn means that capital markets will continue, admittedly imperfectly, to value the equity of firms on the assumption that no changes in capital structures are planned and that prevailing levels of equity income risk will continue.

As a framework for decision making, the most striking feature of orthodox capital structure theory in an imperfect world is the extent to which managerial judgement is required. Capital structure change is both costly and risky, and it is not possible to refer the risk to market forces or to investors. Besides, management is likely to see itself as best placed to weigh the risks involved in trying to improve V by a change in the level of debt.

These conclusions about the importance of capital structure to management are strongly reinforced in later chapters as the static context progressively gives way to assumptions about investment and growth. From here on, management's attempt to interpret market valuation is assumed to focus on identifying c for a given capital structure, with the possible effects of varying that structure receiving little attention: optimising with respect to the size of its investment programme is a more convincing picture of management's activity on behalf of investors than optimising with respect to capital structure. Indeed, a constant c-value — not necessarily a minimum — considerably simplifies the appraisal of sets of investment proposals. The informational requirement for optimising in this new sense is lower than for OCS, and the continuing importance of investment decision making suggests that management will prefer to re-define its concept of market value — related optimising under real-world conditions of capital market disequilibrium and growth.

In spite of this re-definition, this chapter's central elements are neither irrelevant nor unimportant in what is to follow. In later chapters the basic view of valuation imputed to management is retained; and the dividend policy implicit in the static model is progressively adapted to the various growth contexts defined. Indeed, dividend policy, explicitly stated and deliberately implemented, remains throughout the essential link between management's understanding of market valuation and its optimising behaviour. Without a coherent dividend policy market valuation is unintelligible, at least in the terms required for investment decision making to be directed to the maximisation of the market value of investors' wealth. The essential elements of this chapter's valuation model and general optimising approach are carried forward for adaptation to the growth situations defined in the remainder of Part I and in Part II.

3 : 4 (ii) <u>The relevance of the Modigliani and Miller propositions.</u>

Finally, it has not been necessary in this chapter to join or comment on the long-running debate over the Modigliani and Miller (M & M) "irrelevance" propositions, the first of which relates directly to capital structure. [14] The reasons for this abstention are obvious in the light of the indications given of developments in later chapters, but an explicit justification for ignoring the debate can usefully be given here.

If it is true, as M & M asserted, that capital structure is irrelevant to investors under defined equilibrium conditions, management's awareness of the fact may be imperfect; but at least the impression of comparative unimportance will prevail. Correspondingly, interest in optimising in the more realistic sense of investment decision making will predominate. This is precisely the switch of emphasis that has already been indicated.

But there is a more fundamental reason for side-stepping the M & M propositions, including those relating to investment financing, and this should be clearly stated. The basic objection to the M & M propositions here is their irrelevance to management's attitude and approach to financial planning. It is most unlikely, for example, that management actually believes

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(14) The original Modigliani and Miller proposition was stated in "The Cost of Capital, Corporation Finance and the Theory of Investment," <u>American Economic Review</u>, XVIII (1958), pp. 261-97. The extension of the original argument to the growth context was made in M.H. Miller and F. Modigliani, "Dividend Policy, Growth and the Valuation of Shares," <u>Journal of Business</u>, XXXIV (1961), pp.411-33. The debate over the M & M propositions has been conducted at the theory level, where the propositions hold their own quite well; and at the level of practical considerations, where traditional views have not been displaced. A balanced discussion of the M & M contributions is given in A.A. Robichek and S.C. Myers, <u>Optimal Financing Decisions</u>, 1965, chapters III and IV.

capital structure to be completely irrelevant even under ideal static conditions; and in a dynamic context it would be profoundly uncharacteristic for it to accept the irrelevance of financing methods.

Management's financial planning is assumed to focus on the market value of equity; and to provide for its own ability to comprehend valuation it is assumed to plan for the future stream of expected dividends to be subject to a constant level of risk. This intention involves a continuing process of implementation when investments and growth are taking place regularly. The purpose is to ensure that equity market value reflects widespread knowledge of an established dividend policy, which in turn makes possible management's own interpretation of valuation and the derivation of a rational investment criterion. The central role of dividend policy in this scheme, the fact that the policy is defined by management and is intended to be implemented through periodic financing decisions, are arguments against the practical irrelevance to management of financing decisions and the capital structure to which they give rise.[15]

Given a managerial commitment to improve the market value of constant-risk dividend expectations by the appropriate financing of new investments, it is quite irrelevant for a theorist to argue that management's financing decisions are irrelevant. Whatever position may ultimately be agreed, the propositions under discussion in the M & M debate bear little relation to the realities of management's approach to financial planning.

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(15) The importance of established dividend policy to a firm's equity market value is stressed by many writers who have commented on the M & M propositions from a practical standpoint. Solomon, op.cit., 1963, pp.139-144, provides a balanced summary of the two sides to the debate before concluding that the traditional "dividend policy does matter" position is correct. A different but related point is the importance management attaches to following an established dividend policy: the model of dividend policy suggested by J. Lintner in "Distribution of Incomes of Corporations Among Dividends, Retained Earnings and Taxes", American Economic Review (Papers and Proceedings), (1956), remains an authoritative interpretation of the way in which

Footnote (15) (continued)

management expresses and implements its policy. The dividend
policy model to be developed in Chapter 7 shares with Lintner's
the feature of managerial discretion, but appears initially to
contradict it on the important matter of the dividend's ultimate
residual nature. This apparent contradiction is resolved in
Chapter 8 : 4 (ii).

Chapter 4

Constant Risk Class as a Condition and an Expectation

Plan of Chapter

Section 1.   Introduction

Section 2.   The Effects of Single Investments on Risk-Class.

      (i)   The general condition for constant risk class;

     (ii)   Testing the sensitivity of R to deviations from the constant-risk locus.

Section 3.   Conceptual Problems in Predicting Risk Class

Section 4.   Summary and Conclusion

Section 1 : Introduction

In this chapter a particular definition of risk class is chosen and used to examine the effects of investment decisions on the underlying risk of a firm's Q-distribution. The purpose is to test the assumption that investments do not significantly alter the risk of the Q-distribution. If upheld, this assumption implies that the initial rate of valuation of a firm's expected earnings, with a given degree of leverage, remains appropriate after its Q-distribution has been displaced through new investment. Beyond this problem, which is mainly one of numerical illustration, lies the important area of expectations of the future pattern of risks facing a firm. An attempt is made to deal with this in a limited way, though the basic assumption of an unchanging pattern of risks and returns associated with given physical assets is retained.

Throughout this chapter runs the important implicit assumption that the original Q-distribution is directly comparable with the probability distribution for the periodic earnings from a new investment, referred to as the Δ Q-distribution. Here the obvious difficulty lies in assuming comparability between a Q-distribution where shape and position can be taken as confidently accepted by management, and a Δ Q-distribution whose prospective position may require to be estimated. The following discussion concentrates on the effects of an investment on overall risk class once the shape and position of its Δ Q-distribution are viewed with as much confidence as the

original. [1]

Section 2 : The Effects of Single Investments on Risk Class.

4 : 2 (i)   The general condition for constant risk class

Before deriving the acceptance condition and required
financing proportions for a single investment proposal, or
programme, it is necessary to develop the concept of risk
class employed in chapter 3 to provide a rigourous definition
of constant business risk under conditions of a changing
Q-distribution. Such a concept is also required in later
chapters dealing with extended growth situations where the
Q-distribution is expected to shift repeatedly through time
as a series of investment expenditures takes effect. The
first stage is to derive the mathematical condition for an
unchanged risk class to result from an investment decision.

The definition of risk class is obvious in the light
of the analysis in Chapter 3 of the capital structure problem
of the static firm. The risk class of a Q-distribution is
taken to be the ratio of its standard deviation, σ_Q ,

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(1) It is only in Part II that this degree of _ex ante_
confidence in investment prospects is dropped : the concept
of a Q-distribution is retained, but its position is
not known _ex-ante_ with the certainty assumed here.
See Chapter 8 : 1.

to its mean value, $\overline{Q}$. [2] Let this ratio be R :

$$\frac{\sigma_Q}{\overline{Q}} = R \qquad\qquad (i).$$

A firm is deemed to remain in the same risk class when a change to a new Q-distribution leaves the value of R unchanged. An assumption commonly made in cost of capital literature is that the risk class of a firm would be unaffected by the acceptance of an investment proposal : the proportionate changes in $\overline{Q}$ and σ_Q would be equal. While this condition and assumption is easily stated, it is not obvious that investment proposals always conform to this pattern. [3]

The discussion of constant risk class may be generalised by introducing the formal procedure for combining probability

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(2) This definition seems to agree with what various writers have in mind in referring to the concept of "business risk" as opposed to that of financial risk : see A. Robichek and S. Myers, Optimal Financing Decisions, 1965, p.17; J.C. Van Horne, Financial Management and Policy, 2nd ed., 1971, p. 198. The choice of this ratio is suggested by the concept of "standard risk" employed in J.F. Helliwell, Public Policies and Private Investment, 1968, pp.10-11. There are several well-known objections to the use of this ratio, also known as the coefficient of variation, as a representation of risk: one is its failure to take account of any skew characteristics of a distribution; another might be termed the "portfolio aspect" - the likelihood that investors are interested in a particular share's returns as part of an overall distribution of returns on a portfolio of assets.

(3) An example of a definition of the market value weighted-average cost of capital with no proviso about constant risk is given in H. Bierman and S. Smidt, The Capital Budgeting Decision, 3rd ed., 1971, pp.147-8. J.C. Van Horne, op.cit., 1971, p.201, includes the proviso without discussion.

distributions. This allows for the R-value of the ΔQ-distribution to differ from that of the original while permitting the resulting combination to remain in the original risk class. The standard deviation of any probability distribution generated by combining two separate distributions is given by σ_j, where

$$\sigma_j = \sqrt{\sigma_1^2 + 2r_{12}\sigma_1\sigma_2 + \sigma_2^2} \qquad \text{(ii)},$$

in which σ_1 and σ_2 are the standard deviations of distributions 1 and 2 respectively, and r_{12} is the correlation coefficient between variables 1 and 2. [4]

Let $\dfrac{\Delta \overline{Q}}{\overline{Q}} = v$ for a proposed investment. If the investment carries the same risk, as defined, as the Q-distribution, it follows that $\dfrac{\sigma_{\Delta Q}}{\sigma_Q} = v$, so that $\sigma_{\Delta Q}$ may be expressed as $v\,\sigma_Q$. Accordingly, (ii) may be written as follows to give the standard deviation of the new Q distribution, σ_Q^1 :

$$\sigma_Q^1 = \sqrt{\sigma_Q^2 + 2r_{12}\,\sigma_Q\,v\sigma_Q + (v\sigma_Q)^2} \qquad \text{(iii)},$$

which simplifies to

$$\sigma_Q^1 = \sigma_Q\sqrt{1 + 2r_{12}v + v^2} \qquad \text{(iii a)}.$$

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(4) r_{12} can take values between + 1.00 and –1.00, and measures the degree and direction of association between values generated by two probability distributions. For a helpful explanation of the terms used in this section, in relation to the kind of problem considered here, see J.C.T. Mao, <u>Quantitative Analysis of Financial Decisions,</u> 1969, pp.45-52. For a general discussion of covariance see T. Yamane, <u>Statistics: an Introductory Analysis,</u> 2nd ed. 1967, pp.435-443.

If $r_{12} = +1.00$, then $\sigma_Q^1 = \sigma_Q (1 + v)$; and since $\overline{(Q + \Delta Q)}$ $= \overline{Q} (1 + v)$ the constant risk class condition is satisfied. Similarly, whenever r_{12} falls below $+1.00$ the value of σ_Q^1 falls below $\sigma_Q(1 + v)$ and the ratio of σ_Q^1 to $\overline{Q} (1 + v)$ must then fall below the original value of R.

The possibility should now be considered that a Δ Q-distribution not having the same R-value as the original Q-distribution might nevertheless combine with the latter to form a new distribution having the original R-value. As before, let $\Delta \overline{Q} = v\overline{Q}$, so that $\overline{(Q + \Delta Q)} = \overline{Q} (1+v)$. The standard deviation of a new distribution belonging to the same risk class as the original would have to be $(1+v)$ times the old value. Let $\sigma_{\Delta Q} = z \sigma_Q$, where z takes a positive value to be determined. Accordingly, the expression for σ_Q^1 can be written

as $\sigma_Q^1 = \sqrt{\sigma_Q^2 + 2r_{12} \sigma_Q z \sigma_Q + (z\sigma_Q)^2}$ \qquad (iv),

or $\sigma_Q^1 = \sigma_Q \sqrt{1 + 2r_{12} z + z^2}$ \qquad (iv a).

The required value of σ_Q^1 for the new distribution to belong to the same risk class as the original is $\sigma_Q(1 + v)$. That is

$$\sigma_Q^1 = \sigma_Q \sqrt{1 + 2r_{12} z + z^2} = \sigma_Q (1+v) \qquad \text{(iv b)},$$

or $\qquad 1 + 2r_{12} z + z^2 = 1 + 2v + v^2$ \qquad (v)

where the values of r_{12} and v are assumed given.

The locus of all r_{12}, v, z combinations satisfying (v) may be expressed as

$$z^2 + 2r_{12}z - (2v + v^2) = 0 \qquad \text{(vi).}$$

By taking any two variables as given, the value of the third which satisfies (vi) may readily be obtained. As an illustration of this, suppose that r_{12} and v for an investment's Q-distribution are given : the problem is to find the value of z (equal to $\frac{\sigma_{\Delta Q}}{\sigma_Q}$) which will just maintain the firm's overall risk class. Then z is given by

$$z = \frac{-2r_{12} \pm \sqrt{4r_{12}^2 + 4(2v + v^2)}}{2} \qquad \text{(vii)}$$

Because z must be positive it is obvious that whatever the value of r_{12} the solution value of z is given by

$$z = -r_{12} + \sqrt{r_{12}^2 + 2v + v^2} \qquad \text{(viii)}$$

Table 1 presents the results of calculating z from (viii) for the given values of r_{12} and v.

	r_{12}				
	-1.00	-0.50	0.00	+0.50	+1.00
v = 0.05	2.05	1.09	0.32	0.09	0.05
0.10	2.10	1.18	0.46	1.18	0.10
0.15	2.15	1.26	0.57	0.26	0.15
0.20	2.20	1.35	0.66	0.33	0.20

Table 1 : Required values of z for constant risk class, given r_{12} and v.

The pattern of z-values in Table 1 is broadly what one would expect. Whenever r_{12} equals unity, $z = v$ is necessarily the solution. As the degree of positive correlation between the distributions diminishes, there is scope for z to exceed v, implying that the new investment may carry a greater risk than the original Q-distribution without causing a change in the firm's risk class. The strength of this effect depends on the value of v, which expresses the relative importance of the Q-distribution, but there is comparatively little difference between the upper limits to which z may go for different v-values.

4 : 2 (ii) Treating the sensitivity of R to deviations from the constant-risk locus.

Table 1 shows that in general z may exceed v by an amount which increases as r_{12} falls and v rises: the latter effect declines as r_{12} falls and disappears altogether at $r_{12} = -1.00$. From a strictly arithmetic viewpoint the locus of z, v and r_{12} combinations dictated by (vi) suggests a precise and rigid constant risk class condition, and the chance that an investment will have a combination of properties satisfying (vi) is extremely small. It is worthwhile, however, to conduct a simple sensitivity test before concluding that the constant risk class condition is incapable of providing a realistic basis for cost of capital theory and subsequent developments.

The quantitative effect on the value of R in a combined distribution caused by a deviation of one variable from the

constant risk locus (vi) may be judged by taking as a sample the combinations of r_{12}, v and z given in Table 1. Values of z which are high in relation to v can be treated as unrealistic in varying degrees, and the suggested test is to set z experimentally at 1.50 times the level of v and calculate the resulting effect on R. (This hypothetical level of z still represents a risky project in relation to the firms original risk class: for example, an investment for which $v = 0.20$ ($\frac{\Delta \overline{Q}}{\overline{Q}} = 0.20$) is assumed to have a standard deviation equal to 0.30 times that of the Q-distribution.) The ratio of the new R-value to the old can be written as

$$\frac{R_1}{R_o} = \frac{\sigma_Q 1}{(\overline{Q} + \Delta \overline{Q})} \div \frac{\sigma_Q}{\overline{Q}} \qquad \text{(ix)},$$

or

$$\frac{R_1}{R_o} = \frac{\sqrt{1 + 2r_{12} z + z^2}}{1 + v} \qquad \text{(x)},$$

after incorporating (iv a). For the suggested simulation of deviations of z from the values dictated by (vi), equation (x) becomes

$$\frac{R_1}{R_o} = \sqrt{\frac{1 + 3r_{12} v + 2.25v^2}{1 + v}} \qquad \text{(xi)}.$$

Table 2 gives values of $\frac{R_1}{R_o}$ determined by (xi) for the same r_{12}, v combinations used in Table 1.

r_{12}

v	-1.00	-0.50	0.00	+0.50	+1.00
0.05	0.98	0.92	0.99	0.99	1.02
0.10	0.77	0.85	0.92	0.98	1.05
0.15	0.67	0.79	0.89	0.98	1.06
0.20	0.58	0.74	0.87	0.98	1.08

Table 2 : Values of $\dfrac{R_1}{R_0}$ for selected r_{12} , v combinations,

given z = 1.5 v.

Table 2 seems to indicate that while most deviations of z produce a rather small, negative, change in R , some very significant reductions are associated with low r_{12} - high v combinations. In practice, values of r_{12} are likely to occur towards the upper end of the range - 1.00 to + 1.00, and single investments are unlikely to have high v-values. These factors suggest that combinations of r_{12} and v will tend to concentrate towards the top and right of Table 2, where the effects of z deviations on overall risk class are relatively slight and beneficial. [5] If investment is unlikely to result

(5) In Table 2, z is set higher than v in each case in order to make the test more exacting, and with the idea in mind that an individual project might well embody a higher risk (relatively) than the firm's overall q-distribution. The effect of setting z at a level below v is, broadly, to improve risk class relative to the result shown in Table 2 when r_{12} > 0.00, and to worsen it in this sense (though still to improve on the original) when r_{12} < 0. Naturally, many simulations are possible, but the general tendency of results is clear.

in a worsened risk class (and adverse valuation consequences),
why is it necessary to impose such a rigid assumption in
developing an acceptance criterion for investments; especially
as in cases where risk class improves use of the criterion
leads to the recommendation of an unnecessarily high required
rate of return on investment?

The main answer to this question must be the considerable
convenience which, as will be seen, the assumption of constant
risk class permits in deriving both a concept and a measure
of the cost of capital. However, convenience alone cannot
justify constant risk class as an assumption in choosing a
framework for appraising investments. If the assumption is
completely out of touch with reality, and if decision making
on such a basis is known to be erroneous and inefficient, then
the convenience of the approach is no justification. In fact,
most investments are likely to have an insignificant effect
on R; and management should in any case be able to identify
investments with significant potential effects on R and
modify its assumptions and criteria accordingly.

Constant risk class is to be regarded primarily as a
necessary condition for the applicability of the investment
criteria to be described in subsequent chapters. But, as
this section has shown, it may also be seen as reasonable
assumption of management about the normal effects of individual
investments.

Section 3 : Conceptual Problems in Predicting Risk Class.

Discussion of the applicability of the constant risk class assumption in developing a reliable investment criterion has been concerned exclusively with the relationship between the risk properties of the Q and ΔQ-distributions. As far as the once-for-all investment is concerned the discussion need proceed no further: nothing foreseen in the future casts any doubt on the permanence of the risk properties of the new Q-distribution. As shown in Section 2, the assumption of constant risk class in this context seems to stand up quite well to scrutiny: however, to leave the justification for such an important expectation resting on a fairly narrow base is unsatisfactory. Two particular aspects deserve closer consideration before that expectation can be accepted as a worthwhile foundation for a valuation-based cost of capital theory applying to a wide range of investment situations. There is the problem, mentioned earlier, of individual proposals which, in the opinion of management, would have significant and possibly adverse effects on the firm's risk class. Secondly, there is the problem of predicting the effects of future investment decisions on risk class: such effects may eventually alter the market valuation of present earnings prospects, so that a constant-risk effect of current investment proposals only cannot ensure an unchanging risk class in future. These two aspects in a dynamic context are not easy to separate, and will be considered jointly.

The obvious starting point is an extension of the argument that single investments are likely to have minor effects on overall risk class. A similar argument might be applied to a series of investments taking place over an extended period of time. In this context one could argue that the risk properties of the Q-distribution would only alter slowly; or that the effects of different projects might well offset each other, leaving the overall risk class scarcely affected. The limits of this essentially arithmetic argument could be tested by simulation, as was done for the once-only investment.

Another obvious defence of the expectation of constant risk class is that investments made in developing activities similar to those in which a firm is already engaged may be expected to generate additional earnings which are influenced by external events and internal responses in at least the same direction, if not to the same extent, as present earnings. Of course it is equally arguable that a typical firm may seek to diversify its activities, given time, in order to reduce the degree of positive correlation between present and incremental earnings distributions, thus achieving an improved overall risk class.[6]

(6)   The literature on motives for diversification is enormous, and it is clear that a deliberate intention to reduce business risk is far from the only factor at work. In any case, success in reducing business risk by diversification is likely to be limited by the fact that many activities' returns correlate positively with industry or macroeconomic performance: see J.C. Van Horne, op.cit., 1971, pp.168-171. There are of course many examples of the kind of diversification that should have the effect desired: an umbrella manufacturer can add sunshades to his product line; travel agencies offer holidays at home as well as abroad. In most cases these diversifications should already be in effect, and reflected in the existing business risk level.

The possibilities for doing this would require assessment on a case-by-case basis, and even if diversifying investment (in the sense of a low covariance) has definite potential for altering risk class, its relative importance (the first point mentioned above) might still be slight.

A firm may be justified in assuming an unchanging risk class through time if it expects the combined effects of a number of separate investments to leave unchanged the measure of overall risk. Within such a group of investments there would be room for a variety of individual ΔQ-distributions, subject to the single constraint that the firm holds a reasonable expectation of a new combined distribution belonging to the same risk class as the original. Taking this argument to its limit, one could argue that as long as it is the intention of management to maintain a given level of risk in the Q-distribution through such a continuing policy of investment choice, the precise risk characteristics of any individual investment are not important. It would follow that individual proposals should not have to satisfy the constant-risk condition as part of an accept/reject procedure. This suggestion implies that a reasonable view of the investment process and management intentions permits a relaxation of the strict condition for constant risk class applying to a once-only proposal. Far from adding to the difficulty of justifying the expectation of a constant risk class, the introduction of a dynamic and uncertain future appears to make the expectation easier to support. This rather unexpected conclusion deserves further consideration.

It is not hard to circumvent the difficulty created by
conceding management's uncertainty in many (if not most) cases
over the level, profitability and risk properties of future
investments. The important factor is the intention, expectation
or committment on the part of management to achieve in the
long run a constant risk class through the choice of future
investments. The fact that changes in the Q-distribution
(either from the present or from some future date) may not be
accurately foreseen means that overall valuation is a much
more difficult question than was assumed earlier: but because
the purpose of an investment criterion is to compare the
incremental value resulting from an investment with the required
outlays, the problems of overall valuation can be ignored for
present purposes. [7] Management is assumed to know the overall
risk class to which it intends the firm to belong; the desired
amount of debt in relation to any $\bar{Q}$ (the preferred definition
of leverage); and the consequential values of k, i, $\frac{S}{V}$ and $\frac{B}{V}$. [8]
Uncertainty attaches only to the key variable $\bar{Q}$; or, more
accurately, to the pattern of its future movements (apart from
the anticipated effects of investments under consideration).
In this case it is obviously quite sufficient when an investment
does not result in an altered risk class to use its $\Delta\bar{Q}$ to

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(7)   This statement, strictly speaking, is valid only in the
      retained earnings/fully subscribed rights issue method of
      financing, or where the capital market correctly values
      a firm's earnings prospects on the occasion of an issue
      of equity capital to outsiders : see Chapter 6.

(8)   In Chapter 3 it was shown that given the Q-distribution
      and the preferred debt level, i and k are dictated by
      market conditions and so S and V take determinate values.

calculate ΔV and ΔS. The important question remains that
of determining whether an investment may be so treated even
though its risk properties do not satisfy the constant risk
condition, on the grounds of management's intention to main-
tain overall risk class.

This question is answered in the affirmative where
there exists a well-defined investment programme, subsequent
to the expenditure in question, which is expected to restore
the desired level of risk to a firm's Q-distribution. The
main concern is with an interim period, of uncertain duration,
in which the firm's Q-distribution will belong to the "wrong"
risk class. Whatever may be said about this problem must apply
equally to the general issue of uncertainty over the effects
on risk class of future investments.

The straightforward case shades off into others, with
longer and longer interim stages, with investment plans becoming
less and less specific; until at the far end of the spectrum
of possibilities there is simply a vague managerial expectation
of a certain risk class and a feeling that it will be attained
at some uncertain future date through an undefined investment
programme. The important questions are: whether an investment
proposal can be subjected to the same criteria in all situations
falling within such a wide range; or how to divide the range
and suggest modified criteria for the various categories.

A practical answer is that management can and must decide

whether its plans, assumptions and preferences are compatible with any particular frame#work for the appraisal of investment proposals. It must determine whether uncertainty over future risk class is sufficient reason to forsake the convenience provided by the constant-risk assumption, bearing in mind the point that the more unreliable a risk class expectation appears to be, the harder it becomes to discover an alternative assumption on which to base a decision rule.

The use of a constant risk assumption can be justified on the practical grounds of convenience and the absence of an obvious alternative. Yet one should at least consider whether the decision procedure based upon a mistaken risk class expectation is likely to prove so costly as to suggest a wider search for an alternative. A reassuring factor is that the risk class assumption can be kept under review and a mistaken view corrected as soon as it becomes clear. Furthermore, where an error is due to mis-judgement of the effects of investments on risk class, earlier discussion provides re-assurance: the effects of investments on risk class are probably small and/or favourable. The loss incurred during the period of mistaken expectations is difficult to define and measure, but it is hard to believe that within a comparatively short period it could reach a significant size.

Section 4 : Summary and Conclusion.

The main purpose of this chapter has been to define and

examine the expectation of constant risk class in a dynamic context, in preparation for the cost of capital analysis in subsequent chapters. This has been done at some length, in contrast to the brief acknowledgement of the assumption often given in cost of capital literature, and in some ways the exercise has proved unnecessary. The condition of approximately constant risk is likely to be satisfied in many investment situations, and the expectation of constant risk class is both reasonable and likely to appeal to businessmen. Furthermore, one can expect that risk class in practice lacks the precision given to it in this chapter, so that individual investments may be given even more leeway in their individual risk characteristics before being excluded on the grounds of a significant effect on overall risk class. Given uncertainty over the effects of future investments on risk class, management should make the best estimate of future risk class in the light of the sort of investment opportunities expected.

Chapter 5

The Cost of Capital and Financing of Investment Decisions

Plan of Chapter.

1. Introduction

2. The Required Rate of Return and Financing of a Marginal Investment

    (i) The general solution for required rate of return.

    (ii) The equivalence of r^* and the market valuation rate, c.

3. The Market Value Weighted-Average Cost of Capital and the Financing of Investment Programmes

4. Reconsidering the Importance of Capital Structure Optimising

5. Dividend Policy Considerations

Section 1 : Introduction.

This chapter extends Chapter 3's orthodox approach to capital structure theory to the problem of a single, once-only, investment which satisfies the constant-risk condition of Chapter 4. In subsequent chapters the cost of capital concept developed here is adapted to various growth situations and for various methods of equity financing. It should be borne in mind that Chapter 4 concluded that a good deal of leeway might be allowed to individual investments on the question of satisfying the constant-risk condition : firms were assumed to hold long-term views on expected risk-class. The cost of capital concept is shown to have an unambiguous meaning only when it is specified that investment/financing decisions do not alter the value of c - described in Chapter 3 as a cost of capital concept and referred to where appropriate from now on as the overall market valuation rate. The correct financing proportions for investments, consistent with the above conclusion, are shown to depend on their profitabilities.

Section 5 widens the discussion to incorporate dividend policy considerations in so far as these can be brought into a discussion of once-only interference with a policy of 100 per cent pay-out of equity income. This is intended to suggest how management might see the problem of fixing a limit on the retention of equity income for (profitable) re-investment in the light of an assumed commitment to the best interests of long-term investors. The equity financing methods considered in this chapter are confined to the retention of equity income and - regarded as equivalent in the absence of taxation - a rights issue of new shares fully subscribed by present shareholders. In either case financing is seen as a mixture of new debt and equity capital which is intended to maintain the firm's overall market valuation rate, c. It is obvious that firms do not continually adjust capital structures to conform with this theoretical condition as

investment takes place, but it is important to distinguish between the apparent or obvious costs associated with current financing, and the long-run implications of a policy of maintaining the value of c. As far as management is concerned, a commitment to the long-term maintenance of a given market valuation rate implies a commitment to the financing of investment so as to achieve this result : the firm is entitled to treat its cost of capital as constant only as long as it is prepared to operate an appropriate financing policy.

A final point of introduction : the investments considered in this chapter - indeed in Part I as a whole - are alike in having rates of return which are known ex ante to management. An investment's earnings fluctuates about a mean value, but that value, the variability of earnings about it, and the properties of the new Q - distribution are all fully identified by management before the investment is undertaken.

Section 2 : The Required Rate of Return and Financing of a Marginal Investment.

5 : 2 (i)   The general solution for required rate of return.

The market value of a company's equity may now be assumed to consist of two elements: D_o, the dividend to be paid in the near future; and S_o, the ex-dividend market value of the company's shares. Shareholders' wealth, as far as the particular firm is concerned, can be defined as

$$W_o = D_o + S_o. \qquad (i),$$

where $S_o = \dfrac{(\overline{Q} - iB)}{k}$, as defined in previous chapters.

A decision criterion is required to evaluate a hypothetical investment outlay, I, the earnings of which are expected to raise the mean value of Q by $\Delta\overline{Q}$, where $\Delta\overline{Q} = r^*I$, [1] and to have no significant effect on the firm's overall risk-class. Let the investment be financed with equity capital and debt in the proportions $(1-b^*)$ and b^* respectively, and let the rate of interest on additional debt be equal to that already paid by the firm on its present debt. [2] The problem is to establish those values of r^* and b^* for which the hypothetical project is marginally acceptable to the firm's shareholders, and thus to identify the minimum acceptable rate of return, r^*, on an investment which does not alter the firm's risk-class. In considering this hypothetical problem it will become clear that, without further specification, there is no unique pair of r and b values making an investment marginally acceptable in the sense defined.

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(1)   Recall that the return on an investment, expressed as a fraction of original cost, is assumed constant in perpetuity, after making whatever allowance is necessary for maintaining the asset's earning power. See Chapter 2:2.

(2)   The interest rate on the firm's debt is assumed constant for a range of debt levels in relation to the mean of its Q-distribution : the assumption is confirmed by the result obtained in sub-section (ii) below.

There are two aspects to the required solution, given that the investment satisfies the constant risk-class conditions. The first is that equity-holders should be no worse off financially as a result of the acceptance of the project and its financing in the chosen proportions. The second condition, which may initially appear unnecessary in the light of the all-embracing nature of the first, is that the value of k applying to equity income should not change. It is this second condition, in fact, which supplies the additional specification mentioned above as being necessary for obtaining unique solution values of r^* and b^* : its purpose is to ensure that the firm's overall cost of capital is unchanged by the acceptance of the marginal investment proposal and its financing.

The need for, and convenience of, the two-part condition can be demonstrated quite simply, by discarding the second part and leaving the single condition that shareholders must not be adversely affected by the investment/financing decision. This condition may be stated as

$$W_o^1 \geqslant W_o \qquad \text{(ii)},$$

where $W_o^1 = \underline{/}D_o - I(1-b)\underline{/} + \dfrac{\overline{Q} - iB + I(r - ib)}{k^1}$ (iii): [3]

k^1 is the new discount rate on expected equity income and W_o^1 represents the value of equity to the shareholders, given the decision to invest in a proposal offering a return r and financed by equity (retained earnings) and debt in proportions (1 - b) and b respectively.[4] As with W_o, the total consists of the dividend, plus the ex-dividend market value of the equity. The latter has as its numerator the new expected value of dividend in future periods, and as its denominator the new discount rate appropriate to the risk of the equity income distribution.

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(3) This assumes no change in i, for reasons given in footnote (2).

(4) The text assumes that equity finance is obtained from retained earnings, but the argument applies equally to a rights issue of shares fully subscribed by existing shareholders.

Comparing (i) with (iii), the condition for $W_o^1 \geqslant W_o$ may be expressed as

$$I(1-b) \leqslant \left[\frac{\overline{Q} - iB + I(r - ib)}{k^1} \right] - \left[\frac{\overline{Q} - iB}{k} \right] \quad \text{(iv)},$$

which can be conveniently written as

$$I(1-b) \leqslant \left(\overline{Q} - iB \right)\left(\frac{k - k^1}{kk^1} \right) + I(r - ib) \frac{1}{k^1} \quad \text{(iva)}.$$

For a marginally acceptable investment the two sides of (iva) should be equal. The terms on the right-hand side of (iva) are, first, the capital gain or loss on the original equity value resulting from any change in the equity income capitalisation rate; second, the expected value of earnings attributable to the new investment, discounted at the new rate k^1. The left-hand side of (iva) is obviously the net cost to shareholders financing the project, in terms of the immediate sacrifice of dividend income.

If no significant change in the firm's business risk results from undertaking the investment, a change in k can only result from the choice of a value for b. If b lies below a critical value - to be identified - then the risk of the equity income distribution in relation to its mean value will diminish; and the value of k will fall. This will confer a capital gain on the valuation of the whole of the original expected value of equity income, $(\overline{Q} - iB)$. Conversely, if b lies above its critical value the risk on expected equity income will increase and k will rise, causing a corresponding capital loss. The conclusion must be that the value of b is critical in determining the value of r for which (iva) becomes an equation. The required return on a marginally acceptable investment depends on the debt and equity proportions used in its financing.

To leave the condition in this incomplete form is obviously unhelpful, and there are good reasons for restoring the second part of the condition

as originally stated - that k be assumed constant in deriving the minimum
acceptable rate of return which does not alter the firm's overall risk-
class. In the first place, the condition as it stands suggests that an
investment proposal's acceptability depends not only upon its rate of return,
r, but on the proportions of debt and equity chosen for its financing.
Because a given proposal can either be accepted or rejected depending on
the b - value chosen, it is necessary to standardise the financing proportions
assumed in deriving the required rate of return on a marginally acceptable
proposal. Secondly, the need for a standardising procedure becomes clear
on an inspection of (iva), where an element of capital gain/loss arises
whenever k changes as a consequence of an investment/financing decision.
In (iva) an identical capital gain or loss is attainable without the
necessity of undertaking any investment at all, simply by varying the
capital structure of the firm along the lines described in Chapter 3. In
view of this it seems erroneous to attribute to the proposed investment a
result which could have been achieved directly by altering the capital
structure associated with an unchanging Q - distribution. For this reason
it is not only convenient but also completely correct to derive a unique
required rate of return on investment on the explicit assumption that k is
not affected by the investment and its related financing. Any other
assumption about k must involve a confusion between the change in W_o
attributable to the investment itself, and that attributable to a change
in capital structure.

It should be added that if the firm's capital structure is judged
by the management to be optimal, in the maximum V/minimum c sense of Chapter
3, there is every reason to expect new investment to be financed in
proportions that leave unchanged the overall cost of capital c, since only
this offers the firm the full benefits of both optimum capital structure
and the highest level of profitable investment. Given an unchanging interest
rate, if the firm is assumed to optimise with respect to both capital

structure and size of investment programme, the constancy of k follows automatically. More generally, it will be shown that any financing policy which maintains a constant c-value also maintains a constant k-value.

There is one qualification to the general rule that the minimum rate of return required on investment, in the situation defined, should be derived on the assumption of an unchanging cost of capital : it is that an investment provides an opportunity to improve capital structure which may be either too expensive or simply unavailable to a static firm. For example, the fixed cost of an issue of debt may rule out the use of debt until a sufficient capital gain can be realised on its introduction into the capital structure; and this point may only be reached through an expansion of the firm. Where a new investment offers the possibility of moving closer to an optimum capital structure the correct procedure is to compare the increase in V (due to the investment and to the gain made possible by capital structure change) with the combined cost of the investment and the issue cost of debt.
The requirement is

$$I + f(\Delta B) \leqslant \Delta S + \Delta B \qquad \text{(v)},$$

$$\text{or} \qquad I(1-b) + Ib + f(\Delta B) \leqslant \Delta S + \Delta B \qquad \text{(va)},$$

where $f(\Delta B)$ is the cost of issuing debt, ΔS is the expected improvement in the value of equity, ΔB ($=Ib$) is the amount of debt raised, b is the proportion of the cost of the investment covered by new debt. Condition (va) reduces to

$$I(1-b) + f(\Delta B) \leqslant \Delta S \qquad \text{(vi)}.$$

That is, the improvement in the ex-dividend value of equity should be not less than the sum of the cost items on the left-hand side of (vi). In effect, one term has been added to the left side of (iva) to give (vi); and the interpretation is that for a given investment the proposed amount of new debt, bI, will affect the equilibrium value of k depending on the actual value of r . When r and b are known the new k value will determine whether or not condition (vi) is satisfied.

5 : 2 (ii) The equivalence of r^* and the market valuation rate c.

The assumption that k is held constant by an appropriate proportion of new debt finance has been shown as a requirement for establishing a unique and correct measure of the minimum acceptable return on new investment. The implications of the two-part condition can now be set out quite simply.

1. The first condition is that for a marginal investment

$$W_o^1 = W_o \; ; \; \text{or,}$$

$$I(1-b^*) = \frac{I(r^* - ib^*)}{k} \qquad \qquad \text{(vii)}$$

This simplified version of (iva), which recognises the constancy of k, employs asterisks to indicate the required values of r and b. The condition reduces to

$$r^* = k(1-b^*) + ib^* \qquad \qquad \text{(viii)}.$$

Note that the right-hand side of (viii) consists of a weighted average of k and i , and bears a close resemblance to the cost of capital concept, c, defined in Chapter 3: 2 (iii).

2. For constant k , implying no change in the risk-class of the equity income distribution, it is necessary and sufficient that debt (at a constant interest rate) should increase at the same rate as the expected value of Q, where – as assumed – the new Q-distribution belongs to the same risk-class as the original. Thus

$$\frac{\Delta \overline{Q}}{\overline{Q}} = \frac{\Delta B}{B} \qquad \qquad \text{(ix)},$$

$$\text{or} \quad \frac{r^* I}{\overline{Q}} = \frac{b^* I}{B} \qquad \qquad \text{(x)}.$$

This gives $r^* = b^* \dfrac{\bar{Q}}{B}$ (xa),

or $\quad b^* = r^* \dfrac{B}{\bar{Q}}$ (xb).

Substituting (xb) into (viii),

$$r^* = k(1 - r^* \frac{B}{\bar{Q}}) + ir^* \frac{B}{\bar{Q}}$$ (xi)

Collecting terms in (xi) and re-arranging gives

$$r^* \left[\frac{\bar{Q} - iB + kB}{k} \right] = \bar{Q}$$ (xii),

or $\quad r^* (S + B) = \bar{Q}$ (xiii),

or $\quad r^* = \dfrac{\bar{Q}}{V}$ $\qquad$ cf p. 69 (xiv).

Thus r^* has the same value as c , defined in equation (v) of Chapter 3, and it remains to determine b^* . Substituting (xiv) into (xb) gives

$$b^* = \frac{\bar{Q}}{V} \cdot \frac{B}{\bar{Q}} = \frac{B}{V}$$ (xv)

The value of b which on a marginally acceptable investment maintains a constant value of k is given by the ratio of the market value of debt to the total market value of long-term investment in the company. Thus $(1 - b^*)$ equals $\dfrac{S}{V}$, and from equation (viii) it is again clear that r^* equals the weighted average definition of c obtained in the case of a static Q-distribution. Whatever the initial values of k , $\dfrac{S}{V}$ and $\dfrac{B}{V}$, and given the interest rate, i , the minimum rate of return acceptable on an investment which leaves unchanged the firm's overall risk class and whose financing does not affect k , is equal to the initial value of $\dfrac{\bar{Q}}{V}$. This is the case whether or not this value represents an optimum capital structure.

It is important to stress that such an investment with a return

$r^{+} = \dfrac{\bar{Q}}{\bar{V}}$, and financed in proportions $\dfrac{B}{\bar{V}}$ and $\dfrac{S}{\bar{V}}$ of debt and equity respectively,

leaves the firm with an unchanged overall cost of capital. This is because

interest obligations have risen by the same proportion as $\bar{Q}$; so, obviously,

has $\bar{Q} - iB$, the expected equity income. Since both i and k remain constant

the market values of debt and equity each rise by the same proportion,

resulting in constant weights $\dfrac{B}{\bar{V}}$ and $\dfrac{S}{\bar{V}}$: the weighted-average expression

for c must therefore remain unchanged in value.

Section 3: The Market Value Weighted-Average Cost of Capital and the Financing of Investment Programmes.

The last section demonstrated that c is the minimum acceptable rate

of return on a new investment financed in such a way that c itself is

unchanged. It was shown that by financing a marginal investment in such a

way that constant k would result, constant c would also be attained :

the critical values for financing proportions were shown to be $\dfrac{B}{\bar{V}}$ and $\dfrac{S}{\bar{V}}$

for debt and equity respectively. The concept c may be referred to as

a market value weighted-average cost of capital (MV/WACC), or alternatively

as the firm's market valuation rate.

In fact most investment proposals are likely to offer expected

returns higher than a minimum acceptable rate, and it is necessary to consider

the consequences of this for the actual financing of investment programmes,

as opposed to the hypothetical financing associated with a marginally

acceptable proposal. As an illustration of the difference between the

financing of marginal and intra-marginal investments, consider a particular

case where the rate of return, r_j , exceeds the firm's cost of capital, c.

Since the latter may be expressed in the form $c = k\dfrac{S}{V} + i\dfrac{B}{V}$ it is tempting

to conclude that whenever $r_j > c$ the investment should take place and

that the financing proportions should be $\frac{B}{V}$ and $\frac{S}{V}$ for debt and equity respectively. However, the weighted-average method of expressing c is misleading if it conveys the impression that all investments should be financed in the same proportions. When an investment offers a return greater than c , a proportion of new debt equal to $\frac{B}{V}$ would mean a greater proportionate increase in $\bar{Q}$ than in B . [5] This would lower the risk of the equity income distribution and reduce the value of k : S would rise by a greater proportion than $\bar{Q}$, causing a rise in the ratio $\frac{S}{V}$. The result would be a new equilibrium with altered k and (almost certainly) c values. Where a number of proposals are under simultaneous consideration it is obvious that confusion is created by this shifting of the firm's c-value – its minimum acceptable return on investment – during the actual process of selecting acceptable investments. It is easy to imagine a situation in which a proposal might be excluded at one stage of the selection of an investment programme; only to be accepted later, on the grounds that the prospective value of c had fallen as a result of the investment/financing decisions taken since its initial exclusion. So it is essential to identify a reliable and constant measure of minimum acceptable profitability, rather than one which seems likely to change according to the size and profitability of the proposals already selected. (There is also, of course, the possibility that investments already included in a programme might at a later stage be excluded – on the grounds that the prospective value of c had risen in the interim).

The problem is easily handled in practice. $\frac{\bar{Q}}{V}$ is used as the minimum return acceptable on any proposal which fulfils the constant-risk condition.

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(5) When $r_j = c$, setting $b_j = \frac{B}{V}$ implies $\frac{\Delta B}{B} = \frac{\Delta\bar{Q}}{\bar{Q}}$, as shown earlier.
In general, $b_j = \frac{\Delta B}{I} \ne \frac{B}{V}$ implies $\frac{\Delta B}{B} = \frac{I}{V}$; and $\frac{\Delta\bar{Q}}{\bar{Q}} = r_j I \cdot \frac{V}{\bar{Q}} = r_j \cdot \frac{I}{V}$.
Thus, when $r_j > c$, $\frac{\Delta\bar{Q}}{\bar{Q}} > \frac{\Delta B}{B}$ if $b_j = \frac{B}{V}$.

When all qualifying proposals are identified the firm's prospective
Q-distribution is also identified, and it is possible to calculate the
total amount of new debt which will ensure unchanged k and c - values
in the new situation. Once again the rule is simply that if B rises by
the same proportion as $\bar{Q}$, and i is constant, the risk-class of the
equity income distribution is held constant : this ensures constant k.
Since $(\bar{Q} - iB)$ increases by the same proportion as $\bar{Q}$ in this case, the
market values of S and B both increase by this same proportion, given
constant k and i . Therefore, V too increases by this proportion and
all components in the weighted-average expression for c are constant.
The market value weighted-average cost of capital (MV/WACC) derived
originally as a means of identifying a marginal rate of return on investment,
nevertheless retains its validity when intra-marginal investments are
considered. It continues to define the minimum acceptable rate of return,
but does not indicate the correct individual financing proportions for
acceptable projects. Whether one or several projects are involved the
correct overall financing proportions depend simply on the amount of
additional debt necessary to restore the initial ratio of B to $\bar{Q}$, given
the increase in $\bar{Q}$ expected to result from all the investments selected.

Section 4 : Reconsidering the Importance of Capital Structure Optimising.

One of the conclusions stated at the end of Chapter 3 was that, even
for a static firm, optimising with respect to capital structure seemed a
rather unlikely and perhaps unprofitable form of business behaviour; and
that a firm with prospects for investment would probably place relatively
high emphasis on investing as much as possible at a constant cost of capital
and give low priority to reducing the latter's value. In Chapter 3 the main
reasons for doubting the importance of optimum capital structure as a

business goal were : doubts and ignorance about the possibilities; inertia; an impression held by management that any gains would be rather small and unimportant; and the unavoidable cost of changing to a different capital structure. (These separate factors could of course reinforce one another.) An appropriate point has been reached in developing the concept of a valuation-based cost of capital to review this conclusion in the rather different perspective of investment/financing decisions and related changes in capital structure.

Because shifts in the Q-distribution are involved, it should be made clear that the term "optimum capital structure" retains the meaning it had in Chapter 3, but that it is now necessary and convenient to use the ratio $\frac{B}{\bar{Q}}$ as a measure of leverage – whereas in the static case B alone was sufficient. [6] In all situations considered here the risk class of the Q-distribution is not affected by shifts in the distribution through time : the ratio of its mean to its standard deviation is assumed to remain constant, as are any other measures relevant in determining its risk-class. The question now is whether it can be shown that a firm will inevitably move towards its optimum $\frac{B}{\bar{Q}}$ ratio as a result of its experience in making investment/financing decisions. (As specified earlier in this chapter, it is assumed that management sees each set of investments as moving the firm permanently to a new Q-distribution : further net investment opportunities are not foreseen. The financing of each set of investments is seen as establishing a new permanent capital structure, cost of capital and market valuation of the firm.)

Suppose for present purposes that the firm's cost of capital schedule,

---

(6)   See H. Bierman, Financial Policy Decisions, 1970, pp.87-91, for a
       discussion of various leverage measures.

defined now in terms of c and $\frac{B}{Q}$, is U-shaped as shown
in Diagram 1.
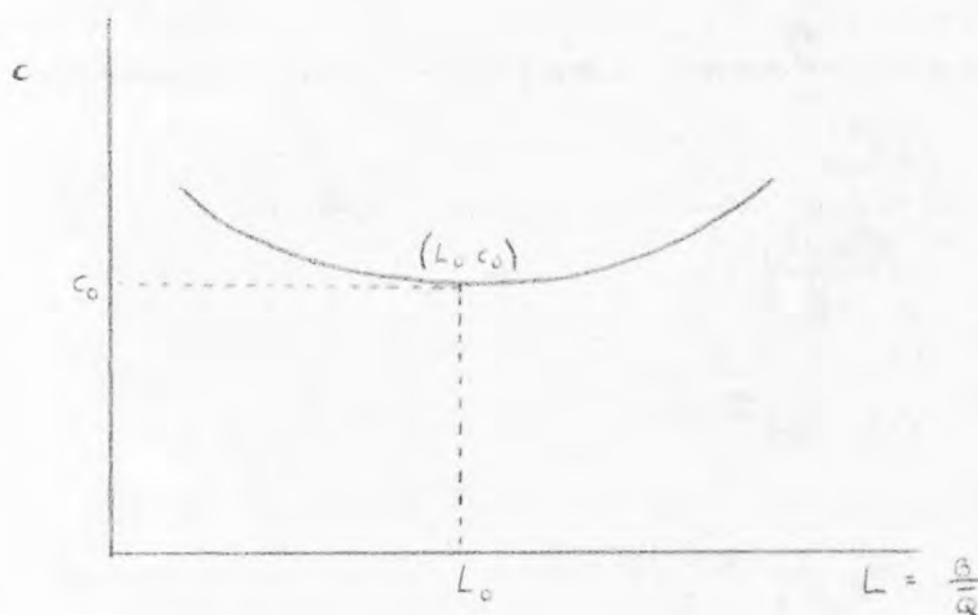
Diagram 1: a conventional cost of capital schedule.

Let optimum leverage be at the point L_o , c_o on the schedule. The
actual position of the firm prior to its investment/financing decisions
may be at any point on the schedule, and the net effect of its decisions
may – in theory – be a move to any other point (or indeed the status quo).

The possibilities are therefore numerous, but one obvious factor may
be important enough to narrow the argument about what happens to c over a
prolonged period during which the firm experiences a number of investment/
financing situations and their results. If the result of any investment
and financing combination has been to raise the value of c , it must be
clear to the firm on the next occasion that a move towards restoring the
original lower value should be possible. Neither ignorance, nor cost, nor
inertia, need prevent the eventual move back towards a lower c value,
given that some additional financing will in any case be required. Indeed,
there is a strong possibility that in attempting to restore its original
cost of capital the firm will overshoot and reach a lower level than the

one intended. Unless the U-shape of the c-schedule is extremely
pronounced and the amount of financing exceptionally large, it is hardly
likely that a single attempt to move down one side of the U towards a
lower value would actually overshoot to the extent of pushing the value
of c all the way round the bottom and some distance up the other side.
It follows that in the great majority of cases the firm is not likely to
misinterpret the experience gained in this limited attempt to clarify the
shape of the c-schedule.

Thus there are strong grounds for believing that firms which are at
all conscious of the importance of the market valuation rate, c, will be
able through time to improve on a performance trend which is recognizably
unsatisfactory. What is perhaps as interesting a question is whether factors
other than recognition of poor performance can be expected to operate on
firms which appear to take no interest in the implications of capital
structure. In considering this it is convenient to deal separately with
cases of excessive and insufficient leverage. In each case the explanation
should reflect the supposed logic of firms which, by definition, have little
awareness of the relevance of c.

A firm's leverage may be described as excessive when its position
on the c-schedule lies to the right of L_o, the optimum ratio of B to $\bar{Q}$
in Diagram 1. In this case a combination of investment and financing
decisions will raise the value of c if

$$b_j > b_j^* = r_j \frac{B}{\bar{Q}} = r_j L \; ;$$

where b_j and r_j are the proportion of a new debt and the rate of return
respectively on the set of investment proposals implemented, and b_j^*
is the proportion of new debt required to maintain a constant ratio of B

to $\bar{Q}$. [7] If $b_j > b_j^*$, a situation in which debt is already above its optimum level will be made worse. A plausible set of circumstances might involve a value of r_j which is low or even unprofitable (in the sense $r_j < c$). If r_j is low there may be a temptation to raise an excessive amount of new debt, because the firm may feel that a high proportion of additional equity investment would be hard to justify. Alternatively, if the firm's investments have a high r_j value, a naive view of the benefits of leverage may suggest that the situation is best exploited by increasing leverage still further. However, a factor which may tend to push even unwilling firms towards lower leverage is the likelihood of a rising interest rate on debt if B is allowed to increase proportionately more than $\bar{Q}$.

Evidently it is not certain that firms with excessive leverage will in the course of financing investment programmes be drawn automatically towards lower c values. In the absence of internal changes in attitudes or the external threat from perceptive takeover bidders, the situation of high leverage, high c , naive decision making and low returns on investment could be self-perpetuating and cumulative. (A.J. Merret and A. Sykes in The Finance and Analysis of Capital Projects, 1963, pp.397-8, describe the advantage enjoyed by a firm or institution which can gain control of the debt and equity of an overgeared company. The present discussion concentrates on internal corrective mechanisms.) The main internal corrective factor would be a management attitude that no further retention of equity earnings could be justified, coupled with a reluctance to offer new equity capital externally. In the short run this could result in a fall in investment or in an even greater reliance on debt financing, or some combination of these developments. In the longer term, however, the position should at least stabilise, with no further worsening of the firm's market valuation rate.

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(7)   On a set of investment proposals $t = 1 \cdots n$, r_j is a weighted average of n individual rates of return using as weights the respective shares of each project in total investment.

Much of what can be said of the possibilities in the low-leverage situation follows from points mentioned above. In this case c worsens if L falls; that is if

$$b_j < b_j^{\;*} = r_j \frac{B}{\bar{Q}} = r_j L \; .$$

This seems likely to occur to firms which are reluctant to take full advantage of leverage, probably a more common attitude than the preference for excessive leverage discussed above. In such cases if r_j is high there may be a reluctance to raise as much debt as will maintain the ratio of B to $\bar{Q}$, causing a further fall in L and rise in c . When this unwillingness to borrow is linked, as seems probable, to a conservative attitude on the minimum acceptable profitability of investment, another type of self-perpetuating and cumulative situation can result. However, as with the high-leverage case, there is a factor common to all low-leverage situations : the importance firms attach to the rate of interest. Here, many firms may feel an incentive to continue raising L when the opportunity occurs, as long as the interest rate on debt does not increase. Given the unsophisticated and naive thinking on capital structure questions attributed to the firms considered in this section, the one component of c which is bound to make an impression is the rate of interest; and if the attraction of a constant rate outweighs the conservative attitude to borrowing, there should be a move towards lower c - values . At worst, as with the high-leverage case, one can expect the forces working for a still-higher c - value ultimately to be held in check by the attraction of a low interest rate.

There seems little point in considering separately the possibilities for a firm which, without any coherent capital structure policy, has arrived at the point of optimum leverage. Obviously, unless any new investment is financed in exactly the correct proportions, c will rise as leverage becomes either too high or too low.

In the light of re-examination in a more dynamic context it seems
that the conclusion of Chapter 3 can be allowed to stand with only slight
amendment. For firms aware of the importance of the market valuation rate
and of the relevance of capital structure in its determination, ignorance
of the precise shape of the c-schedule need not prevent the process of
learning from experience which should at least achieve a roughly constant c
in the long run. On the other hand, for firms without a strong interest
in the importance of c and its determinants, the main reasons for
expecting improved performance in the long run are the incentive effects
of an interest rate schedule which should be horizontal for low-leverage
and rising for high-leverage firms. However, in each case counter-incentives
may also be at work : but if these are too weak to outweigh the interest
rate incentives there remains the question whether the latter alone are
sufficient to draw even naive or disinterested firms towards optimum capital
structures.

The answer to this question depends on what makes the individual c-
schedule turn upwards.
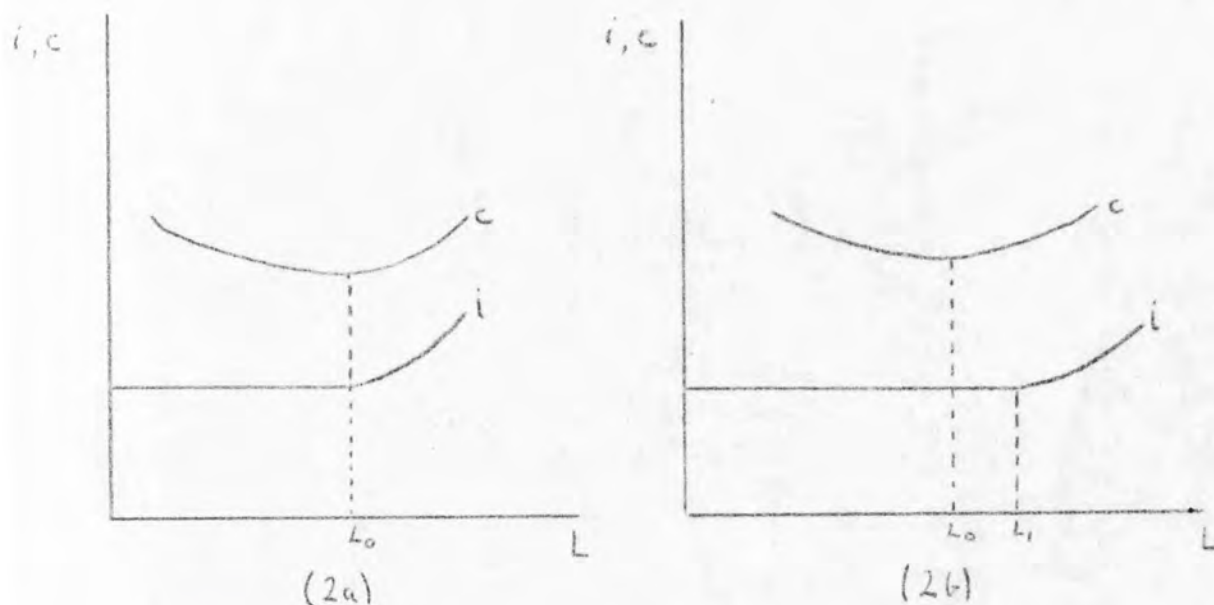
Diagram 2 : Alternative effects of i on c.

If the change from a constant to a rising interest rate is responsible
(Diagram 2 a) then clearly firms from both extremes of leverage would be
drawn to the optimum capital structure. But if c turns upwards before i
does (Diagram 2 b), firms drawn to the kink in the i-schedule emerge with
a capital structure containing too much debt, and a c value higher than
the optimum by an amount depending on the horizontal distance $L_o L_1$ and
the sensitivity of c with respect to a change in L. (The third
possibility - that i begins to rise before minimum c is reached -
must be regarded as improbable; but in any case the conclusion drawn from
Diagram 2 b would apply except that the sub-optimal solution would lie to
the left of L_o.)

Section 5 : Dividend Policy Considerations.

The present context of a once-only investment opportunity represents
a rather unsatisfactory theoretical bridge between static and dynamic situations,
at least as far as the subject of dividend policy is concerned. This
section first describes the nature of management's dividend policy dilemma
in this context and shows that while a solution must exist it must depend
heavily on managerial discretion. Then it is **argued** that the progressive
shift in later chapters to a context of continuous investment decision
making makes possible much more realistic view of management's approach
to dividend policy.

It must be acknowledged that in the conditions defined management
must take into account the effects of an unexpected decision to retain
and re-invest a portion of equity earnings. The difficulty arises in
applying the essentially long-term analysis and criteria of Sections 2
and 3 to the short and medium-term situations likely to exist in the capital

market. The announcement of an unexpected retention of a portion of
equity earnings is virtually certain to produce a period of uncertainty
in the capital market concerning the profit prospects of the investment
that is to be undertaken and the possibility that the interruption of normal
dividend policy (100% payout) may recur. During this period investors
will have time to draw wrong conclusions and make ill-advised decisions
on the basis of misleading short-term price movements and expectations. [8]
Against this management must weigh the likelihood that if external equity
finance is sought instead of the retention of earnings, a portion of the
expected gain will be diverted to "outsiders" who either purchase newly
issued shares directly or who purchase from present shareholders the rights
to buy such shares. Given the great variety of shareholder reactions to
be expected under whatever policy is chosen, it is simply impossible for
management to act simultaneously in the best interests of all long-term
shareholders - even if it knows exactly how different groups will react to
a particular situation.

Evidently the cut-off point between financing investment by (i) retained
earnings and new debt and (ii) external equity (probably rights issue)
and new debt is necessarily a managerial decision, and there is little
prospect of any clear guidance from the capital market. In the long term
the decision is more important the larger the difference in cost of capital
between the two methods and the more elastic the (once-only) investment
demand schedule. The total amount of investment undertaken may depend on
the point at which the cut-off is set, as does the level of cost associated
with external equity finance. The case for retaining earnings may seem quite

(8) One suggestion is that the inconvenience experienced by shareholders
when the established dividend policy is upset may result in downward
pressure on the firms equity value, as shareholders seek to restore
their desired cash positions. Selling pressure may concentrate on
the equity in which the uncertainty and inconvenience originate. For
a review of this and other considerations for dividend policy see
E. Solomon, A Theory of Financial Management, 1963, p .139-44.

strong in the present context because dividend is in any case expected to fluctuate ; so it may be argued that a once-only retention will not cause great additional inconvenience or create much apprehension about the future, causing a temporary setback to the market value of the firm's equity. On the other hand raising external equity minimises the short-term inconvenience of disappointed dividend expectations and this may contribute to more orderly market conditions. A final general point is that it will probably seem easier in most respects to retain and re-invest earnings during a period when earnings are high.

In later chapters the discussion of dividend policy is conducted within the very different context of regular investment decision making — a context in which investment is anticipated by shareholders and by management. When profit opportunities are anticipated the problems of internal financing are quite different from those summarised in this section, though the need to establish a cut-off point between internal and external equity finance may remain. In the following chapters the formulation of dividend policy is discussed in two stages, matching the assumptions to be made about future investment plans. In Chapter 7 it is assumed that management can identify an exact investment level for each future period, so that at the present day future dividend is seen as a risky variable depending on the level of equity earnings — just as in the static model of Chapter 3. Throughout Part II on the other hand, the future level of investment and future earnings are seen as inter-related risky variables, and future dividend expectations must be formulated accordingly. The central concept of planning for a constant ex ante level of dividend risk remains unchanged and indeed provides management with a key to the comprehension of market valuation processes in the more complex and realistic models of expectations to be developed.

Before this main task is begun in Chapter 7 it seems appropriate to present in the following chapter an analysis of the cost of capital in certain external financing situations, to parallel that for internal financing given in this chapter.

CHAPTER 6

The Cost of Capital for External Equity Financing.

Plan of Chapter.

Section 1 :   Introduction

Section 2 :   The Derivation of r^* for a New Issue of Shares to Outsiders.

Section 3 :   The Derivation of r^* for a Rights Issue to Shareholders.

Section 4 :   Conclusion.

Section 1 : Introduction.

This chapter deals with the problem left unconsidered in Chapter 5 , the derivation of cost of capital concepts for investments financed by combinations of external equity and new debt. For this purpose it is assumed here that management reaches a decision on the point at which internal equity financing cannot be increased. Some relevant considerations on this matter were raised in Chapter 5 : 5 (i) and definite identifications of the point of maximum retention will emerge in the two discussions of constant-risk dividend planning in Chapter 7 : 2 (iii) and Chapter 8 : 2 (ii). It remains to determine the minimum acceptable return on a project financed by external equity and new debt, subject as before to the conditions that existing shareholders must remain as well off as before and that the equilibrium value of k remains constant. As shown in Chapter 5 : 2 (i), these conditions ensure a constant cost of capital or market valuation rate, c. [1]

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(1)   This admittedly restricted view of the purposes and scope for external equity financing rests on two considerations. First, throughout Part I management is assumed to desire this result both as an aid in its planning (see Chapter 5 : 3) and to provide maximum information for the market. Second, given a desire to achieve constant c, there is no reason why management should fail to implement it: as will be argued in Chapter 8 : 4 (ii), the length of time management is likely to define as a "period" in its planning is long enough to ensure the occurence of conditions inside the firm and in the capital market which will allow the planned levels and terms of all external finance. The fully realistic versions of the results derived in this chapter are available in a number of works : see A.J. Merret and A. Sykes, The Finance and Analysis of Capital Projects, 1963, pp.82-95 ; J.C.T. Mao, Quantitative Analysis of Financial Decisions, 1969, pp.385-91; E. Solomon, The Theory of Financial Management, 1963, chapters IV, VI, VII. A useful summary of cost of capital results in relation to various sources is given in C.J. Hawkins and D.W. Pearce, Capital Investment Appraisal, 1971, pp.46-51.

Two approaches to external equity financing are considered, each combined with appropriate debt financing, in order to establish an expression for r^* in each case. It is assumed that equilibrium market values of equity prevail both before the announcement of an investment proposal and shortly after the investment/financing operation: the special problems arising in conditions of temporary market disequilibrium are disregarded. [2]

Section 2 : The Derivation of r^* for a New Issue of Shares to Outsiders.

It is convenient to begin with the case of an entire new issue of shares offered to and taken up by outsiders. The requirements can be stated algebraically.

1.
$$\frac{\bar{Q} - iB}{k} = \frac{n}{n + \triangle n}\left[\frac{\bar{Q} - iB + I(r^* - iB^*)}{k}\right] \tag{i}$$

where n and $\triangle n$ are the original number of shares and the number of new shares issued, respectively; and the other variables keep their former meanings. This condition merely states that a marginally acceptable investment/financing proposal does not alter the total value of the original number of shares, at equilibrium prices. Since the second

(2) Cost of capital analysis for market disequilibrium situations has been comprehensively worked out by A.J. Merret and A. Sykes (op.cit., 1963 , Chapter 17). As suggested in footnote (1) above, management expects a good deal of leeway on the timing of financing operations, so the awareness that markets may temporarily be in disequilibrium should not be too worrying in the forward planning context described in Chapter 7 : 2. In Part II management's approach to external financing, like its approach to investment in general, is understood in terms of conditional policy for all foreseeable situations.

condition will ensure constant k, this first requirement states that expected earnings per share should remain constant. Equation (i) reduces to

$$\frac{\Delta n}{n} = \frac{I(r^* - ib^*)}{(\bar{Q} - iB)} \qquad (ii):$$

the proportionate increase in the number of shares and in the expected value of equity earnings should be equal.

2.   Constant k is ensured, as in the earlier formulation of these conditions for internal financing, by letting debt increase by the same proportion as $\bar{Q}$ so that the condition is still

$$b^* = r^* \frac{B}{\bar{Q}} \qquad (iii) \qquad (3)$$

In the expression

$$I = \Delta n P'e + b^* I \qquad (iv),$$

P^1 is the issue price per share ; e is the proportion of P^1 remaining after issuing costs have been met ; other variables retain their earlier meanings. Thus:

$$\Delta n = \frac{I(1 - b^*)}{P^1 e} \qquad (iv\ a).$$

substituting (iii) and (iv a) into (ii) and cancelling I gives :

---

(3)   See Chapter 5 : 2 (ii). In general, constant k is achieved by a debt financing proportion $b = r \frac{B}{\bar{Q}}$, where r is the rate of return on investment (see Chapter 5 : 3).

$$\frac{(1 - r^* \frac{B}{\overline{Q}})}{P^1 e} \ (\overline{Q} - iB) = n(r^* - ir^* \frac{B}{\overline{Q}}) \qquad (\text{v}),$$

which reduces to : *multiply through by $\overline{Q}$*

$$\frac{(\overline{Q} - r^* B)}{P^1 e} = nr^* \qquad (\text{va}),$$

so that :

$$r^* = \frac{\overline{Q}}{nP^1 e + B} \qquad (\text{vi})$$

If P^1 is equal to the pre-issue equilibrium share price, P_o, then $P_o = P^1 = P_1$, the new equilibrium price, since both earnings per share and k remain constant. New shares would be taken up by the market at the prevailing equilibrium price, and in this case if $e = 1$ – that is if issue costs are ignored – the solution for r^* becomes :

$$r^* = \frac{\overline{Q}}{S + B} = \frac{\overline{Q}}{V} = c \qquad (\text{vii}).$$

To the extent that $e < 1$ and /or $P^1 < P_o$, the value of the denominator in (vi) is reduced and r^* increased above c. The greater the cost of a share issue and the lower the price at which it can be sold to outsiders, the higher the minimum rate of return on investment necessary to maintain the equilibrium value of shares at its present level. A general formulation is

$$r^* = \frac{\overline{Q}}{\lambda Se + B} \qquad (\text{viii}),$$

where $P^1 = \lambda P_o$. Given r^*, the value of b^* is derived in the same way as before:

$$b^* = r^* \frac{B}{\overline{Q}} \qquad\qquad (iii)$$

To the extent that r^* exceeds $\frac{\overline{Q}}{V}$ so obviously does b^* exceed the value

of $\frac{B}{V}$. The interpretation of this is that the expense of an equity issue

to outsiders, due either to a low issue price or high costs, ought to

induce a higher proportion of debt in the financing of new investment.

It should also be noted that $P^1 < P_o$ on a marginally acceptable

project implies a capital gain to holders of new shares, but none on

existing shares.

Differing interpretations are possible in connection with the

possibility that P^1 may be less than P_o . If equilibrium in capital

markets is expected to be restored quite soon after the investment/

financing policy is implemented, it is difficult to understand the

necessity to offer new shares at a discount price, lower than they

are expected to be worth in the very near future.

The existence of full capital market equilibrium before and

shortly after a new issue may be regarded as an extreme case, and it

can be shown that with other assumptions the sale of new shares to

outsiders at a price below their expected equilibrium value may be

justified. One obvious example is when the return to correct valuation

is expected to be delayed for an uncertain period of time: the eventual

capital gain to the purchasers of new shares may be looked on as an

appropriate reward for bearing the special uncertainty during the period

of unsettled market value. Of course this "reward" is still discriminatory

in the sense that while all shareholders are affected by the period

of price fluctuation, only holders of new shares enjoy the eventual

capital gain. However, the gain is at least related to some function performed by the purchasers of new shares, without whose support the issue presumably could not have been launched. In any case, as will be suggested shortly, it is quite usual in new equity issues for existing shareholders to be given rights to participate in the issue in proportion to their existing holdings. [4]

Before proceeding to a more general analysis of external equity financing, there is an ther way of formulating the limited problem considered here, in order to ensure that original shareholders also receive some compensation for the uncertainty of price fluctuations following an issue of shares to outsiders. It is quite easy to modify (ii) to

$$\frac{\Delta n}{n} = t \cdot \frac{I(r^* - ib^*)}{(\overline{Q} - iB)} \qquad (ix),$$

where $0 < t < 1$.

This simple device in effect defines a marginal investment/financing proposal as one which raises earnings per share by some minimum percentage, and since the equilibrium value of k is held constant an eventual capital gain should accrue to existing shareholders no matter how the share issue is priced. Substituting (iii) and (iv a) into (ix) and proceeding as before,

$$\frac{(\overline{Q} - r^*B)}{p^1_e} = ntr^* \qquad (x),$$

---

(4)   For the correct perspective (in the present context) on the possibility of temporary market disequilibrium see footnotes 1 and 2 above.

which re-arranges to :

$$r^* = \frac{\overline{Q}}{nP^1 et + B} \qquad (xi).$$

In order to guarantee an eventual capital gain to holders of existing
shares, the value of r^* is increased -- as can be seen by comparing
(vi) with (xi). Needless to say, the decision on the correct value
for t is entirely a managerial one, reflecting management's estimate
of the length of time during which the share price will fluctuate and
of the eventual compensation appropriate for shareholders who retain
their holdings during this period.

Section 3 :   The Derivation of r^* for a Rights Issue to Shareholders.[5]

A company with its equity already quoted on the stock exchange
would normally raise external capital, if the need arose, by means of
a rights issue to existing shareholders. This method apportions the
rights to purchase newly-created shares among the latter in proportion
to their holdings, and the existence of an active rights market allows
individuals a wide choice on whether and how much to invest on the
terms offered.

With most rights issues it is possible to be quite precise about
the market price shareholders should obtain on disposing of rights.

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(5)   This section does not attempt a detailed explanation of the mechanics
      of rights issues, as its main concern is with the valuation of
      rights during the period in which dealings are permitted :
      for this limited purpose the explanation given is adequate. A
      wider analysis of rights issuing activity and of the market in
      shares affected by rights issues is provided by A.J. Merret,
      M. Howe & G.D. Newbould, Equity Issues and the London Capital
      Market, (1967).

In a situation in which the market is imperfectly informed about the profitability of the firm's investment plans it is not surprising that conventional valuation practices hold a powerful attraction as a means of handling temporary uncertainty. For the particular case of a rights issue there is a widely-used and understood method of arriving at a new total value for a firm's equity and its new share price during the fairly brief period of dealings in the rights to purchase the new shares. The method is based on the assumption that after the issue the market value of equity should equal its original value, nP_0, plus the additional equity raised in the new issue, $\triangle nP^1$. Correspondingly, the new share price expected at the close of dealings in rights is

$$P_e = \frac{nP_0 + \triangle nP^1}{n + \triangle n} \qquad \qquad (xii)^{[6]}$$

As stated, the implication of this formula for the share price in the immediate post-issue period is that a shareholder can expect to break even on his additional investment in the short run.

The same result takes on great significance if it should be the case that P_e is equal to the equilibrium post-issue equity price, P_1. If $P_e = P_1$ the implication of (xii) is that the change in the equilibrium market value of equity is exactly equal to the value of

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(6)   See Merret, Howe and Newbould, op.cit., pp.49-50, for an explanation of this formula and for its statement in a more general form. Obviously, a change in general market sentiment, or in views about the issuing firm's prospects, may alter the value of P_e in the same way as these developments would affect P_0 in the absence of a new issue. The expression shows, for given $\triangle n$ and P^1 values, that P_e responds to a change in P_0.

new equity investment. Thus

$$\Delta S = \frac{I\,(1 - b)}{e} \qquad\qquad \text{(xiii)},$$

in which ΔS is the change in equilibrium equity value and $\frac{I\,(1 - b)}{e}$

is the amount of new equity finance raised by the issue. Making the

correct substitution for ΔS in (xiii) yields

$$\frac{I(r - ib)}{k} = \frac{I\,(1 - b)}{e} \qquad\qquad \text{(ixv)},$$

or $\quad r = \frac{(1 - b)\,k}{e} + ib \qquad\qquad \text{(xv)}.$

Given that $b = r\,\dfrac{B}{\overline{Q}}$ as usual, the expression for r, the rate of

return implied on the investment for which $P_1 = P_e$, becomes

$$r = \frac{\overline{Q}\,\dfrac{k}{e}}{\overline{Q} - iB + \dfrac{k}{e}\,B} = \frac{\overline{Q}\,\dfrac{k}{e}}{Sk + \dfrac{k}{e}\,B} \qquad\qquad \text{(xvi)},$$

or $\quad r = \dfrac{\overline{Q}}{eS + B} \qquad\qquad \text{(xvii)}.$

It is easy to compare this formula with that for r^{*} derived in

Chapter 5 : 2 (ii) :

$$r^{*} = \frac{\overline{Q}}{S + B} = c \qquad\qquad \text{(xviii)},$$

and to see that the rate of return implied by $P_1 = P_e$ differs from

r^{*} and c only to the extent of allowing for the cost of raising

equity externally. The difference is not very large as long as e
remains fairly close to unity. [7]

The result $P_1 = P_e$ is in fact the overall property required of
a marginal investment financed by a rights issue and new debt
combination: the increase in the equilibrium value of equity equals
the amount of new equity invested when $P_e = P_1$, or, in terms of a
cost of capital concept, when $r = \dfrac{\bar{Q}}{eS+B}$.

It only remains to show that the outcome for individual share-
holders when $P_e = P_1$ does not depend on their decisions about exercising
or selling their rights to purchase the new shares. The market value
of a right at any time is

$$P_r = P_e - P^1 \tag{xix};$$

that is the market price of a right, P_r, should equal the difference
between the ex-rights share price and the issue price per new share. [8]

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(7)   Merret, Howe & Newbould discuss costs under the headings of administration
      and underwriting. Although costs in the former category are
      difficult to identify precisely, their average level appears to be
      less than one percent of the size of issue. Underwriting costs
      vary considerably, though in only a small fraction of cases does
      the cost exceed two percent of the issue price. (These findings
      relate to a sample survey of new issuing activity in the period
      1959-63 : op.cit., pp.53-55).

(8)   Merret, Howe & Newbould, op.cit., pp.59-64, demonstrate the high
      accuracy of the estimate of P_e derived from the formula compared
      with the actual value of P_e at the start of ex-rights dealings,
      but their analysis does not extend to movements in P_e and P_r
      during the period of dealings in rights. It is obvious from the
      expression defining P_r that it must respond to changes in the
      underlying value of P_e. The assumption in the text, that P_r
      has a unique value, is thus an over-simplification.

On the assumption that $P_e = P_1$, (xix) becomes

$$P_r = P_1 - P^1 \qquad\qquad\text{(xx):}$$

that is, as shown by the overall result in the preceding paragraph, anyone who exercises all of his rights allocation adds to his holding a value exactly equal to his outlay. Anyone selling a portion of his rights - any portion - obtains for it its correct value: he too neither gains nor loses. The individual investor's behaviour does not affect his wealth when the investment to be financed is marginal in the overall sense suggested by $P_e = P_1$.

Finally, for marginal and intra-marginal investments, a managerial decision is obviously required for the combination of Δn and P^1 by which the required new equity, $\dfrac{I(1-b)}{e}$, is to be raised. The governing equilibrium relationships are:

$$\frac{S_1}{n + \Delta n} = P_1 \qquad\qquad\text{(xxi)}$$

and $\qquad P^1 \, \Delta n = \dfrac{I(1-b)}{e} \qquad\qquad$ (iv b)

The important constraint is that the P_1 value implied by P^1 should not permit P_e to fall below a level at which the new share issue can attract buyers. Essentially the expected ex-rights share price should not be less than the amount shareholders are asked to pay for a new share, so the condition to be avoided is $P_e < P^1$. This expands to

$$\frac{nP_o + \Delta nP^1}{n + \Delta n} < P^1 \qquad\qquad\text{(xxii),}$$

or $\quad P_0 < P^1$ $\hspace{6cm}$ (xxiii).[9]

It is easy in practice to ensure that the new shares are priced below P_0 . In the simultaneous solution to (xxi) and (ivb) the effect of varying P^1 (within its upper limit) is to change P_1 in the same direction and Δn in the opposite direction.

Section 5 :   Conclusion.

To conclude this chapter it is appropriate to bring together the two approaches to external equity financing for which cost of capital concepts have now been developed. These are: first, the case of an entire new issue offered directly to the market ; and second, the case of a share issue offered in the first instance directly to shareholders on a pro-rata basis. The relative advantages of these approaches, in terms of issuing costs, convenience and certainty are not considered here : obviously the choice of a financing approach is dictated by a number of factors, including the size of the firm and the size of issue. Nor is the theoretical treatment in this chapter particularly suited to the representation of cost factors. Here the only question to be answered is whether the values of r^* derived in the alternative approaches differ.

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(9)   In their sample of rights issues, Merret, Howe & Newbould found "the average percentage by which the issue price was less than the market price........ was 35.6 per cent." (op.cit., p.56). The situation to avoid is one where the right to subscribe becomes nearly valueless, as would happen with P^1 equal to P_0 at the time of the allotment of rights. Unless a generous allowance is made in planning an issue, a fall in P_0 during the issue could wipe out the value of the right to subscribe.

It was shown that for a non-rights external equity issue a
property of a marginal investment/financing proposal is that the equilibrium
share price should not be affected by its acceptance :   this is necessary
because, without a rights market, the value of a shareholder's invest-
ment can only be maintained if a constant number of shares retains a
constant per unit value. Thus $P_1 = P_o$ is a necessary condition for
a marginal investment financed by a direct external equity issue;
and as argued in Section 2, it follows that the issue price per new
share should also equal P_o. Any higher figure (for a marginal invest-
ment) would result in a capital loss to the new investor, while any
lower figure confers a capital gain on new investors and raises the
value of r^* correspondingly. Thus in Section 2 it was assumed that λ,
the proportion of the full share value at which new shares are offered
to the market, equals unity : on that basis the solution to r^* can be
expressed as :

$$r^* = \frac{\bar{Q}}{eS + B} \qquad\qquad (viiia)$$

For the rights issue method, making the assumption that the
market values rights in the conventional way, it was shown in Section 3
that the expression for r^* is identical with that just quoted for the
other approach. The only difference, therefore, between the r^* values
derived from these alternative approaches is that e may take different
values according to the issue method adopted. The usual belief is that
a rights issue helps to minimise issue costs in several ways, so that
e would take a value closer to unity and r^* a lower value than with
the direct external issue method. Clearly, the importance of any given
difference in costs as a cause of differing r^* values between methods

of external equity finance depends on the relative weights of S and
B in V ; but it is also clear that any difference is of a slight
order of magnitude. Apart from this factor the present framework of
analysis reveals no difference between cost of capital values derived
for different methods of external equity finance. This is because
in each case the value of r^* emerges as the answer to the same question :
namely, what is the minimum acceptable rate of return on an investment -
given the intention of financing it so as to maintain the overall
market valuation rate - such that the increase in the value of equity
and the amount of new equity investment are equal. This way of looking
at the derivation of r^* for the rights issue has been made clear, and
for the direct offer case the point is easily demonstrated. In the
latter case, given that new shares are not sold at a "discount" to
the market, it was shown that neither present shareholders nor new
purchasers incur gain or loss when a marginal investment/financing
proposal is accepted : this means that no overall gain or loss occurs,
and that the fundamental property of r^* mentioned above is applicable
in such a case.

CHAPTER 7

Dividend Policy, Growth and the Cost of Capital

Plan of Chapter

Section 1. Introduction.

Section 2. Constant Business Risk, Growth and the Risk Connected with Future Dividends.

    (i)   Business risk and dividend risk.
    (ii) The financing of planned investment for constant dividend risk.
    (iii) A critique of the planning model.

Section 3. The Cost of Capital and Financing Proportions for a Marginal Investment. (I).

    (i)   The relationship between r^* and b^* for constant k.
    (ii) The equality between r^* and c.
    (iii) The effect on financing plans of acceptance of an additional investment.

Section 4. The Cost of Capital and Financing Proportions for a Marginal Investment (II).

    (i)   Outline of a steady-state growth model.
    (ii) The value of b^* on a marginally acceptable additional project.
    (iii) The equality between r^* and $\bar{Q}/V$ in the static case.

Section 5. The Cost of Capital, External Equity Financing and Growth.

Section 6. Conclusion.

Section 1 : Introduction.

In earlier chapters it was possible to consider investment decision making in almost complete separation from dividend policy, the distinction being possible because of the rather artificial situations assumed. These were : (in Chapter 3) a completely static model with no new investment, in which all equity income was automatically paid out as dividend; and (in Chapter 5) a comparative-static model in which a once-only interference with 100 per cent payout was followed by a reversion to static conditions at a new level. The present chapter extends the cost of capital concept developed earlier to situations in which prolonged or continuous growth in investment and earnings potential is expected to occur, and in which the firm expects to rely heavily on retained earnings for new equity capital. Thus it is no longer possible to artificially separate the financing of investments from the considerations of a policy aiming at the maintenance of a constant level of risk on expected dividends.

Section 2 demonstrates that a constant level of ex ante dividend risk is compatible with a programme of planned investment which generates a rising level of (constant risk) earnings potential. Whereas in the static model of Chapter 3 the firm's capital structure ensures the desired level of dividend risk, it is demonstrated that in a growth context this function is performed by the firm's financing policy - the equity (retained earnings) and debt proportions planned for financing new investments. The desired dividend risk for each future period is obtained by making expected dividend, $\overline{D}$, a constant

proportion of $\bar{Q}$, and the variability of D around this level equal
to that in Q around $\bar{Q}$: this equality is achieved by formulating
in advance investment and debt-financing targets which, together with
interest on inherited debt, are adhered to irrespective of the actual
Q outcome. [1] In later chapters it will be shown that this approach
to the planning of constant-risk expected dividends represents only
the most simple model that can be suggested for a dynamic situation,
and that it is possible to relax the assumptions (of uniquely pre-
determined investment level and Q-distribution for each period)
without necessarily sacrificing the central concept of a constant-risk
dividend policy.

Sections 3 to 5 are concerned with identifying the cost of
capital appropriate to certain defined growth situations which satisfy
the general conditions set out in Chapter 3 : 1 (i) :

(i)   the firm's future investment programme is uniquely determined
        ex ante, but not necessarily known to management or investors;

(ii)   the firm's Q-distribution in each future period is also
        uniquely determined ex ante, but not necessarily known at the
        present, even when the distribution shifts through time;

(iii)   the risk class of the Q-distribution is known to be constant
        through time.[2]

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1.   The variability of Q in relation to $\bar{Q}$ is constant, so that
     dividend risk, as defined, is necessarily constant through
     time when the risk of the Q-distribution is unchanging.

2.   Throughout most of this chapter the future levels of invest-
     ment demand and positions of the Q-distribution are assumed
     known to management. In Section 6 it is argued that even when
     managerial omniscience in this sense is absent, the cost of
     capital results obtained continue to apply - if investments
     have uniquely identifiable r-values.

As predicted in that section, the expression for the cost of capital is shown to be unaffected by the switch to dynamic assumptions; and, as a corollary of this conclusion, it becomes clear that the precise details of these assumptions are also irrelevant to the cost of capital.

The assumption that capital markets value equity assets in terms of their objectively known income and risk properties is retained: only in the concluding section are the possible consequences of its relaxation considered. In each situation the market value of the firm prior to a decision on the investment in question takes into account only the consequences of decisions and plans already known to the market : if an additional project is accepted, the correct adjustment in market value (in the case of a non-marginal project) occurs with a minimum of delay.

Section 2 : Constant Business Risk, Growth and the Risk Connected with Future Dividends.

7 : 2 (i)   Business risk and dividend risk.

In the static and comparative static approaches of Chapters 3 and 5 equity valuation was interpreted as the discounting, at a constant rate compounded over time, of the (constant) expected value of dividend in each future period. The constancy of the Q-distribution, capital structure and dividend risk in all future periods are convenient features of these static cases, but there is no reason why an analogous situation should not be defined for a growth context. This section extends the essential features of the static case to the dynamic, defining conditions in which all expected dividends in a growing stream

can be regarded as equally risky when viewed from the present. In so extending the static analysis it is helpful to assume that growth does not affect business risk : it would be more difficult to attempt to identify a constant-risk time-path for future expected dividend if the underlying risk of the changing Q-distribution were not also constant. The first steps therefore are to recall what is meant by constant business risk, to examine its meaning in a growth context, and to define constant dividend risk.

Business risk was defined for the static Q-distribution as the ratio of the distribution's standard deviation to its expected value, $\overline{Q}$. A constant risk class implied that this ratio (and, of course, other relationships) remained constant when a once-for-all change in the Q-distribution occurred. With continuous change in the Q-distribution, reflecting a steady growth of assets and earnings potential, constant risk class still requires that for each period the ratio between the standard deviation and expected value of the distribution be constant. [3] There is then no logical difficulty in extending the static case definition to the dynamic context. Just as in the static case, risk as defined relates only to the potential deviation of profit from its expected value : other elements of risk, such as the risk of business bankruptcy, are neither measured by nor incorporated into this definition. In concentrating on a risk measure which, in effect, defines the firm as a continuing business entity, there is an implicit assumption that this is the measure most relevant in interpreting market

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(3)   Strictly, all other relationships which may influence the
        market's impressions of risk - and hence the value of k -
        must also be constant.

valuation processes. [4] What applies in the static case has again
been extended to the dynamic, where its applicability is evidently
more suspect. However, the definition of risk and the condition
for i s constancy are retained in what follows : all parties see
the firm as exposed to only one sort of risk.

Dividend risk is defined here in much the same way as the
risk associated with the firm's earnings : indeed, in the static
case with no debt in the capital structure the two measures are
identical. For each future period a standard deviation and expected
value of dividend are known, and when the ratio of the former to the
latter variable is equal for all periods dividend risk is said to be
constant. In a static context with an unchanging capital structure
it is obvious that constant business risk implies constant dividend
risk. In a growth context, however, the constancy of dividend risk
does not result automatically from the constancy of business risk as
growth proceeds :   deliberate policies in respect of the financing
of the firm's investment programme are required to bring it about.

7 : 2 (ii)   The financing of planned investment for constant
                dividend risk.

In what follows it is assumed that management is able to
predict the level of investment by the firm in each period, for as
many future periods as is necessary. The effect of each period's
investment on the firm's Q-distribution in all later periods is also
assumed known. Investment is not expected to alter the firm's

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(4)   The standard models explaining valuation in terms of discounted
         expected dividends disregard all risks other than that
         relating to each period's dividend. The valuation model
         developed in Chapter 10 <u>can</u> accommodate a risk of bankruptcy.

overall risk-class, its business risk. The validity of these
assumptions in this context is debatable, but they can be used
to demonstrate the possibility of simultaneous planning of a constant-
risk dividend policy and the financing of investment over any time
horizon.[5] The basic relationship intended by management to apply
in each period, t, is

$$\frac{v\bar{Q}_t}{\bar{Q}_t - iB_{t-1} - I_t + \Delta B_t} = R, \text{ for } t = 1 \ldots\ldots\infty, \quad (i),$$

where v is the constant parameter which relates the expected value
of Q in each period to its standard deviation, so that $\sigma(Q_t) = v\bar{Q}_t$;

$\bar{Q}_t$ is the expected value of Q in period t ;

i is the rate of interest on debt, assumed constant;

I_t is the unique level of investment outlay intended for t;

B_{t-1} is the level of outstanding debt at end of t-1;

ΔB_t is the amount of additional long-term debt raised during t; and

R is a constant for all t.

The numerator on the left-hand side of (i) is the standard deviation
of the Q-distribution for period t, the denominator is the expected
value of dividend for the same period, $\bar{D}_t$. In this expression for
$\bar{D}_t$ the items I_t, B_{t-1} and $\bar{Q}_t$ all have pre-determined values, and
the equality arises from the residual nature of the periodic dividend

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(5)   For the moment the feasibility of financing planned invest-
ment on all occasions by retained earnings and new debt is
taken for granted. This assumption is questioned and suitably
modified without prejudice to the main points - the possibility
of dividend planning and the uniqueness of the planned growth-
path - in Section 2 (iii) which follows.

as envisaged in this approach. [6] ΔB_t will be determined as
the solution to the firm's financing policy consistent with constant
dividend risk, given the intended investment level for the period, I_t.
Within this framework the only source of variation in the level of
dividend in t is variation in Q_t about its expected value, $\bar{Q}_t$.
Hence, the standard deviation of Q_t, $v\bar{Q}_t$, in relation to the
expected value of dividend, is an adequate measure of the risk attach-
ing to the latter; and constancy in this relationship for all values
of t is an adequate indication of constant dividend risk in this
context. In later extensions of the model it will be shown that
variation in Q_t about its expected value is not the only source of
variation in D_t about $\bar{D}_t$, but the measure embodied in (i) represents
only the first step away from the static and comparative-static frame-
works : at this stage the level of investment in each period, and its
outcome in terms of the effect on the Q-distribution are assumed known
with certainty by management. Accepting for the time being that any
deviation of Q_t from its expected value in any period is entirely
absorbed by the dividend paid out by the firm - just as it was in the
simple static case -- the determination of financing policy consistent

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(6)   Dividend is determined residually as the sum of positive
        cash-flow items (Q_t, ΔB_t) minus the sum of outflows (I_t,
        iB_{t-1}). This ignores the possible use of the company's
        liquidity position, or short-term borrowing, to provide a
        buffer between Q_t and dividend. The main reason for this
        lies in the assumption made about the duration of a "period"
        in the context of dividend planning (see Chapter 8 : 4 (ii)) :
        if, as assumed, a "period" may cover two or three years, a
        certain amount of smoothing of dividend can be achieved
        without any attempt to interfere with its inherently residual
        nature. If it were desired to incorporate liquidity variation
        as a determinant of dividend, this could most realistically
        be done by allowing liquidity change to occur in a predetermined
        relationship to Q_t and/or I_t. The relationship could if
        desired be a probabilistic one. The result would be that
        dividend, taking into account the extent to which management
        would be prepared to use liquidity variation, remained a
        residual variable in exactly the sense assumed in the text.
        The model assuming a unique liquidity requirement for each
        period is retained for simplicity.

with constant dividend risk can be shown.

To simplify the demonstration of how financing policy is determined, suppose that a firm's investment programme is scheduled to end in period n - 1, so that its Q-distribution reaches a static position in the following period and is expected to remain static thereafter.[7] Then constant dividend risk in all future periods requires equation (i) to be satisfied for each period as follows (where the present period is $t = 0$) :

$$t = 1 \quad : \overline{Q}_1 (v-R) + RiB_{-1} + RI_1 = R\Delta B_1 - Ri\Delta B_0$$

$$t = 2 \quad : \overline{Q}_2 (v-R) + RiB_{-1} + RI_2 = R\Delta B_2 - Ri(\Delta B_0 + \Delta B_1)$$

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$$t = (n-1): \overline{Q}_{n-1}(v-R) + RiB_{-1} + RI_{n-1} = R\Delta B_{n-1} - Ri(\Delta B_0 + \Delta B_1 + \ldots + \Delta B_{n-2})$$

$$t = n \quad : \overline{Q}_u (v-R) + RiB_{-1} = R\Delta B_n - Ri(\Delta B_0 + \Delta B_1 + \ldots \Delta B_{n-2} + \Delta B_{n-1})$$

$$t = (n+1): \overline{Q}_u (v-R) + RiB_{-1} = R\Delta B_{n+1} - Ri(\Delta B_0 \ldots \Delta B_n)$$

Note that for each period the re-arrangement of (i) is carried out so that B_{t-1} is broken up into B_{-1}, the level of debt outstanding at the start of the exercise, and the sum of the ΔB_j values, $j = 0, \ldots,$ t-1. This isolates the successive decision variables $\Delta B_0, \Delta B_1 \ldots$ etc., whose values are to be determined.

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(7)   Investments are assumed to add to the firm's earnings
       potential – its Q-distribution – in the period   following
       their purchase by the firm.

At the new equilibrium reached with the conclusion of its investment programme and the emergence of a new static Q-distribution — referred to as Q_u — there should be a new planned level of B, such that the level of debt interest, iB_u, in relation to $\overline{Q}_u$, maintains the desired dividend risk in the static situation then prevailing. The level of B_u is given once $\overline{Q}_u$ is known, and should be reached at end of period n-1, since the Q_u distribution is achieved in period n. Thus the required total change in B, ΔB^*, is equal to $B_u - B_{-1}$, and must be equal to $\Delta B_0 + \Delta B_1 \ldots + \Delta B_{n-1}$. There are n unknown ΔBj values, but since their required total is given one unknown may be treated as a residual. Equation (i) for t = (n-1) can now be re-written as :

$$\overline{Q}_{n-1}(v-R) + RiB_{-1} + RI_{n-1} = R\Delta B_{n-1} - Ri(\Delta B^* - \Delta B_{n-1}).$$

In this expression the only unknown is ΔB_{n-1}, which becomes the first of the unknowns to be determined. This is followed up in the corresponding statement of (i) for t = n-2 :

$$\overline{Q}_{n-2}(v-R) + RiB_{-1} + RI_{n-2} = R\Delta B_{n-2} - Ri(\Delta B^* - \Delta B_{n-1} - \Delta B_{n-2}),$$

in which — given the solution already obtained for ΔB_{n-1} — the only unknown is ΔB_{n-2}. In this way it is possible to work right back to t = 1, where

$$\overline{Q}_1(v-R) + RiB_{-1} + RI_1 = R\Delta B_1 - Ri(\Delta B^* - \Delta B_{n-1} - \Delta B_{n-2} \ldots - \Delta B_1)$$

provides a solution to ΔB_1. Then ΔB_0 becomes the residual ΔB value in the _ex ante_ planning of the financing of an investment programme to achieve a constant level of dividend risk.

7 : 2 (iii)   A critique of the planning model.

Before proceeding to the determination of cost of capital
values in situations such as the one defined here, some features of
this solution are worth commenting on. In the first place, this
solution to the multi-period financing problem depends crucially
on the assumption that management wishes to plan for a uniform level
of dividend risk in all periods, during and after the expansion phase.[8]
This assumption should not be accepted without comment, though as a
partial justification it is obvious that any alternative suggestion
would be at least as arbitrary in its interpretation of management
aims. The obvious advantage of planning for a uniform level of dividend
risk in all future periods is that the market rate of discount applying
to all future dividends may be treated as a constant if management's
understanding of risk coincides with that which lies behind the market
rate of discount.

Secondly, the order in which the ΔB_t solutions emerge suggests
a dependence of more immediate policy decisions upon plans already
determined for later periods; and this may appear as a reversal of the
proper order in which such decisions should be taken. This implication
can be removed by recognising that the planning process described
occurs in $t = 0$ but that the first period actually planned is $t = 1$,
not $t = 0$. In the expression of (i) for $t = 1$ this means that it may
be assumed that ΔB_o is not included in the planning exercise and is

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(8)   The dividend planning model may be used to solve problems with
different specifications to those discussed here. Thus, R
may be given any arbitrary time pattern, and v may not be
a constant for all periods in the firm's expectations. In
the latter case, the dividend planning exercise can, in
principle, be performed with any specified time pattern of
v_t. Naturally these modifications, as well as the original
formulation, must be consistent with feasible solutions of
ΔB_t and $\bar{D}_t$.

in fact not adjustable during t = o. Thus the solution to the ΔB_t

series can begin quite naturally with ΔB_1 in the expression of (i)

for t = 1, and can then proceed forward. There can be different ΔB

time-paths depending on where the solution process is assumed to

begin, and in all that follows it is assumed that management plans

forwards : in planning for period t management knows the value of

B_{t-1}. This view of planning is also the more plausible in not

requiring a final equilibrium situation from which to begin a back-

wards solution to the ΔB_t series : indeed, as will be shown in

Chapter 8 : 2 (i), a minimal amount of forward information available to

management will allow planning to proceed as time passes and to produce

the same ΔB time-path that would have been calculated by a perfectly

informed management acting as described here - beginning with ΔB_1.

Thus a standard version of dividend risk planning is that a value for

B_t is to be chosen during t - 1 to satisfy the equation

$$\frac{v\bar{Q}_t}{R} = \bar{Q}_t - iB_{t-1} - I_t + \Delta B_t \qquad \text{(ia)},$$

in which $\bar{Q}_t$, B_{t-1}, I_t, v and R are all known. The actual

expected value of dividend in t , $\bar{D}_t$, results from the planning

of the desired level of dividend risk.

A third aspect of the method is that all solution values of

ΔB_t are apparently feasible in the sense that management expects to

be able and willing to carry out debt financing (and equity retention)

through time according to the timetable derived in this exercise.

However, it is not certain that suppliers of long-term debt will at

all times be willing to finance the firm's capital expansion on the

scale demanded by the solution values of ΔB_t and at the constant

interest rate assumed in obtaining the solution. Specifically, if the full solution value to ΔB_j <u>might</u> be unattainable under certain possible conditions – low Q_j in particular – the planning of constant risk dividend calls for the excess amount of planned investment to be financed by a planned combination of external equity finance and new debt, or delayed or discarded if this cannot be achieved. In this context the forward solution for the ΔB series is the only one practicable. At each stage the level of investment is determined uniquely – either the original demand or possibly a reduced amount if external equity financing, delay or cancellation has to be allowed for – along with its financing, so that for the following period a unique $\overline{Q}_t$, B_{t-1} and investment demand can be envisaged. Evidently if the need for external equity may result in some investments being delayed or cancelled it is advisable to distinguish between an initial investment plan for a period and the final outlay : both are unique and calculable values, and the feasibility of the forward planning of constant-risk dividend is not in question.

Fourthly, in deriving the firm's debt/equity financing timetable investment demands were assumed pre-determined for all future periods : there was no indication of the method(s) of investment selection employed. For the moment the most satisfactory justification for this is to assume that all pre-determined demands are expected to be sufficiently profitable to make unnecessary the application of a test for their acceptability. The minimum acceptable return on an <u>additional</u> investment in the situation described remains to be derived in the following section, and for the present it is best to suppose that the forecasts of future investment used in deriving a constant-risk dividend policy and related financing programme include no near-marginal projects –

though of course the margin itself remains to be identified.
(This arbitrary distinction between investment decision methods is
shown in Section 6 to be quite unnecessary.)

The model may be criticised on the grounds that the periodic
expected dividend is obtained as a residual, but it is obvious that,
given the commitment to each period's I_t value, and its inherited
level of debt, only a prior commitment to an appropriate amount of
new borrowing can produce the desired _ex ante_ level of dividend risk.
In other words, there is a clear choice between a policy of maintaining
constant dividend risk - in the sense employed - and using financing
policy to help achieve some target _level_ of dividend. The aim assumed
here is the former. [9] The questions of how best to represent dividend
risk and how management may formulate its aims will be considered at
greater length later. At present the position is that debt financing
performs two functions : the obvious one of contributing to the finance
of the investment programme; and the additional one of achieving constant
dividend risk at a particular level - a function which in the static
model was performed by capital structure.

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[9]   A target need not be completely fixed : any kind of _ex ante_
        decision to employ financing policy as a means of cushioning
        dividend against the effect of a difference between Q and $\bar{Q}$
        is equivalent to a deliberate alteration of dividend risk.
        With only one policy instrument - financing policy - it is
        not possible for the firm to achieve desired levels of
        expected dividend _and_ dividend risk, except coincidentally.
        The realism of the chosen view of dividend policy is defended
        in Chapter 8 : 4 (ii).

Section 3 : The Cost of Capital and Financing Proportions for a Marginal Investment. (I).

7 : 3 (i) The relationship between r^* and b^* for constant k.

The case considered in this section is that of a firm faced with an investment proposal not already included in its definite plan for expansion over a limited period.[10] In the following section a similar exercise is performed for the case of a firm which anticipates an indefinite continuation of growth. Not surprisingly, the two approaches yield the same conclusions on the marginally acceptable rate of return on investment, and on the financing proportions appropriate for such an investment, because the two situations are so closely related : a limited period of growth may be extended until it approaches infinity. The reasons for presenting both cases at length are that superficially they differ considerably; and that the second case illustrates the limit to which cost of capital theory can be taken in the context of the view of expectations and plans accepted in this and earlier chapters. [11]

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(10)   Specifically, the situation envisaged is that at t = o , having drawn up the kind of constant-risk dividend plan described in Section 2, the firm becomes aware of one more investment opportunity for t = o. An implication of the dividend planning model is that if unforeseen opportunities arise in the future they must be financed by external equity and new debt rather than by disrupting the programme worked out at t = o. This is an unsatisfactory view, and one of the advantages of the conditions assumed in Part II is that constant risk dividend planning becomes fully compatible with risk in the level of future investment demand : see Chapter 8 : 2.

(11)   The logical next step with a steady-state growth model is to see the growth rate itself (or the profit rate or the re-investment rate) as the variable with respect to which market value is maximised. Here it is still assumed that investments are accepted one at a time by reference to a cost of capital criterion, even if the result is a steady rate of growth of profit, capital, etc. Models in which management can maximise market value through the choice of growth rate are discussed in Chapter 14.

It is assumed that the firm's investment, financing and dividend plan has been prepared for all periods up to the end of the growth phase : thereafter the firm is expected to revert to the static Q-distribution, unchanging debt level and expected value of dividend which were the subject matter of Chapter 3. When dealing with an additional investment proposal it is understood that financing will be achieved by an appropriate combination of new debt and retained equity earnings. (The latter may of course be likened to a rights issue fully subscribed by present shareholders.)

The limited-duration growth case is illustrated here by an example in which investment is expected to take place in periods $t = o$ (the present period) and $t = 1$: the firm's Q-distribution therefore assumes its final position from $t = 2$ onwards. The hypothetical unforeseen investment opportunity occurs at the present time. In the absence of this opportunity the firm's prospects can be represented as shareholders' wealth as follows:

$$W_o = D_o + S_o \qquad\qquad \text{(ii)},$$

where W_o equals the total value of the firm's equity at $t = o$, prior to the payment of a dividend; D_o equals the dividend to be paid in the near future; S_o equals the ex-dividend market value of shares.

For present purposes (ii) can be expanded to :

$$W_o = D_o + \frac{\overline{D}_1 + S_1}{(1 + k)} \qquad\qquad \text{(iii)},$$

where $\overline{D}_1$ is the expected value of dividend in period 1 ; S_1 is the anticipated ex-dividend value of equity at end of $t = 1$; k is the market rate of discount on expected dividend income.[12]

Further expansion of (ii) yields :

$$W_o = (Q_o - iB_{-1} - I_o + \Delta B_o) + \frac{\left[\overline{Q}_1 - i(B_{-1} + \Delta B_o) - I_1 - \Delta B_1\right]}{(1 + k)} + \frac{\dfrac{\overline{Q}_u - iB_u}{k}}{(1 + k)} \quad \text{(iv)},$$

where $\overline{Q}_u$ represents the expected value of the Q-distribution after the completion of the firm's investment programme, that is from $t = 2$ onwards ; B_u is the level of debt which, given the risk class of the Q_u distribution and the value of i , ensures that an equilibrium market rate of discount equal to k applies to the constant level of all future expected dividends: B_{-1} is the level of debt "inherited" at the start of $t = 0$. Investment is assumed to occur, and to be financed, at the close of each period; so that its impact both on the Q-distribution and on the level of debt interest, occurs in the following period. Additional debt incurred in any period is denoted by ΔB, with a subscript to denote the period.

The intended risk associated with the expected value of dividend in long-term equilibrium is given by :

$$\frac{v\overline{Q}_u}{\overline{Q}_u - iB_u} = R \quad \text{(v)},$$

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[12]   The application of the same discount rate to both $\overline{D}_1$ and S_1 was discussed and (with reservations) defended in Chapter 2, Sections 2 (ii) and 2 (iii).

where **v** and **R** retain the meanings assigned earlier. For an increment in $\bar{Q}_u$, such as might be due to the hypothetical unforeseen investment now being considered, the appropriate increment in B_u can readily be obtained :

$$\text{since} \qquad \bar{Q}_u \left(\frac{R - v}{Ri} \right) = B_u \qquad \text{(va)},$$

it follows that

$$\Delta \bar{Q}_u \frac{(R - v)}{(Ri)} = \Delta B_u \qquad \text{(vi)}.$$

When long-term equilibrium is reached in period 2, the firm's debt level should be $B_u + \Delta B_u$ if the new investment proposal is accepted. The original identity for the increase in debt up to the achievement of final equilibrium is :

$$B_u - B_{-1} = \Delta B_o + \Delta B_1 \qquad \text{(vii)},$$

where B_u is obtained from (v) above. The firm's debt financing is timed to coincide with its investment outlays, I_o and I_1 : when an additional outlay at $t = 0$ is considered the identity is preserved through the following modification :

$$\Delta B_u = \Delta \Delta B_o + \Delta \Delta B_1 \qquad \text{(viii)},$$

which is simply the differential form of (vii). The additional borrowing in $t = o$ (that is, borrowing in addition to the previously planned level), $\Delta \Delta B_o$, equals the amount of debt finance for the additional investment in $t = 0$. This hypothetical outlay is now indicated by I_m . Thus :

$$\Delta B_u = b^* I_m + \Delta \Delta B_1 \qquad \text{(ix)},$$

where b^* is the proportion of debt required in financing the additional investment such that the risk associated with subsequent dividends is held at the desired level, and hence k is held constant for all future expected dividends. Thus from (vi),

$$\Delta B_u = \Delta \bar{Q}_u \left[\frac{R - v}{R\,i} \right] = \frac{r^* I_m}{i} \left[\frac{R - v}{R} \right] \qquad \text{(x)},$$

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where $\Delta \bar{Q}_u = r^* I_m$ as in Chapter 5 : r^* is the rate of return required on a marginally acceptable additional investment. Substituting the final expression for ΔB_u in (x) into (ix) and re-arranging :

$$\Delta \Delta B_1 = \frac{r^* I_m}{i} \left[\frac{R - v}{R} \right] - b^* I_m \qquad \text{(xi)},$$

or

$$\Delta \Delta B_1 = I_m \left[\frac{r^*}{i} \frac{(R - v)}{(R)} - b^* \right] \qquad \text{(xia)}.$$

The addition to planned new debt in period 1 is given by (xia). These expressions are essentially definitional, rather than complete solutions, and the next step is to ensure the constancy of k in relation to expected dividend in period 1. Since $\bar{D}_1 = \frac{v}{R} \bar{Q}_1$ states the relationship originally planned for $t = 1$, it is convenient to write the modified version as

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$$\bar{D}_1^1 = \frac{v}{R} \bar{Q}_1^1 = \frac{v}{R} \left[\bar{Q}_1 + r^* I_m \right] \qquad \text{(xii)}.$$

This gives $\dfrac{v \bar{Q}_1^1}{\bar{D}_1^1} = \dfrac{v \bar{Q}_1}{\bar{D}_1} = R$, which expands to give :

$$\frac{v(\bar{Q}_1 + r^* I_m)}{\bar{Q}_1 + r^* I_m - i(B_{-1} + \Delta B_0 + \Delta\Delta B_0) - I_1 + \Delta B_1 + \Delta\Delta B_1} = \frac{v\bar{Q}_1}{\bar{Q}_1 - i(B_{-1} + \Delta B_0) - I_1 + \Delta B_1} \quad \text{(xiii)}$$

This reduces to :

$$\bar{Q}_1 (\Delta\Delta B_1 - i\Delta\Delta B_0) = r^* I_m \left[\Delta B_1 - I_1 - i(B_{-1} + \Delta B_0) \right] \qquad \text{(ixv)},$$

or
$$\frac{r^* I_m}{\bar{Q}_1} = \frac{\Delta\Delta B_1 - i\Delta\Delta B_0}{\Delta B_1 - I_1 - i(B_{-1} + \Delta B_0)} \qquad \text{(ixva)}.$$

The basic identity between sources and uses of funds can be re-arranged :

$$\bar{D}_1 - \bar{Q}_1 = \Delta B_1 - I_1 - i(B_{-1} + \Delta B_0) \qquad \text{(xv)},$$

so that (ixva) can be re-stated as :

$$\frac{\bar{D}_1 - \bar{Q}_1}{\bar{Q}_1} = \frac{\Delta\Delta B_1 - i\Delta\Delta B_0}{r^* I_m} \qquad \text{(xvi)}.$$

Using (xi) and the identity $\Delta\Delta B_0 = b^* I_m$, (xvi) becomes

$$\frac{\bar{D}_1 - \bar{Q}_1}{\bar{Q}_1} = \frac{\dfrac{r^*}{i} I_m \left[\dfrac{R-v}{R}\right] - b^* I_m - i b^* I_m}{r^* I_m} \qquad \text{(xvii)},$$

or,
$$\frac{\bar{D}_1 - \bar{Q}_1}{\bar{Q}_1} = \frac{1}{i}\left[\frac{R-v}{R}\right] - \frac{b^*}{r^*} - \frac{i b^*}{r^*} \qquad \text{(xviii)}.$$

The condition to be established is independent of the actual value of I_m.

The left-hand side of (xviii) may be expressed as $\left[\dfrac{v}{R} - 1\right]$, making use of the relationship expressing the required risk associated with $\bar{D}_1$. Thus, re-arranging (xviii) and collecting terms yields :

$$\left[1 - \frac{v}{R}\right]\left[1 + \frac{1}{i}\right] = \frac{b^*}{r^*} (1 + i) \qquad\qquad \text{(ixx)},$$

or
$$\left[\frac{R - v}{R}\right]\frac{r^*}{i} = b^* \qquad\qquad \text{(xx)}.$$

(xx) yields the value of b required to maintain k on the new expected dividend for $t = 1$, $\bar{D}_1^1$: the solution is in terms of the still unknown value of r^* and the given parameter values R, v and i.

In final equilibrium the risk associated with dividend is given by :

$$\frac{v\bar{Q}_u}{Q_u - iB_u} = R \qquad\qquad \text{(v)},$$

giving $B_u = \left[\frac{R - v}{R}\right]\dfrac{\bar{Q}_u}{i}$, so that (xx) can be expressed as :

$$b^* = \frac{B_u}{\bar{Q}_u} r^* = \frac{B_u}{V_u}\cdot\frac{V_u}{\bar{Q}_u}\cdot r^* \qquad\qquad \text{(xxi)}$$

This may be written as

$$b^* = \frac{b_m \cdot r^*}{r_s} \qquad\qquad \text{(xxii)},$$

where r_s is the required rate of return on investment in the once-only context considered in Chapter 5, and b_m is the ratio between the static case market values of B and V.

7 : 3 (ii)   The equality between r^* and c.

The remaining problem is to identify r^*, given that b^* will be chosen to satisfy condition (xxii). A marginally accept-able investment has the property that the differential of W_o in equation (iv) is equal to zero: this may be written as

$$I_m(1 - b^*) = \frac{\left(\Delta \overline{Q}_1 - i \Delta\Delta B_o + \Delta\Delta B_1\right) + \left[\dfrac{\Delta \overline{Q}_u - i \Delta B_u}{k}\right]}{(1+k)} \qquad \text{(xxiii)}.$$

Employing $\Delta \overline{Q}_1 = \Delta \overline{Q}_u = r^* I_m$, $\Delta\Delta B_o = b^* I_m$, and substituting the right-hand sides of (x) and (xia) into (xxiii) gives

$$I_m(1-b^*) = \frac{I_m\left[r^* - ib^* + \dfrac{r^*}{i}\dfrac{R-v}{R} - b^*\right] + I_m\left[\dfrac{r^* - r^*\dfrac{(R-v)}{R}}{k}\right]}{(1+k)} \qquad \text{(xxiv)}.$$

Substituting the right-hand side of (xxii) for b^* in (xxiv), and cancelling I_m from both sides gives :

$$1 - r^* \frac{b_m}{r_s} = \frac{r^* - ir^*\dfrac{b_m}{r_s} + \dfrac{r^*}{i}\left[\dfrac{R-v}{R}\right] - r^*\dfrac{b_m}{r_s} + \dfrac{r^*}{k} - \dfrac{r^*}{k}\left[\dfrac{R-v}{R}\right]}{(1 + k)} \qquad \text{(xxv)}.$$

This can be re-arranged to give :

$$(1 + k) = r^*\left[1 - \frac{ib_m}{r_s} + \frac{1}{k} + \frac{b_m}{r_s}(1 + k) - \frac{ib_m}{kr_s}\right] \qquad \text{(xxvi)},$$

in which collecting terms yields

$$(1 + k) = r^* \left[\frac{1 + k}{k} + \frac{b_m}{r_s} \left(\frac{1+k}{k} \right) (k - i) \right] \qquad \text{(xxvii)}.$$

Dividing both sides of (xxvii) by $(1 + k)$ and re-arranging gives

$$k = r^* \left[1 + \frac{b_m}{r_s} (k - i) \right] \qquad \text{(xxviii)},$$

or

$$k = r^* \left[\frac{r_s + b_m (k - i)}{r_s} \right] \qquad \text{(ixxx)},$$

from which r^* can be expressed as

$$r^* = \frac{r_s k}{r_s + b_m (k - i)} \qquad \text{(xxx)}.$$

It will be recalled that r_s may be expressed as the following weighted-average concept :

$$r_s = (1 - b_m) k + i b_m \qquad \text{(xxxi)},$$

and on substituting this expression into (xxx) the solution to r^* is shown to be :

$$r^* = \frac{r_s k}{k} = r_s \qquad \text{(xxxii)}.$$

The cost of capital, r^*, in the type of growth situation defined is obtained by using the market value weights, b_m and $(1 - b_m)$, which would be appropriate in the once-only investment situation, and which are given by $\frac{B_u}{V_u}$ and $\frac{S_u}{V_u}$ respectively. This result applies

to the growth context defined earlier, in which the investment programme

is to be completed at the end of $t = 1$, but an identical result would

emerge from a growth phase of longer duration : this will be demonstrated

in Section 4. Thus the cost of capital retains the value it held in

the once-only investment situation, and it is clear that the weighted-

average expression for r^* is more instructive than the market valuation

rate, $\frac{\bar{Q}}{V}$. In the growth context the latter is misleading, unless

$\bar{Q}_u$ and V_u are explicitly used for the purpose. Before equilibrium

is attained $\frac{\bar{Q}_t}{V_t}$ will almost certainly suggest a value too low to be

used as r^*, as the denominator reflects anticipated growth beyond

period t and is higher than it would be if no further growth were

expected.[13]

7 : 3 (iii)   The effect on financing plans of acceptance of an additional investment.

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[13]   MV/WACC has been criticised on the grounds that the financing
proportions it suggests are unrealistic, and some of the
confusion on this matter can be dispelled here. It has been
pointed out that market value weights can move cyclically to
produce absurd implications : see A.J. Merret and A. Sykes,
The Finance and Analysis of Capital Projects, 1963, pp.120-1.
However such implications of price movements are not
inevitable : see H. Bierman and S. Smidt, The Capital Budgeting
Decision, 3rd edition 1971, pp. 159-60. In addition, there
is the suggestion, mentioned in the text, that growth
expectations impart systematic (non-cyclical) bias to the
weights. To both of these criticisms the response is that the
correct weights for MV/WACC as an investment criterion relate
to static equilibrium conditions, and are identifiable : see
footnote (19) below. The more general criticism is that
weights used in measuring cost of capital should reflect
planned financing proportions : see Merret and Sykes, op.cit.,
pp. 120-1 ; J.C. Van Horne, Financial Management and Policy,
2nd edition 1971, pp. 105-8. The results obtained in Chapter
5 : 3 and in this section show that if constant k and c
are the objectives of financing policy, actual financing
weights cannot be predetermined in advance of information on
the rates of return on investments. Merret and Sykes come close
to conceding this point when they recognise (op.cit., p.121)
that debt-raising potentials of particular projects should be
taken into account in deriving cost of capital measures.

Turning to the financing of investments, the first point to note is that $\Delta\Delta B_1$ is equal to zero in the case of a marginal investment undertaken in $t = 0$. Equation (xia) gives

$$\Delta\Delta B_1 = I_m \left[\frac{r^*}{i} \frac{(R - v)}{(R)} - b^* \right] \quad, \text{ and } \quad \text{(xx) gives}$$

$$\left[\frac{R - v}{R} \right] \frac{r^*}{i} = b^* \quad, \text{ so that } \Delta\Delta B_1 = 0 \text{ for any value}$$

of I_m .

The planned financing of I_1 in $t = 1$ is not affected by the provisions made in $t = 0$ for financing an additional expenditure. Similarly, (xia) applies generally to the financing of any investment, I_j , in the form

$$\Delta\Delta B_1 = I_j \left[\frac{r_i}{i} \frac{(R - v)}{(R)} - b_j^* \right] \qquad \text{(xxxiii)},$$

where b_j^* is the value of b required, given the profitability of I_j to maintain the risk-class of $\overline{D}_1$. Thus $b_j^* = \left[\frac{R - v}{R} \right] \frac{r_i}{i}$, from equation (xx), and $\Delta\Delta B_1 = 0$ for any investment regardless of profitability. This may be generalised for planned investment programmes of longer duration : the adjustment in the desired level of B_u resulting from the acceptance of an additional proposal is confined to the period in which the programme revision occurs.

An important conclusion to be drawn from the solutions to r^* and the financing of investments additional to those already accepted as part of a growth programme is that the duration and

pattern of the original programme do not affect r^*, b^* or b_j^*.
Accordingly, there is no difficulty in modifying any stage of a
programme of planned investments : the rate of return required on
any self-contained proposal, I_j, is given by r^*, and its appropriate
financing proportions by $(1 - b_j^*)$ and b_j^*. The correct appraisal
and financing of investments is the same, whether in the context
of planned growth or of a once-only expenditure.

Section 4 :   The Cost of Capital and Financing Proportions for
               a Marginal Investment. (II).

7 : 4 (i)   Outline of a steady-state growth model.

        In recent years mathematical models of sustained growth of
a firm's assets, profit, investment, dividend, debt and equity values
have been used for a variety of purposes, including the identification
of optimal growth and dividend policies. [14] Some of the results
established with the aid of such models, together with their underlying
assumptions, will be considered in Chapter 14 : but the present purpose
is to set up a model of growth as anticipated by management, derive
from it market values of debt and equity, then once again identify
the minimum rate of return acceptable on an investment together with
its correct financing proportions. The assumptions made about future
developments are broadly the same as those employed in Sections 2 and 3
above: the firm's investment for all future periods is taken as uniquely

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(14)   The separate components and structure of such a model are
        clearly described in E.M. Lerner and W.T. Carleton,
        A Theory of Financial Analysis, 1966, esp. Chapters 5, 7.
        The suggestion that investors view a company's dividend
        stream in terms of a steady growth rate of expected
        dividend was first made in M.J. Gordon and E. Shapiro,
        "Capital Equipment Analysis: The Required Rate of Profit,"
        Management Science (1956).

determined; the risk of the changing Q-distribution is constant
for all periods. The additional assumptions are that investment
is expected to grow at a constant rate through time in perpetuity,
and that the average profitability of investment (and of total
assets) is constant through time. Profit fluctuates around its
rising mean value, but the underlying rate of return on investment -
in the sense used in earlier chapters - is constant : in the long
run the average rate of return on assets, taking into account all
variations, is given by $\bar{r}$. It follows from these assumptions
that dividend in each period is determined in the way described
earlier for the limited-duration growth model : dividend is essentially
a residual, differing from its expected value by the same amount
and in the same direction as Q_t differs from $\bar{Q}_t$. The level of
Q_t is not expected at any time to endanger the pre-determined levels
of investment, debt interest and new borrowing. [15]

The model is, therefore, a steady-state growth version of the
situation considered in Section 3 : deriving solutions for r^*
and b^* for this case is worthwhile because it represents the
limit to which the present solution method can be extended. One
feature of all situations covered by the present approach is that
no overall optimising behaviour is necessarily assumed : rather,
the firm is assumed to determine an investment and growth policy by
some undisclosed method, so that interest focusses on the relatively
minor problems of establishing criteria for the acceptance of an
additional investment proposal at some stage of the pre-determined
growth-path, and the financing of such an investment so as to

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(15)   With a completely pre-determined investment programme and no
        external equity financing, the feasibility of planned debt
        financing on all occasions is ensured by assuming that
        dividend risk - and consequently average retention of earn-
        ings - is set at any appropriate level.

maintain the firm's market valuation rate. The problem of formulating a growth model which can be used in an overall optimising exercise is left to Chapter 14, though the present section may be seen as a preliminary to that discussion in introducing the simple mathematical relationships of such models.

The firm's capital stock, K, is assumed to grow at a constant rate, g, per time period. Profit in each period is a random variable with mean $\bar{Q}_t = \bar{r}K_{t-1}$ and standard deviation equal to $v\bar{Q}_t$. Net investment in each period is given by $I_t = gK_{t-1}$, where g is a constant which may be expressed as

$$g = p\bar{r} \qquad \text{(xxxiv)}.$$

In (xxxiv) p is the rate of reinvestment of expected (not actual) profit. The firm finances a fraction b of its capital by issuing new debt at a constant interest rate i, so that the expected value of dividend in period t is given by :

$$\bar{D}_t = \left[\bar{r} - ib - g(1-b)\right] K_{t-1} \qquad \text{(xxxv)}.$$

The major variables K, $\bar{Q}$ and $\bar{D}$ all grow at the same rate, $g = p\bar{r}$, and maintain a constant relationship to each other. Equally important, since the risk class of the firm's earnings remains constant through time (the ratio of the standard deviation of Q to $\bar{Q}$, and other relationships, remain constant) the risk associated with each period's expected dividend is also constant. Equation (xxxv) is thus a special case of the denominator in equation (i), and shares the constant risk property of that general

formulation. It is correct, given these assumptions and relationships, to see the stream of future expected dividends as increasing indefinitely at a constant rate and subject to a constant level of risk.

The valuation of such a stream of expected dividends is easily stated algebraically. Given that

$$S_o = \frac{\bar{D}_1}{1 + k} + \frac{\bar{D}_2}{(1 + k)^2} \cdots + \frac{\bar{D}_n}{(1 + k)^n} \cdots \cdots \infty \quad \text{(xxxvi),} \quad (16)$$

and that $\bar{D}_t$ grows at a constant rate $\overline{pr}$ per period, the summation of a geometric progression to infinity can be used to obtain S_o, the ex-dividend value of the firm's equity at $t = o$. Equation (xxxvi) becomes :

$$S_o = \frac{\bar{D}_1}{(1 + k)} + \frac{\bar{D}_1(1 + \overline{pr})}{(1 + k)^2} \cdots + \cdots \frac{\bar{D}_1(1 + \overline{pr})^{n-1}}{(1 + k)^n} \cdots \infty \quad \text{(xxxvii),}$$

or

$$S_o = \frac{\bar{D}_1}{k - \overline{pr}} \quad \text{(xxxviii).}$$

It is important to note that a finite solution value for S_o requires that the discount rate, k, exceed $\overline{pr}$, the rate at which expected dividend increases. (If this were not so, the terms on the right hand side of (xxxvii) would increase continuously with t, and as t is taken to infinity there would be no finite solution value.) The only respect in which this formulation differs

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(16) See footnote (12) above.

from the approach discussed so far is that it explicitly embodies
a constant growth rate of expected dividend. Therefore, there
is scarcely any more reason to fear that S_o may take an infinitely
high value in this case, where the growth assumption is declared
explicitly, than in any other case which might be represented by
the approach of the preceding two sections - in which growth and
profitability assumptions are not made explicit. [17]

The value of equity, S_o, may be considered from a rather
different angle, emphasising the equity contribution to financing
investment and the share of equity in earnings. S_o is the sum of
all discounted expected dividends, where expected dividend is
defined as expected earnings minus debt interest and retention for
re-investment. Similarly, the rate of retention may be defined
as the proportion of expected equity income re-invested, p_e,
rather than the overall rate p used earlier. To complete the
change of emphasis, the expected return on investment and capital
is defined in relation to equity earnings and the equity contribution
to total assets. Given that the overall expected return on total
assets is $\bar{r}$, and the proportion of investment financed by debt is
b, the expected return to equity investment is :

$$\bar{r}_e = \frac{(\bar{r} - ib)K}{(1 - b)K} = \frac{(\bar{r} - ib)}{(1 - b)} \qquad (xxxix).$$

(17)   When it is recalled that market rates of discount are
        determined by supply and demand forces (though given for
        the individual firm, as assumed here) it becomes obvious
        that rates in general must adjust so that all financial
        assets sell at finite prices : see F. Modigliani and
        M.H. Miller, "The Cost of Capital, Corporation Finance
        and the Theory of Investment : Reply," American
        Economic Review (1959), note (17), p.664.

The rate of retention of expected equity earnings is now denoted by p_e , and the expected dividend for t is given by :

$$\overline{D}_t = (\overline{r} - ib)(1 - p_e)K_{t-1} \qquad \text{(xl)}.$$

Clearly, this variable has a growth rate identical to that of K, since all other terms in (xl) are constants. In steady-state growth the relationship

$$g = \overline{pr} = p_e\overline{r}_e$$

is true by definition, and so the expression for S_o may be written in alternative forms :

$$S_o = \frac{\overline{D}_1}{k - \overline{pr}} = \frac{\overline{D}_1}{k - g} = \frac{\overline{D}_1}{k - p_e\overline{r}_e} \qquad \text{(xli)}.$$

The final expression for S_o in (xli) expands to :

$$S_o = \frac{K_o(\overline{r} - ib)(1 - p_e)}{k - p_e\dfrac{(\overline{r} - ib)}{(1 - b)}} \qquad \text{(xlii)}.$$

7 : 4 (ii)   The value of b^* on a marginally acceptable additional project.

For the particular growth situation considered in this section, the expression $W_o = D_o + S_o$ may be expanded to

$$W_o = D_o + \frac{K_o(\overline{r} - ib)(1 - p_e)}{k - p_e\dfrac{(\overline{r} - ib)}{(1 - b)}} \qquad \text{(xliii)}.$$

For establishing the acceptability of an additional investment project the following formulation is helpful :

$$W_0 = Q_0 - iB_{-1} - I_0 + \Delta B_0 + S_0 \qquad \text{(xliv)}.$$

The definition of a marginally acceptable return on an investment I_m is the same as that developed in Chapter 5 and used in the preceding section of this chapter : it consists of the requirement that $\Delta W_0 = 0$, subject to k remaining constant for all periods. For $\Delta W_0 = 0$ when I_m is undertaken, assuming constant k, the differential of (xliv) becomes :

$$0 = -I_m + \Delta\Delta B_0 + \frac{(r^* - ib^*) I_m}{k} \qquad \text{(xlv)}.$$

This assumes, as in the preceding section, that the additional investment is of the once-only type, having no cumulative effect on profit, investment or dividend in addition to those directly related to it and embodied in the differentials of the terms in (xliv). Since $\Delta\Delta B_0 = b^* I_m$ by definition, (xlv) can be converted into the familiar expression :

$$r^* = (1 - b^*) k + ib^* \qquad \text{(xlvi)}.$$

The condition for constant k is that the additional investment I_m be financed in such a way that the risk-class of total expected dividend in each period is unaffected. This condition is satisfied if:

$$\frac{v\overline{r}K_{n-1}}{(\overline{r} - ib)(1 - p_e)K_{n-1}} = R = \frac{v(\overline{r}K_{n-1} + r^* I_m)}{(\overline{r} - ib)(1-p_e)K_{n-1} + (r^* - ib^*)I_m} \text{ for all n,}$$

$$\text{(xlvii)}.$$

This condition is simply the application of equation (i) to the growth model for each period : the left hand expression reflects dividend risk in all periods in the absence of any addition to the pre-determined growth path of investment; the right hand expression defines an equal dividend risk in all periods allowing for the additional investment and financing occurring at one point in time. The condition simplifies to :

$$r^*(\bar{r} - ib)(1 - p_e) = \bar{r}(r^* - ib^*) \qquad \text{(xlviii)}$$

Evidently the appropriate financing of a marginally acceptable additional investment does not require a continuing adjustment to the financing plans already formulated for future periods : b^* is given in terms of r^* and other parameters of fixed value. Just as in the limited growth phase model, an additional investment affects financing decisions only for the period in which it occurs.

The expression for r^* in (xlvi) may be substituted into (xlviii) to obtain:

$$\left[(1 - b^*)k + ib^*\right](\bar{r} - ib)(1 - p_e) = \bar{r}\,(1 - b^*)k \qquad \text{(xlix)},$$

from which b^* can be obtained in terms of b, k, i, p_e and $\bar{r}$:

$$b^* = \frac{k\left[rp_e + ib(1 - p_e)\right]}{(k - i)\left[\bar{r}p_e + ib(1 - p_e)\right] + i\bar{r}} \qquad (1).$$

Given the relationship (xlviii) between b^* and r^*, this means that r^* can also be found in terms of the fixed parameters of the growth model.

In identifying b^* in (1) as the market value weight for a static Q-distribution, it is better to begin with the latter concept and work from it towards the expression for b^* already obtained. Let b_m be the value of $\frac{B}{V}$ for a static Q-distribution. Then :

$$b_m = \frac{b_s K}{b_s K + \frac{K(\bar{r} - ib_s)}{k}} \qquad (li),$$

where b_s is the proportion of the firm's capital stock financed by debt in a hypothetical static context, and $\bar{r}$ retains the meaning given to it throughout this section. The variables b_s and b (the proportion of investment financed by debt in the growth context) are related by the requirement that k be constant and equal in each context. Thus :

$$\frac{\overline{vr}K_n}{(\bar{r} - ib)(1 - p_e)K_n} = \frac{\overline{vr}K}{(\bar{r} - ib_s)K} = R \qquad (lii),$$

which gives an equal dividend risk in both the growing and static cases. Condition (lii) reduces to :

$$(\bar{r} - ib)(1 - p_e) = \bar{r} - ib_s \qquad (liii),$$

and $$b_s = \frac{\bar{r}p_e - ib(1 - p_e)}{i} \qquad (liia).$$

The book-value weight for debt in the static case, b_s, is given by (liia) which has its right side made up of terms from the growth case. The right hand side of (liia) may now be substituted for b_s in (li) : after substituting and re-arranging of terms the

result is :

$$b_m = \frac{k\left[rp_e + ib(1 - p_e) \right]}{(k - i)\left[\overline{rp_e} + ib(1 - p_e) \right] + i\overline{r}} \qquad \text{(liv)},$$

and as shown by (1) this is also equal to b^*, the correct debt
financing proportion for a marginal once-only investment in the
growth context. Thus:

$$b^* = b_m \qquad \text{(lv)}$$

7 : 4 (iii)   The equality between r^* and c.

Returning to equation (xlvi) for r^*, it is obvious that with
b^* identified as b_m, the market value weight of debt in the static
case, the value of r^* in the steady-state growth context is
exactly the same as in the earlier cases of a once-only investment
in the static context (Chapter 5) and an additional investment
in the limited-growth context (Section 3 above). The result is
artificial, in that b_m refers to a static situation which is
not expected to occur in practice. In another sense too the result
is artificial : the firm's intended financing proportions for its
continuing investment programme are used as parameters for the
solution to b^*, the debt financing proportion on the marginally
acceptable additional investment. There is no indication whether
b itself is the outcome of an optimising exercise or whether it is
simply a conventional or arbitrary value. In either case it may
seem strange that financing proportions for different parts of a
firm's investment programme should be determined according to

different principles. However, if one accepts that a firm may engage in a partial optimising exercise, selecting projects in addition to a main programme whose growth and composition through time are regarded as fixed, the apparent artificiality of the exercise to determine r^* and b^* is less troublesome. Indeed b^* is perhaps a valuable concept for decision-making, when the alternative of arbitrarily chosen financing proportions is considered : in the absence of information about b^*, b itself would presumably be chosen along with some related but equally arbitrary concept of r^* — with no certainty at all about the overall effects on valuation and dividend risk.

Section 5 : The Cost of Capital, External Equity Financing and and Growth.

The investment situation considered here corresponds to that in Section 3 except that new equity capital is to be obtained externally, by means of a rights issue, instead of by retaining earnings. The reasons for such a decision need not be discussed here : the planning of constant dividend risk was shown in Section 2 (iii) above to indicate future occasions on which external equity finance will be required, and if an unforeseen additional investment opportunity arises in the current period the points raised in Chapter 5 : 5 remain relevant.

The expressions for r^* and b^* derived in the preceding section for a specific growth assumption (steady-state growth of investment and expected profit) were shown to be equivalent to those obtained for those concepts in Section 3, where the duration

and profitability of a pre-determined investment programme were not specified. It is therefore both convenient and sufficient to consider the question of an additional investment financed by new debt and external equity in the general context of Section 3, since the result will apply to more specific formulations. Results established in Chapter 6 (on the cost of capital in connection with a once-only investment financed by a rights issue and new debt), and earlier in this chapter (on the effects of financing an additional investment on the future financing plans of a firm), are employed here to shorten the discussion.

In Sections 3 and 4 it was established that the financing of an additional investment with marginal profitability affects the firm's borrowing plans only in the period in which the additional expenditure is incurred, so that the firm's level of debt moves immediately to its correct new value for constant dividend risk in all future periods. This means that for a continuously growing firm, just as for a static firm making a once-only investment, the increase in the ex-dividend value of equity following such an investment/financing decision is given by :

$$\Delta S = \frac{\Delta \overline{Q}_u - i \Delta B_u}{k} \qquad (1vi),$$

where the terms $\Delta \overline{Q}_u$ and ΔB_u have the same meanings as in Section 3. ΔS is therefore the permanent, once-only, increase in the value of the firm's equity, and it embodies the assumption of constant dividend risk. As shown earlier, equation (1vi) may be written as :

$$\Delta S = \frac{I_m(r^* - ib^*)}{k} \qquad\qquad \text{(lvii),}$$

again using the original meanings for I_m , r^* and b^* .

The analysis of rights issues in Chapter 6 concentrated on investments of the once-only type, with no assumption of a continuing investment programme. The result established in Chapter 6 : 3, assuming the conventional market valuation of rights to purchase the new shares, was that r^* is obtained when $P_1 = P_e$; that is when the new equilibrium share price is equal to the value generated by the conventional formula for the post-issue share price,

$$P_e = \frac{P_o n + P^1 \Delta n}{n + \Delta n} \qquad\qquad \text{(lviii).}$$

The purpose of this section is to extend the earlier result to situations in which other investments are taking place and are planned for the future. This can be done with a simple notation, using S for the ex-dividend value of equity – that is, the value of all expected future dividends whether static or growing through time – and using ΔS to indicate the change in S due to the adoption of an additional investment and its financing at the present date.

The expressions used in Chapter 6 for the ex-dividend valuation of equity – with and without the additional investment/ financing – may then be written as

$$W_o{}^1 = \left[\frac{n + \alpha \Delta n}{n + \Delta n}\right]\left[S + \Delta S\right] + P_r(1 - \alpha)\Delta n - P^1\alpha\Delta n \quad \text{(lix)};$$

$$W_o = S \quad \text{(lx)},$$

respectively. (α is the proportion of rights retained by existing shareholders and exchanged for new shares at a unit price of P^1; P_r is the market price of a right to purchase a new share at that price; and Δn is the number of new shares issued by the firm.) By equating $W_o{}^1$ and W_o the investment/financing decision is defined as marginal, and it is understood that financing is arranged to maintain an unchanging value of k through time. Recalling that $P_r = P_1 - P^1$ in the solution to r^* obtained in Chapter 6 : 3, $W_o{}^1 = W_o$ implies

$$0 = \frac{(n + \alpha\Delta n)}{(n + \Delta n)}(n + \Delta n)P_1 + (P_1 - P^1)(1 - \alpha)\Delta n - P^1\alpha\Delta n - nP_o \quad \text{(lxi)},$$

$$\text{or} \quad nP_o = P_1(n + \Delta n) - P^1\Delta n \quad \text{(lxia)},$$

$$\text{or} \quad S = S + \Delta S - P^1\Delta n \quad \text{(lxii)}.$$

Therefore, $S = P^1\Delta n$ is the minimum requirement for an investment financed by debt and a rights issue in such a way that the value of k is held constant.

Since $P^1\Delta n$ equals the total amount of new equity raised, which by definition is equal to $\dfrac{I_m(1 - b^*)}{e}$, the relationship

$$S = \frac{I_m(1 - b^*)}{e}$$

is established. In addition, as shown above, S may be expressed as $\dfrac{I_m(r^* - ib^*)}{k}$ (equation (lvii)), so r^* can be defined in terms of

$$\frac{I_m(1 - b^*)}{e} \quad = \quad \frac{I_m(r^* - ib^*)}{k} \qquad \text{(lxiii)},$$

where $b^* = r^* \dfrac{B}{Q}$ as in Chapter 5 and Section 3 of this chapter. This yields an expression for r^* :

$$r^* = (1 - b^*) \frac{k}{e} + ib^* \qquad \text{(lxiv)},$$

which is identical to that obtained in Chapter 6.

It has now been established that the cost of capital for an additional investment financed by a rights issue and new debt is not affected by the firm's investment and growth plans for subsequent periods : the conclusions of Chapter 6 are of general application.

Section 6 : Conclusion.

The purpose of this chapter has been to extend the applicability of the market value weighted average cost of capital (MV/WACC) to the types of dynamic situation which conform to the assumptions stated. In extending the approach developed earlier for a comparative-static context (Chapters 5 and 6) it has been assumed that it remains valid for management to interpret market valuation as the discounting at an appropriate rate of future dividend expectations; and that the value of k appropriate to

the (constant) risk class of a firm's expected dividend can still be identified. Within this context it has been shown that the weights relevant to MV/WACC are not those derived from actual market values, even where the market correctly values the firm's prospects, but those relating to a hypothetical static situation with the same levels of dividend risk and overall risk as those actually maintained by and affecting the growing firm.

An important conclusion arising from these results is that a precise knowledge of a firm's future investments and growth of earnings potential, as assumed in Sections 3 and 4, is unnecessary in identifying a MV/WACC measure appropriate to any financing decision. For example, the same MV/WACC obviously applies to any investment financed by retained earnings and new debt, regardless of the assumed future pattern of investment and the growth of potential earnings (provided, of course, that the overall and dividend risks are assumed constant). In Section 4 neither the growth rate of expected dividend nor its component elements (the expected profitability of investment and the retention ratio for expected earnings) have any influence on the value of r^*, which is expressed in exactly the same terms as in the once-only investment situation of Chapter 5. Evidently it is sufficient for the identification of r^* that a unique growth-path satisfying the constant risk conditions should exist, not that its details should be known ex ante to management. Since it must always be true, ex post, that a unique growth path existed (though, of course, constant risk would have to be demonstrated), it is tempting to conclude that the expressions for r^* have a

very general validity. It follows that one problem raised in the course of this chapter can now be satisfactorily resolved : that of the contrast in investment decision making between the additional investment (identification of r^*, b^*) and the original unexplained investment programme of the firm. Because r^* can be derived without specific knowledge of future investments and their earnings potential, each investment can be appraised by exactly the same criterion : the whole of a firm's investment programme can be built up in this way, and there ceases to be an unconvincing distinction between the methods used to appraise different parts of the programme. [18]

These conclusions suggest a possible modification of the assumptions about the market valuation of equity and management's interpretation of it which were re-stated at the opening of this section. These assumptions may be too strong to be accepted as representative of the actual communication between the capital market and management, or of the degree of omniscience possessed by the latter about a firm's prospects. The conclusions in the preceding paragraph seem to suggest a possible relaxation of these assumptions, because an important result was obtained without employing the "omniscient management" assumption to the fullest extent. However, possible developments from the relaxation of

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(18)   This remains subject to the obvious condition shown in
        section 2 (iii) above, that planned debt financing must
        be feasible. The firm first identifies all investments
        satisfying the standard MV/WACC criterion : then, in the
        course of drawing up a constant-risk dividend plan,
        any need for external equity (or delay or rejecting of
        projects) becomes clear. In this way a full programme
        of investment, retention, external equity, new debt and
        expected dividend is worked out.

one assumption should be considered : these may occur both
in the behaviour of the capital market and in the interpretation
by firms of that behaviour.

The interpretation accepted so far is that, since both
management and capital market believe in the existence of a unique,
objectively correct, valuation for the firm, that valuation will
prevail through the spreading of information and the convergence
of expectations. (The time taken in reaching ultimate equilibrium
is not important in this context.) Now, if it is acknowledged that
management itself may be uncertain about prospects, and that such
uncertainty is unavoidable, there is no longer a unique level
in the former sense towards which capital market forces can be
expected to steer the firm's market value. In these circumstances
it is necessary to re-examine the supposed thinking and behaviour
of investors, who up till now have been assumed to perform the
simple function of valuing income streams whose objectively
determined mean value and risk for each time period are widely
known. It is not necessary at this point to explore the wealth
of literature on the determination of share values once an initial
move away from the ideal situation is conceded. There is no need,
in fact, to abandon the idea that valuation - to the long-term
investor - represents the discounting of expected future income.
What is more doubtful is the extent to which the decisions of
this group will be dominant in the capital market, as opposed
to the effects of speculative activity on prices; and secondly,
the ways in which even long-term investors may modify their
behaviour in the new circumstances. Long-term investors are still

assumed to think in terms of an objectively correct (but now unknown) market value for the firm's equity.

The assumption of management (and market) omniscience can be relaxed up to a clearly defined point without prejudice to the relevance of the cost of capital results derived in this and earlier chapters. As indicated in Chapter 3 : 1 (i) and repeated in Section 1 above, the firm's future investment programme is assumed uniquely determined ex ante, but not necessarily fully known to management and outside investors. Similarly, the Q-distribution for each period is assumed uniquely determined ex ante, but again not necessarily known to management and investors at the present. The cost of capital results obtained under the assumption of omniscience continue to apply when ignorance of the future is recognised, as long as the ex ante uniqueness of the time-paths of I_t and $\bar{Q}_t$ are retained.

What is at stake if only the strict omniscience assumption is dropped is the ability of management in the resulting capital market conditions to identify k, the discount rate applying in the fully informed market to expected income streams having the same risk as the firm's intended dividend policy. If k can still be measured management should employ the MV/WACC as its investment criterion even when the firm's true value remains unknown. The crucial role of k can be seen by recalling the static case which, as has been shown, provides the correct market value weights for

identifying r^* even in a growth context.[19] It remains to
consider (in Chapter 9) whether k <u>can</u> be derived in the event
that the market accepts an <u>ex ante</u> unique but largely unknown
growth-path, but a positive answer is assumed here.

If k is identifiable in spite of investors' uncertainty
over the objective details of the firm's future dividend stream,
MV/WACC remains an appropriate investment criterion: its results
accord with the basic preferences of long-term investors reflected
in the value of k. This is not to suggest that equity market
value and its fluctuations do not concern investors, but that
because management is able to exercise little control over market
fluctuations the latter can be considered as outside its obligations
to shareholders. In the world of imperfect information about the
firm's future growth-path, management's obligation is simply to
make investment decisions in the interests of long-term investors,
employing MV/WACC for the purpose. Investors benefit from such
decisions in the form of reliable dividend income (subject to the
accepted risk) rather than through less predictable market price
movements.[20]

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(19)   For the static case, $Sk = \overline{D} = (\overline{Q}-iB) = v\overline{Q}$. Thus $\overline{D}$ is
       determined in terms of Q, v, R; R and this gives
       the appropriate B-value. Thus, <u>given k</u>, S is determined
       when D is known; and the weights $\frac{S}{S+B}$ and $\frac{B}{S+B}$ are identified
       in the abstract — as is the MV/WACC itself.

(20)   However, in reverting to a context in which long-term constant-
       risk dividend planning cannot be carried out, management
       finds itself back in the artificial situation described in
       Chapter 5, in which each investment is represented as a once-
       only interference with an established policy of 100% pay-out
       of earnings: see Chapter 5 : 5. The unsatisfactory nature of
       this situation can be seen as a powerful incentive for
       management to adopt one of the views of the future described
       in Chapter 8 : 2.

It is when the belief in an _ex ante_ unique growth-path is recognised as unrealistic that management's interpretation of valuation and its own investment decision making must both be substantially modified. The theory of managerial strategy choice, still based on investment decision making but incorporating both kinds of modification, is the subject matter of Part II of this study.

Part II

Dividend Policy, Valuation Theory and Strategy for Multiple Growth Possibilities

Introduction to Part II

Differences between Parts I and II stem from the fact that investment profitability is now recognised as subject to _ex ante_ risk, so that management must supplement or replace its original cost of capital criterion.

In Part I management's strategy comprises its predetermined policy on dividend risk, which influences the cost of capital through its effect on k, and its conditional rules for retention limits, external equity financing, etc.. Obvious motivations such as security against takeover can be recognised, but value-maximising behaviour is restricted to the application of the predetermined criteria and procedures which in management's view provide a basis for rational and comprehensible valuation.[1]

In Part II on the other hand, management, while retaining a preference for a predetermined investment and financing policy, does enjoy an additional dimension of choice – over the precise expression of its policy towards risky investments – and the effect is that the underlying value of prospects lies at that stage within its influence. In trying to influence market value management operates through its choice of general criteria for investments: the application of the latter to a succession of proposals and situations is as mechanical as in Part I. Unlike Part I, Part II envisages the preliminary stage of decision making conducted as an optimising exercise.

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(1)   For a firm with growth possibilities, minimising cost of capital may not maximise the value of investors' wealth: see H. Bierman and J.E. Hass, An Introduction to Managerial Finance, 1973, p.165. In their version of a steady-state growth model, Lerner and Carleton described the cost of capital as a by-product of a value-maximising solution, but the same model makes it clear that when future investment prospects are unknown neither a value-maximising growth rate/retention policy nor an operational cost of capital can be identified: see E.M. Lerner and W.T. Carleton, A Theory of Financial Analysis, 1966, pp.191-202.

Whatever the nature of the investment decision process and investment risk, management and investors are bound to see future prospects in probabilistic terms; but only in Part II is the range of objective possibilities, and its valuation, taken fully into account in the formulation of strategy. But if this were the only element of difference a curious conclusion might be drawn: management would find itself in a position to follow a value-maximising instinct that was excluded in Part I by the predetermination of policies. The factor that precludes this paradoxical situation and makes a "managerial" decision unavoidable is management's awareness that the unfolding of events must alter even a correct view of future prospects and their probabilities, and so alter correct valuation. Value maximisation loses much of its appeal for management if the strategy offering the highest initial value may later be overtaken by others.

Chapter 8

Multiple Growth-Paths of Constant-Risk Dividend

Plan of Chapter

Section 1.   Introduction

Section 2.   Multiple Growth-Paths of Constant-Risk Dividend

    (i)   Planning without an investment demand model.

    (ii)   Planning with a model of investment demand.

    (iii) A full simulation model.

Section 3.   Combining Multiple Growth-Paths of Constant-Risk Dividend

    (i)   The mathematical conditions for constant risk of the overall dividend distribution.

    (ii)   The meaning and implications of perfect correlation between dividend distributions.

    (iii) An example of perfect positive correlation between dividend distributions.

    (iv)   The logical impossibility of determining the correlation between future dividend distributions.

Section 4.   Conclusions

    (i)   The importance of objectivity in probability distributions.

    (ii)   Reconciling "residual" and "discretionary" dividend models; definition of time period.

Section 1 :   Introduction

In Chapter 7 a combined dividend and investment planning model
was developed, possessing the desired property of constant dividend
risk in the defined sense. Within this context the cost of capital
was identified as being equal to the static firm's market valuation
rate, c. The model was shown to generate a unique growth-path of
expected dividend, given the firm's investment plans and Q-distribution
for each future period and its dividend/financing policy. The unique
nature of this growth-path depends on two assumptions which conflict
with a commonsense understanding of the limited possibilities of business
forecasting and planning, and this chapter considers management's
problems in formulating constant-risk dividend policy when these
offending assumptions are modified or dropped. The assumptions referred
to are, first, that a correct single-valued investment forecast is
available for each future period; and second, that the profitabilities
of all future investments are known with certainty.

This is an appropriate point to introduce a device employed
throughout this chapter to distinguish between two aspects of an
investment's earnings. In the first place, the term _profitability_
refers to the position of the investment's ΔQ-distribution, in
particular to its mean value, $\Delta \overline{Q}$, rather than its earnings in any
particular period. Profitability in this sense is determined once-
for-all at the start of the project's life, and is likely to be subject
to risk factors.[1] These risks may be specified in many ways, and

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(1)   Obviously $\overline{\Delta Q}$ alone is not a measure of an investment's
rate of return: this latter concept might be expressed as the
ratio of $\overline{\Delta Q}$ to investment cost, and its risk shown by the
probability distribution of this ratio.

these are discussed in due course. The second aspect is a familiar
one, that once an investment's ΔQ-distribution is determined, its
periodic earnings over time yield a pattern governed by the distribution.
While the distinction drawn between the two aspects of risk - that
affecting $\overline{\overline{\Delta Q}}$ and that affecting ΔQ, given $\overline{\Delta Q}$ - is perhaps unrealistic,
the purpose is purely expository: the realism of the earlier assumption
about returns on investment is increased by adding to it the risks
affecting investment profitability. It should become clear that the
fundamental characteristics of growth that will be demonstrated would
arise under practically any realistic procedure for representing the
variability of an investment's returns over its lifetime.

Simplifying assumptions can usefully be introduced at this point.
It is assumed that when the ΔQ-distributions of all new investments
simultaneously undertaken are combined with the original Q-distribution,
the result is a new Q-distribution having the same level of business
risk as the original. In other words the condition shown in Chapter
4 : 2 (i) for a single investment to yield unchanged business risk is
extended to a group of investments: the result is not necessarily
assumed to come about through deliberate policy, but is seen as a
reasonable tendency in investment decision making. [2]

The other risk element introduced into the planning model in this
chapter relates to the level of the firm's investment spending.

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(2) The assumption that a typical group of investments leaves business
risk unchanged provides a relatively minor theoretical advantage,
shown in Chapter 11 : 3 (ii); but the expectation is one which
management might reasonably hold (see Chapter 4 : 3).

Previously this was treated as given and known with complete confidence for all periods : now, in stages, investment becomes a probabilistically determined variable not amenable to single-value prediction. As with the formal distinction between the different aspects of risk affecting the returns from investment the model employed to describe how investment demand is determined is comparatively unimportant : it is sufficient to show the consequences for dividend growth of removing the uniqueness of each period's investment level and illustrate various possibilities by simple examples.

The important background to this chapter's treatment of risk is the assumption that investment decisions are taken in accordance with a policy or strategy laid down by management. It is unnecessary here to explore the many ways in which a strategy might be formulated: the general results of its operation are the important subject matter in this chapter. [3] However its strategy is expressed, management is likely to see the level of investment as a probabilistic variable – partly because of the variability of the firm's profit performance and general outlook, and partly because opportunities for investment may vary quite apart from their relationship to the firm's objective situation. It is the assumption that a form of strategy exists that makes it possible to proceed with the kind of analysis developed in this chapter : given the terms of its strategy, management is assumed to be able to make probabilistic statements about the level of investment in given circumstances and about the overall profitability of that investment. These assumptions are the building blocks of what is to follow.

(3)   A simple model showing <u>one important feature</u> of an investment
        strategy is described in Chapter 11 : 3.

When both investment and its profitability are acknowledged to be subject to risk factors, as everyday observation suggests they must be, the unique character of the growth-path as described in Chapter 7 must be replaced by a multiplicity of possible growth patterns; each of which, however, can be made to retain the constant dividend risk property of the original. The latter's uniqueness depended on its single-valued predictions for investment and the Q-distribution in each period : to the extent that these aspects become subject to risk, the possibility of a unique growth outcome is destroyed. In the following section the consequences of moving away from the unique growth-path are derived in stages, working up to a position where a multiplicity of constant-risk growth outcomes can be envisaged. In one sense section 3, which follows this description, prepares the way for an attempt to restore the simplicity of Chapter 7's unique growth-path by considering whether the overall dividend probability distribution for each future period (obtained by combining the dividend probability distributions of all individual growth-paths for that period) could possess a degree of risk equivalent to that of each individual distribution. If such a property could be proved for the overall dividend probability distribution it might then follow that the divergence of actual dividend growth possibilities to be demonstrated in this chapter could be ignored or by-passed in interpreting equity valuation; that an overall average of dividend prospects for each future period would be interpreted by investors in the same way as the series of expected dividends for a unique growth-path. Discussion of this important issue is taken up in Chapter 10, but the mathematical conditions which could justify such a continued simplification of the valuation problem - in spite of the departure from the unique growth-path model - are set out in Section 3.

<u>Section 2 :</u>   <u>Multiple Growth-Paths of Constant-Risk Dividend.</u>

8 : 2 (i)   <u>Planning without an investment demand model.</u>

The first possibility considered corresponds closely to the
dividend planning model outlined in Chapter 7, Sections 2 (ii) and
2 (iii) : the firm's future investment programme is uniquely deter-
mined, as is its Q-distribution for each period. However, management
only becomes aware of these details in time to make the necessary
decision for the immediate future. Thus I_t and $\overline{Q}_{t+1}$ are known before
Q_t is finally determined. This corresponds to the model of Chapter 7
in that each period's investment demand is determined regardless of
the Q outcome for that period.

It follows that the planning of constant-risk expected dividend
for t+1 can only be carried out during t. In the expression

$$D_{t+1} = Q_{t+1} - iB_t - I_{t+1} + \Delta B_{t+1} \qquad (i),$$

I_{t+1} is fixed but unknown to management during t ; the probability
distribution for Q_{t+1} is known to management during t , given I_t ;
B_t is the amount of debt outstanding at the end of t , including intended
borrowing in t ; ΔB_{t+1} is unknown during t and will be determined in
t+1. Since every term except Q_{t+1} on the right-hand side of (i)
will have a unique value, the expected value of D_{t+1} may be written as

$$\overline{D}_{t+1} = \overline{Q}_{t+1} - iB_t - I_{t+1} + \Delta B_{t+1} \qquad (ii).$$

The variability of D_{t+1} about $\overline{D}_{t+1}$ is equal to that in Q_{t+1} about

$\overline{Q}_{t+1}$, and this latter variation can be summarised in terms of the standard deviation of the Q_{t+1} distribution, $\sigma(Q_{t+1})$. Then constant dividend risk requires

$$\frac{\sigma(Q_{t+1})}{\overline{D}_{t+1}} = R \text{ , constant for all } t \qquad \text{(iii),}$$

so that

$$\overline{D}_{t+1} = \frac{\sigma(Q_{t+1})}{R} = \overline{Q}_{t+1} - iB_t - I_{t+1} + \Delta B_{t+1} \text{ (iv).}^{(4)}$$

Given $\sigma(Q_{t+1})$, R, $\overline{Q}_{t+1}$ and iB_t , the remaining terms on the right-hand side of (iv) must have a sum which satisfies (iv). The firm must commit itself in t to a debt financing policy for t+1 such that $(-I_{t+1} + \Delta B_{t+1})$ has the required value. [5]

A second approach adds a further aspect of risk to the model. The outcome of each period's investment – in terms of its effect on the following period's Q-distribution – is not known with certainty when I_t becomes known: it is subject to the risk factors affecting investment profitability. This means that management in t is aware

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(4)   The concept R retains the meaning assigned in Chapter 7 : 2 (ii).

(5)   The feasibility of the solution value for ΔB_{t+1} was discussed in Chapter 7:2(iii). In most cases investment demand can be accommodated by the required mixture of retained earnings and new debt, with resort to external equity financing (with the criteria outlined in Chapter 6) dictated by occasions when the initial solution values for ΔB cannot be relied on. For a discussion of this issue within the wider context of an investment demand probability distribution, see Section 2 (ii) below.

of a number of possible Q_{t+1} distributions, given I_t, each with its own probability, corresponding to the various profitability outcomes for I_t. As before, the level of I_{t+1} is unknown during t, and an _ex ante_ target dividend risk for t+1 requires an appropriate commitment to be made in t. The variability of Q_{t+1} about its expected value must now be derived from the full range of Q_{t+1} distributions. Adapting (iv) to the new situation yields

$$\overline{D}_{t+1} = \overline{\overline{Q}}_{t+1} - iB_t - I_{t+1} + \Delta B_{t+1} \qquad \text{(v),}$$

where $\overline{\overline{Q}}_{t+1}$ is the expected value (during t) of the Q_{t+1} distribution obtained as a combination of all of the separate possible distributions. The mean of this combined distribution is given by

$$\overline{\overline{Q}}_{t+1} = \sum_{i=1}^{n} P_i \, \overline{Q}_{i,t+1} \qquad \text{(vi),}$$

where there are n possible Q-distributions for t+1, each with its own probability.

A commitment to constant-risk dividend requires that (iii) be satisfied for all periods, and here this implies

$$\overline{D}_{t+1} = \frac{\m6(Q_{t+1})}{R} = \overline{\overline{Q}}_{t+1} - iB_t - I_{t+1} + \Delta B_{t+1} \qquad \text{(vii)}$$

in each period. Given I_t, management can again calculate $6(Q_{t+1})$ and $\overline{\overline{Q}}_{t+1}$, as seen from t , and can then resolve that the value of

$(-I_{t+1} + \Delta B_{t+1})$ shall satisfy (vii).[6]

In the planning models considered so far the simulation of growth into the future is impossible, due to a complete lack of information about the level and profitability of investment in later periods. Unique values of investment and unique Q-distributions exist, and the model resembles that of Chapter 7 but for the absence of predictions. The task in the remainder of this section is to present a compromise between the extremes of certainty and ignorance, one which allows the future to be predicted in probabilistic terms.

8 : 2 (ii)   Planning with a model of investment demand.

The planning model is now to be modified to allow for the effects of risk upon the level of investment. It is unnecessary to examine the many possible investment demand models: the purpose here is limited to showing that a constant-risk dividend policy can be followed as long as the investment demand function is known to management. In the examples given the main relationship assumed is between investment demand and profit : this is not intended as a theory of investment behaviour, and it will be obvious that many factors can be allowed for within the

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(6)   The standard deviation of Q_{t+1} is given by

$$\sigma(Q_{t+1}) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} P_i P_j V_{ij}}$$

where n is the number of separate Q_{t+1} - distributions; P_i is the probability of the ith distribution; V_{ij} is the covariance between the ith and j^{th} distributions when $i \neq j$ and the variance of the ith distribution when $i = j$. This definition is borrowed from portfolio selection theory, where the standard deviation of a portfolio's rate of return is expressed in terms of the proportions of an investor's resources invested in various securities, the variances of rates of return on individual securities and covariances between rates of return of all pairs of securities. See W.F. Sharpe, Portfolio Theory and Capital Markets, 1970, p.42.

framework employed. As before, the burden of dividend risk policy falls on the planning of appropriate financing, but such planning must now be seen as conditional rather than unique for each period.

Once it is accepted that there is no unique _ex ante_ time-path of investment, and that investment profitability is subject to risk, it is clear that there is no longer a unique _ex ante_ time-path of growth, or of constant-risk dividend expectation. In period t the firm's investment decisions give rise to a combined Q_{t+1} distribution with known risk properties, and there is one such distribution corresponding to each level and composition of I_t.[7] To identify all of the possible time-paths of constant-risk dividend expectations an additional relationship is required : that existing between Q (and other factors) and I in each period, in probabilistic terms where necessary. If these relationships can be stated it is possible, in principle, to generate all possible Q, I time-paths; and for each one to calculate the required time-path of debt financing which ensures that expected dividend for each period on that time-path is subject to constant _ex ante_ risk.

To illustrate, assume a simple relationship between Q and I in all periods. Management assumes that I_{t+1} is a constant proportion of $\bar{Q}_{t+1}$, so that when the correct Q_{t+1} distribution is identified I_{t+1} is identified accordingly.

$$I_{t+1} = m \, \bar{Q}_{t+1} \qquad\qquad \text{(viii)},$$

where $\bar{Q}_{t+1}$ itself is subject, during t, to a probability distribution with parameters known to management. Given knowledge of the probability

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(7) In the remainder of the chapter it is assumed that the factors determining the level of investment also determine its composition – given the investment strategy adopted by the firm.

distribution of $\overline{Q}_{t+1}$, the probability distribution of I_{t+1} is also known; and in the expression for D_{t+1} both Q_{t+1} and I_{t+1} are now risky variables. It is appropriate to write $\overline{D}_{t+1}$ as

$$\overline{D}_{t+1} = (\overline{Q}_{t+1} - \overline{I}_{t+1}) - iB_t + \Delta B_{t+1} \qquad (ix),$$

and to interpret the variability of D_{t+1}, as seen from t, as due to the variability of $(Q_{t+1} - I_{t+1})$ about its mean value. As before, ΔB_{t+1} is to be fixed during t, given B_t, so that D_{t+1} is subject to the planned degree of _ex ante_ risk.

In these conditions the requirement for constant dividend risk is

$$\frac{\sigma(Q_{t+1} - I_{t+1})}{\overline{D}_{t+1}} = R \text{ for all } t \qquad (x),$$

or $\quad \dfrac{\sigma(Q_{t+1} - I_{t+1})}{R} = (\overline{Q}_{t+1} - \overline{I}_{t+1}) - iB_t + \Delta B_{t+1} \qquad (xi),$

in which ΔB_{t+1} must be given a unique planned value in t to balance equation (xi). This rule differs from those derived earlier in requiring ΔB_{t+1} to be fixed during t, rather than left over to be determined in $t+1$ once I_{t+1} is known. The required committment to a unique ΔB_{t+1} rather than to a provisional financing scheme seems a more inherently probable kind of planning for management to undertake.[8]

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(8) In this respect the planning model reverts to the pattern described
     in Chapter 7 : 2 (iii), where, in the very different conditions of
     a unique investment plan for each period, ΔB for that period
     was fixed in the preceding period.

The conditional dividend planning model takes as its background management's basic committment to obtaining its normal requirement for new equity capital by retaining earnings. Against this background the relationship assumed between dividend risk policy and investment strategy can be clarified. Management is assumed to begin with a target for dividend risk, R, in the expectation that the operation of its investment strategy will always produce a level of investment demand compatible with the debt financing committment for the coming period as determined in (xi). This means that the value of ΔB_{t+1} which balances left and right-hand sides of (xi) must be realistic in that whatever the actual $\bar{Q}$, Q and I outcome in $t + 1$ it will be possible to obtain the value of ΔB_{t+1} planned during t. Management probably anticipates that on most $(Q-I)$ outcomes in $t+1$ the pre-planned level of new borrowing will be possible, but the requirement of the model is that the expected value and target risk of dividend be planned for all occasions, and so the possible financing problem referred to must be allowed for. The problem arises, as stated, when (xi) cannot initially be balanced by planned ΔB_{t+1}, due to the fact that management cannot be sure that all Q, I outcomes in $t+1$ would allow the planned borrowing to be carried out. This possibility means that management must make provisional plans for such occasions, either for capital rationing or raising new equity externally, and so produce a revised probability distribution for I_{t+1} (to be financed by retained earnings and new debt) which will allow (xi) to be balanced by a value of ΔB_{t+1}

on which management can depend in all circumstances. [9] Evidently, management's investment strategy must be understood as incorporating procedures and rules which operate in the way described to ensure that dividend has a known expected value and is genuinely subject to the intended _ex ante_ risk. [10]

8 : 2 (iii)   A full simulation model.

A more complex illustration of the planning problem is given in this sub-section. Again, the investment demand model is expressed in terms of aspects of profit, but many other factors can be assumed to operate in determining its parameters. Let

$$I_{t+1} = m\bar{Q}_{t+1} + \lambda_1 (Q-\bar{Q})^+_{t+1} + \lambda_2 (Q-\bar{Q})^-_{t+1} \qquad (xii),$$

---

(9) This allows dividend risk for t+1 to be planned with complete assurance. Any investment undertaken in t+1 by external equity finance and new debt naturally modifies the picture for t+2, just as internally financed investment does, and the former can be made subject to risk factors in the same way as the latter. The identification of growth-paths of constant-risk dividend expectation is in no way prejudiced by this way of understanding conditional planning.

(10) Clearly, in the overall picture of management's financial decision making, the choice of dividend risk policy precedes that of investment strategy, with the latter assumed to be formulated so that it produces results within the limits set by the former. (The distinction between _subordinate_ and _top-level_ policies was described in Chapter 1 : 3 (ii) and will be developed in Chapters 11:1 and 12: 4   .) Admittedly this is not the only possible view of a relationship between dividend and investment policies, but it should not be taken for granted that the former dominates the latter in an unrealistic way. After all, the object of the plan is to give dividend a _residual_ role; and the scope for framing investment decision policy independently of dividend risk policy remains very great - as will be seen in Chapter 11: 3. The importance of constant dividend risk in the model arises from management's interpretation of equity market valuation and the assumed framework for strategic choice to be explained in Chapters 11 and 12.

where m, α_1 and α_2 are either positive constants or positive parameters subject to independent probability distributions; $(Q-\bar{Q})^+_{t+1}$ indicates and measures a Q_{t+1} outcome greater than $\bar{Q}_{t+1}$; likewise $(Q-\bar{Q})^-_{t+1}$ for outcomes below $\bar{Q}_{t+1}$. In this example the variability of investment and its independence of profit performance are indicated by making m, α_1 and α_2 subject to their own independent probability distributions, as follows:

$p(m)$	m	$p(\alpha_1)$	α_1	$p(\alpha_2)$	α_2
0.4	0.4	0.5	0.6	0.5	0.7
0.6	0.5	0.5	0.4	0.5	0.5

As a result of investments in t management envisages two possible Q-distributions for $t+1$, the details of which are as follows:

		Q_1 distribution (probability = 0.3)	Q_2 distribution (probability = 0.7)
probability of	0.2	6	7
Q, given the	0.6	8	9
Q-distribution	0.2	10	11

Values of Q for period t + 1. (11)

The parameters assumed mean that investment will be most strongly influenced by $\bar{Q}$, when it becomes known, and to a lesser extent (and not symmetrically) by the deviation of Q from its mean value. The fact that parameters are subject to risk factors means that management cannot be certain of the level of investment demand generated

(11) For arithmetical convenience these two Q-distributions differ slightly in their overall business risk: this in no way invalidates the procedures illustrated by the example. It is also convenient in the numerical example to omit the time subscript, t+1, from much of the notation.

by its overall investment strategy even when its profit performance is known: this is the important general characteristic that is assumed to apply to the great majority of planning situations.

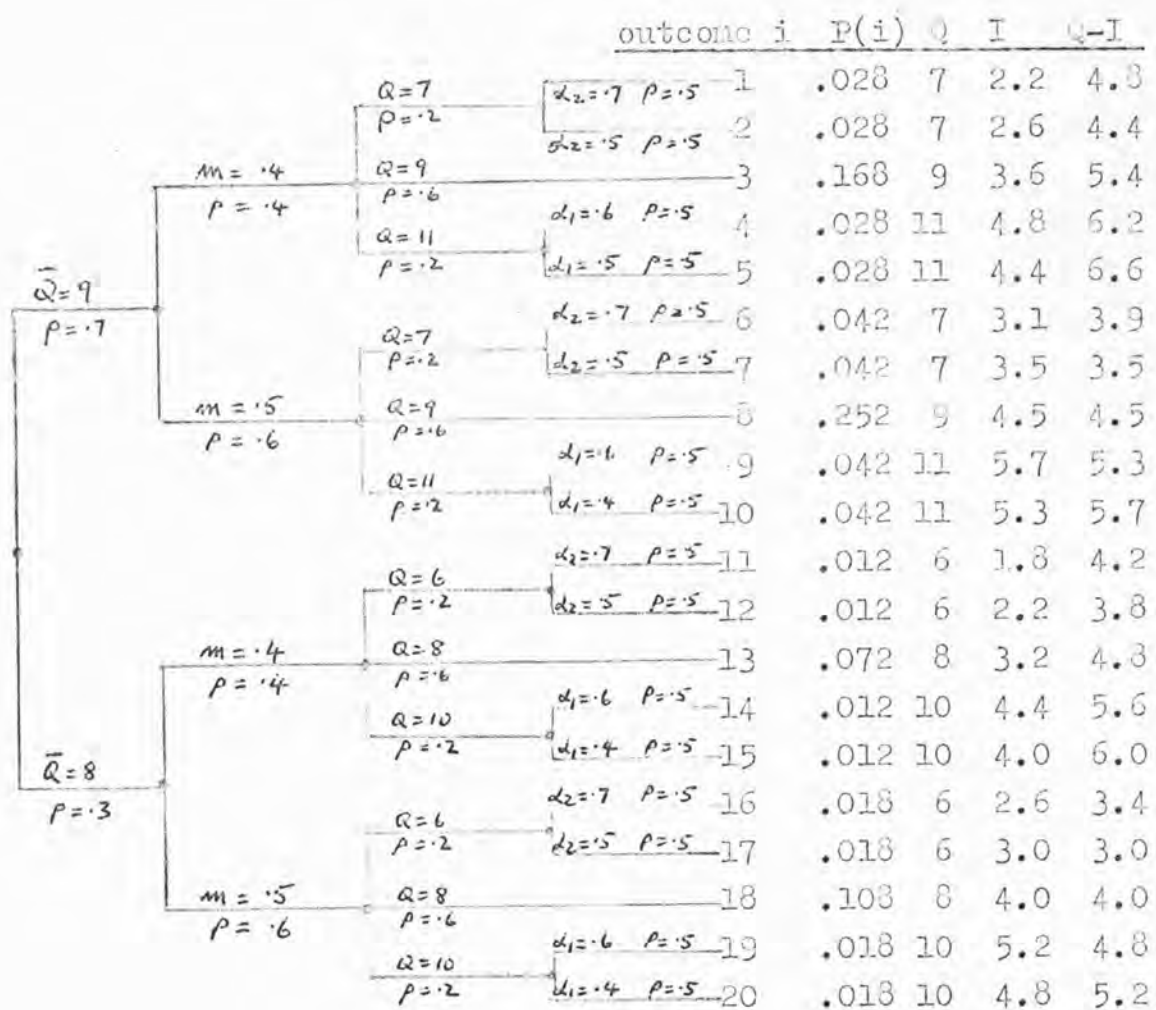The possible combinations of Q and I in t+1, and their respective probabilities, are derived in Diagram 1.

	outcome i	P(i)	Q	I	Q-I
$\alpha_2=\cdot7\ \ p=\cdot5$	1	.028	7	2.2	4.8
$\alpha_2=\cdot5\ \ p=\cdot5$	2	.028	7	2.6	4.4
Q=9, p=·6	3	.168	9	3.6	5.4
$\alpha_1=\cdot6\ \ p=\cdot5$	4	.028	11	4.8	6.2
$\alpha_1=\cdot5\ \ p=\cdot5$	5	.028	11	4.4	6.6
$\alpha_2=\cdot7\ \ p=\cdot5$	6	.042	7	3.1	3.9
$\alpha_2=\cdot5\ \ p=\cdot5$	7	.042	7	3.5	3.5
Q=9, p=·6	8	.252	9	4.5	4.5
$\alpha_1=\cdot6\ \ p=\cdot5$	9	.042	11	5.7	5.3
$\alpha_1=\cdot4\ \ p=\cdot5$	10	.042	11	5.3	5.7
$\alpha_2=\cdot7\ \ p=\cdot5$	11	.012	6	1.8	4.2
$\alpha_2=\cdot5\ \ p=\cdot5$	12	.012	6	2.2	3.8
Q=8, p=·6	13	.072	8	3.2	4.8
$\alpha_1=\cdot6\ \ p=\cdot5$	14	.012	10	4.4	5.6
$\alpha_1=\cdot4\ \ p=\cdot5$	15	.012	10	4.0	6.0
$\alpha_2=\cdot7\ \ p=\cdot5$	16	.018	6	2.6	3.4
$\alpha_2=\cdot5\ \ p=\cdot5$	17	.018	6	3.0	3.0
Q=8, p=·6	18	.108	8	4.0	4.0
$\alpha_1=\cdot6\ \ p=\cdot5$	19	.018	10	5.2	4.8
$\alpha_1=\cdot4\ \ p=\cdot5$	20	.018	10	4.8	5.2

Tree branches (left to right):
$\bar{Q}=9,\ p=\cdot7$ → [$m=\cdot4,\ p=\cdot4$: Q=7 (p=·2), Q=9 (p=·6), Q=11 (p=·2)] ; [$m=\cdot5,\ p=\cdot6$: Q=7 (p=·2), Q=9 (p=·6), Q=11 (p=·2)]

$\bar{Q}=8,\ p=\cdot3$ → [$m=\cdot4,\ p=\cdot4$: Q=6 (p=·2), Q=8 (p=·6), Q=10 (p=·2)] ; [$m=\cdot5,\ p=\cdot6$: Q=6 (p=·2), Q=8 (p=·6), Q=10 (p=·2)]

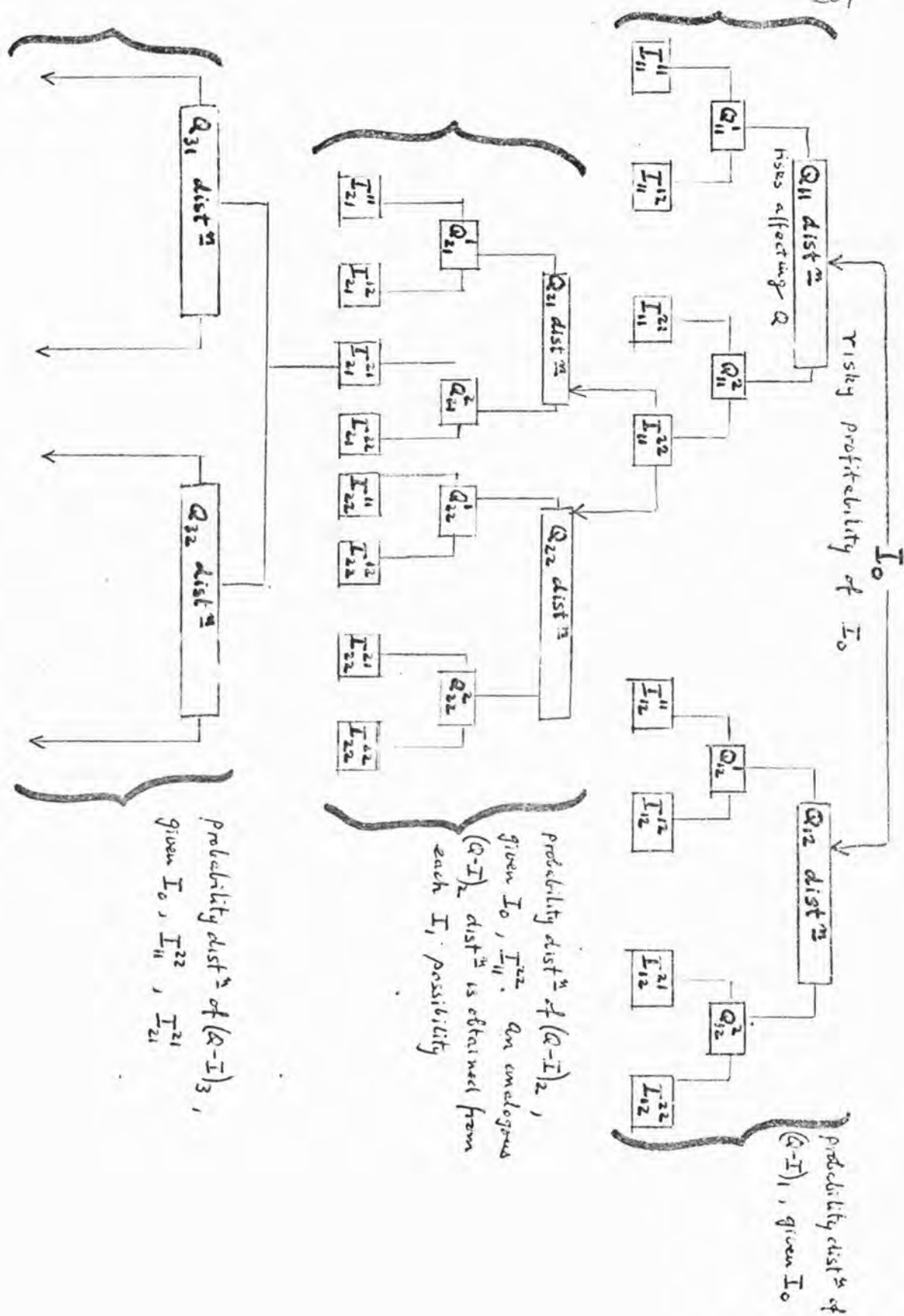Diagram 1 : Probability distributions of Q and I in t + 1.

Diagram 2 : An illustration of multiple time-paths of Q,I.

Beginning with a given level of investment, I_o, in the current period, $t = o$, there are just two Q-distributions for $t = 1$. These are of the simplest form, with only two outcomes on each distribution. Investment demand too is kept to the simplest version of (xii), with only two possible outcomes for each $\overline{Q}$, Q result in $t = 1$. The mechanisms which generate Q and I yield a probability distribution for (Q-I) for each Q-distribution that can arise from I_o. In effect there are two time-paths originating in $t = o$, corresponding to the two levels of investment profitability, and for each of these a (Q-I), distribution can be identified in $t = o$ along with the debt financing that will have to be undertaken in $t = 1$ to create the desired ex ante dividend risk for that period. The diagram traces one time-path as far as $t = 3$ and shows clearly the way in which the number of different time-paths increases as developments proceed farther from the present.[13]

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(13) Although the model developed in this section is primarily intended to show the possibilities for implementing a management policy on dividend risk, it does perhaps invite comparison with the concept of "higgledy piggledy growth" (see I.M.D. Little, "Higgledy Piggledy Growth," Bulletin of the Oxford University Institute of Economics and Statistics (November 1962); I.M.D. Little and A.C. Rayner, Higgledy Piggledy Growth Again, 1966.) First, Little and Rayner measured growth in terms of earnings per share – a measure much closer to expected dividend than, for example, the balance sheet asset values employed in a major study of companies' growth: A. Singh and G. Whittington, Growth, Profitability and Valuation, 1968. Second, the recorded performance summarised by Little and Rayner's title seems fully consistent (a) with the dispersion of the prospects for the firm in period t+1 suggested in the present model by the risks affecting ΔQ, Q and I for t+1, and (b) with changing fortunes from one period to another. (The length of time period assumed in the model is discussed in Section 4 (ii) below.) Singh and Whittington agreed (op.cit., p.108.) that quite different views on the persistence of growth may emerge, depending on the growth measure taken: nevertheless, by their own measures they found that where it was significant at all the actual degree of growth persistence was weak (op.cit., pp 111, 113).

Section 3 : Combining Multiple Growth-Paths of Constant-Risk Dividend.

8 : 3 (i)   The mathematical conditions for constant risk of the overall dividend distribution.

In this section neither the firm's investment strategy nor the form of investment demand function arising from it is in question. It is simply assumed that the latter relationship is such that a debt financing policy to ensure constant dividend risk can be continuously carried out for all possible growth-paths. The main issue in the remainder of this chapter is the interpretation by management of the multiplicity of possible growth-paths of constant-risk expected dividend arising from the operation of its investment strategy. Chapter 10 discusses possible interpretations of the market valuation of equity under these conditions in the light of the alternatives to be identified here. The main question is whether anything more definite can be said on the subject of the risk associated with dividend expectations than has been stated in the preceding section, where the derivation of a multiplicity of constant-risk growth-paths was demonstrated. The choice now is either to leave the matter in this form, with the details of each expected dividend growth-path and its probability known to management; or to attempt to establish conditions under which the separate growth-paths may somehow be unified for conceptual purposes, producing a combined expected value and risk for each period. This more ambitious alternative is considered here.

For each period (as seen from the present) let σ_i be the standard deviation of the objective probability distribution of D_i, the dividend on the ith identified growth-path. The expected value of D_i is given by.

$$\overline{D}_i = \sum_{j=1}^{n} p_j D_{ij} \qquad \text{(xiii)};$$

where D_i may take values $D_{i1} \ldots D_{in}$, with probabilities $p_1 \ldots p_n$ respectively. The separate probability distributions for dividend in t — one for each growth-path — may be combined to give an overall expected value of dividend in t:

$$\overline{\overline{D}} = \sum_{j=1}^{x} p_i \overline{D}_i \qquad \text{(xiv)},$$

where $\overline{D}_i$ is given by (xiii) and p_i is the probability attached to the ith growth path. [14] (It is convenient not to use an additional subscript for t : the argument is assumed to apply separately to each period). The standard deviation of the overall or combined D-distribution is given by

$$\sigma_{DC} = \sqrt{\sum_{k=1}^{x} \sum_{m=1}^{x} p_k p_m V_{km}} \qquad \text{(xv)},$$

where p_k, p_m are the probabilities of the kth and m^{th} growth-paths, respectively; V_{km} equals the covariance between the kth and m^{th} D-distributions when $k \neq m$, and the variance of the kth D-distribution when $k = m$. [15] The general expression (xv) for σ_{DC} may be expanded for the present purpose to

$$\sigma_{DC} = \sqrt{\sum_{k=1}^{x} p_k^2 \sigma_k^2 + \sum_{k=1}^{x} \sum_{\substack{m(\neq k) \\ =1}}^{x} p_k p_m r_{km} \sigma_k \sigma_m} \qquad \text{(xvi)},$$

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(14) The number of possible growth-paths increases as simulation moves farther into the future and divergence occurs: as this occurs the value of x increases in each period.

(15) See footnote (6) above.

in which the following definitions are used:

$$\sigma_k^2 = V_{kk} \quad \text{(variance of the } k^{th} \text{ D-distribution)}$$

$$V_{km} \text{ (Covariance between } k^{th} \text{ and } m^{th} \text{ D-distribution)} = r_{km} \sigma_k \sigma_m$$

The risk embodied in each D-distribution is equal, regardless of the growth-path, due to the basic policy decision of management. Thus

$$\sigma_i = R \, \overline{\overline{D}}_i \quad (i = 1 \ldots x) \qquad \text{(xvii),}$$

(16)

cf $p_{,98}$

and a refinement to (xvi) is possible :

$$\sigma_{DC} = R \sqrt{\sum_{k=1}^{x} p_k^2 \overline{\overline{D}}_k^2 + \sum_{k=1}^{x} \sum_{\substack{m(\neq k) \\ =1}}^{x} p_k p_m r_{km} \overline{\overline{D}}_k \overline{\overline{D}}_m} \qquad \text{(xviii).}$$

Finally, if $r_{km} = 1.00$ for all k, m combinations, expression (xviii) can be progressively simplified as follows:

$$\sigma_{DC} = R \sqrt{\left(p_1 \overline{\overline{D}}_1 + p_2 \overline{\overline{D}}_2 \cdots p_x \overline{\overline{D}}_x \right)^2} \qquad \text{(xix);}$$

$$\sigma_{DC} = R \sqrt{\left[\sum_{i=1}^{x} p_i \overline{\overline{D}}_i \right]^2} \qquad \text{(xx);}$$

$$\sigma_{DC} = R \sum_{i=1}^{x} p_i \overline{\overline{D}}_i \qquad \text{(xxi);}$$

$$\sigma_{DC} = R \, \overline{\overline{D}} \qquad \text{(xxii).}$$

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(16) See Section 2 : (ii) above for the detailed account of the
       derivation of constant-risk growth-paths of expected dividend.

The risk of the overall or combined D-distribution is equal to the policy-determined risk level for each individual D-distribution on condition that $r_{km} = 1.00$ for all k,m combinations. Furthermore, if this situation is representative of all time periods, the growth path of the overall expected dividend, $\overline{\overline{D}}$, is associated with the same constant risk level as each of its constituent possibilities. Obviously the assumption that r_{km} is always equal to unity is crucial in arriving at these conclusions, and it is necessary to examine its implications carefully. This may conveniently be done by way of a numerical example.[17]

8 : 3 (ii)   The meaning and implications of perfect correlation between dividend distributions

Two distributions are said to be perfectly correlated when the pair of observations generated by any probabilistic event, or set of events, lies on a straight line drawn in a two-dimensional diagram in which all such joint outcomes are mapped. Positive perfect correlation requires the straight line to have an upward slope, not necessarily from the origin of the diagram. Let the possible events be denoted by i, where i = 1 .... n; the probability of i be p_i; and the simultaneous outcomes of event i in terms of the variables X and Y be X_i and Y_i respectively. When all such pairs of observations conform to the relationship

(17) One important mathematical consideration can be shown here. The result obtained in (xxii) requires that r_{km} equals unity for all k,m. It is impossible for a shortfall in even one r-value to be made good by an appropriate excess in other cases. Furthermore, if the correlation between the ith (Q-I) distribution and any one of the remainder falls below unity, then even if all pairs of distributions not including the ith are perfectly correlated, r_{ij} must lie below unity for all j.

$$Y_i = c + bX_i \qquad\qquad\qquad\qquad \text{(xxiii)},$$

where c is a constant (possibly zero) and b is a positive constant,
the X and Y distributions are positively and perfectly correlated.
In the present case X and Y correspond to two (any two) D-
distributions for period t, where each D-distribution belongs to one
of the firm's growth-paths of constant-risk expected dividend. The
assumption that r_{km} equals unity for all k,m combinations requires
that a relationship such as (xxiii) apply to any pair of D-distributions
at t.

For any single D-distribution for period t the expected value
in t-1 of dividend, $\bar{D}_t$, is given by

$$\bar{D}_t = \bar{\bar{Q}}_t - \bar{I}_t - iB_{t-1} + \Delta B_t \qquad\qquad \text{(xxiv)},$$

where both the profitability of I_{t-1} and the level of I_t are subject
to probability distributions. (This, the most general form for $\bar{D}_t$
is the one which will be employed throughout this section.) It was
shown in the preceding section that given iB_{t-1}, the level of ΔB_t
would be determined in t-1 in accordance with the requirement of
constant-risk expected dividend in t, so that D_t may be thought of
in period t itself as made up of variable terms, Q_t and I_t, together
with a fixed component equal to $(-iB_{t-1} + \Delta B_t)$. The value of this
fixed component will vary between D-distributions, but within each
distribution it is a constant term. Accordingly, the correlation
between pairs of D-distributions for any period depends exclusively
on the correlation between their respective (Q-I) distributions.
Perfect positive correlation between the D_k and D_m-distributions
for period t requires that the $(Q-I)_{kt}$ and $(Q-I)_{mt}$ distributions
be perfectly and positively correlated - where k and m refer to

any two distributions. It is on the practical and conceptual problems
raised by this requirement that attention must be focussed.

8 : 3 (iii)   An example of perfect positive correlation between
               dividend distributions

The derivation of a (Q-I) distribution has been illustrated by
the example in the preceding section, and it is obvious that an analogous
exercise may be carried out for all D-distributions in any period .
What is not obvious is that the degree of correlation between any two
distributions constructed in this way must be unity as required in the
proposition under consideration : indeed, unless additional assumptions
are made this principal assumption cannot be retained. Additional
assumptions are necessary to make the various $(Q-I)_t$ distributions
identical in shape (but not necessarily in scale), and to ensure that
"equivalent" observations from all distributions are generated by any
given set of events. (Points on different distributions are equivalent
to one another when they are separated from their respective distribution
mean values by an equal number of standard deviations, and result from
the same combination of probabilities). Thus, the additional assumptions
impose restrictions on the structure of the overall Q_t-distribution
for each growth-path, the determination of I_t in relation to Q_t in
each growth-path, and on the probabilistic independence between $(Q-I)_t$
distributions. The whole range of issues raised by these additional
assumptions, together with their precise specifications, may be
conveniently considered in the context of a numerical example.

As in earlier chapters, the level of business risk is assumed
to be constant in the firm's Q-distribution for any period, and for

any growth path. To avoid misunderstanding, the constancy of business
risk applies to the individual Q-distribution possibilities identified
for period t once the probability distribution of the profitability
of I_{t-1} is established. The following numerical example relates to
two growth-paths, A and B, in period t : two possible Q-distributions
for each growth-path are assumed, each corresponding to a particular
profitability outcome for investment in t-1.

growth-path		A		B	
Q-distribution		A1	A2	B1	B2
probability of Q-distribution		0.7	0.3	0.7	0.3
probability of Q, given Q-distribution.	0.6	8	10	0.6	10	12.5
	0.4	12	15	0.4	15	18.75

The constancy of business risk is embodied in each possible individual
Q-distribution: A1, A2, B1, B2, as the ratio of the standard deviation
of each distribution to its mean value is constant. This much is in
accordance with previous assumptions, but the additional element of
uniformity is that between A and B the relationship between possible
Q-distributions is standardised, both as to Q-values and as to the
probabilities assigned.[18] Thus distribution B1 takes values 1.25
times as great as those having an equal probability in A1; and the same
applies to B2 compared with A2. Further, the probabilities of A1
and A2 are the same as those of B1 and B2 respectively. These
specifications mean that the overall Q_A and Q_B -distributions, obtained
in each case by combining the two separate distributions are identical

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(18) The time subscript, t, is omitted in much of the discussion
      relating to this example.

in shape but have differing values: each Q_B value is 1.25 times as great as a Q_A value having an equal probability. The combined distributions are as follows.

$P(Q_A)$	Q_A	$P(Q_B)$	Q_B
0.42	8.00	0.42	10.00
0.18	10.00	0.18	12.50
0.28	12.00	0.28	15.00
0.12	15.00	0.12	18.75

The next stage is to obtain for each growth-path a probability distribution of I , based upon a stated relationship between investment demand and Q . Since a fairly complicated relationship has already been illustrated in the previous section, it is sufficient here to employ the simpler version

$$I_t = m \bar{Q}_t \tag{xxv},$$

with m governed by the following probability distribution:

p(m)	m
0.40	0.40
0.60	0.60

This relationship is assumed to apply to investment demand in each growth-path, so that I and Q probability distributions can be calculated as shown in Diagram 3 (for the case of A).
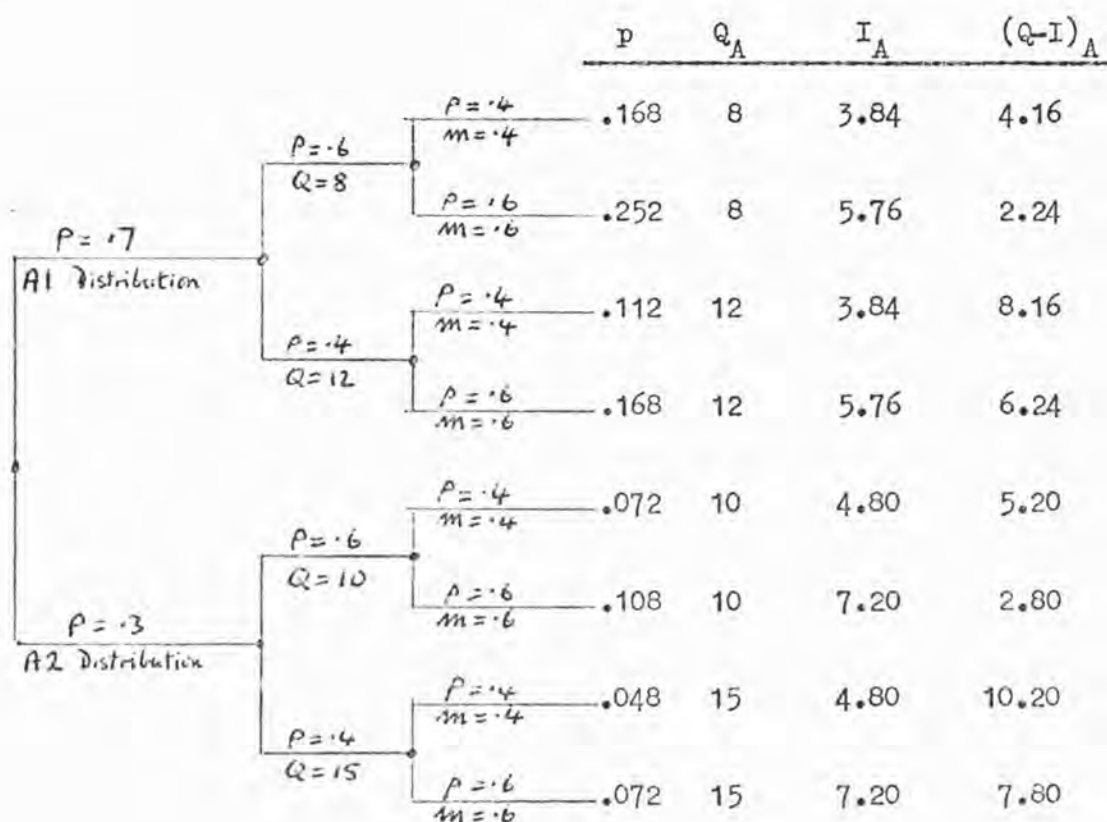
	p	Q_A	I_A	$(Q-I)_A$
$P=\cdot4$ / $m=\cdot4$	.168	8	3.84	4.16
$P=\cdot6$ / $m=\cdot6$	.252	8	5.76	2.24
$P=\cdot4$ / $m=\cdot4$	.112	12	3.84	8.16
$P=\cdot6$ / $m=\cdot6$	.168	12	5.76	6.24
$P=\cdot4$ / $m=\cdot4$	.072	10	4.80	5.20
$P=\cdot6$ / $m=\cdot6$	.108	10	7.20	2.80
$P=\cdot4$ / $m=\cdot4$	.048	15	4.80	10.20
$P=\cdot6$ / $m=\cdot6$	.072	15	7.20	7.80

Tree (left side):

- $P=\cdot7$ A1 Distribution
  - $P=\cdot6$ $Q=8$
  - $P=\cdot4$ $Q=12$
- $P=\cdot3$ A2 Distribution
  - $P=\cdot6$ $Q=10$
  - $P=\cdot4$ $Q=15$

Diagram 3 : Possible outcomes in growth-path A, period t.

An exactly comparable diagram for growth-path B, with distributions B1 and B2 replacing A1 and A2 respectively, and with the same investment relationship as that shown above, generates the following $(Q-I)_B$ distribution:

$P(Q-I)_B$	$(Q-I)_B$
0.168	5.20
0.252	2.80
0.112	10.20
0.168	7.80
0.072	6.50
0.108	3.50
0.048	12.75
0.072	9.75

As expected, each $(Q-I)_B$ value is 1.25 times the value in the $(Q-I)_A$ distribution arising from the same combination of probabilities.

The final stage in establishing a perfect positive correlation between the $(Q-I)_A$ and $(Q-I)_B$ distributions is to assume that the same combination of events, represented by a combination of probabilities, applies to both the A and B situations : in terms of Diagram 2, the probability values of which apply to both A and B, the fact that on one occasion a particular path is actually followed in the A-case means that on the same occasion the same path would be followed in the case of B, leading to the equivalent $(Q-I)_B$ outcome with an equal probability to that in $(Q-I)_A$. [19] Thus, for example, the path A1 (p = 0.70); Q = 12 (p = 0.40); m = 0.40) could be matched only with the path B1 (p = 0.70); Q = 15 (p = 0.40); m = 0.40 (p = 0.40). The probabilities for each outcome are equal because the set of events occuring on this occasion is the same regardless of which growth-path is actually being followed. In other words, it is assumed that whatever the actual $(Q-I)$ outcome on the growth-path followed by the firm, the outcome on any other growth-path would be exactly equivalent and would result from an identical set of circumstances. This very important restriction on the probabilistic independence between the $(Q-I)$ distributions of different growth-paths, together with the assumptions which ensure equivalently structured overall Q distributions in all growth-paths, ensures that all $(Q-I)$ distributions are positively and perfectly correlated. The conceptual problem raised by these restrictive assumptions must now be considered.

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(19) The term "equivalent" continues to have the precise meaning assigned to it at the start of this sub-section.

8 : 3 (iv)   The logical impossibility of determining the correlation
              between future dividend distributions.

It has so far been assumed that the question of correlation

between (Q-I) distributions on different growth-paths for any period

is, in principle, amenable to empirical testing, that information

can be obtained to decide the questions of fact one way or another.

However, as should be obvious, the firm proceeds along only one of the

growth-paths open to it, although paths overlap one another to a greater

extent the closer to the present is the period considered. In other

words, there is no way of knowing whether a given set of events simultaneously

generates an equivalent (Q-I) outcome for each growth-path in a future

period : all that can be known is that for those paths which overlap in

t (not having yet reached the point of divergence) the outcome obviously

satisfies all of the strict conditions because it is common to all.

Looking beyond the immediate future period (in which all growth-paths

must overlap) there is simply no way of confirming at the present time,

or at any later date, whether the crucial conditions for constant risk

of overall expected dividend, $\overline{\overline{D}}$, are or have been satisfied.

The suggestion that a single set of events in each period would

lead to the selection of an equivalent Q-distribution from each growth-

path, an equivalent Q-value from each Q-distribution, and common

parameters for determining investment demand in terms of Q and $\overline{Q}$,

has an obvious attraction and reasonableness. To argue in favour of

any alternative version would be at least as difficult, except possibly

for that version which stands at the opposite end of the spectrum of

possibilities: the complete independence of the events determining

$(Q-I)_t$ in each growth-path from those determining its value in every

other path - in spite of the fact that the $(Q-I)_t$ probability distribution is assumed (in the example) to have the same shape on each growth-path. Expressed in this way there seems to be a marginal logical superiority in the hypothesis that a single set of probabilistically determined events determines the value of $(Q-I)_t$ in each growth-path and that in consequence the $(Q-I)_t$ distributions are perfectly correlated; but there can be no empirical confirmation one way or another. Events in growth-paths other than the actual one(s) (allowing for overlapping) along which the firm travels must remain hypothetical.

In spite of the impossibility of definite proof on the crucial question of $r_{km} = 1.00$, it has been necessary to specify the assumptions upon which it rests because the conclusion resting upon it ($\delta DC = RD$) may prove to be important in understanding management's interpretation of valuation in the situation created by the multiplicity of possible growth-paths. Discussion of this problem is taken up in the Chapter 10.

Section 4 :   Conclusions

S : 4 (i)   The importance of objectivity in probability distributions

The principal concern of this chapter has been to define and illustrate models of multiple growth-paths of constant-risk dividend, and to set out the mathematical conditions under which the overall dividend distribution for each period may be regarded as having an ex ante risk equal to that applying to each individual distribution. The latter discussion is a necessary preliminary to a consideration of investors' valuation of equity in the conditions defined: it would simplify management's task enormously if investors could be assumed

to value the overall expected dividend series $D_1, D_2, \ldots D_n, \ldots$ on the
same basis as has been assumed to apply to the valuation of a unique
growth-path of expected dividend. The following two chapters are
concerned with investors' valuations of the various growth possibilities
(unique and multiple growth-paths), but before this subject is taken
up several matters require brief comment and clarification.

In common with the case of a unique growth-path considered in
Chapter 7, a striking feature of all models developed in this chapter
is the precision they seem to imply in the foresight, planning and
behaviour of management : indeed it may be argued that such implications
invalidate the constructions and/or their relevance in understanding
the valuation process. Specific criticisms are considered shortly,
but a general reaction against implied detailed foresight and planning
is misplaced. The actual situations described in the models are such
that management is aware of a large number of growth possibilities
rather than a single growth-path as defined in Chapter 7. Much emphasis
is placed on the role of partially understood factors and events working
through the random values of the various parameters to determine invest-
ment demand and profitability. Indeed it is precisely this view of the
situation which generates the multiplicity of growth possibilities. The
foresight complained of in such criticism refers to implied knowledge
of probability distributions; and the detailed planning is, after all,
conditional upon the outcome of risky situations. The real difficulty
at this general level involves the methodological question of the use
of probability estimates: the issue is whether an admittedly imperfect
knowledge of probabilities and underlying causes can be assumed sufficient
for the user's purposes. Given the approval in decision theory literature
for a general line of approach in which the decision maker employs the

best probability estimates available (or those estimates which it
seems unprofitable to try to improve upon), there can be little to
quarrel with in the present approach, which can be described as one
by which management clarifies its own views on the question of growth
possibilities. No decisions are yet assumed to depend on the out-
come of this process: indeed, the overall view obtained is the result
of - not the originator of - the firm's previously determined investment
strategy and dividend risk policy. It is not suggested at this stage
that the firm is choosing between different probability distributions
of growth possibilities, each of which is derived from a particular
combination of investment strategy and dividend policy. If that were
the case, then clearly the reliability of each set of probabilities
would be of major importance in any decision on an optimum combination :
but since the picture generated can be described as "for information
only", no particular difficulty arises from the admittedly subjective
nature of some of the probabilities used. [20]

Inaccuracies in identifying the probability distribution of growth
outcomes arise from incomplete knowledge of the factors influencing
the levels of investment demand and profitability and of the ways in
which these factors interact. Management will attempt to formulate
objective probabilistic structures for these variables, but ultimately

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(20) A clear summary of objectivist and subjectivist positions on
probability estimates is given in T.H. Naylor and J.M. Vernon,
Micro-economics and Decision Models of the Firm, 1969, pp.297-303
(part of a chapter contributed by E.T. Byrne). The model of
multiple growth-paths is developed into a strategic decision
framework in Chapters 11 and 12, on the basis of an appropriate
valuation model to be worked out in Chapter 10. Chapter 13
attempts to deal with the obvious line of criticism based on
management's necessarily imperfect understanding of growth
prospects and valuation.

there must remain areas within which neither the forces operating nor their probabilities are understood. Before admitting complete ignorance however, it is possible that while the natures and inter- actions of certain forces are unidentified the probability distribution of their joint effects is known with some confidence. Thus it is not strictly necessary in setting out a probabilistic model for generating growth outcomes that the underlying forces be fully understood: it is sufficient if the probabilities of outcomes generated by unidentified forces are known. As an extreme example, suppose management finds that the best "model" of investment demand is given by $I_t = R_t$; where R_t is a positive random number subject to a specified probability distribution derived from past experience. The attempt to link future investment behaviour - even probabilistically - with other aspects of the firm's performance has failed, and the firm has no good explanation of why the random variable approach works. Understanding is absent, but probabilistic prediction is quite satisfactory for a simulation exercise.

Discussion on the validity of the model of multiple growth-paths may now be summarised. Such a model should not be criticised when its intended use is merely the clarification of managerial thinking on the range of growth outcomes and their probabilities resulting from a given overall investment and dividend strategy. The important questions are whether the same techniques are applicable to the choice between alternative strategies; and whether they offer a means of communication between management and the capital market - in the sense that the former may reasonably assume that investors are primarily concerned with the probability distribution of the firm's dividend growth possibilities in valuing its equity. These matters occupy several of the remaining chapters, but certain specific issues arising from the conditional

planning models require consideration.

8 : 4 (ii)   Reconciling "residual" and "discretionary" dividend models;
              definition of time period.

The outstanding questions relate to the assumption of a managerial

committment to a constant level of dividend risk, the related problem

of defining risk, and the duration of a time period in the simulation

exercise. Subject to the possible objection that the coefficient of

variation alone is an inadequate measure of risk, the chosen definition

of risk is applicable to any situation in which dividend is actually

a residual as described in the models of this and the preceding chapters.

However, there does appear to be a conflict between this model of the

determination of the periodic dividend and the apparently discretionary

or non-residual policies operated by firms in the real world. In the

latter, since dividend is apparently not determined residually, the

whole concept of a dividend probability distribution seems inappropriate

and a different risk concept seems to be required.

Much empirical work on dividend policy is available, and it is

unnecessary to review the findings in detail.[21] The intention is

(21) A contribution of major importance on which much subsequent work
      has been based is J.Lintner, "Distribution of Incomes of Corporations
      among Dividends, Retained Earnings and Taxes," American Economic
      Review (Papers and Proceedings), (1956). Lintner's observations that
      companies make a lagged adjustment of dividend towards a target
      payout ratio and are reluctant to reduce the dividend level are now
      widely accepted (see J.C. Van Horne, Financial Management and Policy,
      2nd edition 1971, pp.268-71; A.J. Merret and A. Sykes, The Finance
      and Analysis of Capital Projects, 1963, pp.80-1) as is a recognition
      that management's discretion on the target payout ratio allows
      investment and financing considerations to influence dividend
      policy (see E. Solomon, The Theory of Financial Management, 1963,
      pp.143-4). On this latter point the target payout ratio concept
      agrees with the dividend model developed in this and the preceding
      chapter: the target level of dividend risk, r, must be compatible
      with investment plans and with financing possibilities, though
      because the dividend risk planning model allows for possible external
      equity finance where necessary (see Section 2 (ii) above) it is
      assumed that r can actually be predetermined by management.

simply to show that the residual dividend model is not necessarily at odds with the real-world behaviour of firms if a correct interpretation of the implied duration of its time period is made. A convenient statement of the dividend policy model implied by empirical work is that dividend will only be increased when management has sufficient confidence in its ability to sustain the new level of payout for a specified minimum number of years. At first glance such an approach appears to contradict the view of dividend's role and behaviour suggested in this study; that is, the principle of allowing actual dividend to be a residual after correctly planning its ex ante level of risk. The apparently realistic interpretation derived from empirical research suggests both a different role for dividend in managerial thinking and a different technical definition of risk. [22] Reconciling the model developed here, with its stress on the inherently residual nature of dividend, and those models which stress managerial discretion and the minimising of risks, requires greater care in the definition of a time period than has so far been necessary. It is obvious that within the space of a company's accounting year management does not leave the determination of dividend to the random forces determining the value of $(Q-I)$ for the period – as suggested by equation (i). The dividend planning model developed in Chapter 7, and assumed to be in

(22) Rather than interpreting risk in terms of the dispersion of a probability distribution, it would be more appropriate to define it in terms of the probability of having to reduce dividend below its established level. An incidental difficulty then arising is that of interpreting investors' valuation of equity; and management's own forecasting of dividend becomes highly problematic, linked as it must be to increases in profit, investment requirements, etc. Management's interpretation of valuation is made no easier under the discretionary theory than in the version to be described in Chapter 10 : 3 which corresponds to multiple growth-paths of constant-risk dividend.

operation in this chapter, can only be made to fit the known facts
and reconciled with widely accepted interpretations of those facts,
if its time period differs in length from the standard accounting year.

The "discretionary" and "residual" interpretations of dividend
policy can be reconciled by recognising that, while management does
make short-run dividend decisions, the level of payment over a "period"
as a whole conforms to the residual model developed in this study.
Thus, for illustration, let a "period" be defined as of three years'
duration: actual payments over the period are then normally divided
into six half-yearly amounts; and it is also possible for the firm to
make occasional extra-ordinary payments which are clearly differentiated
from the regular level of payment.[23]   With such freedom of manoeuvre
in the timing and in the apparent nature of payments it is quite
possible for the firm to be in effect operating the policy defined in
Chapter 7 and assumed throughout this chapter, while giving every
appearance of operating a discretionary or non-residual policy. At
each period's outset management is aware of the dividend probability
distribution for the period as a whole, and half-yearly payments can
begin at a level rather lower than one-sixth of the distribution's
mean value. As the period progresses with payments at this level the
outcome for the whole period becomes increasingly certain, and an
adjustment - admittedly discretionary - may be made in the level of
the remaining regular payments. By setting the initial level of
payment sufficiently low, the discretionary adjustment towards the
end of a period is likely to be in an upward direction; and the final

(23) See J.C. Van Horne, op.cit., pp.271, 279-82

balancing necessary to make the total equal its correct residual value will take the form of a payment which does not have the effect of raising expectations for the future. Such a procedure cannot rule out all possibility of an undesired reduction in the level of the regular payment : the probability of having to reduce the payment can be given as a matter of policy, and the regular payment fixed at the appropriate level, without altering the inherently residual nature of the total dividend for the period. The firm's behaviour as described here for a single period is scarcely modified when a succession of periods is considered : the level of regular payment inherited from the previous period is likely to provide the initial level for the new period : because of expected growth in the new period there is unlikely to be a conflict between this level and one dictated by the kind of prudent planning described earlier. [24]

(24) There are other reasons why management may be expected to simulate developments using a basic "period" of longer than one year. Long-term debt financing is unlikely to be a smooth, unbroken process of adjustment : management probably prefers to complete required adjustments in debt over a period long enough to allow maximum opportunity to find the right combination of internal and external conditions for financing on the terms assumed in the planning model. An investment model of sufficient accuracy can probably be stated satisfactorily in terms of a period of (say) two to three years, with most lags and gestation periods accommodated within the defined period. Bearing in mind the aggregative nature and purpose of the exercise, this is probably the best approach to an investment demand model. Finally, the great reduction in the actual number of events simulated that can be achieved by working with an extended unit of time must be taken into account.

Chapter 9

Management's Unique Growth-Path Valuation Model.

Plan of Chapter

Section 1 : Introduction

Section 2 : Estimating k from Expectations

    (i)   The use of a steady-state growth valuation model.

    (ii)   Improving the model of growth expectations.

Section 3 : Estimating k from Historical Performance.

Section 4 : Market Value and Correct Value.

Management's Unique Growth-Path Valuation Model.

Section 1 : Introduction.

This chapter takes a backward step, to the conditions
in which the capital market is assumed to value equity on the
basis of a unique ex ante expectation about the growth-path
of constant-risk dividend. However, no waste of effort is
involved in briefly considering procedures by which an
observer might try in actual conditions to estimate the market
rate of discount ideally applying to dividend expectations :
the multiple growth-path valuation model to be described in
Chapter 10 requires, for the identification of its parameters
in actual conditions, that management be prepared to make the
kind of assumptions and inferences discussed here, and the
identification of k is no less important than before.

Two distinct approaches are considered. In Section 2
the observer is assumed to infer the market rate of discount
from investors' expectations of dividend growth, while Section
3 discusses the suggestion that historical rates of return on
equity investment can be used to infer the discount rate
applied by investors to their expectations for the future.

Either approach to estimating k inevitably finds itself
at the mercy of observed market values of equity, and Section 4
discusses possible procedures for working from market value to
more satisfactory measures of investors' estimates of ideal or
correct value.

Section 2 : Estimating k from Expectations.

9 : 2 (i)   The use of a steady-state growth valuation model.

A standard approach to the identification of k, the
market rate of discount on expected dividends, is through the
expression for the present value of a stream of dividend
expectations:

$$V_{eo} = \sum_{t=1}^{\infty} \frac{\bar{D}_t}{(1+k)^t} \qquad (i),$$

where V_{eo} is the equilibrium ex-dividend market value of the
stream of dividend expectations $\bar{D}_1$, $\bar{D}_2$ ..... $\bar{D}_n$ ..... ;
and k is the discount rate. Evidently if V_{eo} and the $\bar{D}_t$
series can be identified, k is obtained as the unique value
equating the two sides of (i). Disregarding the enormous
variety of possible time-paths of $\bar{D}_t$, a standard simplification
of (i) can usefully be introduced: this is based on the
assumption that expected dividend is expected to grow at a
constant periodic rate, so that (i) can be written as

$$V_{eo} = \sum_{t=1}^{\infty} \frac{\bar{D}_o(1+g)^t}{(1+k)^t} \qquad (ii),$$

where g is the expected growth rate of $\bar{D}_t$, and $\bar{D}_o$
the expected value (not necessarily the actual value) of
dividend for the present period. This expression can be
simplified to

$$V_{eo} = \frac{\overline{D}_o(1+g)}{k - g} \qquad \text{(iii)},$$

or, as it is usually stated,

$$V_{eo} = \frac{\overline{D}_1}{k - g} \qquad \text{(iv)}.$$

From (iv) it is convenient to isolate k, giving

$$k = \frac{\overline{D}_1}{V_{eo}} + g \qquad \text{(v)},$$

and this form is the basis for numerous attempts to identify k.[1] The intention in this section is to discuss the measurement difficulties inherent in this approach and indicate the directions in which refinements and improvements have been suggested.

Accepting for the moment that the observer believes that the market's growth expectations are as simple as suggested by (ii), the identification of k in (v) requires the values of $\overline{D}_1$, V_{eo} and g. Unfortunately none of these is directly observable, and the resulting measure of k reflects the procedures used to infer the required values. Diagram 1 illustrates the nature of the problem.

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(1) The relationships (iv) and (v) were worked out by Gordon and Shapiro in their important article on identifying cost of capital: M.J. Gordon and E. Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," Management Science (October 1956). The use of D_1 in the text instead of D_o as originally suggested by Gordon and Shapiro is due simply to the definition of V_{eo} as the ex-dividend equity value. The importance of this steady-state growth model lies not only in its simplicity but in its incorporation of a realistic expectations model to provide a measure of g.
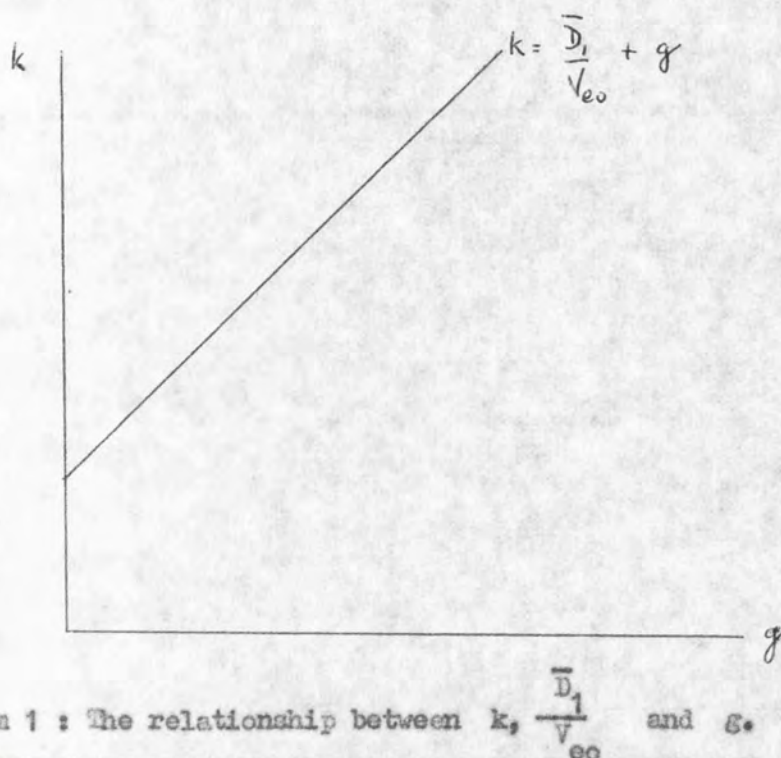
Diagram 1 : The relationship between k, $\dfrac{\overline{D}_1}{V_{eo}}$ and g.

The relationship (v) is a straight line sloping upwards
at an angle of 45° from the vertical intercept $\dfrac{\overline{D}_1}{V_{eo}}$.
If the value of this intercept is known, the problem is the
familiar one of having two unknowns in a single equation:
only if the expected growth rate is identified can the observer
obtain the value of k. There is a one-for-one relationship
between an error in identifying the expected growth rate and
the resulting error in the estimate of k.

To this difficulty must be added that of identifying the
vertical intercept, $\dfrac{\overline{D}_1}{V_{eo}}$. A temptation here is to employ
the readily available measure of dividend yield as a substitute
for the ratio of separately identified values of $\overline{D}_1$ and V_{eo} ,

and the objections to such a simplification should be indicated.
If dividend yield is based on the firm's most recently announced
dividend, D_o , a two-part error occurs: the difference between
D_o and $\overline{D}_1$ is made up of the difference between D_o and $\overline{D}_o$
(original expected value of dividend for the current period)
and the effect of growth over one period. The quantitative
difference between D_o and $\overline{D}_1$ may be quite significant. As
for the denominator, V_{eo} , the procedures which may be employed
by an observer in estimating the equilibrium value of equity
are discussed in Section 4 : it is sufficient to make the
obvious point that current market value is unlikely to be seen
as a completely satisfactory substitute.

The importance of correct estimates of g and $\dfrac{\overline{D}_1}{V_{eo}}$
depends in each case on its relative importance in (v). The
more confident the observer is of a high market expectation of
growth, the less important relatively is his estimate of $\dfrac{\overline{D}_1}{V_{eo}}$
- though of course the growth expectation estimate enters into
the numerator of that ratio. By the same argument, a mistaken
estimate of g affects both terms on the right-hand side of
(v) in the same direction, given $\overline{D}_1 = \overline{D}_o (1+g)$, and there
is no reason to expect an independent estimate of V_{eo} to
tend to stabilise the final estimate of k. The task of
identifying growth expectations, even in the simplified context
presently accepted, may be approached in a variety of ways:
ideally a number of alternative expectations models would be
employed, along with whatever other information and impressions
could be obtained, and a tolerably narrow range of g-estimates
would emerge.

9 : 2 (ii)   Improving the model of growth expectations.

The importance of the growth rate expectation used in
estimating k is obvious, as is the unreality of the
assumption that expectations actually take the simple form
suggested by (ii). For these reasons improved models of
expectations have been suggested, and one possibility is described
here to show that the advantage of simplicity can be retained
at the same time as confidence in the result is improved. The
observer is now assumed to allow for market expectations of a
non-constant growth rate: in its simplest form this modification
allows the market to expect an initial growth rate to be main-
tained over a certain number of periods, followed by a different
constant growth rate in perpetuity.[2]

(2)   Such an interpretation of the market's expectations is
      not necessarily derived from a view of a firm's investment
      prospects in the way assumed by Gordon and Shapiro.
      However, numerous writers have added to the latters'
      simple model a variety of models of non steady-state
      growth patterns based upon changing investment prospects:
      see E. Solomon, The Theory of Financial Management, 1963,
      pp. 59-62, and a useful summary of investment opportunity
      models in J.C.T. Mao, Quantitative Analysis of Financial
      Decisions, 1969, pp.393-406. It is questionable whether
      the market ought to be credited with detailed knowledge
      of investment opportunites, and as Mao shows (op.cit.,
      p.405), very different k-estimates emerge from different
      assumptions about what the market expects. A growth
      model can also be used to identify variables other than
      k in a valuation model: see C. Holt, "The Influence of
      Growth Duration on Share Prices," Journal of Finance
      (1962), and B.G. Malkiel, "Equity Yields, Growth and the
      Structure of Share Prices," American Economic Review
      (December 1963), where implied market expectations of
      growth duration are isolated.

This modification results in the valuation expression:

$$V_{eo} = \sum_{t=1}^{n} \frac{\overline{D}_o (1+g)^t}{(1+k)^t} + \sum_{t=n+1}^{\infty} \frac{\overline{D}_o (1+g)^n (1+m)^t}{(1+k)^t} \quad (vi)$$

where n is the number of periods over which growth occurs at the periodic rate g and m is the long-term equilibrium periodic growth rate expected to apply after period n. This expression reduces to

$$V_{eo} = \overline{D}_o \left[\left(\frac{1+g}{k-g}\right)\left(1 - \left(\frac{1+g}{1+k}\right)^n\right) + \left(\frac{1+g}{1+k}\right)^n \left(\frac{1+m}{k-m}\right) \right] \quad (vii),$$

and given estimates of V_{eo} and $\overline{D}_o$ this can be solved for k with any combination of n, g and m. Instead of a single estimate of growth expectations contributing to the estimate of k, there are now three "growth" dimensions (g, n and m), and this has the advantage that an error in the estimate of any one of the three should be less serious than an error in the single g-estimate for the original version. Nor is there any reason to suppose that errors in g, n and m - estimates will systematically bias the estimate of k.

An example of the sensitivity of the k-estimate to the variation of g, n and m values in (vii) provides some encouragement for the would-be observer. Here the market is believed to expect a long-term growth rate of 3 per cent per annum, but a hypothetical observer is uncertain whether the market expects an initial period of 8 years' growth at 6% p.a. or an initial period of 6 years' growth at 4% p.a. If the observer assumes that V_{eo} is twenty times the value of

$\overline{D}_o$, the estimates of k are 0.092 (for n=8, g = .06) and
0.084 (for n = 6, g = .04). This difference in k-estimates
is not too alarming, particularly as the n, g combinations
chosen may be regarded as drawn from opposite ends of a
spectrum: estimates of k are likely to be drawn closer
together when assumptions about the market's n and g expectations
are paired in mutually offsetting ways - probably a realistic
way of representing the expectations alternatives of the
market in such a situation. [3]

Section 3 : Estimating k from Historical Performance.

Instead of attempting to identify expectations, with all
the attendant difficulties mentioned in Section 2, management
may be able to infer the discount rate employed by investors
by the very different process of measuring and interpreting
historical rates of return on equity investment.

Rates of return achieved on stock exchange investment
can be calculated on many bases. The main respects in which
assumptions can differ are the length of time considered, the
time pattern of purchases and sales during the chosen period
and the breadth of the portfolio. In none of these areas can
a uniquely correct assumption be expected, and any measure
of the historical rate of return on equity investment is bound

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(3)   In this example the observer's uncertainty is confined
       to the duration and rate of growth expected by the market
       in excess of the long-term expectation, m. Obviously
       if he is undecided about whether the market expects
       faster or slower growth (than m) in the immediate and
       near future, his recognition of margin of error is very
       much wider than that shown in the text.

to be arbitrary. The historical performance approach, even when applied to the equity of a single firm, starts at a disadvantage compared to the ideal situation in which a single market discount rate is sought.

The strongest claim on behalf of historical performance is perhaps that made by Merret and Sykes, who argued that

"Where a company has been following a fairly consistent dividend policy ... and where conditions in the industry are stable .... the cost of equity capital in future can reasonably be taken as the yield achieved on an investment in the firm's shares over the last 5-10 years. This is a reasonable presumption in that investors with similar requirements will have been attracted to become shareholders." [4]

The difficulties of accepting and applying this concept can be considered briefly.

The key assumption in using a measure of historical return to investors to identify k is that an ex post result can indicate an ex ante expectation or requirement. Even under ideal conditions, with equity market value always at the correct level, this inference would be improper for periods as short as those suggested by Merret and Sykes: actual dividends are

(4)   A.J. Merret and A. Sykes, The Finance and Analysis of Capital Projects, 1963, p.72.

unlikely to equal, or average out at, their expected values,
and the recorded rate of return would only accidentally equal
the true discount rate. This objection in theory is re-
inforced by practical considerations. The first of these is
the dominant effect on the observed rate of return of actual
market values ruling during the defined period: a tendency
towards a standard rate of return is observable only in the
very long run.[5] Indeed, the results of the historical
performance approach can be made to appear in conflict with
those of the expectations approach outlined in Section 2.
For example, if a share's price has been falling its measured

(5) Merret and Sykes themselves produced evidence to confirm
     this point in their two 1963 articles: A.J. Merret and
     A. Sykes, "Incomes Policy and Company Profitability,"
     District Bank Review (Sept.1963) and "Return on Equities
     and Fixed Interest Securities 1919-63," District Bank
     Review (Dec. 1963). Although their calculations relate
     to a broad portfolio of shares, similar results would
     emerge for individual shares which experienced typical
     price fluctuations over the period covered by the broad
     study. Merret and Sykes's demonstration of relative
     stability in the long-term rate of return seems to depend
     on comparisons between different terminal dates all very
     remote from a common starting date : see Merret and Sykes,
     op.cit., (Sept.1963), pp.21-2 and extending chart. The
     obvious objection to such comparisons is the very great
     length of common experience they all share, and a different
     picture is obtained from the same results pattern if, for
     example, 10-year investment programmes terminating in
     different years are compared. Thus 10-year programmes
     terminating in 1963, 1960 and 1957 showed respective returns
     of 11.8%, 17.7% and 7.6% p.a. (in money terms, after taxes).
     Using U.S. data and rather different assumptions about
     the committment of new funds by investors during a defined
     period, Fisher and Lorie produced results which also
     showed wide differences in the rate of return between
     different post-war periods : L. Fisher and J.H. Lorie,
     "Rates of Return on Investments in Common Stocks," Journal
     of Business (Jan. 1964).

historical rate of return has probably been worsening, and
with it the inferred value of k : yet the fact of the price
fall may actually reflect an increase in the discount rate
presently being applied to expectations. The argument applies
in reverse if the share's price has been rising.

On the simple question of fact, Merret and Sykes's
initial suggestion has been shown to be quite unrealistic. A
firm and its shareholders ought to place very little reliance
on the exact - or even approximate - continuation of recent
growth experience [6] : they ought, indeed, to see future
prospects in probabilistic terms. In Chapter 10 this line of
thought is suggested as the basis for a managerial interpretation
of valuation, but for the moment it is sufficient to observe
that investors would be quite wrong, and unlikely, to base a
unique measure of the appropriate discount rate on a firm's
recent growth experience.

Possibly to circumvent these difficulties, Merret and
Sykes based their own recommendations on the cost of equity
on the historical performance of broad portfolios of shares,
rather than on the faithful application of their initial suggestion
quoted above. This recommendation was further refined by a
switch from historical performance to anticipated performance
of equity investment in the kind of macroeconomic conditions

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[6]   See, for example, the various statistical attempts to
       discover growth consistency in I.M.D. Little and A.C. Rayner
       Higgledy Higgledy Growth Again, 1966, Chapter 1.

predicted.[7] The device of equating an anticipated _average_
rate of return on equity shareholding with the cost of equity
investment capital for an individual firm has been criticised,[8]
but in the present context management's interest in identifying
k is assumed to be the interpretation of market valuation
rather than investment decision making. It is of course convenient
to describe the discount rate as essentially an opportunity
cost and then, by defining a plausible alternative opportunity -
in this case the purchase of a portfolio of shares - to identify
the rate itself. It does seem wiser to avoid, where possible,
an arbitrary definition of the alternative as seen by investors,[9]
and concentrate instead on directly measuring the market rate
itself. If the expectations approach can be made to work, the
problem of identifying investors' alternative uses of funds
can be by-passed.

Section 4 : Market Value and Correct Value.

Neither approach to the estimation of k has provided
a unique and reliable measure for decision making : theoretical
objections and practical problems have not been difficult to

(7)   A.J. Merret and A. Sykes, The Finance and Analysis of
       Capital Projects, 1963, pp.72-3, and A.J. Merret and
       A. Sykes, Capital Budgeting and Company Finance, 1966,
       pp.31-2.

(8)   See C.J. Hawkins and D.W. Pearce, Capital Investment
       Appraisal, 1971, p.48.

(9)   This point was made by L.J. Richenberg in a review of
       Merret and Sykes, op.cit.,(1963) in Economic Journal
       (Sept. 1964).

find. Although the expectations approach is in principle the more satisfactory, it requires at least as much managerial insight into investors' thinking as does that based on historical performance. This section considers the major problem of inference ignored in Section 2, that of working from actual market values of equity to an estimate of investors' views on the ideal or equilibrium equity value.

Two possible lines of approach are suggested, though management may adopt any procedure in which it has confidence. An obvious idea is to look for equilibrium value in the actual pattern of price experience over a relatively short period in which the capital market as a whole is judged to be at about the right level and price relationships within the market seem stable. If such a situation can be recognised, management ought to be capable of interpreting recorded prices to give an estimate of a share's equilibrium value at that time.

A more ambitious approach may appeal to a management prepared to use its advantageous position to observe investors' market behaviour. Inferences drawn from observed behaviour must allow for a number of factors, including short-term speculative inclinations and the fact that at any one time an investor's "best estimate" of correct value may be held with less than complete confidence. A simplified version of an observer's thinking along these lines will indicate the nature of the problems he faces.

After determining what is, in his view, the correct value for the equity in question, the typical long-term investor fixes the price ranges in which he will, respectively, buy, hold or sell. This policy is revised frequently, in the light of any change in his best estimate of correct value, his confidence in that estimate and his view of short-term market prospects.[10] In practice, each price is associated with a conditional quantity decision : investors rarely make a complete switch into or out of a holding at a single price or in one move.

Naturally, it cannot be expected that the weight of long-term investment decisions always moves market price into line with the estimate of correct or equilibrium value. Even if all investors agree on the latter, their short-term policies may differ considerably, and it cannot even be assumed that they always try to close a perceived gap between market and correct prices by buying or selling as appropriate.

From this brief view of an observer's position vis a vis long-term investors, the outlook for management's attempt to identify the latters' estimate(s) of correct value seems unpromising. In effect, returning to equation (v) and Diagram 1, the observer's problem for each group of investors has become one of one equation and three unknowns (four if $\overline{D}_1$ is also considered

(10) The assumption that investors deal separately with the different aspects of risk (that incurred in paying the correct price for equity and that of paying or receiving an incorrect price) forms the basis of a defence of the "ideal" valuation concept in conditions of investor uncertainty about objective dividend prospects: see Chapter 13 : 2 (ii).

as an investors' estimate, to be inferred). However, if
management is prepared to try to reconstruct the parameters
of investors' market policies from observation of behaviour
in buying and selling situations, it may be possible to
uncover the underlying estimates of correct value. As with
the first approach suggested, the most favourable occasion for
an attempt of this kind is when management feels that the
market as a whole is widely recognised as being "about right",
so that speculative elements in long-term investors' actions
are least damaging to the attempt to identify their estimates
of correct value. In fact under such conditions, if the policy
of long-term investors is not tending to disturb price relation-
ships among comparable shares, management may have little
difficulty in accepting the prevailing price as a surrogate for
investors' views of the correct value.

The method of identifying an equilibrium value of V_{eo}
by drawing inferences from the behaviour and assumed attitudes
of investors has one considerable potential disadvantage :
it can lead, as indicated, to the identification of differing
estimates held by different groups of investors. Even with
the market in a state of widely recognised stability, the stability
of a share's price and absence of large-scale switching of
portfolios may be due to the fact that for different groups the
price is either just above the level at which buying would
occur or just below that at which selling would begin. Thus
it is perfectly possible, even in near-ideal conditions, for
long-term investors to be holding conflicting views on the

correct market value of a particular share; and management
must believe, where this occurs, that the differences in
valuation are explained by differences in expectations
rather than by the co-existence of numerous individual valuation
models. The importance of this belief is obvious: if
different discount rates are identified in the expectations
approach the task of identifying the rate which would apply
under ideal conditions will lie beyond the resources of management.
If it is to have confidence in the results of its analysis
management must believe that all groups of investors agree
on the correct value under market equilibrium conditions of
any shared expectation, and that different valuations reflect
only different expectations.[11]

In fact, sustaining this necessary confidence should not
be difficult. Investors expectations and correct value estimates
will not be identified with sufficient confidence to permit the
identification of significant differences in the resulting k-
estimates; and in any case the basic belief in the efficiency
of the market will appear reasonable enough.

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(11) In his exposition of the standard valuation expression
(1), Van Horne defines D_t as the expected value for
investors at the margin and k as the appropriate market
discount rate: see J.C. Van Horne, _Financial Management
and Policy_, 2nd edition, 1971, pp.94-5. The
implication is that it is only the expectations of the
marginal investor at any time that need be identified:
the share's market value at that time represents the
marginal investor's _correct_ discounting of his own
expectations: this by implication confirms the concept
of all investors recognising the correct value of shared
expectations. Using current market value to identify k
still requires the expectations of the marginal investor
to be identified, along with his precautionary / speculative
policy.

To summarise, one or other procedure for interpreting
valuation and expectations will yield an estimate of k.
Confirmation can be sought in repeated observations and the
use of past data, and a final version accepted. The important
point established in this chapter is the necessity for managerial
analysis, assumptions and inferences in arriving at any firmly
held view of valuation, and this will apply with greater force
to the modified valuation model for multiple growth possibilities
that is introduced in the following chapter.

Chapter 10

The Valuation of Multiple Dividend Growth-Paths

Section 1 : Introduction.

Section 2 : Multiple Growth Paths and Market Valuation.

    (i)   Perfect correlation between dividend probability distributions.

    (ii)   The implications for valuation of accepting $r_{km} = 1.00$

    (iii) Rejecting the simple valuation model in the case of multiple growth-paths.

Section 3 : The Interpretation of Valuation and the Identification of k.

    (i)   A simple model of market valuation.

    (ii)   The theoretical possibility of identifying k.

    (iii) The process of identifying k.

    (iv) An improved identification process.

Section 4 : Conclusion.

Section 1 : Introduction.

In Chapter 8 the multiplicity of possible growth-paths of constant-risk expected dividend was demonstrated, and although this property of real-world dynamic situations was ignored in Chapter 9 it is now necessary to consider its implications for management's interpretation of the market valuation of equity.

This chapter has three main purposes. First, to demonstrate for the case of multiple growth-paths the inadequacy of the simple interpretation of valuation adopted in Part 1; second, to attempt a new interpretation which takes into account the conditions defined in Chapter 8; and third, to use this new interpretation to demonstrate the possibility of identifying the value of k , the market rate of discount on expected equity income. The conclusion to the chapter prepares the way for Chapter 11 by considering the relevance and importance for management of identifying k in the conditions which give rise to multiple growth possibilities.

In developing a managerial interpretation of valuation more suitable to real-world dynamic situations the traditional interpretation is not discarded. Indeed, as indicated above, the possibility of identifying k in the new conditions is a major concern. It would be accurate to say that the traditional approach is retained as a basic component of a general valuation model which explicitly recognises multiple growth possibilities in a way which reflects investors' probable attitudes, and which therefore permits a worthwhile interpretation to be reached. This final result contrasts with that of Section 2, where the attempt to incorporate multiple growth possibilities into an unchanged and unsuitable valuation model produces unacceptable conclusions about the expectations and attitudes of investors.

It has been possible in earlier chapters to use the symbol S to represent both the market value of a firm's equity and the discounted value of its stream of future dividend expectations. Having reached a stage where the firm is no longer assumed to possess a unique growth-path of constant-risk dividend, it is now essential to adopt a new distinction between the two concepts, and a new symbolism to match. The simplest change is to use a new symbol, V_e , to represent the market value of the firm's equity capital ; all other additions or changes to the familiar notation will be introduced in the appropriate place.

Section 2 : Multiple Growth Paths and Market Valuation

10 : 2 (i) Perfect correlation between dividend probability distributions

Chapter 9 reviewed the problems involved in identifying k in the context of the model of a unique growth-path of constant-risk expected dividend; and discussion there focussed on the derivation of adequate information rather than on the validity of the model itself. Now it is necessary to return to the subject matter of Chapter 8, where the multiplicity of growth possibilities was demonstrated, to consider the effect of such a model on the validity and identification of k .

Perhaps the most hopeful starting point is the possibility, whose mathematical conditions were derived in Chapter 8: 3(i) that for any t the overall expected dividend, $\bar{D}_t$ (equal to $\sum_{i=1}^{x} p_i \bar{D}_{it}$, where i refers to the growth-path), is associated with a constant level of risk equal to that for each individual $\bar{D}_{it}$. Given that all dividend probability distributions on individual growth paths are of equal risk, it was demonstrated that the overall or combined dividend distribution for a period carries this same level of risk when there is perfect positive correlation between all pairs of D_i – distributions for that period. If these conditions are satisfied it may be possible to continue with the interpretation of market valuation that has served so far throughout this study : the standard discounting model could be assumed to apply to the series $\bar{D}_1$, $\bar{D}_2$, ...$\bar{D}_n$......, all of which would have the same risk – in the defined sense – as the members of any single growth path : $\bar{D}_{j1}$, $\bar{D}_{j2}$ ... $\bar{D}_{jn}$ .... . The fact of a multiplicity of possible growth-paths of constant-risk expected dividend would pose no new difficulties for the interpretation of market valuation ; and the measurement of k would be subject only to those difficulties already considered in Chapter 9. This attractive prospect will now be examined.

In Chapter 8 : 3 (ii) it was shown that the mathematical condition referred to in the paragraph above reduces to the requirement that there must be a perfect positive correlation between all $(Q - I)$ distributions in each period. It is impossible to test whether this requirement has been satisfied for any past period, or is likely to be satisfied in future : this is because all growth-paths but one remain hypothetical, except for overlapping of paths which are due to diverge at later dates. Faced with the impossibility of obtaining confirmation on this crucial assumption, what might management infer about investors' response to the same problem ? A pessimistic conclusion would be that just as there are no grounds for accepting the crucial assumption that $r_{km} = 1.00$ for all k, m , so there are no grounds for accepting any other pattern of r_{km} values ; and therefore no basis for any view whatsoever about the risk of the overall dividend distribution for any given period. In the absence of any acceptable version of r_{km} — for the same reason as the rejection of $r_{km} = 1.00$ — a most disturbing impasse would have arisen. Not only would management be deprived of its useful interpretation of the valuation process, with no alternative version in sight, but investors too would be placed in a position of complete ignorance concerning the objective risks associated with the stream of future dividend expectations; and it would be impossible to decide rationally on the discount rate appropriate to the series of overall expected dividends.

The obvious paradox in this description of the dilemma is that the overall expected values, D_t , are assumed known while almost nothing can be known about the risks of the combined distributions from which these expected values are derived. This in itself would be a highly unusual risk situation, and one is bound to suspect the theoretical model leading to such a conclusion. It is quite unlikely that investors in the real world are - or believe themselves to be - in the kind of situation described, or that they think along the lines suggested by this analysis. Nevertheless,

before turning decisively away from this approach to a managerial interpretation of market valuation, some arguments more favourable to its retention are worth examination.

Neither management nor investors can demonstrate that r_{km} does **not** equal unity : it is a question which cannot be settled conclusively. Therefore, if investors are prepared to treat the assumption as correct, and value the stream of overall expected dividends, $\bar{D}_t$, in the same way as they would a single growth-path of known risk, the resulting situation will be just as if r_{km} = 1.00 (for all k, m and all periods) were capable of positive confirmation. Management would then be able to interpret market valuation along orthodox lines, and k might be derived subject to the considerations raised in Chapter 9. With valuation so readily comprehensible, k might be employed in making decisions on individual investments, or, what is more likely in the model developed in Chapter 8, in assessing the possible consequences of changes in investment strategy. For these reasons management might be inclined to accept r_{km} = 1.00 along with the resulting convenient interpretations of investors' behaviour, expectations and market valuation.

The factors likely to persuade investors that future dividends are subject to known and constant objective risk, and that that risk is equal to the level applying to dividend on individual growth-paths, are rather different. Investors are unlikely to accept that nothing can be known about the risks of overall dividend distributions whose mean values are known ; in the absence of definite contrary information regarding the values of r_{km} in future periods, their understandable caution will lead them to accept r_{km} = 1.00 simply because the risk of the overall dividend distribution is thereby assessed at its highest possible level. [1] By assuming the

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(1)   Equation (xxi) in Chapter 8 gives the risk of the overall dividend distribution.

worst about the risk of the overall dividend distribution for each period,
investors automatically reach the "right" conclusion about the actual
level of risk and its constancy through time. For this reason, it might
be argued, management will continue to be able to interpret the market
valuation of the firm's equity in terms of the valuation model applicable
to the single growth-path, even where the multiplicity of possible growth-
paths is accepted by investors.

10:2 (ii) The implications for valuation of accepting $r_{km} = 1.00$.

Accepting for the moment that investors, prompted by caution and with
no reason to believe otherwise, will regard future dividends – whose expected
values, $\bar{\bar{D}}_t$, are known – as subject to the same risk as would apply in
the single growth-path case, it is instructive to consider the mathematical
relationships given by market valuation along orthodox lines. Given that
valuation is understood to reflect the discounting of constant-risk expected
dividends, $\bar{\bar{D}}_t$, the expression for the equilibrium market value of equity
to long-term investors is

$$V_e = \sum_{t=1}^{\infty} \frac{\bar{\bar{D}}_t}{(1 + k)^t} \qquad (i),$$

which differs from the single growth-path formulation only in the use of
overall expected dividend, $\bar{\bar{D}}_t$, instead of $\bar{\bar{D}}_t$. This form may be expanded
to give

$$V_e = \sum_{t=1}^{\infty} \sum_{i=1}^{\infty} \frac{p_i \, \bar{D}_{it}}{(1 + k)^t} \qquad (ii),$$

in which each period's $\bar{\bar{D}}$ is shown as the sum of the individual probability –
weighted $\bar{D}_i$ values for that period.

Starting from a different position, let S_i be the discounted value of the constant-risk dividend expectations of the ith growth-path, so that

$$S_i = \sum_{t=1}^{\infty} \frac{\bar{D}_{it}}{(1+k)^t} \qquad\qquad (iii).$$

In this expression S_i is what the market value of the ith growth-path (of expected dividend) would be if it were unique and able to be valued separately in the market : it corresponds exactly to the concept of the valuation of a stream of expected dividends employed in earlier chapters. However, S_i is only one such value drawn from the whole set of growth-path possibilities; so that for each growth-path there is an S-value calculated in the same way. There is, therefore, a probability distribution of S-values, each taking the probability of the growth-path to which it corresponds.
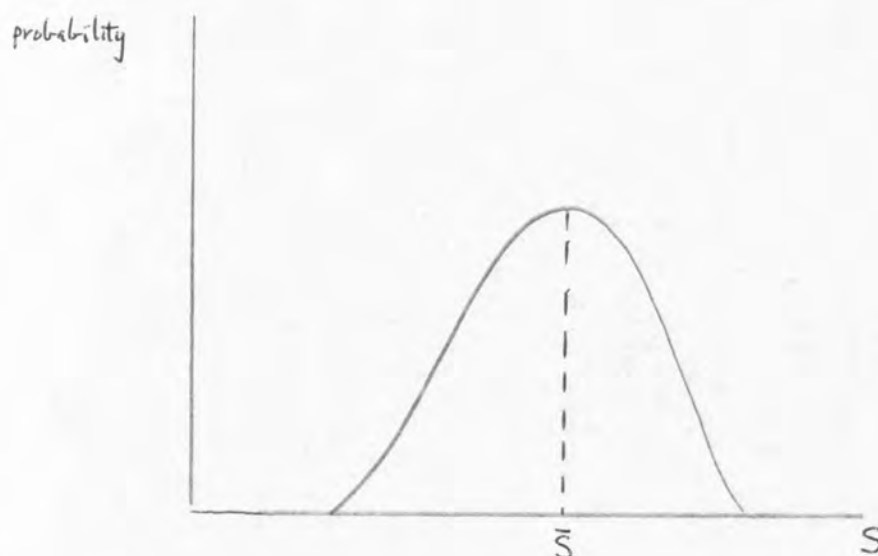
Diagram 1 : Probability Distribution of S-values.

An example of such a distribution, with mean value $\bar{S}$, is shown in

Diagram 1 . Given that

$$\bar{S} = \sum_{i = 1}^{x} P_i S_i \qquad (iv),$$

which can be expanded, using (iii), to give

$$\bar{S} = \sum_{i = 1}^{x} \left[P_i \sum_{t = 1}^{\infty} \frac{\bar{D}_{it}}{(1 + k)^t} \right] \qquad (v),$$

it becomes obvious that $\bar{S}$ is equal to V_e , the equilibrium market value of the firm's equity on the present hypothesis:

$$\bar{S} = \sum_{i = 1}^{x} \sum_{t = 1}^{\infty} P_i \frac{\bar{D}_{it}}{(1 + k)^t} = V_e \qquad (vi).$$

It is this equivalence which gives rise to doubts about the validity of the interpretation of valuation embodied in (i) and (ii). Even where the overall dividend distribution for each period carries - or is thought to carry - the same risk as that affecting the D-distribution on each individual growth-path, the implication that V_e and $\bar{S}$ are necessarily equal can be shown to be extremely damaging to the attempt to interpret valuation along the same lines as in the single growth-path case. This point is developed in the following sub-section.

10 : 2 (iii) <u>Rejecting the simple valuation model in the case of multiple growth-paths.</u>

In the case of a single growth-path of constant-risk expected dividend, the investor's expectation is that in the long run his average dividend will equal the level of expected dividend, with above-average and below-average results largely offsetting each other over many periods. It is on this

expectation of the investor that the orthodox view of valuation is based, with k determined by the dispersion of the typical D_t-distribution about its mean, $\bar{D}_t$. However, in the conditions now defined there can be no comparable expectation about a long-run equivalence between D_t and $\bar{D}_t$ values : dividend income in the long run will fluctuate around the trend values, $\bar{D}_t$, of the growth-path along which the firm actually proceeds. Unless this growth-path happens to co-incide at all periods with that of $\bar{\bar{D}}_t$, the overall average for all growth-paths, the investor cannot receive an average dividend equal to $\bar{\bar{D}}_t$; and (being ignorant of the actual growth path to be followed) he certainly cannot expect _in advance_ that on the average his dividend will equal $\bar{\bar{D}}_t$. For this reason it is obvious that the attempted analogy between the valuations of multiple and single growth-paths breaks down, in spite of the mathematical argument showing that the former may be described as having dividend distributions of constant risk equal to that on a single growth-path. The investor cannot be expected to value the series $\bar{\bar{D}}_t$ in the same way as he would the equally risky series $\bar{D}_{jt}$ if the latter were the only possibility. The same argument might be stated in terms of equity value. With a single growth-path the change in V_e through time is uniquely predictable, whereas in multiple growth-path conditions the change in V_e depends on the growth-path along which the firm moves. One would not expect the firm's prospects to be valued equally in the two situations even if the overall $\bar{\bar{D}}_t$ values were equal to the corresponding $\bar{D}_t$ values on a single growth-path. This argument does not identify the correct valuation of multiple growth possibilities; nor does it help in formulating a general valuation model for such situations. It merely confirms the earlier suspicion that $\bar{S} = V_e$ must be rejected.

The question inevitably arises whether it is reasonable to picture the investor as placing a value upon the probability distribution of S-values, each embodying the concept of k as a discount rate applied to risky expectations. The natural objection to such an interpretation is that the

two-stage valuation process implied is an unnecessarily involved and unrealistic view of investors' reasoning. The first part of such criticism is unacceptable : "rationality" in decision theory often requires some extremely involved calculations to be made by decision makers, and the complexity of the procedure under discussion is quite modest against this background. A more important doubt concerns the validity of continuing with the concept of k which really belongs, as has been shown, to the world of the single growth-path. Might it not be advisable to discard k completely and make a new beginning on the interpretation of investors' valuation of assets with multiple dividend growth possibilities ? Such criticism is difficult to evaluate because it is not clear what alternative interpretation, if any, is being offered : however, a case can be made out for the retention of the k-concept and its incorporation into a revised valuation model.

The main reason for the retention of k in an understanding of valuation is that the investor is still seen as valuing each of the individual constant-risk expected dividend streams in the way assumed throughout this study : if he could know at the present day the growth-path actually to be followed by the firm, the present value to the investor of a unit of its equity would be given by the S-value for that growth-path. No matter which growth-path occurred the investor blessed with this kind of foresight would apply the same valuation procedure. It therefore seems desirable to retain the k-concept in interpreting valuation in spite of the extra complication of a multiplicity of possible growth-paths, because it is relevant to a stage of the valuation process which logically precedes the latter problem. What has been changed by the more complex conditions is not the valuation of any one constant-risk growth-path, but the additional need to consider the joint valuation of all possible individual valuations arrived at in the orthodox manner. Provided the investor is understood to evaluate each possible growth-path separately, by discounting its risky expected returns,

there seems to be no theoretical objection to the retention of k – in spite of the measurement difficulties which obviously lie ahead.

Accordingly, the concept of k is retained in the changed circumstances, but other consequences must be faced immediately. The most obvious is that it is not as simple to estimate k's value as when all that was required was a proper interpretation of the values of the other variables in the general formulation

$$V_e = \sum_{t=1}^{\infty} \frac{\overline{D}_t}{(1+k)^t} \qquad \text{(vii).}$$

The difficulties arising in estimating k are considered in the remainder of this chapter. The other important consequence of accepting the need for a new understanding of valuation while retaining k is the need to re-consider the latter's role in investment decision making. This question is held over until the limits within which k may be estimated have been discussed.

Section 3 : The Interpretation of Valuation and the Identification of k.

10 : 3 (i) A simple model of market valuation

The measurement of k in the conditions defined must now be considered. The inapplicability of the simple discounting valuation procedure expressed in

$$V_e = \sum_{t=1}^{\infty} \frac{\bar{D}_t}{(1+k)^t} \tag{i}$$

or in

$$k = \frac{\bar{D}_1}{V_e} + g \tag{viii}$$

has been amply demonstrated, even where the $\bar{D}_t$ series is thought to embody constant risk. It is clear that without a new interpretation of valuation it will no longer be possible to identify the value of k : identification depends upon an understanding of the valuation process.

The basis for any new understanding of valuation is obviously the probability distribution of S-values, each representing the discounting at rate k of expected dividends on one possible growth-path.

Let

$$\frac{V_e}{\bar{S}} = y \tag{ix},$$

in which y is the market value per unit of $\bar{S}$, the ratio of the firm's market value to the mean of its S-distribution. [3] A firm's y-value is likely to depend on the dispersion of its S-distribution, with a higher value placed on a distribution with a lower standard deviation, σ_s, given the value of $\bar{S}$.

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(3)   The probability distribution of growth-paths of constant-risk expected dividend derives from management's investment strategy, as explained in Chapter 8. That strategy and its range of outcomes are both assumed given for the purpose of this chapter's discussion of valuation.

This suggests the formulation

$$y = h \left(\frac{\sigma_s}{\overline{S}} \right) \qquad \qquad (x),$$

which has the additional advantage of standardising the valuation expression for all values of $\overline{S}$ and V_e . The obvious difficulty in identifying the parameters of (x) in order to identify k is that $\overline{S}$, which appears on each side, is an unknown whose value actually depends on that of k. It is impossible for management to know its own $\overline{S}$ and σ_S until it has identified k, since the S-value for each possible growth-path depends on the value of k.

Faced with such an unpromising prospect for the identification of k, one possibility open to management is to choose various values of k and calculate in each case the properties of the resulting S-distribution : this would yield $\overline{S}$ and σ_S for each assumed k-value. The firm would then possess various conditional combinations of $\overline{S}$ and σ_S , together with the observed market value, V_e; and its task would be to decide which $\overline{S}$, σ_S combination suggested the most plausible set of parameters for (x). The objection to this method is that a single $\overline{S}$, σ_S combination would yield only a single observation from which to attempt an estimate of the parameters of $y = \dfrac{V_e}{\overline{S}} = h\left(\dfrac{\sigma_s}{\overline{S}}\right)$. Instead of simultaneously attempting to identify both k and the form of the valuation relationship, parameters for the latter can be assumed for the purpose of choosing between the $\overline{S}$, σ_S combinations generated by the various trial values of k : the "best" estimate of k would be that which produced the S, σ_S combination fitting the assumed form of (x) most closely. Naturally, there would have to be some provision for confirming management's assumptions about the parameters of the market valuation relationship.

Several comments may be made on this kind of trial-and-error method. In the first place, although the exercise is unlikely to yield a unique and accurate result, the attempt to identify k or to establish a range of its possible values can best be carried out by management, which is the body most able to identify the range of multiple growth possibilities and expectations on whose $\overline{D}_{it}$ values the entire exercise rests. As for the actual procedure suggested, the obvious difficulty lies in management ignorance of the true parameters of (x) and its resulting need to substitute "reasonable" values in order to determine which $\overline{S}, \sigma_s$ combination – and therefore which k-value – makes the closest fit with the estimated form of (x). The best way to appreciate the difficulties and possibilities of the suggested method is to work through a simple numerical example.

10 : 3 (ii) The theoretical possibility of identifying k.

Suppose that management estimates that investors accept the following probability distribution of the growth rate of expected dividend in perpetuity : [4]

probability	growth rate p.a.
0.30	0.03
0.40	0.04
0.30	0.05

(The range of growth possibilities is admittedly very restrictive, but it will serve for illustrative purposes.) If the expected value of dividend in the next period is $\overline{D}_1 = 10$, the application of the formula

$V_e = \dfrac{\overline{D}_1}{k-g}$ for selected values of k produces the following arrangement

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(4) The choice of growth possibilities is admittedly an oversimplification of the real world, but it will serve for illustrative purposes. The important difference between this model and the real world is that here growth-paths diverge only once, whereas the whole basis of the valuation model to be explained here is that at no stage will there be a unique future growth-path to look forward to.

of possible S-values and other relevant information. [5]

g p(g)	S	.07	.075	.08	.085	.09	.095	.100
.03 0.30	$S_1 = \dfrac{10}{k-.03}$	250	222.22	200	181.82	166.67	153.85	142.86
.04 0.40	$S_2 = \dfrac{10}{k-.04}$	333	285.71	250	222.22	200	181.82	166.67
.05 0.30	$S_3 = \dfrac{10}{k-.05}$	500	400	333.33	285.71	250	222.22	200
	$\overline{S}/k$	358.33	300.95	259.99	229.15	205	185.55	169.53
	σ_s/k	98.95	69.67	52.28	40.63	32.53	26.65	22.25
$\dfrac{\sigma_s/k}{\overline{S}/k} =$	z/k	.276	.232	.201	.177	.159	.144	.131
$\dfrac{V_c}{\overline{S}/k} =$	y/k	.558	.665	.769	.873	.976	-	-
$(V_c=200)$								

Table 1 : The derivation of conditional S-distributions from basic data.

The upper half of Table 1 simply records the value of S for each combination of g and k : growth possibilities are limited to the three steady rates already referred to, and seven trial values of k are used.

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(5) It was shown in Chapter 8 that growth paths can be assumed to co-incide for the first period and diverge thereafter : this is the basis for using a common value for $\overline{D}_1$ for all growth paths in this example.

The lower half of the table presents values of the important variables calculated from the basic data in the upper half. $\bar{S}/k$ is the mean value of the S-distribution for any chosen trial value of k, and is obtained by summing the probability-weighted S-values for that value of k. Similarly, σ_s/k is the standard deviation of the S-distribution for a chosen k-value; and Z/k is the ratio of the standard deviation to the mean of the S-distribution for a chosen k-value. The final row of the Table shows the value of y for each assumed value of k : it is based upon a market value $V_e = 200$. (A blank space in this row occurs when the value of y would otherwise exceed unity.)

The example shows that changes in $\bar{S}$ and σ_s brought about by changes in the assumed value of k have the effect of reducing the ratio $\sigma_s : \bar{S}$ when k is increased, and this result may be taken to reflect the general effect of changes in k. Having simulated these possibilities, management's problem is to find the pair of $\dfrac{\sigma_s}{\bar{S}}$ and $\bar{S}$ values with the closest fit to its own best estimate of the relationship,

$$y = \frac{V_e}{\bar{S}} = h\left(\frac{\sigma_s}{\bar{S}}\right) \tag{x}$$

This task is made easier by the fact that, even if the ratio $\sigma_s : \bar{S}$ is fairly constant over a range of k-values, the proposed test is quite discriminatory because $\bar{S}$ itself varies considerably with k : so that for any formulation of $y = \dfrac{V_e}{\bar{S}} = h\left(\dfrac{\sigma_s}{\bar{S}}\right)$ there should be a single value of k which produces the closest fit between $\dfrac{V_e}{\bar{S}}$ and the specified formulation of $h\left(\dfrac{\sigma_s}{\bar{S}}\right)$.

An ideal situation would exist if the solution value of k obtained in this way proved to be comparatively unaffected by variations in the assumed parameters of $h\left(\dfrac{\sigma_s}{\bar{S}}\right)$, but this is perhaps an over-hopeful outlook.

The purpose of the following discussion is to attempt to determine the likelihood that management will be able to confine uncertainty over k within a fairly narrow range. In the absence of additional information on the true parameters of the market valuation relationship $h\left(\dfrac{\sigma_s}{\bar{S}}\right)$, the best that can be hoped for is that a selection of reasonable assumptions about its shape and position will all be found to be fairly well satisfied by values of $\dfrac{V_e}{\bar{S}}$ which represent k-values drawn from a narrow range.

Diagram 2 illustrates three possible general shapes of (x) for comparison.
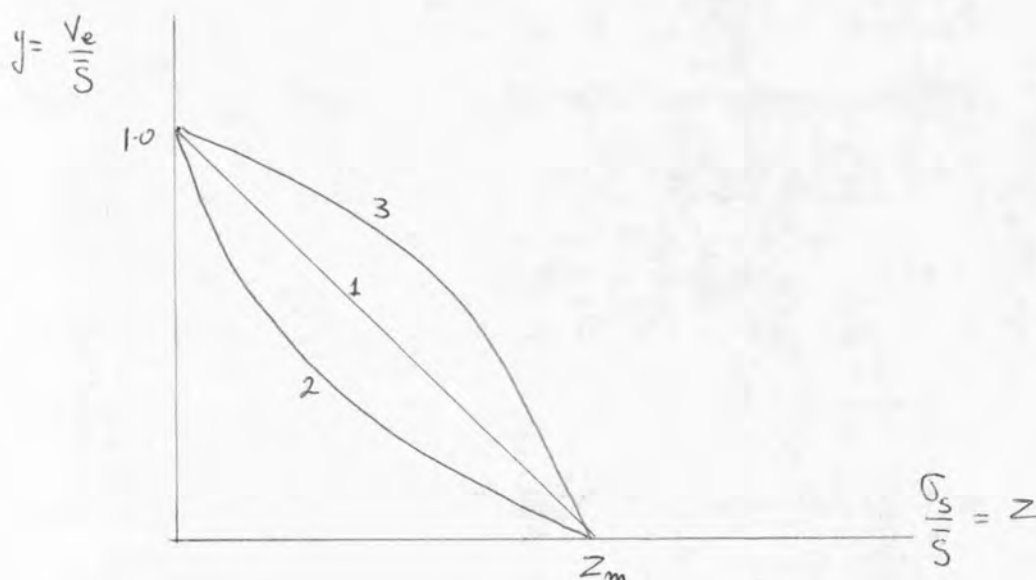
Diagram 2 : Alternative general shapes of $y = h(Z)$

In each case y - the market value per unit of $\bar{S}$ - is assumed to decline with an increase in Z - the risk of the S-distribution as measured by its coefficient of variation : the rate of decline is what distinguishes the different possibilities. The highest y-value attainable on any formulation must be unity, where $V_e = \bar{S}$ and $\sigma_s = 0$: this is the original case of a single growth-path of constant-risk dividend. The explanation for the meeting of all three possibilities at $y = 0$, $Z = Z_m$ is that management may think in terms of a maximum level of risk

at which market value would fall to zero (or, indeed, some other very low value). The position of such a point is assumed independent of the pattern of the decline in y with increasing risk, so that all possible formulations of (x) reach y = 0 at the same risk level. At this stage comparison between possible formulations can begin, and it is probably reasonable to exclude functions with a slope which is convex to the origin – that is, functions similar to number 2 in Diagram 2. The most likely general formulation is one in which the loss of market value is an increasing function of increases in the risk of the S-distribution. Accordingly, the range of likely formulations of y = h(Z) may be considered to lie above the straight-line possibility in Diagram 2, with the latter perhaps retained as a limiting case.

The results of calculations presented in Table 1 can best be used to illustrate the derivation of an estimate of k by means of a diagrammatic construction in which the relationship between y/k and Z/k for the firm is superimposed upon a management estimate of the market valuation relation-ship, y = h(Z). The intersection of the two relationships can then be traced directly to the unique trial value of k to which it corresponds. Diagram 3 employs four quadrants to show the simultaneous relationships existing between k , $\bar{S}/k$, y/k and Z/k, all of whose definitions were given with Table 1 : a market value V_e = 200 is assumed.
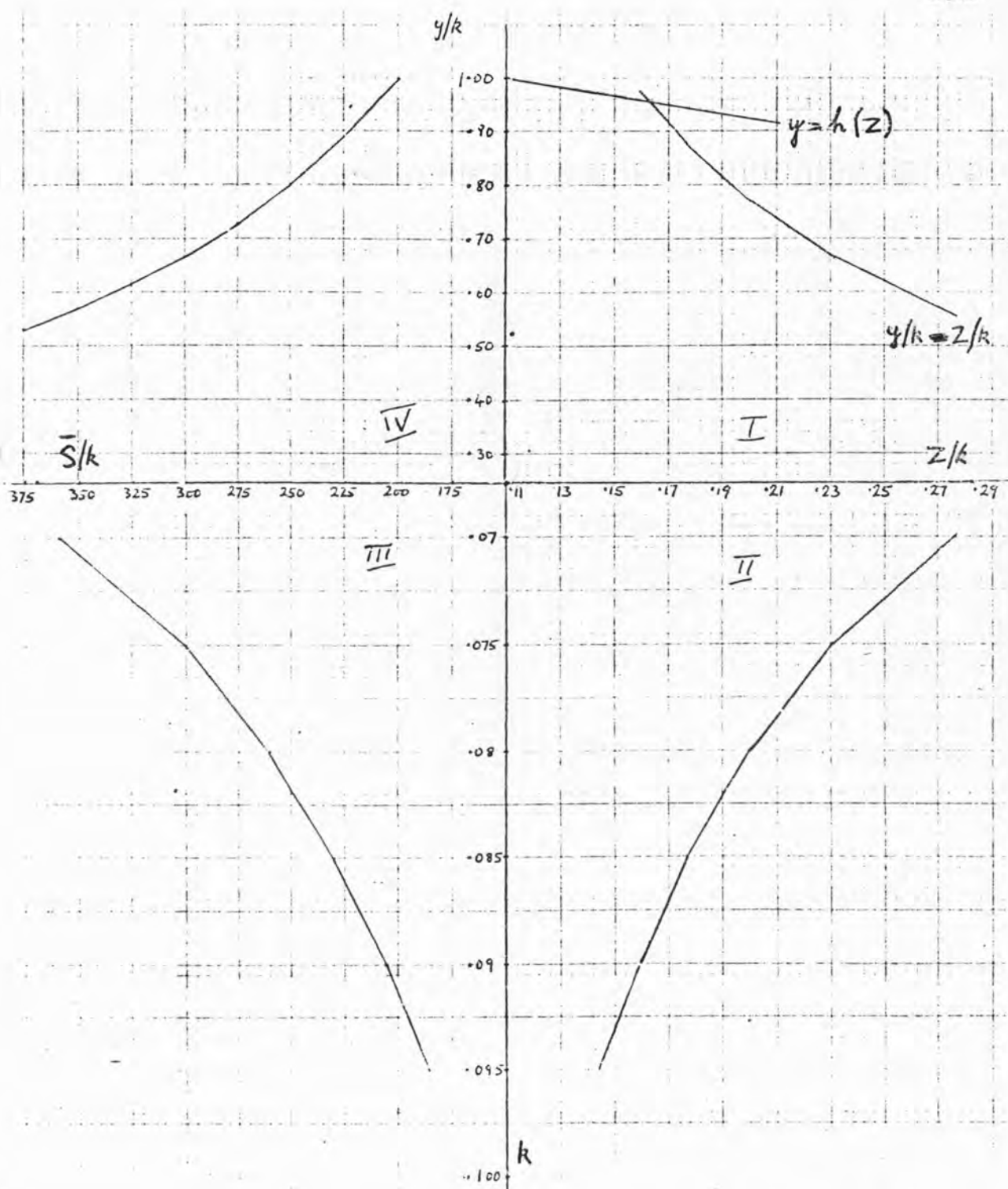
Diagram 3 : Identifying k using data for a single firm.

In the first quadrant of Diagram 3 the axes have been labelled to allow the two relationships between y and Z to be shown simultaneously : y/k as a function of Z/k is simply a graphical presentation of the calculations shown in Table 1 ; and y = h(Z) is the managerial estimate of the market valuation relationship. The first relationship, y/k – Z/k, is derived from the estimated growth possibilities for the individual firm

in question; whereas the second is a managerial estimate of the equilibrium valuation conditions prevailing in the capital market as a whole. It should be recalled that both $\overline{S}$ and σ_s for each trial k-value depend on the probability distribution of growth rates specific to the firm, whereas, in estimating $y = h(Z)$ management is concerned with a general relationship between market valuation and risk.

It is clear from quadrant I that, given the market value V_e of the firm's equity, a unique estimate of k can only be obtained if a unique valuation model is employed. Taking $V_e = 200$, as the ruling market value, the intersection between the y/k - Z/k relationship and a straight-line version of $y = h(Z)$ (chosen only for illustrative purposes) yields a k-estimate of approximately 0.090 in quadrant II. [6] At the same time, quadrant I shows that V_e is at only a slight discount below $\overline{S}$ when the latter is calculated using a discount rate of 0.090. (The possibility of a curvilinear version of $y = h(Z)$, concave to the origin in quadrant I, intersecting the y/k - Z/k relationship in two points can probably be safely ignored). Even if the relationship between trial values of k and $Z/k = \dfrac{\sigma_{s/k}}{\overline{S}/k}$ is such that the latter is constant over a range of the former, the point at which $y = h(Z)$ intersects the resulting vertical section of y/k - Z/k determines a unique estimate of k.

The position now is that if management is fully aware of the market valuation relationship, $y = h(Z)$, it can identify the intersection of that relationship with its own y/k - Z/k function and so derive its own $\overline{S}$, σ_s and k-values. Ordinarily, only one combination of these variables

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(6)   A linear trial version of $y = h(Z)$ is used to illustrate the difficulty of reaching a useful conclusion with only one y/k - Z/k locus to work with. Towards its upper end $y = h(Z)$ is likely to be approximately linear, and its correct slope difficult to guess at without further information.

will be found to satisfy the assumed market valuation relationship. On the other hand, if the latter is not known with certainty it can hardly be employed to identify the k-value which makes the firm's own conditional $\bar{S}$ and σ_s values consistent with the wider forces of market valuation. The firm is then in a position somewhat similar to that described in Chapter 9, dealing with the identification of k in the single growth-path situation : there it was shown that in order to identify k management required to know market expectations of g (or, more generally, of all values in the series $\bar{D}_1$, $\bar{D}_2$ ....). In the present case not only does the firm require the values in the $\bar{D}_{it}$ series for each i — that is, for each growth-path — but it must also know how the valuation of the S-distribution varies with its risk : this latter requirement constitutes the additional dimension of the problem of identifying k in the case of multiple growth possibilities.

10 : 3 (iii) The process of identifying k.

The picture is perhaps not as unpromising as this description suggests, thanks to the possibilities for constructive trial and error implicit in the juxtaposition of the internally generated y/k - Z/k relationship with estimates of the market value relationship y = h(Z). Suppose now that management begins with preconceptions of both y = h(Z) and its own k-value, and attempts to justify each preconception by reference to the other. It is quite likely that such an attempt will fail, but beyond an initial irreconcilability there may lie possibilities for modifying and clarifying ideas.

Diagram 4 : Reconciling inconsistent estimates of k and y = h(Z).

In Diagram 4 the first managerial estimates of k and y = h(Z) are both
indicated : they are k_e and y = h*(Z) respectively. The consequence
of management's acceptance of k_e is that in quadrant I the combination
of y/k and Z/k occurs at point a, the single point on the y/k - Z/k
locus considered relevant by management at this stage. As stated earlier,
there is little reason to expect that management's initial ideas on k
and y = h(Z) will prove mutually consistent in the sense of point a lying

exactly on the function $y = h^*(Z)$. However, it may be possible to achieve at least partial reconciliation of the inconsistency exposed in this way, and to do so in a way which is likely to produce improved estimates of both unknowns in the problem.

Taking as an example the type of discrepancy shown in Diagram 4, with point a lying below $y = h^*(Z)$, one very obvious feature of any formulation of $y = h(Z)$ may indicate to management the probable source of inconsistency between the two sets of ideas. It was suggested earlier that convexity to the origin in $y = h(Z)$ could be dismissed with little argument : it was also suggested that the valuation relationship runs from $y = 1.00$ to a certain Z-value at which y reaches zero or some extremely low value. Taking these two fixed points (the second admittedly less definitely established than the first) it is merely a question of observation whether the line from $y = 1.00$, $Z = 0$ through point a to the lower fixed point is necessarily convex to the origin. If it is impossible to re-draw the assumed valuation relationship to run through point a between $y = 1.00$ and the lower fixed point without breaking the "no convexity" rule, it must be clear to management that k is the main candidate for revision. An ultimate convergence between successive managerial estimates of k and a constant conception of $y = h(Z)$ lies at point b in Diagram 4, but it is quite possible that after k_e has been shifted some distance towards this convergence management will find that $y = h^*(Z)$ can be shifted downwards somewhat without becoming convex as a result of making its new intersection with the $y/k - Z/k$ locus. Such a compromise between the two initial conceptions, is illustrated by point c in Diagram 4, where k_e^1 and $y = h^{**}(Z)$ are found to be mutually consistent. This mutually consistent pairing is not, of course, guaranteed to identify the unknowns correctly : indeed, it is quite possible that more than one pairing exists with the potential for mutual consistency. In other words, it is possible to envisage a locus of points such as c in quadrant 1 :

the one on which management actually settles depends on its reactions to an initial inconsistency and on the extent to which it is willing to modify its original views on both k and $y = h(Z)$. At least it may be assumed that the process of repeated adjustments places the point finally accepted in fairly close proximity to the (unknown) correct pairing in quadrant I.

Although the identification of k in the conditions described was shown to involve an extra dimension of difficulty – the unknown parameters of $y = h(Z)$ – it is evident that a more rigorous process of questioning assumptions is required here than is the case with the unique growth-path model : accordingly, it is not at all certain that the resulting estimate of k will be less accurate under these conditions than it is likely to be when the simple valuation model is employed. In Diagram 4 for instance, it is immediately obvious that mutual consistency is only likely to be achieved at a value of k higher than k_e : this observation by itself is a helpful product of the juxtaposition of the two sets of ideas.

Suppose, however, that an initial discrepancy takes the form of point a lying above $y = h*(Z)$. If the chosen value of k_e produces a point in quadrant I with a y-value in excess of unity then no acceptable revision of $y = h*(Z)$ can resolve the inconsistency : this is an unambiguous pointer to the need for a lower k_e-value. However, apart from this easily identified cause of discrepancy it is less obvious in this case where the main burden of adjustment in ideas must lie. Reducing k_e helps to bring about consistency with a given $y = h*(Z)$; however, management may instead judge that the latter should be adjusted upwards to some extent to help eliminate the inconsistency, while the original k_e is allowed to stand. It seems more likely that management will resolve an inconsistency of this type by making an upward shift in its $y = h*(Z)$ function, provided this move leaves the function within its permitted area, than that its conception of k will be altered to any considerable extent. The justification for this statement

is that management will hold more strongly to its estimates of k, $\overline{S}$ and σ_s, which are internal concerns of the firm, than to a conception of an overall market valuation relationship : although the latter is obviously of great relevance in determining the former values, the divided view described in this section seems likely to prevail, with greater confidence being placed on estimates of the internal variables. Again, as with the opposite type of inconsistency, there can be no guarantee that the correct values of k, $\overline{S}$ and σ_s are identified in this process : indeed, the useful constraint in the earlier situation - the prohibition against convexity in $y = h^*(Z)$ - is scarcely relevant with point a lying initially above the assumed valuation relationship. Nevertheless, this type of inconsistency provides the opportunity for examining each assumption in relation to the other, and the result can hardly be harmful to an understanding of the valuation process.

The suggestion made above - that when point a lies above $y = h^*(Z)$ the main contribution to the search for consistency takes the shape of a modification of the latter function - receives added support from a consideration of the relative slopes of the $y/k - Z/k$ and $y = h^*(Z)$ functions in the relevant area of quadrant I. In Diagram 4 the $y/k - Z/k$ locus is much steeper than the function $y = h^*(Z)$ to the left of the intersection between them, and this means - in quadrant II - that a considerable shift in k_e per unit of change in Z/k is required to move point a down the $y/k - Z/k$ locus towards an unchanging $y = h^*(Z)$. By contrast, a change in $y = h^*(Z)$ in this area of quadrant I is likely to appear to management as a much less fundamental change of view, provided the basic rules governing its position and shape are observed.

10 : 3 (iv) <u>An improved identification process.</u>

The foregoing discussion of a managerial approach to the estimation

of k has assumed that only the y/k - Z/k locus of the firm in question
is tested against the managerial conception of the market valuation
relationship, $y = h^*(Z)$. However, additional useful information could
be generated by calculating y/k - Z/k functions for other firms, where
management felt sufficiently confident of investors' probability
distributions of their respective growth-paths, and where each of the latter
was expected to be of equal risk to those of the firm in question. By
this means a "family" of y/k - Z/k loci could, in principle, be calculated
by the management of a single firm; and for any particular estimate of k
the resulting pattern of points analogous to point a in Diagram 4 would
serve as a rather powerful check on the mutual consistency of a priori
conceptions of k and $y = h(Z)$. The basic problem remains one of reconciling
initially conflicting assumptions in a way likely to bring k_e closer to
its true value, but the mutual consistency of any combination of k_e and
$y = h^*(Z)$ is more easily tested when there exists a schedule of a - points
in quadrant I for a given k_e (one for each firm included in the test)
than when only one such point is available.

This extension of the basic checking procedure contributes in two
obvious ways to the identification of the range within which the true value
of k lies. The locus of a_i points traced out in quadrant I for a trial
value of k must not be convex to the origin; nor may it extend above the
level $y = 1.00$. If either of these rules is broken for a particular
k_e-value it must be rejected as a possibility for the true k-value. In
addition, one would expect the correct value of k to produce a pattern
of a_i points consistent with the accepted general characteristics of the
true valuation relationship: these are an origin at $y = 1.00$ and an increasing
downward slope as Z increases. As the following example confirms, these
requirements constitute an exacting test for any prospective solution to k,
provided a sufficiently widely-spaced variety of a_i points can be obtained.

The graphical construction of Diagram 3 is repeated in Diagram 5, with the addition of two extra sets of conditional equity valuations.
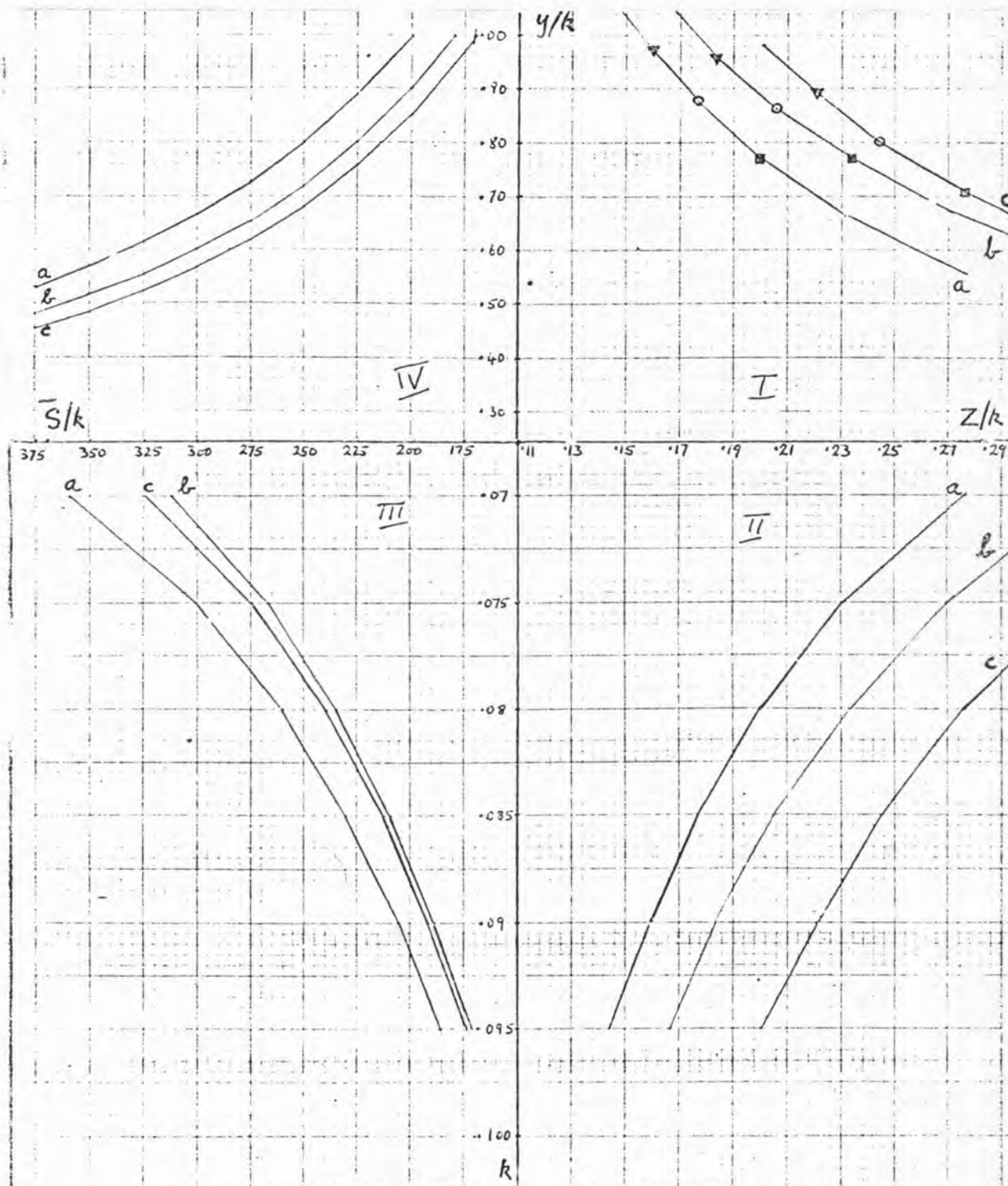

Diagram 5 : Simultaneous identification of k and y = h(Z).

The original probability distribution of growth rates is retained and is indicated by the letter a ; while the new distributions which represent the prospects of other firms are labelled b and c. Table 2 presents the three probability distributions on which the construction is now

based. (7)

a		b		c	
ε	F(ε)	ε	F(ε)	ε	F(ε)
.03	.3	.02	.2	.02	.3
.04	.4	.035	.6	.035	.4
.05	.3	.05	.2	.05	.3

Table 2 : Growth rate probability distributions for firms a, b and c.

The derivation of the y/k – Z/k loci for firms b and c proceeds exactly as for a, except that the market values of the two additional firms are taken to be 180 and 170 respectively : this requires that two new rectangular hyperbolas be drawn in quadrant IV, one for each market value.

Having derived a y/k – Z/k locus from each probability distribution of growth rates, the task of selecting the most likely true value of k is made easier by identifying on each locus the point corresponding to a particular value of k and marking it with a common identifying symbol. Thus in quadrant I of Diagram 5 the point on each locus corresponding to k_e = .08 is indicated by a small square. Similarly, k_e = .085 is indicated by a small circle and k_e = .09 by a small triangle on each locus. These three possible values of k are sufficient to suggest the range of k_e within which the true value of k must lie. Evidently the only set of points traced out in this way which satisfies even the broad conception of the valuation relationship $y = h(Z)$ is that corresponding to k_e = 0.09, the locus identified by the triangular marks on the individual y/k – Z/k loci. With either k_e = .08 or k_e = .085 it is simply not possible to

(7)   As with the firm's own growth prospects, management must infer the views of investors in carrying out this exercise.

connect the identified points (circles or squares, respectively) into a
locus satisfying the requirements of concavity to the origin and vertical
intercept at $y = 1.00$. On the other hand it is obvious that for firms a
and b the values of y for $k_e = .095$ would exceed unity, and so it is
established that the true value of k lies in the region of 0.09.
Finally, it should be noted that this procedure requires no very precise
knowledge of the shape of the true valuation relationship, $y = h(Z)$, and
certainly does not require identification of the value of z at which y
becomes zero or some other defined low value. Provided that several widely-
spaced $y/k - Z/k$ loci can be derived it is clear that the shape and position
of a locus of points representing a particular k_e is extremely sensitive
to changes in k_e, so that the range within which the true value lies is
both narrow and easily discernible. This property represents a distinct
improvement upon the identification procedure outlined in the preceding sub-
section, in which only one set of conditional S-values was assumed to be
available.

Section 4 : Conclusion.

It is now obvious that the recognition in Chapter 8 of the likelihood of multiple growth possibilities of expected dividend necessitates both a radical change in the interpretation of market valuation from that employed up to Chapter 7 and a reconsideration of the practical usefulness to management of knowledge of the value of k. The first of these requirements has been the subject of Sections 2 and 3 of this chapter, while the reason for the second deserves mention here prior to a full discussion in Chapter 11.

The problem of ensuring constant dividend risk in growth situations was solved differently in Chapters 7 and 8. In Chapter 7 a unique growth-path was assumed, for which constant dividend risk could in principle be derived. (It was shown in Chapter 8 : 2 (i) that even when management's information is limited to knowledge of borrowing plans and investment in the present period, together with the Q-distribution and investment level for next period, a constant-risk dividend commitment for next period can be undertaken.) The same model generated the firm's cost of capital to be applied to individual investment opportunities. Given the cost of capital and the limited nature of the risks affecting investment projects (there is no risk associated with the position of the Δ Q-distribution), this single investment criterion was completely adequate. In Chapter 8 this simple view of investment decision making was discarded, and the nature of the risks affecting investment profitability and the probabilistic nature of investment demand were both recognised. It was suggested that the investment process could best be understood as the working of a chosen strategy, and Chapter 11 examines the formulation of such a strategy in simple terms. But the implications for the validity of a cost of capital concept are already clear : the criterion for acceptance of an individual proposal is no longer as simple as was suggested in Chapters 5 and 7.

Recognising the full dimensions of investment risks, a model describing investment choice under risky conditions is a main component of the growth model, and is instrumental in generating the probability distribution of growth-paths of constant-risk expected dividend. [8] There is no longer a need for a simple cost of capital criterion : the parameters of investment strategy replace the cost of capital test, and in contrast to the market-determined nature of the latter, most features of investment strategy are necessarily arbitrarily chosen.

Given the conditions defined, there is no way of avoiding this switch of emphasis away from the identification of a simple criterion for making individual investment decisions. The managerial problem lying behind the planning/simulation models in Chapter 8 was to ensure that, whichever growth path was followed, ex ante dividend risk would always be the same : a probabilistic model of total investment demand was a necessary part of the conditional planning of financing and dividend policy for this risky growth situation. In the light of these developments, with investment decisions taken with reference to a pre-determined strategy, a cost of capital concept as previously understood and employed, has become redundant.

The relevance of k to managerial decision making may also appear to have ended, with investment demand determined as described in Chapter 8. Indeed, the need for any managerial understanding of valuation may be questioned for the same reason : if investment decisions are made with reference to some arbitrary pre-determined general strategy, and if management is already committed to a given level of dividend risk whichever growth-path occurs, the impression that can be gained by simulating growth experience would seem to provide management with as much information as it

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(8)   The formulation of investment decision rules in the conditions described
      is considered at length in Chapter 11 : here it is sufficient to
      assume that rules for choice exist.

desires about the range of possible outcomes of its policy. Unless
some form of optimising in terms of equity market value is intended,
it is unnecessary for management to identify the market valuations of the
various possible growth-paths of expected dividend — a task which requires
the identification of k.

The continuing relevance of market valuation to management depends
therefore upon the latter's assumed desire to optimise (in some sense)
in terms of the value of shareholders' equity — or, at least, upon a desire
to possess the option of doing so. In this event the area in which manage-
ment would exercise choice is the specification of the firm's investment
strategy, not its dividend risk policy. As Chapter 8 showed, these two
features in the conditional planning model are separable, so that a given
investment demand model may be combined with a range of dividend risk/debt
financing options — as long as the implied debt-financing arrangements
remain feasible. However, in the typical situation management will wish to
maintain a constant level of dividend risk simply because of the difficulty
of interpreting the market valuation of its equity if that policy were to
be replaced by one of an indeterminate or changing dividend risk. This
separation of the different aspects of risk allows the underlying risks
of a business — those originating in its investment strategy — to be
deliberately modified by management in an attempt to influence V_e without
at the same time confusing the market and destroying its confidence by
arbitrary shifts in dividend risk. Obviously there must be limits on the
ability of management to achieve compatibility between investment strategies
varying in their acceptance of risks and a constant target level of dividend
risk : in some cases the (conditional) burdens placed upon debt-financing
will exceed what the firm — or capital market — will tolerate. Identifying
the points at which such limits will operate is not the present concern :
it is sufficient to note here that management may alter its investment

strategy, and that as a result the probabilistic investment demand function and the probability distribution of overall investment profitability will both acquire new parameters.

The importance of the valuation process in the new context is now clear. Management can, in principle, choose an investment strategy on the basis of its implied prospect for equity value, subject as before to a constant level of dividend risk on all possible growth-paths. The importance of k is that it applies to each possible stream of expected dividends, regardless of the investment strategy followed, and the parameters of the S-distribution for each strategy depend crucially on its value. If management wishes to choose between strategies on this basis, knowledge of k is a necessary but not a sufficient condition : the wider market valuation relationship (represented here by $y = h(Z)$) must also be understood. The shape of a new S-distribution can be calculated given k, but its market value can only be estimated with the help of some conception of the effect of the dispersion of an S-distribution upon its market value.

The important conclusion drawn from Section 3 is that initially unrelated and inconsistent views of the internal and external aspects of valuation , k and $y = h(Z)$ respectively, may be systematically and fruitfully reconciled by the kind of procedure described. The assumption that management is prepared to review its ideas in this kind of way is a cornerstone of the analysis of strategic decision making to be developed in the following chapters.

Chapter 11

Investment Strategy and Managerial Utility (I).

Plan of Chapter

Section 1 :   Introduction.

Section 2 :   Risks in the Level and Profitability of Investment.

    (i)   The assumption of constant business-risk.

    (ii)   The overall profitability of investment.

    (iii)   The variability of total investment demand.

Section 3 :   An Illustrative Model of Investment Strategy.

    (i)   A simple way of describing strategy.

    (ii)   The possibilities for changing investment strategy.

    (iii)   A classification of states of confidence about strategy outcomes.

    (iv)   The strategy frontier in relation to capital budgeting methods.

Section 4 :   Managerial Utility and Strategy Choice.

    (i)   A value-based managerial utility function.

    (ii)   Expected utility as a criterion for strategy choice.

    (iii)   Applying uncertainty criteria in strategy choice.

    (iv)   The problem of consistency in ranking strategies.

    (v)   A basis for the comparability of all strategies.

Section 5 :   Conclusions.

    (i)   Summary.

    (ii)   Flexibility in investment strategy, (I).

    (iii)   Flexibility in investment strategy, II.

    (iv)   The acceptance of imperfect probability estimates.

Section I : Introduction

In Chapter 10 a managerial interpretation of the equity valuation process was suggested for the situation of multiple growth possibilities. It was shown that management may in principle identify the rate k at which its constant-risk expected dividends on all possible growth-paths are discounted by investors, so that the firm's own S-distribution can be identified. At the same time, as part of the process of interpreting the valuation of its own growth possibilities, management can acquire sufficient understanding to predict the market values of S-distributions other than its present one. [1] Management's ability to reason in this way and its interest in doing so are both taken for granted throughout this chapter.

Throughout Chapters 8 and 10 it has been assumed that the firm operates a fixed policy on the selection of investments, though the terms in which such a policy might be expressed have not been considered. The policy's outcome has two aspects: a probabilistic determination of the level (and composition) of investment demand in each period, given the growth-path the firm has actually followed; and risky investment profitability. These aspects of investment, which are jointly responsible for the multiplicity of growth possibilities under a given investment strategy, are reviewed in Section 2. [2] Section 3 considers the general form in which investment policy is likely to be expressed, the degree of freedom enjoyed by management in arriving at its exact specification, and the possibility that managerial confidence in estimates of S-distributions may vary between alternative policies. Making use of a deliberately naive managerial utility function for the purpose, Section 4 takes up the last of these questions in considering the possibilities for a rational choice of policy.

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(1)   See Chapter 10 : 3 (iv).

(2)   Strictly speaking, either aspect on its own could generate a multiplicity of growth possibilities, but it seems most likely that both are present in the real world.

The managerial utility function employed in Section 4 is admittedly naive. However, because one view of management's over-riding objective is the maximisation of the present-day market value of investors' wealth, one task of Section 4 is to show that even with the simple utility function which appears to correspond to such a clearly stated objective, the unavoidable complexity of policy choice may prevent the adoption of a value-maximising policy. [3]

Here and in the following chapter equity market value is seen as the determinant of managerial utility, and although a lengthy explanation and justification for this approach is given in Chapter 12 in the context of a more realistic utility function and a fully-developed model of decision-making, some preliminary justification is called for at this stage. [4] To begin with there is the point already made, that at least the suggestion cannot be dismissed on the grounds that it oversimplifies management's view of the problem. Two more positive considerations may be mentioned here. The first is that the kind of managerial search hypothesized in Chapter 10 : 3 for an understanding of valuation has as a logical raison d'etre a desire to improve market value (and managerial utility) by means of policy choice.

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(3)   In a static or comparative-static context the maximisation of investors' wealth involves adopting the optimal capital structure and following a simple cost of capital decision rule. These two topics have been fully explored in Chapters 3, 5 and 6. There are, however, suggestions that in a fully dynamic context the maximisation of the present-day value of investors' wealth is a feasible, correct and likely managerial objective. Two examples of the simple view that management can and/or should attempt to maximise in this sense are : J.T.S. Porterfield, Investment Decisions and Capital Costs, 1965, pp. 16-17; and E.M. Lerner & W.T. Carleton, A Theory of Financial Analysis, 1966, Chapters 8 - 10.

(4)   A wide range of theories of the firm, including those which emphasise the ownership/management dichotomy, assume that management is concerned about the market value of equity. The explanations of this concern vary, and the particular approach developed here and in Chapter 12 will be compared with others in Chapter 14.

Second, the conceptual distinction between _subordinate_ and _top-level_ decision-making to be suggested in Chapter 12 : 4 means in effect that management necessarily approaches utility maximisation in two corresponding stages. Subordinate policies are those which must be determined prior to top-level strategy in order to provide a settled background of assumed operating policies against which the latter, which certainly includes investment policy, can be determined. Aspects of performance convered by subordinate policies do not therefore appear in the managerial utility function relating to the top-level combination of policies: the nature of subordinate decision processes need not be explored, but their effect is to leave for possible inclusion in the top-level utility function only those aspects of performance not covered in one way or other at the subordinate level. Prominent among such excluded performance variables, it is assumed, is equity market value; and, given the strong causal link between investment policy (an element in top-level strategy) and equity market value (a top-level performance variable), it seems reasonable to begin by specifying a top-level utility function in terms of that variable, which management attempts to maximise through its choice of investment policy. In thus eliminating other possible components of top-level policy-making and other possible performance aspects determining top-level managerial utility, the top-level decision problem has been arbitrarily simplified: the obvious importance of, and correspondence between the single elements remaining on each side is some justification for this procedure, which in any case is considered at much greater length in Chapter 12, Sections 4 and 5. [5]

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(5)   The reduction of strategy to the single element of investment policy means that the latter can be described in the remainder of this chapter as _investment strategy_: this preserves the distinction between subordinate policies and the set of top-level policies comprising overall strategy.

The important methodological issue which emerges here is whether managerial objectives, thinking and decision making can be legitimately partitioned in the way suggested, so that in effect management's problem is to maximise an objective function in limited terms subject to a varied and lengthy list of constraints. The contrast between this conception and its obvious alternative - in which a decision-maker trades off all desired and undesired aspects of performance against each other and in effect attempts to reach the highest attainable indifference curve in his all-embracing utility function - is considered in Chapter 12 : 4 (ii). For the present the simple concept of managerial utility as a function of equity market value is adopted, on the understanding that no other managerial objectives are adversely affected by the choice of investment strategy.

Section 2 : Risks in the Level and Profitability of Investment.

11 : 2 (i) The assumption of constant business risk.

In earlier chapters the subjects of risk and managerial behaviour in risky conditions have been considered in two distinct contexts. In Part I discussion related entirely to the firm's Q-distribution, while in Part II it is argued that managerial policy on investment influences the nature and extent of the risks involved in the growth process. Before examining the elements involved in this latter context it is worthwhile to summarise the nature of the risks and the role of managerial choice in the essentially static and comparative-static context of Part I.

The derivation of the firm's Q-distribution, its probability distribution of periodic earnings, was demonstrated in Chapter 2. The shape of the distribution indicates the level of business risk confronting the firm, and a conventional measure of this risk is given by the coefficient of variation of the distribution - its ratio of standard deviation to mean value. The

nature of the risks involved and the level of business risk are not matters over which management is unable to exercise choice. First, management chooses a particular line of business activity in which to invest : the competitive situation in its chosen activity becomes a _datum_ only after the committment has been made. Even then the level of risk appearing in the Q-distribution remains partially under control because of management's freedom to choose a set of operating policies for handling the various developments that may occur. Conditional competitive behaviour and the internal running of the business are obviously areas requiring the extensive exercise of managerial choice, and the shape of the Q-distribution will be influenced by the many conditional decisions that must be taken.

Throughout Part I it was assumed that investment makes no difference to a firm's business risk, and that assumption has been retained so far in Part II. The difference between Parts I and II is that in the latter the actual position of the new distribution is not known for certain before investment takes place, whereas this information is given _ex ante_ in Part I investment situations. Following the abandonment of the attempt to apply a simple valuation model to the multiple growth-path situation, there is really no theoretical requirement from here on that business risk should remain constant. There is, however, a slight benefit from retaining the assumption, and in practice it can be expected that business risk will be altered only marginally by a typical mixture of new investments. [6]

11 : 2 (ii) _The overall profitability of investment._ [7]

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(6)   The benefit derived from the assumption of constant business risk relates to the task of attaching precise parameters to management's investment strategy : see Section 3 (ii) below.

(7)   In keeping with the usage adopted in Chapter 8 : 1, the term _profitability_ continues to refer to the mean value of an investment's ΔQ-distribution. In the present context profitability in this sense is subject to _ex ante_ risk.

An extra dimension of risk has been introduced in Part II, in the shape of the combined effects of the factors affecting investment profitability and investment demand. Although constant business risk is still assumed the two additional risk factors have the effect, as shown in Chapter 8, of making the growth-path of expected dividend subject to a probability distribution instead of being uniquely given. In the situation now envisaged the choice of investment strategy affects both the probability distribution of overall investment profitability and the parameters of the total investment demand function. These effects on the basic determinants of the firm's probability distribution of growth-paths are considered in turn, beginning with those relating to investment profitability.

At the outset it is helpful to recall the distinction made in Chapter 8 : 1 between the factors determining once-for-all an investment's profitability - its $\Delta \overline{Q}$-value - and those which in each period of its operation determine the actual ΔQ-value generated from its fixed ΔQ-distribution. The latter aspect of risk has been sufficiently discussed in Chapter 2 and summarised in sub-section (i) above, and its importance here is two-fold. First, as suggested in chapter 8 : 2 (iii), periodic earnings are likely to have a separate effect upon investment demand. Second, the assumption that the firm's investments leave its business risk unchanged implies a relationship between the mean and dispersion of each investment which requires some elaboration.

A fixed set of rules governing the selection of investments - the essence of an investment strategy - results in the choice of a number of projects, for each of which there exists ex ante a probability distribution of ΔQ-distributions. (The total of such investments is itself a probabilistic variable, to be considered in the following sub-section). The assumption made about the joint outcome of all investments is that their actual ΔQ-

distributions in combination with the initial Q-distribution yield an
unchanged level of business risk. This result is not necessarily assumed
to emerge from deliberate manipulation of the mixture of investments chosen,
but is seen as a likely tendency of a typical mix of investments each of
which is chosen on the basis of its own risk and return prospects. Naturally
if management actually aims at this result its attainment is even more
likely. [8]

Apart from the assumption that management expects business risk to
remain constant, there is very little that needs to be specified about the
results of an investment strategy. For example, the expected profitability
– the ex ante mean of all possible $\Delta\bar{Q}$-values – of a typical investment can
be allowed to change through time to reflect expected trends in the rate
of return on investment. Further, such developments need not be expected to
be common to all growth-paths : at a given point in time different
prospective growth-paths might be experiencing quite different probabilistic
structures determining investment profitability. In short, the model could
scarcely be more general in terms of the possibilities for overall invest-
ment profitability permitted to develop in the course of an investment
strategy's operation.

The profitability of an investment in the sense defined may be subject
to ex ante risk for various reasons. Management may achieve variable
success in exploiting potential profitability once an investment has been
selected. Many factors external to the firm may result in an investment's
Q-distribution settling in one position rather than another. Here it is
sufficient to indicate in this very general way the reasons for expecting
management to see investment profitability as subject to ex ante risk.

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(8)   See note [13] below, and note [6] in Chapter 8.

11 : 2 (iii) The variability of total investment demand.

The second major risk factor introduced in Chapter 8 was the probabilistic total investment demand function arising from the operation of a given investment strategy in a particular period on a particular growth-path. This concept replaced that of a unique investment demand for defined circumstances. Having established its investment criteria, management attempts to forecast the total and composition of investments which at any time will satisfy the stated requirements; and as suggested in Chapter 8 : 2 (iii), a relationship between performance and investment demand is likely to be indicated. The relationship may well involve lags, and is likely to incorporate risk factors which make impossible the prediction of a unique investment total. Given its current and recent performance, reflected completely in the growth-path it has been following, and given its investment criteria, it is reasonable for management to allow for dispersion of the possible total of investment demand for any period.

Another interpretation, which does not exclude the first, is that the "stock" of new opportunities at any time is itself a risky variable, given the firm's current and recent performance; and even with constant "efficiency" in the search for new opportunities the quantity of identified opportunities at any time depends upon the variable flow of newly generated opportunities.

Section 3 : An Illustrative Model of Investment Strategy.

II : 3 (i) A simple way of describing strategy.

As indicated in Chapters 8 and 10, the basic assumption made in Part II is the existence of a set of rules whose application in all of the

situations envisaged by management generates probabilistic investment demand
and investment profitability levels. The probability distribution of growth-
paths derives ultimately from the investment criteria observed, and
management is assumed able to obtain a reasonably clear picture of that
distribution, at least for its chosen investment criteria. In this chapter
a major step is taken, with the additional assumption that management is
prepared to estimate probability distributions of growth-paths for a number
of alternative sets of investment criteria (investment strategies). This
section suggests an extremely simple way in which investment strategy may
be stated by management; and although much of the detail of capital budgeting
procedures is inevitably sacrificed, the main purpose - the examination of
the additional assumption referred to - can best be pursued within a simple
framework. In defence of this procedure it should be emphasised that
representing managerial approaches to investment strategy is not the central
purpose here : approaches are bound to vary, but as long as their outcomes
can be described in terms of a probability distribution of growth-paths the
analysis of Part II is applicable.

The elementary approach here is to suppose that management judges
an individual investment in terms of the probability distribution of its
rate of return, with particular importance attached to the distribution's
mean and standard deviation, $\bar{r}$ and σ_r respectively.[9] A frontier of
acceptable combinations of $\bar{r}$ and σ_r is shown in Diagram 1 , to illustrate
the suggestion.

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(9)   A project's rate of return, r , is defined here as the ratio of its
ΔQ value to its investment cost. Accordingly, on a risky investment
r is a risky variable, with $\bar{r}$ its mean value and σ_r the standard
deviation of its probability distribution. In general, of course, one
should envisage a probability distribution of the internal rate of
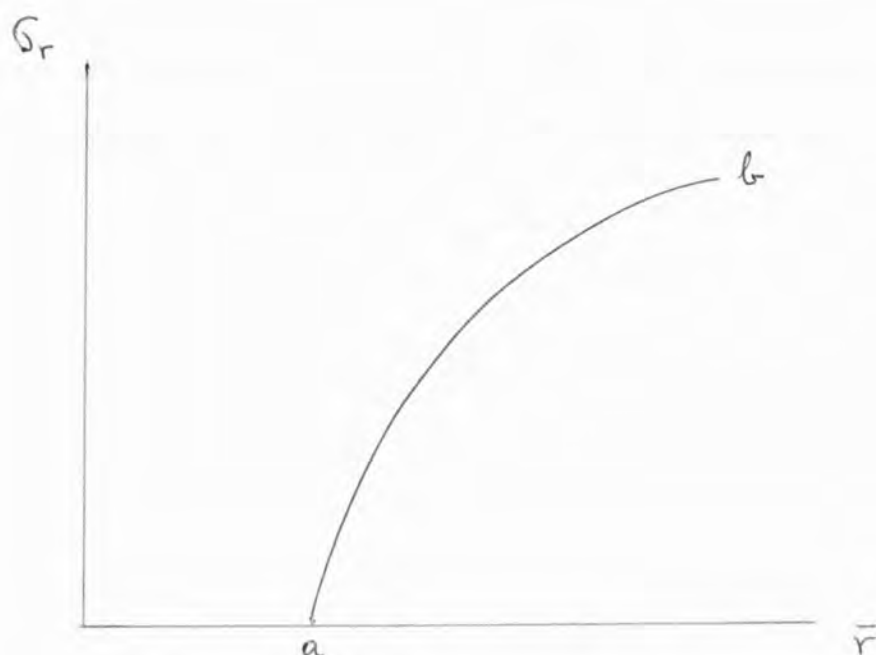return when the project's Q-distribution is not assumed unchanging
through time.

σ_r

b

a

$\bar{r}$

Diagram 1 : A locus of marginally acceptable $\bar{r}$, σ_r combinations.

Acceptable combinations of expected rate of return on investment and risk lie to right of the frontier ab , unacceptable combinations to the left. The shape of the frontier itself needs little explanation, and it is assumed here that management's rules governing investment selection - its investment strategy - are expressed in this form. [10]

(10) The shape of the ab-frontier intentionally resembles that suggested
      in a rather different connection by F. and V. Lutz, The Theory of
      Investment of the Firm, 1951, pp. 188-92. The Lutzes actually
      considered the risk associated with the net present value of a
      programme of investments, rather than that associated with the rate
      of return on a single item of investment, and their intention was
      to describe management's optimising technique with respect to its
      entire capital budgeting problem. In the present context the ab-
      frontier is seen as a preliminary stage in selecting possibilities
      for inclusion in the firm's periodic capital spending programme
      (see footnote 13 below), though its general shape is explained
      in the same terms as the Lutz indifference curves : an aversion
      to risk which can be compensated by higher expected return._ The
      ab-frontier is an indifference curve in the sense that all r, σ_r
      combinations falling upon it are marginally acceptable to
      management at that stage of its capital budgeting exercise.

11 : 3 (ii) The possibilities for changing investment strategy.

A change of strategy is a matter of shifting the frontier of acceptable $\bar{r}$, σ_r combinations, and the ways in which this can be done are shown in Diagram 2.
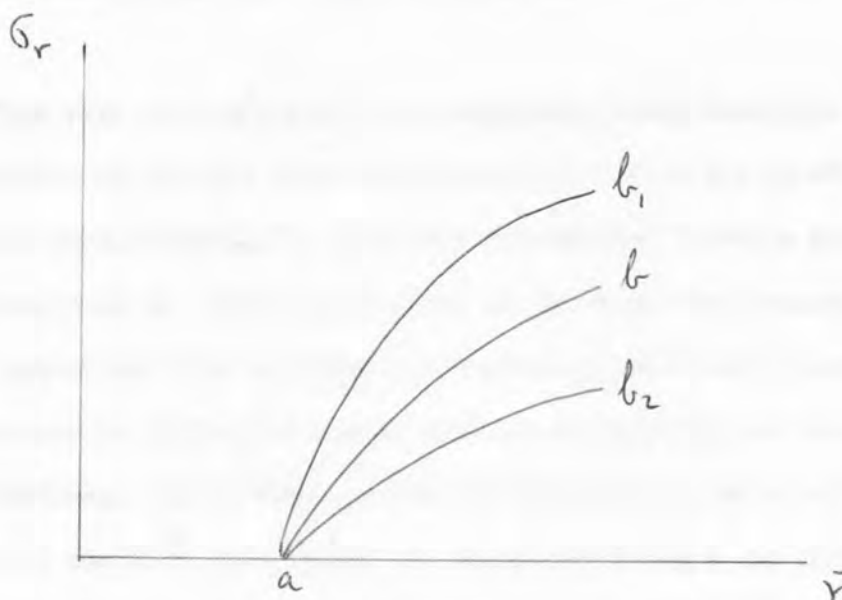

Diagram 2 : Possible changes in investment strategy.

The original frontier ab is retained in Diagram 2, and the changes illustrated are confined to those which maintain the position of point a on the $\bar{r}$ axis.

For the explanation of this convenient simplification the argument in Chapter 7 : must be recalled. The general effect of that chapter was to show that for a firm undertaking "safe" investments the correct measure of the minimum acceptable rate of return was given by its market value weighted average cost of capital. [11] This measure was shown to be correct in

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(11) A "safe" investment is taken to be one for which the ΔQ-distribution is known with certainty before the investment is undertaken. Likewise, a "risky" investment is one for which the ΔQ-distribution is seen ex ante as subject to an identified probability distribution.

principle even when the details of the future paths of investment and expected earnings were themselves unknown to management. Applying this result to the present context, it can be stated that, whatever the firms growth-path may turn out to be, investors would prefer the firm to accept a "safe" investment satisfying the MV/WACC criterion.

This view is confirmed by the valuation model developed in Chapter 10. Irrespective of how the joint valuation of all possible growth-paths is explained or understood, if investors are assumed to value each growth-path individually in the preliminary phase of the valuation process, then a "safe" investment with a satisfactory rate of return will prove acceptable to investors no matter how events develop subsequently as other investments are undertaken. It is thus possible to identify the value of $\bar{r}$ in Diagrams 1 and 2 at which point a should be fixed : no matter how the rest of the frontier is drawn, the minimum acceptable $\bar{r}$ for zero risk ($\sigma_{\bar{r}} = 0$) should equal the MV/WACC. [12]

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(12) The comparative-static cost of capital remains identifiable even in the different valuation context now assumed. Let k be identified as in Chapter 10 : 4 , and let i be given. (All terms retain the meanings assigned in Chapters 5 and 7) Assume any static Q-distribution with business risk, v , equal to that of the firm in question, and dividend risk policy such that

$$v\,\bar{Q} \over \bar{Q} - iB} = R$$

Then $Sk = \bar{D} = \bar{Q} - iB = {v\,\bar{Q} \over R}$. Thus $\bar{D}$ is determined in terms

of $\bar{Q}$, v, R; and this gives the appropriate value of B. Also, given k, S is determined when $\bar{D}$ is known. Thus the weights ${S \over S + B}$ and ${B \over S + B}$ required for the MV/WACC are identified in the abstract, as is the MV/WACC itself.

What has now been established is a clear distinction between, on the one hand, a firm's necessarily arbitrary trade-off between $\bar{r}$ and σ_r for risky investments and, on the other, the logical requirements of its dividend risk policy and the valuation theory relating to "safe" investments. The remainder of this chapter describes the difficulties management may encounter in attempting to establish a consistent order of preference among alternative strategies, taking the position of the a b frontier to be the central element of any strategy. As a preliminary to this important subject, management's ability to identify a strategy's probability distribution of outcomes must be examined.

11 : 3 (iii) A classification of states of confidence about strategy outcomes.

If management can identify the probabilistic investment demand function and related probability distribution of overall investment profitability for any situation arising under an investment strategy, it can in principle identify the complete probability distribution of growth-paths associated with that strategy, and thus the strategy's S-distribution. In such cases there need be no difficulty in adopting whichever strategy yields the S-distribution most highly valued by management. On the other hand, there may be strategies whose consequences cannot be completely worked out in terms of an S-distribution with the same degree of confidence that management attaches to the distribution associated with its present strategy. The ways in which such uncertainty may be defined, and the differing conclusions which follow the definition, can be shown by dividing the area of the $\bar{r}$, σ_r quadrant as shown in Diagram 3.
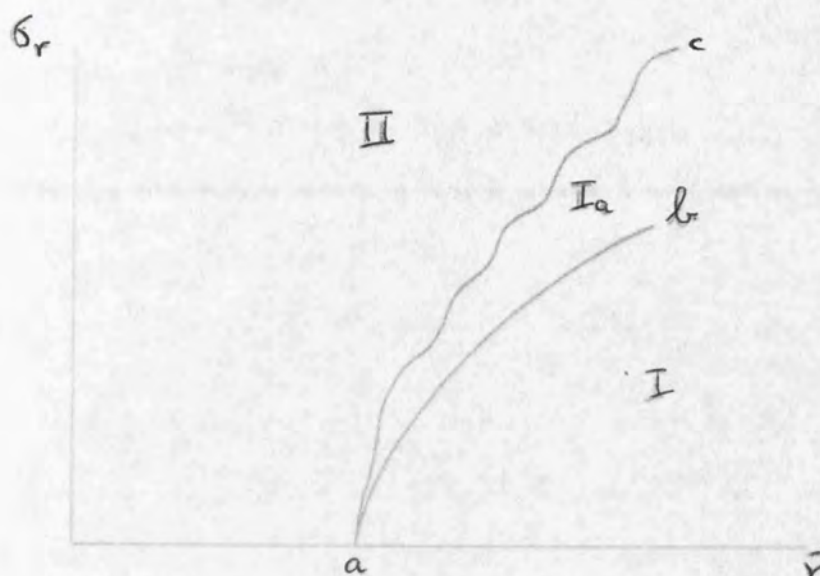
Diagram 3 : Classification of investment strategies.

An area of the $\bar{r}$, σ_r quadrant may be classified according to the confidence management places in its ability to derive an S-distribution for a frontier including combinations within that area . In Diagram 3 the firm's present strategy is represented by the frontier ab, and it is assumed that management has been able to calculate with a satisfactory degree of confidence the associated S-distribution. Given that the frontier ab includes the whole area to its right and below it, it follows that any strategy within this area - designated as area I - can be as confidently converted into its S-distribution as can that of the frontier itself.

The area of high managerial confidence in the ability to derive a strategy's S-distribution may extend to the left of ab, and this possibility is suggested in the area designated as Ia on the diagram. For the reason given earlier, the lowest point of area Ia coincides with point a on the $\bar{r}$ - axis. Finally, the area not classified in either of these ways is designated as area II, and it is sufficient for the moment to say that a

shift of the frontier into this area (still pivoting on point a) would mean a lower level of ex-ante managerial confidence in the estimate of the resulting S-distribution than is the case with the present or area I and Ia strategies. The reason is that the inclusion of an unfamiliar area of $\bar{r}$, σ_r combinations may cause management to feel less certain about the pattern of investment opportunities occuring and about the probability distribution of overall investment profitability than it is in dealing with familiar areas.

Strategies will be designated as area I/Ia strategies if their frontiers run only through areas I and Ia (a mixture of I and Ia is not different in any important respect from either pure type). An area II strategy is one whose frontier runs wholly or partly through area II. This classification presupposes an existing strategy frontier, but is equally applicable if a "strategy vacuum" is assumed — for example in the case of a newly established business.

11 : 3 (iv) <u>The strategy frontier in relation to capital budgeting methods.</u>

As stated in 11 : 3 (i) above, representing the rules governing investment choice in the terms employed here is an over-simplification. Its justification is that it provides a clear picture of the possibilities of differing managerial states of knowledge relating to different positions of the frontier. Reinforcing this practical point is the likelihood that some form of frontier does typically figure in the investment choice procedures of firms; and, indeed, that it is the position of such a frontier which management is most likely to vary in defining a change of strategy.

It was argued that no matter how strategy is formulated, a sufficient condition for the applicability of the managerial theories developed in Part II is that management be able to identify the probability distribution of

growth-paths generated by the interactions of investment demand, investment profitability and financing policy. Nevertheless, having suggested the $\bar{r}$, σ_r frontier as a model of rules governing investment choice, the extent of its oversimplifying ought to be made clear. By itself the frontier cannot completely define the rules for choice, even if for convenience all proposals are assumed subject to the same frontier regardless of their cost or importance. For example, in some situations total investment demand arising from the selection of proposals on this basis may exceed what management expects to be able to finance in its customary ways ; and conditional schemes of capital rationing must therefore be envisaged and their effects taken into account. It is not essential that management's choices in such situations be completely predictable - all that is required, as in all situations, is that probabilistic forecasts of total investment with related forecasts of overall investment profitability can be obtained.

Thus the concept of investment strategy must be understood as subsuming the operation of all aspects of investment decision making in all possible circumstances. [13] The $\bar{r}$, σ_r frontier has been singled out to illustrate clearly the general proposition that management may experience different degrees of confidence in describing the outcomes

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(13) The $\bar{r}$, σ_r frontier is not to be seen as a complete account of how management chooses the total and composition of its investment spending. It is admittedly insufficient by itself to generate the kind of probabilistic relationships between investment and the firm's performance that were suggested in Chapter 8 and are assumed throughout Part II. Rather, the frontier is seen as a preliminary stage - perhaps one of several - in identifying possibilities for the investment programme, with appropriate techniques of capital budgeting then being applied to the preliminary selection in accordance with whatever objectives and conditions management imposes. Capital budgeting theory is well reviewed in J.C. Van Horne, Financial Management and Policy, 2nd Ed., 1971, Chapters 5 and 6; and in J.C.T. Mao, "Survey of Capital Budgeting: Theory and Practice, "Journal of Finance, (1970), pp.349-60.

(in probabilistic terms) of different investment strategies. Because
it is possible to envisage shifts in the frontier forming the basis for
changes in strategy, with the remaining conditional rules for choice
remaining unchanged, investment strategy is from now on taken to be
synonomous with the position of the frontier. The fact this model over-
simplifies reality should be borne in mind, but the model itself is only
intended as an expository device.

Section 4: Managerial Utility and Strategy Choice. [14]

11 : 4 (i)   A value-based managerial utility function.

In order to gain insight into the general nature of management's
problem of strategy choice, this section introduces an extremely simple
managerial utility function in which utility at the present day depends
upon the market value of equity :

$$U = U (V_{eo}) \hspace{3cm} (i)$$

Although not strictly necessary to the following discussion, it can be
assumed that the utility function is conventionally shaped, with first
and second derivatives positive and negative respectively. This utility
function is assumed to be the one consulted by management in its choice of
investment strategy. As Chapter 12 will show, (i) must be seen as only
a special case of a much more general formulation; but the proposition that
management can, should and desires to maximise the present-day market value

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(14) Throughout the discussion of managerial utility functions it is
assumed that management is capable of unifying its preference system.
This assumption is of course commonly made in managerial theories,
and its justification is that top-level or strategic utility is
what is in question. For a discussion of differences in utility
functions attributable to different positions in management structure,
see J.S. Hammond, "Better Decisions with Preference Theory", Harvard
Business Review, (1967), pp.139-40. This problem certainly arises
under a partitioned decision model, and is overcome in practice
either by imposing top-level preferences or by acceptance of
subordinate attitudes. Either way, top management knows what to expect.

of shareholders' wealth is sufficiently widespread to make worthwhile
its examination in the context of a suitably expressed managerial utility
function.

Evidently, a management perfectly informed about the S-distributions
of all investment strategies could immediately maximise its own utility and
shareholders' wealth by choosing the value-maximising strategy. Here the
classification suggested in Section 3 (iii) becomes applicable. Area I/Ia
strategies present no problems of confidence in information or in its
availability, and management can readily identify the value-maximising
strategy. The position is quite different, however, when area II strategies
have to be appraised with a view to maximising V_{eo}. A number of assumptions
may be made concerning the nature of managerial uncertainty in estimating
S-distributions resulting from area II strategies, and indeed the kind of
uncertainty may vary according to the strategy in question. However the
uncertainty is defined, it can be shown that some departure from the simple
objective of value maximising is likely to result when area II strategies
are considered.

11 : 4 (ii)   Expected utility as a criterion for strategy choice.

One possible view is that management feels able to attach probabilities
to various possible outcomes (S-distributions) of an area II investment
strategy. Instead of being able to identify a unique S-distribution and
V_{eo} given the position of the $\bar{r}$, σ_r frontier, management feels itself
confronted with a probability distribution of V_{eo}. Here it is quite
inadequate to describe the strategic decision in terms of utility maximising
by means of value maximising, but the appropriate expression is obvious : the
expected utility value of strategy j is given by

$$EU(j) = \sum_{i=1}^{\infty} p_i U \left[V_{eoi}^j \right] \qquad (ii),$$

where V_{eoi}^j represents the ith possible outcome (S-distribution) of strategy j, and p_i is the probability of that outcome in management's view. The utility derived from V_{eoi}^j, should it occur, is represented by $U\left[V_{eoi}^j\right]$, and the expected utility value of the jth strategy is the probability – weighted average of all such conditional utility values. Given a number of alternative strategies, each with its own probability distribution of V_{eo}, the shape of the managerial utility function is important in determining the strategy which maximises expected utility.[15] Maximisation in this sense is a criterion which applies to strategies in areas I and Ia (where of course it reduces to the single outcome case) and to those in area II about which management is prepared to express its views in probabilistic terms.[16]

11 : 4 (iii) Applying uncertainty criteria in strategy choice.

However, if management feels unable to assign probabilities to the V_{eo} outcomes of a strategy in the way described above, the problem becomes one of "uncertainty" instead of "risk", and decision making has to be re-interpreted accordingly.[17] For the moment it is assumed that management

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(15) Even if management is seen solely as an agent of shareholders, the risk associated with V_{eo} makes value maximising an inappropriate definition of its objective and obliges it to apply a criterion devised by itself.

(16) It is not intended to suggest that maximising expected utility is the only possible description of rational decision making. The conditions in which the decision can be so described are set out clearly in W.J. Baumol, Economic Theory and Operations Analysis, 3rd edition 1972, pp. 587-91.

(17) The distinction between "risk" and "uncertainty" is a standard one in the literature on decision making. A rigourous classification of degrees of knowledge is provided in A. Kaufmann, The Science of Decision-Making, 1968, pp.136-8.

sees all area II strategies in this way. The extensive literature on
decision making under uncertainty makes it unnecessary here to do more than
describe and apply a small selection of criteria. [18] It is assumed that
management identifies the possible S-distributions and corresponding V_{eo}
values for a given area II investment strategy, but that probabilities
cannot be assigned with confidence to these various possibilities. The
great variety of attitudes for which matching decision criteria have been
suggested indicates the futility of pursuing a uniquely correct decision
rule in this situation. Further, even if management establishes a basis
for ranking alternative area II strategies, there remains the important
question whether sufficient comparability exists between such a ranking
and the ranking of area I/Ia strategies - in other words, whether and on what
basis a strategy from one area is comparable with one from the other.

Only three decision criteria are considered here, but these will be
sufficient to illustrate the nature of the comparability problem referred
to. The maxi-min criterion describes the choice between alternatives as
being made in favour of that policy whose worst possible outcome is better
than those of other policies. [19] According to this criterion manage-
ment's preferred area II strategy would be the one with the highest minimum
V_{eo} outcome. The problems of consistency and comparability between the
decision procedures for risk and uncertainty situations might be overcome
in this case by the use of the V_{eo}-value achieved by the best area I/Ia
strategy (which includes present strategy) as a yardstick for strategies

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(18) See Kaufmann, op.cit., Chapter 5, for an account of the various
decision criteria applicable to uncertainty.

(19) This criterion is also described as the mini-max loss or Wald
criterion. The term maxi-min is appropriate here because the decision-
maker envisages positive V_{eo} levels. See Kaufmann, op. cit.,
pp. 156-8.

in area II. Let the maxi-min criterion applied to area II yield V_{eo}^k (min), the minimum V_{eo}-value of the kth area II strategy, as the highest minimum outcome of any area II strategy ; and let V_{eo}^g be the best (unique) value offered by any area I/Ia strategy - the gth strategy in this area. Then if V_{eo}^k (min)$< V_{eo}^g$ no further consideration would seem necessary : according to the standards applied to area II outcomes, no strategy in that area measures up to the undisputed optimum area I/Ia strategy. It is interesting to note that this conclusion depends on an incomplete or "one-way" comparability between rankings of unique (area I/Ia) and uncertain (area II) outcomes : the reverse process of appraising area II strategies in terms of the accepted ranking procedure for area I strategies is not and cannot be attempted. On the other hand, in the unlikely event that V_{eo}^k (min)$> V_{eo}^g$, the results of the different ranking procedures are again sufficiently comparable to give a definite answer : the kth area II strategy is accepted because it offers a better minimum V_{eo} value than the _certain_ outcome of the best area I strategy.

The maxi-min criterion evidently reflects an extremely cautious managerial attitude, unlike the expected utility criterion assumed when probability information is adequate, and it certainly cannot be equated with a value-maximising objective. Its direct opposite, however, comes as close to that objective as the uncertainty of the situation permits : choosing from area II strategies the one offering the highest maximum V_{eo} possibility reflects a commitment to the attempt at value-maximising at any cost. Again it is possible to achieve one-way comparability between the separate appraisals of area I/Ia and area II strategies : the _maxi-max_ value of V_{eo} from the latter group is compared with V_{eo}^g defined above. [20] Let the former be denoted by V_{eo}^P (max), the highest possible

(20) According to the _maxi-max_ criterion the decision-maker chooses the policy with the highest maximum V_{eo} outcome. For an assessment of this rather unlikely criterion see Baumol, _op.cit._, p.577.

outcome of area II strategy p, where strategy p has been chosen because its highest outcome is above the maximum outcome of any other area II strategy. Then if V_{eo}^{P} (max) $\geqslant V_{eo}^{g}$, strategy p is chosen ; and if V_{eo}^{P} (max) $<$ V_{eo}^{g} , management chooses strategy g.

It should be emphasised that with both maxi-min and maxi-max the one-way comparability with area I strategy choice is only possible because choice is based upon a single possible outcome of a strategy - either the best or worst possible outcome. Had management been supposed to appraise area II strategies on the basis of the full range of their respective possibilities, the comparability question would need to be re-considered. It remains an open question which of the two extremes, maxi-min or maxi-max, is less realistic as a reflection of managerial attitudes on the fundamental matter of risk-taking over investment strategy.

A more sophisticated version of a basically conservative approach to decision making under uncertainty is offered by the mini-max regret criterion.[21] Under this rule the decision-maker chooses that policy which, if events turn out badly, will cause him the least amount of "regret" at having made the wrong choice. "Regret" at any policy outcome is measured as the cash (or utility) difference between that outcome and what could have been achieved had the best policy for the prevailing circumstances been chosen. In other words, the decision maker can visualise the extent to which he will, with hindsight, regret the outcome of any policy for any course of external events.

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(21) See Kaufmann, op.cit., pp.160-61, where the situation assumed permits a mixed strategy solution. The same criterion confined to the pure strategy case is discussed in Baumol, op.cit., pp.579-80.

The present case must be treated differently from the simple mini-max regret decision problem found in textbooks, in that whereas it is usual to construct a matrix showing the pay-off for each combination of policy and external events/state of nature/opponents' behaviour, such a version is impossible when, as in the present case, the nature of opposition or external events is uniquely related to individual policies. Accordingly, regret at choosing one investment strategy cannot be measured by identifying what would have been achieved under the same external conditions or determining events by a different strategy. The only comparison possible for each area II strategy is with the unique and certain value of V_{eo} offered by the best area I strategy - that is the value V_{eo}^{e} . Thus the mini-max regret criterion applied to the present problem reduces to the simple maxi-min criterion considered earlier : the identifiable maximum regret for area II strategy k is the excess of V_{eo}^{e} over V_{eo}^{k} (min) : and mini-max regret requires the decision-maker to select from all area II strategies the one with the highest minimum V_{eo} , then to compare this with V_{eo}^{e} . This is exactly the same procedure as for maxi-min.

Once again, one-way comparability between the choices made in different situations is possible in terms of the criterion applied to area II strategies. Mini-max regret, applied in the way made necessary by the limits on comparability between possible strategy outcomes, results in "predictions" of managerial behaviour which may appear to reflect a quite excessive and unrealistic degree of caution. Before rejecting such a criterion on this ground it is as well to recall the fundamental importance of the decision to which it relates : in choosing investment strategy for the foreseeable future a high degree of caution may well be typical of management's approach.

11 : 4 (iv) The problem of consistency in ranking strategies.

Thus far in seeking a common basis for appriasing investment strategy

frontiers in areas I/Ia and II it has been assumed that area II strategies could be treated as though all were subject to risk (sub-section ii) or all were subject to uncertainty (sub-section iii). For each of these states a degree of comparability between the appropriate criterion for area II strategies and that for area I strategies has been demonstrated, although the degree and nature of comparability differ according to the state of knowledge assumed for area II. The obvious questions now are whether all area II strategies are indeed subject to the same kind of doubt in management's mind ; and in the absence of such uniformity, whether comparability between strategies in all three situations (certainty, risk and uncertainty) is possible on the basis of the various decision criteria described. It is entirely possible that management will distinguish between area II strategies, seeing some in terms of probability distributions of V_{eo} and others as involving less quantifiable risks; and so it is necessary to consider whether the results of the decision criteria which may be applied in the three situations are comparable, and in particular whether area II strategies subject to differing kinds of doubt in management's mind can be ranked consistently.

The nature of the difficulty which may arise in an attempt to achieve a completely consistent ranking of all strategies is made plain in the example set out in Table 1. Area II strategies may be uncertain in the sense defined (1 to 3) or risky (4,5), while the best area I/Ia strategy (6) has of course a unique outcome. In uncertainty situations management is assumed to apply the maxi-min criterion, while in risky or certain situations its criterion is expected utility value.

Area	Strategy	Utility of outcomes			Minimum U	EUV
II (uncertain)	1	20	24	26	20	
	2	30	19	25	19	
	3	22	23	27	22	
II (risky)	4	28	25	20	20	24.6
	probability	.2	.6	.2		
	5	19	23	26	19	22.7
	probability	.3	.4	.3		
I/Ia	6		22.2		22.2	22.2

Table 1 : Utility values of alternative strategies.

If the only choices were strategies 4, 5 and 6, where management could maximise expected utility, it would choose strategy 4 with EUV = 24.6. The fact that its minimum utility is inferior to that of strategy 6 would be irrelevant.

If the only choices were strategies 1 to 3 and 6 where it would be appropriate for management to apply the maxi-min criterion, strategy 6 would be chosen : its (certain) utility exceeds the minimum utility of the next-best strategy, 3.

However, when strategies 1 to 3 and 4, 5 are all available, as well as 6, the basis for a non-contradictory choice is no longer obvious. Strategy 4 has the highest EUV of any strategy to which that criterion can be applied, including 6, and would seem to be superior to 6 on that basis. But under the maxi-min criterion 6 is superior to 3 which in turn is superior to 4. Thus, a criterion applicable to all situations, maxi-min, points

to the choice of strategy 6 : but the unresolved contradiction remains, that this "optimal" strategy is inferior to another, 4 , according to a presumably superior criterion. Although it is possible to apply a "lowest common denominator" criterion to all strategies regardless of management's ability to attach probabilities to their outcomes, the choice indicated by that criterion may well be contradicted when a partial comparison is made with the aid of an appropriate and superior criterion.

11 : 4 (v) A basis for the comparability of all strategies.

Two radically different interpretations may now be suggested. The first is that in the absence of probability estimates a logically consistent comparison between area I/Ia , area II (risky) and area II (uncertain) strategies is impossible, and that the firm will simply not contemplate strategies of the third type. The decision can then be made on the familiar and relatively uncontroversial criterion of maximum expected utility value which is of equal validity to I/Ia and II (risky) situations.

The second possible interpretation is that if management wishes to be able to consider all area II strategies it must be prepared to make probability judgements about the outcomes of those strategies hitherto classified as uncertain. Lending support to this interpretation of the response of decision makers to uncertainty is the argument that the state of mind identified here as "uncertainty" is highly unlikely to occur : it is difficult to accept the suggestion that a decision maker is capable of identifying precisely the possible outcomes of a particular policy but is quite unwilling to estimate their probabilities. Admittedly there may be differences in the confidence with which various sets of probabilities are viewed, ranging from complete confidence in coin-tossing probabilities on one hand to insubstantial guesswork expressed in the language of probability on the other ; and anyone intending to compare strategies in

terms of the probability distributions of their respective outcomes must be conscious of the desirability of making equally confident probability judgements for all strategies.

The fundamental question of methodology which lies at the heart of this discussion, namely the usefulness and validity of the concept of probability in decision situations, is discussed with clarity and insight by P.G. Moore. The following selection of remarks provides an admirable summary of the case for a probability approach: [22]

> "It is sometimes argued that results obtained through these numbers (probability estimates) are valueless because they require the decision-maker to attach numbers to the uncertain outcomes (even supposing he can fully enumerate them) and to the consequences. These numbers are, it is argued, so difficult to obtain as to make it hardly worth doing the subsequent calculations on the basis of so much vagueness. ..... But what we are really trying to achieve throughout this kind of approach is consistency, so that the various decision problems faced by a decision-maker will hang together and make a coherent whole. ..... We are not giving the decision behaviour of an idealised, rational and economic man. Rather we are concerned with an approach that will enable the ordinary person to reason and act more systematically and consistently than would otherwise be the case."

To sum up, it is certainly reasonable to expect that management will attempt to attach probabilities to the various V_{eo} outcomes of alternative strategies, thus making it possible to apply a common decision criterion to all area I/Ia and area II strategies. The distinction between risk and uncertainty is somewhat unreal, though it is grounded on real differences in the quantity and quality of available probability information. In spite of such differences management is assumed to be willing to make the best and most objective probability judgements which the situation permits and to employ these in the appraisal

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(22) P.G. Moore, Risk in Business Decision, 1972, pp.224-5.

of all strategies according to a uniform decision criterion. In defence
of this suggestion, it should be added that the rankings resulting from
applying such a criterion can be subjected to sensitivity testing of the
probability values used, and where it seems advisable, procedures for
confirming or revising probability estimates can be employed.

The importance of this conclusion is now clear. The ability of
management to choose consistently from a wide range of investment
strategies has been shown to depend on a willingness to form objective
probability judgements about strategies hitherto classified as "uncertain",
and then to compare all strategies according to a common decision criterion.
Unless this kind of approach can be assumed it is likely that very little
managerial consideration will be given to the fundamental issue of invest-
ment strategy, simply because area II strategies may be thought to have
incalculable consequences and because the present strategy was, at the
time it was chosen, superior to the alternatives in area I/Ia. Above all,
it must be assumed that management is sufficiently interested in the
outcome of its strategy choice to be prepared for the kind of exercises
in probability estimation required.

Section 5 : Conclusions.

11: 5 (i) Summary

The background to this chapter is management's recognition that a
settled investment strategy is important to its ability to comprehend the
valuation process. Investors can only value a range of expected dividend
streams whose probabilities are known. Management is aware of the
informational requirements of investors, and because it desires a workable
interpretation of valuation it attempts to provide the kind of constant or
predictable policy investors must take for granted in valuing equity in

the way assumed. Management's committment to constant-risk dividend policy is an example of its intentions and desires in this respect.

The actual terms in which investment strategy is specified are comparatively unimportant here, but one important general problem has emerged from the particular mode of expression suggested : management may not initially have an equal confidence in predicting the outcomes and probabilities of all strategy alternatives. In spite of this it will wish to describe each strategy in terms of its V_{eo} probability distribution as an essential first step in rational decision making.

Although the utility function employed in this chapter is a deliberate oversimplification, the logic of expected utility value as a decision criterion is accepted both here and in the extended discussion of managerial utility in Chapter 12. An important conclusion obtained with even the simple form (1) is that maximising EUV cannot automatically be equated with a straightforward choice of the strategy with the highest V_{eo}. Only if all strategies are seen ex ante as having unique market value outcomes does maximising EUV imply a straightforward maximisation of V_{eo}.

11 : 5 (ii) Flexibility in investment strategy, (I)

A major difficulty in accepting the above view of communication between management and investors concerns the apparent implausibility of a permanent committment to an unchanging set of rules for investment choice. The fundamental issue is whether management can expect investors to interpret its intentions as it would wish, and this is best considered in two stages – the first of which is the subject of this sub-section. Here the question is that of allowing for flexibility within investment strategy without prejudice to management's belief that investors can value the pattern of growth possibilities that results. This belief is simply

a question of whether management itself can identify all growth-paths and their probabilities under its strategy, because it is still assumed that management and investors eventually hold identical views about the operation of a given strategy.

It is easy to define ideal conditions in which flexibility in investment strategy does not prejudice the concept of an identifiable probability distribution of growth-paths. Management may recognise that future developments may cause it to modify the original strategy. But if at the outset it knows the probability of its doing so at any particular time and for any particular reasons; if the modified strategy which would then be adopted could also be predicted - if necessary as a probability distribution of possible modifications; and if the multiple growth possibilities of each possible modified strategy from the date of its inception could also be identified with probabilities : then it would be possible to widen the concept of strategy while retaining the basic belief that management (and investors) can identify the probability distribution of all possible growth-paths of constant-risk expected dividend.

This daunting condition should not be rejected out of hand as implying an unbelievable degree of foresight and conditional planning by management. The concept of strategy described in Section 3 allows for provisional decision making by management for occasions on which rigid application of rules would involve difficulty or conflict. Probably the best example of this is provisional planning to cover foreseeable capital rationing situations. Evidently built-in flexibility of policy to deal with a limited range of developments can readily be made part of a fixed investment strategy.

The same principle may be extended. If circumstances are foreseen in which new settings of strategy parameters will be chosen, and if the

consequences of the conditional new settings are fully understood, the
resulting model of an evolving strategy remains within the logical
limitation on flexibility.

Evidently a line must be drawn somewhere between types and degrees
of flexibility that can be accepted as possible (even if unlikely) and
those that must be rejected as logically impossible. That line should not
be drawn with reference to the likelihood of one or other type of flexibility:
on this ground alone flexibility would probably be limited to the occasional
bending of a constant set of rules, with conditional shifts in the parameters
of strategy dismissed as too unlikely. In logic, however, the line must
be drawn elsewhere, as the following argument demonstrates.

11 : 5 (iii) Flexibility in investment strategy. (II).

Apart from the question whether management can or does prepare an
enormously wide range of conditional changes of investment strategy, there
is a fundamental logical objection to an interpretation of flexibility
going beyond the possibilities considered in sub-section (ii) above.
Any more ambitious concept would involve predicting future response of
management to the recognition of errors in the assumptions it makes at the
outset. But the extra information which will be available in the future
to guide strategy modification is extremely difficult to visualise.

On the one hand, if management is correct in estimating investment
demand and profitability relationships (both in probabilistic terms) there
will be no new information at all : growth will have followed one of the
paths originally envisaged, and management will have no reason to change
what originally seemed an optimal strategy. This is what would happen
under either of the interpretations given to flexibility in sub-section (ii).
In the limited case the strategy preferred initially would remain superior

to all others as long as events developed along lines which did not give rise to a revision of the estimates and assumptions on which the choice was originally based; and in the "conditional parameter flexibility" case, an evolving strategy would continuously ensure a preferred setting of strategy parameters.

On the other hand, if mistakes are recognised in the initial estimates and assumptions, management will presumably acquire new and relevant information on which to base a modified investment strategy. [23] But supposing this to be so, management's basic requirement must be that the pattern of mistaken original probability estimates and the process of learning from mistakes be predictable, at least in probability terms, at the outset; and this is logically impossible. A probability estimate made at a particular time is by definition the best estimate then available to the decision maker. So, if the results of any future reappraisal of investment demand and profitability relationships cannot be known at the outset even in probabilistic terms, management cannot identify the probabilities of particular strategy modifications being made in the future; and so the complete picture of all possible growth-paths and their probabilities required by management is unattainable.

Management's intentions regarding flexibility in investment strategy must be regarded as extremely limited. What is certainly allowable is the kind of provision for specific situations referred to in sub-section (ii), where such provisions can readily be understood as components of a given

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(23) For the sake of the argument it is assumed in this passage that management is eventually able to decide whether its original probability estimates are correct. This may not be an easy task in the complex situation envisaged: see Chapter 13 : 3.

strategy. Conditional parameter flexibility, with the entire strategy shifting in predictable ways and with predictable results, in response to foreseeable events, is also admissible in logic but must be treated as a highly implausible account of managerial thinking. When flexibility is more ambitiously defined to include a switch to a new set of investment rules on some future occasion in response to the recognition of errors, it is admitted that management cannot forecast a change of strategy or its effect on the firm's probability distribution of growth-paths.

11 : 5 (iv) <u>The acceptance of imperfect probability estimates.</u>

This ultimate limit on foresight need not weaken management's preferred interpretation of the valuation process. Both sides, management and investors, accept that probability estimates are as reliable as the inputs of skill and experience permit, and that the commitment to the chosen strategy is correspondingly strong. The basic premise here is that decision makers on both sides accept the best probability estimates currently available and, where decisions are required, act as though those estimates are correct. Thus management chooses the strategy which on present evidence promises the highest level of expected utility. Investors accept as correct management's estimate of probability distribution of growth-paths of constant-risk expected dividend under the chosen strategy, and because they value equity on the strength of this estimated probability distribution the basis for management's interpretation of valuation remains undamaged by the admission that probability estimates are likely to be imperfect. (24)

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(24) The fact that for an area II strategy management sees the resulting market value as subject to a probability distribution presents no serious problem for this line of reasoning. In such a case management has to decide in advance which probability distribution of market value it prefers ; but following its decision a unique probability distribution of growth paths associated with the chosen strategy will exist, and market value will then be determined as the theory supposes. There is no question of investors having to place a value upon a range of S-distributions.

This "strong" conclusion on the acceptability of probability estimates is admissible in the context of this chapter – a management decision made with reference to its immediate consequence, V_{eo} . Following the suggestions to be made in the following chapter for an extended horizon utility function, Chapter 13 returns to the question of the acknowledged fallibility of probability estimation. The case for accepting probability estimates may weaken if within the decision maker's own horizon the estimates may have to be changed in ways (and with results) which cannot be forecast.

CHAPTER 12

Investment Strategy and Managerial Utility (II)

Plan of Chapter

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   (ii)   Conditional prediction of future equity values.

Section 2 : An Extended Horizon Managerial Utility Function

   (i)   A general formulation for an extended horizon utility function.

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   (iii)   Particular forms of $U = U(V_{eo}, V_{em})$.

Section 3 : The Logical Validity of the "Targets" Utility Function

   (i)   The limited measurability of potential regret.

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Section 4: The Structure of Decision Making

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Section 6 :   <u>Conclusion</u>

  (i)   Summary

  (ii)   The unresolved problem of uncertainty.

Section 1 : Introduction.

12 : 1 (i)   The limitation of the simple managerial utility function.

An extremely simple managerial utility function was employed to establish the main conclusions of the previous chapter : now it is appropriate to consider modifications in that function with a view to improving its realism while retaining the underlying philosophy explained in Chapter 11, Sections 1 and 5 (iv).

The common feature of the various decision criteria discussed in Chapter 11 : 4 was management's concentration on the present-day market value of equity to be achieved through the strategy under consideration. The obvious need now is to find an expression for the managerial utility function which allows for managerial concern over possible future developments in equity market value. To emphasise this need convincingly, it should be obvious that management is unlikely to be indifferent between strategies A and B with respective market value prospects as follows:

A. $V_{eo} = 100$, and V_{e5} (market value after five years) is subject to the probability distribution

$p(V_{e5})$	V_{e5}
0.5	140
0.5	80

B. $V_{eo} = 100$, and V_{e5} is subject to the probability distribution

$p(V_{e5})$	V_{e5}
0.5	180
0.5	50

It is likely that although impersonal market forces value the prospects of A and B equally ($V_{eo}^{a} = V_{eo}^{b} = 100$) management will prefer one set of prospects to the other whatever its underlying motivation; so that the present-day level of equity market value associated with an investment strategy is by itself an incomplete indicator of the utility experienced by management in choosing that strategy.

12 : 1 (ii)   Conditional prediction of future equity values.

Before discussing an expression of the managerial utility function which will allow the prospects for future equity market value to influence present-day managerial utility, it must first be shown that it is possible in principle to identify the probability distribution of V_e for any future date, given the firm's investment strategy. [1] The present-day (t = 0) equity market value, V_{eo}, is assumed to be determined by the kind of valuation process envisaged in Chapter 10 : 3. Given its investment strategy and the value of k, the firm's S-distribution is determined, each S-value and its probability corresponding to one possible growth-path of constant-risk expected dividend. [2] The value of the firm's equity is determined by the market value relationship

$$V_{eo} = H\left(\bar{\bar{S}}, \sigma_s\right) \tag{i},$$

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(1)   It is assumed for convenience that management sees each strategy's prospects as a single S-distribution: this merely simplifies explanation and does not exclude the important possibility of multiple forecasts from the coverage of this chapter.

(2)   A particular value of S may of course represent more than one growth path, but the total probability of that S-value is separable into the probabilities of all growth-paths having that S-value.

the original form from which the standardised expression used in Chapter 10 was derived :

$$\frac{V_{eo}}{\bar{S}} = y = h \left(\frac{\mathscr{S}s}{\bar{S}} \right) = h \ (Z) \qquad\qquad (ii) \ ^{(3)}$$

The S-distribution incorporates all possible growth-paths of expected dividend, so that for any growth-path j the position at any future date conditional upon j being followed up to that date can be identified in the following terms : at $t = m$ on growth-path j the S-distribution of the firm would be S_{m1}^{j} , S_{m2}^{j} , ... S_{mf}^{j} , ... , each S_{m}^{j} value having an identified probability. This conditional probability distribution of S-values for period m represents all possible directions in which the j th growth-path may diverge after m , and includes all growth-paths which coincide with the jth until m . The probabilities attached to the growth-paths from period m are in principle as reliable as those assigned to the diverging growth paths at $t = o$: indeed the former are incorporated in the latter. Thus it is possible at $t = o$ to identify the S-distribution which will obtain at $t = m$ if the jth growth-path has been followed to period m . Since j and m are completely general it follows that management can in principle identify the S-distribution which will apply at any period no matter which growth-path the firm happens to follow from $t = o$ up to that period.

The next step in the argument is the transformation between future possible S-distributions and market values at the dates to which they

(3)   This implies linear homogenity in market valuation. Whatever characteristics "explain" market value, an x-fold change in all of their values changes market value x -fold.

conditionally relate. It has been assumed throughout that capital
market equilibrium price relationships do not change through time,
and although anticipated changes could be allowed for this aspect
is not considered here: market value relationships are assumed to
be the same in all future periods as they are today. It follows
that the transformation from S-distribution to equilibrium market
value will take place through an unchanging relationship at all periods
and on all growth-paths. Given an understanding of that relationship
management can identify equity market value for any date and growth-
path, the probability of that value at that date, as seen from t = o,
being that of the growth-path from t = o to which it relates.

Diagram 1, which assumes that a choice of strategy has been made,
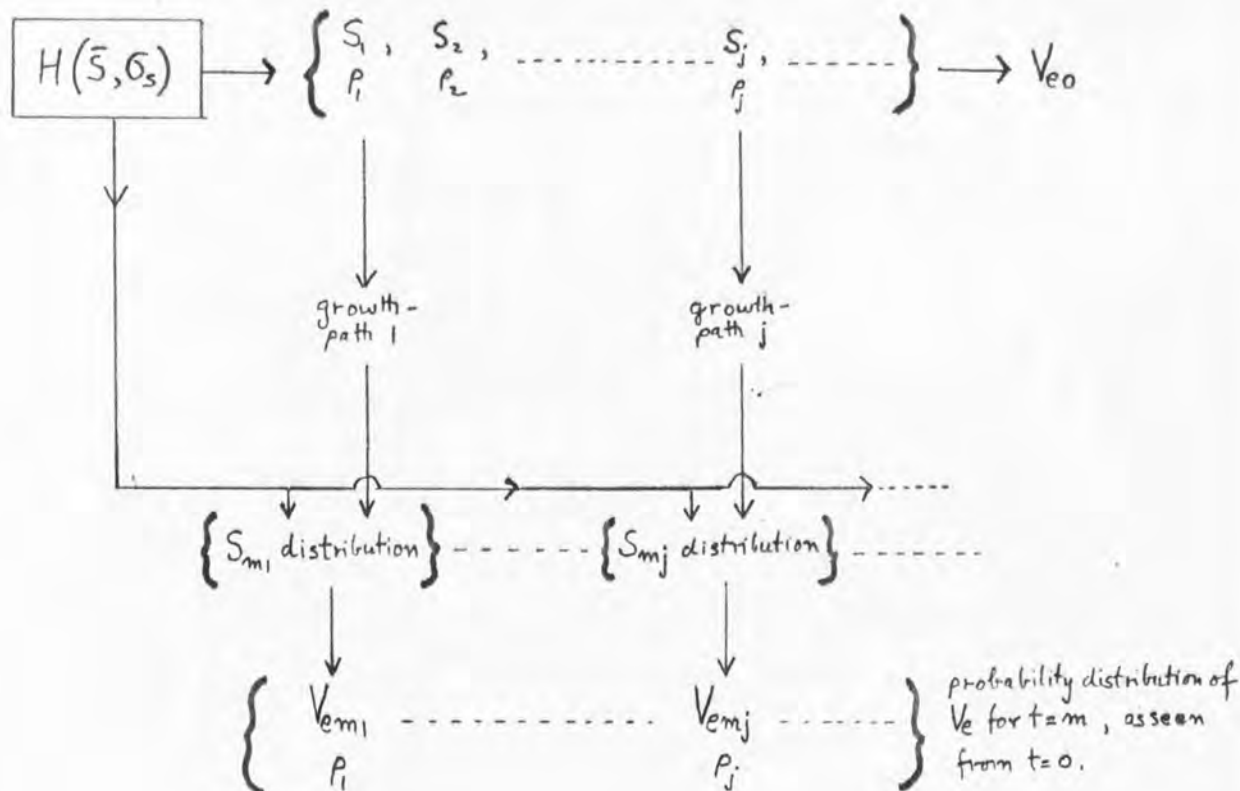illustrates the whole of the above argument.

Diagram 1 : derivation of the V_{em} probability distribution at t = o

Starting with the S-distribution at t=o for the strategy under consideration, the market valuation process symbolised by H $(\overline{S},\ \sigma_s)$ establishes the market value of equity at $t = o$, V_{eo} . For each growth-path, whose probability is given in the present-day S-distribution, it is possible to envisage a probability distribution of growth-paths originating from any future date, exactly analogous to that represented by the S-distribution at $t = o$. In Diagram 1 growth-paths 1 and j are singled out, and for $t = m$ there is for each of these growth-paths a unique S-distribution which will obtain if that path is the one followed up to period m. Just as the valuation process H $(\overline{S},\ \sigma_s)$ generates V_{eo} from the unique S-distribution at $t = o$, so the same process is expected to operate in relation to whichever S-distribution is realised at $t=m$. Thus if growth-path 1 has been followed the S_{m1} - distribution will be realised at $t=m$ and its market value will then be V_{em1} ; and if growth-path j has been followed the S_{mj}-distribution will be realised at $t=m$ and its market value will then be V_{emj}. Seen from the present, for any growth-path r from $t=o$ to $t=m$ there is a known probability, p_r, and this probability is associated with the market value of the firm's multiple dividend growth prospects from $t=m$ onwards, V_{emr}. There exists there-fore at the present day a probability distribution of equity market value for any future period, each conditional future value having the known probability of the growth-path to which it corresponds.

It is clear that, given the multiplicity of growth-paths described in Chapter 8 and the possibility of interpreting valuation along the lines described in Chapter 10, it is possible for management to identify the probability distribution of V_e for any future period. There is so far no conclusion on the appropriate modification to the simple managerial utility function employed in Chapter 11. The importance of what has been

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demonstrated is that no additional information is required by management
in adding an extra dimension - the movement in V_e over time - to its
picture of equity market value prospects. The same understanding that
is required at t=o to generate the probability distribution of growth-
paths - the firm's S-distribution at t=o - and the same interpretation
of the determinants of equilibrium market value that applies at t=o,
remain relevant when management turns its attention from the immediate
consequences of strategy choice to a consideration of the longer-term
possibilities on which its strategy choice is likely to depend.

Section 2 : An Extended Horizon Managerial Utility Function.

12 : 2 (i)   A general formulation for an extended horizon utility
              function.

Turning now to the formulation of managerial utility functions
which take into account possible future developments in equity market
value, the most general expression is

$$U = U\ (V_{eo},\ V_{e1},\ \dots\ V_{em}\ \dots)\ \tag{iii},$$

which simply expresses present-day utility as a function of V_e in all
future periods without specifying any scheme of weighting or discounting
of V_{et} values according to their remoteness in time. The task of
selecting a less general form of (iii) will be attempted shortly, but
first it is necessary to recall the conclusion reached in Section 1 (ii) :
for any investment strategy management must think in terms of an identifiable
probability distribution of V_{em} values for any future period t=m. This
suggests that managerial utility associated with any investment strategy
should be treated as an expected utility value rather than a unique value.

Indeed, only in the particular case of (iii), $U = U(V_{eo})$, is it possible to regard the utility-determining outcome as having a unique value with probability equal to unity. Given its investment strategy, and assuming that the general form of utility function applies, there is obviously a particular growth-path through time of V_{et} corresponding to each of the firm's growth-paths of constant-risk expected dividend, and managerial utility would have to be expressed as

$$EUV = \sum_{i=1}^{x} P_i \; U \; (V_{eo}, \; V_{e1}^i, \; V_{e2}^i \; \dots \; V_{em}^i, \; \dots V_{ey}^i) \quad (iv), [4]$$

with each growth-path ($i = 1 \; \dots \; x$) up to period y being separately converted into a conditional managerial utility value and all such values being weighted by their probabilities before being aggregated to give expected utility for the strategy in question.

For investment strategies which are seen _ex ante_ by management as subject to risk in the sense that more than one possible S-distribution is identified, the definition of expected utility involves an extra stage of computation. For each possible S-distribution an expected utility is obtained as in (iv) above; then the overall expected utility of the strategy is obtained as a probability-weighted average of all such expected utilities, one for each S-distribution envisaged by management. Thus

$$EUV = \sum_{j=1}^{r} P_j \left[EUV_j \right] \quad (v),$$

where the strategy's expected utility value is the probability-weighted average of expected utilities associated with S-distributions 1 ....r.

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(4) Note that V_{eo} is common to all growth-paths under a given strategy with a unique S-distribution. The value of x depends on the remoteness in time of the horizon; in practice, management is not expected to look more than (say) ten years ahead, and probably less.

12 : 2 (ii)   <u>A reduced form of extended horizon utility function.</u>

Having shown in the general expression of utility function that
investment strategies can be ranked according to expected utility value,
it is appropriate now to consider how (iii) can be given greater
precision and be made to reflect obvious managerial attitudes. A
useful beginning can be made by reducing the general formulation to

$$U = U \, (V_{eo}, \, V_{em}) \hspace{4cm} \text{(vi)},$$

where managerial utility at t=o is determined entirely by the combination
of present-day and prospective (t=m) equity market values. This
simplification is attractive from the point of view of handling the
analysis, and in this reduced form the utility function gives a reasonable
reflection of managerial concerns. Whatever the nature of its motivation -
investor-oriented or "managerial" - it is to be expected that early
results carry greater weight with management than remote prospects, and
actual or potential pressures may be represented in its utility function
by assigning great importance to results achieved within a time limit.

This is not the place for a full discussion of various suggested
managerial motivations, but the problem of security from takeover can
usefully be dealt with. It is assumed throughout that the strategies
from which management makes its final choice all have in common a minimum
level of security from takeover, at least as far as their prospective
ratio of market value to asset value at any time is concerned. [5]

(5)   This assumption closely matches that frequently made in the
      context of a steady-state growth model, to the effect that
      strategies offering less than a minimum secure valuation ratio
      are excluded from consideration, regardless of their growth rates:
      see Chapter 14 : 4 (ii).

Although security against an external threat can be provided for in this way, management remains vulnerable to internal threats, some of which may arise in response to an indifferent performance over time: hence the idea of a self-imposed horizon for the evaluation of prospective results is consistent with the generally acknowledged managerial desire for security and survival in office.

All that is implied in the choice of (vi) for the general form of managerial utility function is that various factors point in the direction of a limited horizon being employed in management's comparison of strategic alternatives. For simplicity it may be assumed that the same horizon is chosen whatever the reasons, or that a compromise on the horizon is actually sought and achieved. It will become clear that it is possible to move back towards the general form (iii) without sacrificing any important conclusion derived from the simpler version (vi) if it is felt that separate horizons for different aspects of managerial priorities and attitudes are required.

12 : 2 (iii)   Particular forms of $U = U (V_{eo}, V_{em})$.

Concentrating on the formulation $U = U (V_{eo}, V_{em})$ permits consideration of particular interpretations which appear to reflect the psychological make-up of management. In this sub-section various versions are suggested, each reflecting a particular attitude towards

equity market value and its change over time.[6]

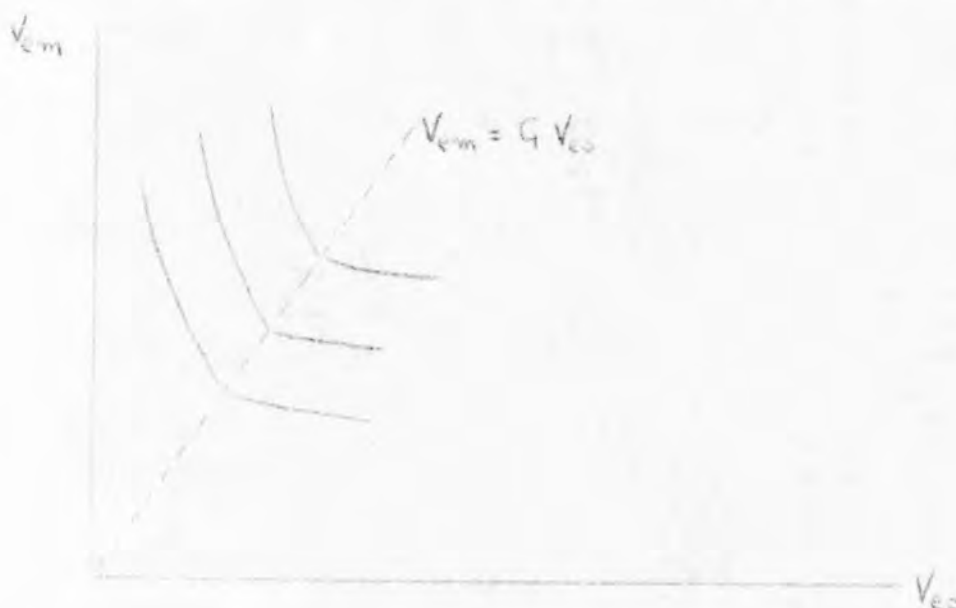The first interpretation is illustrated in Diagram 2.

$$V_{em} = G \, V_{eo}$$

Diagram 2 : Indifference curve system incorporating one target.

V_{eo} and V_{em} are measured on horizontal and vertical axes respectively,
and a system of managerial indifference curves is assumed, with the
property that each curve changes slope abruptly as it crosses the dotted
relationship $V_{em} = G \, V_{eo}$. This system reflects a "target" approach
to the level of V_{em} in terms of the achieved level of Veo. When
prospective V_{em} falls below the target multiple ($G \geqslant 1$) of the achieved

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(6)   Motivations which may underlie the indifference curve systems
      suggested here are considered at length in Sections 5 (iii)
      and 5 (iv) below. If management's plans involve the possibility
      of external equity financing, its attitude to adjusting V_e
      values presumably depends on its underlying motivation. For
      example, a simple adjustment that best measures the market
      performance value of original shareholding is to subtract from
      later V_e values the market value of any new equity contributed by
      "outsiders" and the actual contribution of "insiders" to rights
      issues. Management may prefer, however, to subtract from V_e only
      the total of new equity invested. The fact that different
      adjustments can be suggested merely illustrates the discretionary
      power management possesses to define both the context and the
      criteria for the choice between strategic alternatives.

value V_{eo}, management's reluctance to accept a further decline in V_{em} is sharply increased; this appears as an abrupt increase in the amount of compensation in the form of increased V_{eo} required for a unit decline in V_{em}. The obvious interpretation is that management quite reasonably attaches high priority to a minimum percentage increase in equity market value over whatever base value is achieved. While this version provides an adequate account of managerial attitudes when no existing strategy is available for comparison, it seems unlikely that strategic alternatives would be evaluated without reference to the present-day equity market value under an existing strategy. The latter situation is probably more common than strategy choice in a vacuum, and it is easy to vary the indifference curve system to take account of it.

Diagram 3 illustrates an indifference curve system divided into zones by the operation of two managerially fixed targets.
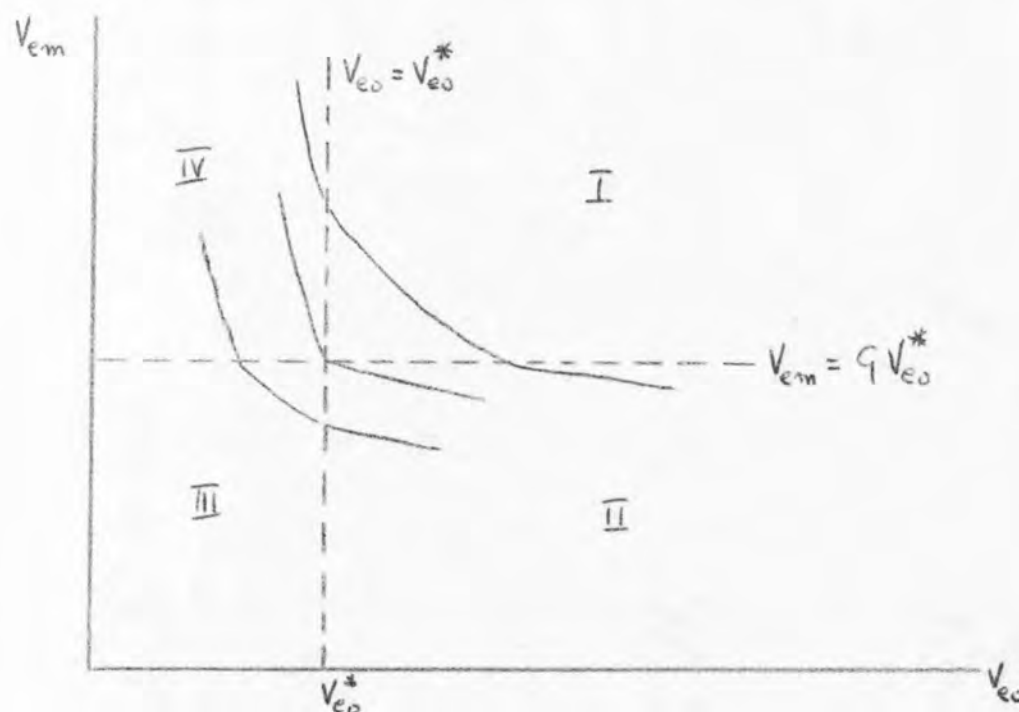

Diagram 3 : Indifference curve system incorporating 2 targets.

The first target applies to V_{eo} and is set by V_{eo}^*, the present-day value of equity under the firm's existing strategy. The second target applies to V_{em} and is expressed as a target multiple of V_{eo}^*. The interpretation is that management feels that V_{eo}^* provides a standard for any alternative strategy for both present and future performance.

The interpretations of the abrupt changes of slope of an indifference curve as it crosses zone boundaries in Diagram 3 are straightforward and in much the same terms as the explanation of Diagram 2. An indifference curve entering zone II from zone I flattens abruptly, indicating a sharply increased requirement for compensation in extra V_{eo} for a prospective unit fall in V_{em} when the latter goes below its target level. Similarly, an indifference curve entering zone IV from zone I steepens abruptly, indicating a sharply increased requirement for compensation in extra V_{em} for a prospective unit fall in V_{eo} when the latter moves below its target level. For an indifference curve entering zone III from zone IV the interpretation of the abrupt flattening of the curve is that a smaller sacrifice in prospective V_{em} will be forthcoming in return for a given increase in V_{eo} when V_{em} goes below its target level. Finally, an indifference curve entering zone II from zone III flattens abruptly, indicating a sharply reduced willingness to accept a prospective reduction in V_{em} for a unit increase in V_{eo} when the latter begins to exceed its target level.

One further version of a double-target managerial indifference curve system is worth illustrating. Diagram 4 is a modification of Diagram 3 with some debt to Diagram 2 : the target for V_{em} is set first of all by a multiple of V_{eo}^*, and if V_{eo} for the chosen strategy exceeds V_{eo}^*, by the same multiple of actual V_{eo}.

V_{em}

$\overline{IV}$ I $V_{em} = q \cdot V_{eo}$

$V_{em} = q V_{eo}^{*}$

$\overline{III}$ $\overline{II}$
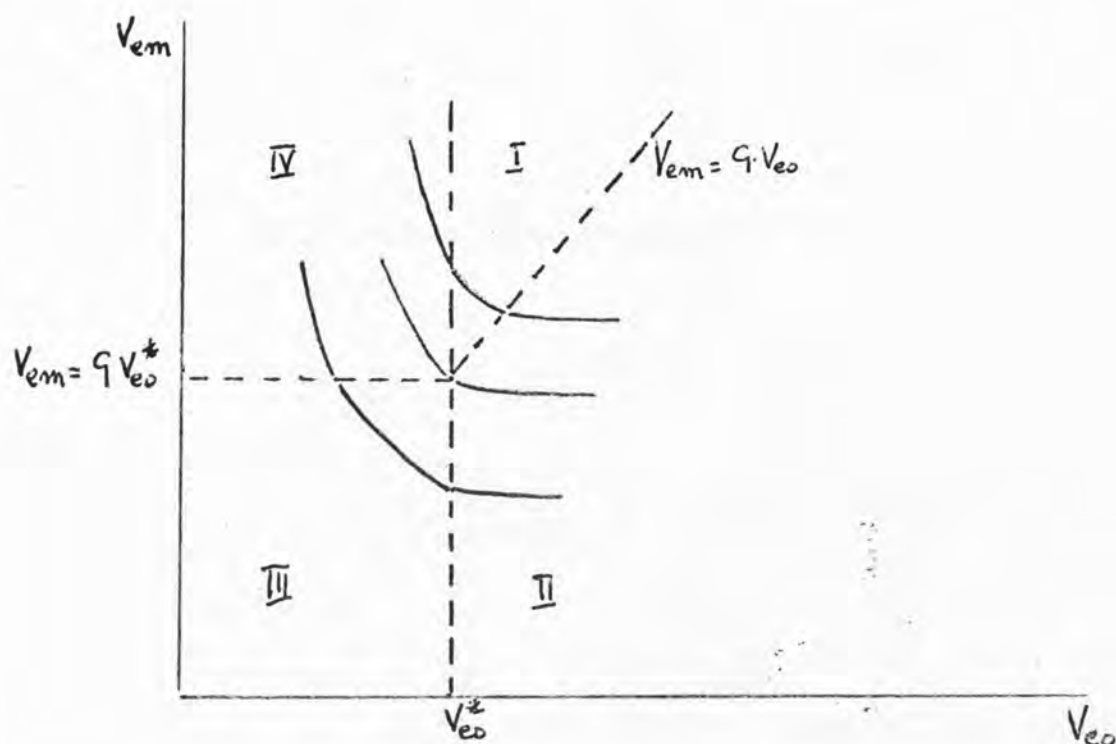
V_{eo}^{*} V_{eo}

Diagram 4 : Revised indifference curve system based on 2 targets.

This results in the enlargement of zone II, one of the zones of "unhappiness", at the expense of zone I, the zone of undiluted "happiness", but no change is required in the interpretations given to the abrupt changes in slope of indifference curves as they cross zone boundaries. This form of indifference curve system is probably the most realistic of those suggested as it incorporates an adjustable target for V_{em} expressed in terms of the better V_{eo} outcome attainable at the present day. (That is, the better of V_{eo}^{*} and V_{eo} for the strategy in question). If it were desired to expand the utility function to incorporate equity market value at more than two dates there would be no difficulty in principle in applying the concept of targets in the larger context thus created: a target for $V_{e(m+h)}$ conditional upon V_{em} could readily be added to the scheme explained here.

Section 3 : The Logical Validity of the "Target" Approach to Utility.

12 : 3 (i)   The limited measurability of potential regret.

Before reaching conclusions on the usefulness and relevance
of the particular interpretations of the general utility function
suggested in Section 2, the important matter of their logical validity
must be considered. In confirming the logic of a utility function
based upon target values of V_e at different dates, this exercise
serves to identify limits on the comparability of investment strategies
and provides essential background for a review in Chapter 14 of some
models of managerial strategy choice.

The general characteristic of the functions considered in Section
2 is that the utility of any prospective V_{eo} , V_{em} combination is
determined by the relationship of the two values to their respective
target levels, the latter being set by a common standard irrespective
of the strategy under consideration. The fact that the target for V_{em}
may be conditional upon the value of V_{eo} actually achieved is not a
criticism of the logic of the approach: provided the V_{em} target is a
given function of V_{eo}^{*} or V_{eo} – whichever is the larger, as in Diagram
4 – the same set of indifference curves applies in the appraisal of
all strategy alternatives.

However, a valid objection in logic to the particular forms of
$U = U (V_{eo}, V_{em})$ suggested would exist if it were possible at $t = o$
to identify the "regrets" which may be associated with any strategy in
future periods more completely than is suggested in Section 2. In the
three indifference curve systems suggested the regret associated with
any V_{eo}, V_{em} combination can be defined as the shortfall of V_{eo} or

V_{em} below its target level. This kind of regret is obviously measurable to the decision maker, and is indeed the concept underlying the three constructions.

The important limitation on the measurability of regret implied in those constructions is that whichever strategy is chosen there is no way of determining the V_{em} value which would have been achieved under a different strategy. This is true despite the fact that, as shown in Section 1 (ii), the probability distribution of V_{em} for any m is completely identified for each investment strategy. It is never possible, before or after the event, to link a particular V_{em} value for one strategy with a unique V_{em} outcome under a different strategy in the sense that management anticipates that a certain combination of circumstances would give rise to one or the other outcome depending on the strategy in operation. This is so because the growth-path which would have been followed up to t = m under a rejected strategy cannot be inferred from that actually followed under the chosen strategy. [7]

Admittedly, whichever strategy is chosen the firm is subject to the same outside developments in the shape of changing macroeconomic and industry-level conditions, and this does suggest the possibility of a degree of inference from actual to hypothetical growth outcomes.

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(7) Nor is it even possible in most cases to establish at t = o the probability that $V_{em}^{a} > V_{em}^{b}$ (for any two strategies A and B) even though the separate V_{em}^{a} and V_{em}^{b} probability distributions are known exactly. (There is the remote and uninteresting possibility that the lowest V_{em}^{a} exceeds the highest V_{em}^{b} value). The reason is that the covariance between the two distributions is essentially unidentifiable. If the question of comparability was raised in this form the obvious, safest, but strictly unjustifiable procedure would be to assume statistical independence between the two distributions and answer the question on that basis.

But even allowing for the fact that the firm's "background conditions" develop along a unique path regardless of the strategy actually chosen, it is not possible that management will be in a position at $t = m$ to compare its record and actual prospects – as reflected in its growth-path and actual V_{em} – with what they would have been under another strategy.

This assertion can be tested by examining the case where _ex post_ comparability would seem most likely to be possible: namely where one strategy (A) involves higher investment in all circumstances than does strategy (B). If the firm is following strategy A and is experiencing a particular development of external or background conditions through time, is it in a position to trace the path that its investment, profit and dividend would have followed under strategy B? To give maximum opportunity for an affirmative conclusion to emerge, assume that in each period management is able to divide its actual investment, profit and dividend into the amounts that would have occurred under strategy B and those attributable to the difference between the two strategies : given its actual experience of A, management can trace a growth-path for strategy B. In spite of having put the question to the test under hypothetical circumstances most favourable to an affirmative answer, the correct conclusion must be that management cannot _uniquely_ identify the _course through time_ that investment under strategy B would have taken. Even if at each period it is able to identify the profits that would have been attributable to strategy B investments made in earlier periods, there remains the admitted element of randomness in investment demand under any strategy in given circumstances: [8] this means that the

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(8)   This point was emphasized in the discussion of the total investment
       demand function in Chapter 11 : 2 (iii), and illustrated in an
       example in Chapter 8 : 2 (iii).

hypothetical strategy B investment demand identified by management
in each period is only one of a considerable number of possibilities
for the circumstances prevailing, and that the hypothetical B growth-
path traced out in parallel to the actual A growth-path is only one
of the growth-paths which the firm might have followed under strategy B.
The fact that a particular historical development of external conditions
will be known to management ex post means only that it will then be
possible to narrow the range of B growth-paths to those which could
have occurred under those conditions: if the experiment is then repeated
using the B strategy rules and identical external conditions there is
only a probability that the actual B investments observed will exactly
match those of the first hypothetical B growth-path. It is not possible
that management will be in a position to uniquely identify a regret
associated with having chosen A instead of B, because an actual V_{em}^{a}
outcome cannot indicate a unique V_{em}^{b} outcome which would with certainty
have been achieved under strategy B.

The limits on the measurability of regret in the future, beyond
those recognised and incorporated in the indifference curve systems in
Section 2, are illustrated by certain rather unhelpful possibilities.
One could compare ex ante as well as ex post the maximum possible values
of V_{em}^{a} and V_{em}^{b}, or their minimum values; or the probability for each
distribution that V_{em} will exceed a certain level. There is also the
remote possibility that for some strategy the minimum V_{em} outcome is
higher than any possible outcome of any other strategy : if such a
strategy could be found there would be little need for further discussion
of managerial utility functions.

12 : 3 (ii)   The dependence of the "targets" approach on the limited
               measurability of regret.

The correct perspective on the conclusions reached above is that
if it _had_ been shown that direct and exact comparability between
prospective V_{em} outcomes for different strategies was possible, the basis
for the indifference curve systems of Section 2 would have been under-
mined. The utility functions represented by those systems are predicated
on the limited present-day measurability of potential regret; whereas
the complete identifiability at $t = o$ of the regret associated with any
particular V_{em} outcome of any strategy - in terms of a unique difference
between that V_{em} and the best V_{em} for the external conditions under _any_
strategy - would necessitate a different structure and interpretation for
the indifference curve system. The central concept of a strategic
decision directed towards maximum expected utility value would not have
to be amended, but the structure of the indifference curve systems would
require substantial modification. The concept of targets set in the ways
described in Section 2 would have to give way to a more general inter-
pretation of the utility function. For any strategy a prospective V_{eo} ,
V_{em} outcome would be directly comparable with the result which would be
achieved through another strategy under the same external developments.
Thus for strategy A each V_{eo}^{a} , V_{em}^{a} outcome would have a utility loss,
defined as the difference (in utility units) between the utility of the
best outcome under any strategy given the external conditions and that
of the actual V_{eo}^{a} , V_{em}^{a} combination in question. Then for strategy A
a measure of expected utility loss can be obtained as

$$EUL\ (A)\ =\ \sum_{j=1}^{\infty}\ p_{j}\ UL\ (A_{j}) \qquad\qquad (vii),$$

where j and p_j refer to the possible growth-paths under strategy A and their probabilities; $U L (A_j)$ is the utility loss experienced when under strategy A the jth growth-path is followed. [9] The value of $U L (A_j)$ is determined, as stated, by the difference between the utility associated with $(V_{eo}^a, V_{en}^a)_j$ and that attainable under the best strategy (highest utility) for the same prospective external conditions.

The proof that expected utility maximisation remains a meaningful objective when the regrets associated with prospective strategy outcomes are identifiable is simply a matter of applying a well-known proposition in decision theory to the present case. [10] Suppose that in the light of the _ex ante_ identifiability of regrets, management expresses its objective as the minimisation of expected utility loss, or expected regret. For strategy k the measure of expected utility loss is

$$E U L (k) = \sum_{j = 1}^{z} P_j \; U L (k_j) \qquad \text{(viii)},$$

just as in (vii) above, and let

$$U L (k_j) = U_j^* - U (k_j) \qquad \text{(ix)}:$$

that is, the utility loss on strategy k growth-path j is given by the difference between U_j^* — the utility achieved by the best strategy for the external conditions governing growth-path j — and $U (k_j)$, the utility of the result actually achieved. Using (ix), (viii) can be

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(9)   When management is considering growth prospects over a finite time horizon it is not necessary to assume that x, the number of different growth-paths, is infinitely large. (This applies also to z, the number of growth-paths for strategy k in the paragraph which follows).

(10)   See P.G. Moore, _Risk in Business Decision_, 1972, pp.107-8.

expanded to give

$$E U L (k) = \sum_{j=1}^{z} P_j U_j^* - \sum_{j=1}^{z} P_j U(k_j) \qquad (x).$$

In minimising expected utility loss management must therefore choose the strategy with the smallest difference between its first and second terms corresponding to those on the right-hand side of (x). What reduces the difficulty of this choice and restores the problem to one of maximising rather than minimising an objective function is the fact that for all strategies the first term on the right-hand side of the equation corresponding to (x) is the same: this term is simply the probability-weighted sum of all utilities when external conditions can be perfectly predicted and strategy chosen accordingly, and as such it has the same value no matter which strategy is being compared with the hypothetical state of perfect information and response. Accordingly, in minimising expected utility loss management is obliged to return to the maximisation of expected utility – the second term on the right-hand side of (x) and of all corresponding expressions. By choosing the strategy with the highest expected utility value management reduces to a minimum the difference between its expected utility under perfect information and its actual expected utility – its expected utility loss.

Apart from its reliance on management's ability to infer from a particular V_{em} outcome the outcomes which would have been achieved under all other strategies – an assumption which has been dismissed as untenable – the approach to utility maximising described here seems less satisfactory than the one based upon the targets approach in Section 2. Whereas the earlier approach relates utility to the levels of V_e at various dates in relation to certain plausible yardsticks, the completely general form

of utility function lacks such an explanatory basis. Yet it is scarcely logical to employ yardsticks or targets in an indifference curve system when regret is to be measured against the best outcome possible in each set of circumstances and not against the fixed targets. From the standpoint of the realistic insight that can be incorporated in an indifference curve system it is fortunate that strict logic limits the measurability of regret to the kind of possibilities described in Section 2.

Section 4 : The Structure of Decision Making.[11]

12 : 4 (i)   Managerial discretion and a model of decision making.

The necessity for managerial discretion on a wide range of matters has been emphasised repeatedly. Examples are : the range of decisions which together determine business risk; dividend risk policy (which logically precedes the estimation of the cost of capital, as shown in Part I); the mode of expression and parameters for investment strategy. In all these areas management decisions substitute for absent, weak or incomprehensible market forces. If alternatives to all management policies were apparent to a wide public, and if the market value effects of those alternatives were also clear and significant, then no

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(11) Preliminary statements of the strategic choice model suggested in this and the following section were made in Chapters 1 : 2(ii), 1 : 3(ii) and 11:1 ; and discussion in this chapter emphasises the role of investment decision rules within the model of decision structure defined in these earlier chapters. The structure of decision making deserves the emphasis it receives here because while managerial theories typically assume a utility function and top-level decision in terms of a limited number of performance variables, the analysis of the processes by which this final simplification is achieved is by no means standardised between theories. On this subject, a comparison between the present account and a Marris-type steady-state managerial theory is given in Chapter 14 : 2 (ii); and the importance of that comparison in a final judgement between alternative accounts of strategic decision making is discussed in Chapter 15 : 2 (iv).

doubt management could be regarded simply as an instrument for putting market-dictated policies into operation and carrying them through efficiently. The scope for discretion would be minimal.[12]

If it is accepted that management typically enjoys greater independence on policy than the above simple view suggests, it follows that its objectives and preferences guide decision making in many areas. A conventional view might be expressed thus : (i) While a high market value is an objective of both types of management, on a realistic view the connection between some policies and market value is too obscure or insignificant to allow all decisions to be directed accurately towards that objective. (ii) Management is likely to have other objectives in addition to the level of equity market value: differences between the strategy decisions of market-oriented and "managerial" managements are to be expected, with a likelihood that market value is to some extent sacrificed by the latter in improving other aspects of performance. An implication of the conventional approach is that it is possible to measure management's "managerialness", the degree of its departure from a completely market-oriented strategy, and that strategy choice can thus be seen in terms of management's compromise between its own and investors' interests – both unambiguously defined. [13] It is this

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(12) This point is confirmed for the case of managerial growth models in C.J. Hawkins, _Theory of the Firm_, 1973, p.80.

(13) The best example of a measurable divergence of management's strategy from that which a market-oriented management would adopt is the concept of a tangency solution between a managerial indifference curve and the objective relationship between valuation ratio and growth rate: The divergence is exactly measured by the extent to which g exceeds that value which maximises v. See R.L. Marris, _The Economic Theory of Managerial Capitalism_, 1964, pp.260-1.

concept of strategies being recognizably more or less managerial
that will be found unacceptable in the account of decision making
given in this chapter.

Three elements combine to produce this conclusion. First,
managerial discretion over many aspects of policy is accepted as
inevitable. Second, the need for a comparatively simple concept of
strategic choice is accepted, in the sense of a managerial utility
function based on a limited number of performance aspects and a
correspondingly limited range of policies making up overall strategy.
The third element adds substance to the second: investment policy is
seen as likely to constitute the main (or only) component of management's
strategy, and is seen also as a natural match for equity market value
as the main (or only) determinant of managerial utility at the strategy
level. These points are developed in the remainder of this section.

12 : 4 (ii)   Subordinate and top-level decision making.

A distinction is drawn between subordinate and top-level (strategic)
decision making in the wide area of managerial discretion. The
distinction is essentially practical in that the effects of the inter-
actions of all aspects of policy cannot be visualised by management, or
are thought in some cases to be of low importance; so the effects of
altering the most important policies will be estimated against an assumed
settled background of other policies. Given the importance of investment
policy in determining the probability distribution of growth-paths, its
inclusion as a component of top-level strategy is accepted without
argument. And given the complexity of the calculations required in
identifying the probability distribution of growth-paths against a settled

background of other policies, it seems unlikely that management will be willing or able to consider variations in other policies as well as investment policy at the top level of decision making.

The analysis of management's (strategic) utility function cannot be separated from the model of decision making, and it is useful at this point to look ahead to see how the two sides of the picture are intended to match one another. Linked to the necessity or preference for a partial approach to an optimum combination of policies is the likelihood that the managerial utility function relevant to top-level strategy is expressed in terms of a limited number of performance variables. If many aspects of policy must be settled individually or by compromise prior to the strategy decision, it follows that for the reduced problem of top level choice a utility function expressed in a correspondingly reduced number of variables is likely to be appropriate. Formally, it is suggested that the managerial utility function has a two-tier structure; with a form of optimising taking place at each level, and with the solution from the subordinate level providing the background against which a top-level maximum is sought. [14]

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(14) Processes of subordinate decision making need not be described in detail here, nor need it be assumed that formal utility maximising characterises all subordinate policy choices (see Chapter 1 : 3 (ii)). Subordinate decisions can be reached in any way without damaging the present model, provided that their conditional workings and interactions are fully understood by top management and can be taken into account when top-level choices are compared. For reviews of theories that might relate to subordinate policy making see G.P.E. Clarkson, "Interactions of Economic Theory and Operations Research," in A.R. Oxenfeldt (ed), Models of Markets, 1963, and C.J. Hawkins, op.cit., 1973, pp.60 - 77, on questions of behaviour in various market conditions and the employment of O.R. in general; and J.R. Wildsmith, Managerial Theories of the Firm, 1973, Chapter 2, on the importance of organisational factors.

It is worthwhile to deal with certain obvious criticisms of the model as outlined above. The suggestion is that a process (or processes) of subordinate decision making is a sensible way to visualise the decision structure, with conflicts between aspects of policy avoided because top-level choice presupposes agreed rules, standards and priorities in and between all subordinate areas. Similarly, conflict between top-level and subordinate policy is precluded because the former takes the latter as its predetermined background. The first criticism recalls the contrast mentioned in Chapter 11 : 1 between this view of decision making and an ideal situation in which all aspects of policy are determined simultaneously in the light of a global, all-inclusive, objective function. The choice between these two concepts of decision making must be made in the light of answers to two questions: whether management can actually perceive all of the trade-offs which may be possible between aspects of policy, and whether an all-inclusive utility function is realistic.

For the unconstrained maximisation concept to be valid, management must be aware in every situation of the effects of a change in one aspect of performance on all other aspects : what might be called the performance possibility frontier must be known in every detail. In reality, many policies are governed by custom and characterised by lack of experiment; and many trade-offs appear quite meaningless. Even supposing a performance possibility frontier to be identifiable in principle, the plausibility of an all-embracing managerial utility function remains doubtful. There are two kinds of objection to the concept. The first stems from the model of a business organisation as composed of interests and groups with differing objectives and priorities, with top management presumably performing a unifying and directing function. Within such a structure top management promotes or imposes compromise where necessary between the aims and policies of conflicting, semi-autonomous, interests; but

it has in addition the inescapable function of determining general strategy or guidelines. In choosing the terms and parameters of its own objective function top management is obliged to think on a different level from narrower interests lower down the firm's structure, and the instruments it employs in pursuit of its objectives are similarly beyond the competence of lower levels to determine. It is realistic to suppose that in determining its own objectives and instruments top management is willing - indeed, obliged - to take as given a great deal of "background" in the shape of policies and attitudes binding the firm at many points and levels.

Second, even if one accepts a corporate personality capable in principle of organising an ideal multi-dimensioned top-level strategy - in effect dismantling the subordinate/top-level distinction suggested here - there remains the question whether such a decision maker can realistically be supposed to express utility in correspondingly multi-dimensional terms. Granting the theoretical justification for an all-embracing utility function, its credibility as an account of real-world processes remains a legitimate subject for discussion if, as is the case, there is at least one competing version of the structure of decision making.

12 : 4 (iii)   Investment policy as the main element of strategy.

Component areas of subordinate policy can be listed in very general terms. A reasonable classification might be: financial policy (dividend risk, debt and external equity policies, etc); operating policy (competitive behaviour, production and inventory and manpower policies); product market policy (general policy on the types of products and markets to aim for, research and development policy, etc.) It is

impossible to be dogmatic about whether all of these areas will in all cases be treated as subordinate, though earlier arguments suggest as much. In some cases it is reasonably certain that a subordinate classification is appropriate: dividend risk policy and general manpower policies are examples that, for different reasons, belong in the subordinate area. However there may be policies which management will wish to evaluate along with investment policy at the top level of decision making: an example might be product market policy, or aspects of it. Again, certain aspects of policy may be so closely tied to investment policy that isolation of the latter would be impossible: in this case investment policy remains effectively alone at the top level, there being only one setting of the "tied" policy for any given setting of investment policy. The general proposition accepted here is that investment policy stands naturally in a separate category from other policies, because of its importance and for the practical reasons given in Section 4 (ii) — that most interactions of policy simply are not visualised by management, which in any case accepts a great deal of policy as "background" in formulating its top-level strategy. [15]

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(15) The argument for separating product market and investment policies and classifying the former as "subordinate" perhaps deserves further clarification. Product market policy is envisaged as the firm's general intentions regarding the development through time of its decision making in this area in response to growth and profit experience, etc. Management determines in a broad, conditional way the features of product market policy prior to its choice of rules governing investment decision making. At the risk of oversimplifying, product market policy amounts to prior and conditional decision making about the kinds of investment opportunities (products and markets) the firm will seek, the policy on research, and the ways in which these decisions will respond to results; while investment strategy controls the selection of opportunities from among those generated by the operation of these prior-determined policies. The distinction merely formalises and reflects the assumption in any model of long-term investment forecasting of background mechanisms which generate investment opportunities.

If management's concern for performance aspects covered by subordinate policies is fully satisfied by the prospective working of those policies, it follows that corresponding to top-level strategy choice is a managerial utility function expressed only in performance aspects not covered by subordinate decisions. The ultimate reduction would be to a simple choice of investment risk policy matching a utility function based only on those performance aspects that are affected by investment policy. While such a neat partitioning and simplification of decision making cannot be taken for granted, there do appear to be good reasons on each side of the model (the structure of policy and the managerial utility function) to expect a considerable simplification along the lines indicated. Only if the top-level utility function includes more than one performance variable is there much reason to expect that top-level strategy involves more than one type of policy. Much of the following section is intended to make the point that equity market value does constitute a performance variable in terms of which management can express various motivations - "managerial" and market-oriented - , thus allowing a correspondingly simplified concept of top-level strategy to take its place on the other side of the model. This conclusion will be welcome at the practical level, where it is difficult (and arguably unnecessary) to imagine a simultaneous choice of investment policy and other policies.

Section 5 : Managerial Motivations

12: 5 (1) Performance variables and policies.

This section deals mainly with the form and interpretation of the top-level managerial utility function, and is intended to develop

the argument that both utility function and policy at the top level are likely to be expressed in terms of limited numbers of variables. As admitted in Section 4 (ii), there is no uniquely correct account of decision making; but a particular version may at least be supported by argument.

It is important to recognise the great variety in the procedures by which firms work out the background conditions (subordinate policy) against which they compare alternative top-level strategies, although the theoretical framework assumed for top-level choice is the conventional one of utility maximising. However, a proposition closely connected with targets-instruments decision theory seems likely to apply at both levels: it is that a decision maker tries to match, or pair, performance variables with appropriate instruments. [16] Accordingly, if top-level strategy does involve only one component – investment policy – there may correspondingly be only one performance variable in the utility function at that level. With two or more performance variables in its utility function management would probably wish to increase the number of effective instruments. In this important sense the separate arguments about the numbers of variables involved on each side of the model (policy variables making up strategy and performance variables determining utility) ought to confirm one another, or at least point in the same direction. In the remainder of this section the case for investment

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(16) This proposition clearly transcends targets-instruments theory and describes a general tendency of efficient decision making in any theoretical framework where utility is derived from a number of separable performance variables. Even if performance variables are inseparable – as when $U = U(X,Y)$ and $Y = y(X)$ – the decision maker looks for an instrument to lessen y's dependence on X, in addition to the single instrument (control of X) already available.

policy as the core of management's strategy is accepted, and it is argued that equity market value can be seen as the determinant of top-level managerial utility.

12 : 5 (ii)   Equity market value and managerial utility.

It is logical to separate the argument that top-level utility may be a function of a single variable from the question of specifying that variable. Preliminary points in favour of equity market value as the determinant of utility are its obvious importance to both management and investors, and its strong association with investment strategy. These considerations can be supported by further argument.

First, the suggestion that managerial utility is determined by equity market value does not imply that management can (or wishes to) operate simply as a value maximiser. Indeed, having stressed the fact of managerial discretion on many subordinate policies, it would be a strange reversal to suggest that at the top level of decision making management is bound to follow without deviation the lead of market forces. As shown in Chapter 11: 4 (ii), when V_{eo} is subject to risk it is generally an oversimplification to describe strategy as directed towards a unique maximum. And in Section 1 (i) above it was argued that management may be in the position of having to choose between strategies on which impersonal market forces place equal, or nearly equal, values. Within wide limits management's discretion on strategy choice must be accepted; but the motivation behind its choice is not dictated by the fact of its discretionary power. A sensible approach therefore is to consider the ways in which different motivations - investor oriented

and "managerial" — are likely to be reflected in strategy choice. [17]

12 : 5 (iii)   Management's motivation : investor-oriented.

There is an obvious attraction in the thought that management,
however discretionary its power, would find itself applying the simple
utility function $U = U (V_{eo})$ if it set out consciously to represent
investors' interests in its choice of strategy. The argument would be
that because V_{eo} incorporates by definition all probability-weighted
expectations of future market value, it is redundant for management to
express its own utility function — a proxy for that of investors — in
terms of both present and future equity values. [18] If management
wishes its strategy to reflect investors' wishes, so the argument might
run, it is quite sufficient to assume the basic version of the managerial
utility function. If all strategies considered have unique V_{eo}-values
the result of applying this logic is not simply a reversion to the
earlier view of managerial utility : managerial discretion in top-level
strategy choice has also been discarded. For this reason it is important

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(17)   At this point the term _market oriented_ as a description of one
conceivable management motivation is replaced by the more accurate
and suggestive term _investor-oriented_. It has been argued at
length that in its subordinate decision making management can
scarcely be described as unswervingly market-oriented, and the
focussing of discussion on top-level motivation will merely
confirm the irrelevance of that term.

(18)   This argument is simply a corollary of what was shown in Section
1 (ii) above: that for all growth-paths there are corresponding
time-paths of market value. However, it will be argued in
what follows that V_{eo} is a poor indicator of the many characteristics
of the pattern of future growth possibilities.

to consider whether management, acting on investors' behalf in strategic decision making, can continue to justify an extended horizon utility function (or some less formal equivalent for looking beyond immediate results) and the greater managerial discretion it allows.

As argued above, the necessity for discretion and for some form of extended horizon utility function are both obvious when a number of alternative strategies have approximately the same V_{eo} but markedly different probability distributions of V_{em}. [19] Discretion is unavoidable, and the only possible interpreter of investors' wishes is management itself – which must in such a case look to an extended horizon. Management may find this task difficult enough, but it is less likely to occur than a situation where rival strategies differ significantly in their V_{eo} prospects : the case for an extended horizon for an investor-oriented management ought to be demonstrated in relation to this latter, more common situation.

The idea that the level of V_{eo} is in some sense a complete measure of investors' well-being is likely to appear to management as an over-

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(19) To simplify discussion it is assumed that management selects a future date, $t = m$, as the focus of its interest in prospects for investors, and later as its own horizon. Of course the two dates need not coincide, and in the case of investors' interests the choice can only be arbitrary. See also footnote 6 above.

simplification. [20] In the valuation model assumed here, it arises from the fact that V_{eo} does incorporate expectations about future levels of market value and cannot therefore be dismissed as a "short-sighted" valuation. However, it is open to management to take the view that after $t = o$ (any time beyond the present) the value of V_{eo} will become an irrelevant byegone – whereas prospects for $t = m$ will remain relevant as long as m remains in the future. In other words, while V_{eo} matters to investors at present, they also have an interest in prospects for V_{em} ; or, if they do not, they will soon enough. V_{eo} is only the market value of equity at a point in time, and as such is hardly a complete or persuasive indicator of investors' well-being in the longer term. But only management can determine longer-term prospects, and in its desire to do so it must develop the criteria to employ on behalf of investors.

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(20)   In a world of unique growth-paths and no taxes, investors' utility is undoubtedly maximised when the present-day market value of their assets is maximised. This proposition lies behind the theory explained in Part I, the traditional financial theory of the firm. However, as will be argued in Chapter 14 : 4 (iii), it is not difficult for management – even in steady-state growth conditions – to define "stock market approval" in its own way. Under multiple growth possibilities even the simple theoretical proposition connecting maximum investors' utility with maximum present-day market value loses its relevance; and management is confronted by intractable problems of investors' horizons, outside opportunities, risk attitudes, tax situations, etc. There is of course a "clientele" theory of shareholding, that investors tend to be attracted by a company's record (of earnings, dividend stability, growth, etc.) which matches their requirements (see, for example, A.J. Merret and A. Sykes, The Finance and Analysis of Capital Projects, 1963, p.72). This idea may provide valuable insight, and if accepted by management it tends to favour an unchanging strategy, but it does assume that investors have all along been aware of the dispersion of the growth-path probability distribution. Even allowing for such awareness among investors, the knowledge that the latter "approve" its present strategy can hardly be helpful to management in deciding whether a different strategy would gain greater or less approval. This point is quite distinct from the question of whether management wishes to satisfy present share-holders or potential shareholders or is simply interested in the market value of its equity.

Investors' well-being is obviously an elusive concept in the conditions assumed, and no standard attitude or procedure can be expected. Procedures may range all the way from an ambitious attempt to infer an investors' utility function of the kind described in 2 (iii) above, with the strategic decision aimed at maximising investors' expected utility,[21] to plain guesswork about the kind of trade-offs between present-day value and risky future prospects that investors would accept. No matter how formal or casual the procedure, however, management remains aware that regret in its technical sense cannot be identified ex ante or ex post : the risky decision it takes on investors' behalf is one whose outcome can never be compared accurately against that of a rejected alternative.

Thus the general picture of strategy choice - extended horizon, managerial discretion, acceptance of the permanence of choice - is little altered in this exercise of assuming an investor-oriented motivation. What remains to be seen is whether the actual choice will be, or is likely to be, predictably different according to the kind of motivation assumed.

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(21) This would mean not only inferring the general shape of the typical indifference curve, but also being prepared to attach a utility value on investors' behalf to each curve in the system. The latter step is clearly an enormous stumbling block to any but the most arbitrary approach: the analogous problem facing management seeking "stock market approval" in the steady-state growth context is considered in Chapter 14 : 4 (iv).

12 : 5 (iv)   Management's motivation : growth.

The alternative view of managerial discretion on strategy choice
is that it is directed to "managerial" objectives, and in particular
to the growth of the enterprise. [22] A strong desire for growth could
be reflected in management's utility function — for example in the
setting of targets in the indifference curve systems described in
Section 2 (iii) — and ultimately in the choice of strategy.

A growth preference is implicit in a managerial utility function
based upon prospective equity values at different dates, the kind of
reference system described in Section 2 (iii) above. The higher the
level of V_{eo} , cet. par., the greater the value placed by the market
on the firm's growth prospects, and the higher the level of managerial
utility. Similarly, the higher is prospective V_{em}, cet.par., the higher
the valuation of growth prospects beyond t = m and the greater the

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(22) This view has a long and impressive history, so much so that
      managerial theories tend to differ on the questions of what other
      objectives to allow for in addition to growth, and on the precise
      factors underlying the desire for growth. On the latter question
      the range of ideas is particularly wide, ranging from a strongly
      organisational, impersonal, motivation (J.K. Galbraith, The New
      Industrial State, 1967, p.177) through "pure" managerial
      motivations such as sales growth maximisation (W.J. Baumol,
      "On the Theory of the Expansion of the Firm," American Economic
      Review (1962)) to "self-interested" managerial motivations arising
      from growth-related rewards and prospects of top management
      (R.L. Marris, op.cit., 1964, Chapter 2).   These different ideas
      should certainly not be seen as necessarily mutually exclusive.
      The present discussion takes a neutral position (among managerial
      theories) on the question of underlying motivation: its concern
      is to show that growth preferences however motivated can be
      represented within the value-based managerial utility function,
      and that the presence of risk adds an extra dimension to
      problem of choice.

improvement since t = o in the value of those prospects. Both of these aspects of growth, prospective and accomplished , contribute to managerial utility.[23]

Thus an equity value-based utility function is consistent with a managerial growth preference : indeed it confirms the nature of management's objective and establishes the dimension in which growth will be measured. Prospective growth which is reflected in high market value, or achieved growth measured in increased market value, are likely to be the kinds of growth preferred by management. Growth prospects without favourable market value implications are almost certainly what management will wish to avoid.

Turning to the question whether management choosing "managerially" in the light of a growth preference is likely to choose noticeably differently from an investor-oriented management, a simple example provides a useful focus. Diagram 5 shows a comparison between strategies A and B in terms of their V_{eo} values and V_{em} prospects.

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(23) In the double-target systems of indifference curves suggested in Section 2 (iii) the implied emphasis is on both prospective and achieved growth. Both V_{eo} and V_{em} reflect growth prospects (and related risks) at their respective dates, and it is easy to interpret a target for improvement in value between the two dates in terms of a managerial growth objective. Similarly, the system reflects managerial trade-offs between growth prospects at different dates.
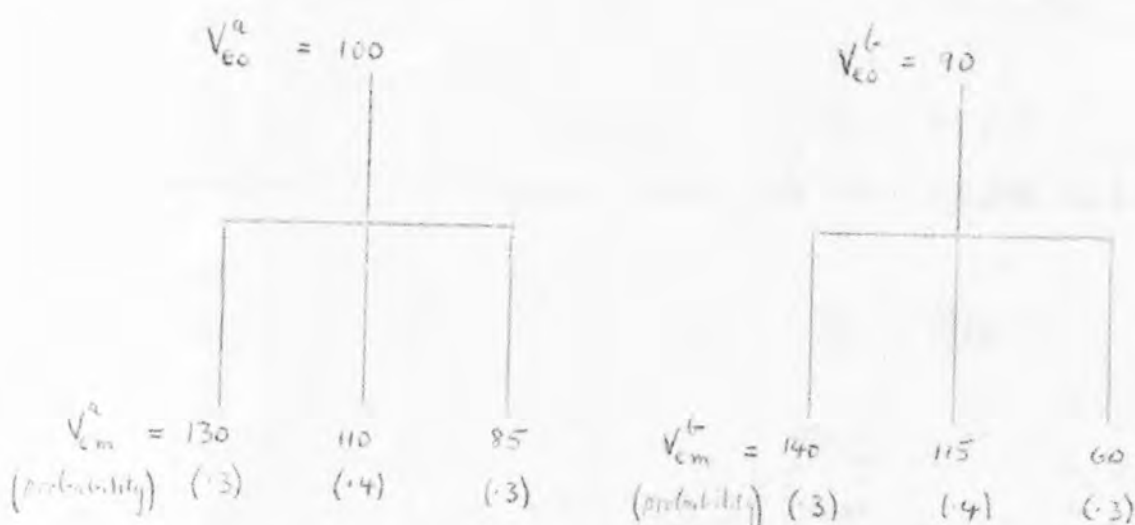
$$V_{eo}^{a} = 100 \qquad\qquad V_{eo}^{b} = 90$$

$V_{em}^{a} = 130 \qquad 110 \qquad 85 \qquad\qquad V_{em}^{b} = 140 \qquad 115 \qquad 60$

(probability) (·3) (·4) (·3)   (probability) (·3) (·4) (·3)

Diagram 5 : Strategies A and B , present and prospective equity values.

The higher present-day value offered by A reflects the market's
"disapproval" of the comparatively wide spread of the V_{em}^{b} distribution,
which in turn reflects a greater divergence of growth-paths from $t = 0$
under B. The choice exemplified by A and B may be seen as the
most awkward management can anticipate, involving as it does significant
differences both in V_{eo} and in the spreads of V_{em}.

Without going to the length of constructing a model managerial
utility function in order to compare A and B formally, certain rather
obvious points about management's dilemma can be noted. A preference
for growth might lead management to choose B - especially if the choice
is made in a vacuum, with no present-day comparison available to investors -
on the grounds that its best and second-best outcomes represent impressive
improvements over the initial market value (and are indeed actually
higher than their respective A counterparts, though of course no
inference can be drawn between unique outcomes in different distributions).

On the other hand if the "preference for growth" is actually a strong
preference to avoid poor growth performance, the choice swings
back towards A which offers a higher initial value and no prospect
of the kind of spectacular growth in value that B offers, but which
safeguards market value much more effectively if the path to t = m
proves unsuccessful.

The point that has been made obvious by the example is that there
can be no such simple motivation as "desire for growth" in the conditions
of multiple growth possibilities. There can be no expectation of a
uniform managerial "growth" motivation on the question of strategy,
just as there are bound to be different managerial interpretations of
the preferences of investors. It certainly cannot be expected that
management's strategy decision will reveal the nature of its motivation
- investor-oriented or managerial. And, as should by now be obvious,
the distinction between types of motivations is itself fictional and
artificial: lacking any means of identifying or divining investors'
preferences on the risks that must be taken, management's decision
on the i r behalf is unavoidably "managerial," and is as likely to be
balanced between A and B-type options as that of an openly "managerial"
management. Indeed, given the impossibility of definitely identifying
investors' preferences in the conditions described, it is most unlikely
that management sees itself as having to choose between the latter and
its own.

Nor can it be argued that the A versus B situation chosen
in the example is exceptional, that in other situations a clearer distinction
between an investor-oriented and a "managerial" decision would be found.

For example, had B offered a present-day value of 100, equal to A's, and prospects for V_{em}^{b} of 110 (probability 0.3), 102 (0.4) and 94 (0.3), the choice at $t = 0$ would have been a straightforward one between two V_{em} distributions, with A's having a higher mean and greater dispersion than B's - the classical dilemma in decision making. That this kind of comparison is indeed typical can be argued briefly. Of two different strategies having the same market value at $t = 0$, one must have a higher $\bar{S}$ and higher σ_s than the other. [24] This difference at $t = 0$ means that for $t = m$ the prospects are for the first strategy's V_{em} distribution to have a greater dispersion and higher mean value than the second's.

Section 6 : Conclusion

12 : 6 (i)   Summary

This chapter has presented the argument for a managerial (top-level) utility function expressed in terms of present and prospective equity values. Two approaches to this proposition have been put forward. First, analysis of the structure of decision making, and in particular the distinction between subordinate and top-level decisions, suggests that investment policy is likely to stand on its own at the top level : if this is so, it provides an appropriate match, or pair, for equity market value as the single element in the managerial utility function. A larger number of utility function variables might require an increase in the number of top-level policies, whereas likely candidates for additional elements of top-level strategy are difficult to suggest.

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(24) See equation (i), $V_{eo} = H(\bar{S}, \sigma_s)$, in which an increase in $\bar{S}$ obviously increases V_{eo}, cet.par., and an increase in σ_s reduces V_{eo}, cet.par.

However, a rigid correspondence between the numbers of policy and utility function variables is not essential, and this latter consideration is suggestive rather than conclusive.

The second method of justifying the proposed form of utility function is to accept for the sake of argument a utility function based upon a single performance variable. Having done this, equity market value appears well qualified to be that variable, whatever the nature of management's motivation. Different motivations are discussed, and in each case an extended horizon utility function based upon equity market value appears suitable. The actual choice of strategy may be affected by the underlying motivation, but it appears to be an over-simplification to suggest any particular direction of bias in strategy choice arising from the nature of management's motivation. This result requires some elaboration.

Typically in managerial theories the utility function embodies management's willingness to compromise between satisfying its own preferences and those of investors. Management is neither wholly market-oriented nor wholly "managerial". This view and the result it produces (for example the tangency solution of Marris, between a managerial indifference curve and the objective valuation ratio-growth rate relationship) are very much at variance with the account given in this chapter. Here all motivations are seen equally as requiring managerial discretion for their implementation: complete investor-orientation of strategy choice is unattainable, and differences between strategies chosen under different motivations are unlikely to fall into any preconceived pattern. One of the purposes of Chapter 14 which employs the framework of the Marris-type steady-state growth model will be to

reassess the familiar concept of strategic choice as a compromise
between clearly identifiable investor and managerial interests. It
will be shown that at least in respect of the discretionary nature of
management's identification of investors' interests and its effect on
the nature of the strategic decision, the steady-state and multiple
growth-path models of decision making are closely related.

12 : 6 (ii)   The unresolved problem of uncertainty.

Throughout this chapter analysis has rested upon two important
assumptions: management's confidence about market valuation processes
and about the growth possibilities inherent in different investment
strategies have both been taken for granted. The strategic decision
taken against this background was treated, by management and investors
alike, as permanent. In fact the impermanence of strategy is recognised
by all, but the logic in relating present-day managerial utility to the
apparent prospects offered by the chosen fixed strategy can be defended
on the same grounds here as it was in Chapter 11 : 5 (iv). These are :
first, that all available information is taken into account and that new
information or action based upon it cannot be anticipated; second, that
on present information management does not foresee that it will wish
to change strategy at any date. The fact that an extended horizon
utility function is now assumed alters neither argument, but is very
relevant in the following chapter where for the first time managerial
doubts about growth and valuation relationships are admitted, and their
effects on the logic of the model of strategic choice are examined.

CHAPTER 13

Uncertainty and the Theory of Strategy Choice.

Plan of Chapter

Section 1 :   Introduction.

    (i)   Summary of "ideal" conditions.

    (ii)   Origins and possible effects of uncertainty.

    (iii) Testing the logic of a strategic decision theory.

Section 2 :   Ideal Valuation and Real-World Conditions.

    (i)   The relevance of conditional value predictions.

    (ii)   Managerial interpretation of the nature of valuation.

    (iii) Identifying the parameters of the ideal value
            relationship.

    (iv)   Justifying the relevance of ideal valuation.

    (v)   Managerial uncertainty and the nature of valuation.

Section 3 :   The Anticipated Durability of Hypotheses.

Section 4 :   Summary and Conclusions.

Section 1 :   Introduction.

13 : 1 (i)   Summary of "ideal" conditions.

Some important questions raised explicitly or by implication
in earlier chapters remain to be considered here. Their common
theme is a recognition that real-world conditions differ from
those depicted in Chapters 11 and 12, and the purpose here is to
consider whether the strategic decision theory of those chapters
is affected by departures from ideal conditions.

The theory of strategic decision making as developed in
Chapters 11 and 12 depends on the assumption that management
correctly interprets the nature of the equity market valuation
process (Chapter 10 : 3) and is able to predict V_e for any
period, conditional upon the strategy chosen and the growth-path
followed to the period in question (Chapter 12 : 1 (ii)). This
ability to make conditional value predictions rests upon an earlier
assumption : that management correctly identifies the probability
distribution of growth-paths of constant risk expected dividend
(referred to now as the g - distribution) for each investment
strategy considered.[1]

In the conclusions to Chapters 11 and 12 it was acknowledged
that neither management nor investors are likely to believe whole-
heartedly in a g - distribution or a valuation model estimated
from available data. Nevertheless, a strong argument was advanced

(1)   It is now assumed that management resolves any doubts about
       which g - distribution is associated with any one strategy :
       see Chapter 11 : 3 (iii).

to the effect that as neither side can predict how present
estimates may change in the future, and as decision making on
the basis of available probabilities is a widely-accepted and
realistic view of many situations, the theory of strategic
decision making should be allowed to stand. This defence was
offered with the proviso that it was likely to apply particularly
to the situation envisaged in Chapter 11, in which a strategic
decision was made with reference only to (the probability
distribution of) equity market value(s) at $t = 0$ - that is, on
the basis of its immediate outcome. Generalising this, a person
is likely to behave as if the probability estimates he employs
are correct when he knows that there will be no opportunity to
discover errors in the estimates, or that there will be no
opportunity to alter the decision based upon them, within the
time horizon relevant to the decision. For such situations the
argument of Chapter 11 : 5 (iv) is appropriate.[2]

In Chapter 12 no more was said about errors in growth and
valuation models or the imperfections of the capital market, and
so the position at this point is that the theory of strategic
decision making proceeds on the assumption - or its effective
equivalent - that ideal estimating and capital market conditions
prevail.[3] The first question then is whether, with an extended

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(2)   A punter who allows himself only one visit to a bookmaker
        may stake £X on the result of a race on the basis of what
        he considers to be the true odds, knowing that once the bets
        are placed - and possibly even before the race is run - he may
        revise his initial views. Even if his views change the £X
        cannot be recovered or reallocated : nor will his approach to
        subsequent gambles change. The punter's situation condemns
        him to the continued use of probability estimates which he
        knows are likely to be altered through time and by events.
(3)   The effective equivalent would be a complete managerial belief,
        whether correct or not, in the prevalence of ideal conditions.

horizon utility function and in spite of doubts about some of
its beliefs and estimates, management is likely to behave as if
ideal conditions prevail. The second question is whether in
the light of modifications which will be shown to be necessary
to preserve the logical consistency of the theory developed in
Chapter 12, some indicator other than equity market value provides
a more realistic determinant of managerial utility in a theory of
strategic decision making.

Before attempting to modify or replace the theory set out
in Chapters 11 and 12 it seems advisable to re-state briefly the
assumptions on which it is based. Management is able to identify
the true g-distribution for any strategy, as well as the value the
capital market will place on the firm's equity at any time on any
growth-path. The market's valuation of equity is based on the
objective properties of the firm's g-distribution, and both the
nature and precise form of the valuation relationship are correctly
identified by management. Finally, with perfect communication
between investors and management, the former possess complete
knowledge at all times of the latter's correct assessment of growth
prospects, so that the market value of equity never differs from
its true value.

13 : 1 (ii) Origins and possible effects of uncertainty

Management may experience various types of uncertainty in
connection with its views of the g-distribution, valuation and the
related decision procedure. While a formal separation of these
types into two categories according to origin may prove helpful

in exposition, it will become apparent that they are unlikely to occur separately. For simplicity it is assumed that management begins at $t = 0$, the present, in a state of complete confidence about the nature and parameters of the valuation process and about the firm's S - distribution.

Uncertainty has an internal origin when it relates to the estimates made by management of the firm's g-distribution and the market value relationship, $V_e = H (\overline{S}, \sigma_s)$. If either of these estimates may change within the period covered by management's utility function, and if it would be possible to alter investment strategy before the terminal date of that function, then such a possibility should not be excluded - as it has been hitherto - from a theory of strategic decision making. This conclusion holds in spite of the logical impossibility of predicting when or how strategy may be changed (Chapter 11 : 5 (iii)). A change in either estimate at any time may cause regret at the strategy choice made at $t = 0$ and lead to the adoption of an apparently superior strategy for the future. If this kind of switch may occur before the terminal date of the utility function, $t = m$, the originally calculated conditional equity values for that date provide a very incomplete picture, even at $t = 0$, of the range of possibilities for $t = m$.

Next, suppose that management acknowledges the possibility that errors in estimates may be discovered before $t = m$, but that no remedial action is expected to be possible before that date. In this case the view from $t = 0$ of the possibilities for $t = m$,

though possibly incorrect, does provide a logically satisfactory
basis for the strategic decision in the absence of any other
expectations about a consequential breakdown in the valuation
process. (Such reactions are to be considered shortly.)
Management's situation here is analagous to that of the punter
described in footnote (2) above: an account of the decision
procedure lacking any provision for "mid-course correction"
is logical because the decision at $t = o$ is taken on the basis
of the best information then available and because no subsequent
interference with the decision or its consequences is expected to
be possible.[4]

Another internal source of managerial uncertainty involves
the assumed basis of investors' valuation of equity. Uncertainty
of this type derives from the first internal uncertainty
considered above, in respect of the stability of investment
strategy, but it looks to the possible effects on investors'
approach to valuation instead of assuming, as above, that this is
not affected. Even if management believes initially that investors
attempt to value equity in the way it prefers to assume, it must
be aware that knowledge of its own uncertainty about the true g-
distribution or about the true valuation relationship may, through
time, cause even rational investors to alter their general
approach to valuation. As with the first uncertainty, the possible

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(4)   This combination of acknowledging the possibility of error
       recognition and denial of all corrective measures is very
       unconvincing in the present context, and from now on it
       is disregarded. Management is expected to take some action
       if it recognises the wisdom of doing so before $t = m$.

instability of investment strategy is foreseen — this time by
investors. Should the latter change their approach to valuation,
the process of valuation would cease to be comprehensible to
management in the way that was possible when both sides regarded
the g-distribution, and management saw its estimate of the valuation
relationship, as objective truth. Not only the current, but the
future basis of valuation too, would be incomprehensible and
unpredictable, and conditional prediction of equity market values
would be impossible. As with uncertainty of the first type, it
is not necessary that management believes that such a development
will occur: a sufficiently strong belief that it may is enough to
invalidate a decision procedure based on confidence in a stable
valuation process.

Both of the effects considered so far, on management's
confidence in maintaining its investment strategy and on its
confidence in the nature of valuation, derive from its recognition
that it may at some time come to suspect its estimates of the
important growth and valuation relationships: one effect relates
to management's own response to that development, the other relates
to investors' response to the mere possibility of that development.
An external and logically separate source of uncertainty is the
behaviour of the capital market: even with complete and continuing
certainty about the g-distribution, management may come to question
its beliefs about the nature and parameters of market valuation.

As long as the capital market behaves in an orderly fashion
and in accordance with conditional predictions there will be no

difficulty in accepting that the interpretation of valuation and the parameter estimates are correct. However, once it is recognised that the market functions imperfectly, uncertainty may take hold in one of several ways. Management may simply believe that the market is inadequately supplied with information, and that its own conception of valuation in ideal conditions and its parameter estimates remain completely correct. It is perhaps more likely that some doubt will be experienced about the parameter estimates, but not about the underlying conception of valuation: instead of attributing all of the discrepancy between actual and predicted market values to market imperfections, management may begin to modify its parameter estimates for the relationship $V_e = H\left(\overline{S}, \sigma_s\right)$ in an attempt to improve predictive power. A final possibility is that confidence in the nature of valuation may be eroded and an interpretation more in keeping with the market's observed behaviour may be sought.

These various reactions may be seen as stages in the transformation of beliefs, but such a progression is not the inevitable outcome of uncertainty. It is wrong to expect that a strictly scientific approach to the formulation, testing and rejection of hypotheses necessarily characterises management thinking about valuation: indeed the intention in Section 2 is to show that management's belief in the relevance of an ideal valuation model can be sustained by rational argument, and even reinforced, in market conditions which are admittedly imperfect.

13 : 1 (iii) <u>Testing the logic of a strategic decision theory.</u> [5]

Evidently the general approach to valuation and the decision

procedure based upon it which have so far been attributed to

management must be seriously questioned, given the uncertainties

described above. This reappraisal is carried out in Sections 2

and 3. First, it is necessary to try to establish a criterion

for the logical consistency of any account of strategic decision

making, with particular reference to the assumed dependence of the

decision maker's present utility (in respect of his strategy

choice) on the probability distribution of performance over a

fairly lengthy period of time. [6]

One rule for checking the logic of a model of decision

making has already been suggested, in Section 1 (i) above: if

a policy and its consequences are alterable before the terminal

date of the decision maker's utility function, any acceptable

account of choice must reflect that fact. Strategy can then no

longer be seen, as in earlier chapters, as a once-for-all

decision, never to be reconsidered; and the possibility that it

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(5)   This sub-section deals with the internal consistency of
        theories of decision making which, like that of Chapter 12,
        involve an extended horizon utility function and require
        the decision maker's confidence in foreseeing all possible
        developments up to a limited horizon. It should be
        emphasised that not all accounts of business behaviour
        need to exclude unforeseen developments or changes of
        policy: see the brief discussion of the behavioural model
        of Cyert and March in Chapter 1 : 1.

(6)   Generality is maintained by not specifying precisely the
        way in which each growth-path in the g-distribution
        generates an indicator which determines conditional
        managerial utility. In particular, management's
        interpretation of valuation and the form of its utility
        function need not be specified.

may be changed within the period covered by the utility function
cannot be disregarded. This conversion to reality is wholly
desirable, but it raises the central problem considered in this
chapter - that of adapting the original theory of strategy choice
to real conditions, and in particular to the impermanence of
strategy.

There is a more exacting test of the logic of an account
of decision making, and although its general validity has in
effect been rejected earlier, the overriding importance of the
strategy decision might be thought likely to produce in manage-
ment an especially careful attitude. The suggested rule is that
for a particularly important decision the decision maker must be
convinced that his choice will not be recognised as mistaken before
the terminal date of his utility function. Under the general rule
mentioned above, an account of a decision procedure is accepted
in spite of the possibility of regret recognition if the remedial
action taken in any situation, and its results, can be foreseen at
$t = 0$: under the present suggestion the mere possibility of regret
recognition is sufficient to exclude an account on logical grounds.
This difference between the two rules is less significant on closer
inspection than it initially appears, because it has already been
conceded that management cannot even identify the nature of the
regret that may become apparent - still less define its actions
and their effects in that event. In practice therefore the two
rules are alike in rejecting the logic of any account of the
strategic decision in which a risk of regret recognition is
present: in the first case the reason is practical and incidental,
in the second it is fundamental.

The rationale of the rigourous test would appear to be
a belief that management will not adopt a decision procedure
whose resulting policy may be recognised as mistaken before $t = m$.
An extreme example - today's decision, irreversible for (say)
five years, may be regretted tomorrow - can make the criterion
look reasonable. It is, however, arbitrary; and if it were a
true reflection of managerial psychology, decision making would
be forced into ineffective and irrational procedures. In the
first place the criterion extends management's quite understandable
dislike of mistakes into an implied managerial rejection of any
decision procedure which involve a recognizable mistake: if true
this would suggest that management favoured procedures equally
(or more) likely to lead to errors, but less likely to reveal them.
The self-defeating nature of the managerial psychology implied
by the test is shown by the ways in which premature recognition
of regret might be avoided. There might be a deliberate imprecision
in defining objectives and/or inefficient monitoring of performance;
or the horizon of the utility function might be shortened, and this
too must seem implausible if a more distant horizon would otherwise
be preferred.

Realistically, there is no satisfactory way of eliminating
completely the possibility of premature regret recognition; and to
suggest that the mere possibility of its occurence necessarily
invalidates a decision procedure in management's eyes is to imply,
as shown above, that managements act irrationally, blindly, and
with deliberate inefficiency in a decision of great importance.

A preferable way of expressing management's natural attitude
is to say that, given a serious and continuing study of the
relationships on which strategy choice is based - and such an
effort reflects an entirely rational self-interest - management
will certainly wish to decide its strategy in a way which gives
it a high degree of confidence in avoiding premature regret
recognition. It is argued in Sections 2 and 3 that management
can realistically enjoy high confidence that it will not
recognise a strategic error prior to $t = m$, whether due to its
interpretation of the nature of valuation, its estimated value
relationship, or its estimated g-distribution.

The original and less rigourous test of the logic of an
account of decision making must also be reformulated in recognition
of the fact that complete certainty is unattainable. According
to that test a decision procedure was judged reasonable even if
mistakes might be recognised, as long as remedial measures and
their consequences could be completely foreseen at $t = o$. Since
this is an impossible condition, management must be assumed willing
to settle for a sufficiently high degree of confidence that errors
will remain undiscovered until $t = m$ at least.

Whichever theoretical test of the logic of models of decision
procedures is thought appropriate - and it has been argued that the
second, rigourous, test implies a most unlikely managerial attitude -
the practical result is shown to be the same. The best management
can hope for in practice, whatever its basic outlook, is that even
after its own diligent efforts premature regret recognition is a
rather remote possibility.

Section 2 :   Ideal Valuation and Real-World Conditions.   (7)

13 : 2 (i)   The relevance of conditional value predictions.

It is useful to envisage two stages of departure from
the ideal capital market conditions, assumed in Chapter 10,
which formed the background to Chapters 11 and 12. In the first
stage management remains convinced of the accuracy of its
estimated g-distribution, but the capital market functions less
perfectly than is assumed in Chapter 10. In the second stage
management concedes that its estimated g-distribution may
subsequently be shown to be incorrect. For the moment it is the
first stage that is of interest.

(7)   Although the subject of this section is not the nature of the
market valuation of equity, but rather with whether manage-
ment can confidently sustain a certain view of its nature
and parameters, it is impossible to ignore the vigourous
debate on fundamentals that has been going on in recent years.
This debate as it then stood was reviewed by Baumol in what
remains a stimulating and helpful contribution: W.J. Baumol,
The Stock Market and Economic Efficiency, 1965. Baumol
concluded (op.cit. p.60) that security prices probably do
follow the developments in company prospects, though competitive
processes cannot be relied on to allocate capital optimally.
This section may be seen as a defence of traditional valuation
theory against the Modigliani and Miller claim that investors
do not and cannot value objective probability distributions
and that the major risk they recognise is that present in
their own subjective views on a share's average earnings
over time: see F. Modigliani and M.H. Miller, "The Cost of
Capital, Corporation Finance and the Theory of Investment,"
American Economic Review (June 1958), esp. pp.265-6. Here
it is argued, as it were on management's behalf, that
investors do make a "best estimate" of objective risks and
an appropriate valuation on that basis; and that the separate
risk of making an incorrect estimate is separately handled
by investors. (It was argued in Chapter 2:4 that objective
risk is not an insignificant factor in determining investors'
valuation of an income stream.)

A degree of failure on the part of the capital market
to correctly value a firm's equity through failure to recognise the
true g-distribution need not prove irreparably damaging to
management's belief that it understands the nature of the valuation
process and can identify its true parameters.[8] If this belief
is sustained, and if investment strategies can be translated into
their respective g-distributions, then conditional prediction of
true equity value at any point on any growth-path will remain
possible - just as described in Chapter 12 : 1 (ii). Both aspects
of management's assumed belief, that the nature of valuation
remains the same as in ideal conditions and that identification
of the ideal market value relationship is possible, are considered
in this section after a preliminary discussion of the relevance of
ideal predictions in real-world conditions.

The ability to predict conditional true equity values
provides no guarantee that the relevant predictions will actually
be realised. Investors may repeatedly fail to perceive the correct
g-distribution so that actual and true market value continue to
diverge. According to one point of view - which, with its opposite,
will be developed later - if management wishes to accept the
relevance of conditional value predictions it must be convinced of
a tendency for investors' views of growth prospects to converge
on its own, the correct version. This requires more than the

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(8)   The terms true and correct used in relation to management's
        beliefs and estimates refer to the relationships and
        parameters which management would expect to observe under
        ideal capital market conditions, when both investors and
        management share the same (correct) expectations and when
        disruptive short-term influences on valuation are excluded.

simple convergence between market expectations and a stationary
set of correct data that might be expected for the static and
comparative-static situations discussed in Chapters 3, 5 and 6.
Instead, the correct g-distribution is different in every period,
so that any convergence of external towards correct internal
assessments is not simply a matter of hitting a stationary target
once : a moving target must be hit at each attempt if convergence
is to occur and be maintained in the way required for management
to be justified in accepting the relevance of its own conditional
value predictions.

Justification for the opposite view, that conditional
predictions remain relevant even when there is little prospect
that convergence will occur, may rest on the absence of a
convincing alternative to management's prediction model, or on
the argument that any alternative prediction model would fundamentally
distort management's views of the prospects for different strategies
and lead to a decline in the quality of decision making and to
reduced confidence in the decisions taken.

These two possible views on the importance of convergence
of expectations are indicative of a fundamental division of opinion
on the question of how management should respond to the capital
market, and this question is considered in sub-section (iv). For
the moment it is assumed that in one way or another management
can justify the relevance of its conditional predictions, given
confidence in its interpretation of the nature of valuation and
in its estimate of the market value relationship. These two
aspects of confidence are the main themes of this section, and

it is now appropriate to return to them.

13 : 2 (ii)   Managerial interpretation of the nature of valuation.

The suggestion was made in section 1 (ii) that management might abandon its attempt to recognise the ideal valuation model at work in real-world conditions if it perceived that investors might recognise (or guess at) the fallibility of its own estimated g-distribution. Management might expect investors to adopt non-ideal valuation practices in the face of the uncertainty over its g-distribution estimate and the possibility that strategy might actually be changed. Now it is clear that an essentially similar situation exists when investors lack complete information, even though management itself experiences no doubts about its estimated g-distribution. In a market characterised by imperfect communications between management and investors, the latter recognise that their own g-distribution estimates may be inaccurate, and this raises for management the same question it may ask itself when it recognises the imperfection of its own estimate : what is the market's basis for valuing equity in the absence of unequivocably objective probability data? Evidently the validity of the original managerial interpretation of valuation has implicitly been in question at any stage from chapter 10 onwards when imperfection of management - investor communications has been suggested. For management the valuation model of chapter 10 : 3 may become suspect not only when it ceases to trust its own g-distribution estimate completely, but at any time when the market fails, or is likely to fail, to perceive management's views perfectly. In the latter situation

management may feel that investors, realising that their views on growth prospects cannot be based on completely objective data, are likely to create a market which is inherently inexplicable and does not lend itself to conditional predictions. This is precisely the nature of management's concern over the effect on the market of possible imperfections _in its own_ g-distribution estimate, and if it can find a way to preserve its preferred interpretation of valuation in the present situation — imperfect market information — the same justification may serve in retaining that interpretation when management uncertainty over the g-distribution is added to the picture.

The justification for continuing with its interpretation of valuation cannot simply be a strong desire on management's part for an objectively based valuation model capable of generating conditional predictions. Confidence in the underlying nature of valuation in real-world conditions, in the conditional predictions made and their relevance, is essential, as will be shown in Section 3. Thus management must be convinced that in spite of the admitted difficulties investors attempt to value equity with reference to the properties of its g-distribution. It would be wrong to dismiss this conviction as an entirely untested and untestable hypothesis about investors' behaviour. (Even if it were, as a strongly held conviction it could still be the basis of management's approach to strategic decision-making.) Granted that management may be predisposed to a certain view of valuation, it would be able to justify its belief by referring to the presence and behaviour of a large number of long-term investors operating in the capital

market. The way in which management, if it were forced to defend its assumed position, might develop an argument on the underlying nature of valuation and on the identifiability and relevance of the ideal market value relationship is set out here and in the following three sub-sections.

According to this argument investors are aware of the risk of error in attempting to value equity with reference to an estimated g-distribution, and this risk appears to them as the risk of paying a wrong price for the equity in question. In principle this aspect of risk is quite different from that accepted by the purchaser in paying the correct price for equity, and the distinction forms the basis of the case for continuing to think in terms of an objectively correct equity value.

The nature of the risk the investor accepts in paying what he considers to be the objectively correct price for equity is that events whose probabilities he can identify will determine the long-term profitability of his investment. Ignorance of the correct price means that an investor risks paying too much or too little for the risks of ownership. (Likewise, he may eventually sell his holding at a wrong price.) If risks of the first type were fully identified (as assumed in earlier chapters), investors would be able to value them correctly; and the extra risk now considered is that the price actually paid will differ from that which a fully informed investor would recognise as correct. Investors may well differ in their attitudes to the gamble presented by the market's uncertainty over the objective risks, but the important

thing is that they do accept the concept of an ideal, objectively
correct equity value, albeit that that value itself is not
established with certainty.

In line with this thinking, investors are assumed to make
their own "best estimates" of the true g-distribution. Consistency
requires that recognition of investors' awareness of an extra risk
dimension be matched by an appropriate interpretation of their
market behaviour. Thus each investor translates his best estimate
of a share's g-distribution into his view of its correct market
value [9], then adopts an individual policy on the prices at which
he will buy, hold or sell the share. This range of prices
obviously depends on the investor's confidence in his g-distribution
estimate, his attitude to the risk of error and his expectations
for general and particular price movements in the short term.
Having argued that investors individually attempt to value equity
in an ideal way, the way is clear for consideration of whether the
parameters of the ideal value relationship are observable in real-
world conditions.

(9)   Of course management finds this a more convenient assumption
        than the alternative: that the investor attaches probabilities
        to different g-distribution possibilities and, in effect,
        places a value upon a probability distribution of possible
        true values. From management's own point of view the
        assumption (that at any one time the investor holds a single
        "best estimate" of the true g-distribution) is defensible on
        logical as well as practical grounds. After all, investors
        are presumed to change their view of the g-distribution when
        they recognize the need to do so (see sub-section (v) below).
        And management itself at any one time holds a single view of
        the g-distribution, and may well expect investors to think
        along similar lines. For an example in a related context
        of the assumption that an investor at any one time holds a
        single best estimate of a future expectation, see A.A.
        Robichek and S.C. Myers, Optimal Financing Decisions, 1965,
        pp. 86-90.

13 : 2 (iii)   Identifying the market value relationship.

In the ideal conditions assumed in Chapter 10 the identification
of the market value relationship $y = h(Z)$ was made possible by the
observation of a sufficient number of different g-distributions
along with their respective market values. [10] These distributions
were converted into S-distributions for different values of k,
and it was shown that a very narrow band of k-values was consistent
with the required general shape and position of $y = h(Z)$.

The conceptual device of separating both the investor's
perception of risks and his approach to valuation into related
components justifies retention of the original concept of ideal
valuation : nor does it neglect the varied individual reactions
of investors to the uncertainty they associate with their estimates
of correct equity value. In these conditions identification of
a market value relationship involves a great deal of managerial
inference of investors' expectations and valuations, but no serious
conceptual problems. It is obviously essential that management
anticipates identifying a single version of $y = h(Z)$, and there
are two ways in which this result may emerge.

The simpler approach would be to repeat the exercise
described in Chapter 10 : 3 (iv) on the assumption that all investors

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(10)   See Chapter 10 : 3 (i). The expression $y = h(Z)$ is the
standardised market value relationship assumed by management,
where $y = \dfrac{V_e}{S}$ and $Z = \dfrac{\sigma_s}{S}$

share common estimates of g-distributions and correct market

value for the different shares studied, thereby deriving a locus

of points representing the function $y = h(Z)$. A more ambitious

approach which may seem more appropriate in real-world conditions

would be to identify different groups of investors between whom

differences in g-distribution and correct value estimates for the

equity in question can be detected. These different estimates

for one firm's equity could be used in the same way as the common

estimates for different firms in the simple approach, again

resulting in a single $y = h(Z)$ relationship.[11] The two approaches

cannot be used to produce directly conflicting views : only one

will seem appropriate at any one time. Whichever is used can also

be applied to data for past periods to yield more observations and

allow greater confidence in the version finally accepted.

The importance of the second possibility is that it allows

management to believe that differences between groups of investors

on estimates of growth prospects and correct market value do not

interfere with the operation or identifiability of an ideal or

standard value relationship. Management is probably unwilling to

accept that different groups of investors can agree on a firm's

g-distribution yet continue to hold differing views of its correct

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(11)   See the construction in Diagram 5, Chapter 10 : 3 (iv) :
        instead of referring to different firms, observations a,
        b and c describe the value and g-distribution estimates of
        three groups of investors in relation to the same firm.
        This interpretation could also have been used in Chapter 10.

value.[12] The presumption will be that any persistent conflicts
of view over the latter are explained by genuine differences on
growth prospects, and that market forces work powerfully enough
to establish the underlying value relationship. In terms of the
elements of the valuation model, management expects market forces
to co-ordinate investors' views on (i) the correct value of k in
relation to dividend risk, and (ii) the correct value of an S-
distribution, given its $\bar{S}$ and σ_s properties.[13]

Procedures for inferring investors' views can be discussed
briefly. In the absence of certain knowledge, plausible methods
can presumably be devised to generate investors' g-distribution
estimates for either of the approaches described.[14] As for

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(12)   The nature of convincing evidence to the contrary can be
         described. Suppose that two groups of long-term shareholders
         can be presumed to continuously accept similar g-distribution
         estimates with high confidence, and that neither group will
         sell at a price below or buy at a price above its view of
         correct equity value. Then if one group is observed to be
         selling while the other is buying it is fair to conclude
         that the two groups employ different valuation models.
         Management's reasonable expectation is that actual price
         movements over time persuade one (or both) groups that their
         valuation model(s) is (are) wrong, and that an approximation
         of views on correct valuation develops.

(13)   This is simply an extension of the managerial belief explained
         in Chapter 9 : 4, that investors recognise the ideal valuation
         model even when they disagree on the objective properties of
         individual shares.

(14)   The different firms, or conflicting g-distribution estimates
         for the same firm, would have to be comparable in dividend
         risk but different in their S-distribution properties.
         Simulation of investors' g-distribution estimates could be
         performed at various levels of sophistication, taking into
         account the kind of information available to investors and the
         inference methods they might employ. An elementary possibility
         for the first approach is that for each firm investors estimate
         a particular probability distribution governing the growth
         rate of expected dividend between any two periods, with the
         result on any occasion expected to have no effect on the
         probability distribution for the next. On this view investors
         would typically estimate a g-distribution with unrealistically
         low dispersion (when translated into S-values), failing to allow
         for cumulative effects and possible changes in underlying
         relationships. To the extent that (some groups of) investors
         allow for such features the dispersion(s) of the simulated
         g-distribution(s) would increase.

identifying investor's estimates of correct value, the processes

of observation and interpretation described in Chapter 9 : 4

remain applicable, though the assumed valuation model differs,

and nothing further need be added here.

Finally, it should be noted that management's confidence

in the usefulness of the identification exercise is likely to be

strengthened by an awareness that a mistake made in simultaneously

estimating k and $y = h (Z)$ affects the measured characteristics

of all strategies in the same ways. The main effects for a typical

strategy are deviations in $\bar{S}$ and σ_s , in the same direction, from

their true values; and a ranking of strategies in terms of perceived

market value prospects is probably quite insensitive to the narrow

margin of error present in the identification of k and $y = h (Z)$

(Chapter 10 : 3 (iv)).

13 : 2 (iv)   Justifying the relevance of ideal valuation.

Management may find support for its central belief, that

imperfect capital market conditions do not alter the underlying

nature of investors' valuation of equity, in the fact that even

under ideal conditions investors would be required -- and would

expect -- to change their estimate of the g-distribution in each

period. [15] This necessity would not affect investors' willingness

to value equity in the way assumed, so that it is likely to appear

that a similar necessity in imperfect market conditions will not by

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[15]   See Chapter 12 : 1 (ii)

itself cause investors to abandon their preferred approach to valuation.

Having satisfied itself on the identifiability of the ideal market value relationship, the remaining problem for management is the relevance of conditional predictions in real-world conditions. Here it is possible to choose from two basic lines of approach. As mentioned in sub-section (i) above, management may believe that convergence between its own and investors' g-distribution estimates is a reasonable expectation. On the other hand, lacking confidence in convergence, it may still try to justify to itself the relevance of its correct conditional predictions for strategic decision making. A choice between these approaches may not be necessary: only if the first possible justification is discarded will it be necessary to consider the radically different view of the capital market implied in the second.

Taking the first and less controversial possibility: given that its own estimates will always be correct, management may well assume that the market's absorption and anticipation of information will improve through time, so that convergence is a reasonable expectation.[16] Additionally - and this consideration hints at

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(16)   The alternatives would be the long-term persistence of under-
         or overvaluation, or fluctuations of increasing or constant
         amplitude around the correct price trend. Of these four,
         only the last is likely to appear at all probable. Depending
         on the constant amplitude anticipated this last alternative
         could qualify as "convergence," given management's dissociation
         from price divergences attributable to investors' uncertainty
         about the true g-distribution (sub-section (ii) above).

the second method of justification - in the absence of obvious
alternatives towards which market expectations might gravitate,
management may regard its own appraisal of growth prospects as
the one towards which opinion must converge.

As stated, the alternative justification of the relevance
of conditional value predictions based on the ideal market value
relationship applies when convergence between management and market
g-distribution estimates appears improbable. Here management may
feel that because its conditional predictions are correct in a
fundamental sense they _ought_ to provide the basis for strategic
decision making, even though market prices are unlikely to match
the relevant predictions. Without attempting a final judgement
on this view it is appropriate to examine the basis for a belief
in the relevance of correct value predictions in actual conditions.
One element in such a belief may be the lack of any satisfactory
alternative theory of valuation capable of generating conditional
predictions; but this is an essentially negative argument, and
stronger support can be found.

The fact that management distinguishes between correct and
market values of equity may reflect its opinion that strategic
decision making should concentrate on the former, and that the
incomprehensible, impredictable and uncontrollable behaviour of the
latter lies outside a proper area of concern. The more certain
management is that investors attempt to value equity in the ideal way,
and that the form of the ideal market value relationship can be
identified, the stronger is likely to be its conviction that strategy

choice should be guided by meaningful prospective values —
those given by the identified ideal relationship.

Such a managerial belief in the relevance of ideal valuation
in strategic decision making is likely to be reinforced by the
expectation that investors retain their holdings over very long periods.
The short-term vagaries of the market then become quite irrelevant
to management in its role as strategic decision maker, and ideal
values function as clear and unwavering guides to rational and efficient
strategic choice. This degree of neutrality on the market price
of equity is in keeping with the managerial view hypothesized in
sub-section (ii) above, that investors are aware of price fluctuations
and of the risk of buying or selling at the wrong price. Management
expects investors themselves, individually, to assess and respond to
this risk.[17]

Arguments of this sort against a wider involvement with the
market value of equity, while not watertight, are strong enough to
appeal to a management predisposed to see greater significance in
correct value than in market value. Nor need the fact that managements'
own interests may be threatened by possible developments in the
market price of equity detract from the significance of the ideal
valuation model and its conditional predictions.

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(17)   Management's interest in equity market value for other purposes
       is not in question here. For example, procedures for
       investment/external financing decisions in conditions of
       market disequilibrium are bound to figure in conditional
       policy. On such occasions management itself assumes
       responsibility for a decision made on shareholders' behalf,
       and it presumably takes into account its own view on the
       extent and duration of the disequilibrium: see A.J. Merret
       and A. Sykes, The Finance and Analysis of Capital Projects,
       1963, Chapter 17.

13 : 2 (v)   <u>Managerial uncertainty and the nature of valuation.</u>

It has now been shown that on either assumption about
convergence between market price and correct equity value a belief
in the relevance of the ideal valuation model can be sustained in
real conditions. To what extent can this belief be sustained when
the second step in introducing real-world factors is taken? Thus
far the only recognised departure from ideal conditions has been
an insufficiency of correct information available to investors :
in spite of this management is confident of its estimate of the g-
distribution, and feels that only communications difficulties
stand in the way of continuously correct valuation. Confidence in
the ideal valuation model and in one of the two views of market
price (convergence or near-irrelevance) remains strong. Now the
second step away from ideal conditions is taken, with management's
recognition that its own estimate of the g-distribution may be
incorrect. This step raises once again the question of how manage-
ment will interpret the nature of the valuation process.

The distinction between aspects of risk which management
assumes investors to make (sub-section (ii)) points to an important
element in its thinking about valuation in the conditions now defined.
If investors are prepared to accept the risk that their estimate
of the g-distribution is wrong, it is unlikely that they will be
concerned over the likelihood that management may find itself in
a similar situation. The fact that management too may be obliged
merely to estimate the firm's g-distribution and correct value
alters nothing in the nature of the investors' problem, or in
their perception and treatment of that problem. Management can

agree that, to investors, nothing is changed directly by an awareness of managerial fallibility: it is the possibility of indirect effects that deserves careful consideration.

It is less easy to infer the reasoning of management about investors' approach to equity valuation if the latter are aware that strategy may be changed at some stage. Nevertheless, management must reach some conclusion on this matter because any acknowledgement that its estimate of the g-distribution may be mistaken implies an acknowledgement that its original strategy may be recognised as unsuitable and may then be replaced. There appear to be two sharply differing arguments.

On the one hand, it is possibly true that few investors have any means of identifying the firm's investment strategy or of detecting any change that may occur. This would suggest that investors have no suspicion that the truth they seek to identify — the correct g-distribution — has changed, so that management's view of their intentions can remain unaffected by the possibility of a strategy change.

However, if strategy is changed the underlying truth sought by investors does alter ; and if true value could be identified for both old and new strategies a difference would be observed. Management must honestly decide whether it believes that investors who are aware that the truth they seek may not be constant at all will continue to attempt to value equity in the way assumed hitherto. Is it not likely that, aware of what may occur, they attempt to value a composite g-distribution made up of estimates of the

implications of different policies management may be expected to adopt under different conditions? Or, perhaps more likely given the greatly increased obscurity surrounding the truth when the strict condition of constant strategy is relaxed, will investors resort to a variety of valuation procedures which have in common only a rejection of the attempt to identify and value a single g-distribution?

Management's choice between these two views of valuation in conditions where the possibility of strategy change is recognised by investors is obviously of crucial importance to its belief in the relevance of ideal valuation. Nothing definite can be said at this stage about the conclusions that management will draw, but the first interpretation - that favouring the original idea of valuation even in the new conditions of uncertainty about the continuity of strategy - cannot be rejected. At any moment there is a single truth (correct g-distribution for current strategy) that investors can attempt to identify and value : and in the nature of things there cannot be information or concrete expectation about a possible strategy change or its implications.[18] This suggests that investors will continue to approach valuation from the standpoint originally assumed. The suggestion that some entirely different basis of valuation will be generally adopted cannot be disproved, but it can be argued that a long-term investor will in his own interest prefer to base his valuation on the most objective data available - his estimate of the g-distribution. If such an investor

(18)   See Chapter 11 : 5 (iii).

is prepared to learn that his first assessment of objective risk was mistaken without modifying his basic approach to valuation, it makes little practical difference whether an error arises from a simple miscalculation about the real situation or from a change in that situation caused by a change of strategy. In either case the investor's response upon realising that things are not as he supposed is to resume his search for the truth and, when he formulates his new ideas, to value them as before.

This argument does not by itself assure management that its own value predictions will remain valid throughout the period covered by its utility function. Its contribution is limited to the assurance that investors will continue to value equity on the same basis, so that neither strategy changes within management's utility function horizon nor anticipated changes beyond it are believed to destroy the underlying philosophy of long-term investors. It remains for management to convince itself that its own conditional value predictions will remain valid at least until the terminal date of its utility function, in the sense that no case for a change in strategy can be recognised before that time. Management's concern need not relate to the underlying nature of valuation, but to the practical questions raised in Section 1 (ii) which may prevent value predictions being used in strategic decision making in the way assumed in Chapter 12. The discussion up to this point has been concerned only with whether management can sustain its original view of valuation, not with whether its confidence in its conditional value predictions is likely to be sufficient to allow the strategic decision to be taken with reference to an extended horizon utility function as

suggested in Chapter 12. This latter question, the final one in demonstrating the practical importance of ideal valuation to management, is considered in the following section.

Section 3 :   The Anticipated Durability of Hypotheses.

Three conditions must be satisfied if Chapter 12's description of strategic decision making is to be applicable in the uncertainty situations defined in this chapter. These relate to the state of management's beliefs at $t = o$ when its strategic choice is made. Management must be confident

(i)   of the underlying nature of valuation, and that in imperfect conditions an ideal relationship can be identified.

(ii)   that within a limited horizon (set by the terminal date of its utility function, $t = m$) its estimate of the ideal market value relationship will not change;

(iii)   that its g-distribution estimate will also survive over at least the same period.[19]

The corollary of these conditions is that management recognises that no hypothesis or relationship can be accepted with absolute confidence, and that new evidence or ideas may lead to new estimates of the central elements in the model. This may even occur within the horizon set by $t = m$; but, for reasons given in Section 2 for condition (i) and in this section for (ii) and (iii), it is reasonable to suppose that management regards such early disproof of its ideas

(19)   Any account of strategic decision making based on an extended horizon utility function requires condition (iii) to be satisfied: (i) and (ii) are obviously essential to equity value-based decision-models.

and estimates as highly improbable, so that the three conditions
can be taken to be satisfied. Nor need the prospect of an
eventual refutation of an estimate or hypothesis alter the nature
of the decision process described in Chapter 12: uncertainty about
the future is an unalterable element in long-term planning, in whatever
terms the plan is expressed, and the assumed form of the managerial
(strategy) utility function leaves open the question of the attitude
to developments beyond $t = m$.

Section 2 dealt exclusively with condition (i), while (ii)
and (iii) are taken together in this section. If sufficient
confidence in either sense is lacking at $t = o$, management will
feel disinclined to trust its view of possible developments to $t = m$.
It acknowledges that if either of the relationships covered by (ii)
and (iii) is recognised as mistaken before $t = m$, the original invest-
ment strategy may have to be altered; and since the direction and
consequences of any such change are incalculable at $t = o$ the suggested
view of strategic decision making requires that management should
see the likelihood of premature error recognition as remote.

Two general points are relevant in assessing the extent
to which (ii) and (iii) are likely to be satisfied. First, a broad
distinction should be made between attitudes towards the further
testing of accepted ideas and relationships. On one side is an approach
favouring constant and rigourous examination of hypotheses in the
light of new evidence and ideas: on the other is a willingness to see
new evidence, at least initially, as confirmation for existing
ideas rather than as potential proof of new ones. The degree to
which a strictly scientific attitude prevails is likely to be

determined by the gains which the decision-maker believes it may yield; and in the present context the prospective gains from frequent revisions of estimated relationships, and consequently of strategy, are likely to appear slight. A less rigourous approach to the checking of assumptions is likely to seem sufficient.

This view is supported by the second general point. The number of observations likely to become available for testing any assumed or estimated relationship during the period $t = o$ to $t = m$ is small. For example, if a dividend/investment/financing planning period covers two years, only four new observations in any series will arise within a horizon of eight years; and that is perhaps as far ahead as prospective results will count in the determination of present-day managerial (strategy) utility.

These two general points apply to both of the relationships covered by conditions (ii) and (iii). Factors affecting management's expectations about individual relationships may be added briefly. When the durability of the estimate of the ideal value relationship is considered, the special nature of the observations from which it is constructed is highly relevant. The estimate is based upon data inferred by management, rather than upon direct observations of investors' valuations and growth expectations. In practice, therefore, a measure of interpretation is bound to occur, and the relatively small amount of new information becoming available between $t = o$ and $t = m$ is unlikely to be sufficient to refute an accepted relationship and conclusively establish another – even supposing the inclination to do so exists. As long as investors' behaviour fits roughly into the pattern anticipated management is likely to

interpret it as confirming its original estimate of the value relationship.

For different reasons it is also likely that management expects its g-distribution estimate to survive until at least $t = m$. The new evidence accumulating after $t = o$, on earnings and investment, does not require the kind of interpretation necessary for quantifying valuation, and it is assumed that investment profitability — the Q — distribution — is identified by the end of the planning period in which investment begins operating.[20] Thus, the quality of the data becoming available for testing the g-distribution estimate is unquestioned, and the important factors determining the estimate's durability are the general ones described above: management's attitude and the quantity of new information.

The g-distribution expresses all possible time patterns of interrelationship between investment profitability and investment demand, and the anticipated durability of estimates of these elements should be considered separately. Investment profitability is treated in the aggregate, and management is assumed to see this aggregate profitability, ex ante, as a probability distribution of outcomes. The actual outcome in each period can then be compared with the distribution from which it is supposed to have been generated. Given that that distribution reflects the firm's own experience and realistic expectations, management is likely to feel at $t = o$ that a short

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(20)   This assumption about investment profitability, while not indispensable to the general argument, seems reasonable and is incorporated in the illustrative examples in Chapter 8.

series of new observations will not shake its faith in the original distribution.[21]

As for the investment demand relationship, its probabilistic structure has already been stressed, in Chapters 8 and 11. Here too management is likely to feel that within a comparatively short time the range of investment demand outcomes will raise no constructive doubts about an estimated relationship which in any case allows random factors and/or unidentified variables to exercise considerable influence.

These considerations suggest that conditions (ii) and (iii) relating to management's confidence at t = o (that all possible developments up to t = m are identified) are satisfied. Management cannot expect to exclude completely the possibility that mistaken estimates may be recognised and acted upon before t = m, but complete confidence is neither attainable nor necessary. The intention has been to establish that at t = o management may feel confident either that it has foreseen all possibilities up to t = m, or that mistakes are very unlikely to be recognised within that time.

Section 4:   Summary and Conclusions.

The various departures from ideal conditions discussed in Sections 2 and 3 are sufficiently important to compel modifications in the original simple accounts of management's conditional equity

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(21)   The probability distribution of investment profitability is not simply a record of the firm's previous experience: all relevant information about future conditions must be assumed to influence management in estimating its parameters.

value predictions and strategic decision making. However, the threat posed by real conditions to the logic of the equity value-based strategy choice has not materialised along the lines envisaged in Section 1 (ii) above. Section 2 showed that a fundamental belief in the relevance of the ideal valuation model can survive the recognition by management that investors are aware of its fallibility in parameter estimation and forecasting and that strategy is unlikely to be permanent. As far as its own expectations are concerned, Section 3 has shown that management can satisfy itself that possible future developments are all identified to the extent that revisions of conditional predictions and of strategy are sufficiently improbable within the period covered by its utility function. A natural concomitant of uncertainty about growth and valuation relationships is a recognition that long-term strategy may at some time have to be changed, and the conclusions of Sections 2 and 3 have made it possible to dispense with the original implication of a once-for-all strategy committment without fundamentally changing the original concept of strategic decision making. The concept of a limited managerial horizon makes as much sense in the real world as in the ideal conditions for which it was originally suggested.

Accordingly, this section is concerned not with the strict logic of whether the original description of strategy choice can still apply in real conditions, but with whether management in those conditions is likely to see some version of ideal valuation as a correct and useful concept and make assumptions and interpretations of the kind described in Section 2 (iii) to overcome obvious difficulties.

No categorical prediction can be made of whether and how management
will employ its own concept of ideal valuation, and all that is
offered here is a summary of points which suggest that in selecting
an investment strategy some attempt will be made to identify an
ideal value relationship. The ways in which information gained from
such attempts may be utilised are not considered here: the description
of strategy choice given in Chapter 12 seems to retain its relevance,
and detailed consideration of alternative concepts of the managerial
utility function and strategy choice is undertaken in Chapter 14.

No matter how the strategy decision is taken it is safe to
conclude that management will employ estimates of the g-distributions
of alternative strategies. This element in the decision is therefore
common to all procedures, so that uncertainty about the estimates
and any resulting uncertainty about the permanency of strategy
cannot be used as a special argument against any one procedure.

Similarly, with any approach to strategy choice management will
be concerned about the market value implications and related dangers
associated with each strategy. This concern will lead to some kind
of attempt to relate particular growth-paths to their market value
implications, but no unique method of achieving this linkage is
implbed.

The paradoxical element in management's position in real
conditions is that while concern about the possible market value
effects of strategy choice is entirely natural and understandable,
management itself is likely to disclaim both an ability to control
equity market value through its policies and an obligation to do so.

The reasons for such a disclaimer require no repetition here, and nothing now recognised about real conditions is likely to alter them. However, the effect is that from one managerial viewpoint it is the market value of equity that can be regarded as irrelevant, and the ideal value -- however defined and measured -- that assumes a corresponding degree of relevance.

A major reason for management to disclaim responsibility for the market value of equity is the latter's unpredictability, even when conditional values of relevant variables such as profit, dividend, investment, etc., are given. Yet a valuation model plays an essential role in strategy choice as the bridge between each strategy's g-distribution and its implied value prospects, and it is important that the model used in this role is accepted with confidence - both in its theoretical approach and in its identified parameters.

Bringing these various points together, conditional predictions derived from an ideal valuation model -- however expressed and identified -- are likely to prove an attractive substitute for unattainable predictions of market value. In conditions of uncertainty about the future behaviour of capital markets the predictions of an accepted ideal model appear to be the best estimates of conditional market values; so that whether management requires conditional ideal values for their own sake -- as inputs in the kind of utility function described in Chapter 12 -- or for limited practical purposes, an ideal valuation model is likely to be attempted. While a conflict may exist between the relevance of ideal and market values as far as present-day value is concerned, such a conflict is considerably less significant when conditional predictions are required.

CHAPTER 14.

The Steady-State Growth Model as a Framework for Strategic Choice.

Plan of Chapter.

Section 1.   Introduction

Section 2.   Management's strategic Choice in a S-S context

    (i)   Product market strategy and the $r - g$ locus.

    (ii)   The nature of strategic decision making.

Section 3.   Market Value, the S-S Model and Managerial Utility

    (i)   Introduction.

    (ii)   Maximising Market Value.

    (iii)   The nature and inevitability of managerial regret.

    (iv)   Applying an extended horizon utility function to S-S conditions.

Section 4.   Analysis of a Timeless Managerial Utility Function

    (i)   Introduction to a timeless managerial utility function.

    (ii)   Representing the security motive.

    (iii)   Determinants of stock market approval.

    (iv)   Quantifying stock market approval.

    (v)   Steady-state growth and managerial utility.

Section 1 :   Introduction.

This chapter explores two aspects of the steady-state (S-S)
growth model which relate to its possible use as a framework for
management's strategy choice. First (in Section 2) it is necessary
to discover the kind of approach to decision making that would be
consistent with a managerial view of growth options as steady-state
paths. Second (Sections 3 and 4), the discussion of managerial utility
functions is resumed against the different background of management's
acceptance of the S-S model. The present chapter therefore parallels
the development in earlier chapters of the multiple growth-path model
and managerial utility functions that relate to it. Criticisms of the
S-S model and its decision making implications and comparisons with
the multiple growth-path model are introduced at appropriate points
in the chapter, but a final summing-up and conclusions on strategic
decision making (growth model and utility function) are held over to
Chapter 15.

A S-S model was introduced in Chapter 7 : 4 (i), and in any case
widespread familiarity with its construction makes a detailed account
unnecessary. Treatment of the model in this chapter concentrates less
on its valuation aspect than on the significance of the observation
that a steady growth rate results from a stable rate of reinvestment
of earnings and a constant rate of return on assets, and is equal to
the product of those terms:

$$g = rp \qquad\qquad (i),$$

where g is the steady-state growth rate, r the constant rate of

return and p the constant rate of earnings reinvestment. Given
constant values of r and p all variables in money units (assets,
investment, profit, dividend) grow at the same rate.

There are two obvious ways in which management's view of valuation
under S-S conditions can be represented. In chapter 7 : 4 (i) steady-
state growth (and its related valuation model) was shown to be
compatible with the variability of earnings around its expected value:
if management wished to achieve a target level of dividend risk it was
necessary to follow an appropriate debt financing policy. A more
popular approach among managerial theories is to disregard the capital
structure aspects of the strategic decision and allow the discount
rate to reflect any assumptions made about risk and its relationship
with growth.[1] No final decision need be taken here on management's

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(1)   In his basic model Marris assumes that dividends and the growth
       rate are certain: a related simplification is that with an all-
       equity capital structure the firm's market value is unaffected
       by its choice between internal financing and new issues: see
       R.L. Marris, "Theories of Corporate Growth," in R.L. Marris and
       A.J.B. Wood, The Corporate Economy, 1971, pp.17-23. Baumol, in
       his simple equilibrium growth model, assumes a constant discount
       rate and the absence of risk as well as, apparently, an all-
       equity capital structure with internal financing: see W.J. Baumol,
       "On the Theory of Expansion of the Firm," American Economic Review,
       (December 1962), pp.1078-81. J.H. Williamson accepts a "Modigliani
       and Miller type of world where.... the purely financial decisions
       of the firm have no impact on its value or on the rate of return
       enjoyed by investors." : see J.H. Williamson, "Profit, Growth
       and Sales Maximization," Economica, (1966), p.8. Lerner and
       Carleton in their all-equity model of steady-state growth do
       allow for a variable discount rate, and they extend the model to
       allow debt in the capital structure: see E.M. Lerner and W.T.
       Carleton, A Theory of Financial Analysis, 1966, Chapters 8,10.

interpretation of valuation, and the all-equity model assumed in this chapter may be entirely suitable for the conditions assumed. However, with little difficulty and no loss of generality the whole discussion could be set against the alternative background of valuation constructed in earlier chapters. The important common ground between all valuation models is management's presumed concern for the market value of the firm's equity capital.

The standard expression for the value of a stream of dividends is

$$V_{eo} = \sum_{t=1}^{\infty} \frac{\overline{D_t}}{(1+k)^t} \qquad (ii),$$

where V_{eo} is the present-day market value of an all-equity firm whose dividend expectations, $\overline{D_t}$, are given for all future periods, and where k is the market rate of discount. When steady-state growth at rate g is expected - based upon constant r and p - equation (ii) becomes

$$V_{eo} = \frac{(1-p)r \, K_o}{(k-g)} \qquad (iii),$$

where K_o is the present-day stock of assets held by the firm. Given (i), equation (ii) can be written as

$$V_{eo} = \frac{(r-g) \, K_o}{(k-g)} \qquad (iv).$$

A willingness on management's part to estimate a relationship between r and g, or between r and p, allows (iv) to become more

than a convenient formula for the valuation of a given dividend stream. The valuation model is transformed into an operational framework for strategic decision making, and can be used as such whenever management's utility function is expressed in the same (or directly linked) terms.[2]

Section 2 :   Management's Strategy Choice in a S-S Context.

14 : 2 (i)   Product market strategy and the r - g locus.

Comparing the steady-state and multiple growth-path models, the obvious difference is also the most important. On grounds of realism alone there is little doubt which version is preferable as an _ex ante_ view of the prospects offered by any strategy. However, comparative realism in the representation of prospects is not directly at issue in this section, where the subject matter is an analysis of the managerial assumptions and decision making that provide a background to the definition of strategic options in S-S terms.

Looking at the S-S model simply as an account of how management may envisage growth alternatives, attention naturally focusses on the central concept of a locus of sustainable combinations of rate of return on assets and rate of growth (of assets, profit, etc.), and in particular on the inverse relationship usually assumed as a characteristic of the locus and on the question whether r, g

(2)   See G.M. Heal and A. Silberston, "Alternative Managerial Objectives: An Explanatory Note," Oxford Economic Papers (July 1972), for a useful summary of the growth model and valuation model components embodied in (iv).

combinations are indeed sustainable. [3] Such a relationship has been suggested by numerous writers. Gordon argued that in valuing equity capital investors would find it convenient to assume that a steady rate of profit would result from the firm's customary retention ratio: this amounts to an expectation of steady-state growth. [4] Whether management reaches a similar conclusion and feels able to identify all such growth possibilities remains to be considered.

A downward-sloping locus of sustainable r, g combinations has been explained or implied in various contributions. However, from the present standpoint of interest in the implications for the firm's product market strategy of different r, g combinations, not all theoretical contributions are equally satisfactory. [5] For example, Lerner and Carleton demonstrated an inverse linear relationship between r and g under steady-state conditions – a result based upon assumptions about technology and management's competitive policy as well as on the effects of general economic growth in offsetting

(3) Marris, who may fairly claim to have explained the growth-profitability relationship most persuasively, was prepared in the later version of his model to limit discussion to the case of an inverse relationship: see Marris, op.cit., 1971, pp.18-19. This compares with his earlier account in which a rising section of the locus was allowed for : see R.L. Marris, The Economic Theory of Managerial Capitalism, 1964, pp.251-2.

(4) M.J. Gordon, The Investment, Financing, and Valuation of the Corporation, 1962, pp.46-7.

(5) The term product market strategy can be formally defined as including the many aspects of policy that together determine the firm's performance in given markets, the types of products it introduces and markets it enters in the course of its growth, and the rates at which it makes these changes.

what would otherwise be a secular fall in r.[6] They admitted that developments might cause the r - g locus so derived to shift through time, but in their analysis of management's value-maximising decision they allowed only for an _ex ante_ risk in relation to the actual position of the locus.[7] Although their r - g locus is worked out in considerable detail, its implications for product market strategy do not all emerge clearly.

Baumol and Williamson in their respective analyses of the implications of managerial objectives were not primarily concerned with the variations in product market strategy which might be associated with different points on the r - g locus. Baumol argued that the so-called "Penrose effect" (increasing managerial diseconomies in growing fast) would operate even on a perfectly competitive firm (constant input and output prices, constant linear - homogeneous production function) choosing an optimal growth rate, and so implied that such a firm faces a downward-sloping r - g locus.[8] Williamson alluded explicitly but briefly to departures from the single product - perfect competition framework, though it is clear that he intended his analysis to apply to a wide range of situations. In Williamson's model the basic Penrose effect was joined by the full complement of growth diseconomies (research and development, sales promotion, etc.) in producing a lower sustainable rate of return the higher the growth rate.[9]

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(6) Lerner and Carleton, _op.cit._, pp.67-88.

(7) _Ibid._, pp.140-3.

(8) See Baumol, _op.cit._, p.1080, and the original statement of the Penrose effect : E.T. Penrose, _The Theory of the Growth of the Firm_, 1959, Chapter IX.

(9) See J.H. Williamson, _op.cit._, pp.4-5.

Moving away from Baumol's heroic simplifications (no external
constraints on growth, with constant input and output prices), it is
in fact difficult to establish a locus that is both downward-sloping
and composed of sustainable r, g combinations. For example, regard-
less of the state of competition within a market it is reasonable to
assume that a firm established in that market can sustain growth at
any rate up to the exogenously determined market growth rate without a
decline in its rate of return. No trade-off between r and g will
appear necessary in this range of growth rates. This view appears
to be consistent with Baumol's assumption that at low growth rates
managerial diseconomies of growth are unimportant.[10] But growth
at a faster rate than that given exogenously for the market does not
qualify, as by definition it is not indefinitely sustainable. Again,
expansion within a number of markets, in each case at a rate equal to
the exogenous growth rate of that market, is sustainable but does not
produce a sustainable r, g combination: the overall values of r and g
will come to be dominated by their counterparts in the fastest growing
market.

Objections of this kind to the concept of growth confined entirely
to markets with exogenously determined growth rates point to the
importance (in establishing the r - g locus) of expansion into and within
markets whose growth rates are not seen in this way by management.
In such a context faster growth requires the diversion of a greater
share of net operating return on assets to the many aspects of generating
new investment opportunities and promoting sales of their outputs -
a process which management may well believe to be sustainable in

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(10) See Baumol, op.cit., p.1080.

different combinations of r and g, yielding an overall downward slope.

More than any other writer, Marris derived a downward-sloping r - g locus from assumptions about the ways in which management would have to adapt its product market strategy to its chosen growth rate.[11] The implications of this close inter-connection between growth policy and product market strategy are discussed in the following sub-section, but two important general considerations are suggested here. One is that the r - g locus - accepting its slope and that its points are sustainable - implies a great deal of managerial calculation of a kind not called for if expansion occurs solely in markets with exogenously determined growth rates. It is at least questionable whether reducing or condensing the many dimensions of product market strategy, and their necessary variations between different r, g combinations, to yield a single locus of strategic possibilities is really a mental process management is likely to undertake.

The other general consideration that should be mentioned at this stage is that a locus of r, g combinations based upon appropriate variations in product market strategy is likely to impress management by the differences in risk between points on the locus. The creation of market demand is itself a risky activity, and the rate of return that can be sustained at any planned growth rate is likely to appear subject to risk - perhaps in some relation to the growth rate intended. This aspect will be relevant later, in considering the S-S model as a

(11)   See Marris, op.cit., 1964, especially Chapter 4. A clear summary of Marris's contribution to managerial theory is provided in J.R. Wildsmith, Managerial Theories of the Firm, 1973, Chapter 7.

framework for the strategic decision.[12] It is not a criticism of
the possibility of steady-state growth _per se_, or of the argument
that in choosing a growth option management must also choose an appropriate
product market strategy. For the moment it is sufficient to note
that the exact location of the $r - g$ locus may not be obvious to
management, or its parameters may only be identified probabilistically.

14 : 2 (ii)   The nature of strategic decision making.

The interpretation of steady-state growth at different rates
which was found to be compatible with the concept of a downward-sloping
$r - g$ locus deserves further consideration. Differences between the
model of decision making outlined in Chapters 11 and 12 and that
implied by the $r - g$ locus of the S-S model relate to the structure of
decision making and to the practical issue of identifying alternative
strategies. These matters are discussed in turn.

As shown in Section 2 (i) above, variations in management's
product market strategy are likely to be required to produce the desired
shape of $r - g$ locus, so that each combination on the locus represents
a distinct strategy in which product market decision making and
investment policy both combine to yield a steady-state growth-path as
the outcome. The obvious and important difference between this account
of decision making and that suggested in Chapters 11 and 12 relates

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(12) A comparison between the accounts of strategic decision making
      linked to the S-S and multiple growth-path frameworks is
      reserved for Chapter 15. For the present it should be noted
      that if risk is related to the growth rate the standard
      assumption of constant k - accepted in this chapter -
      becomes untenable.

to their predicted outcomes : steady-state growth _versus_ a multiplicity of growth possibilities. But this difference in outcomes is not of immediate concern here, where the structure and elements of management's strategic decision are at issue.

It is unnecessary to consider in detail the ways in which management must vary its product market strategy to yield different points on the $r - g$ locus. More rapid growth is attainable at the cost of a lower overall rate of return by means of a greater emphasis on expansion into activities new to the firm. The policies by which a higher re-investment rate is effectively transformed into sustainable faster growth are numerous, extending through all aspects of competitive behaviour (pricing, advertising, marketing, etc.) into research, development and innovation policies. It is this interlocking of all aspects of strategy that is to be compared with the model of Chapters 11 and 12.

The concept of a separation between _subordinate_ and _top-level_ decision making was employed in Chapter 12 in support of the suggestion that management compares investment strategies against a common background of other policies. No detailed reference was made to product market policy, and it was implicit in the model that conditional rules governing behaviour in that area, as with other matters requiring management's decision, could be regarded as predetermined for management's purpose of comparing alternative investment

strategies.[13] This view contrasts sharply with the tight
interlocking of all aspects of a strategy referred to above.

The distinction between the two views centres on the separability
of management's choice of investment rules from its choice of rules
relating to other areas of decision. With the r - g locus no such
separation is possible : the level and composition of investment,
as well as its rate of return, are uniquely interlocked with management's
simultaneous choice of product market strategy, and _vice versa_. A
unique pattern of investments is needed to give effect to decisions
about products and markets. In the model of Chapter 12, however, a
degree of independence between product market policy and investment
is probable. Management may commit itself to a certain product market
strategy - pricing policy, advertising, research, innovation, etc. -
in a separate process (or, indeed, separate processes) from that of
choosing rules to govern investment behaviour. For example, research
may be controlled by different criteria from those employed in selecting
investments: management may double, or halve, its customary research
allocation without altering the rules it plans to observe in finally
choosing between investment opportunities generated by research activity;
or new investment rules could be adopted without a change in research
effort.

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(13) See Chapter 12: 4 (iii). Supporting this interpretation is the
      concept of a partitioned managerial utility function which allows
      management to adopt a preferred mixture of subordinate policies
      and then, separately, determine its (top-level) utility maximising
      investment strategy. Just as the static and comparative-static
      models in Part I presupposed a largely unexplained managerial
      choice of business activity and operating rules as background
      to an appropriate form of maximising behaviour, so in a dynamic
      context not all decisions need be referred to the same managerial
      utility function. See Chapter 1 : 3 (ii).

To summarise, neither of the growth models (considered as an account of managerial thinking) is neutral in its implications for the structure of decision making. The S-S model requires that management make product market strategy the centrepiece of its planning for growth, with the variation of that strategy as the instrument by which the planned growth rate is changed. Investment rules play a very secondary role, and the function of investment is to give effect to product market strategy. In the multiple growth-path model, on the other hand, investment decision rules occupy the centre of the stage - correctly in view of the fact that, whatever its product market policies may be, management associates risk with specific investment proposals. Product market policies are thus subsumed into subordinate policy, though their importance remains great. Like other aspects of subordinate policy, product market behaviour is not formulated inflexibly, but in conditional terms; and one would expect it to develop according to the growth-path actually followed by the firm. Having thus gone some way towards the implication of the S-S model, the model of multiple growth-paths nevertheless remains fundamentally different: a change of strategy is expressed in terms of new investment rules, and while these may entail changes in product market behaviour there is no doubt that the investment rules are the independent instrument.

Turning to a comparison between the S-S and multiple growth-path models on the practical question of identifying alternatives, it is tempting to accept that this process is simpler when management thinks in terms of S-S growth conditions. However, given the many aspects of what has for convenience been called product market strategy, the reduction of alternatives to a single $r - g$ locus is remarkable. Formally, it is correct to represent each interlocked combination of policies resulting in a sustainable r, g combination as a point in
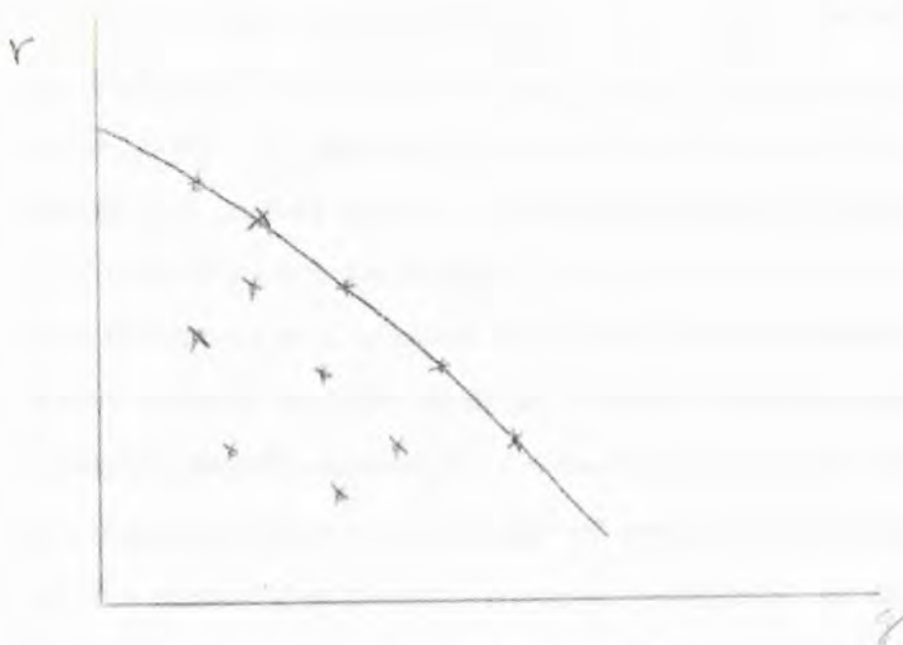
Diagram 1.

Diagram 1 : the r - g locus as an envelope.

Given the manifold aspects of product market strategy, many
of which can be set independently of others, the number of such points
must be very great - even if it includes only those which are sustainable
and which represent policies management would be prepared to operate.
The r - g locus is actually an envelope formed from the outermost
points relating to distinct product market/investment strategies.
Contrary to the impression given by the concept of an r - g locus,
the actual identification of the envelope of S-S growth options
requires the evaluation of all sets of policies resulting in a sustainable
r, g combination.

The comparison between the identification processes of the S-S
and multiple growth-path models has a rather unexpected outcome. In the
former, as shown, all product market/investment strategies must be
evaluated; while in the latter the identification of growth-path
probability distributions for different investment strategies is based

on a common background of subordinate policy. There may thus be much
less of an advantage in employing a S-S framework, in terms of the
difficulty of identifying strategic alternatives, than at first sight
seems likely. In practice, management may not consider all possible
variants of product market strategy, so that the r - g locus it identifies
is a compromise between a desire for profitable growth (with security
from the threat of a takeover bid) and a desire to minimise time and
effort expended in identifying all possible growth options. In the
multiple growth-path model too, there are bound to be short-cuts taken:
the simulation horizon, the number of simulations of anyone strategy
and the number of different strategies considered are obvious instruments
by which the identification process can be kept within planned limits.[14]

It may be concluded that management is able, in principle, to
proceed to the identification of growth options within either of the
frameworks under discussion. The criticism of the S-S model at the
present stage of the analysis is that management simply does not take
the required view of the variability of product market strategy. To
criticise the S-S model on this ground is not to object to the way in
which it describes management's view of the outcome of a strategy,
though that is certainly worthy of comment: the more fundamental
criticism is that management does not even work out its options in the
way assumed by the model.

Section 3 : _Market Value, the S-S Model and Managerial Utility._

14 : 3 (i) _Introduction:_

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(14) The question of simulating growth possibilities is discussed
briefly in Chapter 15 : 2.

Having examined some implications of the proposition that management chooses between sustainable combinations of rate of return and growth, it is appropriate in the remainder of this chapter to review some suggestions as to how this choice is made. For the most part management is assumed to be "managerially" motivated — that is, it fails for one or other reason to pursue the straight forward objective of maximising equity value on behalf of shareholders. The qualification about motivation is particularly important in this section, in which the value-based utility functions considered in Chapters 11 and 12 are applied to the S-S framework, in view of the earlier conclusion that the terms in which management expresses its utility function do not indicate underlying motivation. Section 4, following conventional practice, employs a utility function in which management's weighting of its own and shareholders' interests is intended to be clearly discernible; but in this section it is important to bear in mind that the underlying motivation is similarly at least partly "managerial", in the sense that utility derived from market value prospects is not experienced only on shareholders' behalf.

One guiding principle may be stated at the outset. A decision maker presumably provides himself with the best model of growth prospects he feels it worthwhile to obtain, given the way in which he plans to make his decision and the importance he attaches to it. At some level of managerial interest, in these terms, practically any growth model may be acceptable. For this reason criticism of the S-S model — or any other — on the grounds either of high informational requirements or artificial simplicity cannot be a conclusive argument against the model itself. If management is prepared to work with a rule-of-thumb estimate of the $r - g$ locus, as may be the case, much of the

criticism developed in Section 2 (ii) is wide of the mark. Completely valid criticism can only be made of an overall view of decision making - growth model and managerial utility function - which can be shown to contain irrationality or inconsistency.

In spite of this limitation it is hoped that it will at least be possible to make a judgement on the form and interpretation of the managerial utility function, if not on the growth model to which that function is applied. In this section the attempt to define a managerial utility function for the S-S view of growth options closely parallels that in Chapters 11-13 in relation to the multiple growth-path model, while in Section 4 utility functions specifically intended for the S-S context are discussed.

14 : 3 (ii) Maximising market value.

In addition to its traditional theoretical justification the maximisation of market value in the S-S growth context has the special attractions of mathematical elegance and rigour: specifications of the valuation model which actually produce a maximum can be clearly identified. However, as indicated above, this section is concerned with value-based utility functions for which the underlying motivation is at least partly managerial: maximising shareholders' wealth and utility is not managements' only objective in choosing its growth strategy.[15] As a first step the simple utility function

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(15) Some reasons why the traditional approach to maximising share-
      holders' utility may in any case seem inappropriate are given
      in Section 4 (iii) below.

$$U = U(V_{eo}) \qquad \text{(v)}$$

is considered. Naturally its application results in value maximising as the objective, and the possibility of a value-maximising solution is considered in this sub-section. The implications of the motivation being at least partly managerial rather than entirely traditional will be demonstrated in the following sub-sections.[16]

To simplify discussion an all-equity S-S model is employed, with constant discount rate (regardless of the growth rate) and investment financed entirely from retained earnings:

$$V_{eo} = \frac{(1-p)r \ K_o}{(k - g)} \qquad \text{(iii).}$$

To maximise U management must maximise V_{eo}, subject to the constraint of the r - g locus and the factors determining k- which is assumed constant.[17] Management's only degree of freedom is the choice of a value of r or g and p. Given the r - g locus and the identity

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(16) The sense in which $U = U(V_{eo})$ may represent a mixture of motivations can be explained formally. Let present-day managerial utility be a function of (management's estimate of) shareholders' well-being, W, and management's "personal" satisfaction, Z, with both elements depending on V_{eo}: thus $U = f\left(W(V_{eo}), \ Z(V_{eo})\right)$, simplifying to $U = U(V_{eo})$.

(17) The assumption which yields constant k is that all growth-paths are equally risky - perhaps risk-free. As argued in 14: 2 (i), it is unlikely that where risk is present it will be the same for all r,g combinations, but the use of constant k can be defended on practical grounds. The exact way in which the valuation model should explicitly allow for risk is not obvious, but it seems likely that if variable k is used for the purpose it will vary directly with g. Given such a relationship the behaviour of V_{eo} for different values of g is unlikely to differ much from that shown in Diagram 2 and discussed in the accompanying text.

$$g = rp \qquad (i),$$

fixing any one of these three variables effectively determines the other two. Selecting the decision variable is therefore merely a question of trying to represent managerial thinking.

Suppose management tries to identify the value-maximising growth-rate. Then (iii) may be equivalently stated as

$$V_{eo} = \frac{(r - g) K_o}{k - g} \qquad (iv),$$

in which both k and K_o are given. This relationship offers relevant solutions when $r > g$ and $k > g$, and subject to these conditions the first and second order conditions for maximum V_{eo} reduce to

$$- k (1-r') - gr' + r = o \qquad (vi),$$
and
$$r'' < o \qquad (vii),$$

respectively; where $r' = \frac{dr}{dg}$ and $r'' = \frac{d^2r}{dg^2}$. The $r - g$ locus determines the values and signs of r' and r'', and while $r' < o$ is easy to accept, the second order condition is rather more demanding. If r is linear with respect to g a unique maximum for V_{eo} does not exist even if $r' < o$ as expected.[18]

The relationship between r, k and g can be stated. For positive

(18) See Lerner and Carleton, op.cit., pp.150-1 , where it is also shown that if k is a non-linear function of g a maximum V_{eo} may be found even if $r'' = o$.

V_{eo} r must exceed g and k must exceed g. If interest is limited to situations in which

$$v = \frac{V_{eo}}{K_o} = \frac{r - g}{k - g} > 1 \qquad\qquad (viii),$$

it follows that $r > k > g$ holds whenever $v > 1$, and that $r = k > g$ holds whenever $v = 1$. This relationship holds for any value of g, including that at which V_{eo} is maximised.

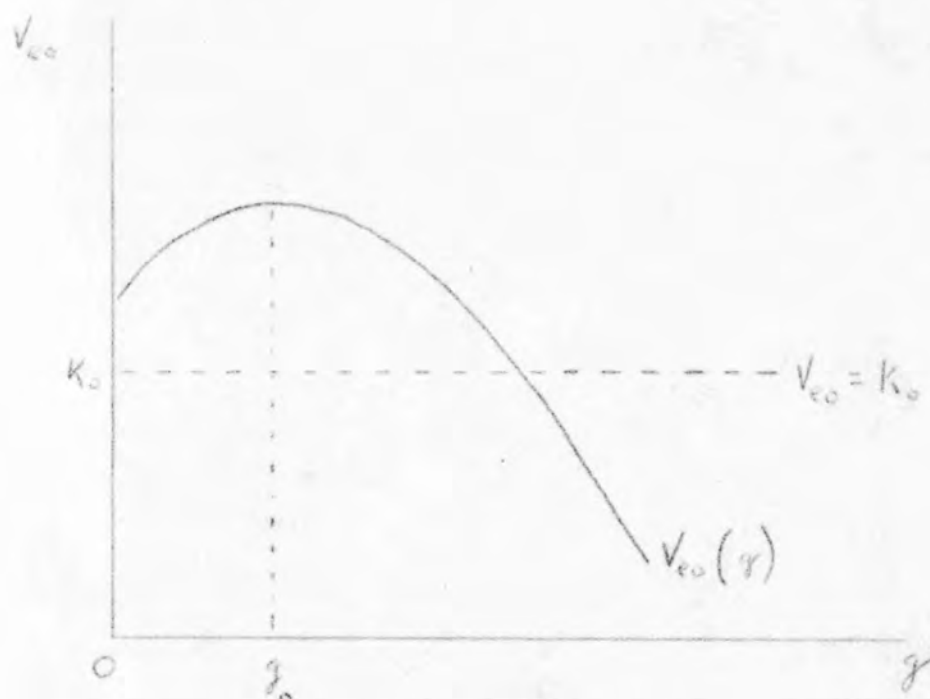Diagram 2 illustrates the relationship between V_{eo} and g on the assumption that a maximum exists.[19]

Diagram 2 : The variation of V_{eo} with respect to growth rate.

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[19] Diagram 2 is essentially the same as Marris's first account of the valuation curve (Marris, op.cit., 1964, p.255) except that absolute market value is used instead of valuation ratio as the vertical axis. In his later version (Marris and Wood (eds.) op.cit., 1971, pp.18-19) Marris pointed out that $v(g)$ — and V_{eo} in the present context — may decline throughout its length.

The value maximising growth rate, g_o , as argued, must lie below the rate of return and the discount rate. When $V_{eo} = K_o$, the rate of return equals the discount rate; and when $V_{eo} = O$ the rate of return equals the growth rate.

In maximising V_{eo} management should encounter no serious problem of financing its investment programme. Given $r > g$, the identity $rp = g$ indicates that p, the rate of profit re-investment, is below unity and that internal financing – or equivalent external plus internal financing – is feasible. Even if management has an aversion to retaining profit on the scale required to achieve the value maximising growth rate, the possibility of selling new equity in a perfect capital market makes such considerations irrelevant and allows the optimal growth rate to be attained.[20] Only if there are artificial constraints on both internal and external financing, or issue costs in relation to the latter, need the optimal solution lie beyond management's reach.

The relationship between V_{eo} and g may not permit maximisation by means of differentiation. The possibility of a linear r – g locus has already been mentioned, and it is also obvious that $V_{eo}(g)$ may fall throughout its length, with V_{eo} maximised at a zero growth rate ($r \geq k > g = O$, according to whether $v \geq 1$).

14 : 3 (iii)   The nature and inevitability of managerial regret.[21]

(20) See Marris, op.cit., 1971, pp.22-3.

(21) For simplicity the utility functions considered in this and the following sub-section are assumed to be entirely managerially motivated, but it will be obvious that exactly the same conclusions would be reached under the kind of divided motivation described in footnote (16) above.

Reservations concerning the maximisation of equity market value
as a strategic decision rule are not related in the first instance
to the S-S model of growth alternatives. It is accepted for the
moment that management sees its choice in terms of the r - g locus and
interprets the valuation of dividends in the conventional way.

Value maximising as a managerial objective has already been
questioned in Chapters 11 and 12, but logical objections to the concept
- as opposed to arguments about what constitutes a likely or probable
managerial attitude - can be shown very clearly when it is applied
to the S-S growth model. An implication of the value-maximising
concept is that management completely discounts _either_ the possibility
that it will ever regret its present choice (in a sense to be defined)
_or_ its state of mind should that regret actually materialise. It can
be shown, however, that the impossibility of future regret cannot be
taken for granted, and that the alternative of deliberate short-
sightedness cannot easily be imputed to management. The rejection of
both supports for the value-maximising concept of strategy, if that
occurs, raises questions about the basis of choice assumed and about
the S-S model as a context for strategic choice.

A comparison between the prospects offered by just two S-S
strategies indicates the sense in which the term _regret_ is used,
and establishes that regret, so defined, will occur if a value-
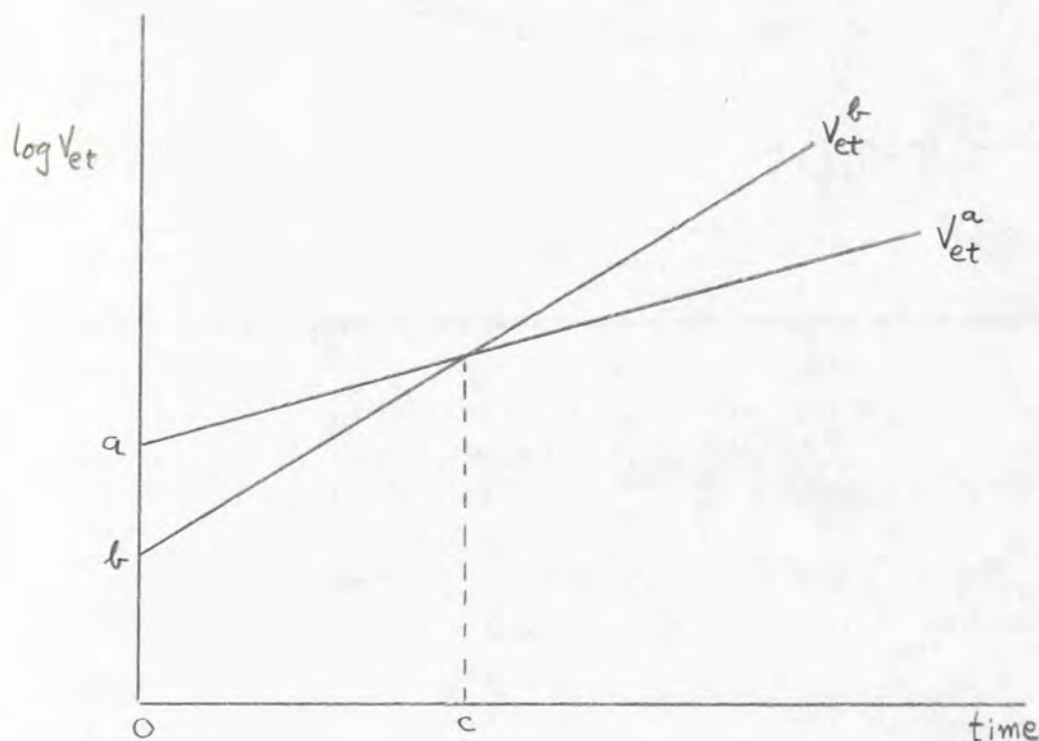maximising strategy is chosen. Diagram 3 illustrates the concept.

$$\log V_{et}$$

Diagram 3 : Alternative growth-paths of equity market value.

Strategies A and B offer present-day market values Oa and Ob
respectively. (A log-scale is used on the V_{et}-axis so that each
steady-state growth-path can be shown as a straight line, with the
growth rate measured as the slope of that line). The lower present-
day value offered by B will grow faster than the initially higher
value under A. A strategy such as B is bound to exist if a value-
maximising strategy exists [22] : a sacrifice of present-day market
value permits an increase in the sustainable growth rate, so that
market value will eventually lie above the level it would have reached
under the strategy which offered maximum (or higher) initial value.
The time taken to achieve equality between the two market values
depends on the gap that has to be closed and on the growth rate difference.
Regret in the present context refers to the hypothetical overtaking
by a rejected strategy of the market value achieved by the chosen

(22) Indeed any growth rate to the right of g_o in Diagram 2 implies
the same dilemma.

strategy. In this sense future regret is predictable at the moment a value-maximising choice is made.

The explanation of the difference between strategies A and B is simple enough. Starting from the same initial stock of assets, higher investment and lower dividends (as a percentage of profit, which in turn is lower due to growth-directed outlays) in B produces faster growth than is achieved under A. At the same time the rewards for investors are more distant and command a lower market value at the present. Taking this to its logical conclusion, no matter how low its (positive) initial market value, the strategy with the highest growth rate would ultimately emerge as the value-maximising strategy if all strategies could commence simultaneously. The date at which this hypothetical ultimate superiority would be achieved may be a long way off, and there is no suggestion that this simple mathematical inevitability obliges or attracts management to act as a growth maximiser.

Management must be presumed to be aware of the future regret implied by a value-maximising choice of strategy, so its response to that prospect must be considered. One response not open to it is a planned change, or series of changes, in strategy in an effort to continually achieve the highest possible market value. It is assumed (to illustrate the point) that at each date the initial choice is repeatable, and that the rankings of alternative strategies will always be identical — that is the r - g locus and market discount rate are un-

changing.[23]   Diagram 4 illustrates the possibilities again with
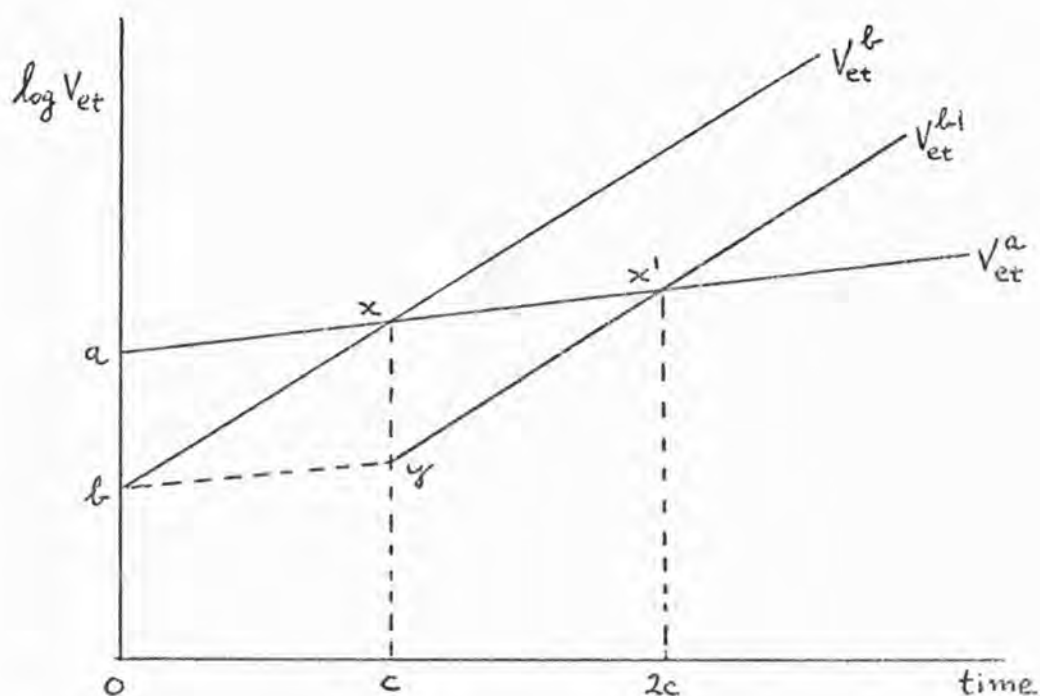reference to strategies A and B.

Diagram 4 : Strategy choices at dates t = o and t = c.

As before, B would overtake A in value at t = c, but if A
is the choice actually made at t = o, and if A and B stand in
the same relationship to each other at t = c as they do at t = o,
then the starting point for B at t = c would be y. It is not open
to the firm to move from a to x and then along V_{et}^{b} : the
alternative to continuing along V_{et}^{a} beyond x would be
to begin moving along V_{et}^{b1} from y, an option rejected
in effect at t = o. The regret foreseen

(23) Given the underlying differences in product market strategy
      between r, g combinations, it must be doubted whether the same
      options would present themselves on a future occasion: in
      this respect the "timelessness" of the S-S version of manage-
      ment's strategic decision is weaker than its equivalent in the
      multiple growth-path model.

in choosing A at the outset cannot be avoided: if A is the strategy preferred at $t = o$ it must remain so at $t = c$ in relation to the prospects which will then be offered by B. In this sense regret experienced at $t = c$ and anticipated at $t = o$ is entirely without a means of avoidance, but it will be argued that this does not lessen its importance in management's decision.

14 : 3 (iv) Applying an extended horizon utility function to S-S conditions.

It cannot be supposed that management will be content to place itself in a position in which it can literally predict the time-path of its regret at its present choice in relation to any strategy offering a faster growth rate. The way to overcome this implication may be to recognise formally that a utility function based entirely on present-day market value does not adequately represent all of the considerations important to management. It appears to be both necessary and sensible to credit management with an extended horizon utility function, and two suggestions for the general form of such a function are

$$U = U (V_{eo}, \ldots, \ldots V_{em}) \qquad \text{(ix)},$$

and $\quad U = U (V_{eo}, g) \qquad \text{(x)},$

where U in each case represents present-day managerial utility. These forms were accepted in Chapter 12 as applicable in the context of multiple growth-path strategies, and their implications in the present context are now considered.

A utility function of the type suggested by (ix) ought to be given a finite horizon for the same reasons that were accepted in Chapter 12 : 2 (ii) in relation to an extended horizon. Maximum utility is not necessarily achieved by maximising V_{eo}, and management deliberately selects a time-path of V_{et} rather than a value ruling at any point in time. An incidental effect of such a utility function is probably a greater emphasis on growth the more distant the horizon.

This concept of managerial utility goes some way towards correcting the implications of a total concentration upon immediate results and complete discounting of medium and long-term prospects. It might indeed be thought that a function such as (ix) removes all difficulties, given the implication that developments beyond the horizon at $t = m$ are of no concern to management - now or ever. As business careers are of finite duration such an interpretation of the horizon seems reasonable; and in any case if management does experience a concern on behalf of its successors, that merely requires an extension of the horizon beyond management's own expected retirement or departure to a date at which such present concern for the future dwindles to zero.

Defining the horizon in this way is, however, not enough to justify (ix) as a valid form of managerial utility function for the choice between S-S growth-paths. Maximising present-day utility, even in respect of an extended horizon utility function, implies a disregard of assessments of the situation that will be made in future. The nature of management's dilemma can be shown, first diagrammatically and then formally.
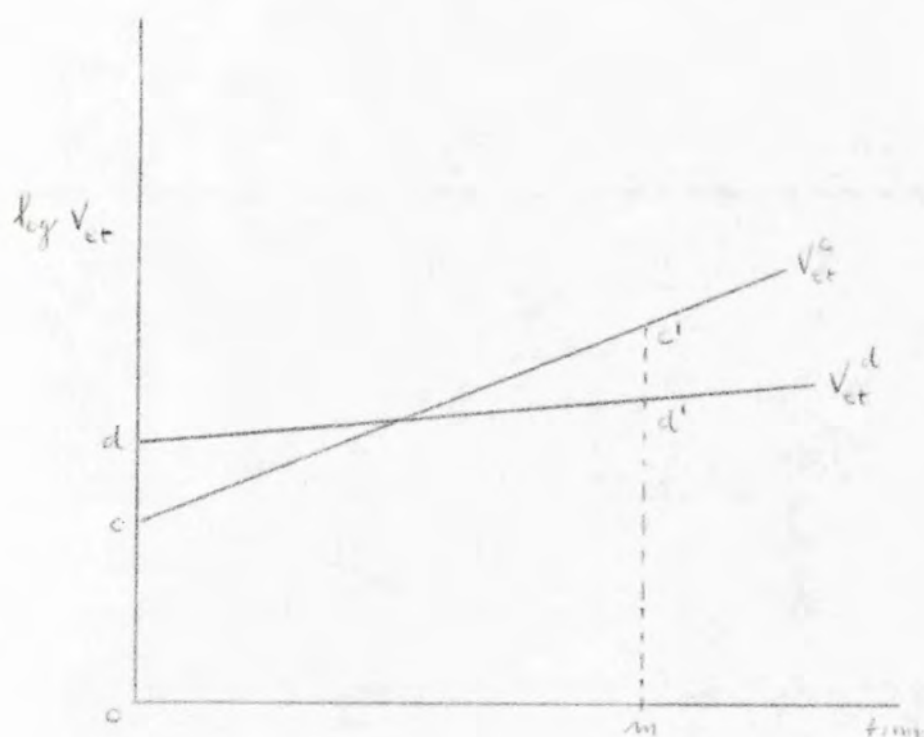
Diagram 5 : Growth-paths of V_{et} for strategies C and D.

In Diagram 5 management is assumed to choose between just two
S-S growth paths, D and C, whose equity market values change through
time as shown. At $t = 0$ a comparison must favour D (growth-path d d')
more than will be the case at any later date. As $t = m$ approaches,
management's utility on path cc' will increase relative to what it
would be on dd'; and at some point before $t = m$ management knows that
its utility on cc' will exceed what it would be on dd'.

Formally, management is aware of the ways in which it will at all
future dates appraise prospects up to its horizon. Thus:

$$U_0 = U_0 \ (V_{e0} , \ldots V_{em}) \ ,$$

$$U_1 = U_1 \ (V_{e1} , \ldots V_{em}) \ ,$$

$$U_m = U_m (V_{em}) \qquad\qquad\qquad\qquad (xi)$$

This step widens the original concept (ix), and defines managerial
utility at any date as a function of market value prospects from that
date to the horizon. Utility ceases to be a one-dimensional concept,
ultimately reducible to a present-day value: instead, a comparison
between alternatives involves the decision maker in comparing his
utility under each alternative for all relevant dates before making
a choice.

This view of managerial utility in the context of a choice
between S-S growth-paths, though certainly necessary in logic, gives
rise to a problem so serious that the very possibility of choice in
such conditions becomes suspect. Evidently a choice between D and
C at t = o involves more than a simple comparison between $U_o(D)$ and $U_o(C)$:
in fact management can compare $U_t(D), U_t(C)$ for all t up to t = m.
Only if $U_t(D) \geqslant U_t(C)$ for all t up to t = m will management feel
completely happy to choose D in preference to C: otherwise its ranking
of D and C must change at some date(s) within the horizon. Yet
the fact that the choice lies between alternative S-S growth-paths
chosen from the V(g) locus of Diagram 2 means that some strategies
are bound to overtake others within any given horizon, giving rise to
precisely this kind of inconclusiveness in establishing a ranking of
alternatives. In the conditions defined a decisive ordering of growth-
paths is literally impossible.

Nor can the decision maker resolve his dilemma by somehow
compressing his prospective feelings at later dates into a meaningful

"permanent" utility value which he will continue to experience up to
his stated horizon, thus securing constant prospective rankings of
all strategies. Utility in this context is essentially and irreducibly
multi-dimensional, and there is no way in which a decision maker can
suppress what he knows will be his sentiments on future occasions. To
prove this point formally, suppose a decisionmaker identifies the feelings
he will experience at each date up to his horizon:

$$A_o = A_o \ (V_{eo} \ \cdots \ V_{em}),$$

$$A_1 = A_1 \ (V_{e1} \ \cdots \ V_{em}), \ \cdots$$

$$A_m = A_m \ (V_{em}) \tag{xii},$$

where A_t is the _attitude_ that will prevail at t. The term _utility_
is deliberately not employed in relation to the decision makers'
attitude at any one date, but is used instead to represent his
present-day state of mind resulting from knowledge of what his future
attitudes will be :

$$U'_o = U'_o \ (A_o \ \cdots A_m) \tag{xiii}.$$

The decision maker's attempt to compress utility into a one-dimensional
concept cannot succeed, as it is perfectly obvious that an analogous
utility concept will apply at every date up to his horizon:

$$U'_1 = U'_1 \ (A_1 \ \cdots \ A_m),$$

$$U'_m = U'_m \ (A_m) \tag{xiii a}.$$

In comparing strategies at t = o the decision maker cannot avoid being aware of what his utility will be at any later date, and in any case the relationship

$$A_t = A_t \ (V_{et} \ldots V_{em}) \quad \text{for all } t \leq m \qquad \text{(xii)}$$

makes each utility value in (xiii) dependent on prospective V_{et} values – just as in (xi) above.

By contrast with the logical impossibility of choosing between S-S growth-paths according to a value-based managerial utility function, it is instructive to recall the successful application of the simple present-day managerial utility function to the choice between multiple growth-path strategies. The factor that prevented a decisive ranking of S-S alternatives such as D and C was the comparability between $U_t(D)$ and $U_t(C)$ for any t. This kind of comparison is impossible between multiple growth-path strategies at any time other than the present; not because future managerial utility cannot be predicted, but because future utility under any strategy is subject to risk in such a way that no useful inference can be drawn from the firm's performance and prospects at any time under one strategy about its success under another.[24] In such conditions management does not have to face the logical necessity of evaluating utility for all dates up to its horizon: the impossibility of comparing at the present day prospective expected utilities at future dates under alternative strategies obliges management to apply the simple type of utility function which is firmly rooted in an exclusively present-day view of

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(24) This argument was developed at length in Chapter 12 : 3 (i).

prospects. Given the conditions worked out in Chapter 13, which provide an adequate guarantee that it will not have to revise its estimates of growth and valuation relationships within its horizon, management behaves rationally in choosing between multiple growth-path strategies by means of an "extended horizon" - present day only" utility function.

The other suggested form of managerial utility function applicable to a choice between S-S growth-paths can be speedily dismissed. The form $U = U (V_{eo}, g)$ appears at first sight to improve on $U = U (V_{eo} \ldots V_{em})$ in that no problem of defining the horizon arises. The utility maximising solution is simply the point of contact between the $V_{eo}(g)$ schedule depicted in Diagram 2 and the highest indifference curve in dimensions V_{eo} and g. In spite of this neat solution there is no essential difference between the two forms of utility function on the main issue of predictability of regret.

Section 4 : Analysis of a "Timeless" Managerial Utility Function.

14 : 4 (i) Introduction to a timeless managerial utility function.

Much of the difficulty encountered in the two preceding sub-sections might have been bypassed had a different managerial utility function been postulated at the outset. This section examines the claims of the valuation ratio, v, and growth rate, g, jointly to replace the time series $V_{eo} \ldots V_{em}$ as determinants of managerial utility when the choice lies between S-S growth-paths. Various interpretations of the function

$$U = U (v, g) \tag{xiv}$$

are considered, with confirmation of the S-S model as a viable

framework for management's strategic choice as the objective. Confirmation must obviously depend on the elimination of the logical difficulties shown in Sections 3 (iii) and 3 (iv) ; but it is also to be hoped that a diversity of possible managerial motivations will be retained.

The valuation ratio relates market value to the book value of the firm's capital stock, and in S-S conditions this ratio remains constant: market value, book value and other variables measured in money terms all grow at the same rate. It follows that a strategic choice in terms of v and g will not be subject to the kind of regret that proved so troublesome in Section 3: such a utility function requires no time dimension, with all strategies remaining in a constant relationship to each other through time.

Little is required to convert the objective aspects of valuation in a S-S context to suit the new form of utility function. In terms of Diagram 2 the vertical axis changes from V_{eo} to $\frac{V_{eo}}{K_o} = v$; and given the value of K_o the change is one of presentation only.[25] The important question is whether the utility function based upon v and g is more or less convincing in this context than one based upon market values was found to be.

Various explanations are available of the roles of v and g in a managerial utility function. Before considering these it is worth-while to recall a general conclusion reached in Chapter 12 : 5, that the terms in which a utility function is expressed do not by themselves

(25) See footnote (19) above.

indicate the motivation and preferences reflected in that function.
In particular, a function based upon prospective equity values,

$$U = U \ (V_{eo} \ ... \ V_{em}) \qquad\qquad (xv),$$

can incorporate as much or as little "managerial" motivation as is
thought appropriate : a complex of preferences about growth, market
value, management's security, etc., can certainly be represented.
It is plainly incorrect to suggest that a truly "managerial" utility
function cannot be based on market value prospects, or that a function
so based can only be associated with simple or "non-managerial"
objectives such as value maximising. The terms suggested for a utility
function must always be supported by positive argument.

14 : 4 (ii) Representing the security motive.

Numerous writers have established a case for reflecting manage-
ment's preference for growth in a managerial utility function, and at
the present general level of discussion no more is necessary. [26]
The arguments used in support of v as a determinant of utility require
more attention. Two such arguments are considered. The first is the
importance of v in giving management security from takeover bids;
the second, discussed in the next two sub-sections, is its importance
as a measure of stock market approval for management's strategy.

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(26) The exact nature of management's growth motivation has not been
universally agreed among managerial theorists, but differences
are not discussed in this chapter: see footnote (22) of Chapter
12.

The validity of v as an indicator of security is not questioned;
but the implication of the security motive itself is important in
view of the hope expressed earlier that a managerial utility function
based upon v and g could be shown to allow a variety of particular
forms and a resulting variety of strategic choices. When v is
interpreted only as management's security indicator that hope is
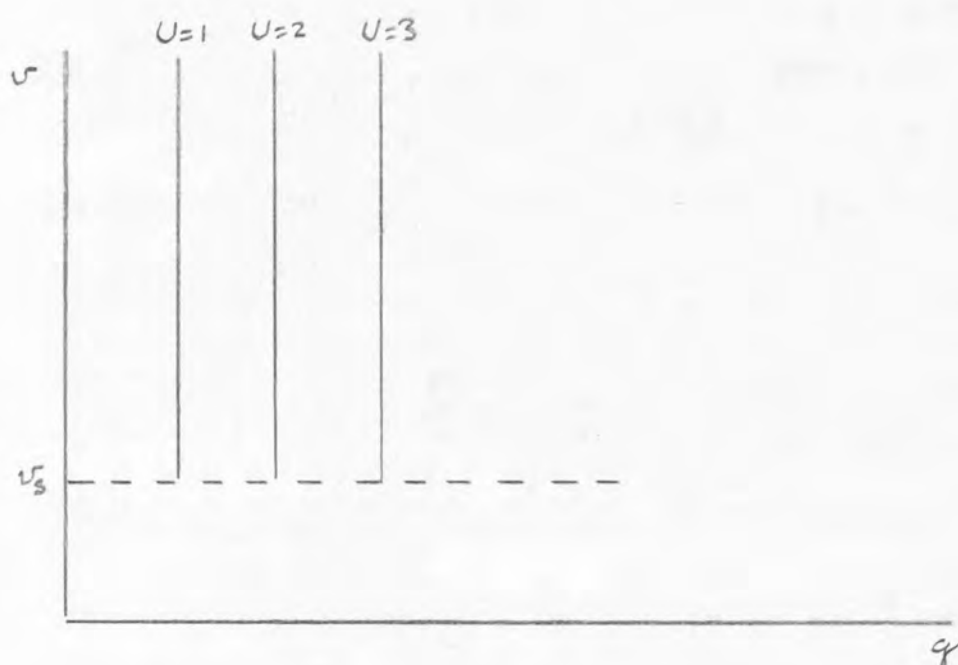likely to be disappointed, as Diagram 6 makes clear.

Diagram 6 : Security as an "absolute" requirement.

In Diagram 6, v features in management's utility function only to
set a minimum acceptable standard, v_s . Provided strategy alternatives
satisfy this security requirement no sacrifice of growth for additional
security will appear worthwhile. (Obviously a portion of the v (g)
schedule must lie above v_s for this approach to be workable, but this
condition can be taken as satisfied.)

The case for describing management's attitude to security in this way rests on the overriding importance of the risk in question. If management's primary motive is indeed survival it is highly probable that the variable representing security in this sense is not treated on a par with other aspects of performance: the latter, after all, are only significant if survival is assured. Concern over security effectively limits the range of choice to those strategies regarded as sufficiently safe, with the choice between qualifying strategies made on other grounds.[27]

Where this attitude applies utility can be seen as a function of g alone. Even if some relaxation is allowed, so that the typical indifference curve embodies some willingness to sacrifice g for increased v above v_s, it remains likely that the curve's slope is very steep and that maximum utility will be found where g is maximised subject to the valuation constraint. The result of stressing security as the rationale for the presence of v in the managerial utility function is that the way is left open for growth maximising (subject only to the valuation constraint) to emerge as the only account of strategy choice. Such a conclusion would disappoint those who see a diversity of preferences and strategies as the hallmark of a truly "managerial" theory. To the extent that the valuation ratio consistent with security is a matter for individual judgement, different choices would in fact

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[27] A managerial security motivation expressed in this way is suggested by Marris (op.cit., 1971, pp.19,32) and in a slightly different form by J.H. Williamson (op.cit., 1966, pp.11-12). Williamson's suggestion that a secure market value should be measured in relation to the maximum attainable market value, M^*, rather than the firm's net assets still allows the security motive to be stated in terms of v once M^* is known.

be observed; but management's objective in each case would be to maximise growth subject to the calculated constraint.

14 : 4 (iii) Determinants of stock market approval.

If the essence of managerial theory is the expectation of a variety of motivations and choices, the main battle is lost when a standardised account of choice is accepted. The second justification for v's inclusion in the managerial utility function may repair the damage, yielding the possibility of varying rates of trade-off between v and g.

According to this second version management is concerned to obtain stock market approval for its strategy, and sees v as the appropriate indicator. Far from contradicting or overriding the first view of v — as an indicator of security — this version may be complementary: that is, beyond the level of v at which security is achieved management may be prepared to trade off growth against stock market approval. The typical indifference curve stops short at v_s as before, but a trade-off between v and g above v_s restores the possibility of a tangency solution between $v(g)$ and an indifference curve. Earlier objections on the grounds of management's foreseeable regret are irrelevant if management feels that its decision should be guided by what it interprets as the market's preferences. In principle a utility function incorporating a "managerial" preference, g, and a "market" preference, v, is perfectly sound; but a closer study of management's understanding of, and attitude towards, the stock market is necessary before the idea is finally accepted.

Whether management sees v as the most appropriate indicator of stock market approval depends on its definition of that term. If the approval of shareholders is sought, management may simply apply the theoretically correct proposition that shareholders' utility is increased by a higher market value: the higher the market value initially achieved by the strategy chosen, the higher the level of shareholders' utility (and approval). Given the firm's stock of assets when the choice is made, the valuation ratio and market value are interchangeable indicators of shareholders' utility. Furthermore, it is a corollary of the proposition relating shareholders' initial utility to initial market value that they - unlike management - will not come to regret a choice made on the grounds of initial market value. And if the choice between alternative strategies is repeatable at a later date the market values of different strategies beginning at that date will then stand in the same relationship to each other as they do initially (that is, the $v(g)$ relationship remains the same); so management will face essentially the same choice at any future date. For these reasons management could infer the permanent approval of shareholders from the level of v, and so could regard an initial choice on this ground as equally permanent.

There are, however, a number of possible "managerial" definitions of stock market approval, any one of which may be preferred to a theoretically correct approach - perhaps because of the practical difficulties posed for the latter by the great variety of tax (and other) circumstances in which different groups of shareholders are placed. Where a simple relationship between the utility of each shareholder and the level of market value is rejected management may consciously try to define and apply an alternative view; or it may take refuge

in an implicit personification of the stock market itself, as distinct
from shareholders. A few examples of possible lines of reasoning, or
substitutes for reasoning, can be suggested.

(i)   Management may simply believe that the market approves of growth —
a belief which is correct in the sense that the market price per unit
of current dividend, $\frac{1}{k-g}$, is a rapidly increasing function of the
growth rate. It might be inferred that the market approves of growth
as well as, or instead of, the valuation ratio; or subject to a valuation
ratio constraint.

(ii) As suggested, instead of looking beyond market value to share-
holders' utility as the source of approval, management may see market
value *per se* as the ultimate indicator. But because a policy of fast
growth overtakes one of slow growth in market value the market's
ranking of each policy may be a function of time. There are two
possibilities.

(iia) If management anticipates its own regret at the inevitable
change in the market's approval of its chosen strategy, the position
is effectively the same as that described in Sections 3 (iii) and
3 (iv) above, when prospective time series of market value were
assumed to be the basis of management's own utility function. Market
approval cannot be assumed constant through time, and a timeless
managerial utility function is unattainable. (There is no suggestion
that management credits the market itself with foresight or hindsight,
so the market's ability to determine price at all times is not in
question.)

(iib) On the other hand, management may define the market's approval
of a strategy at any future date in terms of the value that would
obtain if the strategy were to begin at that date – rather than, as
in (iia), in terms of the value it would have reached by that date
had it been the original choice. The effect of this would be to
maintain a constant relationship between market approval of different
strategies, given a constant $v(g)$ relationship, so that a given value
of v could still represent constant market approval for a strategy
in a timeless managerial utility function – just as it would in a
theoretically correct approach – even if for the wrong reason. However,
this interpretation seems less likely than (iia) because management
probably does not regard the choice of strategy as repeatable at
frequent intervals.

(iii) Perhaps the most realistic view is that management senses **investor**
or market approval of both v and g. Individual concepts of the
determination of approval give rise to corresponding forms of

$$A = A\ (v,g) \hspace{4cm} (\text{xvi}),$$

where A is management's subjective index of stock market or share-
holder approval. As an example, the possible interpretation mentioned
in (i) above – that the market favours growth subject to a valuation
ratio constraint – belongs in this category. Alternatively, manage-
ment may feel that while some groups of shareholders approve of **v**,
others approve of g. High dividend taxation, low capital gains
taxation and the different circumstances of shareholders foster this

attitude.[28]   Accordingly, a locus of v, g combinations represents
management's implicit attempt to identify all situations yielding a
given total of "approval" from all sources. The conceptual problems
of actually doing this kind of calculation are colossal, but if
management recognises the divergent interests of different groups an
implicit calculation is a necessary step in obtaining a one-dimensional
measure of approval.

Even in the case of a firm whose shareholders are completely
homogeneous in their requirements, plans, tax circumstances, horizons,
etc. - the extreme version of a clientele theory of shareholding -
the function $A = A(v,g)$ may apply. Each indifference curve in the
system is a locus of v (V_{e0}) and g combinations between which share-
holders are genuinely indifferent. Lower present-day value and lower
dividends up to their horizon may be satisfactorily offset for members
of the homogeneous group by higher end-value, taking all tax circumstances,
etc., into account. Under ideal, tax-less conditions this could not
occur; but in real-world conditions management may feel that the
interests of all its shareholders can be represented in this way.
Once again, the assumption that management equates market or investors'
approval solely with v proves incorrect. The indifference curve has a
perfectly objective shape in this case, but the point covered by
Section 4 (iv) below remains applicable.

Further examples of managerial inference could be added, but the
important point has already been established. Stock market approval
does seem likely to enter into management's utility function, but it

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(28) See footnote (20) in Chapters 12, and A.J. Merret and A. Sykes,
      The Finance and Analysis of Capital Projects, 1963, p.72.

appears to lack a universally applicable and agreed definition.

14 : 4 (iv) Quantifying stock market approval.

Whatever view management holds of the factors(s) determining
stock market approval - whether the theoretically correct view or one
of many possible "managerial" variations - one further problem of
inference is inescapable if a measured level of approval is to take
its place as an element in management's utility function. This is the
problem of transforming objective data (values of v and g) onto a
scale of "Stock Market Approval" or "shareholders' utility."
Diagram 7 illustrates the problem, using the assumption that manage-
ment actually follows the correct approach in relating shareholders'
utility (and lasting approval) to initial market value or v. This
version of inference is chosen because it is the one commonly assumed
in managerial theory, and because of its comparative simplicity.
Whatever lessons emerge from this exercise will apply a fortiori
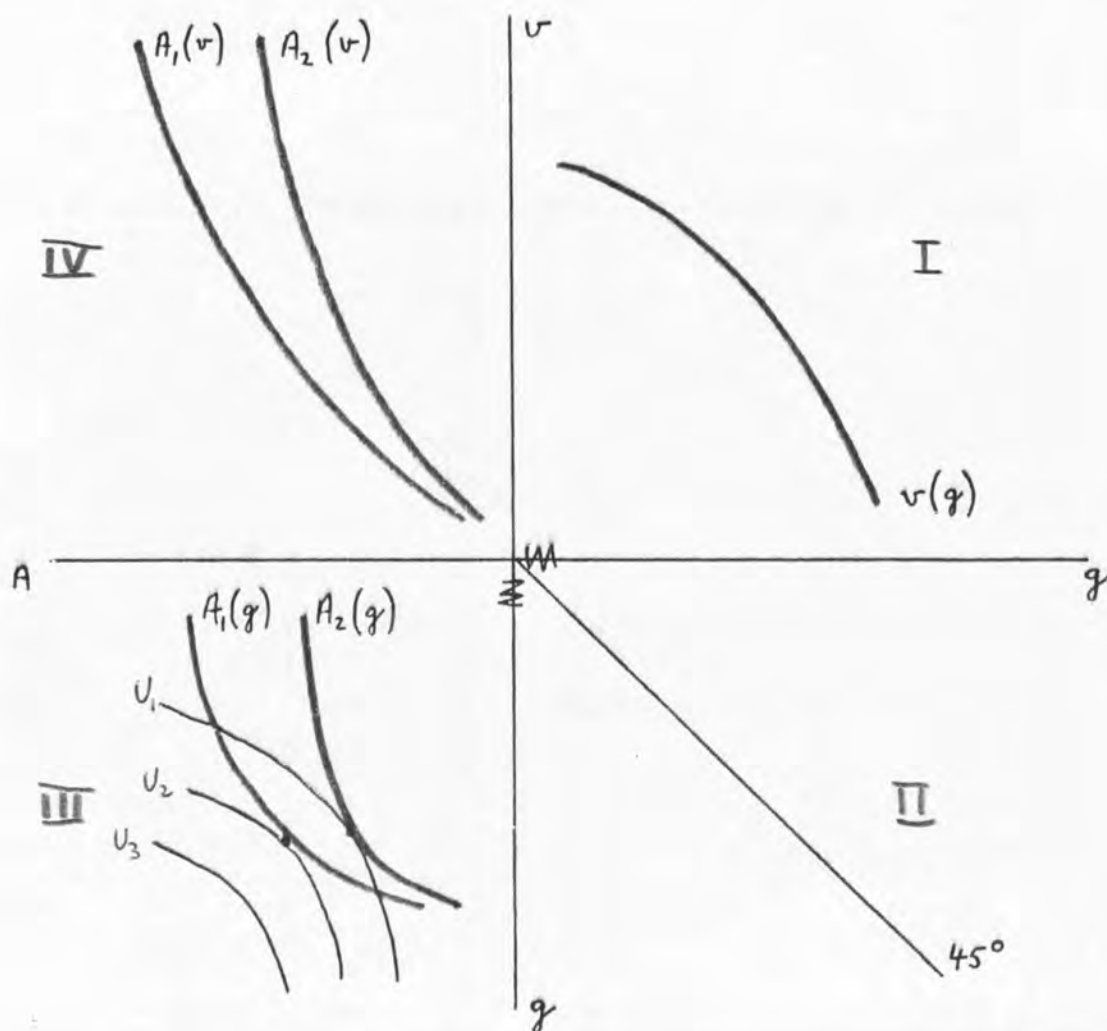to other versions of stock market approval.

Diagram 7 : Constructing the managerial utility function in A and g.

In quadrant I of Diagram 7 the downward-sloping section of a
v(g) relationship is shown. Two different managerial transformations
between v and stock market approval, A, are shown in quadrant IV :
they represent the infinity of equally arbitrary transformations
that might be drawn in this quadrant. By means of the 45° line in
quadrant II and the two A(v) relationships in quadrant IV the
objective v(g) relationship in quadrant I is transformed into two
alternative A(g) relationships in quadrant III. Each A(g) relationship,
based on its particular A(v) transformation, constitutes a frontier of
A,g combinations constraining management's utility maximisation.

Management's choice is made in terms of A and g – not v and g as is usually suggested. The correct general formulation of management's utility function is

$$U = U(A, g) \qquad \text{(xvii)},$$

and the utility maximising exercise takes place in quadrant III where indifference curves drawn from (xvii) are superimposed on whichever $A(g)$ relationship reflects management's view about the relationship between v and A.

The evident arbitrariness of management's $A(v)$ transformation does not demonstrate that management is unable to think in these terms, that it is obliged to retreat to quadrant I and solve its problem at one remove from the terms in which it would ideally prefer to take a decision. Indeed, a retreat to quadrant I offers no escape at all from the problem, because the indifference curve system of quadrant III would have to be reconstructed in quadrant I for the purpose – making use of one $A(v)$ relationship in the process. Instead of finding a number of "objective" $A(g)$ relationships in quadrant III the theorist who prefers the conventional setting of quadrant I finds there the same number of indifference curve systems. No solution is possible in quadrant I without a managerial choice of a particular version of $A(v)$ to be inserted into (xvi), yielding a particular form of

$$U = U \left[A(v), g \right] \qquad \text{(xviii)}.$$

The correct interpretation of the situation is that management implicitly thinks in terms of A and g, and by applying its own conception of A(v) finds no difficulty in seeing both A and g as objective variables. The implicit thought process involved in shifting the decision context into the A,g quadrant cannot be overlooked simply because of its arbitrary and subconscious nature: indeed, ignoring the problem and continuing to depict decision making in the v,g quadrant requires the assumption (in this particular version) that management makes a linear transformation between v and A — a procedure which, when it is made explicit, can attract little theoretical or empirical support. In any case, the very act of treating v as an ultimate variable, directly measuring stock market approval, would constitute a managerial decision on a particular form of A(v), thus confirming the main argument on the inevitability of such a decision.[29]

The problem described here is essentially a "managerial" one. If maximum approval or shareholders' utility is all that management seeks it will simply adopt the growth rate which maximises v. No importance then attaches to the A(v) transformation, and indeed no assumption about its shape is necessary. Only when g enters management's utility function as a "managerial" preference, or when stock market approval is defined in a "managerial" way as influenced directly by g, does the problem arise.

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(29) Marris correctly described the variables in management's utility function as "representing more fundamental utilities" (op.cit., 1964, p.260). The special emphasis given here to the utility derived from A is necessary because of the problem of inferring A's level. This problem does not exist for management in identifying the utility it derives from g.

14 : 4 (v) <u>Steady-State growth and managerial utility.</u>

The position now is that a logically acceptable justification
for v and g as determinants of utility has been reached.
Management's utility function

$$U = U\ (A, g) \hspace{4cm} \text{(xvii)}$$

is made operational by means of a procedure for inferring stock market
approval,

$$A = A\ (v,\ g) \hspace{4cm} \text{(xvi)},$$

so that the utility function can be expressed in general as

$$U = U\left[\ A(v,g)\ ,\ g\ \right] \hspace{3cm} \text{(xviii)}.$$

Two of the stages involved in making (xvii) operational, as in (xviii),
have been covered in the two preceding sub-sections: 14 : 4 (iii)
dealt with the variables and factors that might seem relevant to
management in inferring approval, while 14 : 4 (iv) considered the
necessarily arbitrary business of a scale of approval. The existence
of these stages is of great significance in a comparison between theories
of decision making (growth model plus managerial utility function)
applicable in the different contexts of multiple growth paths and S-S
growth.

From the general expression (xviii) a great variety of indifference
curve shapes in dimensions v and g can be expected, depending (i)

on the weights attached by individual managements to stock market
approval and to growth, (ii) on the variables assumed by management
to determine A, and (iii) on the transformation between objective
data and A accepted by management. The S-S model is confirmed as
a framework from which a variety of decisions can be expected, but
the implications of the result are surprising.

The utility function finally accepted for the S-S context is
closer in its underlying sense to the view reached in Chapter 12 : 5 (iii)
than it is, for example, to Marris's intention. Stock market approval,
as distinct from security, is recognised as an elusive though important
concept requiring arbitrary managerial definition and measurement;
while in the multiple growth-path context investors' preferences and
interests proved incapable of resolution without arbitrary managerial
inferences. In each case the result is effectively the same: in
pursuing "managerial" objectives management lacks a truly objective
trade-off between its own and investors' welfare, and the conventional
distinction between "managerial" and "investor-oriented" strategic
decision making cannot be sustained.

Chapter 15

The Nature of the Strategy Decision.

Plan of Chapter.

Section 1 :   Introduction

Section 2 :   Simulating Multiple Growth-Paths.

Section 3 :   S-S and MG-P Decision Models Compared.

    (i)   Common ground.

    (ii)   The significance to management of multiple growth possibilities.

    (iii)   Managerial utility and the structure of decision making.

    (iv)   Introducing risk into the S-S account.

Section 4 :   Final Review

    (i)   The nature and objectives of strategic choice.

    (ii)   Limits and undeveloped aspects of strategy choice theory.

Section 1 : Introduction

This final chapter is mainly devoted to a comparison
between steady-state (S-S) and multiple growth-path (MG-P)
accounts of the strategic decision. But, as a preliminary,
Section 2 considers briefly some of the principles involved
in the simulation of g-distributions. In Section 3 (i) ...
(iii) the certainty version of the S-S account is accepted
for the purpose of comparison, but in 2 (iv) management's
recognition of risk in relation to growth prospects is
shown to yield a picture of decision making closely resembling
that given by the MG-P account. Finally, Section 4 reviews
what this study has tried to do and, perhaps as important,
what it has not tried to do.

Section 2 : Simulating Multiple Growth-Paths.[1]

This section makes no attempt at a comparison between
the S-S and MG-P models on the question of identifying growth
possibilities: its concern lies exclusively with the feasibility
of strategic decision procedures based on the MG-P model.

Even if management accepts the MG-P framework, its
requirements and procedures are not necessarily standard. On
the one hand its interest may concentrate on the implications

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[1]   A helpful account of simulation applications in business
decision situations is given by G.T. Jones in *Simulation
and Business Decisions*, 1972.

of different strategies for equity value prospects, and
these it may be willing to estimate directly - without
consciously going back to the identification of the
various g-distributions and valuation model. On the other
hand, management may attach the greatest importance to the
g-distribution itself, and may have little interest in its
valuation implications.

Although the particular MG-P based decision procedure
described in Chapters 11 and 12 does not necessarily require
management to go through all the motions of identifying g-
distributions and the valuation relationship, it should at
least be demonstrated that such a "correct" approach is feasible.
The second part of this demonstration has already been given
(Chapter 13 : 2 (iii)), and before the simulation of g-
distributions is considered a number of preliminary points
deserve mention. First, the "period" for dividend planning
purposes is taken to be longer than the standard accounting
year (Chapter 8 : 4 (ii)) : a small number of periods in a
simulation exercise will carry developments quite far into the
future. Second, management probably considers only a small
number of alternative strategies: an initial comparison
between (say) three settings of the r, σ_r frontier (Chapter 11 :3(i))
will probably appear sufficient. Third, the shape of the Q-
distribution is likely to be treated quite separately from the
identification of the g-distribution: it seems safe to assume
that management identifies the Q-distribution's main characteristics
with confidence, irrespective of its views on growth possibilities.
Finally, management's own utility function horizon is taken to

lie in the not-too-remote future, perhaps no more than ten years ahead: this suggests that early periods covered by a simulation exercise will appear crucial, while later stages may appear comparatively unimportant as long as any mistaken assumptions are felt to be common to all strategies examined.

Simulating the g-distribution for any strategy requires a number of simplifying devices;   nevertheless it ought to be possible to make important differences between strategies stand out. Following the classification of factors in Chapters 8 : 1, 11 : 2 (ii) and 11 : 2 (iii), the main elements in a simulation exercise would be :

(i)   the parameters determining investment spending in relation to performance;

(ii)   the resulting level of overall investment profitability;

(iii) shifts in (i) and/or (ii) through time. While not obligatory in simulating the g-distribution, this classification does lend itself to the construction of a simulation routine in which, for each period, small numbers of possible investment levels and profitability levels interact to produce growth-paths of Q and I. Financial planning can also be accommodated in such a routine: for each period on each growth-path the initial (Q-I) distribution for the following period is identifiable (Chapter 8 : 2 (ii), 8 : 2 (iii)), and the routine can if necessary include procedures for the cancellation, delay or external equity financing of investments for which debt financing cannot be fully or confidently anticipated.

Great difficulty arises in the specification of the
third factor enumerated, shifts in investment spending and/or
profitability relationships through time. It is in the
nature of such changes that they are largely unpredictable, and
any attempt at their incorporation can only be made in
probabilistic terms. However, if similar possibilities are
built into the simulation routine for each strategy considered,
the resulting comparison between strategies should be only
minimally biassed by present ignorance of future changes in
the central growth-determining factors. Besides, as already
mentioned, if earlier possibilities are of greater importance
to management than more distant ones, the exact specification
of the possibilities summarised by (iii) will seem comparatively
unimportant.

To summarise : for each strategy under consideration
simulations of experience over any number of periods can be
performed, using the three basic elements suggested. The
number of simulation runs for each strategy must be sufficient
to permit the observation of a frequency distribution of
S-values derived from individual growth-paths.

One major objection in principle to the validity of
simulation in this context is that while growth-paths cannot
be simulated to infinity, the correct individual S-values,
S-distribution and valuation inference all depend on the
discounting of expectations to infinity. Accordingly, interest
focusses both on the practical ways of dealing with the

difficulty and on the importance of any residual discrepancy
between a strategy's true and calculated g-distributions.
Because management's own horizon lies well short of infinity,
and because remote expectations are in any case heavily
discounted in a valuation model, a satisfactory procedure for
simulating the g-distribution can be suggested. A simulation
run can be carried forward for as many periods as is desired,
after which a standard steady-state growth rate can be assumed
to operate from the base given by the final expected dividend
value. This simple device is approximately neutral as between
growth-paths, though the resulting g-distribution and S-
distribution differ from their true counterparts and may be
biassed by the value chosen for the ultimate S-S growth rate.
The result, however, is a limited and manageable number of
growth-paths for each strategy, and the possibility of deriving
a frequency distribution of S-values for different values of k ,
as described in Chapter 10 : 3.

Section 3 : S-S and MG-P Decision Models Compared.

15 : 3 (i) Common ground.

An effective procedure for reviewing the account of
strategic choice developed in Part II is to compare its
essential features with those of decision models based upon
the frame-work of the S-S growth model. Three interrelated
themes have been developed in Part II :

(i) management's view of growth and its related interpretation
     of valuation ;

(ii) the structure of decision making within the firm;

(iii) the managerial utility function — its form and the motivations it embodies. In the following comparison these themes are considered separately, but first it may be useful to indicate briefly the extent of the common ground shared by both versions.

First, the nature of a strategy decision is not in dispute. In each case the decision is intended as a permanent committment, either because relevant conditions do not alter (S-S model, Chapter 14); or because, having made its choice, management believes that it is unlikely within its own horizon to revise its estimates of growth and valuation relationships (MG-P model, Chapter 13). Second, management's interpretation of valuation differs only to an extent dictated by the difference between the two models of growth: differences between valuation models are not the cause of differing accounts of strategy choice. Third, both accounts qualify as "managerial" by their recognition of the discretion over modes of thought, objectives and instruments exercised by an effectively unified managerial personality — at least at the top level of decision making. Fourth, management is not in the position of having to choose between growth models according to its motivation : the MG-P model does allow prospective absolute equity values to determine managerial utility, but broadly speaking most types and mixtures of top-level motivations can be suitably reflected in the utility functions of both accounts.

15 : 3 (ii) The significance to management of multiple growth
              possibilities.

　　　　Notwithstanding this substantial area of agreement on

fundamentals and emphasis, outstanding differences between

the two accounts of strategy choice run deep. In any model

based upon the MG-P framework management accepts the

divergence of growth-paths under any strategy, and the accepted

importance of this divergence leads it to picture growth

in terms of a probability distribution of growth-paths;

while in an account based upon the S-S framework management

accepts that the real world presents alternatives resembling

steady and predictable growth-paths sufficiently closely to

justify acceptance of that view of reality and the approach

to strategy choice it entails. Without abandoning the MG-P

account of the nature of the strategic decision, the question

of management's view of growth is sufficiently important to

merit further consideration.

　　　　The difference between S-S and MG-P models, as models

of growth, may appear comparatively unimportant if it is believed

that under the latter any growth-path generated under a

particular strategy will approximate sufficiently closely to a

predictable S-S growth-path, so that the difference, though

real enough, is of little significance. Whether "approximation"

implies eventual convergence of the actual growth-path to,

or its relatively constant amplitude of fluctuation around,

its predictable steady counterpart, the S-S model may serve

the decision maker as an ideal sufficiently close to reality

to justify its use in his thinking.

To sustain this simplification, management would need to be quite certain as to the precise G-S path which stood as the ideal version of prospective experience under the strategy in question. Confidence at the level required can hardly be expected, given the many changing influences on the level of investment opportunities and on overall investment profitability (Chapter 11 : 2), as well as the ways in which developments may become cumulative over long periods of time. And even if management is prepared to simplify and reduce all possible developments under a given strategy to an approximate equivalence with a unique steady growth-path, the nature of the strategic decision remains as described in the MG-P account - a choice of investment criteria and procedures. Management's view of the real world - its growth and valuation models - may thus be independent of the nature of its strategic decision.

Even if management accepts that all growth-paths under a given strategy ultimately achieve practically the same average growth rate, it still does not follow that the distinction between the two models of growth appears unimportant to it. First, the utility function horizon is surely a long way short of any date at which such a long-term result can be relied on. Second, prospective growth-paths conforming to this assumed long-term tendency are likely to differ significantly in market value according to the time patterns of their dividend streams. Third, even if all growth-paths under a strategy do produce near-identical long-run average growth rates, it is worth

recalling that a small change in the periodic growth rate of
a time series produces a considerable difference in the series'
terminal value over a long run.

In support of the contention that management accepts the
reality and significance of multiple growth possibilities, the
comparative ease with which an orthodox valuation model was
converted into one well suited to the MG-P context can be cited
(Chapter 10 : 3). The ease and logic of the conversion contrast
strongly with, for example, the treatment of the discount rate
suggested by Lerner and Carleton in their attempt to incorporate
the risk associated with a single growth rate expectation into
the same orthodox valuation model. [2] In making a strategic
choice management need not feel hopelessly tied to an inappropriate
framework of valuation theory, or embark on arbitrary modifications
to that theory : the valuation model of Chapter 10 : 3 represents
a managerial extension of the orthodox model to cover the
anticipated results of its own dividend policy under MG-P
conditions, and the input of new or controversial valuation
theory is kept to a minimum.

To summarise : management may prefer a S-S framework for
the consideration of strategic options, but it cannot be argued
that a strategy's multiple growth prospects are in principle all
reducible to an approximate and relevant equivalence to a
predictable steady growth-path. Given this conclusion there

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(2) E.M. Lerner and W.T. Carleton, A Theory of Financial
Analysis, 1966, pp.140-4.

remains the obvious reverse question : whether, when manage-
ment recognises risk in relation to a predicted or expected
growth-path, the S-S view of growth and strategic decision
making is in fact equivalent to the MG-P version. This is
considered in sub-section (iv) below.

15 : 3 (iii) Managerial utility and the structure of decision making.

Given the ability of both frameworks, S-S and MG-P,
to accomodate a variety of managerial motivations in an
apparently permanent strategic committment, and given the
acceptance - up to this point - that management may prefer
either framework for the consideration of strategic options,
only the second theme mentioned at the beginning of this
section - the structure of decision making - remains to
provide a basis for comparing the two accounts of strategic
choice.

It is perhaps surprising to find that the MG-P account
of decision making enjoys something like parity with the S-S
version on the practical question of management's identification
of possibilities. In the former, management compares different
sets of rules for investment choice against a common background
of other policies, with each set of rules generating a probability
distribution of constant-risk dividend growth-paths. In the
latter, the surface simplicity of the r-g locus conceals a
rather formidable problem of identification (Chapter 14 : 2 (ii)).
Nevertheless, management operating in either framework may

believe that it understands the real prospects of different
options well enough to justify the chosen approach; and it
would be wrong to dismiss either as implying a degree of
knowledge or belief that the decision maker must recognise
as impossible.

Where the S-S account does appear illogical is in its
treatment of product market strategy as a utility-yielding
element in decision making.[3] Managerial utility derives
from the chosen v , g combination, with the implied product
market strategy making no direct or indirect contribution:
v and g give utility in their own right, not in any way
as proxies for the product market strategy from which they
derive. It is this aspect of the S-S account that appears
illogical, and it arises from the interlocking of product
market strategy and investment/growth strategy described in
Chapter 14 : 2 (ii). In the MG-F context the strategic
instrument is management's control over investment decision
rules, a purely technical matter that is scarcely likely to
require representation for its own sake in management's top-
level utility function. But in the S-S account the effective
strategic instrument is the heterogeneous and variable mixture
of product and market policies, in respect of which it can
hardly be argued that management is so neutral that no reflection
of them appears anywhere in its utility function.

(3)   In footnote (5) of Chapter 14, product market strategy was
        defined as including the many aspects of policy that
        together determine the firm's performance in given markets,
        the types of product it introduces and markets it enters
        in the course of its growth, and the rates at which it
        makes these changes.

By taking great care in the MG-P account to define
the decision structure so that no decision was excluded from
its appropriate optimising context, a difficulty of this kind
was avoided (Chapter 12 : 4). Conditional product and market
policies are among those that would be predetermined as
subordinate background to the strategic decision, with the
latter rightly concerned with the attainment of an optimum
combination of growth prospects and risk. In the S-S model,
on the other hand, the fact that product market strategy
interlocks with the top-level strategy choice (made in terms
of attributes **v** and g) means that no subordinate process of
optimising in that wide area of decision can be allowed for:
product and market behaviour is dictated by the chosen r, g
combination, not fixed within an optimising procedure directly
involving it.

There are two ways in which the S-S decision model may
be modified to overcome this logical difficulty. First,
product market strategy can be recognised as having attributes
yielding managerial utility at the top-level. The utility
function then expands to

$$U = U (v , g , Y_1, \ldots Y_n) \tag{i}$$

in which Y_i ($i = 1 \ldots n$) is a utility-yielding attribute of
the firm's product market strategy. It is only possible to
re-establish the original utility function, $U = U (v , g)$,
if all Y indicators are definable solely in terms of v and
g, so that, for example,

$$Y_i = Y_i \ (v, \ g) \qquad\qquad (ii), \qquad (4)$$

Such a comprehensive and complete identification of product
market strategy aspects in terms of v and g appears quite
improbable, and the expanded utility function (i) is generally
required.

An alternative modification to the S-S decision model
to handle the problem of product market strategy can be
suggested, though with little enthusiasm. A subordinate
stage of policy making can be envisaged, with, as its outcome,
a range of product market strategies - all of which management
would be equally happy to pursue in its maximisation of top-
level utility, $U = U \ (v, \ g)$. This rather unlikely concept
is in fact the formal equivalent of the actual state of affairs
in respect of the role of product market strategy in the S-S
decision model; and nothing in this formal statement lends any
more credibility to the idea of managerial indifference or
neutrality on the kind of product market performance intended.

15 : 3 (iv) Introducing risk into the S-S account.

Thus far it has been assumed that in employing the S-S
framework for its strategic choice management is certain of the
prospects offered by all strategies under consideration.
This sub-section deals first with the implications for the simple

(4) However, even if this method of deriving the familiar
form of S-S utility function proved valid, the shape
of the typical indifference curve would remain to be
demonstrated.

(certainty) S-S decision procedure of managerial uncertainty about growth prospects; then the nature of a satisfactorily modified S-S account is described.

First, the nature of actual departures from a predicted S-S path can be described. A minimal departure is that described in Chapter 7 : 4 (i), where actual profit (and dividend) fluctuates randomly about its trend expected value; and where investment, the growth of and expected return on assets all maintain their predicted S-S values. There is no reason why this cannot be the type of S-S growth predicted by management : no revision of the valuation model or allowance for the possibility of disappointed managerial expectations need be made in theorising about strategy choice. This version may indeed be seen as a model of a "realistic" S-S growth expectation, and departures from this, rather than from an ideal, will be considered.[5]

One type of departure in this sense occurs when the growth-path of expected dividend is not steady, but "wobbles" about the S-S path originally predicted. A more serious departure, the type considered here, occurs when the original growth-path prediction is disappointed, and fluctuation about an unexpected S-S path develops. (An unexpected S-S path may have a growth rate equal to that of the original S-S prediction). In the event that such a possibility is recognised

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(5) An additional attraction in the version of Chapter 7 : 4 (i) is that dividend risk is convincingly incorporated into a context of certain growth prospects.

by management there arises a need to re-examine the whole concept
of once-for-all strategy choice which proved so suitable for
the case of managerial certainty about the position and
constancy of the r-g locus and the valuation relationship.
As with the questioning in Chapter 13 of the basis for a
once-for-all choice in the MG-P context, no difficulty arises
for the original conception of an unconditional S-S strategy
choice while management believes that within its own horizon
it will have no cause to alter that choice. However, in the
S-S framework it is obviously more difficult for management to
believe that errors will go unrecognised, and it is assumed
here that it does not duck the issues raised by its own admitted
fallibility in prediction (and in interpretation of actual
developments).

Management's recognition that growth may not develop along
(or fluctuate about) the unique path predicted in its strategic
decision has important implications for its behaviour under
actual conditions. The discovery that the predetermined product
market / investment plan is not generating the predicted pattern
of r, p and g values will unavoidably require management to
follow some conditional rule of operation. There may be an
attempt to sustain the predetermined value of p, or to sustain
the intended growth rate or the predicted average return on
assets; or some mixture of revised aims for all three variables.
The need for some response is obvious and inescapable: if no
deliberate response is made management in effect opts for the
continuation of the unanticipated development.

The vital point is that without _some_ conditional policy for the developing situation there is simply no way of describing or predicting the firm's performance under actual conditions. Prior recognition that growth _may_ not follow its predicted path demands prior decision making on policy for the conditions that may occur. It requires only a minimal extension of the argument to show that the necessity for such conditional policy is a continuing one: beyond the first "round" of prediction and disappointment (or confirmation) there lies a second round in which (modified) expectations will either be confirmed or disappointed, and to which new conditional decision making will apply; and so on .... . Evidently the fact that management pictures its strategic options in terms of alternative S-S growth-paths does not affect the requirement that conditional investment (and other) decisions should exist to cover all foreseen possibilities: the concept of strategic choice must be widened to include such decision making. And when the widened picture of conditional responses, all within the scope of a chosen strategy, is examined, its resemblance to the operation (though not to the concept) of the MG-P version of investment strategy is close.

The implication of this line of argument for management's valuation model is obvious. Whenever management recognises that its predicted S-S growth-path may not be attained, and that a succession of unintended developments and responses may occur instead, a more appropriate valuation model must take the place of whatever variant of the basic discounting model

accompanied the S-S certainty assumption. This point carries the discussion to the question of management's willingness to specify probabilities for the possible patterns of development that may occur: investors can be expected to value the firm's prospects at any time only if such probabilities are in principle identifiable, and a desire for effective communication with the capital market would lead management in the same direction - quite apart from its own interest in the consequences of its actions.

A case against the logic of the simple S-S account of strategy choice has now been made, but the argument may seem unfair to those modified accounts which explicitly try to allow for management's recognition of risk. [6] Without examining such models individually, their common implication can be identified and criticised from the standpoint of the Part II interpretation of strategy.

A recapitulation of the relevant aspects of the MG-P account should help clarify the difficulty of accepting even a modified S-S account. The problem any theory faces is that of demonstrating that all eventualities and responses within the decision maker's horizon have been taken into account in his decision; and in particular that at no time prior to his horizon will he make an unforeseen switch of strategy (i) because of events that are foreseeable, or (ii) because of

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(6)   Examples are : J. Lintner, "Optimum or Maximum Corporate Growth under Uncertainty," and R.L. Marris "The Modern Corporation and Economic Theory," both in R.L. Marris and A.J.B. Wood, The Corporate Economy, 1971; E.M. Lerner and W.T. Carleton, op.cit., 1966.

revisions to initial assumptions, beliefs and probability
estimates. The second requirement was considered in Chapter 13,
and the favourable conclusions reached there are accepted here.
The question then is whether management may at some stage wish
to make an unforeseen switch to a different strategy in the
light of events that can actually be foreseen at the outset.

In a discussion of the limits of flexibility in an understand-
ing of investment strategy (Chapter 11 : 5 (ii) ... (iv)) it
was suggested that, realistically, flexibility ought to be
narrowly defined, and should in particular exclude a generalised
"conditional parameter flexibility" or "continuously evolving
strategy" as unnecessarily ambitious. If events follow one
of the courses foreseen at the outset as arising from the
operation of a strategy embodying limited and realistic
built-in flexibility, management should see no reason to change
the predetermined parameters of its investment strategy (Chapter
11 : 5 (iii)).

The MG-P strategy concept envisages the decision maker
reaching a predictable decision, or applying a predictable
procedure, at each point in the evolution of events: a shift
to a new strategy in response to events can make no sense to
the decision maker at the actual moment of decision unless it
also made sense at the planning stage (and is therefore part
of the original strategy's built in flexibility). Strategies
are (and remain) distinct because they embody differing
arbitrary rules for situations in which no uniquely optimal
policy can be defined. Management's initial committment is

to certain arbitrary but flexible criteria and procedures for investment decision making, and different strategies are only compared in terms of their long-term prospects - not in terms of their momentary results in the precise situation existing at a particular time. (Hence the limited value of a concept of continuously evolving strategy: the response to foreseen events is seen as the application of predetermined aspects of overall strategy, rather than in terms of shifts between strategies.)

In the light of this reminder, what can be said of the internal consistency of modified S-S accounts? [7] In the typical modification the position is superficially similar to that described for the MG-P account: management recognises a range of growth prospects under the chosen strategy, and a common implication - described above as applying in the MG-P

(7) Modified S-S accounts do not share the same conceptions of management's assumptions and managerial utility. Lerner and Carleton, for example, adhere to simple value max-imising as a correct and feasible managerial objective, and allow uncertainty over growth to affect the decision only through its effect on the discount rate in the valuation model (op.cit., 1966, pp.140-7, 193-6). Marris retains his more general managerial utility function but, like Lerner and Carleton, allows uncertainty to operate through the valuation model (op.cit., 1971, pp.304-7). Lintner describes managerial utility in modified "growth models" as determined by the expected value and variance of the growth rate (op.cit., 1971, p.216), and retains in his own models the assumption of value maximising behaviour. A number of managerial approaches to the problem of uncertainty about the r-g locus are mentioned in G.M. Heal and A. Silberston, "Alternative Managerial Objectives: An Exploratory Note," Oxford Economic Papers (July 1972).

account - is that whatever development occurs runs its course uninterrupted by unplanned managerial decision.

Yet this interpretation, acceptable for the investment strategy concept of the MG-P account, appears illogical in the S-S context. (If the interpretation is incorrect in respect of management's presumed passivity then, as is clear from the earlier argument, the S-S account in effect approximates to the MG-P version of growth and valuation.) Present day utility does not appear to reflect actions that may be taken by management in response to experience: in general, S-S decision models - simple or modified - are curiously unreal and imprecise on the whole subject of how investment decision making actually proceeds. The central question is whether the prospects determining present day utility are the outcome of a rigid once-for-all setting of policy, or whether they include corrective actions that may be taken in the future. A subsidiary but still important question concerns the exact nature of a once-for-all investment policy decision: does it fix p or a target for r or g? [8]

Specifically, the assumption that management actually confronted by a poor growth and profitability performance would simply accept the outcome, is quite unrealistic. Some kind of "course correction" would undoubtedly be made, and in fact it is

[8] Lerner and Carleton assume that management controls the value of r, letting the retention rate absorb unforeseen effects (op.cit., p.141) ; Lintner sees the growth rate arising from a given retention ratio as uncertain (op.cit., 1971).

probably easy for management to define in advance the type
of remedy that would be applied to each diagnosed problem.[9]
After all, with a simple growth model performance analysis is
easy, types of performance divergence (in relation to expected
values) relatively few, and conditional responses easy to
determine. In evaluating strategic options management is not
condemned to assume that unfavourable developments must run
their course, or - equally illogically - that favourable
situations will not be exploited. All that is required is
that management be prepared at the outset to predict in
probabilistic terms the ways in which future corrective measures
might work in each defined situation ; and since management
is assumed willing to do precisely that for its presently
adopted investment policy, it can hardly be assumed incapable
of it in respect of foreseeable situations in the future.

The similarity between the MG-P and realistically
interpreted S-S accounts is now clear. In the former, manage-
ment will only wish to change strategy if it recognises errors
in its assumptions, etc; foreseeable events will not bring about
a change of strategy because each strategy is defined as
including conditional adjustments to suit prevailing circumstances.
In the latter, on the other hand, the course of events is
actually instructive to management, leading it to change its
ideas about growth prospects - the r-g locus - and take
appropriate actions. In such conditions the concept of strategy

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(9)   The specific remedy selected would vary according to the
      strategy in operation. Just as in the MG-P view of
      decision making, there can be no uniquely correct decision
      for given circumstances.

necessarily embodies continuous evolution, a concept for which
little room exists in the MG-P model. There is no once-for-all
commitment to an inflexible investment policy, however expressed.
The result, however, is a managerial view of growth, conditional
decision making and valuation very similar to that contained
in the MG-P account, though it is reached by a different argument.
Management remains free at each stage to think in terms of an
r-g locus, but its view of growth prospects up to its horizon
is surely one of multiple possibilities. Nor can its valuation
model ignore this reality.

Section 4 : Final Review.

15 : 4 (i) The nature and objectives of strategic choice

The account of strategic decision making presented in
Part II is in keeping with the spirit of managerial theorising
in the sense that it is intended as a minimum departure from the
traditional value-maximising view of the firm's objective;
like its successor in Part II the traditional model, the subject
matter of Part I, recognises managerial discretion over many
subordinate questions including competitive behaviour and the
profit objective. Whether Part II really amounts to a departure
or merely a widening of traditional theory is difficult to
judge: however it is classified it is certainly an essential
development and does recognise new areas of managerial discretion.
Value maximising is discarded not because it is old-fashioned
or "pre-managerial", but because in the conditions described
in Part II it is simply inapplicable. Apart from this necessary

change, however, management's intentions with respect to
dividend policy, its interpretation of valuation, its acceptance
of equity value as a significant performance indicator and its
choice of investment decision making as the central instrument
of strategy all accord closely with the traditional model.

In its simplest terms the problem confronting management
is that of objective risk in relation to future investment,
profitability and growth performance — an area in which capital
budgeting theory is still experimenting with techniques and
trying to define objectives. (It will be recalled that when
all investments are expected to be "safe", even if the growth-
path is unknown, the market value-weighted average cost of
capital can be used as the investment criterion : see Chapters
7 : 6 and 11 : 3 (ii)). From management's own point of view,
however, an objective and a procedure are imperative: a choice
must be made between different sets of investment criteria
offering different growth and risk prospects; and the possibility
of a comprehensible market valuation of the firm's dividend
prospects depends upon investors being able to reach some
conclusions about those prospects — in probabilistic terms if
necessary.

Accordingly, management is portrayed in Part II as deciding
between alternative sets of rules for investment decision making,
for each of which it envisages at the present a different
probability distribution of growth-paths of investment and
constant-risk expected dividend, and a corresponding probability
distribution of equity value for any future date. The

thoroughness with which g-distributions and the valuation model are identified is not specified, but the model of decision making would remain valid even if management simply formed intuitive judgements about the probability distributions of V_{cn} – values that might be generated by an ill-defined investment policy.

15 : 4 (ii) Limits and undeveloped aspects of strategy choice theory.

Turning to matters this study has not attempted to consider, it is helpful to recall the preliminary account given in Chapter 1, where (Section 2 (ii)) the central importance of investment decision making was stressed. Yet in Chapter 1 and at appropriate later stages (Chapters 8, 11) discussion of investment decision making in the MG-P context did not extend beyond a repeated statement of the nature of its results : questions of capital budgeting procedures in general and for particular situations were deliberately bypassed, with the single exception of the concept of a frontier of acceptable combinations of risk and return on individual investments: the inclusion of this was justified on the ground that it appeared to provide management with a convenient device for modifying investment strategy in the planning stage. The important fact, given the capital budgeting procedures employed in the firm, is that in any given situation there exists an ex ante probability distribution of total investment with its related probability distribution of overall profitability.

Secondly, one of the boxes or compartments of managerial

theory mentioned in Chapter 1 : 3 (ii) was reserved for a
model of the firm's decision structure. Here again, the central
property of that structure has played an important part in the
managerial theory as a whole, but there has certainly been no
attempt at a complete organisational model or theory on which
the preferred view of decision structure could have been based.
All that can be said in defence of that view at this point
is that top management is assumed to rely on the conditional
behaviour in all relevant respects of the organisation it
controls : subordinate policy does not have to be made by top
management, but the latter must be in a position to understand
it and rely upon its operation. Given this reliance, the nature
of the organisation is not relevant to the theory of top-level
decision making.

Another disclaimer is in order, though it follows from
what has already been conceded. At no point is the firm's
competitive behaviour in product markets, or any other aspects
of its profit-earning operations, defined. The managerial
theory of both parts of this study is deliberately neutral
on the still unsettled questions arising from the inadequacy of
simple profit-maximising theory.

A final note of caution in relation to the general form
assumed for the managerial utility function in Part II :
the basic justification for this form lies in the attempt to
modify and extend the traditional view of management's financial
objective and thinking (Chapter 1 : 2 (ii)), but it is quite
clear that the form of the utility function and the multiplicity

of growth possibilities are separate aspects of the theory

as a whole. Other utility function forms might have been

suggested, all matching the basic feature of multiple growth-

paths; and in defence of the form chosen it need only be repeated

that when management is assumed to envisage growth in terms

of growth-paths of constant-risk dividend a utility function

based upon equity value prospects is entirely appropriate.

Although it cannot be claimed that the suggested managerial

preference for a market value performance indicator is the sole

possibility for the MG-P context, the fact that a value-based

utility function has been shown to be compatible with a wide

range of managerial motivations strengthens its general

acceptability.

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Investment, Valuation, and the Managerial Theory of the Firm

Common to both parts of this study is an acceptance of
management's discretionary behaviour in product markets,
comparative security against takeover, self-financing
potential and interest in defining a top level of strategy
choice. The study first explores the "managerial" content of
orthodox valuation and cost of capital theory, then modifies
that theory to represent management's framework for strategy
choice under realistic conditions.

Part I develops the orthodox valuation and cost of capital
theory applicable to "safe" investments. Management's
continuing objective is the constrained maximisation of equity
investors' wealth: dividend risk policy – not minimisation of
the cost of capital – takes priority, and the constancy of
cost of capital is only achieved through continuous control
over investment financing aimed at maintaining the target level
of dividend risk.

In Part II investment profitability is assumed subject to
ex ante risk, and this necessitates a further retreat from a
market-determined investment criterion. Given management's
(largely) discretionary criteria for investment – its investment
strategy – performance through time is generated by repeated
interactions of probabilistic investment demand and profitability
functions, and can be described ex ante in terms of a probability
distribution of growth-paths of earnings and investment. With
appropriate conditional planning of investment financing,
management can picture any strategy as a probability distribution
of growth-paths of constant-risk dividend expectations.
A minimal extension of the orthodox valuation model yields an
interpretation of share valuation under multiple growth-path
conditions and allows management to identify the equity value
prospects of alternative strategies.

The resulting model of decision making and managerial utility
permits a full range of realistically interpreted motivations
– from investor-oriented to "mangerial" – and contrasts
strongly with certain implications of models depicting strategy
choice in terms of a choice between steady-state growth-paths.