PRICING, INVESTMENT, AND DEMAND MANAGEMENT IN THE WATER SUPPLY INDUSTRY

BY
HUSSAIN M.A RAJI COBBA

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TO MY FATHER AND MOTHER

ABSTRACT

The subject matter of this thesis is the definition, measurement and use of marginal cost as a tool of analysis to assist the process of decision-making in the water supply industry. Demand management is viewed in broad terms to include the establishment of an optimal structure and level of prices and investment in optimal capacity as well as investment in demand-restraining measures such as leakage detection and control. The study examines the definition of marginal cost as a benchmark for price setting. It provides empirical estimates of the various components of marginal cost of water supply in the Hampshire area, part of the Southern Water Authority. These estimates assume an exogenously determined level of demand and therefore exclude any possible direct interaction between the pricing and investment decisions. Departing from this tradition the study also examines a number of models where, under specific assumptions, optimal prices, output and capacity levels over a chosen planning horizon are simultaneously determined. This allows for direct interaction between the pricing and investment decisions. The study simulates optimal paths of prices, output and capacity expansion in the Hampshire area. This is carried out under various assumptions, one of which admits the potential of staging capacity expansion in order to take advantage of economies of scale in the capital cost function. An analysis of leakage detection and control as a demand management tool is presented in the final part of the study. The purpose of this analysis is to investigate how leakage detection and control may be conducted using either cost-benefit analysis or an appropriately defined tool of marginal cost.

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CHAPTER 1

INTRODUCTION

The subject under consideration in this thesis is the definition, measurement and use of the concept of marginal cost as an analytical tool, to assist decision-making concerning demand management in the water supply industry.

Both the definition and the measurement methodology of marginal cost are not necessarily independent of the final use for which marginal cost is being measured. In the water supply industry one can think of at least two different uses related to demand management:

(1) to contribute to the design and establishment of tariff structures and levels so as (a) to obtain optimal allocation of limited water resources, (b) to obtain optimal utilisation of water supply systems and (c) to enable consumers to send signals to the managers of water authorities which will contribute to the determination of optimal capacity ; and

(2) to establish a benchmark by which specific demand management measures, such as leakage detection and control, may be assessed and evaluated. In this latter case marginal cost may be viewed as a tool for investment appraisal to be used as a substitute for cost-benefit analysis. The appropriate definition and measurement technique may well depend on various considerations relating to these possible end uses.

Thus, the first objective of this study is to examine the definition and measurement methodology of marginal cost as it relates to the design and establishment of tariff structures and levels. The definition of marginal cost for pricing purposes in the water supply industry is complicated by the presence of capital indivisibilities in many components of the supply system. Marginal analysis is not particularly suited to the case of indivisible plant. The types of constraints assumed in the 'model', moreover, further complicate the definition and measurement of marginal cost. Thus we could very well have a multitude of definitions depending on the degree of capital indivisibility, the assumed (desired) level of price variability over time, the assumed (desired) level of price uniformity between different supply areas (with different marginal cost levels) and the degree of assumed (desired) uniformity between different classes of consumers with different marginal costs of supply. The desired level of emphasis on short-run optimal use of existing water supply systems as opposed to providing signals which feed into decisions about additional capacity will also influence the definition and measurement methodology of marginal cost.

The definition of marginal cost may also be closely related to (1) the methodology of demand forecasting adopted by the enterprise for which marginal cost is being measured, and (2) the method of planning for capital expansion adopted by the said enterprise. This highlights the importance of assumptions made concerning, for example, the length of the chosen planning horizon, the demand forecasting model including its degree of disaggregation, and the nature of the capital planning model(s) for the various capital components used by the enterprise in question.

The second broad objective is to investigate the nature of the dynamic relationship between pricing and investment decisions in the water supply industry. Traditional micro economic theory usually examines pricing decisions (marginal cost pricing or otherwise) given a specific inherited amount of capacity or plant. It also usually examines investment policy given prices. Such a dichotomy, useful as it may be because of its operational simplicity, rules out any scope for dynamic interaction between prices and optimal capacity addition(s). Allowing for such an interaction gives rise to potential gains in net social benefits derived from joint decisions about prices over time and the time and size of capacity addition. Such a process involves the construction of an optimisation model capable of solving for the optimum profile of output, prices and capacity addition which maximise net social benefits of the community over the specified planning horizon. Such models may be described as marginal cost pricing models and/or models of optimum capacity addition.

Recently water supply managers have begun to use measurements of marginal cost of water supply for the evaluation of non-price demand management policies such as evaluating investment in different leakage control policies. Economists, on the other hand, would have probably preferred evaluating these policies using the tool of cost-benefit analysis. This observation leads to the third objective of this thesis namely (1) to examine critically these two approaches and (2) to examine the appropriate definition of marginal cost when used for the evaluation of different non-price demand management policies.

The thesis has six main chapters pursuing the above objectives both at the theoretical and empirical levels. Chapter two examines the literature on the definition of marginal cost for pricing purposes in the water industry. The chapter mainly examines the subject with reference to two types of theoretical models found in the literature : (a) marginal cost derived from cost minimisation models and (b) marginal costs (prices) derived from benefit-less-cost maximisation models. The distinction in fact separates, in general, the approach based on a dichotomy between pricing and investment decisions from the approach based on dynamic interaction between investment and pricing decisions. The chapter also has one short section (2.1.2) on the relationship between marginal cost and cost-benefit analysis.

Chapter three prepares the ground for later empirical analysis. It examines important economic aspects of the water supply industry, an understanding of which is necessary for the construction of cost models specific to the industry. Therefore this chapter examines briefly (1) the most salient cost and technological characteristics of the water supply industry in general, (2) salient demand characteristics and a general overview of demand forecasting techniques used by the industry, and (3) the same topics specifically in relation to the Hampshire (Hants) region in southern UK, an area falling within the operation map of Southern Water Authority (SWA). Hampshire will serve throughout this thesis as an area to which many of the models will be applied.

In chapter four we examine the process of capital planning (expansion) in Hampshire. This is carried out with the objective of ultimately estimating the capital component of long-run marginal cost of water in the area. The chapter includes a model for minimising capital expenditure on the expansion of the 'central' part of the water supply system while satisfying the constraint of fulfilling all point forecasts of demand in the area without resorting to price changes or any other rationing device. An analogous analysis of investment on the distribution part of the water supply system is also presented. The second component of marginal cost, namely marginal operating cost, is also examined and estimated for the Hants area. This chapter may therefore be described as an empirical application of an approach to marginal cost measurement when marginal cost is to be used for pricing while ignoring any direct interaction between the pricing and investment decisions. We have found it appropriate to conclude this chapter with a brief examination of the accounting approach to pricing in the water industry and of a discussion of the possible objections to long-run marginal cost pricing as a viable alternative to the former.

It is to be noted here that the estimates of long-run marginal cost of chapter four are used again in chapter seven, which is devoted to the examination of leakage control policy in general with some reference, for illustration, to the Hampshire area. The main purpose of chapter seven is in fact to explore how marginal cost can be used as a proxy for cost-benefit analysis for the evaluation of non-price demand management policies such as leakage control. The potential pitfalls of doing so are noted.

Three dynamic optimum capacity expansion models based on marginal cost pricing are examined in chapters five and six. Chapter five first examines in detail the assumptions made regarding demand and cost conditions common to all models (sections 5.1 to 5.12). Various subjects are examined including the price elasticity of demand, the assumed shape of the dynamic demand curves, non-price determinants of demand, the capital cost function to be included in the models and the operating cost function as well as other matters such as the initial level of inherited capacity, leakage ratios and peak ratios of demand. The discussion of all of these matters is moulded so as to facilitate the construction of our models. All of the simplifying assumptions made must be seen in this light.

The first of the dynamic models, presented in sections

5.12 to 5.17, simulates optimum output, prices and capacity addition in the Hampshire area. Besides the host of simplifying assumptions common to all models, this particular model features the special restrictive assumption that all capacity addition throughout the planning horizon (treatment capacity) must be added at one go.

The second of the dynamic models, presented in sections 6.1 to 6.5, also simulates output, prices and capacity addition over the planning horizon. This time however we remove the restriction that all capacity addition is to take place at one point in time. Introducing the possibility of staging capacity construction opens the potential for improving the value of the net social benefits of capacity expansion as compared with the formulation of the single stage capacity addition model. The improvement, if any, may only be achieved at the 'cost' of perhaps introducing a higher degree of price variability.

The third model, relating to capacity expansion under the constraint of observing a constant price throughout the planning horizon, is presented in section 6.6. The purpose of this model is to throw light on the potential loss in net social benefits that may arise from the imposition of a constant price constraint compared with flexible prices under the two previous models. Results of all three models are critically examined in section 6.7 which also presents some tentative conclusions on the subject of the relationship between the pricing and investment decisions.

Chapter seven is devoted to the evaluation of different leakage control policies where use is made both of the concepts of marginal cost and cost-benefit analysis. Some reference is made to leakage control in the Hampshire area of Southern Water Authority.

Chapter eight presents a summary of the major results derived and conclusions.

CHAPTER 2

ANALYSIS OF MARGINAL COSTS OF PUBLIC ENTERPRISES

(2.1) Introduction

This chapter is about the analysis of marginal costs of public enterprises in general and with reference to those operating in the water industry in particular. The analysis centres on the choice of an appropriate definition of the concept of marginal cost.

Following Turvey (1969)[1] the analysis of marginal costs in this chapter is conducted in the context of an economist trying "to decide what ought to be done." In other words marginal costs are taken to be a tool of analysis. In the water supply industry at least, marginal costs could be used in the analysis of two distinct classes of economic decisions normally faced by managers in the industry : (I) to contribute to the design and establishment of tariff structures and levels and (II) for the assessment of a number of important investment decisions including investment in leak detection and control, investment in the metering of consumers and more generally investment in various demand management measures.

This chapter examines a number of models which investigate the appropriate definition of marginal cost for the said purposes. Section 2.1.2 briefly examines the relationship between marginal cost and cost-benefit analysis. This subject is further examined in chapter 7 in relation to the assessment of leakage control policies. The rest of this chapter is devoted to discussion of marginal cost when used for pricing purposes. Section 2.2 briefly discusses the Neoclassical theory of cost including its limitations. Section 2.3 examines the general relationship between cost minimisation models and marginal cost. This subject is investigated further, in section 2.4, using a general dynamic

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[1] Turvey,R.,1969, 'Marginal cost,' Economic Journal, Vol. LXXIX, PP282-299.

cost minimisation model (Turvey 1971 model) where capacity addition is assumed to be divisible. In section 2.4.3 the analysis is extended to a situation with capital indivisibility. The alternative approach of maximising benefits less costs over time is discussed in sections 2.5 and 2.6. Section 2.5 briefly discusses a model of multi-period unconstrained benefit-less-cost maximisation with capital indivisibility. Section 2.6 discusses a model of multi-period constrained benefit less cost maximisation with capital indivisibility. This particular model suggests a possible link with the family of cost minimisation models of sections 2.3 and 2.4. A number of definitions of marginal cost proposed specifically in relation to the water industry are discussed in the final two sections of this chapter.

(2.1.1) Marginal cost pricing

Economists following the prescriptions of traditional economic theory have advocated, with varying degrees of enthusiasm, some kind of marginal cost pricing for public utilities including water enterprises.[2]

Hirshleifer et.al.,(1960)[3], for example, have strongly advocated the use of marginal costs for water rates : "average-cost pricing is inefficient; it is marginal cost pricing which leads to the best use of resources." In their book, using traditional textbook cost curves, they examine the optimum pricing of water supply and its impact on capacity expansion. Using this traditional framework, involving a high level of abstraction and many simplifying assumptions, they reach the familiar conclusion that " the best short-run solution is to have short-run marginal cost equal price, and the best long-run solution (the optimal scale of plant) is achieved when long-run and short-run

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[2] It is not intended here to survey the huge literature on marginal cost pricing. Our sole concern here is to briefly underline that marginal costs could be used as a pricing standard.

[3] Hirshleifer,J.,Dehaven,J.C., and Milliman,J.W.,1960, 'Water Supply: Economics, Technology and Policy', The University of Chicago Press,1960, PP93 and Chapter V.

marginal cost both equal price."[4]

In a similar fashion Hanke and Davis (1973)[5] argued that "the efficiency with which the nation's water resources are produced and consumed can be improved considerably if the general principles of marginal cost pricing are used as a guide in evaluating water pricing policies."

In a recent report surveying pricing in the water industries of the OECD countries Herrington (1987) [6] stresses that "allocative efficiency... implies that price should reflect incremental costs to the community of satisfying marginal demands" adding that "such a charging system is usually known as marginal cost pricing."

While economic theory provides a rationale for marginal cost pricing and also points to the nature of the assumptions behind the 'model' on which many of the prescriptive judgments just quoted are based [7]it is also true to say that the application of the principle still has its practical and technical problems. These include problems concerning cost computations, present and future; issues relating to price volatility, income volatility, revenue erosion, potential excess revenue and distributional consequences.[8]

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Hirshleifer,et al, op.cit.,PP97.
For a discussion of the limitations of this kind of traditional analysis of marginal cost pricing including the realism of the explicit and implicit assumptions see section 2.2.

5 Hanke,S.,and Davies,R.K,1973, 'Potential for Marginal Cost Pricing in Water resource Management,' Water Resources Research, Vol.(9),No.4,1973, PP805-825.

6 Herrington,P.,R.,1987,'Pricing of Water Services,' OECD, 1987, PP10-13.

7 For example, questions relating to the significance of externalities and the second best considerations or philosophical questions as to the validity of the assumption that prices reflect consumer needs or desires; or, indeed, whether at the technical level it is possible to levy prices at all. These and similar questions are not tackled here.

8 See Mann, P.,C., and Shlenger D.,1982, 'Marginal Cost and Seasonal Pricing of Water Services,' Journal of the American Water Works Association, Vol.74, PP6-11. See also :
(Footnote continued)

Of these problems we will be concerned with only some. We should in particular be addressing the following : (a) What is the appropriate definition of marginal cost when it is used for the design of prices ? (b) How should this definition (and hence measurement technique) be moulded so as to reflect marginal costs, structure and level, over time and over seasons ? (c) How is the definition and measurement of marginal cost amended if we impose some constraints on price variability and/or revenue fluctuation over time? (d) What is the relationship between short-run and long-run marginal costs; operating and capacity costs? (e) How are marginal costs defined in capital-intensive industries, such as that of water, with marked indivisibility of capital assets? (f) How do we define and estimate marginal costs for different consumer classes with different demand characteristics and how do we allow for spatial differences in marginal costs? (g) What is the possible dynamic relationship between marginal cost (price) and capacity expansion? Some of these questions will be examined in this chapter in the context of various theoretical models available in the literature. In later chapters (4, 5, and 6) we will develop and apply specific models to an actual case study with the purpose of further expounding some of these questions.

In general two main approaches to the modelling of pricing, output and investment decisions may be distinguished: (a) marginal cost pricing based on cost minimisation models using point forecasts of demand without immediate price/demand /investment interaction and (b) optimum dynamic models which make simultaneous price/output and investment decisions based on the maximisation of benefits less costs over time.

In this chapter we will consider theoretical models of both approaches highlighting both theoretical and practical distinctions. In chapter 4 we will apply the cost-minimisation approach to an actual case study involving a water supply system. Chapter 5 and 6 on the other hand will

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Herrington,P.R., and Webb,M.G.,1981, "Charging Policies for Water Services", Water Services, Vol.85, No.1025, PP341-347.

outline in detail various models incorporating the approach of dynamic maximisation of benefits less costs, also applied to a water supply system.

(2.1.2) The link Between Marginal Cost and CBA

A number of investment decisions in the water industry could be examined using an appropriately defined concept of marginal cost of water.[9] In reality this is a shorthand version of economic analysis of investment decisions using the well known technique of cost-benefit analysis.[10]

Consider the economic evaluation of a specific leakage[11] control policy which if activated today will result in a reduction in losses by an amount equal to Q million litres per year starting now and lasting forever. The reduction in losses will be matched by an equal reduction in necessary water production, Q ML/year from now and forever. The reduced volume of water production will result in cost[12] savings spread over time, the amount of which expressed in present worth terms is, for example, equal to PWB (present worth of benefits).[13] Moreover let the present worth of all the social costs, from now and forever, arising from the

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[9] See Hanke,S.,H.,1980b, ' A Cost-Benefit Analysis of Water Use Restrictions,' Water Supply and Management Vol.4, PP269-274.

[10] This shorthand version could turn out to be inferior to CBA if marginal cost is incorrectly specified.

[11] For a precise definition of leakage and a discussion of the economics of leakage control see chapter 7. For now leakage may be defined as simply that part of water which escapes from the distribution system without any useful purpose.

[12] Costs should of course be taken to include all external costs associated with water production.

[13] Note that these cost savings need not be evenly spread over time. Indeed with lumpy investment patterns some of the savings (those due to capacity) will be concentrated in few years which are far apart. Also we note that the text abstracts from all the problems that normally must be faced in CBA such as the choice of the discount rate, the valuation of environmental costs and benefits and so on.

implementation of the said policy be PWC. It follows then that this specific leakage control policy is economically viable only if PWB is greater then PWC, ie it is only viable if the present worth of its benefits over time exceeds the present worth of its costs over time.

The same information can be presented in a different way: the specific leakage control policy under consideration can be said to be economically viable if the annual equivalent value of its total discounted benefits over time, B, exceeds the annual equivalent value of its discounted total costs over the same period of time, C, ie. B>C. Expressed in another way, the policy is desirable so long as:

$$Q.(B/Q) > C$$

If we then define B/Q as the 'marginal cost'[14] of providing water supply then it is possible to rewrite the condition as:

$$Q.MC > C$$

$$\text{or} \quad MC > C/Q$$

that is the policy is acceptable so long as the marginal cost of water exceeds the unit cost of implementing the leakage control policy. While this presentation is true it may in fact be better, as we will argue later, to eschew the use of marginal cost of water and evaluate leakage control policies and indeed all investment decisions by the explicit use of the method of cost-benefit analysis.[15]

In analogous fashion discussion of the economics of

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14 Expressed this way the per unit cost saving has been referred to in the literature as the 'long-run marginal cost of water' or sometimes as the 'unit cost of water'. See chapter 7 for a discussion of these issues and for some references in the literature.

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One reason for this preference can be attributed to the danger of assuming that the estimate of MC is independent of Q. In some cases this may be the case but certainly no generalisation is possible. Further discussion of this point will appear frequently in chapters 4 and 7.

metering domestic consumers may be conducted using either the technique of cost-benefit analysis or using the above concept of marginal cost.

In principle leakage control and the metering of consumers may be viewed as selected examples from a wider set of policy instruments for the management of water demands. Many other instruments exist such as the introduction of dual-flush water closets, low water-using washing machines and dishwashers, atomiser sprays for showers and spray taps and so on.[16] Moreover there is no reason why other conservation measures such as education and restrictions[17] cannot be included in the list of demand management tools.[18]

Hanke(1980b)[19] analyses the economics of introducing a 'conservation policy' in general. He explains that a desirable policy is one "which promises incremental benefits which are greater then incremental costs." Incremental benefits according to Hanke are "the savings in resources which are expected to result from the introduction of a water conservation policy." These are calculated "by the product of the reduction in water use resulting from the policy and the marginal cost of water." This approach is clearly identical to the one presented above.

Hence on this approach the marginal cost of water turns to be an 'essential' information required for the assessment of the conservation policy in question. It goes without

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16 See Herrington,P.,R.,1982, 'Escaping From Prison : An Economic Assessment of Rutland Water', in Rutland Water: Decade of Change, ed. Harper,D.,M., University Collections.

17 Restrictions may include measures such as a ban on outside sprinklers and/or a limit on the hours when outside sprinklers can be used .

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Demand management tools need not be confined to conservation policies. If supply is plentiful with no immediate constraints on capacity, it may be economic to encourage consumption by consumers who are willing to pay a price equal to or greater then marginal running cost of production.

19 Hanke,S.,H.,1980b,op.cit.

saying that if we are to follow this approach then the method of calculating marginal cost in each particular case is of critical importance. As a shorthand version of CBA for the evaluation of investment decisions, marginal costs must be estimated in a manner which is consistent with the CBA method itself. Moreover there is no compelling reason why the relevant method for estimating marginal costs for one investment decision should be also suitable for the evaluation of other investment decisions.

(2.2) The Definition of Marginal Cost/ Neoclassical Theory of Cost

Economic textbooks[20] define marginal costs as the cost of producing one extra unit of output per period of time or the cost that can be saved by reducing the output rate by one unit per period of time. To transform this simple concept into a practical tool for the design of prices for public enterprises may prove to be quite difficult.[21] Some of the problems and limitations of the above definition, rendering it (in its present form) to be too impractical for direct application, will be discussed below.

The traditional neoclassical analysis of cost is conducted in comparative static terms. The firm starting from scratch faces constant technical conditions and factor prices. It conducts a series of cost-minimisation exercises, the results of which are summarised in the long-run total cost curve. For each possible level of annual output the total annual costs are calculated, and then divided by the rate of output to give average total costs.[22]

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20 See for example Khan,A.,E., 'The Economics of Regulation: Principles and Institutions', Vol. I, 'Economic Principles,' John Wiley & Sons.

21 Indeed Turvey(1969) argues that the textbook concept of marginal cost is "too simple to be useful." Turvey,1969, op.cit.

22 The choice of the year as the length of the unit period
(Footnote continued)

These long-run planning cost curves (total and average) encapsulate the principle that all factors of production including capacity[23] are variable. Associated with the average long-run cost curve is the long-run marginal cost curve. This is defined as the increment in cost which would arise from a shift from (i) an indefinitely continuous rate of output generated by a plant of optimum design for that rate to (ii) an indefinitely continuous (marginally) higher rate of output generated by means of a plant of optimum design and capacity for this higher rate of output; all carried out under a given set of prices for the various factors of production, constant technology and usually divisible factors of production.[24] If the higher rate of output is anything more than one unit then marginal costs will have to be derived by dividing the change in costs over the postulated increment in output. The resulting marginal cost would thus involve some averaging .

The short-run cost curves on the other hand are derived by supposing the input of one or more of factors of production, usually including capacity, to be fixed. It is sometimes implied that the output increment to be costed is of temporary duration and hence will be produced solely by an increase in the rate of utilisation of the existing plant and equipment. At other times the short-run is taken to imply a resort to more intensive use of existing plant because the change in output was sudden and unanticipated.

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is arbitrary, though it is the widely used convention.

23 No attempt is made here to define capacity rigorously. In general capacity is taken to be a durable input providing services over many years.

24 The long-run textbook concepts refer to the 'stationary' state. The assumptions of constant output through time, constant technology, factor prices, and constant unit running costs over time enables us to express the present worth of the time stream of costs including that of capital into an 'annual equivalent figure.' The cost minimisation associated with each output level is thus transformed into the familiar world of iso-quants. See Millward,R.,1971, 'Public Expenditure Economics',New York, McGraw-Hill, chapter 7.

The distinction between short-run and long-run marginal costs may thus be viewed as a distinction between constant and variable costs. In the short-run capital (and management) costs are excluded from the estimate of marginal cost and as such interest and depreciation charges as well as management salaries and other 'overheads' are excluded. Thus, Mann et.al (1980)[25] identify short-run marginal cost as reflecting increments in operating costs occasioned by increases in output. They take the long-run to include capacity changes and as such long-run marginal costs include marginal capacity costs as well as marginal running costs.

A fundamental theorem of the neoclassical cost theory states that short-run and long-run average costs and their corresponding marginal costs are equal when capacity is optimal. At this optimum size, the additional cost of producing an extra unit of output by a slightly greater utilisation of the present plant will just be equal to the additional cost of this extra unit of output if it were to be produced by an increase in plant capacity. Thus short-run and long-run marginal costs are equal at this point.[26]

(2.2.1) Limitations of Traditional Analysis

Attempting to translate the above concepts into practical tools for decision making (eg for the setting of tariffs) immediately exposes their limitations. These limitations deal with problems of inherited capacity, uncertainty and time, and they will now be discussed in turn.

(a) Starting from Scratch

The relevant cost analysis, as many economists have pointed out[27] refers not to a firm starting from scratch but

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25 Mann,P.,C.,Saunders,R.,J and Warford,J.,1980, 'A note on Capital Divisibility and the Definition of Marginal cost,' Water Resources Research ,VO.16, No.3, PP602-604.

26 In the neoclassical model with divisibility the debate between short-run and long-run marginal cost pricing is therefore only meaningful in disequilibrium situations. See Turvey 1969, op.cit.

in the words of Turvey(1969) to the "costs of running and of expanding or contracting the hotch-potch we have got, the fossilisation of past decisions taken by our predecessors."

Baumol(1971)[28], questioning the usefulness of the long-run 'Vinerian marginal cost', that of a firm starting from scratch, proposed the alternative concept of 'intermediate' long-run. This 'new' concept does not assume that all assets are perfectly liquid (ex-post) with no plant and equipment inherited from the past. He explicitly recognises that in practice the firm can only expand or contract starting from its existing inherited plant. Thus Baumol proceeds to offer us an amended, more practical definition of long-run marginal cost :

> "consider the situation at any moment in time with all current plant and equipment given. Let us then estimate the prospective trends in demand for a particular service and the associated operating and capital costs now and in the future. Suppose that a reduction in price in the service were to bring some increment in its current and future demand and that we could estimate the corresponding changes in present and future operating and capital costs. The difference between these two cost streams _ between the anticipated current and future costs before and after the demand increase _ is the relevant incremental cost corresponding to the change in output in question."[29]

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27 See for example :
(1) Turvey, R., 1971, The Economics of Public Enterprise, Unwin Press, London, chapter 6.
(2) Turvey, R., 1969, op.cit, PP282-299.
(3) Bonbright, J.,C., 'Principles of Public Utility Rates,' Columbia University Press, New York, 1961, chapter 7.
(4) Baumol, W.,J., 1971, 'Rate Making : Incremental Costing and Equity Considerations,' in 'Essays on Public Utility, Pricing Regulation, Trebing,H.,M., (ed), MUS Public Utility Studies, 1971, PP137-150.

28 Baumol,1971, op.cit,PP137-150.

Bonbright (1961)[30] again rejects the traditional long-run concept, which he describes as the limiting case, in favour of a more practical concept similar to Baumol's intermediate marginal cost. For practical rate making he suggests the following :

> "as a rule these are the increments in costs that may be anticipated to result during the next several years from increases in rates of output to be accomplished by whatever plant additions and improvements will be warranted in view of the actual layout and actual capacity of the present plant."

Neither Bonbright nor Baumol elaborate on the appropriate time horizon to be incorporated, the size of the increment, whether the output increment is to be summed over time using discount factors, the choice of the appropriate discount rate and so on.

(b) Forward-looking Marginal Cost/ Technology/ Factor Prices and Uncertainty

There is almost full agreement among economists that the relevant concept of marginal cost for price-setting and other decision making purposes must be based on future rather then historic costs.

Mann and Schlenger(1982)[31] for example stress that marginal cost calculations involve projecting future

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29 Baumol,1971, op.cit. This definition as we will see in section 2.3 is identical to Turvey's present worth incremental system cost corresponding to a permanent increment in output lasting for the stipulated number of periods (years). Presumably for pricing purposes one would take the annuity of the difference between Baumol's two cost streams, the annuity factor being over the stated number of years for which the increment in output lasts. We also note the similarity with the approach outlined in section 2.1.2 concerning the marginal cost of water in relation to CBA. It is rather unfortunate that Baumol does not elaborate his concept any further leaving for example the size of the decrement as well as its duration undetermined.

30 Bonbright,1961, op.cit.

operating and capacity costs for a specified time span, calculating cost changes occasioned by long range future demand increments.[32]

Turvey(1969) stresses that while accounting historical costs may be relevant for 'control' purposes[33] and though they may help to determine revenue requirements yet they are very different from what is required for pricing purposes. This is because pricing affects future decisions and not past ones and hence it is future costs which are relevant.

This immediately brings out the discrepancy between the economist's concept of costs and the traditional one generally followed by public enterprises. The latter is usually based on securing a return on existing assets rather than on new investment. The two will only be equal under static conditions and when the marginal plant is identical to existing plant. Given that this condition is rarely met in real life, Turvey and Anderson(1977)[34] have criticised the traditional approach on the grounds that it "creates the illusion that resources which can be used or saved are as cheap or as expensive as in the past."

The search for the minimum cost solution to meet projected or expected demand must by its very nature be forward looking. Once this is done then not only today's technology and factor prices but also those pertaining to the future become relevant. This reinforces the view regarding the irrelevance of the traditional long-run marginal cost based on today's technology and factor prices.

Moreover when items relevant to decision making relate to the future they need to be forecast. Forecasts are uncertain and subject to revision, whether in relation to demand,

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31 Mann and Schlenger, 1982, op.cit.

32 They leave the length of the planning horizon unspecified.

33 For example checking on the performance of management.

34 Turvey,R., and Anderson,D., 'Electricity Economics : Essays and Case Studies,' A World Bank Publication, The John Hopkins University Press, Balitmore and London, 1977. See also Hanke,S.,1975,"Water Rates: An Assessment of Current Issues", Journal of the American water Works Association, Vol.67, Part 5, 1975, PP215-219.

availability of capacity or to future technology and factor prices.

(c) The Time Dimension

It has been argued that the traditional concept of marginal cost is essentially 'static'.[35] What is needed as remedy is to introduce time explicitly into the analysis so that we have a 'dynamic' concept of marginal cost.[36] This may be done by explicitly dating both inputs and outputs so that one starts thinking in terms of time streams of inputs and outputs.

But once we do that marginal cost ceases to be a simple clearly defined concept. Introducing time transforms the simple marginal concept into a multidimensional one. Thus from a model of intertemporal planning of production, one used by firms to minimise the present worth of a time stream of inputs to meet a time stream of output, one can derive more than one measure of marginal cost. The additional costs due to a unit increment in output will depend, among other factors, on the date the change in output is expected to occur, on the date of the decision taken to change it, on the duration of the change, and also on the date to which cost changes are discounted or compounded in calculating their present worth.[37]

Thus Webb(1976)[38], for example, notes that allowing for changing technology and factor prices and for the fact that some input adjustments take a longer time to implement then others, it follows that the discounted marginal cost associated with an output increment tomorrow may be different from that of meeting a similar output increment in one year's time.

Hanke(1980a)[39] states that "variations in avoidable costs

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35 See for example Webb,M.,G.,1976, 'Pricing policies for Public Enterprises, The Macmilan Press, London,1976.

36 Turvey,1969, op.cit.

37 See Turvey,1971, op.cit, chapter 6, and also Hanke(1981): Hanke,S., 'On the Marginal Cost of Water Supply,' in Water Engineering and Management, Feb. 1981, PP60-86.

38 Webb,1976, op.cit, chapter 6.

will depend on the length of time allowed for the proposed output change. The shorter the time period for the given change in output, the greater will be the avoidable costs." This is presumably because there will be a minimum length of forewarning to adapt the existing productive apparatus so that it can provide the proposed output change in a least cost fashion. If the period of advance warning is less than this minimum, then higher costs will have to be incurred if the change in output is to be met at all. It is quite possible that in some situations the extra output cannot be met without adequate advance warning. Marginal cost in this latter case is then infinite; or more realistically it will show itself, in the context of water supply, in the form of extra costs associated with a higher probability of a water shortage and/or lower supply pressure, ie generally a lower quality of service.

Turvey(1971)[40] points to another subtle reason for multidimensionality of dynamic marginal cost. Marginal cost will change over time; for example the marginal cost of a unit output increment decided upon now to take place in 1990 is different from the marginal cost of the same increment in output at the same date decided upon next year instead of now. This is especially the case in an environment of rapid technological change. Deciding upon a course of action today may rule out choice from among better options that may emerge tomorrow. In this case deciding today on how to meet an output increment to take place sometime in the future will involve extra costs compared with making the same decision next year. In other words "the effluxion of time ...may eliminate certain options by changing the time from future conditional to past definite."[41]

Dating outputs and inputs also helps clear some of the

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[39] Hanke,S.,1980a, "On the Measurement of Marginal Cost : A Practical Guide For Water and Wastewater Services," The Proceedings of the First Technical Seminar on Marginal Cost Analysis and Pricing, Washington, The Inter-American Development Bank, October 1980.

[40] Turvey,1971, op.cit.

[41] Turvey,1971, op.cit.

confusion surrounding the traditional short-run marginal cost. Short-run marginal cost is normally taken to mean the additional operating cost, given constant capacity, incurred to meet an additional unit of output at a specific point of time. Moreover it is sometimes implied that the increment in cost takes place at the same time as the increment in output. This need not always be the case however. Thus, for example, Khan (1971)[42] correctly argues that taking additional business now may involve additional labour and material costs expended now. It may however also involve other expenses which will not materialize until a future date. Future expenditure will arise if additional use now involves more wear and tear of capacity and consequently earlier retirement and replacement. These future costs are surely part of short-run marginal costs.

Turvey(1971) makes the same point in more general terms. He argues that the marginal cost of an output change to take place in, for example, 1990 may or may not be equal to the additional costs incurred in 1990. It will be equal to the additional costs incurred in 1990 if the cheapest way, planned now, of meeting the output increment is by using more inputs in 1990 only. If for example there is spare capacity in 1990 and if higher utilisation will not shorten the life of capacity by wearing it out faster, then only labour and materials will be required. In this case marginal cost would be the same as the change in costs in 1990.

But this is a special case. For example the cheapest way, in present worth terms, of adjusting the time stream of inputs to meet an altered output target in 1990, may involve input changes besides 1990. This may involve building more capacity in 1989 or earlier, or it may involve bringing forward the commissioning date of capacity from 1991 (or any other year) to 1990.

Multidimensionality of marginal cost is also caused by discounting. Since costs are incurred over time, marginal cost must be expressed in terms of a certain base year, thus discounting or compounding costs incurred in other years.

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42 Khan,1971, op.cit.

This in turn implies that marginal cost will now depend on which year all costs are discounted to, possible variations in the discount rate over time as well as the initial choice of the appropriate discount rate itself.

(2.3) Cost Minimisation and Marginal Cost

Turvey(1969,1971,1976,1968)[43] stresses the limitations of the static textbook concept of marginal cost as being too simple to be useful for practical purposes. In particular because output and inputs have a time dimension and because firms rarely start their operations from scratch, Turvey points to the need to depart from traditional models couched in comparative static terms in favour of analysis of costs in dynamic terms. The main characteristics of Turvey's dynamic 'model'[44] may be summarised as follows :

(1) An Approach of Cost Minimisation

Turvey has readily acknowledged that optimal price/output and capacity combinations over time should in principle emerge from a model where these variables are simultaneously determined as a by-product of an exercise maximising social benefits less social costs subject to whatever technical and institutional constraints are imposed both at present and in the future. "In principle, of course, the optimal price output combination should be fixed in the light of predicted demand and cost functions," (Turvey 1971).

But "the practical derivation of prices which maximise benefits less costs is difficult or impossible," according to Turvey(1971). This is because of the constraint on the amount

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43 Turvey,R.,1969, op.cit, PP282-299.
Turvey,R.,1971, chapter 6, PP53-69.
Turvey,R.,1968, 'Optimal Pricing and Investment in Electricity Supply,' Allen and Unwin 1968, chapter 4, PP44-59.
Turvey,R.,1976, 'Analysing the Marginal Cost of Water Supply,' Land Economics, May 1976, PP158-168.

44 The references listed in the previous footnote contain more then one model which retain the general approach of Turvey to the analysis of the concept of marginal cost.

of available information regarding the own- and cross-price elasticities of demand for past and future years. He argues that in general "output forecasts ...cannot be made which give output as a function of future prices except to a very limited extent," (Turvey 1971). Only point estimates of demand can be made in general.

This constraint, it follows, has a direct bearing on the way tariffs can be fixed in practice using the general principles of marginal cost pricing. It is argued that the 'only' practical way, given the information constraint, is to proceed in steps, the first of which is to derive marginal costs from cost minimisation models (and to use these estimate for pricing purposes) thus initially ignoring the possible impact of prices on demand and in turn its feedback on cost calculations and prices.

Outlining this approach Turvey(1969) says :

> " ..in practice the feedback from the pricing decision rule via output and back to marginal cost cannot be taken simultaneously. Marginal costs have to be calculated for a given output forecast. As time passes, the output forecast will be revised and together with other changes, will require new marginal cost calculations. These in turn, with delays, will effect prices via the pricing decision rule. Thus the feedback happens sequentially even though at any point of time a shortage of information makes it necessary to neglect it."

This kind of scenario has the implication that equilibrium or an optimum will never be reached but this is to be accepted since "life is like that....and one might as well try to adjust to it," (Turvey 1971).

(2) <u>Relaxing the Assumptions of Traditional Analysis</u>

Given the information constraint above and the consequent marginal cost pricing strategy, Turvey sets out a general model to analyse the costs of public enterprises in a general framework where costs are forward looking (not historic), in terms of maintaining, expanding or contracting an existing inherited productive apparatus (not starting from scratch), and perhaps with due allowance for dynamic changes in technology and factor prices.

In this framework estimates of marginal costs emerge as a by-product of the search for the minimum cost solution of meeting a series of predetermined point output forecasts stretching over a chosen planning horizon.

(3) Constraint on Marginal Cost (Price) Fluctuation

To the extent that the required marginal cost concept is used for pricing purposes Turvey(1969) would argue that it must be derived with due consideration to the constraint that "prices both have to endure for some years and have an impact on consumers which will frequently endure for some years." This absolute constraint on price fluctuation which Turvey desires to impose arises because (1) in practice there are "institutional hurdles" against frequent price changes which in any case would involve costs (administrative) to the enterprise and inconvenience to the consumer and (2) price stability may be desirable if the correct signals are to be transmitted to consumers investing (on the basis of these price signals) in the acquisition of durable complementary (and also substitute) goods such as the purchase of gas heating appliances or water using machines.[45]

Marginal cost in any one year would measure the extra costs of a one unit increment in output in that year, given the amount to be supplied in other years. Marginal cost for one particular year as defined above need not be equal for different years. If they are not, one would then need to consider the appropriate price structure, reflecting marginal costs, under the constraint that price has to be fixed at a constant level for a number of years. The appropriate price in this case turns out to be some kind of an 'average' of marginal costs for the chosen years.

For the same reasons Khan(1971)[46] states that growing public utilities attempting to set rates as stable as

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45 For further discussion of the advantages and disadvantages of price volatility see section (6.7).

46 Khan,1971, op.cit, chapter 3, PP108.

possible must set them on the basis of some estimate of an average of 'long-run' marginal costs "over some more or less arbitrarily selected planning period of perhaps five years." Vickrey(1968)[47] defines the proper time horizon for the cost determination as the probable interval between rate adjustments.

Of course a structure of uniform tariffs for a number of years ahead helps to satisfy the further requirement that rates must be predictable to consumers so that they can rationally plan their investment in and use of durable complementary or substitute goods and services. Stability is a sufficient but not necessary condition for predictability, however, for it could be argued that fluctuating tariffs can equally well provide information to consumers to plan their permanent consumption decisions, so long as these tariffs are predictable. Whether consumers then have the intelligence, time and farsightedness to react to that information in the 'appropriate' fashion is another question, and one that will not be addressed here.

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[47] Vickrey,W.,1968, "In the Matter of American Telephone and Telegraph Company," Networks Exhibit No. 5, July 1968. The reference is quoted by Khan,1971, op.cit.

(2.4) The Turvey(1971) Model

A simple model formulated in programming terms is used by Turvey(1971) to illustrate the assertion that it is possible to give a general definition of marginal cost set in dynamic terms without going into the details of cost models specific to individual industries.[48]

A number of simplifying assumptions are used; output is produced by one type of capacity which incurs running costs, the construction costs of new capacity and also the running costs per unit of output may change through time, capacity is divisible, and all costs are externally predetermined. A summary of the model is presented below starting with definition of its symbols.

C^v = Present worth now of capital costs of new capacity per unit of output which becomes operational at time t=v , where v stands for the vintage of capacity.

r_t^v = Present worth now of the per unit running costs of output in year t from capacity of vintage v.

Q^v = Number of units of capacity of vintage v.

O_t^v = Output produced in period t by capacity of vintage v.

X_t = Demand to be met at period t .

The objective function to be minimised, so as to achieve efficiency, is an expression encapsulating the total discounted costs of expansion (both running and capital costs) over the planning horizon fixed in this instance to be infinity :

[48] Of course the analysis of marginal costs of a particular industry requires a specific cost model relating to that industry. Thus in chapter 3,4,5 and 6 we develop such models for parts of the water industry.

$$\sum_{v \geq 0}\left[C^{v}\, Q^{v} \; + \sum_{t \geq v} r_{t}^{v}\, O_{t}^{v}\right] \qquad .....(2.1)$$

The cost minimisation is carried out subject to constraints 2.2,2.3 and 2.4 :

$$O_{t}^{v} - Q^{v} \leq 0 \qquad .....(2.2)$$

that is output from capacity of vintage v cannot exceed the amount of such capacity in year t.

$$X_{t} - \sum_{v \geq t} O_{t}^{v} \leq 0 \qquad .....(2.3)$$

that is the demand forecast must be met (rationing by price or otherwise is ruled out).

$$Q^{0} - \bar{Q}^{0} \leq 0 \qquad .....(2.4)$$

that is the amount of capacity acquired from previous periods, $\bar{Q}^{0}$, cannot exceed the existing amount of such capacity, Q^{0}.

The conditions for equilibrium [49] are given in the pairs of relationships 2.5 to 2.10 where in any single relationship if its first part holds with strict inequality then the second part holds with strict equality and vice versa.

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Derived using the Kuhn-Tucker theorem for the solution of nonlinear constrained optimisation problems. See : Glaister,S.,1984, 'Mathematical Methods for Economists,' Third edition, Basil Blackwell.

$$C^v - \sum_{t \geq v} K_t^v \geq 0 \quad ; \quad Q^v \geq 0 \quad \text{for all } v > 0 \quad ......(2.5)$$

$$-\sum_{t > v} K_t^0 + U_0^0 \geq 0 \quad ; \quad Q^0 \geq 0 \quad ....(2.6)$$

$$r_t^v + K_t^v - m_t \geq 0 \; ; \; O_t^v \geq 0 \text{ for all } v \text{ and } t \geq v \; ....(2.7)$$

$$O_t^v - Q^v \leq 0 \quad ; \; K_t^v \geq 0 \quad \text{for all } v \text{ and } t \geq v \; ....(2.8)$$

$$X_t - \sum_{v=0} O_t^v \leq 0 \quad ; \; m_t \geq 0 \quad \text{all } t \quad .....(2.9)$$

$$Q^0 - \bar{Q}^0 \leq 0 \quad ; \quad U_0^0 \geq 0 \quad ...... \quad (2.10)$$

The main conclusions of the model as well as its extension to the case of indivisible plant will be discussed below.

(2.4.1) **The definition of marginal cost/PWISC**

The dual of the output constraint in period t, m_t, may be recognised as the present worth of incremental system cost of a unit output increment (or decrement) in period t, $PWISC_t$. It may be viewed as the difference between (i) present worth of all future costs with the output stream as stated initially and (ii) the present worth of all future costs with the output as in (i) except for X_t which now becomes $(X_t + 1)$. This may be seen as the ceteris-paribus marginal cost or the cost incurred by increasing output in period t by one unit while holding all others constant. If instead of discounting to year t_0 we discount to the year in question we transform m_t into marginal cost in year t expressed in the present worth of the same year.

From equation 2.7, $(O_t>0)$, it is seen that the $PWISC_t$ can be expressed as the sum of discounted marginal running cost r_t^v of capacity vintage v in year t, plus discounted marginal capacity cost in year t of the same vintage of capacity . The latter, K_t^v, is the dual of the capacity constraint. If we abstract from differences in running costs between vintages then we have the simple result that $PWISC_t$ equals marginal running cost in year t plus marginal capacity cost in the same year.

Moreover $PWISC_t$ will be equal to r_t^v (marginal running cost) of that vintage of capacity which has zero marginal capacity cost, $K_t^v=0$. From 2.8 it is seen that partially used capacity has zero marginal capacity cost. Thus we have the simple classical result that whenever we have some vintage of capacity which is used but not fully, discounted marginal cost for that period is equal to its discounted running cost. Again if we abstract from vintages then $PWISC_t$ is equal to running cost in year t whenever there is excess capacity.

In years when new capacity is added, discounted marginal cost is equal to marginal running cost of new capacity plus marginal capacity cost K_t^v. The latter is equal to first year amortisation of new capacity. Amortisation is defined as interest on the value of new capital at the beginning of the year plus depreciation on the incremental capital during the year, defined as the difference in its residual value to the system at the beginning and at the end of the year. The crucial point to note here is that in the presence of expectations of technical progress amortisation cannot be determined by the textbook concept of a constant annuity. "[F]irst year amortisation" must "epitomize the complex of expectations and calculations about the future which are central to the notion of marginal cost," Turvey (1968).

Moreover it can easily be shown that $PWISC_t$ "can in fact be viewed as the effect on present worth of system costs of bringing forward or postponing the acquisition of one unit of capacity by one period," Turvey (1971). Using 2.5 and 2.7 we get :

$$\sum_{t \geq v} m_t = C^v + \sum_{t \geq v} r_t^v \qquad .....2.11$$

And using 2.11 we can write for example for m_4 :

$$m_4 = r_4^4 + C^4 - \sum_{t \geq 5} (m_t - r_t^4)$$

or $$m_4 = C^4 + r_4^4 - (m_5 - r_5^4) - \sum_{t \geq 6} (m_t - r_t^4) \qquad ....2.11a$$

similarly

$$m_5 = C^5 + r_5^5 - (m_6 - r_6^5) - \sum_{t \geq 7} (m_t - r_t^5) \qquad ....2.11b$$

substituting 2.11b into 2.11a for m_5 and rearranging :

$$m_4 = [C^4 + \sum_{t > 4} r_t^4] - [C^5 + \sum_{t > 5} r_t^5]$$

and more generally :

$$m_t = [C^t + \sum_{t} r_t^t] - [C^{t+1} + \sum_{t+1} r_t^{t+1}] \qquad .....2.12$$

As Turvey(1971) points out the last two equations show that for example $PWISC_4$ can be interpreted as "the present worth of capital and lifetime running costs of an extra unit of vintage 4 capacity less the present worth of capital and lifetime running costs saved by dispensing with one unit of vintage 5 capacity."

PWISC in period t is thus seen to depend not only on parameters belonging to period t but also those belonging to future periods. Considerable simplification is however possible if we assume that running costs and the cost of capital are independent of vintage, thus in effect assuming away technical progress, then 2.12 reduces to :

$$PWISC_t = C^t \frac{i}{(1+i)} + r_t \qquad .....2.13$$

and if we further make allowance for an infinite chain of replacements, assumed to take place at fixed intervals of L years, then the right hand side of 2.13 simply reduces to the annuitised value of new capacity plus its running cost. This is demonstrated by replacing C^t in 2.13 by an expression which allows for perpetual replacement, namely :

$$\frac{C^t\ (1+i)^L}{(1+i)^L - 1} = C^t + \frac{C^t}{(1+i)^L} + \frac{C^t}{(1+i)^{2L}} + \ldots\ldots$$

so that :

$$PWISC_t = \frac{C^t\ (1+i)^L}{(1+i)^L - 1}\ \frac{i}{(1+i)} + r_t$$

and this reduces (approximately) to :

$$PWISC_t = \frac{C_t\ i\ (1+i)^L}{(1+i)^{L+1} - 1} + r_t \qquad \ldots\ldots 2.14$$

where the annuity factor is equal to $\dfrac{i\ (1+i)^L}{(1+i)^{L+1} - 1}$.

In other words 2.14 is approximately the equivalent of

$$PWISC_t = r_t + [C^t - \frac{C^t}{(1+i)}] + [\frac{C^t}{(1+i)^L} - \frac{C^t}{(1+i)^{L+1}}] + \ldots \quad 2.15$$

But if, as should be done, allowance is made for future technical progress, the result is to raise marginal cost in the early years. As Turvey(1968) points out "the paradox is due to the fact that providing new plant for year n rather then year n+1 involve the sacrifice of one year's technical progress and this is a cost." Given for example that the annual reduction in the value of C^t is p, ie p is the rate of technical progress, then 2.15 becomes :

$$PWISC_t = C + \frac{C(1-p)^L}{(1+i)^L} + \frac{C(1-p)^{2L}}{(1+i)^{2L}} + \ldots\ldots\ldots\ldots$$

LESS

$$\frac{C(1-p)}{(1+i)} + \frac{C(1-p)^{L+1}}{(1+i)^{L+1}} + \frac{C(1-p)^{2L+1}}{(1+i)^{2L+1}} + \ldots\ldots\ldots\ldots$$

(2.16)

If p=0, ie no technical progress, then we are back to 2.15. If on the other hand p>0 then this will clearly have the effect of raising $PWISC_t$.[50]

Given that future technical progress will lower future marginal costs, given that amortisation should be defined as the excess of marginal cost over running cost of the machine the amortisation of which is in question, and given that running costs will not fall over time, it then follows that amortisation will fall over time. This as Turvey(1969) points out will result in " the amortisation of plant in the first year of its life will be greater than the constant annuity whose present value equals capital cost while in the last year it will be less than this annuity." Thus in the case where technical progress figures markedly the annuity approach will not be adequate.

(2.4.2) <u>Marginal Cost of Temporary and Permanent Output Changes:</u>

The model also helps throw light on the relationship between the present worth of incremental system cost of a one unit output increment lasting for one period and the present worth of incremental system cost of a unit output increment lasting for more than one period. As Turvey(1971) explains

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50 This must be the case since $[C - \frac{C(1-p)}{(1+i)}]$ is greater then $[C - \frac{C}{(1+i)}]$ and the same for other terms.

"it turns out that m_t are additive on present assumptions." Thus the present worth of incremental system cost of a permanent unit increment in output starting at time t, say t=4, and when marginal running costs are ignored for simplicity, is equal to :

$$m_4 + m_5 + m_6 + \dots\dots\dots\dots (2.17)$$

which may be written as :

$$(C^4-C^5) + (C^5-C^6) + (C^6-C^7) + \dots\dots\dots (2.18)$$

It is to be noted that this additivity holds even in the case when capacity is not being acquired in all periods.[51]

Turvey(1969,1968) defines the present worth of incremental system cost of a one period increment in output in year n as the excess of (i) over (ii) where :

(i) is the present worth of the increment of system costs resulting from a permanent load increment staring in year n ;

(ii) is the present worth of the increment of system costs resulting from the same permanent load increment starting at the beginning of year n+1.

In the simplified case summarised in equations 2.17 and 2.18 above we have equation 2.17 standing for (i) in the case of a unit permanent increment in output starting at year n=4. In a similar fashion the present worth of the increment of system costs resulting from a unit permanent load increment starting in year n+1 (year 5) or (ii), is equal to :

$$m_5 + m_6 + m_7 + \dots\dots$$

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[51] The additivity breaks down in the case where new capacity would not have been bought at all were it not for the increment in output, a case probably ruled out if output is growing over time.

Therefore it must follow that (i) less (ii) is clearly m_4 or (C^4-C^5). $PWISC_t$ for a one period output increment is thus seen to be the costs of having one extra unit of capacity in period t less the costs saved by having one unit less of capacity in the subsequent period. This is nothing other then the effect on total discounted system costs of bringing forward the acquisition date of capacity by one period, including bringing forward its replacement dates.

(2.4.3) Definition of Marginal Cost: Case of Indivisibility

Turvey (1976) applies the model sketched above to the case where indivisibilities are significant. He applies the model to the analysis of marginal costs of the 'central system' of a public enterprise engaged in the supply of water.[52] The components of the central system (water sources, treatment plants and major trunk mains) are usually added in large indivisible lumps whose full utilisation occurs only after a considerable time lag.

As before the $PWISC_t$ is defined as "the difference between total system costs with and without the increment or decrement (in output) assumed to take place in a specified year." The size of the increment or decrement for which marginal cost is calculated is not the one unit used in the previous analysis. Rather more realistically it "may well be actually calculated in terms of a large increment or decrement of supply, e.g., as the cost of an increment or decrement of 10^3 m^3 per day, which is divided by $365x10^3$ to express it per m^3." The unit period of time is taken to be the year. Moreover the increment or decrement is taken to be equal to one year's growth in demand. Presumably this particular size of the increment or decrement will depend on the purpose of calculation. If marginal cost is being estimated for pricing purposes then one could justify Turvey's choice of one year's growth on the grounds of convenience since this particular change in output probably satisfies Turvey's (1969) requirement of the need for an

52 For a description of the components of water supply systems see chapter 3.

increment which is postulated as being "large enough to be noticeable but small enough to be marginal." More generally the size of the increment or decrement must be chosen according to the decision in question.

Marginal costs defined as the effect on future system costs of a small increment or decrement to the projected growth in demand will involve marginal operating costs and marginal capacity costs. This is true in those years when "demand is at or near the reliable yield of existing capacity," Turvey(1976). In other words marginal cost includes marginal capacity cost only in those years when demand is near or at the reliable yield of existing capacity. In all other years marginal cost will be equal to marginal operating cost only.

The fact that additions to capacity come in large indivisible lumps exceeding one year's growth in demand implies that in between successive schemes when reliable yield much exceeds demand, marginal capacity costs are zero; $PWISC_t$ for those years will be confined to running costs. This conclusion agrees with the results of the previous model as well as that of traditional analysis.

Marginal capacity costs turn out, as in the 1971 model, to be equal to the cost saving due to postponing the next scheme for one year[53], or the extra costs involved in bringing it forward by one year following an increment in demand. This cost when divided by the size of the increment or decrement gives the present worth of marginal capacity cost.

Turvey(1976) argues that this, the cost saving or extra cost incurred due the postponement or bringing forward of the project by one year, may be expressed in either case as the annuitised value of the cost of the project in question. This again agrees with the 1971 model for the case where technical progress is not a central feature in the industry, an assumption which may well be justified in the mature water industry.

Moreover the relationship between the PWISC of a permanent output increment, one which stretches for many

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[53] Of course including its replacement.

years into the future, and the PWISC of an output increment lasting for one single year holds as in the case of divisibility. In particular the PWISC of a permanent increment in output will be equal to the sum of the individual PWISCs for the one year output increments for those years comprising the permanent change provided that the reoptimisation exercise results merely in the change of timing of projects and not in their sequence or size. One cannot a priori say that this condition will hold at all times. Indeed Hanke and Wentworth (1981)[54] applying a similar approach to the estimate of long-run marginal cost of a wastewater system warn that "in some cases the plans for sequencing and designing facilities might have to be entirely reformulated, and in others ... the effect (of the permanent increment) may be simply to bring forward in time each phase of the investment plan."

In principle there is no reason why the postulated increment or decrement need be constant over time. But Turvey (1968) notes that "it is inconvenient to cost an increment which is itself growing through time" and therefore he "consider(s) a marginal addition to the anticipated load in each future year which is the same for each future year." It goes without saying that if the estimated marginal cost is used for purposes other than pricing, e.g., as a shorthand for CBA of specific investment proposals then the constant increment or decrement over time might be totally inappropriate.[55]

Present worth incremental system cost in years when demand is at or near reliable capacity has an element of marginal capacity cost arising from postponing or bringing forward the particular 'scheme' designed to enhance capacity. As Turvey(1976) stresses the scope of the scheme has to be defined carefully. In particular it must include all other capacity extensions which are necessarily linked to it though not necessarily formally part of it.

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[54] Hanke,S.,H., and Wentworth,R.,W.,1981, 'On the Marginal cost of Wastewater Services,' Land Economics, November 1981, Vol.57, No.4, PP583-567.

[55] See section 2.1.1.

(2.4.4) The Short-Run and Long-Run Definition of Marginal Cost

The present worth of incremental system cost of an output change superimposed on projected output, lasting for a specified period of time, may turn out to be of the short-run or long-run variety (assuming that the short-run refers to additional running costs consequent upon the output increment being achieved by altering capacity utilisation rather then by incurring extra capital expenditure).

It is quite possible that the cheapest way of meeting an increment in output for a specified number of years involves no extra capital costs. Consider the appropriate definition of marginal cost for an increment of demand lasting for a number of years which could be met from existing spare capacity.[56] In this case PWISC of the postulated output change would surely involve no capital costs. Of course there is the complication that accepting the commitment to meet this demand might well also involve the commitment to meet demand at later dates which might require some capital expenditure then. This implies that the exclusion or otherwise of capital expenditure in marginal costs depends not only on the size of the postulated increment but also on its duration. In our example if it can be made clear to consumers that prices based on marginal costs which exclude capacity costs are available for a limited period of time, while spare capacity lasts, then the relevant definition of marginal cost (for pricing purposes) remains of the short-run variety despite the fact that it applies for an output change lasting for perhaps several years.

All marginal costs examined so far, with or without capacity costs, refer to situations where the enterprise has had ample warning of the postulated output change; they all involve optimal adjustment. If demand were to change unexpectedly (ie with short notice), so that capacity ceases to be variable, then there will be other costs which might be

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[56] This situation might well arise if capacity additions are indivisible.

termed 'short-run', in the shape of a change in the probability of supplies that are inadequate, Turvey (1976). If it were possible to include in the cost function a reliable measure of the expected social costs of higher probability of supply failure then in principle optimal capacity levels, both in timing and sizing, as well as the pattern of 'short-run marginal cost' (in the above sense) may be obtained from an optimisation model which minimizes the discounted sum of future operating and capital costs plus the discounted sum of expected losses due to supply failure.

Moreover if instead of a cost minimisation model (in the Turvey tradition) we opt for a benefit less cost maximisation model for the planning over time of capacity additions, output and prices, then marginal costs would take yet another new meaning. Marginal cost (which may also be termed of the short-run variety) in such a framework could mean the marginal opportunity cost of exclusion measured by the value of the service to the excluded customer. This is the market clearing price that rations existing capacity in phases where there is no excess capacity. Such phases may of course be part of the optimal solution given that plant can only be added in indivisible lumps.[57] Turvey's PWISC emerges from a cost-minimisation model and therefore does not admit a demand function; so in consequence it does not produce short-run marginal costs of the latter variety.

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57 For a discussion of this and related matters see chapters 5 and 6.

(2.4.5) Averaging Marginal Costs

Suppose that the purpose of our marginal cost estimation is to determine the prices to be fixed by a public enterprise for the next n years at a constant level.[58] The yearly PWISC will reflect to the consumer the costs incurred or saved by the public enterprise consequent upon the consumer using one more or one less unit that year. Such a pricing schedule is more than likely to show quite a degree of variability depending on the immediate relationship between demand and capacity, a situation prevalent whenever capital is added in large indivisible lumps.[59]

The point to observe is that if tariffs which reflect marginal costs (yearly PWISC) are not to vary from one year to the next, for a period of n years, then they have to be set at some average of the yearly PWISC for each of the n years.

Thus Turvey(1976) states that "the analysis of marginal costs year by year does not necessarily imply that charges which reflect marginal costs should vary from year to year." Moreover Turvey (1976) goes on to illustrate how the yearly marginal costs can be averaged;

> "suppose for the sake of numerical illustration that the marginal cost per m^3, calculated at today's prices over the next ten years are :

[58] For a discussion of the reasons why it may be desirable or /and necessary to constrain prices from fluctuation over time see section 2.3 and also chapter 6 .

[59] See Marcel Boiteux, 'Peak-load Pricing,' reprinted in 'Marginal cost Pricing,' ed., Nelson,J.,R., Englewood Cliffs, Prentice Hall, 1964.

year	MC p/m^3
1	4.0
2	4.0
3	4.0
4	7.0
5	3.5
6	3.5
7	3.5
8	8.5
9	3.5
10	3.5

The present worth of these marginal costs, ie the present worth of their sum is 27.5 pence and this has the same present worth as 4.5 pence per year for ten years."

In other words 'Average' MC = 4.5 p/m^3. It is to be noted that a perhaps more accurate description of this average would be 'weighted average' marginal cost since it is evident that the averaging of the yearly PWISC is conducted using weights equal to discount factors which fall over time at a constant percentage rate equal to the discount rate.[60] The nature of this averaging procedure is explained in the following equations :

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[60] Turvey's averaging is conducted using a 10% constant discount rate. Turvey does not elaborate the reasons for this particular choice. See discussion of section 5.14 .

$$\text{Weighted Average MC} = \frac{\displaystyle\sum_{0}^{n} \frac{MC_t}{(1+i)^t}}{\displaystyle\sum_{0}^{n} \frac{1}{(1+i)^t}} \qquad \text{....(2.19)}$$

$$= \left[\sum_{0}^{n} \frac{MC_t}{(1+i)^t} \right] \times \left[\frac{i(1+i)^n}{(1+i)^{n+1}-1} \right] \quad \text{..(2.20)}$$

$$= \left[\sum_{0}^{n} PWISC_t \right] \times \left[\frac{i(1+i)^n}{(1+i)^{n+1}-1} \right] \quad \text{..(2.21)}$$

It is also interesting to note that the first term of 2.21 may in general be interpreted as the present worth incremental system cost associated with a permanent change of output of particular size, starting now and lasting for n years; the second term is the averaging device where n is the chosen time horizon for which the costing is to be conducted and during which marginal cost is restrained from fluctuation.

Indeed all of the different approaches to the estimation of 'long-run' marginal cost of water supply can be seen in the light of attempts to 'smooth' the peaks and troughs inherent in the 'pure' marginal cost over time. Most of the differences in approaches to marginal cost estimation may be explained in the implicit or explicit choice of the averaging device and/or the choice of the planning horizon over which the costing and averaging is conducted. This is of course in addition to the size of the increment or decrement of output and its variability over time.

(2.5) A Model of Multi-period Unconstrained Benefit less Cost Maximisation With Indivisibility

In this section we depart from the Turvey tradition of cost minimisation models opting instead for the derivation of marginal costs (prices) from a simplified model of dynamic benefit less cost maximisation. If we ignore complications relating to income distribution, externalities and pricing distortions in the rest of the economy the rules for optimal allocation of resources over time may be derived from a general model of multi period maximisation of benefits less costs. The model should give answers regarding the pattern of output, prices and capacity acquisitions over time.

Given the absence of any constraints on price fluctuation or/and constraints on the financial position of the enterprise these rules may be summarised as follows:[61] (1) there is the pricing rule that price should be equal to whichever is the higher of marginal operating cost or the price necessary to restrict demand to existing capacity; and (2) there is the accompanying investment rule that the present worth of the shadow value of capacity should be equated with its marginal capacity cost.[62]

In this section the objective is to demonstrate in a simple theoretical multi period model the interaction of these rules and their implication for the pattern of prices, output and capacity addition over time given the prevalence of significant capital indivisibility. In doing so we will follow a model presented by Rees (1984).[63]

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61 See Rees,R., Public Enterprise Economics, Weidenfeld and Nicolson, second edition, 1984, chapter 5 and Appendix to the chapter, PP85-95. See also Turvey,R., and Anderson,D.,1977, Electricity Economics: Essays and Case Studies, A World Bank Research Publication, The John Hopkins University Press, Baltimore and London.

62 It should be clear that this is a short hand version of the usual present value investment criterion.

63 Rees,1984, op.cit. It is to be noted that in chapters 5 and 6 we will develop actual case studies using models the (Footnote continued)

Consider the following assumptions :(1) A public enterprise the capacity of which is subject to considerable indivisibility; (2) The demand for the output of the enterprise is steadily growing through time, and time is a continuous variable. Deterministic demand is given at time t by $q[P(t),t]$, and consumer benefit, approximated by the area under the demand curve, is given by $B[q(p,t)]$ and ; (3) capacity at present is given by $\bar{Q}$. It is assumed that capacity can only be increased in a discrete step of $\bar{Q}$. It is also assumed that the cost of $\bar{Q}$, expressed per unit of capacity is constant at C. Plants are of the constant efficiency type and of infinite durability. Running costs per unit are equal to v regardless of vintage; and (4) further, it is assumed that capacity increments take place at times t_i, $i=1,2,3,\ldots..$, so that at t_1 capacity becomes $2\bar{Q}$ and at t_2 it becomes $3\bar{Q}$ and in general at t_i capacity becomes $(i+1)\bar{Q}$;[64] (5) there are no constraints on price variability over time and we are given a constant discount rate equal to r.

The exercise is to choose the optimal points in time, t_i, at which to make the discrete capacity additions together with the implied optimal time path of prices (marginal costs)[65] so as to maximise the objective function which is equal to the discounted net benefits (net of running and capital costs) of consumption over the planning horizon.[66]

The objective function to be maximised can be developed

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ontinued) formulation of which is somewhat different from the Rees model.

64 It is to be noted that there are no restrictions on the periods between successive capacity additions which are endogenously determined by the solution of the model. At the other extreme the sizing of capacity additions are severely restricted to be equal to $\bar{Q}$ at any one time. Rees does not offer any rationale for this restriction. In chapter 6 we will develop a multi-stage capacity addition model where capacity addition at any one point of time can take any multiple of a given smallest practical size of capacity addition and where capacity addition can only take place at predetermined points of time.

65 Thus marginal costs will be interpreted as the higher of marginal operating costs or the price necessary to restrict demand to existing capacity.

as follows: over any interval of time (t_{i-1}, t_i) the present value of the net benefits of consumption will be given by

$$B_i = \int_{t_{i-1}}^{t_i} \{ B[q(p,t)] - v\, q(p,t) \} e^{-rt} dt$$

It follows that total net benefits over the entire planning horizon, the maximand of the problem, is given by the sum of B_i over the relevant time periods, net of discounted capacity costs:

$$N = \sum_{i=1}^{h} \{ B_i - C\bar{Q} \} e^{-rt_i}$$

N is to be maximised with respect to the decision variables of prices over time (p_t) and the timing of the fixed size capacity additions, t_i, subject to the capacity constraints :

$$q[p(t),t] < i\bar{Q} \quad \text{when } t_{i-1} < t < t_i$$

Rees states the necessary conditions for the solution of the model as :

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66 Thus the model can be described as a dynamic benefit less cost maximisation as opposed to Turvey type models based on cost minimisation.

$$\{B[q(\overset{*}{p},\overset{*}{t}_i)] - vq(\overset{*}{p},\overset{*}{t}_i) - B[q(\hat{p},\overset{*}{t}_i)] + vq(\hat{p},\overset{*}{t}_i) + rC\bar{Q}\}e^{-rt_i} = 0 \qquad 2.22$$

$$\{p(t) - v\}e^{-rt} - \overset{*}{\emptyset}(t) = 0 \qquad 2.23$$

$$q[p(t),t] - i\bar{Q} \leq 0 \quad ; \overset{*}{\emptyset}(t) \geq 0$$

$$\overset{*}{\emptyset}(t)\{q[p(t),t] - i\bar{Q}\} = 0 \quad \text{when } t_{i-1} \leq t \leq t_i \qquad 2.24$$

where $\overset{*}{\emptyset}(t)$ is the dual of the capacity constraint at time t, $\overset{*}{p}$ is the price at $\overset{*}{t}_i$ <u>without</u> the capacity addition (the high price necessary to ration existing capacity) and $\hat{p}$ is the price at $\overset{*}{t}_i$ <u>with</u> the capacity addition (the low price that will prevail after the capacity addition).[67] Rearranging condition (2.22) with obvious change of notation we get:

$$\hat{B}(\overset{*}{t}_i) - \overset{*}{B}(\overset{*}{t}_i) - v(\hat{q}-\overset{*}{q}) = rC\bar{Q} \qquad 2.25$$

This is now the familiar condition for optimal timing of capacity with indivisibility; when the increase in consumer benefit (measured by the change in the area under the demand curve) made possible by the capacity expansion net of increased operating costs is just equal to the annual cost of the increment in capacity, $rC\bar{Q}$, then the capacity addition is warranted.[68] This will immediately be recognised as the familiar investment criterion expressed in annual terms.

The remaining conditions, concerning the pattern of

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[67] The low price will equal v, marginal running cost, if capacity exceeds demand , otherwise is equal to whatever price necessary to ration the augmented overall capacity.

[68] We note that if, as should be, allowance is made for replacement then the annual cost of new capacity reduces to the annuitised value of the cost of new capacity, given that technical progress is not very marked. The right hand side of (4) is of course marginal capacity cost in those years when capacity is economically desirable.

prices over time, correspond to the pricing rule stated in the opening of this section. If at any time demand is less then available capacity then $\overset{*}{\emptyset}(t)= 0$ and price according to (2.23) is equal to marginal operating cost. If however $\overset{*}{\emptyset}(t)>0$ demand is being restricted to capacity and price exceeds marginal operating cost, v, to the necessary extent to make demand equal to capacity. The value of $\emptyset$ will be, a priori, greater the more inelastic the demand curve for the commodity in question. $\emptyset$ is thus interpreted as the marginal opportunity cost of exclusion.

In this kind of a model of a one-product enterprise following these rules of pricing and investment in an environment of growing demand and indivisible plant, the development through time of prices, output and capacity will be of the sort displayed in diagram 2.1.[69]

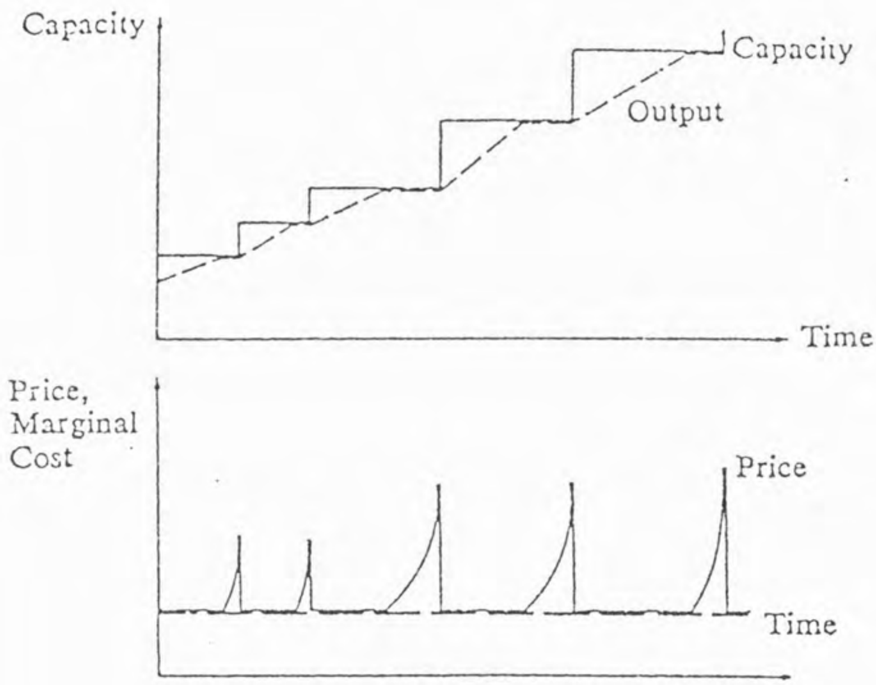
Optimal time-paths of prices, output and capacity implied by the rules stated earlier involve , as the diagrams show, fluctuations in prices which could be marked. The steeper the demand curve and the larger is the size of the indivisible capacity addition[70] the wider will be the gap between the upper and lower limits on prices.

69 See chapter 6 for an actual simulation of prices, output and treatment capacity for a water supply area in the UK.

70 See chapters 5 and 6 where 'indivisibility' emerges as the consequence of economies of scale in the capital cost construction function and not as a technical datum. As such the sizing of capacity addition is another variable to be optimised, besides capacity timing. The two chapters also contain further discussion of the variables that effect the degree of price variability over time.

FIGURE 2.1

PRICES, OUTPUT AND CAPACITY
INDIVISIBLE PLANT



(2.6) A Model of Multi-period Constrained Benefit less Cost Maximisation With Indivisibility

Turvey and Anderson(1977)[71] redefined the optimisation problem to find the pricing rule (for an industry with indivisible capacity) that will maximise the present worth of consumption benefits less the present worth of all costs, subject to the constraint that a uniform price be charged during a considerable period of time. That is the Turvey and Anderson model of this section may be viewed as the extreme opposite of the Rees model of the previous section where price was free to vary at will.

Suppose that a uniform price, P, has to be chosen so as to rule from now, t=1, to some time in the future, t=F well after the next indivisible lump of capacity becomes necessary. This new capacity will have to be commissioned earlier the lower is P since a lower price will increase demand. If we denote q_t as the volume of consumption in period t, then the present worth of the value of consumption brought about by a marginal change in P is :

$$\sum_{t=1}^{t=F} P \frac{\theta q_t}{\theta P} \theta P \frac{1}{(1+i)^t} \qquad \text{....(2.26)}$$

Turvey and Anderson argue that a higher level of demand brought about by a lower price would generate extra capacity costs[72] in the form of having built the last capacity larger or of building the next one sooner. The first is dismissed as relevant only under the unlikely conditions of price being fixed further back in time under conditions of perfect foresight. Thus they argue that the relevant capacity cost manifests itself in the form of either having to bring forward the commissioning date of the next capacity addition or of having to postpone it. If the next capacity addition is

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71 Turvey and Anderson,1977, op.cit.

72 Running costs are ignored for simplicity.

planned for year T, then the effect upon the present worth of costs of a marginal change in the price, P, is :

$$\frac{\theta q_T}{\theta P} \; \frac{\theta T}{\theta q_T} \; \frac{iK}{(1+i)^T} \, \theta P \qquad \text{.....2.27}$$

The first two terms give the rate of change of commissioning date with respect to price. The third term is the rate of change in the present worth of capacity costs with respect to commissioning date. Optimal commissioning date and optimal constant price are found by equating (2.26) and (2.27).

$$\sum_{t=1}^{t=F} P \, \frac{\theta q_t}{\theta P} \, \frac{1}{(1+i)^t} = \frac{\theta q_T}{\theta P} \; \frac{\theta T}{\theta q_T} \; \frac{iK}{(1+i)^T} \qquad \text{...2.28}$$

If $\frac{\theta q_t}{\theta P} = \frac{\theta q_{t+1}}{\theta P}$ for t= 1,2,...F then the equilibrium may be interpreted as saying that the present worth of a permanent stream of output equal to θq lasting from 1 to F, when valued at P, is equal to the present worth of the costs of catering for it . In fact (2.28) gives :

$$P = \frac{iK}{\theta q_T/\theta T} \; \frac{\dfrac{\theta q_T}{\theta P} \; \dfrac{1}{(1+i)^T}}{\displaystyle\sum_{t=1}^{t=F} \frac{\theta q_t}{\theta P} \; \frac{1}{(1+i)^t}} \qquad (2.29)$$

The first term on the right is of course marginal cost at time T.[73] Postponing or bringing forward the commissioning

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[73] This is on the assumption that the asset has infinite life. Otherwise allowance must be made for replacement.

date saves or costs interest per unit time period by an amount equal to iK. This is strictly true only when price is constrained from fluctuation, otherwise the necessary rationing price may be taken as the relevant marginal cost. We can now regard P as a weighted average of marginal costs over the whole period from t=1 to t=F. The weights are the discounted slopes of the demand curves; and marginal cost is zero at all times except time T, the date of planned addition of capacity. As Turvey and Anderson point out this result is an example of the proposition that when a tariff has to be simpler than the cost structure that it is to reflect, it should be a weighted average of the relevant marginal costs. The appropriate weights are proportional to the effects of a divergence between price and marginal cost upon the objective function.

If again we suppose that $(\theta q_t/\theta P)$'s are all the same and if we make due allowance for replacement then (2.29) reduces to :

$$\sum_{t=1}^{t=F} \frac{P}{(1+i)^t} = \frac{1}{(1+i)^T} \quad \frac{AK}{\frac{\theta q_T}{\theta T}} \qquad \ldots.(2.30)$$

where A is simply the annuity factor, ie $A = \dfrac{i(1+1)^L}{(1+i)^{L+1}-1}$.

This again means that the optimal price is such to make the present worth of revenue from a unit increase in consumption from t=1 to t=F, equal to the present worth of the cost of catering for it. The optimal price can now be derived as :

$$P = \frac{i(1+i)^F}{(1+i)^F-1} \times \frac{AK}{\frac{\theta q_T}{\theta T}} \times \frac{1}{(1+i)^T} \qquad ..(2.31)$$

(2.7) The World Bank Definitions of Marginal Cost of Water

In this section we give a brief summary of the discussion regarding the appropriate definition of marginal cost as presented by the widely quoted World Bank's paper (1977).[74]

The World Bank (WB) discusses the definition of marginal cost in the water supply industry with particular reference to marginal cost pricing. The role of the latter is seen as being twofold; (a) to achieve short-run allocative efficiency in the sense of obtaining optimal utilisation of existing capacity and (b) to achieve long-run efficiency in the sense of providing the managers of the industry signals as to the optimal time of capacity addition. Thus the appropriate definition of marginal cost turns out according to the World Bank (WB)) to depend not only on (1) the degree of permitted price (and revenue) variability over time, but also on (2) the desired relative emphasis to be placed on each of the above objectives.

Before we examine the various definitions offered by the WB we discuss the function of the investment-signal price. Our starting point must be the marginal pricing rules as developed in the Rees model of section 2.5 . Without a constraint on price fluctuation, optimality necessitates setting prices equal to marginal running costs whenever there is excess capacity. As demand grows and excess capacity nears exhaustion the public enterprise considers new investment. In principle Cost-benefit analysis should be used to see if new investment is justified at any one particular point in time. If found not justified at that time, presumably price would be allowed to rise to allocate the perfectly inelastic supply. At the point in time when this CBA exercise shows investment to be economically justified it will be undertaken, and price then would have to be set to marginal running cost again (assuming there will be excess capacity immediately after the commissioning of new investment). The

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[74] World Bank,1977, Staff Working Paper No. 259, 'Alternative Concepts of Marginal Cost for Public Utility Pricing : Problems of Application in the Water Supply Sector,' Washington, International Bank for Reconstruction and Development.

pattern of prices show considerable variation over time in the case of indivisible plant. This is shown in diagram 2.1 of section 2.5 .

The practicality of this approach however is disputed by the WB. It is argued that in practice it is very difficult to quantify and value the benefits of water supply (and sewerage projects). In other words it is difficult to specify a demand function for the supply of water from which willingness to pay for water services and thus the benefits of consumption can be ascertained. Accordingly the WB argues that "conventional cost-benefit analysis is rarely a tool that can be applied in practice to the water supply and sewerage sector."

Further it is argued that a mechanism for signalling the economic desirability of investment projects is needed to replace the normal CBA traditional approach. This comes in the form of marginal cost pricing as means of giving the consumers themselves the opportunity to demonstrate to the water supply managers that the value of the incremental output (the project) exceeds its costs. According to the WB "price should be raised to ration existing capacity ....up to the point where consumers reveal their willingness to pay a price for additional output equal to short-run marginal cost plus the annual equivalent of marginal capacity cost." At this point when capacity is fully utilised and consumers are paying a price equal to long-run marginal cost, investment in additional capacity is warranted. Thus the investment signal-price is seen as a way to indicate the economic worth of new investment when CBA cannot be used.

The WB offers four different definitions of marginal cost. These will be reviewed in turn below.

(2.7.1) Textbook Marginal Cost (TMC)

The WB's textbook marginal cost makes use of two concepts: the short-run marginal cost which reflects increments in operating and maintenance costs brought about by the increase in output and marginal capacity costs which reflect necessary capital expenditure. Thus the WB states TMC_t as being given by :

$$TMC_t = \frac{R_{t+1}-R_t}{Q_{t+1}-Q_t} + \frac{AI_t}{Q_{t+1}-Q_t} \qquad (2.32)$$

Where t = Year for which TMC is being calculated.

Q_t = Water produced in year t.

R_t = Operation and maintenance expenditure in year t.

I_t = Capital expenditure in year t.

A = the capital recovery factor (annuity factor).[75]

With lumpy investment TMC for years in which capacity expenditure takes place reflects long-run marginal costs (ie includes capacity costs). [76] For years with no investment TMC will be confined to short-run running costs.[77]

Pricing and investment proceeds according to this model as follows: as with the traditional analysis price is set every year to the higher of marginal running cost or the necessary price to ration available capacity. This is supplemented by the investment rule that the signal to the water utility to add capacity is given when capacity is fully used and price being paid by consumers is equal to or greater then textbook long-run marginal cost thus defined. Such a charging and investment policy would no doubt involve considerable variation in prices and perhaps revenue.

This approach must be implicitly assuming that demand can

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75 The annuity factor was defined earlier as $\frac{i(1+i)^L}{(1+i)^{L+1}-1}$. This compares with the WB's annuity factor of $\frac{i(1+i)^L}{(1+i)^L-1}$.

The difference simply relates to as whether the first annuity payment occurs at the beginning or at the end of the first year. In practice there should be little difference between the two.

76 It is to be noted that the WB's definition of marginal capacity cost when discounted to a chosen base year reduces to Turvey's one year present worth incremental system cost. See section 2.4.1 .

77 There is here the implicit assumption that short-run running cost of year t comprise running costs incurred in year t only. This may not always be the case however. See section 2.2.1 .

respond to prices quickly so that rationing by price is feasible. Also a water enterprise must be able to forecast the demand price elasticity if it were to follow this pricing approach.

More fundamentally it is the investment signal price that raises important questions. Given divisibility the investment signal price is a fully justified approach. With indivisibility however the approach might be questioned. The correct criterion for assessing the justification of investing in additional capacity is derived using cost-benefit analysis comparing the life long benefits of the investment with its life long costs. This investment criterion when expressed in annual terms reduces to condition 2.25 of section 2.5, on the Rees model. In particular the investment in extra indivisible capacity is warranted only when the increase in consumer benefit (measured by the change in the area under the demand curve) made possible by the capacity expansion net of operating cost is equal to or greater then the annual cost of the increase in capacity, AI_t. The increase in consumer benefit per unit of output will be certainly different from the preinvestment rationing price paid by consumers if the change in output following the addition of capacity is not marginal. Given a demand curve with non zero price elasticity and a degree of indivisibility in capacity addition, then the change in output will certainly be more then marginal. In this case the investment-signal price of the WB will be inaccurate.[78]

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[78] For more discussion of this point see the Hanke(1977) model in section 2.8 .

(2.7.2) Textbook long-run incremental cost ($TLRIC_t$)

The World Bank defines textbook long-run marginal cost in the following way;

$$TLRIC_t = \frac{R_{t+1}-R_t}{Q_{t+1}-Q_t} + \frac{AI_k}{Q_{k+1}-Q_k} \quad \text{for } t<k \qquad (2.33)$$

Where the notation is identical to that used for TMC with the exception that k denotes the year in which the next major investment expenditure is completed. As a result during the years t through k the term $AI_k/Q_{k+1}-Q_k$ remains constant, reflecting the annual equivalent marginal capacity cost for the next lump of investment. In year k+1, ie after investment has taken place in year k, k is redesignated to be the year in which the next investment is to take place.

It is clear that this definition of marginal cost when used for pricing purposes implies that prices are constrained from fluctuation for the whole period of time in between successive major investments. Prices would under this definition be pegged to textbook marginal cost calculated from investment expenditure on the next major lump of investment. Thus (a) this definition emphasizes price stability and (b) price stability is achieved at a level which takes into account long-run considerations (giving signals for capacity addition) rather then short-run allocative efficiency (making the best use of existing capacity).

Under this definition prices would be set equal to TLRIC based on the next major lump of capacity addition even if such a price would leave capacity less then fully used for a considerable period of time. The resulting loss in short-run allocative efficiency arises from the fact some consumers who might be inclined, for example, to switch on a water-using appliance if price were set to marginal running cost, are now 'wrongly' dissuaded from consumption by the 'high' TLRIC.

Moreover it is to be noted that the constraint of price stability can be satisfied by any price other then TLRIC so long as it remains constant. The choice of TLRIC is therefore only justified by the desire to facilitate the investment rule. This rule under this pricing scheme would simply be to invest in additional capacity whenever capacity is fully used since price is already equal to textbook marginal cost. The justification of the TLRIC, it follows, depends entirely on the justification of the investment rule under the TMC scheme. If the latter is flawed, as it might be (see above), then the whole TLRIC pricing/investment scheme would also become suspect.

Indeed we have already seen in section 2.6 , on constrained benefit less cost maximisation with indivisible plant that short-run allocative efficiency is achieved when the constant price is set to the weighted average of textbook marginal cost for the years for which price is to remain constant. The weights were the discount factors. The corresponding investment rule would presumably be to invest in additional capacity when capacity is fully exhausted and price is equal to the said weighted average. Therefore it seems that overall economic efficiency is better served by this pricing and investment criterion, in comparison with the TLRIC procedure.

Moreover it is important to note that under both formulations the feedback from prices to demand and in consequence on the timing of investment is ignored. Only after some time when prices are being revised such a feedback may be taken into account. That is it is not until the second round of pricing that the effect of prices), if any, on demand is included.

(2.7.3) <u>World Bank's Present Worth Of Incremental System Cost (PWISC)</u>

It is necessary at the outset to state that the way the World Bank defines present worth of incremental system cost for period t amounts to the present worth of incremental system cost of a change in output equal to $Q_{t+1}-Q_t$ lasting from time t to time k. In other words we think that this definition (given linear demand growth) is effectively equivalent to discounted textbook long run incremental cost of the previous section.[79] This is despite the World Bank contention that its formula captures Turvey's one period PWISC.

The formula given by the WB for $PWISC_t$ is as follows:

$$PWISC_t = \frac{R_{t+1}-R_t}{Q_{t+1}-Q_t} + \frac{\left[\frac{I_k}{(1+i)^{k-t}} - \frac{I_k}{(1+i)^{k-t+1}}\right] + \left[\frac{I_{k+29}}{(1+i)^{k+29-t}} - \frac{I_{k+29}}{(1+i)^{k+30-t}}\right]}{Q_{t+1}-Q_t} \quad ...2.34$$

where the life of the asset in question is assumed to be 30 years[80] and all other terms retain their previously defined meaning.

If we were to reflect continuous replacement every 30 years and if we were to assume no technical progress so that $I_k = I_{k+29}$ then the WB's definition reduces to :

$$(WB)PWISC_t = \frac{R_{t+1}-R_t}{Q_{t+1}-Q_t} + \frac{\frac{AI_k}{(1+i)^{k-t}}}{Q_{t+1}-Q_t} \quad ..2.35$$

(A is the annuity factor)

This is readily seen as nothing other than discounted textbook long run marginal cost as defined above. The point however is that neither (2.34) nor (2.35) fit with Turvey's definition of the present worth of incremental system cost of a one year output change. The WB correctly defines Turvey's $PWISC_t$ as :

> "the present worth of the increment of system costs resulting from a permanent increment in consumption at the beginning of year t minus

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[79] See section 2.7.2 for a discussion of the World Bank definition of TLRIC.

[80] The World Bank assumes for illustration purposes a one year construction period. Therefore if new capacity is to be ready in 30 years, construction must start in 29 years. It also seems that the WB is assuming that all expenditure is incurred at the beginning of the year.

the present worth of the increment in system costs resulting from the same permanent increment in consumption starting at the beginning of year t+1 ."

This correct definition is at variance with both (2.34) and (2.35), for the correct definition implies, as we explained in section 2.4.1, that marginal capacity costs exist only in those years when output is near capacity, being equal to zero in all other years. This means that $PWISC_t$ as defined by Turvey will follow the pattern of TMC, being equal to discounted marginal running cost in years with no capital expenditure.

This comment however, is not to be taken to mean that the WB definition of PWISC is fundamentally wrong. It is just something other than the 'pure' marginal cost of Turvey[81] ; it is in fact the PWISC of a permanent increment in output starting at period t and ending at period k, the increment being of size $Q_{t+1}-Q_t$. The rationale for this definition, as in the case of TLRIC, is perhaps the desire to see prices stable and where the emphasis is placed on the investment signal rather than short-term allocative efficiency.

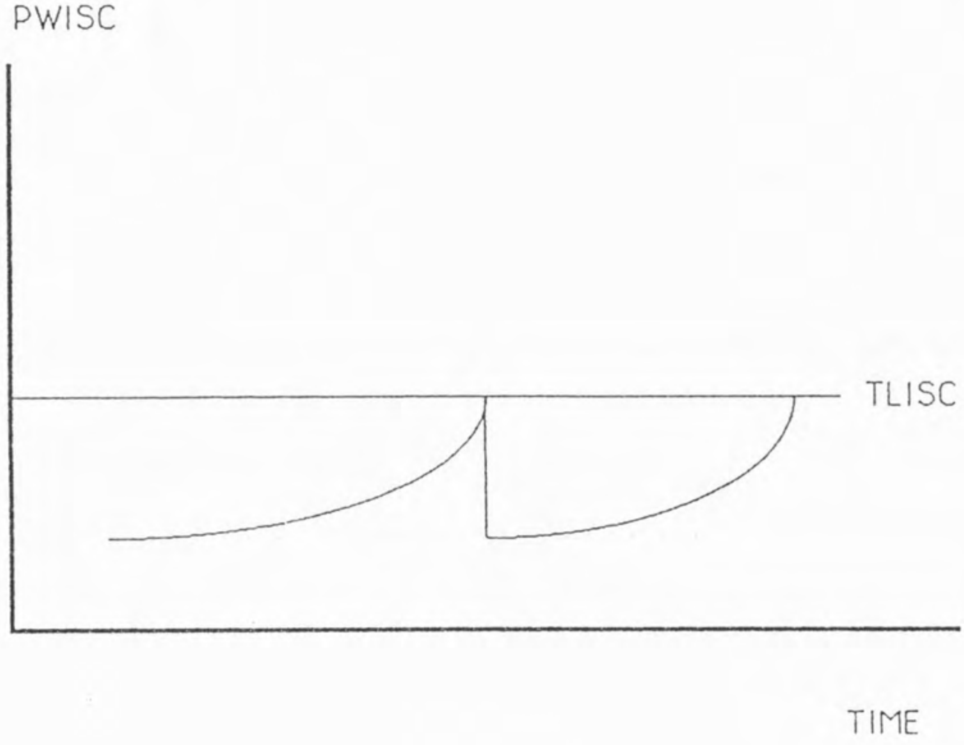
The degree of price stability implied by the use of the World Bank's PWISC is of course less then in the case of textbook long-run incremental system cost. As diagram 2.2 shows the pattern of PWISC over time is such that it peaks in those years with major capital expenditure, dropping to its lowest level immediately afterwards. At its peak PWISC is equal to TLRIC.[82]

81 Which turns out to be discounted textbook marginal cost in the case of no technical progress.

82 The diagram assumes, in line with the WB, that charging according to the estimated PWISC will not change the optimal commissioning date of the next project. In other words one in practice is forced, for a little while, to ignore the feedback from the defined marginal cost to price and thence to quantity demanded, the timing of investment and the marginal estimate itself. If we desire to allow for such an interaction we would need a model that solves for prices, output and investment simultaneously.

FIGURE 2.2

WORLD BANK PRESENT WORTH INCREMENTAL SYSTEM COST



(2.7.4) Average Incremental Cost (AIC)

The final definition of marginal cost proposed by the World Bank assumes that in cases when investment is lumpy, the capital component of marginal cost, or marginal capacity cost, can be estimated as :

$$MCC = \frac{\text{Present worth of the least cost investment stream}}{\text{present worth of the stream of incremental output resulting from the investment}}$$

The actual formula used by the WB to simulate AIC is :

$$AIC_t = \frac{\sum_{\hat{t}=1}^{T} \frac{(R_{t+\hat{t}} - R_t) + I_{t+\hat{t}-1}}{(1+i)^{\hat{t}-1}}}{\sum_{\hat{t}=1}^{T} \frac{Q_{t+\hat{t}} - Q_t}{(1+i)^{\hat{t}-1}}} \qquad (2.36)$$

Where the notation is as before except that T is the number of years for which water expenditure and attributable output are forecast and over which price is being smoothed.

According to (2.36), for any particular year AIC and hence price for that year will be calculated by looking ahead for T years[83] at the costs and amounts of water to be delivered, relating discounted sum of costs to the discounted sum of water supply.

Prices based on AIC will not be constant from one year to the next despite the fact that some smoothing of the pure textbook marginal cost over time is taking place. In fact AIC of the World Bank may be seen as smoothing out the steep variation in marginal costs but nevertheless not stamping them out altogether. This in contrast to the weighted average

83

T is usually the number of years for which reliable data is available.

of marginal costs as in the Turvey model of section (2.5) where the averaging is conducted so as to obtain one price to rule for the duration of T years.

(2.8) Simple Average Marginal Cost: Hanke(1977) Model

We conclude chapter 2 with further discussion of a few more definitions of marginal cost discussed in relation to the water industry. Hanke(1977)[84] argues that it is useful to average marginal costs over some years smoothing out any sharp variation. He goes on to outline how to average marginal costs over a five year period[85] : "therefore in a given year, marginal capital costs are defined as the annuitised value of the investments that are planned for the following five years, divided by the increment in total water use for those five years."[86] According to Hanke this may be expressed as following :

$$\text{Marginal capital Cost in year } t = (1/5) \sum_{j=t}^{j=t+4} \frac{rI_j}{\Delta Q_j} \qquad 2.37$$

Where I_j = Total demand related capital investment that becomes operational in year j, with capital expenditure before the year of operation being discounted forward to year j using a rate of interest equal to i.

r = the annuity factor given by Hanke as

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[84] Hanke,S.,1977, 'A Method of Integrating Engineering and Economic Planning,' Proceedings of the Symposium on Water Services: Financial, Engineering and Scientific Planning, The Institute of Water Engineers and Scientists, London, chapter 5.

[85] No particular reason is given for the choice of five years. It is likely that this is determined according to the number of years for which there is reliable information on investment planning.

[86] Hanke,1977, op.cit.

$$r = \frac{i(1+i)^L}{(1+i)^L - 1}$$

L being the life of the asset.

ΔQ_j = the increment in annual demand for year j.

It is immediately apparent from the above definition that for any year marginal cost is being defined as the simple average of textbook marginal cost for the following five years.[87] No particular reason is given by Hanke for the choice of this particular form of averaging instead of, for example, using the discount factors as the appropriate weights for averaging.[88]

Hanke also introduces a distinction in marginal costs according to season. If demand has peak characteristics which are consistent with certain seasons of the year, e.g. peak demand always falling in the summer months, then it may desirable for efficiency reasons to distinguish between marginal capacity costs according to season. This is necessary so long as some capacity is entirely designed to meet peak demand.

Accordingly we have marginal off-peak capital costs and marginal peak capital costs :

$$\text{Marginal off-peak capital Cost in year } t = (1/5) \sum_{j=t}^{j=t+4} \frac{rI'_j}{\Delta Q_j} \qquad 2.39$$

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[87] For a definition of textbook marginal cost see section 2.7.1 above.

[88] When the averaging is conducted for not a very long period then it may be that it does not matter a great deal whether the simple average formula or the weighted average formula is used. It may be noted in passing that Turvey and Anderson (1977) while advocating in principle the use of the weighted average method when indivisibility is present, in fact use the simple average method in their case study on the electricity industry in Thailand when calculating average marginal cost for lumpy transmission capacity. Turvey and Anderson (1977), op.cit.

$$\text{Marginal peak capital Cost in year } t = (1/5) \sum_{j=t}^{j=t+4} \frac{rI''_j}{\Delta Q_{sj}}$$

Where I'_j = total off-peak capital expenditure, ie capital expenditure designed to increase the overall yield of the system not just peak demand.

I''_j = total peak capital expenditure, ie capital expenditure designed to meet peak demand.

ΔQ_j = the increment in annual demand in year j.

ΔQ_{sj} = the increment in seasonal (summer) demand, ie the increment in peak demand.

Such a seasonal distinction in marginal capacity costs is useful for the purpose of peak-load pricing; peak prices should be equal to off-peak plus peak marginal costs; off-peak prices only to off-peak marginal costs.[89]

According to Hanke marginal costs calculated as above (with or without seasonal distinction) should form in general the basis for pricing consumers of water services. These marginal costs, he argues, are relevant so long as their adoption does not result in part of existing capacity remaining idle. "it may happen, however, that prices computed this way are too high and result in 'excess' capacity ...but we know that marginal cost pricing presupposes demand matching supplies. The apparent inconsistency clears up when we see that water authorities invested too much capital in such systems not long before they adopted marginal cost pricing." And therefore he suggests that when planners are faced with capacities that are not 'proper', capacities that are too large, they should set price just high enough to exhaust probable supplies. That is in the presence of 'excessive' capacity Hanke suggests adopting short-run

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89 This of course presumes that peak load pricing is feasible in the sense that it is technically and economically possible to record accurately the volume of consumption during the various seasons.

marginal cost pricing in the form of using the higher of marginal running cost or the price necessary to ration existing capacity.

The calculated marginal cost is given the role of the investment signal-price. Hanke suggests that when the price paid by consumers "reaches the relevant marginal cost calculated from equation [2.38], the authorities should invest in capacity," and moreover having done so they should "set future prices equal to the total marginal cost."

These conclusions deserve few comments. First, Hanke seems to rule out the possibility of having excess capacity for reasons other than the failure of a water authority to adopt marginal cost pricing earlier on. As we have seen marginal cost pricing in the presence of large indivisibilities does not presuppose demands matching supplies. In fact the discrepancy between probable demands and supplies, an inherent situation in the presence of indivisibility even with marginal cost pricing, is at the heart of the problem of defining marginal cost for pricing and investment planning purposes.

Second, Hanke seems to favour short-run allocative efficiency in that he seems to suggest a pricing/investment model along the lines suggested by the World Bank's Textbook Marginal Cost.[90] The difference from the WB's model seems to relate to the definition of the investment signal price; the WB uses textbook marginal cost whereas Hanke uses a five year average of textbook marginal cost.

Finally in the presence of indivisibility it is not clear why should price be set equal to the 'total marginal cost' immediately after the commissioning of new capacity. If that capacity is large, short-run allocative efficiency dictates setting price equal to marginal running cost.

If on the other hand Hanke is suggesting the use of his average marginal cost for pricing purposes he must be prepared to accept that at times there would be excess capacity in the system; ie immediately after the addition of indivisible plant, an excess capacity which cannot be taken

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90 See section 2.7.1 .

up by inducing additional consumption by the use of low prices.

Hanke (1980a and 1981)[91] further examine the concept of marginal cost for water supply in the context of an empirical example related to the marginal cost for the Spring Valley Water Company which is a private water company serving Rockland county, New York.

Here Hanke adopts a different definition for marginal cost from the one used in Hanke(1977) above. To examine this, and also highlight the impact of the averaging process on MC estimates, we reproduce the information used in his example. Future investment to meet water demand in Spring Valley comprise in the main[92] the 'Ambrey' project whose construction is scheduled to begin in 1980 so that it is ready for commissioning in 1985, the year when its services are expected to be needed. Expected relevant capital expenditure by the Spring Valley company is given in table 1 below.

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91 Hanke,S.,1980a, 'On the Measurement of Marginal cost: A Practical Guide For Water and Wastewater Services,' The proceedings of the First Technical Seminar on Marginal Cost Analysis and Pricing, Washington: The Inter-American Development Bank, October 1980.

Hanke,S.,1981,'On the Marginal Cost of Water Supply,' Water Engineering and Management, Feb. 1981, PP60-86.

92 At least as far as 'central system' costs. For a definition of this and other concepts related to water supply see chapter 3.

Table 2.1

Expected Capital Expenditure
Spring Valley Water Company
$1000

Year	I	Discounted I
1980	2026	2026
1981	3742	3449
1982	7188	6106
1983	5822	4553
1984	4123	2975
1985	0	
1986	0	
1987	0	
1991	0	

The same information may also be written as :

Year	I
1980	0
1981	0
1982	0
1983	0
1984	$2026(1+i)^4+3742(1+i)^3+7188(1+i)^2+5822(1+i)+4123=26489.3$
1985	0
1986	0
1987	0
1991	0

Source: Hanke(1980,1981),op.cit.

Note: Expenditure on the Ambrey project spaced between 1980 and 1984 has been compounded forward to year 1984 using a discount rate i equal to 0.085. It is unclear whether these figures allow for replacement. It is also unclear what is the actual life of the project which consists of a reservoir, treatment plant and a transmission main.

The corresponding expected annual growth in water demand which the Spring Valley Company is expected to meet is also given by Hanke.[93] This is given in table 2.2 below:

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[93] It is however not clear at all whether any price strategy has been incorporated in this demand forecast or indeed at what level of prices this demand forecast is supposed to prevail.

Table 2.2

Expected Annual Demand Growth
Spring Valley Company
1000 cubic feet

Year	ΔQ_A
1979	29,099
1980	29,099
1981	29,099
1982	29,147
1983	29,099
1984	29,099
1985	21,140
1986	21,141
1987	21,189
1988	21,141
1989	21,141
1990	17,820

Source: Hanke(1980,1981),op.cit.

According to Hanke(1980,1981) the total marginal capital cost of water supply (central system) in 1980 in Spring Valley is \$51.45/1000 cf. This is calculated by Hanke by stepping the investment programme, ie the Ambrey project, by one year. The Ambrey project can be stepped by one year if there were a permanent decrement in output in 1980 equal to 29,099 cf. Accordingly Hanke calculates the following:

Change in discounted capital cost =

$$[2026+3449+6106+4553+2975]-1/(1+i)[2026+3449+6106+4553+2975]$$
$$= \$1497 \times 1000$$

That is a permanent (five year) increment (or decrement) in annual demand of the amount of 29,099 cf results in a saving wit a PW of \$1497x1000. Relating the two Hanke thus argues that the annual marginal capital cost in 1980 is:

$$\text{Marginal Capital Cost(1980)} = \$1497/29099$$
$$= \$51.45 \text{ per 1000 cu.ft.}$$

This estimate deserves a few comments. First, the methodology used here is certainly at variance with that of the 1977 model above. Indeed if we assume that the annuity factor of the Ambrey project is simply i/(1+i), ie assuming infinite life, then simple average marginal cost for 1980 is equal to :

$$\text{Average marginal cost} = (1/5)(0.085/1.085)(26489.3/29099)$$
$$= \$14.26 \text{ per } 1000 \text{ cu.ft.}$$

(the figure 26489.3 comes from table 2.1)

Indeed the first estimate, \$51.45, seems to be nothing other than the discounted Textbook long-run incremental cost (TLIC) of the World Bank.[94] This is seen from the following :

$$\text{Undiscounted TLRISC(1980)} = (0.085/1.085)(26489.33/29099)$$
$$= \$71.31 \text{ per } 1000 \text{ cu.ft.}$$

And when discounted to 1980 it becomes :

$$\text{Discounted TLRISC(1980)} = [1/(1+0.085)^4]\ 71.31$$
$$= \$51.45 \text{ per } 1000 \text{ cu.ft}$$

Moreover as argued earlier the discounted TLRIC can be interpreted as the present worth of incremental system costs associated with a permanent increment or decrement in annual output.[95] That is \$51.54 can be interpreted as the present worth of incremental cost of an output increment (or decrement) of size 29099 cu.ft. lasting at least from 1980 to 1985. The latter must surely be different from Turvey's yearly present worth incremental system cost. The yearly PWISC of Turvey as correctly defined by Hanke is :

"to compute the marginal capital cost for any

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[94] See section 2.7.2 for a definition and discussion of the WB's LRISC.

[95] See discussion in section 2.4.2 .

year, y, we must compute the present worth in year y of planned system costs with the increment in annual output starting in year y. We then subtract from this value the present worth in year y of planned system costs without the increment in annual output starting in year y, but rather with it starting in year y+1. This difference is then divided by the annual increment in use to obtain the marginal capital cost per unit of use."[96]

Applying this correct definition of Turvey's yearly marginal cost we get[97]:

year	marginal capital cost
1980	0
1981	0
1982	0
1983	0
1984	71.31

The discussion of the definitions of marginal cost proposed by Hanke has highlighted (a) the seasonal distinction in marginal capacity cost, and (b) the substantial difference, in the presence of indivisibility, between 'pure' marginal capacity cost and average marginal capacity cost.

(2.9) Summary

A summary of the various propositions on the definition of marginal cost for pricing purposes is now in place.

In the framework of cost minimisation 'pure' marginal cost is given by the WB's textbook definition (section 2.7.1). When expressed in discounted terms it is similar to Turvey's yearly present worth incremental system cost (section 2.4.1). With indivisibility 'pure' marginal cost has a capacity component only in those years when capacity is nearing exhaustion. In consequence 'pure' marginal cost, in the presence of indivisibility, displays significant

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[96] Hanke,1980, op.cit.

[97] The postulated permanent output change must be known as early as 1980.

variation. Smoothing the variation in marginal cost is needed if uniform prices over time are required. The averaging method distinguishes some of the definitions we encountered in this chapter. A theoretically attractive averaging method is to use the discount factors as the appropriate weights (section 2.6). Simpler averaging methods (section 2.8) may in practice be used. Seasonal distinction in marginal cost is possible if seasonal peak pricing is sought.

The model of Turvey and Anderson (section 2.6) provided a theoretical link between cost-minimisation models and benefit-less-cost maximisation models. Given an absolute constraint on price fluctuation the benefit-less-cost maximisation model produces an optimal price equal to the weighted average of Textbook marginal cost, the weights being as before the discount factors.

Removing the constraint on price fluctuation as in the Rees model (section 2.5) broadens the definition of marginal cost to include marginal user opportunity cost. In this case optimal prices, output and capacity over time have to emerge from maximising an appropriately defined objective function. Fundamental to this formulation of the problem is the inclusion of a demand function, something which was absent in the cost-minimisation models.

CHAPTER 3

OVERVIEW OF URBAN WATER SUPPLY SYSTEMS

(3.1) Introduction

The present chapter may be divided into two parts. In the first we describe (a) the general technological and cost characteristics of an urban water supply system, (b) the nature of the demands placed on the system, and (c) how to go about the important business of predicting future levels of this demand. The overview will be conducted with a view to setting the ground for later analysis of (I) marginal costs of water supply (in chapter 4) and (II) the analysis of investment and pricing decisions (undertaken in chapters 5 and 6) .

The remainder of the chapter (section 3.5 to 3.8) presents a description of the water supply system and demand characteristics in the U.K. Southern Water Authority (SWA) in general and that of the Hampshire division (an area within SWA) in particular. The situation in Hampshire is described in some detail since it will serve as our case study in each of chapter 4, 5, and 6 .

(3.2) Urban Water Supply : An Overview of the System

An urban water supply system is a complex network of components which carry out the functions of collecting, transporting, treating and distributing water to the final domestic, industrial and commercial consumer. A general understanding of the system and its salient technological and economic characteristics is necessary if we are to proceed in the analysis of the economic decisions concerning, for example, investment planning, pricing, metering domestic consumers and formulating a leakage control policy. While it is true that water supply systems display great variation depending on the particular location under consideration, it is still possible to characterise some of the general and main features of water supply systems.

Water is abstracted from both surface and underground sources. Surface sources include(1) direct abstractions from river intakes operated under gravity or pumped flow; (2) river intakes coupled with upland or low-land storage reservoirs designed to exploit the maximum yield of the surface source (catchment area).[1] Storage is needed to provide secure supply especially when river flow is highly variable. Dams and reservoirs are sometimes built so as to generate hydroelectricity as well as to provide water supply and flood control. Hydroelectricity thus provides a rationale to build storage capacity higher than would otherwise be warranted for water supply alone.

Direct abstraction of water from river intakes without storage implies that the yield of the scheme is determined by the immediate flow of water in the river. Usually demand is at its peak during the summer at a time when river flow is at its lowest. Therefore river intake schemes must be assessed using peak summer demands and dry weather flow of the river.

This highlights the 'reliability' attribute of supply in relation to the assessment of the yield (capacity) of sources whether surface or underground. Water supply must be reliable in the sense that the risk of prolonged disruption due to shortage of supply is not excessive. This concept may be translated into drought criteria in common usage. Thus it is often stated that there must be enough source capacity so that in drought conditions the yield of a source(s) should not fall below the demand being placed upon it (them) with an occurrence of not more than, say 1 in 50 or 1 in 100. The lower the drought frequency criteria the more reliable is the supply. Higher reliability can be achieved by higher capital

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1 Much of this section is drawn from the Manual Of British Water Engineering Practice of the Institution of Water Engineers referred to as the "bible" of water practitioners in the UK. This is in addition to the Institution's new series of Water Practice Manuals.

The Institution of Water Engineers and Scientists,1979, Water Practice Manuals, Book I, " The structure and Management of the British water industry", Dangerfield Bernard J.(ed).

The institution of Water Engineers,1969, 4th edition, Vol.I, Vol.II and Vol.III, " Manual of British Water Engineering Practice".

expenditure on bigger and more expensive water sources and schemes or/and by suppressing peak demand levels. In principle the acceptable drought criteria is itself a variable subject to economic optimisation. In other words the drought criteria in principle can be chosen in such a way as to balance the extra costs of achieving a marginal improvement in reliability through a bigger construction programme against the benefits of this extra reliability. The latter is seen as the reduced likelihood of incurring economic losses due to potential water shortage. The losses arise from the need to adjust to reduce water supply during the drought.[2] It should be noted here that the drought criteria approach has been criticised on the grounds that in practice conservation measures (often in the form of restrictions) are usually used to prevent a water source running out. It is therefore argued that a better approach to the question of reliability can be formulated in terms of intensity, frequency and duration of supply restrictions.[3]

Wells and boreholes are water sources that tap underground natural storage in water-bearing strata. While any realistic description of the technical and economic aspects of tapping underground resources is certainly outside the scope of this section it is perhaps useful to make few comments. The capital cost of developing an underground source varies according to the location of the well or borehole, its capacity, depth and so on. The same can be said about the costs of pumping water out of the well. More than often the quality of the abstracted water from an underground

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[2] For an application of such an approach see : Russel Clifford S, Arey David G, and Kates Robert W, 1970, "Drought and Water Supply : Implications of the Massachustts Experience for Municipal Planning", Resources For The Future, The John Hopkins Press. The approach is based on optimising the timing and sizing of capacity addition which would minimize the discounted sum of construction costs and expected losses from water shortages. The feasibility of the approach of course depends on the availability of information regarding the expected costs of drought to domestic as well as industrial consumers.

[3] See Herrington, P.R.,1987,"Water forecasting in OECD Countries,".

source is such that only minor treatment, such as precautionary chlorination and possibly softening, is required. This is in contrast to river water whose quality is such that full treatment is usually required including sedimentation, filtration and chemical dosing. Therefore the higher pumping costs of underground sources is often more than out weighted by lower initial capital costs and lower treatment running costs compared with surface sources. On balance this tends to tip the cost advantage in favour of underground sources. This perhaps explains why underground sources are tapped first.

As expected the degree of treatment of raw water depends on its quality. The list of treatment processes, required for low quality river water would include screening and straining, sedimentation, coagulation and flocculation, filtration, aeration, sterilization as well as further processes for the removal of possible colour, taste and odor, and the treatment of corrosion problems.[4] Usually treatment works are located near river intakes. Treated water is transported in trunk mains to the service reservoirs usually located nearer to the area of demand. Treatment works are highly capital intensive, usually requiring low manning levels.[5]

Trunk mains are the means of bulk transmission of treated water from the treatment works to the main service reservoir in the supply area. Pipes are made out of various materials depending on their size and the pressure of the water flow. Water flow in a trunk main may be gravity fed or it may be boosted or pumped supply. Like other components of the water supply system trunk mains have a relatively long life. Various economic trade offs between size, location, type of material, thickness and so on arise in the construction and

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[4] For further details see Overman M.,1976, "Water: Solutions to a Problem of Supply and Demand", The Open University Press, London.
See also chapter 3 section 3.10 for further description of the treatment process as well as estimates of the cost function for the construction of treatment works.

[5] Indeed labour costs are treated as fixed costs and not variable costs proportional to the volume of output.

operation of trunk mains. To the water planner these must offer areas of potential cost savings. Routing of mains is another obvious source of potential savings.

The distribution network begins at the service reservoir from which a network of pipes aided, sometimes, by pumping stations take water to the premises of consumers. A service reservoir, called a water tower when built at an elevation, serves to store water so that supply can be maintained when it exceeds the flow out of the trunk main. In addition it provides storage for emergency supply in the case of an interruption in supply due to outage or maintenance of a trunk main or treatment plant.[6] The supply area is normally divided into distribution demand zones, each of which is normally commanded by a service reservoir capable of meeting daily demand fluctuation . Zoning helps the monitoring of pressure requirements as stipulated by statutory obligations. This is normally planned for using network analysis with the help of computer simulations. One data input necessary to conduct these simulations is demand forecasts at the level of the zone. Hence zoning also serve to influence the format of available disaggregated demand forecasts.

Service reservoirs are built to meet peak week demand, ie the average daily demand in the peak week. The distribution mains below the service reservoir, however, are built to meet the within-day peak flow (peak hour, peak quarter hour, etc).

In general design capacity for different components of the water supply system including trunk mains, treatment works, service reservoirs and distribution mains are sized according the following general equation:

Design Capacity = Average daily demand in a future year
X Peak Factor for that year
+ A Margin[7]

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[6] Service reservoirs can provide other functions including the reduction of peaking in mains both above and below the service reservoir which can achieve important cost savings.

[7] Average daily demand is calculated from annual consumption figures divided by 365. A peak factor, a peak
(Footnote continued)

Each component has an appropriate peak factor as in table 3.1 below.

Table (3.1)

Peak Factors for Design Capacity

(1) Water storage	Average demand,(no peak dependence)
(2) Trunk Mains	Peak week factor
(3) Treatment works	Peak week factor
(4) Service reservoirs	Peak week/peak day factor
(5) Distribution Pipes	Peak Hour/peak quarter hour

Source[8] : Cooper & Lybrand, Department of the Environment, 1985, Main Report, (mimeo).

Following Turvey(1969)[9] and others we will distinguish the distribution system (from the service reservoir downwards) from the "central system" consisting of sources, treatment works and major trunk mains. This distinction is necessary because planning and executing capital expansion in the two differ substantially. "The distribution network expands through many small investments, while the 'central system' proceeds more by small number of large additions to capacity, separated by lengthy intervals."[10] It follows that capital investment on the 'central system' is in the nature of 'strategic investment' whose timing and sizing is planned centrally for rather long periods of time and in direct relation to forecasted demand growth. Distribution investment while in principle also related to demand growth is planned for short periods of time perhaps with more decentralisation and with work being undertaken as the need arises more or

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continued) week factor for example, is demand during the daily average of the highest recorded weekly consumption divided by the average daily consumption as in above.

[8]See also Hanke,S.,1975, "Water Rates: An Assessment of Current Issues, Journal of the American Water Works Association, Vol.67, No.5.

[9] Turvey R.,(1969), op.cit.

[10] Turvey R.,(1976),PP158, op.cit.

less on continuous basis. These differences have an impact on the framework of investment planning and in turn on the estimation of the least cost capital expansion programme and consequently on the methods of estimating marginal cost.

One common and salient economic characteristic of water supply systems can be added. Virtually all cost equations for the various components of the water system including storage reservoirs, treatment works, trunk mains and distribution mains display some degree of economy of scale, ie the cost per unit of output decreases as capacity increase.[11] With any positive interest rate and positive rate of growth of demand there arise the potential of considerable savings through large scale construction of components whose size may be far in excess of current need and whose full utilisation occurs only after a considerable time lag. The extra capital costs of the seemingly indivisible large capacity additions are balanced by the reductions in the per unit capacity costs. This trade off between interest payments and economies of scale have led system engineers to suggest some rules of thumb regarding scale of 'water resource projects'; thus Hall and Dracup (1970)[12] suggest as "an approximate rule of thumb that the design capacity should equal to the requirements projected for 30 years after the first water delivery." Different components of the water system would have different 'scale multipliers' depending on the extent of economies of scale displayed in their cost functions. This can in principle be solved for different cost functions and interest rates using modelling techniques common in the literature.[13]

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[11] This observation is cited by numerous authors on the subject. See for example Hall and Dracup (1970): Hall Warren A., and Dracup John A.,(1970), 'Water Resources Systems Engineering',McGRAW-HILL Book Company.

See also TR61(1977) for estimates of cost functions for the various components of the water system for the UK : TR61,1977,"Technical Report TR61: Cost information for Water Supply and Sewage Disposal", Water Research Centre, UK.

See also Chapter 3 for more discussion of the point in relation to the cost of treatment works.

[12] Hall and Dracup,1970,PP19, op.cit.

The fact that economies of scale dictate sometimes the construction of long lived large capital intensive projects which once constructed have little or no alternative use emphasises the need to make accurate forecasts of demand requirements and the associated peak ratios. It also highlights the significance of the interest rate in as much as higher interest rates mitigate the benefits of economies of scale. Another consequence of economies of scale and large scale construction and peaking demand is the observation that many water utilities operate at levels of output well below their reliable capacity.[14] Moreover given that fixed costs are usually large relative to variable costs it follows that whenever the water enterprise is operating below its reliable capacity its short run marginal costs will be substantially below its total average cost.

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[13] See for example the following references:
(1) Manne,A.S.1967, (ed),"Investmets for Capacity Expansion Size, Location and Time Phasing", MIT Press, Cambridge, Mass.

(2) Lauria D.T.,Schlenger D.L.,and Wentworth R.W., 1977, "Models Capacity Planning of Water Systems",Jouranal of Environmental Engineering Division,Am. Soc. Civ. Eng., 103(EE2).

(3) Scarto R. F.,1969,"Time Capacity Expansion of Urban Water Systems",Water Resources Research 5(5).

(4) Chenery H.B,1952,"Overcapacity and the Acceleration Principle",Econometrica 20(1).

[14] See Hanke(1972):
Hanke S.,1972, "Pricing Urban Water", in Mushkin S.J.,ed.,"Public Prices for Public Products",Urban Institute, Washington D.C.

(3.3) Components of the Demand for Water

Public water supplies[15] in the UK comprise metered (measured) and unmetered (unmeasured) supplies.[16] Unmetered supply comprise accounted plus unaccounted-for supplies. Unmetered accounted-for supply consists of unmetered domestic consumption plus a small amount of unmetered commercial use. Unmetered unaccounted-for supply is mainly leakage and small amount of illegal and communal use. Metered supply mainly consist of industrial consumption, see diagram 3.1 .

Almost all of domestic household consumption in the UK is unmetered. Consumers are charged according to the rateable value of their property and not according to the volume of consumption. In recent years consumers have been given the option of paying according to rateable value or according to measure. Several studies of domestic consumption have sought to disaggregate household demand into basic components such as basic use, bathing, toilet flushing, garden watering and other external use and so on.[17] Typical proportions of the various components are given below.

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[15] Public water supply forms the major constituent of total water abstraction in the UK. Other abstractors include power stations, industry, fish farming and agriculture. See Water Industry Review, 1982, Supporting Analysis, National Water Council.

[16] Supply may normally be taken as fulfilled demand. The traditional literature use the two words interchangeably since demand is generally fulfilled (save during an unexpected drought or other 'crises' such as freeze-up, and cut offs due to labour disputes).

[17] See among others the following references :

(1) Archibald,G.G.,1983,"Forecasing Water Demand, A disaggregated Approach," Journal of Forecasing,(2).

(2) Jenking,R.C.,1973,"Fylde Metering, A Research Study," Fylde Water Board.

(3) Thackray,J.E., Cocker,V., and Archibald,G.G.,1978, "The Malvern and Mansfield Studies of Domestic Water Usage," Journal of Proceedings of the Institute of Civil Enginners,(64).

Table (3.2)

Pattern of Domestic Demand in England and Wales 1985

Component	Proportion %
W.C.	28
Bath	11
Shower	5
Washing Machine	15
Waste Disposal Unit	0.2
Dishwasher	1
Garden	3
Other External	0.6
Other and unaccounted	37

Source : Cooper & Lybrand, op.cit.

The average level of domestic consumption in the UK in recent years according to Park(1986)[18] stood at around 125 litres/head/day(l/h/d). The corresponding figure in Southern Water Authority in 1986 was estimated to be around 135 l/h/d.

Among the most important determinants of domestic demand are the demographic features of population growth and its distribution together with average household size. Economic and social factors also influence domestic consumption; rising living standards are usually translated into increased ownership of water using appliances such as washing machines, dishwashers and waste disposal units. Earlier in the postwar period, domestic consumption increased rapidly as baths and inside toilets were installed in older properties. Technical improvements to water taps, toilets and such like appliances economising in the use of water are also expected to influence domestic demand especially if they become cost effective to the consumer.

Unmetered unaccounted-for supply comprises in the main losses from the water system and a small proportion of unmetered supply for communal use such as fire fighting and street cleaning plus a small amount of meter underregistration. Losses or leakage can be found in

18 Park,C.,1986,"Water Forecasting and the Social Sciences,"in Gardiner,V., and Herrington,P.,1986,(ed), "Water Demand Forecasting : Proceedings of a Workshop Sponsored by the Economic and Social Research Council, Geo Books.

different parts of the supply system including service reservoirs, trunk mains and distribution pipes. Leakage is thought to be higher in older parts of the system.[19] Leakage levels are estimated in the U.K. by measuring night flow (during the early hours of the morning) and making some minor adjustments to account for legitimate night consumption.[20] Leakage can be expressed in terms of flow measured by litres/property/hour or in terms of litres/head/day. The aggregate amount of leakage in different parts of the UK varies reaching in some parts 50%-60%[21] of the total volume of water put into the supply network. Its specific amount depends on the state of the water system and how much is invested in leakage detection and control. The national average in England and Wales is thought to be 30%.[22] Leakage therefore forms a substantial source of 'demand' on the system. Indeed the growth of unmetered demand can be attributed in part to the growth of leakage from an ageing water distribution system. Reductions in leakage to reduce losses to an 'optimum' amount can in principle be made with reference to economic cum engineering analysis.[23]

Unmeasured supplies in England and Wales have increased steadily from 6719 to 12036 million litre/day from 1961 to 1985 registering a rise of 79% . The rise has been steady except for the drought year of 1976.(see table 3.3 and diagram 3.2).

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[19] See chapter 7 for a discussion of leakage control. Also see:
STC 26, 1980, "Leakage Control Policy and Practice," National Water Council, Department of the Environment, UK.

[20] If domestic consumption is measured then unaccounted-for water can be estimated as the difference between the water delivered to the distribution system and the sum of meter readings of consumers. This measure would exclude losses beyond the consumers' meters.

[21] See Parker,J.D.,and Penning-Rowsell,E.C.,1980,"Water Planning in Britain," George Allen and Unwin.

[22] Herrington,P.,1987,"Water Demand Forecasting in OECD Countries,".

[23] For a discussion of leakage control see chapter 7.

FIGURE 3.1

CATEGORIES OF DEMAND FOR WATER

Supply category \ Demand category	households	commercial premises	industrial premises	electricity generating industry	agricultural use
public water supply					
potable metered		●	●	●	●
potable unmetered	●	●			●
non-potable			●		
licenced abstraction in England and Wales			●	●	●

● main types of water supplies taken in a particular demand category

Table (3.3)

Public Water Supply England and Wales
1961 to 1985

	Resident Population 000s	Unmetered ML/d	Metered ML/d	Non-Potable ML/d	Total ML/d
1961	46196	6719	3801	213	10733
1962	46640	6920	3884	225	11029
1963	46901	7134	3955	285	11554
1964	47219	7401	4094	320	11815
1965	47540	7409	4190	349	11948
1966	47824	7701	4272	313	12286
1967	48113	7904	4351	329	12584
1968	48346	8084	4510	417	13011
1969	48540	8464	4721	497	13679
1970	48680	8684	4803	517	14004
1971	48854	8876	4749	532	14157
1972	49026	9131	4718	546	14395
1973	49154	9343	4884	579	14806
1974	49159	9555	4801	548	14904
1975	49157	9907	4668	534	15109
1976	49142	9519	4359	539	14417
1977	49120	9961	4211	552	14724
1978	49117	10407	4352	584	15343
1979	49171	10961	4490	643	16094
1980	49244	11083	4247	546	15876
1981	49634*	11224	4029	561	15814
1982	49607	11548	4106	561	16215
1983	49650	11842	3996	516	16354
1984	49760	11956	3990	558	16504
1985	49760	12036	3965	575	16576

* pre 1981 figures exclude residents temporarily overseas and include overseas visitors. This accounted for a 278000 rise in 1981.

Source : Water Facts, 1986, Water Authorities Association.

FIGURE 3.2

PUBLIC WATER SUPPLY IN UK

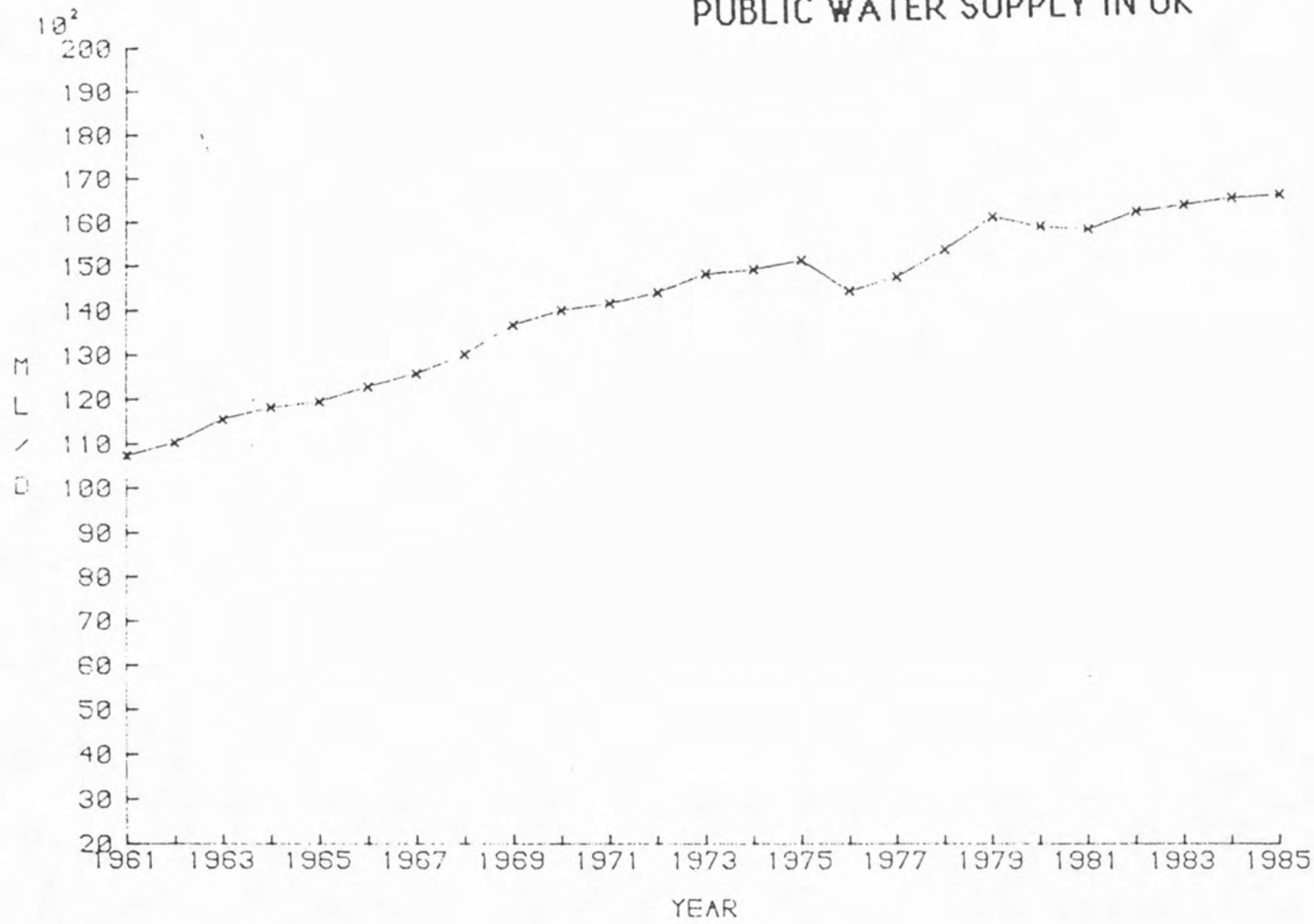
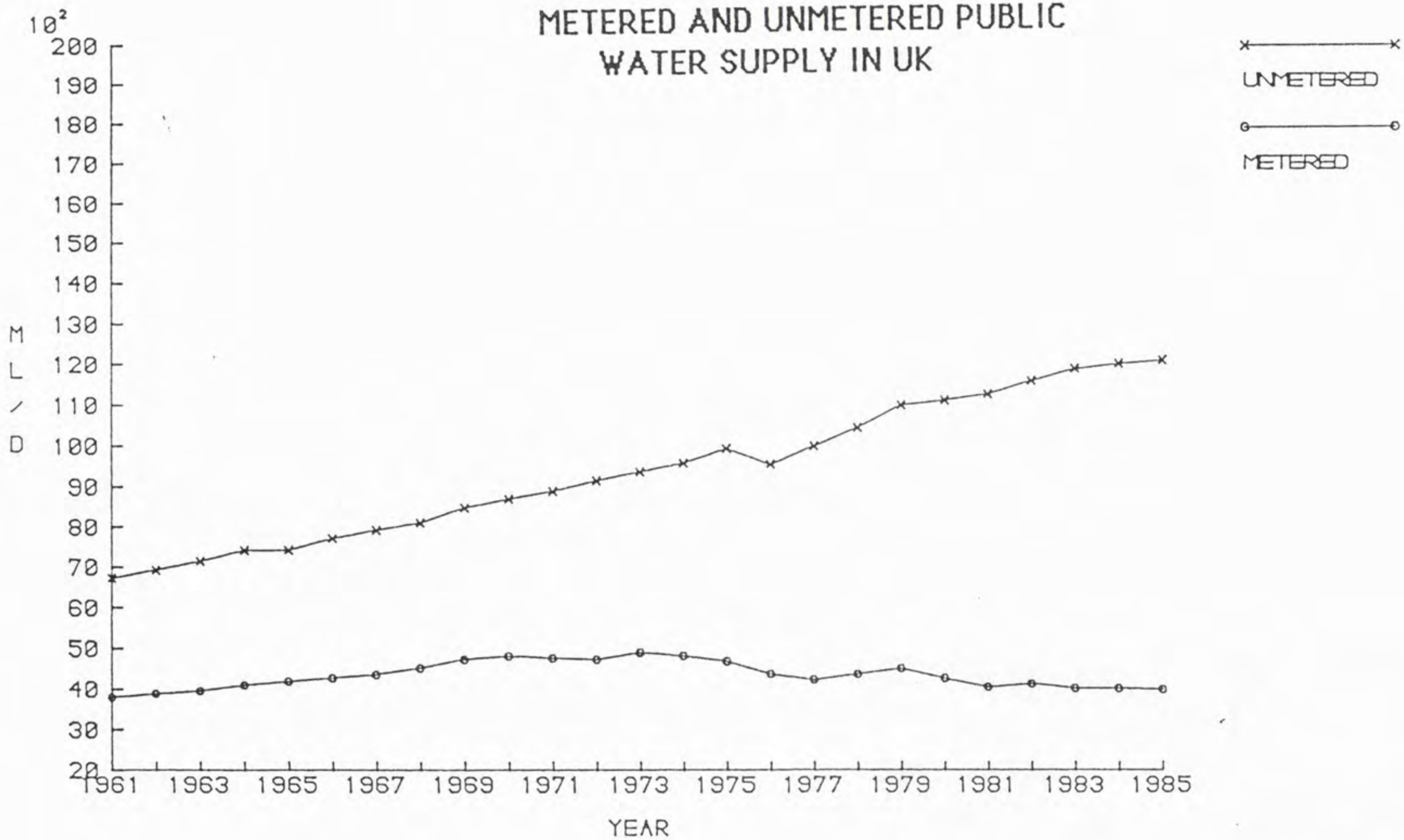


FIGURE 3.3

METERED AND UNMETERED PUBLIC
WATER SUPPLY IN UK



Metered supply goes mainly to industry and commerce, public administration and services (such as schools and hospitals) and some agriculture. Between 1970 and 1985 metered supply fell from 4803 ML/d to 3965 ML/d or by as much as 17%. In the last few years overall metered supply has been fairly steady though it is still falling in the recession hit northern regions, while it is rising in the more prosperous south. The overall decline during the 1970s may be attributed to decline in traditional water using heavy manufacturing and to a general tendency to use water more efficiently as more and more recycling is introduced by industry.[24]

Demand Peaks

Demand is far from uniform temporally. It is a variable which fluctuates from hour to hour, from day to day, week to week and so on. The extreme value it takes during any specified period of time is the peak of demand for that period. Peak characteristics are expressed in terms of peak ratios (factors). A peak factor for a particular period of time is defined as the maximum recorded demand during that period over average annual demand. Each class of demand exhibits its own peak characteristics as expressed by the peak factor.

It is generally accepted that domestic demand has a higher peak then industrial demand and leakage. Domestic demand moreover peaks during the summer months when the weather is dry and hot and when garden watering and external use in general are most required. In holiday resorts tourists contribute significantly to the peak.

Leakage on the other hand generally peaks during the winter months when low temperature takes its toll on mains and pipes. Indeed in some instances the overall system peak

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[24] Water Facts, op.cit.
It would be interesting to investigate whether the 1976 drought in any way induced firms to introduce more recycling technologies. If so then the drought may be said to have helped industry economise its water consumption.

may take place during the winter months if the leakage peak is more pronounced than the domestic summer peak.

Industrial demand may also display seasonal or daily variation, but there is no reason to expect this peak to coincide with that of the system peak as determined (usually) by the domestic peak.

The overall system peak for a particular demand zone can be estimated as the weighted average of the peak factors of the various classes of demand in that particular zone. The weights would be the share of each class of demand in total demand. Some further adjustment is usually made through the use of 'coincidence factors' to allow for the fact that the various peaks may not coincide in time.

Overall peak factors are in practice estimated from analysis of supply records and local experience. Forecasts of peak factors for domestic consumption are even more difficult to make and subject to more judgment because there are few records of domestic consumption since it is in general not metered. Estimates have been made using special surveys of consumers whose consumption was metered for that purpose.[25] Table 3.4 below gives an illustration of the range of values for overall peak week factors as reported by Bland (1986)[26] for various divisions (areas) in Anglian Water Authority.

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See among others:
(1) Bland,A.,1986,"Peak Demand Forecasting," in Gardiner and Herrington, op.cit.
(2) Males,D.B., and Turton,P.S.,1979,"Design Flow Criteria in Sewers and Water Mains," Technical Note NO. 32, Central Water Planning Unit, Reading, UK.

26 Bland,A.,1986,"Peak demand Forecasting," in Gardiner and Herrington,op.cit.

Table (3.4)

Peak Week Factors Anglian Water 1983	
Cambridge	1.25
Colchester	1.27
Lincoln	1.18
Oundle	1.12
Norwich	1.31
All divisions	1.21

Source : Bland (1986).[27]

Typical recent values for peak factors of total demand in the UK for various time periods have been reported by Herrington(1987). These have been reproduced here in table 3.5, below.

Table (3.5)

Total Demand Peak Factors UK	
Peak Week	1.1 to 1.7
Peak Day	1.1 to 2.1
Peak Hour	1.5 to 6.0

Source: Herrington,1987.[28]

[27] Bland,1986, op.cit.

[28] Herrington,1987, op.cit.

(3.4) Demand Forecasting

There is a huge volume of literature on the methodologies of forecasting demand for water.[29] Here we will confine the discussion to a few comments.

Perhaps the most widely used forecasting technique in the industry is based on extrapolation of historic data.[30] The extrapolation may be based wholly on simple judgment, or more commonly by fitting a trend using statistical techniques such as ordinary least squares or other methods of time series analysis. When regression is used the fitted trend may be a simple linear trend or a log-linear trend reflecting a compound rate of growth. Trends incorporating satiatation levels, S shaped curves, may be used. All these approaches have one common feature, namely, they assume a smooth underlying trend with no kinks or sharp bends. Also there is the implicit assumption that factors influencing demand in the past remain the same in the future. This may not be very satisfactory.

The trends of domestic consumption are usually forecast in per capita terms. Once the per capita future estimates are made they are translated into demand forecasts by multiplying them by the forecasted future population levels. Unmeasured demand (with or without unaccounted-for water) has also been estimated using extrapolation techniques of past data. For example Southern Water Authority not long ago used to forecast the whole of unmeasured demand in this way; estimates of unaccounted-for water and unmeasured commercial demand are later substracted with the result that domestic demand is derived as a residual.[31]

Domestic per capita demand has also been extrapolated separately using the 'components' approach, Archibald

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[29] For surveys of the literature see Herrington(1987) and Gardiner and Herrington (1986).

[30] The British Water Industry,1979, op.cit.

[31] Monopolies and Mergers Commission,1986, A Report on Water Supply and Distribution Services of the Authority(SWA) and Companies,HMS.

(1983).[32] This approach involves detailed projections of the individual components of domestic demand such as basic use, toilet flushing, bathing, washing machine and so on. For each component, usually involving a water using appliance, estimates are made of the expected future ownership of the appliance in question. This information together with estimates of future frequency of use and volume of water used give estimates of demand generated by that particular appliance. The sum of demand generated by each single component, usually expressed in per capita terms, give the forecast of overall domestic demand. This approach is said to be attractive on account of the ability to incorporate into the forecast specific expected future developments as regards for example the expected ownership and use of, say, automatic dishwashers. Forecasts of ownership, frequency of use and volume of water used are necessary for this approach. This may prove to be problematic. The 'component' approach can also be used to forecast other classes of demand besides domestic consumption.

Industrial demand (in practice all of metered demand) is often forecasted on the basis of past consumption and expectations regarding future industrial activity.[33] This in practice means extrapolating future levels of metered demand from historical data and supplementing the analysis by judgment and perhaps analysis of the components of industrial demand based on survey and billing data. On occasions multivariate regression analysis with explanatory variables such as manufacturing output and employment have also been used, Smith(1986).[34]

Archibald(1983)[35] commenting on forecasts by the old Water Resources Board notes that the existence of a sizeable percentage of leakage in unmeasured demand was not properly

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[32] Archibald,G.G.,1983,"Forecasting Water Demand, A Disggregated Approach," Journal of Forecasting (2).

[33] Manual of British Water Engineers Practice,1969,op.cit.

[34] Smith,R.,1986,"Forecasting Industrial Demand for Water," in Gardiner and Herrington,op.cit.

[35] Archibald,G.,1986, op.cit.

considered. The WRB extrapolated trends for the whole of unmeasured demand. This implied the same linear rate of growth for per capita leakage as for legitimate per capita domestic consumption. The implication of this is a forecast rise in leakage in proportion to the population increase. This is clearly unacceptable. What is required is incorporating into the forecast the actual leakage detection and control strategy as formulated by the relevant water body responsible for public water supply. This strategy should in principle be determined with economic considerations in mind. This means that passive attitudes to current and future levels of leakage should be rejected in favour of active leakage control policies. More generally passive demand forecasts ought to be rejected in favour of demand forecasts incorporating a degree of active control through leakage control as well as other demand management policies.

This brings us to the final comment of this section. Economists associate demand with prices. Demand is routinely taken as a function of price. It can be regulated up and down using price changes. Yet in the above discussion the role of price has been conspicuously ignored. Perhaps with few exceptions a price strategy is rarely incorporated into the demand forecasts. Behind this neglect lies a 'supply fix'[36] approach, one which sees demand as a 'requirement' which must be catered for, a 'requirement' that is beyond control or at best whose control is difficult and/or costly.[37] The

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[36] The 'supply fix' approach has been discussed by many including :

(1) Rees,J.,1976,"Rethinking Our Approach to Water Supply Provision," Geography 61(4).

(2) Herrington,P.R,"The Economics of Water Supply and Demand," Economics,12(2).

(3) Parker and Penning-Rowsell,1980,op.cit.

(4) Rees.J., "Waste Control in the Water Industry: An Economic Approach," Symposium on Waste Control: Its Importance in the Planning and Management of Water Supply Systems, Institution of Water Engineers, London.

[37] For further discussion of this and related matters see (Footnote continued)

rationale for this approach must ultimately fall back on the often cited argument that demand is fully price inelastic and/or there is an absolute constraint on price fluctuation.[38] 'Opponents' of this approach argue that this is inefficient leading to too big and too early investments in components of the water supply system. The demand for water, they argue, is not totally price inelastic nor is it unresponsive to other demand management tools such as education.

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continued)
chapters 5 and 6.
See also Hanke,S.,1977, op.cit.

[38] Warford,J.J.,1966,"Water Requirements : The Investment Decision in the Water Supply Industry," Manchester School (34).

(3.5) Southern Water Authority: A General Overview

Southern Water Authority (SWA) was formed in the 1974 reorganisation of the water industry in England and Wales. The 1973 Act had resulted in the amalgamation of some 200 small water undertakings into ten large public corporations known as the Regional Water Authorities.[39] These are the Anglian, Northumbrian, North West, Welsh, Severn Trent, Southern, South West, Wessex, Thames and Yorkshire Water Authorities.

The new authorities are different from their predecessors in that they are charged with managing the whole of the water cycle including both the clean water supply side and the sewerage and sewage disposal side. The whole array of functions now covered by the water authorities include:[40]

1. Water resources.
2. Water Supply.
3. Sewerage and sewage disposal.
4. River pollution control.
5. Fisheries.
6. Water-based recreation.
7. Land drainage.
8. Sea defences.

The boundaries of the water authorities are based on river basins and as such cross-boundary river transfers are at minimum (Thorpe 1986).[41] The authorities vary enormously

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[39] The water authorities were in fact created from 29 River Authorities, 157 Water Supply Undertakings and 1393 Sewage Treatment Departments. For detailed information on the 1974 reorganisation of the water industry see Parker and Penning-Rowsel,1980, op.cit.

[40] See Thorpe B.R,1986,"Current Value Rate Base, The Approach of England and Wales," in Annual Conference Proceedings (1986) of the American Water Works Association, Denver,1986.

as regards for example the size of population they serve, the quantity and quality of water resources, the length of coastline, the state of the water and sewerage systems and so on.

Southern Water Authority covers some 10500 KM^2 (This is including the area covered by the companies). It is the eighth largest in terms of area and the sixth largest in terms of population, serving some 3.9 million people (including the water companies). Figure 3.4 depicts the boundaries of the 10 regional water authorities. Table 3.6 below shows the area and population of the various water authorities.

Table 3.6

REGIONAL WATER AUTHORITIES RESIDENT POPULATION AND AREA* 1985/86

	Area KM^2	Resident Population (000)
Anglian	26795	5157
Northumbrian	9274	2619
North West	14415	6866
Severn Trent	21600	8315
Southern	10552	3944
South West	10884	1442
Thames	13100	11565
Welsh	21262	3047
Wesex	9918	2340
Yorkshire	13503	4381

* Figures cover the companies.
source: Water Facts,op.cit.

41 Thorp 1986, op.cit.

Figure 3.4

REGIONAL WATER AUTHORITIES IN ENGLAND AND WALES

Some 29 private water companies now coexist with the regional water authorities across England and Wales. In the area of operation of SWA there are now six such companies. In total these companies supply water to a greater geographical area than SWA and to nearly as many people as SWA. (see table 3.7)

Table 3.7

Statutory Water Companies in SWA Area

	Area KM^2	1985 Population (000)
1. Eastbourn Water Works Co.	826	204
2. Folkstone and District Co.	420	142
3. Mid Kent Co.	2056	505
4. Mid Sussex Co.	1041	243
5. Portsmouth Co.	868	633
6. West Kent Co.	240	136
Total	5451	1863

Source : Water Facts,1986, Water Authorities Association.

There exists a high level of cooperation between the companies and SWA regarding all aspects of the management of the water supply and other functions. The arrangement is such that SWA retains the ultimate responsibility for all aspects of supply in the whole region including those supplied by the companies. This responsibility is discharged by the companies, on behalf of SWA, in those areas served by them.

SWA is itself divided into four multifunctional divisions responsible for all the above listed functions in their respective areas (see figure 3.5). The four divisions are :

1. The Isle of Wight (IoW).
2. Hampshire (Hants).
3. Sussex.
4. Kent.

From the point of view of water supply these divisions are almost independent of each other in that each has its own water supply system. The divisions, in other words, are largely based on independent water catchment areas. This is extremely convenient for economic analysis of cost structures of water supply systems since each division can be looked at independently of the others.

The four divisions display considerable variation as regards water supply and demand characteristics. Sources of supply are different between and within the divisions. Different sources have different characteristics as regards, for example, yield (capacity), quality of abstracted water, cost of abstraction, treatment and pumping to supply. Demand characteristics vary as regards, for example, level, composition, peak properties and expected growth.

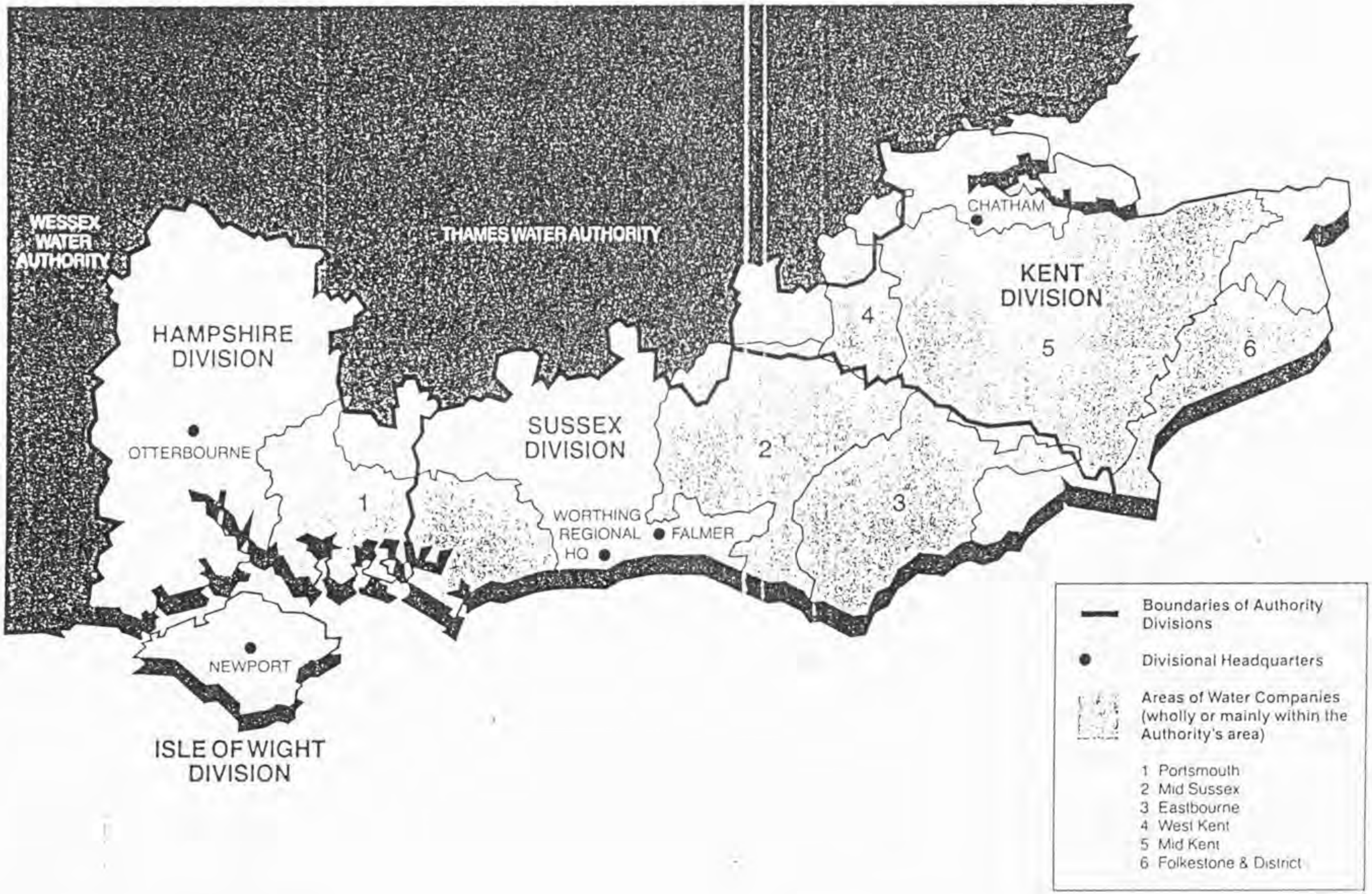
In general SWA draws the greater part of its water supply from local ground water sources. Indeed SWA has only a few major rivers from which water is extracted in any substantial quantity.[42] In 1984 for example ground sources provided some 69% of total water put into the supply system (see table 3.8). This contrasts sharply with the situation in the rest of England and Wales where surface sources account for the overwhelming proportion of supply.[43]

[42] Major river extractions take place in Hampshire division from the rivers Test and Itchen, and in Kent from the river Medway.

[43] For the whole of England and Wales ground sources accounted for 27.9% of total supply. See Performance Review 1984, SWA.

FIGURE 3.5

DIVISIONAL BOUNDARIES OF SWA



There is also some variation in the degree of dependence on ground sources of supply in between the four divisions as can be seen from table 3.8 below.

Table 3.8

Sources of Supply SWA
1984

	Ground water %	Surface water %
1. Iow	70.8	29.2
2. Hants	60.8	39.2
3. Sussex	68.4	31.6
4. Kent	76.6	23.4

Source : Standard Statistics,1984, SWA.

In general as the growth of demand gradually exhausted the available 'cheap' underground sources SWA has been forced to tap more and more the relatively more expensive surface water sources. In Hants for example the full exploitation of underground sources has led the division to resort to increased abstraction from the river Test at the site of Testwood.[44] As noted earlier such a development has cost implications. In Hants for example the cost of supply to Timsbury and Lyndhurst demand zone is £12.06 per ML when supply is drawn from the local underground well (cost of treatment and pumping). This compares with £19.8 per ML when supply to the zone is drawn from the river Test at the Testwood site. Demand at Timsbury and Lyndhurst now exceeds the capacity of the cheaper local supply and therefore needs to be supplemented by the more expensive supply from the marginal source of Testwood.[45]

Other examples can be cited. SWA and the Mid Kent Water Co. have been for some time promoting the construction of a storage reservoir at Broad Oak, near Cantebury. The need for this surface source, as SWA argued, is occasioned by the

[44] See Annual Reports 1981, and 1982, SWA.

[45] See below for a discussion of the supply system in Hants and its costs characteristics.

exhaustion of ground sources in the area which are being pressed by growing demand.[46] Currently the marginal source of supply for East Kent is the ground source of Wingham with production cost per ML equal to £8.9 (for both chemical and power).[47] If the Broad Oak source is developed sometime in the future it will substitute Wingham as the marginal source of supply in east Kent. Production (running) costs of supply when water is drawn from Broad Oak are estimated to be £34.5 to £43.9 per ML.[48] The difference is clearly substantial.

In terms of population Sussex is the largest of the divisions, Iow being the smallest.[49] In 1984 some 34% of total supply went to Sussex, 31% to Kent, 29% to Hants, and only 6% to Iow (see table 3.9 below).

Overall unmetered accounted-for supply[50] formed some 43% of total supplies. Unaccounted-for unmetered supply was about 30%, leaving metered supply with 27%. A more detailed picture of the break up of total demand between metered and unmetered demand for SWA including the divisions is given in tables 3.10 and 3.11 where demand is expressed in terms of l/h/d.

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[46] Annual Plan 1981,1984, SWA.

The development of the said reservoir was turned down by the Secretary of State for the Environment in 1980. This has forced SWA to adopt a policy based on the postponement of the Broad Oak project, proceeding for now with the options of leakage control, trunk main links, further ground water development and lowering the Medway river minimum residual flow requirements.

[47] Cooper and Lybrand,1985, op.cit.

[48] Cooper and Lybrand,1985, op.cit.

[49] Sussex incorporates West Sussex, Sussex Coast, and East Sussex. Kent incorporates Kent Medway, and Kent Thanet.

[50] The disaggregation of unmetered supply into accounted and unaccounted-for water is based on estimates and not actual measurements.

Table 3.9

COMPONENTS OF DEMAND 1984
SWA, DIVISIONS

	Iow	Hants	Sussex	Kent	All
Population(000)	120.9	556.8	771.3	586.8	2035.8
Water supply area KM^2	381	1826	1467	761	4435
Metered supply ML/d	8.4	58.7	49.2	66.1	182.4
Unmetered Accounted ML/d	18.9	78.5	110.7	84.0	292.1
Unmetered Unaccounted ML/d	14.1	62.6	70.0	59.8	206.5
Total supply ML/d	41.4	199.8	229.9	209.9	681.0

Source : (1) Division and Company Demand Forecasts Compatible with Annual Plan 1986 Regional Forecast, Part A, 1986, Directorate of Technical Services, SWA, mimeo.

(2) Standard Statistics 1984, Directorate of Technical Services, SWA.

Table 3.10

Components of Unmetered Demand
Accounted and unaccounted
SWA 1984

	SWA excluding companies l/h/d	SWA including companies l/h/d
Domestic	135	135
Holiday	3	3
Commercial unmetered	5	5
Total Accounted	143	143
Communal Use	3	3
Leakage etc	98	91
Total Unaccounted	101	94

Source : 1. Divisional and Company Demand Forecasts Compatible with Annual plan 1986 Regional Forecasts, Part A, SWA, mimeo.

2. Annual Plan 1986, SWA.

Table 3.11

Components of Unmetered Demand
SWA Divisions 1984
l/h/d

	Iow	Hants	Sussex West	Sussex Coast	Sussex East	Kent Med.	Kent Than.	All
Unmetered Accounted	156	141	142	143	149	142	146	143
Unmetered Unaccounted	116	112	89	104	28	112	79	101

Source : Division and Company Demand Forecasts Compatible with Annual Plan 1986 Regional Forecast, Part A, Mimeo.

It can be seen from the tables above that estimates of accounted-for water (overwhelmingly domestic consumption) for the various divisions are evenly spread around the authority average of 143 l/h/h. The variance around this mean seems to be small. The exception is Iow, mainly due to it having a more than average share of holiday makers whose consumption would normally be grouped with that of the resident population.

Unaccounted-for water, overwhelmingly leakage, showed a great variation between the divisions in 1984, stretching from as little as 28 l/h/d in Sussex East to 116 l/h/d in Iow. It is perhaps important to stress again that these figures are estimates which make them sensitive to the estimating procedure, errors and a great deal of judgment. Nevertheless variations are also a reflection of geological differences between areas, differences relating to the state of the distribution system and the amount of resources expended on leakage detection and control in the various divisions.[51]

Water supply in the region of operation of SWA, including the companies, has grown steadily over the years, except for the drought year of 1976 (see table 3.12 and figure 3.6). Between 1975 and 1985 water supply in the region, including the companies, had gone up by 15%. The growth reflects growth in population as well as per capita consumption.

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51 See The Monopolies and Merger Commission, Southern Water Authority,(The Companies), A Report on Water Supply and Distribution Services of the Authority and Companies, 1986, HMSO.

Table 3.12

ACTUAL WATER SUPPLY IN SWA REGION
1975-1985

Year	Supply ML/d
1975	1088
1976	1034
1977	1050
1978	1114
1979	1153
1980	1153
1981	1162
1982	1223
1983	1234
1984	1248
1985	1244 (provisional)

Source : Monopolies and Mergers Commission (MMC), 1986, op.cit.

Table 3.13

Components of Water Supply
SWA Excluding Co.'s
1981-1985

	Unmetered ML/d	Metered ML/d	Total ML/d	Population (000)
1981	461.83	176.49	638.32	2027.4
1983	492.60	175.20	667.80	2032.5
1984	498.60	182.40	681.00	2035.8
1985*	500.00	183.00	683.00	2051.5

* This is an estimate drawn from 1986 Annual Plan.

Sources : 1. Appendices to 1981-1982 Annual Report and Accounts, SWA.

2. Annual Plan 1985, and Annual Plan 1986, SWA.

(3.6) Regional Demand Forecasts of SWA

For the purposes of planning, SWA forecasts demand at the regional level, ie at an aggregate level combining the demands of the divisions and the water companies. The forecasts are taken to year 2011. Forecasts are made for overall demand as well as of its constituent parts, ie demand of the various classes such as metered and unmetered demand including accounted and unaccounted for demand. The forecasts are reviewed yearly and presented in aggregate form (for the whole region) in the yearly annual plan.

The first forecast was made in 1977. Since then two major revisions have been made, in 1984 and 1986. The early forecasts were entirely based on 'naive' extrapolation methods which are thought to be, at least in part, behind the overestimates of demand in those early days.[52]

The regional forecasts of SWA are based on projected regional population, projected unmetered demand components (in l/h/d), and trend extrapolation of the metered component (in ML/d and not l/h/d).[53]

Population projections of the Structure Plans of the Local Authorities are used. The latest of these forecasts used by SWA stretched to year 2001. These were extended by SWA to year 2011 using simple extrapolation. Table 3.14 gives population forecasts for the whole region as contained by

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[52] See:
1. Parker and Penning-Rowsel, 1980, op.cit.
2. MMC, 1986, op.cit.
3. Herrington P.R, 1979, Broad Oak (Canterbury) Reservoir Proposal, Proof of Evidence, London : Council for the Protection of Rural England.

[53] See :
1. Annual Plan 1984, SWA.
2. Annual Plan 1986, SWA.
3. MMC, 1986, op.cit.
4. Hampshire, Isle of Wight, and Portsmouth RACS study, 1985, op.cit.

SWA's 1984 and 1986 demand forecasts.

Table 3.14

Population Forecasts*
SWA Region

	(1984)^	1991	2001	2011
AP 1984	3896.7	3988.3	4130.6	4211.6
AP 1986	3896.7	3994.5	4143.0	4230.0

* including water companies.
^ actual.

Source : Annual Plans 1984 and 1986, SWA.

The 1986 plan incorporate a rate of growth of population for the period 1984 to 2011 equal to 8.5%. This compares with a national average rate of growth of 4.5%. The difference undoubtedly can be attributed to expected net immigration into growth areas in the region such as Hampshire. Hampshire is expected to have the highest rate of growth, Kent the lowest. Table 3.15 indicate the range of expected population growth in the various regions.

TABLE 3.15

POPULATION FORECASTS
SWA DIVISIONS
AP 1986

	1984 (000)	2011 (000)	Change %
1. Iow	120.9	137.0	13.3
2. Hants	556.8	658.0	18.1
3. Sussex	771.3	826.0	7.1
4. Kent	586.8	623.0	6.1

Source : Division and Company Demand Forecasts Compatible with Annual Plan 1986, op.cit.

The projection of metered demand (in terms of ML/d) is based on trend extrapolation. A linear rate of increase is fitted to overall historical growth. And in order to allow for uncertainty a plus and minus 5% range is allowed for to form an upper and lower forecast. Table 3.16 shows both the upper and lower metered demand forecasts for SWA (divisions only) as obtained from 1986 Annual Plan of SWA.

SWA seems to recognize the inherent uncertainty in predicting the general economic environment, the industrial structure and activity in the region, factors which ultimately determine the level of future metered (industrial) demand. It may also be inferred that SWA is perhaps aware of the limitations of the simple extrapolation technique used to forecast metered demand. According to the report of the Monopolies and Mergers Commission (1986)[54] SWA has plans to carry out more detailed analysis of metered supply based on a postal survey of metered consumers.

Forecasts of unmetered demand are fraught with even more difficulty. Essentially they are also based on trend forecasts; trend estimates of per capita (l/h/d) demand together with population forecasts. Unmetered demand is broken by SWA into accounted-for and unaccounted-for demand; each is forecast separately. Baseline figures for regional per capita estimates of each of accounted and unaccounted-for water are obviously needed. The per capita domestic component of unmetered accounted-for consumption in the baseline year has been estimated in two different ways. The less than satisfactory approach, contained in the 1984 Annual Plan, based the domestic baseline component "on the experience obtained by metering experiments of individual households by other Water Authorities and enhanced by 10% to allow for under-registration of small domestic meters."[55] This domestic component was put at 129 l/h/d for the base year of 1984 as

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[54] MMC,1986, op.cit.

[55] Division and Company Demand Forecasts Compatible with Annual Plan 1986 Regional Forecast, Part A, Directorate of Technical Services, SWA, mimeo.

estimated in the Annual Plan 1984. Total accounted-for demand was put at 137 l/h/d for the same year; the 8 l/h/d difference was taken up by tourist demand and a component allowing for commercial unmetered use.

It has to be noted however that the MMC (1986) report on SWA suggests that the authority's forecasting method for unmetered accounted-for demand is rather different from the method outlined above. According to MMC the experience of other water authorities was used by SWA only in as much as a cross check on its own estimates. The procedure followed by SWA according to MMC was as follows : estimates of UFW (unaccounted-for water) and of the small amount of umneasured commercial demand are substracted from unmeasured requirements, forecasted as a whole, with the result that figures for unmeasured domestic consumption in total and of domestic consumption per head are derived as a residual.[56]

In any case the 1986 Plan containing a revision of the demand forecasts of 1984 adopted a new approach as regards estimates of domestic per capita consumption. The baseline (1984) estimate of accounted-for consumption has been revised up to 143 l/h/d and that of domestic consumption revised up to 135 l/h/d. The new estimates were based on some early results from the Authority's newly formed control areas in Kent, Sussex, and Isle of Wight.[57] These areas involving the metering of domestic consumers were set up for the purpose of making some reliable estimates of domestic consumption. Table 3.17 show the 1984 per capita levels of the various components of unmetered demand as contained in the 1986 demand forecast together with the values of these components as projected by SWA up to 2011.

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[56]Monopolies and Mergers Commission, 1986, A Report on Water Supply and Distribution Services of he Authority (SWA) and Companies, HMS.

[57] For more information on the control areas see Annual Plan 1985, SWA.

TABLE 3.16

S.W.A. - ANNUAL PLAN 1986

FORECAST OF DIVISIONAL POPULATION AND WATER SUPPLY

	1984	1985				1986				1991				2001				2011			
POPULATION ('000)	2 044.8	2051.5				2062.5				2109.0				2198.0				2 244.0			
Ml/d		UPPER		LOWER		UPPER		LOWER		UPPER		LOWER		UPPER		LOWER		UPPER		LOWER	
MEASURED DEMAND	182.4	186		183		189		184		206		187		228		206		248		224	
UNMEASURED DEMAND	292.4	298		295		303		297		331		310		378		356		415		393	
		A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
UNACCOUNTED FOR	206.5	205	193	205	193	204	177	204	177	196	105	196	105	204	110	204	110	209	112	209	11[illegible]
TOTAL POTABLE DEMAND	681	689	677	683	671	696	669	685	658	733	642	693	602	810	716	766	672	872	775	826	7[illegible]
NON-POTABLE DEMAND	3	3		3		3		3		16		16		16		16		16		16	
COMPONENTS l/h/d																					
Domestic	135	137		136		139		136		149		139		163		153		176		166	
Holiday	3	3		3		3		3		3		3		4		4		5		5	
Commercial unmetered	5	5		5		5		5		5		5		5		5		4		4	
TOTAL ACCOUNTED	143	145		144		147		144		157		147		172		162		185		175	
Communal use	3	3		3		3		3		3		3		3		3		3		3	
Leakage, etc.	98	97		91		96		83		90		47		90		47		90		47	
TOTAL UNACCOUNTED	101	100		94		99		86		93		50		93		50		93		50	

A - ASSUMES MAINTAINING 1981 LEVELS OF LEAKAGE

B - ASSUMES LEAKAGE TARGET ACHIEVED BY 1991 AND MAINTAINED

FIGURE 3.6

PUBLIC WATER SUPPLY IN SWA (INCLUDING COMPANIES), ACTUAL 1961-1985 FORECAST 1986-2011

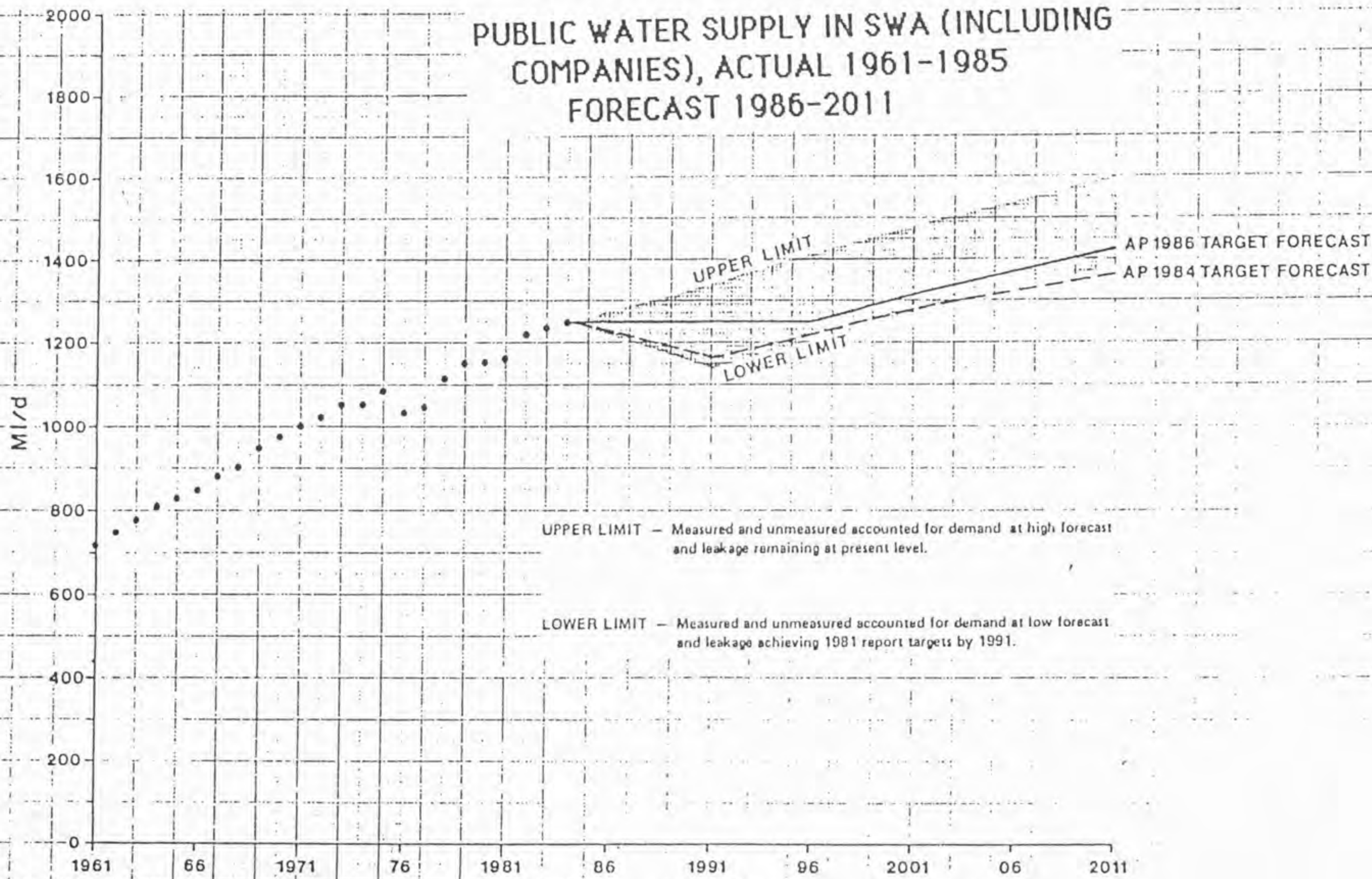

FIGURE 3.7

PUBLIC WATER SUPPLY IN SWA (EXLUDING COMPANIES), ACTUAL 1961-1985 FORECAST 1986-2011

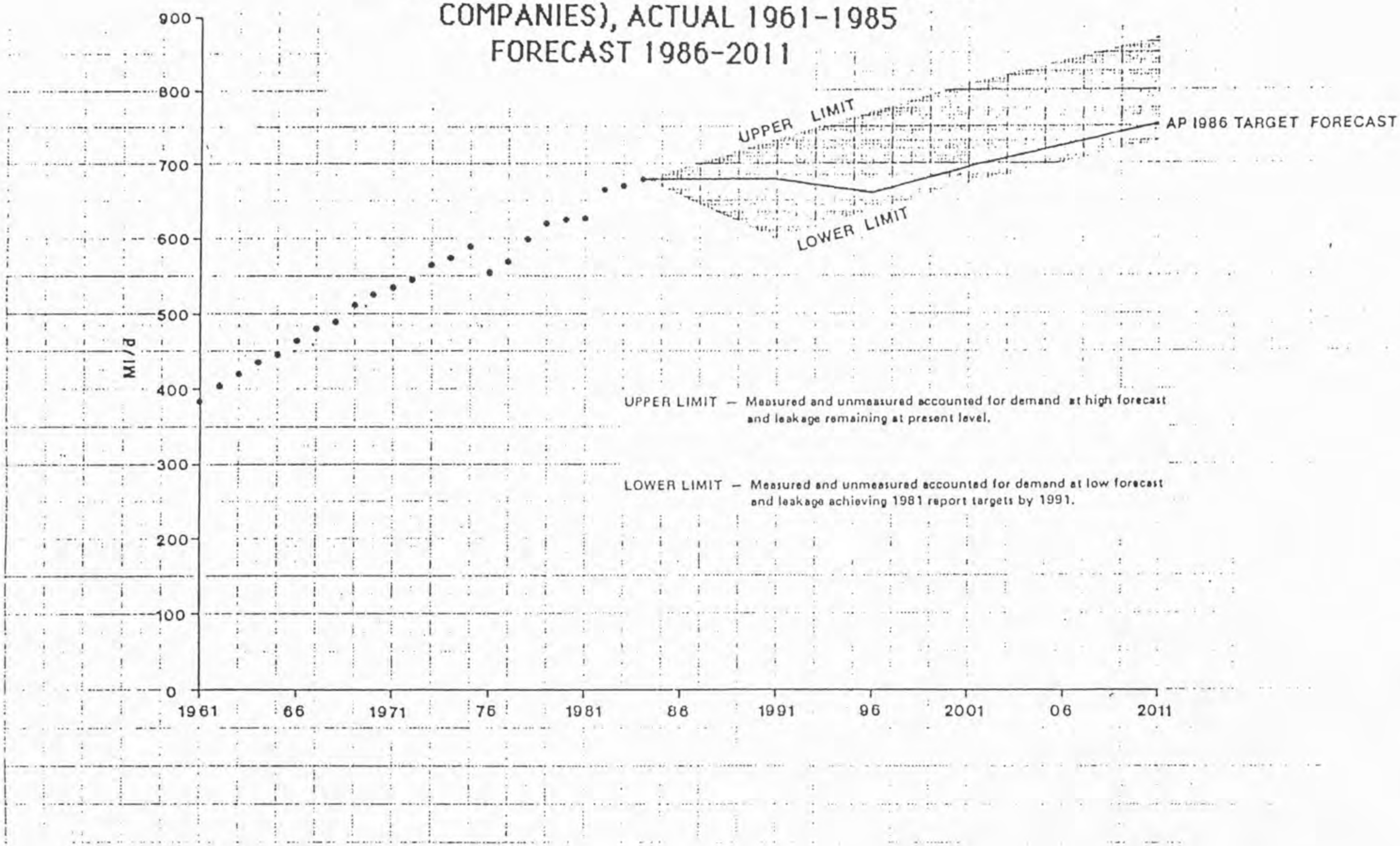

Table 3.17

SWA ANNUAL PLAN 1986
COMPONENT BASED PROJECTIONS OF UNMETERED PER CAPITA DEMAND*
litres/head/day

	1984	1991		2001		2011	
		Upper	Lower	Upper	Lower	Upper	Lower
Domestic	135	149	139	163	153	176	166
Holiday	3	3	3	4	4	5	5
Commercial unmetered	5	5	5	5	5	4	4
Total-Accounted	**143**	**157**	**147**	**172**	**162**	**185**	**175**
Communal	3	3	3	3	3	3	3
Leakage,etc	91	87	54	87	54	87	54
Total-Unaccounted	**94**	**90**	**57**	**90**	**57**	**90**	**57**
Overall	237	247	204	262	219	275	232

* For region as a whole.

Source : Division and Company demand Forecasts Compatible with ANNUAL PLAN 1986 REGIONAL FORECAST, PART A, SWA, MIMEO.

A range of upper and lower forecasts has been derived by SWA by adding and substracting 5 l/h/d from projected percapita domestic demand. It is however unclear from SWA documentation what was the basis of deriving the projections from the estimated baseline figures. It is likely that a considerable amount of judgment is used. Per capita domestic consumption for the region as whole is forecast to grow from its 1984 level at 135 l/h/d to 171 l/h/d in 2011.[58]

Unaccounted-for demand is the second constituent component of unmetered demand. This is a significant constituent whose level will crucially effect overall future levels of demand. Unaccounted-for demand is estimated by SWA to be equal to 94 l/h/d in 1984. The authority had formulated a leakage detection and control policy in 1981. This was reviewed in later years. The 1986 forecast, containing the latest review of leakage policy, makes two assumptions about future levels of leakage (and consequently two assumptions about unaccounted-for demand). The upper forecast incorporate the assumption that leakage levels, at the regional level, will by 1991 be reduced to their 1981 level (90 l/h/d including communal use) and thereafter maintained at this level by an active leakage control policy, one requiring no doubt some capital and other expenditure in the years to come. For the lower forecast it is assumed that leakage levels, regionally, will by 1991 be reduced to a target level of 54 l/h/d excluding communal use (57 l/h/d including communal use), and thereafter be maintained at this level. These assumptions are shown in table 3.17.[59]

The overall demand forecast for the whole of SWA (excluding the companies) is given in table 3.16 and diagram 3.7. On the upper forecast demand (excluding the companies) is expected to grow from its 1984 level equal to 681 ML/d to

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[58] 171 is the mid point of 176 and 166 l/h/d reported in table 3.17 in the text.

[59] The corresponding figures used in 1984 Annual Plan are 90 l/h/d for maintaining 1981 leakage level, and a target of 50 l/h/d for the 'ambitious' target. This together with the lower base (1984) domestic consumption make the 1984 forecast lower in general than the 1986 forecast. The 1986 forecast moreover has an upper and lower range, apart form leakage, which was absent in the 1984 forecast.

a level equal to 872 Ml/d in 2011 registering thus growth for the said period equal to 28%. On the lower forecast demand is expected to grow by 7% from 681 ML/d in 1984 to 729 ML/d in 2011. Other combinations are presented in the table. The range of demand forecasts are shown diagrammatically in figure 3.7.

Disaggregated forecasts at the level of the divisions and companies are made at the same time when the overall regional forecast is made. A high level of consultation is maintained between the divisions, companies and SWA so that the various disaggregated forecasts are chosen in such a way to be consistent with the regional aggregate forecast. The consistency objective means ensuring that the sum of divisional and company forecasts add up to the regional forecast.

Each division in turn decomposes its overall demand forecast into a second tier of disggregated figures representing demand of the zones of the division, ensuring in the process that the forecasts are consistent again.[60]

(3.7) The Division of Hampshire: Overview of Water Supply System

Hampshire (Hants) in terms of area is the largest of the divisions of SWA (see table 3.6). Almost one in four of the people supplied by SWA live in Hants. In 1984 the division accounted for 29% of total water supply in the authority. The division moreover has one of the fastest growing populations in the authority (table 3.15).

Hants has three networks of water supply; the northern network, the central network and the southern network. This is in addition to the network belonging to the Portsmouth water company south east of the division.

The northern network is almost independent from the rest of the system. It mainly covers rural areas with population centers concentrated in the towns of Andover, Whitchurch, Overton and further north Kingsclere. The network has five demand zones; Broughton, Ibthorpe, Andover, Whitchurch and Overton, and finally Kingsclere.

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[60] See discussion below regarding demand forecasts for the zones that make up Hampshire division.

Local groundwater sources provide all the required supply to the network. These sources, listed in table 3.18, are sufficient to meet average and peak week demand in the network both for now and for the foreseeable future. The implication of this is that SWA does not expect to require any investment in capital works upstream of the service reservoir in order to meet future demand. Moreover there seems no possibility, according to SWA,[61] of the area 'exporting' any excess supplies to the rest of the division.

Table 3.18

**Sources of Supply Hants Division
Northern Network**

Source	ADO ML/d	PDO ML/d
1. Kingsclere	5.7	5.7
2. Eastwoodhay	5.0	5.0
3. Overton	1.6	1.6
4. Whitchurch	1.6	1.6
5. Ibthorpe	2.7	3.4
6. Faberstown	0.4	0.5
7. Chilbolton	0.5	0.5
8. Andover	19.9	19.9
9. Broughton	4.4	4.4
10. Horsebridge	5.0	5.0
11. West Tytherly	0.5	0.5

Notes:
(1) ADO is annual drought output defined as the output in a drought year at the time of minimum groundwater levels. PDO is peak drought output defined as the output in a drought year during the time of peak demand.
(2) All listed sources are groundwater. The quality of the abstracted water moreover is such that it requires no major treatment. Water from these sources is only subjected to chlorination before being sent to supply.

Source : Hampshire, Isle of Wight and Portsmouth Water Company RACS study, 1985, Directorate of Operations,SWA.

The southern network of Hants has the main centers of

61 Hampshire, Isle of Wight and Portsmouth Water Company RACS study, 1985, Directorate of Operations,SWA.

population in the division as well as the main growth areas. Besides the city of Southampton it includes areas such as Twyford and Rosmey to the north and the Waterside Parishes south including Totton and further south Marchwood, Dibdon, Buttsash and so on. The southern network accounts for about 75% of total supply in the division and as such is the most important area from resource development point of view. The network has seven demand
zones :

1. Rounhams .
2. Southampton Common .
3. Otterbourn direct .
4. Twyford .
5. Moorhill (sometimes combined with
   Twyford as Twyford and Moorhill demand zone)
6. Timsbury and Lyndhurst .
7. Yew Hill .

Unlike the northern network this area draws its water supply from both underground and surface (river) water sources. Groundwater is abstracted from the sources of otterbourn, Timsbury, Twyford and Twyford Moors. Ground water is again of high quality requiring only minor disinfection. Running costs are therefore dominated by power costs of abstraction and pumping to supply in the case of underground sources. Table 3.19 lists all sources of supply in the southern network of Hants together with their yields.

Table 3.19

Sources of supply Hants Division
Southern Network

Source	ADO ML/d	PDO ML/d
1. Timsbury (ground)	6.8	6.8
2. Otterbourn (ground)*	68.2	68.2
3. Twyford (ground)	18.2	19.1
4. Twyford Moors (ground)*	11.3	11.3
5. Testwood (river Test)	129.0	129.0
6. Otterbourn * (river Itchen)	53.0	53.0

* There is a maximum limit of 24 MG/d (109 ML/d) on the aggregate of abstraction from Otterbourn river and Otterbourn ground; Also a limit of 25 MG/d (113.7 ML/d) on the said Otterbourn output plus that of Twyford Moors.
Source : RACS,1985, op.cit.

It is considered that there is little further potential of developing more conventional underground sources of supply in southern Hants. Demand growth therefore will have to be met from marginal river sources in the area.

Southern Hants already draws a significant proportion of its supplies form two river sources; the river Itchen and river Test. Abstraction from the Test takes place at the site of Testwood just to the north of Totton. The 2% drought yield of the river Test at the Testwood site is 129 ML/d. The abstraction licence from the Test is 30 MG/d or 136.2 ML/d.

According to SWA documentation the yield of the river Test in the long run can be raised , if so required, by a groundwater augmentation scheme in the upper reaches of the river.[62] The scheme according to SWA could be designed to give a net increase in yield of an estimated 50 ML/d. The cost of construction of the scheme, at 1981 prices, is put by SWA at £3.167 million, a figure which includes an allowance for perpetual replacement.[63]

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62 In reality this is only one alternative, though it is the preferred one. See RACS,1985, op.cit.

63 The reported data does not specify the life of the
(Footnote continued)

Water from the Test needs to be fully treated before it can be pumped to supply. Both potable and non-potable supplies are demanded from the Test. Potable supplies need considerably more treatment then non-potable supplies. All treatment is carried out at the divisional treatment works at Testwood near the river intakes. The treatment processes for potable supply include the processes of coarse and fine screening, coagulation and sedimentation, rapid gravity filtration, and chlorination. The potable treatment capacity is 15 MG/d or 68.2 ML/d, which is considerably lower than the yield of the river or abstraction license. The non-potable capacity is 6 MG/d or 27.3 ML/d.

SWA has considered the cost of expansion of treatment capacity at Testwood. An eventual total treatment capacity of 40 MG/d (181 ML/d) is a possibility which has been investigated by SWA. This expansion would be staged; the first stage involves the construction of extra capacity of 9 MG/d taking up total capacity at Testwood from the existing total of 21 MG/d to 30 MG/d (136.2 ML/d). The second stage would involve raising total capacity up to 40 MG/d or 181 ML/d. The first stage would provide capacity to an amount equal to current abstraction licence. The second expansion would provide capacity up to the full abstraction potential from the river when the augmentation scheme is in operation. The first stage expansion can take several forms.[64] Among these there is the option of having only potable water treatment capacity, ie the existing 6 MG/d of non-potable treatment capacity is transformed into potable capacity and another 9 MG/d is added to give a total of 30 MG/d. The cost of this expansion, as estimated by SWA, is £2.33 million expressed using 1981 prices.[65] The cost of the second stage expansion, up to 40 MG/d, is estimated by SWA to be equal to

continued)
various components of the capital works. In other places civil works are assumed to have a life of 60 years, machinery and equipment is assumed to have a life of 20 years. The cost of replacement is discounted using a rate of discount equal to 5%. For a discussion of issues concerning the rate of discount see section (14) of chapter 5.

64 See Testwood, Stage 3 Feasibility Study,1985, Hampshire Division, SWA.

£4.04 million, again at 1981 price level and allowing for perpetual replacement.[66]

At present potable water from Testwood mainly supplies the demand zone of Rounhams. Treated water is pumped to Rounhams service reservoir which commands part of the supply in the zone. The rest of the zone, the Waterside Parishes, receives its supply direct from Testwood pumping station. Testwood supply meets both average and peak week demand.

During peak time Testwood water is used to supplement supplies to Southampton Common zone. A cross connection between the Rounhams zone and the Southampton Common zone exists via a 15 in valve at Gover road, Redbridge with a capacity of 12 ML/d. Any future supplies in excess of the current capacity of the connection would require the construction of a new direct link, trunk main, between Testwood and Southampton Common zone. The cost of such a trunk main is estimated by SWA to be equal to £ 0.72 million, at 1981 price level and allowing for perpetual replacement.

Testwood water is also used to supplement supplies to the Timbsbury and Lyndhurst demand zone. This zone normally taps first the cheaper local underground source of Timsbury. But when water levels at Timsbury wells is low and/or the pressure of water supply to the Lyndhurst part of the zone is low, then water is drawn from Testwood to supplement local supply. Treated water is transferred from Rounhams zone to Timsbury and Lyndhurst zone via two connections with the Rounhams zone.[67]

Partially treated non-potable water is pumped from Testwood to two destinations. The first destination is the Esso refinery at Fawley whose maximum daily requirement, as

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65 This figure is based on estimates cited by the RACS study. A slightly different figure is reported by Testwood feasibility study. It is assumed that civil works has a life of 60 years, machinery and equipment has a life of 20 years.

66 The cost of the first stage expansion is considerably lower because unlike the second stage it can be carried out by adapting the existing plant.

67 For further details on this point see Testwood Feasibility Study.

contracted with SWA, is 5 MG/d or 22.7 ML/d.[68] Testwood non-potable supply is also 'exported' to the Isle of Wight (Iow) in order to supplement local supply. Twin 200 mm diameter undersea pipelines across the Solent are used to transport the partially treated water. Full treatment of supplies is conducted at the treatment works of Iow at Broadfields. The pipelines capacity is 15 ML/d. Water has to be pumped several times before it reaches its ultimate destination.

Demand for potable supplies from Testwood also emanate from other demand zones not directly connected to Testwood. For example by increasing supplies from Testwood to Southampton demand zone less would need to be drawn from Otterbourn sources by Southampton (see below) thus freeing some extra resources in Otterbourn to meet increased demand in zones such as Twyford, Yew Hill, Moorhill, and Winchester. Increased demand in these latter zones is thus essentially satisfied, by substitution, from Testwood.

The second source of surface water in southern Hants is the river Itchen. Raw water is abstracted from the river Itchen at two locations. The first is at Otterbourn where SWA has a licence to abstract 45 ML/d.

Otterbourn river water undergoes full treatment at divisional works situated at Otterbourn. The capacity of the treatment works is slightly greater than the abstraction licence (the treatment capacity is 54 ML/d and a intake capacity of 49.5 ML/d). Treated river water is mixed with underground water, also from Otterbourn, before it is pumped to supply.

The combined aggregate abstraction from Otterbourn surface and underground sources has a maximum limit of 24 MG/d (109.1 ML/d). There is also an upper ceiling of 25 MG/d (113.7 ML/d) on aggregate Otterbourn abstraction plus abstraction from the underground source of Twyford Moors.

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68 Esso also receive a supply from the West Hampshire Water Company which despite its name actually falls within the boundaries of the Wessex Water Authority and not in that of SWA. It is to be noted that the average daily supply to Esso is in fact less than the contracted maximum demand by Esso.

Recently river flow in the Itchen has been supplemented by groundwater augmentation schemes in the river's upper reaches; the Candover and FIRAS schemes. These projects have increased available supplies from the lower Itchen.

Raw water is also abstracted from the Itchen at Gaters Mill (lower Itchen) by Portsmouth Water Company. The company has a licence to abstract 45 ML/d from this source. Water is treated by the company at its works at Gaters Mill (capacity 68.2 ML/d) before it is transported in bulk eastwards via a large trunk main to service reservoirs closer to the company's demand centers.

According to SWA documentation the upper Itchen resources (Otterbourn) are now fully developed and as such offer no potential for further expansion. Expansion is possible however at the lower Itchen where it is thought that some 90 ML/d is available for further abstraction.

The additional source at the lower Itchen could be used by both SWA and Portsmouth Water Company. Arrangements for abstraction and treatment from the lower Itchen, if required, can take many different forms. One possibility is to construct a new divisional abstraction and treatment works at Gaters Mill. The capital cost of this proposal is estimated by SWA, using 1981 price level and allowing for perpetual replacement, is £4.51.[69] This option presents the division with a second potential marginal source of water supply, that is in addition to or as an alternative to expansion of abstraction and treatment from the river Test at the Testwood site.

The bulk of Otterbourn underground and river water goes to the demand zones of Otterbourn Direct and Southampton Common (Otterbourn Indirect). The combined supply to Otterbourn Direct and Southampton Common zone has an upper limit of 75 ML/d.

Otterbourn river and groundwater is also pumped to Yew Hill which could in future include supplies to Winchester demand zone in the central part of Hampshire.

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69 Other possibilities include expansion of the existing Portsmouth treatment works which can then be shared with SWA.

Otterbourn groundwater and river water is also pumped to Twyford and Moor Hill demand zone(s). These receive their supply in the first place from the ground sources of Twyford and Twyford Moors. The combined capacity of these two sources is 30.4 ML/d during the time of peak demand (see table 3.19). This is already short of current demand levels. Supplies are therefore supplemented by Otterbourn combined groundwater and river supply. These supplies are subject to an upper limit of 13 ML/d. Twyford and Moor Hill zone(s) cover one of the fastest growing areas in the division. Peak demand in 1986 already exceeded the combined capacity of Twyford, Twyford Moors and the 13 ML/d from Otterbourn supply.[70] Current and projected future deficit mean that Twyford and Moor Hill need to be provided with an extra source of supply. Given that there is no potential for further groundwater development in the area, extra supply to Twyford and Moor Hill can only be secured by recourse to one of two options. The first option would involve relaxing the constraint on supplies from Otterbourn to Twyford and Twyford Moor from its current level of 13 ML/d to at least 37 ML/d.[71] This would enable bridging the existing and potential deficit of supplies with extra supply from Otterbourn. Under this option one is ultimately falling back on the Test river for additional supplies. As more supplies are drawn form the sources of Otterbourn, the full capacity of these resources is reached sooner with the consequence that more supplies are drawn at an earlier date from the Testwood source to meet demand at Southampton Common. Relaxing the 13 ML/d constraint therefore means that ultimately extra supplies to the Moor Hill and Twyford area will have to come, by substitution, from the river Test. Such a solution would in turn entail an earlier date for the development of Test resource projects, that is an earlier date than otherwise for the development of extra treatment capacity at Testwood, an earlier date for the

[70] From available SWA documentation we were unable to understand the exact nature of this constraint.

[71] The cost of doing so is not known.

groundwater augmentation scheme and indeed an earlier date for the new link between Testwood and Southampton Common demand zone.

Under option two, extra supplies to Twyford and Moor Hill can come from the lower Itchen. Under this option the 13 ML/d constraint will be maintained and extra supplies to the area can come from new divisional abstraction and treatment works at Gaters Mill.

Each of these options presents Hants division with a different package of central resource projects. Each option involves different total capital cost. Each option also presents different magnitudes and profile of running costs (see below for a discussion of central investment planning).

The central Hampshire supply network comprise the demand zones of Abbotstone and Tottford, and Winchester. Water supplies to these demand zones come at present in full from the underground sources of Barton Stacey, Tottford and Easton (table 3.20).

Table 3.20

Sources of Supply Hampshire Central Network

Source	ADO ML/d	PDO ML/d
1. Tottford(ground)	4.5	4.5
2. Barton Stacey (ground)	1.1	1.1
3. Easton	17.4	27.3

Source : RACS study,op.cit.

The combined capacity of these three underground sources is now (1986) sufficient to meet demand (average and peak demand). However, given projected demand growth, and given the lack of potential of local underground sources, a deficit is expected to develop sometime in the future in Winchester demand zone.

To bridge this deficit supplies would need to be drawn from Otterbourn. To this end a trunk main would need to be built linking Winchester to Yew Hill (in southern Hants) in order to link central and southern Hants, thereby ending the present isolation of Winchester and Abbotstone and Tottford demand zones. The estimated cost of constructing this trunk main, at 1981 prices and also allowing or perpetual replacement, is £0.514. This is according to SWA documentation.

(3.8) The division of Hampshire: Zonal demand Forecasts

For the purpose of investment planning in the division of Hampshire we require the overall demand forecast for the division disggregated into forecasts at the zone level. In principle there should be no difficulty in deriving the demand forecasts for each of the relevant demand zones ourselves. In practice such an exercise has not been possible given our limited knowledge of the necessary detailed information on for example zone boundaries, zone population levels and projected growth, leakage levels present and future and so on. Nor is there any compelling reason why we should carry out these forecasts ourselves given such forecasts are normally carried out by SWA by forecasters who have intimate knowledge of local conditions.[72]

Forecasts of demand, both average and peak week, for the Hampshire division were made available to us by SWA. These were however forecasts compatible with 1984 Annual Plan and not the more recent one (and therefore not compatible with 1986 Annual Plan). The 1984 forecast for the whole of SWA is generally slightly lower than the more recent one (lower by about 1%), though both show similar trends.[73]

For each demand zone we have two forecasts; an upper forecast, A, based on the assumption of reducing leakage levels by 1991 across the division to a level (l/h/d) equal to that estimated for 1981, and thereafter maintained at that

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[72]Possible limitations in the forecasting techniques used by SWA as well as some ambiguities have been discussed above in section 3.6 .

[73]Data collection took place prior to the appearance of 1986 Plan. Moreover the latest strategic investment planning study conducted by SWA available to us uses the 1984 Annual Plan forecast. It was therefore convenient for us to use the 1984 forecast as well.

level ; the lower forecast, B, assumes achieving a more ambitious target level of leakage (l/h/d) across the division by 1991 and also to be maintained at this level thereafter.

Table (3.21) give the actual levels of average aggregate demand for the whole division for the period 1976-1984 as well as the upper and lower forecast up to year 2011 according to Annual Plan 1984. Diagram 3.8 displays the same information.

Table 3.21

Potable Water Supply
SWA Hampshire Division
Actual 1976-1984
Forecast 1986- 2011
(Annual Plan 1984)

Year	Average demand ML/d
1976	158.26
1977	162.49
1978	172.17
1979	176.86
1980	183.64
1981	183.30
1982	197.85
1983	205.08

Year	Forecast A ML/d	Forecast B ML/d
1986	206.4	206.4
1991	218.8	186.1
2001	244.8	209.2
2011	263.2	224.1

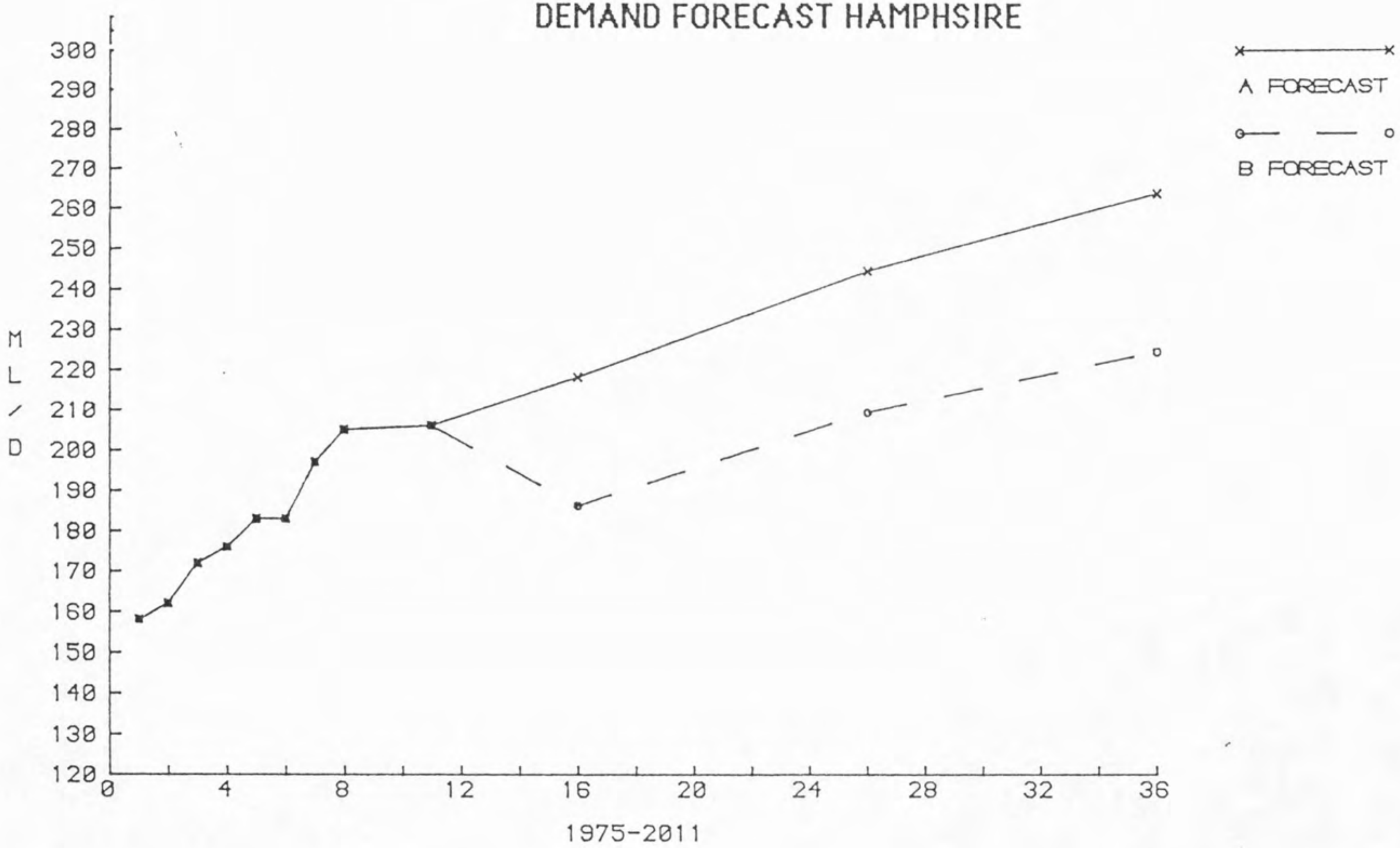
Notes :

1. Forecast according to 1984 Annual Plan.
2. The upper forecast, A, maintains leakage from 1991 at 1981 level. Lower forecast, B, achieves a lower leakage target by 1991.
3. Excluding exports to Iow+Esso

Sources : SWA documentation, mimeo.

FIGURE 3.8

DEMAND FORECAST HAMPHSIRE



The upper forecast A, for each of the demand zones in southern and central Hampshire, is given in table 3.22. In addition to the said demand zones the table also includes a row for the combined demand of Esso refinery and 'exports' to the Isle of Wight, both of which now draw partially treated water from Testwood. Esso demand has been fixed at a constant level of 5 MG/d or 22.7 ML/d throughout the period 1986 to 2011, both for average and peak week demand. This is equal to the maximum Esso can demand from SWA at any single day according to arrangements between them and SWA. Imports by Iow from Hants make up the rest of the of column.[74] The Esso demand and Iow requirement appear in overall demand for southern and central Hants because they will have an impact on the timing and perhaps the sizing of resource projects there.

We have not, however, reported demand forecasts for the demand zones in the northern network since, as discussed earlier, expected demand in these zones is thought to have no impact on the development of capital projects upstream of service reservoirs in the autonomous northern network.[75]

Table 3.23 give the corresponding peak week demand for each of the demand zones. The table follows SWA's assumption that peak week factors for each of the demand zones remain constant throughout.

Table 3.24 contains demand forecast A, the upper forecast, for each of the demand zones, this time however the forecast is disaggregated into components of average metered demand, average unmetered accounted-for demand and average unmetered unaccounted-for demand.[76] Table 3.24 also breaks down peak week demand forecast A, into its various components

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[74]Import requirements of Iow have been drawn from RACS,1985. RACS,1985, op.cit.

[75]The implication of this is that an output change in northern Hants, whether temporary or permanent, has no impact on expected future investments in the 'central' part of the water supply system.

[76]It is to be noted that because of rounding errors these components may not add up exactly to those reported in table 3.22.

in the same fashion of average demand. The disaggregation of the peak demand was however conducted by us using the assumption that each of metered demand and unmetered unaccounted demand have a peak week factor equal to 1.[77] This assumption means that each of metered demand and unmetered unaccounted-for demand have a level of demand during the time of system peak no greater than their average levels respectively. This in turn implies that unmetered accounted-for consumption during the peak week can be found as a residual.

Table 3.25 contains average and peak week demand for each of the zones according to the lower forecast, B, ie according to the forecast of SWA which assumes the achievement of the more ambitious leakage control targets.[78]

We have also ourselves derived three further demand forecasts which will be used for the purpose of sensitivity analysis regarding both central investment planning and estimates of long run average incremental cost of central capacity. Table 3.26 presents the first of these, Forecast C . This forecast assumes that following the introduction of domestic metering, in 1986, average domestic demand as proxied by average unmetered accounted-for demand, is to fall

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[77] See Cooper & Lybrand, 1986, op.cit.
This assumption allows for the fact that what matters is peak behavior of leakage and metered demand during those summer weeks when the system has its overall peak as determined by domestic demand. In other words to assume a peak week factor for leakage equal to 1 (for the purpose of our peak demand forecasts) is not the same thing as saying that leakage does not have a peak, occurring as it does in the winter months.

[78] Forecast B has not been disaggregated into its constituent parts as in table 3.25 since this is not required by the analysis to follow. It is to be noted that for each of the zones the overall peak week factor for each of forecast A and forecast B are assumed, by SWA, to be the same. This seems rather odd since if unmetered accounted-for demand and metered demand remain the same, including their respective peak factors, it must follow that a reduced level of unmetered unaccounted-for demand, following a more active leakage control policy, must lead to a change (rise) in overall zonal peak factor if the latter is derived as a weighted average of the peak factors of the various classes of demand. This point however will not be pursued any further.

by 12.5%. The corresponding fall in peak week domestic demand is assumed to be higher at 15.5%.[79] This follows Coopers and Lybrand (1985) who based their estimates of the percentage fall in domestic demand in the event of metering on some British and international evidence.[80]The reduction in domestic peak week demand by 15.5% following metering is based on the assumption of a reduction in external domestic use of 30% compared with 12% reduction of in-house consumption.[81] Because of the uncertainty regarding this estimate it is perhaps prudent to take a range estimate of the likely fall in domestic demand in the event of consumers being metered. Demand forecasts D and E in tables 3.27 and 3.28 give a 20% range in both directions around forecast C. In other words forecast D assumes a fall in average domestic demand of 10% and in peak domestic demand of 12.4%. Forecast E on the other hand assumes a fall in average domestic consumption of 15% and in peak domestic consumption of 18.6%.[82]

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[79] It is assumed here that both metered and unmetered unaccounted-for demand (leakage), both average and peak week, remain as in forecast A. In one respect this may be unrealistic in that leakage levels may be related to the level of legitimate consumption. Indeed in a later chapter we will assume that a 10% reduction in legitimate consumption leads to a 10% reduction in leakage.

[80] Coopers and Lybrands, 1985, op,cit.
We also note that Iow demand figures have been excluded from any adjustment. In effect it is being assumed that Iow is not covered by metering. This is done so as to concentrate the analysis on the southern and central parts of Hants.

[81] See Herrington, P.,R.,1985, "The Role of the Water Industry in the Economy as Seen by Economists, or Marginal Costs Rule, O.K ? ," Proceedings of the 1985 Symposium on the Impact of Financial Constraints on the Level of Service in the Water Industry, The Institution of Water Engineers and Scientists, London.

[82] Peak demand is assumed to be more sensitive to the introduction of metering because it contains more external use which is taken to be more sensitive to metering than internal use. It is also worth noting that overall peak factors change after the introduction of metering. They actually fall.

Table 3.22

AVERAGE DEMAND FORECAST A (UPPER)
SOUTHERN AND CENTRAL HANTS
ZONE LEVEL
1986-2011
ML/d

		1986	1991	1996	2001	2006	2011
1.	Abbotstone	5.04	5.11	5.17	5.24	5.30	5.35
2.	Winchester	16.50	17.10	17.70	18.30	18.80	19.30
3.	Yewhill	13.10	13.60	14.00	14.40	14.80	15.20
4.	Twyford	6.50	7.07	7.76	8.46	8.90	9.30
5.	Moorhill	31.00	33.93	37.24	40.64	42.70	44.70
6.	Otterbourn	21.50	22.30	23.00	23.80	24.40	25.10
7.	Southampton	40.50	42.00	43.60	45.20	46.60	47.90
8.	Rounhams	37.10	40.70	44.40	48.10	50.50	53.00
9.	Timsbury & Lyndurst	9.60	10.20	10.60	11.10	11.50	11.90
10.	Iow & Esso	35.80	38.10	39.30	40.50	41.70	42.70

Source : SWA documentation, mimeo. Also RACS, 1985, op.cit.

Table 3.23

PEAK WEEK DEMAND FORECAST A (UPPER)
SOUTHERN AND CENTRAL HANTS
ZONE LEVEL
1986-2011
ML/d

		1986	1991	1996	2001	2006	2011
1.	Abbotstone	6.40	6.40	6.50	6.60	6.70	6.70
2.	Winchester	20.60	21.30	22.10	22.90	23.50	24.10
3.	Yewhill	16.40	17.00	17.50	18.00	18.50	19.00
4.	Twyford	8.05	8.79	9.75	10.60	11.03	11.63
5.	Moorhill	38.75	42.41	46.56	50.08	53.37	55.86
6.	Otterbourn	26.90	27.80	28.80	29.70	30.50	31.30
7.	Southampton	50.60	52.50	54.50	56.50	58.20	59.90
8.	Rounhams	49.30	54.20	59.00	63.90	67.20	70.50
9.	Timsbury & Lyndhurst	12.50	13.32	13.80	14.50	15.00	15.50
10.	Iow & Esso	41.50	43.00	44.40	46.50	49.20	52.00

Source : SWA documentation, mimeo. Also RACS,1985, op.cit.

Table 3.24

COMPONENTS OF DEMAND FORECAST A (UPPER)
SOUTHERN AND CENTRAL HANTS
ZONE LEVEL
1986-2011
ML/d

Abbotstone (average)

	1986	1991	2001	2011
Unmetered Accounted	2.16	2.19	2.25	2.30
Unmetered Unaccounted	1.17	1.19	1.22	1.24
Metered	1.71	1.73	1.77	1.81
Total	5.04	5.11	5.25	5.35

Abbotstone (peak)

	1986	1991	2001	2011
Unmetered Accounted	3.52	3.52	3.60	3.65
Unmetered Unaccounted	1.17	1.19	1.22	1.24
Metered	1.71	1.73	1.78	1.81
Total	6.40	6.44	6.60	6.70

Winchester (average)

	1986	1991	2001	2011
Unmetered Accounted	7.09	7.35	7.87	8.30
Unmetered Unaccounted	3.80	3.94	4.21	4.47
Metered	5.61	5.81	6.22	6.56
Total	16.50	17.10	18.30	19.30

Winchester (peak)

	1986	1991	2001	2011
Unmetered Accounted	11.21	11.61	12.43	13.11
Unmetered Unaccounted	3.80	3.94	4.21	4.47
Metered	5.61	5.81	6.22	6.56
Total	20.62	21.36	22.86	24.14

Table 3.24 (cont.)

Yewhill (average)

	1986	1991	2001	2011
Unmetered Accounted	7.00	7.23	7.91	8.45
Unmetered Unaccounted	3.83	3.95	4.00	4.02
Metered	2.30	2.40	2.50	2.70
Total	13.13	13.58	14.41	15.17

Yewhill (peak)

	1986	1991	2001	2011
Unmetered Accounted	10.29	10.64	11.54	12.27
Unmetered Unaccounted	3.83	3.95	4.00	4.02
Metered	2.30	2.40	2.50	2.70
total	16.42	17.00	18.04	19.00

Twyford & Moorhill (average)
(combined)

	1986	1991	2001	2011
Unmetered Accounted	19.11	21.22	26.82	30.33
Unmetered Unaccounted	10.46	11.59	13.56	14.44
Metered	7.88	8.16	8.70	9.24
Total	37.45	40.97	49.08	54.01

Twyford & Moorhill (peak)
(combined)

	1986	1991	2001	2011
Unmetered Accounted	28.47	31.47	39.10	43.84
Unmetered Unaccounted	10.46	11.59	13.56	14.44
Metered	7.88	8.16	8.70	9.24
Total	46.81	51.22	61.36	67.52

Otterbourn (average)

	1986	1991	2001	2011
Unmetered Accounted	9.59	9.95	10.91	11.69
Unmetered Unaccounted	5.25	5.45	5.52	5.57
Metered	6.67	6.90	7.35	7.81
Total	21.51	22.30	23.78	25.07

Table 3.24 (cont.)

Otterbourn (peak)

	1986	1991	2001	2011
Unmetered Accounted	14.99	15.45	16.83	17.92
Unmetered Unaccounted	5.25	4.45	5.52	5.57
Metered	6.67	6.90	7.35	7.81
Total	26.91	27.80	29.70	31.30

Southampton (average)

	1986	1991	2001	2011
Unmetered Accounted	13.56	14.02	15.39	16.48
Unmetered Unaccounted	7.42	7.66	7.78	7.85
Metered	19.53	20.32	22.05	23.61
Total	40.51	42.00	45.22	47.94

Southampton (peak)

	1986	1991	2001	2011
Unmetered Accounted	23.70	24.52	26.67	28.44
Unmetered Unaccounted	7.42	7.66	7.78	7.85
Metered	19.53	20.32	22.32	23.61
Total	50.65	52.50	56.50	59.90

Table 3.24 (cont.)

Rounhams (average)

	1986	1991	2001	2011
Unmetered Accounted	17.53	19.66	24.48	27.78
Unmetered Unaccounted	9.59	10.74	12.38	13.22
Metered	9.94	10.33	11.19	11.97
Total	37.06	40.73	48.05	52.97

Rounhams (peak)

	1986	1991	2001	2011
Unmetered Accounted	29.79	33.20	40.35	45.35
Unmetered Unaccounted	9.59	10.74	12.38	13.22
Metered	9.94	10.33	11.19	11.97
Total	49.32	54.27	63.92	70.54

Table 3.24 (cont.)

Timsbury & Lyndhurst (average)

	1986	1991	2001	2011
Unmetered Accounted	4.08	4.37	5.06	5.57
Unmetered Unaccounted	2.23	2.38	2.56	2.65
Metered	3.30	3.45	3.50	3.70
Total	9.61	10.20	11.12	11.92

Timsbury & Lyndhurst (peak)

	1986	1991	2001	2011
Unmetered Accounted	6.97	7.37	8.44	9.15
Unmetered Unaccounted	2.23	2.38	2.56	2.65
Metered	3.30	3.45	3.50	3.70
Total	12.50	13.20	14.50	15.50

Source : Average demand figures were obtained from SWA, mimeo.
Peak demand was derived by us as explained in text.

Table 3.25

AVERAGE DEMAND FORECAST B (LOWER)
SOUTHERN AND CENTRAL HANTS
ZONE LEVEL
1986-2011
ML/d

		1986	1991	1996	2001	2006	2011
1.	Abbotstone	5.00	4.40	4.50	4.50	4.60	4.60
2.	Winchester	16.50	14.10	14.70	15.20	15.70	16.20
4.	Yewhill	13.10	11.40	11.80	12.20	12.50	12.90
5.	Twyford	6.50	5.87	6.54	7.21	7.64	8.30
6.	Moorhill	31.00	28.23	31.46	34.69	36.76	38.50
7.	Otterbourn	21.50	19.20	20.00	20.70	21.30	21.90
8.	Southampton	40.50	37.70	39.30	40.90	42.20	43.50
9.	Rounhams	37.10	34.70	37.90	41.10	43.30	45.50
10.	Iow & Esso	35.80	35.70	36.90	37.90	39.00	40.00

PEAK DEMAND

		1986	1991	1996	2001	2006	2011
1.	Abbotstone	6.40	5.60	5.60	5.70	5.80	5.80
2.	Winchester	20.60	17.60	18.30	19.00	19.60	20.20
3.	Yewhill	16.40	14.20	14.70	15.20	15.70	16.10
4.	Twyford	8.05	7.37	8.17	9.04	9.55	10.38
5.	Moorhill	38.75	35.33	39.33	43.36	45.95	48.12
6.	Otterbourn	26.90	24.00	24.90	25.90	26.60	27.40
7.	Southampton	50.60	47.10	49.10	51.10	52.70	54.40
8.	Rounhams	49.30	46.10	50.40	54.60	57.60	60.60
9.	Timsbury	12.50	11.50	12.00	12.60	13.10	13.60
10.	Iow & Esso	41.50	40.00	41.40	42.80	44.60	47.20

Source : SWA documentation,(mimeo) and RACS (1985),op.cit.

Table 3.26

POST METERING DEMAND FORECAST C
SOUTHERN AND CENTRAL HANTS
AVERAGE DEMAND ML/D
1986-2011

	1986	1991	2001	2011
1. Abbotstone	4.77	4.83	4.96	5.06
2. Winchester	15.61	16.18	17.31	18.29
3. Yewhill	12.25	12.67	13.42	14.11
4. Twyford	6.03	6.58	7.86	8.63
5. Moorhill	29.03	31.73	37.72	41.58
6. Otterbourn	20.31	21.00	22.41	23.60
7. Southampton	38.81	40.24	43.29	45.68
8. Rounhams	34.86	38.27	47.24	49.49
9. Timsbury	9.10	9.60	10.48	11.22
10. Iow & Esso	35.80	38.10	40.50	42.70

PEAK DEMAND

	1986	1991	2001	2011
1. Abbotstone	5.81	5.89	6.05	6.17
2. Winchester	18.88	19.56	20.93	22.10
3. Yewhill	14.82	15.34	16.25	17.08
4. Twyford	7.29	7.97	9.50	10.44
5. Moorhill	35.09	38.36	45.78	50.27
6. Otterbourn	24.59	25.40	27.09	28.52
7. Southampton	47.00	48.69	52.36	55.49
8. Rounhams	44.70	49.12	57.66	63.51
9. Timsbury	11.41	12.05	13.19	14.08
10. Iow & Esso	41.50	43.00	46.50	52.00

Source : Derived from table 3.24 .

Table 3.27

POST METERING DEMAND FORECAST D
SOUTHERN AND CENTRAL HANTS
AVERAGE DEMAND ML/D
1986-2011

	1986	1991	2001	2011
1. Abbotstone	4.82	4.89	5.02	5.12
2. Winchester	15.79	16.36	17.51	18.50
3. Yewhill	12.43	12.85	13.61	14.32
4. Twyford	6.11	6.68	7.97	8.76
5. Moorhill	29.42	32.16	38.42	42.19
6. Otterbourn	20.55	21.25	22.68	23.90
7. Southampton	39.15	40.59	43.68	46.29
8. Rounhams	35.30	38.76	45.60	50.19
9. Timsbury	9.20	9.71	10.61	11.36
10. Iow & Esso	35.80	38.10	40.50	42.70

PEAK DEMAND

	1986	1991	2001	2011
1. Abbotstone	5.88	6.00	6.17	6.29
2. Winchester	19.22	19.92	21.31	22.51
3. Yewhill	15.14	15.67	16.61	17.47
4. Twyford	7.44	8.14	9.72	10.67
5. Moorhill	35.83	39.17	46.79	51.40
6. Otterbourn	25.06	25.88	27.61	29.07
7. Southampton	47.71	49.46	53.19	56.37
8. Rounhams	45.62	50.15	58.92	64.91
9. Timsbury	11.64	12.28	13.45	14.37
10. Iow & Esso	41.50	43.00	46.50	52.00

Source : Derived from table 3.24

Table 3.28

POST METERING DEMAND FORECAST E
SOUTHERN AND CENTRAL HANTS
AVERAGE DEMAND ML/D
1986-2011

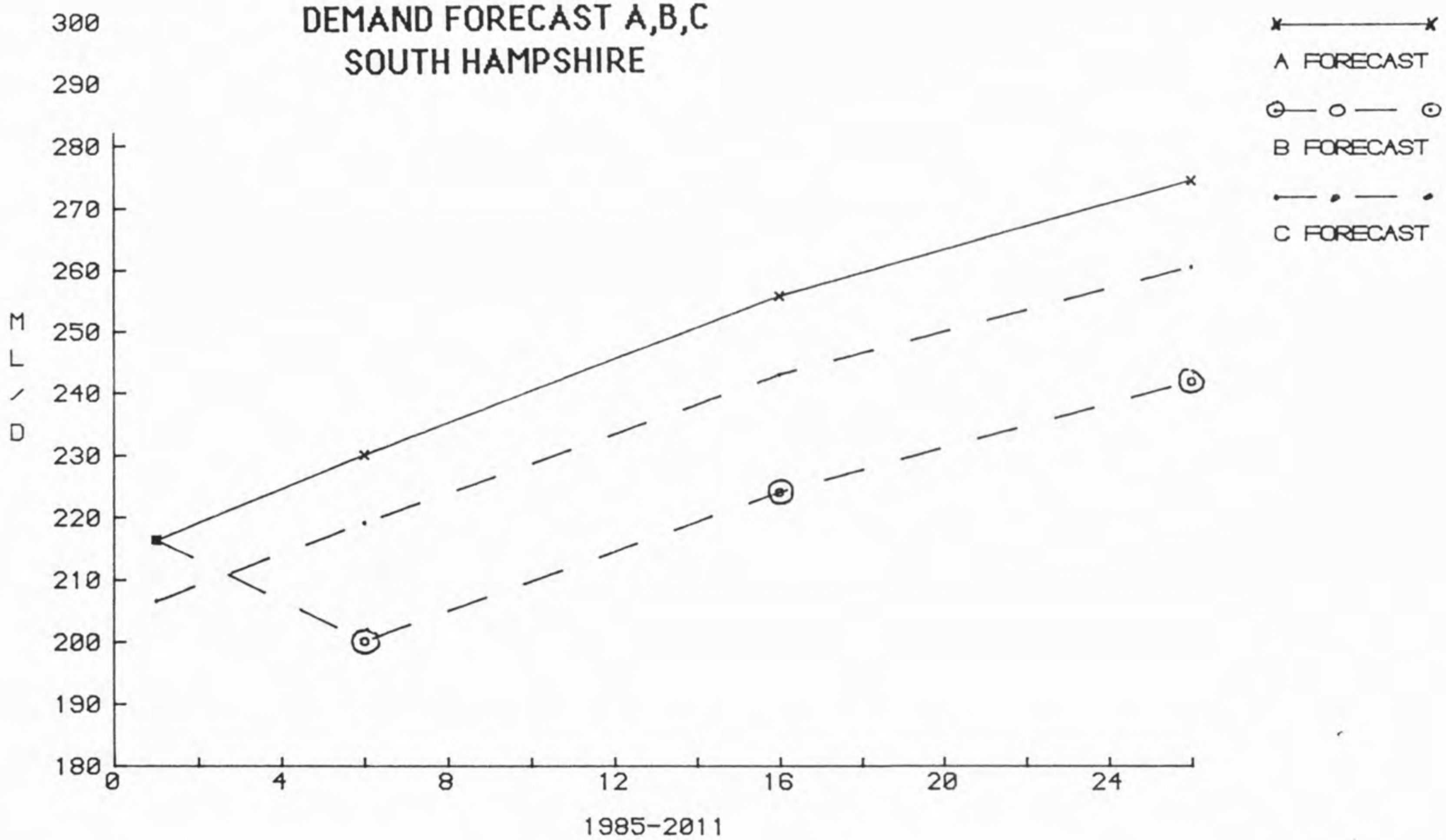
	1986	1991	2001	2011
1. Abbotstone	4.71	4.78	4.91	5.00
2. Winchester	15.43	16.00	17.12	18.08
3. Yewhill	12.08	12.49	13.22	13.90
4. Twyford	5.95	6.50	7.75	8.51
5. Moorhill	28.63	31.29	37.31	40.95
6. Otterbourn	20.07	20.76	22.14	23.32
7. Southampton	38.47	39.89	42.91	45.46
8. Rounhams	34.43	37.78	44.05	48.80
9. Timsbury	9.00	9.49	9.77	11.08
10. Iow & Esso	35.80	38.10	40.50	42.70

Peak demand

	1986	1991	2001	2011
1. Abbotstone	5.70	5.78	5.94	6.06
2. Winchester	18.52	19.20	19.20	20.55
3. Yewhill	14.50	15.01	15.90	16.70
4. Twyford	7.14	7.80	9.30	10.21
5. Moorhill	34.37	37.55	44.77	49.15
6. Otterbourn	24.13	24.92	26.57	27.97
7. Southampton	46.24	47.94	51.54	54.61
8. Rounhams	43.78	48.09	56.41	62.10
9. Timsbury	11.21	11.83	12.93	13.80
10. Iow & Esso	41.50	43.00	46.50	52.00

Source : Derived from table 3.24 .

FIGURE 3.9


DEMAND FORECAST A,B,C
SOUTH HAMPSHIRE
A FORECAST
B FORECAST
C FORECAST
M L / D
300
290
280
270
260
250
240
230
220
210
200
190
180
0
4
8
12
16
20
24
1985-2011

CHAPTER 4

LONG-RUN MARGINAL COST OF WATER SUPPLY HAMPSHIRE DIVISION OF SWA

(4.1) Introduction

This chapter centres on the estimation of a forward looking long-run marginal cost of water supply in the Hampshire division of Southern Water Authority. In the process of estimation the meaning of the long-run marginal cost of water supply as well as the way of estimating it will be examined. We will make use of most of the information concerning the Hampshire division developed in chapter 3. Reference to some of the concepts regarding marginal cost developed in chapter 2 will be made.

The very close relationship between expected investment on the various capital components of the water supply system over the planning horizon and long-run marginal cost of water supply will be emphasised. A considerable amount of attention will therefore be devoted to investment planning including that of what has been termed earlier as the central part of the system as well as the distribution part. The point will be emphasised that different planning procedures for different components of the system may dictate different methodologies of estimating the corresponding capital component of long-run marginal cost.

We will also underline the impact of different expected trends of demand on the estimates of long-run marginal cost of water. In so doing we will highlight the significance of peak characteristics of demand.

The marginal capital cost of water supply to different classes of demand may not be the same and as such may require separate estimates. This issue will be examined while attempting to isolate the long-run marginal cost of domestic demand, as opposed to the long-run marginal cost of supply to a unit of system average demand, (domestic demand is therefore being distinguished as a separate class of consumers with specific demand characteristics).

In a chapter devoted to the estimate of long-run marginal cost it is difficult not to make frequent references to pricing. It must be emphasised however that this reference is only incidental since our main focus of attention here is the concept of marginal cost and not pricing as such.[1] Having said so it must be pointed out that the long-run marginal cost we estimate is very relevant to pricing decisions as well as to the analysis of other issues such as formulating leakage control policies and assessing the metering of domestic consumers.

The long-run marginal cost of water supply normally has two constituents : (1) marginal operating cost (referred to frequently as short-run marginal running cost) reflecting increments in operating costs brought about by increases in output and (2) marginal capacity costs reflecting increments in capital expenditure (capacity) which are necessary to increase output.[2] Much of this chapter will be devoted to the latter.

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[1] An investigation of the structure of marginal costs, however, must be the first step to take when studying pricing policies according to marginal cost principles.

[2] But see discussion regarding the long and short run in section (2.4.4). An increment in output extending many years into the future may sometimes be provided without any need for additional capacity. The long-run costs of meeting the extra output will be confined to operating costs in this case.

(4.2) A Model of long run Planning of Water Supply 'Central System' Capital Investment

In this section we develop a model for long term planning of capital investment in the central part of a water supply system. As outlined earlier, the central part of the water supply system refers to components of the system upstream of the service reservoir including source works, treatment works, and major trunk mains.

Long term planning of capital expenditure on the central part of the water supply system is conducted in such a way as (1) to derive the optimum capital expenditure programme, in the sense of choosing a feasible programme which is least cost (neglegting externalities) in terms of the sum, up to the end of the planning horizon, of total discounted capital costs and total discounted running costs, (2) to ensure that the chosen programme is feasible in that the 2% drought capacity of each of the various sources is not exceeded at any moment of time up to the end of the planning horizon, (3) to ensure that all demand, average and peak week, is met at each of the demand zones in the area up to the end of the planning horizon, and (4) to ensure that the chosen programme must be feasible in the sense that it takes account of all of the peculiarities of the water supply system in the area under investigation including, for example, existing and potential water resources, existing capacities of treating raw water and so on.

The cost minimising central investment programme must be capable of meeting all projected demand and as such the model does not admit demand management using prices, for example, as a viable alternative to capacity expansion. [3]

In principle the cost minimising model for planning central investment should give predictions regarding both the optimum sizing and timing of each of the constituent

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[3] The cost minimising model therefore is in the Turvey tradition, see section 2.3 and Turvey (1969). For an alternative approach to pricing and investment planning based on benefit less cost maximisation see chapters 5 and 6.

components of the central system.[4] In the model of this section however we confine the analysis to the question of timing of potential projects whose 'optimal' size, we assume, has been determined beforehand by SWA using either some complicated rules of optimisation or more likely relying on some rules of thumb based on the practical experience of the industry as a whole.

The model of this section may be described as a cost minimisation exercise which could be set up in a linear programming framework. When set up in this framework the model essentially attempts to allocate water resources available at existing and potential sources to the various demand centers in such a way as to minimise running costs while at the same time not violating any capacity and other constraints representing the system.

This is done by specifying an objective function representing total operating costs of meeting demand at any particular moment in time. Operating costs are composed of power costs of abstraction and pumping to supply as well as chemical costs of treatment. The exercise is thus to allocate water supply resources in any one moment of time, both for average day demand and peak week day demand, so as to minimise total running costs in that day and while satisfying the various constraints of the then existing water supply system. The objective function representing total running costs of meeting demand in a particular day, either average day demand or peak week day demand, is assumed to be linear. This implies that the costs of abstracting, treating and pumping water from a particular water source to a particular demand zone are linear with respect to output.

When peak week demand levels have grown to levels exceeding those which could be met without violating the various capacity constraints no feasible solution is possible without the introduction (commissioning) of some new project(s) adding to capacity and relaxing the binding constraint(s). Thus our cost minimising model when solved for peak week demand levels serves also to indicate the time

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4 See section 3.2.

when necessary capital projects must be commissioned.

All too often there is more than one way of relaxing the constraints to meet demand growth. Different packages of capital investment programmes (strategies) are possible to meet growth in demand.[5] Each strategy would involve the introduction of a specific package of capital projects, to relax some constraints, whose dates of introduction are determined by peak week demand and the optimal (least cost) allocation of water resources to demand zones as solved by the cost minimising model described above. Each strategy will involve meeting demand in a particular way involving different total running costs.

The cost minimising model thus can serve to help us choose the most cost effective from among the feasible strategies. The way to proceed is as follows; for each candidate strategy we find the optimum allocation of water resources to demand zones, both during average day demand and peak week day demand, and for every single year up to the end of the planning horizon. The solution for peak week day demand will indicate the latest possible dates of commissioning of the projects in the strategy. The said solutions will contain the necessary information to determine the total discounted system cost of that particular strategy. This is simply the sum of total discounted capital cost and total discounted running cost, not for a single year but for all years up to the end of the planning horizon. The former is determined by the commissioning dates of the projects contained in the strategy. The latter is simply the discounted sum of the least cost value of the objective function as determined by the solution for average day demand for each of the years in the planning horizon.

The most cost effective strategy is of course the one with the least total discounted system cost. To be sure the whole exercise can be repeated using different demand forecasts. If the most cost effective strategy is insensitive to the demand forecast all the better since the chosen

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[5] The packages may well have one or more projects common to all of them.

strategy could then be described as robust. This is important since a robust strategy implies that reasonable changes in demand would not alter the composition of the least cost capital expenditure programme perhaps merely changing the timing of its projects.

We have applied the above sketched framework to the analysis of central investment planning to the southern and central parts of the Hampshire division of SWA. A full description of the existing supply system including sources and their capacities at the time of average demand and the time of peak week day demand, as well as existing links between sources and demand zones, treatment capacities and other characteristics of the system was given in section 3.7 above and therefore will not be repeated here. Part of this information is contained in diagram 4.1 showing the various demand zones and supply sources.

Section 3.7 also described in some detail the potential projects for resource development in the future in order to meet demand growth. These projects have been grouped in table 4.1 below .

FIGURE 4.1

SOUTH AND CENTRAL HAMPSHIRE SYSTEM NETWORK

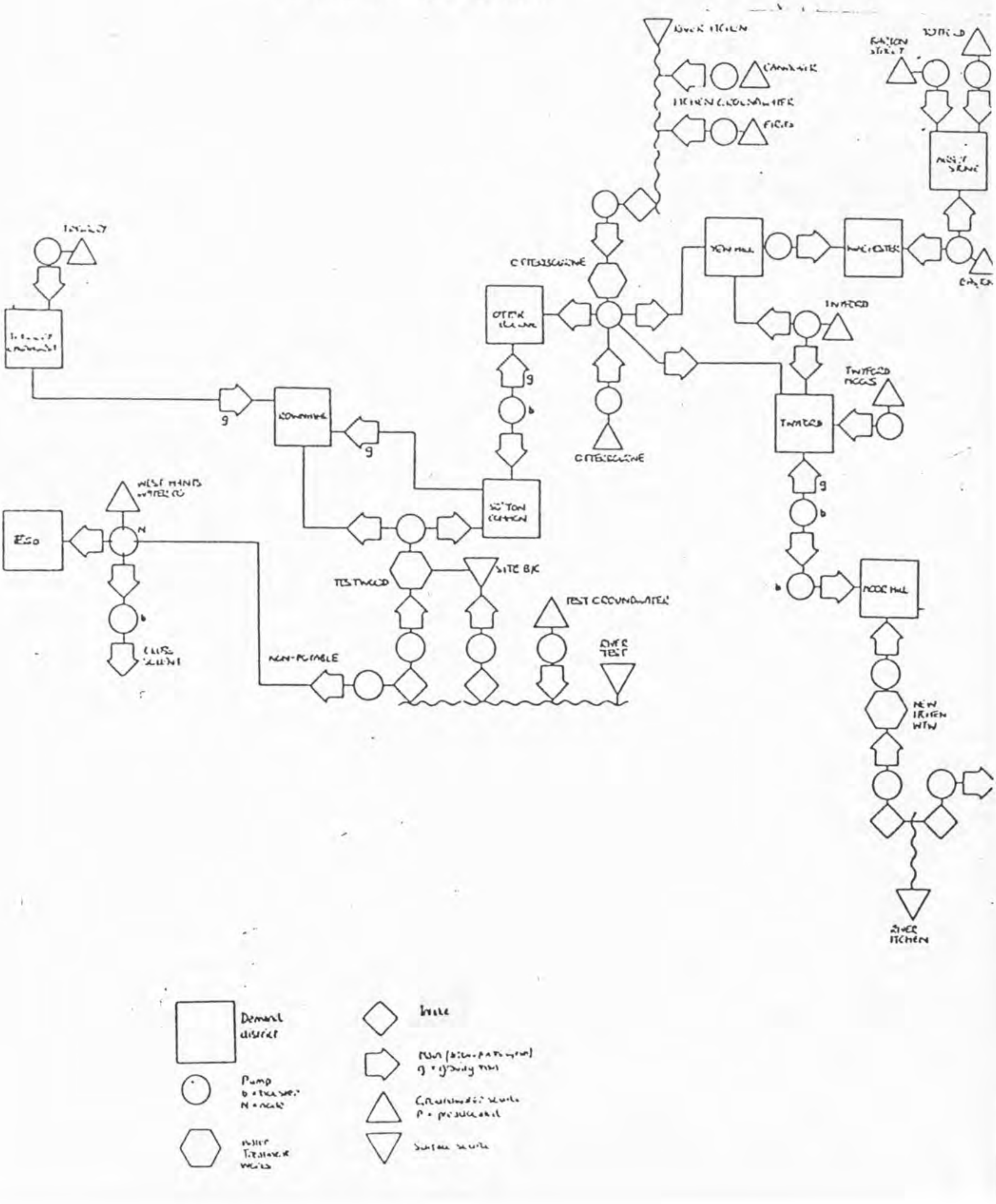

Table 4.1

Potential Central System Capital Projects in Hants

Name of Project	Cost £million*
1. Yew Hill to Winchester trunk mains	0.514
2. Testwood to Southampton trunk mains	0.720
3. Testwood water treatment works (30 MG/d)	2.330
4. Testwood water treatment works (40 MG/d)	4.040
5. Test ground water augmentation	3.167
6. New divisional water treatment works at lower Itchen including trunk mains	4.510

* All costs are at 1981 price level and also allow for perpetual replacement.
Source : RACS,1985, SWA, op.cit.

Discussion in section 3.7 has identified two possible strategies for the development of capital projects in southern and central Hampshire. Strategy I was based on the relaxation of the constraint on the maximum possible volume of supplies from otterbourn ground and river sources to the combined demand zones of Twyford and Moorhill from 13 ML/d to at least 37 ML/d.[6] Strategy II involves maintaining the 13 ML/d restriction on supplies from Otterbourne ground and river sources to the zones of Twyford and Moorhill. The two strategies are different to the extent that each involves different combination and timing of some or all of the projects listed in table 4.1 as well as different allocation of sources to demand zones and consequently different total running costs.

Table 4.2 defines 27 variables representing the volume

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[6] It has to be said that we have not been able to discern the exact nature of this constraint from available SWA documentation. We are not sure whether there are some costs involved in relaxing this constraint or whether it is simply identified in order to distinguish between two alternative strategies. We have assumed the latter, that is there are no costs involved in relaxing the constraint from 13 ML/d to 37 ML/d.

of possible supplies, for a particular year t, t = 1,2,,,,26, from each of the 10 supply sources in the area (as listed in tables 3.20 and 3.21 and including potential Lower Itchen) to the 10 demand zones in the area including Iow and Esso.[7] The table also presents unit operating costs of power and chemicals for the abstraction, treatment and pumping to supply from each of the different supply sources to the various demand zones. These costs are expressed in 1981 price level and are derived from SWA documentation which in turn are based on actual divisional operational records. Unit operating costs are assumed linear with output and constant in real terms up to the end of the planning horizon, year 2011.

Demand levels, both average day and peak week day, for each of the 10 demand zones for the period 1986-2011 have already been presented in tables 3.23 to 3.28. These tables contain demand forecasts A,B,C,D and E. The nature of each of these forecasts was discussed at length in section 3.8 above. In brief, they reflect the following assumptions:

A : demand forecast with high leakage

B : demand forecast with low leakage

C : demand forecast with high leakage and metering of domestic consumers; fall in average domestic demand assumed 12.5%, and in corresponding peak demand 15.5%.

D : demand forecast with high leakage and metering of domestic consumers; fall in average domestic demand assumed 10%, and in corresponding peak demand 12.4%.

E : demand forecast with high leakage and metering of domestic consumers; fall in average domestic demand assumed 15%, and in corresponding peak demand 18.6%.

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7 In total there are 100 variables to cover all possible theoretical connections between the 10 sources and 10 demand zones. The 27 variables listed in table 4.2 cover those connections already in existence plus possible connections between potential Lower Itchen river source and Twyford and Moorhill demand zones and Otterbourn to Winchester via Yew hill. Thus for example though supplies from Timsbury wells could in theory be taken to say Abbotstone demand zone yet the capital and running cost of so doing is that high to rule it out.

Table 4.2

Variables of Cost Minimisation Model

Source		Demand Zone	X_i	Running Cost P/M^3
Totfford	to	Abbotstone	X_1	1.10
Barton stacey	to	Abbotstone	X_2	0.89
Easton	to	Abbotstone	X_3	1.55
Easton	to	Winchester	X_4	1.55
Ottr. river	to	Winchester	X_5	2.01
Ottr. river	to	Yewhill	X_6	2.01
Ottr. river	to	Twyford	X_7	1.58
Ottr. river	to	Moorhill	X_8	1.58
Ottr. river	to	Ottr. direct	X_9	1.22
Ottr. river	to	Southampton common	X_{10}	1.36
Ottr. ground	to	Winchester	X_{11}	1.95
Ottr. ground	to	Yewhill	X_{12}	1.95
Ottr. ground	to	Twyford	X_{13}	1.53
Ottr. ground	to	Moorhill	X_{14}	1.53
Ottr. ground	to	Ottr. direct	X_{15}	1.16
Ottr. ground	to	Southampton common	X_{16}	1.30
River Itchen	to	Twyford	X_{17}	1.98
River Itchen	to	Moorhill	X_{18}	1.98
Twyford	to	Twyford	X_{19}	1.29
Twyford	to	Moorhill	X_{20}	1.29
Twyford Moor	to	Twyford	X_{21}	1.49
Twyford Moor	to	Moorhill	X_{22}	1.49
Testwood	to	Southampton	X_{23}	1.58
Testwood	to	Rounhams	X_{24}	1.58
Testwood	to	Timsbury	X_{25}	1.58
Testwood	to	Esso+Iow	X_{26}	0.73
Timsbury	to	Timsbury	X_{27}	1.20

* All cost figures at 1981 price level.

Source : (1) RACS, 1985, SWA, op.cit.

(2) Testwood Feasibility Study, op.cit.

The linear programming model can be expressed as follows :
Given that c_i , i= 1,2....27, stands for the unit cost of X_i then the problem may be stated as

$$\text{Minimize} \quad \sum_i C_i \, X_i(t) \qquad t = 1,2,,,,,,,26$$

subject to :

Demand

$X_1(t)+X_2(t)+X_3(t)$	$= D_1(t)$	---1	Abbotstone
$X_4(t)+X_5(t)+X_{11}(t)$	$= D_2(t)$	---2	Winchester
$X_6(t)+X_{12}(t)$	$= D_3(t)$	---3	Yew hill
$X_{13}(t)+X_7(t)+X_{19}(t)+X_{21}(t)+X_{17}(t)$	$= D_4(t)$	---4	Twyford
$X_8(t)+X_{14}(t)+X_{20}(t)+X_{22}(t)+X_{18}(t)$	$= D_5(t)$	---5	Moorhill
$X_9(t)+X_{15}(t)$	$= D_6(t)$	---6	Ottr.
$X_{10}(t)+X_{16}(t)+X_{23}(t)$	$= D_7(t)$	---7	Southampton
$X_{24}(t)$	$= D_8(t)$	---8	Rounham
$X_{25}(t)+X_{27}(t)$	$= D_9(t)$	---9	Timsbury
$X_{26}(t)$	$= D_{10}(t)$	---10	Iow+Esso

Sources

$X_1(t)$	$\leq$ 4.5	---11	Totford
$X_2(t)$	$\leq$ 1.1	---12	Barton stacey
$X_3(t)+X_4(t)$	$\leq$ 17.4,27.3	---13	Easton
$X_5(t)+X_6(t)+X_7(t)+X_8(t)+$ $X_9(t)+X10(t)$	$\leq$ 49.5	---14	Ottr. river
$X_{11}(t)+X_{12}(t)+X_{13}(t)+X_{14}(t)+$ $X_{15}(t)+X_{16}(t)$	$\leq$ 68.2	---15	Ottr. ground
$X_{17}(t)+X_{18}(t)$	$\leq$ 199.9	---16	Lower Itchen
$X_{19}(t)+X_{20}(t)$	$\leq$ 18.2,19.1	---17	Twyford
$X_{21}(t)+X_{22}(t)$	$\leq$ 11.3	---18	Twyford Moor
$X_{23}(t)+X_{24}(t)+X_{25}(t)+X_{26}(t)$	$\leq$ 180	---19	Testwood
$X_{27}(t)$	$\leq$ 6.8	---20	Timsbury

Other

$X_7(t)+X_8+X_{13}(t)+X_{14}(t)$	$\leq$ 13,37	----21	Strategy II,I
$X_9(t)+X_{10}(t)+X_{15}(t)+X_{16}(t)$	$\leq$ 75	----22	Ottr+Southam.
$X_5(t)+X_6(t)+X_7(t)+X_8(t)+X_9(t)+$			

$X_{10}(t)+X_{11}(t)+X_{12}(t)+X_{13}(t)+$
$X_{14}(t)+X_{15}(t)+X_{16}(t)+X_{21}(t)+X_{22}(t) \leq 113.7$ ----23 Ottr.+ Twyfor moor

Notes:
1. Constraints 13 (Easton) and 17 (Twyford) have two figures in the right hand side; the first is capacity during average demand and the second is capacity during peak week demand.
2. The right hand side of constraint 16 (potential Lower Itchen) has been set at an arbitrarily high level so that if Lower Itchen is used it should not run out before the end of the planning horizon.
3. The right hand side of constraint 19 (Testwood) is set at 40 MG/d (180 ML/d), that is it has been relaxed to cater for demand growth. The actual time of relaxing the constraint will emerge from the runs of the model.
4. The right hand side of constraint 21 depends on the choice of strategy I or II.
5. Constraint 22 reflects the top limit on possible supplies from Otterbourn sources to the zones of Otterbourn direct and Southampton Common.
6. Constraint 23 reflects the top limit of the combined capacity of Otterbourn ground and river sources plus the ground source of Twyford moor.
7. $D_i(t)$ reflect demand at zone i at time t. This is either average day demand or peak week day demand depending on the required run.

It can seen from above that the objective function of the model is a linear combination of the 27 supply variables with the parameters being the unit running cost of supplying a M^3 from the various sources to the various demand zones. Other running costs such as power costs of the Candover and FIRAS schemes in the upper reaches of the Itchen are excluded on the grounds that these are essentially fixed costs whose level is invariant to the level of demand and output.[8] Labour costs are excluded for the same reason. Rates on the other hand are excluded because they are in the nature of financial costs and do not constitute real economic costs.

Unit running cost of industrial non potable water from Testwood to Iow and Esso is taken to be equal to £7.30 per ML for all years up to 2011. This is despite the fact that sometime in the future supplies to Esso and Iow become fully potable and thus command a higher unit running cost equal to £15.80 per ML. The reason for adhering to the lower unit running cost is explained by the observation that the increment in unit running cost of supplies to Iow and Esso when Testwood become fully potable will be matched by an equivalent decrement (saving) in cost achieved by Iow and Esso who need not incur any more extra treatment costs themselves. Since we are interested in efficiency considerations and not distributional ones the changed incidence of the financial costs that may ensue is of no consequence to our discussion here.

For each demand forecast the solution of the model is derived from solving the linear programming model 26 times for the average day level of demand (for each of the years from 1986 to 2011) and likewise 26 times for peak week day demand for the same years (after 2011 demand as assumed constant). This is carried out either under strategy I by specifying constraint No. 21 equal to or less than 37 ML/d or solved under strategy II by specifying the same constraint equal to or less than 13 ML/d.

Each run of the linear programming model has been solved

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[8] Indeed the said two schemes would be in continuous operation regardless of the level of demand from the Itchen. See RACS,1985, Op.cit.

using the standard simplex method.[9] Using this method the model has been solved with the aid of a computer programme obtained from the Nag Library (Subroutine H1ADF).[10]

Results of the runs are reported in tables 4.3 to 4.17 . Tables 4.3 and 4.4 contain the solution for demand forecast A (the highest) and for strategy I. Table 4.3 contains the allocation of sources to demand zones, for each of the years from 1986 to 2011, both for average and peak week day demand. Table 4.4 contains a list of all the capital projects required under the said scenario including their timing, discounted cost (5% interest rate). It can be seen that under strategy I the marginal source of water for southern and central Hampshire is exclusively the Test river with abstraction and treatment at the Testwood site. The second potential marginal source of supply, the Lower Itchen, is not used. This implies a relatively early development of Testwood water treatment facilities, relatively early development of Test groundwater augmentation scheme and likewise early development of the Testwood to Southampton trunk mains. Moorhill and Twyford demand zones under this strategy draw an ever increasing amount of their supplies from the Otterbourn ground and surface water sources. Less and less water is pumped from Otterbourn to Southampton Common demand zone which instead gradually draws more and more supplies from Testwood. The resulting running costs, as in average day demand, for demand forecast A under strategy I is reported, together with other running costs, in table 4.17.

The solution under demand forecast A, strategy II, with the restriction of supplies from Otterbourn to Twyford and

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9 See any Operations Research textbook such as Wagner (1975).
Wagner,M.Harvey,"Principles of Operations Research ," second edition,1975, Prentice Hall.

10 It must be said that if we abstract from differences between Otterbourn ground and river supply and likewise between Twyford and Moorhill demand zones, treating them as one, then the modelled network becomes simple enough for one to work his way through it thus finding the correct answers to the problem without having to resort to the simplex method.

Moorhill demand zones maintained at 13 ML/d, is reported in tables 4.5 and 4.6 . It can be seen that under this scenario we have two marginal sources of supply namely, the Lower Itchen (for Twyford and Moorhill) and the Test for the remainder of the demand zones. This means that new works at the Lower Itchen need to be constructed as early as 1990. But it also implies savings arising from comparatively late development of the Test resources; the expansion to 30 MG/d of the water treatment works at the Testwood site under strategy II is needed in 1991 compared with 1988 under strategy I ; the expansion of the same works to 40 MG/d is not required till 2003 compared with 1997 under strategy I ; the Test ground water augmentation scheme and the Testwood to Southampton trunk mains are likewise delayed.

Comparison of total discounted capital cost of strategy I and strategy II (tables 4.4 and 4.6) indicate that strategy I offers a lower cost option from the point of view of capital costs.[11] This advantage is strengthened by the observation that in fact strategy I is cheaper than strategy II even from the point of view of running costs. It can be seen that running costs for each of the years 1986 to 2011 under strategy I is equal to or lower than the corresponding costs under strategy II.

The cost advantage of strategy I is confirmed by examining the results of the model under demand forecast B. These are given in tables 4.7 to 4.10 . The same broad conclusions derived under forecast A are applicable under demand forecast B. It is noted however that the timing of projects under demand forecast B is later then the corresponding commissioning dates under demand forecast A. This is to be expected since demand forecast B is lower than demand forecast A. Also not all projects required under A are

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11 It is however not as clear cut as it might first seem if instead of constructing new divisional water treatment works at the Lower Itchen we expand the existing water treatment works also at the Lower Itchen at the Gaters Mill site belonging to the Portsmouth Water Company. We have not discussed this option on the grounds that SWA, having examined these options in some detail, seems to prefer what we have termed as strategy I.

needed under demand forecast B.[12]

Using the preferred strategy, I, we have also simulated optimal allocations of sources to demand zones and the implied timing of needed projects together with the resulting total discounted capital costs for each of demand forecasts C,D, and E. The results are reported in tables 4.11 to 4.16.

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[12] Thus we can now safely assert that as far as Hampshire is concerned a permanent decrement in demand will lead not to a major restructuring of the optimal capital expenditure plan but merely to a delay in the execution of the projects within this plan.

Table 4.3

WATER RESOURCE ALLOCATION: SOUTHERN & CENTRAL HANTS
DEMAND FORECAST "A" OPTION I 1986-2011

	1986	1991	1996	2001	2006	2011
1- Timsbury & Lyndhurst						
AVERAGE DEMAND	9.60	10.20	10.60	11.10	11.50	11.90
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	2.80	3.40	3.80	4.30	4.70	5.10
PEAK DEMAND	12.50	13.20	13.80	14.50	15.00	15.50
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	5.70	6.40	7.00	7.70	8.20	8.70
2- Rounhams						
AVERAGE DEMAND	37.10	40.70	44.40	48.10	50.50	53.00
SOURCES:						
Testwood	37.10	40.70	44.40	48.10	50.50	53.00
PEAK DEMAND	49.30	54.20	59.00	63.90	67.20	70.50
SOURCES:						
Testwood	49.30	54.20	59.00	63.90	67.20	70.50
3- ABBOTSTONE/TOTFORD						
AVERAGE DEMAND	5.04	5.11	5.17	5.24	5.30	5.35
SOURCES:						
Easton/Totford/Barton Stacey	5.04	5.11	5.17	5.24	5.30	5.35
PEAK DEMAND	6.40	6.40	6.50	6.60	6.70	6.70
Easton/Totford/Barton	6.40	6.40	6.50	6.60	6.70	6.70
4- Winchester						
AVERAGE DEMAND	16.50	17.10	17.70	18.30	18.80	19.30
SOURCES:						
Easton	16.50	17.10	17.40	17.40	17.40	17.40
Ottr. via Yewhill	--	--	0.30	0.90	1.40	1.90
PEAK DEMAND	20.60	21.30	22.10	22.90	23.50	24.10
SOURCES:						
Easton	20.60	21.30	21.80	22.00	22.10	22.20
Ottr. via Yewhill	--	--	0.30	0.90	1.40	1.90

Table 4.3 Cont.

	1986	1991	1996	2001	2006	2011
5- Yewhill						
AVERAGE DEMAND	13.10	13.60	14.00	14.40	14.80	15.20
+ Supply to Winchester	--	--	0.30	0.90	1.40	1.90
Total	13.10	13.60	14.30	15.30	16.20	17.10
SOURCES:						
Otterbourn (ground & surface)	13.10	13.60	14.30	15.30	16.20	17.10
PEAK DEMAND	16.40	17.00	17.50	18.00	18.50	19.00
+ Supply to Winchester	--	--	0.30	0.90	1.40	1.90
Total	16.40	17.00	17.80	18.90	19.90	20.90
SOURCES:						
Otterbourn (ground and surface)	16.50	17.00	17.80	18.90	19.90	20.90
6- Otterbourn						
AVERAGE DEMAND	21.50	22.30	23.00	23.80	24.40	25.10
SOURCES:						
Otterbourn (ground and surface)	21.50	22.30	23.00	23.80	24.40	25.10
PEAK DEMAND	26.90	27.80	28.80	29.70	30.50	31.30
SOURCES:						
Otterbourn (ground and surface)	26.90	27.80	28.80	29.70	30.50	31.30
7- Twyfords & Moorhill						
AVERAGE DEMAND	37.50	41.00	45.00	49.10	51.60	54.00
SOURCES:						
Twyford/Twyford Moors	29.50	29.50	29.50	29.50	29.50	29.50
Otterbourn (ground and surface)	8.00	11.50	15.50	19.60	22.10	24.50
Lower Itchen	--	--	--	--	--	--
PEAK DEMAND	46.80	51.20	56.31	60.68	64.40	67.40
SOURCES:						
Twyford/Twyford Moors	30.40	30.40	30.40	30.40	30.40	30.40
Otterbourn	16.40	20.80	25.91	30.28	34.00	37.00
Lower Itchen	--	--	--	--	--	--
8- Southampton						
AVERAGE DEMAND	40.50	42.00	43.60	45.20	46.60	47.90
SOURCES:						
Otterbourn (ground and surface)	40.50	42.00	43.60	43.70	39.70	35.70
Testwood	--	--	--	1.50	6.90	12.20
PEAK DEMAND	50.60	52.50	54.50	56.50	58.20	59.90
SOURCES:						
Otterbourn (ground and surface)	42.70	36.80	29.89	23.52	18.00	13.20
Testwood	7.90	15.70	24.61	32.98	40.20	46.70

Table 4.3 cont.

	1986	1991	1996	2001	2006	2011
Total Supplies From Otterbourn						
AVERAGE DAY:						
To Ywehill	13.10	13.60	14.30	15.30	16.20	17.10
To Twyford/Moorhill	8.00	11.50	15.50	19.60	22.10	24.50
To Otterbourn & Southampton	62.00	64.30	66.60	67.50	64.10	60.80
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	94.40	100.70	107.70	113.70	113.70	113.70
PEAK DEMAND:						
To Ywehill	16.40	17.00	17.80	18.90	19.90	20.90
To Twyford/Moorhill	16.40	20.80	25.91	30.28	34.00	37.00
To Otterbourn & Southampton	69.60	64.60	58.69	53.22	48.50	44.50
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	113.70	113.70	113.70	113.70	113.70	113.70
8- SUPPLIES FROM TESTWOOD						
AVERAGE DAY:						
To Rounhams	37.10	40.70	44.40	48.10	50.50	53.00
To Timsbury/lynhurst	2.80	3.40	3.80	3.40	4.70	5.10
To Southampton	--	--	--	1.50	6.90	12.20
TOTAL	39.90	44.10	48.20	53.93	62.10	70.30
PEAK DAY:						
To Rounhams	49.30	54.20	59.00	63.90	67.20	70.50
To Timsbury/lynhurst	5.70	6.40	7.00	7.70	8.20	8.70
To Southampton	7.90	15.70	24.61	32.98	40.20	46.70
SUB TOTAL	62.90	76.30	90.61	104.58	115.60	125.90
NON-POTABLE:						
To ESSO & Iow	41.50	43.00	44.40	46.50	49.20	52.00
TOTAL	104.40	119.30	135.01	151.08	164.80	177.90

Table 4.4

Water Supply Central Investment Programme
Demand Forecast A (Hants)
Strategy I
1986 - 2011

Project	Commissioning Date	Discounted Cost £m	Undisacounted Cost £m
1. Yew hill to Winchester trunk main	1995	0.315	0.514
2. Testwood to Southampton trunk main	1989	0.591	0.720
3. Testwood water treatment works up to 30 MG/d	1988	2.010	2.330
4. Testwood water treatment works up to 40 MG/d	1997	2.246	4.04
5. Test ground water augmentation	1995	1.941	3.167
6. New Itchen water treatment works and trunk main		not required	
TOTAL		7.103	

* all costs are in terms of 1981 price level and include allowance for perpetual replacement.

Table 4.5

WATER RESOURCE ALLOCATION: SOUTHERN & CENTRAL HANTS DEMAND FORECAST "A" OPTION II 1986-2011

	1986	1991	1996	2001	2006	2011
1- Timsbury & Lyndhurst						
AVERAGE DEMAND	9.60	10.20	10.60	11.10	11.50	11.90
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	2.80	3.40	3.80	4.30	4.70	5.10
PEAK DEMAND	12.50	13.20	13.80	14.50	15.00	15.50
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	5.70	6.40	7.00	7.70	8.20	8.70
2- Rounhams						
AVERAGE DEMAND	37.10	40.70	44.40	48.10	50.50	53.00
SOURCES:						
Testwood	37.10	40.70	44.40	48.10	50.50	53.00
PEAK DEMAND	49.30	54.20	59.00	63.90	67.20	70.50
SOURCES:						
Testwood	49.30	54.20	59.00	63.90	67.20	70.50
3- ABBOTSTONE/TOTFORD						
AVERAGE DEMAND	5.04	5.11	5.17	5.24	5.30	5.35
SOURCES:						
Easton/Totford/Barton Stacey	5.04	5.11	5.17	5.24	5.30	5.35
PEAK DEMAND	6.40	6.40	6.50	6.60	6.70	6.70
Easton/Totford/Barton	6.40	6.40	6.50	6.60	6.70	6.70
4- Winchester						
AVERAGE DEMAND	16.50	17.10	17.70	18.30	18.80	19.30
SOURCES:						
Easton	16.50	17.10	17.40	17.40	17.40	17.40
Ottr. via Yewhill	--	--	0.30	0.90	1.40	1.90
PEAK DEMAND	20.60	21.30	22.10	22.90	23.50	24.10
SOURCES:						
Easton	20.60	21.30	21.80	22.00	22.10	22.20
Ottr.via Yehill	--	--	0.30	0.90	1.40	1.90

Table 4.5 Cont.

	1986	1991	1996	2001	2006	2011
5- Yewhill						
AVERAGE DEMAND	13.10	13.60	14.00	14.40	14.80	15.20
+ Supply to Winchester	--	--	0.30	0.90	1.40	1.90
Total	13.10	13.60	14.30	15.30	16.20	17.10
SOURCES:						
Otterbourn (ground & surface)	13.10	13.60	14.30	15.30	16.20	17.10
PEAK DEMAND	16.40	17.00	17.50	18.00	18.50	19.00
+ Supply to Winchester	--	--	0.30	0.90	1.40	1.90
Total	16.40	17.00	17.80	18.90	19.90	20.90
SOURCES:						
Otterbourn (ground and surface)	16.50	17.00	17.80	18.90	19.90	20.90
6- Otterbourn						
AVERAGE DEMAND	21.50	22.30	23.00	23.80	24.40	25.10
SOURCES:						
Otterbourn (ground and surface)	21.50	22.30	23.00	23.80	24.40	25.10
PEAK DEMAND	26.90	27.80	28.80	29.70	30.50	31.30
SOURCES:						
Otterbourn (ground and surface)	26.90	27.80	28.80	29.70	30.50	31.30
7- Twyfords & Moorhill						
AVERAGE DEMAND	37.50	41.00	45.00	49.10	51.60	54.00
SOURCES:						
Twyford/Twyford Moors	29.50	29.50	29.50	29.50	29.50	29.50
Otterbourn (ground and surface)	8.00	11.50	13.00	13.00	13.00	13.00
Lower Itchen	--	--	2.50	6.60	9.10	11.50
PEAK DEMAND	46.80	51.20	56.31	60.68	64.40	67.40
SOURCES:						
Twyford/Twyford Moors	30.40	30.40	30.40	30.40	30.40	30.40
Otterbourn	13.00	13.00	13.00	13.00	13.00	13.00
Lower Itchen	3.40	7.80	12.91	17.28	21.00	24.09
8- Southampton						
AVERGE DEMAND	40.50	42.00	43.60	45.20	46.60	47.90
SOURCES:						
Otterbourn (ground and surface)	40.50	42.00	43.60	45.20	46.60	47.2
Testwood	--	--	--	--	--	0.70
PEAK DEMAND	50.60	52.50	54.50	56.50	58.20	59.90
SOURCES:						
Otterbourn (ground and surface)	46.10	44.60	42.80	40.80	39.00	37.20
Testwood	4.50	7.90	11.70	15.70	19.20	22.70

Table 4.5 cont.

	1986	1991	1996	2001	2006	2011
Total Supplies From Otterbourn						
AVERAGE DAY:						
To Ywehill	13.10	13.60	14.30	15.30	16.20	17.10
To Twyford/Moorhill	8.00	11.30	13.00	13.00	13.00	13.00
To Otterbourn & Southampton	62.00	64.30	66.60	69.00	71.00	72.30
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	94.40	100.70	105.20	108.60	115.50	113.70
PEAK DEMAND:						
To Ywehill	16.40	17.00	17.80	18.90	19.90	20.90
To Twyford/Moorhill	13.00	13.00	13.00	13.00	13.00	13.00
To Otterbourn & Southampton	73.00	72.40	71.60	70.50	69.50	68.50
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	113.70	113.70	113.70	113.70	113.70	113.70
8- SUPPLIES FROM TESTWOOD						
AVERAGE DAY:						
To Rounhams	37.10	40.70	44.40	48.10	50.50	53.00
To Timsbury/lynhurst	2.80	3.40	3.80	4.30	4.70	5.10
To Southampton	--	--	--	--	--	0.70
TOTAL	39.90	44.10	48.20	52.40	54.75	58.80
PEAK DAY:						
To Rounhams	49.30	54.20	59.00	63.90	67.20	70.50
To Timsbury/lynhurst	5.70	6.40	7.00	7.70	8.20	8.70
To Southampton	4.50	7.90	11.70	15.70	19.20	22.70
SUB TOTAL	59.50	68.50	77.70	87.30	94.60	101.90
NON-POTABLE:						
To ESSO & Iow	41.50	43.00	44.40	46.50	49.20	52.00
TOTAL	101.00	111.50	122.10	133.80	143.80	153.90

Table 4.6

Water Supply Central Investment Programme
Demand Forecast A (Hants)
Strategy II
1986 - 2011

Project	Commissioning Date	Discounted Cost £m	Undiscounted Cost £m
1. Yew hill to Winchester trunk main	1995	0.315	0.514
2. Testwood to Southampton trunk main	1997	0.400	0.720
3. Testwood water treatment works up to 30 MG/d	1991	1.738	2.330
4. Testwood water treatment works up to 40 MG/d	2003	1.676	4.040
5. Test ground water augmentation	2000	1.523	3.167
6. New Itchen water treatment works and trunk main		3.531	4.510
TOTAL		9.183	

* all costs are in terms of 1981 price level and include allowance for perpetual replacement.

Table 4.7

WATER RESOURCE ALLOCATION: SOUTHERN & CENTRAL HANTS DEMAND FORECAST "B" OPTION I 1986-2011

	1986	1991	1996	2001	2006	2011
1- Timsbury & Lyndhurst						
AVERAGE DEMAND	9.60	8.80	9.30	9.70	10.10	10.40
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	2.80	2.00	2.50	2.90	3.30	3.60
PEAK DEMAND	12.50	11.50	12.00	12.60	13.10	13.60
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	5.70	4.70	5.20	5.80	6.30	6.80
2- Rounhams						
AVERAGE DEMAND	37.10	34.70	37.90	41.10	43.30	45.50
SOURCES:						
Testwood	37.10	34.70	37.90	41.10	43.30	45.50
PEAK DEMAND	49.3	46.10	50.54	54.60	57.60	60.60
SOURCES:						
Testwood	49.30	46.10	50.54	54.60	57.60	60.60
3- ABBOTSTONE/TOTFORD						
AVERAGE DEMAND	5.00	4.40	4.50	4.50	4.60	4.60
SOURCES:						
Easton/Totford/Barton Stacey	5.00	4.40	4.50	4.50	4.60	4.60
PEAK DEMAND	6.40	5.60	5.60	5.70	5.80	5.80
Easton/Totford/Barton	6.40	5.60	5.60	5.70	5.80	5.80
4- Winchester						
AVERAGE DEMAND	16.50	14.10	14.70	15.20	15.70	16.20
SOURCES:						
Easton	16.50	14.10	14.70	15.20	15.70	16.20
Ottr. via Yewhill	--	--	--	--	--	--
PEAK DEMAND	20.60	17.60	18.30	19.00	19.60	20.20
SOURCES:						
Easton	20.60	17.60	18.30	19.00	19.60	20.20
Ottr.via Yehwhill	--	--	--	--	--	--

Table 4.7 Cont.

	1986	1991	1996	2001	2006	2011
5- Yewhill						
AVERAGE DEMAND	13.10	11.40	11.80	12.20	12.50	12.90
+ Supply to Winchester	--	--	--	--	--	--
Total	13.10	11.40	11.80	12.20	12.50	12.90
SOURCES:						
Otterbourn (ground & surface)	13.10	11.40	11.80	12.20	12.50	12.90
PEAK DEMAND	16.40	14.20	14.70	15.20	15.70	16.10
+ Supply to Winchester	--	--	--	--	--	--
Total	16.40	14.20	14.70	15.20	15.70	16.10
SOURCES:						
Otterbourn (ground and surface)	16.50	14.20	14.70	15.20	15.70	16.10
6- Otterbourn						
AVERAGE DEMAND	21.50	19.20	20.00	20.70	21.30	21.90
SOURCES:						
Otterbourn (ground and surface)	21.50	19.20	20.00	20.70	21.30	21.90
PEAK DEMAND	26.90	24.00	24.90	25.90	26.60	27.40
SOURCES:						
Otterbourn (ground and surface)	26.90	24.00	24.90	25.90	26.60	27.40
7- Twyfords & Moorhill						
AVERAGE DEMAND	37.50	34.10	38.00	41.90	44.40	46.80
SOURCES:						
Twyford/Twyford Moors	29.50	29.50	29.50	29.50	29.50	29.50
Otterbourn (ground and surface)	8.00	4.60	8.50	12.40	14.90	17.30
Lower Itchen	--	--	--	--	--	--
PEAK DEMAND	46.80	42.70	47.50	52.40	55.50	58.50
SOURCES:						
Twyford/Twyford Moors	30.40	30.40	30.40	30.40	30.40	30.40
Otterbourn	16.40	12.30	17.10	22.00	25.10	28.10
Lower Itchen	--	--	--	--	--	--
8- Southampton						
AVERGE DEMAND	40.50	37.70	39.30	40.90	42.20	43.50
SOURCES:						
Otterbourn (ground and surface)	40.50	37.70	39.30	40.90	42.20	43.50
Testwood	--	--	--	--	--	--
PEAK DEMAND	50.60	47.10	49.10	51.10	52.70	54.40
SOURCES:						
Otterbourn (ground and surface)	42.70	47.10	45.70	38.30	35.00	30.80
Testwood	7.90	--	3.40	11.80	17.70	23.60

Table 4.7 cont.

	1986	1991	1996	2001	2006	2011
Total Supplies From Otterbourn						
AVERAGE DAY:						
To Ywehill	13.10	11.40	11.80	12.20	12.50	12.90
To Twyford/Moorhill	8.00	4.60	8.50	12.40	14.90	17.30
To Otterbourn & Southampton	62.00	56.90	59.30	61.60	63.60	65.40
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	94.40	84.20	90.90	97.50	102.20	106.90
PEAK DEMAND:						
To Ywehill	16.40	14.20	14.70	15.20	15.70	16.10
To Twyford/Moorhill	16.40	12.30	17.10	22.00	25.10	28.10
To Otterbourn & Southampton	69.60	71.10	70.60	65.20	61.60	58.20
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	113.70	108.90	113.70	113.70	113.70	113.70
8- SUPPLIES FROM TESTWOOD						
AVERAGE DAY:						
To Rounhams	37.10	34.70	37.90	41.10	43.30	45.50
To Timsbury/lynhurst	2.80	2.00	2.50	2.90	3.30	3.60
To Southampton	--	--	--	--	--	--
TOTAL	39.90	36.70	40.40	44.00	46.60	49.10
PEAK DAY:						
To Rounhams	49.30	46.10	50.40	54.60	57.60	60.80
To Timsbury/lynhurst	5.70	4.70	5.20	5.80	6.30	6.80
To Southampton	7.90	--	3.40	11.80	17.70	23.60
SUB TOTAL	62.90	50.80	59.00	73.05	81.60	91.00
NON-POTABLE:						
To ESSO & Iow	41.50	40.000	41.40	42.80	44.60	47.20
TOTAL	104.40	90.80	104.40	115.85	126.20	138.20

Table 4.8

Water Supply Central Investment Programme
Demand Forecast B (Hants)
Strategy I
1986 - 2011

Project	Commissioning Date	Discounted Cost £m	Undiscounted Cost £m
1. Yew hill to Winchester trunk main	not required		
2. Testwood to Southampton trunk main	2002	0.313	0.720
3. Testwood water treatment works up to 30 MG/d	2000	1.120	2.330
4. Testwood water treatment works up to 40 MG/d	2011	1.135	4.04
5. Test ground water augmentation	2008	1.029	3.167
6. New Itchen water treatment works and trunk main		not required	
TOTAL		3.597	

* all costs are in terms of 1981 price level and include allowance for perpetual replacement.

Table 4.9

WATER RESOURCE ALLOCATION: SOUTHERN & CENTRAL HANTS
DEMAND FORECAST "B" OPTION II 1986-2011

	1986	1991	1996	2001	2006	2011
1- Timsbury & Lyndhurst						
AVERAGE DEMAND	9.60	8.80	9.30	9.70	10.10	10.40
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	2.80	2.00	2.50	2.90	3.30	3.60
PEAK DEMAND	12.50	11.50	12.00	12.60	13.10	13.60
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	5.70	4.70	5.20	5.80	6.30	6.80
2- Rounhams						
AVERAGE DEMAND	37.10	34.70	37.90	41.10	43.30	45.50
SOURCES:						
Testwood	37.10	34.70	37.90	41.10	43.30	45.50
PEAK DEMAND	49.3	46.10	50.54	54.60	57.60	60.60
SOURCES:						
Testwood	49.30	46.10	50.54	54.60	57.60	60.60
3- ABBOTSTONE/TOTFORD						
AVERAGE DEMAND	5.00	4.40	4.50	4.50	4.60	4.60
SOURCES:						
Easton/Totford/Barton Stacey	5.00	4.40	4.50	4.50	4.60	4.60
PEAK DEMAND	6.40	5.60	5.60	5.70	5.80	5.80
Easton/Totford/Barton	6.40	5.60	5.60	5.70	5.80	5.80
4- Winchester						
AVERAGE DEMAND	16.50	14.10	14.70	15.20	15.70	16.20
SOURCES:						
Easton	16.50	14.10	14.70	15.20	15.70	16.20
Ottr. via Yewhill	--	--	--	--	--	--
PEAK DEMAND	20.60	17.60	18.30	19.00	19.60	20.20
SOURCES:						
Easton	20.60	17.60	18.30	19.00	19.60	20.20
Ottr.via Yewhill	--	--	--	--	--	--

Table 4.9 Cont.

	1986	1991	1996	2001	2006	2011
5- Yewhill						
AVERAGE DEMAND	13.10	11.40	11.80	12.20	12.50	12.90
+ Supply to Winchester	--	--	--	--	--	--
Total	13.10	11.40	11.80	12.20	12.50	12.90
SOURCES:						
Otterbourn (ground & surface)	13.10	11.40	11.80	12.20	12.50	12.90
PEAK DEMAND	16.40	14.20	14.70	15.20	15.70	16.10
+ Supply to Winchester	--	--	--	--	--	--
Total	16.40	14.20	14.70	15.20	15.70	16.10
SOURCES:						
Otterbourn (ground and surface)	16.50	14.20	14.70	15.20	15.70	16.10
6- Otterbourn						
AVERAGE DEMAND	21.50	19.20	20.00	20.70	21.30	21.90
SOURCES:						
Otterbourn (ground and surface)	21.50	19.20	20.00	20.70	21.30	21.90
PEAK DEMAND	26.90	24.00	24.90	25.90	26.60	27.40
SOURCES:						
Otterbourn (ground and surface)	26.90	24.00	24.90	25.90	26.60	27.40
7- Twyfords & Moorhill						
AVERAGE DEMAND	37.50	34.10	38.00	41.90	44.40	46.80
SOURCES:						
Twyford/Twyford Moors	29.50	29.50	29.50	29.50	29.50	29.50
Otterbourn (ground and surface)	8.00	4.60	8.50	12.40	13.00	13.00
Lower Itchen	--	--	--	--	1.90	4.30
PEAK DEMAND	46.80	42.70	47.50	52.40	55.50	58.50
SOURCES:						
Twyford/Twyford Moors	30.40	30.40	30.40	30.40	30.40	30.40
Otterbourn	13.00	13.00	13.00	13.00	13.00	13.00
Lower Itchen	3.40	--	4.10	9.00	12.10	15.10
8- Southampton						
AVERGE DEMAND	40.50	37.70	39.30	40.90	42.20	43.50
SOURCES:						
Otterbourn (ground and surface)	40.50	37.70	39.30	40.90	42.20	43.50
Testwood	--	--	--	--	--	--
PEAK DEMAND	50.60	47.10	49.10	51.10	52.70	54.40
SOURCES:						
Otterbourn (ground and surface)	46.10	47.10	49.10	48.30	47.10	45.90
Testwood	4.50	--	--	2.80	5.60	8.50

Table 4.9 cont.

	1986	1991	1996	2001	2006	2011
Total Supplies From Otterbourn						
AVERAGE DAY:						
To Ywehill	13.10	11.40	11.80	12.20	12.50	12.90
To Twyford/Moorhill	8.00	4.60	8.50	12.40	13.00	13.00
To Otterbourn & Southampton	62.00	56.90	59.30	61.50	63.50	65.40
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	94.40	84.20	90.90	97.50	100.30	102.60
PEAK DEMAND:						
To Ywehill	16.40	14.20	14.70	15.20	15.70	16.10
To Twyford/Moorhill	13.00	12.30	13.00	13.00	13.00	13.00
To Otterbourn & Southampton	73.00	71.10	74.00	74.20	73.60	73.30
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	113.70	108.90	113.70	113.70	113.70	113.70
8- SUPPLIES FROM TESTWOOD						
AVERAGE DAY:						
To Rounhams	37.10	34.70	37.90	41.10	43.30	45.50
To Timsbury/lynhurst	2.80	2.00	2.50	2.90	3.30	3.60
To Southampton	--	--	--	--	--	--
TOTAL	39.90	36.70	40.40	44.00	46.60	49.10
PEAK DAY:						
To Rounhams	49.30	46.10	50.40	54.60	57.60	60.80
To Timsbury/lynhurst	5.70	4.70	5.20	5.80	6.30	6.80
To Southampton	4.50	--	--	2.80	5.60	8.50
SUB TOTAL	59.50	50.80	55.60	63.20	69.50	75.90
NON-POTABLE:						
To ESSO & Iow	41.50	40.000	41.40	42.80	44.60	47.20
TOTAL	101.00	90.80	97.00	106.00	114.10	123.10

Table 4.10

Water Supply Central Investment Programme
Demand Forecast B (Hants)
Strategy II
1986 - 2011

Project	Commissioning Date	Discounted Cost £m	Undiscounted Cost £m
1. Yew hill to Winchester trunk main	not required		
2. Testwood to Southampton trunk main	not required		
3. Testwood water treatment works up to 30 MG/d	2006	0.834	2.330
4. Testwood water treatment works up to 40 MG/d	not required		
5. Test ground water augmentation	not required		
6. New Itchen water treatment works and trunk main	1990	3.531	4.510
TOTAL		4.365	

* all costs are in terms of 1981 price level and include allowance for perpetual replacement.

Table 4.11

WATER RESOURCE ALLOCATION: SOUTHERN & CENTRAL HANTS DEMAND FORECAST "C" OPTION I 1986-2011

	1986	1991	1996	2001	2006	2011
1- Timsbury & Lyndhurst						
AVERAGE DEMAND	9.10	9.60	10.04	10.48	10.85	11.22
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	2.30	2.80	3.24	3.68	4.05	4.42
PEAK DEMAND	11.41	12.05	12.62	13.19	13.63	14.08
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	4.61	5.25	5.82	6.39	6.83	7.28
2- Rounhams						
AVERAGE DEMAND	34.86	38.27	41.63	44.99	47.24	49.49
SOURCES:						
Testwood	34.86	38.27	41.63	44.99	47.24	49.49
PEAK DEMAND	44.70	49.12	53.39	57.66	60.58	63.51
SOURCES:						
Testwood	44.70	49.12	53.39	57.66	60.58	63.51
3- ABBOTSTONE/TOTFORD						
AVERAGE DEMAND	4.77	4.83	4.89	4.96	5.01	5.06
SOURCES:						
Easton/Totford/Barton Stacey	4.77	4.83	4.89	4.96	5.01	5.06
PEAK DEMAND	5.81	5.89	5.97	6.05	6.11	6.17
Easton/Totford/Barton	5.81	5.89	5.97	6.05	6.11	6.17
4- Winchester						
AVERAGE DEMAND	15.61	16.18	16.74	17.31	17.80	18.29
SOURCES:						
Easton	15.61	16.18	16.74	17.31	17.40	17.40
Ottr. via Yewhill	--	--	--	--	0.40	0.89
PEAK DEMAND	18.88	19.56	20.24	20.93	21.51	22.10
SOURCES:						
Easton	18.88	19.56	20.24	20.93	21.11	21.21
Ottr.via Yewhill	--	--	--	--	0.40	0.89

Table 4.11 Cont.

	1986	1991	1996	2001	2006	2011
5- Yewhill						
AVERAGE DEMAND	12.25	12.67	13.04	13.42	13.76	14.11
+ Supply to Winchester	--	--	--	--	0.40	0.89
Total	12.25	12.67	13.04	13.42	14.16	15.00
SOURCES:						
Otterbourn (ground & surface)	12.25	12.67	13.04	13.42	14.16	15.00
PEAK DEMAND	14.82	15.34	15.80	16.25	16.67	17.08
+ Supply to Winchester	--	--	--	--	0.40	0.89
Total	14.82	15.34	15.80	16.25	17.07	17.97
SOURCES:						
Otterbourn (ground and surface)	14.82	15.34	15.80	16.25	17.07	17.97
6- Otterbourn						
AVERAGE DEMAND	20.31	21.00	21.70	22.41	23.00	23.60
SOURCES:						
Otterbourn (ground and surface)	21.50	22.30	23.00	23.80	24.40	25.10
PEAK DEMAND	24.59	25.40	26.25	27.09	27.81	28.52
SOURCES:						
Otterbourn (ground and surface)	24.59	25.40	26.25	27.09	27.81	28.52
7- Twyfords & Moorhill						
AVERAGE DEMAND	35.06	38.31	41.94	45.58	47.89	50.21
SOURCES:						
Twyford/Twyford Moors	29.50	29.50	29.50	29.50	29.50	29.50
Otterbourn (ground and surface)	5.56	8.81	12.44	16.08	18.39	20.71
Lower Itchen	--	--	--	--	--	--
PEAK DEMAND	42.39	46.34	50.81	55.29	58.00	60.72
SOURCES:						
Twyford/Twyford Moors	30.40	30.40	30.40	30.40	30.40	30.40
Otterbourn	11.99	15.94	20.41	24.89	27.60	30.32
Lower Itchen	--	--	--	--	--	--
8- Southampton						
AVERGE DEMAND	38.81	40.24	41.76	43.29	44.48	45.68
SOURCES:						
Otterbourn (ground and surface)	38.81	40.24	41.76	43.29	44.48	43.09
Testwood	--	--	--	--	--	2.59
PEAK DEMAND	47.00	48.69	50.53	52.36	53.92	55.49
SOURCES:						
Otterbourn (ground and surface)	47.00	45.72	39.94	34.17	29.92	25.59
Testwood	--	2.97	10.59	18.19	24.00	29.90

Table 4.11 cont.

	1986	1991	1996	2001	2006	2011
Total Supplies From Otterbourn						
AVERAGE DAY:						
To Ywehill	12.25	12.67	13.04	13.42	14.16	15.00
To Twyford/Moorhill	5.56	8.81	12.44	16.08	18.39	20.71
To Otterbourn & Southampton	59.12	61.24	63.46	65.70	67.48	66.69
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	88.23	94.02	100.24	106.50	111.33	113.70
PEAK DEMAND:						
To Ywehill	14.82	15.34	15.80	16.25	17.07	17.97
To Twyford/Moorhill	11.99	15.94	20.41	24.89	27.60	30.32
To Otterbourn & Southampton	71.59	71.72	66.19	61.26	57.73	54.11
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	113.70	113.70	113.70	113.70	113.70	113.70
8- SUPPLIES FROM TESTWOOD						
AVERAGE DAY:						
To Rounhams	34.86	38.27	41.63	44.99	47.24	49.49
To Timsbury/lynhurst	2.30	2.80	3.24	3.68	4.05	4.42
To Southampton	--	--	--	--	--	2.59
TOTAL	37.16	41.07	44.87	48.67	51.29	56.50
PEAK DAY:						
To Rounhams	44.70	49.12	53.39	57.66	60.58	63.51
To Timsbury/lynhurst	4.61	5.25	5.82	6.39	6.83	7.28
To Southampton	--	2.97	10.59	18.19	24.00	29.90
SUB TOTAL	49.31	57.34	69.80	82.24	91.41	100.69
NON-POTABLE:						
To ESSO & Iow	41.50	43.00	44.40	46.50	49.20	52.00
TOTAL	90.81	100.34	114.20	128.74	140.61	152.69

Table 4.12

Water Supply Central Investment Programme
Demand Forecast C (Hants)
Strategy I
1986 - 2011

Project	Commissioning Date	Discounted Cost £m	Undiscounted Cost £m
1. Yew hill to Winchester trunk main	2003	0.213	0.514
2. Testwood to Southampton trunk main	1997	0.400	0.720
3. Testwood water treatment works up to 30 MG/d	1996	1.360	2.330
4. Testwood water treatment works up to 40 MG/d	2005	1.519	4.04
5. Test ground water augmentation	2002	1.380	3.167
6. New Itchen water treatment works and trunk main		not required	
TOTAL		4.872	

* all costs are in terms of 1981 price level and include allowance for perpetual replacement.

Table 4.13

WATER RESOURCE ALLOCATION: SOUTHERN & CENTRAL HANTS
DEMAND FORECAST "D" OPTION I 1986-2011

	1986	1991	1996	2001	2006	2011
1- Timsbury & Lyndhurst						
AVERAGE DEMAND	9.20	9.71	10.16	10.61	10.98	11.36
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	2.40	2.91	3.36	3.81	4.18	4.56
PEAK DEMAND	11.64	12.28	12.87	13.45	13.91	14.37
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	4.84	5.48	6.07	6.65	7.11	7.57
2- Rounhams						
AVERAGE DEMAND	35.30	38.76	42.18	45.60	48.35	50.19
SOURCES:						
Testwood	35.30	38.76	42.18	45.60	48.35	50.19
PEAK DEMAND	45.62	50.15	54.54	58.92	61.92	64.91
SOURCES:						
Testwood	45.62	50.15	54.54	58.92	61.92	64.91
3- ABBOTSTONE/TOTFORD						
AVERAGE DEMAND	4.82	4.89	4.95	5.02	5.07	5.12
SOURCES:						
Easton/Totford/Barton Stacey	4.82	4.89	4.95	5.02	5.07	5.12
PEAK DEMAND	5.88	6.00	6.08	6.17	6.23	6.29
Easton/Totford/Barton	5.88	6.00	6.08	6.17	6.23	6.29
4- Winchester						
AVERAGE DEMAND	15.79	16.36	16.93	17.40	18.00	18.50
SOURCES:						
Easton	15.79	16.36	16.93	17.40	17.40	17.40
Ottr. via Yewhill	--	--	--	--	0.60	1.10
PEAK DEMAND	19.22	19.92	20.62	21.31	21.91	22.51
SOURCES:						
Easton	19.22	19.92	20.62	21.31	21.31	21.49
Ottr.via Yewhill						

Table 4.13 Cont.

	1986	1991	1996	2001	2006	2011
5- Yewhill						
AVERAGE DEMAND	12.43	12.85	13.23	13.61	13.96	14.32
+ Supply to Winchester	--	--	--	--	0.60	1.10
Total	12.43	12.85	13.23	13.61	14.56	15.42
SOURCES:						
Otterbourn (ground & surface)	12.43	12.85	13.23	13.61	14.56	15.42
PEAK DEMAND	15.14	15.67	16.14	16.61	17.04	17.47
+ Supply to Winchester	--	--	--	--	0.60	1.10
Total	15.14	15.67	16.14	16.61	17.64	18.57
SOURCES:						
Otterbourn (ground and surface)	15.14	15.67	16.14	16.61	17.64	18.57
6- Otterbourn						
AVERAGE DEMAND	20.55	21.25	21.97	22.68	23.29	23.90
SOURCES:						
Otterbourn (ground and surface)	20.55	21.25	21.97	22.68	23.29	23.90
PEAK DEMAND	25.06	25.88	26.74	27.61	28.34	29.07
SOURCES:						
Otterbourn (ground and surface)	25.06	25.88	26.74	27.61	28.34	29.07
7- Twyfords & Moorhill						
AVERAGE DEMAND	35.53	38.84	42.61	46.39	48.67	50.95
SOURCES:						
Twyford/Twyford Moors	29.50	29.50	29.50	29.50	29.50	29.50
Otterbourn (ground and surface)	6.03	9.34	13.11	16.89	19.17	21.45
Lower Itchen	--	--	--	--	--	--
PEAK DEMAND	43.27	47.31	51.91	56.51	59.28	62.07
SOURCES:						
Twyford/Twyford Moors	30.40	30.40	30.40	30.40	30.40	30.40
Otterbourn	12.87	16.91	21.51	26.11	28.88	31.67
Lower Itchen	--	--	--	--	--	--
8- Southampton						
AVERAGE DEMAND	39.15	40.59	42.16	43.68	44.99	46.29
SOURCES:						
Otterbourn (ground and surface)	39.15	40.59	42.16	43.68	44.99	41.63
Testwood	--	--	--	--	--	4.66
PEAK DEMAND	47.71	49.46	51.32	53.19	54.78	56.37
SOURCES:						
Otterbourn (ground and surface)	47.71	43.94	38.01	32.07	27.54	23.09
Testwood	--	5.52	13.31	21.12	27.24	33.28

Table 4.13 cont.

	1986	1991	1996	2001	2006	2011
Total Supplies From Otterbourn						
AVERAGE DAY:						
To Ywehill	12.43	12.85	13.23	13.61	14.56	15.42
To Twyford/Moorhill	6.03	9.34	13.11	16.89	19.17	21.45
To Otterbourn & Southampton	59.70	61.84	64.13	66.36	68.28	65.53
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	89.46	95.33	101.77	108.16	113.31	113.70
PEAK DEMAND:						
To Ywehill	15.14	15.67	16.14	16.61	17.64	18.57
To Twyford/Moorhill	12.87	16.91	21.51	26.11	28.88	31.67
To Otterbourn & Southampton	72.77	69.82	64.75	59.68	55.88	52.16
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	112.08	113.70	113.70	113.70	113.70	113.70
8- SUPPLIES FROM TESTWOOD						
AVERAGE DAY:						
To Rounhams	35.30	38.76	42.18	45.60	47.89	50.19
To Timsbury/lynhurst	2.40	2.91	3.36	3.81	4.18	4.56
To Southampton	--	--	--	--	--	4.66
TOTAL	37.70	41.67	45.54	49.41	52.07	59.41
PEAK DAY:						
To Rounhams	45.62	50.15	54.54	58.92	61.92	64.91
To Timsbury/lynhurst	4.84	5.48	6.07	6.65	7.11	7.57
To Southampton	--	5.52	13.31	21.12	27.24	33.28
SUB TOTAL	50.46	61.15	73.92	86.69	96.27	105.76
NON-POTABLE:						
To ESSO & Iow	41.50	43.00	44.40	46.50	49.20	52.00
TOTAL	91.96	104.15	118.32	133.19	145.47	157.76

Table 4.14

Water Supply Central Investment Programme
Demand Forecast D (Hants)
Strategy I
1986 - 2011

Project	Commissioning Date	Discounted Cost £m	Undiscounted Cost £m
1. Yew hill to Winchester trunk main	2002	0.224	0.514
2. Testwood to Southampton trunk main	1996	0.420	0.720
3. Testwood water treatment works up to 30 MG/d	1994	1.500	2.330
4. Testwood water treatment works up to 40 MG/d	2003	1.676	4.04
5. Test ground water augmentation	2000	1.523	3.167
6. New Itchen water treatment works and trunk main		not required	
TOTAL		5.343	

* all costs are in terms of 1981 price level and include allowance for perpetual replacement.

Table 4.15

WATER RESOURCE ALLOCATION: SOUTHERN & CENTRAL HANTS DEMAND FORECAST "E" OPTION I 1986-2011

	1986	1991	1996	2001	2006	2011
1- Timsbury & Lyndhurst						
AVERAGE DEMAND	9.00	9.49	9.63	9.77	10.42	11.08
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	2.20	2.69	2.83	2.97	3.62	4.28
PEAK DEMAND	11.21	11.83	12.38	12.93	13.36	13.80
SOURCES:						
Timsbury	6.80	6.80	6.80	6.80	6.80	6.80
Testwood	4.41	5.03	5.58	6.13	6.56	7.00
2- Rounhams						
AVERAGE DEMAND	34.43	37.78	40.91	44.05	46.42	48.80
SOURCES:						
Testwood	34.43	37.78	40.91	44.05	46.42	48.80
PEAK DEMAND	43.78	48.09	52.25	56.41	59.25	62.10
SOURCES:						
Testwood	43.78	48.09	52.25	56.41	59.25	62.10
3- ABBOTSTONE/TOTFORD						
AVERAGE DEMAND	4.71	4.78	4.84	4.91	4.96	5.00
SOURCES:						
Easton/Totford/Barton Stacey	4.71	4.78	4.84	4.91	4.96	5.00
PEAK DEMAND	5.70	5.78	5.86	5.94	6.00	6.06
Easton/Totford/Barton	5.70	5.78	5.86	5.94	6.00	6.06
4- Winchester						
AVERAGE DEMAND	15.43	16.00	16.56	17.12	17.60	18.08
SOURCES:						
Easton	15.43	16.00	16.56	17.12	17.40	17.40
Ottr. via Yewhill	--	--	--	--	0.20	0.68
PEAK DEMAND	18.52	19.20	19.87	20.55	21.12	21.70
SOURCES:						
Easton	18.52	19.20	19.87	20.55	20.92	21.02
Ottr.via Yewhill	--	--	--	--	0.20	0.68

Table 4.15 Cont.

	1986	1991	1996	2001	2006	2011
5- Yewhill						
AVERAGE DEMAND	12.08	12.49	12.85	13.22	13.56	13.90
+ Supply to Winchester	--	--	--	--	0.20	0.68
Total	12.08	12.49	12.85	13.22	13.76	14.58
SOURCES:						
Otterbourn (ground & surface)	12.08	12.49	12.85	13.22	13.76	14.58
PEAK DEMAND	14.50	15.01	15.45	15.90	16.30	16.70
+ Supply to Winchester	--	--	--	--	0.20	0.68
Total	14.50	15.01	15.45	15.90	16.50	17.38
SOURCES:						
Otterbourn (ground and surface)	14.50	15.01	15.45	15.90	16.50	17.38
6- Otterbourn						
AVERAGE DEMAND	20.07	20.76	21.45	22.14	22.73	33.32
SOURCES:						
Otterbourn (ground and surface)	20.07	20.76	21.45	22.14	22.73	33.32
PEAK DEMAND	24.13	24.92	25.74	26.57	27.27	27.97
SOURCES:						
Otterbourn (ground and surface)	24.13	24.92	25.74	26.57	27.27	27.97
7- Twyfords & Moorhill						
AVERAGE DEMAND	34.58	37.79	41.42	45.06	47.25	49.46
SOURCES:						
Twyford/Twyford Moors	29.50	29.50	29.50	29.50	29.50	29.50
Otterbourn (ground and surface)	5.08	8.29	11.92	15.56	17.75	19.96
Lower Itchen	--	--	--	--	--	--
PEAK DEMAND	41.51	45.35	49.71	54.07	56.71	59.36
SOURCES:						
Twyford/Twyford Moors	30.40	30.40	30.40	30.40	30.40	30.40
Otterbourn	11.11	14.95	19.31	23.67	26.31	28.96
Lower Itchen	--	--	--	--	--	--
8- Southampton						
AVERGE DEMAND	38.47	39.89	41.40	42.91	44.18	45.46
SOURCES:						
Otterbourn (ground and surface)	38.47	39.89	41.40	42.91	44.18	44.54
Testwood	--	--	--	--	--	0.92
PEAK DEMAND	46.24	47.94	49.74	51.54	53.07	54.61
SOURCES:						
Otterbourn (ground and surface)	46.24	47.52	41.90	36.26	32.32	28.09
Testwood	--	0.42	7.84	15.28	20.75	26.52

Table 4.15 cont.

	1986	1991	1996	2001	2006	2011
Total Supplies From Otterbourn						
AVERAGE DAY:						
To Ywehill	12.08	12.49	12.85	13.22	13.76	14.58
To Twyford/Moorhill	5.08	8.29	11.92	15.56	17.75	19.96
To Otterbourn & Southampton	58.54	60.65	62.85	65.05	66.91	67.86
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	87.00	92.73	98.92	105.13	109.72	113.70
PEAK DEMAND:						
To Ywehill	14.50	15.01	15.45	15.90	16.50	17.38
To Twyford/Moorhill	11.11	14.95	19.31	23.67	26.31	28.96
To Otterbourn & Southampton	70.37	72.44	67.64	62.83	59.59	56.06
+ Supplies from Twyford Moors	11.30	11.30	11.30	11.30	11.30	11.30
TOTAL	107.28	113.70	113.70	113.70	113.70	113.70
8- SUPPLIES FROM TESTWOOD						
AVERAGE DAY:						
To Rounhams	34.43	37.78	40.91	44.05	46.42	48.80
To Timsbury/lynhurst	2.20	2.69	2.83	2.97	3.62	4.28
To Southampton	--	--	--	--	--	0.92
TOTAL	36.63	40.47	43.74	47.02	50.04	54.00
PEAK DAY:						
To Rounhams	43.78	48.09	52.25	56.41	59.25	62.10
To Timsbury/lynhurst	4.41	5.03	5.58	6.13	6.56	7.00
To Southampton	--	0.42	7.84	15.28	20.75	26.52
SUB TOTAL	48.19	53.54	65.67	77.82	86.56	95.62
NON-POTABLE:						
To ESSO & Iow	41.50	43.00	44.40	46.50	49.20	52.00
TOTAL	89.69	96.54	110.07	124.32	135.76	147.62

Table 4.16

Water Supply Central Investment Programme
Demand Forecast E (Hants)
Strategy I
1986 - 2011

Project	Commissioning Date	Discounted Cost £m	Undiscounted Cost £m
1. Yew hill to Winchester trunk main	2005	0.193	0.514
2. Testwood to Southampton trunk main	1999	0.363	0.720
3. Testwood water treatment works up to 30 MG/d	1998	1.234	2.330
4. Testwood water treatment works up to 40 MG/d	2007	1.377	4.04
5. Test ground water augmentation	2004	1.250	3.167
6. New Itchen water treatment works and trunk main		not required	
TOTAL		4.426	

* all costs are in terms of 1981 price level and include allowance for perpetual replacement.

Table 4.17

Optimum Central System Operating Costs
Average day in Hants
£ per day

Year	Strategy I A	Strategy I B	Strategy I C	Strategy II A	Strategy II B	Strategy II C
1986	2847	2847	2694	2847	2847	2694
1987	2884	2804	2729	2884	2804	2729
1988	2921	2747	2763	2921	2747	2763
1989	2959	2697	2797	2959	2697	2797
1990	2996	2647	2833	2996	2647	2833
1991	3036	2597	2869	3036	2597	2869
1992	3072	2633	2903	3072	2633	2903
1993	3110	2669	2938	3110	2669	2938
1994	3146	2705	2972	3150	2705	2972
1995	3185	2739	3007	3192	2739	3007
1996	3222	2776	3041	3232	2776	3041
1997	3261	2810	3075	3274	2810	3076
1998	3299	2845	3109	3315	2845	3113
1999	3337	2875	3144	3357	2875	3151
2000	3376	2914	3179	3399	2914	3189
2001	3418	2948	3214	3441	2948	3227
2002	3448	2974	3239	3471	2974	3253
2003	3478	2999	3265	3500	3001	3281
2004	3508	3025	3290	3530	3029	3308
2005	3538	3051	3316	3560	3056	3336
2006	3567	3076	3341	3589	3084	3363
2007	3597	3102	3367	3618	3111	3390
2008	3626	3127	3392	3647	3138	3417
2009	3656	3152	3418	3676	3165	3444
2010	3685	3177	3446	3705	3192	3471
2011	3714	3202	3473	3735	3219	3499

* All cost figures are in terms of 1981 price level.
* Cost figures are to be taken in comparative terms rather than in absolute terms since some 'running' costs have been excluded (see text).

(4.3) Average Incremental Marginal Capital Cost of 'Central' Capacity of The Water Supply System in Southern and Central Hampshire

What is the effect upon future central system capital costs of water supply of a specific increment or decrement in projected annual average daily demand starting and ending at specified dates? Questions of this nature are asked by planners involved in the analysis of wide ranging supply and management issues in the water industry. Similar questions are asked for example when formulating (1) leakage detection and control policies, (2) examining the benefits and costs of metering domestic and other consumers, (3) formulating forward looking 'marginal cost' pricing policies, and (4) assessing other demand-management options such as regulation and education policies.

The cost minimising model of the previous section provides a suitable framework for the analysis of the impact of changes in forecast demand for water supply on total discounted capital costs of the central part of the water supply system. It also serves to focus attention on the kind of questions to ask about the increment or decrement to be costed such as its size, duration and peak characteristics and so on.

The optimal capital expenditure programme of the central part of the water supply system for each of the demand forecasts A,B,C,D, and E in Hampshire has already been established using the cost minimising model of the last section. These results are reported in tables 4.4, 4.8, 4.12, 4.14 and 4.16 .

We now can restate these results in a fashion suitable for answering the question in the opening paragraph of this section. It can be argued that a 26 year (1985 to 2011) decrement in average day demand as that in the difference between demand forecast A and C, with the corresponding reduction in peak week day demand, results in a reduction in total discounted capital cost from £7.103 million to £4.872 million achieving a saving of £2.231 million.[13] Adjusting the figure from 1981 to 1985 price level (assuming a 3% overall rise)[14] we get £2.297 million. In other words a saving equal to £2.297 million would arise, due to delaying the project construction programme, if demand were to fall from the A forecast to the C forecast, a reduction which could arise following the metering of domestic consumers resident in the southern and central parts of Hampshire.[15]

An alternative way of presenting the same information is possible in terms of what was termed in chapter 2 as the average incremental marginal capital cost (AIMC) (of central capacity[16]. AIMC can be defined as :

$$\text{AIMC} = \frac{\text{Annual equivalent change in central investment}}{\text{Annual equivalent change in average demand}}$$

$$\text{or} \quad = \frac{\text{Change in discounted capital cost of central inv.}}{\text{Discounted sum of average demand change}}$$

Both the numerator and denominator of either of the above definitions are readily available. The former, reduction in discounted capital cost, has been estimated for A-C to be equal to £2.297 million and the latter, the discounted sum of average day demand reduction A-C is equal to 168.97 ML/d.[17] Accordingly AIMC of central investment when based on A-C demand reduction is :

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13 We note that the investment figures allow for replacement. Postponing for a year a new water treatment plant will postpone its retirement date for a year and in turn postpone all its replacement dates for a year.

14 This follows Cooper and Lybrand,1985, op.cit.

15 Other savings arising in from reduced distribution investment and reduced investment in sewerage and sewage disposal may of course also arise.

16 In this section this will be taken for granted without having to specify central part of the water supply system every time.

17 Of course using the same discount rate (5%) as that used for costs. The figure of 168.97 ML/d has been derived as the discounted difference between demand forecasts A and C for Hants given in tables 3.23 and 3.27.

$$\frac{2.297 \times 10^8}{168.97 \times 1000 \times 365} = 3.72 \text{ P/M}^3 \text{ (37.20 £/ML)}$$

This will then be the proxy measure of marginal cost of central capacity a proxy which overcomes the definition problem of marginal cost in the presence of indivisibility. In a similar way an average incremental marginal capital cost of central capacity can be found for demand reduction as in A-D, A-E and A-B. The results are reported in table 4.18 below.

Table 4.18

Average Incremental Marginal Cost of Central Capacity in Hants

	Change in discounted Capital Cost £m	Discounted Sum of Water Change M^3	AIMC P/M^3
A-C	2.297	168.97x1000x365	3.72
A-D	1.812	134.08x1000x365	3.70
A-E	2.757	204.56x1000x365	3.69
A-B	3.611	359.97x1000x365	2.74

* All costs are at 1985 price level.
* Discounted capital costs are based on strategy I.

AIMC of central capacity is almost identical when based on A-C, A-D, and A-E decrements in average day demand and the corresponding peak week day demand. One is tempted to conclude from this that in the context of southern and central Hants AIMC of central capacity based on a reduction in domestic consumption from 1985 to 2011 is insensitive to the size of the said reduction. It is however our belief that this particular conclusion is perhaps a special case, peculiar to our case, which could not safely be generalised.

AIMC of central capacity of the water supply system when based on the A-B reduction in demand is seen to be markedly different from the other estimates. Indeed if this was not the case then we would have argued that the results were suspect. First we note that the B forecast is based on the achievement of the ambitious leakage control targets. As such the AIMC of central capacity when based on the A-B demand reduction measures the per unit cost saving due to the reduction in average day leakage. AIMC based on A-C on the other hand measures the per unit cost saving due to a reduction in average day demand in domestic consumption. Given that domestic consumption has a higher peak week factor than leakage and given that the commissioning of components of central capacity is timed according to peak week day demand and not average day demand, it follows that AIMC when based on A-C must be higher than AIMC when based on A-B. Put otherwise an increment in average demand of 1 M^3 superimposed on the expected trend of demand would cost more to provide if it emanates from domestic consumption compared with it emanating from leakage.[18] The former would require the provision of more capacity compared with the latter. If domestic demand and leakage have the same peak week factor we would then expect them to have equal AIMC. More generally different classes of demand have different peak characteristics and as such have different costs. This implies that when costing an increment or decrement in demand

[18] Of course the same can be said about other classes of demand with lower peak week factor then domestic consumption such as industrial demand.

it is not sufficient just to specify its size and duration. It is also necessary to specify its peak characteristics.

Further it is also important to note that the estimated AIMC of central capacity is appropriate for year-round uniform pricing and not for peak load seasonal pricing. In other words it is a marginal cost per unit of average (year-round) demand. It does not represent the marginal cost of meeting that part of demand which occurs during the peak. The latter we would a priori expect to be considerably greater then our average estimate. But to the extent that seasonal peak load pricing is still to date ruled out by metering limitations, then our AIMC per unit of average demand would be perfectly suited as input when setting prices based on long-run marginal cost principles.

(4.4) Average Incremental Cost of Distribution Capacity of The Water supply System in Hampshire

The distribution system, from the service reservoir downstream, usually claims a substantial proportion of the capital expenditure of a water supply enterprise. As such capacity costs of the distribution network must form an important part of overall long-run 'marginal cost' of water supply.

In principle long-run average incremental capital cost of distribution capacity of water supply should be estimated using the same broad principles applied in the estimation of AIMC of central capacity (see previous section). In particular we need to use a planning model to ascertain a change in total discounted cost of distribution investment in relation to a specific increment or decrement in average (and peak) demand imposed on the expected trend of demand.

The execution of such a procedure is not usually possible however. The nature of distribution investment precludes the kind of planning whereby a direct and clearly defined relationship between demand and investment can be readily identified. As pointed out in section 3.2 the distribution network differs from the central part of the system in that it expands through time through many 'small' investments, more or less on a continuous basis and with more decentralisation, giving the growth of the system as a whole a 'biological character'.[19]

The implication of this is that distribution investment is (a) planned only for a short period of time and (b) it is not immediately possible to define clearly a demand investment relationship for modelling purposes.[20]

In order to estimate long-run marginal cost of

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[19] Turvey,R.,1976,'Analayzing the marginal cost of water supply,' Land Economics, Vol.52, No.(2).

[20] In principle it should be possible, at least in some cases, to use network analysis for the purpose of planning distribution investment. We have not however come across such studies.

distribution capacity one must therefore proceed using a different methodology from the one used in the previous section for the estimation of average incremental marginal cost of central capacity. The proposed methodology(s) is essentially based on relating proposed expenditure on the distribution components of the water supply system (for whatever years for which data is available) to the expected growth in demand to be met by the system. The resulting estimate is in the nature of average incremental capital cost (AIC) rather then average incremental marginal cost (AIMC). These two measures are in principle different though in practice may well give the same answers.

Before outlining how to go about the estimation we have to point out that not all future capital expenditure on distribution is relevant to marginal cost. In particular some of this investment is invariant to the level of expected demand and as such should be left out. For example quality related and much of replacement investment would normally take place regardless of the level of expected growth in demand.

To isolate that part of the investment programme which is demand related is perhaps more easily said then done because it is usually the case that investment projects serve more than just one purpose. Thus a pipe due for replacement may very well be replaced by a larger one serving to meet expected demand growth. Cost in such instances is joint, between two (or more) purposes.

It is fortunate however, at least in the UK, that water authorities (including SWA) break up their total distribution investment according to purpose. Thus if a new main is judged to be 50% to meet demand growth and 50% to for replacement, a division using a great deal of judgment no doubt, then its capital cost would be divided between the two said purposes accordingly. There might also be a case for distinguishing between investment to reinforce the existing distribution system to cope with demand growth from existing consumers and investment to extend the distribution system to new consumers.

Demand related capital expenditure[21] on the distribution system for the Hampshire division, as estimated by SWA for the period 1986 to 1990 to meet demand forecast A (the central forecast), is reported in table 4.19 below. The table also reports average demand forecast, A, for the whole of the division for the years 1985 to 1990.

Table 4.19

Demand Related Distribution Investment in Hants Demand Forecast A

	Demand Related Capital Investment £1000	Average Demand A ML/d
1985		203.92
1986	929	206.40
1987	825	208.99
1988	1239	211.36
1989	1125	213.84
1990	1294	216.32

Source : SWA documentation, mimeo, and Cooper and Lybrand op.cit.

It is to be noted that SWA plans its distribution capital expenditure for five years ahead at any one moment of time. Also the reported figures in the table above represent about 30% of total distribution investment, the rest being for purposes other than to meet demand growth.

Average incremental capital cost of distribution capacity of water supply may be estimated from the data in table 4.19 using either of the following two methodologies .

[21] excluding extention investment.

AIC Methodology I

Average incremental capital cost of distribution may be estimated as the simple average of the World Bank's textbook marginal capacity cost for the years for which data is available.[22] Textbook marginal capacity cost was defined earlier :

$$MCC_t = \frac{A\ K_t}{Q_{t+1} - Q_t} \qquad (1)$$

where A is the annuity factor defined as before as:

$$A = \frac{i(1+i)^L}{(1+i)^{L+1}-1}$$ (L being the life of the asset and i the interest rate)

and K_t is investment in year t, Q_t is supply in year t. AIC is in turn defined as :

$$AIC = \frac{1}{n} \sum_{t=1}^{n} MCC_t$$

Assuming that the overwhelming part of the distribution assets have very long lives (40 years and over, see table 4.20) then the annuity factor may be approximated by i/(1+i). Using data in table 4.19 we can calculate AIC of distribution capacity per unit of average system demand as:

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22 See section (2.8) of chapter 2. It is to be noted that the same method was used by Turvey and Anderson for the estimation of average incremental cost of distribution and transmission for the Thai electricity industry. See : Turvey R. and Anderson,D.,1977, 'Electricity Economics, Essays and Case Studies,' A World Bank Research Publication, The John Hopkins University Press, Baltimore and London.

$$AIC = \frac{(5412)(10^8)(0.047)}{(5)(2.48)(10^6)(365)}$$

$$= 5.68 \text{ P/M3}$$

It must be stressed that the estimated unit cost is an average of marginal costs not only over a number of years but also over various demand zones, thus averaging any differences in distribution costs as between for example rural and urban areas, low and high areas and so on. Such an averaging process may be rationalised, when the resulting estimate is used as an input in a tariff structure, by the argument that metered charges more than often are constrained to be uniform. When the estimate is used for other purposes, eg in the evaluation of the economics of domestic metering, such an averaging process may be inappropriate. This would be so if for example the AIC of distribution capacity for the overwhelming proportion of domestic consumers is higher than the average estimate. This may be the case if the overwhelming number of domestic consumers are town dwellers and if distribution costs of meeting demand growth are high on account of for example high reinstatement costs.

Table 4.20

Typical Asset Lives of Water Supply Components

Water Mains		
	Tunnels	150 years
	Mains not made from PVC, lives depending on diameter	80-125 years
	All PVC	40 years
Operational Structures		
	All dams	150 years
	Service reservoirs	80 years
	Reinforced concrete water towers	40 years
Plant and Equipment		
	Non-powered river barges	60 years
	Hydraulic and electro-magnetic meters	40 years
	Static pumps etc	20 years
	Mechanical pumps	15 years

Source : Thorpe,B.R.,1986, 'Current Value Rate Base: The Approach in England and Wales,' Annual Conference Proceedings of the American Water Works Association,1986,AWWA publication,1986.

AIC Methodology II

This methodology may be described as simply deriving the weighted average of textbook marginal capacity costs, the weights being the discount factors. Thus given a linear demand growth (as is the case for Hants for the period 1985 to 1990) we have:

$$MCC_t = \frac{A\,K_t}{\Delta Q}$$

$$AIC = \frac{\dfrac{A}{\Delta Q} \displaystyle\sum_{1}^{n} \frac{K_t}{(1+i)^t}}{\displaystyle\sum_{1}^{n} \frac{1}{(1+i)^t}}$$

and if we let

$$\sum_{1}^{n} \frac{K_t}{(1+i)^t} = TDCC$$

then

$$AIC = \frac{A\ TDCC}{\displaystyle\sum_{1}^{n} \frac{\Delta Q}{(1+i)^t}}$$

$$\text{or } AIC = \frac{\text{Annuitised value of total discounted capital cost}}{\text{Discounted sum of annual growth}}$$

We also note that if the annuity factor is taken to be equal to i/(1+i) then AIC becomes :

$$AIC = \frac{TDCC - TDCC/(1+i)}{\sum_{1}^{n} \frac{\Delta Q}{(1+i)^t}}$$

The last equation offers another line of interpretation of average incremental cost of distribution capacity. The numerator may now be interpreted as the reduction in total discounted capital cost following the stepping back of demand by one year.[23] The denominator is the discounted sum of the demand change. This approach, it might be argued, is similar to that used for the derivation of AIMC of central capacity the difference being the duration of the long-run.

The AIC of distribution capacity according to this methodology and using the available Hants data is equal to 5.53 P/M^3 . It is noted that this is not markedly different from the earlier estimate.

One final remark is in order before concluding this section. We have seen in the previous section that an increment of 1 M^3 superimposed on the expected demand trend of demand would cost more to provide if it emanates from domestic demand compared with it emanating from another class of demand with smaller peak factors since the former would require the provision of more capacity. The same argument must surely apply in the case of average incremental costs of distribution capacity; the AIC of distribution capacity is higher for those classes of demand with high peak ratios compared with those with lower peak ratios.

The above argument implies that the estimates derived so far, based on overall average levels of demand, must be adjusted if they are to reflect the cost of providing supplies to different consumer classes with different peak

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23 In principle stepping demand back one year would require reoptimisation to derive a new optimum capital expenditure programme which could in principle involve changes other than simple delay of all projects in the programme by one year.

characteristics. The relevant peak factors for distribution capacity depend on the particular components (see table 3.1); service reservoirs are driven by peak week/ peak day factors, and pipes by peak hour/quarter hour factors.

To adjust estimates of AIC of distribution capacity to reflect the cost to the various classes the estimates derived above must be multiplied by the following ratio (for the case of domestic demand) :[24]

$$\frac{\text{the relevant domestic peak factor}}{\text{the corresponding system peak factor}}$$

For example purposes we assume that for Hants domestic and system peak factors are as those reported below :

	peak week	hour peak
domestic	1.65	3.0
system	1.24	1.90

We further assume that 90% of distribution investment is composed of pipes the installation of which is driven by peak hour factors and the remainder 10% comprises investment driven by the peak week factor[25] then

$$\begin{aligned} \text{AIC (domestic)} &= (5.53)(0.9)(3.0/1.9) + (5.53)(0.1)(1.65/1.24) \\ &= 8.58 \text{ P/M}^3 \end{aligned}$$

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24 This procedure assumes that the class of consumers for which we are deriving AIC has its peak at the same time as the system peak overall. If this is not the case then we must adjust the peak factors to allow for the fact that what matters is consumption at the time of the system peak.

25 All these assumptions are highly speculative and are essentially reported here for illustration.

(4.5) Marginal Operating Costs of Water Supply in Southern and Central Hampshire

The discussion so far has been confined to those capacity costs of water supply the timing and sizing of which is a function of maximum (peak) demand. These costs form the basis of marginal capacity costs of supply. Capacity costs however are only one component of long-run marginal cost of water supply. The second component is the marginal running (operation) costs of supply.

Marginal running costs of supply comprise the extra power and chemical cost expended on the provision of an extra unit of supply. These costs include only those operation costs whose level is a function of output. 'Operation' costs such as labour and maintenance, the level of which is invariant to the level of supply, must therefore be excluded. Running costs in the nature of financial transfers with no allocative consequences must also be excluded.

The estimation of marginal running costs of supply is complicated by the fact that marginal cost is a multi-dimensional concept. Marginal costs will depend on where in the system the additional unit of supply is demanded, during which season of the year it is being required and in which year it is to take place. If marginal running costs are to be estimated as the expected increase in annual running costs of the whole system in a particular year divided by the expected increase in the annual quantity of supply for the whole system for the same year then we would be averaging out possible variations in marginal running costs arising from : (a) seasonal variation in running costs caused by variations in running costs as between average and peak demand, (b) zonal variation in marginal running costs.

Seasonal and geographical variations in marginal running costs would need to be averaged out to the extent that a uniform marginal running cost is required, say for pricing purposes. Marginal running costs may vary from one year to another because of (1) changes in the real prices of chemicals and power, (2) running costs are not linear with output and (3) changes over time in the marginal source(s) of supply. Changes in marginal running costs over time, if any, would need to be averaged if a uniform marginal cost, for a number of years, is required. Averaging out differences in the yearly marginal running costs can be carried out in a similar fashion to the way of averaging out differences in yearly textbook marginal capacity costs as derived in the previous section. That is we can either derive a simple average or a weighted average of the yearly marginal running costs, the weights in the latter being the discount factors. These definitions are illustrated in the following equations.

$$MOC_t = \frac{R_{t+1} - R_t}{Q_{t+1} - Q_t} \qquad 4.1$$

$$AIC = \frac{\sum \frac{MOC_t}{(1+i)^t}}{\sum \frac{1}{(1+i)^t}} \qquad 4.2$$

$$\text{or} \qquad AIC = 1/n \sum MOC_t$$

These complications are demonstrated in the case of central and southern Hamspshire. Indeed marginal running costs in Hants depend on where in the system the marginal unit of supply is being demanded, at which year it is required and during which season of the year it is to take place. Several examples will serve to illustrate the nature of this multi-dimensionality. Marginal running cost of average day demand depends on the demand zone in question. Thus in 1987 for example marginal running costs for the demand zones in southern and central Hants are reported in table 4.21 below.

Table 4.21

Zonal Marginal Running Costs Hants
Average Day Demand 1987

Demand Zone	Marginal Source of Supply	MC £/ML
(1) Timsbury & Lyndhurst	Testwood	15.80
(2) Rounhams	Testwood	15.80
(3) Otterbourn	Otterbourn	11.60-12.20
(4) Twyford & Moorhill	Otterbourn	15.30-15.80
(5) Yew hill	Otterbourn	19.50-20.10
(6) Southampton	Otterbourn	13.00-13.60
(7) Tottford & Abbotstone	Tottford	11.00
(8) Winchester	Easton	15.50

Thus an increment in of 1 ML in average day demand would cost £11 in power and chemicals if it were to arise in Tottford and Abbotstone demand zone ; £15.50 if it were to arise in Winchester demand zone ; £15.80 in Rounhams and so on. These differences would be averaged out if marginal running cost is derived for the whole system in 1987 as in equation 4.1 above. This average estimate would be appropriate only if uniformity is required.

Marginal running costs for peak week demand for some of the demand zones in central and southern Hants would be different from the corresponding marginal costs for average day demand. This seasonal dimension of marginal costs is illustrated by examination of table 4.22 containing the marginal running costs during the peak week.

Table 4.22

Zonal Marginal Running Costs Hants
Peak Week Demand 1987

Demand Zone	Marginal Source of Supply	MC £/ML
(1) Timsbury & Lyndhurst	Testwood	15.80
(2) Rounhams	Testwood	15.80
(3) Otterbourn	Testwood(indirectly)	14.40
(4) Twyford & Moorhill	Testwood(indirectly)	18.00
(5) Yew hill	Testwood(indirectly)	22.30
(6) Southampton	Testwood	15.80
(7) Tottford & Abbotstone	Easton	15.50
(8) Winchester	Easton	15.50

The difference in running costs between average and peak demand is apparent, the latter is generally higher then the former (at least no less). It is also clear that supply during the peak week is dominated by the marginal source of river supply from the river Test at the Testwood site. It is directly the marginal source of supply to each of the demand zones of Southampton, Rounhanms and Timsbury and Lyndhurst. In these cases marginal running costs are simply the extra running cost of supply from Testwood to the respective demand zones. There are no complications of system interdependence to take account of.

A 1 ML increment in peak week demand in for example the demand zone of Otterbourn will be met from the marginal source of Testwood indirectly. Such an increment will in the first place be met by extra supply from the Otterbourn sources. But in doing so it denies Southapmton Common demand zone a 1 ML of supplies from Otterbourn, forcing it to fall back on Testwood for the 1 ML cut. Thus the extra 1 ML of peak week demand in 1987 at Otterbourn is met by substitution from the marginal source of Testwood. In situations like this some care must be taken when deriving the relevant marginal running cost. Because of this kind of system interdependence[26] it is always safer to estimate marginal running costs using a cost minimisation model of the type used in section 4.2 . First we find the minimum total running cost, for 1987, given peak week demand forecast A. In the next step we repeat the same exercise also using demand forecast A but with, for example, Otterbourn demand increased by 1 ML. The difference in total running costs as between the first and the second run is obviously the extra running cost of meeting a 1 ML of supply in Otterbourn during the peak week in 1987. This is of course the required estimate of marginal running cost of peak supply in Otterbourn in 1987. This has been estimated to be equal to £14.40 per ML. Indeed this figure can be seen as simply :

(£11.60 - £13.00) + £15.80 = £14.40

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[26] Of course the degree of system interdependence in our case is rather simple. In other cases it could be much more involved.

or the cost of 1 ML from the Otterbourn ground source to Otterbourn demand zone (£11.60) less the cost (saved) of 1 ML less supplies from Otterbourn ground to Southampton Common demand zone (£13.00) plus the extra costs of 1 ML of supplies from Testwood to Southampton to substitute for the Otterbourn cut (£15.80).

Using similar analysis the rest of marginal running costs of peak week demand for the other demand zones can be easily ascertained. The results are listed in table 4.22.

Moreover marginal costs for average day demand for the different demand zones will also depend on the year in question. The resources of Otterbourn (ground , river and Twyford moor ground) are expected to be exhausted according to demand forecast A and central investment option 1[27] by year 2001 (see table 4.3). From then onwards Testwood becomes the main marginal source of supply (either directly or indirectly) in southern and central Hants. This will cause variations in marginal costs of average demand in between the years, for at least some of the zones. The relevant marginal cost for each of the demand zones for different years can be estimated using the same method outlined above for peak demand. The point to note that even after averaging the differences in marginal running costs between the demand zones for a specific year, differences between the estimates of this average for various years will not disappear.

A selection of average system-wide marginal running cost of average demand for a number of randomly selected years, calculated as in equation 4.1 above, are reported in table 4.23 below.

27 See discussion in section 4.2.

Table 4.23

System Marginal Cost of Average Demand
Southern and Central Hants

Year	Marginal Running Cost £/ML
1987	13.80
1996	15.04
2000	15.71
2006	15.84
2010	15.84

As expected the reported system marginal running costs of average demand increase with the pass of time (at least up to a point). This reflects the fact that with the passage of time (growth of demand) more and more of the expensive sources of supply will need to be brought into the system or/and be used more extensively. We also note that some time in the future (around year 2000) the system wide marginal running costs of average demand are not markedly different from that of abstraction, treatment and pumping to supply at Testwood. This is to be expected since Testwood dominates as almost the exclusive marginal source of supply around year 2000. The last observation indicates that the simplifying assumption that running costs of Testwood represent the system wide marginal running cost is less damaging (1) the earlier Testwood dominates as the marginal source of supply, (2) the higher the weight of the demand zones directly supplied from Testwood and (3) the less are the differences in the running costs of pumping supplies from Testwood to the various demand zones.

Long-run marginal cost can now be derived as the sum of estimates of its various components. Thus an estimate of the long-run marginal cost of water for system-wide domestic consumers in Hants (an estimate averaging out variations in between demand zones and in between different years) may be taken to be equal to (3.70 + 8.58 + 1.58 = 13.86 P/M^3). This estimate may serve as a benchmark for long-run marginal cost pricing of domestic consumers.[28]

Estimates of the long-run marginal cost of leakage can also be made for the purpose of assessing leakage control policy. Again the various components would need to be added up. The relevant capacity component is equal to 2.74 P/M^3 (table 4.18). The distribution capacity component may be taken to be 5.53 P/M^3.[29] The operating cost component is 1.58 P/M^3. The overall long-run marginal cost would thus be equal to 9.85 P/M^3.

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[28] Estimates of the long-run marginal cost of water for industry can likewise be derived. This would entail estimating its various components using similar methods to those used for domestic demand. Because demand by industry in general has smaller peak factors compared with domestic demand , its long-run marginal cost would be lower.

[29] This is not absolutely correct since a downward adjustment of this system wide figure may be called for to allow for the fact that leakage has small peak factors.

(4.6) The Accounting Approach to Water Pricing

Pricing policies of many water enterprises are based on financial rather than economic criteria.[30] The financial approach is mainly concerned with price levels designed so as to recover accounting (usually historical) costs from consumers in an equitable manner.

Accounting costs are generally assessed in four categories[31]: capacity, customer, commodity and finally other costs. Capacity costs are sometimes derived by the application of a certain depreciation rule to the total value of the assets of the water enterprise (based on historical or current cost).[32] A financial target is sometimes added and taxation may also be included. The financial target may be set so as to enable the enterprise achieve a surplus on top of its cash outflow. This surplus may then be used to supplement historical cost depreciation (if current cost depreciation is not used), ensure a certain self-financing ratio and/or provide the treasury with a source of taxation.

Commodity or volume related costs comprise running costs, usually energy costs of pumping and chemical costs of treatment, the level of both is a function of the volume of output. That part of maintenance costs which is a function of output is also added.

Customer costs are those costs associated with customer connections which are not related to capacity or volume. They usually include customer connection costs, metering and billing costs. 'Other costs' is a catch all term for remaining costs which are not capacity, customer or demand

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[30] See among others:
(1) Keller,C.,W.,1977, "Pricing of Water," Journal AWWA, January 1977.
(2) Hanke,S.,H.,1975, "Water Rates: An Assessment of Current Issues," Journal AWWA, vol.67, part 5, May 1975.
(3) Herrington,P.,R., and Webb,M.,G.,1981, "Charging Policies for Water Services," Water Service, Vol.85, No.1025.

[31] Sometimes referred to as the 'Hopkinson' classification. See Hirshleifer et al, op.cit.

[32] See Department of the Enviroment,1974, The Water Services: Economics of financial Policies. Third Report to the Secretary of State for the Environment, London, Her Majesty's Stationary Office.

related, e.g. 'certain' central management costs.

These categories, including any subdivisions, are evaluated to form the revenue requirements of the water enterprise over a specified period of time. The next step in the accounting approach is to allocate these costs in an 'equitable' manner among consumers. This usually involves allocating these categories of costs to classes of consumers, where a class groups consumers with broadly similar demand characteristics.

Various notions of equity can be found both at the theoretical and practical level.[33] In general the broader concepts refer to income distribution while the narrower concepts refer to the equal treatment of individuals or classes of consumers (parity and equalisation), eg. charges per unit of consumption are equal for all, or charges per unit of rateable value are equal for all. Other concepts of equity may be advocated such as ability to pay and the benefit principle.

The use of average accounting costs is consistent with a multitude of tariff structures. Accounting costs are recovered in practice using, for example, one of the following structures (or a variant of one of these structures):

1. A fixed charge only.
2. A volume charge only.
3. A multi-part tariff consisting of a fixed or minimum charge per period of time plus one of the following;
   (i) a single volume charge (thus creating a two-part tariff);
   (ii) decreasing block volume charge;
   (iii) increasing block volume charge;
   (iv) seasonal volume charge.

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[33]See among others:
(1) United Nations,1980, "Efficiency and Distribution Equity in the Use and Treatment of Water: Guidelines for Pricing and Regulations."
(2) Frankham,J.,and Webb,M.,G.,1977, "The Principle of Equilisation and the Charging for Water," Public Finance and Accountancy, Vol.4, No.6.
(3) Herrington,p.,R.,1987, op.cit.

A system of fixed charges (flat rate charges) is simple to understand, cheap to run and, provides stable and predictable revenue to the water enterprise. The flat rate charge can, as in the UK, be based on estimated property value (rateable value). From an economic point of view the price of water at the margin in this system is zero. It is an equitable principle to the extent that it may broadly accord with the ability to pay principle.

Historical accounting costs can be recovered using commodity charges only. Capacity, customer and other costs are added up and divided by the expected number of units to be supplied, to give average unit cost. The scheme is equitable to the extent of charging all consumers a uniform price (irrespective of the cost of supply).

Perhaps the most common charging scheme (but not in the U.K.) combines a fixed charge per unit of time with a volumetric charge per unit consumed. The former is usually related to customer costs and a portion of capacity costs. The volumetric charge is usually related to operating and maintenance costs.

Declining block volumetric charges have been justified by the arguments that (1) larger consumers typically contribute less to capacity costs, (2) such a system taxes consumer surplus helping to achieve revenue requirements and (3) it achieves positive discrimination in favour of big business for developmental objectives.

Increasing block volumetric charges have been justified, especially in developing countries, as a means of taxing richer consumers who tend to have higher water consumption. Progressive rates will conserve more water and achieve bigger resource savings if larger consumers have more price elastic demands especially when their consumption is concentrated at times of system peaks.

Comparison of these tariff structures from the point of view of efficiency is somewhat difficult. A few comments can be made. A single volumetric charge (with or without a fixed part) is generally thought to be superior to both increasing and decreasing block tariffs.[34] For example it is grossly inefficient that a water-intensive manufacturer is treated harshly in comparison with a small domestic consumer the supply of which may usually be far more expensive due to high peak capacity costs. This situation would arise with increasing block tariffs.

Moreover it is usually claimed that a system of purely fixed charges is inferior to other charging systems on Efficiency as well as equity grounds.[35] Leaving the equity argument aside, it is however difficult to see why a fixed charge system should necessarily be inferior to all other systems including for example a volumetric charge covering all accounting costs. Indeed a zero price (as in the fixed charge system) might, under some circumstances, perform better than a system based on an accounting cost volumetric charge only.

Suppose the total benefits function of consumption (over a specified planning horizon) net of all operating and capacity costs is as in diagram 4.2 .[36] In this case an average cost price higher then P^* would perform worse than a zero price.[37]

Irrespective of the particular structure chosen, the accounting approach to the pricing of public water supply can be characterized as inferior to a marginal cost pricing structure stemming from the economic criterion of efficiency. Prices based on accounting costs are inefficient because:

(a) They are backward looking reflecting historical costs whereas efficiency dictates the use of forward looking costs.[38]

(b) charges in all the above structures are derived

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[34] See for example Hirshleifer et al, op.cit.
[35] See for example Herrington and Webb, op.cit.

[36] See section 6.6 for a discussion of this function. See in particular diagram 6.2.

[37] A zero price would generate overconsumption and hence extra capacity and operating cost not warranted by corresponding consumer benefits. On the other hand it is possible that a too high a price equal to average accounting cost will cut consumption by amounts in excess of what is warranted by corresponding cost savings. The point made in the text is simply that there is no a priori reason to suppose that the former is worse than the latter.

[38] See section 2.1.1

by spreading total accounting costs among consumers. From the point of view of resource allocation what is required is not this average measure but instead a marginal or incremental measure.

(c) From the point of view of economic efficiency what is needed is a rate structure mirroring, in principle at least, the dimensions of incremental or marginal cost, ie the marginal cost of connecting one more customer to the supply system, the marginal cost of additional unit of supply in a particular year, the marginal cost of additional unit of supply in winter, the marginal cost of additional unit of supply in summer, the marginal cost of additional unit of supply in rural areas, in urban areas and so on. Only a structure of charges reflecting the dimensions of marginal cost can achieve efficiency, since consumers are thereby provided with incentives to shape their consumption patterns in line with the costs they impose on the water industry and the community at large.

Moreover prices based on accounting costs have been criticized as failing to achieve equity[39] or fairness if equity is interpreted as charging consumers according to the costs they impose on the water supply system and on the community at large. It is therefore sometimes claimed that marginal cost pricing is fundamentally more equitable than other charging systems since it does just that.[40]

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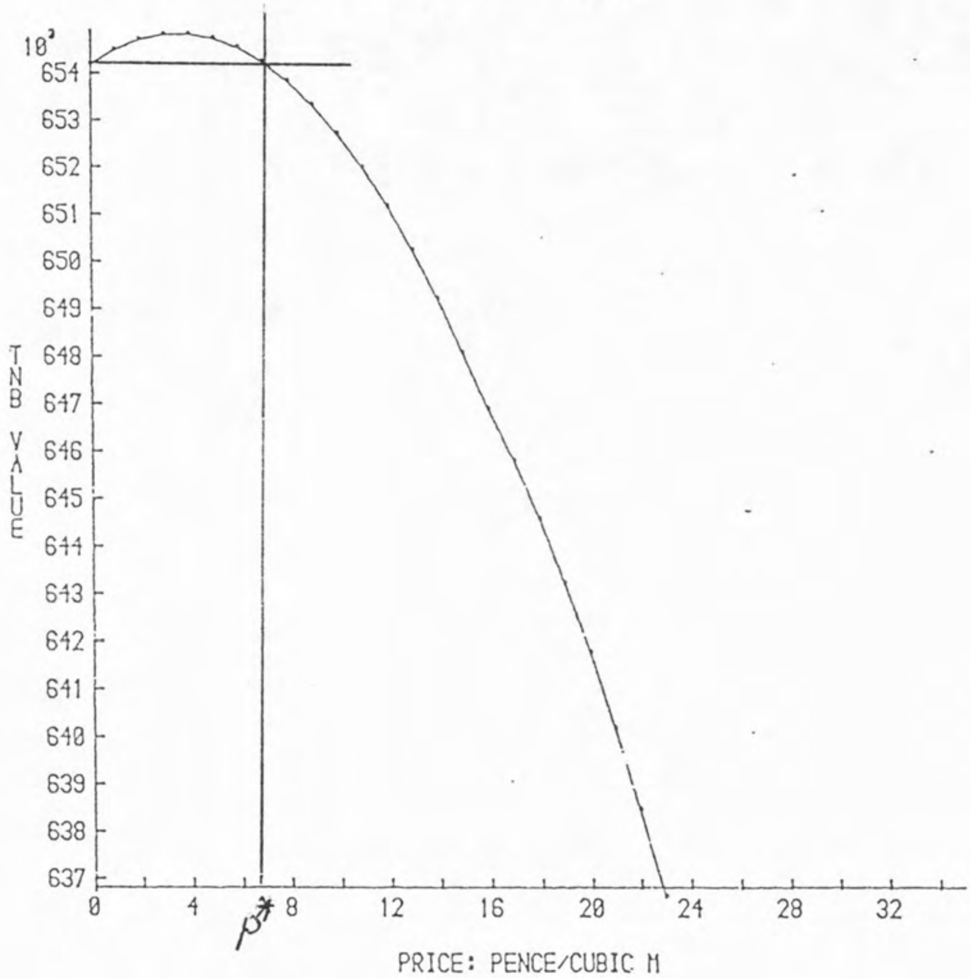
[39] Equity here is taken in the narrow sense. Broader income redistribution objectives can be achieved, at least in the developed part of the world, using other instruments such as income taxation and social security transfer payments.

[40] The 'equalisation' principle for example will equate unit charges of different consumer irrespective of variation in the cost of supplying them.

FIGURE 4.2

A NET BENEFIT FUNCTION FOR WATER CONSUMPTION



(4.7) Objections to Marginal Cost Pricing of Water Supply

The apparent gains that can be achieved in moving from a system of charges based on accounting costs to a system of charges based on marginal costs have been questioned.[41] Rejection of the recommendation that economic efficiency and equity are better served by designing tariff structures according to marginal costs is based on the following arguments.

(1) Metering Consumption

The imposition of any nonzero price for water supplies (marginal cost based or otherwise) is impossible if consumption is not metered. A move to marginal cost pricing, in this case, will normally be preceded by the installation of meters. It may be that any gains in allocative efficiency arising from the move to marginal cost pricing fall short of the associated costs of installing, running, and servicing meters. Of course there is no a priori reason why this should always be the case, but if it is then marginal cost pricing and indeed any positive volumetric price is ruled out.

(2) Inelastic Demand

The gains in allocative efficiency from marginal cost pricing as compared with flat-rate charges are smaller the lower is the price elasticity of demand. If demand is perfectly inelastic then there is no allocative case for marginal cost pricing. Indeed this is the rationale for the 'supply fix' traditional approach to pricing and investment planning found in the water industry.[42] Evidence on price

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[41]See for example:
Rees,J.,A.,1981, "An Economic Approach to Waste Control: A Second Look," Symposium: An Understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, London.
Rees rejects the 'conventional economic approach' to pricing arguing that 'further debate on pricing is largely irrelevant to present day decision-making within the water industry.'

[42]See Warford,J.,J.,1966, "Water Requirements: The Investment Decision in the Water Supply Industry," Manchester School, Vol.24.

elasticity of demand for water, presented in section 5.2, firmly indicates that it is significantly different from zero.

(3) <u>Marginal costs are difficult to determine</u>

Vickrey(1948)[43] noted that marginal costs are difficult to estimate with accuracy and can result in calculations that are no more than approximations to theoretical marginal costs.

One reason for this stems from the fact that the relevant marginal costs are derived from uncertain forecasts of future cost and demand levels, and not from certain historical cost and demand levels. Dealing with future levels of demand, future technology and factor prices would inevitably introduce some degree of uncertainty. Uncertainty apart, chapter 2 has shown that the estimation of marginal costs presupposes that water enterprises carry out some demand forecasting and investment planning. If not, then resources will have to be devoted to these tasks. The extra resources needed to facilitate the estimate of marginal costs will be resources well used since any credible management would in any case need to engage in demand forecasting and investment planning to run the industry efficiently.

Another possible source of difficulty arise from the multi-dimensionality of marginal costs. First we note that while this certainly makes the estimate of 'marginal cost' more difficult yet it does not in itself constitute a serious objection to the concept of practical marginal cost pricing. Multi-dimensionality does not mean, as it is sometimes implied, that economists cannot agree on the definition of marginal cost.

What is perhaps more difficult to agree upon is what dimensions of marginal costs ought to be reflected in the structure of tariffs and which, if any, ought to be averaged out and how the averaging is to be conducted. A clue to answering this question lies in the observation that tariffs

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43 Vickrey,W.,1948, "Some Objections to Marginal Cost Pricing," Journal of Political Economy, Vol.56, Part 3, June 1948.

have to be simple to administer, cheap to enforce and easy to understand by consumers. Therefore, in general, tariffs have to be simpler then the cost structure they represent, incorporating only few of the many dimensions of the full cost structure.[44] Which particular dimensions are we to choose? While it is true that no general answer suitable to all circumstances can be given, one can still make some general observations. Variation in marginal costs over time, due to indivisibility of capital assets, need to be averaged out if price variability over time is to be avoided as impractical or too costly.[45] The appropriate averaging method has already been discussed at both the theoretical and practical levels in chapters 2 and 4. Diurnal variations in marginal costs due to peaking demand would presumably be averaged out on account of the limitations of metering possibilities. Seasonal variation in marginal costs may be translated into seasonal charges if the labour and other costs of frequent meter readings are not excessive.[46] Averaging out spatial variations in marginal costs has been criticized on the grounds that it leads to overinvestment in system capacity, encouraging premature development of land at the rural urban fringe.

It is perhaps also of some importance to note that the traditional categorisation of costs as capacity, volume, and customer related need not be inconsistent with marginal cost pricing provided that they refer to forward looking properly defined marginal and not average costs. Volume related costs

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[44] See among others:
(1) Turvey,R.,1971, op.cit.
(2) Turvey,R.,1976, op.cit.
(3) Hanke,S.,1975, op.cit.
(4) Herrington,P.,R., and Webb.,M.,G.,1981, op.cit.
(5) Turvey,R.,and Anderson, 1977, op.cit.

[45] For a discussion of models with 'indivisible' plant and where price variability is not ruled out see chapters 5 and 6.

[46] See for example:
Sewell,W.,R.,D.,and Roueche,L., 1974, "Peak Load Pricing and Urban Water Management: Victoria,B.C., a Case Study," Natural Resources Journal, Vol.13, Part 3.

may be taken to refer to marginal operating costs. Marginal capacity costs, as defined for the case when averaged over time, may be grouped together with volume related costs.
If a two part tariff is used then its volume related part will embrace both marginal operating and marginal capacity costs. Its fixed part on the other hand would be confined to customer and other costs.

(4) Financial and Distributional Consequences

Marginal cost pricing may produce too little or too much revenue in comparison with the stated financial target. Likewise marginal cost pricing may produce unacceptable distributional consequences. Both of these matters cannot be ignored since they are relevant to the design of any practical and acceptable structure of tariffs. However this observation need not constitute a decisive case against marginal cost pricing. The way to proceed is first to design tariffs according to marginal costs and second to modify these tariffs in accordance with any desired financial requirement and equity consideration.[47]

The point to note here is that any adjustment to marginal cost based tariffs must be carried out with minimum adverse effect on allocative efficiency. This means concentrating most of the 'tax' or 'subsidy' additions on marginal costs on those components of demand which are least responsive to price and on those parts of the price structure that least affect consumer behavior.[48]

It has been pointed out that a two-part tariff structure provides enough flexibility to meet the financial requirement and any equity consideration without inflicting too much damage on allocative efficiency. Any additional revenues would be collected by additions placed on the fixed part of the tariff which would result in little or no damage at all provided that they are not so high as to price

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[47] Of course as noted earlier it might strongly be argued that marginal cost pricing is in itself equitable (from the narrow view point).

[48] See Rees,R.,1984, op.cit.

consumers out of the market. An analogous case can be made for reducing the fixed part of the two-part tariff when there are excess revenues.[49] Alternatively the excess revenue might simply be treated as extra tax revenue to the treasury.

(5) Second Best Considerations

Marginal cost pricing will fail to produce allocative efficiency if marginal conditions do not obtain in other sectors of the economy. If, as is usually the case, non-marginal cost pricing exist in related sectors of the economy, then allocative efficiency requires 'second best' pricing rules. To develop such pricing a water enterprise would have to develop a considerable data base on the way that prices of inputs, of close substitutes and of complements deviate from their respective marginal costs. The practicality as well as the net benefit of such a procedure is doubted. Hence it is argued that the whole case for marginal cost pricing becomes questionable.

Proponents of marginal cost pricing have countered by arguing that (1) the case for second best pricing depends on how important are the links between the water industry and other sectors in the economy; the weaker the links are the less is the strength of the second best argument; to the extent that water does not appear to have any close substitutes or complements, second best pricing is not an important issue, and (2) there should be no presumption, in the absence of any empirical evidence, that marginal costs in second best situations distort resource allocation more then average historical costs.[50]

(6) Technological Efficiency

Allocative efficiency cannot be attained without first achieving technological efficiency in the sense of employing least cost production techniques for the given output levels. Moreover it has been argued [51]that the water industry (at

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[49] If this means, in extreme cases, negative fixed charges (payments to consumers to connect to the water supply system), such a system might be objectionable.

[50] See for example Mann and Shlenger, 1982, op.cit.

[51] Rees, J., 1981, op.cit.

least in the UK) is not technologically efficient and "attempts to reduce (such inefficiency) are likely to produce greater and more speedy financial returns than efforts to revise pricing policies to conform to a notionally 'optimal' system."

The force of this argument against marginal cost pricing must be doubted. Assuming that the net social economic (and not financial) benefits of moving to a marginal cost pricing system are positive, then in the absence of financial constraints such a move can be coupled with other efforts to achieve further economic benefits by also improving technological efficiency.

(7) Problems of Transition

We suspect that one reason for some opposition of some water managers to a move to marginal cost pricing stems from their apprehension of the possible controversy that is likely to arise from such a move due to its possible distributional impact. If a move to marginal cost pricing does involve considerable distributional consequences then opposition may be lessened if the move is made more gradual instead of it being sudden and full.

(8) Other Objections

Other less serious objections to marginal cost pricing include for example the need to determine an appropriate discount rate for the calculation of marginal cost.[52] While this is obviously true, as we have seen in chapters 2 and 3, yet it surely cannot form a major obstacle to marginal cost pricing. An appropriate discount rate would need to be determined in any case for the assessment of investment projects in the water industry.[53]

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[52] Herrington, P., R., 1987.
[53] For a discussion of the appropriate discount rate see section 5.14.

CHAPTER 5

SHORT-RUN MARGINAL COST PRICING AND SINGLE STAGE CAPACITY ADDITION MODEL

(5.1) Introduction

In this chapter we shall develop an optimisation model incorporating the traditional economic rules of short-run marginal cost pricing. The model we develop is in the framework of maximising benefits less costs over time. The pricing rule is used in conjunction with an investment criterion which seeks to maximise social welfare. A distinguishing feature of the model is that it tackles the investment and pricing decisions simultaneously. In contrast traditional economic theory discusses pricing policy given capacity and investment policy given prices. The model we seek to construct should give answers to questions regarding output and prices over time as well as capacity additions given future demand conditions, capacity constraints, production functions, constraints on price variability and so on. This is to be carried out simultaneously. Therefore the model may be described as a marginal cost pricing model and/or a model of optimum capacity addition.

The model uses a dynamic demand function incorporating, besides the price variable, the influence of growing population and real income on demand. Traditional textbook theory usually assumes a static demand function. Inclusion of the price variable distinguishes this approach from the cost minimisation approach where it is implicitly assumed that demand is completely inelastic. We believe that it is difficult to rationalise any marginal pricing rule, short-run or long-run if demand is completely inelastic. However as we will see below there is ample evidence to suggest a price elasticity for demand for water different from zero. This therefore justifies the approach of this chapter and the next where we operate in a world of simultaneous planning of pricing and investment decisions. Another feature of the model is the absence of any constraints on price variability.

The approach of this chapter is also underlined by the assumption we choose to make that capacity addition can only take place at one point in time in the planning horizon. In other words staging of capacity addition is ruled out. This assumption is relaxed in the next chapter.

In sections 5.2 to 5.14 we examine the required data for the application of our theoretical models of this and the next chapter. Our case study shall be the division of Hampshire in SWA. Data regarding demand characteristics including growth, and peak factors and data regarding supply conditions including capacity constraints, cost of capacity additions and so on will be examined first. This will always be carried out keeping in mind the ultimate objective of using this data in our models. This will influence some of the simplifying assumptions that we make in the process. The rationale for some of the simplifying assumptions may not be very clear until later sections in this chapter when we examine the model properly.

(5.2) The price elasticity of demand

There is general agreement that studies of the demand for the public water supply, carried out in the past for different parts of the world and at different times, show that the price elasticity of various classes of demand as well as total demand to be significantly different from zero.

These studies have invariably attempted to regress demand for water (absolute or per capita) against such variables as price, income, wealth (or some surrogate measure of wealth such as property value), number of persons per dwelling, presence of public sewers, climate ...etc.

Demand has been either an aggregate measure including the components of domestic, industrial and commercial demand (with or without losses) or representing individual components only. Domestic demand is sometimes decomposed into indoor and outdoor consumption, the latter reflecting in the main lawn sprinkling. Seasonal distinctions in demand are sometimes made.

The studies have used either cross-sectional data taken at specific points in time from different locations or time series data taken from a specific location over many time periods.

The estimated function has generally been either of the linear or log-linear type. When a log-linear function is used the price elasticity of demand is constant along the demand curve. With a linear demand curve the price elasticity will vary along the curve. Reported elasticity values must in this case refer to specific points on the demand curve; usually the reported value refers to a price equal to that of the latest year in the time series or to that prevailing in a particular location in the case of cross sectional data.

A summary of the findings of a number of recent studies of the price elasticity of demand for water is provided by Herrington (1987a)[1]. As the table shows the results seem to be clear at least in one respect: the price elasticity of demand for water is different from zero.

The relevant value of the elasticity naturally depends on the location in question, the time being considered, the class of demand being examined and the season to which it refers. Nevertheless a general estimate of the range of the values of the price elasticity of aggregate year-round demand for water may be possible. Indeed following Herrington (1987a) we will take it that in general it is reasonable to assume, in the absence of better information, that the said elasticity lies somewhere in the range of 0.00 to -0.3. This will be rationalised in the paragraphs that follow.

What is required for the purpose of our models is a specific estimate of the price elasticity of aggregate demand in Hampshire, UK, which is the study area of our models. Unfortunately such specific information is not available. It is also not possible to conduct a study now to estimate this elasticity simply because at present domestic consumption in Hampshire as in the rest of the UK is almost wholly subject to a flat rate charging system and not a volumetric charge. In consequence there exists no data on the response of domestic consumption to different prices.

All that is possible is an educated guess relying in the first place on whatever relevant British data is available

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[1] Herrington, P.R.(1987a), "Pricing of Water Services", OECD Publication.

and also drawing on the international evidence .

Rees (1971)[2] examined the price elasticity of the demand for water in Malvern, England which is the only area in the country with a substantial number of metered households who pay for their water a volumetric charge. Using time series data (14 years) she regressed daily domestic demand against price, rainfall, and time. The estimated winter price elasticity was -0.13 and the summer elasticity was -0.16. The model used a linear demand curve and the price variable was in money rather than real terms, thus implying money illusion on the part of consumers.

Herrington(1982)[3] reports a price elasticity of metered industrial demand in England and Wales of -0.3 . This coincides with two price elasticity estimates obtained in the Severn-Trent Water Authority area derived from (a) time series analysis and (b) an analysis of the economics of water saving schemes introduced or shelved by all firms in the 1972 to 1978 period.[4]

The limited UK evidence seems to be in line with the broad findings of the international studies. Therefore given the lack of any other evidence it seems reasonable to assume that the price elasticity of aggregate demand for water in Hampshire to be somewhere in the region of -0.3. Aggregate demand is taken to include the components of domestic, commercial and industrial demand, losses, and other smaller items such as public use for fire fighting purposes and other similar uses.

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[2] Rees,J.(1971),Factors Effecting Metered Water Consumption,Final Report To The Social Science Research Council,Great Britain.

[3] Herrignton,P.R.(1982),"Water: A consideration of conservation,"Journal of The Royal Society Of Arts,Vol.cxxx,No.5310.

[4] See Thackray,J.E,and Archibald,G.G,"The Severn Trent Studies of Industrial Water Use," Proceedings Of The Institute of Civil Engineers, part 1,vol.70,1981.

'able (5.1)

A Summary of findings of Demand Price Elasticity

Author	e	Comments
Conley(1967)	-1.02 to -1.09	US, California x-sectional
Derooy(1974)	-0.345 to -0.894	US, industrial D x-sectional
Elliot&Seagraves(1972)	-0.70	US, industrial D x-sectional
Ethridge (1970)	-0.40	US,5 poultry plants pooled time series
Flack (1965)	-0.12 to -1.0	US, 54 waterworks X-sectional
Fourt (1958)	-0.39	US, 34 waterworks
Gallagher & Robinson,(1977)	winter :-0.36	Australia,metered,
Gallagher et al., (1981)	short term :-0.26 long term :-0.75	Australia,Queensland 137 households,pooled
Gardener&Schick(1964)	-0.77	US, 43 waterworks Utah, x-sectional x-section and time series
Grima,(1972)	winter : -0.75 summer : -1.07	Canada, urban demand x-sectional
Gottlieb (1963)	-0.66 to -1.24	US, Kansas x-sectional
Hanke & Mafe, (1982)	year round :-0.15	Sweden,Malmo 69 domestic residents pooled sectional and time series
Herrington,(1982)	year round :-0.3	England & Wales industrial metered time series
Howe,(1982)	winter : 0.06 (east)summer:-0.57 (west)summer:-0.43	US, eastern & western residential x-sectional

Laukkanen,(1981)	year round :-0.11	Finland, Helsinki municipal demand time series
Martin,Ingram, Laney & Griffin (1983)	year round :-0.25	US,Tucson,Arizona domestic use time series
Metcalf (1962)	-0.65	US, 29 waterworks x-sectional
Morgan (1973)	-0.25 to -0.45	US, California residential use x-sectional
Renshaw (1958)	-0.45	US, 36 waterworks x-sectional
Ridge (1972)	-0.30 to -0.60	US, Brewing and Milk plants, x-sectional
Seidal & Bauman (1957)	-0.12 to -1.0	US, waterworks x-sectional
Thackray & Archibald,(1981)	-0.3	England & Wales, 411 firms, water saving investment
Thomas, syme & Gosselink,(1983)	in-house: -0.04 ex-house: -0.31 overall : -0.18	Australia,Perth, x-sectional, 315 houselhods
Ware & North (1967)	-0.61 to -0.67	US, Georgia x-sectional
Wong et al (1963)	-0.01 to -0.72	US, Illions x-sectional
Wong (1972)	-0.02 to -0.28	US, Illions time series 1951-1961
Young (1973)	-0.41 to -0.60	US,Arizona time series

Sources:
(1) Herrington(1987a),op.cit.
(2) Hanke,S.,H.,(1977), "A Method for Integrating Engineering and Economic planning", proceedings of Symposium on Water Services: Financial, Engineering and Scientific Planning, The Institute of Water Engineers and Scientists, 1977.

(5.3) Peak Factors

Peak factors are important because they directly affect the sizing of various components of the water supply system including source works, trunk mains, service reservoirs, treatment works and distribution networks. Each type of investment is driven by a particular peak ratio. It is for example generally accepted that the capacity of treatment works is driven by the peak week factor. This means that the design capacity of treatment works are planned such as to meet the peak week demand with some allowance for reserve.[5]

The standard practice seems to project the size of the "design population"[6]; then a projected average per capita level of demand together with design population are used to derive total average demand to be met by new capacity ; and finally the average demand is scaled up to arrive at the estimate of the peak week demand of the "design population" which determines the size of design capacity.[7]

Thus it is clear that in deciding upon design capacity it is as important to forecast average aggregate demand, which is to be met in some future year, as it is to forecast the relevant peak factor in that year. In our case what is relevant is of course the peak week factor of aggregate demand for water for the Hampshire area for the period covered by our model.

Herrington (1987b)[8] reports that seven out of nine econometric models estimated within the UK water industry since 1979 revealed no trend increase in monthly or weekly peak ratios. This evidence coincides with the assumption of constant peak week ratios for the period of 1985 to 2011 used in one study at least by Southern Water Authority in

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[5]See Cooper&Lybrand (1985),op.cit.

[6] This simply the expected level of population in some future date. Design capacity is to be adequate to meet demand up to this date.

[7]See Dangerfield,Bernard j.(ed)," Water Practice Manuals : The Structure and Management of the British water industry",IWES.

[8] Herrington,P.,1987b, Water Demand Forecasting in OECD Countries, OECD Publication.

relation to the Hampshire area.[9]

This has convinced us that an assumption of a constant peak week demand factor over time may not be unreasonable provided we can make the further assumption that peak ratios in general are substantially unaffected by variations in the level of charges which may well result from the working of the marginal cost pricing models which we seek to build.[10]

There is evidence[11] indicating that the peak ratios may indeed be sensitive to the charging structure. In particular a move from a flat rate charge to a volumetric charge would be expected to entail a reduction in peak ratios.[12] While this is readily accepted it can perhaps also be argued that given a fixed volumetric charging structure it would take a very sharp variation in the level of prices to produce significant changes in the level of peak factors. This is more so in the case of a charging system with uniform price throughout the year.

Accordingly it will be assumed that the peak week factor for the Hampshire division of Southern Water Authority will remain constant for the period of the study at a level which is invariant to any price changes that may emerge from the working of the model.[13]

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[9]This assertion is based on the following study: "Hampshire, Isle of Wight and Portsmouth RACS study",Southern Water,Directorate of Operations,1985.(mimeo)

[10]The same type of assumption was made by Roirdan(1969); see Roirdan,C,"Towards the Optimisation of Investment and Pricing Decisions: A model of urban supply treatment facilities," Ph.D thesis,Cornell University.

[11]See How,C.W., and Linaweaver,F.,P.,1967, "The impact of Price on Residential Water Demand and Its Relation to System Design and Price Structures", Water Resources Research, Vol.3, No. 1 .

[12]Indeed this was reflected in the derivation of demand forecasts in the post metering situation, thus under demand forecast C average domestic demand fell be 12.5% whereas peak week domestic demand fell by 15.5%.

[13] In the move from the no metering to the metering situation we have allowed for a change in the peak factors. Once meteres are installed however we invoke the assumption of constant peak factors regardless of the ruling price level.

Table (5.2) sets out the 1986 peak week factors of aggregate demand as expected to prevail in Hampshire after the introduction of domestic metering. The peak factors relate to each of the demand zones in the study area. The weight attached to each demand zone is also stated. These weights are based on each zone's share in the total of the demand of all zones. The figures are based on the assumption that the metering of domestic consumers results in a 12.5 percent reduction in average domestic demand and a 15.5 percent reduction in peak domestic demand.[14]

The area's overall weighted average peak ratio is 1.24 and this will be used throughout the models.

Table (5.2)

Demand Peak Week Factors for Southern Hampshire

Demand Zones	PWF(1986)	WEIGHT
(1) Rownhams & Waterside Parishes	1.28	0.23
(2) Southampton Common	1.22	0.26
(3) Otterbourn Direct	1.21	0.14
(4) Twyford & Moorhill	1.21	0.24
(5) Timsbury & Lynhurst	1.26	0.06
(6) Yewhill	1.21	0.08
OVERALL	1.24	

Source: Based on forecast "c" of chapter three table(3.26).

[14] Therefore these peak ratios are lower than they would be in the case of no metering.

(5.4) Non-Price determinants of aggregate demand

Forecasts of percapita domestic demand carried out by the various water authorities responsible for water supply indicate that while the rate of increase of percapita consumption is slackening somewhat it is still far from being at or indeed near a satiation level. Evidence from countries such as the USA, Sweden and Switzerland indicate that per capita levels of consumption tend to level off reaching a satiation level at 'high' levels of consumption.[15] Forecasts of per capita domestic demand for the Hampshire division carried out by SWA indicate rising per capita domestic consumption at least up to year 2011.[16] Table (5.3) indicates the expected pattern of growth of per capita unmeasured but accounted-for demand for water in the whole of Hampshire division. This is based on SWA forecasts. Domestic per capita consumption is expected to rise from its 1984 level of 141 l/h/d to 173 l/h/d by 2011. This amounts to an average annual growth of 1.18 l/h/d .

The growth in domestic consumption however is counterbalanced by an expectation of an improved leakage performance to be achieved by 1991. SWA hopes that, on the least optimistic forecasts, it should be able to hold leakage at a level no higher than 98 l/h/d. Leakage stood at an estimated 112 l/h/d in 1984.

Measured demand, mostly representing industrial demand shows some slight growth when looked at in absolute terms (million litres/day). However it declines when expressed in per capita terms. Demand for water by industry has seen a marked decline in the England and Wales in general (metered consumption fell by 17 percent between 1970 and 1983).[17] Hampshire's slight growth in measured consumption is perhaps explained by the general buoyancy of the local

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[15] Herrington, P.R, (1987b), reports an apparent satiation level of domestic percapita consumption in Switzerland equal to 260 litres/head/day, op.cit.

[16] Domestic consumption is taken to be unmeasured but accounted-for demand as displayed in table (5.3).

[17] Water facts, 1985, op.cit.

economy as opposed to the general economic recession in the UK that has been experienced in the recent past.

Table (5.3)

Demand Forecasts for Hampshire (Pre metering)

(l/h/d)	Actual 1984	1986	Forecast 1991	2001	2011
(1) PCUM (Accounted-for)	141	142	145	160	173
(2) PCUM (Unacounted-for)	112	108	98	98	98
(3) Metered (industrial)	105	103	99	99	102
Total	358	353	342	357	373

Notes:

(a) PCUM Accounted-for is per capita unmetered legitimate consumption, the overwhelming part of which is accounted for by domestic consumption. The remainder is commercial unmetered consumption.

(b) PCUM Unaccounted-for is per capita unmeasured leakage. The estimate assumes that by 1991 a target level of 98 l/h/d will be achieved and maintained there throughout. Other scenarios are possible. The stated one in fact is the least optimistic. PCUM would normally also include a small proportion of 'public' use such as for fire-fighting purposes.

(c) Metered consumption is by and large industrial demand. The cited estimates have been converted by us to l/h/d so that they become compatible with other components.

(d) All forecasts do not allow for active pricing policy. In fact the forecasts assume that domestic demand continues to be unmetered.

(e) Figures for 1986 have been interpolated by us from 1984 and 1991 figures.

Source: Division and Company Demand Forecasts Compatible with Annual Plan 1986 Regional Forecasts.,Part A, table D, Southern Water, Directorate of Technical Services, 1986,(mimeo).

(5.5) Metering Domestic Consumers

For the purpose of our models it will be necessary to assume that all 'useful' water consumption including the domestic sector is charged according to measure. In other words we shall assume that a decision to meter domestic consumers had already been taken and implemented, at least for that part of SWA area covered by our study.

It is therefore required to assume that all public supply is subject to a volumetric charge by 1986, the starting date of the models.[18] Our models of marginal cost pricing are therefore applicable to situations where the decision to meter domestic consumers has already been taken and implemented. To that extent the choice of 1986 as a starting point is arbitrary and can easily without any loss of generality be replaced by a later year to fit other more realistic situations.

Of more significance is the profile of expected demand after the metering of domestic consumers. The Watts report assumed, drawing on international and some UK evidence[19], that metering would reduce domestic average daily consumption by 12.5 percent to 15.5 percent.[20] The reduction was assumed to be once and for all, and there was no discussion of the

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[18] Such an assumption is necessary if we were to have a meaningful marginal cost pricing model. Moreover the plausibility of this assumption is made stronger on the grounds that the water industry in England and Wales is considering such a move. See the findings of the 1985 metering report; The Watts Report,1985, Joint Study of Water Metering, Report of Steering Group, London, HMSO.

[19] Two UK studies were used. One refers to the Malvern/Mansfield study by Severn Trent Water Authority in 1976 comparing domestic consumption at Malvern (with metered consumers) with that estimated for unmetered consumers in Mansfield. The second refers to an experiment in Fylde, the old (before reorganisation) Fylde Water Board area, comparing the consumption of metered consumers who have agreed to experimental testing with the consumption of unmetered consumers.

See (1) Jenkin,R.C,(1973),Fylde Metering, Blackpool, Fylde Water Board, UK. (2) Thackray,J.E., and Archibald,G., 1978,"The Malvern and Mansfield Studies of Domestic Water Usage: Discussion", Proceedings of the Institution of Civil Engineers, Vol.64.

[20] Peak demand was assumed to fall by a higher percentage than average demand. See Cooper & Lybrand (1985),op.cit.

impact of different price levels on the amount of water saved nor of the specific price level that was to produce the postulated 12.5/15.5 percent reduction in domestic consumption. The implicit assumption made by the Watts report seems therefore to be that the demand curve is kinked as in the following diagram, (diagram 5.1).

Thus for any price minutely greater than zero, demand becomes totally inelastic at A. The distance AB in diagram 5.1 represents the 12.5/15.5 percent reduction in consumption expected to result from metering.

But as discussion in the previous section has shown the evidence indicates that the price elasticity of demand for water in the domestic as well as other sectors is nonzero. If one accepts this evidence, as we do, then the impact of metering on domestic consumption cannot be determined independently of price. Put differently, the postulated 12.5/15.5 percent reduction in domestic demand must be associated with a precise price or perhaps a range of prices. Because evidence on this specific point does not exist we shall assume that the postulated reduction is associated with charging domestic consumers in Hampshire the existing (1986) volume charge for measured consumption, which applies for both domestic and industrial metered consumers. This is equal to 25.7 pence per cubic meter (P/M^3) or, equivalently, £257 per million litres (ML) or £1166 per million gallons (MG).[21] Moreover attention must also be given to the impact of metering on the amount of leakage, since presumably the amount of leakage is partly determined by how much water is being pumped into the system to meet domestic and industrial demand.

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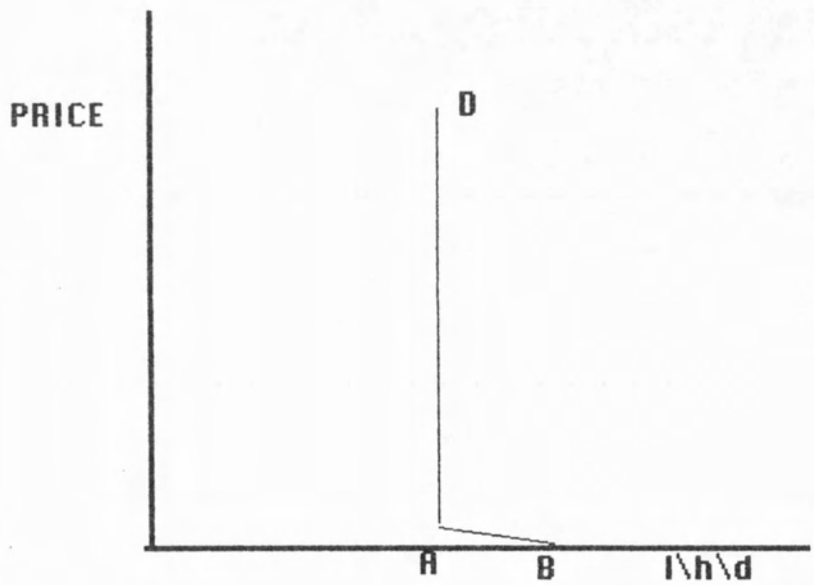
21 The assumed price of 25.7 P/M3 compares with 37 P/M3 which was the rate ruling in Malvern in 1976 at the time when the Malvern/Mansfield study was conducted. The rate has been adjusted to be presented in 1985 prices. Some 1400 households are currently(1986) charged according to measure in Hampshire.

figure 5.1



Kinked Demand Curve

For the purpose of the models in this and the forthcoming chapters we make the assumption that a 12.5% reduction in domestic consumption will lead to a 12.5% reduction in that part of leakage attributed to domestic consumption.[22] Further, the amount of leakage we attribute to domestic consumption is in proportion to the ratio of domestic demand over the sum of domestic and industrial demand. In this fashion and provided the leakage control policy remains the same, the share of leakage in aggregate demand (including leakage) remains the same regardless of the level of consumption.[23] Given these assumptions the post-metering average demand figures (l/h/d) for a 12.5% reduction in average domestic demand and a price equal to 25.7 P/M^3 will be as in table (5.4). The complete forecast, up to 2011, given that domestic demand is metered and charged a uniform constant real price equal to 25.7 P/M^3, is given in table (5.5).

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[22] We note that this assumption is contrary to that of section 3.8 where it was assumed that leakage is independent of legitimate consumption. Moreover this would also be at variance with the assumptions built into the derivation of the 1986 peak week factor for Hampshire reported above in table 5.3. This slight inconsistency in assumptions arise because of our desire to formulate the model with constant share of leakage in overall supply and with a constant overall peak week ratio regardless of the ruling level of prices and the consequent level of demand.

[23] It is convenient to set up our model in such a way that variations in prices leave the overall peak week factor constant. If the shares of each class of 'demand' is constant and if the peak week factor of each is constant then it must follow that the overall peak week factor will remain constant regardless of the level of prices. Moreover a constant share of leakage to overall demand at all level of prices is particularly convenient when it comes to measuring the total consumer benefits of 'useful' consumption net of leakage, (see section 5.15).

Table (5.4)

Impact of Metering on Average demand (1986)
Uniform Price of 25.7 P/M^3
l/h/d

	1986 Pre Metering	1986 Post Metering		Change
(1)Domestic	142	124.25		12.5%
(2)Leakage	108	100.17	on (dom)	12.5%
	62.6+45.4	54.77+45.4	on (ind)	0.0%
	(dom) (ind)	(dom) (ind)		
(3)Industrial	103	103		0.0%
Total	353	327.42		

Source : Constructed from table (5.3)

Table (5.5)

Post Metering Average Demand Forecast
uniform Price 25.7 P/M^3
l/h/d

	1986	1991	2001	2011
(1)Domestic	124.25	126.87	140.00	151.37
(2)Leakage	100.17	90.72	90.43	90.29
(3)Industrial	103.00	99.00	99.00	102.00
Total	327.42	316.59	329.43	344.00

Source : Constructed from table (5.3)

We note that over time the ratio of leakage to aggregate demand falls slightly. This is because of the initial assumption that SWA is to maintain leakage (before metering) at a level not exceeding 98 l/h/d as from 1991 despite the growth of consumption. This implies that more active leakage control policies will have to be adopted over time. The implied fall in the leakage ratio f for the post metering forecast (at a price of 25.7 P/M^3)is as follows :

	1986	1991	2001	2011
f	0.30	0.28	0.27	0.26

As the table shows the fall in f is only marginal and it is perhaps not too unrealistic to assume it constant for a period equal to the planning horizon chosen for the models at a level equal to the average of its values over the period 1986 to 2011.[24]This will be particularly attractive since in this way a great deal of analytical simplification in the model can be achieved seemingly without too high a cost in terms of lost realism. It is therefore assumed that f remains constant at a level equal to 0.27. This particular figure has been derived as the simple average of the forecasted leakage ratios for the period 1986 to 2011.

[24]A constant f may be said to define a constant leakage policy if the latter is expressed in terms of constant l/h/d. The usual practice however is to define a constant leakage policy in terms of constant litres lost per property per hour.
See chapter 7.

(5.6) Growth in per capita Aggregate Demand in Southern Hants

A great deal of simplification in the marginal cost pricing model can be achieved if the forecast growth of per capita aggregate demand over time can be approximated by a constant average arithmetic rate. Table (5.5) above indicates that between 1991 and 2011 it is not unreasonable to represent the forecast growth of aggregate per capita demand by a constant average annual rate. The growth between 1991 and 2011 is in fact contrasted by a forecast decline for the period 1986 and 1991. The decline is again due to the initial assumption that SWA was to increase the intensity of leakage control activity.

However, to avoid some of the complexities that arise with a variable growth rate it is convenient for the purpose of the models that the demand growth can be represented by a linear trend. The remaining question is: a constant demand growth at what rate ? Using the figures of table (5.5) two possible values may be derived. The higher figure is 1.35 l/h/d, based on growth from 1991 and 2011. A lower figure of 0.68 is derived when based on the average growth for the period 1986 to 2011. Of course neither represents true expectations. And as a working compromise we have adopted the average of these two rates giving an approximate average rate of 1 l/h/d per annum. This figure will be used in the models and like other parameters will be subjected to sensitivity tests.

(5.7) <u>Population Growth in Southern Hants</u>

One of the essential ingredients of the models is the size of the population to be served by the distribution network as well as its growth over time. The marginal cost pricing model is formulated so as to be applied in the southern network of SWA's Hampshire division where the major binding constraint on output is treatment capacity at the Testwood works where the marginal source of water supply from the river Test is located.

There are two more system networks in Hampshire; the Northern and Central networks. Although both are connected to the Southern network we have excluded them from the model because they have their own independent local sources of supply and therefore do not draw any water from the Testwood works. As such consumption in those areas is not constrained by the treatment capacity at Testwood. This situation is thought to hold true for the foreseeable future.

Therefore the area of study whose population concerns us here includes the following demand zones falling within the defined area:[25] Rounhams & Waterside Parishes, Southampton Common, Otterbourn, Twyford and Moorhill, Timsbury and Lyndhurst and Yewhill.

Winchester demand zone has been excluded from the study despite the fact that SWA plans to connect it to the Southern network by a trunk main via Yewhill. The exclusion is caused again by the fact that Winchester draws the bulk of its water supply from the groundwater source at Easton and therefore has little effect on treatment capacity requirements at the marginal source of Testwood which is the focus of our model from the point view of capacity.

Population data for the area of the study defined above has been drawn from Southern Water Authority

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[25] See Testwood stage 3 Feasibility Study, Southern Water, Hampshire Division, Centre of Operations, May 1985,(mimeo)

[26] Testwood Stage 3 Feasibility Study,op.cit.

documentation.[26] Initial population of the study area is 458000, mainly resident in the Southampton and adjacent areas. This population is expected to grow at an average constant rate of approximately 3720 persons per annum up to year 2011.[27] Thus the population is expected to increase by some 93000 persons (20%) by 2011, representing just under 1% simple average rate of growth.

Table (5.6)

Population Growth Southern Hampshire

Demand Zone	1986	1991	1996	2001
(1)Rownhams & Waterside Parishes	122900	130590	138280	145972
(2)Southampton	85230	85806	86382	86958
(3)Otterbourne	60277	60721	61164	61608
(4)Twyford & Moorhill	119932	128762	137592	146422
(5)Timsbury & Lyndhurst	25632	26460	27288	28116
(6)Yewhill	44042	44272	44503	44733
Total	458013	476611	495209	513809

Source : Testwood Stage 3 Feasibility Study, Southern Water, Hampshire Division.

[27] The available figures from SWA for the disggregated forecasts only go as far as 2003. The same pattern of growth has been extended by us to year 2011. The 1986 Annual Plan for SWA gives population growth in aggregated form up to 2011.

(5.8) Absolute growth in aggregate demand

For a constant population of size 458000 persons aggregate total would rise by a constant amount of 458000 litres/day per annum if per capita consumption grows by 1 l/h/d as postulated earlier. This growth is independent of the charged price, ie in diagrammatic terms it can be represented by the demand curve shifting outwards in a parallel fashion. This is shown in figure 5.2 .

Incorporation of the income effect in this fashion would simplify our marginal cost pricing models without losing too much realism. A parallel shift in the demand curve would indeed be consistent with a gradual decline in the income elasticity of demand as income rises, which in turn accords well with the satiation hypothesis.[28]

However, as has been explained earlier, population is not constant over time but in fact growing by some 3720 persons per annum reaching some 551000 persons by the year 2011. In order to maintain the simplifying assumption[29] of constant absolute growth of demand per annum we have used the average of 458000 and 551000 (or 504000) as the basis from which the absolute yearly increment in demand is estimated. Therefore the absolute increment in demand (D) turns out to be equal to 0.504 ML/d which is equal to 40.5 MG/year. This is the figure that will be used in our models.

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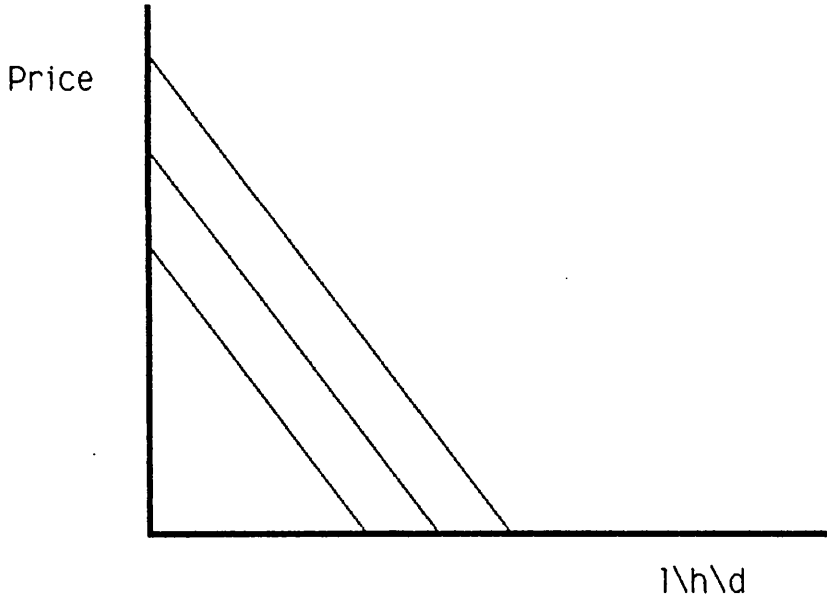
[28] The income elasticity of demand is defined as (dq/q)/(dy/y) where q stands for aggregate consumption and y for the community's income. Our assumption above indicate that dq/q will fall over time (dq is constant and q is growing). At the same time it is reasonable to assume that dy/y will be constant over time. Hence the elasticity must be falling over time.

[29] A constant growth rate simplifies the mathematical formulation of the models.

Figure 5.2



Parallel Shift in Demand

(5.9) The Dynamic Demand Curves

In the preceding sections we have developed all the essential ingredients necessary to specify a dynamic aggregate demand curve for water which can be used in our marginal cost pricing models. These findings may be briefly summarised as :[30]

(1) A price elasticity of aggregate demand for water equal to about -0.3 .

(2) An absolute annual increment in aggregate demand for water equal to D where D is assumed to be equal to 40.5 MG/year.

(3) With uniform annual pricing the peak week to average demand ratio is invariant to changes in prices. A peak week factor of 1.24 is assumed.

(4) A constant proportion of aggregate demand, f, equal to 0.27, is lost out of the system in the form of leakage. This ratio is invariant to the price level.

(5) One point on the demand curve at the starting time (t=0) is a per capita consumption of 328 l/h/d (or the same thing 0.02637 MG/head/year) associated with a uniform price of 25.7 P/M^3 (or 1166 £/MG). And this is combined with (1) above by stipulating that at this particular point the price elasticity of demand is -0.3. Admittedly there exists some degree of arbitrariness in this choice. However this may be justified on the grounds that the models can be easily respecified and run with fresh and more accurate

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[30] These assumptions are close to those of Roirdan(1969). Roirdan however assumed a zero income elasticity of demand. He also did not discuss or allow for leakage in his models, the implicit assumption being zero losses which we know to be at variance with the existing situation in most if not all of the UK water supply industry and indeed the USA as well. This is in addition to other differences regarding the assumption concerning the initial capacity of the system, a subject which we tackle later.
Roirdan(1969),op.cit.

information regarding the magnitude of the elasticity. This will have to wait until metering is actually installed and enough time has elapsed for the accumulation of the necessary data.

And furthermore we will later argue that the solutions of the model imply a range of elasticity from 0.0 to -0.3 which accords well with the initial values assumed.

We first develop the dynamic demand curve assuming a zero income effect (D=0). This will help throw light on the nature of the demand curve which in a later step is reformulated to incorporate the postulated income effect.

At any point in time knowledge of one point on the aggregate demand curve (price and quantity) together with the assumed price elasticity at that point enables one to completely specify the demand curve if the latter is assumed to be linear. This can be done as follows : first the slope of the demand curve can be derived as

$$dp/dq = \frac{1}{e}\,\frac{p}{q} \qquad (5.1)$$

Where dp/dq is the slope of the demand curve, e is the price elasticity at the particular point (p,q). At time zero, one point on the demand curve is a price equal to 1166 £/MG and a total aggregate demand of 328 l/h/d x 458000 (or 0.02637 MG/head/year x 458000) with a price elasticity of -0.3.

Thus the linear demand curve may be expressed as in equation (5.2) below.

$$P(q,t) = -(dp/dq)q + P' \qquad (5.2)$$

Where p' is the intercept of the demand curve and dp/dq is the slope of the demand curve .

The same demand curve may be expressed in a slightly different way. Suppose now that price is set equal to a constant[31] marginal running cost (MC). If we let r = per capita consumption in MG/head/year when price is equal to MC. This will be constant over time provided MC is constant and the income effect assumed away,

Z(t)= population at time t,

and P' = the intercept of the demand curve at time zero. This will be constant over time if there is no income effect,

then dp/dq, the slope of the demand curve at time t, can be expressed as :

$$dp/dq(t) = \frac{P' - MC}{r.Z(t)}$$

and consequently the whole demand curve at time (t) may be expressed as:

$$P(q,t) = \frac{P' - MC}{r.Z(t)} q + P' \qquad (5.3)$$

Therefore with zero income elasticity (this will be relaxed later) the outward shift in the demand curve over time will be completely determined by population growth and only the slope of the function changes. This implies that the demand curve pivots outwards around the same intercept P' as shown graphically in diagram (5.4).

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[31] A constant marginal running cost is empirically justified in section 5.11.

Figure 5.3

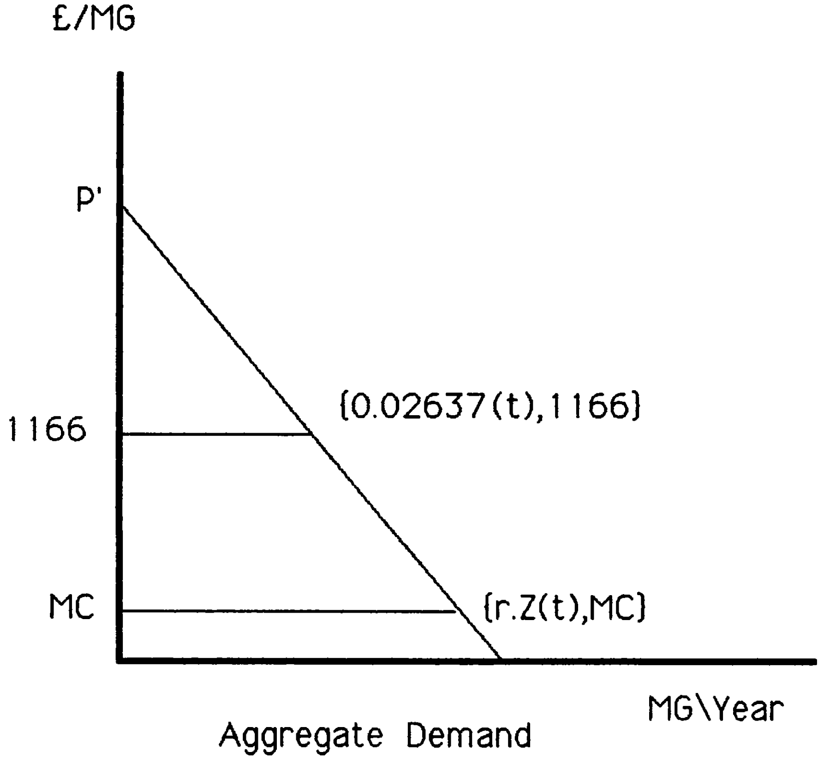
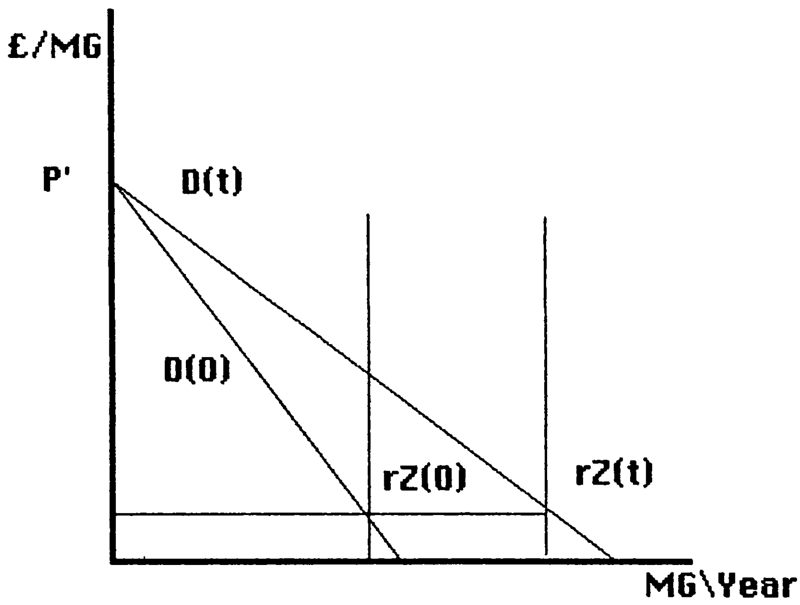


Figure 5.4



Pivoting Demand

The implication of the pattern of aggregate consumer behavior is that at a constant price (eg. equal to MC) per capita consumption over time (eg. r) remains constant over time. In this way a zero income elasticity is maintained.

The demand curve over time in this case is completely determined given P', MC, r, and Z(t).

In our case MC will be specified in a later section as 79 £/MG. P' can easily be calculated for our Hampshire case, at t=0 we have :

$$dp/dq = \frac{1}{e}\,\frac{p}{q}$$

$$= \frac{1}{0.3}\;\frac{1166}{(0.02637)(458000)}$$

$$= 0.32$$

and

$$0.32 = \frac{P' - 1166}{(0.02637)(458000)}$$

this gives

$$P' = 5030 \quad (£/\ MG\ )\ ,\ \text{or } 110.79\ P/M^3$$

and r can easily be derived since

$$0.32 = \frac{P' - MC}{r.\ Z(0)} \quad \text{or}$$

$$0.32 = \frac{5030 - 79}{r\ .\ 458000}$$

$r = 0.03378$ MG/head/year (or 420 l/h/d)[32]

[32] The implied price elasticity of the aggregate demand curve at the point where price equal MC is -0.015 . And since a price less than MC will not be admissible as a solution to any of our models and given that the actual solutions as we will see later never admit a price higher than 25.7 P/M then it follows that the price elasticity of demand will range between -0.015 and -0.3. This seems to fit well with the empirical findings regarding the range of values e takes in developed urban economies.

And finally population at time t, Z(t) has already been specified to grow at an arithmetic rate of 3720 per annum so that

$$Z(t) = Z(0) + gt \quad \text{where} \quad Z(0)=458000 \text{ and } g=3720$$

Population will grow for a period starting at time zero , t=0, and terminating at time t=T. T is decided as the latest point in time for which population growth can be safely extended. This is taken to be 25 years hence with population growing from year 1986 to year 2011. Growth in absolute demand, D, will also be terminated after T years. This is again a simplifying assumption made for convenience.

The demand curve can now be expressed as

$$P(q,t) = - \frac{P' - MC}{r.Z(0) + rgt} q + P'$$

and if we let

$r.Z(0) = K$ ie the required initial capacity to meet demand if price is equal to MC.

$rg = R$ ie the total increment in demand per annum given price equal to MC.

Then the dynamic demand function can be expressed as

$$P(q,t) = - \frac{P' - MC}{K + Rt} q + P' \qquad (5.4)$$

Extending the analysis so as to incorporate in the dynamic demand function an income effect in the form of a constant absolute increment equal to D per annum is straightforward. As diagram (5.5) below shows the shift in the linear demand function over time may be decomposed into two steps : The first step involves a parallel outward shift in the pivoted demand curve by a distance equal to the constant D in every period of time. This is the effect on demand due to income. The second, involving the population effect, manifests itself by pivoting the demand curve around the Y axis intercept, (P'+SDt). The demand curve developed so far incorporates this movement. The first shift implies an upward move in the location of the intercept P' in every time period by an amount equal to SD where S is the slope of the demand curve at time t=0. Therefore the intercept in period t can be written in the general case as (P'+SDt). The slope of the dynamic demand curve on the other hand, as diagram (5.5) indicates, becomes :

$$\text{slope} = \frac{P' + SDt - MC}{K + Rt + Dt}$$

where all the terms retain their meaning as before.

The overall dynamic demand curve incorporating the postulated income effect therefore can be expressed as :

$$P(q,t) = -\frac{P' + SDt - MC}{K + Rt + Dt}\, q + (P' + SDt) \qquad (5.5)$$

This is a more general demand function than (5.4) and the latter formulation clearly reduces to (5.4) in the case when D=0. Equation (5.5) will be used in our models. It is specified for the parameters P', MC, S, D, R, K, and T.

It remains to be pointed out that the demand curve we use is deterministic. Moreover the demand function above assumes that quantity demanded responds instantaneously to price variations without any significant time lag. This of course is a simplifying assumption which may well be violated in real life. This is especially the case when the response to a price change requires some investment or disinvestment in water-using appliances rather than simply turning the tap on or off. However it may also be argued that by using an argument of this type one could claim that the elasticity estimates we have incorporated into the demand function represent the short-run response and that the long-run elasticity allowing for an adjustment time lag may well be higher than the one we have used.[33] If this is true then it may act to counter balance the weakness arising from the use of an instantaneous response in the demand function.

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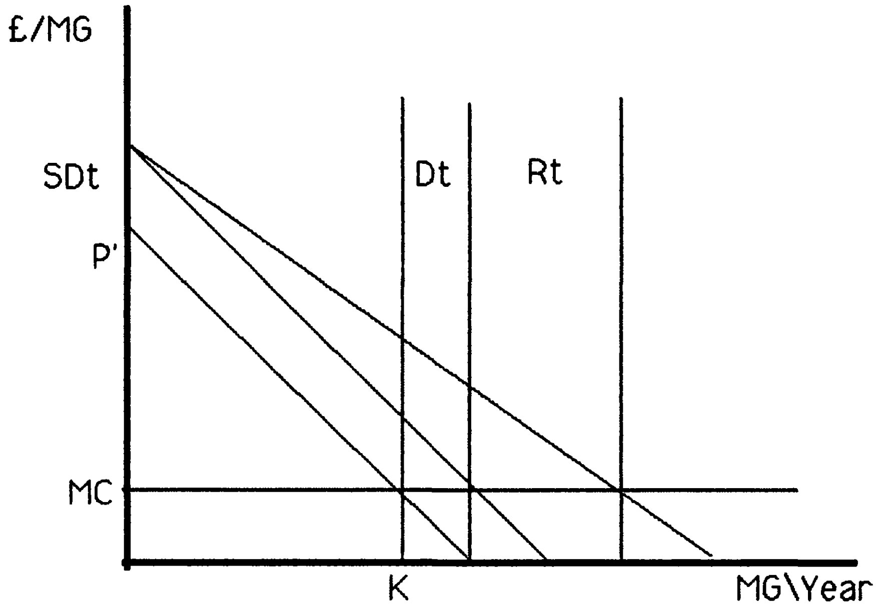
33 Of course our dynamic demand function would need to be adjusted according to the assumed lagged response.

Figure 5.5



Decomposing The Dynamic Demand Movement

(5.10) The Capital Cost Function

The objective of this section is to derive an expression which displays as accurately as possible the total capital cost of constructing conventional water treatment works as a function of design capacity. This function is to be applied in the area of our study, Hampshire.

In our search for such a function we have heavily relied on the substantial and comprehensive work conducted by the Water Research Centre (UK) on cost functions of various components of the supply system including water treatment. This work was published in a document known as Technical Report 61 (TR61).[34] At a later stage we also make some comparisons between the British results and some results obtained in the USA.

Two possible approaches to the estimation problem may be identified :

(a) "components " approach

(b) "whole works" approach

Under (a), a specific cost function is estimated for each particular treatment process and where each cost function is expressed in terms of engineering variables such as 'plan area for filtration' or 'volume of sludge thickening tanks'. To estimate the total costs of a particular treatment works it becomes necessary under this approach to have detailed knowledge of the specific components, their sizes and configurations. Total costs of complete works are built up from the estimated cost of specific components.

Under (b) however we have at our disposal cost functions for complete treatment plants (not components) which are especially useful when the planner is in a preliminary situation without the detailed specifications necessary for the components approach above.

While TR61 develops both of these approaches we have

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[34] TR61.,(1977),"Cost Information For Water Supply and Sewage Disposal,"Water Research Centre, WRC Environment, UK.

found that (b) is the only practical approach in our case. This is because of lack of the detailed information regarding planned treatment works necessary for the application of approach (a).

Complete treatment works models relating total cost to throughput have been developed by TR61 for conventional types of treatment works. These cost functions cover the cost of treatment including the following processes :

(1) preliminary works (inlet structures such as grit settling, intake and screen chambers, intake screens and microstrainers,....etc.
(2) chemical plant and control equipment
(3) sedimentation, flotation
(4) filtration; rapid gravity filtration, pressure filtration, slow sand filtration
(5) disinfection
(6) sludge processes
(7) water storage tanks
(8) other work items such as earthworks, drainage, roads, footpaths, fences, sewers and sewage works within the treatment works, pipelines with works, interconnecting process pipework ..etc.
(9) pumping and power.

Excluded from the costs are items that are not primarily related to treatment such as raw water storage and staff housing.

TR61 related the total cost function to volume of throughput and also to the basic type of conventional treatment, ie whether pressure filtration, rapid gravity filtration or sedimentation filtration. Accordingly a cost function for total treatment works for each type of these basic types of treatment processes was reported. Moreover it was assumed that the cost of treatment would be influenced by the ease of treatment processes as represented by the extent of additional treatment processes. The ease of treatment variable was introduced into the function by the means of a dummy variable "SCORE". The value of "SCORE" is determined in each specific case by counting 1 for each of the following processes:

filtration (pressure or gravity)
sedimentation
microstraining
slow sand filtration
activated carbon filtration
softening (precipitation, ion exchange)

"SCORE" can bee regarded as a measure of the complexity of the treatment, and for the 55 cases included in TR61's sample 'SCORE' took the values 1 , 2 or 3 .

The general functional form employed by TR61 is one which reflects the presumed presence of economies of scale in capital construction. In particular the chosen function is similar in form to that originally used by Chenery (1952)[35];

$$C = a(X)^b(Y)^c$$

where C= the total cost (£000) in 1976 prices
X= rated capacity in 1000 M^3/d
Y= the 'SCORE' variable defined above

In this type of a function economies of scale will prevail so long as b is less than 1, indicating that the bigger the works the less is the average cost per unit of capacity.[36] A scale factor of less than one would explain the phenomenon of constructing large seemingly indivisible treatment works with capacity sufficient to meet requirements for many years to come instead of just the immediate future. Of course the bigger the capacity the longer it remains underutilised and therefore the economies of scale of constructing bigger capacity has to be balanced against the extra cost of building larger capacity that would remain idle for longer periods of time. Hence the trade off between scale economies and the extra construction cost of partly used capacity.

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35 Chenery,H.B.,(1952),"Overcapacity and the Acceleration Principle,"Econometrica,20,1028.

36 b is the scale factor and usually applies to other components of the water supply system such as service reservoirs, sewage treatment works, water supply pumping stations,....etc. See Dangerfield Bernard J. op.cit.

TR61 reports three cost functions for three types of basic treatment :

(1) <u>Gravity filtration</u>

$$C = 108(X)^{0.69} (Y)^{0.54}$$

This is based on 11 observations and where C is in (£000) and X in (1000) M^3/d. Y, the score variable had a mean value of 1.64 . The cost is expressed in terms of 1976 prices.

(2)<u>Sedimentation with Filtration</u>

$$C = 68.2(X)^{0.69} (Y)^{1.07}$$

This function was based on 35 observations and a mean value of SCORE equal to 2.29 .

<u>(3)Pressure filtration</u>

$$C = 64.8(X)^{0.76} (Y)^{0.81}$$

This was based on 20 observations and a mean value of 1.22 for SCORE .

The capital cost data for the estimation of the above equations originated from projects constructed in various parts of the UK during the 1960's and early 1970's. All costs were adjusted for inflation and expressed in terms of 1976 prices.

In principle one would like to adjust the estimates so that, inflation apart, the cost function reflect more recent construction experience. While this can be done in principle it clearly falls outside the scope of this study. It is therefore assumed that a cost function holding up to 1976 is not significantly different from the same cost function prevailing in more recent years. Such an assumption may well be justified if one accepts that since 1976 there has not been major advance in the technology of conventional treatment plants.

A more serious weakness is the following : TR61 cost functions are based on data of different size new treatment works constructed at different cites in the UK. What is required for our purposes, however is a cost function of constructing different size treatment plants at the specific location of Testwood in Hampshire.[37] Moreover in the case of staged expansion, which our dynamic multi-stage model of the next chapter will introduce, we need a cost function which represents staged expansion as well as adding whole new works.

The required cost function may well turn out to be different from the 'average' cost function reported by TR61. Moreover there is some grounds to indicate that the cost function of extensions may offer different economies of scale to that when starting from scratch.

While these difficulties are readily acknowledged it has unfortunately been impossible to surmount them on account of availability of data and scope of study. We were therefore left with the only practical option of accepting TR61 cost functions as rough approximations to the true but unknown cost functions that should apply to our specific case.

As indicated earlier the cost function of treatment works reported by TR61 are in terms of 1976 prices. Data used in the estimation of the treatment plant functions were converted into 1976 prices using the New Construction Wholesale Price and Output Index.[38]

Therefore it follows that adjusting the reported cost functions to a price level of 1986 should be carried out using the same index.

In deriving the cost function it will be assumed that the type of treatment work to be constructed at Testwood will be of the sedimentation with filtration type. A 'SCORE' value equal to the mean value of the sample used by TR61 for this

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[37] This is where the existing works is located. We are therefore assuming that further capacity can in fact be built there. This accords with existing plans of SWA.

[38] In fact TR61 experimented with several indices for each particular cost function. A preferred index was chosen for each function on the basis of statistical and other criteria .

type of treatment work will be used. Carrying out these adjustments and expressing the throughput in terms of million gallons day as opposed to M^3/d we get the following cost function :

$$C(X) = 970\ (X)^{0.69} \qquad (5.6)$$

where C is in £000 and X in million gallons per day.

Table (5.7)

Cost Functions of Conventional Treatment Works

Name of study	b	Comments
(1) TR61	0.69	UK,1960's and 1970's
(2) Orlob & Lindorf	0.67	US,1950's
(3) Roirdan	0.61	US,based on engineering estimates by Metcalf & Eddy.
(3) Koenig	0.68	US based. Study conducted 1967.

Source: Compiled from various references.

It is of interest to point out the striking similarity of the functional form and indeed the value of the exponent between the estimates of TR61 and some international estimates. This is shown in table 5.7 above. The similarity is particularly striking given the big difference in location and the time period. For example Orlob and Lindorf (1958)[39] as far back as 1958 reported the following cost function for California :

$$C(Q) = 257(Q)^{0.67}$$

where C(Q) is total capital cost in (000) dollars and Q is the design capacity in million gallons day. The cost function was estimated using cost figures of treatment plants constructed in the early and mid 1950's and where costs were expressed in terms of 1954 dollars.

[39] Orlob,T,Gerald and Lindorf,R.F.,(1958),"Cost of Water Treatment in California ," JAWWA.,vol.50,jan.1958.

Roirdan (1969)[40] presented the following cost function based on study by Metcalf and Eddy Engineers (1967)[41]:

$$C(Q) = 633(Q)^{0.61}$$

where C was in (000) dollars and Q in million gallon day. Koenig (1967)[42] used data on 21 plants in the USA and obtained the following estimates:

$C(Q) = 307(Q)^{0.68}$ where C was in ($1000) 1964 prices.

It remains to be pointed out that the postulated function will be assumed to hold into the future. This in turn indicates an implicit assumption of constant technology, and constant quality of raw water to be treated and that conventional plant is the chosen technology throughout.

Finally it is assumed that the installed capacity has infinite life and as such requires no replacement. This assumption, perhaps at odds with real life, is again made for convenience. An alternative approach would be to allow for replacement by assuming that capacity has a specific predetermined life; and in this way the capital cost function can then be scaled up to allow for replacement. In our formulation we have adopted the first approach. The second approach could also be easily be used without any major adjustment to the model formulations.

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40 Roirdan(1969),op.cit.

41 Metcalf and Eddy Engineers (1967),"Comprehensive Water Supply Study for Orange County,vol.II :Appendices, CPWS-4 (Albany : State of New York, Department Of Health,1967)

42 Koenig,L.,(1967),"cost of Water treatment by Coagulation, sedimetation and rapid Filtration, Journal of American Water Works Association,59(3),1967.

(5.11) Short-Run Marginal Cost Function

An essential ingredient of our capacity expansion and marginal cost pricing models is the short-run marginal cost function which must incorporate all variable costs of operating the system , ie those costs that are a function of output given capacity. Focus is therefore centered on both treatment and power costs of pumping supplies to consumers.

Care however must be taken to exclude "variable" costs that are not a function of output. In this category is labour costs of operating the system which are usually assumed to be invariant to the level of output.[43] Also excluded from the short-run marginal cost function are such items as administration costs, meter reading, billing and maintenance costs. All these costs are generally thought to be invariant to output levels. Also excluded are items of cost which thought variable with output yet do not represent a resource cost but merely a financial one with distributional but no allocative consequences. Rate payments by water authorities fall in this category and therefore are excluded from short-run variable costs.

Real total variable costs of chemical materials and power including pumping to supply at the existing water treatment plant at Testwood, Hampshire, SWA, indicate that these costs can approximately be taken to be proportional to output. The relevant information is displayed in tables (5.8) and (5.9).

Unit chemical costs at Testwood for the years 1980-1985 fluctuated around an average level of 4.25 £/ML by about 15% in each direction. More to the point perhaps is the absence of indications of the possibility of pronounced change in unit costs as output increases, thus giving some substance to the assumption of constant average (and marginal) unit chemical costs of treatment.

Table (5.9) indicates that by a similar argument it may be possible to assume a constant average and marginal power costs (covering power consumed at the treatment plant as well as power costs of pumping to supply). This will be taken to

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[43] See Cooper & Lybrand (1985), op.cit, where it is reported that Southern Water Authority expect labour costs to remain the same despite planned increase, almost doubling, in treatment capacity.

be equal to 13 £/ML at 1985 prices .

When each of the unit power and chemical costs are adjusted to 1986 prices we arrive at a total unit running cost equal to 17.48 £/ML or, the same thing, 79 £/MG. This estimate is based on evidence from existing treatment and pumping costs at the Testwood cite. Using this result in our marginal cost pricing models would implicitly be assuming that the same constant unit running cost applies to other scales of capacity then the existing one at Testwood. Direct evidence on the plausibility of this assumption is hard to find. There may well exist some economies of scale [44] in which case a specific operating cost function would be required for each scale of output. Such information is just not available [45] and therefore it is necessary to assume that the unit running costs is constant for different levels of output and for different scales of plant.

The short-run marginal cost function is therefore assumed constant at a level equal to 79 £/MG. Moreover the short-run marginal cost function will be assumed to apply up to design capacity, thereafter becoming infinite as diagram (5.6) illustrate. The functional form of the short-run marginal cost relation may be expressed as :

$$MC(q,\bar{Q}) = 79 \quad q < \bar{Q}$$
$$MC(q,\bar{Q}) = \text{infinity} \quad q > \bar{Q} \quad (5.7)$$

The case for using unadaptable type of plant, ie one with rigid capacity may be justified on several grounds: (1) In our model we rule out the possibility of quality deterioration. The assumption of infinite short-run marginal cost function at design capacity ensures that this condition is in fact met; (2) it is likely that the short-run marginal cost function does rise very steeply beyond design capacity.

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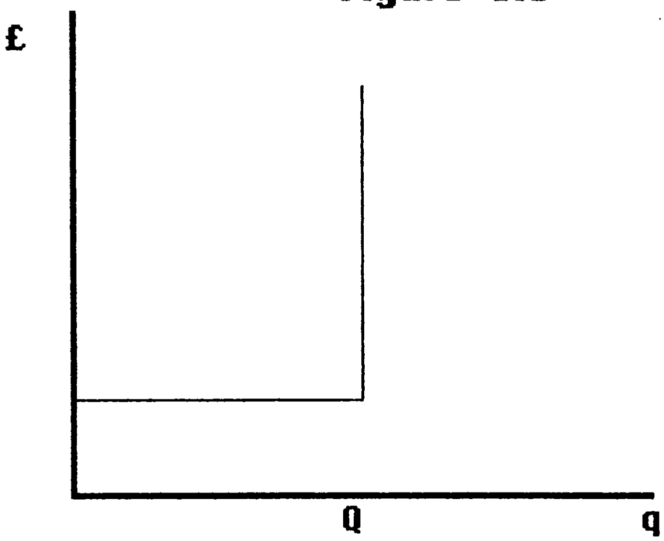
[44] TR61 indicates the existence of such economies of scale. TR61(1977).,op.cit.

[45] Even TR61 information referred to in the previous footnote would not be adequate for it does not specify the cost function applicable for different scales of capacity.

Figure 5.6



Short Run Marginal Cost Function

Table (5.8)

Unit Chemical Costs at Testwood, Hampshire

Year	Annual(1) Potable Output M^3	Cost £(2) Chemicals	Unit Cost £/ML
1980	13809520	66211	4.80
1981	15134470	75065	4.90
1982	16541340	69310	4.20
1983	17633420	63371	3.60
1984	15972770	62656	3.90
1985	15860200	64839	4.10

(1) Output in cubic meteres and excluding industrial consumption.
(2) Cost figures are in real terms (1985).The adjustment conducted by us using the Chemical Industry Price Index.

Source: Compiled by us using figures from SWA,(mimeo).

Table (5.9)

Unit Power Costs at Testwood, Hampshire

Year	Annual(1) Output M^3	Cost £(2) Power	Unit Cost £/ML
1980	16990800	267288	15.73
1981	19214210	265815	13.83
1982	20009430	246647	12.32
1983	21773560	258061	11.85
1984	19630930	231039	11.81
1985	20135070	250488	12.44

(1) Output includes non-potable industrial demand.
(2) Cost figures are in real terms (1985). The adjustment conducted by us using the Fuel and Light Price Index.

Source. Compiled by us using figures from SWA,(mimeo).

(5.12) Initial Capacity of the Model

Total "treatment"[46] design capacity at the study area (and hence water supply capacity) in 1986 stood at 207 ML/d. This was made up as following:

Table 5.10

Initial "Treatment" Capacity

Name of Source	Capacity ML/d
(1) Otterbourn (ground) plus Otterbourn (river) Plus Twyford Moors (ground)	113.7
(2) Testwood (river) (potable only)	68.2
(3) Timsbury(ground)	6.8
(4) Twyford (ground)	19.1
Total	207.8

Source: Various SWA documents.

Total initial capacity is equivalent to 45.77 MG/d or 16706 MG/year. This level of capacity can accommodate a maximum of peak week demand, for a community of 458000 persons, equal to 453.7 l/h/d with a corresponding maximum average daily demand of 366 l/h/d given a peak week factor of 1.24 as estimated for the Hampshire division in a previous section.

[46] Treatment capacity is taken here in the broad sense to include not only the treatment of river water but also that of ground water. The latter may need only minor treatment such as disinfection.

(5.13) The Objective Function and Model's Environment

We shall assume a water enterprise, publicly owned and operated, the objective of which is to supply water to consumers in such a fashion as to secure optimal allocation of resources. The management of the enterprise therefore makes its pricing and investment decisions with the objective of maximising the community's net social benefits over time.

In doing so a partial equilibrium approach is followed where the water industry is seen in isolation from the rest of the economy. This enables us to ignore price distortions in the rest of the economy and all ensuing second best considerations. Likewise all externalities, if any, are ignored.[47]

The water enterprise is assumed to take a neutral stand regarding income distribution. This implies that as far as pricing and investment decisions of the enterprise are concerned we are assuming that a £1 of benefit or cost has the same weight in the objective function irrespective of who receives it or who, ultimately, bears it. This is a common and extremely useful assumption frequently made in welfare economics which conveniently dichotomizes efficiency considerations from distributional ones.[48]

The objective function to be maximised is net social benefits over the planning horizon. Total benefits at a particular point of time, given the partial equilibrium setting[49], are measured by the area under the aggregate

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47 Or we assume that all of the Pareto optimum exchange and production conditions are satisfied elsewhere in the economy.

48 See Webb(1975),op.cit. The traditional argument often cited by economists is to the effect that pricing and investment decisions by the water enterprise ought not to be used as instruments of achieving distributional objectives. The instrument of lump sum transfers is often advocated as a substitute. Of course it may well be argued that pricing and investment may in practice be the only feasible instruments.

49 Output/input changes in the water industry will be treated as if they have no significant effect on the real incomes of consumers and that of factor suppliers.

demand curve for water at that time. This area is readily identifiable as the sum of consumers' surplus plus revenue to the enterprise from the volumetric charge.

Total economic social costs comprise production running costs and capacity costs. Both refer to 'opportunity costs' and in the case of capacity costs it is assumed that once capacity is built it has no alternative use or significant scrap value. Historical capacity costs are 'sunk' costs, ie they have zero opportunity costs.[50]

As was demonstrated by the Rees model[51],maximising net social benefits can be achieved by a policy of short-run marginal cost pricing. The water enterprise therefore is assumed to follow such a policy where price at any point of time is set to whichever is the higher of short-run marginal cost or the price necessary to ration demand (including peak demand) to a level equal to total capacity.

Seasonal peak load pricing is not admitted on the grounds of present metering impracticality. For peak load pricing to be effective the water enterprise must be able to measure with reasonable accuracy the level of consumption at the time of the peak. With traditional meters (and a rotating system of meter reading so as to save on labour costs) it is not possible to do so. The required technology for the kind of advanced telemetry to measure consumption at times of the day, week, or year by means of remote meter reading is available now. However the introduction of such advanced technology in the near future seems unlikely .[52]It therefore seems reasonable to assume that uniform pricing will prevail after the introduction of traditional metering.

It is also assumed that political and administrative constraints do not rule out price fluctuations that may

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50 See Millward(1971),op.cit.

51 See section 2.5.

52 The Watts report(1985) concluded that the additional costs of customer telemetry would greatly exceed the additional economic benefits for the average consumer, unless a high proportion of the costs were born by other utilities. The Watts report however does not provide explicit estimates of benefits and costs.

emerge from the solution of the marginal cost pricing and investment decision models. The degree of fluctuation in prices will in fact be examined later including the welfare implications of this fluctuation. In particular we will examine, in chapter 6, the welfare implications of charging a uniform price throughout the planning horizon. Also we will tentatively examine the presumed costs to the consumer of expected and unexpected price fluctuation.

As for balancing the books it is assumed that a system of standing charges will be used to bridge shortages of revenue to cover capacity and running costs that may emerge from the short-run marginal cost pricing policy. This is not unrealistic given the existing (UK) flat charging system and the practice in other countries using volumetric charging. In effect this means that the existing system of flat charges is supplemented by a volumetric charge instead of being scrapped altogether in favour of a wholly volumetric charge.

It is assumed in the models that treatment plant capacity is the only critical constraint on the volume of production. This implies that once an optimal capacity expansion schedule for treatment works is decided upon, the capacity of the other components of the water supply(ie source works, trunk mains and distribution networks) will always be maintained at levels sufficient to meet demand as dictated by the capacity of the treatment works.

Such a highly restrictive assumption is justified on the following grounds : (1) For Southern Hampshire, ie the study area of the model, this assumption is not as damaging as might first seem to be. This is because for the next 25 years or so the situation is such that indeed treatment capacity may be taken to be the only major constraint on output from the point of view of 'strategic' investment which excludes distribution capacity. Therefore if we abstract from distribution capacity our assumption may well be acceptable; (2) unless all components of the water supply system are constructed simultaneously, and as such effectively can be treated as 'one', the incorporation of many components of capacity vastly complicates the solution of the marginal cost cum capacity investment optimisation models which we are dealing with.[53]

Finally the model of this chapter assumes, somewhat arbitrarily, that all additional treatment plant is to take place at one point of time, ie staging of capacity addition is ruled out. Relaxation of this assumption leads us into the dynamic programming formulation of the optimisation model.

(5.14) The Discount Rate

Our marginal cost pricing models require the specification of a discount rate to be used for the aggregation of benefits and costs spread over time.

One possibility is to use 'the' market interest rate or perhaps more specifically the water enterprise's borrowing rate of interest. This would be correct if we were conducting a financial analysis and appraisal. However we are not and therefore we must search for other discount rates more suitable for our economic rather than financial analysis.[54]

For economic analysis economists usually advocate one of two rates; the social time preference rate (STP) and the social opportunity cost of capital rate (SOC).[55] The STP rate measures the marginal rate of substitution between present and future consumption as seen by society. It is customary to assume for operational convenience that this rate is constant over time, although there may not be much theoretical justification for this assumption.

The intertemporal measure of STP is impossible to observe and measure directly. Indirect methods based on some

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[53] This is especially the case when we move into a dynamic programming version of the optimisation model where the inclusion of more than one type of capacity increases the number of the 'state' variables creating what is known as the problem of dimensionality. See Hamdy,A.Taha,(1982), "Operations Research, An Introduction," Third edition, Macmillan Publishing Co.,INC.1982.

[54] Several reasons can be given as to why the public sector discount rate cannot be observed from the capital market. These include that government sector investment has some public good attributes. See Sen,M,(1967),"Isolation,Assurance and the Social Rate of Discount," Quarterly Journal of Economics, Vol.81,1967.

[55] See Layard,R.,1972, Cost-Benefit Analysis: Selected Readings, Penguin Books, London.

hypothesis regarding the elasticity of the marginal social utility of consumption with respect to consumption, together with a forecast of expected growth of income[56] have been used to derive estimates of STP.

Using such an approach Scott[57] estimated that STP in the UK falls in the range of 4.5% to 6%. Herrington(1977)[58] reports that this estimate is consistent with figures suggested by Balmer[59] for use in the UK for the water industry. A Treasury working paper (1978)[60] elaborating on the methodology of the 1978 White Paper[61] suggests that economic considerations of the type discussed above could be used to justify a planning rate (STP) of around 5 percent.

The social opportunity cost rate is a measure of the value to society of the next best alternative use to which funds employed by the project might otherwise have been put.[62] In principle the opportunity cost of the capital funds could be the displacement of one or more of the following : public sector consumption, public sector investment, private sector consumption and private sector investment. Which of

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56 This approach can be found in Eckstein(1961). The STP rate is estimated as the product of the expected rate of growth, g, and the 'estimated' (guessed) elasticity of the marginal utility of consumption with respect to a change in consumption. A 'pure' time preference rate is sometimes added.
See Eckstein,O.,(1961)," A Survey of the Theory of Public Expenditure Criteria," in Houghnton,R.,(ed.),"Public Finance, Penguin Books, London.

57 Scott,M.F.G,(1977),"The Test Rate of Discount and Changes in Base Level Income in the United Kingdom,"Economic Journal,Vol.87,1977.

58 Herrington,P.R.(1977),"Choices Within the Water Industry: Does Economics Help ? in Proceedings of the Symposium on Water Services: Financial,Engineering, and Scientific Planning, The Institute of Water Engineers, London.

59 Balmer,R.(1975),JIWES 29,390

60 Treasury Working Paper No.9,(1978), " The Test Discount Rate and the Required Rate of Return on Investment," Proceedings of the Seminar held at the Civil Service College , January 1979.

61 Cmnd.7131,London,HMSO,1978

62 See Feldstein,M.(1964)," The Social Time Preference Rate in Cost Benefit Analysis." Economic Jounal,Vol.74,1964.

these or combination of these is appropriate depends on the nature of the political, administrative, and economic constraints effecting the sources of public sector investment funds as well as the investment opportunities available to the public sector (constraints on the use of public funds).

The official view in the UK, as expressed in the 1967 and 1978 white Papers seems to have favoured the assumption that the opportunity cost of public funds is wholly displaced private sector investment. Such an interpretation would be correct if following Flemming(1977)[63]one accepts the view that there is no boundary on the extent or variety of public sector investment opportunities, ie there is no marked constraint on preventing public enterprises from taking up investment opportunities in the private sector.

In such circumstances the opportunity cost of public funds can be expressed as the foregone real rate of return that could have been earned in the private sector had the funds been invested there. The real rate of return on low-risk marginal private investment was expressed up to 1978 by what was known as the Test Discount Rate (TDR). The TDR, 10%, was based on evidence of the ex-ante rate of return in the private sector. The 1967 White Paper[64]recommended that public investment projects be appraised using a discount rate equal to TDR. The 1978 White Paper however, marked the demise of the TDR rate and the birth of the Required Rate of Return (RRR). This rate which was to be required not from individual projects but from all new investment was also meant to be based on the principle of the opportunity cost of capital. A figure of 5% was given for RRR based on the ex-post real rate of return not on the marginal private sector project but on the rate of return achieved by the private sector on its entire stock of capital. As Heald(1978)[65]and others have

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63 Flemming,J.S.,(1977), "What Discount Rate for Public Expenditure," In "Allocation Between Competing Ends,"Posner,M.V.,(ed.),Cambridge University Press,London.

64 Cmnd.3437,London,HMSO,1967

65 Heald.,D.,(1978),"The Economic and Financial Control of UK Nationalised Industries: A Critique of the White Paper,"Discussion Paper,No.30,University of Glasgow.

pointed out the RRR rate is not comparable with the TDR rate since they measure different things. Which is more acceptable for discounting purposes in the public sector is perhaps a matter of conjecture. However it is worth noting that the significant drop from 10% to 5% seems according to Rees[66] to have been in part justified (by the Treasury) using time preference considerations; the desire to see the discount rate at a level closer to the expected range of STP.

The discussion so far indicates the general nature of the debate regarding the STP and SOC discount rates. It also indicates the uncertainty regarding the values of both the STP and SOC rates. This uncertainty seems to make a third approach to discounting rather difficult to apply. A theoretically attractive approach, based on the use of STP and properly defined SOC, has been provided by Marglin(1963)[67]. This approach amounts to using the STP rate for discounting benefits and costs over time but with due adjustment to the capital costs of the public project using a 'shadow price' for capital funds. This shadow price depends on the SOC as well as the STP rate and is usually taken to be greater than one so long as investment in the economy is thought to be below what is socially desirable.

Our approach to the question of discounting in the marginal cost pricing cum investment model would be to use a range of 3% to 10% for the discount rate. In particular benefits and costs of expanding capacity will be assessed using four different discount rates; 3%, 5%, 7% and 10%. No adjustment will be made to allow for the shadow price of capital funds.[68]

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66 See Treasury Working Paper No.9, op.cit. It contains a comment by Rees,R. which the text refers to.

67 Marglin,S.A,(1063),"The opportunity Cost of Public Investment," Quarterly Journal of Economics, Vol.77,1963. See also UNIDO,(1972),"Guideline For Project Evaluation," United Nations Industrial Development Organisation,New York,UN.

68 The discounted sum of net benefits over time will be referred to as net social benefits. This strictly speaking is only true if the discount rate is taken to be equal to the STP rate and due allowance is made for shadow pricing capital funds.

(5.15) Model formulation

Given the assumptions of the previous sections we are now in a position to build an optimisation model incorporating the short-run marginal cost pricing rule and the accompanying investment criterion of traditional economic analysis;

> (1) price should be equal to whichever is the higher of marginal operating cost or the price that rations demand to match existing capacity;
>
> (2) the investment criterion is to the effect that extra capacity should be added only when the benefits of extra output net of running costs over time exceeds the capital cost of new capacity.

The optimisation model, seen in the framework of maximising benefits less costs over time, tackles the pricing and investment decisions simultaneously. It should give answers regarding optimal output, optimal prices and optimal capacity addition over the planning horizon. These answers are found simultaneously.

The distinguishing feature of this chapter's single stage capacity addition model is the constraint that capacity can only be added at one point in time.

When constructing the model we will not explicitly include all the variables listed above (prices over time, output over time, and capacity addition). As will be seen below the same answers can be found from a model expressed in terms of two variables only; size of the once and for all capacity addition and its timing. This together with the short-run marginal cost pricing rules implicitly defines all the other variables.[69]

First we introduce the concept of effective capacity as opposed to design capacity used in the capital cost function (eq. 5.6). Effective capacity is defined as design capacity divided by the peak week factor (1.24 according to the analysis of section 5.3).

Demand in the model is expressed in terms of aggregate annual average levels in million gallons per year. The marginal cost pricing rule in the model can now be formulated in terms of average annual demand and effective capacity as

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[69] This formulation as well as that of the next chapter are based on the work of Roirdan (1969), op.cit.

defined above. So long as average aggregate demand at a price equal to short-run marginal cost is less than effective capacity, price would be set to short-run marginal cost (MC) and output would be that dictated by the postulated demand relationship at P = MC. If at a price equal to MC average demand exceeds effective capacity then price would have to be raised to whatever level necessary to reduce average demand to a level equal to effective capacity.

The use of this pricing mechanism together with effective capacity ensures that an analogous mechanism operates during time of peak demand, this time using design capacity. This must be the case since peak demand is 1.24 of average demand and likewise design capacity is 1.24 times effective capacity. That is, price at the peak is set equal to MC when peak demand at a price equal to MC is less than or equal to design capacity. Price is set to whatever level necessary to ration design capacity otherwise; this will be the same price that rations design capacity during the time of average demand.

The use of the concept of effective capacity together with a peak week factor which is independent of prices, ensures that our uniform pricing policy does not require explicit consideration of the peak. The benefit expressions of the model can be expressed in terms of aggregate average demand and effective capacity. Of course the initial capacity level of 16706 MG/year derived in section 5.12 will have to be converted into an equivalent effective capacity. This is equal to 13472 MG/year. The only place where design capacity considerations need enter into the model is of course in the cost of capacity addition function (5.6). When the model solution requires the addition of Q additional units of effective capacity, the cost of that addition is clearly equal to the cost of adding (1.24 Q) units of design capacity.

The once and for all possible addition of capacity could lead to one of three possible situations depicted in diagrams (5.7), (5.8), (5.9). In all these figures as in the model formulation the beginning of the analysis (year 1986) is designated time zero. This is the time to which all benefits and costs will be discounted, ie all benefits and costs will be expressed in terms of 1986 base year.

At time zero existing effective capacity is 13472 MG/year (K1=13472, see section 5.12). This capacity can support an average demand of 366 l/h/d for a community of 458000 persons. This is equivalent to 0.029416 MG/head/year.

Aggregate demand at time zero on the other hand is equal to 15471 MG/year (K=15471) when price is set equal to MC (79 £/MG) This is equivalent to 0.03378 MG/head/year, 420 l/h/d.

Given the existing effective capacity K1 (13472) is less than the required effective capacity consistent with MC pricing at time zero (K1=15471), it follows that price at time zero must go above MC. Using the demand equation

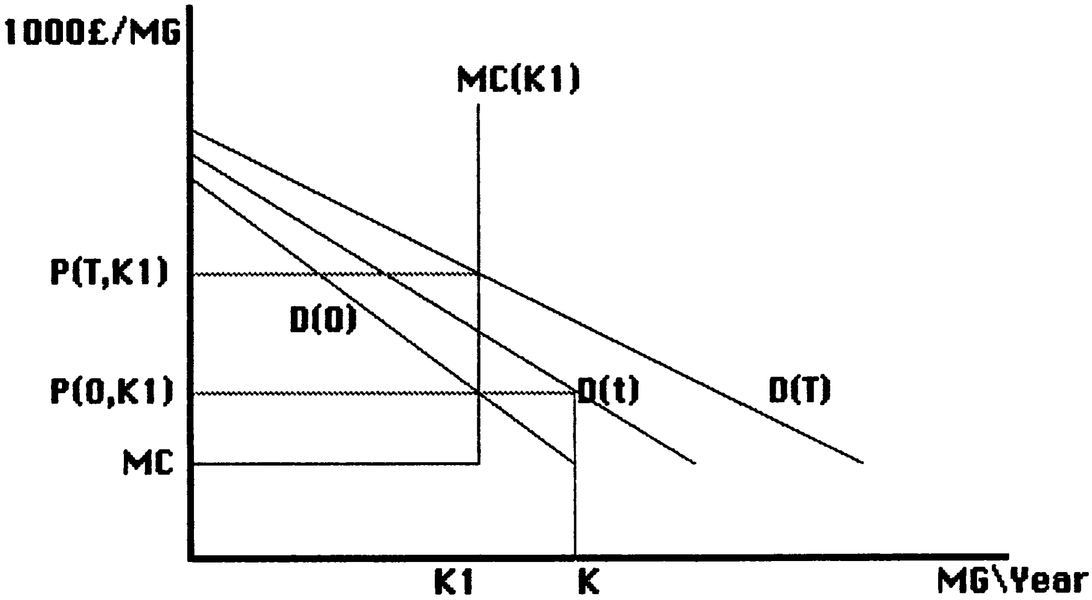
$$P(t,q) = (P'+SDt) - \frac{P'+SDt-MC}{K + Dt + Rt}\, q \qquad (5.5)$$

and the assumed values for the parameters P', S, D, MC, K and R, we find the required level of price at time t=0 to ration the existing effective capacity of K1 or 13472 MG. The required price is equal to 719 £/MG, (or 158.3 £/ML or 15.83 P/M^3).

Our first case depicted in diagram (5.7) shows a situation in which the solution of the model is such that no capacity addition at all is warranted throughout the planning horizon. Total effective capacity throughout is equal to the initial existing capacity K1. The output solution of the model is likewise equal to K1 throughout.

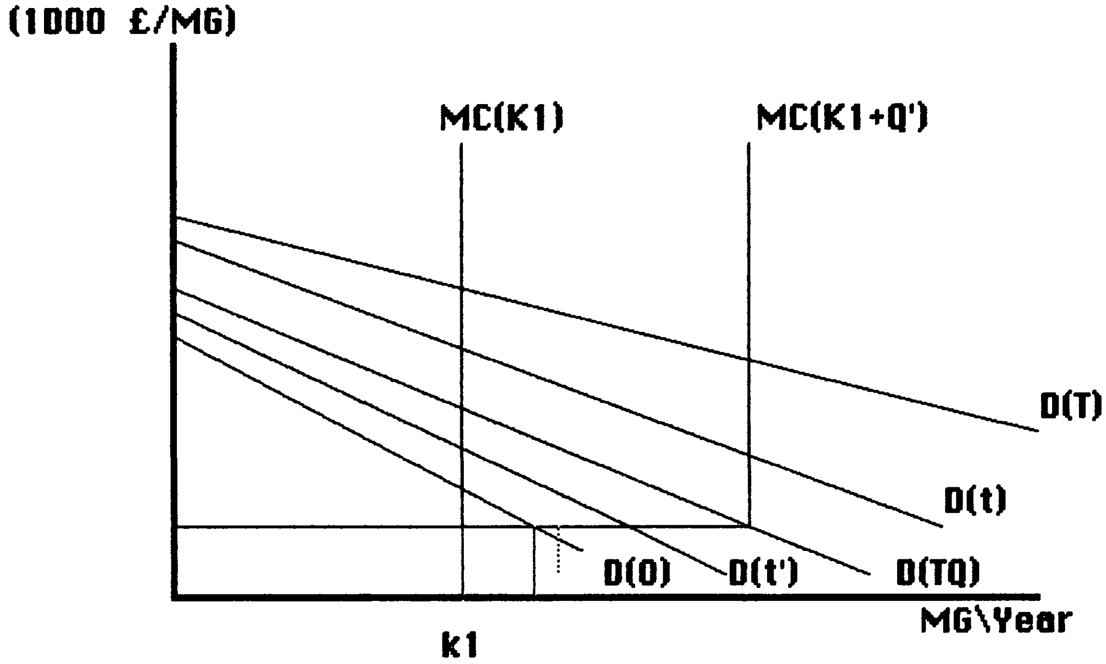
Price in such a situation would need to be increased continuously from its initial level at time zero. The rise continues up to year T in order to ration effective capacity in the face of growing demand. At time T population and absolute growth in demand are assumed to stabilize. It follows therefore that at time T price also stabilizes at a level equal to P(T,K1). The level of prices over time can

Figure 5.7



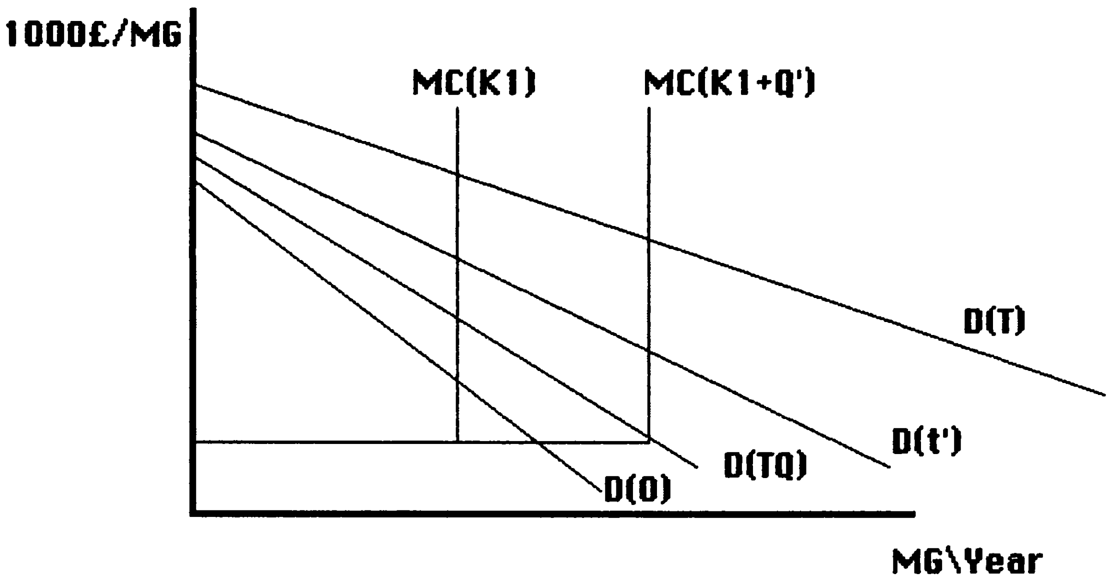
Single Stage Model
Case I

figure 5.8



Single Stage Model
case II

Figure 5.9



Single Stage MODEL
Case III

readily be found out from the dynamic demand equation by simple substitution.

In figure (5.8) we have a situation in which the solution of the model involves the addition of capacity size $\hat{Q}$ at time $\hat{t}$. The size of the capacity addition and its timing are such that output and price pass through four phases; from time zero to $\hat{t}$ capacity is at its initial size of K1 and so is output, but prices would be increasing gradually to check growing demand in line with capacity; at time $\hat{t}$ capacity is incremented by $\hat{Q}$ to a total of $K1+\hat{Q}$ which is sufficient to meet all demand at a price equal to MC (ie K+Rt+Dt). Price at this phase is maintained at MC. This phase ends at time TQ when new capacity $\hat{Q}$ is just exhausted given a price equal to MC. Time TQ marks the beginning of a third phase when price at any time t, $P(t,K1+\hat{Q})$ has to be chosen as the lowest possible price to ration capacity of size $K1+\hat{Q}$ in the face of growing demand. Price in this phase will grow continuously while output remains constant at $(K1+\hat{Q})$. The price rise terminates at time T, when demand stabilises, settling at $P(T,K1+\hat{Q})$ up to the end of the planning horizon.

In case III, figure (5.9) we have a situation of the model also involving the addition of capacity of size $\hat{Q}$ and at time $\hat{t}$. This time however, the size of the capacity addition and its timing is such that at no time is there sufficient capacity to meet all demand at a price equal to MC. In this case we identify three phases for output and prices; between time zero and $\hat{t}$ price would have to rise to ration existing capacity K1; at time t' capacity is augmented by $\hat{Q}$ to a level of $K1+\hat{Q}$, in consequence price would fall to a level, greater then MC, and high enough to ration demand to a level equal to $K1+\hat{Q}$; this phase of rising prices will prevail from time $\hat{t}$ to time T, ie during the period when demand is growing; as from T when demand stabilises, price will remain constant at a level equal to $P(T,K1+\hat{Q})$; output during this period would in turn remain constant at $K1+\hat{Q}$.

Of course which of these cases holds depends entirely on the values of $\hat{t}$ and $\hat{Q}$. This may be expressed algebraically :

Case I

$\hat{Q} = 0$ then case I applies.

Case II

This case applies if the magnitude of the expansion in effective capacity $\hat{Q}$ is such that there is for a time spare capacity in the newly-enlarged treatment works { $K1+\hat{Q}$ is greater than $(K+D\hat{t}+R\hat{t})$ which is aggregate demand that would result at time $\hat{t}$, time of capacity expansion, if a price equal to MC is set then }. This condition may be expressed as:

$$K1+\hat{Q} > K+D\hat{t}+R\hat{t}$$

or
$$\frac{\hat{Q}-(K-K1)}{R+D} > \hat{t}$$

or
$$\hat{t} < \frac{\hat{Q}-(K-K1)}{R+D}$$

where it is evidently clear that $\frac{\hat{Q}-(K-K1)}{R+D}$ is equal to the time when the new capacity $\hat{Q}$ is just exhausted given a price equal to MC. This is equal to the variable TQ in the text above. Hence case II applies when:

$$\hat{t} < TQ$$

where $TQ = \frac{\hat{Q}-(k-k1)}{R+D}$

Case III

The condition for case III can be derived in analogous way to case II, the condition being :

$$\hat{t} \geq TQ$$

where TQ is defined as before.

In summary the three possible cases for the solutions of the model are :

$Q' = 0$ case I applies

$t' < TQ$ case II applies

$t' \geq TQ$ case III applies[70]

The objective function to be maximised can be specified as follows. Gross benefits at any point in time, as represented by consumers' willingness to pay is given by the area under the aggregate demand curve. (See diagram 5.10 below). Thus at time t given the demand curve P(t,q) the gross benefits of supplying $\bar{q}$ output is the shaded area between 0 and $\bar{q}$. This is equal to :

$$GB = \int_0^{\bar{q}} \{ (P'+SDt) - \frac{p'+SDt-MC}{K+Rt+Dt} q \} \, dq \quad (5.8)$$

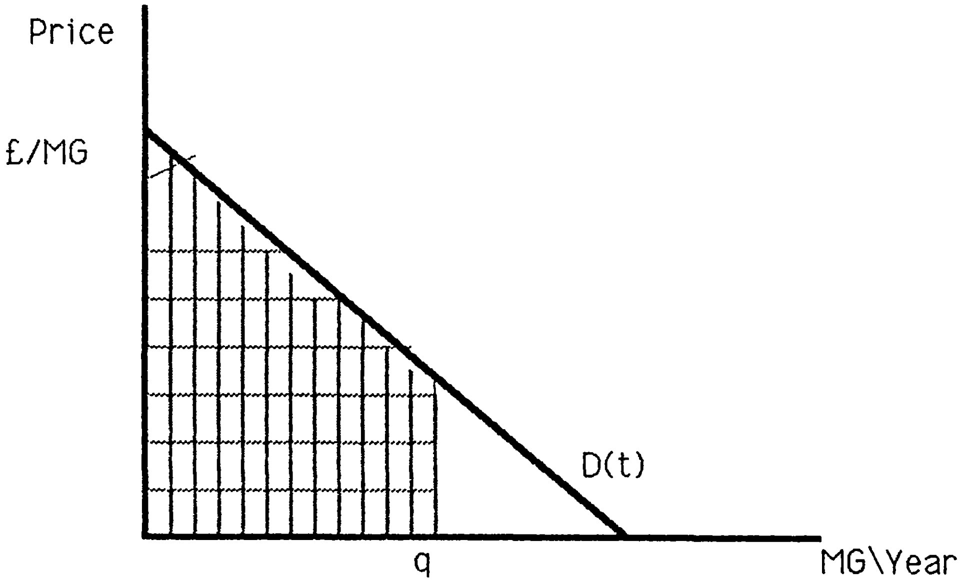
$$GB = (p'+SDt)\bar{q} - \frac{P'+SDt-MC}{K+Rt+Dt} \frac{\bar{q}^2}{2} \quad (5.9)$$

The gross benefit expression (5.9) would correctly represent gross benefits to the community only if leakage from the system is zero. However we have already established that losses from the system do in fact occur in the study area at a constant proportionate rate equal to (f) or as established earlier 27%. Accordingly the gross benefit function will need to be corrected by a factor equal to (1-f). The corrected gross benefit function is simply given by equation (5.10):

$$GB = (1-f)\{ (P'+SDt) - (1-f) \frac{P'+SDt-MC}{K+Rt+Dt} \frac{\bar{q}^2}{2} \} \quad (5.10)$$

[70] We have switched to t' and Q' instead of $\hat{t}$ and $\hat{Q}$ so that the latter notation is reserved for the optimum solution, ie those Q and t that maximise the objective function.

Figure 5.10



Area Under Demand Curve

Equation (5.10) can also be derived by integrating an amended demand equation between an upper limit of $(1-f)\bar{q}$ and a lower limit of zero. The amended demand equation is :

$$P(t,q) = (P'+SDt) - \frac{P'+SDt-MC}{K+Dt+Rt}\,\frac{1}{1-f}\,q$$

and the integral then becomes:

$$GB = \int_{0}^{(1-f)\bar{q}} \left\{ (P'+SDt) - \frac{P'+SDt-MC}{K+Dt+Rt}\,\frac{1}{1-f}\,q \right\} dt$$

and where f is the leakage ratio. Integrating this expression between the limits gives the results above.

Out of gross benefits we have to subtract the corresponding running costs. Running costs, given our constant MC formulation, are simply $MC\bar{q}$. Net benefits at time t for output level $\bar{q}$ are then given by :

$$NB = (1-f)(P'+SDt)\bar{q} - (1-f)\frac{(P'+SDt-MC)}{K+Rt+Dt}\,\frac{\bar{q}^2}{2} - MC\bar{q} \quad (5.11)$$

Next we derive expressions of total net benefits over an infinite planning horizon. The expression of total net benefits over the planning horizon would depend on which of the above cases (I,II,III) applies since the relevant expression is dependent on the particular configuration of Q' and t', the size of capacity addition and its timing. That is the net benefit expression is unknown being either as in case I or II or III depending on the values of Q' and t'. It is therefore necessary to develop the total net benefit function over time for each of the three defined cases.

<u>Case I</u> when Q' = 0

$$TNB = \int_0^T \left\{ (1-f)(P'+SDt)K1 - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\frac{K1^2}{2} - MC\ K1 \right\} e^{-it}\,dt$$

$$+$$

$$\int_T^H \left\{ (1-f)(P'+SDT)K1 - (1-f)\frac{P'+SDT-MC}{K+RT+DT}\frac{K1^2}{2} - MC\ K1 \right\} e^{-it}\,dt \qquad (5.12)$$

where H stands for the planning horizon assumed by us to be infinity.

We note that the above expression corresponds to the discussion before and the diagrammatic representation of case I.

The first integral in equation (5.12) above contains the phase where demand is growing and output is limited to the initial effective capacity K1. The second integral on the other hand contains the phase from time T to infinity, a period when demand is stationary and where output like before stands at K1.[71]

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[71] Continuous discounting has been used instead of discrete discounting since the former is more convenient. i in our notation stands for the discount rate,0.03 to 0.10.

Case II when t' < TQ and $TQ = \dfrac{Q'-(K-K1)}{R+D}$

$$TNB = \int_0^{t'} \left\{ (1-f)(P'+SDt)K1 - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\,\frac{K1^2}{2} - MC\,K1 \right\} e^{-it}\,dt$$

$$+$$

$$\int_{t'}^{TQ} \left\{ (1-f)(P'+SDt)(K+Rt+Dt) - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\,\frac{(K+Rt+Dt)^2}{2} - MC(K+Rt+Dt) \right\} e^{-it}\,dt$$

$$+$$

$$\int_{TQ}^{T} \left\{ (1-f)(P'+SDt)(K1+Q') - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\,\frac{(K1+Q')^2}{2} - MC(K1+Q') \right\} e^{-it}\,dt$$

$$+$$

$$\int_{TQ}^{H} \left\{ (1-f)(P'+SDT)(K1+Q') - (1-f)\frac{P'+SDT-MC}{K+RT+DT}\,\frac{(K1+Q')^2}{2} - MC(K1+Q') \right\} e^{-it}\,dt$$

$$- C(1.24\,Q'/365)\,e^{-it'} \qquad (5.13)$$

Each of the integral expressions above represents one of the four stages identified in the discussion above regarding case II. Output for each of these stages is defined by K1, (K+Rt+Dt), (K1+Q') and (K1+Q') respectively. The discounted capital cost of design capacity of size 1.24Q' is netted out.

Case III when $t' \geq TQ$ and $TQ = \dfrac{Q'-(K-K1)}{R+D}$

$$TNB = \int_{0}^{t'} \left\{ (1-f)(P'+SDt)K1 - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\frac{K1^2}{2} - MC\ K1 \right\} e^{-it}\,dt$$

$$+$$

$$\int_{t'}^{T} \left\{ (1-f)(P'+SDt)(K1+Q') - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\frac{(K1+Q')^2}{2} - MC(K1+Q') \right\} e^{-it}\,dt$$

$$+$$

$$\int_{T}^{H} \left\{ (1-f)(P'+SDT)(K1+Q') - (1-f)\frac{P'+SDT-MC}{K+RT+Dt}\frac{(K1+Q')^2}{2} - MC(K1+Q') \right\} e^{-it}\,dt$$

$$- \quad C(1.24\ Q'/365)\ e^{-it'}$$

(5.14)

As before each of the integrals corresponds to a particular phase in time with output being equal to K1, (K1+Q'), and (K1+Q'), and where the capital cost of the design capacity is like before.

The next step is to conduct the integration of the relevant terms in the above equations. Upon conducting the integration and some algebraic manipulations we get the following expressions for total net benefits over time for each of the three cases identified.[72]

Case I

$$TNB = [(1-f)P'K1 - MC\,K1]\left[\frac{-e^{-it}}{i}\right]_0^T$$

$$-(1-f)\frac{K1^2}{2}\int_0^T \frac{P'+SDt-MC}{K+Rt+Dt}\,e^{-it}\,dt$$

$$+\,(1-f)S\,K1\,D\left[\frac{-(1+it)}{i^2}\,e^{-it}\right]_0^T$$

$$+\left[(1-f)(P'+SDT)K1 - (1-f)\frac{P'+SDT-MC}{K+RT+DT}\,\frac{K1^2}{2}\right]\left[\frac{-e^{-it}}{i}\right]_T^H$$

(5.15)

[72] See appendix for derivation of equations.

<u>Case</u> <u>II</u> when t'< TQ

TNB =

$$[(1-f)P'K1 - MC\ K1]\left[\frac{-e^{-it}}{i}\right]_0^{t'}$$

$$-(1-f)\frac{K1^2}{2}\int_0^{t'}\frac{P'+SDt-MC}{K+Rt+Dt}\,e^{-it}\,dt$$

$$+\ (1-f)S\ K1\ D\ \left[\frac{-(1+it)}{i^2}\,e^{-it}\right]_0^{t'}$$

$$+\ 0.5\ K\ [(1-f)P' - (1+f)MC]\ \left[\frac{-e^{-it}}{i}\right]_{t'}^{TQ}$$

$$+\ [0.5(1-f)(P'R+P'D+SDK) - 0.5(1+f)(R\ MC+D\ MC)]$$

$$\left[\frac{-(1+it)}{i^2}\,e^{-it}\right]_{t'}^{TQ}$$

$$+\ 0.5(1-f)(SDR+SD^2)\left[\frac{-(i^2t^2+2it+2)}{i^3}\,e^{-it}\right]_{t'}^{TQ}$$

$$+\ (1-f)P'(K1+Q')-MC(K1+Q')\left[\frac{-e^{-it}}{i}\right]_{TQ}^{T}$$

$$+\ (1-f)(K1+Q')\ SD\ \left[\frac{-(1+it)}{i^2}\,e^{-it}\right]_{TQ}^{T}$$

$$-\ (1-f)\ \frac{(K1+Q')^2}{2}\int_{TQ}^{T}\frac{P'+SDt-MC}{K+Rt+Dt}\,e^{-it}\,dt$$

$$+ \left[(1-f)(P'+SDT) - (1-f)\frac{P'+SDT-MC}{K+RT+DT}\,\frac{(K1+Q')^2}{2} - MC(K1+Q')\right]\left[\frac{-e^{-it}}{i}\right]_T^H$$

$$- \; 970\,(1.24\,Q'/365)^{0.69}\,e^{-it'}$$

$$(5.16)$$

Case III when $t' \geq TQ$

$$TNB = [(1-f)P'K1 - MC\,K1]\left[\frac{-e^{-it}}{i}\right]_0^{t'}$$

$$-(1-f)\frac{K1^2}{2}\int_0^{t'} \frac{P'+SDt-MC}{K+Rt+Dt}\, e^{-it}\, dt$$

$$+ (1-f)S\,K1\,D \left[\frac{-(1+it)}{i^2}\, e^{-it}\right]_0^{t'}$$

$$+ (1-f)P'(K1+Q')-MC(K1+Q')\left[\frac{-e^{-it}}{i}\right]_{t'}^{T}$$

$$+ (1-f)(K1+Q')\,SD \left[\frac{-(1+it)}{i^2}\, e^{-it}\right]_{t'}^{T}$$

$$- (1-f)\frac{(K1+Q')^2}{2}\int_{t'}^{T} \frac{P'+SDt-MC}{K+Rt+Dt}\, e^{-it}\, dt$$

$$+ \left[(1-f)(P'+SDT) - (1-f)\frac{P'+SDT-MC}{K+RT+DT}\,\frac{(K1+Q')^2}{2} - MC(K1+Q')\right]\left[\frac{-e^{-it}}{i}\right]_T^H$$

$$- 970\,(1.24\,Q'/365)^{0.69}\, e^{-it'}$$

(5.17)

We note that the integral below appears several times. The analytic solution to this integral is set below. It was derived using integration by parts twice as well as by referring to a standard table of integrals.

$$I = \int_b^a \frac{P'+SDt-MC}{K+Rt+Dt} e^{-it}\, dt$$

The solution is equal to :

$$I = \left[\frac{P'-MC}{R+D} e^{iK/(R+D)}\right] [sum(t)]_b^a$$

$$-\left[\frac{SD}{i} \frac{te^{-it}}{K+Rt+Dt}\right]_b^a$$

$$-\left[\frac{SDK}{i(R+D)} \frac{e^{-it}}{K+Rt+Dt}\right]_b^a$$

$$-\frac{SDK}{(R+D)^2} e^{ik/(R+D)} [sum(t)]_b^a$$

and where $[sum(t)]_b^a$ is defined as follows

$$[sum(t)]_b^a = \left[LOG\left(\frac{K}{R+D} + t\right) - \frac{i\left(\frac{K}{R+D} + t\right)}{1.!} + \frac{i^2\left(\frac{K}{R+D} + t\right)^2}{2.2!} - \frac{i^3\left(\frac{K}{R+D} + t\right)^3}{3.3!} + \ldots\ldots\ldots\ldots \right]_b^a$$

It is to be noted here that the solution of the integral involves the convergent series sum(t). Care must be taken so that enough terms of the series sum(t) are counted to ensure accuracy. The number of terms required before truncating the series might well depend on the relative values of i, D, K, R and t. In our case we took 70 terms. This was based on experimental calculations; less terms proved inadequate for some particular configurations of the parameters.

(5.16) Solution of the Model

A typical method for solving a problem of unconstrained maximisation involving a function with several variables is to partially differentiate the objective function with respect to each of the variables; set the partial derivatives to zero and then solve the resulting equations for the unknown optimum values of the variables.

Such an approach however is not immediately possible for the solution of our optimisation problem. In our case the problem is complicated by the presence of three separate expressions for the objective function depending on the relative values of the two variables involved, Q' and t'. An analytical solution may be contemplated only if it were possible to specify in advance which of the three expressions contains the optimum solution. This information cannot easily be established by analytical methods. It is also not easy to establish analytically the concavity of the relevant net total benefit function even if it were possible to be identified. Indeed each of the three expressions of the net benefit function is sufficiently complex to rule out a practical solution of the problem along the classical analytical method suggested in the above few lines.

In consequence we had to look for another more practical numerical solution method. The numerical method chosen simply involved systematic calculation of the value of the net benefit function for different values of Q' and t' in a methodological search for the pair of Q' and t' with the highest (maximum) value of the objective function.

Besides its simplicity this method can in fact prove to be quite accurate in indicating that the function is indeed concave in the region of the located maximum and thus giving some assurance that (a) the located maximum is not a saddle point and (b) that it is indeed a global and not a local maximum. This confirmation can be achieved once the sampling is carried out by graphical methods. Plotting the surface area of the total net benefit function would be useful for this purpose.

This sampling approach is considerably facilitated by the fact that some meaningful range for the decision variables Q' and t' can be established analytically. This range once established forms the limits on the values of Q' and t' that have to be sampled. Clearly neither Q' or t' can take a value lower than zero. Moreover t' can never be greater than T.[73] Thus a range of values for t' is established as (0,T).

An Upper limit can also be established for Q'. This is equal to (RT+DT+K)-K1 , representing the net addition to initial effective capacity required if we were to meet all demand at a price level equal to MC and for all periods including time T. The inefficiency of having bigger capacity then RT+DT+K-K1 is demonstrated by the argument that any extra capacity would require a selling price lower then short run marginal cost, MC, with undoubtedly negative effect on the value of the objective function. Therefore the first stage of the sampling procedure involves scanning the value of the objective function for values of t' and Q' in the range of (0,T) and (0,RT+DT+K-K1) respectively.

Given that the first stage of the sampling procedure produces convincing evidence that the net benefit function is in fact concave and an initial maximum is located, then one can proceed safely into a second step of the solution. This involves a second round of sampling of the values of the total net benefit function for values of Q' and t', this time the variation in their values being in the vicinity of the first round solution. If the second round of sampling fails to improve on the value of the objective function then we conclude that the first round solution was indeed the

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73 This point is proved by Roirdan(1969). The proof goes like this; if capacity is to take place after T then it means that it pays to postpone capacity addition at T. But if this is so then it surely must also pay to postpone capacity addition for ever since nothing after T is different from what was prevailing at T. Conversely if it is desirable to invest in additional capacity after T it must be even more desirable to invest that capacity at time T or earlier. See Roirdan(1969),op.cit.

maximum, as accurately as one can get using this technique. If on the other hand we improve on the value of the objective function then we may proceed to third round of even a finer sampling grid in the hope of improving the value of the objective function. This process can continue so long that an improvement in the value of the objective function can be achieved, though in practical terms the search would probably be terminated after several iterations when hopefully the value of the function converges or the improvement is small enough to be ignored.

The equations (5.15), (5.16), (5.17), for the total net benefit function specify the objective function of the single stage capacity / marginal cost pricing model. Those equations contain two variables Q' and t'. The equations are fully specified when the following parameters are given specific values :

S = The slope at time t=0 of the aggregate demand function.

P'= The intercept at t=0 of the aggregate demand function.

MC= Marginal running cost.

D = The absolute yearly increment in aggregate demand, due to income growth.

R = The absolute yearly increment in aggregate demand, due to population growth, given a price equal to MC.

K1= Initial effective capacity at t=0.

K = The required initial capacity at time t=0 if price is equal to marginal running cost.

f = The constant proportional level of leakage.

i = The discount rate.

T = The time when growth in demand halts.

H = An infinite planning horizon.

And also the following parameters which are used indirectly;

Pop = Population of the study area at t=0.

g = The arithmetic growth rate of population.

r = Percapita annual aggregate average demand given a price equal to short-run marginal cost.

e = The price elasticity of demand.

A summary of the values used for each of these parameters which have been established and/or discussed in various parts of this chapter is given below in table (5.11) :

Table (5.11)

Summary of parameter values of Single Stage Model

Parameter	Value	Unit
S	0.00032	(000) £/MG Year
P'	5.03	(000) £/MG
MC	0.079	(000) £/MG
D	40.50	MG Year
R	125.60	MG Year
K	15471	MG Year
K1	13472	MG Year
f	0.27	
i	0.03 to 0.10	Per Year
T	25	Year
H	infinity	
Pop	458000	persons
r	0.03378	MG/Year/Head
g	3720	persons

In order to carry out the suggested solution technique we wrote a specific computer programme which conducts systematic evaluations of the net benefit function as specified in equations 5.15, 5.16, and 5.17, the calculations being conducted for different configurations of Q' and t'.

The range of values for t' and Q' used in the actual solution of the model was as follows : t' ranged between 0 and 25 (T=25), while Q' ranged between 0 and 6151, the upper limit being equal to (R+D)T+(K-K1).

In the sampling procedure t' was incremented by a value equal to 1 ; Q' on the other hand was incremented by a value equal to 410. Therefore we ended up with 26 points on t, including one point where t'= 0 , and 16 points on Q' including one point where Q'= 0. Since Q'= 0 can only be paired with t'= 0, this meant that we had a total sample of 391 points of different combinations of t' and Q' for which the net benefit function had to be evaluated.

These points were used to draw the surface area of the net benefit function. This was done using a standard graphical computer routine.[74] The surface area of the objective function has been drawn in two standard formats : (1) using a graph of three dimensions as in figure 5.11 and

[74] We have used the Ghost/80 library routines for this purpose. The library is readily available in the computer centers of most British universities.

(2) using contours as in figure 5.12. Of course both diagrams convey the same information, and therefore the inclusion of both is for the purpose of clarification.

The second round of sampling also involved the sampling of 391 points of different configurations of Q' and t', this time however the increment in the values of each variable was considerably smaller then the first round of sampling.[75] Moreover the variation in each variable were centered around the Q' and t' values of the maximum of the previous round of sampling.

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75 The size of the increment thus was lower for each successive sampling round.

(5.17) Results of the Model

The model has been solved for the values of the parameters as specified in table (5.11) and initially for a discount rate of 5%. The first round of sampling produced a surface area for the net benefit function as displayed in figures 5.11 and 5.12.

There seems to be little doubt that the diagrams, as well as visual inspection of the results, indicate that the net benefit function is indeed concave and possesses a maximum which may be described as a global one. Moreover the first round of sampling indicated that this maximum can be safely said to lie in the vicinity of t'= 0 and Q'= 5330 MG/year. This is evident from either of diagram 5.11 and 5.12 as well as from a visual inspection of the results of the first round sampling.

Using the second stage of our sampling procedure a maximum for the objective function was pinpointed at the second iteration, thereafter no further improvement in the value of the objective function could be achieved. In this way a maximum equal to £655.111 million was recorded for an effective capacity addition of 5300 MG/year (14.52 MG/day), and for construction time of t=0, ie 1986, or the starting date of the model.

An optimal addition of effective capacity of 5300 MG/year implies the construction of 6572 MG/year (18 MG/day) of design capacity as specified by the model formulation.

The construction at t=0 of 18 MG/day of design capacity permits an immediate drop in charges to a level equal to short-run marginal cost of 79 £/MG (or 1.74 P/M^3). Total effective capacity following the construction of new capacity in year 1986 rises to 18772 MG/year (51.43 MG/day). Average aggregate annual demand in 1986 at a price equal to short-run marginal cost is equal to 15471 MG/year (42.38 MG/day). The new level of capacity can more than meet this level of demand.[76]

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[76] Although a higher level of demand can be met yet it is inefficient to do so since it would require a price level lower than short-run marginal cost.

FIGURE 5.11

SURFACE AREA OF THE NET BENEFIT FUNCTION OF THE SINGLE-STAGE MODEL

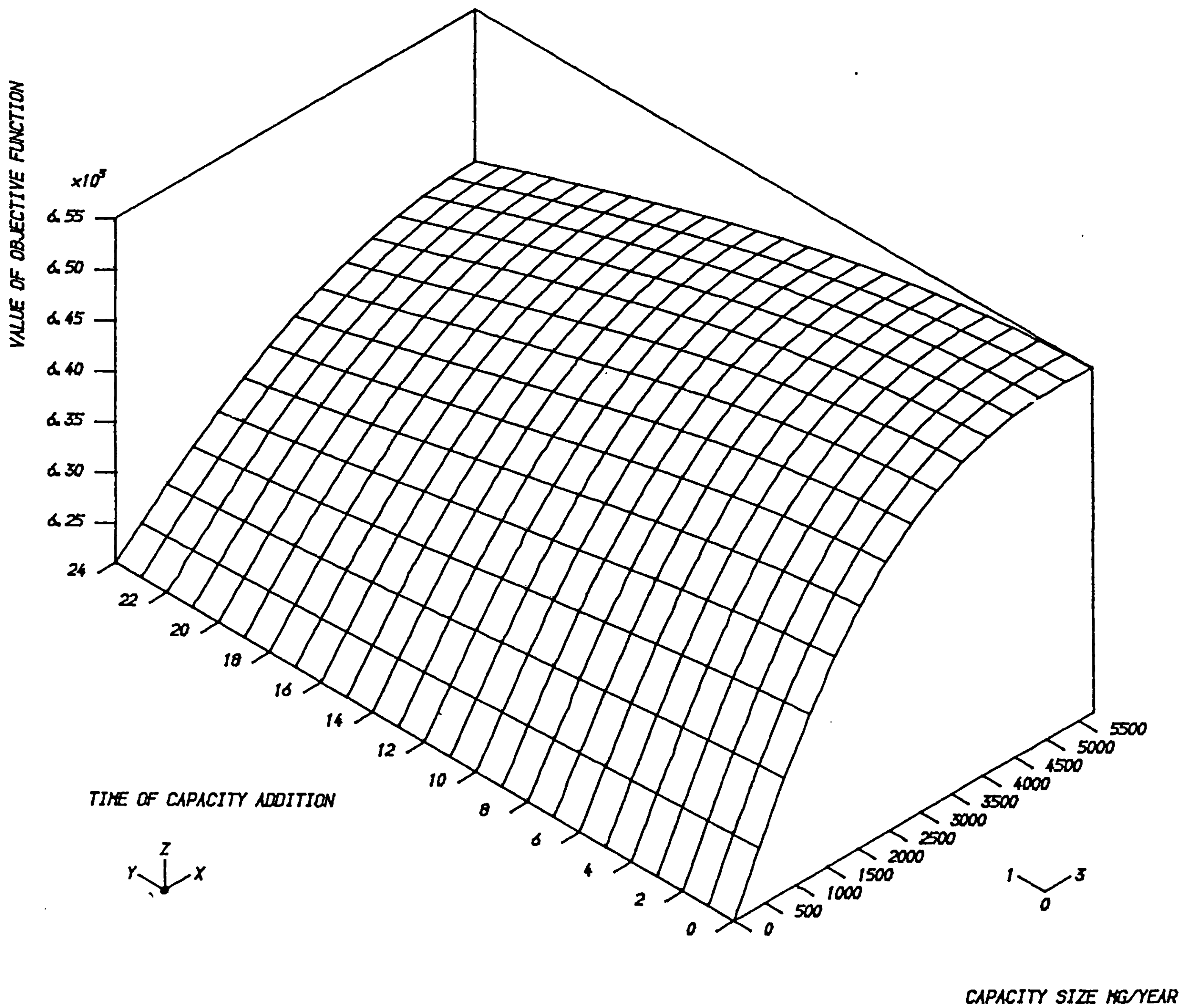
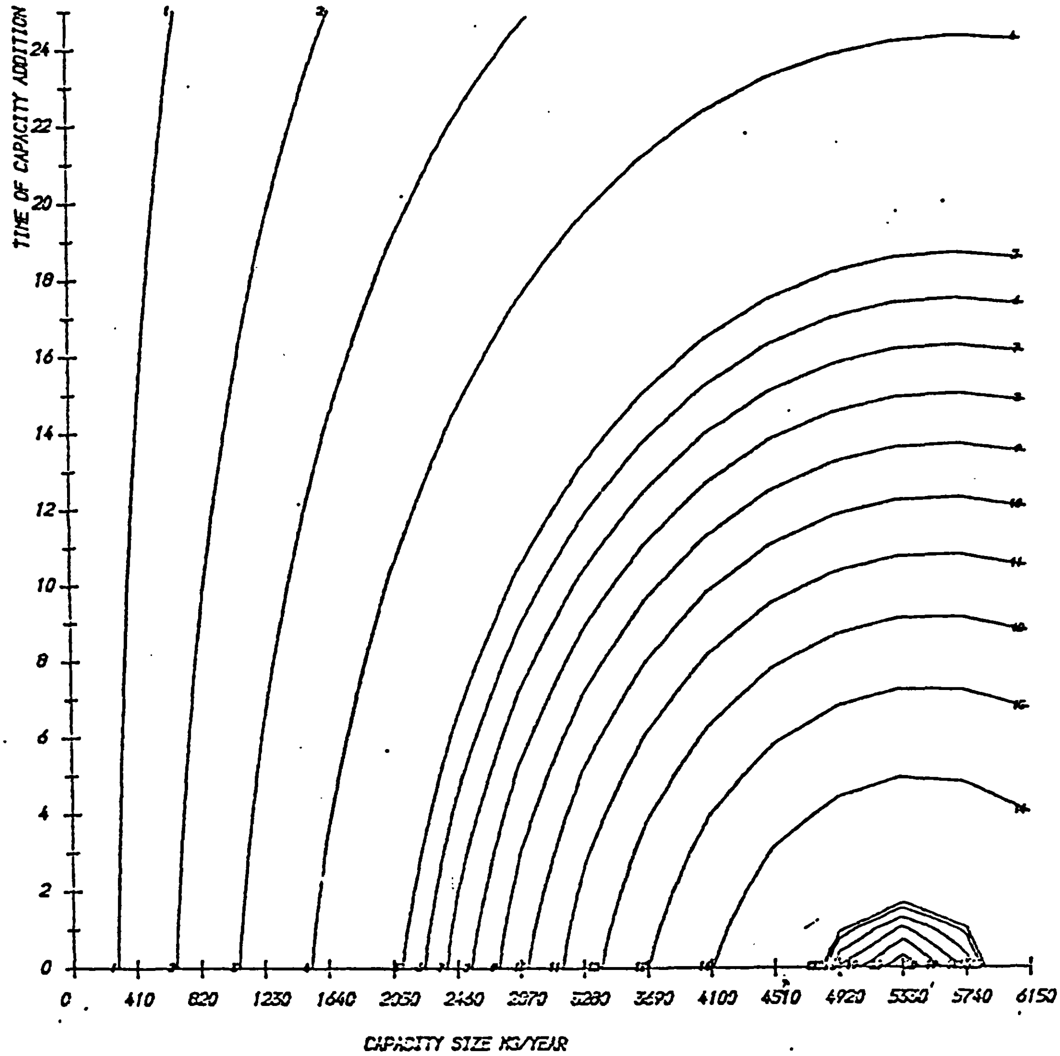


FIGURE 5.12

CONTOUR REPRESENTATION OF THE SINGLE-STAGE NET BENEFIT FUNCTION



The corresponding design capacity would likewise be more than adequate to meet all peak week demand at a price equal to short-run marginal cost. A year-round uniform price equal to short-run marginal cost would therefore rule in 1986 according to the optimal solution.

The optimal solution of the model implies the construction of enough capacity to meet all demand at a price equal to short-run marginal cost for all years up to 2007.[77] Thereafter capacity runs out, implying that price would need to be raised to whatever level necessary to keep average annual aggregate demand at a level equal to total effective capacity of 18772 MG/year. According to this sketch prices would need to go up as from 2007 up to 2011 in order to check growing demand to a level equal to constant capacity. Table 5.12 sets out the profile of output and prices for the period 1986 to 2011. During the period 2007 to 2011 price would have to rise from a level equal to short-run marginal cost of 1.74 P/M^3 to a level equal to 6.76 P/M^3.[78] The price level moreover stabilises by year 2011 when demand growth comes to a halt. Several comments on the value of the objective function seem useful to make. We first point out that the net benefits comprise in the main consumer surplus.[79] We also note that most of this consumer surplus accrues, year after year, from consumption from the existing initial capacity (equal to 13472 MG/year), and as such represents an amount of benefits that is common to all alternatives of capacity addition. Therefore when comparing different policies it makes sense to compare the incremental

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77 A more accurate statement is capacity can support all demand at short-run marginal cost for 19.87 years. This has been approximated in the text to 20 years,

78 This represents a lofty average rate of growth equal to 57% per annum over the five year period. It is to be noted however that despite this steep rise the price level never reaches the level actually charged in 1986 in Southern Water area equal to 25.7 P/M^3 .

79 It is clear that when price is equal to short-run marginal cost, and given constant marginal running costs, the net benefit is entirely consumer surplus. It is only in those years when price is higher that MC that the water enterprise makes some surplus.

benefits net of this constant amount achieved on the inherited capacity.

The relevant value of this constant simply equals the area under the demand curve between output levels of zero and initial effective capacity, K1. The discounted sum of this area over time is the constant that must be netted out from the value of the objective function. This is clearly nothing other than the value of the net benefit function for the case of zero addition of capacity, ie case I, as represented by equation 5.12. The value of the net benefit function under case I, $Q' = 0$, has already been calculated as part of the first round sampling procedure. This is equal to £621.071 million.

Therefore the incremental net benefit associated with the optimal pricing and investment policy, as derived under the single stage capacity addition model and for the parameter values reported is equal to £34.04 million. In other words a net increment in welfare equal to £34.04 million can be achieved by the investment in year 1986 of some 18 MG/day of design capacity and thereafter sticking to a policy of short run marginal cost pricing.

On a priori grounds one would expect the discount rate to have an important influence on the size and timing of capacity addition. In particular, in line with traditional analysis of 'static' project appraisal, one would expect that a higher interest rate to favour lower capacity addition and a later date of construction, and vice versa. These predictions should be readily verifiable using runs of our model for different discount rates. The results of such runs are reported below in table 5.13.

Table (5.12)

Output and Price simulation 1986 to 2011
Single Stage Model
A discount rate of 5%

Time	Average Annual output MG/year	Average Annual Output ML/day	Price £/MG	Price P/M^3
1986	15471	192.43	79	1.74
1987	15637	194.49	79	1.74
1988	15803	196.56	79	1.74
1989	15969	198.62	79	1.74
1990	16135	200.69	79	1.74
1991	16301	202.75	79	1.74
1992	16467	204.82	79	1.74
1993	16634	206.89	79	1.74
1994	16800	208.96	79	1.74
1995	16966	211.02	79	1.74
1996	17132	213.09	79	1.74
1997	17298	215.15	79	1.74
1998	17464	217.22	79	1.74
1999	17630	219.28	79	1.74
2000	17796	221.35	79	1.74
2001	17962	223.41	79	1.74
2002	18128	225.48	79	1.74
2003	18294	227.54	79	1.74
2004	18460	229.61	79	1.74
2005	18626	231.67	79	1.74
2006	18772	233.49	79	1.74
2007	18772	233.49	129	2.84
2008	18772	233.49	175	3.85
2009	18772	233.49	220	4.84
2010	18772	233.49	264	5.81
2011	18772	233.49	307	6.76

Table (5.13)

Optimal Capacity addition and timing
for different i

i	$\hat{Q}$ (MGY)	$\hat{t}$	TNB (£Million)
0.03	5753	0	1140.163
0.05	5300	0	655.111
0.07	4645	0.44	452.348
0.10	3661	1.69	305.136

The table readily confirms the a priori conclusions; the higher the interest rate the lower the optimal capacity expansion and the later (slightly later in our case) its construction date. It is also interesting to note a corollary of the above conclusion, namely that the higher the interest rate the more the degree of price variation. This must be the case since a lower capacity addition and later date of construction would require greater dependence on the price variable as a rationing device. This is readily seen from the simulated price and output profiles associated with a 10% discount rate as shown in table 5.14 .[80]

Table (5.14)

Output and Price Simulation 1986 to 2011
Single Stage Model
A discount rate of 10%

Time	Average Annual output MG/year	Average Annual Output ML/day	Price £/MG	Price P/M^3
1986	13472	167.56	650	14.32
1987	13472	167.56	675	16.85
1988	15803*	196.56	79	1.74
1989	15969	198.62	79	1.74
1990	16135	200.69	79	1.74
1991	16301	202.75	79	1.74
1992	16467	204.82	79	1.74
1993	16634	206.89	79	1.74
1994	16800	208.96	79	1.74
1995	16966	211.02	79	1.74
1996	17132	213.09	79	1.74
1997	17133	213.10	127	2.79
1998	17133	213.10	175	3.85
1999	17133	213.10	223	4.92
2000	17133	213.10	269	5.93
2001	17133	213.10	316	6.97
2002	17133	213.10	362	7.97
2003	17133	213.10	407	9.95
2004	17133	213.10	451	9.95
2005	17133	213.10	495	10.92
2006	17133	213.10	539	11.87
2007	17133	213.10	580	12.77
2008	17133	213.10	619	13.64
2009	17133	213.10	666	14.67
2010	17133	213.10	707	15.58
2011	17133	213.10	748	16.48

* Capacity addition is to take place approximately at year 2.

[80] The implications of price variability will be discussed in the next chapter.

Next we conducted sensitivity tests for the values of the various parameters for the case of a discount rate of 5%. The tests are conducted by holding the values of all parameters constant with the exception of the one parameter being tested.

First we examined the results of 10% variation, either way, in the value of the parameter R, the absolute growth in consumption due to the population growth. The results are presented in table 5.15 below.

Table (5.15)

Single Stage Model
Sensitivity to R

R(MG)	$\hat{Q}$ MGY	$\hat{t}$	TNB £Million
113	4987	0	648.636
125.6	5300	0	655.111
138.1	5615	0	661.539

It seems clear that variation in R has no impact on t, the time of capacity addition. And as expected high R leads to more capacity addition.

Table (5.16) indicates that similar results apply for variation in D.

Table (5.16)

Single Stage Model
Sensitivity to D

D MG	$\hat{Q}$ MGY	$\hat{t}$	TNB £Million
36.45	5207	0	650.507
40.50	5300	0	655.111
44.55	5411	0	659.736

Testing the model for different leakage rations produced the following results :

Table (5.17

Single Stage Model
Sensitivity to 'f'

f	$\hat{Q}$ MGY	$\hat{t}$	TNB £Million
0.24	5343	0	683.468
0.27	5300	0	655.111
0.30	5255	0	626.756

The results again seem to be in the expected direction.

Higher leakage ratios lead to a decline in the optimum addition of capacity. They seem however to have no impact on capacity timing. The latter result seems to be consistent with the results of the sensitivity tests of the other parameters.

Finally we ran the model for a time horizon of 25 years (H = 25) instead of the infinite planning horizon, thereby we are discarding the benefits to consumers occurring after 25 years. The results are presented in (5.18):

Table (5.18)

Single Stage Model
Sensitivity to H

H	$\hat{Q}$ MGY	$\hat{t}$	TNB £Million
infinity	5300	0	655.111
25	3887	0	442.028

As expected shortening the planning horizon has led to a sharp reduction in optimum capacity addition because all the benefits that accrue after 25 years are ignored. Under this solution new capacity can only meet demand at short-run marginal cost for 11.36 years compared with 20 years under the infinite planning horizon solution. It is to be noted that the solution of the model with H = 25 would be appropriate if we assume that the life of capacity addition is 25 years as opposed to the infinite life assumption which we have been using throughout.[81]

81 An infinite time horizon together with a capital cost function that allows for replacement, say every 25 years for ever, should produce the same results as a 25 year planning horizon with a cost function without allowance for replacement.

CHAPTER 6

A MODEL OF SHORT-RUN MARGINAL COST PRICING AND MULTI STAGE CAPACITY EXPANSION

(6.1) Introduction

In this chapter we reexamine our model of capacity investment/marginal cost pricing. The objective is to reformulate the model so as to solve an optimsation problem incorporating all the assumptions of the previous chapter with the exception of one. This concerns the assumption that capacity addition can only be added once throughout the planning horizon. In this chapter we replace this assumption by a less restrictive one to the effect that capacity additions can be staged. In particular we assume that capacity can be added at any one of n+1 predetermined points of time t_j , $j = 0,1,2,....n$. Solution of the model with staged capacity addition requires reformulation of the problem in a way conducive to the solution technique of dynamic programming. The results we get from the model with staged capacity addition will be critically compared with those of the model of single stage capacity addition. The improvement in the value of the discounted net benefits over time, if any, will be assessed in the light of the resulting output and price profiles as compared with those of the single stage model. Ultimately we hope to draw some conclusions regarding the benefits and costs of staging capacity addition.

Later in the chapter, we review the results of the single and multi stage capacity addition and marginal cost pricing models of this chapter and the one before. This review is conducted with the objective of assessing the desirability as well as the cost of the price variability dictated by the solution of the models. The analysis will be aided by some empirical estimates of the net benefit function resulting from policies based on charging a constant price (in real terms) throughout the planning horizon. Several such prices will be sampled. Such a procedure should help shed some light on the advantages and disadvantages of various pricing and investment policies.

(6.2) Basic Assumptions of the Dynamic Programming Model

The multi stage capacity addition/marginal cost pricing model of this chapter employs the same assumptions used in the single stage model of the previous chapter; this is with the exception of one.

The exception being that we shall assume here that capacity can be added more than once. Capacity addition is assumed here to be possible at any of n+1 points of time t_j, $j = 0,1,2.....n$. The initial point of time t_0 is taken to be year zero (1986). The last possible point of capacity addition is assumed to be T, the time when demand growth halts.[1]

Moreover these predetermined points in time are assumed to be spaced by a constant period of time equal to five years. That is :

$$
\begin{aligned}
& t_0 = 0 = 1986 \\
& t_1 = 5 = 1991 \\
& t_2 = 10 = 1996 \\
& t_3 = 15 = 2001 \\
& t_4 = 20 = 2006 \\
t_n = \; & t_5 = 25 = 2011
\end{aligned}
$$

and we let:

$$t_{n+1} = \quad t_6 \quad H = \text{infinity}$$

Of course other intervals may be used. We however have chosen the five year interval since it is our opinion that this period may reasonably be taken to represent the minimum practical time separating two consecutive capacity additions.[2] The effect of a shorter interval on the model's

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1 This assumption is justified by the argument in section 5.16 .

2 In principle the minimum period separating two consecutive capacity additions could be a matter of economic
(Footnote continued)

solutions will however be tested as part of the sensitivity analysis of the model.

It is to be noted that the (n+1) points of possible capacity addition define n+1 time periods. The first n periods are defined by t_j to t_{j+1} where j= 0,1,2,...n-1. The last period, the (n+1)th period is defined by an interval stretching between t_n=T and H, where H stands for the planning horizon, infinity in our case.

In one respect the multi stage model of this chapter utilises a more restrictive assumption than the single stage model of the previous chapter. In the single stage model the once and for all capacity addition was taken to be a continuous variable. In the multi stage model however the capacity increment at any one point of time is assumed to take some specific discrete values equal to some multiple of a constant equal to dQ. The latter, dQ, is the size of the minimum practical capacity addition at any point of time. This is assumed, somewhat arbitrarily, to be equal to 0.5 MG/day or 182.5 MG/year. Other sizes of the step increment in capacity addition will be tested as part of the sensitivity analysis.

Using an argument similar to that used in section 5.16, we may establish the highest possible warranted total addition of effective capacity. This is equal to (R+D)T+K-K1, where all the terms retain their previously defined meaning and values. The highest warranted total addition of effective capacity is thus established to be equal to 6151 MG/year. We may now define a new parameter m as the nearest integer value satisfying the relation :

$$m \, dQ \leq (R+D)T+K-K1$$

m is thus equal to 34.[3]

(continued)

consideration if for example shortening this period can only be achieved at a cost. On the other hand this minimum period could be governed by technical limitations.

[3]In other words m is the highest integer multiple of dQ representing the size of possible capacity addition at any one moment of time. The reported value for m is in fact associated with a total warranted capacity slightly over 6151 MG/year.

The other assumptions of the model, those similar to the single stage model, may be summarised as follows; The dynamic demand relation of the model is defined by equation 5.5 ; the capital cost function of the model is defined by equation 5.6 ; the short-run marginal cost function is defined by equation 5.7 ; and the following values of the parameters of the model will be used (see table 6.1 below).

Table 6.1

S	0.00032	(000) £/MG/year
P'	5.03	(000) £/MG
MC	0.079	(000) £/MG
D	40.50	MG/year
R	125.60	MG/year
K	15471	MG/year
K1	13472	MG/year
f	0.27	
i	0.03 to 0.10	per annum
H	infinity	
T	25	year
pop	485000	persons
g	3720	persons
r	0.03378	MG/head/year
dQ	182.5	MG/year
n	5	
m	34	

(6.3) The Net Benefit Expressions

For the purpose of the model we must have expressions measuring the value of net benefits for each of the n+1 periods of the model spanning a time horizon from t_0 to t_6 or from time 0 to time H. In a fashion analogous to the discussion of chapter 5 the net benefit expression for the time periods t_j to t_{j+1} for j= 0,1,2...n-1 (ie the first five periods, (or all periods with the exception of the last) depends on how much capacity has been installed up to and including t_j (or the time marking the beginning of the period). It also depends on the length of the time period as defined by t_j and t_{j+1} as well as the rate of growth of demand as defined by R and D.

Three possible forms may be defined for the net benefit function for the period t_j to t_{j+1} for values of j= 0,1,2...n-1 . These cases are as follows:

Case A

This case applies when total effective capacity throughout the period concerned falls short of meeting average aggregate demand at a price equal to MC. First let us define a variable $\bar{Q}_j$ as the sum of all capacity additions up to and including time t_j. Then for this case to apply we must have :

$$K+t_j(R+D) \geq K1+\bar{Q}_j$$

or

$$t_j(R+D) \geq \bar{Q}_j-(K-K1)$$

or

$$t_j \geq \frac{\bar{Q}_j - (K-K1)}{R+D}$$

$$t_j \geq TQ_j$$

where

$$TQ_j = \frac{\bar{Q}_j - (K-K1)}{R+D}$$

Case A therefore involves a situation where price would need to be increased throughout the period in order that growing demand is checked at a level equal to prevailing total effective capacity, $K1+\bar{Q}_j$.[4]

The algebraic expression of the total net benefit during the period t_j to t_{j+1} for j= 0,1,2 ..... n-1 may be expressed as follows :[5]

When $t_j \geq TQ_j$ and $TQ_j = \dfrac{\bar{Q}_j - (K-K1)}{R+D}$

$$R_j = \int_{t_j}^{t_{j+1}} \left\{ (1-f)(P'+SDt)(K1+\bar{Q}_j) - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\,\frac{(K1+\bar{Q}_j)^2}{2} - MC(K1+\bar{Q}_j) \right\} e^{-it}\, dt$$

$$-970(1.24\ Q_j/365)^{0.69}\ e^{-it_j}$$

where j = 0,1,2 ....n-1. **(6.1)**

and where Q_j is capacity addition at time t_j.

<u>Case B</u>

Case B applies when the following condition applies:

$$t_j < TQ_j < t_{j+1}$$

where $TQ_j = \dfrac{\bar{Q}_j - (K-K1)}{R+D}$

4 And of course to check peak demand at a level not exceeding prevailing design capacity.

5 We note that the cost of capacity addition, Q_j, at the beginning of the period is netted out. The capital cost of other additions of capacity are referred to their corresponding periods.

In this case the output and price variables go through two phases during the period t_j to t_{j+1} : (a) the first phase lasts from t_j to time TQ_j where effective capacity $(K1+\bar{Q}_j)$ can support all demand at a price equal to short-run marginal cost. Output during this phase is equal to $K+(R+D)t$; (b) the second phase applies during the period TQ_j to t_{j+1}, during which time output will need to be restricted to a maximum level of $K1+\bar{Q}_j$. This will need to be carried out by resort to higher prices. Prices during this phase will go up by whatever is necessary to keep demand at a level equal to existing effective capacity. Algebraically the net benefit expression applicable to this case for the period t_j to t_{j+1} can be written as follows :

$$R_j = \int_{t_j}^{TQ_j} \left\{ (1-f)(P'+SDt)(K+Rt+Dt) - (1-f)\frac{(P'+SDt-MC)}{K+Rt+Dt}\,\frac{(K+Rt+Dt)^2}{2} - MC(K+Rt+Dt) \right\} e^{-it}\,dt$$

$$+$$

$$\int_{TQ_j}^{t_{j+1}} \left\{ (1-f)(P'+SDt)(K1+\bar{Q}_j) - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\,\frac{(K1+\bar{Q}_j)^2}{2} - MC(K1+\bar{Q}_j) \right\} e^{-it}\,dt$$

$$-970(1.24\,Q_j/365)^{0.69}\;e^{-it_j}$$

where $j = 0,1,2 \ldots.n-1.$ **(6.2)**

<u>Case C</u> Case C applies when the following condition applies:

$$t_{j+1} \leq TQ_j \qquad \text{and} \qquad Q_j = \frac{\bar{Q}_j - (K-K1)}{R+D}$$

In this case total effective capacity, $K1+\bar{Q}_j$, during the period t_j to t_{j+1} is adequate to meet all demand at a price equal to short-run marginal cost. Output therefore, during this period is equal to K+Rt+Dt.

The relevant algebraic expression for net benefits is therefore given as below :

$$R_j = \int_{t_j}^{t_{j+1}} \left\{ (1-f)(P'+SDt)(K+Rt+Dt) - (1-f)\frac{(P'+SDt-MC)}{K+Rt+Dt}\frac{(K+Rt+Dt)^2}{2} - MC(K+Rt+Dt) \right\} e^{-it}\,dt - 970(1.24\,Q_j/365)^{0.69}\,e^{-it_j}$$

for j = 0,1,2, ...n-1 **(6.3)**

<u>The final period</u>

And finally we have an expression of net benefits for the final period in the model, that stretching from t_n to t_{n+1} or from the same thing from T to H. Total net benefits during this period can be calculated using the following expression:

$$R_j = \int_{T}^{H} \left\{ (1-f)(P'+SDT)(K1+\bar{Q}_n) - (1-f)\frac{P'+SDT-MC}{K+RT+DT}\frac{(K1+\bar{Q}_n)^2}{2} - MC(K1+\bar{Q}_n) \right\} e^{-it}\,dt - 970(1.24\,Q_n/365)^{0.69}\,e^{-iT}$$

$T = t_n$ **(6.4)**

Upon conducting the integration and some algebraic manipulation equations 6.1, 6.2, 6.3, and 6.4 can be expressed as follows.

Case A When $t_j \geq TQ_j$

$$TQ_j = \frac{\bar{Q} - (K-K1)}{R+D}$$

$$R_j = \{ (1-f)P'(K1+\bar{Q}_j) - MC(K1+\bar{Q}_j) \} \{ \frac{-e^{-it}}{i} \}_{t_j}^{t_{j+1}}$$

$$+$$

$$(1-f)(K1+\bar{Q}_j)SD \{\frac{-(1+it)e^{-it}}{i^2}\}_{t_j}^{t_{j+1}}$$

$$-$$

$$(1-f) \frac{(K1+\bar{Q}_j)^2}{2} \int_{t_j}^{t_{j+1}} \frac{P'+SDt-MC}{K+Rt+Dt} e^{-it} dt$$

$$-$$

$$970(1.24\ Q_j/365)^{0.69} e^{-it_j}$$

(6.5)

<u>Case B</u>

When $t_j < TQ_j < t_{j+1}$

$$TQ_j = \frac{\bar{Q}_j - (K-K1)}{R+D}$$

R_j =

$$\frac{1}{2} K \{ (1-f)P' - (1+f)MC \} \left\{ \frac{-e^{-it}}{i} \right\}_{t_j}^{TQ_j}$$

$$+$$

$$\{ 1/2(1-f)(P'R+P'D+SDK)-1/2(1+f)(RMC+DMC) \} \left\{ \frac{-(1+it)}{i^2} e^{-it} \right\}_{t_j}^{TQ_j}$$

$$+$$

$$1/2(1-f)(SDR+SD^2) \left\{ \frac{-(i^2t^2+2it+2)}{i^3} e^{-it} \right\}_{t_j}^{TQ_j}$$

$$+$$

$$\{ (1-f)P'(K1+\bar{Q}_j) - MC(K1+\bar{Q}_j) \} \left\{ \frac{-e^{-it}}{i} \right\}_{TQ_j}^{t_{j+1}}$$

$$+$$

$$(1-f)(K1+\bar{Q}_j)SD \left\{ \frac{-(1+it)e^{-it}}{i^2} \right\}_{TQ_j}^{t^{j+1}}$$

$$-$$

$$(1-f) \frac{(K1+\bar{Q}_j)^2}{2} \int_{TQ_j}^{t_{j+1}} \frac{P'+SDt-MC}{K+Rt+Dt} e^{-it} \, dt$$

$$-$$

$$970(1.24\ Q_j/365)^{0.69} e^{-it_j}$$

(6.6)

Case C

When $t_{j+1} \leq TQ_j$

$$TQ_j = \frac{\bar{Q}_j - (K-K1)}{R+D}$$

$R_j =$

$$1/2\ K\ \{ (1-f)P' - (1+f)MC \}\{ \frac{-e^{-it}}{i} \}_{t_j}^{TQ_j}$$

$$+$$

$$\{ 1/2(1-f)(P'R+P'D+SDK)-1/2(1+f)(RMC+DMC) \}\{\frac{-(1+it)}{i^2}e^{-it}\}_{t_j}^{TQ_j}$$

$$+$$

$$1/2(1-f)(SDR+SD^2)\ \{ \frac{-(i^2t^2+2it+2)}{i^3}\ e^{-it} \}_{t_j}^{TQ_j}$$

$$-$$

$$970(1.24\ Q_j)^{0.69}e^{-it_j}$$

(6.7)

<u>The final term</u>

$$R_n =$$

$$\left\{ (1-f)(P'+SDT)(K1+\bar{Q}_n) - \frac{(P'+SDT-MC)}{K+RT+DT} \frac{(K1+\bar{Q}_n)^2}{2} - MC(K1+\bar{Q}_n) \right\} \left\{ \frac{-e^{-it}}{i} \right\}_T^H$$

$$- 970(1.24\ \bar{Q}_n/365)^{0.69}\ e^{-iT}$$

(6.8)

We note that as in the case of the single stage expansion model we encounter the recurrent integral

$$\int \frac{P'+SDt-MC}{K+Rt+Dt}\ e^{-it}\ dt$$

the analytic solution of which may be found in section (5.15).

It is to be noted that the net benefit in the periods t_j to t_{j+1} is a function of :

(1) Q_j or the capacity addition at the beginning of the period;

(2) $\bar{Q}_j$ or the total (cummulative) effective capacity addition at the beginning of the period;

(3) t_j and t_{j+1}, time marking the beginning and end of the period.

(6.4) The objective function and model formulation

The objective function to be maximised is of course not the net benefit in a single period t_j to t_{j+1} but rather net benefits over a time period starting from time t_0 and ending at time t_{n+1} or, the same thing, the end of the planning horizon at H. The relevant net benefit function is therefore nothing other than the sum of the net benefits over the n+1 consecutive time periods defined by the n+1 possible investment points (t_j, j= 0,1,2, ...n) and the planning horizon H. Total net benefits over the entire planning horizon, TNB, is thus defined as :

$$TNB = SUM(R_j) \qquad j = 0,1,2 \ldots\ldots n$$
$$= R_0+R_1+R_2+{-}{-}{-}{-}{-}{-}{-}{-}{-}R_n$$

Of course the addition is quite legitimate since all the terms of net benefits are discounted to the same base year of t_o or year 1986. All the t_js are specified (in our case the points of 0,5,10,15,20,and 25). Moreover $\bar{Q}_j$, cumulative capacity addition up to and including time t_j, is simply a function of Q_j, capacity addition at time t_j, so it must be possible to express TNB as a function of the variables Q_j (six in our case), $Q_0, Q_1, \ldots\ldots, Q_n$. These variables are the optimal capacity additions at each and every predetermined point of possible capacity addition. The determination of these values implicitly define the optimal profile of output and prices over time.

Like the single stage capacity addition model the maximum addition of capacity $\bar{Q}_n$ will not exceed (R+D)T+K-K1 and therefore the multi stage capacity expansion / marginal cost pricing model may be expressed as a maximisation problem involving the following objective function:

$$\begin{aligned} &\text{Max}[R_n(\bar{Q}_n,Q_n)+R_{n-1}(\bar{Q}_{n-1},Q_{n-1})+\ldots\ldots\ldots \\ &\quad \ldots\ldots\ldots+R_1(\bar{Q}_1,Q_1)+R_0(\bar{Q}_0,Q_0)] \end{aligned} \qquad \textbf{(6.9)}$$

$$\text{s.t} \quad \bar{Q}_n \leq (R+D)T+K-K1 \qquad \textbf{(6.10)}$$

In other words the multi stage capacity expansion model involves the choice of Q_j, j= 0,1,2,.....n, so as to maximise the total net benefits as defined by equation 6.9 and 6.8, 6.7, 6.6, and 6.5, and subject to the constraint that total capacity addition by time t_n should not be greater than is necessary to meet total demand at a price equal to short-run marginal cost.[6]

The model in this form can be solved using the technique of dynamic programming.[7] In particular we can use a forward-moving discrete dynamic programming algorithm. The model is thus characterised as a one-dimensional dynamic programming problem involving n+1 stages, with a 'state' variable $\bar{Q}_j$ defined as before as the total effective capacity addition at the beginning of stage j+1, j=0,1,2,...n ; and the state variable takes the discrete values of 0,dQ,2dQ,......mdQ where dQ is defined as the minimum practical capacity addition at any one point of time and m the nearest possible integer such that :

$$mdQ \leq (R+D)T+K-K1$$

Because total optimal capacity addition may take any multiple value of dQ up to mdQ, it follows that we have not one dynamic problem to solve but rather m+1 problems, one problem for each possible multiple of dQ. To find the optimal solution therefore we must solve m+1 different dynamic programming problems, each problem involving different total addition of effective capacity. The separate problems however

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6 Our formulation of the model is similar in form to that of Roirdan(1971). But as with the single stage model we have amended his formulation in the following respects:
(1)Our demand function includes an income effect, his does not;
(2)we have allowed for leakage in the system, he did not ;
(3)the initial capacity of the system, K1, is less than the required capacity with a price equal to short-run marginal cost,K, Roirdan's model assumed the two to be equal;
(4)we have used peak week ratios while Roirdan used peak day ratios ;
(5)we apply the model to an actual situation while Roirdan used hypothetical data;
(6)all our equations in general are different because of the points 1 through 4.

7 See Taha,Hamdy,H.,(1982),"Operations Research : An Introduction,"3rd edition,Macmillan,New York.

are of the same nature so once one is solved the rest are a repeat of the same exercise for different values of total capacity addition.

The solution to any one of the dynamic programming problems above can be achieved using the 'Principle of Optimality.'[8] Accordingly for our problem of n+1 stages we can write a dynamic programming recursive relationship as follows :

$$f_0(\bar{Q}_0) = R_0(\bar{Q}_0, Q_0)$$

$$\begin{aligned} f_j(\bar{Q}_j) = \underset{\substack{\text{all feasible } Q_j \\ 0 \leq Q_j \leq \bar{Q}_j \\ j=1,2,3,\ldots\ldots n}}{\text{Max}} [R_j(\bar{Q}_j, Q_j) + f_{j-1}(\bar{Q}_j - Q_j)] \end{aligned} \qquad (6.11)$$

$f_j(\bar{Q}_j)$ measures the maximum value of net benefits at stage j+1 and for a particular value of the state variable $\bar{Q}_j$, and where $R_j(\bar{Q}_j, Q_j)$ is as defined earlier, ie net total benefits for the period t_j to t_{j+1} when total effective capacity at the beginning of the period is $\bar{Q}_j$ and capacity addition at the beginning of the period is Q_j.

This standard recursive relationship is perhaps best understood when presented in the analogous fashion of a network model.[9] A typical problem of dynamic programming can be formulated as a network problem for n=5, ie six points of possible capacity addition including time zero. These will be taken to be equally spaced with a five year interval:

$t_0 = 0$

$t_1 = 5$

$t_2 = 10$

---

8 The principle of optimality revolves around the property that "all future decisions are selected optimally without recourse to information regarding previously made decisions." See Taha(1982), PP332/364, op.cit.

9 We have found the network model particularly useful especially when writing the necessary computer program to conduct the required computation of the algorithm.

$t_3 = 15$
$t_4 = 20$
$t_5 = 25$

Such a formulation defines a six stage dynamic programming problem. m on the other hand will be taken purely arbitrarily, to be equal to 3, thus giving four possible values of total effective capacity at any one stage and likewise four possible increment sizes of capacity. These are 0,dQ,2dQ, and 3dQ.

The problem to be solved is a search for the optimum capacity addition (0,dQ,2dQ, or 3dQ) at each possible point of time of capacity addition, t_j where $j=0,1,...n$, ($t_0=0$, $t_1=5$, $t_2=10$, $t_3=15$, $t_4=20$ and $t_5=25$). The maximisation problem is under the constraint that exactly 3dQ, in this example, must be built at most by time t_n or t_5 , (year 25).

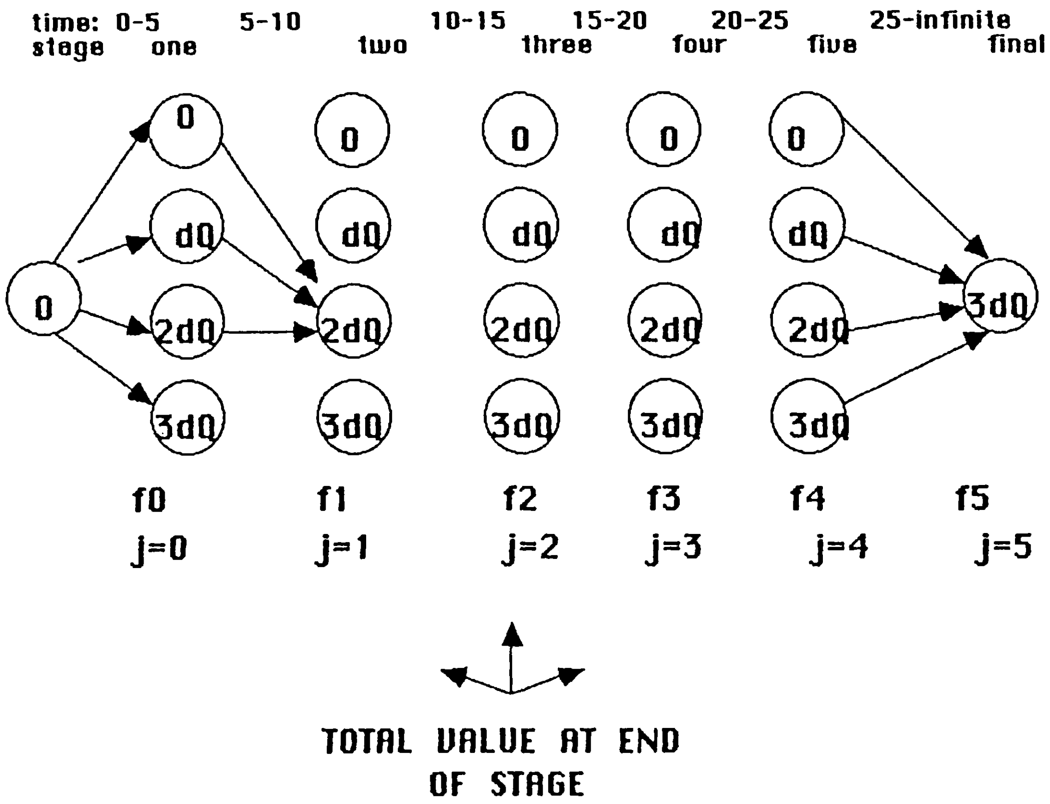
Figure 6.1 presents the network formulation of the problem. Each possible value of $\bar{Q}$ is represented by a node at the end of each of the stages of the model (stage 1,2,3,4,5,6). So that for example at the end of stage 1 ($j=0$) we could have a total effective capacity of 0,dQ,2dQ, or 3dQ. A similar situation prevails at the end of the next four stages. At the beginning of the final stage however, ie by time 25, we must have a total effective capacity of 3dQ . This is the constraint on the maximisation problem.

The difference in the value of any two nodes between successive stages must by definition represent capacity addition at the beginning of the second. Thus for example moving from $\bar{Q}_2=0$ to $\bar{Q}_3=3dQ$ implies an addition of 3dQ at the beginning of stage 4 or at t_3, or year 15. The condition that $0 \leq Q_j \leq \bar{Q}_j$ defines the feasible alternatives, ie routes, to arrive at a particular node at any stage. For example we cannot have $\bar{Q}_2=3dQ$ and $\bar{Q}_3=2dQ$ since that would imply $Q_3<0$.

The first stage computation, as defined by the first part

FIGURE 6.1

NETWORK FORMULATION OF THE MULTI-STAGE CAPACITY ADDITION MODEL



f0 measures net benefits at theend of stage 1 (year 5), 0 dq 2dq 3dq at the colunm marked f0 represent the total amount of capacity at the beginning of stage 1 ie at time 0 of equation 6.11, involves evaluating the value of the total net benefit function by the end of the first stage (year 5) and for four possible values of the state variable $\bar{Q}_0$:

$$f_0(0) = R_0(0)$$
$$f_0(dQ) = R_0(dQ)$$
$$f_0(2dQ) = R_0(2dQ)$$
$$f_0(3dQ) = R_0(3dQ)$$

R_0 is of course evaluated according to one of the equations 6.5, 6.6, 6.7, depending on the relative value of dQ in relation to and t_{0+1} . We note here that in the first stage there is only one possible route to each of the values of the state variable $\bar{Q}_0$.

Once the four possible values of the net benefit function at the end of the first stage are evaluated we can proceed into the evaluation of the value of the net benefit function by the end of the second stage $f_1(\bar{Q}_1)$. This again takes four values depending on the value of the state variable. Using the recursive relationship we can write :

$$f_1(\bar{Q}_1) = \mathrm{Max}[R_1(\bar{Q}_1,Q_1)+f_0(\bar{Q}_1-Q_1)]$$
$$\text{for } \bar{Q}_1 = 0, dQ, 2dQ, 3dQ$$
$$\text{and } 0 \leq Q_1 \leq \bar{Q}_1$$

that is for each possible value of the state variable $\bar{Q}_1$, the total capacity installed by the beginning of stage two and lasting up to the end of the period or from t_1=5 to t_2=10, we find the best route of arriving there. This is done utilising information regarding f_0 of the previous stage.
For example for the particular value of the state variable $\bar{Q}_1$ equal to 2dQ there are three possible routes. We must chose the best of them, the best in the sense of giving the highest $f_1(2dQ)$:

$$f_1(2dQ) = \text{Max of : } \begin{array}{l} R_1(2dQ,0) + f_0(2dQ) \\ R_1(2dQ,dQ) + f_0(dQ) \\ R1(2dQ,2dQ) + f_0(0) \end{array}$$

Thus the decision on the optimal route to arrive at node 2dQ at stage two is reached by enumerating all the possible routes from the stage before, inspecting the results and choosing the route with the highest value of net benefit. This is done for every possible value of the state variable and at each stage. This is essentially what the recursive relationship does.

The final stage, beginning at time t_n, must begin and end with a unique value for the state variable, in our example 3dQ. $f_5(3dQ)$ therefore must represent the maximum overall value of the net benefit of capacity addition of 3dQ ; this maximum comes from a particular route from the end of stage 5 which is readily identifiable. Also by now we should have already established the best route to arrive at each and every node at the end of stage 5, stage 4, stage 3, stage 2 and stage 1. This information thus enables us to trace our way back from the final stage right to the start. In the process of doing so not only do we establish the best value of the net benefit function at the end of each stage but also the amount of effective capacity added at the beginning of the stage. This is precisely the information we need to identify the solution to the initial maximisation problem.

Having found the optimum expansion path of capacity (and implicitly of output and prices) for one possible maximum total capacity addition, $\bar{Q}_n$, equal to 3dQ in the example, we proceed by repeating the exercise for other possible values of maximum total capacity addition. The overall total benefit of capacity addition over the planning horizon, $f_n(\bar{Q}_n)$ for each possible total capacity addition would have to be listed. The search would of course be confined to values of $\bar{Q}_n$ in the range $0 \leq \bar{Q}_n \leq (R+D)T+K-K1$. This involves solution of the dynamic programming problem m+1 times. The best $\bar{Q}_n$, the one with the highest overall value of the discounted net benefit function, will then be picked up together with the implied staging schedule and output and price profiles.

(6.5) Results of the dynamic programming model

The execution of the dynamic recursive relationship 6.11, containing the discrete dynamic programming algorithm requires a specific computer program. The computational effort of any problem of reasonable size, $n \geq 5$ and $m \geq 30$, cannot be handled other than with the aid of a computer.

Therefore a computer program incorporating the forward moving discrete dynamic algorithm, equation 6.11, together with the net benefit expressions 6.5, 6.6, 6.7, and 6.8 was written specifically for this purpose.[10]

The model was first solved for the parameter values reported in table 6.1. The minimum practical capacity addition at any one point of time is assumed to be 0.50 MG/day or 182.5 MG/year, and where capacity addition is assumed to be separated by a minimum of five years. The model solution for a discount rate of 5% is presented in table 6.2 below .

Table (6.2)

Optimum Capacity Addition With Staging

Year	Effective Capacity Addition MG/year	Design Capacity MG/day
1986	3832.5	13.02
1987	0	0
1988	0	0
-	-	-
2006	1825.0	6.20
2007	0	0
-	-	-
2011	0	0
Total	5657.0	19.22

TNB = £655310 (1000)

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[10] The programme we have written included some efficiencies to minimise the computational effort. The algorithm was decomposed in such a way that if some expression enters the calculation x times, it need only be worked out once, stored and retrieved x times. This is instead of being calculated unnecessarily x times.

Table 6.3

Output and Price simulation 1986 to 2011
Multi Stage Model
A discount rate of 5%

Time	Average Annual output MG/year	Average Annual Output ML/day	Price £/MG	Price P/M^3
1986	15471	192.43	79	1.74
1987	15637	194.49	79	1.74
1988	15803	196.56	79	1.74
1989	15969	198.62	79	1.74
1990	16135	200.69	79	1.74
1991	16301	202.75	79	1.74
1992	16467	204.82	79	1.74
1993	16634	206.89	79	1.74
1994	16800	208.96	79	1.74
1995	16966	211.02	79	1.74
1996	17132	213.09	79	1.74
1997	17298	215.15	79	1.74
1998	17304.5	215.23	125.70	2.76
1999	17304.5	215.23	173.36	3.82
2000	17304.5	215.23	220.86	4.86
2001	17304.5	215.23	267.48	5.89
2002	17305.5	215.23	313.49	6.90
2003	17305.5	215.23	358.90	7.90
2004	17305.5	215.23	403.72	8.89
2005	17305.4	215.23	447.98	9.86
2006	18793	233.75	79	1.74
2007	18959	235.81	79	1.74
2008	19125	237.88	79	1.74
2009	19129	237.93	123.10	2.71
2010	19129	237.93	167.71	3.69
2011	19129	237.93	211.79	4.66

The solution involves the addition of effective total capacity of 15.5 MG/day or 19.22 MG/day of design capacity. This is 6.74% higher than the optimum capacity addition under the single stage model. Moreover the capacity addition takes place in two stages : stage one involves the addition of 13 MG/day of design capacity (10.5 MG/day of effective capacity) at time zero, ie immediately or in 1986; stage two, involving the addition of 6.2 MG/day of design capacity (5 MG/day of effective capacity) takes place at 2006 or at t=20. This compares with the addition all at one go of some 18 MG/day of design capacity at time zero under the single stage model of the last chapter.

The value of the objective function under the multi stage capacity expansion model was found to be £655310 (1000). Subtracting the constant part of £621071, which is that part of the net benefit function common to all solutions, we arrive at an incremental value of the net benefit function equal to £34239 (1000). This compares with the incremental value of the net benefit function under the single stage model equal to £34040 (1000). Therefore the improvement in the incremental value of the net benefit function due to staging amounts to no more than 0.58%, an amount which can hardly be described significant.[11]

It is to be noted that staging involves a reduction in the costs of capacity construction. Under the single capacity expansion model the discounted value of capacity costs is £7.128 million. The corresponding figure for the multi stage model is £6.95 million. The saving is a £178000. To this one must add any benefits that may arise from the additional consumption associated with the extra capacity of the solution of the multi stage model.

Table 6.3 provides the solution as regards to prices and output. It is clear that the slight improvement in the value of the net benefit function is achieved at the expense of more price fluctuation when compared with the single stage solution. If price fluctuation is deemed to be costly, particularly if price changes are unexpected or difficult to perceive by consumers, then the increment in the net benefit value would to have to be reduced accordingly.[12]

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[11] See the discussion below regarding the improvement for a 10% discount rate.

[12] It is to be noted that there is nothing to prevent the value of the net benefit function from actually declining due to staging if due allowance is made for the costs of extra price variation. However staging will be more advantageous in situations where elements of supply in addition to treatment form the constraints on supply.

As part of sensitivity analysis we have solved the dynamic programming model for discount rates other than 5% while retaining the values of all other parameters. The results of the runs were as follows:

Table (6.4)

Total Capacity Addition
Multi Stage Model
various i

i	Total Effective Capacity Addition MG/year	Total Effective Capacity Addition MG/d	Total Design Capacity Addition MG/d	Comparison with Single Stage Model
0.03	5840.0	16.0	19.84	+1.5%
0.05	5657.5	15.5	19.22	+6.3%
0.07	5292.5	14.5	17.98	+13.9%
0.10	4927.5	13.5	16.74	+34.5%

Table (6.5)

Total Net Benefits
Multi Stage Model
Various i

i	TNB £1000	Constant £1000	Incremental Mult.Stage	Incremental Sing.Stage	Change on Sing.Stage
0.03	1140152	1063423	76729	76740	- 0.01%
0.05	655310	621071	34239	34040	+ 0.58%
0.07	452854	434535	18319	17813	+ 2.70%
0.10	305616	297344	8272	7792	+ 6.00%

Table (6.6)

i	Optimal Staging of Design Capacity MG/day	Total
0.03	(1) 19.84 at 1986	19.84
0.05	(1) 13.02 at 1986 (2) 6.20 at 2006	19.22
0.07	(1) 9.30 at 1986 (2) 8.68 at 2001	17.98
0.10	(1) 8.06 at 1986 (2) 6.68 at 2001	16.74

Table (6.4) clearly shows the inverse relationship between the discount rate and total effective capacity ; the higher the discount rate the lower the optimal total addition of total capacity. This is in line with traditional conclusions of micro economics.

Table (6.5) indicates the performance of the multi-stage model in terms of the recorded value of the net benefit function, and for each of the sampled discount rates. The performance is judged in terms of the value of the incremental benefit function in comparison with that recorded for the single stage model. The incremental net benefit is defined as before: it is net benefits of the optimal multi stage solution less that part of total net benefit which is common to all solutions and achieved on account of inherited capacity at 1986. Table 6.5 shows that the multi stage solution improves on the single stage solution for all discount rates with the exception of 0.03%. For a discount rate of 0.03% the multi stage model not only fail to improve the value of the objective function but actually registers a minute decline. The decline can readily be explained : the multi stage model essentially arrives at a solution which is identical to that of the single stage model, ie no staging of capacity. This in turn must mean that the total addition of capacity must be identical to the single stage solution. However because capacity addition is formulated as a discrete variable with a minimum step increment of 182.5 MG/year the multi stage solution will only get as near as possible to the 'true' solution found under a continuous variable formulation, (as in the single stage model) . The two solutions will be identical only by coincidence, but in our case it can be seen that the solutions are indeed so close that they may be regarded as equal and the value of the net benefit function declines only by an insignificant amount.

For discount rates equal and greater than 5% the multi stage model does improve on the solutions of the counterpart single stage model. The improvement moreover is higher in percentage terms for higher discount rates, rising from a mere 0.58% for a 5% discount rate to 6% for a 10% discount rate. One can therefore safely conclude that both the change in total capacity addition as well as the improvement in the value of the net benefit function seem to be sensitive to the chosen discount rate.

Further, the actual staging is also sensitive to the discount rate. Table (6.6) shows the staging of capacity addition for various discount rates. One pattern at least seems clear; there is always some capacity addition at time zero (1986) the size of which is inversely related to the discount rate. Moreover for discount rates of 5% and over there is always one further addition of capacity taking place: at the year 2006 for i=5% and year 2001 for i= 7% and 10%.

We have also tested the model for variations in the parameter R, the annual growth of demand due to population growth, given a price equal to short-run marginal cost. The results are shown in tables (6.7), (6.8), and (6.9).

Table (6.7)

Total Capacity Addition
Multi Stage Model
various R

R	Total Net benefits £(1000)	Total Effective Capacity Addition MG/y	Total effective Capacity Addition MG/d	Total Design Capacity Addition MG/d
113.0	648811	5292.5	14.5	17.98
125.6	655310	5657.5	15.5	19.22
138.1	661758	6022.0	16.5	20.46

Table (6.8)

R MG/year	**Optimal Staging of Design Capacity MG/d Various R**	
113.0	(1) 12.40 at 1986	(2) 5.58 at 2006
125.6	(1) 13.02 at 1986	(2) 6.20 at 2006
138.1	(1) 13.64 at 1986	(2) 6.82 at 2006

The results are as expected in that higher R leads to investment in higher total additional capacity. Furthermore the timing of additional capacity seems to be invariant to variations in R (at least within the sampled range) ; higher R involves the addition of higher capacity at each and every stage of the capacity investment schedule.

Table (6.9)

R MG/year	Change in TNB on Single Stage Model	Change in Capacity
113.0	+ 0.56%	+ 6.1%
125.6	+ 0.58%	+ 6.7%
138.1	+ 0.59%	+ 7.2%

Table (6.9) shows that the percentage improvement in the incremental benefits of the multi stage solution on the single stage solution and for a discount rate of 5% is small for all values of R. The size of the improvement rises with the value of R.[13]

Next we investigated the sensitivity of the model's results to variation in D, the absolute annual growth in demand due to income growth. The results are given in tables 6.10, 6.11 and 6.12.

Table (6.10)

Total Capacity Addition
Multi Stage Model
various D

D MG/year	Total Net Benefits £(1000)	Total Effective Capacity Addition MG/y	Total effective Capacity Addition MG/d	Total Design Capacity Addition MG/d
36.5	650754	5475.0	15.0	18.60
40.5	655310	5657.5	15.5	19.22
44.5	659883	5840.0	16.0	19.84

Table (6.11)

D MG/y	Optimal Staging of Design Capacity MG/d Various D
36.5	(1) 12.40 at 1986 (2) 6.2 at 2006
40.5	(1) 13.02 at 1986 (2) 6.2 at 2006
44.5	(1) 13.02 at 1986 (2) 6.8 at 2006

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13 Of course the size of the improvement will be greater for each R given a discount rate greater then 5% and vice versa.

Table (6.12)

D MG/y	Change in TNB on Single Stage Model	Change in Capacity
36.5	+ 0.75%	+ 5.1%
40.5	+ 0.58%	+ 6.7%
44.5	+ 0.41%	+ 7.9%

The tables above indicate that a higher D increases the optimal size of capacity addition. The multi stage solutions show a slight improvement in the incremental benefits on the single stage model. This time, however, the improvement diminishes in size with higher D.

We also tested the model using different values for f, the leakage ratio. The results are presented below in tables 6.13, 6.14, and 6.15.

Table (6.13)

Total Capacity Addition
Multi Stage Model
various f

f MG/year	Total Net Benefits £(1000)	Total Effective Capacity Addition MG/y	Total effective Capacity Addition MG/d	Total Design Capacity Addition MG/d
0.24	683666	5657	15.5	19.22
0.27	655310	5657	15.5	19.22
0.30	626959	5657	15.5	19.22

Table (6.14)

f	Change in TNB on Single Stage Model	Change in Capacity
0.24	+ 0.55%	+ 5.8%
0.27	+ 0.58%	+ 6.7%
0.30	+ 0.63%	+ 7.6%

Table (6.15)

f	Optimal Staging of Design Capacity MG/d Various f
0.24	(1) 13.02 at 1986 (2) 6.2 at 2006
0.27	(1) 13.02 at 1986 (2) 6.2 at 2006
0.30	(1) 12.40 at 1986 (2) 6.82 at 2006

Table (6.14) above indicates that the multi stage capacity addition model improves on the single stage capacity addition model for all values of f. The improvement is associated with a higher total capacity addition compared with the single stage solution. We also note that using the single stage model a higher leakage ratio reduces the size of total capacity addition. In the multi stage model there is no such consequence. However, a leakage rate of 0.3 results in a slight change in the staging as compared with f= 0.27 . In particular the higher rate seems to result in the postponement of 0.62 MG/d of capacity addition from 1986 to 2006. The overall size of capacity addition remains the same .

We have also examined the solution of the model for T=30, that is extending the duration of demand growth from 25 to 30 years. The results of the modeling were as in tables 6.16, 6.17, 6.18.

Table (6.16)

Total Capacity Addition
Multi Stage Model
various T

T	Total Net Benefits £(1000)	Total Effective Capacity Addition MG/y	Total effective Capacity Addition MG/d	Total Design Capacity Addition MG/d
25	655310	5657	15.5	19.22
30	665434	6387	17.5	21.70

Table (6.17)

T	Optimal Staging of Design Capacity MG/d Various T	
25	(1) 13.02 at 1986	(2) 6.2 at 2006
30	(2) 12.40 at 1986	(2) 9.3 at 2006

Table (6.18)

T	Change in TNB on Single Stage Model	Change in Capacity
25	+ 0.58%	+6.7%
30	+ 1.16%	+7.4%

One interesting aspect of the solution is the relatively small increase in the value of the discounted net benefit function despite the increase in the duration of the period of demand growth. The reason for this must be that the extra benefits take place during the period 2011 and 2016. Such benefits are discounted to year 1986 and hence the absolute increment in discounted net benefits would be smaller than would be the case were these benefits to occur earlier.

We have also solved the model for a time horizon of 25 years, ie instead of a planning horizon of infinity. The result of such a formulation is that all capacity is added immediately without any staging. As such the solution of the model in this case should in principle be exactly the same as in the single stage formulation. Our results, as in table 6.19, indicate that when allowing for the constraint of the step increments of Q, the two solutions are indeed very close.

Table (6.19)

H=25	Single Stage	Multi Stage
TNB(£1000)	442028	442027
$\bar{Q}_n$ (effective capacity)	10.64 MG/d	10.50 MG/d
$\hat{t}$	1986	1986

The final part of the sensitivity test concerns the assumption regarding the minimum practical size of capacity addition and the minimum period of time separating two consecutive capacity additions. Both of these were more or less arbitrarily specified as 182.5 mg/year of effective capacity addition and five years respectively. Other specifications can perhaps be advanced with equal justification. This can be an important issue if the solution is very sensitive to these assumptions.

In order to investigate the point we have solved the model under a new specification namely that the minimum effective capacity addition is 91.25 MG/year and a minimum period separating two capacity additions of 2.5 years. This give N = 10 and M = 68. The results of the solution are reported in table 6.20 below.

Table (6.20)

Optimum Capacity Addition
With Staging
N=10 M=68

Year	Effective Capacity Addition MG/year	Addition of Design Capacity MG/day
1986	3467.5	11.78
1987	0	0
1988	0	0
-	-	-
2003.5	2190.0	7.44
2007	0	0
-	-	-
2011	0	0
Total	5657.5	19.22

TNB = £655326 (in units of 1000)

The solution can be seen to have several remarkable similarities with that of the original formulation. First we note that total additional capacity addition is identical to the original formulation. This seems to indicate that optimal total addition of capacity is perhaps insensitive to the assumptions regarding the spacing of successive capacity additions and minimum practical capacity additions. Secondly the improvement in the value of the discounted net benefit function is only minute indicating that decreasing the size of the discrete step below that of the original formulation does little in the way of improving the value of net benefit function. Thirdly the staging of capacity addition is not radically altered. The difference in capacity addition in year 1986 is only 1.24 MG/day. This is postponed to year 2003.5 where all the rest of capacity addition is to take place.

(6.6) A Model of Capacity Expansion with Constant prices

Before we draw some final conclusions regarding optimum investment and pricing policies that may be drawn from our models we have to investigate one further area. This is the value of the net benefit function under a policy of maintaining a constant real price throughout the planning horizon and ensuring no shortages.

In order to do so we build a model where price fluctuation is ruled out altogether. Once a price is set at the beginning of the planning period, year 1986, it has to be maintained there up to the end of the planning horizon. Capacity expansion takes place at a single stage as soon as average aggregate demand is equal to total effective capacity inherited from the past at the beginning of the planning period. With a price sensitive demand the timing of capacity addition is thus determined by the initial price set at year 1986, when t=0. The volume of capacity addition is likewise determined by the price set in 1986 which would prevail throughout since capacity addition must be such that total effective capacity at year T is equal to all demand generated at that price. We retain all previous symbols and introduce the following new ones :

P = The constant price level set at t=0

K2= the demand level at t=0 when price is set equal to P

R2= the absolute demand growth due to population growth given a price equal to P

t2= the time when additional capacity is to be built.

We can now develop an expression for net benefits from time t_0 to H, ie from 1986 to the end of the planning horizon.

$$TNB = \int_0^T \Big\{ (1-f)(P'+SDt)(K2+R2t+Dt) - (1-f)\frac{P'+SDt-P}{K2+R2t+Dt}\,\frac{(K2+R2t+Dt)^2}{2} - MC(K2+R2t+Dt) \Big\} e^{-it}\,dt$$

$$+$$

$$\int_T^H \Big\{ (1-f)(P'+SDT)(K2+R2T+DT) - (1-f)\frac{P'+SDT-P}{K2+R2T+DT}\,\frac{(K2+R2T+DT)^2}{2} - MC(K2+R2T+DT) \Big\} e^{-it}\,dt$$

$$-$$

$$970(1.24Q/365)e^{-it2} \qquad (6.12)$$

Where

$$Q = (R2T+DT) + K2-K1$$

and K1 is initial inherited capacity as before.

In other words total net benefits, given a real constant uniform price of P up to the end of the planning horizon, is decomposed into two parts. The first captures net benefits for the period of demand growth from t_0 to T. The second spans the period from T to H. During the first period output is equal to (K2+R2t+Dt), and during the second it is stationary at (K2+R2T+DT). Capacity addition takes place at t_2, ie when the initial capacity K1 is exhausted. The volume of capacity addition is (R2T+DT)+K2-K1, that is by an amount sufficient to meet all demand growth at a price equal to P up to time T so that price need not be increased.

Conducting the integration and some algebraic manipulation we get the following expression for TNB :

$$TNB = \left[\frac{(1-f)}{2} P'K2 + \frac{(1-f)}{2} PK2 - MCK2\right]\left[\frac{-e^{-it}}{i}\right]_0^T$$

$$+$$

$$\left[\frac{(1-f)}{2} P'R2 + \frac{(1-f)}{2} P'D + \frac{(1-f)}{2} SDK2 + \frac{(1-f)}{2} PR2 + \frac{(1-f)}{2} PD - MC\,R2 - MC\,D\right]\left[\frac{-(1+it)}{i^2}\right]_0^T$$

$$+$$

$$\left[\frac{(1-f)}{2} SDR2 + \frac{(1-f)}{2} SD^2\right]\left[-\frac{(i^2t^2 + 2it + 2)}{i^3} e^{-it}\right]_0^T$$

$$+$$

$$\left[(1-f)(P'+SDT)(K2+R2T+DT)-(1-f)(P'+SDT-P)(K2+R2T+DT)/2 - MC(K2+R2T+DT)\right]\left[-\frac{-e^{-it}}{i}\right]_T^H$$

$$-$$

$$970(1.24Q/365)^{0.69}\, e^{-it_2} \qquad \mathbf{(6.12)}$$

Because we are interested in the value of the net benefit function not just for one value of P but for all possible prices, and ultimately for the one with the highest TNB, we have written a computer program in order to conduct repeated evaluations of the function for different levels of prices. The value of the net benefit function, given the values of the parameters as in table 6.1, for a selection of prices and a discount rate of 5% is given in table 6.21 .

FIGURE 6.2

NET BENEFIT FUNCTION FOR THE CONSTANT PRICE MODEL

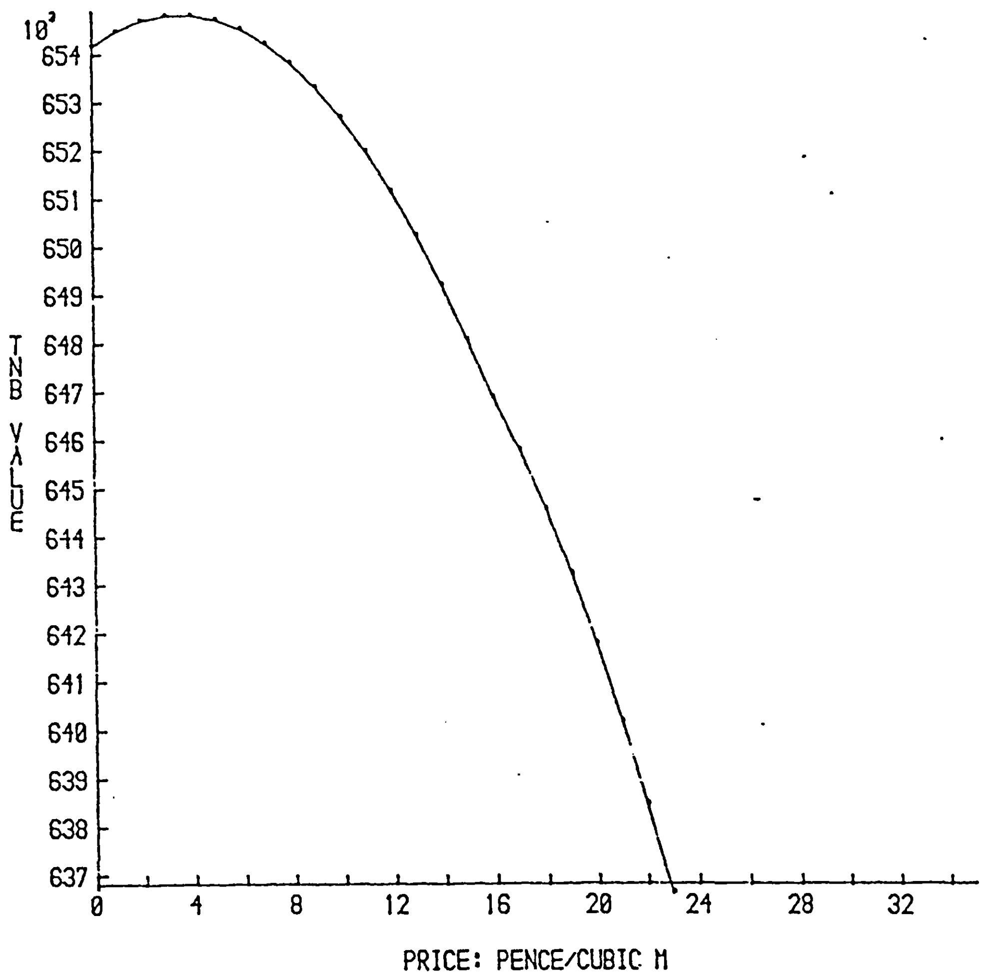

Table (6.21)

Constant Price Model
i=5%

Price P/M^3	Price £/MG	Effective Cap Add MG/y	Design Cap Add MG/d	time of Add	TNB £1000
0	0	6451	21.91	0	654267
1.00	45.4	6280	21.33	0	654583
1.74	79.0	6152	20.89	0	654751
3.52	160.0	5850	19.87	0	654921
6.00	272.4	5427	18.43	0	654611
24.50	1116.0	2255	7.66	8.86	633575

This sampling procedure can serve to identify the maximum of the TNB function. Figure 6.2 displays the shape of the function when expressed in terms of P. It is evident that the function is concave. The price which maximises the function can in fact be identified from the sampling procedure. This was carried out, using our computer program, by sampling the TNB function for fine increments in P.[14] The maximum of the net benefit function was identified as £654921 (1000) for a uniform real price of 160 £/MG or 3.52 P/M^3. The required capacity addition when this price is charged at time t=0 is 5850 MG/year of effective capacity equivalent to 19.87 MG/day of design capacity. This is to be installed immediately, at t=0 or 1986.

Another run of the model for the same parameters but for a discount rate of 10% produced the following results: a value of the net benefit function of £303792 (1000) for a price equal to 240 £/MG which is equivalent to 5.28 P/M^3. This requires the addition of some 18.85 MG/day of design capacity to be installed at time t=0.

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14 The size of the increment depends on the degree of accuracy required.

(6.7) Some Tentative Conclusions Pricing & Investment Models

The purpose of this section is to bring together results of our modeling, examine them critically while pointing to the limitations of the models, and draw some tentative general conclusions. This will be carried out by examining the results for the two discount rates, 5% and 10%.

For each discount rate we have results from three different models :

(1) Constant price model with single stage capacity addition ;

(2) Variable price with single stage capacity addition ;

(3) Variable price model with multi stage capacity addition.

The last two models employ short-run marginal cost pricing policy while the first picks that price which maximises total net benefits under the constraint of no price fluctuation. All three models assume that treatment capacity is the only constraint on output; all models assume a deterministic demand curve of the linear type with no built-in time lags. The models differ as regards the following : (a) whether admitting price variability or not and (b) whether admitting staging or not. Model (1) above is the most restrictive as regards these points while model (3) is the least restrictive. Accordingly the recorded value of discounted total net benefits of model (3) should be the highest, that of model (1) the lowest.[15] This is indeed verified by our results summarised again in table 6.22 below.

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[15] This is of course discarding the costs of price variability. See below.

Table (6.22)

TNB Value of The Models

i=5%

Model	TNB £1000	Incremental TNB £1000	Improvement on Model(1)
1- Constant Price	654921	33850	-
2- Variable Price Single Stage	655111	34040	0.56%
3- Variable Price Multi Stage	655310	34239	1.14%
	<u>i=10%</u>		
1- Constant Price	303792	6448	-
2- Variable Price Single Stage	305136	7792	20.8%
3- Variable Price	305616	8272	28.2%

The improvement in the value of the TNB for the case of a discount rate of 5% is small as can be seen from the table. For a discount rate of 10% however the improvement is more pronounced thus indicating that the results and conclusions are sensitive to the choice of the appropriate discount rate; the higher the discount rate the more attractive the less restrictive models become. This is ignoring for a moment the associated costs of price variation. Moreover one would expect the improvement to be more pronounced the more elements of capacity are included.

The various parameters of the models also effect the relative performance of the various models. Previous results have shown that the higher R, the absolute growth rate of demand at a price equal to MC, the better is the relative performance of the multi stage model. The same applies for f, the leakage ratio and for T, the duration of demand growth. Moreover one would expect the performance of the multi stage model to improve the higher is the degree of economies of scale in the cost of capacity addition function, ie the lower is the scale exponent.

The improvement in the value of TNB is achieved only at the expense of price variation. In principle there is no reason why one should have any prejudice against price variability. This is, of course, if such variability entails no costs either in terms of efficiency (overall resource or utility loss) or distributional consequences. However one often encounters the view that price fluctuation is politically unacceptable. If unacceptablity of price variation is an overriding and unalterable political constraint then the planner would have to accept such a constraint, grudgingly perhaps. The solution to the modelling exercise would then be along the lines of model No (1). It has to be said however that objection to price variability may not be so strong if the fluctuation, though perhaps still sharp, takes place around a mean value which would be considerably lower then the constant but high price level that may rule otherwise. That is, a high but constant price may be more objectionable then a variable but low price.[16]

Aside from these political objections one must naturally examine the real resource costs of price fluctuation. If a charging system by volume does exist then the extra administrative costs of implementing price changes ought not to be so excessive as to warrant too much concern here. The major potential cost of price variability comes from the possible misallocation of resources on the part of the consumers who might be induced to make the wrong investment decisions in water-using and water-saving durable appliances, investments based on the wrong perception of future price levels.[17] Thus for example a consumer who has been charged a price equal to 1.74 P/M^3 for some twenty years might choose to invest in a water intensive production technology on the expectation that this price will carry on into the future. This consumer would have good reason to complain if having done so he discovers later that prices have shot up to 7 P/M^3

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16 It has to be said that there is a good possibility that once a low level rules for some time the public may soon forget the old high levels. Objection to price rises may still be strong then even if we were starting from a low base.

17 This point is frequently stated in the literature. This reason is given for example by Turvey(1969) as to why " prices both have to endure for some years and have an impact on consumers which will frequently endure for some years ". Turvey(1969), PP282/299, op.cit.

as would happen if we were to follow the scenario of figure 6.3. At this price the consumer, a 'firm', might find itself with the 'wrong', expensive, type of production technique. This of course would involve the wasting of resources which would have to be set against any extra benefits that may arise from the optimum expansion with no constraint on price variability. Of course all this depends on the assumption of failure to convey to the consumer enough information regarding future price movements. Such failure could arise because the water enterprise does not announce in advance the likely trends of future prices and/or failure of the consumer to understand such announcements if they are indeed made. The first case is very likely to happen in the real world, one with considerable uncertainty as well as severe constraints on the availability of reliable information required for precision modeling. It is therefore not difficult to appreciate the reasons for the reluctance of water and other enterprises to commit themselves to such a framework of decision making regarding prices and investment.

Thus when evaluating the relative performance of the different models the net benefit value of each must in principle be adjusted downwards by an amount which is probably proportional to the degree of price fluctuation involved in each.[18] The degree of price variability, together with the associated demand profiles for each of our models may be seen in figures 6.3 to 6.8.

For a 5% discount rate the multistage model has the most pronounced price variability (see figures 6.3, 6.5 and 6.7. The single stage variable price model at 5% discount rate has an optimal solution a constant price equal to short-run marginal cost (1.74 P/M^3) from t=0 to t=20. Thereafter price rises continuously from this level up to 6.76 P/M^3 in the span of five years. From t=25 when demand growth ends price stabilises at its highest level equal to 6.76 P/M^3. The constant price model on the other hand gives as a solution a constant price equal to 3.52 P/M^3.

---

[18] The amount of the downward adjustment is unfortunately unknown. We have not been able to find any attempts to quantify the losses that are supposed to accompany price variation.

Although the resource cost of price variability is not known it seems safe to draw some tentative conclusions about the 'optimum' pricing and investment policy given all our assumptions and in the case of a discount rate of 5%. This is possible in the case of 5% because the improvement on the constant price model by the variable price models is so little that in all probability it is more than counterbalanced by the extra costs occasioned by the variable price solutions. If this conclusion is correct then the optimal policy would be to invest immediately in some 19.87 MG/day of design capacity; set the price as from now to 3.52 P/M^3 and keep it there up to the end of the planning horizon.[19] It is to be noted that the charging of a price greater than 3.52 P/M^3, so as for example to raise extra revenue, rapidly erodes the value of the net benefits as can be seen from figure 6.2. The temptation of a water enterprise to recover all its fixed costs through a volumetric charge would have no efficiency consequences only if the demand curve was fully price inelastic. Even a small positive elasticity, as in our case where e=-0.3, can vividly demonstrate the inefficiency of restricting demand by charging a higher price than is warranted by the optimal solution.[20] Any deficiency in revenue that may arise because of the suggested pricing policy[21] can be recovered in perhaps a better way through standing charges. It is also to be noted that revenue collected from the suggested pricing policy need not necessarily fall short of total costs including that of capacity and administration. The suggested price is twice as

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[19] Extra benefits might be possible given the constant price solution if capacity is staged in such a way as to keep price constant. This has not been modeled by us however.

[20] Of course a higher price than that of our solution would be warranted when allowance is made for distribution capacity investment and other elements of the supply system not considered by us. The constant price model can easily be expanded to allow for other components of capacity so long as the functional form of these costs are known. We have not followed up this point because of space limitations.

[21] We also note that the suggested pricing policy assumes no discrimination between various classes of consumers and as such a uniform price for all.

high as the short-run marginal (and average) cost and in consequence the water enterprise will make a surplus over and above running costs. Whether this is sufficient to recover fixed costs depends on a host of factors including past investment and depreciation accounting conventions.[22]

Are these conclusions valid for a discount rate other then 5% ? The answer seems to be no, at least not for a discount rate of 10%. The price and output profiles for the various models for a discount rate of 10% are presented in tables 6.4, 6.6 and 6.8. It can be seen that considerable variation in prices over time is present in the solutions of the variable price models of the single and multi stage types, though the variation are of different nature.[23] The constant price model provides a maximum of total discounted net benefits at a price equal to 5.28 P/M^3 . When compared with the variable price models it seems that the stability of prices is achieved at a cost equal to £1.284 million in comparison with the multi stage model and £1.344 million when compared with the single stage model. When the distribution capacity costs and those of other elements of supply ignored in this analysis are added then it becomes rather difficult to draw any tentative conclusions since the situation is not as clear as was the case when the discount rate was equal to 5%. The costs of price variability, the administrative feasibility of price variability and its political admissibility would need to be examined carefully before any final conclusions are drawn.

The validity of our conclusions and analysis depends on the validity of our models whose legitimacy depends on the validity as well as realism of the assumptions behind them.

It goes without saying that economic models are meant to help us understand and analyse real life problems. This however does not mean that these models have to incorporate

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[22] We note here that the water enterprise would in fact make a larger surplus on those intramarginal units of supply whose running cost of production is less then 1.74 P/M. These units include all water abstracted from ground sources which does not need major treatment as in the case of river water.

[23] Therefore the revenue implications of the two solutions are different.

in them all the complex details of real life situations. Indeed the model's usefulness may judged by its ability to simplify what otherwise may be a very complex situation that defy straightforward analysis. Simplifying assumptions are thus often necessary.

It is our opinion that the main force of the analysis is not undermined by the list of criticisms and limitations of these models. These limitations include :

(1) Demand is assumed to respond to price without time lags. While this is true, yet it may be countered by the argument that in this case the long-run price elasticity of demand would be greater than that we have assumed. Using the long run elasticity together with lagged adjustment would in all probability strengthen our results. More analytical complexity would be added to the models but perhaps without any added insight.

(2) The omission of capacity elements other then treatment works may be said to be unrealistic. This is true but the inclusion of other elements would not alter the main conclusions above, thought it would certainly give different simulations from ours.

(3) It has been unrealistically assumed that demand is a deterministic function while in fact demand is a stochastic variable[24]. Again while accepting the force of this argument we do not see how it would alter the main conclusions above. It would certainly make precision modelling that much harder.

(4) Various criticisms may be levelled at our choice of the cost and demand functions. Some of these have been tackled in the course of the various sections and will not be repeated here. It is to be said however that like above, the main thrust of our conclusions are not changed, though the details of implementation might well be altered.

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24 The same of course can be said about supply.

FIGURE 6.3

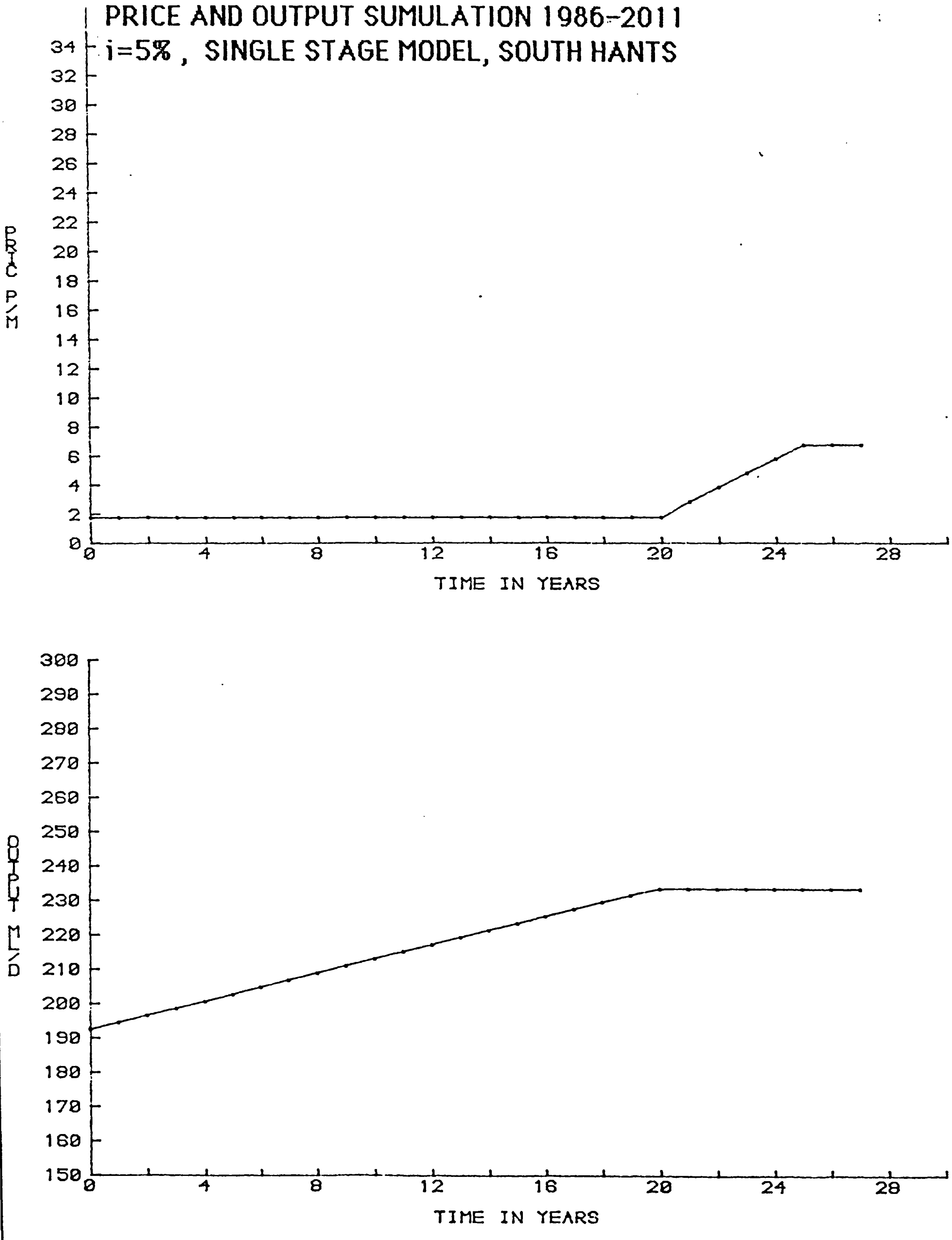
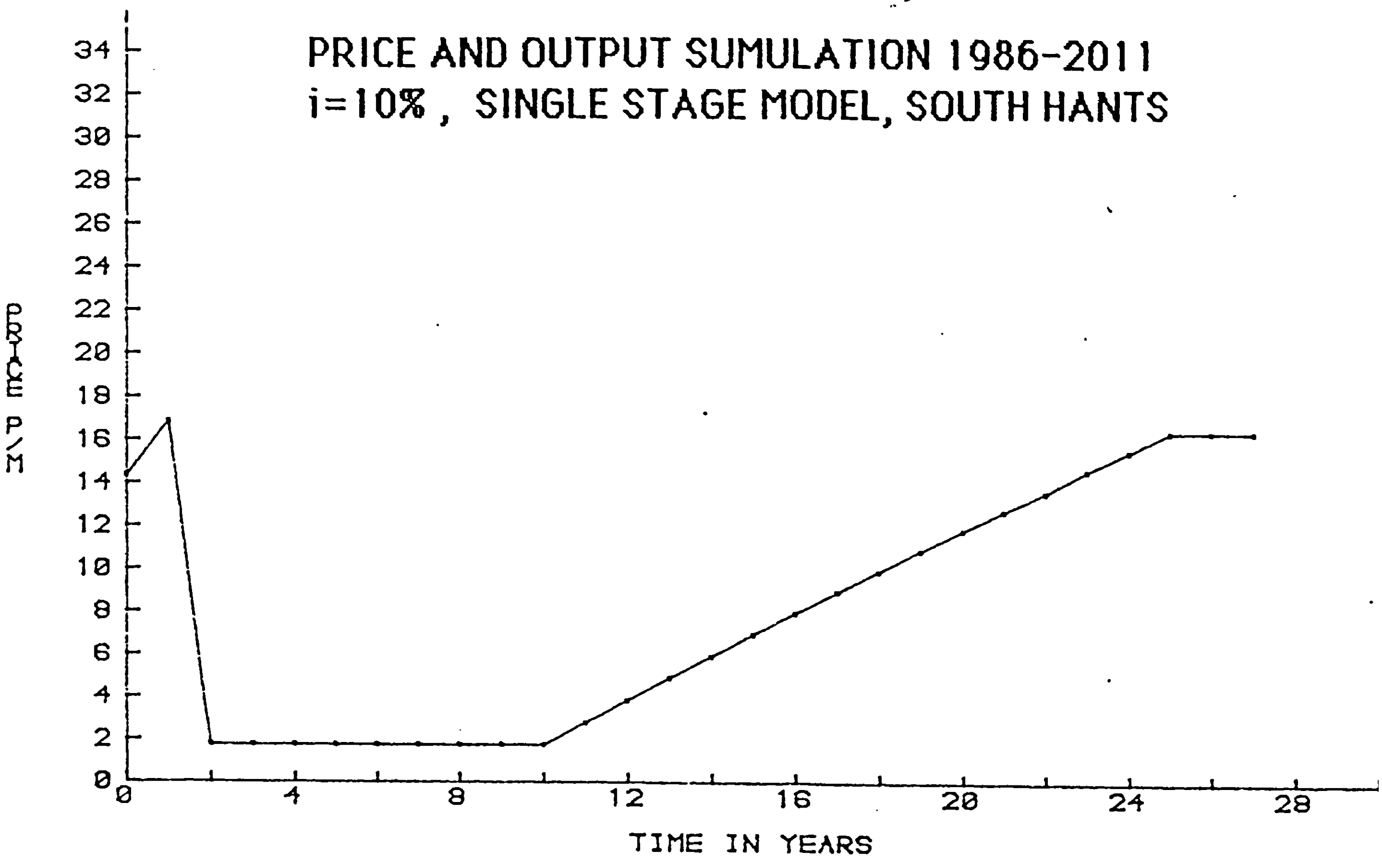

PRICE AND OUTPUT SUMULATION 1986-2011
i=5% , SINGLE STAGE MODEL, SOUTH HANTS
PRIC P/M
TIME IN YEARS
OUTPUT ML/D
TIME IN YEARS

FIGURE 6.4


PRICE AND OUTPUT SUMULATION 1986-2011
i=10% , SINGLE STAGE MODEL, SOUTH HANTS
PRICE P/M
TIME IN YEARS

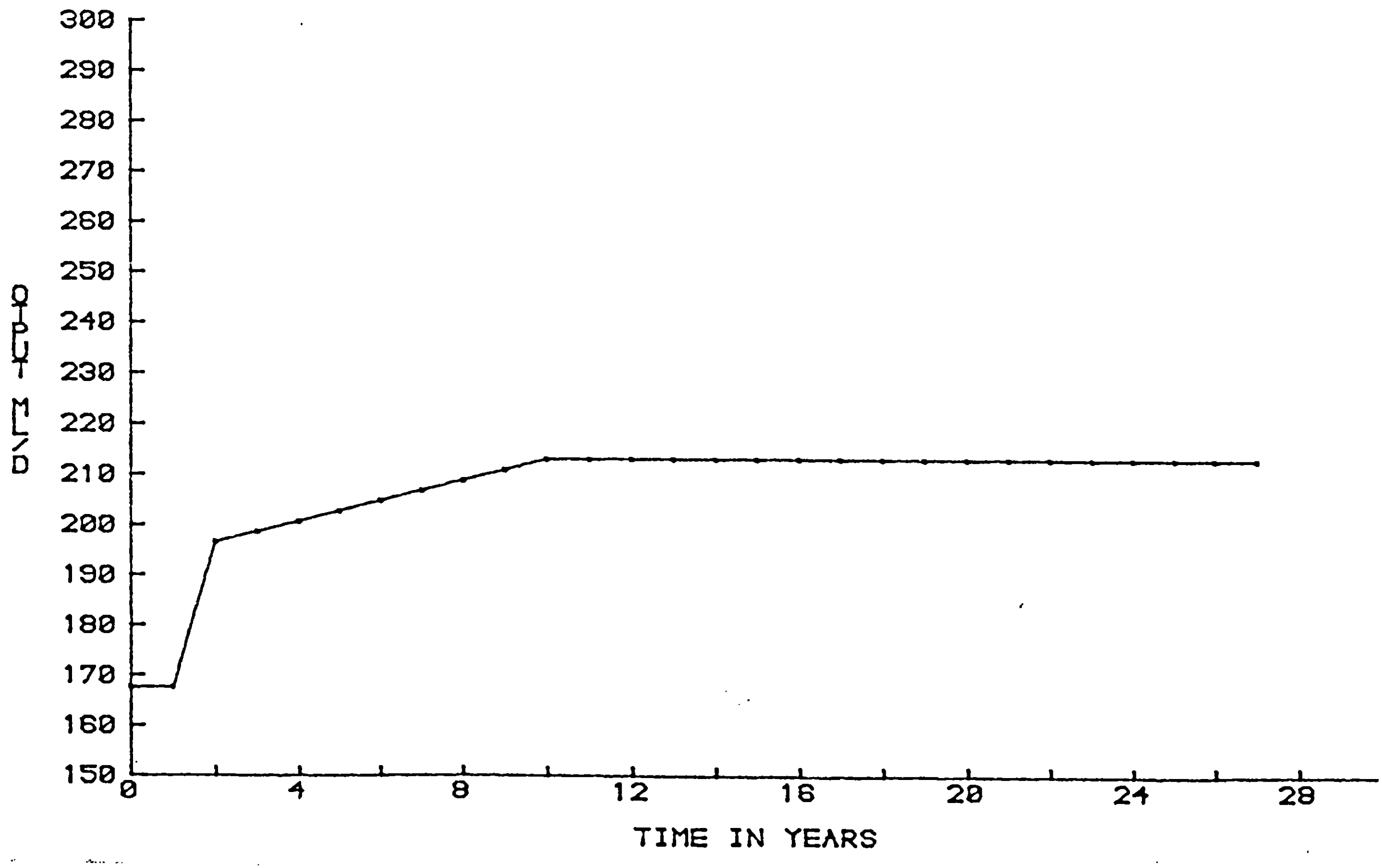
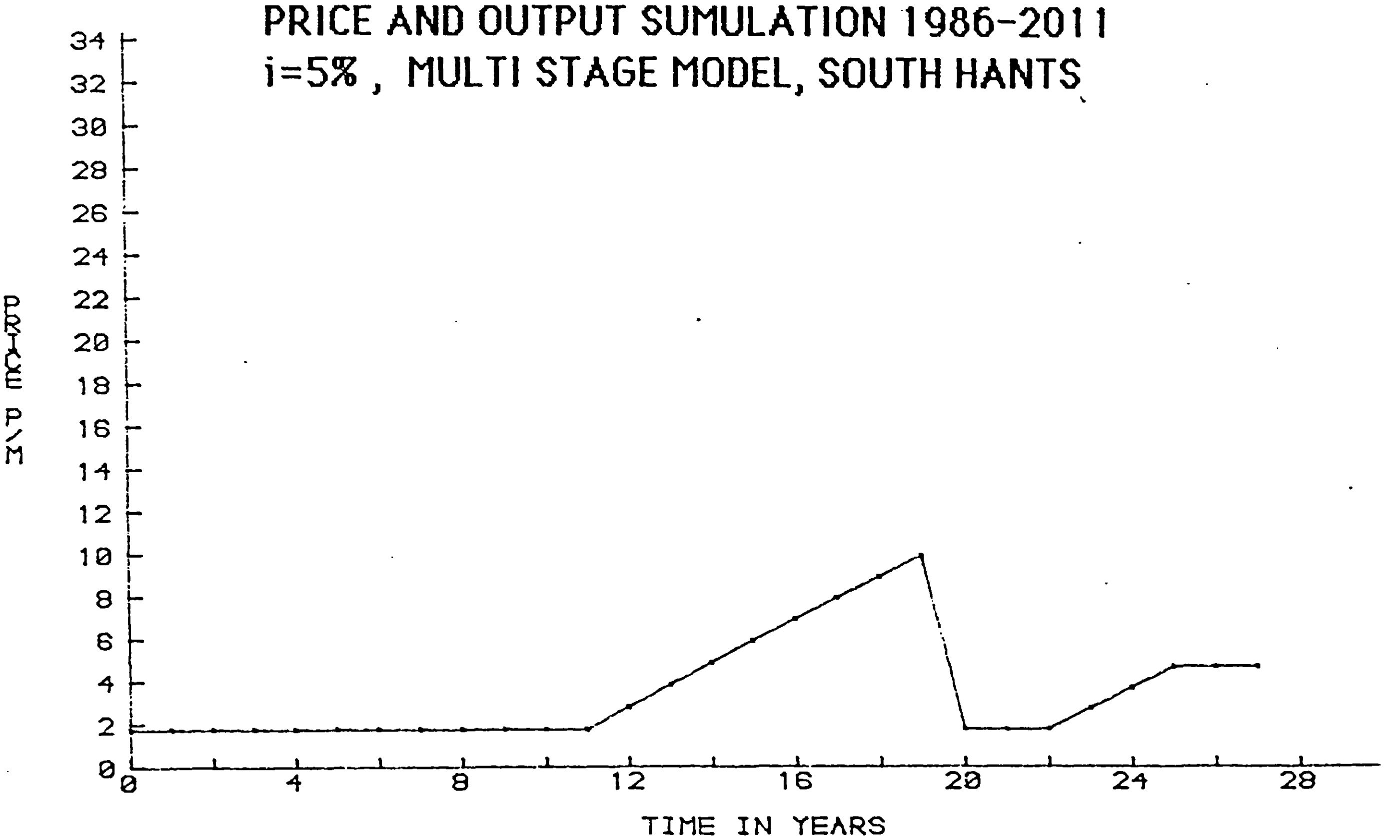

OUTPUT ML/D
TIME IN YEARS

FIGURE 6.5

PRICE AND OUTPUT SUMULATION 1986-2011
i=5% , MULTI STAGE MODEL, SOUTH HANTS



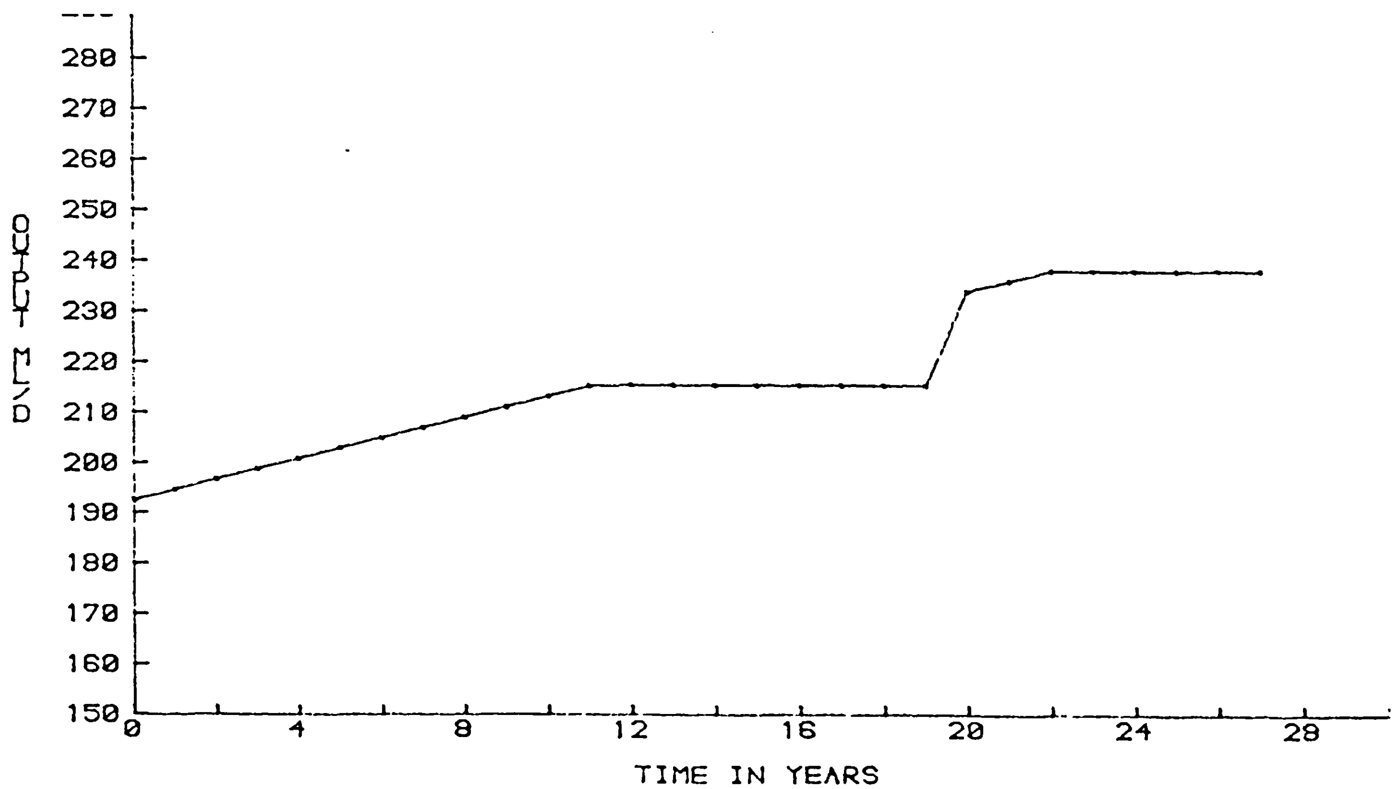


FIGURE 6.6

PRICE AND OUTPUT SUMULATION 1986-2011
i=10%, MULTI STAGE MODEL, SOUTH HANTS

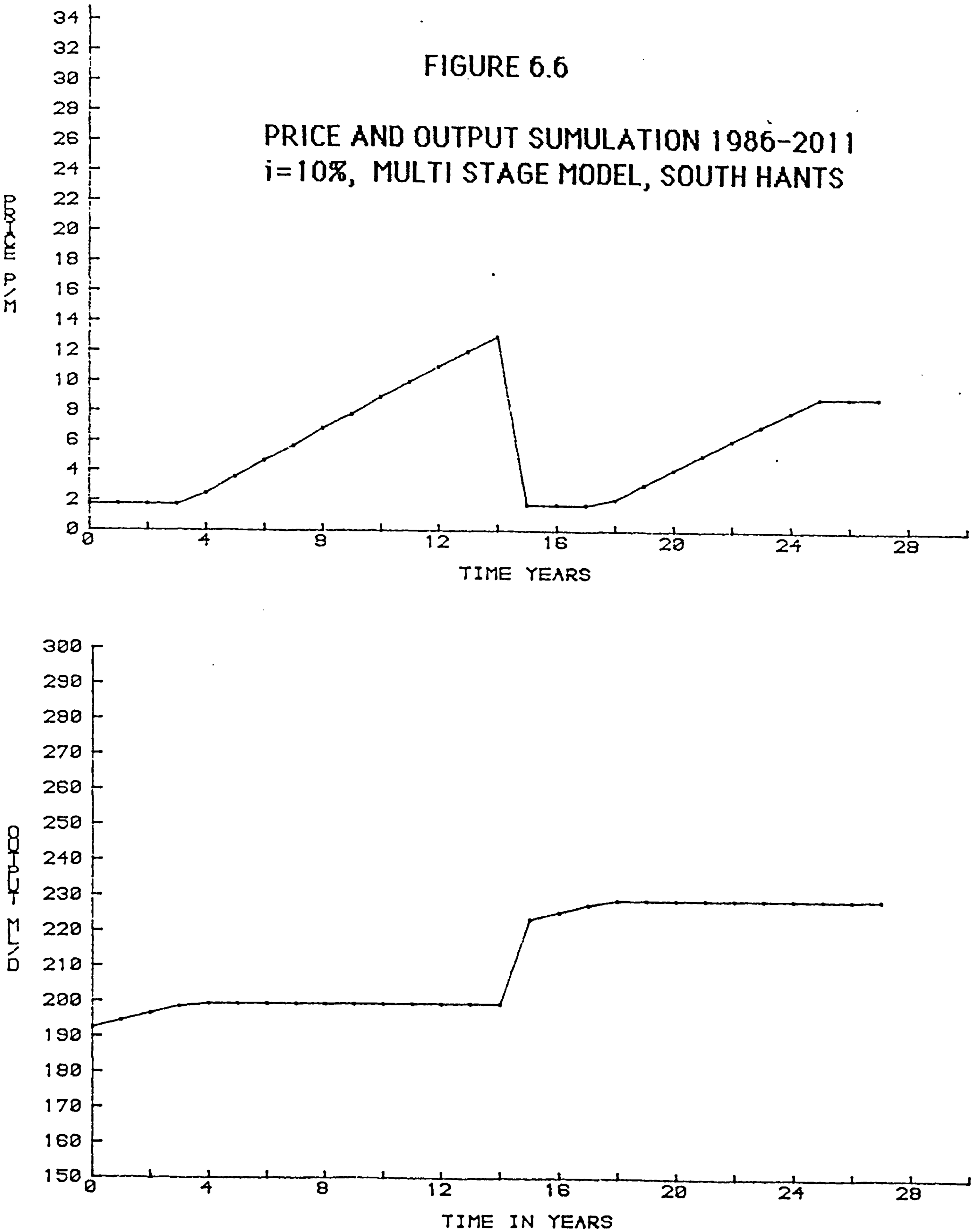
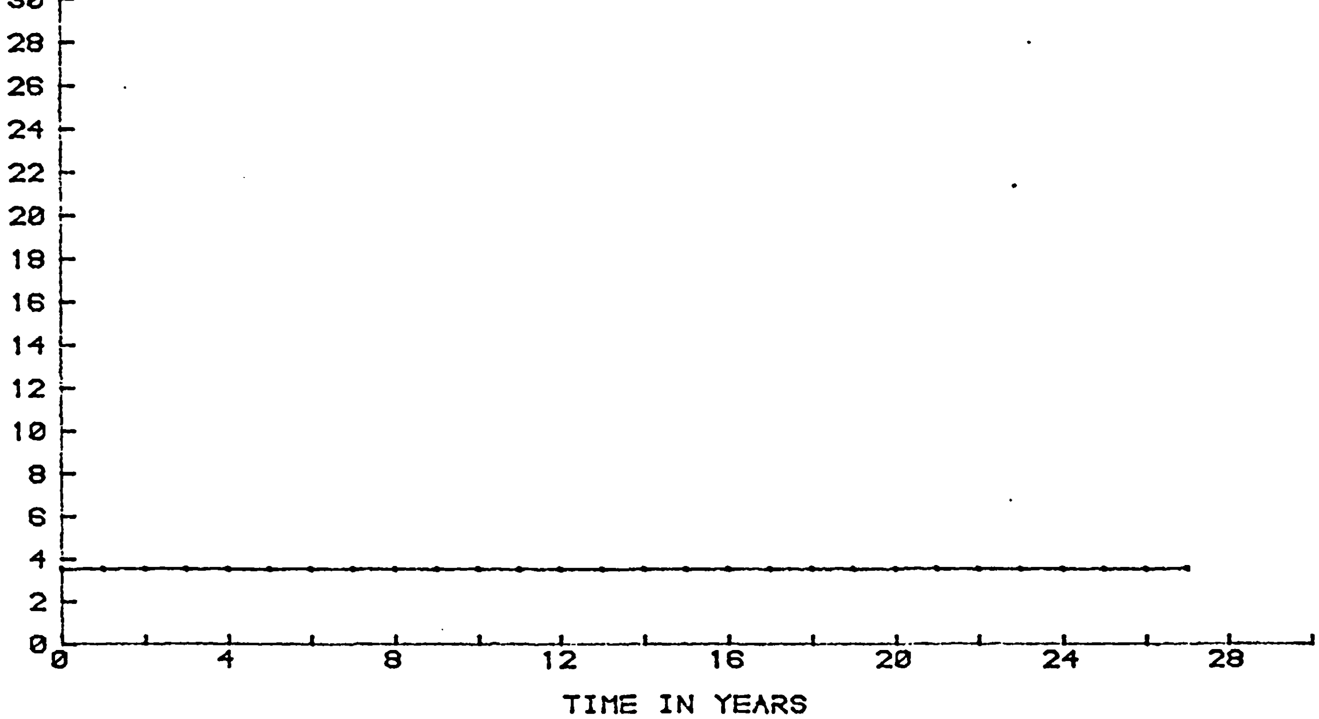


FIGURE 6.7

PRICE AND OUTPUT SUMULATION 1986-2011
i=5% , CONSTANT PRICE MODEL, SOUTH HANTS



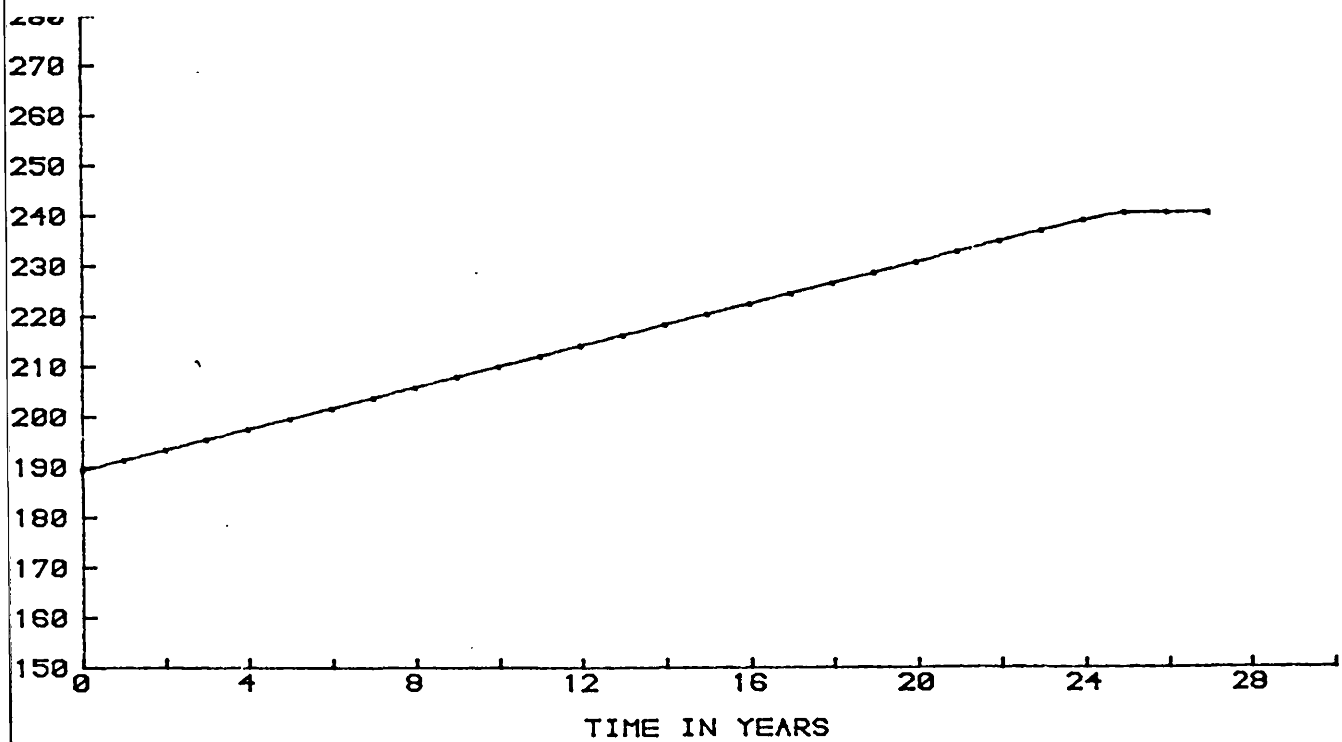
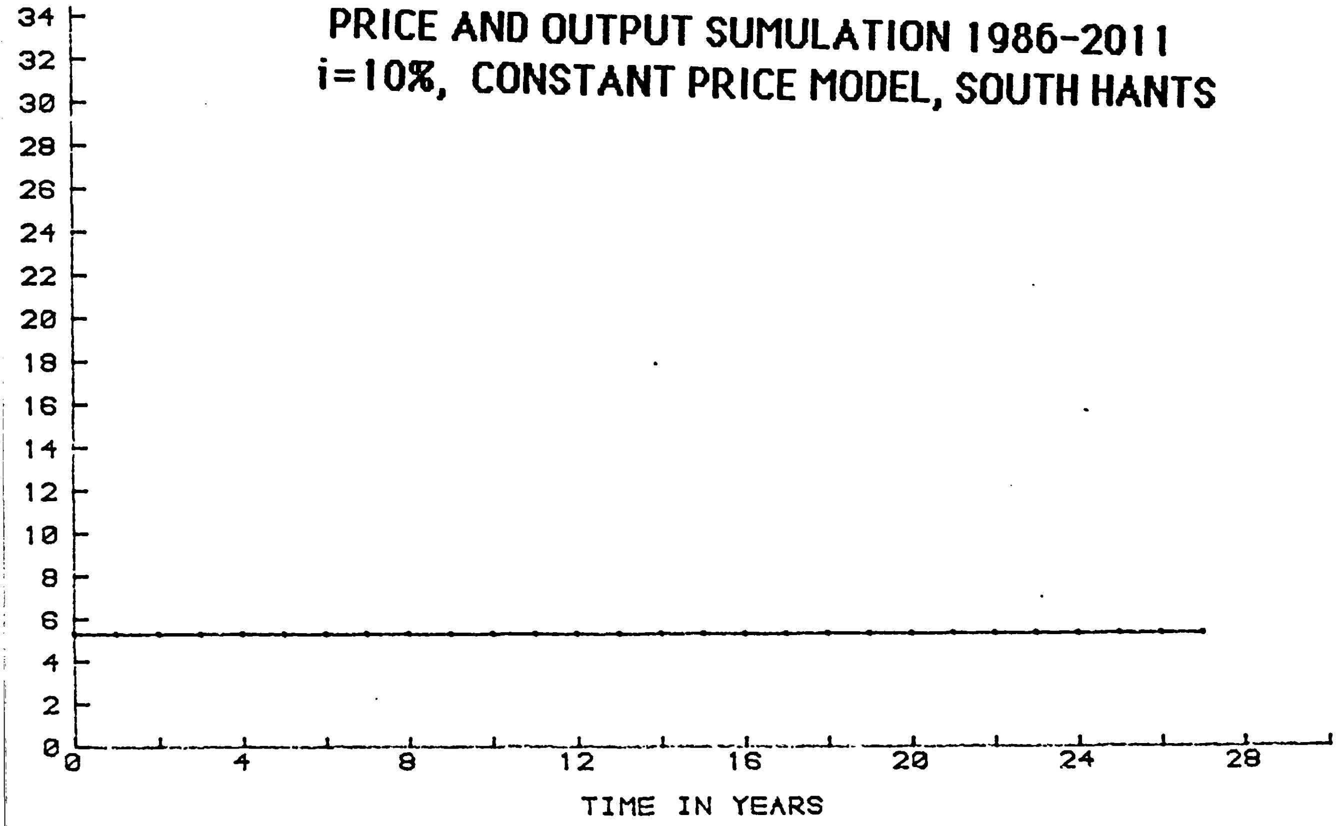


FIGURE 6.8


PRICE AND OUTPUT SUMULATION 1986-2011
i=10%, CONSTANT PRICE MODEL, SOUTH HANTS
34
32
30
28
26
24
22
20
18
16
14
12
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4
2
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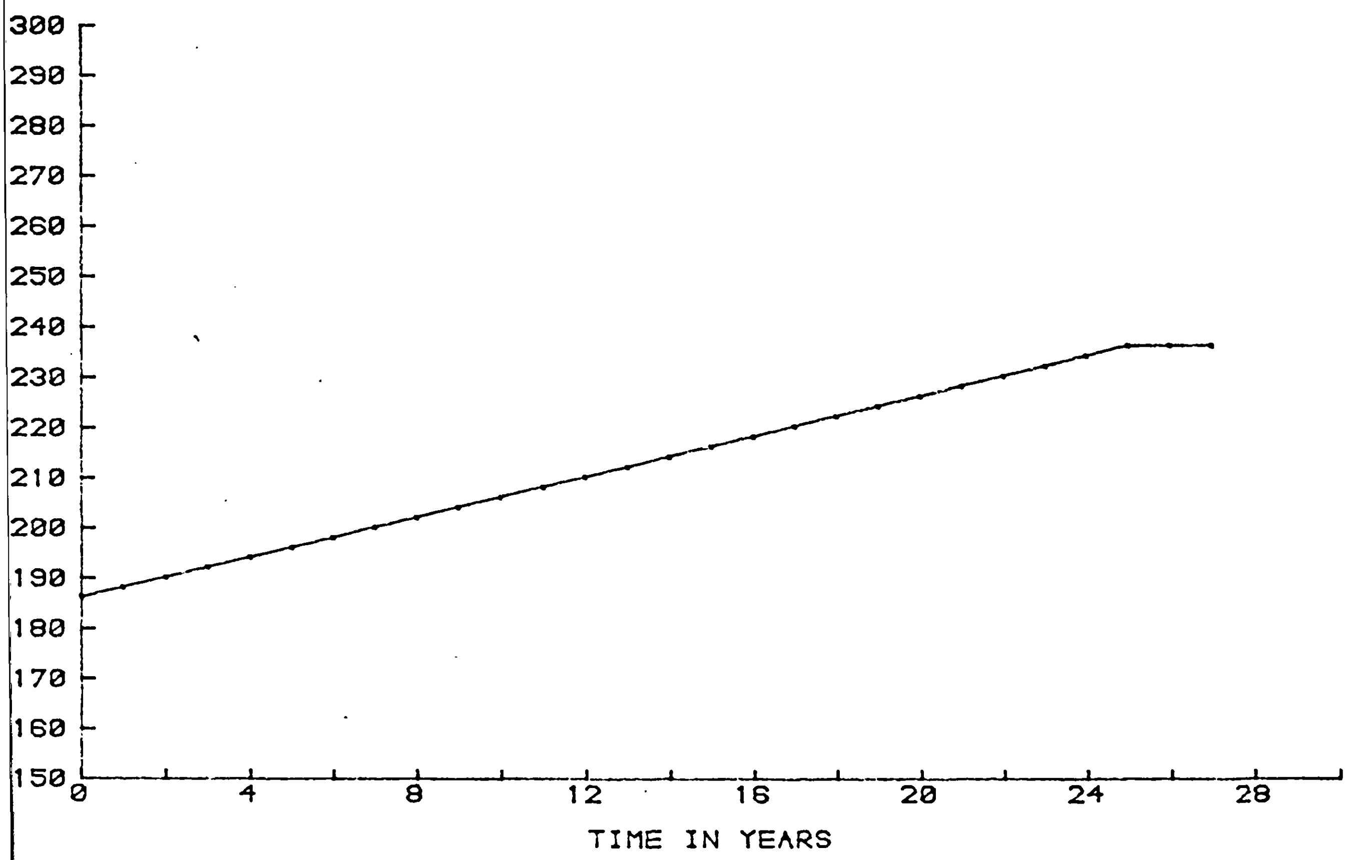
24
28
TIME IN YEARS


300
290
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260
250
240
230
220
210
200
190
180
170
160
150
0
4
8
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16
20
24
28
TIME IN YEARS

Chapter 7

ASSESSMENT OF LEAKAGE CONTROL AS A DEMAND MANAGEMENT TOOL

(7.1) Introduction

Pricing policy may be viewed as one component of a package of demand-management tools designed to offer means of bridging any discrepancy between demand and supply. Such tools give water supply managers the scope to reduce their dependence on supply-fix policies relying mainly on capacity expansion.

At least four additional types of measures can be listed under the general heading of demand-management tools. These include waste control, the introduction of water saving technology in water using an consumer durables, regulation and education.[1]

Of the four we will examine only waste control, concentrating on the methodology for the assessment of the benefits and costs of curbing waste where the definition of waste is confined to losses from the distribution system.

As argued in section 2.1.2 investment in any of these demand management tools (including leakage control) should normally be evaluated using cost benefit-analysis. It was also shown that these investment decisions may alternatively be examined using appropriately defined marginal cost of water supply.

The main objective of this chapter it follows is to examine how the technique of cost-benefit analysis can be used to facilitate the choice of an economic leakage control policy and how this compares with the use of marginal cost of water . One objective of this chapter is to show that the method currently recommended and most often in use for the evaluation of leakage control in the UK water industry, based

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[1]Domestic metering is another category. It naturally comes under pricing policy, however.

on the use of marginal cost of water, may be viewed as a special (and rather restrictive) case of the more general approach of cost-benefit analysis. Discussing the application of cost-benefit analysis in the evaluation of leakage control should also serve to illustrate how the technique may be applied in the evaluation of other demand management policies.

In sections 7.2 and 7.3 we examine the definition and measurement of waste and leakage. Established methods of leakage control and the amount of water savings associated with them is discussed in sections 7.4 and 7.5. The capital and running costs of implementing these methods is discussed in section 7.6. In section 7.7 on the other hand we examine the benefits of implementing leakage control as well as its relationship to marginal cost of water supply. The assessment of leakage control policies is discussed in section 7.8. Section 7.9 presents some concluding remarks.

(7.2) Definition of Waste and Leakage

Various definitions of the terms waste and leakage can be found in the literature of the water industry. The broad definition of waste embraces all water which is "not effectively used".[2]

Rees (1974)[3] offers a listing of the items she would include in this wide definition of waste. They include :

(1) leakage from pipelines and other components of the water supply system;

(2) excessive use of water, ie when the cost of the last gallon of water taken by a consumer exceeds its value to her or him;

(3) 'unduly' high quality of water being taken for any purpose;

(4) and, finally, waste arising from not allocating existing resources to their most useful productive use.

Leakage may thus be viewed as a subset of waste as very broadly defined above. Borrows(1983)[4] therefore defines leakage as that part of waste which leaks or escapes or is lost from the water supply system other then by deliberate or controllable action. This definition, in other words, usefully narrows down the meaning of leakage by distinguishing it from items 2,3 and 4 above. The meaning of leakage is confined to that part of water which leaks from components of the supply system including mains, reservoirs,

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[2] Reid J.,1974,"Waste Prevention: The Design and Operation of Distribution Systems," Symposium on Waste Control: Its importance in the Planning and Management of Water Supply Systems, The Institution of Water Engineers, September 1974, PP99-115.

[3] Rees J.,1974,"Waste Control in the Water Industry: An Economic Approach," Symposium on Waste Control: Its Importance in the Planning and Management of Water Supply Systems, the Institution of Water Engineers, September 1974, PP45-55.

[4] Borrows P.F,1983,"Water losses - Recent Thames Water Studies," Water Supply, Vol.2, Brussels, Pergamon Press Ltd, PPB131-141.

consumer service pipes and other fittings such as taps and ball valves.[5]

Factors affecting leakage

Leakage from the water supply system had been recognised as early as Roman times.[6] The problem of water losses because of leakage is still with us today despite the tremendous improvement in the design and construction specifications of water supply systems. Indeed it is in the very nature of water supply systems that 'some' water leaks away from unsuspected holes and cracks in underground pipes and buried service reservoirs.

Many factors influence leaks from the water supply system. They include:

1- Water conditions including its pressure, temperature and aggressiveness;

2- soil characteristics and movement;

3- condition of pipes and mains including type of material used and joining methods;

4- poor quality of workmanship;

5- stress from traffic vibration and other external factors;

6- age of the supply system.

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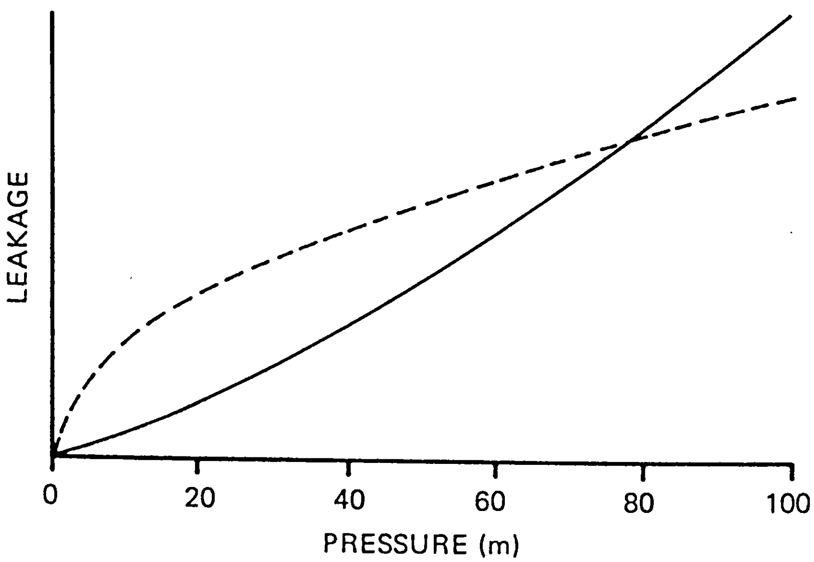
[5]Giles,H.,J., 1974,"An Assessment of the Causes of and Extent of Waste and Undue consumption in Distribution Systems and on Consumer Premises," Symposium on Waste Control: Its importance in the Planning and Management of Water Supply Systems, The Institution of Water Engineers, September 1974, PP9-21.

[6]Borrows (1983), op.cit.

FIGURE 7.1

PRESSURE LEVELS AND LEAKAGE



Pressure affects both the number of leaks in the system and the rate of flow of water from these leaks. It is now an established fact that increases in pressure lead to higher numbers of bursts and also increase the rate of loss from an individual leak. Diagram 7.1 above shows the now established practical relationship between pressure levels and leakage rates. The diagram shows that, contrary to the presumed theoretical relationship, small reductions in pressure result in correspondingly larger falls in leakage rates.[7]

Changes in temperature, frosts and subsidence cause soil movement which in turn lead to breaks in pipes, movement in joints and failure of fittings, all causing leakage.

Stenberg(1982)[8] provides a description of how galvanic corrosion (rust) and erosion (wear) are generally responsible for leaking pipes. Corrosion occurs in iron and steel pipes leading eventually to the emergence of holes and/or pipe fractures. Erosion is generally caused by the action of a jet of water whose erosive power is aggravated if the water is under high pressure and if it contains sand particles.

Prevention is always better then cure. Thus it is always important to pay attention to the initial design, choice of construction materials, and quality of work when constructing water supply systems.

The type of soil effects leakage in several ways. For example leaks in pipes buried in highly permeable soil can go on for a long time before being noticed. In other types of soil the leaking water will soon surface to the ground indicating a leaking pipe needing repair.

Other characteristics of the water supply system may also prove to be important. For example leakage from water towers is easily detected and repaired whereas leakage from buried service reservoirs is usually more difficult to detect and repair.[9]

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[7]See :
(1) Stenberg,R.,1982, "Leak Detection in Water Supply Systems," VAN, Scandiaconsult, Stockholm, Sweden.
(2) STC 26,1981, op.cit.

[8]Stenberg(1982), op.cit.

[9]Giles,1974, op.cit.

(7.3) The measurement of Leakage

When all consumption is by measure then leakage is usually easily indicated by reference to unaccounted-for water, ie the difference between the amount produced by the water utility and the amount of water sold to consumers as is registered on their meters.[10] Leakage forms the overwhelming part of unaccounted-for water. Estimates of unaccounted-for water when adjusted for any meter under-registration and any unmetered legitimate consumption give estimates of leakage.

The situation however is more problematic when there is a substantial amount of unmetered consumption, as it is in the UK where almost all domestic consumption escapes metering. In this case we have the problem of identifying each of (1)unaccounted-for water and (2) accounted-for (legitimate) consumption, ie items which make up unmetered supply.[11] Two approaches to the estimation of leakage in this case has been followed. These are:

1- The total integrated flow .
2- The total night flow rate.

(1) The total integrated flow method

STC26(1981) and Field(1981) suggest the following formula for the estimation of leakage:

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[10]In this case water losses beyond consumer meters do not enter into unaccounted-for water and leakage. Some allowance must be made of course for meter inaccuracy.

[11]Discussion of ways of estimating leakage in the system can be found in several references including:
(1) STC26, 1981, op.cit.
(2) Stenberg,1982, op.cit.
(3) Giles,1974, op.cit.
(4) Field,D.B,1981,"Understanding Unaccounted-for Water," Symposium: An understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, 1981, London.
(5) Moyer,E.E,1985, Economics of Leak Detection, American Water Works Association, Denver, U.S.A.
(6) Reid,J.,1974, op.cit.

$$u = s - (m+ap)$$

where:

u = unaccounted-for water

s = sum of all water inputs into the system

m = sum of all water accounted for by measure

a = average domestic consumption per capita of population plus an allowance for any unmetered commercial supply

p = population supplied.

All terms must naturally refer to the same accounting period, usually a minimum of three months is needed so as to overcome seasonal variation in demand.

Estimates of leakage using this approach are subject to a great deal of uncertainty and error, which in turn is difficult to assess. The overall uncertainty surrounding the estimate of u emanate from uncertainties associated with each of the terms s,m,a and p.

One source of error arises from estimates of unknown average domestic consumption and unmetered commercial consumption. This source of error is higher if national average figures are used instead of locally based figures derived by monitoring actual consumption levels, thus allowing for any local features such as holiday demand and other relevant factors.[12]

Measurement of both s and m is complicated by uncertainties arising from inaccurate meter readings. Mechanical meters are particularly subject to error as are older meters. Large meters are particularly prone to err on the low side when registering low flows.[13] Field (1981)[14] notes that even in a well managed system overall uncertainty

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[12]See Philip,J.E,1985, "Better Accounting for Unaccounted for Water," Four Papers on Aspects of Leakage Control, JIWES.

[13]Philips,E.J, notes that at low flows meter under-registration may on occasions reach as high a level as 60%.

[14]Field,1981, op.cit.

in m may range between 3% and 6%. Giles(1974)[15] notes that on a trunk main metered at two ends and with no intermediate draw-offs in between, an error of 5% on each meter, but in opposite directions, could result in a 18.2 M^3/d leak being undetected for a nominal flow of 4544 M^3/d through the main. Meter calibration ought to reduce this source of error but such exercises are often quite expensive to conduct. Moreover it is important to ensure coincidental meter reading relating to the same accounting period. This may be impossible when requiring the reading of many meters of consumers with metered supply.

Uncertainty in u can also arise from uncertainties regarding population levels especially if they are subject to significant seasonal variation. And last but certainly not least the uncertainty regarding two large numbers (s and m+ap) make the relatively small difference between them even more uncertain.[16]

Thackray(1981)[17] argues that uncertainty regarding the estimate of u using the total integrated flow formula does not mean that this method of estimation has no useful applications. So long as the factors giving rise to the uncertainty in u remain stable over a period of time then the estimates of u can be used to monitor changes in leakage over that period of time. The total integrated flow measure is more useful when applied in a small area where changes in population levels, legitimate metered or unmetered consumption can be monitored with a reasonable degree of accuracy. But care must be exercised in not using these estimates for comparison between different areas since the underlying degree of uncertainty in the respective estimates of u may well be widely different.

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[15]Giles,1974, op.cit.

[16] Field (1981) using a statistical technique for the measurement of uncertainty, together with plausible values for the uncertainty associated with s, m, and ap, believes that the overall degree of uncertainly associated with u can be as high as $\mp$37%.
Field,1981, op.cit.

[17]Thackray,J.E,1981, Discussion of Field's(1981) paper. See reference for Field(1981) above, PP2.22.

(2) The total night flow rate method

More accurate and reliable estimates of leakage rates can be obtained using measurements of total night flow rate, ie flow rates during the early hours of the morning when legitimate metered and unmetered consumption is presumed to be at its lowest.[18] In this case the relevant formula is as follows:

$$u' = s' - (m' + a'n')$$

where u' = unaccounted-for night flow rate

m' = total night flow rate for all trade and commercial users (metered users)

a' = average domestic night flow rate per property

n' = number of properties supplied.

s' = night flow rate.

s' can usually be measured by turning off the inlet(s) to controlling service reservoirs for the night period and supplying the system from these reservoirs so as to carry drop tests of water levels in these reservoirs to indicate the flow rate of s'. According to STC26(1981)[19] a' is approximately 2 liters/property/hour but regional variations must be taken into consideration.[20] m' may be obtained by reference to actual measurements of individual large industrial and other consumers while the consumption of the large number of smaller metered consumers would need to be estimated.[21] The number of properties served, n', should be readily available.

The biggest advantage of this method of estimation stems from the fact that both m' and a'n' are likely to be small in relation to s'. Thus the degree of error in u' arising from uncertainties regarding m' and a'n' is reduced.[22]

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[18] See STC26(1981) and Ried(1981) among others.

[19] STC26,1981, op.cit.

[20] Thus in the Wessex region it is estimated that a' is at least 20% higher then the STC26 figure. Hence one source of uncertainty is already apparent. Philips,1985,op.cit.

[21] Hence a second source of uncertainty.

The volume of leakage often is expressed as a percentage of the total volume of water put into the supply system. For example the Monopolies and Mergers Commission Report on the Severn Trent Water Authority[23] noted in 1981 that more effort should be made to control leakage, having observed that leakage there stood at 27% of total supply. The North West Water Authority estimated leakage in its region in the early 1980's at 34% of supply.[24] Rees(1981),[25] on the other hand, while noting the inherent difficulties of such measures expressed the view that leakage at the national level in the UK was 25% of total supply. In fact some authors go as far as suggesting that leakage ratios may be taken as a measure of system performance. Thus, for example, Moyer(1985)[26] states that 10% to 15% losses from the distribution system are considered to be reasonable.

Several writers however have questioned such unqualified use of percentages, especially for comparison between different areas and for the setting of standards. Giles(1974), STC26(1981) and Reid(1974) for example have convincingly argued that such ratios can be quite misleading

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[22]In Anglian Water Authority it was found from a pilot study that the inaccuracy of the source work meters (measurement of s) could be of the same order of magnitude as the possible volume of leakage. Therefore it was decided to use night flows for the assessment of leakage instead. See Bullocks (1981).
Bullocks P.F.,1981, Discussion of paper by Field,D.D,1981, op.cit, Symposium: An understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, 1981, London.

[23]Monopolies and Mergers Commission Report on Severn Trent Water Authority, June 1981.

[24]Thomas,D.L.,1981, Discussion of paper by Howarth,F,1981, "Alternative Waste Control Methods",PP2.23, Symposium: An understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, 1981, London.

[25]Rees,J.,1981,"An Economic Approach to Waste Control: A Second Look," Symposium: An understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, 1981, London.

[26]Moyer (1985), op.cit.

because of the various ways they could be derived (for example should leakage from trunk mains be included or excluded). Therefore it has been argued that it is better to express leakage as a flow rate, eg liters per hour or as a flow rate per property or per head, eg liters/property/day. Moreover it is certainly not possible to state norms for acceptable levels of leakage without making reference the value of the product (ie the marginal cost of water).

(7.4) Established methods of controlling leakage

All methods of leakage control with the exception of one require various amounts of resources expended on the detection of leaking pipes and other apparatus. The exception being reduction of leakage through pressure control. The list of established methods of leakage control include[27]:

1- Pressure control
2- Passive control
3- Regular Sounding
4- District metering
5- Waste metering
6- Combined district and waste metering

Pressure control

A minimum level of pressure in the water distribution system is one necessary attribute of quality which the consumer has become accustomed to. But it is also true 'excessive' pressure means waste in the form of excessive leakage and an excessive amount of undue consumption because of "flow uses" of water such as teeth cleaning and some hand washing and dish rinsing.[28] Therefore it has been suggested

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[27]There are many references on this subject. Four references are particularly useful, both on the economic and technical aspects of leakage control. These are :
(1) Symposium: An understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, 1981, London.
(2) Symposium on Waste Control: Its importance in the Planning and Management of Water Supply Systems, The Institution of Water Engineers, September 1974.
(3) STC26,1981, op.cit.
(4) Stenberg,1982, op.cit.

that one approach to the reduction of leakage is to keep pressure levels no higher than those which would stop consumer complaints.[29] The use of the technique of pressure control, moreover, can proceed alongside other more conventional methods.

The 'success' of pressure control in reducing leakage levels in the UK has been demonstrated by the experience of several water utilities including the Wessex Water Authority and the Bristol Water Works Company.[30] Howarth and Olner(1981)[31]report on an automated pressure control system pioneered by the Wessex Water Authority in the late 1970s. The project used computer based remote monitoring and control, using electronically controlled pressure valves. As a result the night line flow for Weymouth, which had suffered from many old and poor condition pipes, was cut from 14.4 to 6.1 litres/prop/hour. This was achieved following a reduction of pressure by some 15M, to a level equal to 19M. The authors report a saving in pumping costs in Weymouth of some £9000 per annum, but do not refer to the corresponding costs of implementing the telemetry scheme.[32] Bessey(1985)[33] reports on the endeavour of the Bristol Waterworks Company to use pressure control to supplement combined district and waste metering so as to cut leakage levels. The author points out

[28]See discussion in section 7.2 above.

[29]Giles,1974, op.cit.

[30] It is necessary to point out here that the effectiveness of any leakage control policy must be judged by reference to its economic viability in comparison with other options including the option of doing nothing. For more on this point see section 7.8 .

[31]Howarth,F., OLner,P.,1981, Symposium: An understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, 1981, London, PP4.1-4.27.

[32]Of course the telemetry scheme produced other benefits beside energy savings. These may include the provision of continuous data on how the distribution system behaves including pressure and flow levels, thus serving operations and network analysis. These benefits must considered in any economic assessment of such schemes.

[33]Bessey, S.G,"Some Developments in Pressure control" Four Papers on Aspects of Leakage Control, JIWES.

that in cases where investment in costly remote control telemetry with microprocessor controlled pressure reducing valves is not justified one should examine the alternative of using hydraulically controlled valves.

The high cost of installing pressure reducing valves is often cited as one reason for shunning away the method of pressure control.[34] Other authoritative sources such as STC26(1981)[35] have claimed that many systems in the UK and Wales are operated at optimum pressures and that in some systems high pressures cannot be decreased without creating supply problems. This explains why STC26 seems to place more emphasis on the other methods of control.

(2) Passive Leakage Control

This method, sometimes described as the laissez-faire approach (Reid 1974)[36], involves the least amount of detection effort and therefore permits the 'highest' leakage levels. The method essentially restricts repairs to those leaks that become self-apparent by water showing at the surface or when falling pressure levels, because of the leak(s), cause consumer complaints. In general this method is quite adequate when leakage is confined to major pipe breaks causing a substantial gush of water to the surface. Given that leakage can be caused by a 'large' number of small leaking holes, it follows that a considerable part of leakage can pass unnoticed. Setford(1985)[37] reporting on a study by the Mid Kent Water Company underlines the conclusion that in the area of operation of the company passive leakage control was economically inferior to active control. The main advantage of passive leakage control must be its low cost. It is therefore best suited to areas where the cost of water

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[34]Giles,1974, op.cit. See also Speed,H.D.M,"Water Losses," C.I.P.F.A/IWES Seminar.

[35]STC26,1981, op.cit.

[36]Reid,1974, op.cit.

[37]Setford, M.T.,1985, "An Economic Appraisal of Active and Passive Methods of Leakage Control," Four Papers on Aspects of Leakage Control.

production is low.

(3) Regular Sounding

This traditional labour-intensive method of leakage control relies completely for the detection of leaks on listening methods.[38] The method involves the deployment of team(s) of expert inspectors using 'listening sticks' or more sophisticated equipment. Their job would involve the regular sounding of mains, hydrants, valves and stopcocks in search for the characteristic noise made by leaking water.[39] The frequency of sounding varies between water undertakings. According to STC26(1981) this method is most effective when the cost of water is low and where soil characteristics are such that leaks show on the surface fairly quickly.

In general, because of its limitation in early detection of leaks, this method is giving way to others such as waste metering which use sounding only to the extent of final pinpointing of the exact location of leaks before excavation for repair takes place.

(4) District Metering

This method of continuous metering and water accounting detects leakage by monitoring any unexplained deviations in total consumption levels from flow standards established earlier for the area under investigation.[40] The application

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[38] See among others:
(1) Stenberg,1982, op.cit.
(2) Reid,1974, op.cit.
(3) Pocock,J.S.,1981,Symposium: An understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, 1981, London,PP3.1-3.21.
(4) STC26,1981, op.cit.
(5) Heim, P.,M.,1979,"Conducting Leak Detection Search," AWWA, Vol.71, No.2, PP66-69.

[39] The use of listening sticks to locate hidden leaks is a very old practice. By the Victorian era they were used systematically in conjunction with step tests (see below): Pocock(1981), op.cit.

[40] Again various references on this subject may be seen including:

(Footnote continued)

of this method involves the division of the supply area into smaller zones of about 2000 to 5000 properties. These zones are metered in such a way that cumulative supply in each is monitored over regular periods, weekly or monthly.[41] The readings from these meters are analysed in search for departures from established flow patterns.[42] Areas with unexplained 'high' levels of consumption suspected of having significant leakage are identified. Inspection sounding teams are sent to those areas in search of the presumed sources of leakage. Sounding is thus confined to suspect areas only. In fact the pinpointing of the location of leaks can be carried out using several methods including :

1- Various sounding methods
2- Leak Correlation method
3- Gas Tracer techniques
4- Other techniques such as the cut and cap method and trial excavation.

Normally methods 2 to 4 are used for the detection of more difficult leaks which escape the sounding method.

The method of district metering suffers from a number of limitations. These are summarised below:

a- District metering does not determine the position of a leak as effectively as for example the method of waste metering.
b- The method does not use night line flows in the monitoring of leakage. Therefore it does not measure leakage as accurately as waste metering.

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continued)
(1) STC26,1981, op.cit.
(2) Stenberg,1982, op.cit.

[41] These meters are usually of the integrating type used to record the total quantity of water passing through them.

[42] An example of district metering is provided by Pepper(1985). He reports that the South Shields Water Company has 240 leakage control districts in its supply area. Water accounting is computer aided, analysing data obtained from telemetered and manually read meters in these districts. Pepper,R.A.,1985,"District Metering," Four Papers on Aspects of Leakage Control, JIWES.

c- There is the inherent difficulty of establishing norms. This difficulty is compounded by the need to allow for changes of norms over time and over seasons.

(5) Waste Metering

This method is a 'fine mesh' version of district metering. It involves the setting up of areas of about 1000 to 3000 properties where each is supplied via a single pipe fitted with a flow meter capable of measuring minimum night flow rates.[43]

Leakage in a particular area is indicated when there are deviations in minimum night flow from previously recorded norms for night flow.[44] Inspection of an area suspected of having leakage can be conducted using one of two methods; (a) sounding the whole area or (b) using the method of successive valve closing (step testing), conducted at night, to determine the particular length of main from which leakage is taking place. Sounding is then confined to that particular length of main to determine the exact location of the leak where excavation is to take place.

The main advantage of this method is that it is sensitive to small changes in leakage and as such can detect leaks which other methods either fail completely to detect or detect only after a longer period of time. Another advantage of this method is that it relies less on sounding for the pinpointing of leaks.[45]

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[43]This meter may be either permanently installed (in an underground or overground position) or carried on mobile trailer and connected temporarily to the system via hydrants. Each of these methods has its technical and economic advantages and disadvantages. For details on this matter and on waste metering in general see:
(1) STC26,1981,op.cit.
(2) Stenberg,1982, op.cit.

[44]The fact that minimum night flow is not all leakage (ie includes legitimate consumption) is not as important as it might first appear. Provided there is no cause for believing that legitimate night consumption has changed over a period of time, then any recorded deviations will be probably caused by changes in leakage.

(7.5) How Much Water is Saved by Leakage Control ?

It is a widely held view that the level of leakage in a particular area is inversely related to the amount of effort being expended on the detection and repair of leakage.[46]

The Water Research Centre (in conjunction with a Technical Working Group on Waste of Water) conducted in the late 1970s an extensive experimental data collection programme to establish factual information on leakage levels in various parts of the UK associated with different control policies. Results of this programme, as reported by STC26, give "typical" values of net night flow associated with different control policies for large urban areas consisting of many districts. Reported leakage levels (net night flows) for each control policy stretched in a range covering low, medium and high levels. These figures, given in table 7.1 below, were based on national averages, it must be stressed. Thus in effect it is being postulated that, say, in an area thought to have medium leakage levels, a permanent move from a passive control policy to, for example, a policy of waste metering would result in a permanent water saving (reduced leakage) of, on average, 12 litres/property/hour. Likewise, in the same area, a permanent move from regular sounding to waste metering would result in a permanent saving of 4 litre/prop/hour. The same information contained in table 7.1 is produced graphically in diagram 7.2 below.

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[45] An example on the application of waste metering is provided by Borrows (1983). The example is from the area of Hounslow within the Thames Water Authority. Waste metering was conducted in 28 districts with the result that leakage decreased from 18.2 l/prop/hour to 6.7 l/prop/hour. Borrows,1983, op.cit.

[46] See among others :
(1) Pepper,1985, op.cit.
(2) STC26,1981, op.cit.
(3) Stenberg,1983, op.cit..
(4) Borrows,1983, op.cit.

FIGURE 7.2

"EMPIRICAL" GRAPH OF LEAKAGE

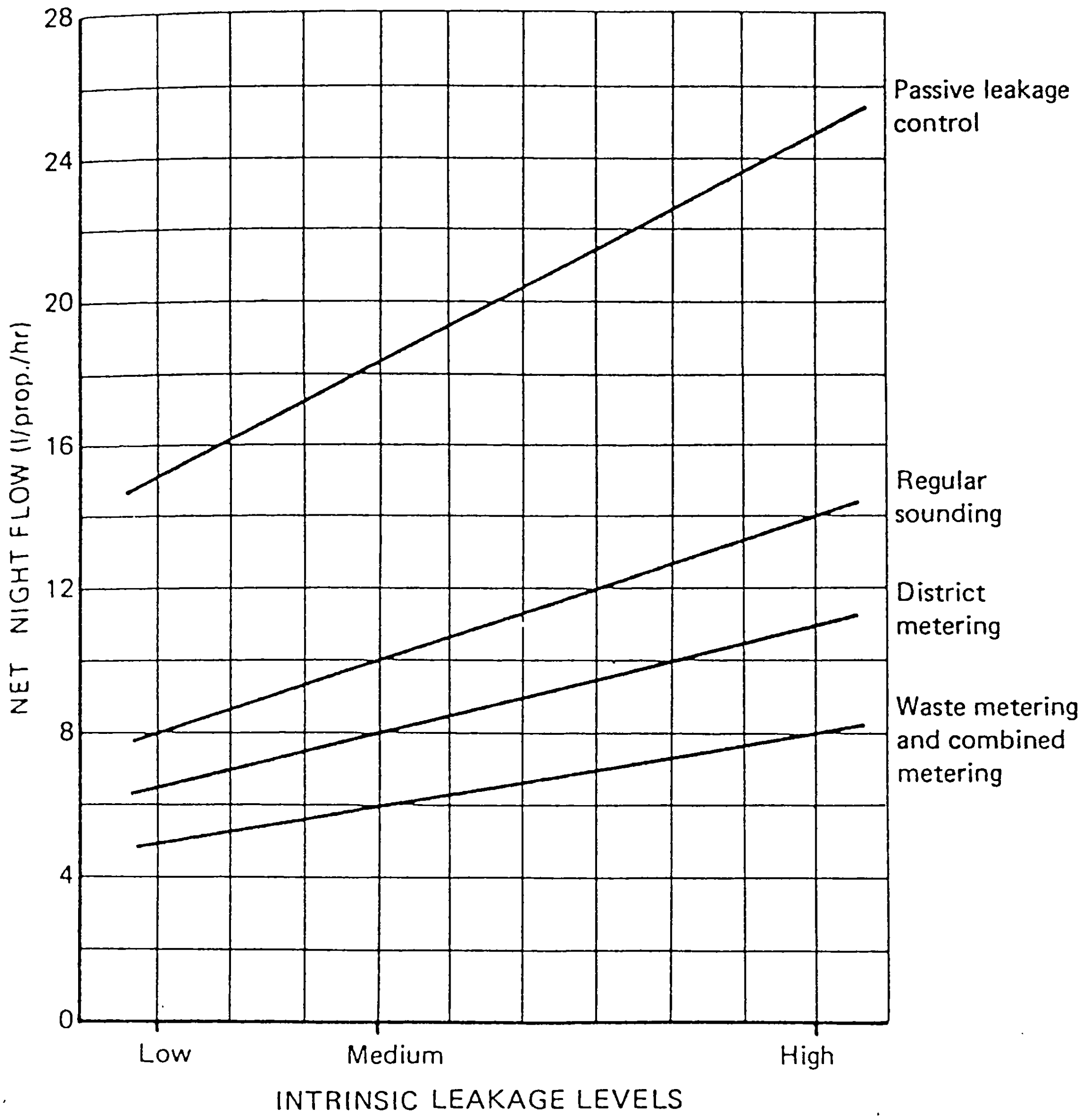

Table 7.1

TYPICAL LEAKAGE LEVELS IN LARGE URBAN AREAS (UK)
(litres/property/hour)

	LOW	MEDIUM	HIGH
(1) Passive Control	15	18	25
(2) Regular Sounding	8	10	14
(3) District Metering	6.5	8	11
(4) Waste Metering	5	6	8
(5) Combined District and Waste Metering	5	6	8

Source: STC26,1981, op.cit.

Implicit in the above proposition is the following:

1- A move from one control policy to another is a long term decision which cannot easily be reversed. Once chosen the water undertaker would need to invest in manpower training and other capital expenditure. Hence it is assumed that once a policy is chosen it will be maintained for a long period of time.[47]

2- It is being assumed that so long as the more intensive policy is maintained, the water saving, compared with the abandoned less intensive policy, would remain constant. That is, in the first example above, the initial reduction of 12 l/prop/hour would carry on into the indefinite future. This assumption is consistent with leakage levels associated with various control policies behaving over time as in diagram 7.3 or 7.4.

3- A move from one leakage control policy to another more

[47]This may prove on some occasions to be too restrictive an assumption. See discussion in section 7.9 .

intensive one would result in an initial period of falling leakage levels. Leakage levels will not continue declining forever. After the initial period they will stabilise at the level typical to the leakage control policy being practiced.[48]

4- A move from one leakage control policy to another more intensive one would lead to a short-term increase in the number of leaks requiring repair. The adoption of the more intensive control policy will help find leaks which the old policy failed to detect. But once this is done the number of leaks requiring repair revert to its long-term steady rate which is independent of the leakage control policy adopted.[49]

5- The assessment of various leakage control policies as candidates for long-term adoption in a specific area requires ex-ante prediction of the level of leakage associated with each in that area. This, no doubt, requires intimate knowledge of local conditions. Nevertheless, according to STC26(1981) some insight may be gained by making the heroic assumption that table 7.1 and diagram 7.2 above, concerning national average figures, may be applicable to individual areas. In this case knowledge of existing net night flows plus the existing leakage control policy is sufficient to place the position of the area being examined on diagram 7.2. From there an ex-ante prediction of the likely order of permanent water saving that would arise from a permanent move from the existing policy to another can easily be made using the empirical graph, figure 7.2.[50]

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[48] See STC26(1981) and Borrows(1983).
[49] STC26,1981, op.cit.
Speed, op.cit.

[50] The North West Water Authority, for example, used this "empirical graph" in their extensive study of leakage control in the North West region. Nevertheless some reservations were expressed as to the validity of applying the empirical graph to the situation in the North West. See Thomas(1981).
Thomas,D.L.,1981, Discussion of Paper by Howarth and Olen, op.cit.

FIGURE 7.3

LEAKAGE OVER TIME: GROWING LEVELS

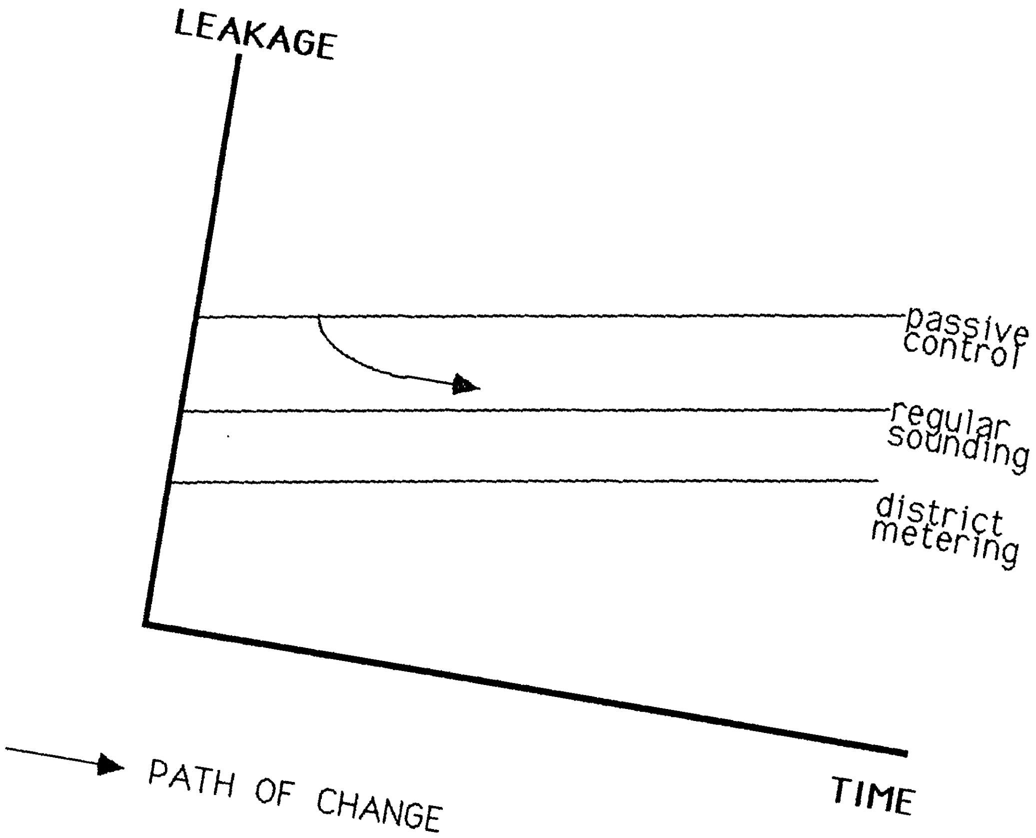
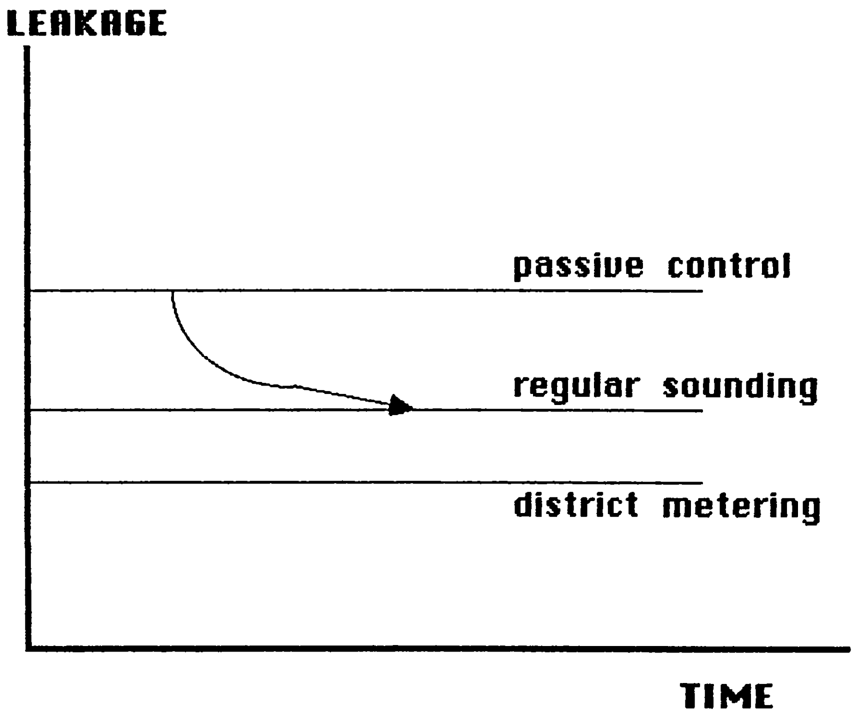


FIGURE 7.4 LEAKAGE OVER TIME STEADY LEVELS



(7.6) Cost of Leakage Control

The cost of leakage control programmes are, in general, higher the more intensive the leakage control policy (see diagram 7.5). Costs naturally depend on the location under examination as well as the time of investigation. Moreover, one expects to encounter problems of joint costs when attempting to identify the costs of a particular leakage control policy. For example waste detection inspectors are also normally expected to carry duties unrelated to leakage control.

These difficulties apart, STC26(1981) provides empirically based UK countrywide averages of the cost of the implementation of various leakage control policies. Details of these cost figures, using 1979 prices, are reproduced here in table 7.2, 7.3, and 7.4. The tables give an idea of the type and order of labour and capital expenditure involved.

Generally speaking the costs of any leakage control policy fall into two groups. In the first we put (in present worth terms) all the capital and other nonrecurring costs.[51] The present worth of these capital and any other costs will be referred to as PWC_c. The second category of costs is of the recurring annual type incorporating labour and any other running costs for maintaining the control policy in question. If these costs are constant from one year to another at a level equal to R, then their discounted sum over an infinite time horizon, PWC_o, can be simply calculated as R/i, where i is the appropriate discount rate.[52]

Total capital and running costs of implementing a particular leakage control policy can then be expressed (1) in present worth terms as $PWC = PWC_c + PWC_o$ or (2) in terms of annuitised annual costs, U, is equal to $i.PWC_c + i.R/i$ or simply $iPWC_c + R$, when the time horizon of the stream of

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[51] These costs are not, strictly speaking, nonrecurring since if a long horizon is taken they may include recurring replacement costs.

[52] Of course we need not necessarily take an infinite time horizon. Also there is nothing inherently correct about the assumption of constant running costs over time. Both of these assumptions, especially the latter, are made for simplicity and/or for not knowing better.

costs is infinite.

Table 7.2

COSTS OF COMPONENTS OF LEAKAGE CONTROL METHODS
UK and Wales (1979) (£)

Component	Cost	Notes
1- Install Waste or District Meter	1650	capital cost of installation £1400; £250 for planning & checking district
2- Record MNF	36	8 man hours including 3 at overtime
3- Perform a step test	85	13 man hours at overtime rate
4- Sound 1000 houses	150	20 properties per hour
5- Read 100 district meters	80	1 man and van reads 40 property/day
6- Repair backlog of leaks (per 1000 properties)	300	
7- Locate reported leaks with passive leakage control per 1000 properties	60	

Source : The results of experimental programme on leakage and leakage control, STC2, op.cit.

Table 7.3

INITIAL COSTS OF DIFFERENT METHODS OF LEAKAGE CONTROL

Leakage Control Method	Activity	Costs(£)	Average Cost per property
(1) Passive Control (1000 properties)	none	-	-
(2) Regular Sounding (1000 properties)	Sound 1000 prop	150	
	Repair backlog of leaks	300	
		450	0.45
(3) District Metering (3000 properties)	Install meter	1650	
	Read meter	1	
	Inspect district	450	
	Repair backlog of leaks	900	
	Read meter (iii)	1	
		3002	1.00
(4) Waste Metering (2000 properties)	Install meter(vi)	1650	
	Record MNF	36	
	Perform step test	85	
	Sound whole district (vii)	300	
	Repair backlog of leaks	600	
	Record MNF	36	
		2707	1.35
(5) Combined District and Waste Metering (4000 properties in 2 waste meter districts)	Instal meters (x)	3800	
	Read district meter (iii)	1	
	Sound whole district (vii)	600	
	Repair backlog of leaks	1200	
	Record MNF (iii)	72	
	Read district meter	1	
	Record MNF	72	
		5916	1.48

Source : The results of experimental programme on leakage and leakage control, STC2, op.cit.

Table 7.4

ANNUAL COST OF LEAKAGE CONTROL POLICIES
(£)

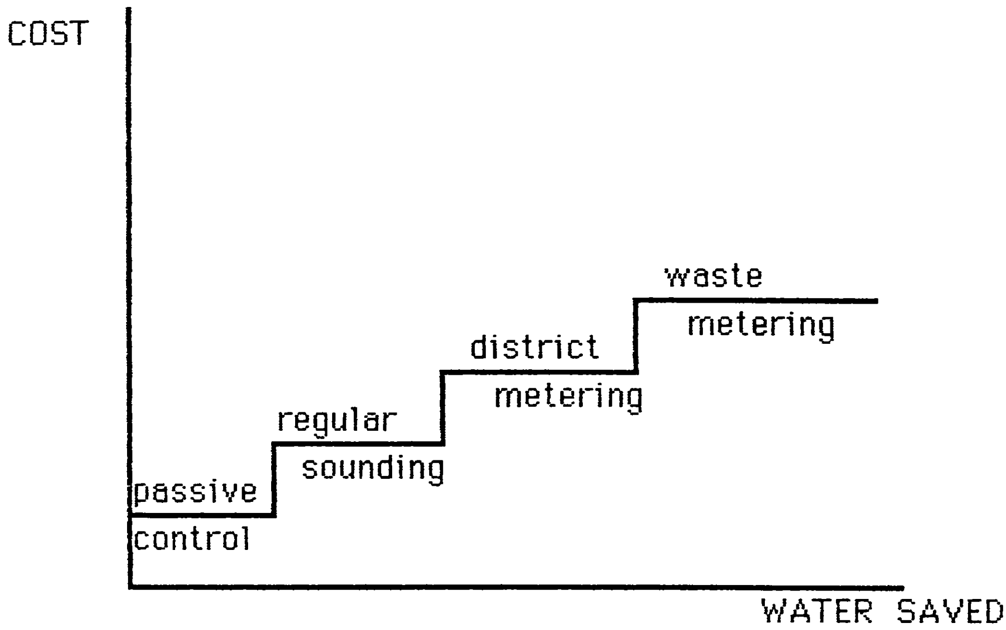
Control Method	Activity	Cost	AV. cost/prop
1- Passive Cont. (1000 prop)	Locate reported leaks	60	0.06
2- Regular Sounding (1000 prop)	Inspect prop. (i)	150	0.15
3- District Metering (3000 prop)	Read meters iv	42	
	Inspect districts (iv)	450	
		492	0.16
4- Waste Metering v (2000 prop)	Record MNF viii	216	
	Perform step test (viii)	212	
	Inspect district (ix)	488	
		916	0.46
5- Combined District and Waste metering (4000 prop)	Read district meter (iv)	42	
	Record MNF viii	432	
	Perform step test (viii)	425	
	Inspect district (ix)	975	
		1874	0.47

Source : The results of experimental programme on leakage and leakage control, STC2, op.cit.

Notes :

(i) Frequency of regular sounding is once a year.
(ii) A typical district of 3000 properties.
(iii) An initial reading as part of the procedure of district metering.
(iv) District meter reading at weekly interval.
(v) Waste districts containing 2000 properties.
(vi) it sometimes possible for one meter to monitor several districts resulting therefore in lower costs.
(vii) The initial inspection covers the whole district.
(viii) The proposed frequency is 6 a year.
(ix) These costs are at day sounding rates for 65% of the steps sounded 2.5 times a year.
(x) The cost of two waste meters and setting up these districts plus £500 for the district meter installed together with one of the waste meters.

FIGURE 7.5 COST OF LEAKAGE CONTROL



(7.7) Benefits of Leakage Control

The benefits of leakage control may be divided into two broad groups. The first incorporates all non-quantifiable benefits and the second includes all quantifiable benefits.

Non-quantifiable and difficult-to-quantify benefits

The list of benefits which are difficult to quantify and/or value in monetary terms are listed below.

(1) The conservation of water resources in general may in certain situations be a desirable objective independently of the use value of water. This may arise because of the social costs of water resource utilisation.

(2) Controlling leakage in a disciplined manner should, in the long run, achieve saving in the resources expended on repair and replacement of pipes and fittings.[53]

(3) Knowledge and information gained about the distribution system from leakage control should allow improved control of the system. one such advantage is checking the accuracy of various key meters in the system. A second advantage may arise in helping to gain information on the urgency and priority of replacement work.

(4) Leakage control may prevent damage to properties caused by leaking water. Leakage control may prevent soil movement caused by leaking water and any consequent damage to property.[54] Leakage control may also help prevent damage

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[53] see among others:
Lior,S.,K., and O'day,D.,K.,1986, "Economic Model for Leak Detection and Repair," 1986 Annual Conference Proceedings, Water Key for Life, American Water Works Association, 1986, PP1755-1761.

[54] Kingston, W.,L.,1979, "A Do-it-yourself Leak Survey Benefit-Cost Study," AWWA, Vol.71, No.2, PP70-72. Kingston in fact goes as far as suggesting that benefits arising from the prevention of damage to property, improved meter reading and better public relations may be given estimated monetary
(Footnote continued)

inside properties, given that even small leaks if left undetected may cause substantial damage.

(5) One of the indirect benefits of leakage control may in fact be better public relations. Thus control by regular sounding ensures that leaks on service pipes and other fittings inside consumer properties are detected and consumers are promptly notified.[55] Sounding inspectors would often give advice to consumers on the likely sources of leaks. Some water undertakings would repair free of charge all leaking valves and taps in consumers premises. These exercises are thought to enhance public relations and promote positive consumer attitudes towards water consumption.

(6) Reduced risk of contamination is another indirect benefit of leakage control arising, from reduced risk of backsiphonage of groundwater being sucked into a fractured pipes.

(7) Other benefits include less wear and tear of pumps, plants and other components of the distribution system as less water is handled. In addition there is the potential for benefits arising from an unknown reduction in the size of wastewater flowing to sewage treatment plants.

<u>Quantifiable Benefits</u>

Quantifiable benefits arising from reduced leakage can be expressed as the value of the stream of water saved over the time period when the leakage control policy is in force. An alternative but equivalent expression of these benefits is the saving in (1) operating costs and (2) capital costs

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:ontinued)
values. While in principle this may indeed be desirable yet we do not see how we can place objectively determined monetary value on all these benefits.

[55] Ingham,G.,L.,1981,"Water Losses and the Consumer," Symposium: An understanding of Water Losses, Proceedings, Institution of Water Engineers and Scientists, 1981, London, PP5.1-5.33.

arising from the introduction of the leakage control policy in question. These will be examined in turn.

Operating costs:

Savings in operating costs due to the introduction of a leakage control policy comprise (i) saving in energy costs,(ii) saving in chemical materials and (iii) perhaps a small (and probably insignificant) reduction in labour and maintenance costs.

As was discussed in section 4.5, the reduction in both energy and chemical costs due to a reduction in demand (leakage) will depend on (1) in which part of the system the reduction takes place and (2) when this reduction takes place. The first point arises because different parts of the system (demand zones) receive their water supplies from different sources thus incurring different marginal energy and treatment costs.[56] It is therefore particularly important to try to identify where the leakage is taking place and the appropriate marginal source of supply and the relevant pumping and chemical costs. Moreover, as we have seen in sections 4.2 and 4.5, it is also important to make proper allowance for interdependence between various parts of the system.[57] Thus a saving of 1 ML/d in demand zone X may at first appear to result in a saving of 1 ML/d from supply source A. Closer inspection, however, may reveal that because of substitution effects, the saving in fact is taking place at supply sources B, C or D (or any combination of them).[58]

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[56] In Hampshire, for example, Tottford and Abbotstone receive their supply from the underground source of Tottford at a cost of 11 £/ML. Rounhams demand zone on the other hand receives its supply from Testwood at a cost of 15.8 £/ML. See discussion in sections 3.7 and 4.5.

[57] In a system where the supply sources and demand zones are isolated this point loses its relevance.

[58] Therefore what is required is a simulation of the least cost way to operate the whole system with and without the proposed leakage control policy together with the accompanying total operating costs. In practice such simulation may not be necessary given that local expertise can usually identify the 'best' practical operating arrangements and resulting system operating costs under different demand levels.

The second point arises from a consideration of the often made assumption that the chosen leakage control policy (and hence the water saving) is of a long term nature. Operating costs in a particular location in the system may change over time. For example, as explained in section 4.5, operating costs in Southern and Central Hampshire were expected to rise with the pass of time as more expensive sources are tapped to augment exhausted cheaper sources.[59] This in turn implies that the value of savings due to the leakage control policy will increase over time, which must in turn be reflected in the evaluation of leakage control.[60] To assume that these costs are constant over time is a simplification which may distort the true magnitude of savings of operating costs.

Furthermore, even if the marginal source of supply remains the same, operating costs may rise or fall depending on changes in the relative real prices of fuel and chemicals. Of course the analysis of leakage control is concerned with real savings associated with each leakage control policy. It follows therefore that we need not concern ourself with uniform movements in the general price level (inflation or deflation) when assessing the benefits and costs of each policy.

Let PWB_o stand for the present worth of the stream[61] of benefits arising from savings in operating costs due to the introduction today of a particular leakage control policy. Let Q_t stand for the saving in water in year t arising from the introduction of the said control policy. Let m_t stand for the relevant marginal operating cost (per unit) of water in a particular location in year t. We may express PWB_o as:

$$PWB_o = \sum \frac{m_t \, Q_t}{(1+i)^t}$$

If m_t and Q_t are constant over time and n is infinite, then

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[59]See section 4.5.

[60] It must be noted that STC26,1981, does not seem to allow for a consideration of this type.

[61]Over a specified period of time.

$$PWB_o = m.Q/i$$

and we can now say that the annuitised value of benefits arising from saving in operating costs, expressed per unit of water saved is simply equal to m :

$$PWB_o \ i/Q = m$$

m will henceforth be referred to as the unit operating cost of water supply.[62]

Capacity costs

In a water supply system facing growing demand the benefits of leakage control will not be confined to savings in operating costs. A permanent reduction in overall demand due to a permanent saving arising from reduced leakage will generate savings in capacity costs.

A reduced demand forecast because of leakage control will have an impact on the future capital requirements of the water undertaking. This impact can be of two forms: (1) mitigating the need altogether for some parts of capacity additions to various components of the system, and, more likely, (2) a reduced demand forecast (as in diagram 7.6) will result not in the removal of the need altogether for the said capital projects but instead for a deferment of their commissioning dates by a period of time proportional to the amount of leakage saved. Thus in diagram 7.6, t_1 is the latest possible commissioning date of a particular capacity enhancing project given demand forecast A, ie without the leakage control policy. t_2 is the corresponding commissioning date given demand forecast B, associated with reduced leakage, (S in the diagram is the permanent saving arising from the introduction of the more active leakage control policy).

In more general terms let TDCC be the total discounted capital cost of the least cost capital expansion programme associated with demand profile A.[63] This will be the least cost capital expansion programme

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[62] So long as m and Q are constants then the unit operating costs is always equal to m regardless of the chosen time horizon of the savings.

associated with existing leakage control policy. Let TDCC' be the total discounted capital cost of the least cost capital expansion programme associated with demand forecast B, ie that associated with reduced leakage following the introduction of a more active control policy. TDCC-TDCC' would then measure the saving in capital costs, in present worth terms, arising from the introduction of the more active leakage control policy. We note here that the derivation of each of TDCC and TDCC' would require ,in principle, a cost minimisation exercise. In its simplest form the difference between TDCC and TDCC' would involve only the postponement of each and every project in the programme (including the dates of their future replacement).[64] Let us arbitrarily assume the following:

(a) The demand profile faced by the water undertaking is linear and extends into the infinite future;

(b) a constant permanent reduction in demand, due to the saving arising from better leakage control, is equal to 'd' where d happens to be the annual rate of growth of demand;[65]

(c) more crucially it is assumed that the lower demand trend associated with the lower leakage level results only in the deferment of capital projects contained within the original capital expansion programme of the higher demand trend. The introduction of the leakage control policy does not result in any reshuffling or restructuring of capital projects contained in the original capital programme.

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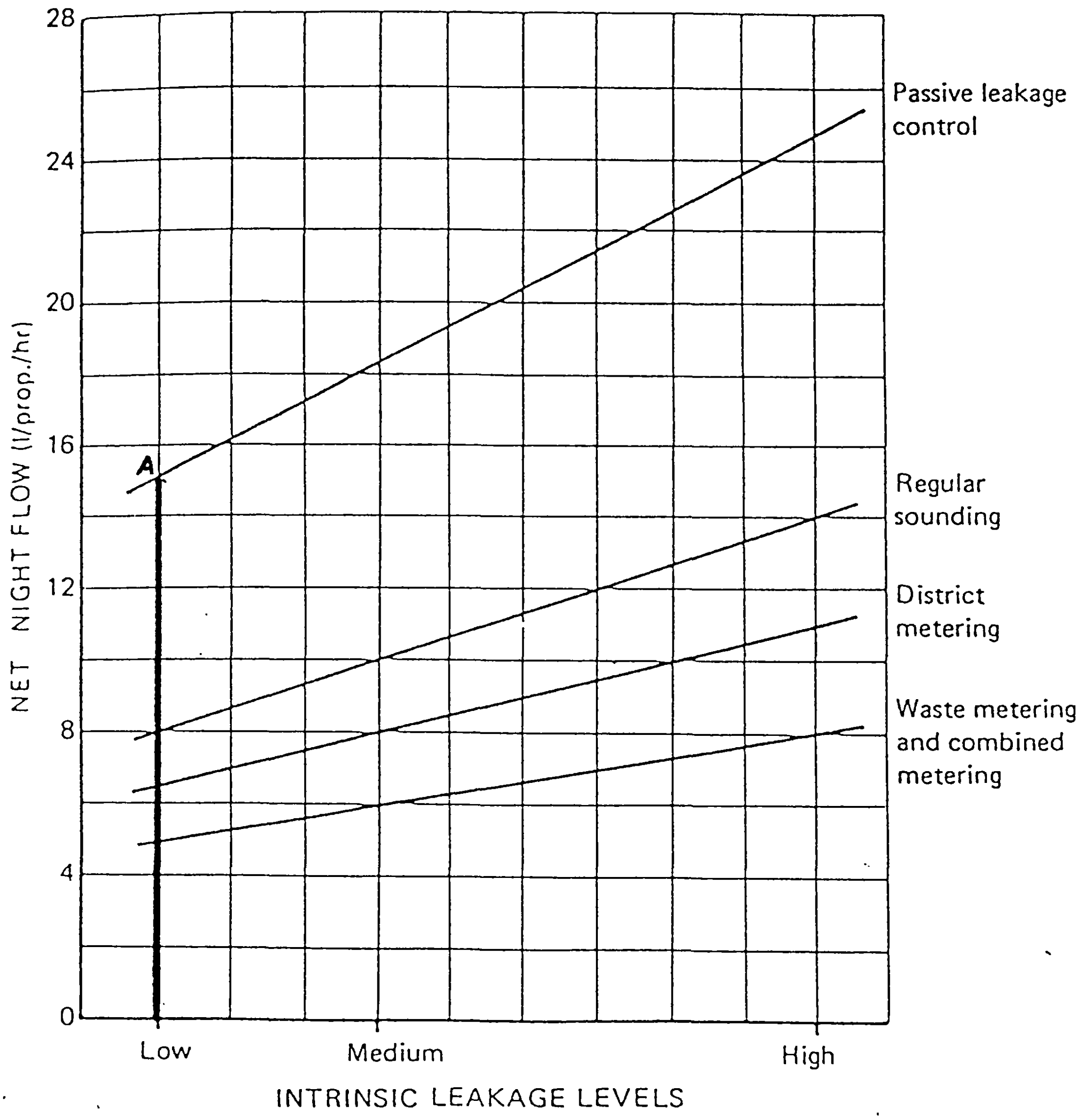
[63]The time horizon of the demand forecast will vary between water undertakings. The least cost capital expansion programme will cover various periods of time, again depending on the nature of capital planning and demand forecasting practised by the water undertaking. STC26 assumes that water undertakings face linear demand trends which extend into the infinite future. Furthermore it assumes a rather crude planning procedure based on extending five year capital expenditure programmes into the infinite future using what STC26 terms 'capacity multipliers'. See STC26,1981, op.cit.

[64]But note that capital expenditure on the replacement of existing assets does not enter into TDCC.

[65]This very strong assumption is not necessary to develop the next argument in the text. It can safely be dropped so long as assumption 'c' is maintained. Assumption 'b' simplifies the exposition however.

FIGURE 7.7

HAMPSHIRE ON THE "EMPIRICAL" GRAPH



With these assumptions the savings in capital costs, TDCC-TDCC', simplifies to :

$$\begin{aligned} PWB_c &= TDCC - TDCC' \\ &= TDCC - TDCC/(1+i) \\ &= TDCC\ i/(1+i) \end{aligned}$$

As with operating costs of water supply we may now define 'c' (unit capital cost of water supply) as the per unit annuitised value of the savings in capital costs arising from the introduction of leakage control. Then

$$'c' = \frac{TDCC\ i^2}{(1+i)\ d}$$

Moreover, given our assumptions, it is possible to prove that 'c' is, contrary to the impression given by the last equation, independent of the size of the permanent reduction in leakage. Thus consider a leakage policy which results in the deferment of the demand trend by two years instead of one year as in above (Q=2d). If, as above, we assume that the new leakage control policy has its impact in the form of 'stepping forward' by two years all capital schemes contained in the initial capital expansion programme (including the replacement of these projects at future dates), then :

$$'c' = \frac{\text{annual equivalent of TDCC less TDCC pushed ahead 2 years}}{\text{annual equivalent of decrements in demand of d,2d,2d,..}}$$

$$= A/D$$

where $A = i\,[\,TDCC - TDCC/(1+i)^2\,]$

$$= TDCC\ i^2\ (2+i)/(1+i)^2$$

and D is given by :

$$\frac{D}{(1+i)} + \frac{D}{(1+i)^2} + \frac{D}{(1+i)^3} + \ldots\ldots = \frac{d}{(1+i)} + \frac{2d}{(1+i)^2} + \frac{2d}{(1+i)^3} + \ldots\ldots$$

Therefore,

$$\frac{D}{i} = \frac{2d}{(1+i)} + \frac{2d}{(1+i)^2} + \frac{2d}{(1+i)^3} + \ldots\ldots\ldots - \frac{d}{(1+i)}$$

It follows that

$$\frac{D}{i} = \frac{2d}{i} - \frac{d}{(1+i)}$$

and $D = d(2+i)/(1+i)$

so that :

$$'c' = \frac{TDCC\ i^2\ (2+i)}{(1+i)^2} \quad \frac{(1+i)}{d(2+i)}$$

$$'c' = \frac{TDCC\ i^2}{(1+i)d}$$

which is the same thing as 'c' if it were calculated assuming a reduction in leakage equal to one year's of growth, d. The total discounted value of benefits arising from capacity costs PWB_c in this case is simply :

$$PWB_c = \frac{TDCC\ i}{(1+i)} \quad \frac{Q}{d}$$

Thus if we adopt assumptions a and c above, it seems possible to arrive at an estimate of the present worth of total capacity savings without having to reoptimise the capital expenditure programme regardless of the size of the achieved saving in leakage. In this case knowledge of the total discounted capital cost as it stands today without the new leakage control policy, TDCC, together with the annual rate of growth of demand, d, the expected constant reduction in leakage due to the new policy, Q, and the relevant unit operating cost, m, is sufficient to determine the overall saving in resources arising from the new leakage control policy.[66] In other words we have:

$$PWB = PWB_o + PWB_c$$

$$= \frac{m\,Q}{i} + \frac{TDCC\ i}{(1+i)}\ \frac{Q}{d}$$

Moreover the same information can be expressed in annuitised terms in the form of the "unit cost" of water, UC, referred to in some literature:[67]

$$UC = i\ PWB/Q$$

$$= \frac{i}{Q} \left\{ \frac{mQ}{i} + \frac{TDCC\ i}{(1+i)}\ \frac{Q}{d} \right\}$$

$$= m + c$$

Thus the annual saving (benefit) of the particular leakage control policy in question is simply Q(m+c). Q will vary according to the control policy in question whereas (m+c), under assumptions (a) and (c) above, is independent of the leakage control policy.

[66] This is the implicit position taken by STC26. STC26,1981, op.cit.

[67] See for example STC26 for example, op.cit.

(7.8) Cost-Benefit Analysis of Leakage Control

The choice of the optimal leakage control policy for a particular location can be carried out using the standard method of cost-benefit analysis. In particular cost-benefit analysis can be used to answer the following question: Which is the most economic control policy a water undertaking should choose from among the established methods of control given that the choice has to be made now and given that the chosen policy would have to be maintained for a long period of time ? Since the established methods of leakage control involve different intensities of effort and correspondingly different levels of expenditure, one may reformulate the question and enquire about the appropriate level of resource commitment on leakage control in a particular area.

Any leakage control policy has to pass two tests before it can be preferred to other policies as the most economic. To pass the first test the policy in question must be capable of producing positive net economic benefits, ie the present worth of all its benefits (PWB) must exceed the present worth of all its costs (PWC):

$$PWB > PWC$$

The same test can of course be expressed differently in terms of annuitised benefits and costs. In this form the test may simply be stated as requiring the annual benefits of the policy to exceed its annual costs,[68] ie we must have :

$$UC \times Q > U$$

$$\text{or} \qquad UC > U/Q$$

where UC is the annuitised benefits of the policy per unit of annuitised total water savings, Q, and U is the annuitised cost of implementing the leakage control policy in question.

[68]But one must always remember that the annual quantities are annuitised values of present worth sums. Except for special cases, the annual sums cannot be derived independently of the present worth sums. Working with present worth figures is always the safer of the two approaches.

Expressed this way the first test reduces to the familiar statement that a leakage control policy is acceptable if the cost of saving a unit of water (U/Q) is lower than the cost of producing that unit of water (UC).

Test one above is a necessary but not sufficient condition for a policy to be the most economic. In order to be truly the best from among other options, the chosen policy must, in the absence of any resource or financial constraints, have the greatest positive total net benefits, ie its PWB must exceed its PWC by the greatest margin compared with other policies. Test two can therefore be expressed as choosing the policy with maximum PWB-PWC, or choosing the leakage control policy with maximum (UCxQ - U). Again the second formulation is more common in the literature despite its potential pitfalls.

A Hypothetical Case Study

To demonstrate the application of the outlined CBA approach we shall use a mixture of actual and hypothetical data and a number of assumptions relating to the division of Hampshire in Southern Water Authority. The list of actual figures and additional assumptions are discussed below.

(1) Existing leakage levels in Hants

The actual level of unaccounted-for water in the division of Hants in 1984 as estimated by SWA (and reported by us in table 3.11) is equal to 112 litres/head/day. This level will be taken as the level prevailing in the initial situation in 1984/1985. Allowing for 3 litres/head/day for communal use leaves 109 litre/head/day of leakage. To transform this figure to its equivalent in terms of litre/head/hour we have followed STC26(1981) using a factor of 20 for the transformation. This produces a figure of 5.45 litre/head/hour.[69] Moreover assuming an occupancy rate of

[69] STC26 (1981) used a figure of 20 to transform hourly night flows into daily flows. 20 is used instead of 24 to allow for variation in pressure levels between day and night. STC26,1981, op,cit.

2.62 persons per property[70] we arrive at an approximate figure of 14.5 litre/property/hour for the net night flow rate. In addition it will be assumed that the Hants division has in operation in 1985 a policy of passive control.

(2) Expected level of leakage

We further assume that the 'empirical graph' of leakage levels, as in STC26, applies to the Hants division. Knowledge of the division's existing net night flow (14.5 l/p/h), associated with passive control, positions Hants on the empirical graph at point A in diagram 7.7. Next we can use the empirical graph to establish the level of leakage that we may expect to result in the long run with the application of different leakage control policies. The resulting expected leakage levels are reported in table 7.5 below. The five control policies are assumed to be mutually exclusive. Once the initial choice is made switching from one policy to another is ruled out.

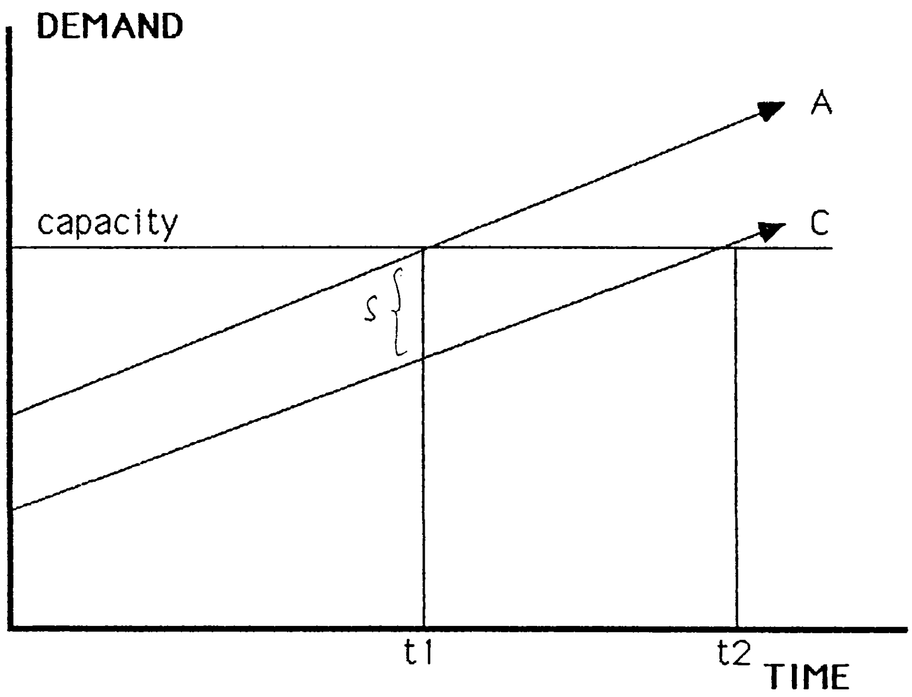
Table (7.5)

Expected Leakage levels in Hants

Leakage Control Policy	Net Night flow Litre/prop/hour
1- Passive control	14.5
2- Regular Sounding	8.0
3- District Metering	6.2
4- Waste Metering	4.5
5- Combined Metering	4.5

[70] Water Facts,1986, op.cit.

FIGURE 7.6 SAVING IN CAPACITY COST



(3) Cost of Various leakage Control Policies

It is again assumed that national average figures for the initial and annual costs of leakage control as reported by STC26, are applicable in the Hants division.[71] These figures are reproduced in table 7.6. The figures in table 7.6 are in terms of annuitised costs of leakage control where the annuity factor (at 5% discount rate) for a duration of 26 years is 7%.[72] STC26 cost figures have been increased by 60% to transform them from 1979 to 1985 base year.[73]

Table (7.6)

Total Cost of Leakage Control £/property

Leakage Control Pol	PW[i] £/property	Annuitised £/property/year
1- Passive Control	1.37	0.1
2- Regular Sounding	4.14	0.2
3- District Metering	5.25	0.37
4- Waste Metering	12.67	0.89
5- Combined Metering	13.11	0.92

(i) Present Worth of 26 years of annual costs.

71 After adjustment of the base year of prices from 1979 to 1985 .

72 We are annuitising over 26 years because the demand forecast for Hants and the corresponding capital expenditure programme cover 26 years.

73 The 60% rise is based on the Price Index for All Manfuactured Products and the Index of Wages and Salaries per Unit of Output for the whole Economy. Both indices are found in Economic Trends, Central Statistics Office, HMSO, Jan. 1986.

(4) Benefits of Leakage Control

It will be assumed that the quantifiable benefits of the various leakage control policies can be derived from knowledge of (1) unit operating cost, (2) unit capital cost of central and distribution investment and (3) the amount of permanent water saving achieved by the introduction of each of the established leakage control policies.

Estimates of the relevant unit capital cost of central investment and of distribution investment which are consistent with those required for the calculation of the benefits of leakage control have already been derived in chapter 4.[74] These costs are reproduced in table 7.7 below.

Table (7.7)
Unit capital and Operating Cost of Water in Hants (1985 prices) P/M^3

1- Unit capital cost of central investment	2.74 (a)
2- Unit capital cost of distribution investment	5.53 (b)
3- Unit operating cost	1.58 (c)
TOTAL	9.85

a- Based on A-C demand forecasts. See table 4.18 and section 4.3.
b- See section 4.4.
c- see section 4.5 and table 4.23.

Using tables 7.7 and 7.5 we can derive the quantifiable benefits of the various leakage control policies, both in annual terms and in the form of the present value the stream of gross benefits stretching for 26 years. These benefits are stated in table 7.8 for each of the policies of regular sounding, district metering, waste metering, and combined metering. Passive control is the existing policy and thus it forms the benchmark from which savings of the other four policies are measured.

[74] It was argued in chapter 4 that a reduced demand forecast in Hampshire would merely result in delaying the commissioning dates of capital projects associated with the original (higher) demand forecast. Therefore condition c above is met.

Table (7.8)

Gross Benefits of Leakage Control policies in Hants

(PW 26 years

Policy	Water Saved l/prop/hour	Saving M^3/prop/year	Unit Cost of Water p/M^3	Saving £/prop/year	Saving £/prop
1-Regular Sounding	6.5	47.45	9.85	4.67	66.71
2-District Metering	8.3	60.59	9.85	5.96	85.14
3-Waste Metering	10.0	73.00	9.85	7.19	102.71
4-Combined Metering	10	73.00	9.85	7.19	102.71

(a) The hourly water saving is converted into daily saving using a factor of 20.

<u>Net benefits of the various leakage control policies</u>

In table 7.9 we report the net benefits of moving from the existing policy of passive control to any one of the four alternative options. Net benefits are expressed as (1) a total Present worth sum of recurring annual benefits for a period of 26 years and (2) as a constant annual figure of recurring net benefits.[75]

[75] But note that actual spread of net annual benefits is radically different from the annuitised figures in the table.

Table (7.9)

Net Benefits of Switching to Alternative Leakage Control Policies

Policy	Annual Net Benefit £/prop	PW of 26 years of Benefits £/prop
1- Regular Sounding	4.47	62.57
2- District Metering	5.59	79.89
3- Waste Metering	6.30	90.04
4- Combined Metering	6.27	89.60

Thus given our assumptions the highest quantifiable net benefits can be achieved by switching now from the existing policy of passive control to a policy of waste metering. With approximately 190000 properties in Hants a switch to waste metering, given the validity of our assumptions[76], should achieve a net saving over a 26 year period of time equal to £17.1 million. A switch from passive control to any other policy[77] will also produce net quantifiable benefits, though by an amount less then in the case of waste metering. In this instance waste metering produces 43% more benefits than regular sounding and 13% more benefits than district metering. Indeed the margin of difference in net benefits between waste metering and the other less intensive leakage control policies is so pronounced that for all reasonable sensitivity tests waste metering should remain the optimal policy.

Generally speaking one can state that the lower is the money value of the product the more a water authority can tolerate waste, and the less should it be inclined to spend

[76] We would not claim that any of our assumptions truly reflect the situation in Hants.

[77] A move to another policy less expensive to implement then waste metering might be forced upon the water undertaking because of financial (eg capital) and manpower constraints.

on waste control. This proposition can readily be tested with reference to the Hants example sketched above. Suppose that system capacity, central and distribution, in Hampshire is so abundant that a reduction in leakage will lead to no foreseeable savings in capital costs. In this case the benefits of leakage control will be confined to savings in operating costs. The present worth of benefits of leakage control, the present worth of costs of leakage control, and the present worth of net benefits of the various leakage control policies are stated in table 7.10 below.

Table (7.10)

Net Benefits of leakage Control in Hants with Abundant System Capacity

Policy	PW of Benefits £/prop	PW of Costs £/prop	PW of Net Benefits £/prop
1-Regular Sounding	10.71	4.14	6.57
2-District Metering	13.67	5.25	8.42
3-Waste Metering	16.47	12.67	3.80
4-Combined Metering	16.47	13.11	3.36

The results of table 7.10 indicate that when water is cheaper the optimal policy becomes district metering involving less intensive leakage control and less expenditure compared with waste metering.

The actual choice of control policy should of course also take into account (1) non-quantifiable benefits and (2) financial constraints on expenditure on leakage control.[78] As

[78] Borrows(1983) and Rees(1981) report that in the UK and Wales leakage control activity has in general been hit by attempts to cut immediate financial costs. The financial short-term gains of reducing manpower expenditure (cutting leakage control) are immediately obvious, whereas the financial long-term gains of leakage control are not, nor are the financial long-term penalties of cutting leakage control.
(Footnote continued)

far as the first point is concerned a general description of the non-quantifiable benefits of each policy could be attached to the quantifiable net benefit calculation. Inferior options can be dropped outright, whereas the more difficult choice between conflicting options can perhaps be left to the subjective judgment of the concerned water managers. The usual sensitivity analysis for the more crucial parameters such as the unit cost of water and the unit cost of leakage control should routinely be carried out.

(7.9) Concluding Remarks

(1) The Choice of leakage control policy depends crucially on the host of implicit and explicit assumptions made in the various stages of the analysis. Some of these assumptions have to be handled with care. For example the outcome of any leakage control analysis crucially depends on the assumptions made regarding both existing levels of leakage and those expected to prevail with alternative policies. Statistics relating to the latter are difficult to obtain. The use of national averages is likely to create major errors. Locally based estimates must be preferred since they ought to reduce the potential for mistakes. The same applies to the costs of implementing the individual leakage control policies.

(2) A vital statistic for the calculation of the benefits of leakage control is the 'value' of water saved. Under a specific set of assumptions, spelled out above, the total benefits of leakage control may indeed be estimated using the 'unit cost' of water in conjunction with the volume of expected water savings. Thus it may seem that one can remove the need to reoptimise the capital expenditure programme according to the overall demand level associated with each leakage control policy. The unit cost of water may in this situation simply be derived by stepping back the capital expansion programme for one year and relating it to the

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continued)

Barrows,1983,op.cit.

Rees,1981, op.cit.

annual rate of growth of demand.[79] The pitfall of this approach lies in the existence of situations where this procedure is not correct, namely when fundamental changes in the capital expansion programme may result following the leakage reduction. It is always safer to explicitly reconsider the capital expansion programme associated with different demand levels resulting from different control policies. This latter approach has the further advantage that it easily permits variable savings in operating costs over time.

3- In line with the above we would further advocate the use of explicit present value calculations in preference to shortcut annuitised value calculations (using the unit cost of water and the unit cost of leakage control). The traditional approach of present value calculations has the advantage of forcing one to explicitly take into account the time dimension of the stream of benefits and costs involved in the assessment of various leakage control policies. Thus it focuses attention on the expected size of water savings over time and their potential variation from one year to the next.[80] The present value calculation, moreover, focuses attention on the starting and end dates of the potential benefits and costs. In this way it makes sure that the implicit time horizon for benefits matches the time horizon of all costs. Moreover the traditional PW approach has the advantage of focusing attention on the fact that the benefits and costs of leakage control need not be synchronized over time, the costs being concentrated in the early years of the adoption of the policy while the benefits tend to be spread over time. Finally the present value approach focuses more attention on the significance of the chosen discount rate and indeed readily allows for the shadow pricing of some resources should there be any need to do so.

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[79] This may then be termed long-run marginal cost of water.

[80] In other words it readily permits that water savings may not be constant over time.

4- As with traditional literature on leakage control the procedure outlined in this chapter examined the choice from among five rigid leakage control options. Each option represented the adoption of a particular leakage control policy which is supposed to prevail without change from the time of its inception into the infinite future. As at least one writer has pointed out[81] this procedure is far too restrictive as far as the range of options it examines. Broadening the list of options takes on particular importance in view of the fact that because of marked indivisibility in the capital components of a water supply system, the unit value of water may vary sharply over time, giving cheap water when capacity is plentiful and expensive water when capacity is nearing exhaustion. This implies that we ought to examine more than five leakage control options. In particular we ought to examine two extensions to the five standard options; (1) We add the options of starting each of the five policies at different dates. Comparing the five standard options (starting each of the leakage policies now) may produce passive control as the most economic choice if we make the comparison now when water is cheap. Yet if we also at the same time compare, besides the five standard options, the option of introducing leakage control not now but say in ten years time, it may well turn out that the overall most economic option is district metering initiated in ten years time. (2) The second extention comes in the form of increasing the number of potential choices by the inclusion of any feasible and realistic combination of the five leakage control policies. For example we could add the possibility of starting with limited regular sounding for a number of years (achieved by overtime pay and limited training of additional personnel) to be followed when capacity nears exhaustion by a more intensive leakage control such as waste metering. The listing of all possible options, including those that permit some partial or complete switching between the five traditional approaches would not alter in any way the general

[81] Twigg,R.,D.,H.,1986, "Developments in Policy for Leakage Control," M MacDonald & Partners, Consulting Engineers, mimeo.

approach to the calculation of net present values of any of the single options.

(5) Finally one must also draw attention to the fact that at the operational level it is not economic to repair every and each single leak no matter how small it is. The economic resources needed for excavation, repair and reinstatement may well exceed the expected benefits of repairing that particular leak. This will especially be the case if the amount of leaking water is small. Therefore, long run decisions apart, inspectors face the additional question of whether to repair or not detected individual leaks. The method of cost-benefit analysis can also be used to evaluate such a decision.[82] The general approach is the same as before, though the analysis is probably on a smaller scale. Here we need a quick method of establishing the amount of leaking water, its current and expected future level if left unattended. Engineering methods may be used to estimate this information.

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[82] Stenberg(1982), op.cit.

CHAPTER 8

SUMMARY AND CONCLUSION

The central and recurrent theme in this study has been the concept of marginal cost. The meaning of this concept, in its various shades and colours, as well as its estimation in the capital intensive water supply industry, occupied a substantial part of the study. It was, however, quickly established that the definition as well as the estimation methodology of marginal cost could not be logically separated from the purpose for which marginal cost was being defined and estimated.

We have established and illustrated in the course of the analysis two broad areas where marginal cost may be used as instruments of decision making in the water supply industry: (a) for the design and establishment of tariff structures and levels, and (b) to be used as a shortcut for full cost benefit analysis for the evaluation of investment in demand management policies.

Thus chapter 2 dealt with both issues in theoretical terms whereas chapters 4, 5 and 6 examined empirically and in depth the relationship between the measurement of marginal cost and the establishement of prices. Chapter 7 dealt empirically with the relationship between marginal cost and cost-benefit analysis.

The rest of this chapter will follow the (a) and (b) distinction above.

(8.1) Pricing and Marginal Cost

The simple, static and one-dimensional textbook concept of marginal cost is too simple to be useful for the practical design of a tariff structure based on marginal cost. Time must properly be allowed into the analysis. Other factors must also be explicitly introduced such as the fact of indivisibilities and inherited plant as well as spatial and seasonal variations in cost conditions.

We have distinguished two general approaches to the analysis of marginal costs and optimal price/output determination :

(a) Cost-minimisation Models, and (b) Benefit-less-cost maximisation models.

Cost minimisation models

One practical approach to the definition and estimation of marginal cost for the purpose of tariff setting is based on cost minimisation models. In this tradition we have examined, in chapter 2, several approaches to and measures of marginal cost. These included definitions arising from the Turvey 1971 model (section 2.4) as well as several variants presented by the World Bank (section 2.7) and Hanke 1977 (section 2.8). None of these models admit a demand function. Capital and running costs expected to be incurred over a planning horizon are minimised subject to the constraint of meeting specific point forecasts of demand, forecasts which may or may not allow for the impact of price on demand. Prices are thus fixed according to the marginal costs derived from these cost minimisation models. Any possible feedback via the pricing rule to demand forecasts and the cost minimisation exercise and back to marginal cost estimates is ruled out at any one round of estimation.

Even within this restrictive formulation of the problem the definition of marginal cost is not straightforward. The definition is influenced by many factors. Among these are the following:

1- The nature of the constraint on price variability over time, which in turn is related to institutional and financial constraints as well as the resource allocation costs of price variability, if any. This is also linked to the frequency of price revisions. The distinction between 'pure' marginal cost as in Turvey's present worth incremental system cost (section 2.4.1) and, among others, Hanke's simple average marginal cost (section 2.8) reflects this constraint. The empirical estimates of long-run marginal cost of 'central' capacity and of distribution capacity for Hampshire (sections 4.3 and 4.4) assumed an absolute constraint on price variability.

2- The extent of invisibility present in the capital cost functions of the various capital components of the industry. The speed of technical progress may also be an important consideration. We have illustrated (sections 3.5 and 5.10) that indivisibility may be the result of economic considerations rather than technical ones.

3- The nature of the capital planning procedure and the desired emphasis on the investment signal price.

4- Temporal variation in pure marginal cost is compounded by spatial variations, as well as variations in the marginal cost of supplying distinct classes of consumers. Thus the definition of marginal cost will depend on the extent of the desired averaging out of differences in each of these dimensions. The same of course can be said about seasonal variations in marginal cost.

Moreover the definition and estimation methodology of marginal cost specific to the water supply industry is influenced, in addition to the above, by the following considerations:

1- The distinction between investment in the central part of the system and investment in the distribution part. More generally the estimation method of marginal cost corresponding to different components of the water supply system depends on the nature of the capital planning procedure for that part of the system, specifically on how capital expenditure is driven by demand forecasts. This was illustrated in the case of Hampshire in chapter 4 where the long-run marginal capital cost of the central and distribution parts of the water supply system were estimated.

2- Demand characteristics take on special importance in the estimation of marginal cost in the water supply industry. In particular peak characteristics, both for overall demand and for specific demand classes, in thepresent and in the future, are very important. This was illustrated (in chapter 4) by deriving separate estimates of marginal capital cost for the class of domestic demand and for leakage (considered as a separate class of 'demand'). The former has higher than average peak and the latter has below average peak ratio.

3- Uncertainty in the estimates of marginal cost arises from:
   (a) uncertainty surounding the demand forecasts (section 3.4);
   (b) uncertainty associated with capital expenditure planning including uncertainty about yields of water supply sources, future prices and technical progress;

4- Multidimensionality of marginal cost in the water supply industry is caused by :
   (a) Variation in the duration and magnitude of the postulated incremental output change being costed, though in some cases marginal cost estimates may turn out to be insensitive to the postulated magnitude of

the change, as in the case of the incremental capital cost of the central system in Hampshire. This was discussed and illustrated in chapter 4 ;

(b) Spatial variation in marginal cost due to differences in cost conditions both from the point of view of operating costs and capital cost. This was demonstrated empirically in the case of Hampshire in chapter 4. The analysis must also carefully take into consideration system interdependence and changes in the marginal source of supply over time (chapter 4).

(c) Seasonal variation in marginal cost, both short-run and long-run, is caused by peaks in demand. Such differences, in the case of operating costs, were illustrated empirically for Hampshire (section 4.5).

Despite these complications, it is our belief, as demonstrated in chapters 3 and 4, that estimates of marginal cost, both short-run and long-run, are possible with a reasonable level of accuracy and confidence. This is subject to the condition that the framework of analysis must be defined very carefully, including: (a) the objective of marginal cost estimation and, (b) the required degree of price uniformity over time, space and consumer classes. Without such a framework it is difficult, if not impossible, to state what is the meaning of marginal cost.

Benefit Less Cost maximisation Models

Once we admit a demand function with a non-zero price elasticity into the analysis, marginal cost would then need to be defined broadly so as to include the price that would ration inherited capacity among consumers. That is, marginal cost, would sometimes measure opportunity costs in terms of the willingness to pay of the marginal excluded consumer, or marginal user opportunity cost.

Conducting the analysis in dynamic terms we introduced a specific dynamic demand function (sections 5.2 to 5.9) incorporating the effect over time on demand of prices, growing population and an income effect. Using appropriately defined cost functions (sections 5.10 and 5.11) we simulated profiles of prices (marginal costs), output and capacity levels over a chosen planning horizon. These output, prices and capacity profiles maximise an appropriately defined net social benefit function (section 5.13) over the planning horizon.

Besides the common assumptions neutralising income distribution effects and excluding externalities, we have first assumed, in two of our models, the absence of any constraint on price variability over time (chapters 5 and 6). We however have excluded the possibility of spatial and seasonal variation in prices. The first of the said models (in chapter 5) ruled out the staging of capacity addition while the second (chapter 6) admitted such a possibility. The third and final model (section 6.6) assumed an absolute constraint on price variability over time together with no staged capacity addition.

A summary of what we believe to be the major conclusions that may be drawn from the analysis of these models is now appropriate. The major conclusion of the models is that the price variable should not immediately be dismissed as a legitimate candidate to be included in the list of policy options that may be advanced as alternatives to capacity expansion in a general package of demand management. In general we conclude that when there is spare capacity in the system the optimal policy would be to lower price so as to encourage consumption since so long as price is not lower then short-run marginal cost then the benefits of extra consumption would exceed the corresponding costs. As demand nears full capacity the water enterprise should not be inhibited from raising prices so as to check demand and thereby delay the construction of new capacity. This policy would generate more benefits the greater is the discount rate, the greater is the degree of economies of scale in the capacity cost function(s), the higher is the rate of growth of absolute demand, the higher is the leakage ratio, the higher are peak ratios, and the longer is the duration of expected growth. All of these conclusions were illustrated empirically in the case of Hampshire in chapters 5 and 6. The required price changes to implement such a policy, and hence their cost, are found to be lower the higher (in absolute terms) is the demand price elasticity.

Moreover the numerical simulations of the optimal prices, output and capacity additions (chapter 5 and 6) may be used to draw the following conclusions specific to the case of Hampshire:

(1) For a discount rate of 5% the variable-price\single-stage capacity addition model achieved only a small improvement in total net benefits over the constant-price\single-stage capacity addition model, of a value equal to £0.190m. This represents an improvement in terms of incremental net benefits of less than 1%. It must be stressed however that the comparison is based on an optimal constant price and not just any other constant price such as one based on average cost. Indeed section 6.6 indicated that the total net benefit function, for the case of constant price model, falls sharply as the constant price increase. Bearing in mind the limitations of our models, discussed in section 6.7, the above conclusion indicates that for a discount rate of 5% and single-stage capacity addition, Hampshire water managers may consider (once meters are installed) adopting a constant (optimal) price in preference to an optimal profile of variable prices over a 25 year horizon. This would be correct provided that forgone net total benefits, in comparison with the variable price scenario, are outweighed by the likely resource and administrative costs and inconvenience associated with price variability.

(2)The simulations have also indicated the degree of sensitivity of the relative performance of the models to the chosen discount rate. Comparison of the single-stage\variable-price model with the single-stage\constant-price model using a discount rate of 10% indicates that the former outperforms the latter to the tune of £1.344m, or by 20.8% when expressed in incremental terms (section 6.7). Of course the improvement is only achieved at the expense of considerable price variability (figure 6.4). A water authority in this situation has the unenviable task of having to carefully weigh the extra costs against the extra benefits of price variability. The difficulty lies in estimating the former.

(3) The simulation also gives some insights as to the advantages of staging capacity, treatment capacity in the case of Hampshire. This may be carried out using results of single-stage\variable-price model and the multi-stage\variable-price model. The results, grouped in section 6.7, for a discount rate of 5%, shows that staging achieves an

improvement equal to £0.199m, or less than 1% in incremental terms. It must be stressed however that the comparison is being conducted assuming optimal variable prices in both cases. The comparison would need to be reexamined if it were to be conducted under the assumption of charging a constant price, which might be equal to zero as is the case at present in the UK. The point to note is that staging cannot be examined only in terms of savings in capacity construction costs and therefore independently of consumer benefits and of prices.

(4)Moreover the simulation has shown that, at least in the case of Hampshire and given the assumed price elasticity of demand and a variable price model, staging would introduce a considerable degree of price variability in comparison with the single-stage capacity addition solutions. This in turn would reduce the size of the gains in net total benefits by an amount equal to the likely resource and administrative costs of price variability.

(8.2) Marginal Cost and CBA

A general package of demand management policies in the water supply industry should normally include the options of investing in policies such as leakage control, introduction of water saving technology, regulation and education.

Investment in each of these may normally be evaluated using the traditional tool of cost-benefit analysis. Thus the choice of the most economic leakage control policy in a particular water supply area would entail conducting a CBA exercise for each of the policy options, evaluating for each the discounted total of its benefits in comparison with the discounted sum of its costs over a uniform time horizon. The most economic policy, in the absence of financial constraints, is that one with the highest margin between its benefits and costs.

Using the case of leakage control we have also demonstrated that there is an equivalent assessment method to the CBA approach just outlined. In particular we have shown that each leakage control policy option may be evaluated by comparing its unit cost of implementation with the unit value of water saved by it. The value of water saved may be found according to the 'unit cost' or long-run marginal cost of water. Thus for each candidate leakage control policy we compare the long-run marginal cost of water with the unit cost of implementing this particular leakage control policy. The most economic policy is the one having the highest margin between its unit cost of implementation and the long-run marginal cost of water.

However we have also argued that the CBA approach may in general be said the safer of the two approaches. We have argued that (1) the long-run marginal cost of water may not be independent of the amount of water saved and hence of the leakage control policy option itself and, (2) the use of explicit present value calculations, as in CBA, has the advantage of bringing to the forefront the time dimension of benefits and costs, ensuring uniformity of the time span of benefits and costs and highlighting the critical role of the discount rate.

Appendix 1

SECTION 1 : STANDARD INTEGRALS

In this section we show the derivation of three integrals which we encounter in the derivation of the total net benefit functions of the various models. The integrals are:

1- $\int te^{-it}\,dt$

2- $\int t^2e^{-it}\,dt$

3- $\int \frac{P'+SDT-MC}{K+Rt+Dt}\,e^{-it}\,dt$

where all terms are constants with the exception of t, the time variable.

Derivation of the first integral $I = \int te^{-it}\,dt$

let $u = t$ and $v = \frac{-1}{i}e^{-it}$

thus $\frac{du}{dt} = 1$ and $\frac{dv}{dt} = e^{-it}$

integrating by parts we have :

$$I = uv - \int \frac{du}{dt}v\,dt$$

$$I = t\,\frac{-1}{i}e^{-it} - \int \frac{-1}{i}e^{-it}\,dt$$

$$= \frac{-1}{i}e^{-it} + \frac{1}{i}\int e^{-it}dt$$

$$= -e^{-it}\left(\frac{t}{i}+\frac{1}{i^2}\right)$$

$$= -e^{-it}\left(\frac{it+1}{i^2}\right)$$

$$= -e^{-it}\left(\frac{1+it}{i^2}\right)$$

thus in general we have :

$$\int_a^b te^{-it}\,dt = \left[\frac{-(1+it)}{i^2}\,e^{-it}\right]_a^b \qquad .....(1)$$

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The second integral is : $\int t^2e^{-it}dt$

let $u = t^2$ and thus $\frac{du}{dt} = 2t$

let $v = \frac{-1}{i}e^{-it}$ and thus $\frac{dv}{dt} = e^{-it}$

(2)

Integrating by parts we get :

$$I = \frac{-t^2}{i} e^{-it} - \int 2t\left(\frac{-1}{i}\right) e^{-it}\, dt$$
$$= \frac{-t^2}{i}e^{-it} + \frac{2}{i} \int te^{-it}\, dt$$

and using result (1) above we get :

$$I = \frac{-t^2}{i}e^{-it} + \frac{2}{i}\left[\frac{-(1+it)}{i^2}\right]e^{-it}$$

and when simplified and adding the limits of the integral

$$I = \left[(-e^{-it})\frac{i^2t^2+2it+2}{i^3}\right]_a^b \qquad ....(2)$$

Derivation of the third integral is the most involved of all;

$$I = \int \frac{P'+SDt-MC}{K+Rt+Dt}\, e^{-it}dt$$

$$= \int \frac{P'-MC}{K+Rt+Dt}e^{-it}dt + \int \frac{SDt}{K+Rt+Dt}e^{-it}dt$$

(first term) (second term)

We first examine the first term :

$\int \frac{P'-MC}{K+Rt+Dt}e^{-it}dt$ can be written as $\frac{P'-MC}{R+D} \int \frac{e^{-it}}{\frac{K}{R+D}+t}\, dt$

$$= \frac{P'-MC}{R+D}\, e^{i\frac{K}{R+D}} \int \frac{e^{-i\left(\frac{K}{R+D}+t\right)}}{\frac{K}{R+D}+t}\, dt$$

Using the standard integral of the form :

$$\int \frac{e^{ax}}{x}\, dx = \mathrm{Log}_e x + \frac{ax}{1!} + \frac{a^2x^2}{2.2!} + \frac{a^3x^3}{3.3!} + - - - - -$$

Then by analogy we have $x = \left[\frac{K}{R+D} + t\right]$ and $a = -i$, thus :

$$\int_a^b \frac{P'-MC}{K+Rt+Dt}e^{-it}dt = \frac{P'-MC}{R+D}\, e^{\frac{iK}{R+D}}\, [\mathrm{Sum}(t)]_a^b$$

where $[\mathrm{Sum}(t)]_a^b = \left[\log_e\left(\frac{K}{R+D}+t\right) - \frac{i\left(\frac{K}{R+D}+t\right)}{1!} + \frac{i^2\left(\frac{K}{R+D}+t\right)^2}{2.2!} - \frac{i^3\left(\frac{K}{R+D}+t\right)^3}{3.3!} + \text{-------}\right]_a^b$

Now we examine the second term :

$$\int \frac{SDt}{K+(R+D)t}e^{-it}dt = SD\int \frac{t}{K+(R+D)t}e^{-it}dt$$

the integration of $\int \frac{t}{K+(R+D)t}e^{-it}dt$ can be carried out by parts :

let $u = \frac{t}{K+(R+D)t}$ so that $\frac{du}{dt} = \frac{K}{[K+(R+D)t]^2}$

let $v = -\frac{1}{i}e^{-it}dt$ so that $\frac{dv}{du} = e^{-it}$

and $\int_a^b \frac{t}{K+(R+D)t}e^{-it}dt = uv - \int_a^b \frac{du}{dt}v\, dt$

$$= \left[\frac{t}{K+(R+D)t}\right]\left[\frac{-1}{i}e^{-it}\right] - \int_a^b \left[\frac{K}{[K+(R+D)t]^2}\right]\left[\frac{-1}{i}e^{-it}\right] dt$$

$$= \left[\left(\frac{t}{K+(R+D)t}\right)\left(-\frac{1}{i}e^{-it}\right)\right]_a^b + \frac{k}{i}\int_a^b \frac{e^{-it}}{[K+(R+D)t]^2}dt$$

Now we take the integral $\int_a^b \frac{e^{-it}}{[K+(R+D)t]^2}dt$

again using the parts technique :

let $u = e^{-it}$ so that $\frac{du}{dt} = -e^{-it}$

let $v = \frac{-1}{R+D}\frac{1}{K+(R+D)t}$ so that $\frac{dv}{dt} = \frac{1}{[K+(R+D)t]^2}$

$$\int_a^b \frac{e^{-it}}{[K+(R+D)t]^2}dt = uv\int_a^b \frac{du}{dt}v\, dt$$

$$= \left[\frac{-e^{-it}}{(R+D)[K+(R+D)t]}\right]_a^b - \int_a^b \frac{(-1)(-ie^{-it})}{(R+D)[K+(R+D)t]}\, dt$$

(4)

$$=\left[\frac{-e^{-it}}{(R+D)[K+(R+D)t]}\right]_a^b-\frac{i}{(R+D)}\int_a^b\frac{e^{-it}}{K+(R+D)}\,dt$$

as before the solution of the last integral involves the series sum(t), so that the last expression may be simplified as follows :

$$=\left[\frac{-e^{-it}}{(R+D)[K+(R+D)t]}\right]_a^b-\frac{i}{(R+D)}\frac{e^{\frac{iK}{R+D}}}{(R+D)}\int_a^b\frac{e^{-i\left(\frac{K}{R+D}+t\right)}}{\frac{K}{R+D}+t}\,dt$$

$$=\left[\frac{-e^{-it}}{(R+D)[K+(R+D)t]}\right]_a^b-\frac{i}{(R+D)}\frac{e^{\frac{iK}{R+D}}}{(R+D)}[\mathrm{sum}(t)]_a^b$$

the analytic solution of the expression we called term two, ie $\int_a^b \frac{SDt}{K+(R+D)t}e^{-it}dt$ can now be written as :

$$SD\left\{\left(\frac{t}{K+(R+D)t}\right)\left(\frac{-e^{-it}}{i}\right)+\frac{K}{i}\left[\frac{-e^{-it}}{(R+4)[K+(R+D)t]}-\frac{i}{R+D}\frac{e^{i\frac{K}{R+D}}}{R+D}(\mathrm{Sum}(t))\right]\right\}$$

$$=\left[\frac{-SD}{i}\frac{te^{-it}}{K+(R+D)t}\right]_a^b-\left[\frac{SDK}{i(R+D)}\frac{e^{-it}}{[K+(R+D)t]}\right]_a^b-\left[\frac{SDK}{(R+D)^2}e^{i\frac{K}{R+D}}\{\mathrm{Sum}(t)\}\right]_a^b$$

Now combining the integration expressions for term(1) and term(2) we get :

$$\int_a^b\frac{P'-MC+SDt}{K+Rt+Dt}\,dt=$$

$$\frac{iP'-MC}{R+D}e^{i\frac{K}{R+D}}[\mathrm{Sum}(t)]_a^b-\left[\frac{-SD}{i}\frac{te^{-it}}{K+(R+D)t}\right]_a^b-\left[\frac{SDK}{i(R+D)}\frac{e^{-it}}{[K+(R+D)t]}\right]_a^b-\left[\frac{SDK}{(R+D)^2}e^{i\frac{K}{R+D}}\{\mathrm{Sum}(t)\}\right]_a^b$$

where $[\mathrm{Sum}(t)]_a^b=\left[\log_e\left(\frac{K}{R+D}+t\right)-\frac{i\left(\frac{K}{R+D}+t\right)}{1!}+\frac{i^2\left(\frac{K}{R+D}+t\right)^2}{2.2!}-\frac{i^3\left(\frac{K}{R+D}+t\right)^3}{3.3!}+\text{-------}\right]_a^b$

SECTION 2 : THE SINGLE STAGE MODEL

Case I : Q' = 0

$$TNB = \int_0^T \left[(1-f)(P' + SDt)K1 - (1-f)\frac{P' + SDt - MC}{K + R + Dt}\frac{K1^2}{2} - MCK1 \right] e^{-it}\, dt$$

$$+ \int_T^H \left[(1-f)(P' + SDT)K1 - (1-f)\frac{P' + SDT - MC}{K + RT + DT}\frac{K1^2}{2} - MCK1 \right] e^{-it}\, d$$

written alternatively as TNB = (1) + (2)

$$(1) = \int_0^T (1-f)P'K1e^{-it}\, dt + \int_0^T (1-f)SDK1te^{-it}\, dt - (1-f)\frac{K1^2}{2}\int_0^T \frac{P' + SDt - MC}{K + Rt + Dt} e^{-it}\, dt$$
$$- MCK1 \int_0^T e^{-it} dt$$
$$= [(1-f)P'K1 - MCK1] \int_0^T e^{-it}\, dt + (1-f)SDK1 \int_0^T te^{-it} dt$$
$$- (1-f)\frac{K1^2}{2}\int_0^T \frac{P' + SDt - MC}{K + Rt + Dt} e^{-it}\, dt$$

$$(2) = \left[(1-f)(P' + SDT)K1 - (1-f)\frac{P' + SDT - MC}{K + RT + DT}\frac{K1^2}{2} - MCK1 \right] \left[\frac{-e^{-it}}{i} \right]_T^H$$

All remaining integrals in the expressions above have already been derived in section 1 of this appendix.

SINGLE STAGE MODEL CASE II : $t' < TQ$ and $TQ = \dfrac{Q' - (K - K1)}{R + D}$

The TNB function in this case has four integrals which may be expressed as follows :

$$TNB = \int_0^{t'} (\ )dt + \int_{t'}^{TQ} (\ )dt + \int_{TQ}^{T} (\ )dt + \int_T^H (\ )dt$$

corresponding output K1 K + Rt + Dt K1 + Q' K1 + Q'

Next we examine each of these four integrals in turn :

(6)

$$(1) = \int_0^{t'}\left[(1-f)(P'+SDt)K1 - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\frac{K1^2}{2} - MCK1\right]e^{-it}$$

$$= [(1-f)P'K1 - MCK1]\left[\frac{-e^{-it}}{i}\right]_0^{t'} + (1-f)SDK1\int_0^{t'} te^{-it}dt$$
$$-(1-f)\frac{K1^2}{2}\int_0^{t'}\frac{P'+SDt-MC}{K+Rt+Dt}e^{-it}dt$$

again expressions for the last two integrals have already been derived .

$$(2) = \int_{t'}^{TQ}\left[(1-f)(P'+SDt)(K+Rt+Dt) - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\frac{(K+Rt+Dt)^2}{2} - MC(K+Rt+Dt)\right]e^{-it}dt$$

with the appropriate manipulation we get :

$$(2) = \frac{1}{2}K[(1-f)P' - (1-f)MC]\left[\frac{-e^{-it}}{i}\right] + \frac{1}{2}(P'R + P'D + SDK) - \frac{1}{2}(1+f)(MCR + MCD)\int_{t'}^{TQ} te^{-it}dt$$
$$+\frac{1}{2}(1-f)(SDR + SD^2)\int_{t'}^{TQ} t^2e^{-it}dt$$

again expressions in the last two integrals have already been derived.

$$(3) = \int_{TQ}^{T}\left[(1-f)(P'+SDt)(K1+Q') - (1-f)\frac{P'+SDt-MC}{K+Rt+Dt}\frac{(K1+Q')^2}{2} - MC(K1+Q')\right]e^{-it}dt$$

$$= [(1-f)(K1+Q') - MC(K1+Q')]\left[\frac{-e^{-it}}{i}\right]_{TQ}^{T} + (1-f)SD(K1+Q')\int_{TQ}^{T} te^{-it}dt$$

$$-(1-f)\frac{(K1+Q')^2}{2}\int_{TQ}^{T}\frac{P'+SDt-MC}{K+Rt+Dt}e^{-it}dt$$

again expressions for the last two integrals have already been derived.

$$(4) = \int_T^H \left[(1-f)(P' + SDT)(K1 + Q') - (1-f)\frac{P' + SDT - MC}{K + RT + DT}\frac{(K1 + Q')^2}{2} - MC(K1 + Q') \right] e^{-it} dt$$

$$= \left[(1-f)(P' + SDT)(K1 + Q') - (1-f)\frac{P' + SDT - MC}{K + RT + DT}\frac{(K1 + Q')^2}{2} - MC(K1 + Q') \right] \left[\frac{-e^{-it}}{i} \right]_T^H$$

By grouping expressions (1), (2), (3), and (4) we derive equation 5.16 of the text in chapter 5.

SINGLE STAGE MODEL : Case III $t' \geq TQ$

$$TNB = \int_0^{t'} (\,)dt + \int_{t'}^{T} (\,)dt + \int_T^H (\,)dt$$

output K1 K1 + Q' K1 + Q'

each of these integrals is examined next :

$$(1) = \int_0^{t'} \left[(1-f)(P' + SDt)K1 - (1-f)\frac{P' + SDt - MC}{K + Rt + Dt}\frac{K1^2}{2} - MCK1 \right] e^{-it}$$

the derivation of (1) has already been carried out.

$$(2) = \int_{t'}^{T} \left[(1-f)(P' + SDt)(K1 + Q') - (1-f)\frac{P' + SDt - MC}{K + Rt + Dt}\frac{(K1 + Q')^2}{2} - MC(K1 + Q') \right] e^{-it} dt$$

likewise the derivation of (2) has been carried out.

$$(3) = \int_T^H \left[(1-f)(P' + SDT)(K1 + Q') - (1-f)\frac{P' + SDT - MC}{K + RT + DT}\frac{(K1 + Q')^2}{2} - MC(K1 + Q') \right] e^{-it} dt$$

again this integral has already been derived. Grouping the relevant terms we get equation 5.17 in the text.

SECTION 3 : MULTI-STAGE MODEL

CASE "A" : $t_j \geq TQ_j$ where $TQ_j = \dfrac{\bar{Q}_j - (K - K1)}{R + D}$

$$R_j = \int_{t_j}^{t_j+1}\left[(1-f)(P' + SDt)(K1 + \bar{Q}_j) - (1-f)\frac{P' + SDt - MC}{K + Rt + Dt}\,\frac{(K1 + \bar{Q}_j)^2}{2} - MC(K1 + \bar{Q}_j)\right]e^{-it}dt$$

This simplifies to :

$$\left[(1-f)P'(K1 + \bar{Q}_j) - MC(K1 + \bar{Q}_j)\right]\left[\frac{-e^{-it}}{i}\right]_{t_j}^{t_j+1} + (1-f)(K1 + \bar{Q}_j)SD\int_{t_j}^{t_j+1} te^{-it}dt$$

$$-(1-f)\frac{(K1 + \bar{Q}_j)^2}{2}\int_{t_j}^{t_j+1}\frac{P' + SDt - MC}{K + Rt + Dt}e^{-it}dt$$

All remaining integrals have already been derived. This gives eqaution 6.5 after adding the capital cost function.

CASE "B" : $t_j \leq TQ_j \leq t_{j+1}$

$$R_j = \int_{t_j}^{TQ_j}(\;)dt + \int_{TQ_j}^{t_j+1}(\;)dt$$

output $\quad K + Rt + Dt \quad K1 + \bar{Q}_j$

R_j in this case simplifies to :

$$R_j = \frac{k}{2}[(1-f)P' - (1+f)MC]\left[\frac{-e^{-it}}{i}\right]_{t_j}^{TQ_j}$$

$$+\left[\frac{1}{2}(1-f)(P'R + P'D + SDK) - \frac{1}{2}(1+f)(MCR + MCD)\right]\int_{t_j}^{TQ_j} te^{-it}dt$$

$$+\frac{1}{f}(1-f)(SDR + SD^2)\int_{t_j}^{TQ_j} t^2e^{-it}dt$$

$$+\left[(1-f)P')(K1 + \bar{Q}_j) - MC(K1 + \bar{Q}_j)\right]\left[\frac{-e^{-it}}{i}\right]_{TQ_j}^{t_j+1}$$

$$+(1-f)(K1 + \bar{Q}_j)SD\int_{TQ_j}^{t_j+1} t^2e^{-it}dt$$

(9)

$$-(1-f)\frac{(K1+\bar{Q}_j)^2}{2}\int_{TQ_j}^{t_{j+1}}\frac{P'+SDt-MC}{K+Rt+Dt}\,e^{-it}dt$$

This gives equation 6.6 after adding the capital cost function.

CASE C : $t_{j+1} \le TQ_j$

$R_j = \int_{t_j}^{t_{j+1}} (\ )dt$ where output is $K + Rt + Dt$

$$R_j = \frac{1}{2}K[(1-f)P' - (1+f)MC]\left[\frac{-e^{-it}}{i}\right]_{t_j}^{t_{j+1}}$$

$$+\left[\frac{1}{2}(1-f)(P'R + P'D + SDK) - \frac{1}{2}(1+f)(MCR + MCD)\right]\int_{t_j}^{t_{j+1}} te^{-it}dt$$

$$+\frac{1}{2}(1-f)(SDR + SD^2)\int_{t_j}^{t_{j+1}} t^2e^{-it}dt$$

This gives equation 6.7 of the text after adding the capital cost function.

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ABSTRACT

The subject matter of this thesis is the definition, measurement and use of marginal cost as a tool of analysis to assist the process of decision-making in the water supply industry. Demand management is viewed in broad terms to include the establishment of an optimal structure and level of prices and investment in optimal capacity as well as investment in demand-restraining measures such as leakage detection and control. The study examines the definition of marginal cost as a benchmark for price setting. It provides empirical estimates of the various components of marginal cost of water supply in the Hampshire area, part of the Southern Water Authority. These estimates assume an exogenously determined level of demand and therefore exclude any possible direct interaction between the pricing and investment decisions. Departing from this tradition the study also examines a number of models where, under specific assumptions, optimal prices, output and capacity levels over a chosen planning horizon are simultaneously determined. This allows for direct interaction between the pricing and investment decisions. The study simulates optimal paths of prices, output and capacity expansion in the Hampshire area. This is carried out under various assumptions, one of which admits the potential of staging capacity expansion in order to take advantage of economies of scale in the capital cost function. An analysis of leakage detection and control as a demand management tool is presented in the final part of the study. The purpose of this analysis is to investigate how leakage detection and control may be conducted using either cost-benefit analysis or an appropriately defined tool of marginal cost.