

**TRANSPORT PROPERTIES IN
HIGH TEMPERATURE GASES**

Thesis for the Degree of Doctor of Philosophy

by

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Table of Contents

Chapter 1 : Introduction

- 1.0 Historical Background
- 1.1 Arc Literature
- 1.2 Scope of the Study

Chapter 2 : Measurement of the Transport Properties of Partially Ionised Gases

- 2.0 The Nature of the Problem
- 2.1 Measurement Techniques on Arc Columns
- 2.2 The Arc System
- 2.3 Arc Measurements
- 2.4 Measurement of Axial Field Strength
- 2.5 Measurement of Radiation Energy Transport
 - 2.5.1 Measurement Details. Radiation
- 2.6 Measurement of Radial Temperature Distributions
- 2.7 The Measurement of Thermal Conductivity
 - 2.7.1 The Qualifying Assumptions
- 2.8 A First Evaluation of Thermal Conductivity. Radiation Neglected.
- 2.9 Evaluation of Thermal Conductivity. Non Negligible Radiation

Chapter 3 : The Transport of Radiation Through a Gas

- 3.0 Theoretical Background
- 3.1 The Generalised Spectral Distribution Equations
- 3.2 Radiation for an Optically Thick Gas
- 3.3 The Absorption Coefficient Equations
 - 3.3.1 Absorption of Line Radiation
 - 3.3.2 Absorption of Continuum Radiation
- 3.4 Radiation Intensity Calculation
 - 3.4.1 Bound-Bound (Line Radiation)
 - 3.4.2 Bremsstrahlung and Recombination Radiation
- 3.5 Radiation Transfer in the Arc Column
- 3.6 Comparison of Measured and Calculated Radiation Intensity

Continued.....

Chapter 4 : Measurement of the Viscosity of High Temperature Plasmas

4.0 Theory

4.1 Measurements

Chapter 5 : The Calculation of Plasma Transport Coefficients

5.0 Introduction

5.1 Summary of Methods of Calculation

5.1.1 The Transport Coefficients

5.2 An Alternative Approach

5.3 Some Possible Simplifications to the Calculations

5.4 The Diffusion Coefficient

5.5 The Coefficient of Viscosity

5.6 Thermal Conductivity

5.7 Calculation of Collision Frequencies

5.8 The Viscosity Coefficient at Ionisation (Monatomic Gas)

5.9 The Diffusion Coefficient at Ionisation (Monatomic Gas)

5.10 The Thermal Conductivity of Ionised Monatomic Gases

5.11 Diatomic Gas at Dissociation and Ionisation

5.11.1 Viscosity at Ionisation

5.11.2 Diffusion Coefficients at Ionisation

5.11.3 Thermal Conductivity of Diatomic Gases

Chapter 6 : The Experimental Determination of Electron Collision Cross Sections in a High Density Plasma

6.0 Introduction

6.1 Theoretical Background

6.1.1 Refractive Index

6.2 Measurement of Absorption Coefficient

6.2.1 Techniques of Measurement

6.2.2 Direct Absorption Measurement

6.2.3 A Nulling Absorption Measuring Set

6.2.4 Noise Reduction

Continued.....

- 6.3 Experimental Results
- 6.4 Experimental Difficulties
 - 6.4.1 Optical Absorption and Re-emission
 - 6.4.2 Poor Selectivity of the Detector System

Chapter 7 : Discussion of Experimental and Theoretical Results

- 7.0 The Measurement of Thermal Conductivity
- 7.1 The Calculation of Radiation Intensities
 - 7.1.1 Comparison of Results for Thermal Radiation Calculations
- 7.2 The Measurement of Viscosity
 - 7.2.1 Possible Sources of Error
- 7.3 Theoretical Predictions of Transport Properties
 - 7.3.1 Frozen Composition Plasmas
 - 7.3.2 Plasma Properties
- 7.4 The Experimental Determination of Collision Cross Sections
 - 7.4.1 Comparison of Calculated and Measured Absorption

Chapter 8 : Conclusions and Recommendations

Appendices.

Bibliography

Appendices

- (I) Measurement of Temperature Distribution in the Arc.
- (II) Details of the Thermal Conductivity Calculation
 - Table 1 Energy Balance N_2
 - Table 2 Simpson
 - Table 3 2 Temp + Measured Radiation
- (III) Abel Integral Inversion
- (IV) The Meaning of Thermal Equilibrium
- (V) Transport Calculations "Dobbs" Method
- (VI) Collision Cross Section $e - N$
- (VII) Collision Cross Section $e - N^+$
- (VIII) Detail of Collision Frequency Measurement from Absorption Coefficients. Method of Calculation

List of Figures

1. The Arc Column. Exploded View
2. Photograph of Complete Column
3. Argon. Temperature in the Free Burning Arc
4. Argon. Temperature in the Constricted Arc.
5. Schematic Radiation Measurement
6. Radiometer Schematic
7. Measured Specific Radiation Intensity as a Function of Arc Temperature. Argon
8. Nitrogen. Measured Specific Radiation Intensity as a Function of Temperature
9. Argon. Radial Distribution of Arc Temperature
10. Nitrogen. Radial Distribution of Arc Temperature
11. Argon. Measured Radial Temperature Gradient
12. Nitrogen. Measured Radial Temperature Gradient
13. Argon. Arc Column Field Strength.
14. Nitrogen. Arc Column Field Strength
15. Measured Argon Thermal Conductivity. No Radiation
16. Measured Nitrogen Thermal Conductivity. No Radiation
17. Argon. Electrical Conductivity
18. Nitrogen. Electrical Conductivity
19. Argon. Measured Thermal Conductivity. Including Measured Radiation.
20. Measured Nitrogen Thermal Conductivity. Measured Radiation Included.
21. Argon. Measured Thermal Conductivity. Demonstration of the Importance of Thermal Radiation
22. Nitrogen. Measured Thermal Conductivity. Demonstration of the Importance of Thermal Radiation.
23. Argon. Calculated Total Radiation Intensity
24. Nitrogen. Calculated Total Radiation Intensity
25. Argon. Comparison of Measured and Calculated Radiation Intensities.
26. Nitrogen. Comparison of Measured and Calculated Radiation Intensities
27. Velocity Measuring Schematic
28. Argon. Measured Velocity Profiles
29. Nitrogen. Measured Velocity Profiles
30. Argon. Measured Viscosity
31. Nitrogen. Measured Viscosity
32. Argon. "Frozen" Composition Calculated Thermal Conductivity

Continued.....

33. Argon. "Frozen" Composition. Calculated Diffusion Coefficient
34. Argon. "Frozen" Composition. Calculated Viscosity
35. Argon Plasma. Calculated Diffusion Coefficient
36. Argon Plasma. Calculated Viscosity
37. Argon Plasma. Calculated Thermal Conductivity
38. Nitrogen Plasma. Calculated Viscosity
39. Nitrogen. Calculated Thermal Conductivity
40. Argon and Nitrogen. Refractivity
41. Schematic Direct Optical Absorption Measurement
42. Photograph of Direct Absorption Measurement Apparatus
43. Block Schematic. Synchronous Detector/Amplifier
44. "Nulling" (Differential) Absorption Measurement
45. Diode Laser Pulse Generator
46. Photograph of Ga As Laser Unit
47. Argon Measured Integrated Absorption. $h = 0.5$ cm
48. Argon Measured Integrated Absorption. $h = 1.0$ cm
49. Nitrogen. Measured Integrated Absorption. $h = 0.5$ cm
50. Nitrogen. Measured Integrated Absorption $h = 1.0$ cm
51. Argon. Measured Absorption Coefficients $h = 0.5$ cm
52. Argon. Measured Absorption Coefficients $h = 1.0$ cm
53. Nitrogen. Measured Absorption Coefficients. $h = 0.5$ cm
54. Nitrogen. Measured Absorption Coefficients. $h = 1.0$ cm
55. Argon. Plasma Jet Temperature Profiles
56. Nitrogen. Plasma Jet Temperature Profiles
57. Argon. Calculated and Measured Optical Absorption Coefficients
58. Nitrogen. Calculated and Measured Optical Absorption Coefficients
59. Argon Collision Frequencies Electron Atom
60. Nitrogen. Collision Frequencies Electron Atom.

Figures in Appendices

- AI Spectrographic Temperature Measuring System. Schematic
- A2 Section Through Cylindrical Plasma at Right Angles to Major Axis

List of Principal Symbols

Where duplication occurs, the context is sufficient to identify the symbol.

A_{mn}	Transition Probability
B	Planck's Black Body function
c	Velocity of electromagnetic radiation in free space
D	Diffusion coefficient
e	Electronic charge
E	Electrical Field Strength
f_{mn}	Oscillator strength
f_c	Collision frequency
g	Gaunt factor and statistical weight
h	Planck's constant
I	Electrical current
j	Reaction energy
K	Thermal Conductivity
k	Boltzmann's constant
m_n	Mass of n^{th} species
N_n	Number density of n^{th} species
N_0	Loschmidt's number
P	Pressure
Q_{rad}	Electromagnetic radiation density
r	Radial co-ordinate
T	Temperature
v	Linear velocity
W	Maxwellian velocity
x	Linear co-ordinate
Z	Nuclear charge
α	Absorption coefficient
ϵ_0	Dielectric constant of free space
λ	Electromagnetic wavelength
μ	Micro (10^{-6}) and refractive index
ν	Frequency
η	Absolute viscosity
σ	Electrical conductivity,
ω	Angular velocity

Chapter 1

Introduction

1.0 Historical Background

Gases are normally accepted to be good electrical insulators although it has been known for many years that the application of a sufficiently high electric field can cause breakdown of the gas after which a current will flow. Various types of electrical discharge in gases are known. However, at a sufficiently high current, discharges are obtained having certain important common characteristics which allow them to be classified as "Electric Arc" discharges.

The electric arc is first recorded as having been observed by Sir Humphrey Davy in about 1808. This was an electric arc burning between carbon rods and its main interest for Davy at that time was the intense light which was produced.

By 1815, the high temperatures generated in the arc were well known and the device was used for melting materials which were otherwise difficult to melt. From the middle of the 19th century systematic studies were made on metallic arcs both at high pressures and in vacuum. Further systematic physical examinations of the arc

established the complex nature of the various processes which constituted this type of discharge, The discovery, towards the end of the last century, that the arc potential drop consisted of three separate parts - the cathode voltage drop, the anode voltage drop and the column voltage marked the beginning of the detailed theoretical study of arcs.

The first extensive study of the carbon arc in air was made by Mrs Ayrton in 1902.⁽¹⁾ Since that time numerous other workers have published arc theories and studies of the processes occurring in electric arcs. Detailed accounts of much of this work are to be found in the monograph entitled "The Electric Arc" by J.M. Somerville⁽²⁾ and the book "Gaseous Conductors" by J.D. Cobine.⁽³⁾

In more recent times the arc has been developed into a powerful light source and a source of very high temperature, ionised gas for scientific research, particularly in the aerospace industries for simulating the re-entry conditions of space vehicles and for the fabrication industries. In the purely scientific field there is a continuing interest in the measurement of the electrical, optical and thermal properties of arcs as a means of obtaining data on the transport properties of gases at high temperatures.

1.1 Arc Literature

It is not the purpose of this study to produce a bibliography of arc literature although during subsequent sections of the work a considerable volume of references

appear. As valuable surveys, the books of Somerville and Cobine have already been mentioned. A few additional titles, however, are worthy of note in that they produce wide ranging general information on electric arcs. A description of arc developments up to the year 1923 together with a literature survey is contained in the article by A.Hagenbach, entitled "The Electric Arc in "A Handbook of Radiology"⁽⁴⁾. A similar work by Engel and Steenbeck⁽⁵⁾ entitled "Electrical Discharges in Gases" brings the work further up to date. The classical discourse on the high pressure arc column is that published by Elenbaas in 1951⁽⁶⁾; this work being the foundation of many recent developments in arc column theories. Even a brief survey of arc and plasma literature cannot be considered complete without mentioning the very detailed work of Finkelnburg and Maecker published in 1956⁽⁷⁾. In addition to this, a very large volume of modern studies is available from the Office of the American Space Agency, these works deal with every aspect of arcs and thermal plasmas^{(8),(9),(10)}

1.2 The Scope of this Study

This work is concerned primarily with the properties of the gases heated by an arc to temperatures ranging from 8000-20000°K as opposed to the mechanism of the arc itself. In particular much effort is devoted to the measurement and calculation of the transport properties (that is the means whereby energy is transported in the arc). This

has involved a detailed experimental and theoretical study of transport processes in high temperature gases whose properties are not frozen, that is gases which contain and continually generate reacting species. Various methods of estimating the species temperature and concentration in high temperature gases have been examined and following from this, measurements have been made of the thermal conductivity and the viscosity of gases in argon and nitrogen arcs. Because the measured and calculated values of the transport coefficients, both in this study and in many others, diverge very markedly at high temperatures, the reasons for these discrepancies have been examined and a considerable amount of practical and theoretical work has been carried out on the transport of energy by electro magnetic radiation from a hot gas.

Because of the considerable difficulty normally encountered in making theoretical calculations of the values of the transport coefficients, a study has been made with the object of simplifying the calculations so that recourse is not required to intractable, non linear equations containing numerical coefficients whose values may be doubtful. The results of this work are compared with rigorous theoretical calculations of other workers and with measured values obtained in this study and other published works.

It should be emphasised that the work reported in

this thesis is aimed, not at producing further detailed, rigorous theories of arc and plasma physics, but rather of gaining an understanding of the difficulties encountered in obtaining realistic information on high temperature gases in engineering applications.

Chapter 2

Measurement of The Transport Properties of Partially Ionised Gases

2.0 The Nature of the Problem

During the past 10 to 15 years, wide ranging studies have been made of the transport properties of gaseous plasmas at atmospheric pressure in the temperature range from 1000-50000°K. These studies have been motivated by the rapid developments in one of a number of fields such as:

- (i) Electric arcs as sources of light and heat
- (ii) Arc extinction in powerful switches
- (iii) Arc switching devices such as plasmatrons..
- (iv) The protection of space vehicles moving at high speeds in the dense atmosphere of the earth and other planets.
- (v) Plasma power conversion, that is plasma magnetohydrodynamic converters (MHD) and plasma rocket engines.

In principle the kinetic theory allows the calculation of any property of a plasma of pre-determined composition when the collision cross sections of particles in the plasma are known. These cross sections can be obtained from experiments with electron, atomic and molecular beams at

low pressure, and the properties of gases and their transport coefficients at low temperatures are well established. Here for the purpose of measurement, low temperature and pressure is taken to mean $p \approx 1 \text{ mm Hg}$, $T \approx 300^\circ \text{K}$.

However, at high temperatures, wide ranging extrapolations are required from this low temperature data and difficulties have been encountered in applying these data to dense high temperature plasmas.

A major difficulty arises in using the kinetic theory when dissociation and ionisation takes place, for neither the observed thermal conductivity and viscosity nor the approximations to the diffusion coefficients agree with the results for a non-reacting plasma or a fully ionised plasma both of whose properties can be accurately evaluated^{(11), (12)}. Other workers have recognised this limitation and modified the expressions to obtain agreement at the fully ionised limit⁽¹³⁾. Again, other workers^{(14), (15)} have ignored these difficulties and made approximations to the cross sections to obtain engineering predictions. In such cases accuracy was not considered important.

As already pointed out, a weakness of these methods is the long range extrapolations which are required to obtain the collision data. In principle, reliable results can be obtained by means of statistical mechanics and kinetic theory using precise inter-molecular force

information. If, at high temperatures, information concerning the repulsive part of the interaction energy curve is available, then satisfactory calculations can be made⁽¹⁶⁾. It should be noted that at high temperatures only the repulsive energy is required since the high temperature properties are relatively insensitive to attraction⁽¹¹⁾. As already pointed out, information on atomic and molecular interactions can be obtained from the scattering of molecular beams and this can be used to calculate the transport properties of the gases at high temperatures.

The temperature range in which the calculated gas properties are valid is determined by the range of validity of the potential energy of interaction obtained from scattering experiments and this is determined by the geometry of the scattering experiments and the kinetic energy of the beam particles.

Amadur and Mason⁽¹¹⁾ report on the calculation of gas properties with experimental results of interaction potential energies in the range 0.2 to 1.5ev. These two authors calculate gas properties over the range 1000°K to 15000°K; this upper limit of temperature corresponding to the temperature at which kT was equal to the maximum interaction energy then known experimentally.

These calculations have also been extended to molecular

gases and gas mixtures. However, errors in the results at high temperatures when dissociation and ionisation occur are ever present and very obvious when compared with the results obtained even from crude experiments.

The calculations made by Amadur and Mason on the transport properties of the rare gases can be compared with calculated properties of the same gases made by Cann⁽⁸⁾ who has taken into account ionisation effects and whose calculations seek, unsuccessfully, to simplify the very tedious algebra required for the evaluation of multicomponent gas transport properties using the method of Hirschfelder, Curtis and Bird et al⁽¹⁷⁾.

The calculations on molecular gases with frozen composition made by Amadur and Mason and by Mason and Weissman⁽¹⁸⁾ who include dissociation in hydrogen can be compared with the theoretical calculations of the transport properties of nitrogen by K Penski⁽¹⁹⁾ and Krinberg⁽¹⁴⁾ who examine in addition hydrogen, argon and oxygen, and also, the measured and calculated values of Burhorn⁽²⁰⁾ for nitrogen. The wide discrepancies between the calculated and measured values show the magnitude of the problems encountered.

These wide variations occur in well characterised gases. When the same problem is considered for more

THE ARC COLUMN

FIG 1

Exploded View (not to scale)

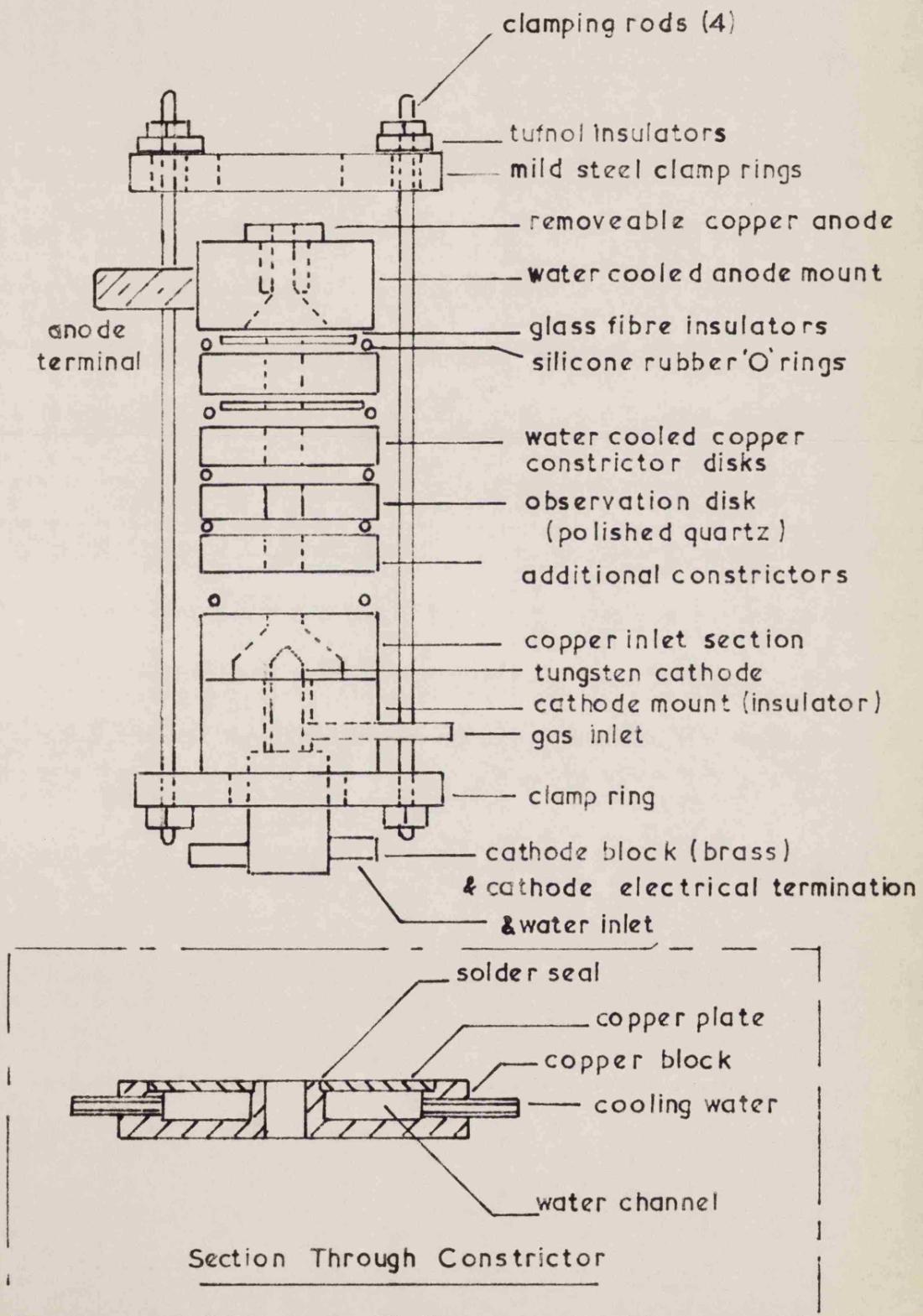
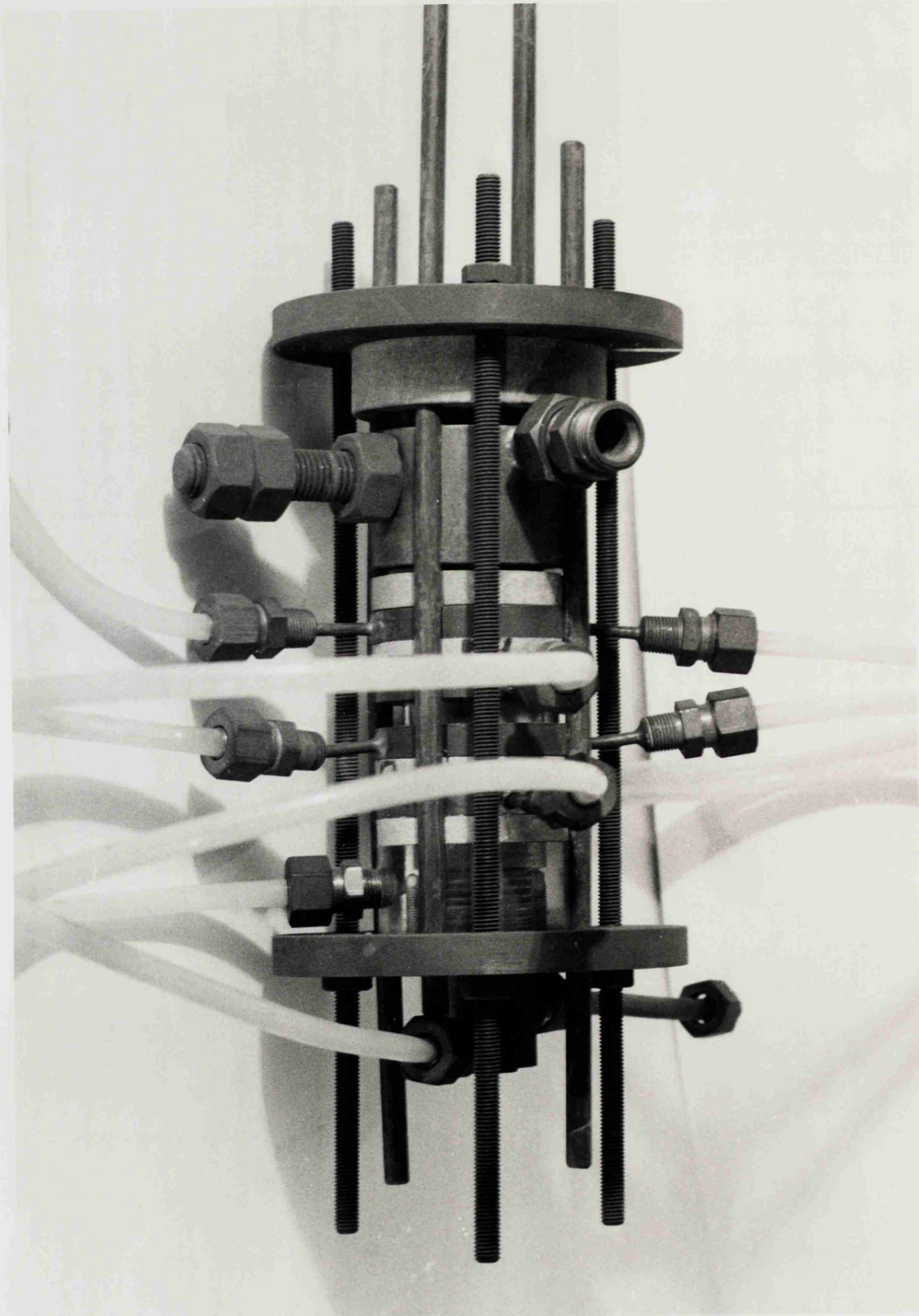


FIG 2



ARC COLUMN

complex gases now being used in high temperature arc technology, the problem assumes proportions of a new magnitude.

This section of the study seeks to examine various techniques for the direct measurement of the transport coefficients, in particular viscosity and thermal conductivity, in partially ionised gases.

2.1 Measurement Techniques on Arc Columns

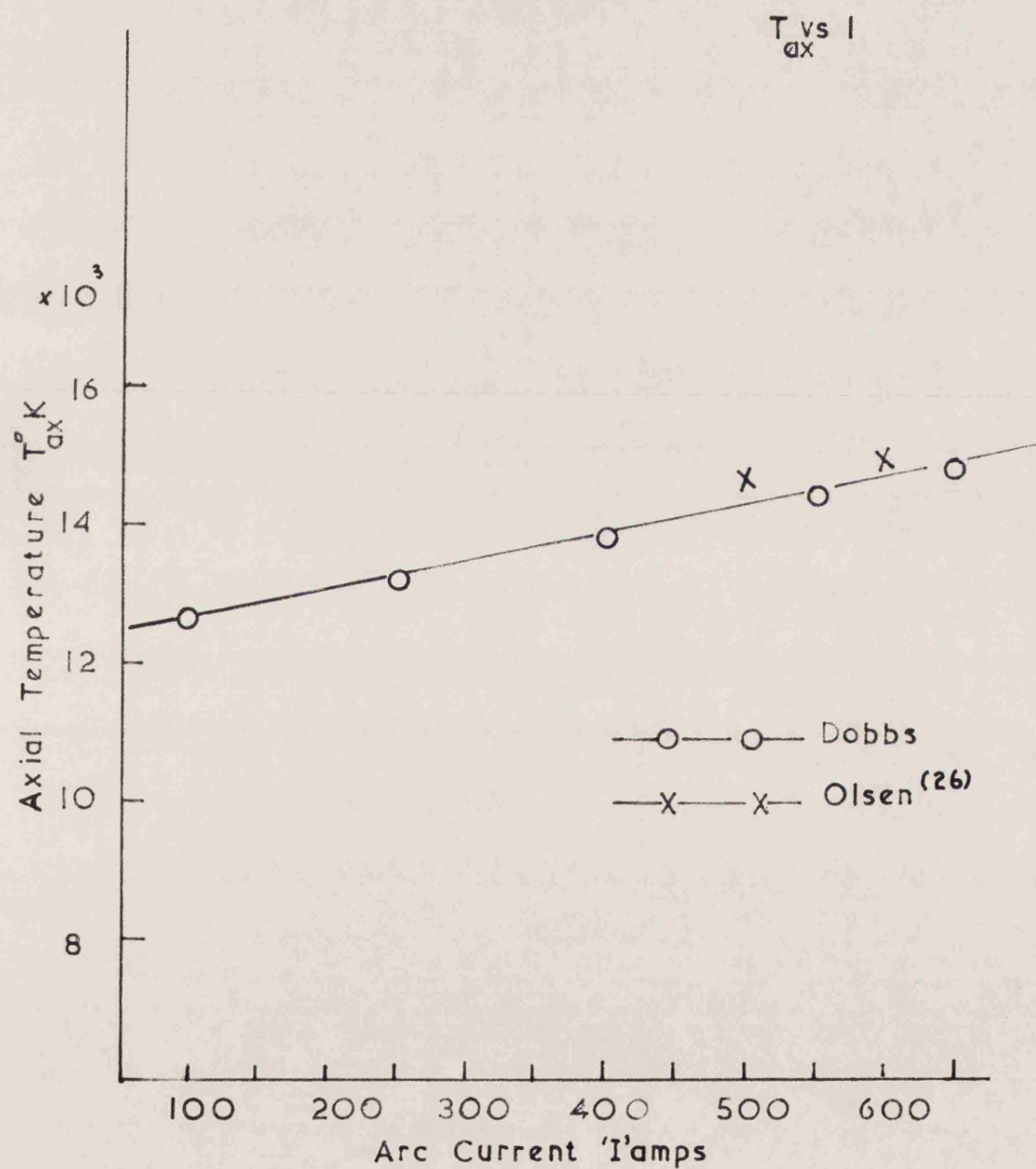
The first series of experimental investigations carried out in this study has been made to determine the difficulties inherent in making accurate measurements of the transport coefficients, particularly thermal conductivity and viscosity, of gases in the temperature range 1000° - 20000° K. The actual experimental methods used are well documented and many sets of transport properties determined by various workers have been published^(20,21,22,23,24). The measure of agreement is, however, not very good and various techniques of measurement and data analysis are examined.

2.2. The Arc System

Measurements made on a wall stabilised electric arc, that is an arc constrained in a cylindrical cavity with cooled walls, have been described by several investigators as a means of determining high temperature gas properties. The construction of such a column as used in this study

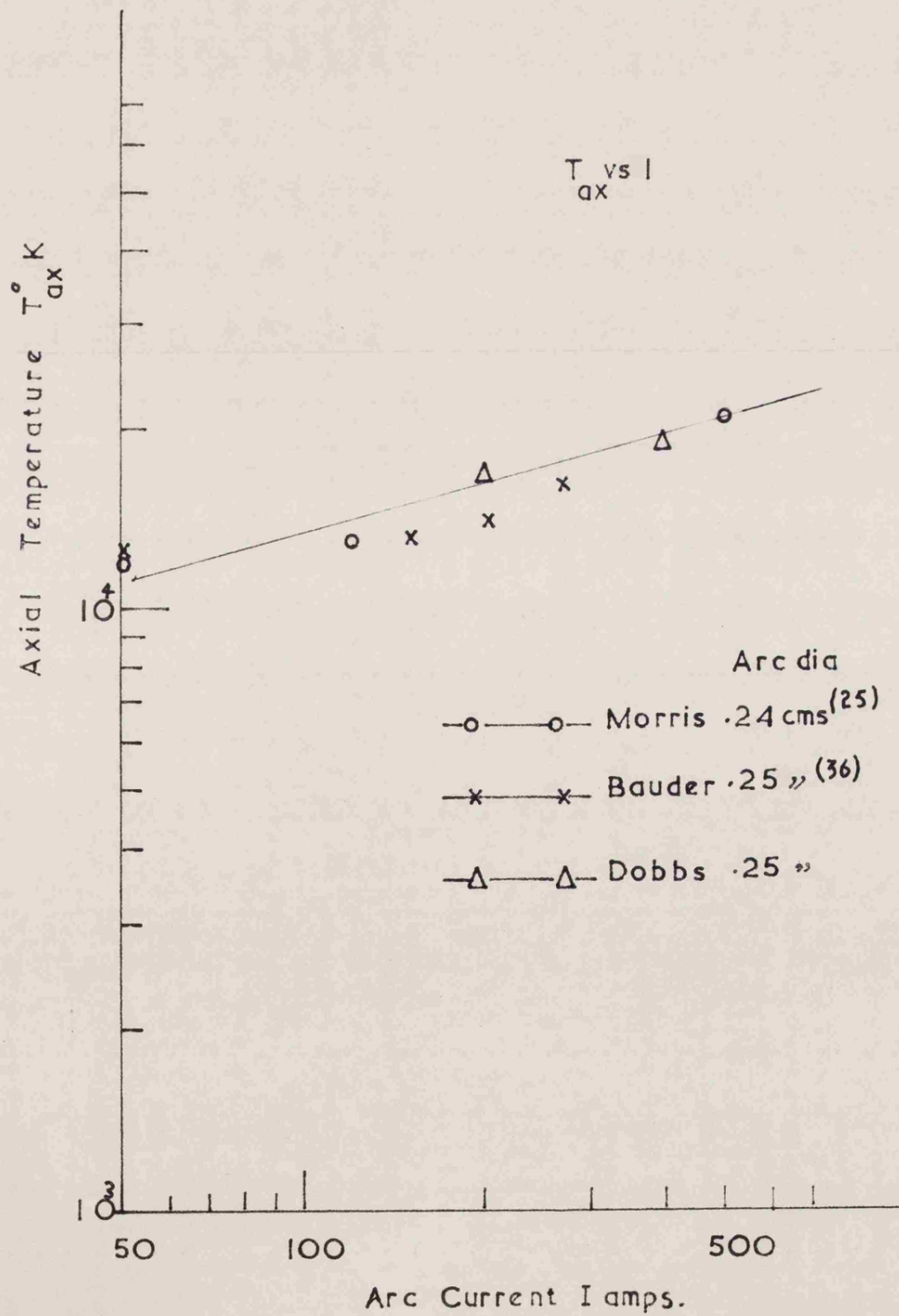
ARGON
TEMPERATURE IN THE FREE
BURNING ARC

FIG 3



ARGON
TEMPERATURE IN THE
CONSTRICTED ARC

FIG4



is shown in Figure 1 and 2. Although a free burning arc is usually considered to be symmetrical about a longitudinal axis, its stability and uniformity may be questioned. The presence of the constraining wall establishes a high degree of symmetry about the longitudinal axis and further establishes the boundary conditions in the equations describing the arc. Lastly, but not least important, the cooled constraining wall makes variation of the arc axial temperature a more sensitive function of current⁽²⁵⁾ with a resultant, desirable wider temperature range for a given change in arc current. This permits examination of arc properties as a function of temperature over a wide range. Figures 3 and 4, which compare measurements of arc temperature as a function of current in the column of a constricted arc and a free burning arc demonstrate this. A current change of 50 to 500 amperes in the free burning arc results in a temperature change of approximately 2000°K. In the constrained arc the same current change results in a 10,000°K temperature swing. In the free burning arc, an increase in current tends to increase the arc radius rather than increase its temperature⁽²⁶⁾. The presence of the cooled wall prevents an increase in arc cross section and an increase in current therefore, results in an increase in current density with the attendant increase in power dissipation and temperature rise.

2.3 Arc Measurements

To obtain the transport properties of the plasma from the energy balance equations it is necessary to measure the following parameters:

- (i) The electrical characteristics, E, I and σ of the arc
- (ii) The power lost by radiation transport, Q_{RAD}
- (iii) The radial temperature distributions at different currents in the arc
- (iv) The pressure drop along the column and axial velocity profile as a function of radius.

2.4 Measurement of Axial Field Strength

The axial field strengths in the arc column is obtained from electrical potential measurements made between the copper discs which form the central region of the arc column. Since the high conductivity core of the plasma comes close to the walls of the column chamber, particularly at high currents, the discs have a low resistivity connection to the plasma and assume a potential close to it. The cooling water and insulation between the discs both have a resistivity which is very high compared with that of the plasma-disc connection and have no discernible effect on potentials. A graph was plotted of disc potential against distance from the arc anode for each experiment made and only

measurements on the linear part of this characteristic were used. This technique eliminated effects due to the anode and cathode potential falls and axial gradients caused by the gas injection and extraction near to the electrodes.

The voltage gradient obtained by measuring difference in potential between successive discs in the arc column will be accurate provided a high impedance measuring device is used. The absolute value of the potential at any point, however, will not be known due to the wall-plasma sheath resistance.

The consumption of gas was measured with calibrated rotameters, measurements being made to an accuracy of $\pm 1\%$. The pressure drop along the column was obtained from a knowledge of the atmospheric pressure, the input gas pressure and the pressure obtained by manometers inserted at two very small (0.1 mm diameter) holes in two of the discs along the arc column.

The power transported to the constricting wall through the plasma per unit length of column may be obtained by conventional calorimetry. However, due to noise, insulation problems and the long time constants involved in obtaining equilibrium, this method was abandoned in favour of measuring the electrical power per unit length, EI. This measurement can be made very accurately in the column when axial temperature

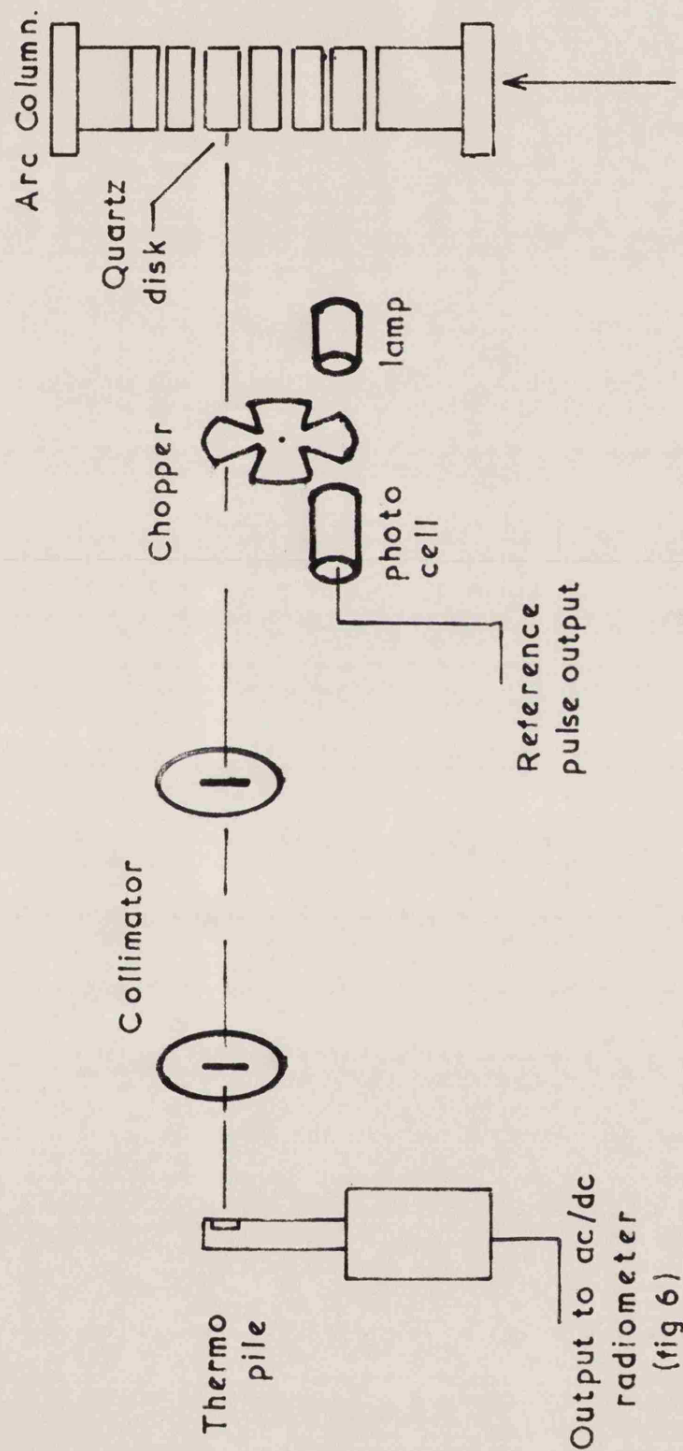


FIG 5

SCHEMATIC RADIATION MEASUREMENT

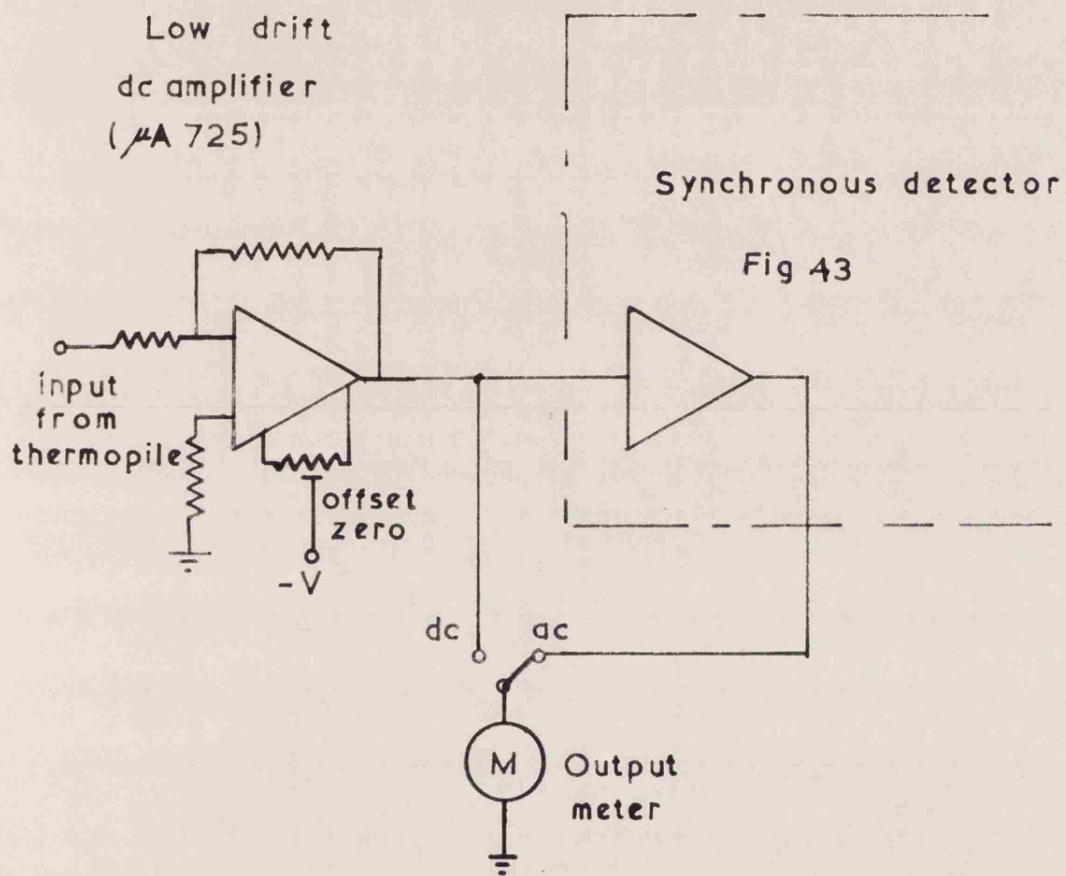
gradients are absent. The electrical power input is balanced by the radial power flow (axial flows being reduced to negligible levels). This radial flow is comprised of conductive and radiative components which must be separated experimentally.

2.5 Measurement of Radiation Energy Transport

An experiment was set up as shown in Figure 5. A Hilger-Watts thermopile was arranged so that radiation from the arc column could be scanned across it. The radiation from the arc was allowed access to the thermopile by replacing one of the copper discs in the arc column with a transparent quartz disc with optically polished faces.

Although the receiver size of the thermopile was very small, high resolution measurement of $Q_{\text{RAD}} = f(r)$ were not entirely successful due to considerable uncertainty in the use of optical collimators over the wide range of wave lengths which have to be considered. A mechanical collimator so attenuated the radiation intensity that the radial distribution of intensity could only be obtained with a relatively coarse spatial resolution. By using an AC technique, higher sensitivity and hence higher spatial resolution was obtained but a calibrating source suitable for the thermopile was not available and hence the manufacturer's steady state calibration had to be relied upon. Ultimately, the

FIG 6



RADIOMETER SCHEMATIC

total integrated intensity (along the arc diameter) was measured with a DC system and four "off-axis" points measured with a high sensitivity AC system, the radiation being chopped mechanically. As the arc radius was 0.25 cms, a collimating slit 0.05 cms wide delineated four strips along chords through the arc column, each strip having the same dimensions as the thermopile receiver. To provide a calibration under AC conditions a subsidiary measurement of the total integrated radiation was made with the AC system and compared with the DC measurement. The ratio of these two values then gives a calibrating factor. Figure 6 shows a block diagram of the electronics which form the radiometer circuit. *The agreement between the AC & DC measurements was better than $\pm 5\%$.*

The radial energy distribution, albeit at low spatial resolution does enable a comparison to be made with theoretical predictions of radiation transport.

A further difficulty was encountered due to the fact that after a short exposure to the arc plasma, the initially transparent quartz disc very quickly devitrified and became progressively more opaque. This was not overcome with the existing arc column. Other workers have used constricting discs with small holes or slits to permit examination of the arc.

2.5.1 Measurement Details. Radiation.

The thermopile used in the measurements had a quartz window which acted as a bandpass filter for radiation between the wavelength limits of 0.25 and 2.5 μm .

The receiver dimensions were 0.2 cm x 0.05 cm. A collimator (Figure 5) consisting of two metal plates with rectangular apertures the same size as the receiver, delineated a zone of the plasma from which the radiation emanated.

The sensitivity "s" as quoted by the manufacturer was 1 μV per mW of incident radiation. To record the total radiation per unit length of arc a numerical factor

$$K = \frac{2\pi R}{d} \cdot \frac{1}{\ell} \cdot S$$

was generated and used as a multiplier for the thermopile output voltage.

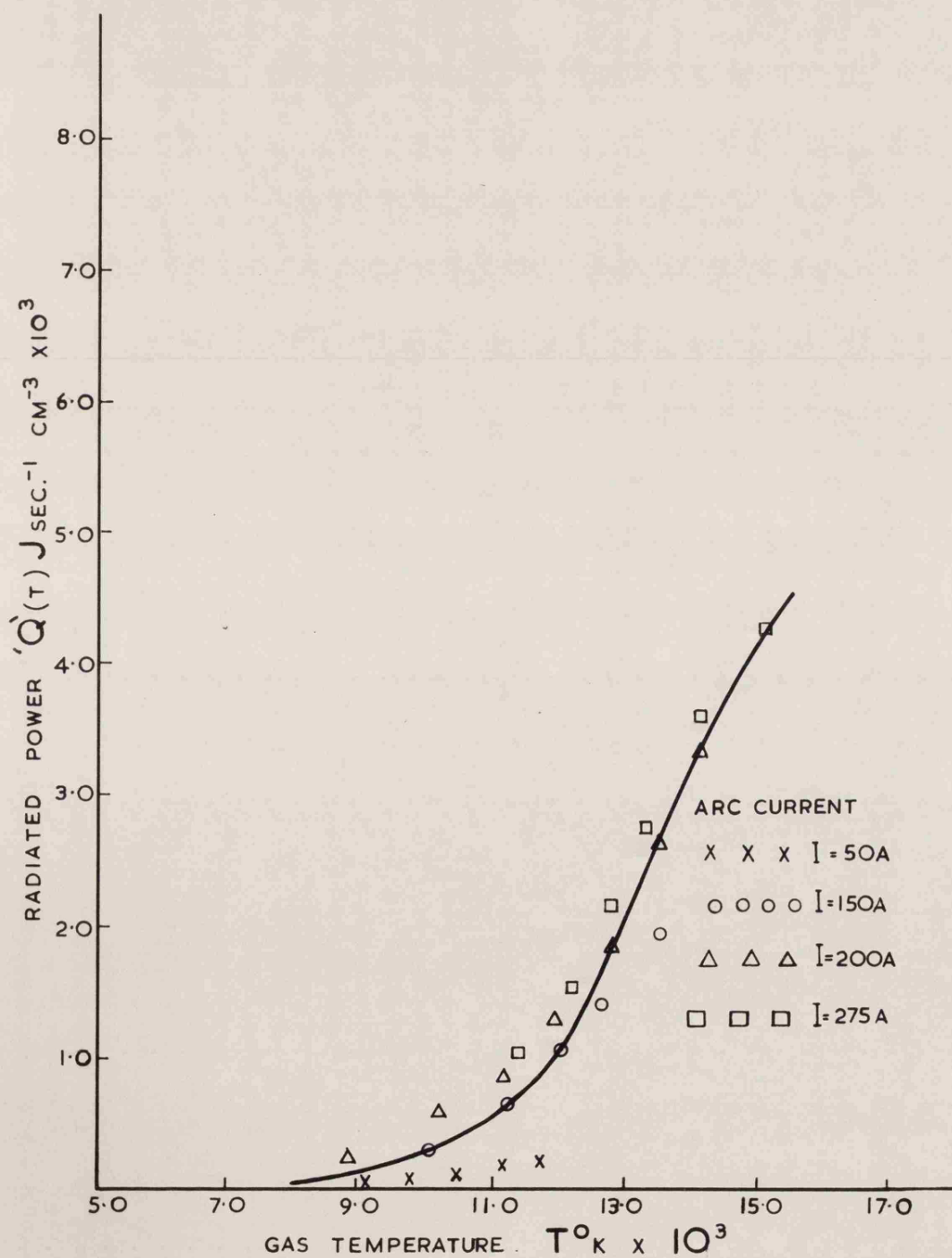
In the above equation, "R" is the radius of the arc column, "d" the receiver width, " ℓ " the receiver length and "S" the sensitivity.

For the equipment described K is then equal to 157.

Then the output voltage multiplied by K equals the power per unit length of the arc in watts transported by radiation.

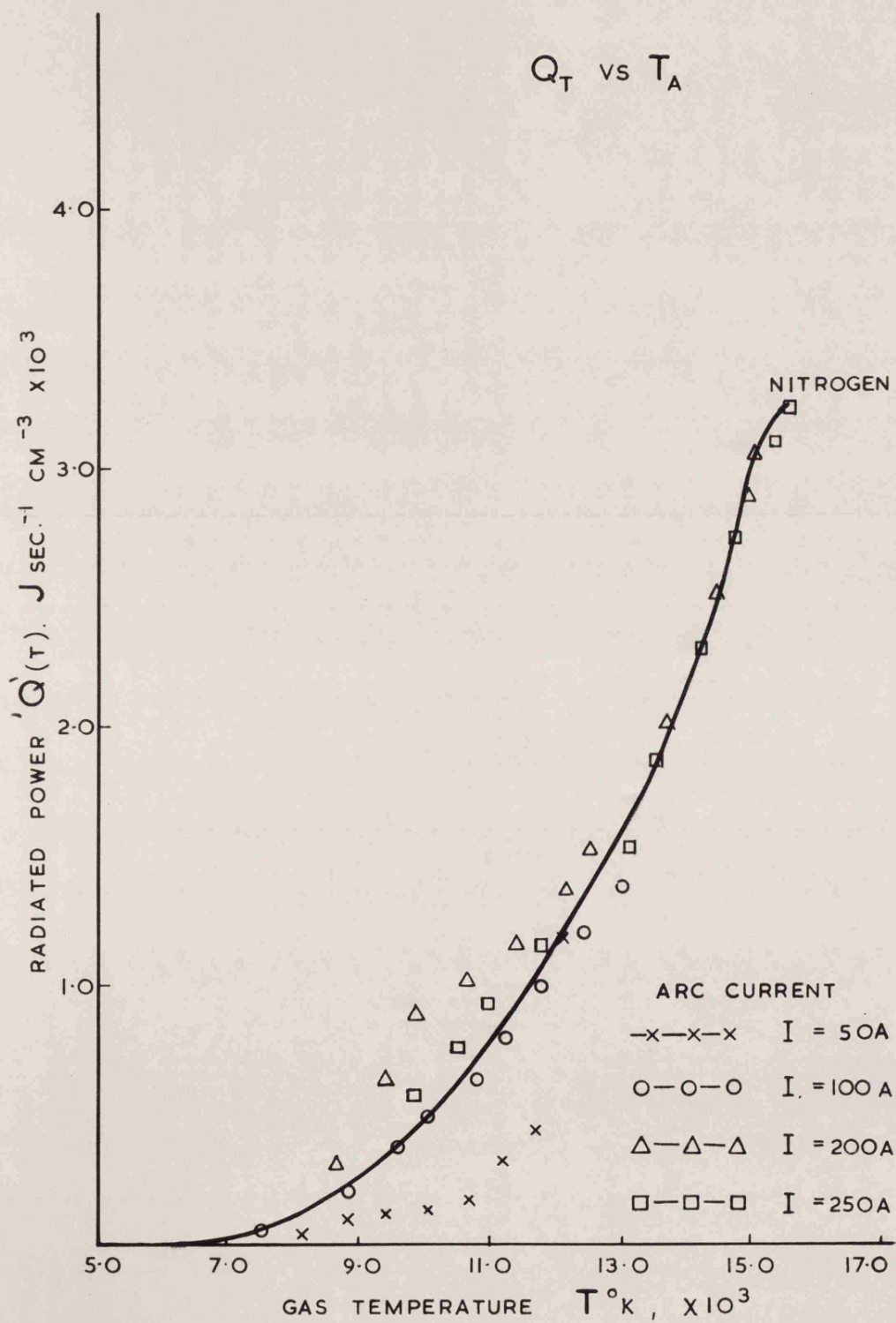
MEASURED SPECIFIC RADIATION FIG.7
INTENSITY AS A FUNCTION
OF ARC TEMPERATURE

ARGON
 Q_T vs T_A



NITROGEN
MEASURED SPECIFIC RADIATION
INTENSITY AS A FUNCTION OF
ARC TEMPERATURE.

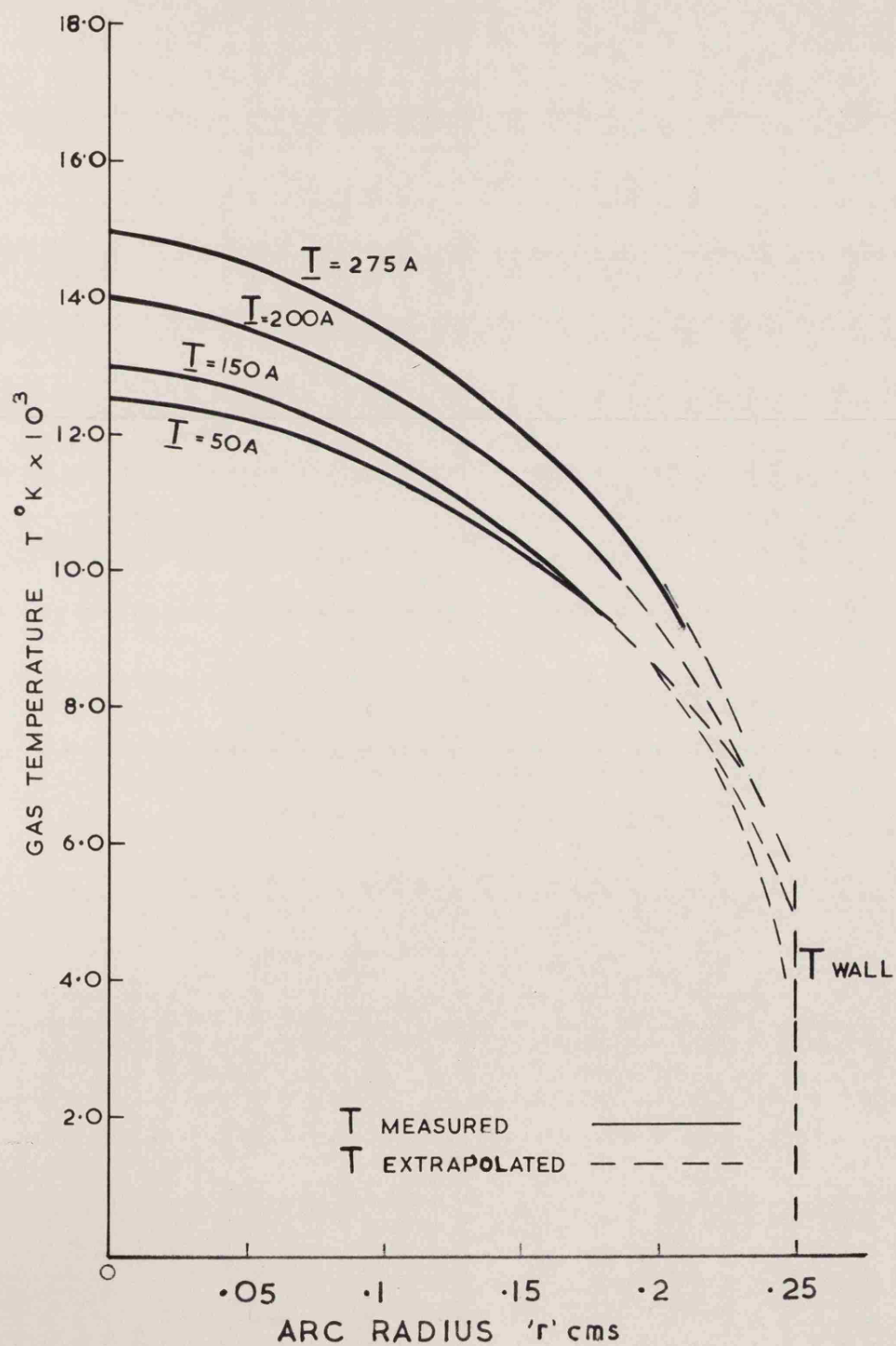
FIG 8



ARGON
RADIAL DISTRIBUTION OF
ARC TEMPERATURE.

FIG.9.

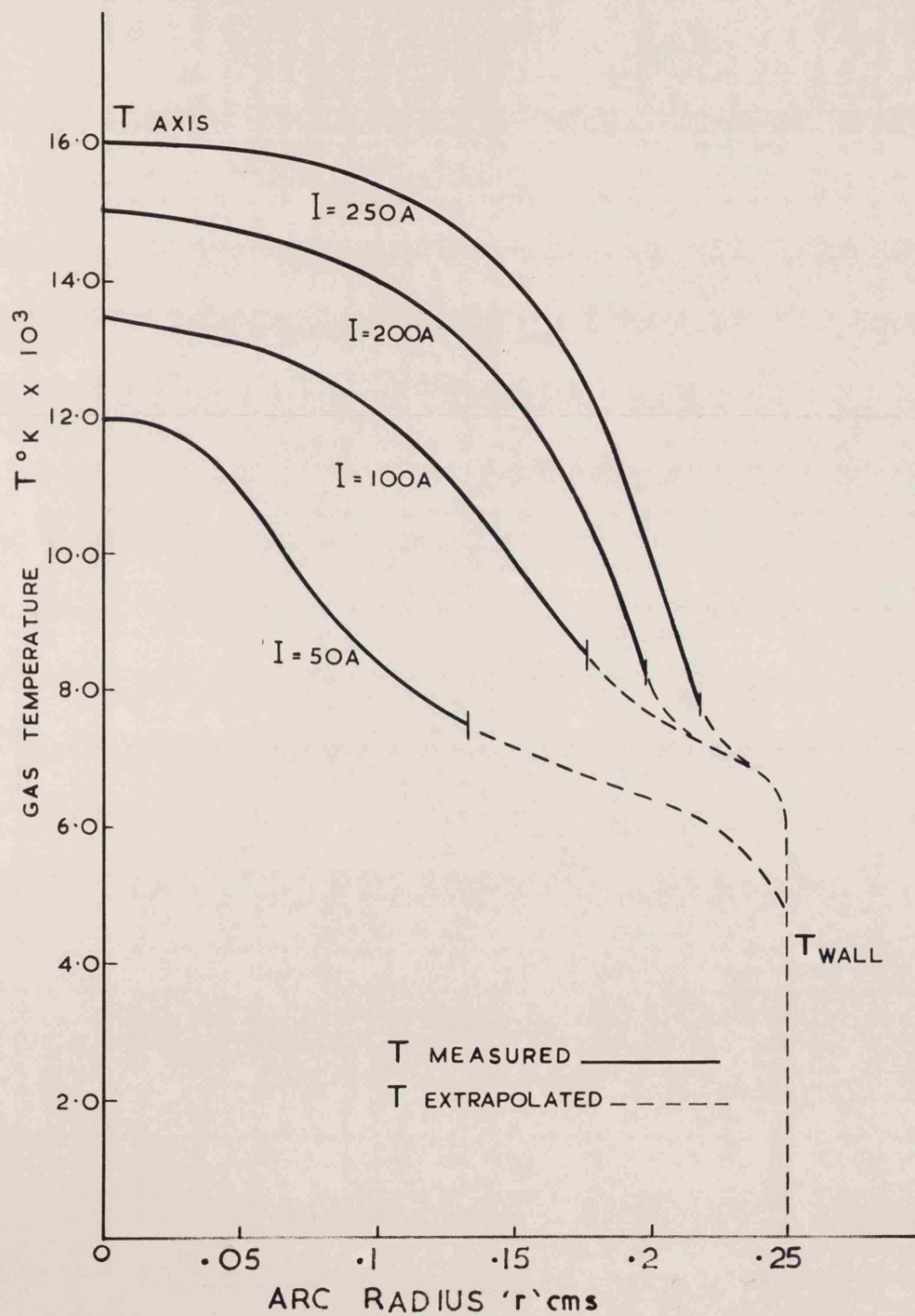
T vs r



NITROGEN
RADIAL DISTRIBUTION OF
ARC TEMPERATURE

FIG. 10

T vs r



The off axis measurements of radiation, as already stated, were made using chopped radiation and an AC synchronous amplifier-voltmeter, (Figure 6), the off axis intensity being too low for the DC measurements to be distinguishable from the noise.

The radiation falling onto the thermopile is the integrated intensity along a chord through the cylindrical plasma. To obtain the intensity as a function of radius, the "Abel" integral inversion^{(27), (28)} explained in Appendix III was used. The radiation per unit length of arc associated with each radius was then evaluated by inverting the integrated radiation. The sensitivity factor for these AC measurements " S_{ac} " was obtained by measuring the total radiation under DC conditions and comparing it with the same measurement made with the chopped radiation. The difference in sensitivity between AC and DC measurements arises because of the relatively long time constants associated with the thermopile.

As the calculation of thermal conductivity with a correction for radiation requires the intensity as Joules, $\text{sec}^{-1} \text{cm}^{-3}$, the radiation per unit length was converted to this form.

Using the experimentally obtained temperature profiles for the arc shown in Figures 9 and 10, the specific radiation intensity was plotted as a function of gas temperature. These results are shown in Figures 7 and 8.

2.6 Measurement of Radial Temperature Distributions

The temperature distribution in the arc column was determined spectroscopically. Appendix I gives a detailed treatment of the method used and examines some of the difficulties. A thorough analytical and experimental study of this problem is reported by Ruddy of Leicester University⁽²⁹⁾ Figures 9 and 10 show the variation in column temperature of argon and nitrogen as a function of radius with arc current as a parameter, obtained in this study. In both cases the flow was very laminar with a differential gas pressure across the column of approximately 0.05 to 0.10 cm Hg. Depending upon temperature and velocities measured in the column, the Reynold's numbers were in the range 2 to 500.

As the cooled wall is approached, the temperature gradient becomes very steep and the spatial resolution of the spectrometer employed in making the measurements was such that, coupled with the very low emission intensity of the spectral lines used for making measurements in the temperature range in question ($T = 300 - 8500^{\circ}\text{K}$), meaningful measurements could not be made. The graphs (Figures 9 and 10) show this region with extrapolated dashed lines. As will become apparent later, this extrapolation is justified in these experiments. Although errors may well exist, the contribution made to the thermal conductivity in the high temperature zone of the plasma by these, albeit inaccurate, lower temperature regions is small and is overridden by other theoretical and experimental considerations.

2.7 The Measurement of Thermal Conductivity

Consider a volume element of a heated fluid in motion. A complete energy balance on this element can be written as

$$\rho C_v \frac{DT}{Dt} = -p \nabla \cdot \mathbf{V} + \nabla \cdot \mathbf{K} (\nabla T) - \nabla \cdot \mathbf{Q}_{RAD} + \mu \phi + Q_{int}$$

Under steady state conditions the rate of gain of internal energy $\rho C_v \frac{DT}{Dt}$ is zero. For the low velocity system to be considered, the frictional dissipation $\mu \phi$ can be set to zero. By considering the fluid to be incompressible (valid for Mach numbers < 0.3) the term $p \nabla \cdot \mathbf{V}$ is also equal to zero. Writing the remaining terms in cylindrical co-ordinates and retaining only the radial components we obtain the Elenbaas-Heller⁽⁶⁾ energy equation for an arc

$$\frac{1}{r} \frac{d}{dr} \left(r K \frac{dT}{dr} \right) - Q_{RAD} + \sigma E^2 = 0 \quad (1.2)$$

with boundary conditions $\frac{dT}{dr} = 0$ at $r=0$ and $T_{wall} = \text{constant}$ where σ = electrical conductivity, K = thermal conductivity, E the axial electrical field strength, Q_{RAD} the radiation source strength and T the temperature.

This equation is admissible if the following hold true:

- (i) The gas density is sufficiently high to permit the plasma to be treated as a continuum
- (ii) The optical depth of the plasma is not of the same order of magnitude as the tube diameter for wavelengths which transport a significant quantity of energy to the tube walls.
- (iii) The properties $Q(T)$, $\sigma(T)$ and $k(T)$ exist as a function of the temperature only. Thus the

gas must be in local thermodynamic equilibrium otherwise the transport properties which are sought will be functions of additional variables.

(iv) There are no axial gradients in the apparatus.

This equation, as a means of obtaining transport coefficients is not directly applicable if the absorption of radiation plays an important part in the energy exchange equation. In this case the radiation term Q_{RAD} , represents the balance between the energy Q_E lost from a volume element by radiation emission and the energy Q_A gained by absorption; an equation

$$Q_R = Q_E - Q_A \quad (2.2)$$

can be written to describe this process.

The emission, $Q_E(T)$ depends only on T whilst the energy absorbed, $Q_A(T, T_{AX})$ depends also on T_{AX} , the axial temperature which is a function of I , the arc current. Since no direct measurement of the energy absorbed by the plasma appears possible, the thermal conductivity must be deduced from measurements obtained by considering the energy balance together with a detailed measurement of the radiation transport through the column.

2.7.1 The Qualifying Assumptions

- (i) For atmospheric pressure arcs the first assumption as to gas density would appear to be quite satisfactory as the particle densities are in the range $10^{14} - 10^{19} \text{ cm}^{-3}$
- (ii) For plasmas in which the ratio of optical depth to the column diameter is small compared to unity, the radiant energy is transferred directly to the wall of the arc column so that the absorption by the plasma is negligible. When the ratio is large compared to unity, energy is radiated from a volume element, absorbed and re-radiated, this process continuing in microscopically small distance steps within the tube. Processes of this type are part of the overall energy transport which forms part of the measured thermal conductivity. The measured value of thermal conductivity will therefore encompass any effects due to absorption and re-radiation. To separate the apparent from the real thermal conductivity the magnitude of the radiation transported must be known.
- (iii) Previous experience of the author⁽³⁰⁾ and other workers^{(31), (32)} has shown that gases such as argon, nitrogen and hydrogen are in close approximation to thermal equilibrium in the arc chamber. It is reported, however, that the case of helium, this assumption fails⁽³³⁾.
- (iv) A major source of axial gradients within the arc

column is the entry area for the gas at the cathode and the exhaust region near the anode. Considerable work has been done on the design of gas inlet devices for plasma generators to ensure uniform flow and provided that a length equal to some five or six times the chamber diameter is left at the inlet and exit point of the arc column the axial flow gradients are reasonably assumed to be negligible over the central region of the arc.

2.8 A First Evaluation of Thermal Conductivity. Radiation Neglected

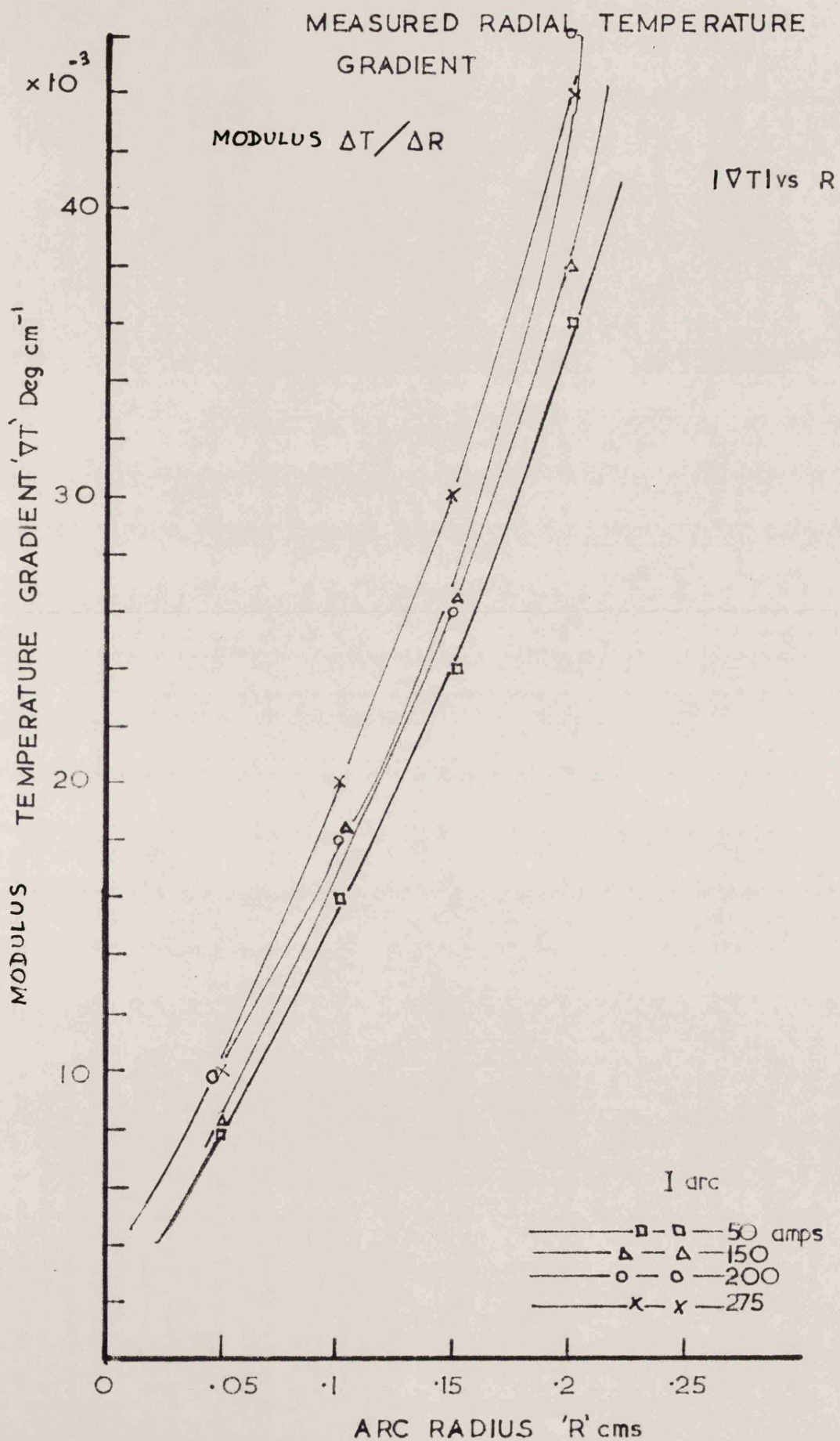
It is seen from equation (1.2) that if the electrical power dissipation per unit length (σE^2) is measured together with the temperature as a function of radius, then provided that the radiation transport is negligible the thermal conductivity K can be immediately calculated by integrating equation (1.2) over the radius. This results in equation (3.2). with y as a dummy for 'r'.

$$K = \frac{E^2 \int_{y=0}^{y=r} \sigma y dy}{\left(y \frac{dT}{dy}\right)_{y=r}} \quad (3.2)$$

However, although the mathematical manipulation is relatively easy, a large error source is always present in the temperature term. Although high accuracy of temperature measurement is claimed by various workers, the gradient of temperature, dT/dr is very sensitive to errors.

ARGON

FIG 11



NITROGEN

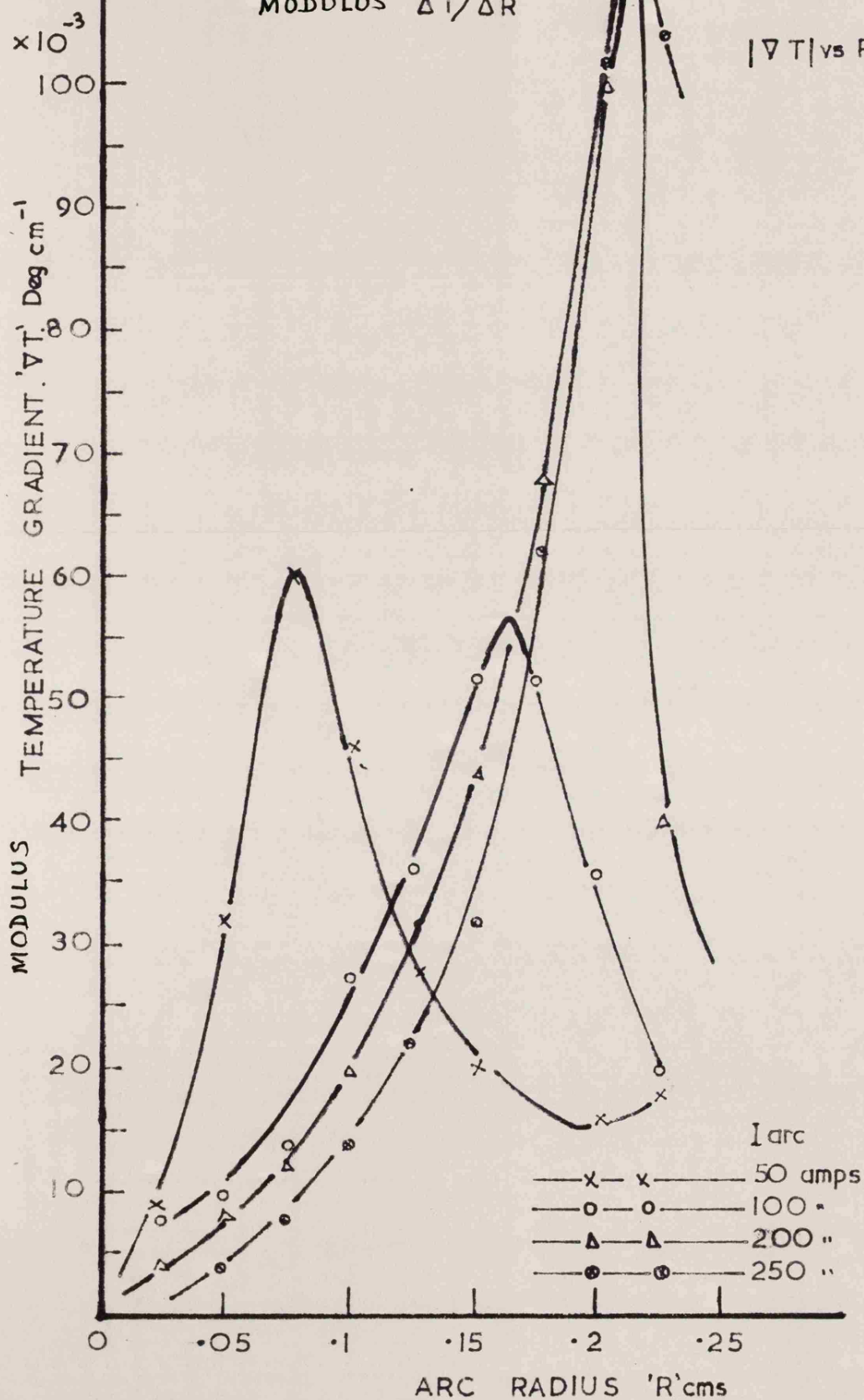
FIG 12

MEASURED RADIAL TEMPERATURE

GRADIENT

MODULUS $\Delta T / \Delta R$

$|\nabla T|$ vs R



Figures 9 and 10 show measured radial temperature distributions in argon and nitrogen arcs with arc current as a parameter. Figures 11 and 12 show the temperature gradients obtained from these graphs and indicate the errors which can occur when small changes in profile appear either as measurement errors or real variations. In the high intensity axial region of the arc column temperature as a function of radius is measured in this study to better than 10% as determined from consistency checks on repeated measurements using a range of spectral line frequencies.

It is suggested by Burhorn⁽²⁰⁾ that the values of $\frac{dT}{dr}$ should be experimentally adjusted until integration of the terms produces a result in agreement with the initial temperature distribution. Such errors are particularly noticeable near to the wall when the temperature measurement becomes very uncertain and near the edge of "cores" which appear in molecular gases (See in particular, Figure 12).

The electrical power input $E^2 \int_0^R \sigma r dr$ is obtained directly from the measurements of electrical field strength and total arc current (shown graphically in Figures 13 and 14) since:

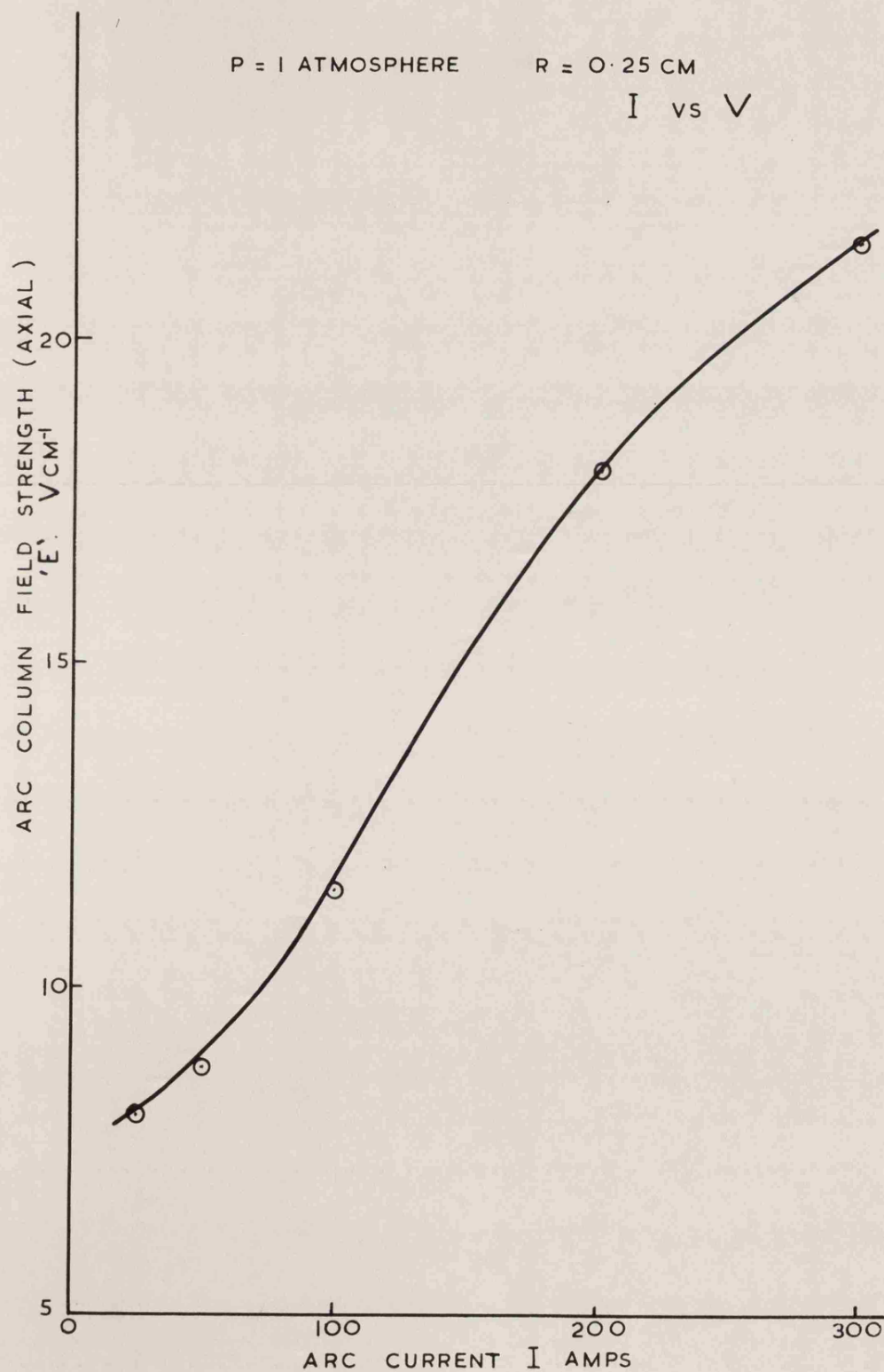
$$E^2 \int_0^R \sigma r dr = EI \quad (4.2)$$

The electrical conductivity σ , in equation 4.2 is

ARGON ARC COLUMN
FIELD STRENGTH

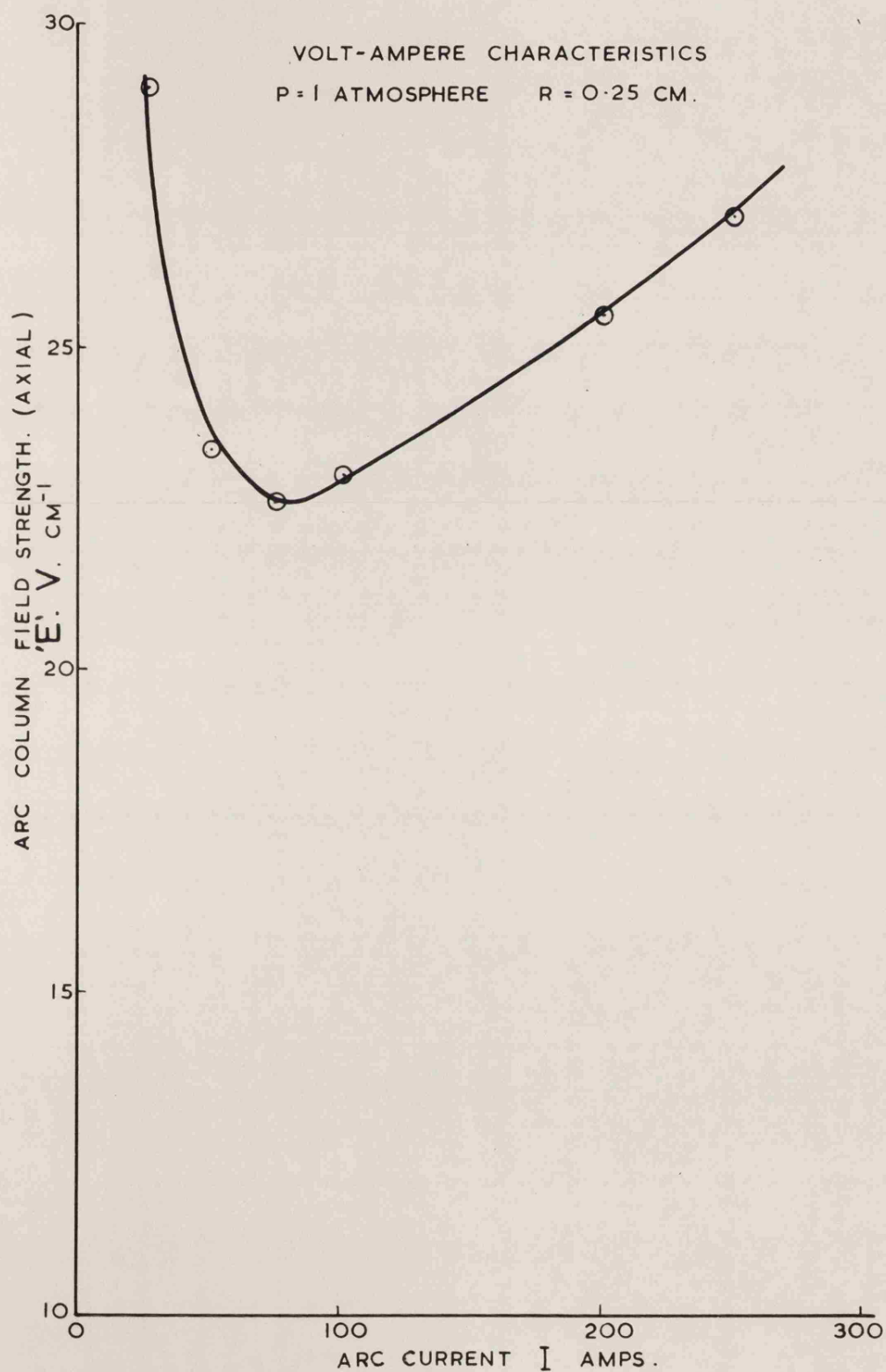
FIG.13.

VOLT-AMPERE CHARACTERISTIC

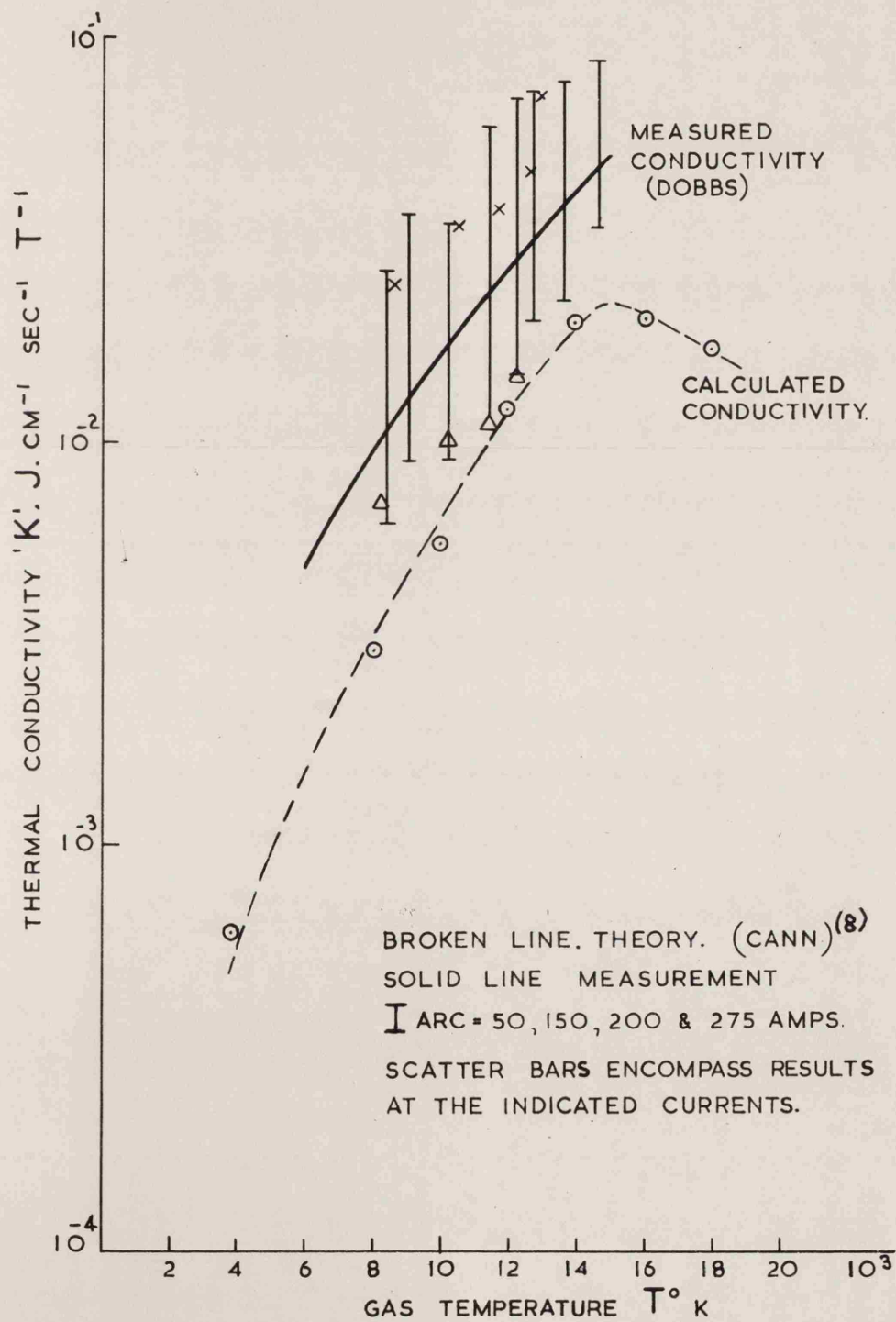


NITROGEN ARC COLUMN FIELD
STRENGTH.

FIG. 14.

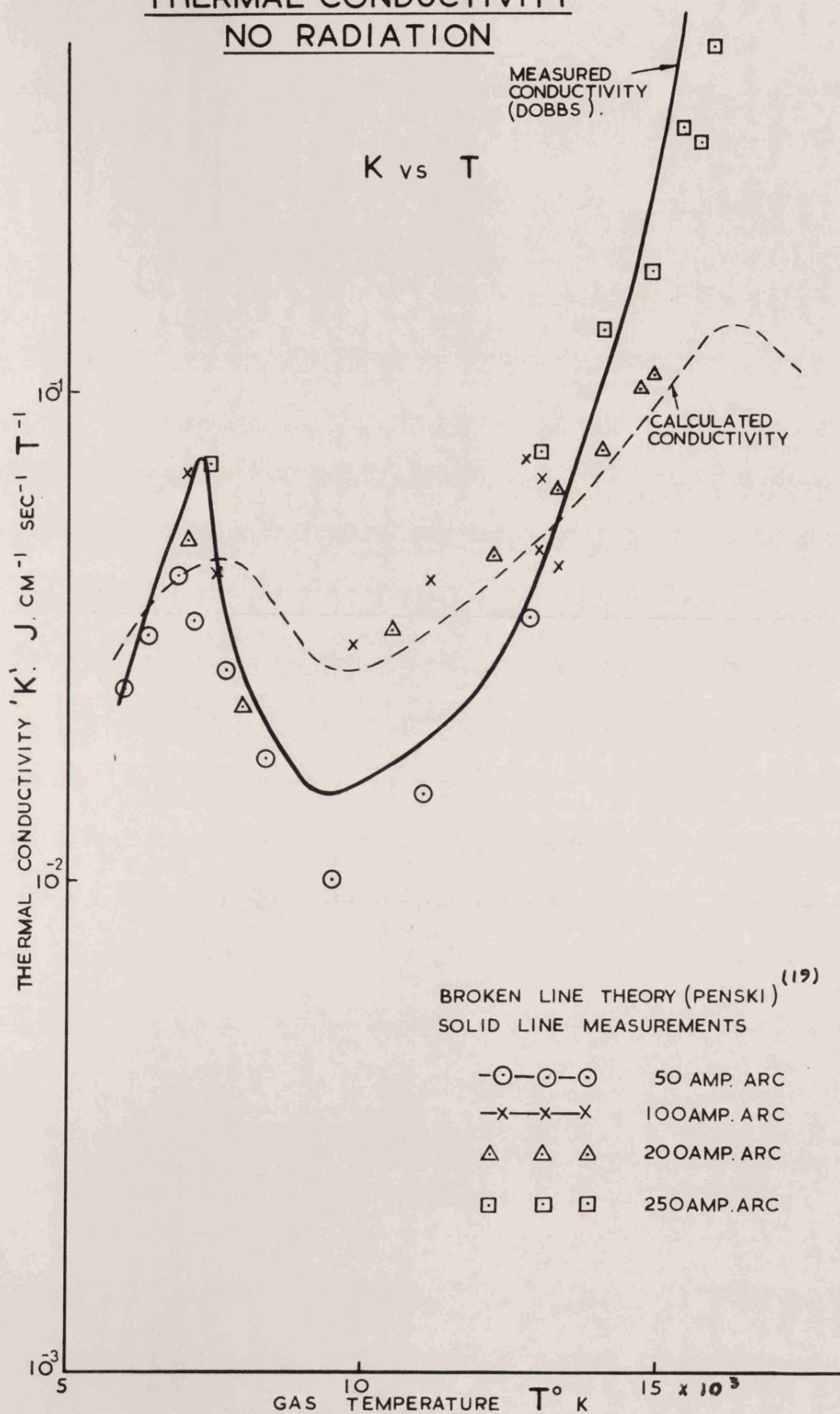


MEASURED ARGON THERMAL CONDUCTIVITY. NO RADIATION. FIG. 15



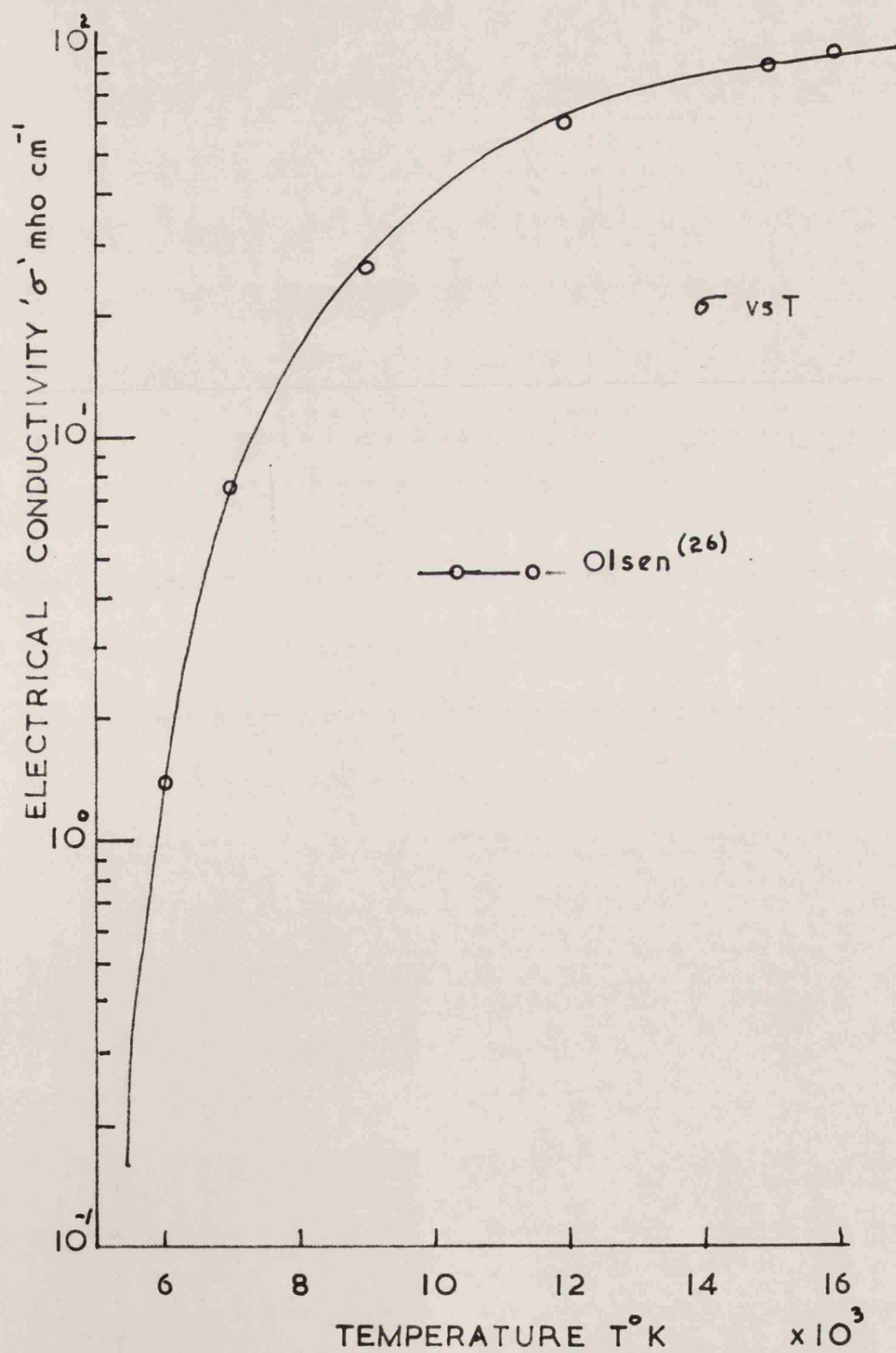
MEASURED NITROGEN.
THERMAL CONDUCTIVITY
NO RADIATION

FIG.16



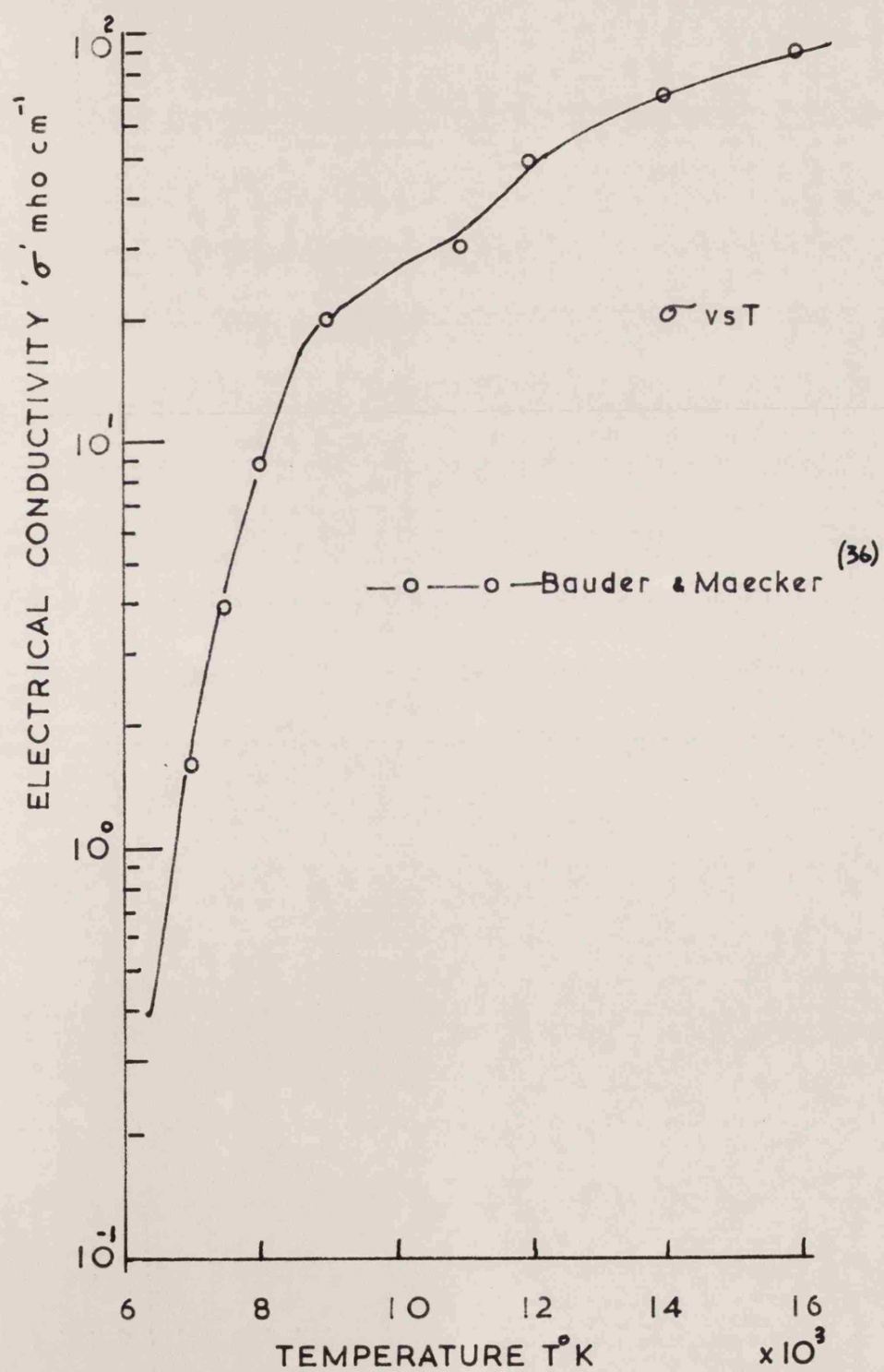
ARGON
ELECTRICAL CONDUCTIVITY

FIG 17



NITROGEN
ELECTRICAL CONDUCTIVITY

FIG 18



required as a function of temperature. This data is plotted in Figures 17 and 18, references (26) and (36).

Figures 15 and 16 show the thermal conductivity of argon and nitrogen, calculated with the aid of equations (3.2 and (4.2) as a function of gas temperature. Plotted on the same graphs are theoretically derived values of this same property^{(8),(19)}. Similar disparities between the measured and theoretical values are reported by Burhorn⁽²⁰⁾, Durand⁽³⁴⁾ and others⁽³⁵⁾. The discrepancy is attributed to the transport of energy by radiation at high temperatures.

Table 1 in Appendix II shows detailed calculations of apparent thermal conductivity for nitrogen using equation (3.2). The evaluation of $\int_0^R \sigma r dr$ (equation (4.2)) was made by the simplest numerical integration as the greater accuracy offered by the use of Simpson's rule did not justify the extra computational effort with the data available. Table 2 in Appendix II compares the calculation of $\int_0^R \sigma r dr$ for a 200 ampere nitrogen arc using a simple trapezoid and "Simpson".

The accuracy of the above calculation was monitored by making the summation $2\pi \sum \sigma_r dr$ over the range $0 \rightarrow r=R$ and comparing it with the measured electrical conductivity $\frac{I_{arc}}{E_{arc}}$ obtained from Figures 13 and 14. The agreement between measured values of σ and the summation $\sum \sigma_r dr$ is sufficiently good ($\approx 10\%$) to justify the acceptance of apparent weaknesses

in certain aspects of the experiments particularly as the published data on electrical conductivity as a function of temperature is also subject to error.

2.9 Evaluation of Thermal Conductivity with Non Negligible Radiation

Although a complete, rigorous solution of the energy balance equation, including radiation energy transport may be possible, the experimental and computational difficulties involved in the evaluation of the thermal conductivity by such methods are very considerable and a simple alternative is examined here.

This evaluation treats the plasma column as a two temperature system and allows the calculation of K as a function of T for varying axial temperatures. This method, although not being a complete solution of the energy equation nevertheless produces values of thermal conductivity as a function of temperature which are in better agreement with theory than those obtained when the full temperature profile is brought into the solution but radiation is neglected. In this method the apparent thermal conductivity K is given as before by equation (3.2). To avoid the difficulties of considering the radiation loss as a function of radius, the apparent thermal conductivity on the arc axis is examined.

Taking the Elenbaas-Heller equation and making the radiation non negligible we have, in radial co-ordinates

$$\frac{1}{r} \left(\frac{d}{dr} \left[rK \frac{dT}{dr} \right] \right) = -\sigma E^2 + Q_{\text{radiation}} \quad (5.2)$$

Multiplying (5.2) through by r we have

$$\frac{d}{dr} \left(rK \frac{dT}{dr} \right) = \sigma r E^2 + r Q_{\text{radiation}} \quad (6.2)$$

Integrating (6.2) over r

$$rK \frac{dT}{dr} = -E^2 \int_0^r \sigma r dr + \int_0^r r Q dr \quad (7.2)$$

which on re-arranging gives

$$K = \frac{-E^2 \int_0^r \sigma r dr}{r \frac{dT}{dr}} + \frac{\int_0^r r Q dr}{r \frac{dT}{dr}} \quad (8.2)$$

Now since the radiation is most significant at the high temperatures found near the arc axis, consider a zone with $r \rightarrow 0$. Equation (8.2) is now indeterminate as $r \rightarrow 0$.

With K , σ and Q as functions only of T , and applying L'Hopital's rule by differentiating (8.2) w.r.t. " r " we have

$$K = \frac{-E^2 \sigma r}{T^1 + r T^{11}} + \frac{r Q}{T^1 + r T^{11}} \quad (9.2)$$

Equation (9.2) is still indeterminate due to "r" in the numerator and denominator.

Applying L'Hopital's rule again we obtain

$$K = \frac{-E^2 \sigma}{T^{11} + T^{11} + rT^{111}} + \frac{0}{T^{11} + T^{11} + rT^{111}} \quad (10.2)$$

This is now determinate and since $rT^{111} \rightarrow 0$ we can ignore it and the thermal conductivity can be written near to the arc axis as

$$K = \frac{-E^2 \sigma}{2T^{11}} + \frac{0}{2T^{11}}$$

In finite difference form $(2T^{11})_{\text{axial}}$ is given by

$$2T^{11} = 4 (T_{\text{ax}} - T_r) / r^2 \quad (11.2)$$

This can be evaluated graphically from the measured radial distribution of temperature.

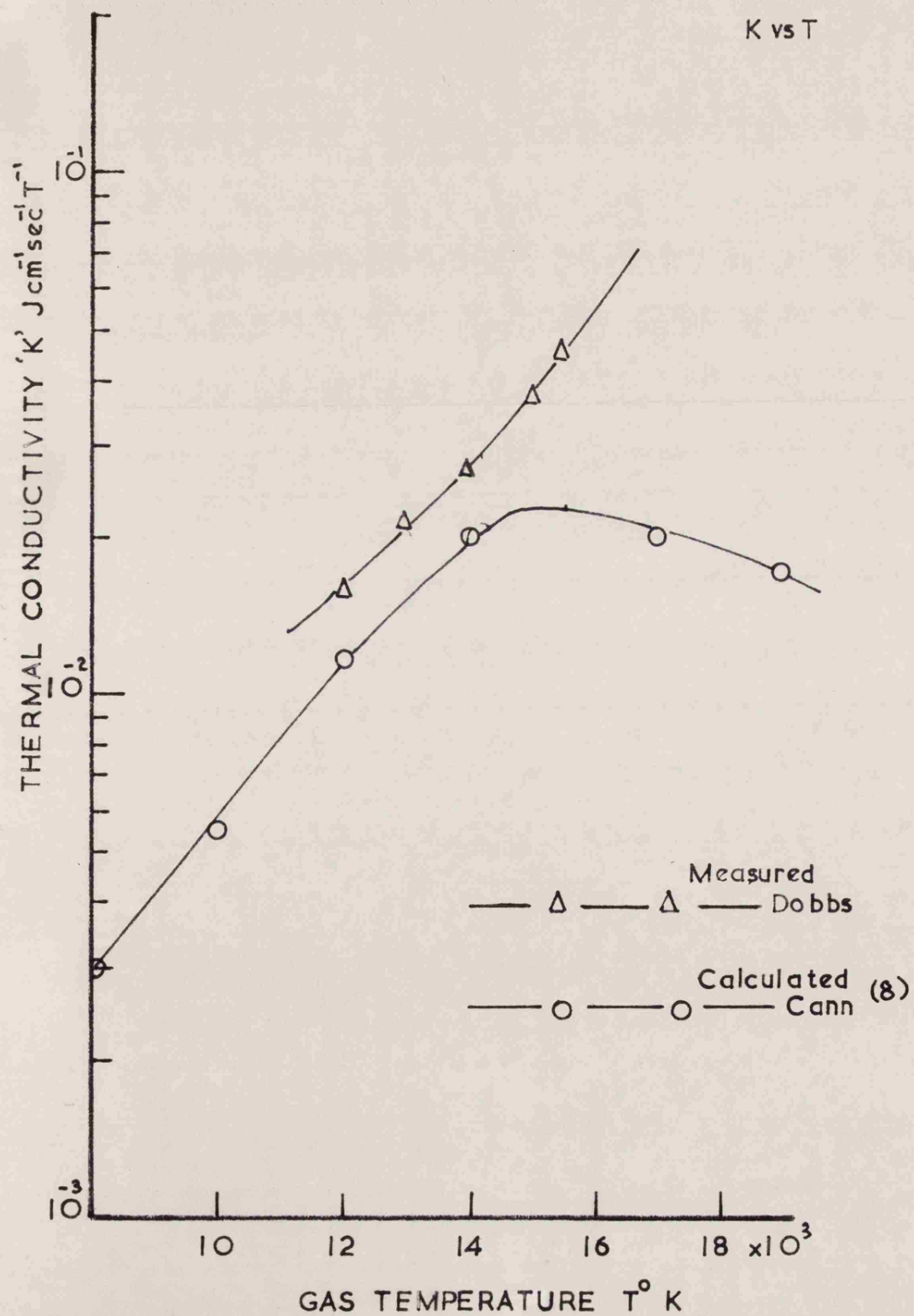
The energy transported by radiation, $Q_{\text{radiation}}$, is obtained from measurement of the total flux of radiation at the

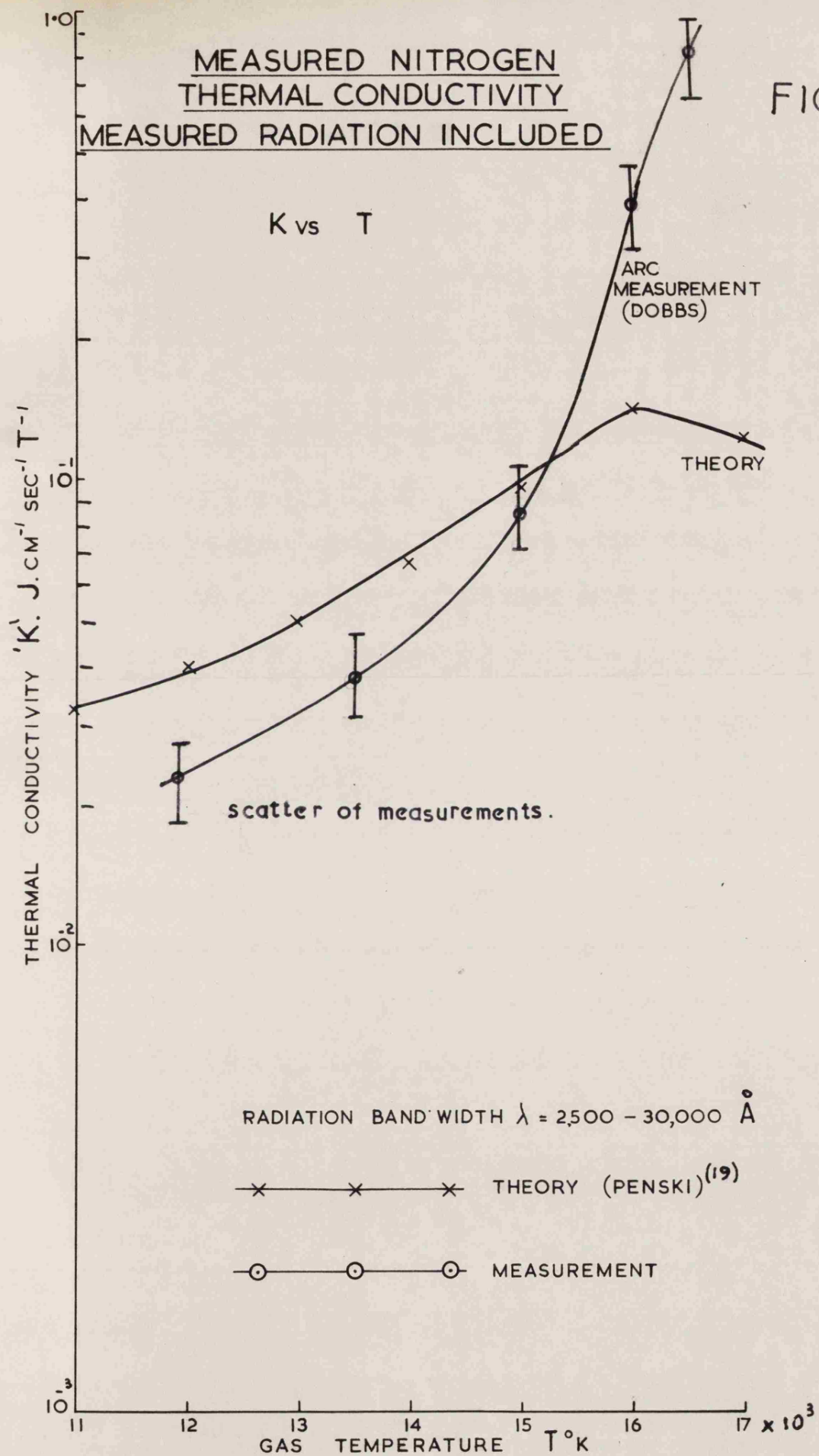
ARGON

FIG 19

MEASURED THERMAL CONDUCTIVITY (AXIAL)

Including Measured Radiation .
(Bandwidth 2500 25000 Å)





arc exterior (Section 2.5)

Figures 19 and 20 show the thermal conductivity of argon and nitrogen obtained from the experimental results from the arc column using the two temperature method including the measured radiation. These results are compared with theoretical calculations and it is seen that at high temperatures where radiation of energy becomes significant, the measure of agreement is only slightly improved, the error is still large and very significant.

As in the case of the arc in which radiation transport is ignored, a source of inaccuracy exists in the determination of the second derivative of temperature w.r.t. radius. However, the zone over which this differentiation must be made is small and is in a region of the arc column where the determination of temperature is relatively easy. Further, the temperature can be determined by one series of measurements. The matching of profiles obtained from different emission lines with their inherent transition probability, uncertainty is eliminated when the temperature range is small.

Table 3 in Appendix I shows details of the calculation of thermal conductivity using the "Two Temperature" method for nitrogen using axial temperature data taken from Figure 10 and radiation intensity measurements from Figure 8.

ARGON

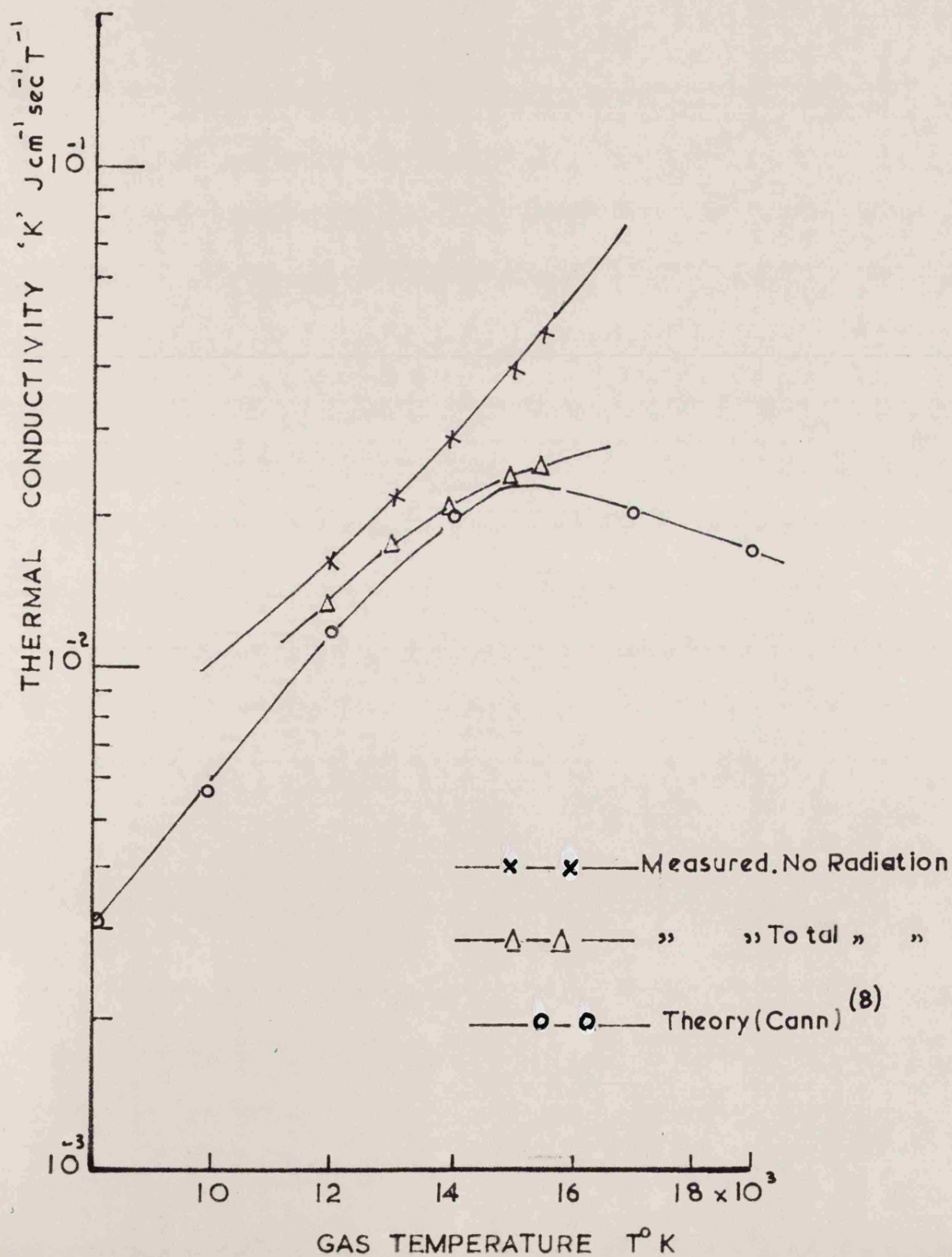
FIG 21

MEASURED THERMAL CONDUCTIVITY (AXIAL)

Demonstration of the

$K \propto T$

importance of Thermal Radiation



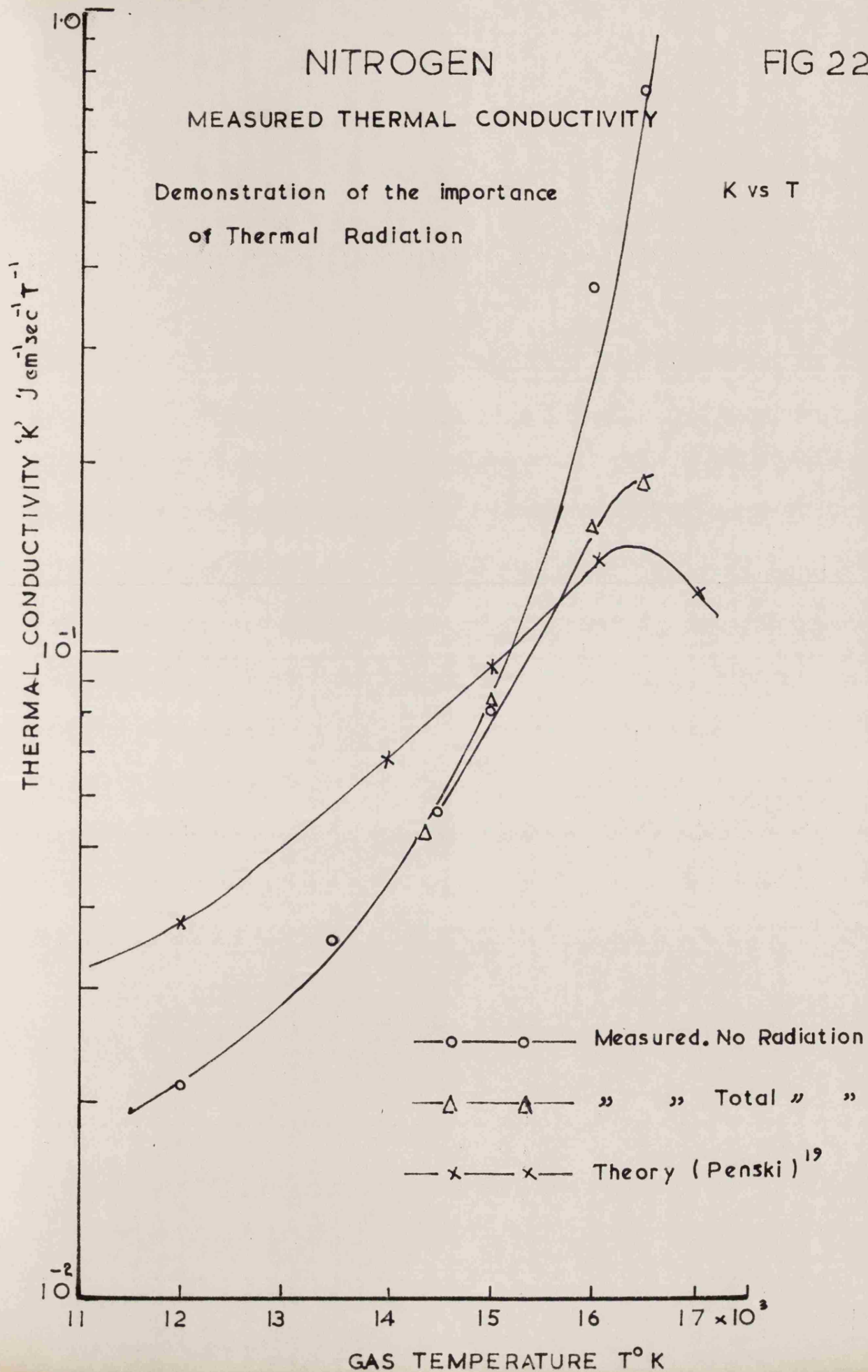
NITROGEN

FIG 22

MEASURED THERMAL CONDUCTIVITY

Demonstration of the importance
of Thermal Radiation

K vs T



A possible source of error may lie in the measurement of the energy transport by radiation. Taking for example, the 250 ampere nitrogen arc, with an axial temperature of 16000°K the measured radiation intensity was 350 w/cm³. To bring the measured thermal conductivity into line with the theoretical predictions, the specific intensity (w/cm³) in equation(11.2) would have to be much larger than this measured value.

In these experiments the radiated power is

$$Q_{TOT} = \int_{\lambda_1}^{\lambda_2} Q_{\lambda} d\lambda \quad (14.2)$$

where λ_1 and λ_2 are limits set by the optical system of approximately 0.25 μ m to 3.0 μ m.

A large amount of radiation is generated at much shorter wave lengths than 0.25 μ m and this may be significant.

The calculation of total radiation intensities is considered in the next chapter.

Using these total specific radiation intensities, the real thermal conductivities of argon and nitrogen were re-calculated using the "Two Temperature" method. The results obtained are shown in Figures 21 and 22.

Chapter 3

The Transport of Radiation Through a Gas

3.0 Theoretical Background

Problems concerned with the radiative transport of energy in high temperature, high pressure plasmas can be divided into two classes. The first is concerned with radiation transport through and out of an optically thin plasma. The second is the transport of radiation through an optically thick plasma. For this latter condition, energy is transported through the gas by an absorption-re-emission process. In this mode, the mean free path of the radiation is small, but the magnitude of the transferred radiation may be large where thermal gradients exist such as in arc columns. Whether the radiation is emitted under optically thick or thin conditions the following basic equation may be used to describe its transfer

3.1 The Generalised Spectral Distribution Equations

The radiation from a slab of gas in thermodynamic equilibrium at a temperature T , and x cm thick can be predicted from (47,94)

$$\frac{dQ}{dA d\lambda d\Omega} = \left(1 - e^{-\alpha'_{\lambda} x}\right) \left(\frac{dB(\lambda, T)}{dA d\lambda d\Omega}\right) \quad (1.3)$$

where $B(\lambda, T)$ is the Planck function for the black body intensity.

$$\frac{dB(\lambda, T)}{d\lambda d\Omega} = \frac{2hc^2}{\lambda^5} \left(e^{hc/\lambda kT} - 1 \right)^{-1} \quad (2.3)$$

and $1 - e^{-\alpha'_\lambda x}$ is the Emissivity of the radiating gas, α'_λ is the apparent absorption coefficient corrected for the radiation added by induced emission. The ratio of the apparent absorption coefficient to the true absorption coefficient is given by

$$\frac{\alpha'_\lambda}{\alpha_\lambda} = \left(1 - e^{-hc/\lambda kT} \right) \quad (3.3)$$

α is sometimes defined as the probability that a photon of energy $\frac{hc}{\lambda}$ will be absorbed in unit path length of the gas. By combining equations (1.3), (2.3), and (3.3) and using the first term for the series expansion of $e^{-\alpha'_\lambda x}$ thus requiring $\alpha'_\lambda x \ll 1$, the following commonly used formula for the radiation emitted from an optically thin layer is obtained.

$$\frac{d\mathcal{O}}{d\lambda d\Omega} = \alpha_\lambda x \frac{2hc^2}{\lambda^5} \exp(-hc/\lambda kT) \quad (4.3)$$

The basic difficulty in accurately calculating the radiation emitted from a volume of gas lies in estimating the wavelength dependent absorption coefficient .

3.2 Radiation for An Optically Thick Gas

In equation 4.3 the radiation at a fixed temperature is treated as being linearly dependent upon the absolute number of active species present. As the number increases, however, the radiating system becomes optically thick and the linear dependence of equation 4.3 breaks down. In addition, the new dependence of the radiation intensity is found to be quite different for the different types of transition producing the radiation: (free-free), (free-bound), (bound-bound). To theoretically predict the density or depth dependence of the total radiation for either line or continuum radiation, an accurate knowledge of the wavelength dependence of the absorption coefficients must be known.

3.3 The Absorption Coefficient Equations

For a complicated radiating system of lines and continuum, α_λ is the sum of the absorption coefficients of all radiating processes resulting from molecular, atomic, and ionic lines and free-bound and free-free, continuum radiation. The following sections give a brief theoretical description of the absorption coefficients for the various types of transitions.

3.3.1 Absorption of Line Radiation (Bound-Bound Transitions)

The absorption coefficient for the bound-bound type

of transition is given by⁽⁴⁷⁾

$$\alpha'_{\lambda} = 2\pi^2 r_o^2 c f_{mn} N_n \left[1 - \exp \left(- \frac{hc}{\lambda kT} \right) \right] L(w) \quad (5.3)$$

Where N_n is the number of absorbing atoms in the upper energy level, r_o is the classical electron radius, c is the velocity of light, f is the oscillator strength for the particular transition, and $L(w)$ is the normalised line shape. For high density and high temperature plasmas, the line shape is predominantly collision broadened and has a form closely described by a Lorentzian profile, namely

$$L(w) = \frac{W(\frac{1}{2}, \frac{1}{2})}{\pi} \frac{1}{(\nu - \nu_o + d)^2 + (W(\frac{1}{2}, \frac{1}{2}))^2} \quad (6.3)$$

where d is the line shift frequency $\Delta\nu$ and $W(\frac{1}{2}, \frac{1}{2})$ is the half-width of the line at half of the peak intensity. The three important parameters for determining the absorption coefficient are the f -value, the shift d , and the half-half width w of the line. These three parameters are difficult to calculate theoretically and for those calculations which do exist, relatively few experiments are available to support them.

3.3.2 Absorption of Continuum Radiation

(a) Free-Bound Transitions. The absorption coefficient for this type of transition may be analytically represented by⁽⁴⁷⁾

$$\alpha'_{\lambda} (fb) = \sum_{n,h} \sigma_{n,L} N_{n,L} (1 - \exp(-hc/\lambda kT)) \quad (7.3)$$

where σ is the continuum photo-excitation cross-section for the state n_l . N is the number of atoms in state n_l , λ is the wavelength, T is the temperature and h, c and k are the well known constants.

(b) Free-Free Transitions. This class of radiation, often referred to as Bremsstrahlung radiation results from the interaction of a free electron with an ion in which the electron is decelerated but not captured. The radiation is the energy which the decelerating electron is required to expend by classical and quantum mechanical theory.

The free-free linear absorption coefficient is given by (47)

$$\alpha_{\lambda}^{(ff)} = \frac{4\pi e^6 \lambda^3}{3\sqrt{3} hc^4 m^2 v} \cdot gZ^2 \frac{N_e N_i}{T^{\frac{1}{2}}} \quad (8.3)$$

where v is the electron velocity, g is the Gaunt factor, N_e and N_i are the electronic and ionic number densities respectively and Z is the effective nuclear charge.

An examination of the above equations reveals a number of fundamental parameters which must be determined experimentally to test theoretical predictions or to predict the radiation transfer in a high temperature, high density plasma. These parameters are the f value, half width, shift and line shape, and the re-combination cross-section and gZ^2 for the continuum

3.4 Radiation Intensity Calculation

As an alternative to assuming the optically thick gas to be

a Planckian radiator with selective absorption occurring, the individual intensities of radiation for the three processes, free-free, free-bound, and bound-bound can be calculated as a function of temperature and frequency. If we assume that all the radiation generated crosses the plasma in a series of "absorption-re-emission" steps and only the total radiation reaching the boundary is required, this approach may possibly be considered simpler.

3.4.1 Bound-Bound Radiation

As an estimate of the upper value of radiation emitted from a line in which no absorption takes place, that is from an optically thin line in a band pass of width $\lambda_2 - \lambda_1$, Q is given by (37)

$$Q = \frac{hc}{4\pi\lambda} Ag \frac{N}{U(T)} \left[\exp \left(-E_n/kT \right) \right] \int_{\lambda_1}^{\lambda_2} L(\lambda) d\lambda \quad (9.3)$$

where A is the transition probability, and g the statistical weight of the level. The intensity Q being in Watts.cm⁻³. sec⁻¹.steradians⁻¹. The time dimension being unit frequency interval for the emitted radiation. The line profile is taken to be Lorentzian which is the shape most dominant for lines at the conditions existing in the arc plasma.

Morris and Garrison⁽³⁷⁾ show graphs of vacuum ultra-violet line and continuum intensity in nitrogen against electron density for a range of wavelengths using their

experimental best values of Stark half-half widths, absorption 'f' numbers, and the Burgess and Seaton⁽³⁸⁾ recombination cross sections. Their results are compared with theoretical values of Wilson and Nicolet⁽³⁹⁾ for line radiation and Burgess and Seaton for the continuum.

3.4.2 Bremstrahlung and Recombination Radiation

The recombination and Bremstrahlung continuum radiation is a dominant spectral feature of a high temperature high density arc plasma. The measurement of the energy transport in high temperature plasmas is also strongly affected by the contribution accounted for by radiation energy transfer. As already stated the continuum radiation comprises two parts, that is the free-free and free-bound contributions.

Free-Free Radiation

The intensity of free-free radiation is given by the following equation.⁽⁹²⁾

$$Q_{\text{ff}} = 5.44 \times 10^{-46} \frac{n_1 n_e}{T^{\frac{1}{2}}} Z^2 g_{\text{exp}} \left(-\frac{C_2}{\lambda T} \right) \cdot \text{W.cm}^{-3} \text{sec}^{-1} \text{Sr}^{-1} \quad (10.3)$$

where n_1 = ion number density

n_e = electron number density

T = temperature

C_2 = second radiation constant

λ = wavelength

Z = the nuclear charge

g = the Gaunt factor

The Gaunt factor 'g' in equation (10.3) is a measure of the deviation of the charge structure of the atom or ion from the Hydrogenic model. The free electron is presumed to be influenced not only by the nuclear charge Z , but also by the electron shells through which it may pass during its collision trajectory. In passing through the outer shells the free electron is exposed to strong forces. Maecker and Peters⁽⁴⁰⁾, in extending the Krammer's theory⁽⁴¹⁾ of continuous absorption introduce a term $(Z + s)^2$ to allow for this effect where $(Z + s)^2$ lies between

$$(Z + s)^2 = n^2 \frac{E_i - E_n}{E_{i(H)}} \quad (11.3)$$

and $(Z + s)^2 = Z^2$

where Z = nuclear charge

E_i = ionisation energy of the atom

E_n = excitation energy of emitted photon

$E_{i(H)}$ = ionisation energy of hydrogen ion

The value of $(Z + s)^2$ is for argon and nitrogen, close to unity for the single ionised species as are the Gaunt factors for the same conditions^{(42), (43)}

Free-Bound Radiation

The intensity of radiation at any wavelength for free-bound

transitions for the i^{th} species in a gas is given by⁽⁹³⁾

$$Q_{fb} = 40.03 \times 10^{-24} \frac{\exp(-C_2/\lambda T)}{\lambda^3} \sum \sigma N_{i-1} \quad (12.3)$$

where the factor σ is the photo excitation cross section for the level contributing at the particular wavelength. It should here be noted that the species number density is N_{i-1} . That is, if radiation from photons generated by ions is considered then the number density of atoms is the value required for N .

3.5 Radiation Transfer in the Arc Column

It has already been pointed out that the calculation of thermal conductivity from a constricted arc requires that proper account be taken of energy transfer via radiation. To simplify the calculations of radiation transfer, a division was made in the wavelength spectrum at $2500\text{\AA}$. For wavelengths greater than $2500\text{\AA}$ the column is assumed to be optically thin and no radiation is absorbed. Therefore, any radiation generated reaches the walls of the arc column directly. Radiation at wavelengths less than $2500\text{\AA}$ was assumed to be absorbed and re-radiated within the column as it crossed to the walls of the generator.

The short wavelength cut off point for the radiation is justified by reference to the optical bandpass of the quartz windows in the plasma column and the thermopile. Ideally,

a second wavelength limit of 25000 Angstrom units should also be considered but here the cut off was very indeterminate and a sharp delineation could not be made.

3.6 Comparison of Measured and Calculated Radiation Intensity

The calculation of the radiation intensity from a heated volume of gas is in principle straightforward and the basic equations are set out in Section 3.6. Provided that the number density of the active species, photo excitation cross sections and Gaunt factors for the continuum and transition probabilities or oscillator strengths and statistical weights of the various energy levels together with line widths for the line radiation are available as functions of temperature, the calculations reduce to a large volume of relatively simple arithmetic.

In the case of nitrogen, the photo excitation cross sections, line widths and transition probabilities for atomic and singly ionised species are tabulated in considerable detail by Wilson and Nicolet⁽³⁹⁾. The wavelength range considered being from about 600Å to 20000Å.

Additional information on nitrogen is provided by Morris and Garrison⁽³⁷⁾ for the strong vacuum UV lines. For optically thin nitrogen radiation, the oscillator strengths of Bates and Damgard⁽⁴⁴⁾ are tabulated by Griem⁽⁴⁵⁾

The Gaunt factors for singly ionised nitrogen are, as

already indicated in Section 3.3, set at unity so that $z^2 g = 1$.

The data available permits calculation of the total radiation and radiation in the optically thin pass band to be compared with each other and with the experimental measurements made in this study and the calculations made by other investigators.

For argon the amount of available information is much less. Photo ionisation cross sections are given by Samson⁽⁴⁶⁾ and some limited data on the infra-red line and continuum coefficients is provided by Morris⁽⁴⁷⁾ of the AVCO Corporation. For the line spectrum, the absence of line half width and transition data makes the calculation of the total line contribution susceptible to guesswork and a theoretical summation of the total line contribution made by Yakubov⁽⁵⁰⁾ was used in this study, no independent calculation being made. For the spectral lines generated in the following transitions:

4p $\rightarrow$ 4s

3d $\rightarrow$ 5p

5s $\rightarrow$ 4p

Yakubov's summation takes the following form using the 4p $\rightarrow$ 4s series as an example

$$\epsilon(4p \rightarrow 4s) = B\nu - \frac{\pi e^2}{m_e c}, N_a \cdot (\exp \bar{U}_m / kT) \sum_{i=1}^{30} f_i g_i$$

where ϵ is the total radiation as a function of temperature,

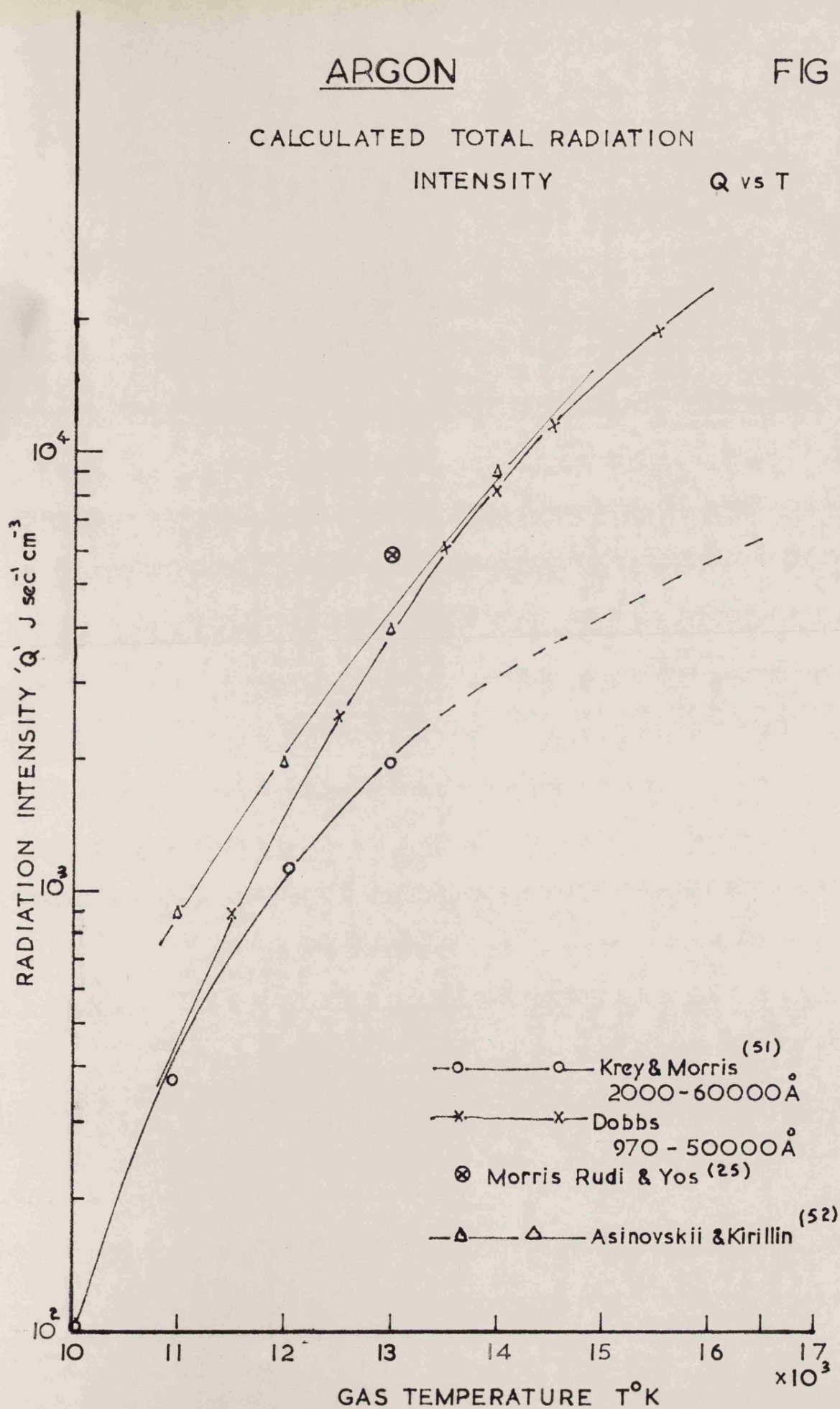
ARGON

FIG 23

CALCULATED TOTAL RADIATION

INTENSITY

Q vs T



NITROGEN

FIG 24

CALCULATED TOTAL RADIATION
INTENSITY

Q vs T

RADIATION INTENSITY 'Q' J sec⁻¹ cm⁻³

10²

10³

10⁴

10

11

12

13

14

15

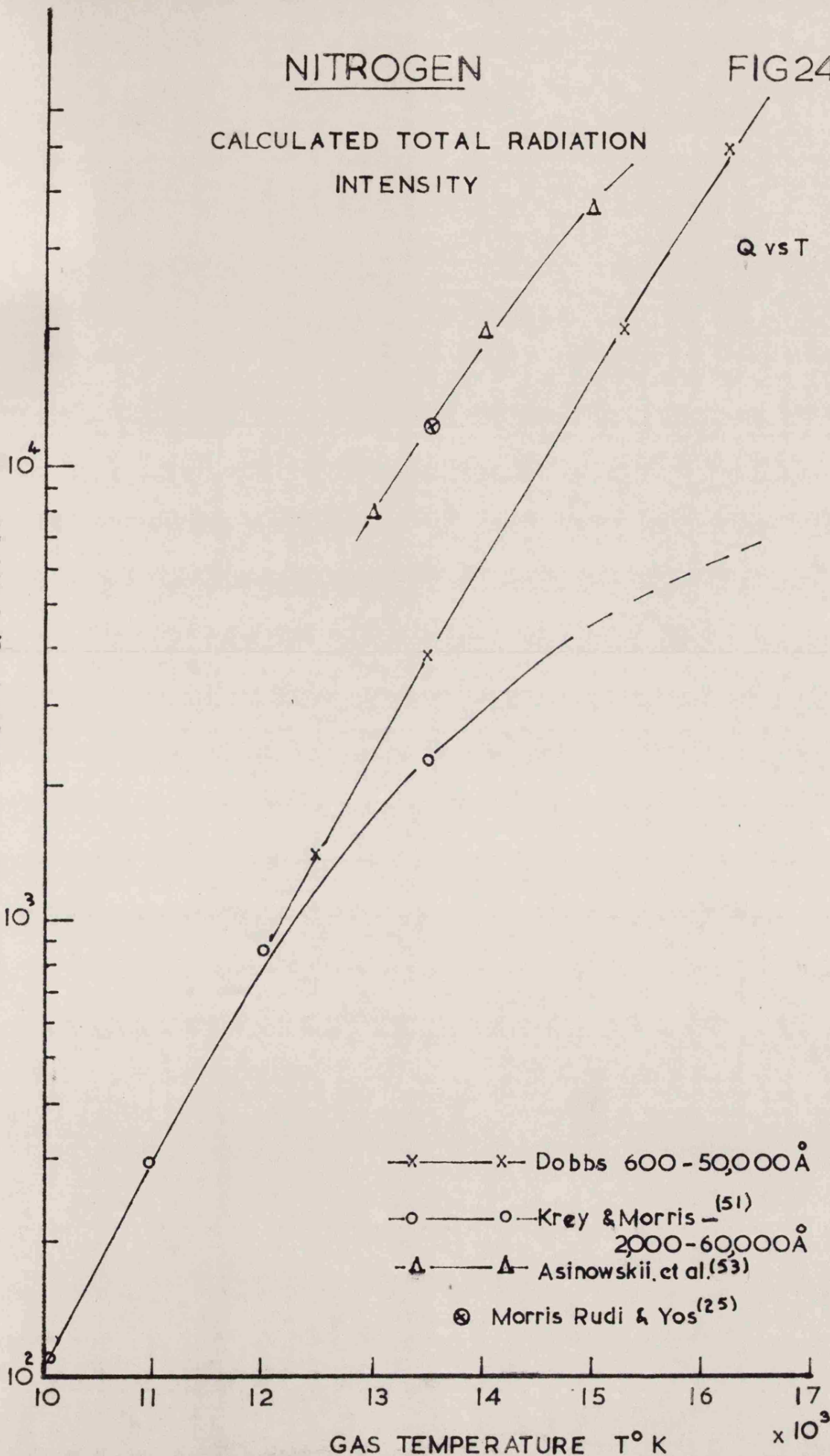
16

17

GAS TEMPERATURE T° K

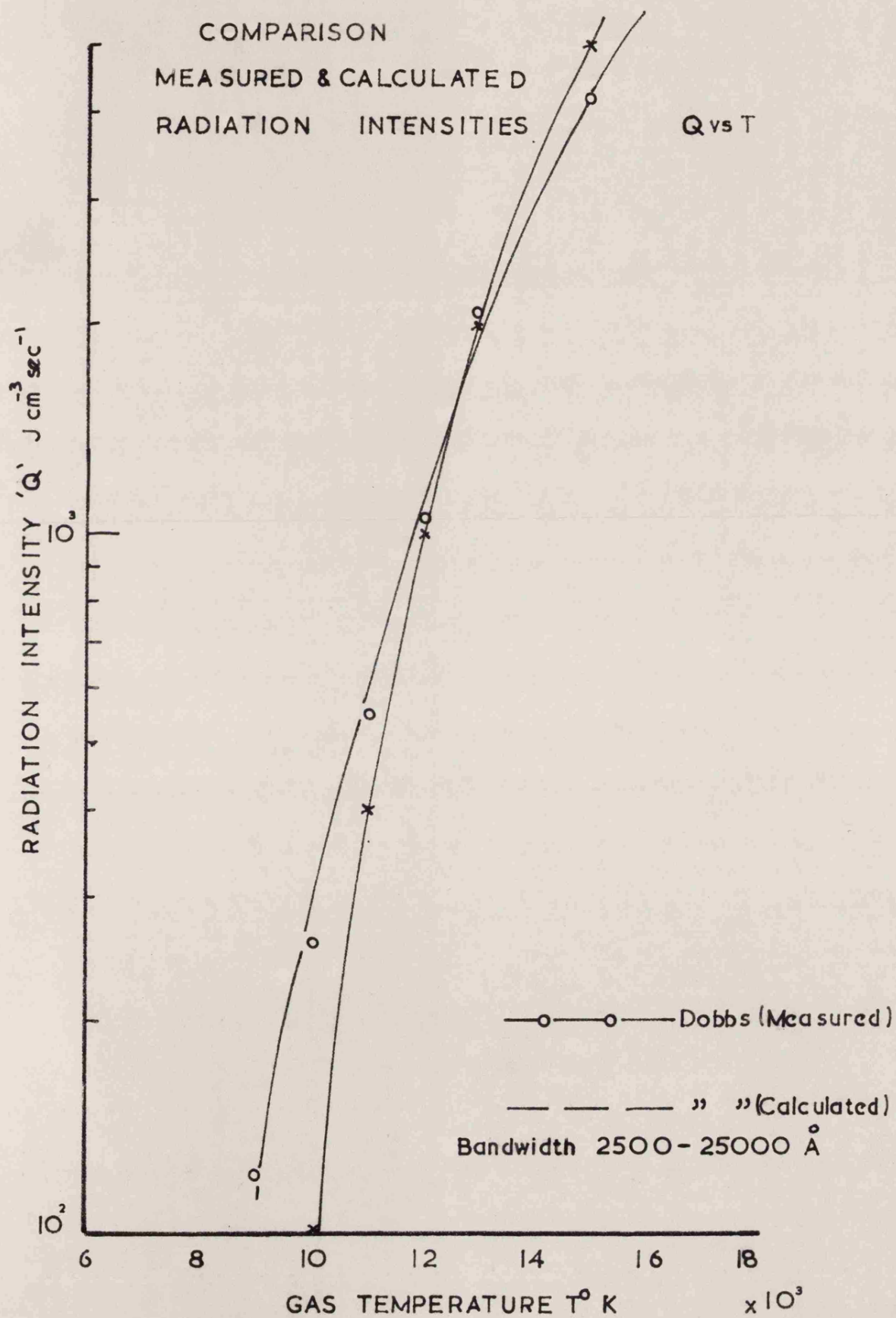
x 10³

- x-x- Dobbs 600 - 50,000 Å
- o-o- Krey & Morris⁽⁵¹⁾
2000 - 60,000 Å
- Δ-Δ- Asinowskii, et al.⁽⁵³⁾
- ⊗ Morris Rudi & Yos⁽²⁵⁾



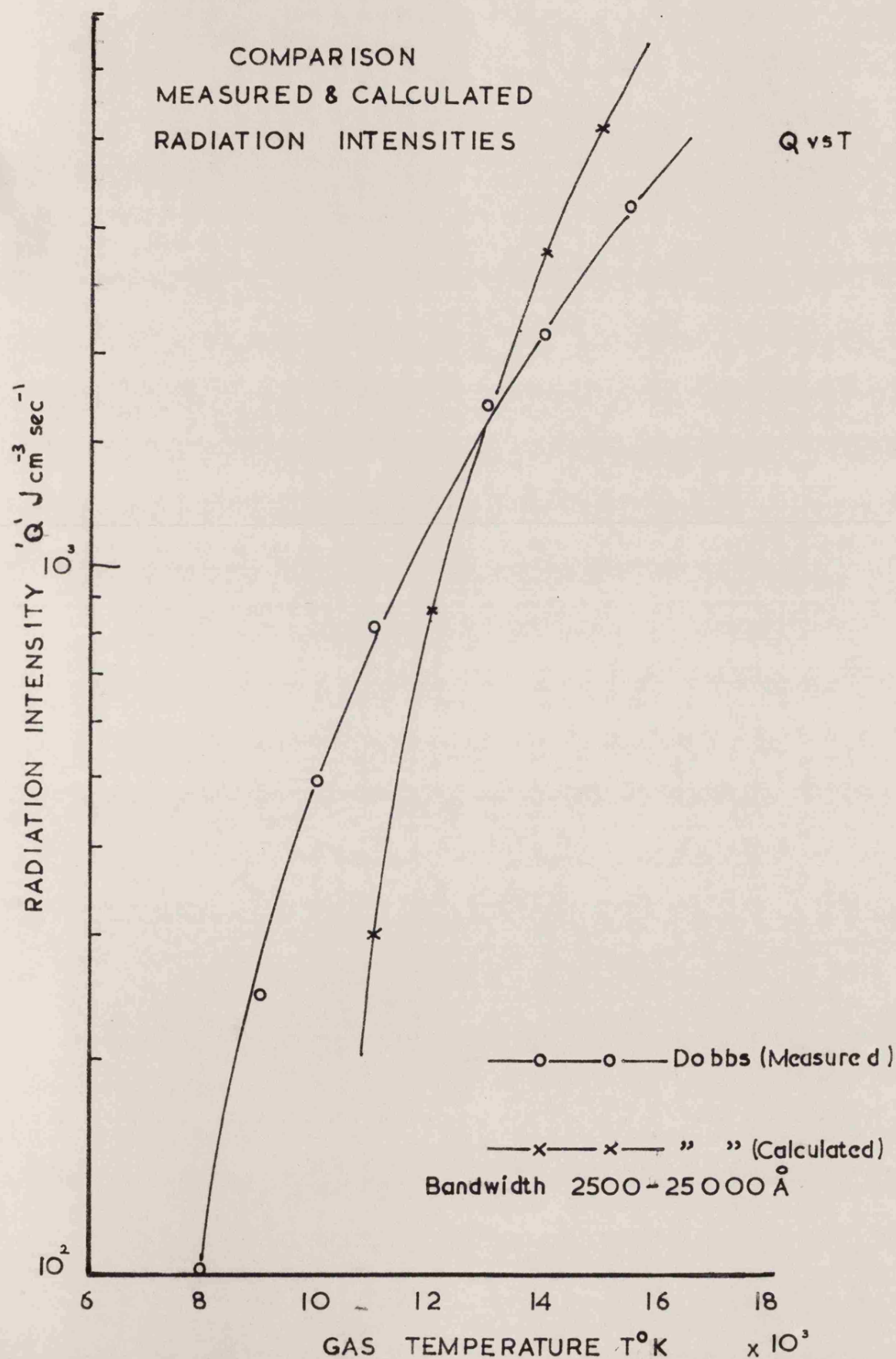
ARGON

FIG 25



NITROGEN

FIG 26



B_ν , the black body emission, $\bar{U}_m$ is the average excitation energy of the 4s levels and f_i and g_i the absorption oscillator strengths and statistical weights. The summations $\sum f_i g_i$ are open to some dispute. In the above example, for instance, Olsen⁽⁴⁸⁾ makes the value to be 13, whereas Razumovskaya⁽⁴⁹⁾ computes it to be 27. The Yakubov calculations have, however, been compared with measurements of the line contribution and found to be in good agreement. This was done by the author for the optically thin region of the spectrum by measuring the total radiation in the range 2500Å - 25000Å and subtracting the calculated continuum contribution..

Figures 23 and 24 show the calculated total radiation intensities for argon and nitrogen as a function of temperature. On the same graphs the results of other investigators are included for comparison^{(42), (51), (52), (53)}

Figures 25 and 26 show the calculated "optically thin" radiation as a function of gas temperature compared with the measured intensity over the same wavelength range.

Chapter 4

Measurement of the Viscosity of High Temperature Plasmas

4.0 Theory

If the cylindrically symmetric central portion of an arc plasma column, well away from the perturbations of electrodes, is considered, a differential equation similar to the thermal energy balance can be written. The distribution of the velocity V_r is given by the balance of the forces, pressure drop and viscous shear stress.

$$P = - \frac{1}{r} \frac{d}{dr} \left(r \eta \frac{dV}{dr} \right) \quad (1.4)$$

In this equation V is the velocity, η is the viscosity of the fluid and P is the axial component of the pressure gradient. Radial mass flows are assumed not to exist.

This equation is a form of the Poiseuille pressure drop law. It can only be applied as it stands to laminar gas flows and this therefore limits the range of application of the experiment to very low gas mass flows and relatively high arc currents. The restriction on mass flow rate and hence velocity is, however, not too severe as at the high temperatures in the arc, the kinematic viscosity (η/ρ) is very high and low Reynold's numbers with laminar flows are obtained at relatively high velocities.

As a check that the flow approximated to a laminar

condition, a fine tungsten wire was inserted into the plasma column and the vapourised tungsten emerging from the plasma generator orifice showed a laminar pattern of stream lines for a considerable distance beyond the orifice. Also, the Reynolds number calculated from the mean viscosity at the operating temperature of the plasma is only about 20 (Dobbs M.Sc. thesis 1969). With a Reynolds number of this magnitude one might expect the flows to be laminar. However, this mean Reynolds number could be widely erroneous as no account is taken of the velocity and temperature profiles across the plasma jet.

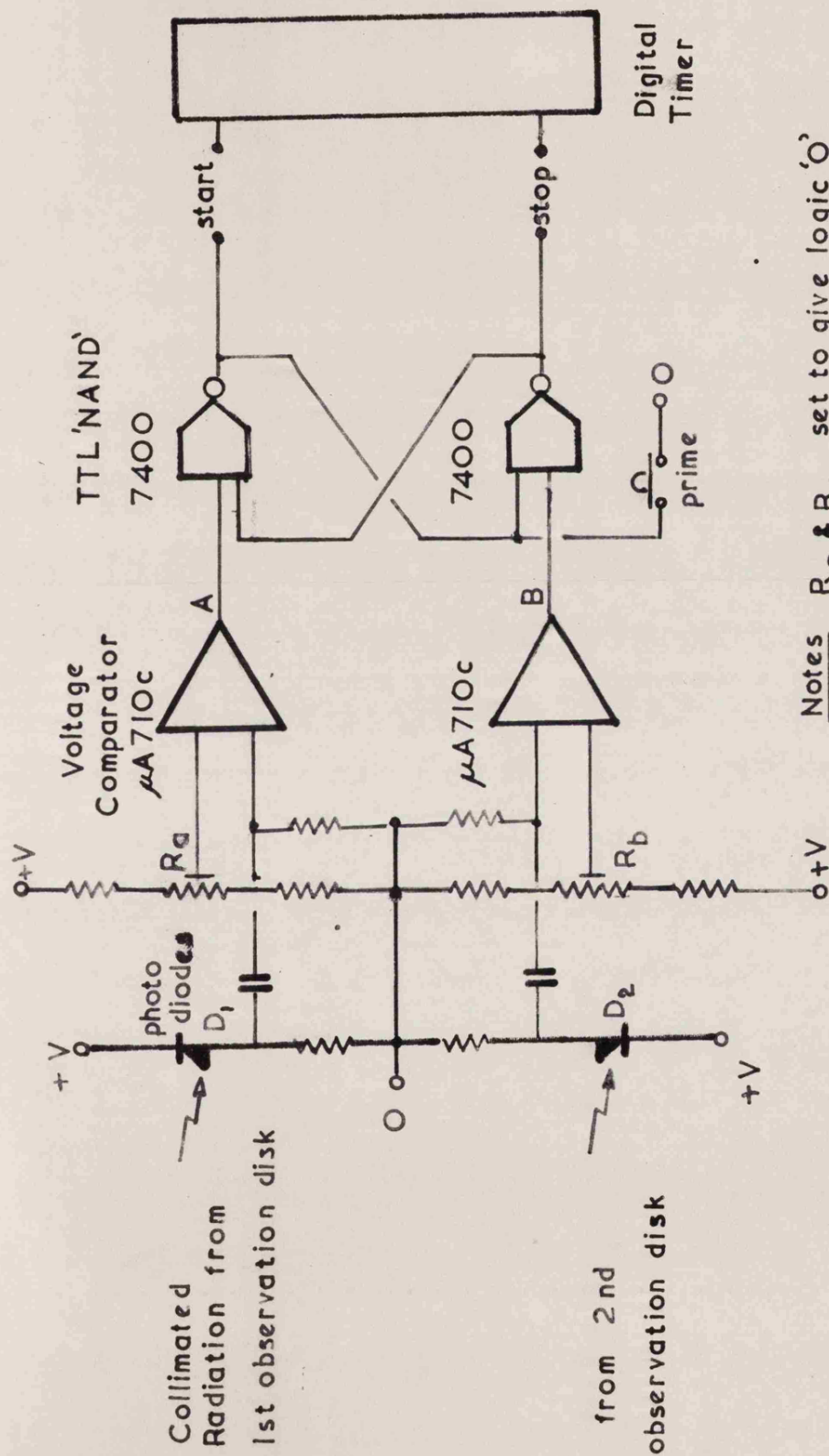
Integration and re-arrangement of equation (1.4) results in the following with y as a dummy for ' r '.

$$\eta = -P \frac{\int_{y=0}^{y=r} y \, dy}{\left(y \frac{dV}{dy} \right)_{y=r}} \quad (2.4)$$

If the pressure drop across the column and the plasma velocity as a function of radius is measured, then the viscosity can be calculated. Since temperature is also measured as a function of radius in the same apparatus it is possible to produce a relationship between temperature and viscosity.

4.1 Measurements

The pressure drop across the plasma column was measured by the use of manometers inserted into two of the column segments. Measurement of the radial velocity distribution requires additional instrumentation and this was achieved



Notes R_a & R_b set to give logic '0'

at 'A & B' when D_1 & D_2 are illuminated by the steady state arc column.

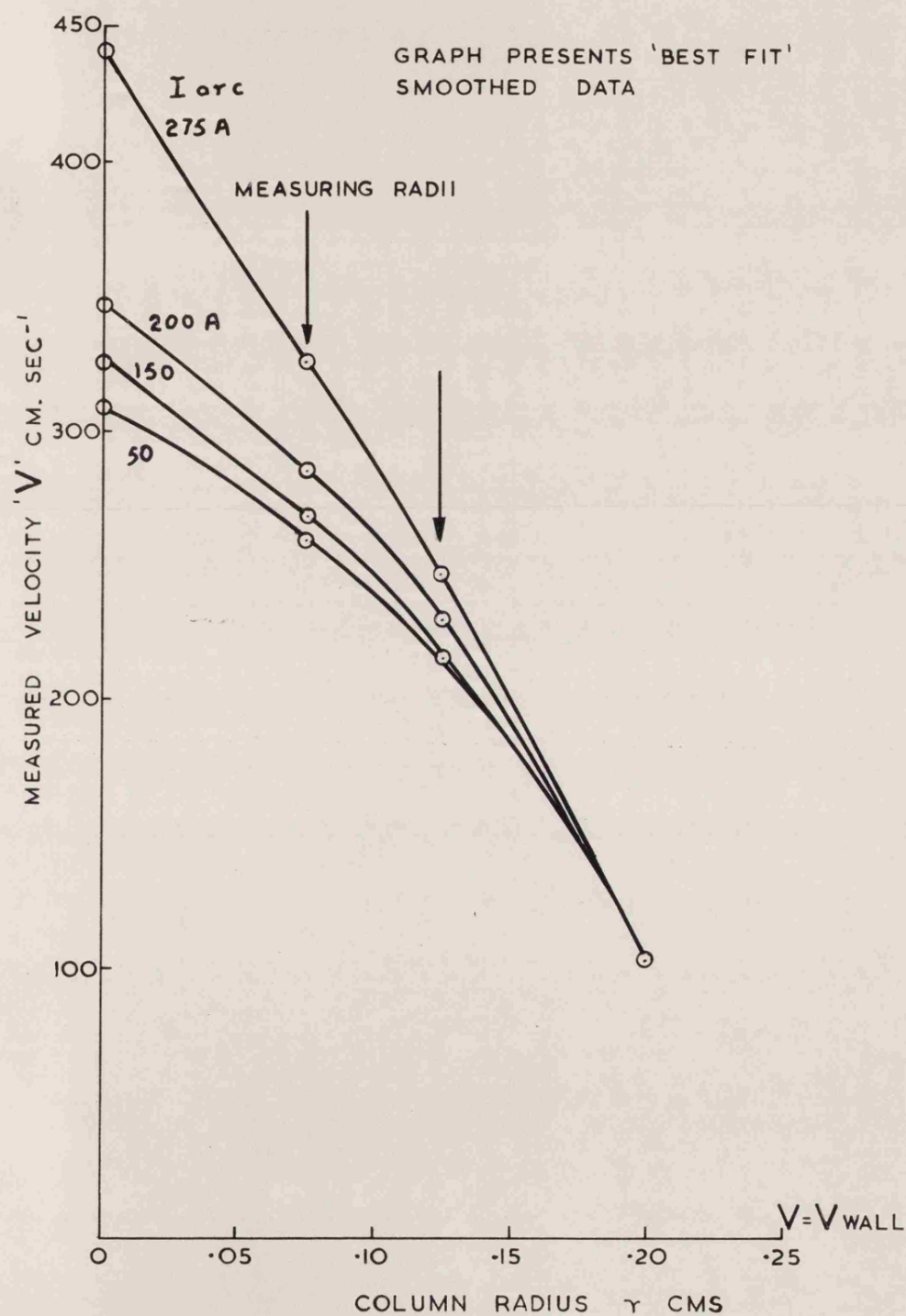
VELOCITY MEASURING SCHEMATIC

FIG 27

ARGON
MEASURED VELOCITY PROFILES

FIG. 28.

V vs r

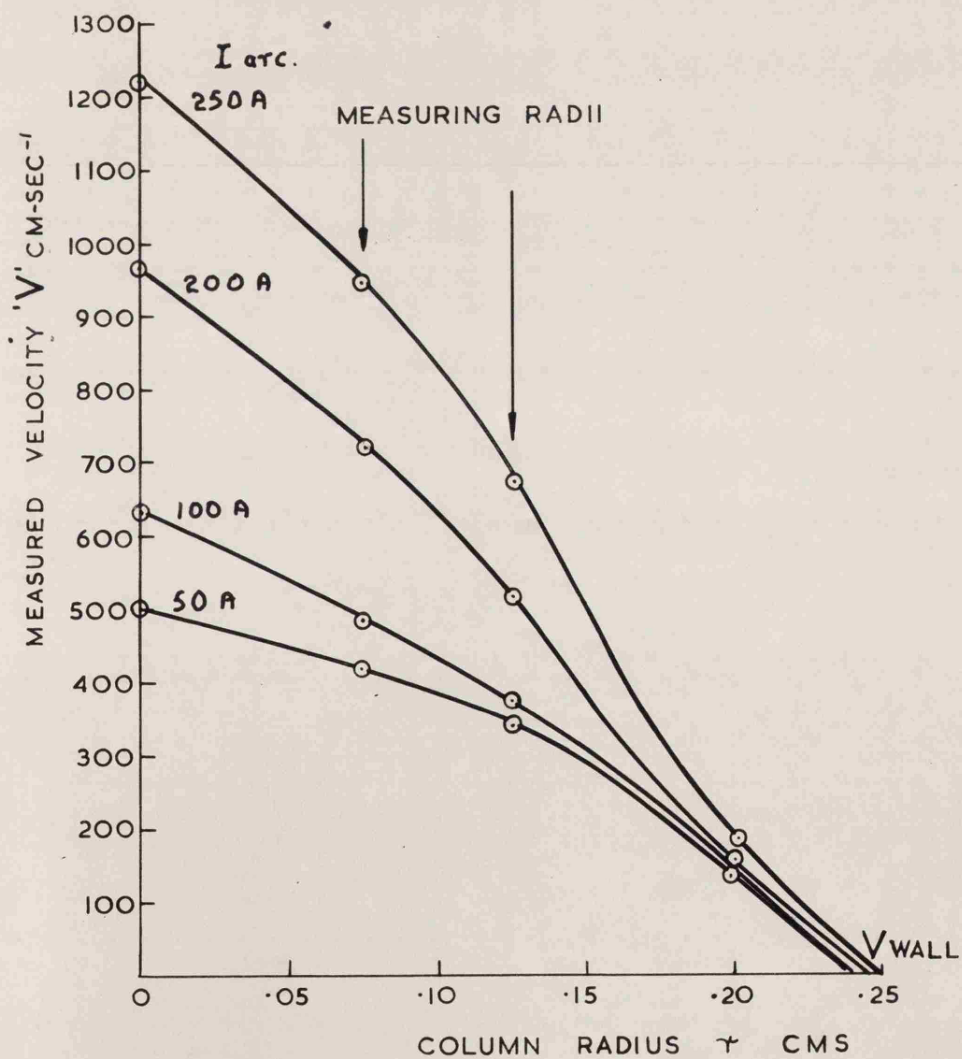


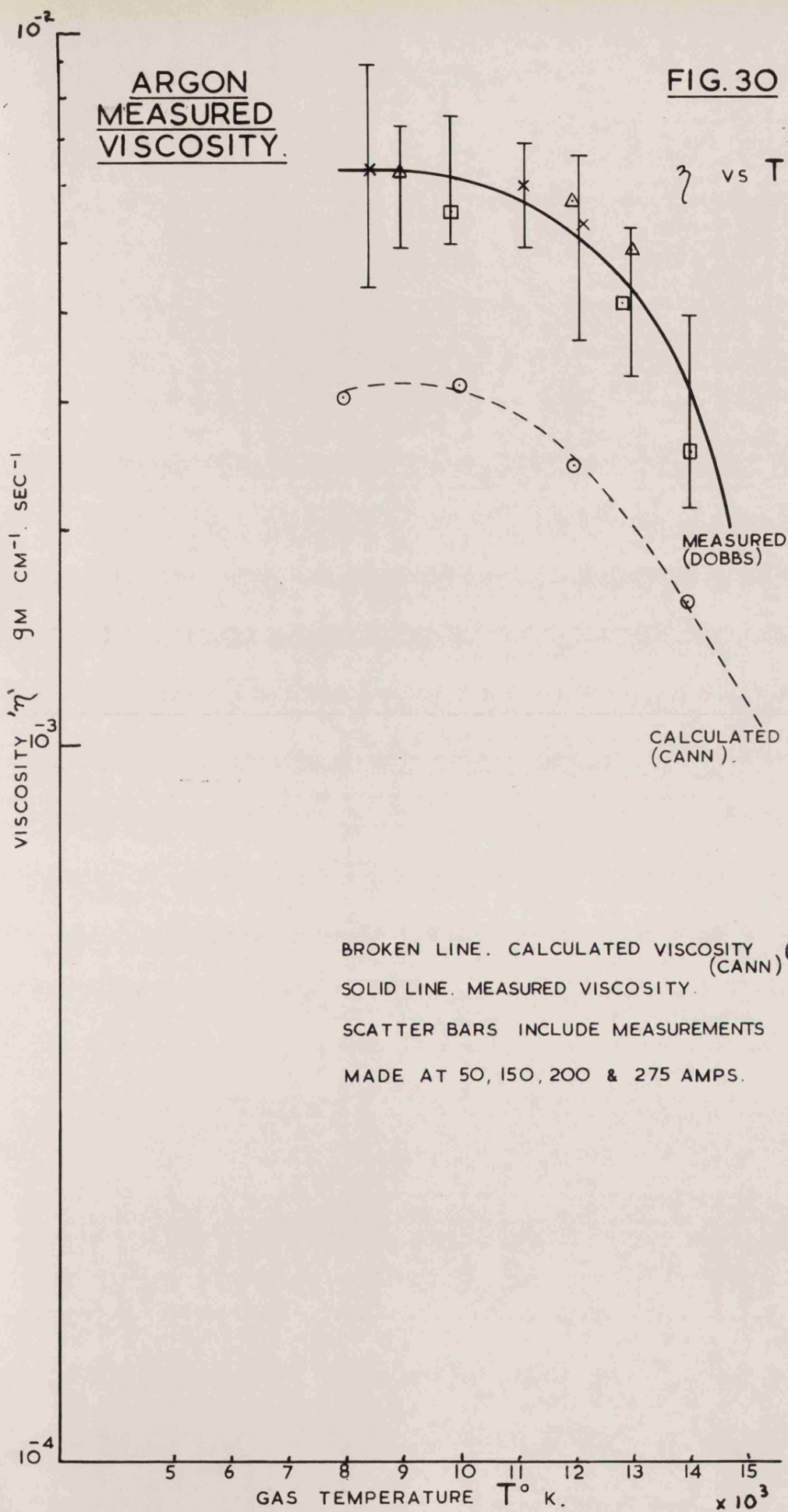
NITROGEN
MEASURED VELOCITY PROFILES

FIG. 29

V vs T

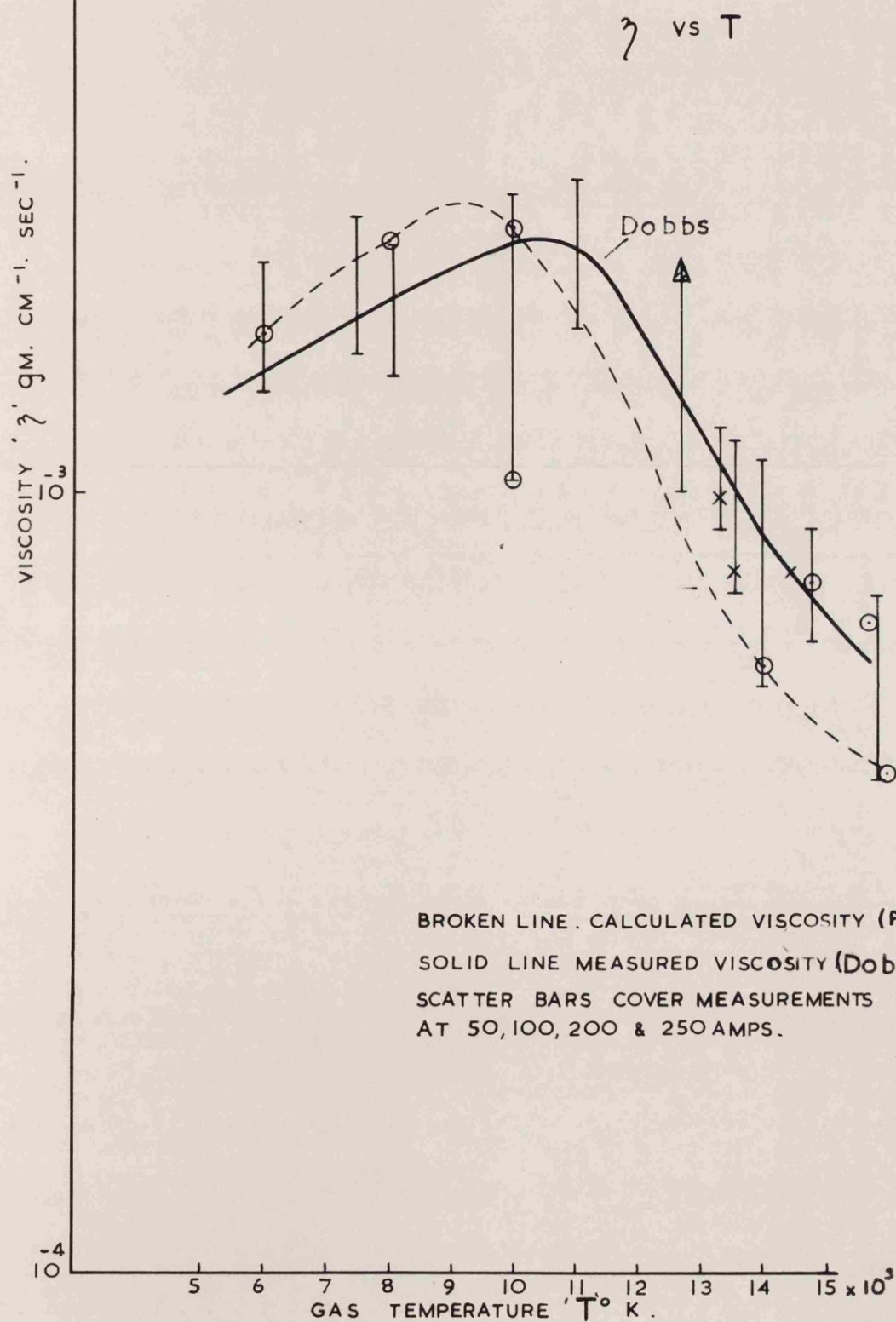
GRAPH PRESENTS 'BEST FIT'
SMOOTHED DATA.





NITROGEN MEASURED VISCOSITY

FIG. 31



by creating within the plasma a small intermittent discontinuity in the form of a high temperature zone produced by an electrical pulse injected into the steady state arc current. The hot discontinuity was seen to take up a characteristic shape in the plasma jet as it emerged from the orifice of the generator and this shape is assumed to be indicative of the velocity distribution. Two photoelectric cells with narrow angle collimators were set 1cm apart in the vertical plane. Each photo cell was connected to a level comparator which was set so as to be just below its operating point when the cell was illuminated by the normal plasma radiation. When the bright zone passed one or other photo cell the comparator changed its level. The first photo cell initiated a high speed electronic timer, the second cell stopping the timing process. Therefore, by scanning the two photo detectors across the plasma a measure of the axial velocity as a function of radius can be obtained. This is an extension of the method of velocity measurement described in the M.Sc. thesis of D.J. Dobbs, 1969. The circuit schematic for the velocity measurement is shown in Figure 27. To obtain effective operation of the photo-cell-timer-counter equipment, high spatial resolution could not be obtained but velocities on the arc axis and at the wall of the column were obtained with two intermediate values. However, even with this limited velocity data, measurements of viscosity made were in good agreement with the calculated values. Figures 28 and 29 show velocity profiles for argon and nitrogen as functions of arc radius with arc current as a parameter. Figures 30 and 31 show graphs of viscosity

versus arc temperature for argon and nitrogen obtained from the measured velocities shown in Figures 28 and 29. Also included on these graphs are calculated viscosity values published by other workers,^{(8),(19)}

In connection with the measurement of velocity, other techniques for high accuracy measurement were considered, in particular spectrographic velocity profile measurement using the Doppler shift of spectral lines in the plasma jet. A paper by Beth and Kling⁽⁵⁴⁾ reports on this technique. For velocities of the order of 100,000 cm/sec., the Doppler shift in the 4348Å line in argon only amounted to 70mÅ. In view of the much lower velocity anticipated in a laminar plasma jet, the making of such a measurement even with a high precision Fabry-Perot interferometer was a most unlikely process. Therefore, this method was not considered further.

Results reported by Freeston and Kelk⁽⁵⁵⁾ at the University of Sheffield using a microwave technique for velocity measurement must be discounted as due to a relatively long wavelength of the radiation compared with the dimensions of the plasma, no spatial resolution could be expected.

Chapter 5

The Calculation of Plasma Transport Coefficients

5.0 Introduction

As already pointed out in an earlier chapter of this study, the prediction of flow velocities, pressures and heat transfer rates from hot gases flowing over, or impinging on, cold surfaces requires a knowledge of the transport properties, i.e. density, viscosity, thermal conductivity, of the gas in question as a function of temperature. For the common gases, that is oxygen, hydrogen, nitrogen, argon etc., experimental values over a wide temperature range are available^{(8), (18), (19), (21)}₍₅₆₎ and these are supplemented with detailed rigorous calculations of the same properties. In many cases the measure of agreement between measurement and calculation is not good⁽²⁾. Complex experimental methods, some of which have been described in Chapter 2 and a variety of different theoretical approaches produce widely differing results.

A method for the estimation of transport properties is suggested which although open to criticism on the grounds of scientific rigour, does offer a rapid means of obtaining properties of gases for which the necessary collision cross-section data is not available. For purposes of comparison with available data, some transport properties for argon and nitrogen are calculated. The lack of rigour in this

approach is more than justified on examination of the results obtained and as a means of obtaining data for the solution of engineering problems the equations produced demonstrably have considerable value.

5.1 Summary of Methods of Calculation

Using the methods of statistical mechanics, a function $f(r.v.t.)$ defining the velocity and position of all the particles in a gas is postulated.

If the function "f" is known, than in principle, the physical properties of the gas are calculable provided that the number-density, temperature and velocity of the particles in the gas can be specified.

The distribution function "f" can be obtained from the kinetic equation obtained by considering all the forces acting on the gas. The collisionless kinetic equation may be written as

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{f}}{m} \frac{\partial \mathbf{f}}{\partial \mathbf{r}} = 0 \quad (1.5)$$

where the first term on the left-hand side is the time dependent variation of the function f.

The second term represents the presence of gradients in the distribution of particles.

The third term represents the influence of macroscopic fields.

If particle collisions are considered, then the kinetic equation becomes

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m} \frac{\partial f}{\partial \mathbf{r}} = \left(\frac{\partial f}{\partial t} \right)_{\text{collisions}} \quad (2.5)$$

where the right-hand term in the equation is the rate of change of the function f due to collisions.

This collision term can vary in form but in gas kinetics the simplest form of this term is the Boltzmann collision integral which considers particles making elastic, binary collisions.

If the right hand side of equation (2.5) is written as

$$\left(\frac{\partial f}{\partial t} \right)_{\text{coll}} = \sum_j \left(\frac{\partial f}{\partial t} \right)_{\text{coll } j} \quad (3.5)$$

the equation is called the Boltzmann equation⁽⁵⁷⁾

Unfortunately, in real gases, particularly at high temperatures, the collisions are not all binary and momentum is not conserved. This can lead to serious errors in

calculating gas properties from this equation.

Provided that the gas is in thermal equilibrium (that is external forces are not great enough to mask the collisional equilibrium in the gas) the distribution function f is normally considered to be "Maxwellian" and is given by equation (4.5)

$$f_{\text{maxwell}} = N \left(\frac{m}{2\pi kT} \right)^{3/2} \exp - (mv^2/2kT) \quad (4.5)$$

Thus, the application of rigorous gas kinetic theory leads to the Boltzmann collision equation, an integro-differential equation, which describes the changes in the distribution function for the constituent species in the gas. Solving the Boltzmann equation for this distribution function leads to the calculation of the transport coefficients. Unfortunately, this solution has only been evaluated in a few cases and demands very considerable mathematical effort. Solutions have been provided by Enskog⁽⁵⁸⁾, Chapman and Cowling⁽⁵⁹⁾, Waldmann⁽⁶⁰⁾ and others.

5.1.1. The Transport Coefficients

The physical properties of a gas, i.e. electrical conductivity, thermal conductivity, diffusion coefficients and viscosity are referred to as transport coefficients. These are coefficients relating external forces acting on the gas to the transport of some associated property, thus:

Electrical conductivity relates the external electrical

field and the transport of charge.

Thermal conductivity relates the thermal gradient and the transport of heat.

Diffusion coefficient relates density gradient and mass flow.

Viscosity relates the velocity gradient and momentum flux.

The major obstacle to be overcome in calculating the value of these coefficients is the solution of the complete Boltzmann equation with the necessary collision integrals. For hypothetical gases containing only rigid spheres making elastic collisions, the terms describing the interaction of the particles can be obtained from classical mechanics. The resulting equation describing two body elastic collisions for spherical particles, although non-linear, can be solved analytically and at moderately low temperatures a reasonable approximation to the observed transport properties of real gases can be obtained.

For molecular gases having multiple internal degrees of freedom, the two particle model does not apply although corrections (Eucken)⁽¹⁷⁾ can be made which give a more realistic picture. At high temperatures, however, real

gases will consist of molecules, atoms, ions and electrons all interacting in a complicated manner with both elastic and inelastic collisions occurring. In this case, the collision integrals can only be obtained from experiment. The methods of statistical mechanics are not able to provide a solution to the problem although complex force laws can be postulated which will account for some interactions. These laws involve functions of interparticle separation and are generally applicable only over a limited temperature range^{(11),(16),(61)}.

Under such conditions any simple treatment of the Boltzmann equation can cause gross inaccuracies in the calculated value of the transport coefficients. For example, the solution of the linearised Boltzmann equation given by Boyd and Sanderson⁽⁵⁷⁾ with a simplified treatment of the collision term predicts a monotonic variation of thermal conductivity and viscosity with temperature. Experimental observations and measurements show that this is not the case when dissociation and ionisation occur.

To obtain meaningful high temperature transport coefficients, it is necessary to use a collision term representing all the possible interactions in a multi-component gas. This problem is of great mathematical complexity and gas transport properties are usually evaluated with severe restrictions imposed on the number of possible interactions which may occur.

Typical of this approach is the Enskog series solution of the Boltzmann equation^{(58),(17)}. By a perturbation technique, a series solution of the kinetic equation is obtained. The transport coefficients can then be expressed in terms of the coefficients of an expanded polynomial series for the perturbed distribution function and the flux vectors describing the flux of mass, momentum and energy. The number of terms in the finite Sonine polynomial series determines the order of approximation of the solution. Even this mathematical technique only considers mixtures of simple non-reacting molecules making elastic collisions. Modifications to the theory are required to handle multi-component mixtures where reactions between, for instance, ions and electrons are present and even then, unless the precise force laws relating to the particle interactions are known, recourse must be had to intelligent guesswork or experiment.

5.2 An Alternative Approach

In an ionised gas or plasma, a range of simultaneous interacting energetic processes occur including the transport of electrical charge by the diffusion of electrons relative to ions; ambipolar diffusion, that is the flow of charged particles relative to neutral particles; energy transfer by thermal conduction and species diffusion; re-combination of electrons and ions and the transport of momentum in the gas.

Analytic solutions to the Boltzmann equation for systems of atoms, ions and electrons by, for instance, the method of Chapman or Enskog is, as already stated, fraught with mathematical difficulties. With this type of solution it is extremely difficult to account for all the interactions which occur. The thermodynamics of irreversible processes serves to demonstrate the full range of the possible particle interactions. However, its use is not a panacea for all difficulties and should really be considered only as a book-keeping system for accounting for the interactions between the many processes occurring in the gas or fluid under examination.

To fully comprehend the motion of all the various species in an ionised gas relative to the centre of mass of the gas, and of the energy transport within the gas, it is necessary to list all the phenomena for consideration before commencing an analysis. Although not exhaustive, the list given below indicates some of the processes occurring in a partially ionised plasma.

- (i) The diffusion of electrons relative to ions (the flow of electric current)
- (ii) The flow of charged particles relative to neutrals (ambipolar diffusion). These flows occur when there are gradients in either species density, pressure, temperature or electrical potential.
- (iii) Transfer of energy by thermal conduction and

species diffusion.

- (iv) The transfer of energy to the gas from an external electric field.
- (v) Recombination processes between electrons and ions and the ionisation of neutral species in the gas.
- (vi) The rate at which the various processes described above take place are all characterised by reaction or relaxation times. For the establishment of equilibrium these relaxation times are of the utmost importance.

A complete solution to the relevant Boltzmann equation considering all of these processes is not yet available. Spitzer and Härm⁽¹²⁾ have considered the problem for a fully ionised gas and amongst many others Chapman and Cowling⁽⁵⁹⁾ have investigated various aspects of partially ionised gases with severe restrictions placed upon them, in particular restrictions which remove the majority of the possible interactions in the gas.

To apply the methods of irreversible thermodynamics as described by De Groot⁽⁶²⁾ and others the following scheme must be pursued.

- (i) The rate of entropy production per unit mass of gas is derived.
- (ii) All terms in this production equation representing reversible entropy changes are separated from

those representing irreversible production of entropy.

(iii) All terms that involve the irreversible production of entropy are written down as linear equations involving the product of a force and a flux. All fluxes and forces that are of the same tensor rank are linearly inter-dependent. Tensors of rank 2 and higher representing forces and fluxes may be contracted to produce new terms.

(iv) The resulting system of 'm' linear equations each have "n" conjugate coefficients. These $m \times n$ coefficients, by the application of the Onsager reciprocity relations, are then reduced in number.

It is at this point that the strength of the method begins to fail since any coefficients that are left after applying the Onsager relations must be evaluated, as in the case of the Boltzmann equation solutions, from kinetic theory or experiment. However, the procedure does have a great advantage in that a picture of the transport properties of a reacting gas mixture can be built up from the results of a number of simple analyses.

Finkelburg and Maecker⁽⁷⁾ have examined a three component plasma which is not restricted as to the possible interactions. The equations produced are extremely complicated and are not applied to any specific problem. Furthermore, it is claimed by Cann⁽⁸⁾ that algebraic errors

occur in their work. A very complete exposition of this technique is given by Cann in a research memorandum⁽¹⁸⁾ and applied to evaluating the transport properties of argon and helium.

5.3 Some Possible Simplifications to the Calculations

The calculation of transport properties of some monatomic and polyatomic gases has been carried out by Amadur and Mason⁽¹¹⁾ using theoretical force laws for the species interactions. These calculations have been made up to very high temperatures but the gas has been considered as having a frozen composition so that only binary elastic collisions are considered. It is obvious even from relatively simple experiments and observations that the high temperature predictions in this work are very erroneous. At low temperatures, however, up to 1000°K the predictions are very accurate, as indeed they should be, since the equations used accurately describe the processes occurring. Similar accuracies, however, can be obtained by the application of simple kinetic theory as described in the classic work of Jeans⁽⁶³⁾. To improve the accuracy of gas models based on hard spheres, the simple kinetic theory is often modified by the application of the Sutherland correction⁽⁶⁴⁾. This is an empirical correction based upon the hypothesis that the particle size increases with temperature. The Sutherland correction is usually considered as being valid only over a relatively small temperature range. However, if the Sutherland correction is applied to simple kinetic theory, remembering that the gas must

be considered frozen, then up to very high temperatures such as those considered by Amadur and Mason⁽¹¹⁾, the Sutherland theory can be shown to be applicable. This correction is used in Section 5.7 to improve the accuracy of collision frequencies computed from room temperature mean-free-path data. However, this approximation to the transport coefficients is still erroneous at high temperatures when dissociation and ionisation occur, that is when the gas is no longer "frozen".

In view of the difficulties and apparent failings of rigorous methods for calculating the transport coefficients, it was felt worthwhile to look for some simpler technique of calculation which would give a working guide to realistic values. The next section of this report considered the derivation of some basic equations without recourse to the Boltzmann equation. An attempt is made to obtain the equations for non-reacting transport coefficients using the techniques of dimensional analysis and induction.

5.4 The Diffusion Coefficient

The mathematical relationship between the flux of particles and diffusion coefficient is given by

$$J_i = - D \frac{dp_i}{dz} \quad (5.5)$$

where D is the diffusion coefficient

J_i = mass flux per unit area in direction z

and $\frac{dp_i}{dz}$ = density gradient of species i

Then dimensionally equation (5.5) becomes

$$nMT^{-1}L^{-2} = - DnML^{-3}L^{-1} \quad (6.5)$$

or on re-arrangement

$$D = \frac{T^{-1}L^{-2}}{L^{-4}} = L^2T^{-1} \quad (7.5)$$

Now, recognising that the kinetic energy of a thermal particle is $k\theta$ which has the dimensions ML^2T^{-2} we can multiply the right hand side of equation (7.5) by MT^{-1}/MT^{-1} without affecting its dimensional properties

Then

$$D = \frac{ML^2T^{-2}}{MT^{-1}} \quad (8.5)$$

or

$$D = \frac{k\theta}{MT^{-1}} \quad (9.5)$$

where k = Boltzmann constant

θ = temperature $^{\circ}K$

For a hypothetical hard sphere gas, kinetic theory demands that the diffusion coefficient be controlled by the frequency of elastic collisions. The term T^{-1} in equation (9.5) has the dimensions of frequency, therefore, we can write

$$D = \frac{k\theta}{nmf_c} \quad (10.5)$$

where f_c is the collision frequency of particles of mass m and n the number of interacting particles per unit volume.

It is worthwhile noting that this equation for the diffusion coefficient agrees exactly with that obtained by the solution of the linearised Boltzmann equation given by Boyd and Sanderson⁽⁵⁷⁾, albeit for a fully ionised gas in which an electron mass predominates.

The equation for the diffusion coefficient can be written for the j^{th} species as

$$D_j = \frac{k\theta}{m_j n_j f_c} \quad (11.5)$$

which for a gas of atoms at S.T.P. becomes

$$D_a = \frac{k\theta}{m_a N_o f_c} \quad (12.5)$$

where m_a = atomic mass

N_o = Loschmidts Number

Finally to correct for change in species number density as the gas is heated we write

$$D_a = \frac{k\theta}{m_a \frac{n_a}{n_o} \cdot f_c} \quad (13.5)$$

where n_a = number density of atoms at temperature θ

5.5 The Coefficient of Viscosity

The mathematical formulation relating shear stress and the coefficient of viscosity is

$$S_{xy} = -\eta \frac{dV}{dz} \quad (14.5)$$

where S_{xy} = shear stress between unit planes in the direction z

η = coefficient of viscosity

$\frac{dV}{dz}$ = velocity gradient in direction z

Thus, dimensionally for n interacting particles of i^{th} species, per unit volume

$$n_i ML^{-1} T^{-2} = \eta \frac{LT^{-1}}{L} \quad (15.5)$$

$$\text{or } \eta = n_i ML^{-1} T^{-1} \quad (16.5)$$

Once again, for a thermal particle, its kinetic energy may be written as $k\theta$ which has the dimensions $ML^2 T^{-2}$. This can be introduced into equation (16.5) by multiplying the

right hand side by $L^3 T^{-1} / L^3 T^{-1}$ without affecting its dimensions.

Then

$$\eta = \frac{n M L^2 T^{-2}}{L^3 T^{-1}} \quad (17.5)$$

which becomes

$$\eta = \frac{n k \theta}{L^3 T^{-1}} \quad (18.5)$$

As in the previous case of the diffusion coefficient, the viscosity of a hypothetical hard sphere gas is determined by the collision frequency, therefore the dimension T^{-1} can be written as f_c which is the collision frequency.

Since the number of particles n has been defined for unit volume, equation (18.5) can finally be written as

$$\eta = \frac{N_o k \theta}{f_c} \quad (19.5)$$

which once again agrees with the equation obtained from the Boltzmann equation, where N_o = Loschmidts number = 2.7×10^{19} atoms cm^{-3}

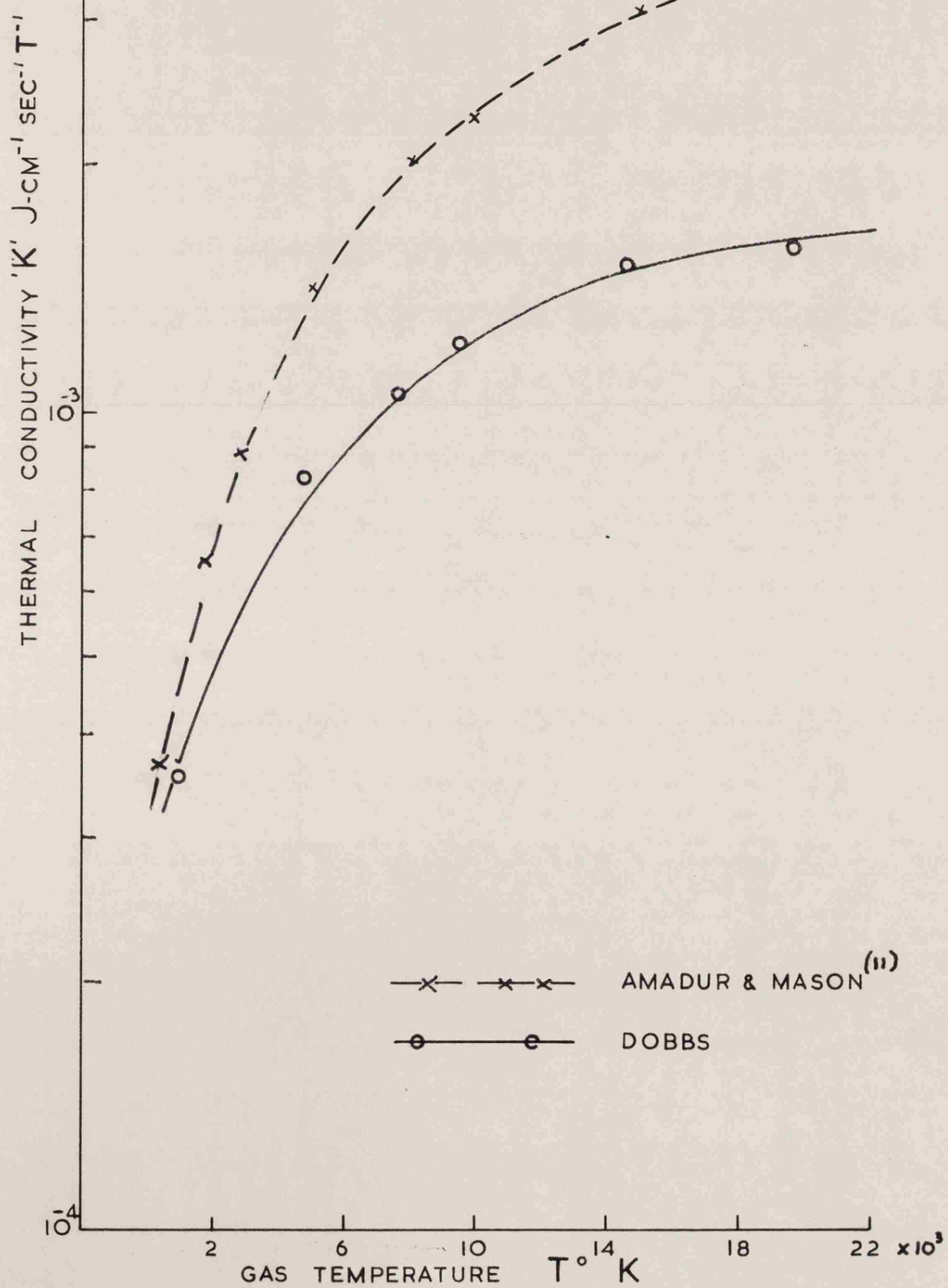
5.6. Thermal Conductivity

By a process of induction and analysis similar to that

FIG. 32.

$$K \vee T$$

CALCULATED THERMAL CONDUCTIVITY.

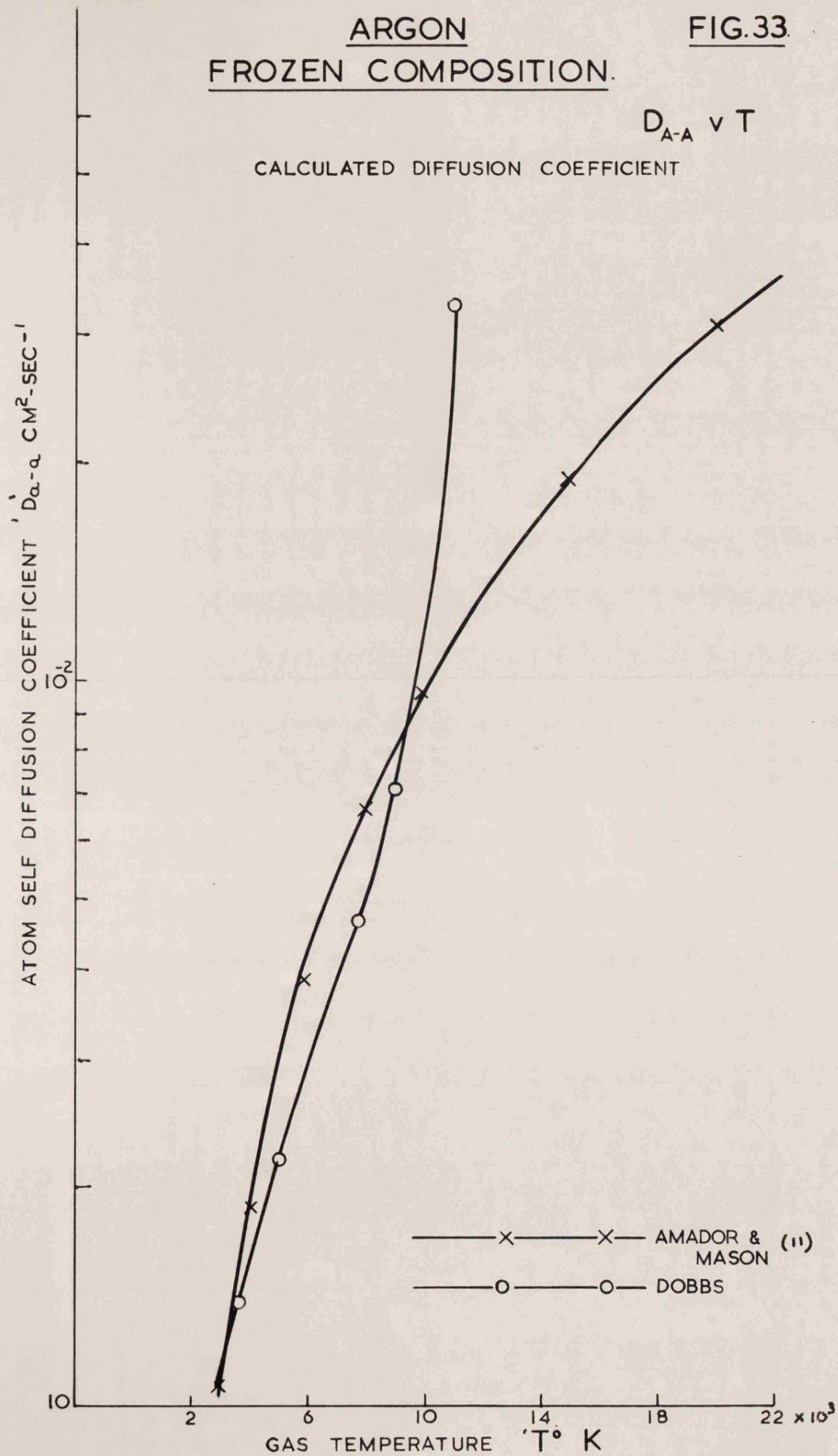


ARGON
FROZEN COMPOSITION.

FIG.33

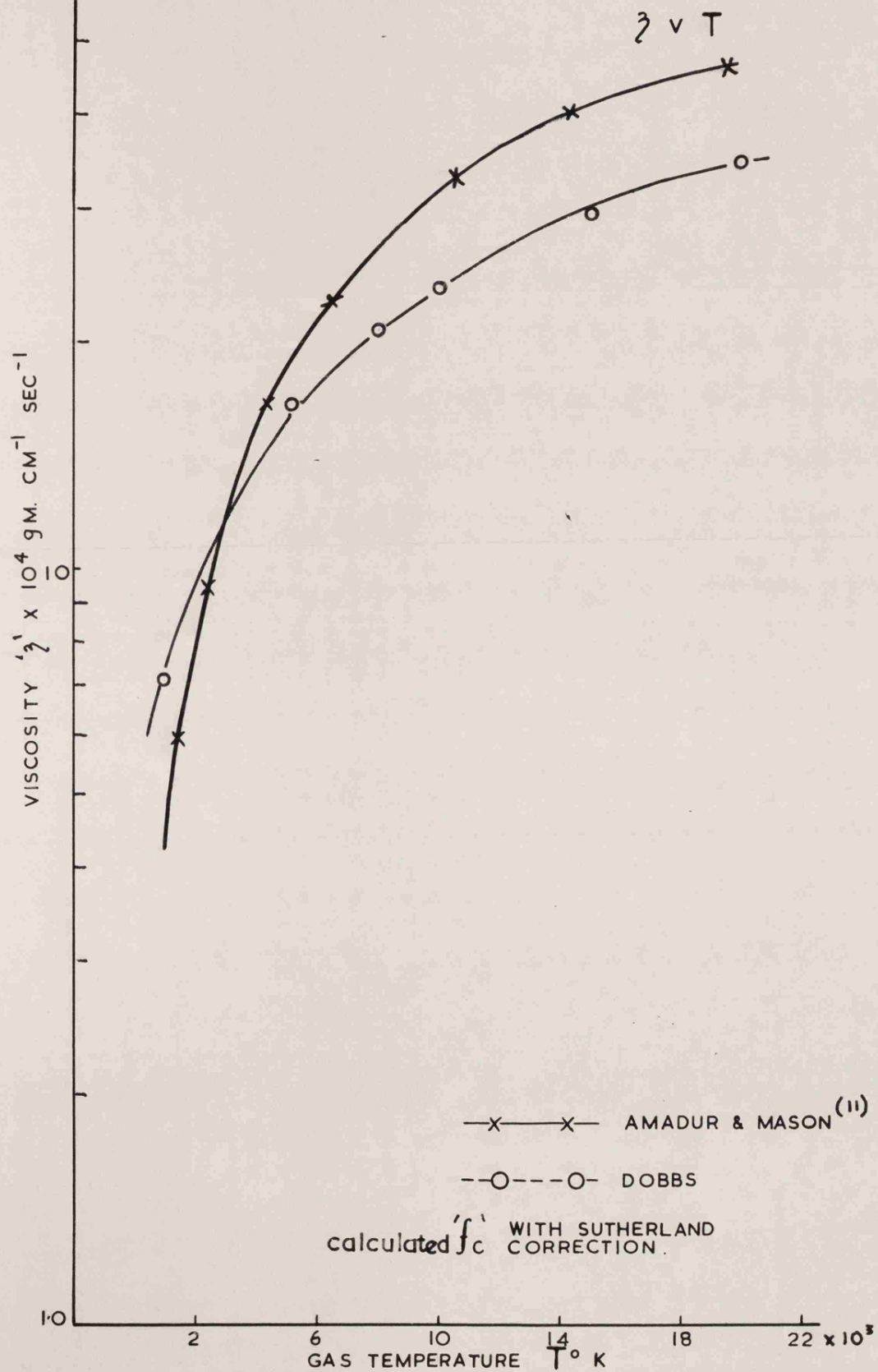
$D_{A-A} \propto T$

CALCULATED DIFFUSION COEFFICIENT



ARGON
FROZEN COMPOSITION
CALCULATED VISCOSITY.

FIG.34



set out for viscosity and diffusion, the thermal conductivity equation can be obtained. Alternatively by using the results obtained from simple kinetic theory⁽¹⁷⁾,⁽⁶³⁾ which give thermal conductivity in terms of specific heat and viscosity the thermal conductivity of a simple hard sphere gas can be written as

$$K = \frac{5}{2} \frac{N_o k \theta}{m f_c} \quad (20.5)$$

Figures 32, 33 and 34, show the frozen thermal conductivity, diffusion and coefficient and viscosity of argon calculated from these equations, the collision frequency f_c being obtained from a consideration of simple kinetic theory⁽¹⁷⁾. For comparison frozen composition coefficients calculated by Amadur and Mason⁽¹¹⁾ are also shown.

5.7 Calculation of Collision Frequencies

Assuming thermal equilibrium, then with a gas of atoms, the particle velocity at temperature θ can be obtained from Maxwell's distribution equation

$$W_{RMS} = \sqrt{\langle W^2 \rangle} = 1.224 \sqrt{\frac{2k\theta}{m}} \quad (21.5)$$

The collision frequency $f_c = \text{velocity}/\text{M.F.P.}$ The room temperature mean free path (MFP) for the gases under consideration being obtained from standard reference

works^{(3),(64)}. Since, however, we have already indicated that the mean free path will vary with temperature, a modification to these room temperature figures must be obtained. It has been shown by examination of the work of Amadur and Mason⁽¹¹⁾ that the Sutherland correction is valid up to high temperature, therefore this is used in adjusting the mean free path for the collision frequency calculation.

The ratio between the mean free path L_1 at a reference temperature θ_1 and L_2 at some elevated temperature θ_2 is given by

$$\frac{L_1}{L_2} = \frac{1 + T^1/\theta_1}{1 + T^1/\theta_2} \quad (22.5)$$

where T^1 is Sutherlands constant.

Table 1 in Appendix V sets out the calculation of "frozen" viscosity for argon by way of example. In this table the argon atomic mass is

$$m = 6.7 \times 10^{-23} \text{ grm}$$

$$N_o \text{ (Loschmidts number) is } N_o = 2.7 \times 10^{19} \text{ atoms/cm}^3$$

$$\text{and Boltzmann's constant is } k = 1.4 \times 10^{-6} \text{ ergs/k}^\circ$$

5.8 The Viscosity Coefficient at Ionisation (Monatomic Gas)

In an ionising monatomic gas, heavy particles (atoms)

still control viscosity since atom/electron and electron/electron collisions, due to the small mass and collisional size of the electron have very small momentum interchange. However, when ionisation commences, the number of atoms decreases as the number of electrons and ions increases. Therefore, per unit volume of gas the momentum transfer will decrease as the effective number density of particles taking part in collisions is reduced. It may be argued that since the ion has a mass only slightly less than that of the atom the total number of "heavy" particles involved in collisions is not changed, however, screening effects due to the charge on the ion make elastic collisions far less frequent.

It is proposed therefore that we write for the effective number density of the ensemble of particles in an ionising monatomic gas

$$n_{\text{eff}} = \frac{n_e + n_a}{n_a} \quad (23.5)$$

The viscosity η then becomes

$$= \frac{N_o k\theta}{f_c \frac{n_e + n_a}{n_a}} \quad (24.5)$$

n_e = number density of electrons as a function of temperature

n_a = number density of atoms as a function of temperature.

Table 2 in Appendix V shows this correction term for

ARGON
PLASMA

FIG.35.

D_A vs T

CALCULATED DIFFUSION COEFFICIENT

ATOMIC SELF DIFFUSION COEFFICIENT D_{A-A} $\text{cm}^2 / \text{SEC.}$

10

2

4

6

8

10

12

14

16

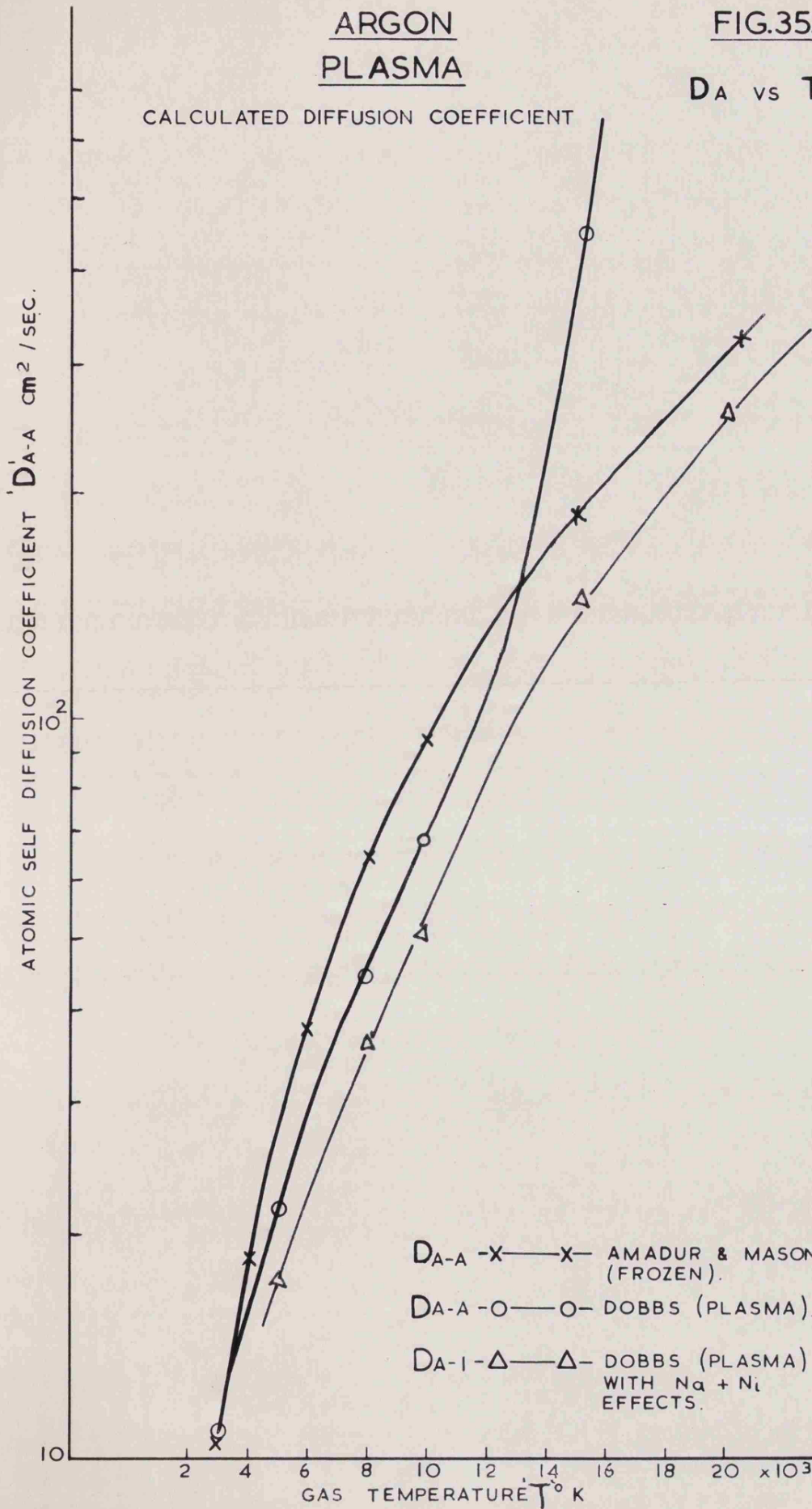
18

20

$\times 10^3$

GAS TEMPERATURE $T^\circ \text{K}$

- D_{A-A} -X-X- AMADUR & MASON⁽¹¹⁾
(FROZEN).
 D_{A-A} -O-O- DOBBS (PLASMA).
 D_{A-I} - Δ - Δ - DOBBS (PLASMA)
WITH $N_a + N_i$
EFFECTS.



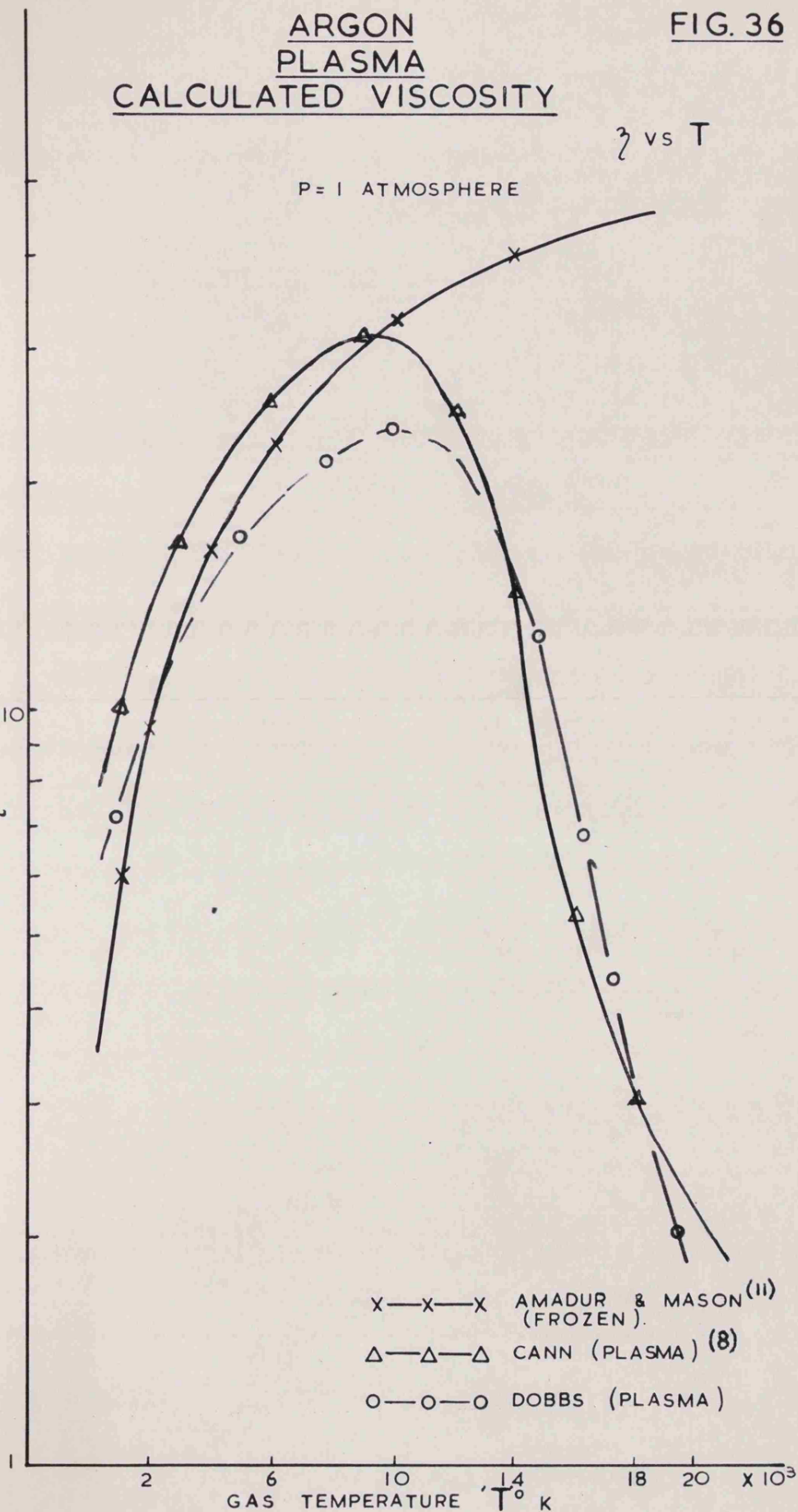
ARGON PLASMA CALCULATED VISCOSITY

FIG. 36

η vs T

P = 1 ATMOSPHERE

VISCOSITY $\eta \times 10^4$ gm/cm-sec.



argon as a function of temperature. If the momentum is reduced by the factor $\frac{n_e + n_a}{n_a}$ then the viscosity will be reduced by the same factor.

5.9 The Diffusion Coefficient at Ionisation (Monatomic Gas)

From Figure 33 it can be seen that above about 12,000K the atom-atom diffusion coefficient rises very rapidly. This point corresponds to the onset of ionisation. Now the ion will have a mass only 1/1800 less than the atom and thus the total mass of particles transported due to concentration gradients is scarcely reduced. It is also postulated that the ionic charge need not be taken into consideration in a first approximation. Since the plasma is electrically neutral, charge separation does not occur and the flux of ions will appear, macroscopically as a flux of neutral particles.

It is therefore suggested that the diffusion coefficient for atoms and ions be written as

$$D_{ai} = \frac{k\theta}{m_a \frac{n_a + n_i}{N_0}} \cdot f_c \quad (25.5)$$

Figures 35 and 36 show D_{ai} and η for argon as a function of temperature compared with results published by Cann⁽⁸⁾ and Amadur and Mason⁽¹¹⁾.

5.10 Thermal Conductivity of Ionised Monatomic Gases

In considering the thermal conductivity of a gas a number of contributions to energy transport must be taken into consideration. Whilst the gas is undissociated or unionised, the classical contact thermal conductivity model applies and the thermal conductivity coefficient can be calculated from the equation derived in Section 5.6 with the mean free path Sutherland correction being applied. This calculation is set out in detail for argon in Table 3 of Appendix V.

When the gas commences to ionise the number of particles contributing to the contact conductivity is reduced in a manner already suggested for the case of viscosity. However, energy can also be transported by reaction conductivity. This transport of energy is caused by ionised particles moving under the influence of a temperature or concentration gradient to cooler regions in the gas and these, giving up their ionisation energy. This term may be written as

$$K_R = n_i j_i D_i / \theta \quad (26.5)$$

where K_R = reaction energy coefficient ($\text{J.cm}^{-1}\text{sec}^{-1}\text{O}^{-1}$)

j_i = energy transported/particle (J/particle)

For a singly ionised atom

$$j_i = E_i \text{ (ionisation energy)}$$

The diffusion coefficient used in the above calculation requires some explanation. A plasma must be electrically neutral. Due to the large difference in the ion and electron masses (1830 : 1) these particles would have very different diffusion coefficients and it would appear that charge separation would occur in the presence of temperature gradients. The large internal forces created by any tendency to separate charges ensures that in a plasma ions and electrons diffuse at the same rate and it is shown by Boyd and Sanderson⁽⁵⁷⁾ that this "ambipolar" diffusion coefficient is given by

$$D_{\text{ambipolar}} = 2D_{\text{ion}} \quad (27.5)$$

Table 4 in Appendix V shows the detailed calculation of the reaction thermal conductivity for argon.

A further source of energy transport is due to the diffusion of electrons which may be calculated in a manner similar to the transport of ion reaction energy. For the energy per particle we write

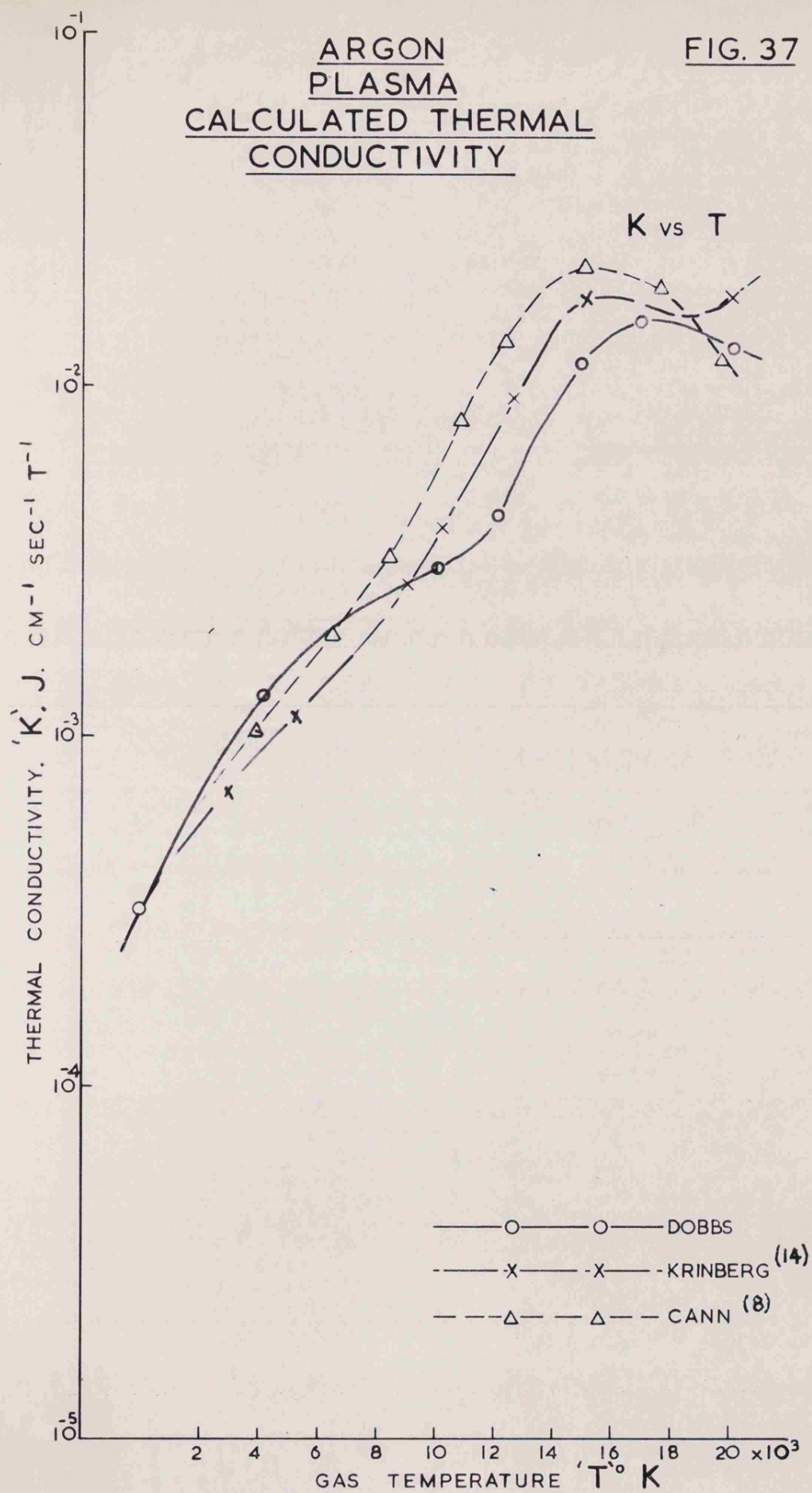
$$j_e = 3/2.k\theta.\text{Joules/electron} \quad (28.5)$$

which is the classical heat content of the electron and for this contribution to the coefficient of thermal conductivity we write

$$k_e = n_e j_e D_{\text{amb}} / \theta. \quad \text{J.cm}^{-1}\text{sec}^{-1}\theta^{-1} \quad (29.5)$$

ARGON
PLASMA
CALCULATED THERMAL
CONDUCTIVITY

FIG. 37



Finally we consider the energy transported by diffusion of atoms. Here the energy is carried by the classical specific heat $C_p\theta$. For the specific heat we can write $\frac{5}{2}C_p\theta$ for each atom. Then at a temperature θ , the classical thermal energy content is given by

$$j_a = \frac{5}{2} n_a k\theta \quad \text{J/atom} \quad (30.5)$$

whence the atom diffusion thermal conductivity equals

$$K_d = \frac{5}{2} n_a kD_a / \theta \quad (31.5)$$

Detailed calculations of these contributions to the thermal conductivity of argon are shown in Tables 5 and 6 of Appendix V.

Finally, the total thermal conductivity may be written as

$$K_T = K_C + K_R + K_e + K_d \quad (32.5)$$

(CLASSICAL)(REACTION)(ELECTRON)(DIFFUSION)

The results for argon are plotted in Figure 37 and compared with measurements and published calculation.

5.11 Diatomic Gas at Dissociation and Ionisation

In the case of a dissociating diatomic gas such as nitrogen where the gas will contain a mixture of atoms and

molecules, no correction for viscosity with changing composition is required. This is because at dissociation the total number of neutral particles is conserved if the negative ion is ignored. As the density of negative ions is very low, this is admissible.

Thus, the total momentum transfer is unchanged at dissociation.

For the atom-molecule diffusion coefficient the correction required must account for dissociation and we write

$$D_{\text{MOL-ATOM}} = \frac{k_0}{m_{\text{mol}} \cdot f_c \frac{n_2 + n_a}{N_0 - n_2 - n_a}} \quad (33.5)$$

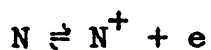
where N_0 = Loschmidts number

n_2 = number of density of molecules

n_a = number of density of atoms

5.11.1 Viscosity at Ionisation

At ionisation the reaction, considering nitrogen as an example, of interest for the correction to viscosity is



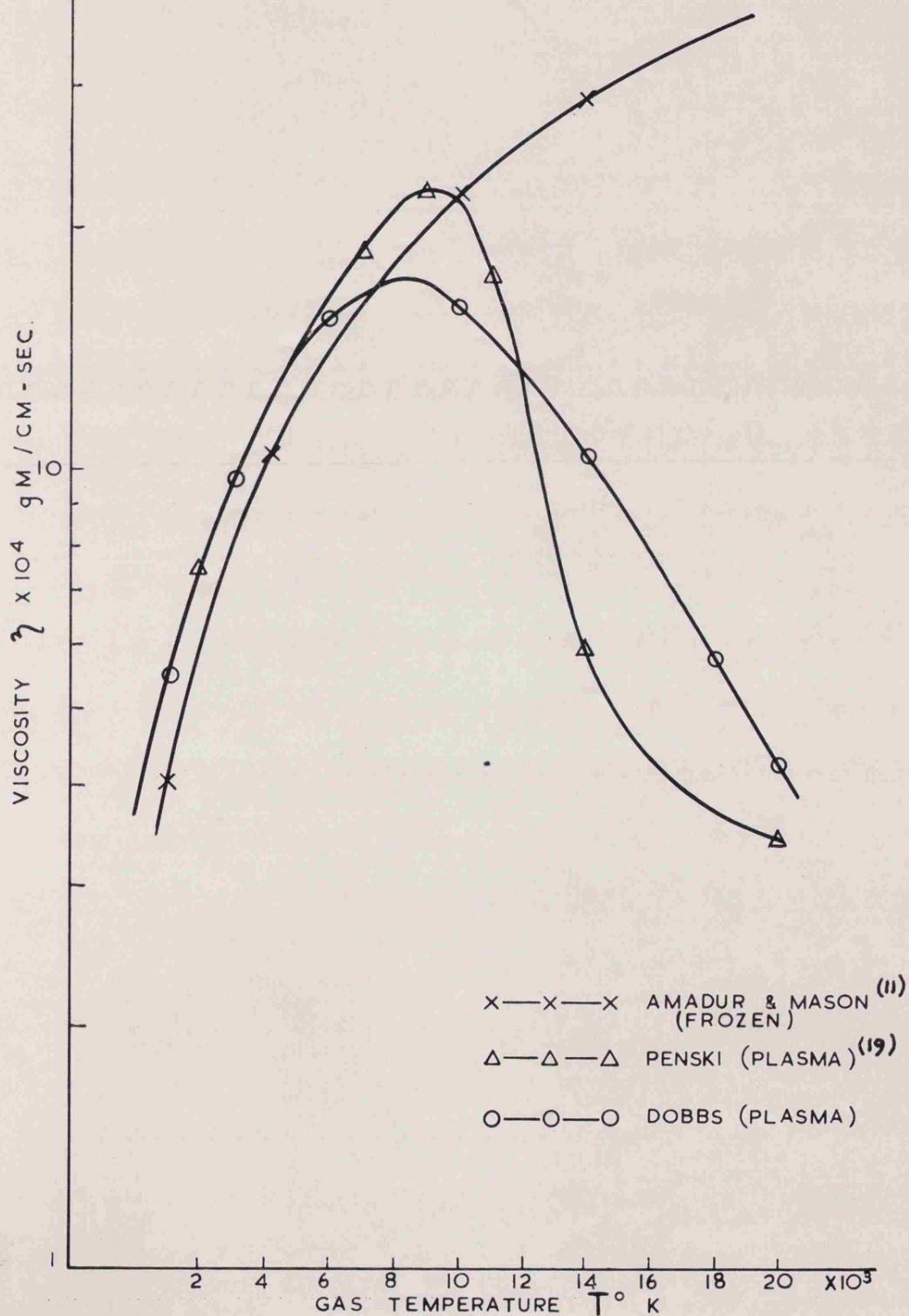
We assume that the separation between the dissociation

NITROGEN
PLASMA
CALCULATED VISCOSITY

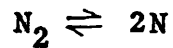
FIG. 38

η vs T

P = 1 ATMOSPHERE



and ionisation energies is such that the dissociation reaction



is complete. Then the viscosity correction can be made in the same manner as for the monatomic gas, i.e.

$$\eta = \frac{N_o k\theta}{f_c \frac{n_e + n_a}{n_a}} \quad (34.5)$$

Figure 38 shows the variation of viscosity with temperature for nitrogen and compares the calculated results with results published by Penski⁽¹⁹⁾

5.11.2 The Diffusion Coefficient at Ionisation

Assuming that the gas now consists of molecules, atoms, ions and electrons, the correction applied to the basic equation for the diffusion coefficient is simply an extension of the treatment used for dissociation equation (25.5)

We now write

$$D_{A-A} = \frac{k\theta}{m_a f_c \frac{n_2 + n_a + n_i}{N_o - n_2 - n_a - n_i}} \quad (35.5)$$

It would appear here that a new density correction factor is being applied which differs from that used for

monatomic gases, thereby spoiling the generality of the method. This is not so as the correction for argon $\frac{n_a + n_c}{N_o}$ should be written more correctly as

$$\frac{n_a + n_i}{N_o - n_a - n_i} \quad (36.5)$$

Table 7 in Appendix V. however, shows that the numerical difference between these two correction factors is negligible, hence the simpler form is used for the monatomic gas.

5.11.3 Thermal Conductivity of Diatomic Gases

For these gases the following terms must be taken into consideration.

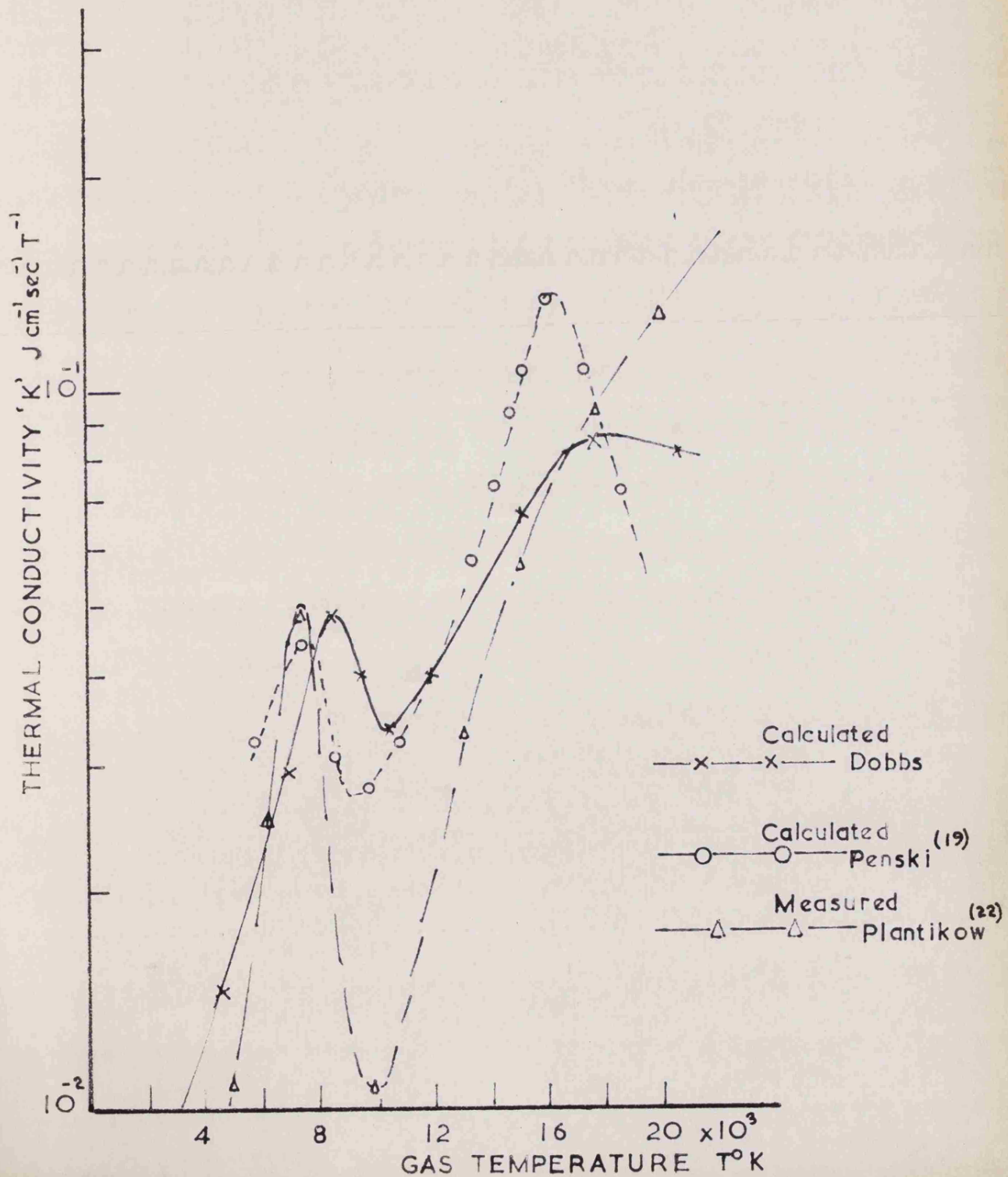
- (a) The classical thermal conductivity. This is calculated in exactly the same way as for argon except that to account for the additional internal degrees of freedom of the diatomic molecule, the factor $5/2$ is replaced by a factor $7/2$. No correction such as that of Eucken has been made at high temperatures for vibrational effects. The reaction conductivities to be taken into account are those associated with dissociation energy and transport of energy by electrons and ions. These calculations are made in exactly the same manner as for the monatomic gas previously discussed. When ion and electron transport is being considered it must be remembered

NITROGEN

FIG39

CALCULATED THERMAL CONDUCTIVITY

K vs T



that the diffusion coefficient "D" used in the general transport equation for reacting species,

$$K_R = j_R n_R D / \theta \quad (37.5)$$

is the "ambipolar" coefficient and for the diatomic gas is equal to

$$2D_{\text{atom-atom}} \quad (38.5)$$

Outline calculations for these properties are shown in Table 8 of Appendix V and Figure 39 shows the thermal conductivity of nitrogen as a function of temperature compared with published measurements and calculations.

Chapter 6

The Experimental Determination of Electron Collision Cross Sections in a High Density Plasma

6.0 Introduction

In Chapter 5 of this study, the calculation of the transport coefficients of a gas as a function of temperature was examined. A full treatment of the problem requires a detailed knowledge of the interactions between all the particles which make up the gas or plasma, that is atoms, electrons and ions. These interactions, collision cross sections or collision probabilities are usually determined from experiments made with beams of electrons or ions at low pressure and low temperature. To use this data in the high temperature, high density plasmas such as exist in electric arcs, these values must be extrapolated up into the range of interest. Details of the measurement and calculation of collision cross sections at low pressures and temperatures are given in Appendices VI and VII together with a review of the meaning of collisions in the context of a mixture of reacting species.

6.1 Theoretical Background

The magneto ionic theory of Ratcliffe⁽⁶⁵⁾ and the theory of absorption in ionised gases proposed by Spitzer⁽⁶⁶⁾ give relationships for the absorption of photons in passing

through an ionised gas in terms of electron density, temperature, wavelength of the incident radiation, collision cross section and refractive index. Spitzer gives the coefficient of absorption for photons of wavelength λ , via the mechanism of inverse Bremsstrahlung for

$$\frac{hc}{\lambda} < kT$$

where h and k are the Planck and Boltzmann constants as,

$$\alpha = 1.6 \times 10^{-37} \lambda^2 n_e^2 T_e^{-3/2} \text{ cm}^{-1} \quad (1.6)$$

with λ measured in microns and T_e in electron volts.

Ratcliffe under similar conditions gives the relationship between absorption coefficient, refractive index and collision cross section assuming electron-neutral collisions as the dissipative mechanism as

$$\alpha = \frac{f_c}{\mu} \cdot \frac{4 \pi N_e \cdot e^2}{2c \cdot \epsilon_0 m (\omega^2 + f_c^2)} \quad (2.6)$$

where f_c is the collision frequency, μ the refractive index of the gas, ω the illuminating frequency and ϵ_0 the dielectric constant of free space.

These two equations therefore facilitate the determination of an electron-neutral collisions cross section for an ionised gas provided that the other stated parameters are known and the imposed constraints respected.

From the Spitzer equation (1.6), provided that the temperature of the gas is known, the measurement of absorption coefficient permits the determination of electron density. For this, an independent measurement of temperature is required, a spectrographic technique which depends upon the emission intensity of spectral lines being usually adopted.

The applicability of equations (1.6) and (2.6) is governed by certain constraints. These have been examined and the validity of the equations demonstrated.

The following criteria must be met before the magneto ionic equations are used:⁽⁵⁷⁾

- (a) External magnetic fields (macro fields) are absent, i.e. $B = 0$. This is the case in the plasma torch free jet.
- (b) The incident radiation energy must be less than the thermal energy of the plasma to avoid excitation phenomena.

$$h\nu < kT$$

This inequality is just satisfied for $\lambda = 6328\text{\AA}$

- (c) The incident radiation frequency must be higher than the plasma frequency to avoid reflections.

$$\omega_i > \omega_p$$

This inequality is very easily satisfied at

$$\lambda = 6328 \text{ and } 9100\text{\AA}$$

- (d) The plasma must be "Cold". This last criterion would not appear at first to be satisfied as $10,000^{\circ}\text{K}$ can hardly be said to be cold. However, a condition for wave propagation in a "cold plasma" is given as

$$\frac{\omega}{g} > \left(\frac{kT_i}{m_i} \right)^{\frac{1}{2}}$$

where ω/g is the phase velocity of the incident radiation, g = propagation coefficient. This is satisfied in the experimental programme described.

6.1.1 Refractive Index

Equation (2.6) requires, among other data, the refractive index of the gas " μ " as a function of temperature. Only a very limited amount of experimentally derived information is available^{(67), (68)} and recourse must be had to calculation.

It is convenient to work in terms of "refractivity" rather than refractive index.

$$\text{Refractivity} = (\text{Refractive index} - 1)$$

For a partially ionised gas consisting of several species, atoms, ions, electrons etc. the refractivity may be written

as

$$\mu - 1 = \sum_i K_i N_i$$

where K_i is the refractivity of the i^{th} component of density N_i .

In the temperature range of interest in this study (5000-15000°K) the components of refractivity considered to be of interest are, free electron, singly ionised atoms and neutral atoms. Then

$$(\mu - 1) = (\mu - 1)_e + (\mu - 1)_i + (\mu - 1)_a \quad (3.6)$$

At relatively low collision frequencies ($T \approx 1\text{eV}$) the electron refractive index reduced to

$$\mu_e^2 = 1 - \omega_p^2 / \omega_i^2 \quad (4.6)$$

where ω_p the plasma frequency is given by

$$\omega_p = \left(N_e e^2 / \pi m_e \right)^{\frac{1}{2}} \text{ radians sec}^{-1} \quad (5.6)$$

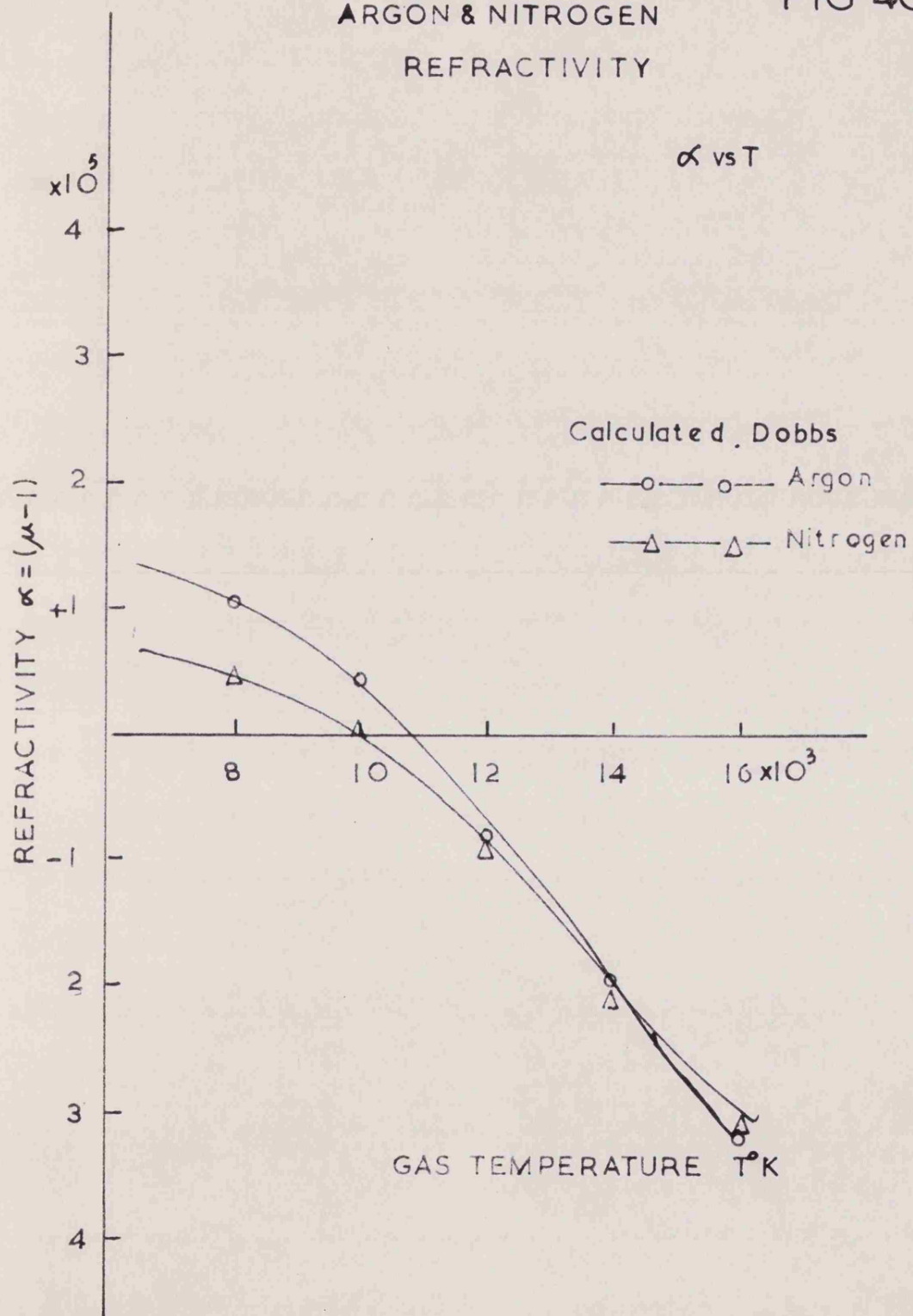
and ω_i the incident radiation frequency is given by

$$\omega_i = 2\pi c / \lambda \quad \text{radians sec}^{-1} \quad (6.6)$$

At low temperatures $\approx 5000^\circ\text{K}$ the atomic contribution to the refractive index is significant although as the temperature increases this contribution falls. The refractivity of neutral atoms is given by Born & Wolf⁽⁶⁹⁾ as

ARGON & NITROGEN REFRACTIVITY

FIG 40



$$\mu - 1 = A + B/\lambda^2$$

The coefficients A and B are given by Allen⁽⁷⁰⁾.

The ionic component of refractivity becomes more important as the temperature of the gas increases and in terms of ion number density N_i and polarizability a_i , is given by

$$\mu - 1 = 2\pi a_i N_i$$

The polarisation term being developed from the method of Kirkwood⁽⁷¹⁾

The refractivity of argon and nitrogen was calculated over the temperature range 5000-16000°K and is shown graphically in Figure 40.

It is interesting to note the change in sign of the refractivity at about 10,000°K when the electronic component becomes dominant. This is seen in a later phase of the study to have a considerable effect on the experimental techniques employed for measuring the optical absorption coefficients.

As the refractive index, over the temperature range considered, only deviates from unity by a few parts in 10^5 , subsequent calculations of collision frequencies using the Ratcliffe equation (2.6) were made with μ_p set as unity and invariant with temperature.

6.2 Measurement of Absorption Coefficient

An electro magnetic wave is propagated in an ionised gas by its interactions with electrons in the medium. In a perfect electron gas the propagation process would be non-dissipative and the incident electro magnetic wave would propagate without loss. In a gas containing heavy particles such as neutrals and ions, the electrons in the gas will make collisions with these particles, the nett result being that the incident photon loses energy by a process referred to as the "Inverse Bremsstrahlung" effect^{(72),(73)}. If the photon beam loses energy the relationship between the incident and emergent intensities is given by

$$I = I_0 \exp (-\alpha x) \quad (7.6)$$

where x is the path length traversed by the photon.

The factor α is termed the absorption coefficient. Its magnitude gives an indication of the frequency of electron interactions with heavier particles.

6.2.1 Techniques of Measurement

For the absorption of energy by the inverse Bremsstrahlung mechanism, the quantum-mechanical inequality $\frac{hc}{\lambda} < kT$ must be satisfied. For plasmas with an electron temperature of around 1eV, ($T = 11000^\circ K$) this means that the wavelength must be greater than 1 micron to be absolutely confident of no other processes occurring.

A second condition for the applicability of absorption coefficient measurements is that there is no effect due to plasma emission. This means that the intensity of the incident source must be greater than the black body brightness of the plasma. Low power lasers fulfil this criterion without any difficulty. At temperatures of 10000°K the black body brightness of a plasma is around $1\text{W}\mu^{-1}\text{cm}^{-2}$ whilst the He-Ne laser is a factor 10^{10} times greater. In the above, " μ " is a unit wavelength interval in micrometres.

During the course of this study, the only lasers available were the gaseous He-Ne laser operating at a wavelength of 0.63 microns and a Ga-As solid state laser operating at approximately 0.9 microns. For these wavelengths the quantum mechanical inequality is satisfied but $\frac{hc}{\lambda}$ is only just less than kT . As will be seen, however, the results obtained indicate that spurious effects did not give inaccuracies, the discrepancy between measured and calculated values of α being explained with reference to electron-ion collisions (Appendix VII). Ideally, since the absorption coefficient varies as the square of the wavelength, a long wavelength laser such as the continuous wave carbon dioxide system at $10.6\mu\text{m}$ would be preferable to give increased absorption and accuracy.

6.2.2 Direct Absorption Measurement

Two techniques of measurement were attempted. Figure

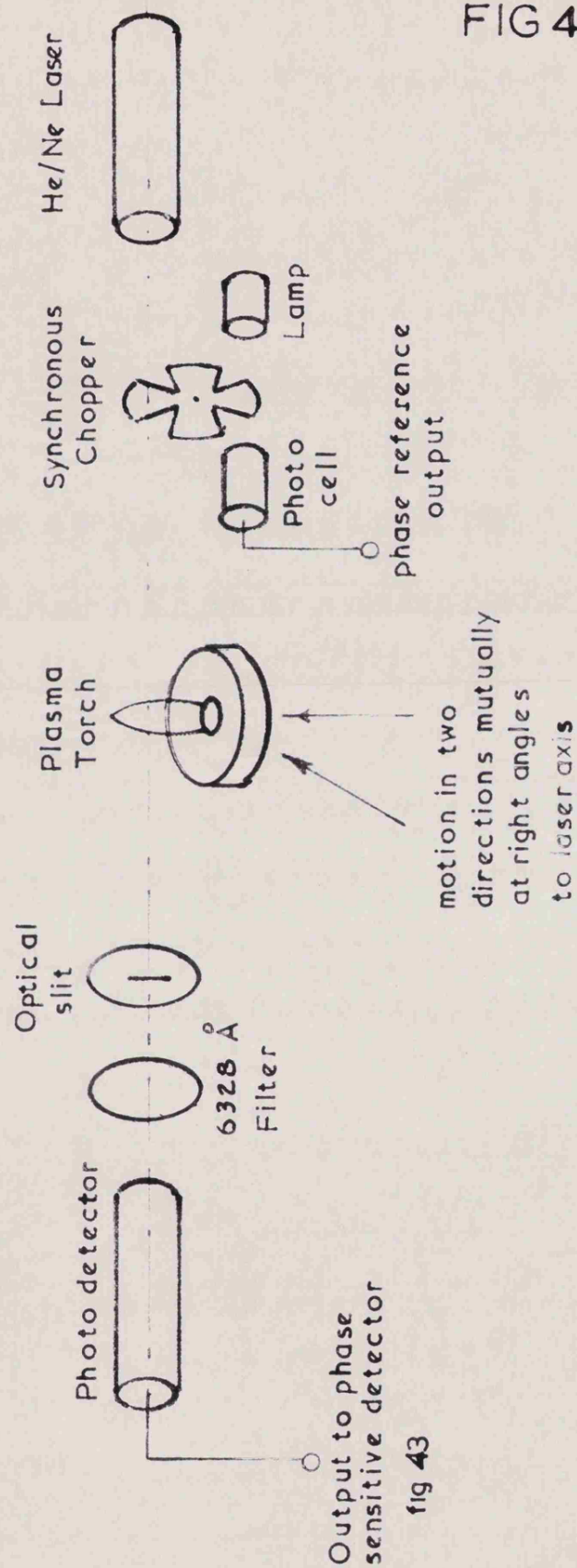
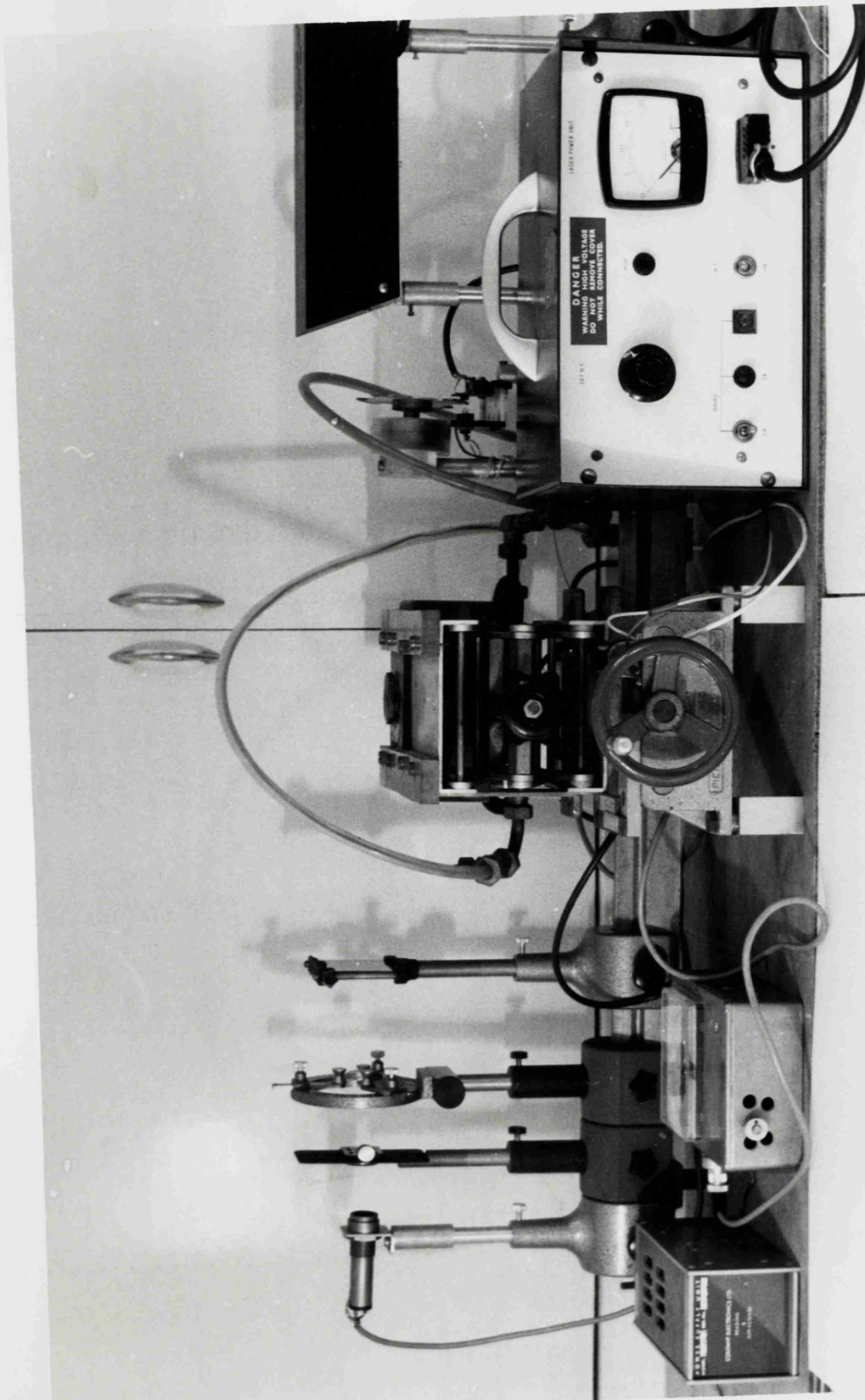


FIG 41

SCHEMATIC-DIRECT OPTICAL ABSORPTION MEASUREMENT

FIG 42



DIRECT
ABSORPTION APPARATUS

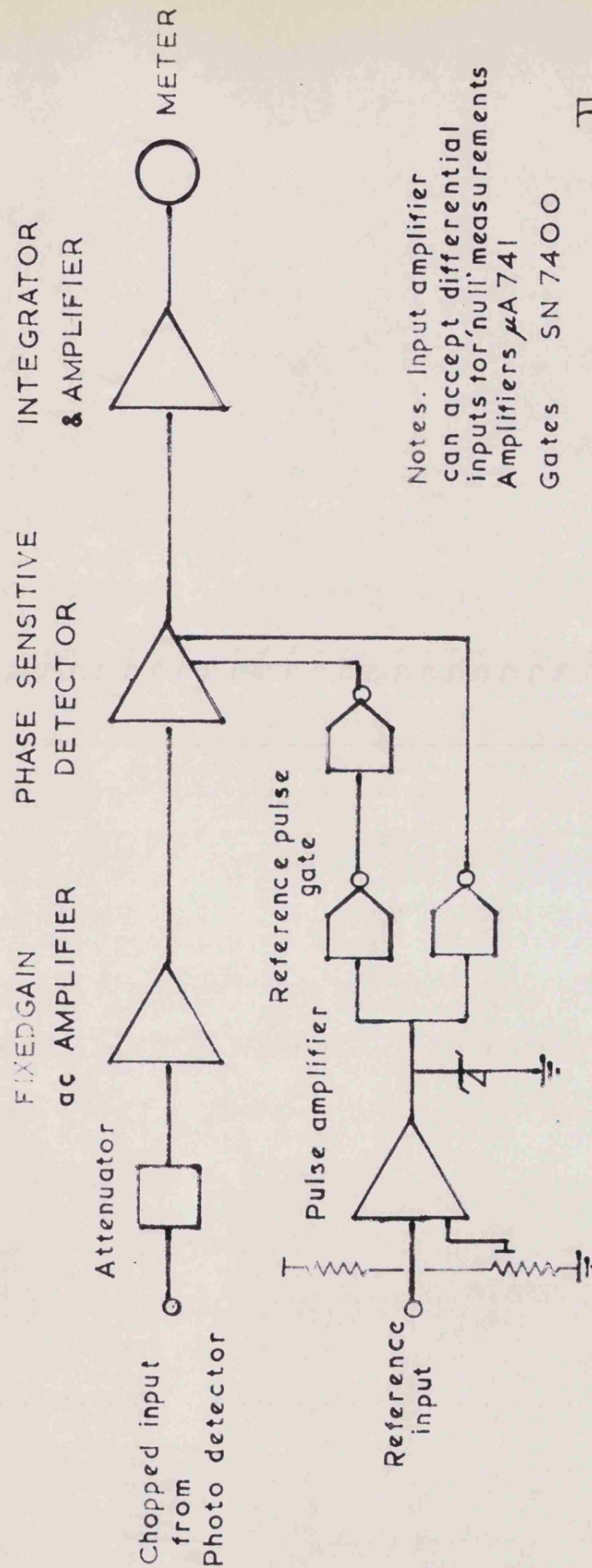


FIG 43

BLOCK SCHEMATIC SYNCHRONOUS DETECTOR /AMPLIFIER

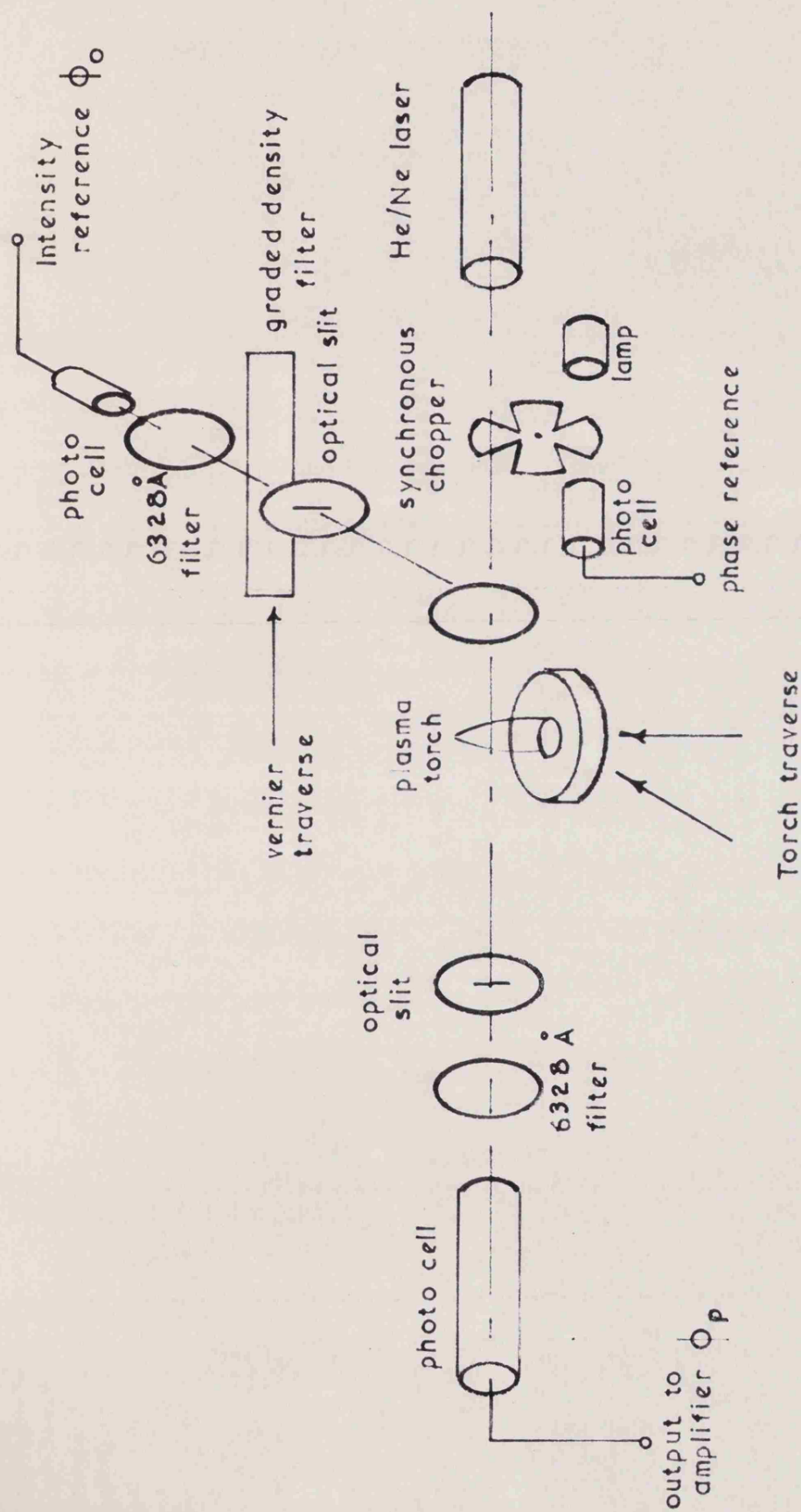
41 shows a schematic layout of the optical and electrical apparatus for the direct measurement of absorption, whilst Figure 42 shows a photograph of the apparatus.

In this method the path taken by the radiation from a 6328A He-Ne laser through the plasma jet can be varied by movement of the plasma generator in two planes mutually at right angles to the laser beam. The laser radiation is modulated with a mechanical chopper at 40 Hz. After passing through the plasma the beam passes through a vertical 0.1mm slit in front of a narrow pass optical filter whose function is to reduce the optical bandwidth of the system and hence reduce noise pick-up from the plasma jet. The filtered radiation is then incident upon a silicon photoelectric cell operating in the photo-voltaic mode. The output from the detector is amplified and fed to a synchronous detector with a moving coil meter as an indicator.

A block diagram of the electronic system is shown in Figure 43.

First results with the He-Ne laser produced absorption coefficients in the range $1.0 - 10 \text{ cm}^{-1}$. The apparatus described is quite satisfactory for making measurement in this range, however, these early results (Dobbs/Maxwell)⁽⁷⁴⁾ have since been shown to be spurious and due to an optical artefact of the system. The optical system was, therefore, modified and the apparatus shown in Figure 41 was then unable to detect any absorption from the He-Ne laser (See

FIG 44



NULLING (DIFFERENTIAL) ABSORPTION MEASUREMENT

Section 6.4ff for a discussion of the difficulties).

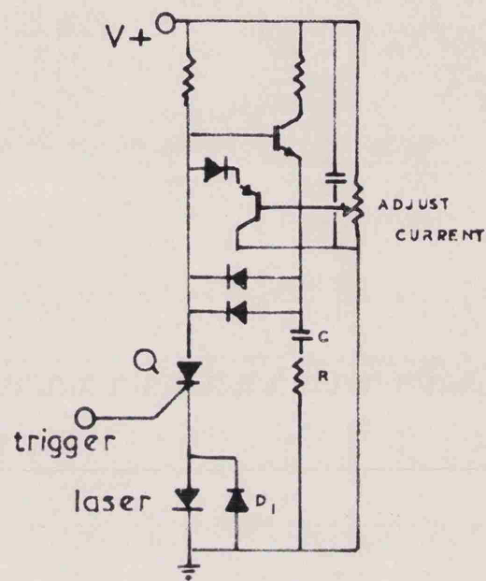
6.2.3 A Nulling Absorption Measuring Set

Figure 44 shows an improved electrical and optical apparatus for the measurement of very small absorption coefficients. Here the laser radiation is split into two beams before reaching the plasma. These are incident on similar silicon photo detectors operating in the photo voltaic mode and fed to a differential amplifier system. The first stage of the amplifier consisted of a thermally compensated differential amplifier with an integral temperature control oven. The photo cells when normally illuminated by the laser produced an output of ≈ 200 mV, thus for a change in illumination intensity of -0.001 , a change of 20 V would appear at the input of the amplifier. This was well within the noise and drift specification of the system.

To overcome the large magnitude, fluctuation noise spectrum from the plasma, the DC amplifier following the demodulator stage had a built in time constant, switchable from 0.5 to 10 seconds.

In the absence of the plasma, the output of the differential amplifier was nulled by a graded density filter on a vernier traverse in the amplitude reference arm of the apparatus (Figure 44). By the use of a second calibrated density filter in the place of the plasma

FIG 45

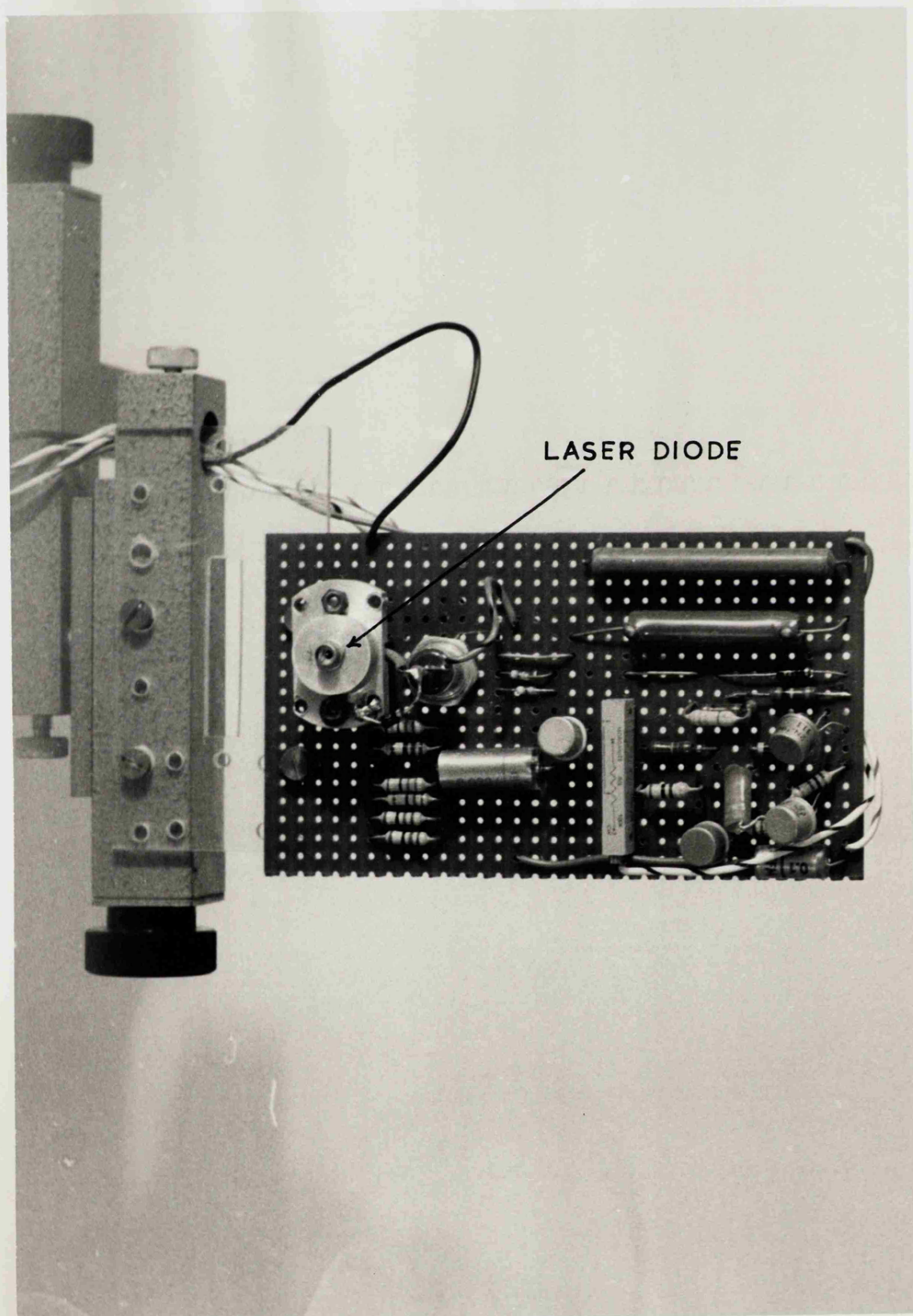


Notes

- 1 Thyristor Q is rapid turn off
for 5 kHz operation
- 2 Pulse network C & R $0.02\mu\text{F}$ & 2.0Ω
- 3 V+ 300 volts

DIODE LASER PULSE GENERATOR

FIG 46



GaAs LASER

it was demonstrated that this system, when allowed to become thermally stabilised, could easily resolve an absorption change of 0.001 reciprocal centimetres..

In an attempt to obtain even greater sensitivity, since $\alpha = f(\lambda^2)$ a solid state Gallium-Arsenide laser working at a wavelength of 0.9 microns was also used. At room temperature this device could not be used in a continuous mode and was pulsed at 1000Hz with a pulse width of 0.2 micro-seconds.

When this pulsed laser was used the radiation was not mechanically chopped and the synchronising circuit for the detector was driven directly from the pulse generator driving the laser. The circuit for this generator is shown in Figure 45. Figure 46 shows a photograph of the Ga/As laser pulse and drive unit.

6.2.4 Noise Reduction

In the case of the He-Ne laser the radiation is chopped with a 1:1 mark-space ratio. The synchronous detector has inherent noise cancelling properties when operated in this 1:1 ratio regime. In the case of the Ga/As laser the mark-space ratio was less than 1:1000 and a design was produced for a detector system with a variable aperture gate width. However, in practice this complex detector was not required as the noise spectrum seen by the two detectors is, over a period of time, identical and in the differential amplifier this resulted in a zero output.

6.3 Experimental Results

The plasma torch was operated with a constant argon or nitrogen flow at currents of 100, 200 and 260 amps. In the case of the argon plasma, a power supply giving 70 volts open circuit was used. For the nitrogen plasma a higher open circuit voltage was obtained and also a higher running voltage, by connecting three identical sections of a BOC power supply in series.

Measurements were made on both laminar and turbulent jets in argon and turbulent jets only in nitrogen.

The beam from the laser was set to pass through the plasma jet at a "stand off" height of 0.5 and 1.0 cms above the orifice of the jet. The plasma jet was moved across the axis of the laser beam in steps of 0.05 cm and the emergent beam intensity recorded at each step.

It should be noted here that a further reason for using the Ga/As laser was that it was hoped to obtain a much higher spatial resolution with this device over the He-Ne laser as its beam dimensions were approximately 2 x 30 microns whereas the beam width at the half power points of the gas laser was 0.1 cm.

Designating the intensity of the incident beam as I_0 and the intensity of the attenuated beam as I , we can write the following equation

$$I = I_0 \exp (-\alpha x) \quad (8.6)$$

FIG 47

ARGON
MEASURED INTEGRATED
ABSORPTION

A vs R

- $\times$ — $\times$ — $.91\mu$ } 260 Amp
 $\circ$ — $\circ$ — $.63\mu$ }
 Δ — Δ — $.63\mu$ 100 " "

height . 0.5 cms

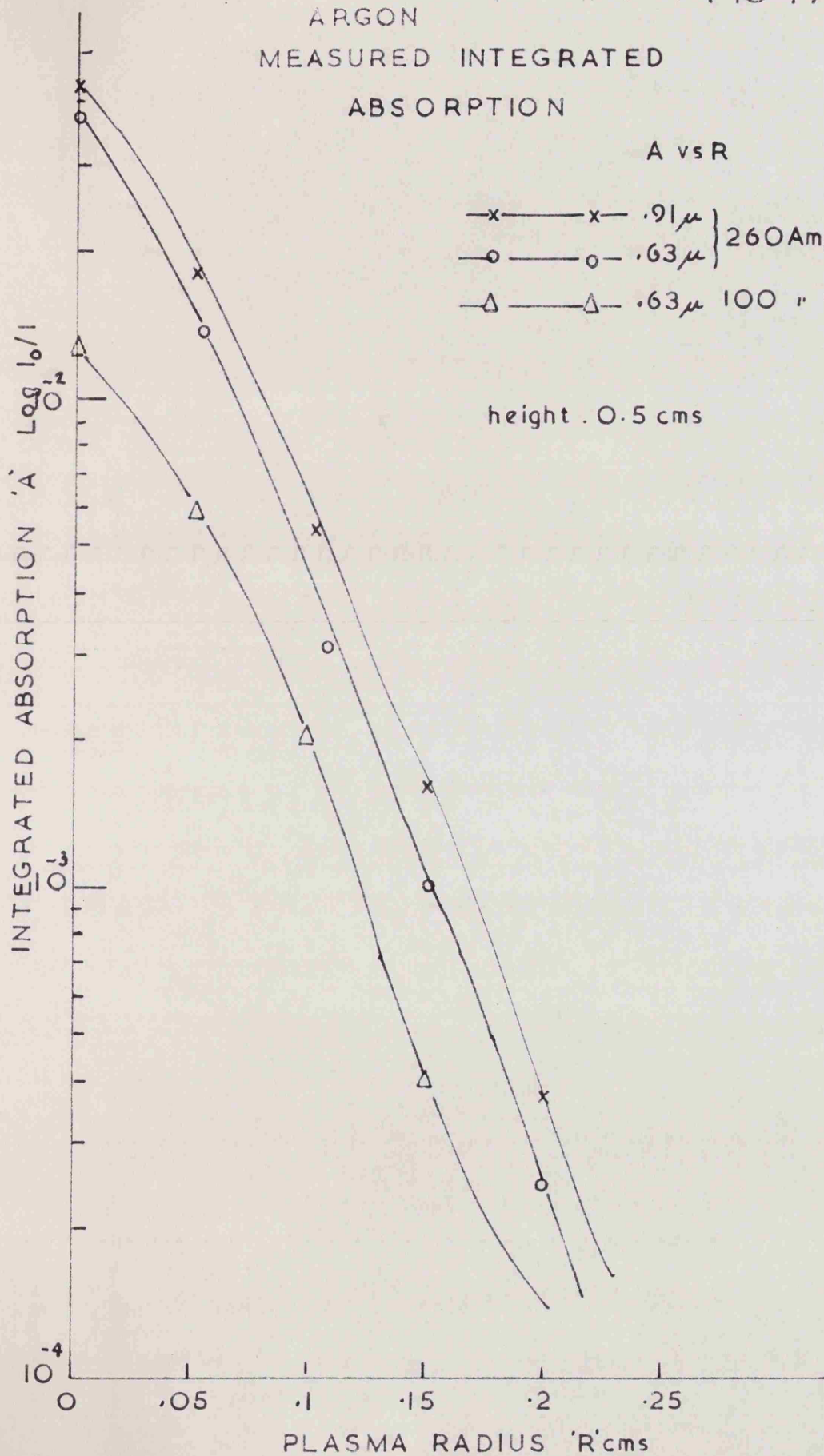


FIG48

ARGON
MEASURED INTEGRATED
ABSORPTION

A vs R

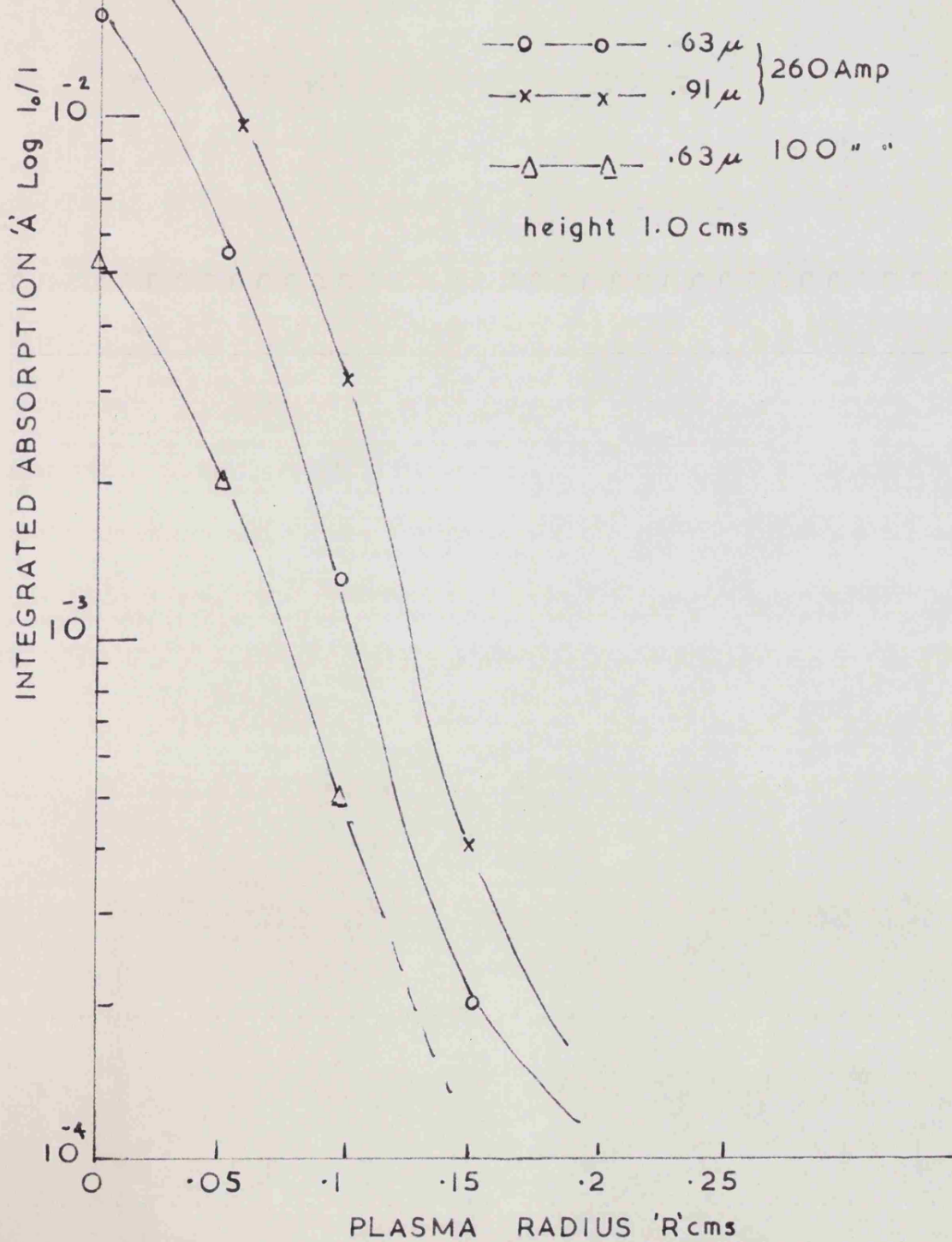


FIG 49

NITROGEN
MEASURED INTEGRATED
ABSORPTION

A vs R

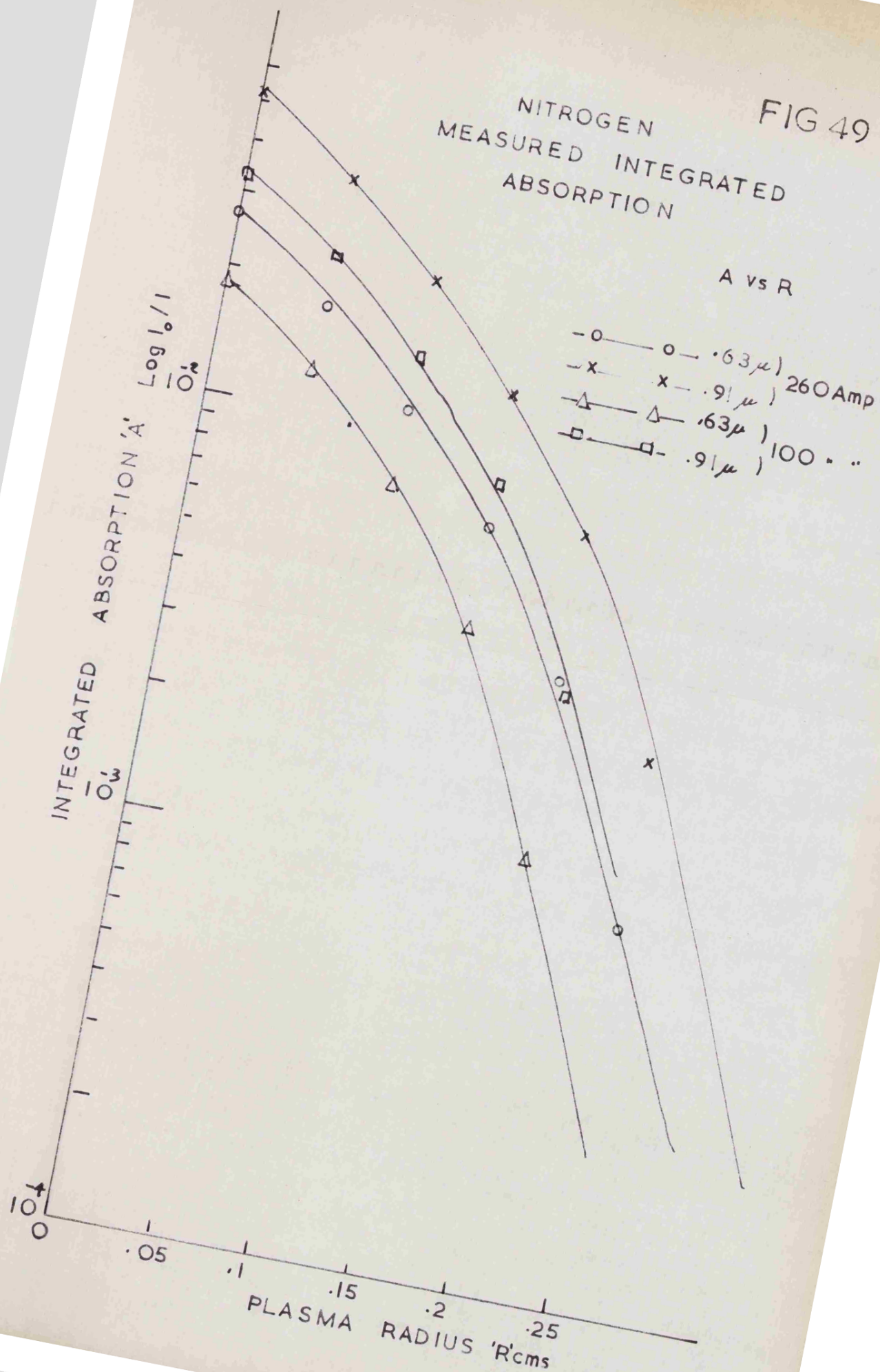


FIG 50

NITROGEN
MEASURED INTEGRATED
ABSORPTION

A vs R

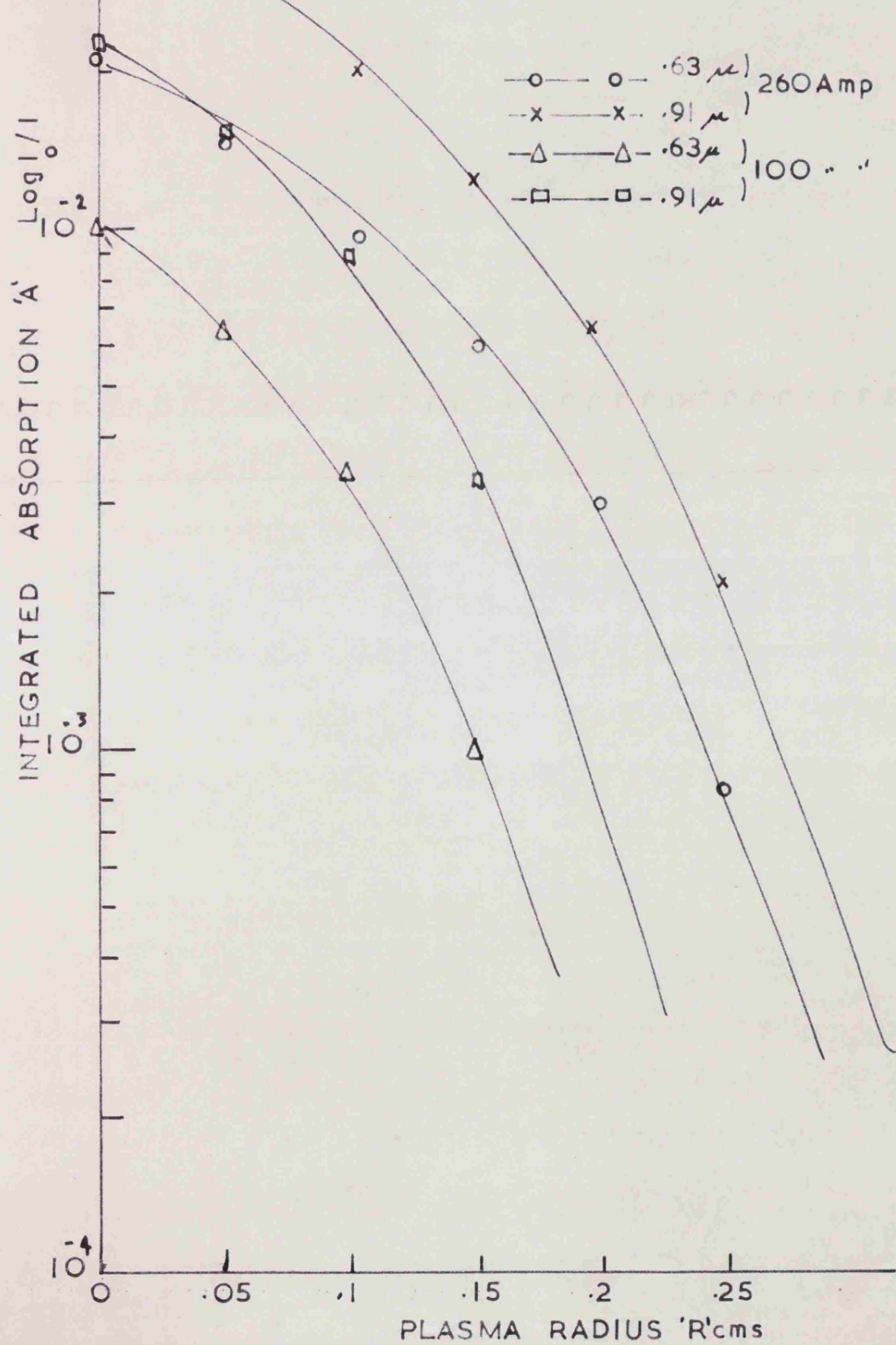


FIG 51

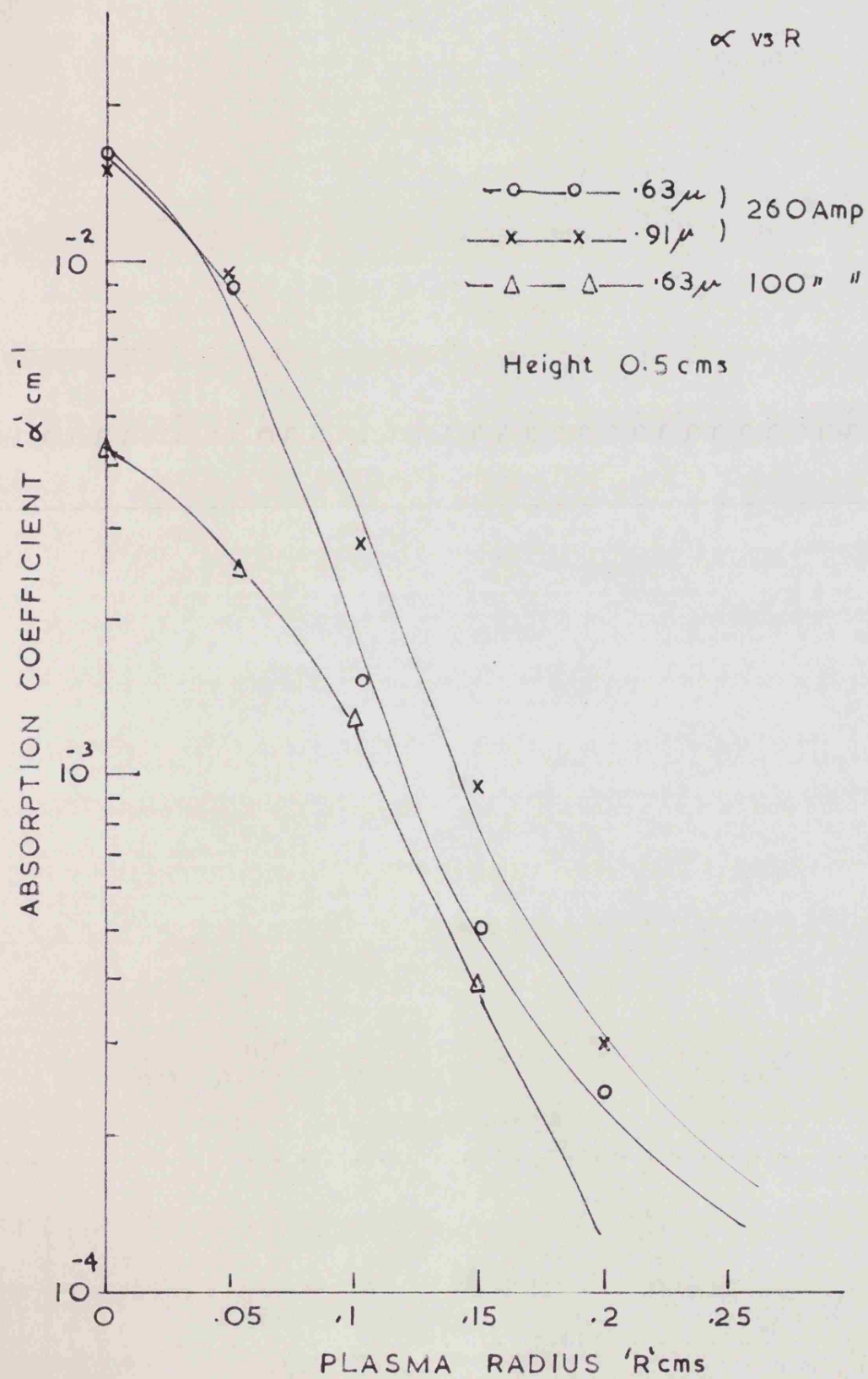
ARGON
MEASURED ABSORPTION
COEFFICIENTS

FIG 52

ARGON
MEASURED ABSORPTION
COEFFICIENTS

α vs R

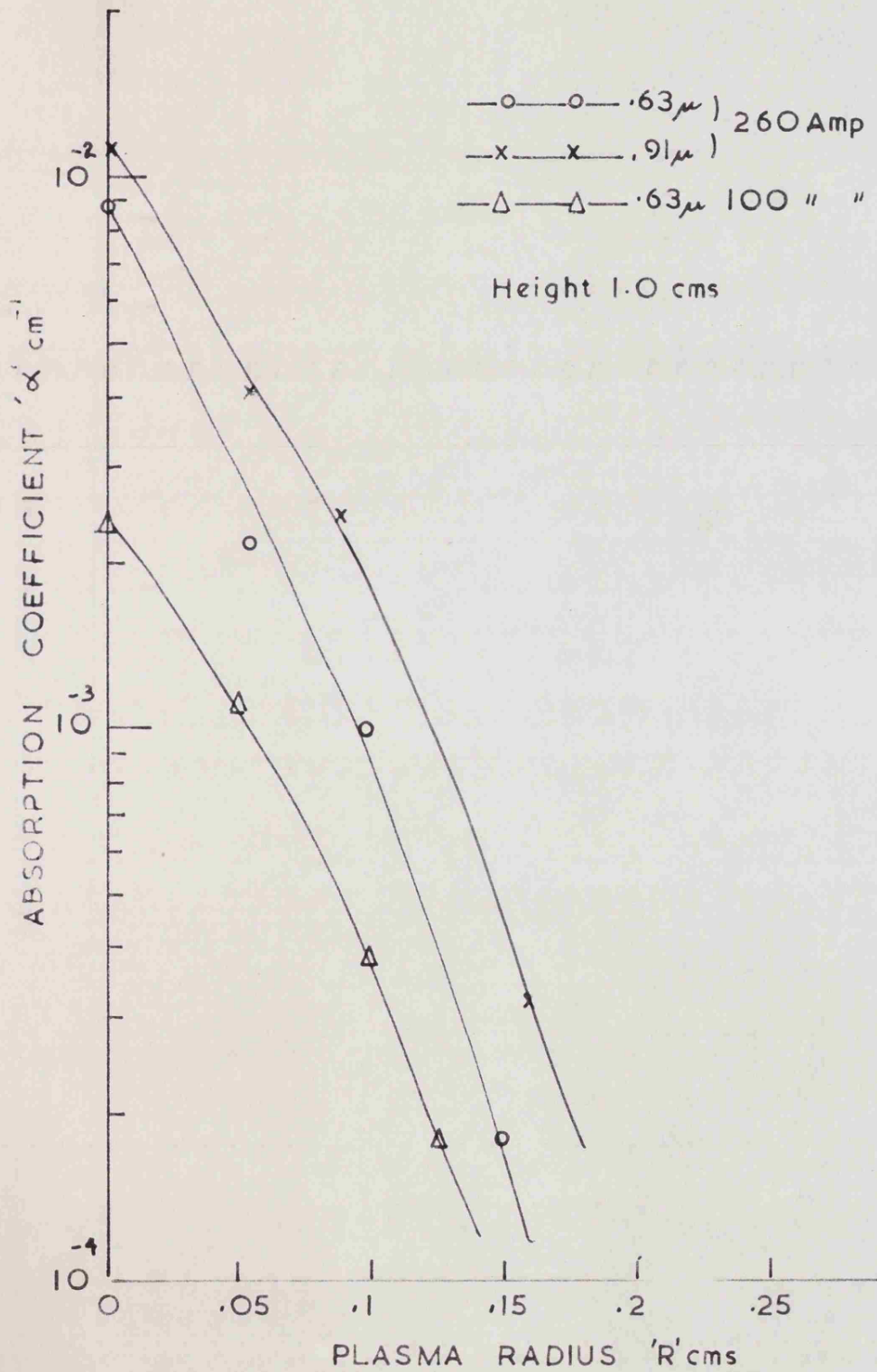


FIG 53

NITROGEN MEASURED ABSORPTION COEFFICIENTS

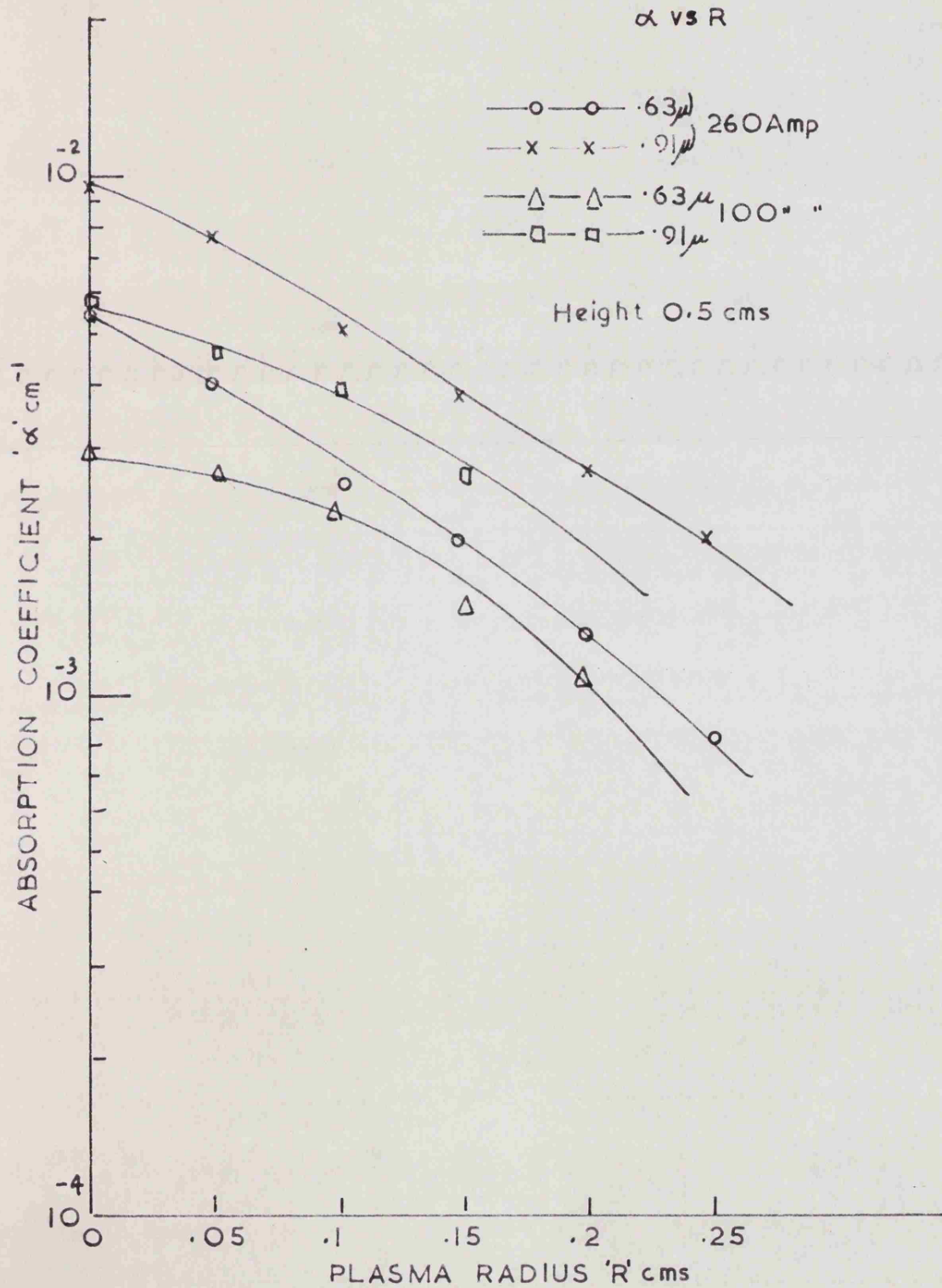
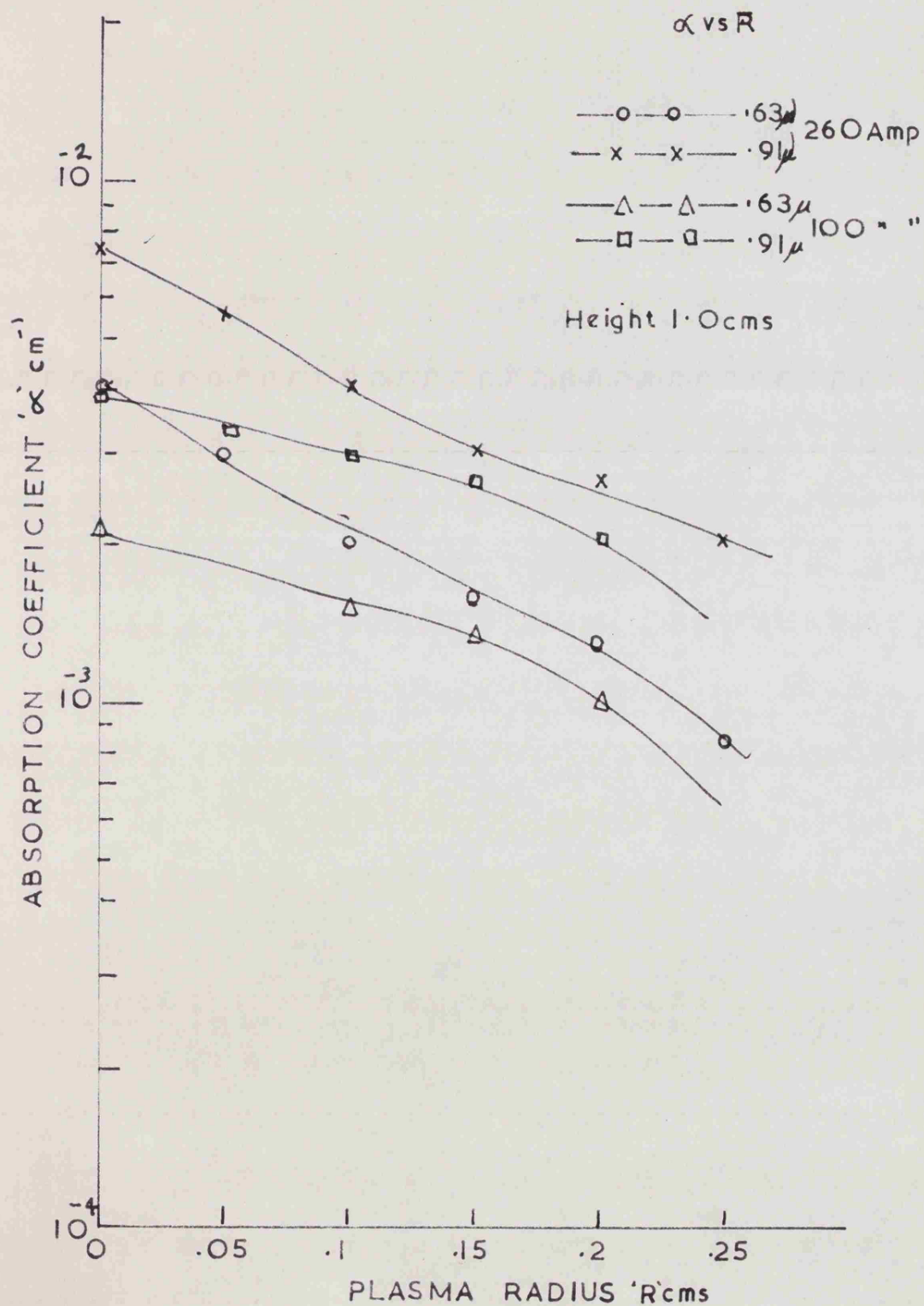


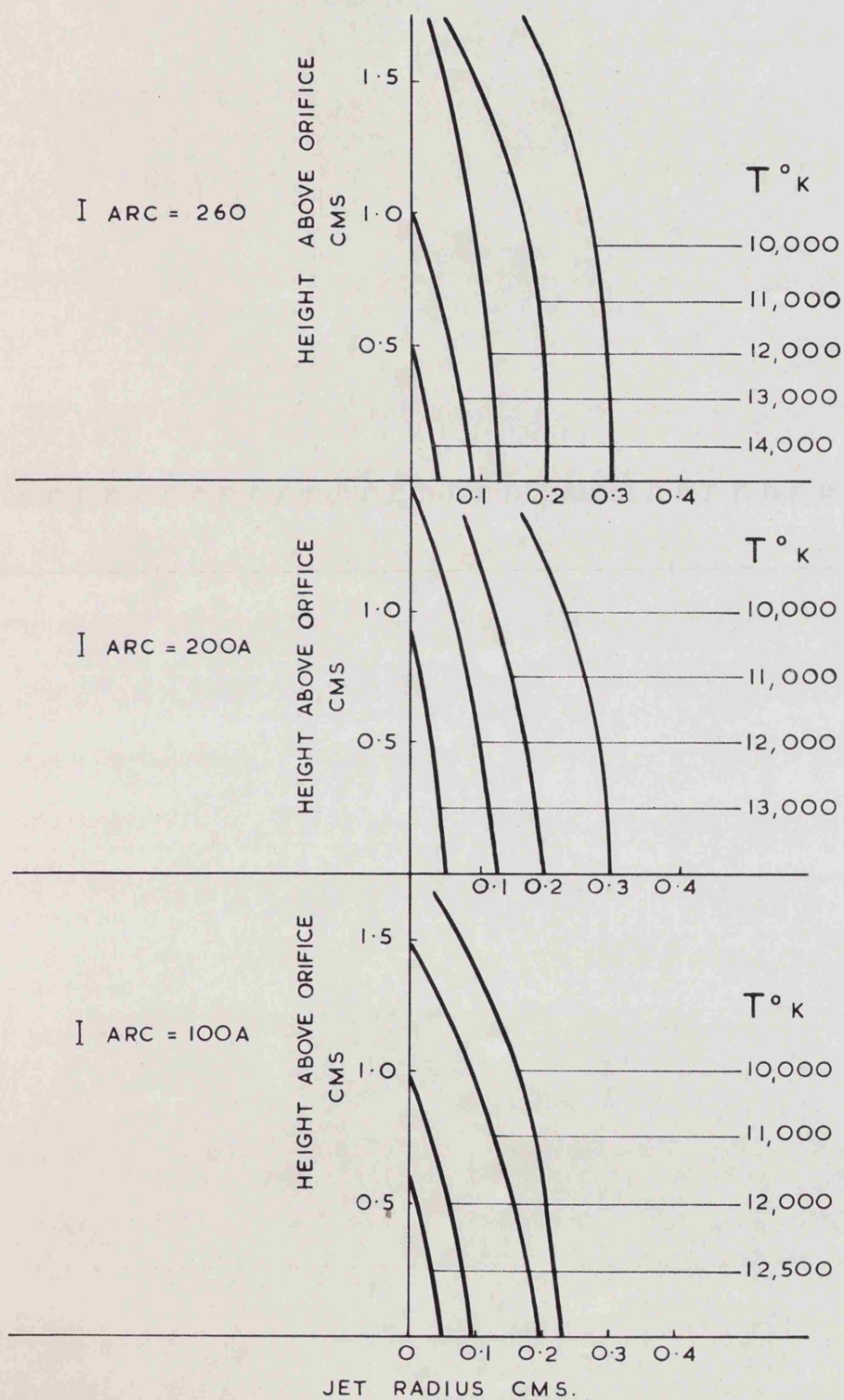
FIG 54

NITROGEN
MEASURED ABSORPTION
COEFFICIENTS



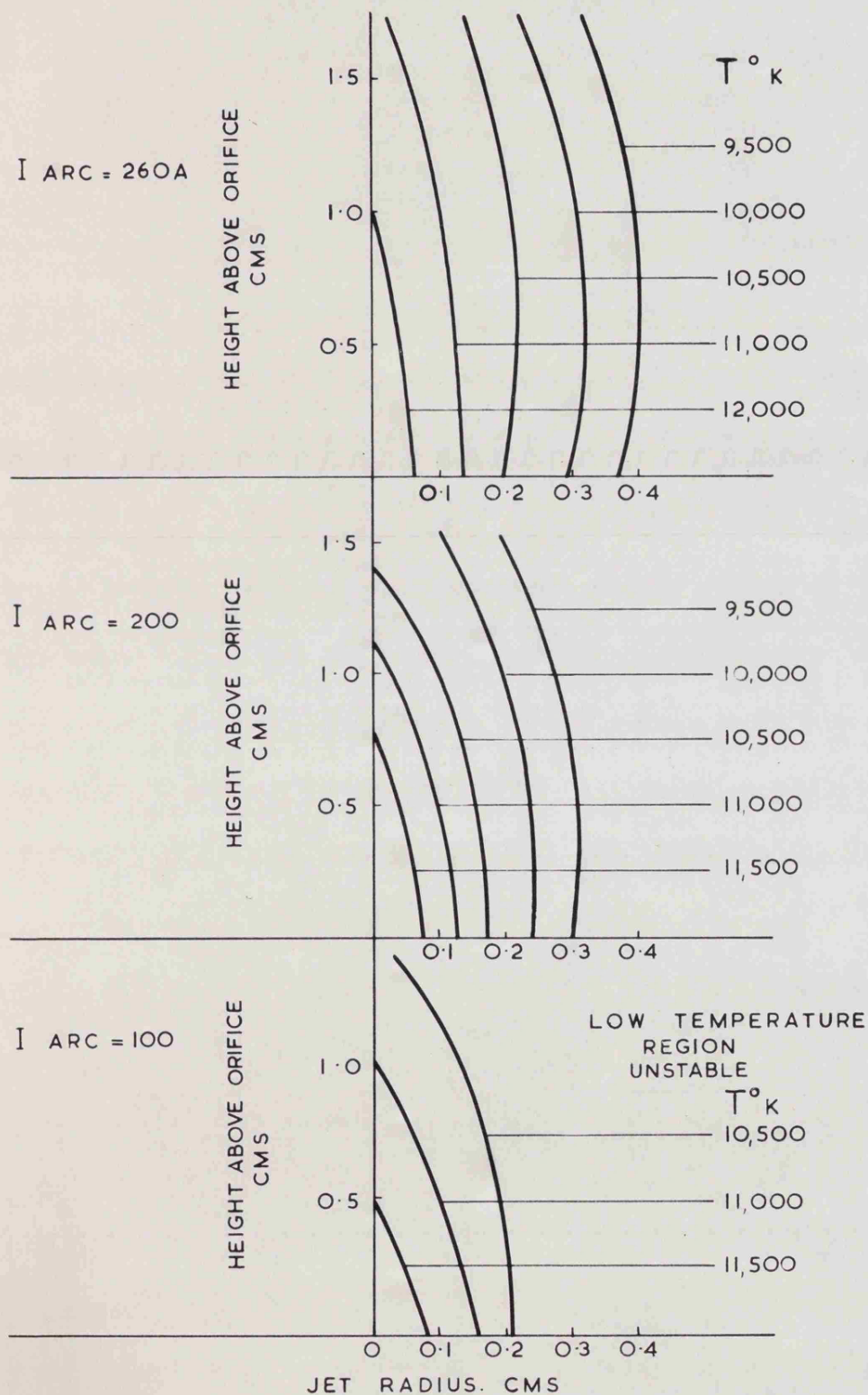
ARGON PLASMA JET TEMPERATURE PROFILES

FIG.55



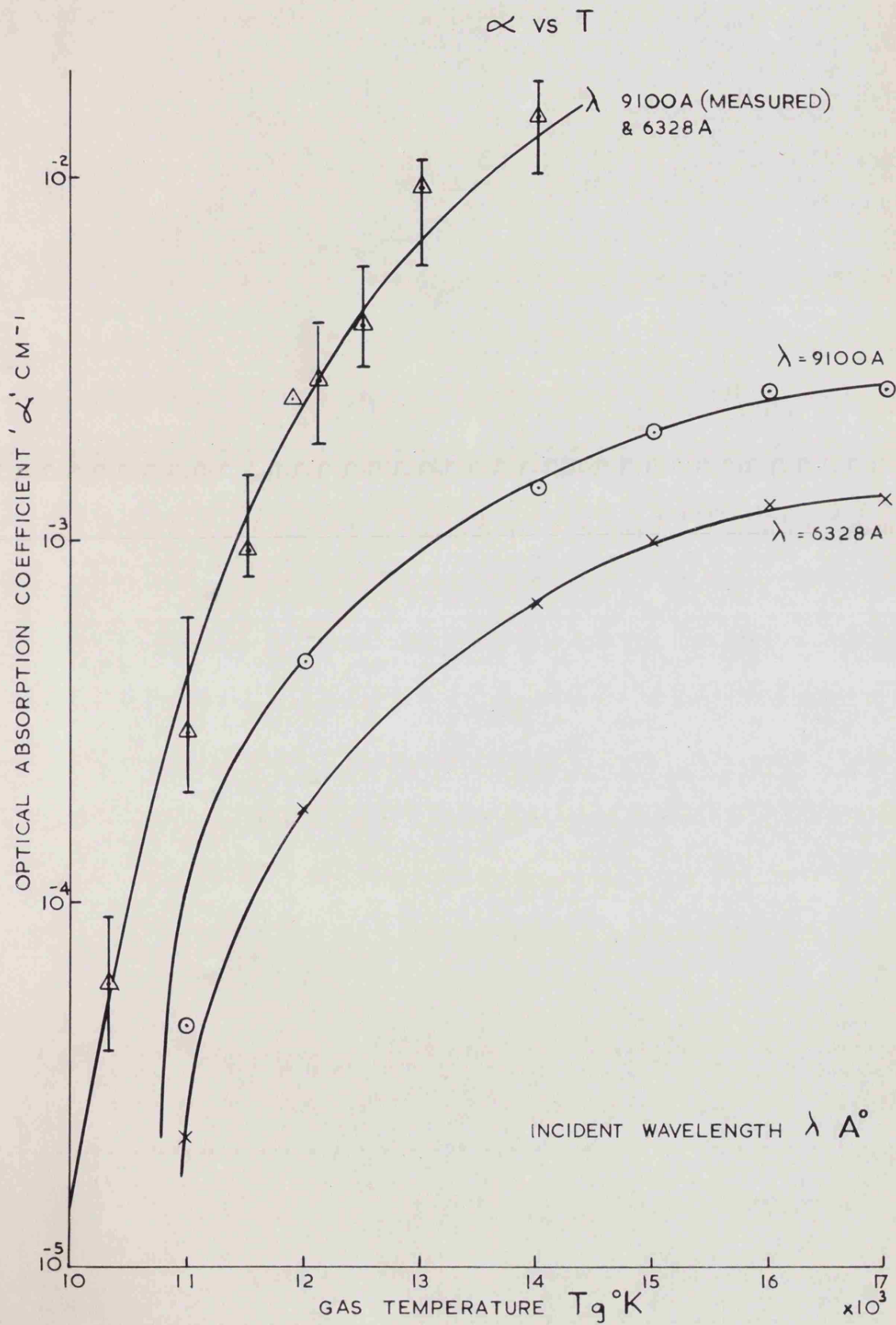
NITROGEN PLASMA JET TEMPERATURE PROFILES

FIG.56



ARGON
CALCULATED & MEASURED
OPTICAL ABSORPTION COEFFICIENTS

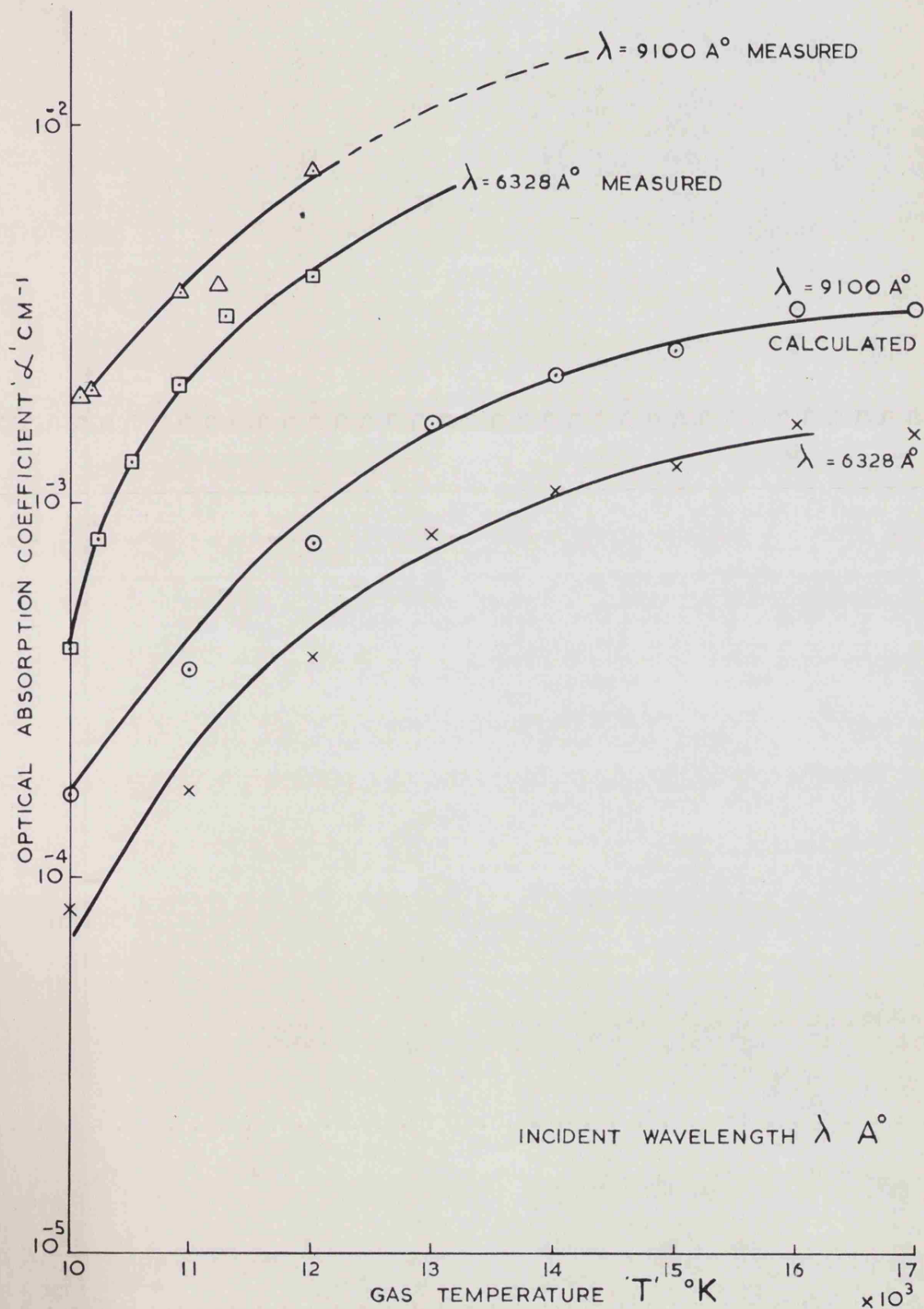
FIG.57.



NITROGEN
CALCULATED & MEASURED
OPTICAL ABSORPTION COEFFICIENTS

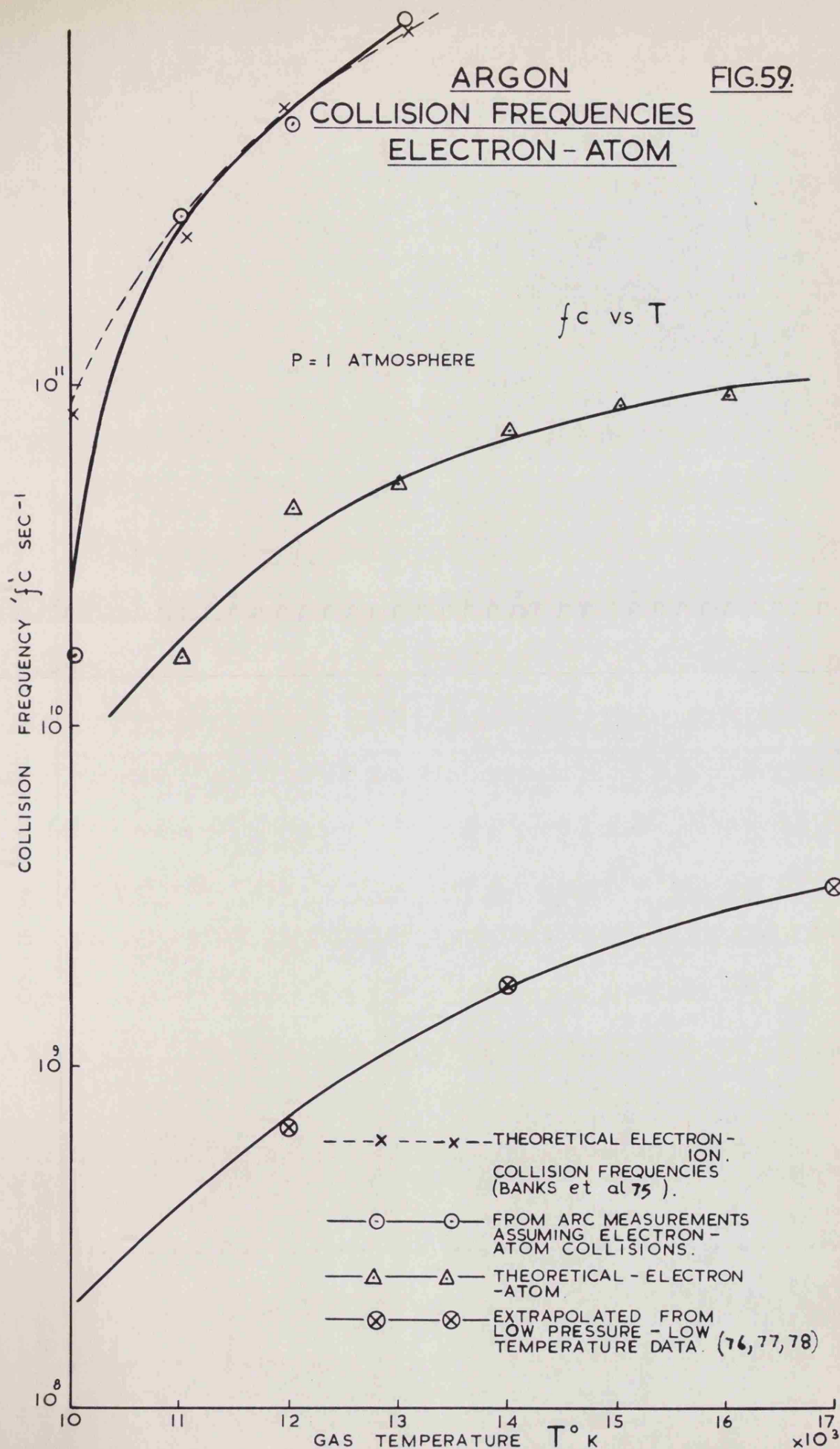
FIG.58

α vs T



ARGON COLLISION FREQUENCIES ELECTRON - ATOM

FIG.59.

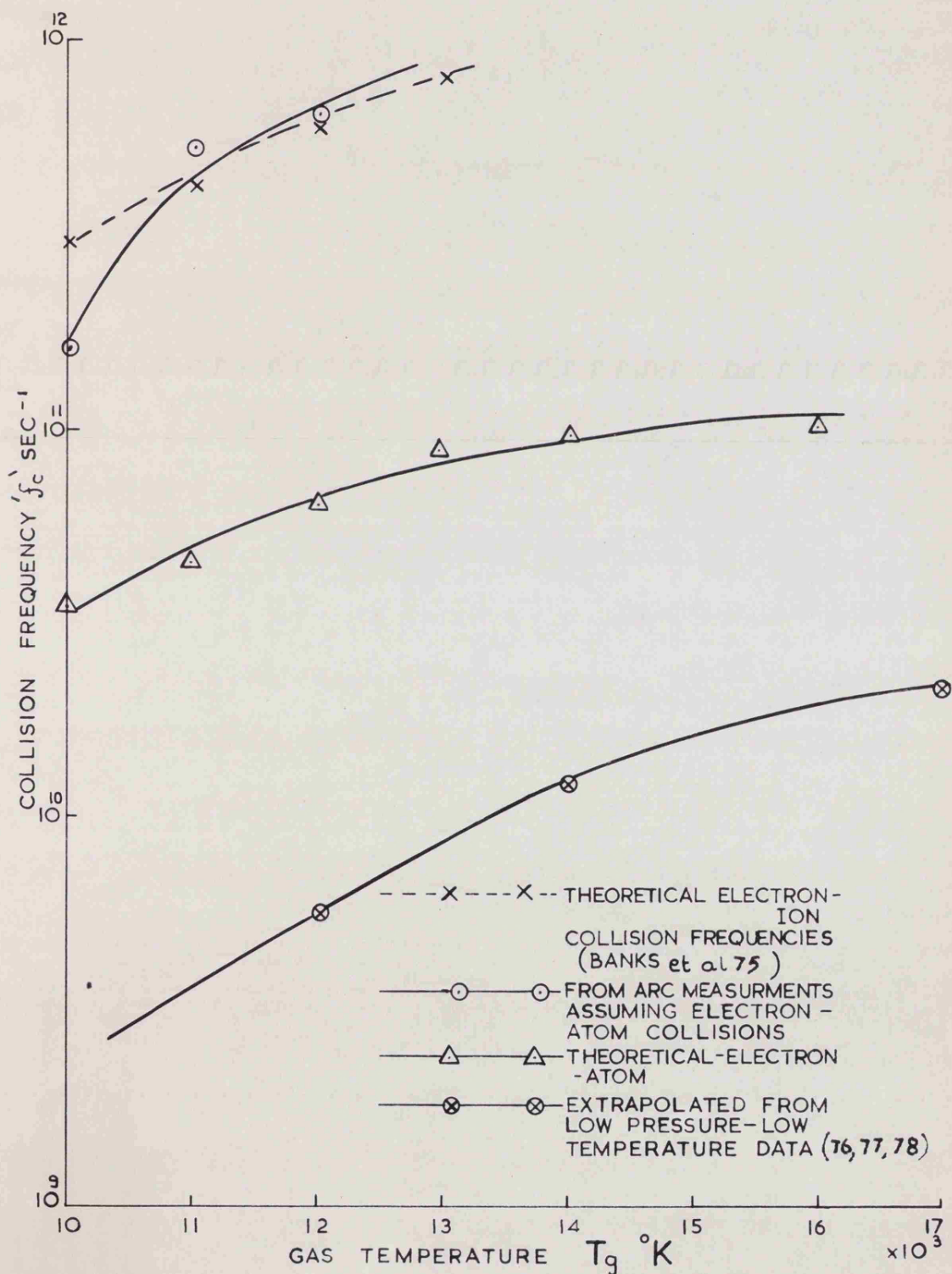


NITROGEN COLLISION FREQUENCIES ELECTRON-ATOM

FIG.60.

f_c vs T

P=1 ATMOSPHERE



Taking logarithms and re-arranging the result gives

$$\ln I_0/I = \alpha x \quad (9.6)$$

The averaged results of a number of experiments made at each current level and at two stand off heights of 0.5 and 1.0 cms above the jet orifice are shown graphically in Figures 47, 48, 49 and 50 for argon and nitrogen.

These results show the integrated absorption coefficient when the laser beam passes along a chord through the plasma of length x . To obtain the value of α at any radius r , recourse is had to the Abel integral inversion. The application of this mathematical technique is discussed in Appendix III. This integral, applied to the data shown in Figure 47, 48, 49 and 50, produces the radial distribution of absorption coefficient shown graphically in Figure 51, 52, 53 and 54. Since temperature is also measured as a function of radius, the absorption coefficient may be displayed as a function of temperature as shown in Figures 57 & 58. The plasma jet temperature profiles required to effect the conversion from a radial distribution to a function of temperature are shown in Figures 55 and 56.

Finally, the collision frequencies from equation 2.6 are shown as a function of temperature in Figures 59 and 60. These frequencies are compared with data extrapolated from low temperature, low density measurements. Details of the calculation of the collision frequencies from absorption data are set out in Appendix VIII. Theoretical derivations of the electron atom and electron ion collision frequencies are shown

in Appendices VI and VII.

6.4 Experimental Difficulties

The magneto ionic theory indicates that the absorption coefficient should increase monotonically with temperature. Spectroscopic measurements of temperature made on the plasma jets also indicated a monotonic increase in temperature from the edge to the axis. The first experimental results with the He-Ne laser did not show this form of variation for the absorption; instead of a peak in the absorption coefficient on the axis of the jet, symmetrical off axis peaks occurred whose magnitude was some 10 times greater than the values expected.

These anomalous results were considered in terms of optical absorption and re-emission and poor selectivity of the detector system.

6.4.1 Optical Absorption and Re-Emission

If an emission line in the plasma with a frequency slightly lower than the laser has a partly populated ground state then absorption of the laser radiation and subsequent re-emission is possible. Although a spectral line of the precise frequency of the laser is not reported in the argon spectrum it may be that a line of slightly lower energy can absorb some of the laser radiation. As the laser probe approaches the centre of the plasma jet the optical density of

this line will become very great and could result in the complete absorption of the laser radiation. Re-emission at a slightly lower frequency within the optical pass band of the system at a high intensity could explain the negative absorption coefficients obtained in the experiment. (A negative absorption coefficient can be interpreted as an enhancement of the incident radiation). The bandwidth of the optical filter in the experimental system was approximately $100\text{\AA}$. This would permit line frequencies very near to that of the laser to be accepted by the detector system. These lines would, of course, be modulated at the chopping frequency,

A search with a Hilger prism spectrometer across the plasma jet near to the laser frequency did not reveal any spectral lines enhanced by, or modulated at the laser chopping frequency.

6.4.2 Poor Selectivity of the Detector System

To test the discrimination of the detector amplifier system against optical and electrical noise from the plasma, the plasma jet was scanned across the detector, the laser being switched off. No output was obtained from the amplifier. However, an oscilloscope examination of the input to the first wideband amplifier (fixed gain AC) showed a high level high frequency signal which was causing a malfunction of the stage. The introduction of a low pass filter with a 100 Hz 6 db/octave "roll-off" removed the high frequency, interfering signal. The expected monotonic variation in absorption was then obtained.

With this improvement to the electronic system the Ga/As laser radiation apparently experienced no measurable absorption. However, the values obtained with the He-Ne laser were between 10 and 100 times higher than expected from the absorption theory. Two possible explanations were offered for this. The first was that the quantum mechanical inequality was being violated and the local thermal equilibrium of the plasma was being disturbed. The second possibility was that since the plasma column had a refractive index slightly less than unity over part of its diameter in the region where $(\mu - 1)$ is negative (Figure 40) it was acting as a negative lens. This meant in practice that the beam from the plasma column would diverge on passing through the plasma. Since the detectors were only of a sufficient size to just accommodate the emergent beam from the He-Ne laser, the introduction of a negative lens would cause the initially parallel radiation to diverge and spill over the sides of the detector and hence indicate apparent attenuation in the beam.

As this was the easier of the two theories to check, a convex, quartz lens was interposed between the photo detector and the plasma column to converge the possibly diverging laser beam and thereby ensure that all the radiation was incident upon the detector.

With the introduction of this lens the absorption coefficient for the He-Ne radiation fell to a level below the resolving power of the simple amplifier-detector

system (Figure 43) and as described in Section 6.2.2, the "hulling" differential system was designed and built to provide a resolution and sensitivity commensurate with the magnitude of absorption predicted from theory (section 6.2.3).

Chapter 7

Discussion of Theoretical and Experimental Results

7.0 The Measurement of Thermal Conductivity

The success of the work depends upon the validity of the energy balance equation. As set down in Chapter 2, equation 1.2 is incomplete as it only considers conduction and radiation as elements in the heat transfer process. A more complete equation is given by Bird, Stewart⁽⁹⁵⁾ and others which includes a convective heat transfer term. The convective term, however, is small compared with the conduction process as the radial velocity components in the plasma column are essentially zero. This is achieved by ensuring that the flows in the constricted arc chamber are highly laminar. Very low mass flow rates (0.1-0.2 grm/sec) with the resultant low velocities, a low Reynold's number and a long (10-15cm) column which produces a well established laminar profile in the measuring region all contribute to the reduction in radial convection.

The low axial mass flow rate also ensures that axial transfer of thermal energy is very low. By way of example, consider the 200 Amp argon arc column. The total power input to a 10 cm column is about $50 \times 10^3 \text{ sec}^{-1}$.

Assuming an average column temperature of 10^4 K (a very

high estimate) and a mass flow of 0.2 gm/sec the axial flow of power, assuming an atomic weight for argon of 40 and total enthalpy of 50 K cal/mole/sec at 10^4 K, will be approximately 10^3 j. sec^{-1} . Thus it does not seem unreasonable to neglect this heat transfer process.

As may be seen from the development of Chapter 2, the reduced balance equation, considering only the conduction term, is not accurate at high temperatures. The radiation transfer term must be included even though the contribution made by convective processes may safely be neglected. It is quite obvious from Figures 15 and 16 that the determination of thermal conductivity using the reduced balance equation is accurate for argon and nitrogen up to about 12 000 deg. Kelvin. Above this level, however, there is considerable divergence in the results particularly in the case of nitrogen, the measured conductivity rising at a high rate whilst the calculated conductivity (both reported and calculated by the author) shows an actual fall in real thermal conductivity.

If it is assumed that terms other than conduction may be neglected, then sources of error are worth considering in more detail. An obvious source of error could lie in the measurement of the temperature profile in the arc column or more important, the temperature gradients. At high temperatures (greater than $10,000^\circ$) strong atomic and ionic emission lines are present in the arc spectrum and it is not very difficult to make accurate measurements of temperature at a high spatial resolution. Furthermore, temperature

comparisons can be made from measurements on different species such as emitting atoms and ions, both first and second ionisation being considered. However, at lower temperatures, as the cooled wall is approached, the density of electrons and ions falls very rapidly as predicted by the Saha equation and line radiation intensities are low. Furthermore, with the presence of the cooled arc chamber wall, very steep gradients are encountered over a small change in radius and this can lead to very considerable inaccuracies in predicting low temperature conductivities.

It is important to notice here, however, that the inaccuracies in the measurements do not occur at the low temperature end of the scale and, in fact, agreement between measurement and theoretical calculations is good. This indicates that the extrapolation towards the wall is valid and also suggests that, since the thermal conductivities in the arc are a summation of the values at discrete intervals in radius, the inaccuracies in the high temperature high conductivity region should not be unduly influenced by errors in what are relatively low conductivities at low temperatures.

A second source of error could possibly be the electrical conductivity which is used in computing the power input to the system. Here a ready check is to be had between the published data which is used and the values measured in the arc. A direct measurement of arc current and field strength gives an integrated value for arc conductivity. Over the temperature range measured, a summation of the published

electrical conductivities as a function of temperature gives close agreement with the measured values (within 10%) thereby tending to eliminate this error source.

In a plasma torch at high mass flow rates, a strong convective energy transfer process is known to occur, in fact the plasma torch depends upon this. In the wall stabilised arc, however, the mass flow rates are very low (a small fraction of a gram of gas per second). A simple calculation shows that the errors resulting from ignoring this term are quite insignificant.

Section 2.9 indicates a method of including radiative transfer from the arc axis where measurement of temperature gradient is relatively easy and where radiation energy transfer might be expected to be highest, (the Planck and Wien radiation laws demonstrate this).

As can be seen from Figures 19 and 20, when measured arc radiation energy is included in the balance equation, very little change is made in the divergence between theory and measurement. A simple prediction of how much radiation should flow from the arc axis is not easy and this subject is dealt with in considerable detail in Chapter 3. An estimate can be made of the radiation which might be expected if the plasma was assumed to obey Planck's radiation law but here difficulties are encountered in knowing what emissivity should be ascribed to the gas, particularly as this will be a function of temperature and wavelength. Before examining the radiation from the plasma in greater detail, the experimental

technique is worth considering. The thermopile and radiometer system used did not give a high spatial resolution but sufficient data was collected to indicate the distribution of radiation across the arc. Furthermore, there was relatively good agreement between the total radiation measured from the arc and the summation of the radiation from different zones. At low temperatures near the wall of the arc chamber the radiation density was negligible. At low temperatures, however, as already pointed out, there is no great discrepancy between the measured and calculated conductivity results; it is therefore at the high temperatures where a high level of radiation might be expected and is, in fact, needed to correct the divergence in results. Unfortunately, only a relatively low level was measured. To check the accuracy of the thermopile and the associated radiometry equipment, a measurement was made using a Nernst "glow-bar". Assuming this glow-bar to have an emissivity of 1.0, the Stephan-Boltzmann formula was used to compute the bar temperature from the absolute radiation intensity. Agreement was had to within 5% of the temperature measured with a platinum - platinum-rhodium thermocouple.

The accuracy of the thermopile being established, other sources of error were examined and in particular detailed calculations made of the radiation density.

Two sources of error are readily discovered. The first error source is the non-transparency of the plasma to short

wavelength radiation. Wiens displacement law predicts a shift in the peak radiation intensity towards the ultra violet end of the spectrum as the temperature increases. In the centre of the arc where the temperature is highest, the frequency of radiation at the peak of the intensity curve, determined by Wiens law is in the region of $1,500 \text{ \AA}$ units. This wavelength is far shorter than the normally accepted transparency limit for gases at normal pressures in the ultra-violet. Therefore strong absorption is expected in passing through the plasma and in passing through the atmosphere to the radiometer.

A second error source was found in the window of the thermopile which was made of fused quartz. This material has a pass band of 2,500 to $25,000 \text{ \AA}$ units. Thus not only is the thermopile selective in rejecting short wavelength radiation but also in filtering out much longer wavelength radiation. Thus, unless radiation measurements can be made in vacuum in plasmas which are both optically and geometrically thin, the use of a thermopile is of little value in obtaining data on radiation for making corrections to thermal conductivity measurements.

When the total radiation intensity, from the ionisation edge of the gas under examination out to the far infra-red, is calculated and these data incorporated in the energy balance equation, good agreement between theory and measurement is obtained.

7.1 The Calculation of Radiation Intensities

At relatively low temperatures, gases are transparent to thermal radiation over a wide part of the spectrum. As the temperature of the gas rises, energetic processes occur which result in the emission of electro-magnetic radiation at discrete frequencies. Emission of radiation also implies absorption and unfortunately, from the point of view of calculating the quantity of radiation emitted from the gas the wavelength dependence of the absorption must be calculated as the gas is a selective absorber. In a gas, not only the length of the radiation path but also the number of particles encountered by the radiation, that is the product of density and length are important. After travelling a distance x in an absorbing medium the intensity of radiation I_x may be written as

$$I_x = I_0 \exp - \alpha x$$

Where I_0 is the initial intensity and α is an absorption coefficient.

In a gas this absorption coefficient is a function of wavelength and it is this dependence that provides a major difficulty in calculating the intensity of radiation emitted by a hot gas.

When the optical thickness of the layer becomes infinite, that is as αx approaches unity, the absorption and hence the

emissivity of the gas is then the same as that of a black body. Unfortunately, because of selective absorption, emitting gases do not obey the Stephan-Boltzmann law.

The general spectral distribution equations for an emitting gas are given in Chapter 3 equations (1.3 to (4.3). Equation (4.3), however, requires that the optical thickness of the gas be much less than 1. At high temperatures the optical depth, however, can no longer be considered negligible and the gas is no longer transparent to radiation at all frequencies. The radiation intensity in non-transparent plasmas is critically dependent upon the type of radiative process being considered and for the high temperature plasmas under examination in this study three mechanisms must be taken into account, that is the free-free, free-bound and bound-bound transitions. Essentially these mechanisms cover line and continuum radiation. The details of these calculations are given in Chapter 3, Section 3.3. Although the arithmetic required is trivial, if time consuming, the data required for the calculations is not always readily available.

The calculation of the radiation intensity was divided into two regions.

- (a) From the ionisation edge to $2,500 \text{ \AA}$ units and
- (b) $2,500 \text{ \AA}$ units to the far infra-red.

This division at $2,500 \text{ \AA}$ units was made on two counts. The first being that the optical band pass of the window in

the thermopile was bounded at the high frequency end of the spectrum by strong absorption at $2,500 \text{ \AA}$ and secondly, the vacuum ultra-violet is normally accepted as starting at $2,500 \text{ \AA}$, very high absorption coefficients being measured in air.

7.1.1 Comparison of Results for Thermal Radiation Calculations

Where possible the calculations and measurements of thermal radiation made in this study have been compared with published data with a strong measure of agreement.

The good agreement is not altogether surprising as due to the limited amount of basic data on such things as photo-excitation cross sections, transition probabilities and statistical weights etc., practically all workers in the field are left to rely upon the same data sources.

Using the absorption and radiation data obtained from these calculations good agreement is seen between previously divergent results of theory and measurement of thermal conductivity at high temperatures.

7.2 Measurements of Viscosity

Using the elementary theory of viscous flow under streamline conditions, (the Poiseuille equation) a balance equation (e.g. (1.4)) similar in form to that generated for the arc energy balance can be obtained. This equation gives

pressure drop along the column in terms of the radius, viscosity as a function of temperature and the velocity gradient. Of these variables the pressure drop is relatively simple to measure by small manometers introduced into the wall constricting discs of the arc chamber. The velocity gradient, however, is considerably more difficult to obtain. Various techniques including the photographic profile following of carbon particles introduced into jets, and the deflection or disturbance of small particles passing through the jet have met with varying degrees of success and failure.

For very high current high velocity arcs, a relatively recent paper by Bowman described a velocity measurement obtained by dropping small steel ballbearings through the arc and observing their deflection. However, application of these techniques requires a high velocity flow, which with the apparatus available was not considered satisfactory. Neither was the spectrographic technique using the Doppler effect. A technique, described in Chapter 4 was developed during previous work at the University of Leicester for the bulk velocity measurement of a plasma and has subsequently been improved and modified to permit the measurement of velocity profiles.

7.2.1 Possible Sources of Error in the Velocity Measurements

One source of error which may be significant is that when a high current pulse is injected into the plasma column, the measurement relies upon the resultant high temperature

zone moving outwards along the arc axis with the same velocity as the arc gases. Because the bright zone observed represents a high concentration of electrons and ions, these will be diffusing at a velocity determined by the diffusion coefficients of the particular species at the prevailing temperatures in the arc. As can be seen from later work on calculation of diffusion coefficients in argon and nitrogen (Chapter 5) the measured values of gas velocity are commensurate if not lower than the diffusion velocities, evidence of this is also seen in the form of the hot zone which when it passes the observation windows is already very diffuse after only 2 cm of travel from the cathode. However, the coherence of this bright zone is still sufficient to make a measurement and as will be seen later the agreement in the calculated and measured viscosities is sufficiently good to indicate that errors in the velocity measurements were not too significant.

A second source of error associated with the injection of a high current pulse is local heating and compression waves which affect the velocity distributions. No attempt has been made to estimate the magnitude of such errors as the agreement between measured and calculated viscosity profiles is sufficiently good to allay fears on this count.

Both the problem of diffusion and pressure disturbances require an accurate knowledge of the transient temperature induced by the current pulse. A time resolved temperature measurement in the arc column was not possible during the course of this study, but using temperature data on pulsed

arcs reported in work by Morris⁽⁴⁷⁾ an estimate of the temperature in the hot moving zone could possibly be made. This would entail deductions as to the transient electrical conductivity of the arc and of the circuit constants which form the pulse.

Due to the limited number of velocity measurements which could be made across the profile of the arc, the spatial resolution of velocity is not high; however, as can be seen from the graphs shown in Figures 30 and 31, the agreement, particularly in the case of argon, between calculation and measurement of viscosity is good. In the case of nitrogen the agreement in profile is not quite so good although the magnitudes of calculation and theory are sufficiently close to be acceptable. A reason for the poorer correlation in the case of the nitrogen arc could well be put down to the considerable difficulties in obtaining a stable arc even in the long, cooled wall column.

7.3 Theoretical Predictions of Transport Properties

It is essential here to point out that the calculations made on transport properties in high temperature gases using the method derived from dimensional analysis and induction is only claimed to be valid over a relatively limited range, that is up to dissociation and first ionisation at atmospheric pressures. The method has not been tested at higher levels of ionisation nor at reduced or enhanced pressures. In all probability it would fail at much lower pressures as reliance is placed on the existence of short

mean free paths and frequent collisions.

From the graphical presentation of the results, however, it is obvious that a very good measure of agreement between rigorous theoretical calculations as published by other workers and this simplified scheme can be obtained. Not only are the magnitudes of the properties obtained very close to those published over a wide range of temperature but also over this range the fine detail or structure of the variation of a given property with temperature is also well displayed. As a means of providing transport property data for engineering applications, therefore, the method has much to recommend it over complex experiment and rigorous calculation.

7.3.1 Frozen Composition Plasmas

For gases whose composition is assumed to be frozen, that is no new species are generated or destroyed, the form of the equations obtained by induction from dimensional analysis and those obtained from the rigorous application of the kinetic theory are identical and the graphs of frozen composition properties are in extremely good agreement even at high temperatures. One thing which is brought out in this study of frozen composition is that the Sutherland correction which is normally applied only over a limited, low temperature range, appears valid up to very high temperatures of the order of 18 to 20,000°K.

7.3.2 Plasma Properties

The real success of the simplified method developed during this study is brought out on examination of the transport properties as a function of temperature for gases whose composition is not frozen, that is for gases where new species are continuously generated. Once again the magnitudes of the calculated results are in very good agreement and the detail at dissociation and ionisation is well brought out. In the case of the thermal conductivity of the nitrogen plasma, the agreement between rigorous theory and the reduced simplified theory is far better than between any theoretical calculations and measured values obtained from simple experiments. In the case of the simplified theory, however, for nitrogen, there is a small shift in the maxima at dissociation and ionisation towards higher temperatures. These points appearing some 600 or 700° higher than in the rigorous case. The reason for this has not been investigated in any detail but in view of the fact that all the possible interactions between particles in nitrogen have not been considered and, in fact, some particles have been neglected completely for instance, the nitrogen negative ions, the results are considered to be more than acceptable when engineering calculations are required.

7.4 The Experimental Determination of Collision Cross Sections

A major reason for developing a simplified procedure for the calculation of transport properties was the difficulty in obtaining reliable data for the interaction parameters or collision cross sections of the particles in the gas or plasma. Usually this data is obtained from low temperature low pressure data and extrapolated upwards to very high temperatures at high densities. The intention behind this phase of the study was to examine the problems inherent in obtaining collision cross sections from somewhat more realistic experiments.

A seemingly ideal system for the measurement of electron collision frequencies under realistic conditions would be to produce a high density electron beam and measure its attenuation in passing through a high density, high temperature plasma. The generation of such high density collimated beams, however, was well beyond the scope of this study and was therefore not considered further. Provided that certain constraints are observed, however, an alternative technique of measuring the attenuation of an electro-magnetic wave in traversing the plasma can reveal information on collision frequencies. This is based on the theory of propagation of electro-magnetic waves in plasmas. For the plasma under consideration the following conditions were required to be observed.

- (a) External magnetic fields were zero

(b) The quantum mechanical inequality must be satisfied, i.e. $h\nu < kT$

(c) The plasma must be considered as a "cold plasma"

Using these constraints on the theory, electron-atom collision frequencies can be deduced from experiments made on the attenuation of electro magnetic radiation from a laser in passing through a plasma jet. Provided that sufficient spatial resolution can be obtained, the absorption coefficient as a function of radius can be obtained by inverting the experimental integrated absorption data with the Abel integral transform. Similarly if the temperature in the arc as a function of radius is known, it is possible to demonstrate the variation of absorption as a function of temperature.

7.4.1 Comparison of Calculated and Measured Absorption

Using the equation (1.6), the absorption coefficient was measured at two wavelengths as a function of radius and hence temperature. A theoretical prediction was also made of the absorption coefficient using data obtained from a direct measurement of temperature in the plasma jet and application of the Saha equation to obtain electron density. The comparison of these results is shown in Figures 57 and 58. In both cases, that is for argon and nitrogen, the measured absorption coefficients were considerably higher than the calculated values. This discrepancy in the values was not entirely attributable to experimental error and at high temperatures greater than $10,000^{\circ}\text{K}$ the order of disagreement was a factor of 10. Although considerable experimental difficulties have been recorded,

the noise measured on the radiometer with the plasma generator operating in the absence of the electro-magnetic radiation probe (laser beam) was several orders of magnitude lower than the signal strength from the probe.

Similarly the measured temperature profiles have been demonstrated to be satisfactory when used for determining other properties in the arc.

Using equation (2.6), with the previously obtained experimental and theoretical absorption data, a possible explanation of the discrepancy between theory and calculation is obtained. Inserting realistically predicted values of refractive index (confirmed by the measurements of Sturrock⁽⁶⁸⁾) and the radial frequency of illumination into equation 2.6, the collision frequencies obtained are shown once again to be far higher than those predicted by theory. The discrepancies here are even more marked, there being almost a factor of 10 difference between the electron-atom collision frequencies as obtained from a low temperature low pressure plasma and the theoretical results predicted by equation (2.6). Between these theoretical collision frequencies and the collision frequencies calculated from the absorption coefficients measured in a high density plasma there is once again a further factor 10.

This work, however, has assumed that the plasma electron density is far higher than that of other species. It is known, however, that at the temperatures under

consideration electron and ion densities are very similar. Using a modified version of the electron-ionic theory produced by Banks and others⁽⁷⁵⁾ to calculate the collision frequencies between electrons and ions a very close agreement is obtained with the measured frequencies of what were considered to be electron-atom collision. In other words, the assumption that the electro-magnetic radiation only loses energy in passing through an electron-atom gas is not valid and the plasma must be considered to contain a high density of ions. Further theoretical and experimental work is required to separate the electron-atom and electron-ion collision frequencies in the experimental results.

Chapter 8

Conclusions and Recommendations

The discussion of the theoretical and experimental results in the previous chapter leads naturally to the following conclusions.

8.1 Transport Property Measurement

Common to both the measurement of thermal conductivity and viscosity of high temperature gas plasmas is a requirement for generating a stable, high temperature volume of gas whose geometry is accurately known. The constricted arc column serves this purpose but has obvious weaknesses. To obtain very high temperatures, high electrical power inputs must be provided for the arc column. These high powers pose very considerable problems of dissipation in the column, failure to provide adequate cooling leading to destruction. Alternatively, the column diameter can be made very small to cause strong constriction of the arc with the ensuing high temperatures. This, however, leads to problems of temperature measurement. The small diameter makes high spatial resolution measurement of temperature very difficult.

8.1.1 Temperature Measurements

Moving now to the measurement of temperature in the

arc, in principle this is a well established technique employing the relationship between emission intensity of a spectral line and the temperature of the energetic electrons producing the emission. Straight away a difficulty arises; is the electron temperature the same as the temperature of the gas whose properties are being measured? Problems of thermal equilibrium are now raised and confidence in any result can be low unless considerable experimental sophistication is introduced. Much more important, however, is the problem of interpreting the measured line intensities. For the common gases, argon, hydrogen, nitrogen, etc., the physical constants required to convert intensity into temperature are well documented, albeit with considerable choice as to which are the correct ones. In the case of many gases used in engineering applications, the Freon range of gases for instance, no such abundance of data is available, and unless thermal equilibrium in the arc can be guaranteed, the practice of mixing traces of well characterised gases with the gas under investigation for temperature determination can lead to very questionable results.

8.1.2 Radiation

If the problem of temperature measurement is satisfactorily solved the way is still not clear to making reliable measurements. This is obvious from the comparison of the measured and calculated thermal conductivities as a function of temperature. The very significant discrepancy between measurement and theory has

been shown to be due to thermal radiation enhancing the value of the "Conduction Conductivity". Two courses are apparently open to separate the radiation contribution from the apparent conductivity:

(a) Measurement

(b) Calculation

Although measurement of the absolute intensity of radiation emanating from the arc column presents some experimental difficulty the real problem is not immediately apparent. Measurements made at atmospheric pressure where radiation from the arc axis first must traverse layers of cooler plasma, then an air gap to the measuring system, finally passing through some form of window to reach the measuring system have been shown to produce gross underestimates of intensity. This is due to strong absorption of the short wavelength ultra-violet and long wavelength infra-red radiation. To overcome this, the only experimental technique available is to make the measurements in a vacuum with windowless detectors, the radiation passing through hollow electrodes, viewing the plasma end on to avoid the filtering action of the plasma. Such an experiment raises the experimental difficulties to a level far above those already experienced in the system used in this study.

The calculation of the total radiation intensity from a volume of hot gas would be simple if the gas could be considered to be a black or even grey body. Unfortunately,

radiating gases are selective radiators, that is the emissivity is a function of frequency; this function is not continuous and analytic. These frequency dependant emissivities or alternatively absorption coefficients can in principle be calculated but once again the physical constants required to make the calculations are rarely easily available, if at all. The information required is in part the same as that required for temperature measurement, that is, statistical weights and transition probabilities. In addition line widths, Gaunt factors and photo excitation cross sections are required. Even for the common gases this information is not over abundant and for the less common ones is non existent. Thus, for other than the well characterised gases the choice is to make intelligent guesses for the radiation constants or invest in very sophisticated equipment to make reliable measurements.

Provided that the total radiation intensity can be quantified the real, conduction thermal conductivity can be measured with reasonable accuracy as has been shown in the comparison of the experimental and theoretical results in this study.

8.2 Transport Property Calculations: Established Theory

The theory of energy transport in gases, both simple and multi-component is a well established mathematical science, therefore in principle, very accurate calculations of transport properties as a function of temperature can

and have been made. A more detailed examination of the theory, however, shows that apart from the mathematical complexity, which can be handled with modern computing techniques, temperature and pressure dependent kinetic and compositional data is required some of which is either readily available or easily calculable (composition as a function of temperature) whilst other data on complex particle interactions (collision cross sections) must be measured or arrived at by intelligent guesswork.

As with the optical properties required for radiation calculations, collision data is available for the common gases, but when complex molecular gases are considered, data is sadly lacking.

8.2.1 Measurement of Collision Cross Sections

As part of this study, the measurement of the collision cross sections required for calculating the transport coefficients was examined.

Although the constricted arc column might have provided the most suitable means of generating the plasma required for the measurements, it was abandoned in favour of the free jet produced by a constricted arc plasma torch on the following grounds.

- (a) As small absorption coefficients were anticipated (< 0.1) the rapid "devitrification" and clouding of the quartz observation windows in the column was considered to be an experimental

artefact which would obscure the true result.

- (b) The long constricted column is a complex device with attendant experimental difficulties. One aim of this study was to find relatively simple ways of arriving at the desired result and it was felt that the free jet, although not having the well defined boundaries of the enclosed column, offered a satisfactory means of providing a plasma, provided that measurements were made relatively close to the orifice where cylindrical symmetry existed.

Apart from the problem of measuring the small changes in absorption of the laser radiation on passing through the plasma, solved with relatively standard differential measuring techniques, this part of the experimental study was straightforward. However, the interpretation of the results leaves certain questions unanswered.

The electron-atom collision frequencies derived from the measured absorption coefficients were a factor ten times greater than those predicted by the magneto-ionic theory which were a similar order higher than experimental results obtained from low pressure, low temperature measurements. Even allowing for errors in the measurement of temperature and absorption coefficient in the jet, together with inaccuracies in the other data required to calculate the collision frequencies, these differences cannot be fully explained.

Further examination of the theory of absorption, however, does offer some guidance for improving the agreement between the plasma jet measurements and the magneto-ionic theory. The absorption mechanism in the plasma jet has been considered to be due to electron-atom collisions. In the jet used in the experiments, the atom density in the hot central zone was very much less than the ion density and it is possible that the electron-ion collision mechanism should have been considered in the calculation of collision frequencies. This view is supported by two pieces of evidence.

- (a) In the cooler regions of the jet the agreement between measurement and calculation improves considerably as might be expected if electron-atom collisions predominated.
- (b) An extension of the magneto-ionic theory by Banks⁽⁷⁵⁾ and others gives equations for the calculation of electron-ion collision frequencies. Calculating the expected frequencies using these equations gives extremely good agreement between measurement and theory and strongly supports the suggestion that a large part of the measured absorption was indeed due to electron-ion collisions.

No attempt has been made in this work to separate the electron ion and electron atom collisions and this could form the basis of further experimental and theoretical studies.

8.3 An Alternative Simplified Method of Calculating the Values of the Transport Coefficients

The rigorous mathematical theory of transport calculations has been criticised for its complexity and, more important, difficulty in application in the absence of physical data on the gas under consideration.

The theory developed in this study offers a means of deriving some of the transport coefficients in the absence of detailed physical constants, combined with a simplicity of application which reduced the complexity from a digital computer problem to one suitable for a slide rule.

Obviously some data is required but it is of a much more basic nature than collision cross sections for the various species. Provided that the equilibrium composition of the gas as a function of temperature can be obtained together with ionisation and dissociation energies, then very good approximations to the transport coefficients as functions of temperature can be obtained. The method may be criticised for lack of rigour, but the demonstrated comparisons of transport coefficients obtained by this method (The Dobbs method), rigorous theory and measurements show its value.

The range of applicability to low and high pressures and an extended temperature range to cover multiple ionisations are subjects which have not been examined and could well form the basis of a further study.

There is no doubt in the mind of the author that compared with the experimental difficulties associated with the measurement of the transport coefficients, together with the pitfalls, for instance, of accounting for radiation that this simplified scheme for calculation is of considerable engineering value even if it throws little light on the physics of ionised gases.

APPENDIX IAI.0 The Spectrographic Determination of Temperature

The emission of electro-magnetic radiation at discrete frequencies in gases at high temperatures (in an arc) is due to the quantum-mechanical behaviour of the gas in an excited state. When energy is supplied to an atom, electrons in normally low energy states can be raised to discrete quantized higher energies. Ultimately an electron can be completely removed from the atom then the atom is said to be ionised. The removal of one electron produces a singly ionised atom, the removal of two and more electrons leads to doubly and triply ionised atoms etc. Electrons in these higher energy states are quasi stable and will spontaneously return to their normal state. In order to maintain the detailed energy balance, the excited electron must lose its excess energy which it can do by emitting a packet of radiation called a photon. If the energies associated with the normal and excited states of E_n and E_m respectively, the energy of the photon emitted by an electron making the transition $E_m - E_n$ is given by

$$E_m - E_n = h\nu \quad (AI.I)$$

where h is Planck's constant and ν is the frequency of the radiation.

The absolute intensity of a spectral line corresponding to transitions between excited and normal states in a volume V is given by

$$I_{mn} = \Delta V \cdot A_{mn} h \nu_{mn} N_m \quad (A2.I)$$

where N_m is the number of particles (atoms or ions etc) in the higher energy state. "A" is the Einstein transition probability of a particle in a state m , spontaneously making a transition to the state n . The transition probability may be considered to be a function of the time a particle spends in an excited state.

The number of particles N_m in an excited state at an absolute temperature T is related to the total population of that particular particle N_0 by the distribution function

$$N_m = N_0 (g_m/Z) \exp(-E_m/kT) \quad (A3.I)$$

where Z is the internal partition function for the particle in question, g_m the statistical weight of the m^{th} level and k is Boltzmann's constant.

The distribution function and the intensity function can be combined to give the following result for the radiative intensity of a transition

$$I_{mn} = \Delta V \cdot A_{mn} h \nu_{mn} \cdot N_0 g_m \frac{\exp(-E_m/kT)}{Z} \quad (A4.I)$$

In principle, if the absolute intensity of a spectral line is measured the temperature of the system emitting the line can be calculated. This raised complex, difficult experimental problems and for temperature measurement recourse is usually had to somewhat simpler techniques which require only the measurement of relative line intensity.

Temperature Measurements from Relative Line Intensities

Consider radiation from two discrete spectral lines emanating from the same volume element of a gas, conditions of local thermal equilibrium prevailing. If the lines have upper energy levels E_p and E_q and radiate with frequencies ν_p and ν_q then the ratio of their relative intensities I_p and I_q can be written using equation A4.I

$$\frac{I_p}{I_q} = \frac{\nu_p \cdot A_p g_p}{\nu_q \cdot A_q g_q} \cdot \exp - (E_p - E_q)/kT \quad (A5.I)$$

Finally, replacing the frequency ν with the wavelength λ , this equation can be rearranged to give the temperature as

$$T = \frac{\frac{1}{k} (E_p - E_q)}{\log_n \frac{I_q \lambda_q A_p g_p}{I_p \lambda_p A_q g_q}} \quad (A6.I)$$

For this relatively simple method both lines should belong to either the spectrum emitted by the neutral atom or to the ionic spectrum of the element.

In equation A6.I, T is the absolute temperature, k the Boltzmann constant and g_p and g_q the statistical weights of the levels p and q . The statistical weight or degeneracy is given by the values of the magnetic quantum number M_j

$$g_p = M_j = 2J_p + 1 \quad (A7.I)$$

J_p is the inner quantum number and specifies the total angular momentum of the atom.

For hydrogenic levels, the statistic weight is given by

$$g_p = 2 \sum_{l=0}^{p-1} (2l + 1) = 2p^2 \quad (A8.I)$$

In the above equation the term $2l + 1$ gives the orientation of the orbital momentum and the factor 2 accounts for the two spin directions.

Energy levels and values of J_p are comprehensively tabulated by Charlotte Moore⁽⁷⁹⁾.

To clarify confusion which occasionally exists in the literature, in place of the transition probability, a quantity called the absorption oscillator strength " f " is used. This term was introduced by Ladenburg⁽⁸⁰⁾ to relate the classical and quantum theory of atomic absorption. By definition, the oscillator strength f for the transition $m - n$ is the number of classical oscillators that produce the same absorption as one

atom in the state n . A relationship between transition probability and oscillator strength is given by Glennon and Wiese⁽⁸¹⁾ who give the following

$$A_{mn} = 6.6702 \cdot 10^{15} \frac{(g_n/g_m) f_{mn}}{\lambda_{mn}} \quad (\text{A9.I})$$

AI.1 Sources of Error in Temperature Measurement

A study of the literature and tabulated values of transition probability shows considerable variations in the magnitudes to be ascribed to various lines. To alleviate this difficulty to some degree, the temperature of any source should be measured with as many spectral line intensity ratios as possible. In this manner confidence in the results can be obtained when consistency between pairs of measurements is obtained.

Data on transition probabilities and oscillator strengths is given by the following authors

Argon. Argon I spectrum H.W. Drawin⁽⁸²⁾ W.L. Wiese⁽⁸³⁾
 Nitrogen. Neutral and N I spectrum. J. Richter⁽⁸⁴⁾
 and H. Motschmann⁽⁸⁵⁾

although this is not in any way comprehensive. For argon a detailed examination of temperature measurement has been made by Ruddy⁽²⁹⁾ of Leicester University.

A second weakness of the method in taking energy ratios is the very small energy difference between two excited

states in the same system, that is between two states in the atomic spectrum or two in the spectrum of the singly ionised atom. This difference is rarely more than the normal thermal energy kT . This makes the method somewhat insensitive to temperature changes. Improvement can be made by using lines from successive ionisation levels where the separation $E_m - E_n$ can be much greater than kT . Unless very high temperature arcs are being examined the intensity of lines in the second ionisation spectrum may be very low. For this reason this modified technique was not pursued in this study.

AI.2 Optical Thinness

The arcs and plasmas under examination possess cylindrical symmetry; radiation from the centre of the system must therefore pass through layers of gas at different temperatures before reaching the spectrometer. It is essential that the degree of absorption of a spectral line is small if meaningful results are to be obtained. The plasma must then be optically thin.

Provided that the lines under examination do not originate from the system ground state, the number of unexcited particles available to absorb an incident photon will be small in the visible part of the spectrum. The conditions for optical thinness of an atmospheric argon plasma have been examined by ⁽²⁶⁾⁽⁸⁶⁾Olsen who concludes that the absorption

coefficients are insignificant. In the same reference, criteria for establishing the transparency of a plasma are given. It is worth pointing out here that lines in the ultra-violet part of the spectrum are often very strongly absorbed as has been pointed out in the course of this study on the measurement of radiation intensities.

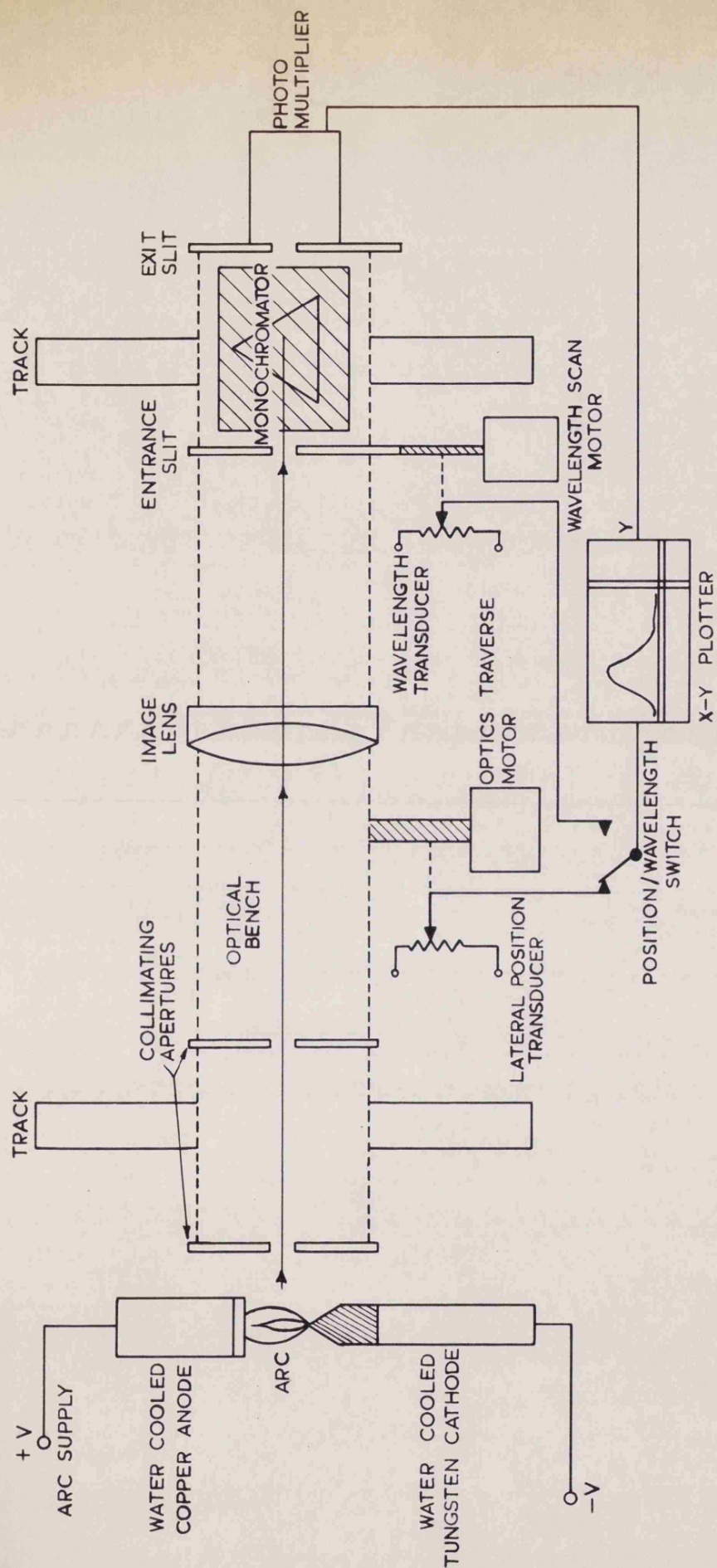
AI.3 Local Thermal Equilibrium

The intensity of an emitted line is determined by the number of particles making transitions between energy states. This transition rate is governed by the temperature of the particles. It is possible for electrons and ions in partially ionised gases to have temperatures considerably higher than the bulk temperature of the gas. Electrons for instance, accelerated through steep potential gradients can have very high effective temperatures in a relatively cold body of gas. Measurements made on emission lines from such particles will produce very erroneous estimates of gas temperature. It is essential therefore that electrons, ions and neutral gas particles shall be at the same temperature that is in local thermal equilibrium.

In atmospheric plasmas, the collision frequencies between excited and neutral particles can be expected to be high and energy interchange, leading to local thermal equilibrium conditions should prevail. This has been discussed by H.R. Griem⁽³¹⁾ who establishes theoretical criteria for establishing L.T.E. conditions. A suggestion has been made

by Olsen⁽²⁶⁾ that the temperatures in a plasma should be measured using two sets of emission lines from successively higher excitation states. If measurements from, say argon I and argon II spectra give the same temperature, then it is reasonable to assume that L.T.E. conditions will apply. For reasons of experimental difficulty this technique is not always easily applicable due to the relatively low intensity of lines in the higher ionisation states. The Russian investigators Gurevich and Podmoschenski⁽⁴⁵⁾ have investigated L.T.E. conditions in arcs and suggest an experimental technique for deciding whether such conditions apply. In this method the arc under investigation is short circuited by heavy current switch and the rate of decay of a particular spectral line is displayed on an oscilloscope. Cooling of the bulk plasma takes typically 10^{-3} to 10^{-2} seconds whereas electrons lose their surplus energy in 10^{-7} to 10^{-6} seconds. If the electron temperature is higher than the gas temperature, a two stage intensity-time profile is obtained. First, the electron temperature rapidly drops to the gas temperature then the gas cools to ambient. These two phases are claimed to be very easily distinguishable, differences between gas and electron temperature of 0.5% being resolved. This experimental technique has not been used during this study. On arcs and plasmas of a similar intensity and geometry investigated during experimental work previously undertaken, only a single phase cooling curve was seen, suggesting that the electrons and neutral gas particles were in thermal equilibrium.

FIG. A1.



**SPECTROGRAPHIC
TEMPERATURE MEASURING SYSTEM
SCHEMATIC**

AI.4 Experimental Details of the Spectrographic Temperature Measurement Technique

Many published descriptions of spectrographic temperature determinations of arcs and plasmas show an image of the arc being focused onto the entrance slit of the spectrometer, the slit being used to delineate the section of the arc selected for examination. Examination of the geometry of such systems, however, reveals that the section through the arc image is not, as believed, a parallel sided strip. Further, an exercise in geometrical optics shows that radiant energy from areas remote from the selected region can enter the dispersing optics and appear as noise on the output with resultant errors in the arc temperature profile.

To overcome this difficulty, referring to Figure A1, the arc is viewed through two coaxial, circular apertures mounted on a common optical bench. A colour corrected doublet produces an image of the second aperture on the entrance slit of the monochromator. The object and image distances for the lens are carefully adjusted to give unity magnification.

The apertures, lens and spectrometer are all mounted on a common, rigid, movable base so that the arc or plasma can be scanned by the complete optical test set which is shown in detail in the schematic.

An electrical signal representing the position of the optical system with respect to the major axis of the arc was

produced from a linear potentiometer actuated by the monochromator traverse.

The intensity of a selected spectral line was monitored by a photo-multiplier set at the focus of the dispersing optics in place of the monochromator eye piece. For spectral lines in the red-green spectral regions ($7,000\text{\AA} - 5,000\text{\AA}$) a Mullard photo-multiplier type 150 CVP was used. Wherever possible, lines of a very similar wavelength were used for measurement to overcome the objection to the variable response of the photo-multiplier.

In making a temperature measurement, the selected line image was first examined optically to ensure that the whole line was accommodated in the monochromator slit. An intensity plot was then made of the line by scanning the arc image across the monochromator field and then of the continuum radiation on which the emission line is superimposed. The line intensity at any point in the plasma is the difference between the two intensities. The total energy under the line profile was obtained graphically using a planimeter.

Since the intensity of an emission line measured "side-on" from a cylindrical source is the integrated intensity of the line after passing through successive regions of plasma at different temperatures, provided that the plasma is optically thin, the radial intensity can be obtained from the integrated intensity by use of the Abel integral transform. This is described in detail in Appendix III.

APPENDIX IIDetails of the Calculation of
Thermal Conductivity from Arc DataAII.1 The Full Temperature Profile

Neglecting the possibility of energy transport by radiation, the energy balance for the wall stabilised arc can be written, considering only the radial component of the cylindrical co-ordinate system, as

$$\sigma(r)E^2 = -\frac{1}{r} \frac{d}{dr} (rK \cdot \frac{dT}{dr}) \quad \text{A1.II}$$

with boundary conditions $T_{\text{wall}} = \text{constant}$

$$\frac{dT}{dr} = 0$$

$$r = 0$$

Integrating equation (A1.II) over "r" gives

$$E^2 \int_0^r \sigma(r) r dr = Kr \cdot \frac{dT}{dr} \quad \text{A2.II}$$

which on rearranging gives

$$K = -E^2 \int_0^r \sigma(r) r dr / (r dT/dr) \quad \text{A3.II}$$

For each set of arc measurements made at a constant current, the following data are required:

Arc potential gradient E v/cm from graph of
arc potential gradient vs current. Figure 13
and 14.

Gas electrical conductivity as a function of r and
 T . Figures 17 and 18.

The temperature gradients across the arc column
 $\left(\frac{dT}{dr}\right)_r$. Figures 11 and 12.

In order to obtain the apparent thermal conductivity " K " as
a function of radius (and ultimately temperature) the
integration of

$$- E^2 \int_0^r \sigma(r) r dr$$

must be made for each value of " r " for which $\sigma(r)$ is
recorded. In view of the accuracy of the data available
it was not considered worthwhile to fit a polynomial to
the electrical conductivity curves and so the integration
was made numerically.

For ease of manipulation on a desk calculator the simplest
numerical integration, a two-point trapezium was adopted.
(A test using Simpson's 3-point interpolation did not improve
the accuracy of the final result in spite of the increased
computational effort).

For each pair of adjacent values of $\sigma(r)$, i.e. σ_i , σ_j ,
the products $\sigma_i \cdot r_i$ and $\sigma_j r_j$ were calculated.

The area of the enclosed trapezium is, using

$$I = \sum ydx$$

$$A(r) = \frac{(\sigma_i r_i + \sigma_j r_j)}{2} (r_j - r_i)$$

the summation $\sum A(r)$ is then made and tabulated.

Table A2(1) shows the complete set of calculations for the nitrogen arc over a current range of 50-250 amperes.

Table A2(2) shows a comparison of the integration $\sum ydx$ by Simpson's rule, a trapezoid and the measured value of the total electrical conductivity for the 200A arc.

The thermal conductivity obtained is shown graphically in Figure 16 as a function of temperature.

As a check on the accuracy of the numerical integration, the total electrical conductivity of the arc column given by

$$\sigma_{\text{total}} = 2\pi \sum A(r) \quad (\text{A4.II})$$

can be compared with the measure of value obtained from arc voltage gradient and current measurements, E and I , $\sigma_{\text{total}} = I/E$

Burhorn⁽²⁰⁾ suggests adjusting the terms in the integration until the computed and measured conductivities agree. Since the calculation relies on the accuracy of the temperature-radius measurements, the validity of such

an exercise is dubious.

AII.2 The Two Temperature Model

A major source of inaccuracy in the determination of the thermal conductivity of a gas lies in the difficulty of measuring the temperature gradients away from the arc axis.

In Chapter 2 of this study an equation was developed for the thermal conductivity of the gas, considering only the axial zone of the arc. Neglecting the radiation term again this gave

$$K = \frac{\sigma_{\text{axial}} E^2}{2 d^4 T / dr^2} \quad (\text{A5.II})$$

which in finite difference form becomes

$$K = \frac{\sigma_{\text{axial}} E^2}{4 (T_{\text{ax}} - T_r) / r^2}$$

Thus only two temperatures, close to the arc axis where measurement accuracy is higher, are required.

For each value of arc current the arc potential gradient is recorded.

The axial temperature and the temperature at $r = 0.025$ cms together with the electrical conductivity at T_{axial} is the only additional experimental information required.

Table AII(3) gives the full details of the calculations for nitrogen.

AII.3 Calculation of the Thermal Conductivity Including Radiation Energy

The two previous sections of this Appendix show results which ignore energy transported by radiation, this gives an "apparent" thermal conductivity.

The two temperature calculation, however, is easily adapted to account for the radiation, assuming that all the radiation emanates from the arc core at a temperature T_{axial} , and absorption, if any, occurs in the remaining relatively cool arc column, and in the intervening air space between the column and the measuring system.

The real thermal conductivity k , making the above assumption is then given by

$$k = -\frac{E^2 \sigma_{axial}}{2T''} + \frac{Q_{radiation}}{2T''} \quad A7.II$$

Table AII(3) includes the modified thermal conductivity of nitrogen using the radiation from the arc measured with a thermopile as described in Section 2.5ff.

Table A2(1) : The Nitrogen Arc. Detailed
Thermal Conductivity Calculations

The Integral $\int_0^R \sigma(r) r dr$ is evaluated as the sum
of the trapezia $\sum_0^R \sigma_i r_i + \sigma_j r_j \Delta r$

Arc Condition 1

I = 50 amps

E = 23.4 v/cm

r cm	T °K	$\sigma(r)$ mho/cm	$\sum ydx$ mho/cm	$E^2 \sum ydx$ Watts/cm	$\Delta T / \Delta r$ °K/cm	$K = \frac{E^2 \sum ydx}{r \Delta T / \Delta r}$ W.cm ⁻¹ °K ⁻¹
0	12,000	48	-	-	-	-
0.025	11,800	40	0.0125	6.9	8,000	0.035
0.05	11,000	30	0.0437	24.1	32,000	0.015
0.075	9,500	23	0.0840	46.4	60,000	0.01
0.1	8,350	16	0.147	81.2	46,000	0.018
0.125	7,650	4	0.173	95.5	28,000	0.027
0.15	7,150	2	0.183	101.1	20,000	0.034
0.175	6,800	0	0.187	103.2	14,000	0.042
0.2	6,400	0	0.187	103.2	16,000	0.032
0.225	5,950	0	0.187	103.2	18,000	0.025
0.25	T _{wall}	6	0.187	103.2	-	-

Continued....

Table A2(1) (continued)

Arc Condition 2

I = 100A

E = 23 v/cm

r cm	T °K	σ (r) mho/cm	$\sum ydx$ mho/cm	$E^2 \sum ydx$ Watts/cm	$\Delta T / \Delta r$ °K/cm	$K = \frac{E^2 \sum ydx}{r \Delta T / \Delta r}$ $W \cdot cm^{-1} \cdot ^\circ K^{-1}$
0	13,500	55	-	-	-	-
0.025	13,300	53	0.017	8.99	8,000	0.045
0.05	13,050	51	0.065	34.4	10,000	0.069
0.075	12,700	50	0.144	76.2	14,000	0.073
0.1	12,000	45	0.247	130.7	28,000	0.047
0.125	11,100	33	0.355	187.8	36,000	0.042
0.15	9,800	24	0.452	239.1	52,000	0.031
0.175	8,500	18	0.536	283.5	52,000	0.031
0.2	7,600	4	0.585	309.5	36,000	0.043
0.225	7,100	0	0.585	309.5	20,000	0.068
0.25	T _{wall}	0	0.585	309.5	-	-

Table A2(1) (Continued)

Arc Condition 3

I = 200A

E = 25.5 v/cm

r cm	T °K	σ (r) mho/cm	$\sum ydx$ mho/cm	$E^2 \sum ydx$ Watts/cm	$\Delta T / \Delta r$ °K/cm	$K = \frac{E^2 \sum ydx}{r \Delta T / \Delta r}$ W.cm ⁻¹ °K ⁻¹
0	15,000	74	-	-	-	-
0.025	14,900	72	0.0225	10.78	4,000	0.108
0.05	14,600	70	0.066	43.1	8,000	0.107
0.075	14,500	64	0.104	93.2	12,000	0.104
0.1	14,000	62	0.137	159.4	20,000	0.079
0.125	13,300	52	0.156	234.9	28,000	0.067
0.15	12,200	43	0.162	313.5	44,000	0.047
0.175	10,500	38	0.164	392.8	68,000	0.033
0.2	8,000	15	0.121	451.4	100,000	0.023
0.225	7,000	0	0.037	469.2	40,000	0.052
0.25	T _{wall}	0	-	-	-	-

Table A2(1) (continued)Arc Condition 4

I = 250A

E = 27 v/cm

r cm	T °K	$\sigma(r)$ mho/cm	$\sum ydx$ mho/cm	$E^2 \sum ydx$ Watts/cm	$\Delta T / \Delta r$ °K/cm	$K = \frac{E^2 \sum ydx}{r \Delta T / \Delta r}$ $W \cdot cm^{-1} \cdot K^{-1}$
0	16,000	90	-	-	-	-
0.025	16,000	90	0.056	40.82	-	-
0.05	15,900	88	0.139	101.33	4,000	0.506
0.075	15,700	86	0.275	200.48	8,000	0.334
0.1	15,350	82	0.458	338.88	14,000	0.342
0.125	14,800	73	0.675	492.1	22,000	0.178
0.15	14,000	62	0.905	659.7	32,000	0.137
0.175	12,450	44	1.12	816.48	62,000	0.075
0.2	9,900	25	1.77	1290.33	102,000	0.063
0.225	7,300	3	1.84	1341.36	104,000	0.051
0.25	T _{wall}	0	1.84	-	-	-

Table A2(2)

Comparison of Methods of
Integration of $\sigma(r)r \cdot \Delta r$

Arc Condition 3

I = 200A

E = 25.5v

Simpson's rule (3 point interpolation)

r cm	σ mho/cm	y_o $r\sigma(r)$	y_n	y_{even}	y_{odd}	$\frac{\Delta r}{3} (2y_{\text{odd}} + 4y_{\text{even}})$
0	74	74(0)=0				
0.025	72			1.8		
0.05	70				3.5	
0.075	64			4.8		<u>0.978</u>
0.1	62				6.2	
0.125	52			6.5		
0.15	43				6.45	
0.175	38			6.65		
0.2	15				3.0	
0.225	0			0		
0.25	0		0.25(0)=0			

$$\sum \sigma(r)r \quad (\text{Simpson 3 point interpolation}) = 2 \times 0.978$$

$$= \underline{6.14 \text{ mho.cm}}$$

$$\sum \sigma(r)r \quad (\text{Two point linear interpolation}) = 2 \times 0.969$$

$$= \underline{6.09 \text{ mho.cm}}$$

$$\text{Measured Arc Conductivity}' = \frac{I}{E} = \frac{200}{25.5} = \underline{7.8 \text{ mho.cm}}$$

Table A2(3)

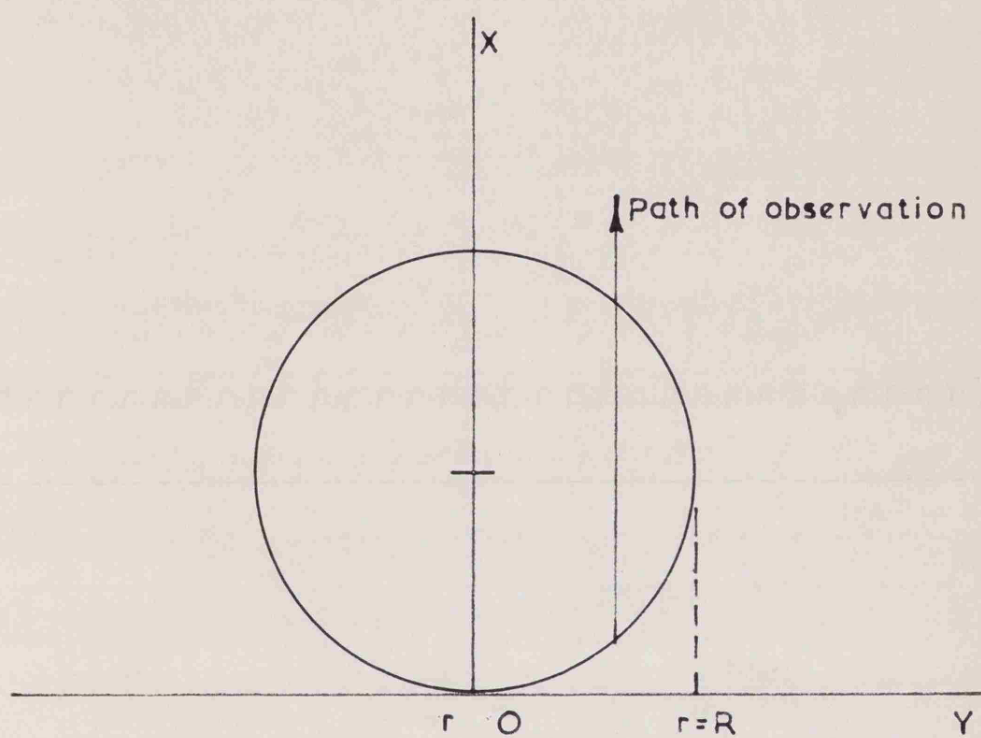
The Two Temperature Method. Measured Radiation Included

$$K_{\text{axial}} = \frac{E^2}{2T''} + \frac{Q_R}{2T''}$$

I _{arc} Amps	T _{ax} °K	T _r °K	σ _{axial} mho/cm	K _{ax} J.cm ⁻¹ sec ⁻¹ T ⁻¹	Q _R Watts/cm ³	$\frac{Q}{2T''}$	K _{ax} J.cm ⁻¹ sec ⁻¹ T ⁻¹
50	12,000	11,800	48	0.02	1,100	0.008	0.02
100	13,500	13,300	55	0.023	1,500	0.001	0.022
200	15,000	14,900	74	0.075	2,900 _m	0.004	0.071
250	16,000	15,900	90	0.41	3,200	0.02	0.39

At the higher temperature (16,000°K) "K" apparent is still large compared with the most recent rigorous calculations and this divergence is considered to be due to the measured radiation intensity failing to account for the shorter wavelength contribution. This total radiation contribution is considered in Chapter 3.

FIG A 2



SECTION THROUGH CYLINDRICAL PLASMA
AT RIGHT ANGLES TO MAJOR AXIS

APPENDIX IIIApplication of the Abel Integral to the
Determination of Absorption Coefficients
in a Cylindrical Plasma

With reference to Figure A2 which is a plane through the plasma jet taken at right angles to its longitudinal axis, considering the propagation of a beam of radiation of intensity I_0 , we can write:

$$I_x = I_0 \exp - \int_{-}^{+} \alpha(x, y) dy \quad (A1.III)$$

where α is the optical absorption coefficient.

Taking logarithms

$$\text{Log. } I_0/I_x = \int_{-}^{+} \alpha(x, y) dy \quad (A2.III)$$

If $\alpha(r) = 0$ for $r \geq R$, then introducing a function $\phi(x)$ we have

$$\phi(x) = \text{Log. } I_0/I_x = 2 \int_0^R \alpha(x, y) dy \quad (A3.III)$$

ϕ_j being the integrated value of all $\alpha(x)$ along any chord.

Then by the Abel inversion

$$\alpha(r) = \frac{1}{\pi} \int_r^R \frac{\phi(x) dx}{(x^2 - r^2)^{\frac{1}{2}}} \quad (A4.III)$$

In view of the problems which have previously arisen using the Abel integral inversion with unsmoothed data and data having off axis maxima, a new programme has been written based on a method of Maecker⁽⁸⁷⁾. In this method (r) is assumed to be constant over a small interval $\Delta_x = a$, so that the resultant series of integrals may be integrated. This yields after manipulation

$$a_j(r) = \left(\frac{\phi_j(x)}{2a} - \sum_{n=j+1}^N \alpha_n(r) A_{j,n} \right) / (2j-1)^{\frac{1}{2}} \quad (\text{A5.III})$$

where

$$A_{jn} = \left(n^2 - (j-1)^2 \right) - \left((n-1)^2 - (j-1)^2 \right)^{\frac{1}{2}}$$

Equation (A5.III) has been adapted for use on a Wang 700 desk calculator as the availability of a large digital computer (IBM) was severely restricted during the course of this study.

This inversion is applicable to a number of the problems arising in this study where only an integrated value of a function along a chord of a cylindrically symmetric distribution can be measured. The measurement of arc temperature from spectral line intensities and the determination of total radiation densities as a function of radius are examples.

APPENDIX IVThe Meaning of Thermal Equilibrium

Depending upon the nature of the gas under examination, the pressure of the system and the current, the temperatures in an arc column lie between 2000K and 50000°K. In this temperature range the degree of ionisation "x" lies between $0.01 < x = 100\%$. Thus, the arc column will consist of a mixture of unexcited atoms, electrons, positive ions and more or less highly excited neutral atoms which are strongly radiating.

For a plasma to be in perfect thermal equilibrium all kinetic processes, that is interactions between molecules, atoms, ions and electrons will be a specific function of temperature. Similarly the particle velocities, radiation density, electrical conductivity and heat conductivity will all be functions of temperature. This thermal equilibrium only exists approximately because it is continuously disturbed by radiation, the movement of charge carriers and heat conduction processes. It can be said that there is a good approximation to thermal equilibrium if the perturbations or disturbances are small. That is, the ionisation processes are cancelled by recombination; only a small number of charge carriers are lost by diffusion; collisional processes dominate the excitation of atoms and only few excited atoms lose their radiation energy irreversibly. Additionally, in the presence of steep thermal and electrical field gradients, the number of collisions between electrons and heavy plasma particles must be sufficiently great that the electrons in spite of the small amount of energy transferred at each collision, can transfer

all of their surplus energy taken from the field to the heavy particles so that a uniform plasma temperature is established.

In the past, much importance has been attached to the disturbances in equilibrium by diffusion processes and the classical thermal conductivity of a plasma and it is only recently that attention has been paid to the part played by radiation in disturbing and setting up equilibrium. Radiation quanta produced in the plasma move to cooler regions in an arc column thereby transferring energy out from the arc and disturbing the L.T.E.

Complete thermal equilibrium only exists when the distribution of all possible energy states obeys the Boltzmann function. Such a state will exist for example in a hollow radiator (black body) the interior of which is at a uniform temperature. Alternatively it can exist in a hot gas mass of sufficient magnitude so that numerous energy exchange processes, in particular radiation - absorption and re-emission occur. Any process which attempts to remove or supply energy to the plasma will be unfavourable to the setting up of thermal equilibrium. Therefore, in an arc where steep gradients can exist the conditions may be unfavourable. This, however, will only be so when energy exchange processes or collisions in the system are few, which at the pressures used in these experiments is not the case. Thus, if only electrons, which can easily absorb energy from the electric field in an arc, are present, then thermal equilibrium could easily be disturbed.

The presence of heavier particles such as neutrals and ions make possible the conversion of the kinetic energy of the electrons into an irregular temperature field. Complete disorder, that is, a uniform electron temperature is easily obtained after relatively few collisions at atmospheric pressure.

The setting up of equilibrium is favoured when excitation and ionisation processes are accounted for. If these are opposed by the reverse processes that is collisions and recombination, equilibrium is maintained. If, however, radiation occurs as the result of energetic processes in the gas and the radiation can leave the plasma then the energy loss is not transferred from the electrons to heavy particles and this represents a disturbance in local thermal equilibrium. In the past, radiation losses have been ignored but when one considers the effect of radiation on, for instance, measurement of thermal conductivity, it can be seen that this process is not insignificant. Since it is the high velocity electrons in the Maxwell distribution which predominantly supply radiation energy, it might be expected that the high velocity side of the Maxwell distribution will be greatly disturbed by this loss of energy. However, this does not seem to be the case at atmospheric pressure as the surplus of slow electrons tends to be rapidly equalised by the numerous collisions in the gas.

As already stated thermal equilibrium exists when the Boltzmann distribution equation is satisfied. That is, electron temperature T_e = gas temperature T_g . This condition

appears to be relatively easily satisfied in the arcs under consideration. If, however, the ionisation temperature T_i is considered then conditions appear less favourable. By ionisation temperature is meant that temperature which will be put into the Saha equation for the determination of the species concentration in the gas.

Once again if the gas were in perfect equilibrium with black body radiation as in a hollow radiator T_i would naturally equal T_g .

A detailed study of this problem is made in a paper by Oxenius entitled "An Approach to Thermal Equilibrium Through Interaction with Black Body Radiation"⁽⁸⁸⁾. It is demonstrated that a gas immersed in black body radiation will approach a state of complete thermodynamic equilibrium even if collisions between particles are neglected. This paper asked the question, how long will it take the kinetic degrees of freedom of atoms, ions and electrons to reach a Maxwellian distribution; the excitation degrees of freedom to reach a Boltzmann distribution and for the ionisation degrees of freedom to reach a Saha distribution where all the distributions correspond to a temperature T_g .

The Disturbance of the Temperature Equilibrium by a Temperature Drop

Of particular interest in the application of plasma jets is the incidence of a jet on a cold wall. In the boundary layer formed at the wall very steep temperature gradients may exist. It can be demonstrated that the disturbance of

equilibrium is small when the temperature difference over a mean free path length λ is small compared with the temperature itself⁽⁷⁾, i.e.

$$\frac{\lambda \Delta T}{T} \ll 1 \quad (\text{A1.IV})$$

For the path length λ , a value is always used which is appropriate to the equalisation process in question. If the equalisation of the electron temperature T_e or the gas temperature T_g is in question, then correspondingly the free path length corresponding to these conditions must be inserted.

Using equation A1.IV the magnitude of the temperature gradient required to significantly disturb equilibrium in an atmospheric pressure plasma at 10,000°K can be readily assessed. For argon, taking a mean free path of $\lambda = 10^{-5}$ cms at 10,000°K, for the inequality to be violated, $\nabla T = 10^9$ °K/cm. It can be seen from this example that the magnitude of the gradient is far higher than could be reasonably anticipated in practice. However, a more detailed study of the fluid dynamics of plasmas at cold walls is necessary to investigate just how closely the temperature gradient approaches a disturbing level.

APPENDIX V

A Suggested Simplified Approach to
the Calculation of Transport Coefficients
in a High Temperature Gas. The "Dobbs"
Method.

This Appendix contains the basic calculations of Thermal Conductivity, Viscosity and Diffusion coefficients of high temperature gases as a function of temperature. Dissociation and Ionisation are accounted for. The derivation of this method is explained in Chapter 5.

The equilibrium particle densities are taken from reference (3) (Leicester University M.Sc. Thesis by D.J. Dobbs).

The Sutherland constants are to be found in "Ionised Gases"⁽⁶⁴⁾ together with the S.T.P. mean free path data.

Table A5(1)Calculation of "Frozen" Viscosity of Argon

$$\eta = \frac{n_o k \theta}{f_c}$$

Basic Data

n_o	Loschmidt's number 2.7×10^{19} atoms. cm^{-3}
k	Boltzmann's constant 1.4×10^{-16} erg. K^{-1}
m_a	Argon atomic mass 6.7×10^{-23} grms.
L_1	Argon M.F.P. at S.T.P. = 10^{-5} cm
T^1	Sutherland constant. Argon = 169.9°K

A. Calculate Collision Frequency f_c

$$f_c = \frac{\text{Velocity}}{\text{Mean Free Path}}$$

Mean Free Path

$$\frac{L_1}{L_{\theta_2}} = \left(\frac{1 + T^1/\theta_2}{1 + T^1/\theta_1} \right)$$

L_{θ_2} = M.F.P. at required temperature θ_2 ($\theta_1 = 273^\circ \text{K}$)

L_1 = M.F.P. at S.T.P.

R.M.S. Maxwellian Velocity

$$W_{\text{RMS}} = \sqrt{\langle W \rangle^2} = 1.224 \sqrt{\frac{2k\theta}{m_a}}$$

Table A5(1) (continued)

$\theta^\circ\text{K}$	$\sqrt{\langle W \rangle^2} \text{ cm/sec}$	$L_2 \text{ cm}$	$f_c \text{ sec}^{-1}$
1,000	7.9×10^4	1.38×10^{-5}	5.3×10^9
5,000	17.7	1.57	11.3
8,000	22.4	1.58	14.2
10,000	25.0	1.6	15.6
15,000	30.6	1.61	19.0
20,000	35.4	1.62	21.8

B Calculate Viscosity

$\theta^\circ\text{K}$	$n_o k\theta \text{ ergs}$	$\eta = n_o k\theta / f_c$
1,000	3.78×10^6	7.1×10^{-4}
5,000	18.9	1.7×10^{-3}
8,000	30.2	2.1
10,000	37.8	2.4
15,000	56.8	3.0
20,000	75.7	3.5

Table A5(2)Argon Viscosity Correction at Ionisation

$$n_{\text{eff}} = \frac{n_e + n_a}{n_a}$$

n_e = Electron density

n_a = Atom density

θ	n_e	n_a	n_{eff}	Viscosity η (frozen)	Viscosity η (plasma)
1,000	-	2.5×10^{19}	1	7.1×10^{-4}	7.1×10^{-4}
5,000	1.3×10^{12}	1.5×10^{18}	1	1.7×10^{-3}	1.7×10^{-3}
8,000	1.4×10^{15}	9.2×10^{17}	1	2.1	2.1
10,000	1.5×10^{16}	7.0×10^{17}	1.02	2.4	2.4
15,000	1.8×10^{17}	1.3×10^{17}	2.38	3.0	1.26
20,000	1.8×10^{17}	8.8×10^{15}	22	3.5	0.16

Table A5(3)Calculation of atomic "contact" thermal conductivity of argon

Frozen Conductivity $K_a = \frac{5}{2} \frac{n_o k \theta}{m_a f_c} \text{ Jcm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{K}^{-1}$

Correction for Ionisation $K_a = K_a / \frac{n_e + n_a}{n_a}$

The collision frequency f_c is calculated in a manner identical to Table 1.

$^\circ\text{K}$	$K_a \text{ joule cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{K}^{-1}$ Classical (Frozen Composition)	K_a (plasma)
1,000	3.83×10^{-4}	3.83×10^{-4}
5,000	8.8	8.8
8,000	10.96	10.96
10,000	12.7	12.1
15,000	15.7	6.6
20,000	16.1	0.73

Note that the classical "contact" thermal conductivity falls as the gas ionises. This is due to the reduction in n_a with increasing temperature.

Table A5(4)

Calculation of Reaction Conductivity
for Argon Ion Diffusion

$$\text{Reaction Conductivity } K_R = n_i h_i D_{\text{amb}} / \theta \text{ J.cm}^{-1} \text{sec}^{-1} \theta^{-1}$$

$$D_{\text{amb}} = \text{ambipolar diffusion coefficient} = 2 D_a$$

$$D_a = \frac{k\theta}{n_a f_c \frac{n_a + n_i}{n_o}}$$

f_c is calculated as shown in Table 1

$$h_i = \text{energy/ion} = 15.5 \text{eV} = 2.48 \times 10^{-18} \text{J}$$

$\theta^\circ \text{K}$	n_i	D_{amb}	K_i
1,000	10^{10}	-	
5,000	1.3×10^{12}	35.7	21.4×10^{-9}
8,000	1.4×10^{15}	69.4	29.9×10^{-6}
10,000	1.5×10^{16}	101.9	37.8×10^{-5}
15,000	1.8×10^{17}	294	87.9×10^{-4}
20,000	1.6×10^{17}	518	102.8×10^{-4}

Table A5(5)Calculation of Reaction Conductivity
for Argon Electron Diffusion

Thermal Energy per electron $h_e = \frac{3}{2} k\theta$

Electron Conductivity $K_e = n_e h_e D_{amb} / \theta \text{ Jcm}^{-1} \text{sec}^{-1} \theta^{-1}$

$D_{amb} = 2D_a$ (See Table 6)

$\theta^\circ\text{K}$	n_e	K_e
1,000		
5,000	1.3×10^{12}	0.78×10^{-9}
8,000	1.44×10^{15}	2.1×10^{-6}
10,000	1.48×10^{16}	3.2×10^{-5}
15,000	1.79×10^{17}	1.06×10^{-3}
20,000	1.8×10^{17}	1.99×10^{-3}

Table A5(6)Calculation of Diffusion
Conductivity of Atoms

Energy carried by "Classical" Specific Heat = $C_p \theta$

$$C_p = \frac{5}{2} k \theta \quad (\text{Specific Heat at constant pressure})$$

Diffusion thermal conductivity (atomic)

$$K_a = \frac{5}{2} n_a k D_{\text{atom}}$$

θ	n_a	D_{atom}	K_d
1,000			
5,000	1.5×10^{18}	16.7	0.875×10^{-3}
8,000	9.2×10^{17}	34.7	0.97×10^{-3}
10,000	7.0×10^{17}	51.0	1.26×10^{-3}
15,000	1.3×10^{17}	146	0.67×10^{-3}
20,000	8.8×10^{15}	259	0.077×10^{-3}

Total Plasma Thermal Conductivity $K_T = K_a + K_i + K_e + K_d$

θ	K_T
1,000	3.83×10^{-4}
5,000	1.75×10^{-3}
8,000	2.098×10^{-3}
10,000	2.88×10^{-3}
15,000	11.2×10^{-3}
20,000	12.39×10^{-3}

Table A5(7)

Argon Atom-Atom Diffusion
Coefficient. Density Correction.
Comparison of Complete and
Reduced Terms

Complete Correction $\frac{N_o - n_a - n_i}{n_a + n_i}$

Reduced Correction $\frac{N_o}{n_a + n_i}$

θ	$\frac{N_o - n_a - n_i}{n_a + n_i}$	$\frac{N_o}{n_a + n_i}$
1,000	0.35	-
5,000	15.6	18
8,000	28.8	29.8
10,000	37	38.0
15,000	86	87
20,000	153.7	154

Table A5(8)Calculation of Thermal Conductivity of Nitrogen

$$\text{Total conductivity } K_T = K_C + K_D + K_i + K_e$$

$$\underline{\underline{K_C}} = \text{"Classical" contact conductivity}$$

$$= \frac{7}{2} \frac{n_o k^2 \theta}{m_{fc}}$$

$$= \frac{7}{2} \frac{n_o k^2 \theta}{m_{fc} \frac{n_{N_2} + n}{n_{N_2}}} \quad (\text{dissociating gas } N_2 \rightleftharpoons 2N)$$

$$n_o = \text{Loschmidt's number}$$

$$n_{N_2} = \text{Number of Nitrogen molecules}$$

$$n_N = \text{Number of Nitrogen atoms.}$$

The main free path for calculating f_c is obtained as in Table 1 using:

$$L_i (N_2 \text{MFP}) = 9.44 \times 10^{-6} \text{ cm}$$

$$T^i \text{ Sutherland Constant} = 110.6^\circ \text{ K}$$

$$\underline{\underline{K_D}} = \text{Dissociation Reaction conductivity}$$

$$= K_D = R_a h_a D_a / \theta$$

$$h_a = \text{energy per atom (dissociation energy} = 7.9 \text{ eV)}$$

$$D_{a-a} = \text{atom-atom diffusion coefficient calculated from}$$

$$D_{a-a} = \frac{k\theta}{m_a f_c \left(\frac{n_{N_2} + n_{N_a} + n_i}{n_o - n_{N_2} - n_a - n_i} \right)}$$

Table A5(8) (continued)

K_i = Ionisation Reaction conductivity

$$K_i = n_i h_i D_{amb} / \theta$$

Where h_i = energy per ion = 14.54 ev

D_{amb} = ambipolar diffusion coefficient = $2D_{a-a}$

K_e = Electron Diffusion conductivity

$$K_e = n_e h_e D_{amb} / \theta$$

Where h_e = mean thermal energy/electron = $\frac{3}{2} k\theta$

Calculation of Thermal Conductivity of Nitrogen

θ	K_C	K_D	K_i	K_e	K_T J.cm/sec/ θ
1,000	4.9×10^{-3}	-			4.9×10^{-3}
2,000	7.5	-			7.5
3,500	10.2	-			10.2
5,000	11.6	2.7×10^{-3}			14.3
7,000	7.3	2.1×10^{-2}	0.94×10^{-3}		2.19×10^{-2}
8,500	2.8	4.3	1.6		4.46
10,000	0.28	2.9	5.9		3.49
12,000		2.4	17.0	0.19×10^{-3}	4.19
15,000		0.99	58.0	0.8	6.87
17,000		0.14	85.0	1.3	8.87
20,000		0.05	84.0	1.7	8.62

APPENDIX VIElectron-Atom Collision FrequenciesAVI.0 General

Consider a gas to be a mixture of atoms with a radius r_a and electrons of negligible radius, then by definition a collision is said to occur when the centre of the electron approaches within a distance r_a of the centre of an atom, so that the mean free path of the electron λ_e , is given approximately by

$$\lambda_e = (\pi r_a^2 n_a)^{-1}$$

The collision frequency of electrons with neutral particles is then determined by the relation

$$\nu = v / \lambda_e$$

where in its simplest form the electron velocity is given in terms of the gas temperature by the relation

$$v = (3kt/m)^{1/2}$$

We then obtain for the collision frequency

$$\nu = \pi r_a^2 n_a (3kt/m)^{1/2}$$

AVI.1 Numerical Example

Consider the argon atom, the radius of the atom is approximately 10^{-8} cm⁽⁹⁶⁾

With $T = 13,000^{\circ}\text{K}$ and $N \approx 10^{17}$, the simple kinetic approach yields a value for the collision frequency of $8 \times 10^9 \text{ sec}^{-1}$

AVII.2 Collision Cross Sections

An alternative approach to calculating the electron atom collision frequency is to use the collision cross section obtained from measurements of electrons scattered in a neutral gas.

Electron-atom collision data is usually presented in one of two ways, either as a probability of collision P_c or a collision cross section σ_c . The collision probability is defined as the average number of collisions that occur when an electron travels a distance of 1 cm. at a pressure of 1 torr at 0°C . The probability of collisions is determined (in principle) by measuring the change in current when an electron beam passes through a gas at different pressures. With an initial current I_0 of electrons with a velocity v and a final current I after passing through a distance x of a gas at pressure p , the relationship

$$I = I_0 \exp -k, (p.v)x$$

is obtained.

k , is a constant function for constant p and v .

If the pressure is varied keeping the distance constant we can write

$$I = I_0 \exp -k_2(x,v)p$$

Where k_2 is a function of x and v . Combining these two expressions gives

$$I = I_0 \exp - p_0 P_c(v)x$$

This expression defines the probability of collision P_c .

Here, $P_0 = 273 \text{ p/t}$. The dimensions of P_c are cm^2/cm^3 per unit pressure so that P_c is also called the effective collision cross section Q_c . The effective collision cross section Q_c is related to the total scattering cross section q by the relationship

$$Q_c = qN$$

q = scattering cross section cm^2 at 1 torr and 0°C .

N = number of neutral particles at temperature T and pressure p .

From the foregoing it can be seen that the mean free path of an electron λ_e is given by $\lambda_e = 1/Q_c$

Data on the scattering cross sections q for argon is

given in reference (76). From the data for argon

$$q = 2 \times 10^{16}$$

$$N \text{ at } 13,000^{\circ}\text{K} = 3.5 \times 10^{17} \cdot \text{cm}^{-3}$$

Thus, $Q = 70 \text{ cm}$, therefore $\lambda_e = 1.4 \times 10^{-2} \text{ cm}$

Assuming a Maxwellian velocity distribution the velocity of the electrons can be calculated at this temperature. The r.m.s. velocity $v = 7.55 \times 10^7 \text{ cm sec}^{-1}$

The collision frequency is then given by

$$\nu = v/\lambda_e$$

and with the data quoted, $\nu = \underline{5.5 \times 10^9 \text{ sec}^{-1}}$

Alternatively, the mean free path can be obtained from the collision probabilities given in references (77) and (78).

Here

$$\lambda_e = 1/P_o P_c$$

From this data for argon, $P_c = 5$. The reduced pressure

$$p = 273/P/T = 16$$

Therefore

$$\lambda = 1.25 \times 10^{-2} \text{ cm and } \nu = 6.0 \times 10^9 \text{ sec}^{-1}$$

The three values obtained from different sources of collision data are in close agreement and a mean value of $6.5 \times 10^9 \text{ cm}^{-1}$ is used in the calculation of the optical absorption coefficient at $13,000^\circ \text{K}$.

APPENDIX VIIElectron-Ion Collision FrequenciesAVII.0 A Simple Debye Correction (Dobbs)

The gas ion, having a positive charge, can exert a perturbing influence on electrons in the gas at distances far greater than the neutral particle. It is necessary therefore, to ascribe an effective radius for collisions to the ion.

The Debye shielding distance or Debye length suggests a value for this effective radius. The Debye length is the distance from the ion at which its Coulomb potential becomes substantially ineffective and is a function of gas temperature and ion density N . This shielding distance h is given by

$$h = 6.9 (T/N)^{\frac{1}{2}} \text{cm}$$

For an argon plasma at $13,000^{\circ}\text{K}$

$$h \approx 2.5 \times 10^{-6} \text{cms}$$

Using this value as a radius for collision calculations a collision frequency of 10^{14}sec^{-1} is obtained using the simple

kinetic theory described in Appendix VI. The optical absorption coefficients obtained using this correction are $\approx 10^3$ times greater than the measured values.

AVII.1 Charged Particle Interactions

Banks⁽⁷⁵⁾, and Schwenn⁽⁸⁹⁾ both derive equations for the electron-ion collision frequency based upon the interaction parameter described by Spitzer⁽⁶⁶⁾.

Considering the motion of a test particle whose mass m , is much less than the mass of the field particles m_f through which it moves, Spitzer shows that it is deflected through an angle of 90° or more when the potential energy of this test particle at a distance 'p' from a heavy neutral particle is twice its original kinetic energy. This value of impact parameter is defined as P_o

$$P_o = \frac{Z_1 Z_2 e^2}{m_1 v_1^2} \quad (A1.VII)$$

where Z_1 , Z_2 are the charges on the interacting particles, m_1 and v_1 being the mass and velocity of the test particle.

The collision cross section for such encounters is πP_o^2 .

The relaxation time or mean time between collisions for this neutral gas

$$t_c = \frac{1}{\pi P_o^2 n_2 v_1} \quad (A2.VII)$$

where n_2 = number density of field particles

v_1 = velocity of test particles.

The above treatment applies to close encounters for a neutral gas. For gases composed of charged particles the mean free path given by p_0 is far larger than experiment indicates. This is because the force field of charged particles decreases very slowly compared with that of neutral particles. Thus, when two charged particles pass at a distance large compared with p_0 the deflection angle ψ is not negligible. Obviously, the number of such events will be great in comparison with the less frequent "close" encounters. When $p \gg p_0$ the deflection ψ of a particle varies as $2p_0/p$. If all such encounters produced a unidirectional effect, these "distant" collisions would have a very great effect on the gas properties since the number of collisions with an impact parameter between p and $p + dp$ varies as $2\pi p dp$. Since the direction of all the deviations is random many will cancel out and the resultant effect will be smaller. Because of this randomness an accurate mathematical description of the effect requires the application of statistical theory.

Assuming a Maxwellian velocity distribution function:

$$f(v) = \frac{n}{\pi^{3/2}} (m/2kT)^{3/2} \exp - (m/2kT)v^2 \quad (A3.VII)$$

Spitzer shows that the average rate of change of velocity of an ensemble of particles in a direction at right angles to their original velocity is given by

$$\langle (v_i)^2 \rangle = 8\pi n_f v^3 p_o^2 / n(p_m/p_o) \quad (A4.VII)$$

where n_f = number density of field particles.

The impact parameter p_m is set equal to "h" the Debye radius

$$h = (kT/4\pi n_e e^2)^{1/2} \quad (A5.VII)$$

Replacing the kinetic energy term $m_i v_i^2$ in equation A1.VII by the mean thermal energy $3kT$ to give the average value of p_o gives:

$$p_m/p_o = h/p_o = 3/2 Z Z_f e^3 (k^3 T^3 / \pi n_e)^{1/2} = \Omega \quad (A6.VII)$$

Using a $\log_n \Omega$ correction for charged particles interactions Spitzer calculates electron-proton gas relaxation times.

From this work Banks⁽⁷⁵⁾ and Schwenn⁽⁸⁹⁾ derive the following collision frequency equations

$$\text{Banks } \nu_{ei} = \frac{4}{3} \sqrt{2\pi m_2} \frac{(Z_i e^2)^2 \log_n \Omega}{m_e^{1/2} (kT)^{3/2}} \quad (A7.VII)$$

$$\text{Schwenn et al } \nu_{ei} = \frac{\pi^2}{16} \sqrt{\frac{8kT_e}{\pi m_e}} \frac{n_e e^4}{(kt_e)^2} \log_n \Omega \frac{1}{0.582} \quad (A8.VII)$$

The numerical factor in the Schwenn equation is to correct for electron-electron collisions in a singly ionised gas.

Both equations A7.VII and A8.VII give a collision frequency

for argon with $N_e \approx 10^{17} \text{ cm}^{-3}$, $T_g = 13,000^\circ \text{K.}$ of $2 \times 10^{-12} \text{ sec}^{-1}$.

To check the validity of these theoretical calculations the effective electron-ion collision frequency in argon with N_e and T_g as above was derived using the relationship

$$\sigma = N_e e^2 / m \nu_{ea+ei}$$

applied to the electrical conductivity data of an argon plasma measured by Fischer et al⁽⁹⁰⁾. This experimental value of $4 \times 10^{-2} \text{ sec}^{-1}$ is in good agreement with the theory.

APPENDIX VIIIDetails of the Derivation of Electron-Atom Collision Frequencies from Experimental Data and TheoryA VIII.0 Basic Derivation of f_c

In Chapter VI, equation 2.6, derived by Ratcliffe⁽⁶⁵⁾ is presented. This relates the optical absorption coefficient to the electron atom collision frequency

$$\alpha = \frac{f_c}{\mu} \cdot \frac{4\pi N_e e^2}{2c \epsilon_0 m_e (\omega^2 + f_c^2)} \quad (\text{Ratcliffe})$$

If the equilibrium electron density N_e and refractive index μ are determined by measurement together with the absorption coefficient at a frequency ω (incident radiation frequency) then f_c can be calculated.

N_e is obtained from the Saha equation^{(91), (30)}, the gas temperature being measured spectrographically. The refractive index is either measured as a function of temperature⁽⁶⁸⁾ or calculated as described in Section 6.1 and the absorption coefficient is obtained from measurements of the attenuation of intensity of a beam of electromagnetic radiation passing through the plasma under investigation (Section 6.2).

Since at the wavelengths selected in this study $\omega^2 \gg f_c^2$ (anticipated), the Ratcliffe equation may be re-arranged to give f_c as

$$f_c = \frac{2\alpha\mu c\epsilon_0 m_e \omega^2}{4\pi N_e e^2} \quad (\text{A1.VIII})$$

which with

$$\epsilon_0 = 8.85 \times 10^{-2} \text{ coul}^2 - \text{sec}^2 \cdot \text{cm}^{-3} \text{gm}^{-1}$$

$$c = 3 \times 10^{10} \text{ cm sec}^{-1}$$

$$m_e = 9.1 \times 10^{-28} \text{ gm}$$

$$e = 1.6 \times 10^{-19} \text{ coulomb}$$

reduces to

$$f_c = \frac{1.5\alpha\mu \omega^2}{N_e} \quad (\text{A2.VIII})$$

Within the limits of accuracy obtained in this study, the refractive index may be written as 1.0, as over the temperature range considered, its value is only less than unity by about 0.1%

AVIII.1 Comparison of Measured and Calculated Absorption Coefficients

To check the validity of measurements made on the plasma, the absorption coefficient was calculated, assuming electron-atom collisions from the Spitzerequation (Section 6.1).

$$\alpha = 1.6 \times 10^{-37} \lambda^2 n_e T_e^{\frac{1}{2}} \text{ cm}^{-1}$$

with λ measured in microns and T_e in electron volts.

For the $H_e N_e$ laser at $\lambda = 6328\text{\AA}$ the above equation reduces to

$$\alpha = 6.4 \times 10^{-38} N_e^2 T^{-3/2}$$

For the $G_a A_s$ laser at $9100\text{\AA}$

$$\alpha = 1.3 \times 10^{-37} N_e^2 T^{-3/2}$$

Tables A8(1A) and A8(1B) list this calculation for Nitrogen and Argon at two wavelengths. The comparison of these calculated values and the measured absorption coefficients is shown in Figures 57 and 58.

AVIII.2 Comparison of Measured and Calculated Collision Frequencies with Published Low Pressure, Low Temperature data

The most usual source of collision frequencies is the collision cross section data obtained from low temperature, low pressure experiments described in Appendix VI.

The extrapolation from this data up to the high temperature atmospheric pressure arc conditions is also described in Appendix VI.

Tables A8(2A) and A8(2B) list the details of the derivation of the electron atom collision frequencies in argon and nitrogen using the "Spitzer" theoretical absorption

coefficient and extrapolated low temperature data.

The comparison of these two sources of collision frequencies and those obtained from the arc measurements are shown in Figures 59 and 60. The systematic differences between the three sources of collision frequency are examined in the discussion associated with Chapter 6 where it is demonstrated that the arc measurements of absorption are very significantly influenced by electron-ion collisions.

Table A8(1A) Theoretical Estimation of Optical Absorption. Nitrogen

T_{gas} $^{\circ}\text{K}$	T_{gas} eV	N_e	N_e^2	$t_e^{-3/2}$	α at 6328Å cm^{-1}	α at 9100Å cm^{-1}
6×10^{-3}	0.52	1.3×10^{15}	1.69×10^{30}	2.66	2.87×10^{-7}	5.8×10^{-7}
7	0.6	3.2	1.02×10^{31}	2.15	1.4×10^{-6}	2.83×10^{-6}
8	0.69	9.0	8.1	1.74	9.02	1.83×10^{-5}
9	0.77	1.8×10^{16}	3.2×10^{32}	1.48	3.03×10^{-5}	6.16
10	0.86	3.2	1.02×10^{33}	1.25	8.16	1.65×10^{-4}
11	0.95	5.0	2.5	1.09	1.7×10^{-4}	3.54
12	1.03	8.0	6.4	0.96	3.9	7.98
13	1.	1.2×10^{17}	1.44×10^{34}	0.87	8.02	1.63×10^{-3}
14	1.2	1.5	2.25	0.76	1.09×10^{-2}	2.2
15	1.29	1.7	2.89	0.68	1.25	2.55
16	1.39	2.0	4.1	0.62	1.62	3.3
17	1.46	2.1	4.4	0.57	1.60	3.26
18	1.55	2.2	4.84	0.52	1.61	3.27

The comparison of the theoretically estimated optical absorption coefficients at 6328Å and 9100Å and coefficients measured in the plasma jet are shown in Figures 55 & 56

Table A8(1B) Theoretical Estimation of Optical Absorption. Argon

T_{gas} $^{\circ}\text{K}$	T_{gas} eV	N_e	N_e^2	$T_e^{-3/2}$	α at 6328Å cm^{-1}	α at 9100Å cm^{-1}
6×10^3	0.52	1×10^{14}	1.0×10^{28}	2.66	10^{-5}	10^{-5}
7	0.6	2.4	5.7	2.15		
8	0.69	4.0	1.6×10^{29}	1.74		
9	0.77	6.0×10^{15}	3.6×10^{31}	1.48		
10	0.86	1.0×10^{16}	1×10^{32}	1.25		
11	0.95	1.8	3.2	1.09	2.23×10^{-5}	4.5×10^{-5}
12	1.03	5.5	3.0×10^{33}	0.96	1.8×10^{-4}	3.7×10^{-4}
13	1.1	7.0	4.9	0.87	2.7×10^{-4}	5.5
14	1.2	1.2×10^{17}	1.4×10^{34}	0.76	6.8×10^{-4}	1.4×10^{-3}
15	1.29	1.5	2.3	0.68	1.0×10^{-3}	2.0
16	1.39	1.8	3.2	0.62	1.27	2.7
17	1.49	1.9	3.6	0.57	1.3	2.7
18	1.55	2.0	4.0	0.52	1.33	2.3

The comparison of the theoretically estimated optical absorption coefficients at 6328Å and 9100Å and coefficients measured in the plasma jet are shown in Figures 55 & 56

Table A8(2A) Electron-Atom Collision Cross Sections Derived from Theory and Published Data. Nitrogen

P = 1.0 atm.

T °K	n_e	V_e cm sec ⁻¹	f (calculated) ₁ sec ⁻¹	q cm ⁻²	Q=qN _e	$\lambda_e = 1/Q$ cm	f_c cm ⁻¹
6x10 ³	1.3x10 ¹⁵	5.2x10 ⁺⁷	2.9x10 ⁺⁹	8.2x10 ⁻¹⁶	1.06	0.94	5.5x10 ⁷
7	3.2	5.7	5.8				
8	9.0	6.1	1.3x10 ⁺¹⁰				
9	1.8x10 ¹⁶	6.4	2.2	9.0	16.2	6.1x10 ⁻²	1.05x10 ⁹
10	3.2	6.6	3.4				
11	5.0	7.1	4.5		76.8	1.3x10 ⁻²	5.7x10 ⁹
12	8.0	7.4	6.4	9.6			
13	1.2x10 ¹⁷	7.7	8.8				
14	1.5	8.0	9.6	10.2	153.0	6.5x10 ⁻³	1.2x10 ¹⁰
15	1.7	8.3	9.8				
16	2.0	8.6	1.07x10 ¹¹				
17	2.1	8.8	1.01	11.5	241.5	4.14x10 ⁻³	2.12x10 ¹⁰
18	2.2	9.1	9.7x10 ¹⁰				

The comparison of the collision frequencies obtained from published low pressure low temperature data (76,77,78), theoretical derivation assuming only electron-atom collisions and frequencies obtained from arc measurements are shown in Figures 59 & 60

TableA8(2B) Electron-Atom Collision Cross Sections Derived from Theory and Published Data. Argon

P = 1.0atm

T °K	n_e	v_e cm sec ⁻¹	f_c (calculated) cm sec ⁻¹	q cm ⁻²	$Q=qN_e$	$\lambda_e = 1/Q$	f_c cm ⁻¹
6×10^3	1×10^{14}	5.2×10^7		1.4×10^{-16}	1.4×10^{-2}	71	7×10^5
7	2.4	5.7					
8	4.0	6.1		1.5×10^{16}	6×10^{-2}	16.6	3.6×10^6
9	6.0×10^{15}	6.4					
10	1.0×10^{16}	6.8					
11	1.8	7.1	1.64×10^{10}				
12	5.5	7.4	4.4	1.7×10^{-16}	9x35	0.11	6.7×10^8
13	7.0	7.7	5.13				
14	1.2×10^{17}	8.0	7.54	1.9×10^{-16}	22.8	0.44	1.8×10^9
15	1.5	8.3	8.87				
16	1.8	8.6	9.4				
17	1.9	8.8	9.1	2.0×10^{-16}	38	0.026	3.4×10^9
18	2.0	9.1	8.8				

The comparison of the collision frequencies obtained from published low pressure, low temperature data (76,77,78), theoretical derivation assuming only electron-atom collisions and frequencies obtained from arc measurements are shown in Figures 59 & 60

Bibliography

1. Ayrton. H. The Electric Arc. The Electrician. Series D. Van Nostrand. New York 1902
2. Somerville J.M. The Electric Arc. Methuen Monograph 1959
3. Cobine J.D. Gaseous Conductors. Dover Publications. New York 1958
4. Hagenbach. A. The Electric Arc. A Handbook of Radiology. Leipzig 1924
5. Engel A.V. and Steenbeck M. Elektrische Gasentladungen, ihre Physik und Technik. Volumes 1 and 2. Springer Verlag
6. Elenbaas W. The High Pressure Mercury Vapour Discharge. North Holland 1951.
7. Finkelburg W and Maecker H. Electric Arcs and Thermal Plasmas. Handbuck der Physik. Volume 22. Springer Verlag Berlin 1956.
8. Cann G.L. Energy Transfer Processes in a Partially Ionised Plasma. Hypersonic Research Project. CALTEC. Memorandum No. 61. June 1961.
9. John R.R. and Bade W.L. Recent Advances in electricl Arc Plasma Generation Technology. A.R.S. Journal. Volume 31 No. 1 pp 4-17. January 1961
10. Magregor W.K. Performance of a D.C. Arc Excited Plasma Generator. U.S.A.F. A.E.D.C. TN60-112. August 1960
11. Amadur I and Mason E.A. Properties of Gases at very High Temperatures (Frozen Composition). Phisc. Fluids. Volume 1 No. 5. pp 370-383 September-October 1958.

12. Spitzer L and Harm R. The Fully Ionised Gas. Phys. Rev. 89. pp 077. 1953.
13. Wilke C.R. J. Chem. Phys. 18 pp 517f 1950.
14. Krinberg A. Calculation of the Thermal Conductivity of Some Gases at Temperatures of 1000-20,000°K. High Temperature. Volume 3 No. 4 pp 6-6-609. August 1965
15. Hanson C.F. N.A.S.A. Technical Report No. R50 1960
16. Devoto R.S. Transport Properties of Ionised Monatomic Gases. Phys. Fluids. Volume 9, No. 6 pp 1230-1240 June 1966
17. Hirschfelder J.O. Curtis C.F. and Bird R.B. Molecular Theory of Gases and Liquids. John Wiley & Sons Inc. New York 1964.
18. Mason E.A., Weissman S., Vanderslice J.T and Fallon R.J. High Temperature Transport Properties of Dissociating Hydrogen. J. Phys. Fluids. Volume 5 No. 2 p 155. Feb. 1962
19. Penski K. Theoretical Calculation of Transport Properties in Nitrogen Equilibrium Plasma. A.S.M.E. Proc. 4th Symposium on Thermo Physics. pp 189-202 April 1962
20. Burhorn F. Berechnung und Messung der Warmleitfähigkeit von Stickstoff bis 13,000°K. Zeits fur Physc. 155 pp 42-48 1959.
21. Plantikow U and Steinberger S. Elektrische und Thermische Leitfähigkeit von H. bis 27,000°K. Zeits fur Phys. Volume 231 pp 109-118. 1970
22. Plantikow U. Transport Properties of the N₂ Plasma. Ninth International Conference. Phenomena in Ionised Gases. Bucharest pp 421 1969.

23. Monterde-Garcia. Alfonso. Materialfunktion von Bogenplasmen aus Charakteristik messungen. Zeits fur Phys. 181 pp 317-330 1964.
24. Herman W & Scade E. Transportfunktionen von Stickstoff bis $26^{\circ}000\text{K}$. Zeits fur Phys. Volume 223 pp 333-350 1970.
25. Morris J.C. Rudis R.P. & Yos J.M. Measurement of Electrical and Thermal Conductivity of H_2 , N_2 and Ar at High Temperatures. Phys. Fluids Volume 13 No. 3 pp 608 March 1970.
26. Olsen H.M. Thermal and Electrical Properties of An Argon Plasma. Phys. Fluids. Volume 2. No. 6 pp 614-623. November-December 1959.
27. Cremers. C.J. and Berkebak R.C. Applied Optics. Volume 5. No.5 and No. 6. pp 1057-1064. June 1966
28. Maxwell R.W., Ruddy M.J. and Dowson M. An Abel Transformation. University of Leicester Engineering Department. Report 70-12. April 1970.
29. Ruddy M.J. Ph.D. Thesis to be submitted 1973.
30. Dobbs, D.J. The Arc Plasma Generator. M.Sc. Thesis. University of Leicester 1969.
31. Gurevich D.B. and Podmoshenski I.V. The Relationship between Excitation Temperature and Gas Temperature in the Positive Column of an Arc Discharge. Optics and Spectroscopy. 15 p.319 1963.
32. Plantikow U. Bestimmung von Transportfunktionen aus den Charakteristiken Zylindrischer Bogen (Wasserstoff) Zeits fur Phys. Volume 227 pp 271-290 1969
33. Uhlenbusch J.F. and Fischer E. Influence of Diffusion and Non-Equilibrium Populations in Noble Gas Plasmas. Proc. I.E.E.E. Volume 59. No. 4 April 1971.

34. Durand J. Zeits Naturforschung. A. Volume 18, pp 281. 1963.
35. Neumann W. Beitrag. Plasma Phys. Volume 2. p.80 1962
36. Bauder U.H. and Maecker H. The Determination of Transport Properties from Arc Experiments. Proc. I.E.E.E. Volume 59. No. 4 pp59 April 1971
37. Morris J.C. and Garrison R.L. Measurement of the Radiation Emitted "f" Values and Stark Half Widths for Strong Vacuum Ultra Violet Lines of O_I and N_i . Phys. Reg. Volume 188 No. 1. pp 112f
38. Burgess A and Seaton M.J. A General Formula for the Calculation of Atomic Photo-Ionisation Cross Sections. Monthly Notices of the Royal Astronomical Society. Volume 120 No. 2 pp 121ff 1960
39. Wilson K.H. and Nicolet W.E. Spectral Absorption Coefficients of Carbon, Nitrogen and Oxygen Atoms. J. Quant. Spect. and Radiative Trans. pp 905ff. Volume 7 No. 6. November-December 1967.
40. Maecker H and Peters T. Das Elektronenkontinuum in der Saule des Hochstromkohlebogens und in anderen Bogen. Zeits fur Phys. Bild 139 pp 448-463. 1954
41. Kramers H.A. Phil Mag. 46. pp 836 1923
42. Morris J.C. Rudis R.P. and Yos J.M. Measurements of Electrical and Thermal Conductivity of Hydrogen, Nitrogen and Argon at High Temperatures. Phys. Fluids Volume 13. No. 3 pp 608 ff. March 1970.

43. Peach G. Monthly Notices. Royal Astronomical Society 124. pp 372 1962.
44. Bates D.R. and Damgaard A. Phil. Trans. Royal. Soc. London A.242 pp 101 1949.
45. Griem H.R. Plasma Spectroscopy. McGraw Hill Book Co. New York 1964.
46. Samson J.A.R. Experimental Photo Ionisation Cross Sections in Argon. J. Opt. Soc. America. Vol. 54 No. 3 pp 420 March 1964.
47. Morris J.C. and Morris R.U. Radiation Studies in Arc Heated Plasmas. AVCO Corp. AVATD-0207-69-CR. January 1970
48. Olsen H. Optical Measurements of High Temperatures (ed P. Dickerman). Chicago 1961.
49. Razumovskaya L.P. Optika i Spektroskopia. 14. pp 189 1963.
50. Yakubov I.T. Energy Emitted by Argon Plasma in Spectral Lines. Optika i Spektroskopia 19. pp 497-500. 1965.
51. Krey R.U. and Morris J.C. Experimental Total and Total Line Radiation of Nitrogen, Oxygen and Argon Plasmas. Phys. Fluids. Volume 13, No. 6 pp 1483. June 1970.
52. Asinovskii E.I. and Kirillin A.V. Determination of the Thermal Conductivity of an Argon Plasma. Teplofizika Vysokikh Temperatur. Volume 3 No. 5. pp 677-685 October 1965.
53. Asinovskii E.I. Drokhanova E.V. et al. Experimental and Theoretical Investigation of the Thermal Conductivity Coefficient and of the Total Emission Coefficients.

- Translation of Teplofiz Vysok. Temperatur. Volume 5
No. 5 pp 736-750. October 1967 (Ref. U.D.C. 533.932.
535. 23,537. 523.5)
54. Beth M.U. and Kling M.G. Spectroscopically Measured
Velocity Profiles of an M.P.D. Arc. Jet. A.I.A.A.
Journal. Volume 7 No. 11 pp 2181-82. November 1969.
 55. Freeston I.L. and Kelk J.E. private communication.
 56. Uhlenbusch J. Berchnung der Materielfunktionen eines
Stickstoff und Argon Plasmas. Zeits fur Physik 179.
pp 347-66. 1964.
 57. Boyd T.J.M. and Sanderson J.J. Plasma Dynamics.
Thomas Nelson London 1969.
 58. Enskog D. Archi fur Matematik, Astronomi och Fysik.
Volume 16. p 16f 1922 (Detail in Hirschfelder, Curtis
and Bird Ref. 17)
 59. Chapman S and Cowling T.G. The Mathematical Theory of
Non Uniform Gases. Cambridge University Press 1939.
 60. Waldmann L. Zeits Naturforschung. Volume 5A pp 322 1950.
 61. Saxena S.C. and Bahethi O.P. Transport Properties of
some Simple Non-Polar Gases. Molecular Physics. Volume
7. No. 2 pp 183-189. 1963-1964.
 62. De-Groot S.R and Mazur P. Non Equilibrium Thermo
Dynamics . North Holland Publishing Co. Amsterdam
and London 1969.
 63. Jeans J. Kinetic Theory of Gases. Cambridge University
Press 1940
 64. Von Engel A. Ionised Gases. Oxford Clarendon Press.
2nd Edition 1965.
 65. Ratcliffe J.A. The Magneto Ionic Theory. C.U.P. 1959.

66. Spitzer L. Jr. Physics of Fully Ionised Gases.
Wiley (Interscience). New York 1956.
67. Jahoda F.C. and Sawyer G.A. Optical Refractivity of
Plasmas. Methods of Experimental Physics Volume 9.
Part B.
68. Sturrock J. Estimation of Temperature Profiles in an
Argon Plasma Jet. M.Sc. Thesis. University of Leicester
(To be submitted 1973).
69. Born M and Wolff E. Principles of Optics. Pergamon
Press. New York 1970.
70. Allen C.W. Astrophysical Quantities (University of
London Press) 2nd Edition 1963.
71. Kirkwood J.G. Zeits fur Physik. Volume 33 p.57. 1932
72. Akasu A.Z. and Wald L.H. Bremsstrahlung of Slow
Electrons in Neutrals Gases and Free-Free Absorption
of Microwaves. Physc. Fluids. Volume 10 No. 6 pp.1327
June 1967.
73. Ohmura T and Ohmura H. Continuous Absorption due to Free-
Free Transitions in Hydrogen. Phys. Rev. Volume 121.
No. 2. pp 513. January 15th 1961.
74. Dobbs D.J. and Maxwell R.W. Determination of Electron
Densities from the Optical Absorption of a Plasma.
Report 71-14. April 1971
75. Banks P. Collision Frequencies and Energy Transfer. Part
1. Electrons. Part II Ions. Planet Space Science.
Volume 14 pp 1085-1103 and Volume 14 pp 1105-1122. 1936
76. Landolt-Bornstein. Tabellen. Springer Verlag. Berlin
1959.
77. Laborie P. Rocard J.M. and Rees J.A. Sections Efficaces
Electroniques. Coefficients Macroscopiques. I.Hydrogene

et Gaz Rares. Dunod. Paris 1968.

78. Brode R.B. Rev. Modern Physics Volume 5 p. 257. 1933.
79. Moore C. Atomic Energy Levels Volume 1. Sections 1 and II. National Bureau of Standards. U.S.A. 1958.
80. Ladenburg R. Die Quanten Theoretische Deutung der Zahl der Dispersionselektronen. Zeits fur Physik. Volume 4, pp 23 1921.
81. Glennon B.M. and Wiese W.L. Bibliography on Atomic Transition Probabilities. M.B.S. Monograph. pp 18-23 1962.
82. Drawin H.W. Zeits fur Physik. Volume 146. pp 295-313 1956.
83. Wiese W.L. Transition Probabilities for Prominent Ar.I lines. Eighth International Conference. Phenomena in Ionised Gases. pp 447. Vienna 1967.
84. Richter J. Zeits fur Astrophys. Volume 51 pp 177-186 1961
85. Motschmann H. Messung der Oszillatorenstarke einiger N.I. Linien. Zeits fur Physik. volume 143 pp 77-92 1955
86. Olsen H.M. The Electric Arc as a Light Source for Quantitative Spectroscopy. J. Quant. Radiative Transfer Volume 3 pp 305-333 1963.
87. Maecker H. Zeits fur Physik. 136 pp 119. 1953.
88. Oxenius. J. Approach to Thermal Equilibrium Through Interaction with Black Body Radiation. J. Quant Spectt. Radiative Transfer Volume 7. pp 837-855. 1967.
89. Schwenn R. et al. Electrical Conductivity of an Argon-Potassium Plasma at Low Current Densities. Plasma Physics. Volume 10 No. 12 December 1968.

90. Fischer E., Hackmann J and Uhlenbusch J. Transport Coefficient Determination of Rare Gases. IX International Conference on Phenomena in Ionised Gases. pp 414. Bucharest 1969.
91. Saha M.N. Elements in the Sun. Phil Mag 5.6 Volume 40 No. 240. Paper A. pp 472-488. Paper B pp 809-834 1920. (Results for Ar and N₂ tabulated by Dobbs, Ref. 30).
92. Morris J and Krey R.U. Argon Continuum Radiation. J. Quant Spect. Rad. Volume 9. 1969
93. Morris J., Krey R.U. and Garrison R.L. Bremsstrahlung and Recombination Radiation of Neutral and Ionised Nitrogen. Phys. Rev. Volume 180 No. 1 April 1969
94. Gröber H, Erk S and Grigull U. Fundamentals of Heat Transfer. McGraw Hill, 6th Edition (1967).
95. Bird R.B., Stewart W.E. and Lightfoot E.N. Transport Phenomena. Wiley (New York) 1962.

ABSTRACT

Transport Properties in High Temperature Gases

Leicester University Ph.D Thesis 1973

D.J. Dobbs

The experimental techniques for the measurement of thermal conductivity of gases at very high temperatures were examined. Using a wall stabilised arc the thermal conductivity of argon and nitrogen was measured in the temperature range 8000-16,000°K and compared with published experimental and theoretical data. Reasons for the very significant discrepancies between theory and experiment at the higher temperature end of the range examined were sought and the importance of thermal radiation demonstrated. The problems involved in including the thermal energy contribution to the arc energy balance were examined together with the measurement and calculation of the radiation density.

A method for the direct measurement of viscosity using the wall stabilised arc was examined and a new method for the measurement of velocity in high temperature gases was developed.

Following the experimental measurement of gas properties, the problems involved in their theoretical prediction were examined. A new method of calculation of thermal conductivity and viscosity of gases with non-frozen composition was developed which, whilst maintaining a high level of accuracy in the temperature range up to about 18,000°K in argon and nitrogen at atmospheric pressure offered a very considerable simplification and reduction in labour over other published methods.

The final section of this study was concerned with the evaluation of electron-proton and electron-ion collision data in the arc. The absorption of electromagnetic radiation produced by low power H_e-N_e and $G_a A_s$ lasers was used to make the measurements; the importance of the electron-ion collisions being clearly demonstrated in this work.