

PROFESSIONAL DEVELOPMENT OF TURKISH PRIMARY MATHEMATICS
TEACHERS WITHIN A COMPUTER-INTEGRATED LEARNING
ENVIRONMENT: AN EXPLORATION OF CHANGES IN BELIEFS

Thesis submitted for the degree of

Doctor of Philosophy

at the University of Leicester

by

Umit Kul

School of Education

University of Leicester

2013

Abstract

The curriculum for primary mathematics in Turkish schools was revised in 2005, and one of the aims of this reform was to introduce constructivist approaches to the learning and teaching of mathematics. When instigating changes in the Turkish educational context, only minimal attention was given to the professional development (PD) of mathematics teachers. Thus, in this study a PD course was created to offer participants an opportunity to experience the role of the learner in a computer-integrated setting. The intention was to provide them with better theoretical and practical comprehension of mathematics teaching and learning.

The study investigated six Turkish primary teachers' beliefs pertaining to the following fields: the nature of mathematics, its teaching and learning, the use of technology, prior to and following their involvement in a PD course designed using a Dynamic Geometry Systems based on a constructivist approach. The objective of this study was to examine how they in such a professional learning setting formalise their beliefs.

A multiple case study design was used to explore mathematics teachers' beliefs and to examine the dynamics of teachers' belief shifts. The data generation instruments used in the study included observation, semi-structured interviews, participants' writings and questionnaires. The qualitative analysis of the data revealed that teachers' beliefs were transformed to some extent in support of fallibilist views regarding mathematics along with constructivist views about teaching and learning of mathematics. In fact, the connection between stated beliefs and intended teaching is complex and the social contexts of teaching were very influential on teachers' pedagogical decisions, participants' world views about the nature of mathematics serving as a primary source of their beliefs about pedagogy and student learning; this connection was not clear. The research findings also reveal a substantial change in participants' beliefs in favour of the use of technology in general and in particular in the use of GeoGebra, in their teaching.

The findings from this study have implications for Turkish primary mathematics education and teacher belief literature. Further research is needed in order to capture the complexity involved in the cultural dimensions that influence teaching mathematics.

Acknowledgments

This thesis would not have been completed without the help and support of many people. I would especially like to thank the following people.

I would like to acknowledge and thank Prof Janet Ainley for her expert guidance, time and effort. I particularly appreciate her careful attention to detail, her patience in clarifying things in the simplest terms and for her promptness in giving feedback to my papers and responding to my email queries.

All the teachers who participated in my research project and gave their time for this study. I thank them for their time, patience and willingness to share their stories and experiences with me. I would like to also thank my colleagues Dr.Nural, Mr.Uslu and Mr. Kutay, for the push they have given me to dig deeper and think harder; and I would like to thank for their encouragement and wisdom.

I reserve my deepest thanks to my dearest wife, Medine who always with me through this PhD journey and giving me a lovely daughter and son. This thesis would not have been possible unless she had continuously encouraged me with her sincere optimism on this work. I would like to show gratitude my father and my mother, as well as to my brother and sisters. I love you all and I am grateful for the bounty of a loving and supportive family.

Last but not least, I am indebted to many of my friends from the Leicester Turkish community for their support. I will always remember the enjoyable time spent together with Sinan, Mustafa, (Arabali) Sinan, Ugur, Dursun, Serkan, Ilhan, Suleyman abi and Leicester Turkish society. I also would like to thank respectable person Dr.Tuncay for inspiring me.

Dedication

To my wife Medine; and my lovely children Elif Sena and Enis Safa

for their love

Table of Contents

Abstract	i
Acknowledgments.....	ii
Dedication	iii
Abbreviations	xi
CHAPTER 1: INTRODUCTION	1
1.1 Background of the Study	1
1.1.1 Personal Motivation for Undertaking this Research	3
1.2 Significance of the Present Study	4
1.3 The Rationale for using Technological Artefact (GeoGebra) in PD course	6
1.4 Aims of the Study	8
1.5 Research Questions	8
1.6 Organisation of the Thesis	9
CHAPTER 2: THE CONTEXT OF STUDY	11
2.1 Introduction	11
2.2 A Brief History of the Turkish Education System.....	11
2.2.1 Control of Education	11
2.2.2 General structure of the Turkish National Educational System.....	12
2.2.3 Examinations	14
2.3 Developments in In-Service Teacher Education in Turkey	15
2.4 Reform in Primary Mathematics Education in Turkey	17
2.4.1 Changes in Primary Mathematics Education	17
2.4.2 An Overview of Technology Integration in Turkish Schools	19
2.5 Summary	21
CHAPTER 3: LITERATURE REVIEW	23
3.1 Introduction	23
3.1.2 The Islamic Conceptualisation of Knowledge and Education	24

3.2 Constructivism and Its Influence on Mathematics Education	27
3. 2. 1 The Theory of Constructivism	27
3. 2. 2 Piaget's Constructivism	29
3. 2. 3 Vygotsky's Social Constructivism	32
3. 2. 4 Constructivism in Mathematics Education	34
3. 2. 5 Summary	38
3. 3 The Importance of Teachers' Beliefs	39
3. 3. 1 Introduction	39
3. 3. 2 Beliefs	40
3. 3. 3 Shifts in Beliefs	44
3. 3. 4 Teachers' Beliefs about the Nature of Mathematics	47
3. 3. 5 Teachers' Beliefs about Mathematics Learning and Teaching	50
3. 3. 6 Summary	56
3. 4 Teachers, Dynamic Geometry Systems and Mathematics	58
3.4.1 Potential for Using Technology within Mathematics Education	58
3.4.2 The Potential of DGS in Mathematics Education	60
3.4.3 Professional Development of Mathematics Teachers and Their Experiences with Technology	62
3.4.4 Summary	70
3. 5 Chapter Summary	71
CHAPTER 4: METHODOLOGY	74
4.1 Introduction	74
4.2 Philosophical Perspectives of this Study	75
4.2.1 Paradigm	75
4.2.2 Ontology	76
4.2.3 Epistemology	76
4.3 Research Design	77

4.3.1 Case Study Design and Rationale	78
4.3.2 Generalization in Case Studies.....	80
4.3.3 Researcher's Role.....	81
4.4 Design of the Professional Development Course	82
4.4.1 Rationale and Principles for a GeoGebra PD Course.....	83
4.4.2 Structure and Content of the PD Course.....	86
4.5 Sampling and Access	92
4.6 Data Collection Methods.....	94
4.6.1 Interviews.....	95
4.6.2 Observation	99
4.6.3 Questionnaires	101
4.6.4 Participants' Writings	102
4.7 Data Analysis.....	104
4.7.1 Interviews/reflective writings and observational (field) notes.....	104
4.7.2 Analysis of Questionnaires	108
4.8 Reliability and Validity	109
4.9 Ethical Issues and Concerns	109
4.10 Chapter Summary	111
CHAPTER 5: FINDINGS	112
5.1 Six Profiles	112
5.1.1 The Case of Eysun.....	112
5.1.2 The Case of Emin	114
5.1.3 The Case of Muslum.....	116
5.1.4 The Case of Musti	118
5.1.5 The Case of Celal	119
5.1.6 The Case of Asim	121
5.2 The Participants' Initial Beliefs	124

5.2.1 Nature of Mathematics.....	124
5.2.2 The Importance of Mathematics.....	128
5.2.3 Approaches to Teaching and Learning Mathematics	132
5.2.4 Computers in Mathematics Education.....	138
5.3 Participants' Experiences with GeoGebra-Based Activities	143
5.3.1 Participants' Expectations of and Reactions towards the PD Course	143
5.3.2 Reflections on the PD Course Activities	146
5.3.3 Reflections on Social Interaction and Pedagogy	153
5.3.4 Linking GeoGebra with Mathematics	157
5.3.5 Time as a Key Issue.....	160
5.4 Changes in the Participants' Beliefs	162
5.4.1 Changes in the Participants' Beliefs about Computer	162
5.4.2 Changes in the Participants' Mathematical Beliefs.....	165
5.5 Summary	170
CHAPTER 6: DISCUSSION	174
6.1 Introduction	174
6.2 What beliefs do Turkish primary mathematics teachers hold about the nature of mathematics, its teaching and learning before participating in the PD course?.....	175
6.2.1 Beliefs about the Nature of Mathematics	175
6.2.2 Beliefs about Teaching and Learning of Mathematics	178
6.3 How do they change their existing beliefs while engaging in GeoGebra-based mathematical activities within the course as learners about mathematics, its teaching and learning?	182
6.3.1 The Influence of the Context on Teachers' Beliefs regarding Technology Use in Mathematics.....	182
6.3.2 The Importance of Social Engagement within a Professional Learning Environment.....	183
6.3.3 Dilemmas between Expectations and Reality	185

6.3.4 Changes to Teachers' Mathematical Beliefs	188
6.4 Chapter Summary	190
CHAPTER 7: CONCLUSION.....	192
7.1 Introduction	192
7.2 Summary of the Study.....	193
7.3 Implications	196
7.4 Limitations of the Study.....	200
7.5 Recommendations and Directions for Future Research.....	202
7.6 Final Remarks	202
Appendix 1A: Approval Letter (Turkish Version)	204
Appendix 1B: Request Letter (Turkish version).....	205
Appendix 2: Participant Consent Letter	206
Appendix 3: Mathematical Belief Questionnaire	207
Appendix 4: Interview Protocols	210
Appendix 5A: An Example of Coding/Categorisation Process.....	213
Appendix 5B: Extracts of coded data	214
Appendix 6A: The Purpose of the Each Worksheet	215
Appendix 6B: Introduction Booklet	218
Appendix 6C: Worksheets	222
Appendix 6D: Home-based activities.....	240
REFERENCES.....	243

LIST OF TABLES

Table 4.1: The content of the PD course.....	87
Table 4.2: The characteristics of the participating teachers.....	94
Table 4.3: Data collection schedule.....	103
Table 4.4: Initial themes.....	107
Table 4.5: Main themes.....	107
Table 5.1: A schematic overview of changes in participants' belief systems.....	123

LIST OF FIGURES

Figure 2.1: The structure of the Turkish national education system.....	14
Figure 3.1: Piaget's adaptation theory.....	31
Figure 3.2: Zone of Proximal Development.....	33
Figure 4.1: A Scale for Determining Agreement and Disagreement.....	108
Figure 5.1: A summary of participants' initial beliefs about computer.....	142

Abbreviations

BOED: Board of Education and Discipline

CAS: Computer Algebra Systems

COHE: The Council of Higher Education

DGS: Dynamic Geometry Systems

EU: European Union

FOE: Faculties of Education

FSA: Faculty of Science and Arts

GSSE: Government Civil Servant Selection Examination

I1: First Interview

L: Starting line number of interview transcript

MONE: Ministry of National Education

PD: Professional Development

SSEE: Secondary School Entrance Exam

SSPC: Student Selection and Placement Centre

UEE: University Entrance Exams

ZPD: The Zone of Proximal Development

CHAPTER 1: INTRODUCTION

1.1 Background of the Study

Since Turkey became an official candidate for accession into the European Union, reformations have been encouraged in an attempt to concur with EU standards; these have affected various aspects of the national educational system (Aksit, 2007). The Turkish Ministry of National Education (MONE) has been working to develop the educational structure. This undertaking has involved revision to the curriculum, including revisions in 2005 to primary (pupils aged 10-14) mathematics education. The purpose of the reform to the mathematics curriculum has been to modify the content and focus of the entire mathematics curriculum, by moving away from the previous content-based curriculum that stipulated how to teach, and imposed on students received mathematical knowledge, sets of formulas and procedures. The changes to mathematics teaching and learning suggested in the new curriculum involve transference to a constructivist style of learning, which privileges student-centred classrooms and reflective teaching philosophies (Bulut, 2007). Moreover, mathematics teachers have been encouraged to integrate technology in their teaching. However, to date, there has been little interest directed towards the professional development of teachers (Babadogan and Olkun, 2006; Bulut, 2007).

Prior to the changes outlined above, emphasis in Turkish mathematics classrooms was on the use of explanatory models of teaching. Typically, teachers employed a whole-class lecturing style and concentrated on routine activities and procedures to introduce techniques for determining facts and solving problems according to certain steps and algorithms (Aysel, 2012). This methodology was predicated on the belief that repetition and rote learning form the basis of effective skill acquisition; thus, students were not required to participate actively and mathematics teachers were not required to diversify their activities or methods (ibid). This approach forced learners to learn mathematics to achieve instrumental understanding (Skemp, 1976), thereby it was natural that it emphasised students' successful memorisation of mathematical facts, procedures and their application. As in many similar educational systems, the foundation of learning was the prioritisation of exams comprising multiple choice questions; the most successful candidates were those who accurately answer as many questions as possible

as quickly as possible (Aysel, 2012, Guven, Cakiroglu and Akkan, 2009). There was no need for teachers to encourage their learners to think independently to pass these exams and progress. Ekici (2005) argues that the washback from exams is one of the most significant agents directing teaching activities in Turkey. Although the new curriculum has been designed to enable and encourage teachers to employ different teaching approaches and techniques, such as a discovery approach, the explanatory model of learning and teaching continues to dominate mathematics education in Turkey (Temizoz and Ozgun-koca, 2008). In the absence of professional development to promote the development and implementation of new innovations, teachers are struggling to teach the curriculum effectively making use of new innovations.

The role of teachers in reforming education is vital, and the responsibility for teaching mathematics differently rests ultimately on teachers (Wilson and Cooney, 2002). Thus, it seems logical to expect that before implementing reforms teachers should be made aware of, and come to believe in, the benefits underlying the initiative to reform mathematics education. This is of great importance if we accept the view of Handal and Herrington (2003), that educational reform can be driven forward or held back depending on the nature of teachers' commonly held beliefs. However, the different pedagogical approaches to teaching and learning and the use of technology in mathematics instruction were introduced to the Turkish educational arena before the teachers were made fully aware of how to integrate these principles and philosophies and utilise them practically. In fact, change was imposed using a top-down model that was not concerned with teaching practice and the beliefs of teachers (Norton et al., 2002; Perry et al., 1999). Consequently, developments are not being successfully implemented and a number of misunderstandings and misinterpretations exist within the educators in the Turkish education system. Many teachers preferred to rely on familiar and traditional methods of teaching, as they have confidence in these (Handal, 2003) and to a certain extent are actively resisting changes to their mental schema (Clarke, 1997).

The recent reform in school mathematics that incorporates mathematics success for students, learner-based activities, and constructivist approaches in classrooms, is reliant on a significant shift in teaching practices and a similar significant shift in teachers' beliefs about the nature of mathematics, its teaching and learning (Ernest, 1989). The

current number of Turkish in-service teachers working in schools is huge, and even before 2005 their development constituted an important component of the Turkish educational system. At present the professional development of teachers, in particular when aiming to employ more contemporary pedagogical practices, is a field, which calls for more study.

1.1.1 Personal Motivation for Undertaking this Research

One of the rationales that motivated me to choose this topic and undertake the research project is my personal interest. In my experience as a student in Turkish schools, I was a passive recipient set in the role of accepting an independently produced body of truths. My main complaints concerned the examination-oriented learning activities I undertook based on memorisation and practice. Although I was successful at memorising and practicing mathematical rules and procedures, I did not really enjoy studying mathematics in this way, as there was no opportunity to explore the rationale and the meaning behind mathematical ideas and concepts. Through my experiences, I became bored by the teacher-centred classroom, since this type of environment did not allow me to produce my own methods and construct my own understanding.

During the pursuit of my bachelor's degree in mathematics, I attended a variety of computer-based courses based on programming languages such as MATLAB, C++, PASCAL, and SQL. When I began to work as a teacher in a private school, I became more interested in integrating technology into mathematics education, i.e. 'active mathematics', rather than relying on routine activities and ready-made mathematical formulas. I made great efforts to alter my existing teaching approach. At this point I was unable to employ any educational tools to my teaching approaches since I did not have any hands-on experience of how to integrate innovative teaching ideas into the existing mathematics curriculum.

Upon obtaining scholarship from MONE, I was enthusiastic about exploring educational tools and different learning strategies, for instance the constructivist approach when associated with technology. At my first meeting with Prof Ainley, I was recommended to read Skemp's article, and his distinction between relational and instrumental mathematical knowledge compelled me to dig deeper and think harder. Two key concepts formed my philosophy about mathematics education, and my journey began as I probed the body of literature related to teachers' beliefs, constructivist

principles and the use of technology in mathematics education. Consequently, I attempted to design and deliver a professional development (henceforth PD) course in line with the constructivist approach, employing GeoGebra as a pedagogical tool, in order for groups of mathematics teachers to obtain a new perspective of mathematics teaching and learning. This was based on the notion that allowing teachers to gain enough experience in studying with Dynamic Geometry Systems would provide a partial remedy.

1.2 Significance of the Present Study

In recent years, beliefs have become more important as key concepts worthy of investigation in the field of education. This is largely because a strong link between teachers' beliefs and their teaching practice has been reported in many studies (e.g. Cross, 2009; Stipek et al., 2001; Thompson, 1992; Zakaria and Maat, 2012). Teachers are asked to take the responsibility for creating and designing the learning environment; they make decisions about what and how to teach, not always relying on curriculum guidelines. For the most part, teachers' beliefs about teaching are derived from their personal experiences as students and later through information they gather on teacher education courses and through the observation of other teachers (Lortie, 1975). If one of the aims of reform is to improve and develop teachers' teaching practices in mathematics education, one must also probe the nature of mathematics, exploring what it means to teach and to learn from a teacher's perspective. Before attempting to comprehend how teachers change their beliefs, it is essential to study teachers' conceptions about the subject matter as well as about its teaching and learning. Swan (2006) illustrates that any attempt to develop mathematical teaching practices of teachers must pay attention to the beliefs of mathematics teachers and to shifts in those beliefs. In this way, many scholars have sought to understand and explore teachers' changing practices, and teachers' beliefs and the opinions that inform their pedagogical decisions and classroom practice (e.g. Chapman, 2002; Liljedahl, 2010; Stipek et al., 2001; Wilson and Cooney, 2002).

Teachers' mathematical beliefs determine their use of teaching activities, thus ensuring that these are reflected in classroom practice (Ernest, 1989). Chief attention should be directed towards the study of teachers' beliefs as potentially important to educators in terms of reforming teaching practice (Handal, 2003). In order for teaching approaches,

such as a constructivist pedagogy, to be implemented in the classroom, it is crucial that shifts must be made in beliefs about mathematics, as well as its teaching and learning (Ernest, 1989; Swan, 2006). The present study classifies such beliefs about the nature of mathematics following a variety of viewpoints suggesting that a set of absolutist and fallibilist beliefs can be discerned through the manner in which mathematics is taught (Thompson, 1992; Roulet, 1998). According to the former belief, mathematics is seen as a certain, naive, fixed, and set of rigid rules. According to latter, mathematics is viewed as uncertain, fallible, complex, integrated and devised by human beings. Beliefs about mathematics teaching and learning are described in the main study in terms of a traditional and constructivist viewpoint. These terms will be elaborated on in section 3.3. When reforming mathematics education or changing the nature of mathematics, its learning and teaching, one has to challenge absolutist beliefs of the subject matter and traditional beliefs of teaching and learning. That is, apply a constructivist pedagogy and inquiry-based learning method that presents a new view of mathematics as a social invention resulting from the continuous process of human enquiry; a process of inquiry that is always open to revision. The constructivist view of learning reflects the fact that the construction of knowledge is the responsibility of the students and encourages learners to understand the subject through self-discovery, using open-ended activities in a learner-centred setting.

This research puts forward the argument that teachers need to promote, through adoption, an approach based on the constructivist perspective utilising technology in practice. Although there is already, a well-established mathematics curriculum based on a constructivist approach to teaching and learning in Turkey, teachers' previous learning experiences with mathematics were based on traditional approaches. When instigating changes in the Turkish educational context, little attention was paid to mathematics teachers in primary school's beliefs about mathematics, its teaching and learning, and how PD courses might impact those belief structures and systems, and their teaching practice. Little was also known about whether, or how, teachers alter their mathematical beliefs. This is a significant knowledge gap when attempting to implement a new curriculum that is reliant on pre-service and in-service teachers welcoming a constructivist learning environment. By presenting such an environment within a PD course, more opportunity to improve teaching practice would be anticipated (Hart, 2002; Guven et al., 2009; Mewborn, 2003; Thompson, 1992). Guskey (2002) states that

the professional development available on teacher programs “are systematic efforts to bring about change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of students” (p.381). It is critical, therefore, to recognise that when a teacher is supported to internalise new methods into her/his framework confidence is improved and changes can be effected.

With the Turkish Ministry of National Education’s expectation from teachers in reference to implementing constructivist pedagogy, the present research provides insight into the dynamics and issues associated with this process within the professional learning environment. The work has the potential to describe teachers’ teachers’ beliefs about mathematics as well as about its teaching and learning and explain the dynamics of the changes in teachers’ beliefs. From the perspective of the present study, exposing teachers to new pedagogical approaches as learners is argued to be a positive way to assist them in confronting and changing their beliefs. By providing teachers with alternative models for teaching, through a professional development program, they can be inspired to think about teaching and learning critically and in depth. Therefore, the focus of this study has the potential to provide a thorough understanding of learning experiences among Turkish teachers, which has significant implications for developing the quality of their teaching and learning.

1.3 The Rationale for using Technological Artefact (GeoGebra) in PD course

The focus of the present research was to engage Turkish in-service mathematics teachers in a professional development course to expose them to new pedagogical practices and, therefore, seek to change their beliefs about what it means to do mathematics. The tool (GeoGebra) was employed to facilitate this exploration and contributes a further unique element to the work at a time when digital technologies are playing a crucial role in mathematics education worldwide, but are not yet widely used within Turkey. The National Council of Teachers of Mathematics noted that “technology is essential in teaching and learning mathematics; it influences the way mathematics is taught and enhances students’ learning” (2000, p. 11). The incorporation of technology in mathematics classrooms will be beneficial, and can best be obtained by altering teachers’ beliefs and views about teaching pedagogy and the roles of teachers and students in the classroom. Software that is currently available represents the specialised aspects of computing that lend themselves to mathematics teaching:

“learning from feedback; observing patterns, seeing connections, developing visual imagery, exploring data and ‘teaching’ the computer” (Becta, 2009: p.2). In view of this, many researchers concur with the view that the incorporation of digital tools into mathematics education could be used to support constructivist learning by motivating students to become more open to exploring, examining, conjecturing, discovering principles and making generalisations (Oldknow and Taylor, 2003; Turner, 1999).

Dynamic Geometry Systems are amongst the most readily approved of educational software tools since they were first conceived of for classroom use (Ruthven, 2008b). Pratt and Ainley (1997) emphasise that DGS introduce new approaches to the teaching and learning of geometry. Ruthven (2005) mentioned that DGS allows learners to investigate and visualise the geometrical features of shapes using dragging tool and transforming shapes to an extent that is beyond the scope of traditional activities. Hoyles and Noss (2003) highlight that DGS constitutes a pedagogic tool for creating a context in which “students can construct and experiment with geometrical objects and relationships” and for the examination of “a mathematical domain” (p.333). GeoGebra is a form of open source software that has the potential to combine desirable features of DGS, CAS and spread sheets into a single application (Preiner, 2008). GeoGebra is also a free software application that can be downloaded from the internet; therefore, it is available to all primary teachers who wish to use it in their classrooms. The rationale for using GeoGebra-based mathematical activities is that it makes it possible to accomplish learning through active exploration, discovery and making conjectures.

The present study concerns the need for professional development courses to assist mathematics teachers in promoting a critical standpoint on teaching and learning and the use of technological artefact in the mathematics context. Its purpose is to challenge absolutist-oriented traditional beliefs associated with mathematics and its teaching and learning, thereby offering a constructivist route that teachers can follow to teach mathematics. I shall argue that in order for teachers to understand the learning experiences of their students in the new environment, they need to experience it for themselves. This is an approach to professional development that can help teachers to understand the purpose and rationale behind the incorporation of computers into mathematics education. In summary, this study explores the possibility of effecting

changes in Turkish primary teachers' mathematical beliefs through their involvement in a PD course that was designed using GeoGebra, based on a constructivist approach.

1.4 Aims of the Study

The main aims of the research can be summarised as twofold:

- To expand the knowledge base of mathematics teaching and teacher cognition by determining primary mathematics teachers' beliefs about mathematics, its teaching and learning.
- To ascertain the degree to which a professional development (PD) course based on a constructivist approach in using GeoGebra affects the beliefs of a group of Turkish primary school teachers.

To achieve these aims, a group of mathematics teachers participated in a setting in which they could interact with GeoGebra-based mathematical activities consistent with a constructivist approach within the PD course, so as to gain a better understanding of the subject matter. This setting promotes the interactive processes of conjecture, feedback, critical thinking, investigation and collaboration.

That is, the PD course involved the researcher and the teacher participants within a collaborative setting in an exploration of mathematical circumstances, communication and application of new thoughts through individual and small group examination, debate and negotiation. To promote mathematical thinking beyond the routine, they worked on mathematical activities using GeoGebra. The intention of the PD course was to challenge teachers' mathematical beliefs. The above aims are reflected in the following research questions.

1.5 Research Questions

The central question in the present study is how has involvement in a Professional Development course, that was designed using GeoGebra based on a constructivist approach, influenced the beliefs of Turkish primary mathematics teachers about mathematics, its teaching and learning? This question can be subdivided as follows:

Sub-Questions

- 1) What beliefs do Turkish mathematics teachers in primary schools hold about the nature of mathematics, its teaching and learning before participation in the PD course?
- 2) How do they change those beliefs about the nature of mathematics, its teaching and learning, while engaging in GeoGebra-based mathematical activities within the course as learners?

The data was collected from six Turkish mathematics teachers prior to, during and following their involvement in a PD course that incorporated GeoGebra-based mathematical activities, at 22 hours of workshop sessions during the spring semester from April 2011 to June 2011 in the city of Kahramanmaraş in Turkey.

1.6 Organisation of the Thesis

This thesis has been organised into seven chapters. The introduction illustrates the overall description of the study, which includes background, the rationale and its general focus and the research questions. Chapter 2 provides a brief historical background and general structure of the Turkish educational system as well as the developments of in-service teacher education programs and recent reforms to the country's primary mathematics education. Chapter 3 introduces a review of literature pertaining to the field of enquiry. It consists of three sections regarding constructivism and its influence on mathematics education; investigating the teachers' beliefs about mathematics, its teaching and learning and the teachers' learning experiences with technology respectively.

Chapter 4 provides a detailed outline of methodological concerns associated with research design, data collection instruments, data analysis, trustworthiness and the ethical considerations of the present research. This chapter also discusses the rationale and principles of a GeoGebra-based PD course for Turkish mathematics teachers, and outlines brief descriptions of the activities and strategies employed in the course. Chapter 5 presents the findings of the study about changes in teachers' beliefs throughout a GeoGebra Professional Development course, and consists of two parts as

follows: 1) The first part introduces six-mini profiles of course participants and describing the distinctive characteristics of each teacher's beliefs and detecting the changes in his/her belief structures. 2) The second presents a report on teachers' beliefs about mathematics, its teaching and learning and use of computers in mathematics both before and after the PD course, as well as their experiences and reflections on the course activities when identifying themes relevant to all six informants through cross-case analysis. Chapter 6 focuses on the discussion of the findings. As a structural basis for the discussion, this chapter examines the research findings in reference to the research questions. Chapter 7 is the conclusion, which includes the summary and the limitations of the study, as well as some suggestions and directions for further research and educational implications.

CHAPTER 2: THE CONTEXT OF STUDY

2.1 Introduction

The purpose of this chapter is to provide a brief overview of the general structure of the Turkish education system as well as focussing on the developments of in-service teacher education programs and recent reforms of the country's primary mathematics education. In addition, an overview of technology integration in Turkish schools will be discussed.

The first section is made up of the following sub-sections; control of education, school structure and examinations. The system was established using a top-down approach and is centrally directed by the Ministry of National Education (MONE). As a result, most decisions about pedagogical approaches are specified by the MONE and teachers are expected to follow them.

The second section of this chapter will introduce the current patterns of teacher education in Turkey. The last section will provide background information about recent reforms of primary mathematics education in Turkish schools as well as examining recent developments in the integration of technology within Turkish schools.

2.2 A Brief History of the Turkish Education System

2.2.1 Control of Education

Turkey's location between Asia and Europe means it serves as a bridge between the East and the West, and also has a rich historical background. Turkish history shows how the first education system was formalised by the Ottoman Empire (Turan, 2000). This system was very strongly influenced by Islamic traditions because the Ottoman Empire was ruled by Islamic values and principles. After the Ottoman Empire collapsed, the Republic of Turkish was founded in 1923. Many attempts were made to centralise the Turkish education system; thus, the MONE was established in 1924, all educational institutions and activities in the country fell under its control, and the government became responsible for the funding of education (Cayir, 2009).

In 1928, Turkey renounced the Arabic alphabet and began to use the Latin alphabet: this dramatic change markedly influenced the educational system (Akkoyunlu, 2002). ‘Westernisation’ was the fundamental objective of the new government during the early years of the Republic, and the aim was to follow the lead of Western countries in every field without any consideration for Turkey’s own cultural heritage (Turan, 2000). ‘Westernisation’ was regarded as a modernisation process which changed the new Turkish Republic in line with Western models (ibid). The belief was that following the educational trends of developed countries would provide a useful mechanism for turning an ‘Islamic society’ into a ‘modern society’, thus becoming part of European civilisation and culture (Gok, 2006: p. 248).

MONE plays a crucial role in decision-making in all areas pertaining to education, such as the appointment of teachers and school headteachers and the designing of the curriculum (Karakaya, 2004). The centralised nature of the education system does not allow educators to personalise their own teaching in response to students’ individual learning styles (Gok, 2006). The school curriculum, textbooks and guidelines are prepared by the Board of Education and Discipline (BOED) which is overseen by the MONE (Yildirim, 2003). Karakaya (2004) stated that the central authority makes decisions about what to teach and how this content should be taught in the classroom. A teacher works for the government as a civil servant, therefore s/he must rationalise his/her practice to the school headteacher, and the headteacher in turn reports these to MONE. Consequently, teachers feel that they are accountable to MONE instead of being principally concerned with their students’ development (Yildirim, 2003).

2.2.2 General structure of the Turkish National Educational System

The Turkish education system consists of pre-primary school, primary, secondary and higher education levels. These stages of general education are shown in figure 2.1.

Pre-school

Although pre-primary school education is not compulsory for pupils between the age of 3 and 5, since the recent education reform movement in Turkey these schools have become very popular (The Council of Higher Education, 2010). Services associated with pre-primary education are provided by nurseries, kindergartens, and practical classes are provided by the MONE and private institutions.

Primary education

The first compulsory level of education in primary school lasts eight years. This stage includes pupils between the age of 6 and 14, it is compulsory for all society members, and education is provided without any charges in public schools. It is equally possible for parents to send their children to private institutions where they have to pay tuition fees. The fundamental aim of primary education is to provide students with the necessary knowledge, skills and behaviour to become a good citizen (National Education Statistics, 2012).

The primary school curriculum is defined by BOED and is therefore the same in every primary school. For years 1 to 5, individual classroom teachers are responsible for their class, whereas pupils in years 6 to 8 are taught by different teachers for each subject. The selection of guidebooks for teachers is not optional, and although teachers are free to follow their own methods, the approaches outlined in the guidebooks force them to stick to certain teaching models. The first five years of primary school are referred to as 'First School', and the term 'Middle school' is used to describe the last three years of primary education. Teachers track the achievements of primary schools students through each school year by monitoring their achievement in projects, exam results, homework, classroom participation, attendance and behaviour etc. When they have successfully completed the eight years of compulsory education, students receive a diploma of primary education.

Secondary School

Secondary education is offered by a variety of vocational and technical institutions which provide four years of education after primary school. This level of education is not compulsory, but all students between the age of 14 and 18 can attend public schools free of charge. However, progression to a better secondary school is based on students' achievement in primary school and depends on the type of school they intend to go to; for example, Anatolian secondary schools. These provide a better education and offer more foreign language lessons, and they accept their students through a centrally organised examination called the 'Secondary School Entrance Exam' (SSEE).

Higher education

In order to enter to university in Turkey, students must have a high school diploma and have been successful in the University Entrance Exams (UEE) which are organised by the Student Selection and Placement Centre (SSPC). Transition to a variety of faculties is highly competitive and requires students to attend a preparatory school (in preparation for the exams) since their future depends on the UEE. All higher education institutions are controlled by the Turkish Council of Higher Education (COHE).

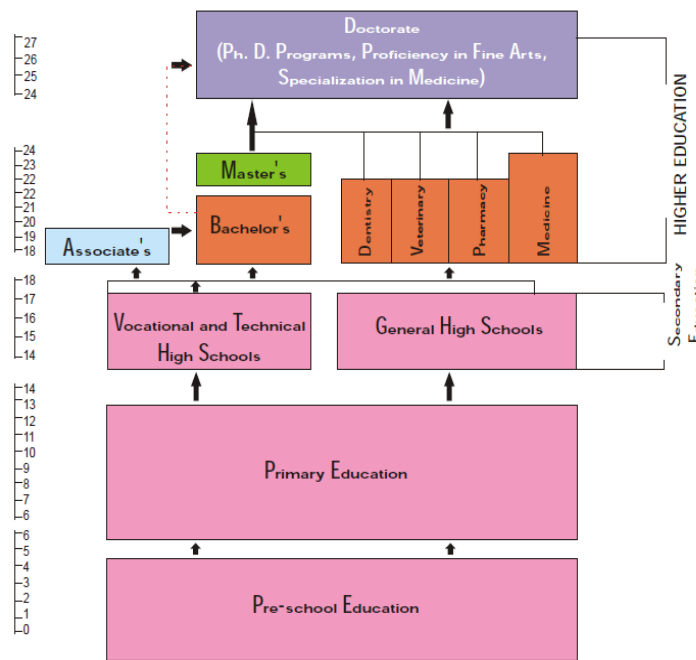


Figure 2.1: The structure of the Turkish national education system

2.2.3 Examinations

The education system in Turkey is mainly examination-oriented, whereby students' examination results are important in order for them to reach the next level of education. Public examinations are important for progression to the next stage at each level of education in Turkey (Aysel, 2012). Children who want to progress to further education must pass exams at the end of the compulsory education and secondary school levels, and these exams consist of multiple choice questions (Aksit, 2007). Ekici (2005) argues that the washback from exams is one of the most significant agents directing teaching activities in Turkey. The focus in the exams is on assessing the students' achievement levels and measuring the amount of knowledge they have amassed, rather than how much in-depth knowledge they have developed in specific subject areas. Primary

students whose scores fall below a certain level are not able to progress to a better secondary school, thus making it difficult for them to gain admission to a good university. In these exams, pupils are required to answer as many questions as possible as quickly as possible.

It could be argued that this assessment system only provides limited insight into learners' overall understanding of a subject, and the result is that the focus of teaching is only to prepare students for the national examinations. As a result, teachers tend to impose their existing knowledge on students rather than allowing them to think for themselves, and the system appears to attach importance to the memorisation of facts and the application of received ideas (Guven et al., 2009). Examination pressure may force teachers to follow the lecture mode of teaching rather than using modern pedagogical approaches which require more time. Teachers come to view their role as being solely focused on preparing students for the examination, therefore traditional approaches to teaching and learning constitute an important part of the Turkish educational system.

The national exams are not only for students but also for qualified teachers. After completing pre-service education, teachers who want to work in state schools are required to sit the Government Civil Servant Selection Examination (GCSSE). This exam covers the following subject areas; General Culture, General Knowledge and Skills and Educational Sciences. This exam is necessary since a huge number of qualified teachers apply for posts in schools.

2.3 Developments in In-Service Teacher Education in Turkey

The developments of new digital technology and knowledge have led to changes in every aspect of life. As a result, there is a need for qualified individuals to sustain their development through a lifelong learning process. In this sense, teachers have been given more responsibility, since they have an important function in the development of society and in educating the individuals governing society; therefore, teachers require support during their teaching years and in order to remain highly qualified. In order to implement new educational changes in schools, the current in-service teachers programs need to be improved since the professional development of teachers, in particular the development of teaching methods, are vital for the successful integration of changes in

schools. This notion is corroborated by Guskey (2002) who notes that the professional development of teacher programs “are systematic efforts to bring about change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of students” (p.381).

Currently, the MONE in Turkey is responsible for providing in-service teacher education programs at pre-school, primary and secondary education stages in order to foster teaching practice. The main focus of these programs is to allow individuals to be successful in their professions, to adapt to the changes in their professional life and to improve their qualifications in order to meet students’ needs (Kucuksuleymanoglu, 2006). Teachers are obliged by the Public Servants Law (1965) to take part in in-service teacher education courses either at home or abroad. Since 1960, regular in-service courses have been run by the Department of In-Service Education and a few universities. The Ministry recognised the need for teachers’ on-going development and there have therefore been many attempts to improve the amount and quality of in-service education. The department provides teachers with written and audio-visual materials concerning new content and methods (MONE, 1997). Since 1995, the MONE’s Department of In-Service Education focussed on the education of probationary and novice teachers and a one-year teacher education course named the ‘probationary education system’ have been implemented. This course covers three issues: basic education (50 hours), preparatory (110 hours) and practical education (220 hours) (MONE, 1995). This course calls for each novice educator to undertake teaching practice at an appointed school under the supervision of a veteran teacher (MONE, 2002). In-service education courses were intended to fulfil the needs of initial school, primary and secondary school teachers and the courses have mostly been run as short seminars. They cover subjects and activities which are related to the new curriculum; for example, materials development, general computer applications, internet, foreign language, and classroom and time management (ibid). In-service courses for primary teachers are run with the aim of ensuring that teachers have the opportunity to learn the new content, rather than to develop teaching practice. Teachers are required to fulfil the stipulations laid down by the MONE, which results in stressful working conditions (Karakaya, 2004). Furthermore, teachers are not able to take into account their students’ needs because the requirements of the MONE have to be their priority (ibid). Teachers

are not given enough freedom in how to teach; they have to use prescribed guidelines which have changed very little in recent years.

2.4 Reform in Primary Mathematics Education in Turkey

2.4.1 Changes in Primary Mathematics Education

As stated above, the Turkish education system consists of pre-primary, primary, secondary and higher education. Primary education includes the teaching of pupils aged 6 to 14 and is compulsory for all citizens, whereas the higher levels of education are optional. The results of international studies such as those by TIMSS (1999) and PISA (2003) have highlighted that the quality of mathematics education at primary level in Turkey is not satisfactory and students' scores are below the international average. The results of these studies prompted a comprehensive and large-scale overhaul of the primary curriculum (Babadogan and Olkun, 2006). During the last decade, much effort has been made to improve the educational system in Turkey. Since being listed as an official candidate for entry into the European Union (EU), Turkey has been presented with a set of complex requirements with which it must comply. Typically, it has responded by attempting to change in every aspect of its social, economic, educational and political sectors. However, such change was essential for a developing Turkish education system (Cayir, 2009). Two proposals for reform to primary education were made in an attempt to comply with the standards of current EU members (Bulut, 2007). The first was associated with the development of the National Curriculum and the second concerned the education system's structure, namely with the aim of decentralising educational administration (Aksit, 2007). However, the second proposal was not approved when it was put forward in 2004 (ibid).

Having considered both proposals, in light of the new demands, the Turkish primary mathematics curriculum was revised and implemented in 2005 by the MONE which is responsible for primary education regulations. At the start of this academic year, the new mathematics curriculum was introduced to all primary schools in Turkey. The idea behind the curriculum change is to transfer from a subject-centred model to a student-centred constructivist model which includes the processes of exploration, discussion, interaction and conceptualisation and learning through classroom activities instead of teachers presenting a set of formulae and procedures in the traditional way. The aim of this reform is to substantially modify the content and focus of the entire mathematics

curriculum, moving away from the previous mathematics curriculum which was content-based, stipulated how to teach, and imposed on students received mathematical knowledge, skills and procedures. This encouraged teachers to transfer mathematical knowledge to students without placing importance on understanding; in short, the teacher was seen as the only decision-maker, knowledge provider and the central authority in the classroom (Isiksal et al., 2007). However, the new curriculum places the student at the centre of the learning process. Activities are designed in a constructivist style and take into consideration individual differences when learning, and leave room for activities. The basic characteristics of the newly developed mathematics curriculum can be described as follows:

- pursues a conceptual approach so as to provide the students with the opportunity to understand abstract mathematical concepts by using their initiative and experience,
- relies on the idea that the pupils should actively engage in the learning process,
- allows pupils to articulate their individual differences and skills through projects and particular homework,
- creates an environment where pupils attempt to investigate, discover and also discuss their findings,
- encourages teachers to integrate technology into the teaching and learning process (MONE, 2005b).

The new curriculum follows the spiral method which is attributed to the constructivist perspective and this is made more effective by hands-on activities and various assessment strategies and techniques (MONE, 2005b). The core tenet of the newly developed curriculum is that ‘every student may learn mathematics’ (ibid). The curriculum follows a conceptual approach which concentrates on inquiry-based mathematics teaching and which strives to improve mathematical reasoning as well as mathematical understanding and other important abilities (including problem-solving and communication). Students come to recognise that mathematics not only consists of procedures and rigid rules, but it is also an entertaining, meaningful and logical discipline. Yet another objective of the newly developed curriculum is to reorganise the learning and teaching environment in line with the constructivist perspective, and also to encourage teachers to integrate ICT into their teaching (MONE, 2005b). This is to

say that computer applications should be integrated into the learning process so as to facilitate various developments in the new curriculum. However, the practice of teachers still maintains the dominant traditions of explanatory teaching and memorisation in mathematics education (Temizoz and Ozgun-koca, 2008). Therefore, the integration of technology into mathematics education in Turkey calls for essential changes in teaching practice (Aksit, 2007; Babadogan and Olkun, 2006).

Research studies exploring teachers' views on the effectiveness of the new curriculum have reported that it has had a favourable impact on teachers, students and parents. Students now play an active role in lessons, which they enjoy, and activities develop their thinking and mathematical skills (Bulut, 2007; Temiz, 2005). The teachers studied by Ozdas et al., (2005) believed that newly introduced topics such as tessellation, patterns and fractals in mathematics, are productive and interesting for students. These teachers also held the belief that these topics can be related to daily life. However, some studies suggest that there are also problems associated with the implementation of the new curriculum in the classroom (e.g. Toptas, 2006). Their participants believed that the main problems related to inadequate resources, lack of instructional tools, not enough time for teaching and evaluation, and a limited variety of activities. Babadogan and Olkun (2006) pointed out that the Turkish national education system had already been significantly altered, but the professional development of teachers has been neglected. Successful implementation of new curriculum ideas into schools depends on teachers' decisions. However, they do not have hands-on experience about how to integrate constructivist teaching ideas into their mathematics classroom (Ozdas et al., 2005; Toptas, 2006).

2.4.2 An Overview of Technology Integration in Turkish Schools

In 1992, in order to reinforce computer education on a large scale and formalise it in primary and secondary schools, an official institution was established, called the General Directorate of Computers Education and Services (GDCES). The Computer Experiment Schools (CES) project was carried out by the GDCES with financial support from the World Bank. Between 1992-1995, the MONE selected teachers from 208 schools and teachers from 23 cities to attend summer courses run with the universities' corporation. These courses were delivered by computer specialists, and academic staffs were provided by the universities' computer departments to teach

certain technological applications. Altun (2002) studied the participants in order to assess to what extent schools incorporated technology into their teaching. He concluded that although there had generally been progress in the use of technology, most teachers were at the early phases of implementing it, and pedagogical use of technology was yet to be successfully integrated. When the period of compulsory education was increased, the MONE decided to set up technology-equipped classrooms in primary schools during the academic year of 1998–1999. 2,541 schools in 80 cities were provided with new computer laboratories.

In parallel with schools, the COHE reviewed the need for change and development in teacher education programs. As part of the National Basic Education Program (BEP), in 1998 education faculties were given technological tools and the necessary hardware and software services. In order to fulfil the need to educate teachers in Turkey, two modules, ‘Computer Literacy’ and ‘Instructional Technologies and Material Development’, became mandatory in 1998 in both all teacher education programmes designed for university level. Prospective mathematics teachers are trained essential computer literacy courses, which include two hours a week of basic computer applications over a year (Güven et al., 2009). The purpose of the instructional technology material development course is to provide pre-service teachers with the necessary knowledge and skills to develop and examine technology-based instructional materials (including spread sheets, transparencies, slides, videos and computer-based material). However, little interest has been given to the use of Computer Algebra Systems (CAS) and Dynamic Geometry Systems (DGS) in mathematics education in the context of these courses (ibid). In fact, in university mathematics teaching departments there are no mandatory modules designed to improve mathematics teaching by using software, although a few have encouraged mathematics students to use computers by offering some courses. As with any other innovation introduced to education, teacher education has always been a crucial factor in the integration process; it is proposed that teachers should have hands-on experience of the new innovative methods, equipment and tasks which they are expected to use in class.

Since the introduction of the new curriculum in 2005, computers have re-gained importance in almost all areas of the Turkish education system as a pedagogical tool (i.e. their value in teaching and learning) which might improve conceptual

understanding and also offer students with a new angle and vision (Guvén et al., 2009). However, obstacles still remain which are affecting the integration of computer technologies into the Turkish education system and recent studies highlight these; for example, planning and pre-service education and technical supports (Bikmaz, 2006). The limited number of research studies on the use of computers in mathematics education in Turkey implies that teachers are the main barrier in this integration process since they view it negatively and the reason for this is teachers' limited knowledge and experience of computer usage in mathematics classes (Baki and Celik, 2005; Guven et al., 2009). Baki (2000) pointed out that these beliefs could be changed with teacher education courses concentrating on the long-term constructivist approach. After observing professional development courses designed for teachers, he concluded that many teachers developed positive beliefs about the use of computer in mathematics. In fact, efforts to incorporate computer technology into mathematics education in Turkey are increasing and these go far beyond meeting the EU requirements. Computer technology is entering schools slowly and this process is taking more time than expected. For this reason, the professional development of teachers in more contemporary pedagogical practice is an area which merits more attention, and giving teachers the opportunity to gain confidence and experience working with Dynamic Geometry Systems may be a solution.

2.5 Summary

This chapter provided an overview of the historical background of the Turkish national education system and information on recent reforms to the mathematics curriculum. In order to illustrate the current situation, it was necessary to understand the establishment of the educational system. In order for its schools to become modernised, the Turkish education system was altered during the foundation of the Turkish Republic, and during this period of change, the educational system became totally centralised.

Since the foundation of the Turkish Republic, the education system had not significantly transformed until recently (Aksit, 2007). However, Turkey's attempt to become part of the EU triggered some developments in 2005 and constructivist teaching notions were introduced to the educational mainstream (ibid). Yet those changes were accompanied by an autocratic approach which disregarded the practice and attitudes of teachers

(Norton, McRobbie, and Cooper, 2002). Hence, teachers' beliefs about mathematics with regard to classroom practice are to be the focus of this study.

On the other hand, the review shows that the main obstacle for innovative teaching and learning is the national examinations, which assess only technical knowledge and do not provide enough scope for students to learn mathematics through exploration. Technology based courses in teacher education do not focus on the instructional use of computers in school mathematics. The pedagogical content of knowledge as a component of teacher education programs is low virtually. From the point of view of the current study, the computer is seen as a vehicle for new ways of teaching and learning mathematics. Introducing the computer to mathematics teachers through PD course might provide solutions to the system's current shortcomings since they need to have practical experience of using computer in their teaching and they need teaching models. Consequently, the existing literature relating to teachers' mathematical beliefs and their PD regarding the use of technology in mathematics education will be examined in the next chapter.

CHAPTER 3: LITERATURE REVIEW

3.1 Introduction

The overarching purpose of this current study is to explore the possibility of effecting changes in Turkish primary teachers' mathematical beliefs through their involvement in a PD course that was designed using GeoGebra, based on a constructivist approach. This purpose is further broken down into the following two sub research questions: 1) What beliefs do Turkish mathematics teachers in primary schools hold about the nature of mathematics, its teaching and learning before participation in the PD course? 2) How do they change their existing beliefs about the nature of mathematics, its teaching and learning, while engaging in GeoGebra-based mathematical activities within the course as learners? To understand this purpose and answer these questions fully, a PD course was created to offer the participants an opportunity to experience the role of the student in reference to the computer-incorporated context by engaging with mathematical tasks designed in line with the constructivist approach. The GeoGebra software application served as a cognitive and pedagogic tool with which to assist constructivist learning and teaching. This was expected to provide teachers with better theoretical and practical comprehension of mathematics teaching and learning. This chapter reviews the areas of literature pertinent to my research as follows: the nature and description of beliefs and teachers' mathematical beliefs; teachers' experiences with the use of digital tools in mathematics education; and constructivism and its influence on mathematics education.

To cover these points, this chapter is structured into three sections: constructivism and its influence on mathematics education; the importance of teachers' beliefs, including changes in beliefs relative to both mathematics itself, and its practice; and, teachers, Dynamic Geometry System (DGS) and mathematics, also referring to the potential and general features of GeoGebra. To provide some contextual background I will also focus on the concept of knowledge and education in Islamic epistemology and constructivism, since this provides a context for some of the findings of this study. The third section sheds light on research related to the exploration of teachers' mathematical beliefs so as to understand how they learn and change. To support the theoretical points, studies measuring teachers' experiences with technology-based model courses will be included in this review. The studies and research referenced in this chapter formed the foundation

for my own research methodology and also provide a comparative point for discussion of my results.

3.1.2 The Islamic Conceptualisation of Knowledge and Education

The Turkish education system provides an insight into education throughout the Islamic world, and therefore its unique characteristics must be considered, as cultural perspectives and attitudes are intrinsic to the belief systems of the teachers that I wish to explore. The current approach to primary education in Turkey dates to 2005, when the new primary mathematics curriculum was developed by The Turkish Ministry of National Education based on constructivist theory. MONE (2005a) stated that

“There seem to be some important developments in population structure, characteristics of family, social fabric, conceptions of consumptions, political domain, science and technology in Turkey. These developments need to be reflected in the Turkish educational system. The solution lies in the constructivist approach” (p.3).

This constructivism and the teaching and learning approaches it indicates must be understood if we are to accurately consider teachers’ interpretations of tasks throughout this study. In this era of globalisation there is an increasing interest in the Islamic world in the educational theories espoused by western civilizations. Western pedagogy mainly derives from philosophical hypotheses about the nature of learning and knowledge (Ernest, 1991; 1998). Constructivism, with its interpretive epistemological position, is one of the learning theories which have a large number of underlying assumptions (see sub-section 3.2.1). Constructivists have endeavoured to find an answer to the problem of objective truth. Islamic thinkers accept that the whole world is precisely constructed with regard to properties, entities and relationships. It is believed that objective truth exists eternally and independent from humankind and the role of human beings is to seek knowledge as an integral part of the Islamic tradition (Alkanderi, 2001). This view implies that better education is obtained by a motivated pursuit of knowledge; this can be attained through pursuit of knowledge in accordance with a constructivist method.

Islam emphasises the concepts of divine knowledge that is given by God and based on spiritual senses, acquired knowledge that is to be discovered by human beings themselves using their physical faculties and senses (Nasr, 1988). In this case constructivism is supported by the need for the human being to establish a personal

connection between the physical and spiritual knowledge. Therefore, the objective of education is then to uncover the truth via this linking process. Islam regards both concepts to be of vital importance and directs Muslims to go and seek out objective truth. The Holy Quran is regarded as the main source of knowledge, being the key power or force to legitimise, produce, and operationalize truth in society (Nasr, 1988); separating the intellect from those agents who may prevent human beings from engaging in the search for truth (Iqbal, 1932).

According to the Islamic perspective, the aim for acquisition of knowledge requires “the recognition of the proper place of God in the order of being and existence” (Al-Attas, 1990, p. 7). Islam posits a crucial belief that all human beings need to fully understand the aim of their existence and their exact place within the whole system. Knowledge either from God (divine knowledge) or discovered by human beings themselves (acquired knowledge) therefore plays an important part in Muslim life. According to the traditional Islamic approach, God has the knowledge of all hidden things and omniscience surrounds the whole universe. For instance, Al-Khwarismi (780-847), an Islamic mathematician and astronomer, made major contributions to mathematics, with his book about the systematic solution of equations, giving examples to explain the problem of the exact truth about how to calculate the circumference of a circle. This approach was translated by Rosen (cited in Baki, 1992):

“This is an approximation, but not to the exact truth itself; nobody can ascertain the exact truth of this and find the real circumference, except the Omniscient. The best method here given is that you multiply the diameter by three and one seventh, for it is the easiest and quickest. God knows best” (p. 226).

The knowledge of π relies on human presumption and might not address the certain truth. It means God can know His creation as He created it all. In fact, Islamic epistemology has two main sources of knowledge; first, the Holy Quran which is certain and revealed by God, and second, human conjecture. Epistemologically, the truth refers to God Himself. According to this approach the only certain knowledge is in the Holy Quran, all others can be mistaken such as human conjecture. However, this approach does not completely dismiss human conjecture. Many Islamic thinkers argue that human interpretation can also reflect the truth. They give an example as *ijtihad* (a process of interpretation of divine knowledge carried out by qualified Islamic scholars); the result

of any *ijtihad* is generally accepted as human conjecture. Knowledge is considered a science in Islam. The Holy Quran provides knowledge of natural phenomena, history, human psychology, and how to navigate the world daily. Mohd explains this by saying that there is no separation between secular and religious science, either in theology or in their utility in the world (WanMohd, 1989). The definition of education in Islam can be explained by clarifying one Arabic word “*Tarbiyah*” (Halstead, 2004). This term is extensively used by Islamic scholars, and refers to education and development of individual potential and the process of nurturing and assisting the pupil to maturity (ibid). The main purpose of education in Islam is to offer assistance to learners so as to help them to become good people. Halstead (2004) implies that the fundamental aim of education is to guide, as a person cannot obtain her skills and realise her potential automatically. Within this perspective, teachers have been given different responsibilities and roles such as an educator (*murabbi*), a knowledge provider (*mu'allim*) and a trainer of personality (*muaddib*) (Nasr, 1987 cited in Kasim, 2012).

As stated in the previous chapter, the Turkish education system has been very strongly influenced by Islamic traditions because the Ottoman Empire was ruled by Islamic values and principles. Hence, the educational system was based on traditionally teaching practices in the Turkish classrooms where memorisation with understanding has also been privileged as a common approach within traditional Islamic education. Therefore, many of the Turkish primary teachers now believe that the most appropriate method of teaching is the teacher-fronted method because of certain factors (including overcrowded classrooms). Ozen (2006) argued that this type of teaching method often led to students who were passive learners with regards to constructing their own knowledge; and that they were unable to comprehend the concepts. She claimed that effective teachers in the Turkish context could be characterised as those who asked questions during their teaching, because discussion and interaction between teachers and students in the Turkish context typically began with questions which were triggered by the teacher. Therefore, the questioning strategy in the Turkish classroom becomes most important so as to have interaction and discussion between students and the teacher. This belief appears to be in concordance with constructivist ideas of teaching which support the use of questioning techniques to help the teacher to gain access to learners' ideas and existing knowledge.

3.2 Constructivism and Its Influence on Mathematics Education

3.2.1 The Theory of Constructivism

A considerable amount of research has focused on constructivist learning theory; it has become a central element of educational research. Reformers, educators, policy makers and researchers are actively involved in supporting the utilisation of constructivist ideas for creating and implementing new curricula or activities to develop student learning. It should be noted that constructivism is not a single concept, but rests on three fundamental characteristics which are: a set of epistemological beliefs (about the nature of reality), a set of psychological beliefs (about cognition and learning) and a set of educational beliefs (about pedagogy) (Kanselaar, et al., 2000). That is to say, epistemology, in a broad sense, deals with the nature of knowledge, which informs constructivism. Considering mathematical epistemology, there has been much debate about whether mathematics is a process of invention, creating a way to define the world, or a process of discovering truth (Davis and Hersh, 1981). In contrast to the Islamic perspective, constructivists claim that knowledge is not independent of the individual; on the contrary, it takes its final form from the effects of social and cultural values. It is, in fact, not very easy to draw a clear distinction between psychology and epistemology because the relationship between the two is not straightforward enough to do so.

Knowledge is understood from a constructivist perspective to be based on the learner's construction defining their experience, and does not define or imply an objective real world (von Glasersfeld, 1996; Robins, 2005). For constructivists, the core idea of constructivism is that the individual is the source of meaning. This is distinct from the Islamic perspective, wherein knowledge is independent of the individual; that is, the knowledge already exists and is out there, but it is the individual's responsibility to seek it out. Lerman (1993) critiqued the basis of this distinction by offering an alternate viewpoint, derived from an epistemological perspective:

“[k]nowledge is not in the individual's mind, nor ‘out there’ in objects or symbols. Knowledge is as people use it, in its context, as it carries individuals along in it and as it constructs those individuals. Knowledge is fully cultural and social. And so too is what constitutes human consciousness. Communication drives conceptualisation” (p. 23).

From a further perspective regarding the nature of knowledge, Lakotos (1978) emphasised that mathematical knowledge was regarded as quasi-empirical in nature. Thus mathematical knowledge could be fallible and could evolve through communication, inquiry and modification within the community (Ernest, 1998). In this sense, constructivists suggest that there is a variety of theoretical account of truth and knowledge and its development. For Lakotos, when a theory had gained more recognition; there were good reasons for choosing it. He believed that mathematical knowledge was affected by human activity since it was constructed by human beings rather than the independence of a necessary set of rules. Constructivist theorists argue that knowledge should no longer be seen either as right or wrong, but in terms of whether it fits with experience. The only thing that matters is whether the knowledge we build functions adequately in the context it emerges (Bodner, et al, 2001). Therefore, constructivism sees mathematics as a creation of human endeavour, and it is based on learner activity and mathematical knowledge has a social dimension.

Constructivists claim that knowledge cannot be transferred to the individual's brain in a passive way. It can only be made by the person himself and his own experiences. That is, construction of knowledge, concepts, and experiences can be achieved through their recent and previously gained knowledge and experiences. From this point of view, while traditional teaching regards the individual as a 'sponge' that absorbs knowledge, constructivism considers the individual to be like a 'growing tree' (Wheatley, 1991). Therefore, teachers for whom the constructivist philosophy is dominant are likely to use more open-ended questions and expect students to generate new concepts. The constructivist approach argues against direct teaching, suggesting it is a restrictive method, as learners are only exposed to information that is fed to them. Constructivism allows for knowledge to be acquired as a result of interaction between teachers and students, allowing students to draw conclusions from the learning experience.

According to constructivist learning theory, three fundamental assumptions apply:

- Knowledge cannot be passively taken from external sources, but it is actively built on by the cognising subject.
- Understanding appears in the form of adaptation where a person understands a subject by utilising his own experiences and previous knowledge.

- Knowledge is enhanced as a consequence of interaction; the language and the social environment play an important role in such interaction (Kilpatrick, 1987).

If one agrees the first principle only s/he will be regarded as a simple (trivial) constructivist (Jaworski, 1988). If one agrees all principles s/he might be regarded as a radical constructivist (Kilpatrick, 1987). The most significant misconception about constructivism is that constructivism is a teaching method rather than a learning theory (Richardson, 2003); in fact it is a means of describing the world that can be used inform teaching methodologies.

Two types of constructivism have emerged as the most effective (Confrey and Kazak, 2006): cognitive constructivism (Piaget's constructivism) and social constructivism (Vygotsky's constructivism). Therefore it is beneficial here to consider how these theories form the basis of the constructivist methods currently being developed. Piaget argues that learners construct understanding through a process of active involvement and interpretation of experience with their environment, and Vygotsky stresses that the construction of knowledge is a consequence of social interaction and language usage among learners. Both forms are discussed below.

3. 2. 2 Piaget's Constructivism

Piaget, regarded himself as a 'genetic epistemologist', and was interested in how one comes to know things. His main research focused on the process of constructing knowledge considering cognitive development of children. He attempted to investigate a connection concerning a model of the building of knowledge and the judgement of the validity of knowledge, thereby creating a dynamic interpretation of the progress of knowledge.

“I think human knowledge is essentially active. To know is to assimilate reality into a system of transformations. To know is to transform reality in order to understand how a certain state is brought about” (Piaget, 1972: p. 15).

For Piaget, individual knowledge is generated through actions. The world does not present facts passively. The individual gathers knowledge of the world through taking action on it. That is, individuals first infer new experience and events with regards to pre-existing cognitive structures and then existing knowledge is extended and modified by integrating novel and typical experiences. This action helps the individual to change

his/her cognitive structure so as to create a new mental organisation. There are three important notions in Piaget's stages of cognitive development; 'assimilation', 'accommodation', and 'equilibration'. For him, the organisation of these experiences into meaningful structures was labelled as a 'scheme'. This concept helps us to understand patterns of thought and behaviour in our environment. Namely, learning is an active process by which individuals build meanings or scheme in terms of their new experiences. Piaget also claimed that adaptation and organisation is a mechanism in which shifts in cognition and understanding take place. According to Piaget, the construction of knowledge is based on the adaptation process which becomes possible through assimilation and accommodation.

Through the assimilation process, new ideas or information are incorporated with existing information or past experience (existing schema). Assimilation is a kind of adaptation allowing new concepts to emerge. When one is faced with a new object and events, s/he tries to understand the newly received information on the basis of one's existing schemas. To some extent, this process is subjective, since the learner is inclined to modify or extend experience or knowledge so as to consolidate with his/her pre-existing schema. Accommodation is the concept of modifying and adapting pre-existing information based on new information in terms of the existing schema. Through the accommodation process, if one's existing schema is not enough to understand new information or experience s/he faced, existing ideas are more likely to replace or change with regards to received knowledge. In this way, the process of changing schemas is called accommodation. Piaget (1972) defines accommodation as a process of applying general schemes to the particular contents and discusses that if a thing cannot be assimilated, it may be either disregarded by the individual or accommodated through alterations of schemes so that assimilation takes place. In short, accommodation suggests modifying existing schema with regards to new facts and predominant experiences calling for a restructuring of the existing system. Assimilation indicates integrating new concepts or situations into existing schema; these two such concepts assimilation and accommodation are interconnected to one another. In this sense, accommodation is not possible without having relevant prior knowledge to which the individual can assimilate new ideas and information and all accommodation follows assimilation. Knowledge is built through actions in this recursive cycle of assimilation

and accommodation from pre-existing schema to the new. Thus, these two successive processes make adaptation possible throughout life.

Eggen and Kauchak, (2007) stated that the newly developed schema would accord with the pre-existing schema; generate “equilibrium”. If the individual can make sense of the actions around themselves, it is regarded as a state of cognitive balance, harmony and stability. When equilibrium is not accomplished, the individual is confronted with conflicts (disequilibrium). According to Piaget, the underlying force that drives cognitive development is equilibrium. Therefore, equilibrium is essential for the construction of knowledge process (Hills, 2007). Learning does occur if equilibrium is achieved. In cognitive development, the individual is adapting to the environment through assimilation and accommodation in order to reach a dynamic equilibrium process. This process then takes place in individual’s interaction with his/her environment. When the individual encounters conflict during interactions, there are two options for him/her; either s/he does not take into account the problem, or the accommodation process occurs with some alterations. Hence, these experiences allow the individual to promote one’s new knowledge with former knowledge. At this point, one can incrementally and continuously adapt, driving conceptual development.



Figure 3.1: Piaget’s adaptation theory

For Piaget, the socio-conflict theory is primarily applied for informal tasks and deals with disequilibrium, the significant mechanism for the building of new knowledge. Vygotsky has a counter argument against Piaget suggesting that the disequilibrium mechanism is not the only way that individuals accommodate new cognitive structures. For Vygotsky, learning is a kind of social process in which individuals slot themselves into the intellectual life of those around them. Concepts and realities are cooperatively

constructed by a community of practice. Therefore, Vygotsky concentrates on the question of how one individual together with other individuals comes to know culture rather than on the question of how the individual reorganises cognitive schema due to conflict.

3. 2. 3 Vygotsky's Social Constructivism

Vygotsky was a Russian psychologist who put social interaction at the centre of education. Vygotsky, like Piaget, argued that construction of knowledge relied on individuals' own efforts predicated on their previous knowledge, but he regarded social interaction as a central tenet of the individuals' intellectual development.

The social constructivist model accepts that socialisation plays an important role in individuals' learning. That is, social interaction and culture provide the basis for learners' thinking and activities in a given environment. Social interaction influences the individual's cognitive improvement by "explaining reality, transmitting cultural messages and mediating the learning of environmental rules" (Kouzes and Prasseisen, 1995, p.69). From Vygotsky's perspective, culture also catalyses cognitive development, that is, human actions occur in cultural environments and could be incomprehensible outside of these environments (Woolfolk et al., 2008). Thus, culture offers individuals the tools to think and also directs them how to think.

The most important theory, which was introduced by Vygotsky into the world of educational theory, is the idea of "The Zone of Proximal Development (ZPD)" so as to cope with problems of the assessment of pupils' intellectual skills and the evaluation of instructional behaviour. Vygotsky (1978) defined this theory as the "the distance between the actual developmental level as determined through independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). According to his theory, an engagement with more capable peers is an effective way of developing abilities and approaches. In this regard, the term 'scaffolding' is regularly used in the literature instead of ZPD. This term refers to the context that provides appropriate support (questioning and positive interactions) by teachers, or more able peers or the use of technological tools in order for pupils to achieve their potential (Bruner, 1985; Stuyf, 2002). ZPD is about learning, and would be controlled by the individual's stage of development (Palincsar, 1998). With the purpose of understanding this relationship,

it is essential to determine the individual's 'actual and potential development'. Actual development indicated individual's attainment by him/herself without help. Potential development is the maximum level that individuals can accomplish with support (ibid). Hence, communication is an important part of this process (Bruner, 1985). At this juncture, teachers and more skilful peers provide the scaffolding (ibid). Therefore, collaboration becomes an important aspect of learning between pupils and teachers so as to construct knowledge and skill. Furthermore, this scaffolding (assistance) should come through guidance instead of knowledge transmission (Daniels, 2001).

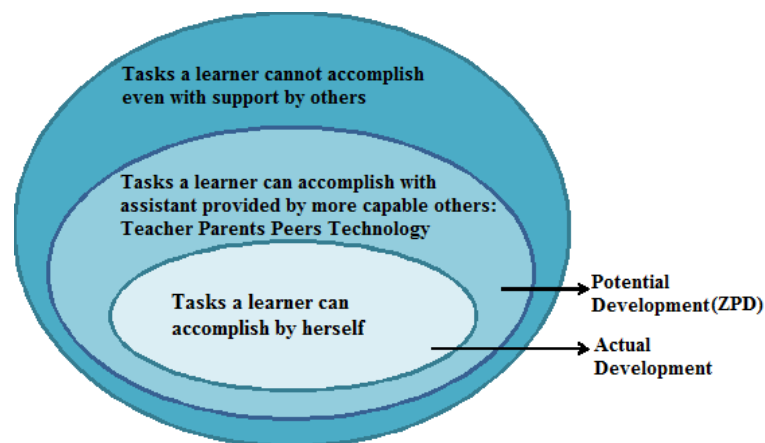


Figure 3.2: Zone of Proximal Development

It can be seen from Figure 3.2 that the scaffolding would occur in the ZPD. According to Vygotsky, it was accepted that the individual had to perform in the ZPD, which is regarded as a social period. However, there are some points that need to be monitored during this period. Some of the most important points are as follows:

- Students' ZPD should be carefully tested.
- Questions, which will be asked to students, should not be above the level of students' understanding.
- Students should solve the problems by external support and using their own existing knowledge.
- Students should develop some new concepts concerning the subject and they should review the topic.

It is argued that if the problem is complex and beyond the understanding of the student s/he will become more reluctant to solve it, so problems should be set at the right level. Moreover, the constructivist approach aims to form a bridge that focuses on the

teacher's pedagogy and mathematics teaching, which relies heavily on the conceptual dimension of mathematics. This conceptual approach is reliant on students using conceptual thinking to make a mathematical definition transform from abstract mathematical thinking to concrete mathematical thinking.

To develop their students' ZPD process, teachers need to consistently design innovative tasks; the ability to achieve this is based in part on the teacher's knowledge and beliefs, and also on the beliefs prevalent outside the teacher, e.g. the school, society, etc. (Nielsen, Barry and Staab, 2007). This research will explore how using GeoGebra-based mathematical activities within the PD course as learners and teachers could alter their beliefs about the teaching approaches that teachers will use with their students.

3. 2. 4 Constructivism in Mathematics Education

Designing a constructivist context for learners to re-evaluate their mathematical ideas

Lerman (1993) argues that constructivism has been overwhelmingly approved within mathematics education research, since it seems to meld with existing teaching strategies. At the root of mathematics teaching in a constructivist context is the teacher's perspective on the role of the learner. From this perspective, the teacher should facilitate the learner to take on a central role in constructing his/her own experience and knowledge rather than in imposing knowledge on him/her. Therefore, the acquisition of mathematical knowledge becomes a learner-based activity rather than a passive activity involving the memorisation and acceptance of an independent body of truths. Thompson (1992) stated:

“Students engage in purposeful activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation” (p. 128).

It has been suggested that when teaching learners should be directed towards ‘mindful activities’ which enable students to articulate their ideas (Adams, 2006) and allow them to construct their knowledge. In this sense, the mathematics teacher's role can be regarded as that of ‘facilitator’ and ‘coach’ for students' learning rather than ‘expert’. From the constructivist perspective, teaching involves designing new environments in which learners' intellectual structures can appear and change (Joyce et al., 2004). Technology allows constructivist teachers of mathematics to design learning

environments that foster interest and promote experientially based understanding acquired by means of collaboration and possibly also through quality social interaction (Pateman and Johnson, 1990). An ideal constructivist context is a “place where learners may work together and support each other as they use a variety of tools and information resources in their guided pursuit of learning goals and problem solving activities” (Wilson, 1996, p. 5). Jaworski’s (1991, 1994) work implies that good mathematics teaching takes place only by questioning, experimenting, discovering, constructing, conjecturing, reflecting, and discussing. To suit the rationales of the present study, technology-based professional learning was created in a context wherein participants could engage with investigational mathematical tasks by means of an interactive process of conjecture, criticism, critical reflection, investigation and collaboration.

The constructivist environment is a place where the teacher considers students’ activities, thus facilitates students’ activities so as to help them make mathematical relationships and patterns, and discuss mathematical meanings, instead of acting as an expert imposing fixed information. In response to von Glaserfeld’s assumptions, Jaworski (1996) implied that “the power of constructivism for mathematics education is encapsulated in this second principle” (p. 2). In this principle, viable mathematical knowledge which fits with experience and evolves through modification and social interaction is based on a learner’s experience of the world. Learners can only know what they have built through new experiences, and can modify with reference to their further experience. This argument can be applied to mathematics teaching: the teacher’s role should be to guide and support students’ creations of viable mathematical notions instead of conveying correct ways of doing mathematics.

The teacher may simplify this process by designing a constructivist environment where the learner works with conceptual problems and involves in a conversation with both peers and teacher. In this situation, rather than just content and prescribed tasks, thinking of students’ activities becomes important in establishing effective mathematical learning environments. The evidence from research on psychology illustrated that learning cannot take place by passive absorption of information only, but instead, in many situations, learners approach each innovative task with previous knowledge, fit in the new knowledge and create their own meaning (Reynolds, 1999). This approach allows learners to establish a link with what they already know with new

ideas. The research that will be conducted and described in the following chapters suggests that technology can be used to create an active experientially based learning environment.

The negotiation of meaning through mathematical communication

Jaworski (1988) suggests how constructivism offers a way of perceiving teaching and learning, illustrating this with an anecdote which includes a dialogue between the teacher and pupils. In this anecdote, the figure looked like a trapezium, but the student saw it as a square. The teacher did not give the formal definition of a square and students might expect the teacher to give right answer, but the teacher lets the boy explain what he understood from this figure and sees if he identified it as a square. It might be possible to acquire a new perspective on what mathematics can offer for learners who both thought the figure as trapezium or as a square. This approach suggests that each learner has her/his own individual perspective on mathematics objects. Although, the teacher may try to communicate mathematically with pupils, it is important that pupils should be taking into consideration their needs and experience which they try to connect with what they do. This anecdote refers to the second principle, and would also be a method that could be suited to a virtual learning environment.

Communication is an important aspect of mathematics instruction. For example, OMOE (2005) stated:

“Mathematical communication is an essential process for learning mathematics because through communication, students reflect upon, clarify and expand their ideas and understanding of mathematical relationships and mathematical arguments” (p. 17).

From constructivist perspective, teachers should initiate discussions in their own classroom and look for understanding students’ mathematical ideas. Kilpatrick (1987) mentioned that linguistic communication becomes an important tool in the process of guiding a student’s learning and construction, not a process for transmitting knowledge. Believing that students are more likely to negotiate mathematical meanings when they share what they think and know about the activity in the classroom. Mathematical meaning varies from one individual to another, and communication relies on the ability to share others’ meaning (Jaworski, 1988). In this sense, mathematical learning

becomes an interactive as well as constructive activity. Social interaction appears to have a crucial role to play in this process since when students work with peers they are inclined to be stimulated to reflect on their ideas which can lead them to revise and modify their ideas.

Constructivist principles call for a subtle change in perspective for the individual who is teaching in the school setting. So that teachers who adopt the constructivist approach, display a shift to teaching by negotiation from teaching by the imposition (Bodner et al., 2001). Therefore, the teacher's role shifts from that of being the authority in classroom to being its facilitator. In a regular classroom, the teacher usually tries to limit students' activities so as to carry out her procedures and routines. Mathematical content introduced by the teacher generally determines the teachers' purposes and plans, and this makes little sense of learning. However, mathematics teaching should be conceptualised as controlling the organisation of the classroom so as to share and develop mathematical meaning. Therefore, such an approach to teaching mathematics would encourage communication and the view that mathematical understanding improves through interaction (Bishop, 1985).

Therefore, mathematics learning and teaching is the negotiation of meaning through social engagement, and challenges the conjecture that meaning is situated in words, practice and objects independently of an interpreter (Jaworski, 1991). She refers to the term "negotiation" which can be defined as a kind of interaction that takes place between teacher and students. This implies that such process is principally essential in the classroom to make mathematical communication possible in the classroom and to support learners' mathematical activity. Teachers and pupils are considered to be 'active meaning makers' who offer contextually based meanings to each other's words and practice as they engage (Booker, 1996, p. 382). On account of this negotiation process, the teacher and students have deep appreciation of each other's roles and responsibilities which involves clarifying and justifying solutions, attempting to comprehend explanations given by others. From the point of view of the present study, group work, negotiation and tasks were the main ideas covered by the PD course. This was grounded on engagement with computer-based investigative mathematical tasks. The PD course involved the researcher and the teacher participants in an exploration of mathematical circumstances to discover how best to promote communication and the

application of new ideas. This was made possible through individual and small group investigation, dialogue and negotiation, all within the context of collaborative enquiry. To illustrate this, the problem below is an example of mathematics project taken from year 8 curriculums which is based on the constructivist approach.

Learning domain: Measuring,

Sub-learning domain: Measuring length,

Skills: Reasoning, relationship, communication, problem solving,

Gains: Solving problems that relate to measuring length,

Problem situation: With the torch in his hand, Ali can illuminate the distance/length of 2.6 m. When he walks 5 steps further from the point where he stands, he can see a rabbit from the reflection of the torch's light. What is the distance/Length between Ali's first position and the place where the rabbit stands?

- What needs to be known to solve this problem?
- Complete the missing information according to your measurements and solve the problem by following the steps outlined below:
 - A. Summarize the question and find information you are given.
 - B. Plan how the solution can be found and explain your strategy.
 - C. Check your solution.
 - D. Arrive at the correct result by discussing the answer with your peers.

3. 2. 5 Summary

This section emphasised the challenge of constructivism by indicating the significance and the practical value of its approach for fostering mathematical learning and teaching in educational context. In brief, the constructivist approach is predicated on the belief that the individual makes her/his knowledge through active involvement in a process of meaning making and knowledge construction and consequently s/he either acquires or is unable to do so. From Piaget's perspective, construction of knowledge is predicated on the adaptation process which becomes possible with the help of assimilation and accommodation. For him, knowledge is a product of interactive accommodative exchanges which take place at all points where the individual experiences are gained.

However, Piaget's constructivism does provide an inadequate theory to explain how new organisations of concepts and new cognitive process are made.

Social constructivism seems to offer a variety of ways to acquire mathematical knowledge. For instance, social constructivism agrees with the idea of mathematical knowledge production through individual interpretation, but highlights the social and cultural dimension of generating mathematical knowledge. It also discusses that mathematics is cooperatively generated by the community of practice and individual interpretations are formed by social engagement (Ernest, 1998). This implies that mathematical knowledge arises from experience, but that the role of social interactions is dominant. Therefore, classrooms should be perceived as a learning context in which students take part not only in individual interpretations, but in a social discourse, including investigation, cooperation and sharing. This reflects an alternative model of teaching contrary to conventional model of teaching which is based on repetition and memorisation. Furthermore, constructivism includes several theories of knowledge acquisition that present a focus for understanding learning, and for analysing why the conventional approach to teaching and learning is not sufficiently successful. They oblige us to reject the notion that teachers can convey knowledge stressing that by students needs to view mathematics as a decision making process (Jaworski, 1988). In accordance with this view, constructivists stress that the teacher should facilitate the learner to take on a central role in constructing his/her own experience through communication, reasoning and problem solving.

3. 3 The Importance of Teachers' Beliefs

3. 3. 1 Introduction

In recent years, there has been increasing attention given to the role of teachers in the teaching and learning process. Teachers often take the responsibility for creating and designing the learning environment and thus make decisions about what and how to teach in classroom instead of relying only on curriculum guidelines. There is an important determinant affecting the role of teachers in the classroom; this is their personal beliefs (Nespor, 1987). Thus, many scholars have made great efforts to understand and explore the beliefs informing teachers' pedagogical decisions and classroom practice (e.g. Cross, 2009; Liljedahl, 2010; Rolka, Rösken and Liljedahl, 2007; Stipek et al., 2001; Swan and Swain, 2010; Wilson and Cooney, 2002). Teachers'

cognitions, including their knowledge, beliefs and attitudes have become an important focus for teacher effectiveness research and become more important key constructs worthy of investigation. Wilson and Cooney (2002) pointed out that “what teachers believe is a significant determiner of what gets taught, how it gets taught, and what gets learned in the classroom” (p. 128). Moreover, Chapman (2002) considered mathematics teachers’ beliefs to be decisive parameter for developing their teaching practice.

Given the recognition of the importance of studying teachers’ beliefs, Wilson and Cooney (2002) emphasise that the role of teachers is crucial because changing or reforming mathematics education mainly depends on them. That is, before implementing reform ideas and efforts regarding changing teaching practice in a managerial way, teachers should be aware of underlying reform initiative in mathematics education. Unsurprisingly, many teachers behaviour is not contingent on the latest pedagogical activities but is governed by their own beliefs when attempting to integrate the curriculum in their practice (Handal and Herrington, 2003). In the context of this research, this would call for substantial shift in the teaching practices of most Turkish mathematics teachers in order to actualise the recent reform movement in Turkey. Therefore, we must consider when answering sub-research questions. These beliefs may vary generationally and in terms of classroom experience. There is also a need for better understanding of what type of beliefs teachers have and how change in their beliefs is demanded for effective teacher reform (Richardson, 1990). The purpose of the next sub-section is to provide definitions of beliefs, theoretical aspects of beliefs, and shifts in beliefs.

3. 3. 2 Beliefs

This sub-section of this chapter considers how we understand the term ‘belief’ as it relates to the educational setting. As Beswick (2005) mentions, although the term ‘belief’ is popular in educational literature today, many researchers acknowledge that there has been no consensus/unanimous definition. Goldin et al (2009) recently emphasised that belief systems comprise very unsettled constructs; therefore, there is a lack of agreement over a precise and distinctive definition. In fact, some researchers agree that the definition of mathematical beliefs is best interpreted as teachers’ personal philosophies about the subject matter as well as its learning and teaching (Ernest, 1989;

Thompson, 1992). Hence, it necessary that the present study indicates those characteristics of beliefs that are frequently discussed in the literature.

Beliefs act as an important factor in the professional development of teachers and hence, the construction of beliefs has been explored in a number of different ways. Differences also exist in the use of terminology. Namely, literature involving terms such as ‘conception’, ‘view’, ‘knowledge’, ‘attitudes’, ‘values’, ‘perspective’, ‘dispositions’, etc., have regularly been applied almost interchangeably with the construct of ‘belief’ (Pajares, 1992). For example, Thompson (1992) addressed both ‘beliefs’ and ‘conceptions’ in her study and her definition of conceptions encompasses both beliefs and knowledge. She described teachers’ conceptions as a more general mental structure, encompassing both beliefs and any relevant knowledge including meanings, concepts, propositions, rules, mental images and preferences (p. 130). Cross (2009) described beliefs as equally “embodied conscious and unconscious ideas and thoughts about oneself, the world, and one’s position in it, developed through membership in various social groups; these ideas are considered by the individual to be true” (p. 326); whereas, Sigel (1985) described beliefs as “mental constructions of experience” (cited in Pajares, 1992: p. 351).

The relationship between knowledge and beliefs has become a controversial issue in educational inquiry, especially as knowledge alone is insufficient to explain and clarify the differences among teachers in the classroom (Ernest, 1989; Speer, 2005). One of the main reasons for regarding beliefs as a complex and messy construct emerged from a discussion of the absence of definitions and the differentiation of knowledge and beliefs (Pajares, 1992). Much has been said about the distinction between knowledge and beliefs. For instance, Philipp (2007) described knowledge as a “belief held with certainty” (p. 259). Beliefs differ from knowledge; they can be carried with various degrees of conviction as well as not necessarily being consensual (ibid). In addition, belief systems do not call for common agreement concerning trustworthiness and suitability (Nespor, 1987). Beliefs do not even demand internal validity within a belief system (ibid). This is what distinguishes beliefs from knowledge.

Other researchers have tended to argue for the equivalence (or close connection) of beliefs and knowledge. For instance, Scheffler (1965) argued that “the close connection that exists between beliefs and knowledge; distinguishing between them is fuzzy” (cited

in Thompson, 1992, p. 129). Likewise, Beswick (2012) claims that “beliefs are taken to be indistinguishable from knowledge” (p. 128). Conceptions concerning mathematical content and instruction naturally consist of mathematics knowledge for teaching (Ball et al., 2008). From this perspective, Beswick (2011) interprets beliefs as formed in the way that knowledge is formed.

Furthermore, some researchers have argued that it is not essential to uniformly define constructs such as ‘knowledge’, ‘conception’, ‘beliefs’ to move the field forward. For instance, Philipp (2007) argues that definitions of the constructs such as teachers’ beliefs, knowledge and practices change, while different researchers benefit from different research methodologies. In the context of this research, beliefs are considered as aspects of conceptual knowledge, in line with Beswick’s (2011) definition of beliefs as what individuals believe to be true about mathematics, frequently founded upon a person’s own experiences as a learner of mathematics. The above terms were reviewed from the experiential context stemming from their exposure through the use of GeoGebra software application in the mathematics education, set within the social-cultural setting of Turkish primary mathematics teachers. In view of the flexibility of these terms, I employ the terms such as ‘belief’, ‘knowledge’, ‘conception’, and ‘view’ interchangeably in the context of this study, since the distinction between words may not be very important (Thompson, 1992).

Belief systems

To understand shifts in belief in response to an intervention, it is crucial to see each participant as an individual with a unique belief system. In order to fully comprehend the notion of belief, many scholars have attempted to explore the notion of a belief system. A combination of beliefs refers to a belief system. Shahvarani and Savizi (2007) mentioned that the range of teachers’ beliefs is wide because each individual possesses different views about syllabus, their own practice, pupils’ knowledge, and so on, concerning mathematics. In fact, belief systems are more complex than belief. Green (1971) suggests that belief systems have offered a convenient structure for exploring and understanding how one’s beliefs are accommodated. This system consists of three dimensions and has been important in clarifying many research findings on teachers’ beliefs (e.g. Cross, 2009): quasi-logicalness, psychological strength and cluster structure.

Green's (1971) first dimension (quasi-logical) is related the structure of beliefs that identifies how beliefs are organised. This organisation is regarded as a quasi-logical structure since particular beliefs may be regarded as either primary or derivative beliefs. The subsequent is the 'core-peripheral' dimension which deals with psychological strength of a belief and also how beliefs are organised. That is, this particular dimension suggests that beliefs can be either peripheral, which indicates less strongly held, or core, which refers to beliefs which are strongly held and difficult to shift. The cluster structure manages beliefs according to how they are clustered. This dimension refers to the notion that beliefs usually rely on other beliefs and that they are formed into different bundles. Aguirre and Speer (2000) refer to this as the concept of 'belief bundles' which they describe as "a particular manifestation of certain beliefs at a particular time" (p. 333). As stated by Green (1971), "beliefs are held in clusters", with a variety of belief bundles existing within a belief system of an individual (p. 48). Clustering can take place when beliefs emerge in differing environments. For instance, beliefs about learning derived from a person's experience of learning as a school student and their beliefs about learning that are shaped in the context of professional development programs could be present in different bundles. The belief bundles protect against mismatches or contradictions within beliefs. The three dimensions above are considered as crucial parameters in understanding the process of changes in beliefs. Green (1971) notes that teaching itself challenges belief systems, by affecting how they form and modifying existing ones. The susceptibility of a student to such modification is responsible in part for their ability to acquire new information, assess and adopt new ideas and to connect novel experiences with previous knowledge (p. 48).

Mismatches between curriculum objectives and teachers' belief system are one of the factors that affect present curriculum reform in mathematics education. Additionally, Hollingsworth (1989) pointed out that the style a teacher uses to employ externally imposed innovative approaches in her class links with whether teachers' beliefs are aligned with the intentionally innovative approaches. That is to say, the teacher's role becomes more critical especially when teachers are expected to introduce innovations in educational system. When the teacher does not believe that innovations can benefit his or her students and classroom practice, it is doubtful that the effort put into introducing innovations in their working environment will yield positive outcomes. According to this emphasis on individualism in the literature, it is supposed that the proposition to use

the GeoGebra system in the classroom to support curriculum change and an inclination towards constructivist pedagogy is likely to receive mixed responses from teachers.

3. 3. 3 Shifts in Beliefs

To answer the research questions in this study an understanding of what constitutes and also alters teachers' beliefs systems is essential. Compatible with a constructivist perspective, beliefs are considered as true (Beswick, 2011), and can be independent of knowledge only in regard to the degree of consensus they attract (Guba and Lincoln, 1989). Ernest (1989) believed that to create meaningful and effective shifts in teacher's beliefs, it highly important to identify, understand and challenge individual philosophies held in reference to mathematics education (p.98). Teachers' beliefs "seem to be manifestations of unconsciously held views of expressions of verbal commitments to abstract ideas that may be thought of as part of a general ideology of teaching" (Thompson 1984, p. 112). They echo how an educator actualises his/her position in the classroom, developed during school times, incidents or actions in the past and experiences in different social groups. They emerge during interaction and often are made stronger by the existing tradition of the educational system (Nespor, 1987). For the most part, teachers' beliefs about teaching are derived from their personal experiences as students and later through information they gather on teacher education courses and through the observation of other teachers (Lortie, 1975). It is these beliefs that regularly shape the groundwork on which each teacher will ultimately construct their own teaching as mathematics teachers (Skott, 2001). McLeod (1992) concluded that beliefs were inclined to grow incrementally and that cultural elements are important in facilitating their development. Once beliefs are fully formed and accepted, it becomes problematic to engineer a shift without intentionally challenging them (Pajares, 1992). In fact, Green (1971) notes that one of the roles of professional development courses is to change those beliefs and misconceptions that might otherwise hinder effective mathematics practice.

Swan (2006) illustrates that any attempt to develop mathematical teaching practices of teachers must pay attention to the beliefs of mathematics teachers and to shifts in those beliefs. Although constructivist teaching ideas were recently introduced to the Turkish educational mainstream (Aksit, 2007), teachers' previous learning experiences with mathematics as pupils and teachers were based on traditional approaches. This creates

and maintains their traditional beliefs about mathematics teaching and learning. Yet those changes were accompanied by an autocratic approach which disregarded the teaching practice and beliefs of teachers (Norton, et al., 2002) and the indispensable modifications were essential to the approved innovation (Perry et al., 1999). However, it takes time to alter a teacher's long-held, conceptions since changing beliefs is not simple, but rather it is a challenging process (Chapman, 2002). However, Ertmer (2005) notes that the substantial shifts in conceptions could take place through experience. Furthermore, change does occur under such situations where one confronts novel information and experiences that oppose deeply-held beliefs (Philippou and Christou, 2002). It is necessary to change, or at least take into consideration, teachers' beliefs so as to implement curriculum changes. Hence, teachers' beliefs with regard to mathematics teaching practice and the potential shift in their beliefs constitute the focus of the present study.

The common approach of many research studies concentrates on the relationship between the shift in beliefs and shift in practices. That is, a shift in one's beliefs *precedes* a shift in one's practice (Chapman, 2002). In other words, changes in what teachers do are contingent on changes in their beliefs. This view is supported by Ernest (1989) who claims that "[t]eaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change" (p. 249). However, some studies have revealed that beliefs may change without necessarily altering practice. They discussed that this is a significant but complex phenomenon (Wilson and Cooney, 2002; Hart, 2002). There seems to be a cyclical process between shifting practice and shifting beliefs; wherever one begins they influence each other (Lerman, 2002). Foss and Kleinsasser (1996) also argue that teachers' beliefs and their practices exist in a 'symbiotic relationship'; that is, beliefs are both formed by and form ensuing interactions. They concluded that the link between beliefs and practice is exposed as more complicated than discussed earlier in the literature.

In order to be successful in curriculum reform, it is important to pay attention to teachers' underlying beliefs, and to challenge these through professional development if they do not align with the theoretical basis of the reform. In fact, the shift in teaching practice does not equate to completely abandoning beliefs, but of incrementally replacing them with more relevant beliefs (Nespor, 1987). Beliefs come into existence

through personal experience (*ibid*). In recognition of this, Guskey (2002) argued that if beliefs are personal and emerge from personal experience, the shift in beliefs may be catalysed through new experience. This could be achieved by introducing new practices through professional development. This would provide an opportunity with many teachers to obtain new practical, specific, and concrete experience (Guskey, 1986). The main idea of professional development of teachers is to increase the quality of education by providing an opportunity for teachers to foster their skills and knowledge. In order to implement new educational changes in Turkish schools, the current in-service teachers programs need to be improved since the professional development of teachers, in particular the development of teaching methods, are vital for the successful integration of changes in schools. The implication for professional development of teachers is crucial at this point: when a teacher is supported to internalise new methods to her teaching and learning, the individual's professed beliefs may also change as her confidence is enhanced.

A common method for producing a shift in teachers' beliefs is that teachers need to participate in an environment where they can interact with mathematics and its pedagogy as a learner from constructivist perspective (Ball, 1988). Exposing teachers to new pedagogical approaches as learners is a way to confront and change their beliefs. By providing teachers with alternative models for teaching, through a professional development program, means teachers can be inspired to think about teaching and learning more deeply and critically. This idea corroborated with von Glasersfeld (1993) who writes that "if one succeeds in getting teachers to make a serious effort to apply some of the constructivist methodology, even if they do not believe in it, they become enthralled after five or six weeks" (p. 29). Within the professional development setting, teachers need to be provided with on-going support, intervention, collaboration as well as opportunities for critical reflection (Pennington, 1995). A number of authors found that reflection takes as a central role in the growth of teachers and teacher development (Jaworski, 2003; Richards, 2004). For the purposes of the present study, teachers need to experience directly working on particular innovative and investigational tasks so as to accommodate change. This research will build on those studies that take teacher education as a basis for educational reform. Therefore, a collaborative professional learning environment was established in this research that enable teachers to become responsible for their own learning. Using artefacts in this environment could act as

catalyst for modifying teachers' mathematical beliefs by providing opportunities for them to experience the role of the learner, and could also allow them to reflect on and change their mathematical beliefs. The potential of technology in teaching and learning will be explored in the next section.

In this study, learning was considered according to a constructivist approach, which encouraged participating teachers to enhance their knowledge and change their beliefs. In fact, a model (conceptual change) was described by Appleton (1997) who addressed constructivist theory (relative to Piagetian language) to determine what occurs when learners, in this case teachers, encounter new ideas and experiences. According to him, there are three possibilities namely 'identical fit', 'approximate fit' and 'incomplete fit'. In the case of identical fit, new information is compatible with prior information; however, this does not mean that learners' accounts are correct. Regarding the possibility of approximate fit, new information seems to be approximately compatible with prior information. However, learners need to clarify the details of new information. Hence, it is important to obtain further details and share new ideas. However, new information is accepted without giving up previous information and the learner may make only modest shifts to her knowledge. In regards to the possibility of incomplete fit, new information conflicts with prior information. In such a situation, the feelings of learners, such as dissonance and disappointment play a crucial role in dealing with conflict. In order for new learning to take place a learner is required to dissatisfy with previous ideas. In sum, cognitive conflict is the main point that all three possibilities share when learners encounter new information. Rolka et al. (2007) believe that this mechanism can also be used as a decisive tool supporting a context for belief change. The constructivist approach appears predominantly sufficient for challenging mathematical beliefs and instigating a feeling of dissonance, thus to instigate 'an incomplete fit'. Teachers will experience a personal discrepancy and will deal with it in a fruitful way during PD course. The next sub-section discusses the classifications which include teachers' beliefs about the nature of mathematics, its teaching and learning and their relationship with practice.

3. 3. 4 Teachers' Beliefs about the Nature of Mathematics

"The issue, then, is not, what is the best way to teach? But, what is mathematics really all about?" (Hersh, 1986, p. 13)

In order to answer above question, Hersh (1986) gave the following response: “mathematics copes with ideas, not marks made with pencils or chalk, not physical triangles or physical sets” (p.22). The notion of his beliefs about the subject matter is that ‘knowing mathematics is making mathematics’, this means its creative activities and processes. In an attempt to comprehend how teachers are changed through their learning experience, it is essential to study teachers’ personal theories about mathematics and label them. Accessing teachers’ beliefs is a challenging task (Pajares, 1992) which is echoed in the various vocabularies used to depict the teacher’s relations with the nature of mathematics (Cross, 2009). For the objective of the current study, it is essential to comprehend the distinctions between different categorisations of mathematical beliefs. An individual’s personal philosophies about the nature of mathematics can be classified as either absolutist-oriented or fallibilist-oriented (Lerman, 1986; Phillips, 2000).

According to absolutist beliefs, mathematics is a totally absolute, rigid, indisputable and objective. This authoritative philosophy of mathematics suggests how mathematics should be recognised (Ernest, 1992). In this regard, mathematics teachers hold the belief that mathematics consists of an independent body of knowledge waiting to be discovered (Lerman, 1986). From this perspective, in order to acquire knowledge, attention should be given to product rather than process (Ernest, 1989). According to fallibilist-oriented beliefs, mathematics is seen as uncertain, incomplete, fallible, and modifiable (Ernest, 1992). In this sense, knowledge acquisition is based on the process more than the product. Individuals construct their own unique perceptions of mathematics. Therefore, mathematics is constructed by human action and understanding of the world, and as such is constantly developing and changing.

Lerman (1986) conducted a study with mathematics teachers to discover if their beliefs about the nature of mathematics influenced their practice, also, as a result he identified two alternative beliefs of mathematics; absolutist and fallibilist. Lerman went even further and argued that these personal theories related to the philosophies of Euclidean and Quasi-empirical. Euclidean mathematics is based on absolute foundations whereas Quasi-Empirical is based on the theory that mathematics is uncertain and changeable. Quasi-empiricism proposes that mathematics is not a hard science in so much as there are no discoveries to be made; rather mathematics is a human creation constantly

developing and changing, open to revision and challenge. Lerman came to the conclusion that it was not clear whether studying these traditional methods made any sufficient contribution to the teachers' understanding or teaching of mathematics. Thompson (1992) identified two concepts: firstly, some teachers carried the belief that mathematics is a discipline characterised by algorithms and objectively right answers. On the contrary, other teachers have seen mathematics as an intellectual action, a social construction including conjectures, proofs, rejections, and its consequences are subject to an open change and validity and should be reviewed with regard to a social and cultural environment. The first concept reflects 'absolutist-oriented' beliefs about mathematics (Lerman, 1986) which is described as a "paradigm of knowledge, certain, absolute, value-free and abstract, with its connection to the real world perhaps of a Platonic nature" (p. 54). The second concept echoes 'fallibilist-oriented' beliefs about mathematics which see mathematics as relying on conjecture, proof and reflections and said that certainty is not absolute, thus the emphasis on the practice and the rebuilding of mathematical knowledge.

Other categorisations have been driven by different perspectives. Three types of beliefs about the nature of mathematics: mathematics as 'training the mind with its logic'; and as 'a tool'; and as 'a criterion for selection' was identified among French, German and English teachers by Pepin (1999). In this research, German and French teachers regarded logic as the primary element of mathematics while admitting the "transcendent nature of mathematics (training of the mind with its logic)" (p. 139). Likewise, English teachers viewed the nature of mathematics as pure logic. They also thought that logic and reasoning in mathematics would assist in the development of logical thinking which led them to view mathematics as a 'tool' or utensil. Some French teachers saw mathematics as a tool in science and others considered mathematics as a criterion to find a job ('a criterion for selection'). Furthermore, Ernest (1989) distinguished three main philosophical beliefs of mathematics among teachers: 'the Problem-solving view', the 'Platonist view' and the 'Instrumentalist view'. The first view of mathematics is seen as a process of human endeavour; a continuous process of inquiry, always open to revision. From Platonist perspective, mathematics is absolute and unified body of knowledge and truth which are out there waiting to be discovered and not created. This view concentrates on mathematical content focused on the basis of the student having an understanding of mathematical principles and procedures underlying the content. The

last view of mathematics (Ernest, 1989) proposes that mathematics is composed of useful and largely unrelated collection of truths, operations and abilities that are to be applied for the attainment of a particular goal.

Parallels can be drawn between Lerman's and Ernest's classification of beliefs about the nature of mathematics. Beliefs described in the present study as absolutist-oriented are paralleled with Ernest's Platonist beliefs and beliefs described as fallibilist-oriented are paralleled with Ernest's problem-solving beliefs. Among other beliefs of mathematics, absolutist and fallibilist-oriented beliefs about the nature of mathematics are discerned due to their occurrence in the teaching of mathematics (Roulet, 1998; Thompson, 1992). In short, to realise the objectives of the current study teachers' beliefs about mathematical knowledge will be conceptualised to consider the nature of the mathematics continuum. Individual teachers will be labelled as holding absolutist-oriented beliefs of mathematics and of believing that mathematics is viewed as found, absolute and unchanging, as well as simplistic and isolated. In contrast, individual teachers will be labelled as holding fallibilist-oriented beliefs of mathematics and as believing that mathematics is viewed as it is constructed; i.e. evolving, complex and integrated.

3.3.5 Teachers' Beliefs about Mathematics Learning and Teaching

As noted earlier, teachers' knowledge of is not adequate in itself to explain the differences among teachers that arise in the classroom, beliefs clarify some of the differences that arise between those who are teaching mathematics (Pajares, 1992). As well as the discussion above about a variety of beliefs concerning the subject matter, there seems to be a link between teachers' beliefs about the nature of mathematics and their beliefs about pedagogy (Kuh and Ball, 1986; Speer, 2005). That is, beliefs about how a student learns and about how a teacher teaches in classroom are related to beliefs about the subject matter. Golafshani and Ross (2006) discuss how differences in teachers' beliefs about mathematics may provide evidence for the differences in their beliefs about mathematics teaching and that their beliefs about mathematics teaching probably may reflect their beliefs about mathematics learning. They stressed that it is problematic to think of teaching approaches without some underlying theory of student learning because there appears to be a coherent and natural link between the two. This is supported by Chan and Elliott (2004), who highlight the view that beliefs about

teaching prescribe a role for teachers and learners, and the meaning of teaching. Therefore, beliefs about teaching and learning, as they appear in the literature, can be classified in terms of traditional and constructivist views (Ernest, 1991). Thus, this study will consider this angle with reference to other studies of this topic, which appear below.

Ernest (1989) argued that among a number of primary aspects, there are three most influential factor that affect the ways of teaching mathematics which are described as follows:“(i) the teacher’s belief systems regarding mathematics and its teaching and learning; (ii) the social context of the teaching circumstances including limitations, opportunities; (iii) the teacher’s level of thought process and reflection” (p. 249). Ernest identified teachers’ beliefs about nature and the significance of mathematics as one of the main influences on their teaching practice; the other was their mental models of learning and teaching mathematics.

As far as teachers’ general beliefs about teaching and learning are concerned, Kuhs and Ball (1986) summarised four models: “learner-focused”, “content-focused with emphasis on conceptual understanding”, “content-focused with an emphasis on performance” and “classroom-focused”. The “learner-focused” model concentrates on one’s own creation of mathematical knowledge through active participation in making mathematics which corresponds to the constructivist view of learning (von Glasersfield, 1987). From this perspective, the teacher’s role is that of facilitator of student learning, asking purposeful questions for investigation and challenging students to think. The second model, “content-focused with emphasis on conceptual understanding”, links to idea that teaching is determined by the content itself but underscores the logical relations among mathematical concepts. Differently from the previous model where students’ thoughts and interests are the main issue, this model might be rooted in teacher’s traditional beliefs. The third model, “Content-focused with an emphasis on performance,” is similar to the previous one in that highlights mathematical content but stresses student performance and mastery of rules and procedures. From this model, the teacher’s role is viewed as a demonstrator who provides ready-made mathematical knowledge and presents procedures to enable students’ learning by imitating these exercises through practice. This model can be linked naturally to the traditional view of mathematics teaching. The last model, “classroom-focused”, centres on effective

organisation of classroom activities, does not concentrate on content or learning, and so does not reflect any particular beliefs about the subject matter.

In surveying 249 secondary teachers, Perry et al. (1999) identified two alternative beliefs of mathematics teaching and learning which they named the “transmission view (traditional view)” and “constructivist view”. Teachers with the transmission view focussed on verification of knowledge in which memorisation of rules and procedures are important. In this sense, mathematics teaching meant transmitting the correct information to students. This group was higher in number than the constructivist profile, where teachers consider that students are able to build their own mathematical knowledge through negotiation of meaning. Teachers holding a constructivist view of learning believe that solutions to new tasks could be investigated by learners.

To summarise, according to traditional beliefs about teaching and learning, individuals hold a belief that teaching includes telling, or providing clear, step-by step demonstrations of mathematical procedures, and that pupils learn by watching and listening to a teachers’ demonstrations and practicing them. The knowledge of mathematics is transferred directly from authority to novice by lecturing and requiring that all students work on the same kind of tasks (Chan and Elliott, 2004). According to constructivist beliefs, individuals hold a belief that teaching is a provision and facilitation of knowledge acquisition (ibid). The rationale of this approach offers enough opportunities for learners to engage in the critical thinking process, including problem solving, communication, and making connections. From this perspective, teachers see their role as facilitators, encouraging learners to share their knowledge with others and to actively build on their own knowledge by giving them sufficient opportunity to understand this independently. For more information, a constructivist perspective can be found in section 3.2.

Mathematics can be taught by targeting meaning-making broadly through a creative and exploratory process that leads to comprehension and unification of knowledge. Alternatively methods can be narrow, focused on instruction and instrumental success, emphasising basic skills and accurate responses (Ernest, 1989). Chan and Elliott (2004) highlighted that teachers’ pedagogical decisions about teaching in the classroom are underscored partly by their beliefs about teaching and learning. Therefore, when the interconnectedness of teaching and learning beliefs is regarded, they are then classified

in terms of the traditional and constructivist views, as in the present research. In Turkey, recent reform to the mathematics curriculum has clearly shown support for incorporating learner-based activities, and constructivist approaches in its classrooms.

The relationships between teachers' beliefs and their teaching practice

The conjecture that teachers' beliefs might affect their pedagogical decisions and so teaching practice has been investigated by a considerable number of researchers to examine this relationship (Cross, 2009; Chapman, 2002; Stipek et al., 2001; Wilson and Cooney, 2002). Thompson (1992) states that the connection between teachers' beliefs about subject matter, pedagogy and enacted beliefs is not a straightforward cause-effect correlation but a complex and dialectical one, while a number of studies about beliefs imply that there is a link between teachers' beliefs and their classroom practice, causality is difficult to elucidate in this link.

Given the recognition of the importance of the teachers' role in classroom, some research has been conducted on exploring descriptively mathematics teachers' beliefs about mathematics, and their connection with intended or enacted mathematics teaching. While some studies have identified consistencies amongst beliefs and enacted beliefs, others have described discrepancies. Roulet (1998) and Ernest (1989) noted that teachers, who see mathematics as certain and fixed (absolutist), then teach utilising traditional methods based on traditional beliefs about teaching and learning. Teachers who see mathematics as changeable (fallibilist) teach utilising constructivist methods and hold constructivist beliefs about teaching and learning. Likewise, Yadav and Koehler (2007) found that in some cases, although some teachers continued to hold fallibilist beliefs while teaching employing a constructivist approach this occurs less frequently. The fact is, there is more of a connection among absolutist-oriented beliefs and traditional teaching than fallibilist-oriented beliefs regarding mathematics and constructivist pedagogy. The reason behind this might be that traditional approaches to teaching and learning have dominated longer than constructivist pedagogy.

For instance, Cross (2009) conducted a collective case study to examine five in-service teachers' belief structures, and reported on an exploration of the correlation between mathematics teachers' beliefs and their enacted beliefs. Some of the teachers studied by Cross (Ms Reid, Mr Henry and Mr Brown) appeared to believe that mathematics is a set of rules, concepts, and procedures that can be applied to solve mathematical problems in

order to get correct answer. They put little emphasis on intellectual and problem solving skills and more on practice. For them, the nature of mathematics was certain, consisted of established body of ideas that was inflexible and absolute and their classroom practice replicated these conceptions. They believed that the role of the teacher is as a "knowledge provider" whose responsibilities involve presenting procedures and algorithms to enable student's learning by imitating these through practice, and hence, mathematical understanding arose from practicing these procedures. For these participants, this can only be achieved through repeated practice and memorisation of these procedures and content. The second teacher in Cross' study, Mr Simpson, held beliefs about mathematics that tend to vary substantially from the other teachers and saw mathematics as a thinking and a powerful tool for solving problems, prioritising meaning making and finding appropriate methods to reach the certain results. He also held the belief that mathematics is a social construction. Mr Simpson described his role as the one responsible for creating tasks that promote knowledge construction. He felt more that effective learning occurs when pupils are enabled to exploit various routes in solving problems, with the teacher acting as a coach who allows the students to explore maths with their own intuitions and who creates classroom activities that focus on constructive sense-making. The last participant, Ms Jones, believed that mathematics was a fixed body of knowledge that originated in numbers. At the same time, she also viewed mathematics as developing a critical perspective on problem solving. In a sense, she believed that the teacher's role is to facilitate students' learning and encourage, stimulate, and support them, but simultaneously believed that teachers' priority is to provide knowledge and show students how to do with mathematics. Like the rest of the teachers, her beliefs about pedagogy pertained to her beliefs of mathematics as a discipline. The explanation for Ms Jones's case is that the teacher's beliefs of mathematics might contain more than one belief – even apparently conflicting beliefs. As mentioned earlier, the clustering quality of belief system may help clarify the occurrence of conflicting beliefs. Cross (2009) concludes that "there was greater alignment than misalignment between teachers' mathematics-related beliefs and their instructional practices" (p. 341).

Thompson (1984) adopted a case study approach to explore three in-service mathematics teachers' beliefs about mathematics and the teaching of mathematics. She observed teachers teaching secondary school mathematics and interviewed them so as to

elucidate the connection between teachers' stated beliefs about mathematics and events that occurred during their teaching. A teacher named Kay in Thompson's study, carried a problem solving view of mathematics, based on the belief that effective learning takes place when the learner investigates mathematics through social interaction with peers in the class, with the teacher acting as a guide who allows the learner to discover mathematics with conjectures. In contrast, Jeanne's stated beliefs and teaching practice was consonant with a Platonist view. She believes that the teacher's role is a "director" who provides mathematical content in clear, logical, and precise manner. Though the third teacher, Lynn, expressed somewhat inconsistent views, it was understood from her teaching practice and from her beliefs that she held an instrumental view of the subject. This participant believes that the teacher is to be the transmitter of mathematical knowledge with very little involvement of the students. The important conclusion reached by Thompson (1984) from exploration of these case studies was that the teachers' beliefs and conceptions about mathematics were reflected in their teaching practice. This suggests that what teachers believe about mathematics influences their teaching practices in a rather direct way. Lerman (1986) revealed that there was a correlation between a teachers' beliefs about mathematics and their beliefs about mathematics teaching, although not as strongly as Thompson (1984).

On the other hand, some researchers have discovered a misalignment amongst educators' mathematics-related beliefs and their teaching or beliefs about teaching mathematics. This inconsistency is described in the literature, as teachers uphold absolutist-oriented beliefs about mathematics while applying constructivist methods related to educational reform or when teachers uphold fallibilist beliefs about mathematics despite applying traditional methods. Liljedahl, Rösken, and Rolka (2006) conducted a study with 39 prospective teachers, about mathematical beliefs held, during a method course. The aim of their study was to understand if a shift in beliefs occurred as a result of participating in the course. The researchers illustrated an inconsistency among seven of the participants' beliefs about mathematics and their intended beliefs as reported after the course sessions. The course participants sustained their preference for a rule-based (absolutist) belief system regardless of the course; but their beliefs about how mathematics should be taught transferred from a traditional approach to a constructivist approach. The remaining participants showed consistency in their beliefs at the end of the course. Ernest (1989) stressed that even though many teachers have an

absolutist perspective towards mathematics and the elements in absolutist (instrumentalist and Platonist) and fallibilist perspectives differ remarkably, teachers are inclined to use the elements of each perspectives in their teaching practices. Likewise, Chan and Elliott (2004) concluded that those of the participants in their research who considered absolutist oriented-beliefs, advocated both traditional and constructivist teaching methods.

Andrews and Hatch (1999) observed some inconsistencies among the conceptions possessed by teachers. For them, the contradictory views can be held in isolation of one another. This inconsistency was also supported by Lerman's (1986) study. It showed that teachers with differing beliefs about mathematics showed similar classroom behaviours as a result of contextual constraints and demands. Lerman considered particularly the school context as significant in affecting the way that teachers act in their teaching practices. This idea is supported by Thompson (1984) who mentioned that the connection between beliefs and practice is weakening since teachers have to work under some external constraints such as school conditions, busy schedules, time limitations and lack of materials, etc. In most cases, such discrepancy can be explained through some social factors, including a lack of time, materials and losing classroom control, all of which can be affected by parental and administrative pressures to follow a traditionally orientated teaching approach (Handal, 2003). Therefore, before beginning a study of teachers' beliefs some of the implications of this on my study's design need to be highlighted. For example, examination of teachers' mathematical beliefs should also incorporate consideration of the 'belief system' in which they are operating, a system described in detail in section 3.1.2.

3. 3. 6 Summary

The literature has shown that researching teachers' beliefs is an important part of understanding their actions in classroom. This supports the introduction of "teachers' beliefs" as a central issue of concern in this study. While some studies have explored teachers' mathematical beliefs descriptively, others have explored the correlation among teachers' beliefs and their teaching practices (see Handal, 2003). There seems to be an agreement amongst the researchers, based on their studies, that teachers' beliefs about mathematics and about its teaching and learning were shaped during their school years and formed by their own experience both as students and as teachers of

mathematics (Handal, 2003 and Lortie, 1975). Some studies in this section imply that in order to be successful in curriculum reform, it is important to pay attention to teachers' underlying beliefs, and to challenge these through professional development if they do not align with the theoretical basis of the reform. The literature suggests that it is rather difficult to change teachers' beliefs, however; it might be possible that changing beliefs through involvement with professional development course, from a constructivist perspective would be feasible.

The literature also illustrates that teachers' beliefs about the nature of mathematics were categorised using many different viewpoints. It was possible to identify beliefs as placed on a continuum with absolutist at one end and fallibilist at the other. According to absolutist beliefs, mathematics follows certain, simplistic, fixed, and set of rigid rules. According to fallibilist beliefs, mathematics is uncertain, fallible, complex, integrated and devised by human beings. As with the beliefs about mathematics, beliefs about teaching ranged from the traditional to the constructivist. The traditional view of teaching and learning is described as telling, or offering clear, step-by step demonstrations of mathematical procedures to pupils, so that they can learn by watching and listening to the teachers' demonstrations and practicing them. Knowledge of mathematics was transferred directly from authority to novice through lecturing. The constructivist view of teaching and learning was described by teachers who see their role as facilitator, encouraging learners to actively build on their own knowledge through being provided with enough freedom.

While some studies have identified consistencies amongst beliefs and enacted beliefs, that is the ways in which beliefs affect practice, others have described discrepancies. The relationship between teachers' stated conceptions and their teaching practice, as noted in the literature, was somewhat found to be related to social issues affecting classroom practice; thus it is important to consider this influential factor in our communications with participants. The rationale for this study is two-fold. One is an attempt to define and/or shift the connection between teachers' existing views and beliefs and the formation of new ones; a transformation that is irreducibly complex. The other is that, changing teachers' belief systems can create a state of tension: a tension, which includes a range of dichotomous pairs, for instance, absolutist/fallibilist beliefs about mathematics combined with traditional/constructivist approaches to teaching and

learning. It is the study of these tensions within context that will provide the most useful findings from this study.

3. 4 Teachers, Dynamic Geometry Systems and Mathematics

3.4.1 Potential for Using Technology within Mathematics Education

The access to technology for educational purposes worldwide has important implications for the future of mathematics teaching; therefore, investigating these ramifications in reference to a Turkish context is one unique aspect of this research. The tool (GeoGebra) that will be used to facilitate this investigation contributes a further unique element to the work. There is little doubt that in all educational settings that offer access to technology, and which promote Piagetian and Vygotskian approaches, incorporating information and communication technology into mathematics teaching will become increasingly prevalent. Some studies (Diković, 2009; Erez and Yerushalmy, 2006; Oldknow and Taylor, 2003) report that today's digital technologies such as computers, mathematical software, graphics calculators, and the internet have the potential to be powerful tools for learning and teaching; many are indeed being increasingly used as such. NCTM (2000) stated that "technology is essential in teaching and learning mathematics; it influences the way mathematics is taught and enhances students' learning" (p. 11).

In recognition of this a number of digital technologies, specifically designed for use by mathematics educators, have emerged. The software to date reflects the specialised aspects of computing that lend themselves to mathematics teaching: "learning from feedback; observing patterns; seeing connections, developing visual imagery, exploring data and 'teaching' the computer" (Becta, 2009: p. 2). Some educators support the idea that integration of technology into mathematics education will encourage students to become more open to exploring, examining, conjecturing, discovering principles and making generalisations, all of which support a constructivist approach (Hoyles and Sutherland, 1989; Oldknow and Taylor, 2003; Turner, 1999). For this reason, many countries have invested in such innovations in order to infuse technology into every level of their educational system. However, Ruthven (2008a) argues that:

“Despite official encouragement and enormous investment across the developed world, the global movement to integrate digital technologies into school mathematics has had limited impact on mainstream classrooms” (p. 1)

This reveals that, although many attempts have been made to provide access to technological tools for teachers, little attention has been given to the need for sustained and appropriate professional development in such technology in order to keep pace with innovations (Forgasz, 2006; Guven et al., 2009). The reasons for this are part of the concern of this thesis.

In particular it is noted that as plans for reforming schools call for the increased use of innovative technology in mathematics, it is essential for the teachers themselves to recognise the potential of such technologies for improving mathematics teaching methods; hence the emphasis on teachers’ perceptions in this work. Adopting technology to promote teaching and learning is a long process, and might require teachers to change their practice—that is, traditional mathematics environment controlled mainly by studying with pen and paper (Pierce and Ball, 2009) and make it meaningful to individual learners (Jung, 2005). Within this process, teachers have been given additional responsibilities and new roles to master. It could also be argued that the effectiveness and efficiency of technological applications depend on teachers and curriculum concepts. Any problem appearing in either of these might decrease the quality of education. In most cases, teachers seem to act as the primary mediator between technology and its integration in the educational system (Zhao, Hueyshan, and Mishra, 2001). Drier (2001) argues that there is a need for qualified teachers who can ‘utilise technology as an essential tool to developing a deep understanding’ (p. 173) in the mathematics and pedagogy for their students. Simply providing the technology itself to teachers does not always result in the successful incorporation of that technology in their teaching (Cuban, et al., 2001); this is the reason why the aim of this research is to provide a PD opportunity as part of the methodology, giving mathematics teachers not only access to in technology but also discussing appropriate pedagogical approaches. This work will thereby support or contradict the assertion that technology usage in education is beneficial in encouraging teachers to seek out new teaching approaches that can be integrated at different academic levels.

3.4.2 The Potential of DGS in Mathematics Education

Of the computer applications that have emerged over the last 20 years the well-known applications, Dynamic Geometry Systems (DGS) have focused on relationships between points, segments, line circles and Computer Algebra Systems (CAS) that concentrate on manipulation of expressions (Sangwin, 2007). These two categories consist of many mathematical software packages including GeoGebra (the subject of this study), Geometer's Sketchpad, Derive and so on. Particular attention is given in this thesis to DGS. This type of software has prompted much interest and enjoyment amongst mathematics educators. DGS are amongst the most readily approved of educational software tools since it was conceived of for classroom use (Ruthven, 2008b). One of the most fundamental features of DGS is that it provides an opportunity for learners to move objects on the screen by controlling primarily with the mouse (Hoyles and Noss, 1994). Within these programs, learners can examine geometrical objects quickly and accurately by dragging with them with the tool of mouse, and changing their dimensions, objects maintain their properties (Preiner, 2008). Healy and Hoyles (2001) point out that "dynamic geometry systems provide access to a variety of geometrical objects and relations with which users can interact in order to construct and manipulate new objects and relations" (p. 235). When the object dragged, dynamic geometry software might turn the geometrical objects into different positions, enabling pupils to give meanings for new geometric construction. Ruthven (2005) mentioned that DGS provides an opportunity for learners to investigate and visualise geometrical features of shapes by using dragging tool and transforming shapes in ways beyond the scope of traditional pen-and-paper activities. DGS may provide teachers of geometry with new approaches (Pratt and Ainley, 1997).

DGS are conceived to offer a learning place where learners could research and attain an understanding of great mathematical concepts independently. DGS can be viewed as a pedagogic tool for creating a practical milieu in which "students can construct and experiment with geometrical objects and relationships" and for the investigation of "a mathematical domain" (Hoyles and Noss, 2003: p.333). These software packages including GeoGebra and their activities allow pupils to check conjectures, seek patterns, to make connections, to investigate and to work with dynamic figures by building their own sketches (Edwards and Jones, 2006). Engström (2004) described DGS as giving students "the possibility to explore, propose, make conjectures and try to demonstrate,

with teacher and students work[ing] together” (p. 4 cited in Ruthven, Hennesst, and Deaney, 2008). Furthermore, Erez and Yerushalmy (2006) imply that student-based and active participation could be supported by integration of DGS into mathematics education successfully.

DGS can be used for visualisation of mathematical conjectures by allowing learners to generate many concrete examples on the computer monitor than is possible with pen and paper. Reviewing existing literature, some of key reasons for teachers who want to integrate DGS in their teaching were summarised by Mainali and Key (2012) as follows: Firstly, DGS are designed for teachers to have great *flexibility* in what they can do and how they do it. Secondly, DGS give an opportunity for teachers and students to work on mathematical concepts together by interacting pupils with discussions about explorations. Therefore, teachers and students would become *co-learners*. Thirdly, DGS encourage teachers to adopt learner-based teaching. That is, DGS provide the basis for the accomplishment of student-directed approach and collaborative working with open-ended mathematical activities through which teachers can provide students opportunity to discover and generate their own knowledge. This idea is associated with a Vygotsky perspective; a social interaction act as a critical function in the development of individual cognitive structure. Lastly, DGS may help students to develop their thinking skills (p. 3).

GeoGebra is open source software with the potential to combine certain features of DGS, CAS and spreadsheets into one application (Preiner, 2008). In the GeoGebra environment, representations of the same mathematical objects are linked dynamically in different ways, enabling learners to move forward and backwards between them, thus making relationships more easily understandable for learners (Ozgun-Koca, 2000). Whenever one of the representations is transformed, all others adapt automatically so as to preserve the relations between the different objects. New objects in GeoGebra can be produced by using toolbars or an algebraic keyboard input. These objects also have different representations in both the dynamic and graphic windows. The learner can adjust the value of the object through either representation or dragging the geometric figure using the mouse to change the algebraic representation through keyboard. Likewise, through dragging object, the algebraic expressions changes consequently. One may criticise GeoGebra in a way that the user needs to become familiar with arcane

syntax so as to illustrate dynamic text or calculation when using the DGS feature of the software. Moreover, using the software requires a fairly high threshold of knowledge in geometry in order to find appropriate tasks (Ainley and Pratt, 2007). In Geometer's Sketchpad the user can display dynamic calculations without having to learn any syntax. The reason behind this is that GeoGebra still remains a relatively new application (Preiner, 2008). In fact, the some important features of GeoGebra are listed by Diković (2009) and summarised as follows:

- GeoGebra could provide a good opportunity for cooperative learning.
- GeoGebra could provide opportunities for learners to visualise mathematics, to investigate mathematics, to make mathematics classrooms interactive.
- GeoGebra could assist students to obtain a better understanding of mathematics, and students could manipulate variables straightforwardly by dragging or using sliders.
- GeoGebra activities allow learners to generate circumstances that will develop their making the indispensable mental productions.
- Using the algebra input bar, student could produce or modify new objects.
- Students can personalise their own creations through GeoGebra features (p.192).

GeoGebra has been quickly gaining popularity amongst educators worldwide, since it is easy-to-use dynamic mathematics tool that integrates many features of different mathematical programs. Moreover, as a result of its open-source nature, there is a wide-ranging user community supporting it. Hohenwarter and Preiner (2007) argue that teachers need a support system and professional development to develop their skills in teaching mathematics using GeoGebra. All these reasons explain why the decision was taken to introduce GeoGebra to the participants in this research study. It is anticipated that as it is a popular tool this will increase the level of applicability and interest in the findings of this work.

3.4.3 Professional Development of Mathematics Teachers and Their Experiences with Technology

This section touches on those research studies that have been conducted to date to identify teachers' beliefs about using educational tools in mathematics teaching and learning with the aim of designing in-service or pre-service courses (Karatas, 2011;

Mainali and Key, 2012; Ozyildirim et al., 2009; Sulaiman, 2011; Tharp, Fitzsimmons and Ayers, 1997). In fact, previous research into Turkish in-service mathematics teachers' beliefs, and how these can be affected by involvement in a short term professional development (PD) course based on the use of DGS, does not exist. These above studies are useful for informing the requirements of this study, as well as for revealing where gaps in knowledge in this area might still remain. As posited in section 3.3 innovation into teacher education, teachers' beliefs and attitudes should be considered, as these are likely to represent obstacles to adoption.

Karatas (2011) conducted a study of 41 Turkish pre-service teachers to find out how they would respond to discovery and investigational geometry activities using Cabri and Derive software programs in a course environment. In the study, 'a computer education course' (UCME) was designed to provide participants with the opportunity to engage in hands-on experience, exploring mathematical ideas and relationships, and conjecturing on the basis of several examples. The main aim of the UCME course was to teach the pre-service teachers about how to use the various features of the educational software for teaching geometry within a course setting. Their formal education had involved eleven weeks of class sessions lasting 33 hours. In order to recognise the potential of learning geometry using Cabri and Derive, the students were encouraged to work on tasks in groups and were also asked to articulate their solutions to activities and their resultant understanding of mathematics. In the final two sessions, the student teachers were given an opportunity to present their own projects. The author observed their performance and made additional field notes regarding their project related experiences. The results of this study showed that student teachers believe that computer-based activities make abstract concepts more concrete and visually accessible. This then supports the student to understand such ideas more fully. The outcome of Karatas' (2011) study was support for the idea that computer technology makes it possible for pre-service teachers to accomplish learning aims through active exploration, discovery and conjecture. The study illustrated that the computer-supported environment based on investigation and discovery may help to enhance learners' confidence, mathematical understanding and problem-solving skill.

Bulut and Bulut (2011) studied with 47 Turkish pre-service teachers in order to investigate their views about the dynamic geometry software (GeoGebra). They were

involved in GeoGebra-based course during their formal education. During the course, the student teachers were taught how to use arcane syntax so as to illustrate dynamic text or picture through dynamic worksheets. Interviews were mainly used to collect data in their study. According to the findings of this study, student teachers favoured to utilize integrate pictures with the background of worksheets to link geometry with real life samples. They also believed that GeoGebra can be used for creating test questions, constructing web pages, calculating algebraic equations. However, in this study the author provided limited detail about the actual course design. Similar to this study, Ozyildirim et al. (2009) investigated views of fourth year pre-service teachers about teaching geometry and incorporating dynamic geometry software (including GeoGebra, Geometry Sketchpad) into geometry teaching during a 14 week course (42 hours). At the beginning of the course 75 participants answered three questionnaires; their responses were then used as the basis of semi-structured interviews conducted on a voluntary basis, at the end of the course. The total number of interviewees was nine. The course presentation was divided into two phases; (i) introductory, (ii) project development. In the introductory phase, general information about the features of DGS was presented to familiarise the participants with it. In the project's development phase, the participants were encouraged to develop new geometry activity worksheets. They worked on these activities in groups. [1] Two of the researchers also took the role of instructors responsible for delivering the course activities. The students who reported their experiences stressed that to use DGS as a sole means of teaching mathematics would be inadequate, and that lessons should be reinforced with concrete manipulatives. The course participants reported that the language issue and the complexity of the commands acted as a barrier to using the system. The authors concluded that the participants had realised the significance of using DGS in geometry teaching as it offers an experiential learning environment and enjoyable activities for students.

Once the participants had qualified as teachers, follow-up interviews were conducted in order to identify whether their views had changed or not. The follow-up study revealed that although participants were willing to employ DGS in their current geometry classes, they were worried about not having the sufficient number of computers for their students and envisaged a lack of financial support when they became available for teachers. This would need to be on-going since DGS was constantly advancing. Therefore, the evidence of this study showed that practical limitations are instrumental

in inhibiting the use of the computer in the current classrooms. For these participants, the future professional development course was considered to be essential because they did not feel confident of their abilities to continue to use the DGS in their classroom.

However, researchers in the aforementioned studies provided little emphasis regarding the exploration of student teachers' experience with regards to the pedagogical aspects of using DGS in their learning and teaching of mathematics. They were taught in the form of lecture type and followed structured DGS-based worksheets. Arguably this characteristic of the course was unrelated to a constructivist pedagogy and inappropriate to a technology based-open-ended environment. These studies suffered from some methodological weaknesses (lack of social interaction) and this could be overcome by incorporating pedagogical aspects of technology mathematics teaching and learning. The researchers did not closely observe participants' ideas about particular tasks and how they interacted with their peers due to the large sample of participants. This study, like other technology-based studies in Turkey, was conducted with pre-service teachers as part of the requirement for their course work and my research conducted with in-service teachers on a more voluntarily basis.

Hoyles, Noss, and Sutherland (1991) conducted a study with 20 in-service mathematics teachers in order to examine their attitudes and beliefs about mathematics and its teaching. They designed and implemented a thirty day Microworlds course for these teachers. These projects aimed at providing first-hand experience for those teachers involving the educational use of the computer in mathematics. The researchers observed the participants in two different settings which included learners having initial experience about the use of Logo in mathematics education within the course environment and their professional life while teaching in normal classrooms. A variety of data collection methods such as three semi-structured interviews, classroom observation, questionnaires, course-data were used in the study in order to apprehend teachers' beliefs and return rich and useful data sets. The project was founded on the notion that the teachers should be engaged from the outset in exploring mathematics for its own sake, before considering classroom activity. The participants initially investigated mathematics using pre-designed opening points and creating their own tasks. Discussion and collaboration were thought to be a necessary component of the course design. According to findings of this study, the researchers developed a series of

caricatures which portrayed the major actions of identifiable bundles of participants through their initial analysis. They concluded substantial changes in the participants' beliefs about mathematics and about the role of the computer in mathematics context. The new experiences that the participants gained through their involvement with the course that enables them to implement these experiences into teaching and to make decisions about teaching practice in their classrooms. The authors emphasised that developing a sense of confidence in teaching mathematics with the computer is an important aspect of integration of the computer into mathematics in creative ways. Some participants found difficulties in integrating the innovation in the classroom. Lastly, they pointed out that teachers need follow up support and guidance until they reach an intended level of implementation.

Mainali and Key (2012) designed and implemented a GeoGebra-based professional development course for fifteen secondary school mathematics teachers in Nepal so as to examine their beliefs, feelings regarding the software, technological difficulties faced and mathematics. The purpose of the researchers was to promote participants' skill and confidence by interacting with GeoGebra. They were required to integrate GeoGebra in their teaching. The first author played three different roles during the workshop sessions: director, instructor and researcher. Besides introducing the software, Mainali also monitored and assisted the participants when they experienced difficulties using either GeoGebra or the computer. The four day session lasted 26 hours. The first three days covered technical activities that were essential to insuring the participants had the basic skills needed for independent use of GeoGebra. Roundtable demonstrations and discussions were employed in the morning sessions with the help of a laptop and projector) to collect participants' views about GeoGebra tools and the potential of GeoGebra for learning. In the afternoon session, the participants engaged in more practical tasks, using the GeoGebra tools they had learned about in the morning session. The final day was given over to the participants to work on independent construction tasks, on the basis of worksheets presenting twelve GeoGebra activities. Data collection methods such as questionnaire, interview, and field notes were used to generate data in this study. After the course, one of the least experienced participants came to believe that GeoGebra activities are easy and appropriate for the teachers to use as well as for students and he held the belief that teaching with GeoGebra would be easier. Other participants thought that this software was really stimulating and useful in mathematics

learning and teaching, and that they were very excited by GeoGebra since it offered a graphical and algebraic view at the same time. Before participating in these sessions, participants in this study had never thought about the possibility that mathematics could be learnt in open-ended way. At the beginning of the sessions, mathematics was seen as a rather static subject. The participants were highly excited by the visual aspects of GeoGebra and described GeoGebra as a very useful tool for practical mathematics learning. The findings indicated that the participants showed enthusiasm and positive attitudes towards the use of GeoGebra in teaching even though constraints and difficulties in connection with accessing advanced technology exist.

The study of changes in Kuwaiti elementary pre-service teachers' beliefs about ICT including Logo, the nature of mathematics and its teaching and learning was conducted by Sulaiman (2011). In order to understand the changes in participants' beliefs, Logo based mathematical course, in line with constructivist perspective, was developed and delivered by the researcher. The author provided the participants with the opportunity to experience the role of the learner within a Logo-based open-ended environment to increase their awareness of the potential of this learning process as promoted by Logo. One of the main objectives of this course was to deliver an opportunity for the participants to learn about how to teach mathematics using GeoGebra as a pedagogical tool for mathematics education. There were four main stages in the model course; Introduction to Logo (8 hours), mathematical based activities (8 hours), preparation of Logo-based session (3 hours) and Practice with the use of Logo (3 hours). The introductory session, lasting 8 hours, was designed to familiarise the participants with Logo. The purpose of the second stage was to immerse the participants in a practical environment, to explore GeoGebra-based mathematical tasks from the perspective of a learner. In the third stage, the pre-service teachers were guided in how to design a lesson plan and how to integrate Logo as a cognitive tool. In the final stage, the participants were given the role of teacher and encouraged to practice teaching a mathematics lesson using Logo. Throughout the workshop sessions, opportunities for collaboration and discussion between the participants were arranged. The author took the role of facilitator during the workshop sessions, giving the participants the opportunity to be accountable for solving the Logo based mathematical activities. A mixed methodology was conducted and two data collection tools, interviews and questionnaires, were used for quantitative and qualitative data to examine the

participants' beliefs and to follow their progress during the Logo based course. The findings of this study illustrated that a strong shift in beliefs in support of the use of technology in general and in particular the use of Logo in their future mathematics teaching, along with using constructivist teaching approaches.

In another study, a fifteen hour in-service course was designed for mathematics teachers and science teachers to assist them to integrate graphing calculators into their teaching (Tharp et al., 1997). The aim of the study was to determine whether the use of graphing calculators would have an effect on the teachers' beliefs about technology and on their teaching pedagogy. As part of the programme teachers were required to watch a video of experienced teachers using calculators in their teaching, and advising how to implement graphing calculators into their own teaching methods. The data collected through pre-post course questionnaires and journal writings indicated that teachers' attitudes towards graphing calculators and mathematics changed significantly because of the new teaching method and the use of technology. The findings also pointed out that there was a positive correlation between teachers' beliefs of mathematics and their beliefs about the use of calculators in the classroom. Those teachers holding a more procedural view of mathematics were inclined to support the idea that calculators do not develop teaching. However, those teachers who held a less procedural view of mathematics were inclined to see calculators as a component of mathematics and science practice. It can be understood from this study that teachers' beliefs about the suitable use of technology for students are affected by their beliefs about mathematics and by their beliefs about teaching and learning mathematics. Tharp et al. (1997) concluded that calculators can be employed to promote shifts in teachers' teaching practice, pursued by shifts in teachers' beliefs.

Technological investigation could be considered as a tool for teaching and learning mathematics is a central issue of concern in the literature. As a result of participating on a technology-based course, the majority of the research illustrated that many participants had developed a positive attitude towards the use of technology in mathematics education and showed enthusiasm for learning mathematics alongside technology, for example, the studies by Mainali & Key (2012) and Bulut & Bulut (2011). In Mainali and Key's study, when their participants faced difficulties, they provided detailed explanations about what the participants were working on, rather than letting

them experience a trial-and error approach. Therefore, the participants received information about how to use computers directly from the researcher. This might have adversely affected their confidence and knowledge, but, instead, the participants in the study developed positive feelings towards the use of technology in mathematics education, showing enthusiasm for learning mathematics in combination with technology. Although Karatas' (2011) study produced positive outcomes in terms of the pre-service teachers' mathematics learning, and their attitudes towards the use of Cabri and Derive, they did not state whether they intended to use computers in their future teaching. Therefore, one could criticise the objectives of the programme as it failed to combine technology education with suitable pedagogical approaches.

Tharp et al. (1997) revealed that the program which they had designed promoted changes in participants' views regarding mathematics and the calculator and they concluded that there was an alignment between teachers' beliefs about mathematics and those surrounding the use of the calculator. However, the study by Ozyildirim et al. can be subjected to methodological criticism. The DGS-based environment which the researcher created did not allow the teachers to reflect upon their pedagogical beliefs about mathematics and experiences with the use of DGS in learning and teaching mathematics. Thus, attention was only given to technology itself rather than providing participants a better theoretical and practical understanding of mathematics teaching and learning, through engaging them with technology-based mathematical activities that were consistent with a constructivist paradigm and they were subsequently expected to use these experiences in their future classroom. In this study, after the course, participants experienced difficulties in integrating DGS into their classrooms.

In the Microworlds projects, the researchers confirmed noticeable shifts in teachers' attitudes towards mathematics and mathematics teaching with Logo. This result could be related to the method employed by the study which highlighted the non-routine Logo-based mathematical activities. This method enabled the participants to experience the innovative ideas of Logo through investigating mathematical concepts and articulating their own ideas. Hoyles et al's project seems longer than Sulaiman's study and some similarities and differences can be identified regarding both their results and sample. Thus the former study conducted follow up support. Sulaiman's participants were student teachers and therefore could be considered to be novice in mathematics

whereas participants in the Microworlds projects were expert in their field of mathematics as well as being motivated to teach it. Both studies have reported positive outcomes in terms of changes in participants' beliefs with regard to mathematics and the use of technology. However, Sulaiman's study would have been more convincing if his participants had been invited to further reflect on the beliefs they hold as they enact them in their actual classroom. These aforementioned studies suggested that reinforcing shifts in teachers' beliefs through teacher involvement in a PD course could be an instrument for affecting beliefs and providing an effective means for studying teachers' beliefs and the dynamics of their change.

3.4.4 Summary

In this section, research on teachers' experiences with digital technologies in learning and teaching mathematics and the potential of DGS in mathematics education were discussed. The literature has shown that teacher plays an important role in integrating of technology into mathematics education. Within this process, teachers have been given additional responsibilities and new roles to master. It could be said that teachers need support in adopting an approach which relies upon a constructivist perception and incorporating technology in their practice. In this regard, professional development should be based on the concept of teacher as a learner who engages in mathematical tasks in meaningful ways so as to gain a better making sense of learners' role in a technology-incorporated environment.

The researchers agree that DGS might be considered as a pedagogic tool for learning and teaching mathematics in classroom. They have the potential to develop pedagogy and reinforce student learning. DGS can be used to support teaching and learning. They offer a learning setting where teachers and pupils can explore aspects of mathematics together. In this environment, students can independently construct their own geometric objects using the principles of constructivism to make knowledge. The attention should be given to the issue of the mathematics teachers in the classroom with technology alongside appropriate pedagogical approaches rather than focussing on the issue of the technology itself.

Some research considered teachers' views and feelings about the use of technology in mathematics education before introducing innovation. As a result of teachers' involvement with technology-based courses, they have developed a positive attitude

towards mathematics and technology. Since the teachers' beliefs play a key role in the process of integrating new technologies into teaching and learning, experiences with digital technologies within unstructured environment might allow teachers to reconsider their beliefs about mathematics and the role of technology. Technologically-oriented professional development of teachers is required to more pedagogical support and guidance in order to sustain their confidence and ability to teach mathematics with technology until they reach an intended level of implementation.

Considering the Turkish educational context, the majority of the technology-based studies have been conducted with pre-service teachers; however, there are a limited number of studies on the exploration of the experiences of in-service mathematics teachers with technology within a professional learning environment. Thus, more attention has been directed in general towards participants' perception of the use of technology in the mathematics context. These studies somewhat differ from the present study which is based on an interaction with computer-based investigational mathematical tasks. This challenge involves the researcher and teachers in the exploration of mathematical situations, communication and the application of new ideas. The present study also included the investigation of changes in participants' pedagogical beliefs about mathematics and the use of technology.

3. 5 Chapter Summary

In the first part of this chapter, concept of knowledge and education in Islamic epistemology and constructivism was discussed. From the Islamic perspective, God created the whole universe and has the knowledge of all hidden things. It is believed that objective truth exists eternally and independent from humankind and the role of human beings is to seek out objective truth. The importance of a theory of constructivism and its impact on mathematics education was also examined. The effects of individual and social constructivist approaches to learning and teaching of mathematics were described, revealing that for Piaget, knowledge is constructed in a learner's mind, whereas for Vygotsky, knowledge is constructed in social engagement within the environment. Despite the fact that constructivist learning highlights the reliance of human learning on understanding, instead of knowing, which is theoretically challenging within an Islamic educational context, it was explained how and why this approach has been adopted by Turkish curriculum. However, in view of quasi-empirical

philosophy of mathematics being consistent with the idea that construction of knowledge is based on negotiation in a mathematical society, constructivism was seen to lend itself well to mathematics curriculums. Construing mathematical knowledge as a learner based activity with understanding developed through social engagement, the use of technology as a facilitator was made evident in Piaget's "knowledge as an individual construction" and Vygotsky's "knowledge as a social construction". The first part of this chapter provides a framework to be able to discuss conceptualisation of knowledge and education from constructivist and Islamic perspectives. This would help me to link these perspectives with the findings pertaining to the nature of mathematical knowledge and would provide a basis for discussion of beliefs about teaching and learning of mathematics.

The second section of the chapter discussed literature pertaining to the complex and personal nature of teachers' beliefs. The process whereby beliefs alter was also investigated and a cyclical relationship between changes in beliefs and changes in practice, whereby one begins to affect the other, was noted. Teachers' beliefs about mathematics were identified as typically categorised into two theoretical constructs. Most of the studies reviewed in this thesis support the notion that there is alignment amongst teachers' beliefs about the nature of mathematics and their intended and enacted teaching practice: they have not, as yet, generated conclusive results. In addition to the subject matter knowledge and epistemology of mathematics, other issues should be regarded (such as social context). This leaves scope for this study to more fully define how teachers' beliefs and stated teaching practices in the area of mathematics are intertwined. The second section of this chapter provides a context for discussion of beliefs which address the following research question: what beliefs do Turkish primary mathematics teachers hold about mathematics, its teaching and learning before they participate in the PD course. To answer the research questions in this study an understanding of what constitutes and alters teachers' beliefs systems is also needed.

In the final section, the research on the role of technology including DGS in mathematics education and the role of teachers in this process embraced a wide range of issues regarding the meanings of teachers' experiences in technology-based mathematical settings. This examination demonstrated that a number of researchers in

this area support the notion that teachers should experience the approaches they are expected to use when they are teaching in a technology-based classroom. To this end, they have emphasised an interaction with technology-based investigational mathematical tasks. This sets a challenge to the researcher to develop a research method that involves teachers' engagement with exploring mathematical situations, communicating and applying new ideas. Therefore, as will be described in more depth in the following chapter, a professional development course was designed to provide teachers with a better theoretical and practical understanding of mathematics teaching and learning through engaging with computer-based mathematical activities (using GeoGebra) that were in line with the constructivist approach. The third section of this chapter would help to understand how teachers interact and experience with technology within professional learning environment. I will discuss methodological issues and theoretical foundations of the present study in the next chapter.

CHAPTER 4: METHODOLOGY

4.1 Introduction

The preceding chapters were concerned with literature exploring teacher change and their professional development with the use of technology; particular attention was paid to teachers' beliefs in relation to mathematics, its teaching and learning. Also highlighted in the literature was the apparent need for research on engaging mathematics teachers, both pre-service and in-service, in a constructivist learning environment within the professional development course so as to improve teaching practice (Guyen et al., 2009; Hart, 2002; Mewborn, 2003). Incorporating the literature review and research questions, I sketch out my conclusions pertaining to my research design to address these issues. In this chapter, I discuss methodological considerations and theoretical perspectives employed to conduct this research.

This chapter covers the following sections: the philosophical perspectives, the research design, design of the PD course, sample and access, data generation methods, data analysis followed by reliability and validity and ethical considerations. The aim of the first section is to make explicit the ontological and epistemological positions that are used and justified within the research. The second section describes detailing the overall design and the choice of research approach for the present research. The case study approach is explained and the strengths of using this approach are also examined. Moreover, issues related to generalisation are illuminated. The third section discusses the rationale and design of the GeoGebra-based PD course. The process of sampling and accessing teachers are described and discussed. The selection of appropriate data collections tools is also defined. In this research, I employed some data generation techniques such as interview, observation, participants' writings and questionnaire to explore the experiences of the in-service mathematics teachers who participated in an eight workshops sessions. Furthermore, the data analysis strategy and trustworthiness are also discussed in detail. The final section of this chapter includes ethical considerations.

4.2 Philosophical Perspectives of this Study

4.2.1 Paradigm

The term ‘paradigm’ is defined as the researcher’s view point that is based on a set of beliefs, assumptions and values (Johnson and Christensen, 2010). Bryman (2008, p.696) describes the term as belief clusters which guide researchers to choose what discipline they explore, the way in which they need to study this discipline and how they approach the interpretation of their evidence. These definitions suggest that the paradigm of a particular field of inquiry has an influence on almost every decision that researchers make throughout their study, including the nature of chosen topic of interest, data collection and analysis. As Bogdan and Biklen (2007) warn, there should be a consistency between methods and “the logic embodied in the methodology” (p.35).

The approach adopted in this study is a constructivist and interpretivist paradigm. This particular paradigm follows the foundations of “relativism”, a notion which suggests that reality is subject to change from one individual to another depending on their unique perceptions and conceptualisations (Cohen, Manion, and Morrison, 2007; Guba and Lincoln, 2005). Interpretivist social scientists intend to capture “the subjective meaning of social action” (Bryman, 2004, p. 13). This paradigm understands the “world of human experience” in which the participants’ view of the world is privileged (Cohen, et al., 2007). The interpretive paradigm emphasizes that there are a number of factors which affect the way things are in the social world. Studies adopting the interpretivist paradigm often aim to explore individuals’ characteristics, different human behaviors, opinions, and attitudes and to understand them from within (Cohen, et al., 2007). The current study investigated the topic from the teachers’ perspective within the professional development (PD) setting. This perspective enabled the researcher and participating teachers to enter into the research frame and collaborate in sharing professional notions. In this respect, the PD course acted as a means for change and provided a lens for the teachers to see different ways for teaching and learning, and allowed the researcher to teach and intervene as a facilitator as well as to monitor the participants’ interactions within the PD course. This led to a form of qualitative data relating to the context, activities and experience of the researcher and participants within this specific context.

4.2.2 Ontology

Ontology is concerned with the nature of the existence or phenomenon being studied (Gray, 2004, p.16). According to the ontological perspective of interpretivist paradigm, “reality is socially and discursively constructed by human actors” (Grix, 2004, p.61). Individuals differ in the way they make sense of the world and the way they construct meanings from objects through their interaction and engagement with them (Bryman, 2004). Interpretivist ontology postulates the idea that research pursues multiple realities since individuals construe realities subjectively in different ways (Creswell and Clark, 2007). It needs to be acknowledged that these multiple realities are shaped by the knowledge of people as participants in social world, their practice and understandings (Robson, 2011, p.24).

Interpretivist researchers often attempt to interpret reality by extrapolating abstract statements from concrete evidence, rather than the other way around (Punch, 2009). That is why a set of hypotheses are not simply confirmed or disconfirmed in a linear way in the current study but the evidence is based on the informants’ views on the topic under investigation (the learning and teaching of mathematics). For example, the constructed realities of teachers as professionals with distinct belief systems are explored in the form of teacher beliefs about the subject matter and the pedagogy of mathematics. Since the study aims to investigate a phenomenon associated primarily with teachers’ views, perceptions, and beliefs within an educational context, the diversity in the participating teachers’ realities needs to be recognised. My role as a researcher is to understand teachers’ interactions with the GeoGebra-based mathematical activities and changes the extent to which the Turkish primary mathematics teachers’ beliefs are affected by their participation in a computer-based professional development course in relation to their beliefs about mathematics, its teaching and learning. Their conception of ‘reality’ could be largely different depending on what they understand from ‘mathematics’.

4.2.3 Epistemology

Epistemology refers to the manner in which knowledge is acquired in a particular discipline (Bryman, 2008). Epistemology is concerned with how the reality of the research subjects can be known and their personally, culturally and socially “situated interpretations of the social life-world” (Crotty, 1998: p.67). What needs to be

recognised is that the researchers' epistemological stance has an influence on how they approach the phenomenon of their interest. In interpretivism, the notion of knowledge is considered to be constructed as a result of people's interactions with each other as they rely on unique experience with the world (Bassey, 1999). It is their individual experience and different quality and degree of knowledge that perceive the same phenomenon differently. As Bryman (2008, p.16) explains, since human beings tend to "act on the basis of the meanings that they attribute to their acts and the acts of actors", it is necessary to construe their thoughts and actions from their viewpoint. Otherwise, "social reality" that has a particular relevance to them cannot be grasped.

In the case of the present study, knowledge was generated about the Turkish primary teachers' initial beliefs in relation to the nature of mathematics, its teaching and learning practices, and about the extent to which the teachers' mathematical beliefs are influenced by their participation and involvement in the PD course designed using GeoGebra. This reflects my epistemological position that knowledge exists through human interactions and negotiations with other peers, social environment and technological tools. It is assumed that knowledge is dependent on and outcome of social construction to a large extent. Accordingly, concerns such as "how digital tools may assist the teachers to build a comprehension of mathematics" and "how these tools could be employed in an interactive manner to scaffold the building of mathematics knowledge", were directed to the participants. Through these questions, the current study primarily concentrates on the process of the social phenomena, as is the case with the majority of qualitative studies (see Robson, 2011). More specifically, in this study, an attempt was made to gain insights into mathematics teachers' learning and reflection processes within the PD course. Since the current study seeks to understand the teachers' beliefs regarding mathematics and its pedagogy which are inevitably subjective in nature, the teachers are given the potential opportunity to change their beliefs following their exposure to technological and pedagogical tool (i.e. a software application called GeoGebra).

4.3 Research Design

The purpose of this section is to describe and justify the choice of research design. A research methodology based on the interpretivist paradigm is related with qualitative data collection methods such as interviews, often used with the intention of gaining

understanding into an identifiable phenomenon within its particular social context (Cohen et al., 2007). Bogdan and Biklen (2007) also state that qualitative researchers aim to make sense of “the processes by which people construct meaning and to describe what those meanings are” (p.43). During the course of documenting these processes, researchers are supposed to provide as much detail as possible. However, the difficulty in following a strict set of stages needs to be acknowledged. This difficulty stems from the fact that, as Bryman (2008) suggests, qualitative research has an approach in which “categorization emerge out of the collection and analysis of data” (p.369). The present project is a qualitative study which concentrated mainly on ‘how’ and ‘why’ questions in order to investigate the changes in the primary mathematics teachers’ beliefs about the nature of mathematics as well as its pedagogy and learning. This requires an in-depth understanding and a thorough account of the primary school teachers’ experiences with the PD course in which they are introduced to an educational tool.

4.3.1 Case Study Design and Rationale

In this section, the use of a case study approach will be justified. In addition, concerns associated with generalisation in qualitative research will be discussed. A case study is described by Merriam (1998) as an “examination of a specific phenomenon such as a program, an event, a person, a process, an institution, or a social group” (p.7). The issue or activity related to one or more individuals is intensively investigated by the researcher (Creswell, 2009). While the flexibility and openness of a case study serve to identify potentially important features and issues in a particular phenomenon, case studies may suffer from some of the weaknesses. For example, since case studies may be affected by the researchers’ bias in selection and observation of informants, they do not easily lend themselves to “cross-checking”.

A case study is a preferred research design of the present study because the researcher’s point of interest lies in the “how” question (Yin, 2009) which is reflected in the research question: *how* do the teacher participants’ mathematical beliefs evolve through their participation on the eight-week PD course as a learner of mathematics? Yin (1994) defined three types of case studies: a) exploratory, b) descriptive and c) explanatory. The present study has been characterised by these types of case study. As Yin (1994) put it, “there are large areas of overlap” between the types of case studies (p.4). Descriptive case studies aim to offer a detailed narrative description of the actions and

interactions related to the inquiry. The study is of descriptive type as it presents the portraits of the teachers' initial beliefs prior to their exposure to the intervention. Being exploratory, this study investigates the impact of professional development, in this case PD course, on teachers' beliefs and the extent to which they respond to the course activities, and they come to modify their beliefs as a result of their participation. Being descriptive, the small-scale study provides a detailed description and analysis of each teacher's expressed beliefs and experience in order to identify changes in their beliefs over eight week workshop sessions. By being explanatory, some causal relationship between the PD course and belief changes is sought. Taken together, an emphasis is placed on the nature of the participating teachers' interactions in the course setting, in which six teachers worked in cooperation with the researcher.

In multiple case studies, "researchers study two or more subjects, settings or depositories of data" (Bogdan and Biklen, 2007, p.69). Multiple-case efforts involve an empirical examination of a particular contemporary phenomenon by drawing upon a variety of various sources of evidence (Robson, 2011). As a multiple-case study, the current study examines mathematical beliefs of six teachers within the PD course as they engage with activities and interact with their peers and the researcher. The phenomenon is teachers' interaction with the PD course and the context is the technology-based environment. Since it is important to contextualise the nature of social interactions within the PD course setting, a case study serves to explore teachers' beliefs and the way they change. As far as the context is concerned, analysing the effect of physical and cultural environments on people and their purposive interaction closely within these environments is the key to understanding participants' sense-making (Noddings, 1990, p.15).

According to Brewer and Hunter (1989), there are six types of units, which could be applied in educational research: persons; attributes of persons; activities and interactions; residues and artefacts of behaviour; settings, incidents and occasions; and collectives. Any of these may constitute the core of case study research. The case or 'unit of analysis' (Miles and Huberman, 1994) is defined in this study as teachers' mathematical beliefs and every individual teacher is a sub-unit of analysis (see their mini profiles in section 6.4). Since the present study is a collective case study (Levin and Wadman, 2005), the teachers are treated both as individuals and as a group. In

other words, I treated each of the six teachers as individual participants who may have distinct belief systems separately and at the same time I treat them collectively where they act as a group. This is different from a type of the study where an experimental group and control group are compared and contrasted in terms of the observable influences of the effectiveness of an innovative pedagogical tool. Since the present study seeks to track changes in primary mathematics teachers' beliefs about an inquiry-based approach embedded in the PD course, to study this particular phenomenon is not conducive to use data collection methods used in experimental research. As Goldin (1990) argues, case study methodology needs to be employed in mathematics education research since the complexity of individual teachers' understanding of mathematics pedagogy cannot be revealed in experimentation which is characterised by strictly-controlled environment.

4.3.2 Generalization in Case Studies

Generalizability is related to the extent to which the finding of one's work can be applied to other situations. Another synonymous term coined by Guba and Lincoln (1989) is 'transferability' which also refers to the application of research findings to different contexts. Transferability is described as the scope to which the research findings can be replicated beyond the case studies (Miles and Huberman, 1994).

According to Williams (2001), generalisation is inevitable characteristic of interpretative research; otherwise the research would be meaningless. On the other hand, some researchers put forward an argument in favour of a different kind of generalisation. For example, Yin (2009) suggests that unlike positivist research, case study research perform analytic generalization in which particular results are generalized into a broader *theory*. This could be accomplished by means of the employment of a multiple-case studies methodology and by comparison of evidence (Lincoln and Guba 1985; Miles and Huberman, 1994). Likewise, Ellis and Bochner (2000) stated that "generalization is constantly tested by readers as they determine if it speaks to them about their experiences or about the lives of other they know" (p.744). Multiple-case studies enable the researcher to enhance analytic generalization through replication logic and/or approval of results to accomplish external validity (Yin, 2009). The researcher could aim for expanding theories rather than sampling (Donmoyer, 2000). According to Yin (2009), generalisation of results from case studies, from either

single or multiple designs, are based on theory rather than on populations. The readers may attempt to draw similarities and parallels between cases' personal and professional experiences and their own. To enhance the level of analytic generalizability, researchers can also employ techniques such as providing rich data and thick descriptions, cross-case analysis and the use of methods for coding and analysis (Lincoln and Guba, 1985).

The research findings generated from a case study may not be generalizable to all subjects and groups in different settings. For example, the cases of mathematics teachers under investigation could be different teachers working in other primary schools in Turkey. That is, they cannot be considered to represent all Turkish school teachers of mathematics because teachers may have different personal philosophy of mathematics as well as different pedagogical experiences within the PD course setting.

4.3.3 Researcher's Role

Qualitative studies require researchers to clarify the rationale behind the choice of a proposed research study, their views on the topic of inquiry and the relationship between the researcher and the researched (i.e. participants) (Schram, 2003). It is therefore important for the researcher to become aware of any biases in arriving at the conclusions from the evidence presented in the study. Nevertheless, any qualitative study is subject to researcher bias to some extent, and it should be noted that the findings and explanations are as trustworthy as possible (see section 4.7 for comments on the issue of reliability and validity).

I took the role of a facilitator (i.e. facilitate their engagement with constructivist pedagogical activities) during the PD course which gave the opportunity to evaluate how I employed my intervention strategies to examine dynamics of the teachers' belief change. Through the workshop sessions, collaboration and discussions among the participants before, during, and after the activities enabled me to elicit their reflections on the tasks in terms of student's learning, curriculum and pedagogical approach. Enabling the participants to see different theoretical perspective for learning and teaching, and to reflect on what they saw and obtained through the PD course, provided me with an opportunity to see how they change their beliefs. This reflective characteristic of the methodology provided me with a clear outlook of what occurs in the setting and a greater awareness of my own role. This then enabled me to draw conclusions based on what I saw through the course and what I heard from the

participants within that frame. The extent to which the participants gained a good view through the course depended on how I sustained my position as a constructivist, how I designed the tasks, and how I created a professional learning environment consistent with a constructivist perspective which includes providing and enabling communication and negotiation in a group discussion or in individual interchange.

Steffe, Thompson and Glasersfeld (2000) argue that the role of the researcher shifts during the course from one where s/he regards him/her selves as either an actor or observer as individuals who interact on the basis of his/her knowledge and beliefs. I was able to observe the way in which the participants might see different techniques of learning and teaching mathematics through the PD course. This provided me with the opportunity to understand the way in which participants change their beliefs about mathematical pedagogy and learning through the use of computers. I hope to be seen by the course teachers as a colleague rather than an “expert”. My position in the present research was that of a participant observer, and partly also that of a teacher educator delivering a professional development course. It should be noted, however, that I am not a member of the school where the study took place but acted as a teacher who was in charge of introducing the participants to the GeoGebra-based professional course. I admit that my involvement in the PD course might have influenced my interpretations of teachers’ beliefs and their interactions with peers and myself. My role as a novice teacher educator role in the study might have had a negative impact on the uptake process. However, the point that I carried out the research for the objectives of my study as a postgraduate student, and not as a figure of any authority in the Turkish context helped me sustain a collegial relationship and rapport with the participating-teachers. The following section describes the overall design of technology-based professional development course and the choice of appropriate mathematical tasks for this course and the strengths of teaching (intervention) approaches adopted by researcher.

4.4 Design of the Professional Development Course

This study was designed to investigate the degree to which a PD course, based on the use of GeoGebra, influences the beliefs of a group of Turkish primary mathematics teachers with reference to following fields: the nature of mathematics, its teaching and learning, the use of technology. To achieve these aims, a PD course was created to offer the participants an opportunity to experience the role of the student in reference to the

computer-incorporated context by engaging with mathematical tasks designed in line with the constructivist approach. The intention was to provide them with better theoretical and practical comprehension of mathematics teaching and learning. It was anticipated that this course could act as a catalyst for modifying teachers' mathematical beliefs. Since teachers' beliefs about mathematics, its teaching and learning are acquired, theoretically they can be altered. Ertmer (2005) proposes that the discernible shifts in beliefs could take place through experience. Therefore, teachers need to participate in an environment where they can interact with mathematics and its pedagogy as a learner from constructivist perspective (Ball, 1988). Exposing teachers to new pedagogical approaches as learners is a way to confront and change their beliefs. According to Putnam and Borko (2000), teachers actively take part and participate in workshop sessions where researchers introduce activities and materials, the teachers are expected to employ innovative ideas in their own learning.

Pennington (1995) states that within the professional development setting, teachers must be provided with on-going help, intervention, and collaboration besides being offered facilities for reflection. Reflection takes as a central role in the growth of teachers and teacher development (Jaworski, 2003; Richards, 2004). For the purposes of the present study, teachers need to experience directly working on particular innovative and investigational tasks so as to accommodate change. This research will build on those studies that take teacher education as a basis for educational reform. Therefore, collaborative professional learning environment in this research was established that enable teachers to become responsible for their own learning.

4.4.1 Rationale and Principles for a GeoGebra PD Course

Rationale for the PD course

At the present time digital technologies are playing a crucial role in mathematics education worldwide. Both the teaching and learning of mathematics can be enhanced by the incorporation of technology, thereby shifting teachers' beliefs and views about the classroom, the roles of teachers and students, and pedagogy. Moreover, incorporating technology can assist learners in becoming independent and active individuals engaged in the learning process (Erez and Yerushalmy, 2006).

A variety of models for PD courses were evaluated and discussed in the literature (Hoyle et al., 1991; Mainali and Key, 2012; Ozyildirim et al., 2009), but it is difficult

to identify a common approach to the pedagogical use of the software applications in mathematics education, since the organisation of professional development courses may vary from country to country and individual differences also arise within the same culture. In recognition of this, the first step in this research was to create a setting based on the employ of GeoGebra, to assist mathematics learners and teachers in improving their conceptual understanding by internalising an approach that was rather different from their previous experiences as pupils or teachers.

There seems to be an agreement amongst researchers that changes in teaching practice demand teachers to continually examine their own beliefs and practices when teaching mathematics (Ernest, 1989; Hart, 2002; Philippou and Christou, 2002). This can be achieved simply in particular situations in which an individual is confronted with new information and experiences that ‘disagree’ with traditional-oriented beliefs. (Philippou and Christou, 2002); this assertion informed the decision making process when designing the PD course. I designed a professional learning environment in which teachers can engage in situations using reflective practices and actively construct mathematical knowledge with the use of GeoGebra, as well as being motivated to evaluate and reflect on their conceptions and beliefs. The main rationale for GeoGebra-based mathematical activities in this study is that it would provide mathematics teachers with opportunities to experience the role of the learner in GeoGebra-based mathematical environment, and could also allow them to reflect on and reconstruct their mathematical beliefs. In addition, GeoGebra can be used as a pedagogical tool, is a free software application, and easy to download from the internet; this makes it accessible to all primary teachers who wish to use it in their mathematics classrooms. The three major objectives of the PD course were:

- to introduce and familiarise primary mathematics teachers with the use of GeoGebra as a pedagogical tool that can assist a constructivist perspective for the teaching and learning mathematics;
- to deliver an intervention opportunity to reflect on and re-examine their existing expectations, views and beliefs about the nature of mathematics, its learning and teaching, and the use of GeoGebra in mathematics education; and
- to deliver an opportunity to learn how to teach mathematics with the use of GeoGebra as a pedagogical tool for mathematics education.

The core expectation was as follows: if primary mathematics teachers experienced using GeoGebra as a tool for learning and teaching of mathematics, its qualities for communicating mathematics' principles would be more extensively appreciated by them. It would also reveal to teachers how GeoGebra can be effectively incorporated into the broader teaching and learning environment. By so doing this then opens up the opportunity to help learners to develop a broader understanding of mathematical ideas when engaging in GeoGebra activities. In order to achieve this aim, teachers needed to experience and perform GeoGebra-based mathematical activities in the workshop sessions. They would play the role of learners during these sessions, and thereby identify the most useful elements to incorporate into their current mathematics teaching.

To prompt the primary mathematics teachers to become actively involved in the process, the researcher provided them with opportunities to engage with the PD course, by taking on the role of independent learners actively engaged in their own learning. First, the teachers were asked to compare their existing teaching experience and knowledge with the experiences offered by GeoGebra-based mathematical activities, from the learners' perspective. Second, they were required to perform some of the activities in the GeoGebra environment as teachers during the home-based stage. Third, they were asked to identify the strengths of the learning process, focusing on GeoGebra's innovative and powerful ideas, which offer a context for cognitive development. Last, the researcher requested that they reflect on and re-evaluate their existing beliefs and views as to the role of the mathematics teacher in the traditional school setting. The research setting also provided an opportunity to more generally investigate participants' perceptions, views, and expectations about mathematics, and of teaching and learning using computers.

General principles for the PD course

According to the principles identified in the constructivist method, the educator need to be a facilitator, promoting pupils to take accountability for their learning and playing a dynamic role in their knowledge acquisition process. This process can be catalysed in a computer based environment through the interactive process of conjecture, feedback, critical thinking, investigation and collaboration (Jaworski, 1994). Hence, the PD course involved the researcher and the teacher participants in an exploration of mathematical circumstances, communication and application of new thoughts. This was found to be

possible through individual and small group exploration, discussion and negotiation, all within the context of collaborative enquiry. To promote mathematical thinking beyond the routine, they worked on mathematical activities by means of GeoGebra.

To succeed pedagogically mathematical tasks should involve students and teachers in the investigation of mathematical circumstances, communication and application of new thoughts in a way that represents a challenge to students' understanding (Anthony and Walshaw, 2009). This idea is associated with the notion of the zone of proximal development (Vygotsky, 1978), where individuals who are beginners and have difficulties in understanding tasks without outside helps. This concept highlights the importance of bringing to bear teachers' previous experiences and beliefs about mathematics, stressing the need to develop new approaches to mathematics teaching and learning. Additionally group work would help participants to communicate with each other and to articulate their concepts about mathematical understanding.

Discussions were mainly based on the identification of problematic tasks that the teacher found difficult while teaching and that encouraged them to come up with their own solutions. During pair work, participants are persuaded to share their existing knowledge. If that knowledge is built on by interaction with others (Vygotsky, 1986), this interaction and communication promotes the reinforcement of knowledge. Through reflection on their own experiences in small groups, the participants were encouraged to question their fundamental philosophies about learning and teaching mathematics. In brief, group work, negotiation and tasks were the main ideas covered by the PD course. The next sub-section describes those content based and structural elements of the PD course that were examined.

4.4.2 Structure and Content of the PD Course

After determining the objectives of the PD course, the next step was to identify its structure and content. I made use of investigational mathematical tasks, which were adapted from the new Turkish primary mathematics curriculum and the GeoGebra website. Primary mathematics textbooks on relevant topics were reviewed when preparing the mathematical tasks. After development of the GeoGebra-based mathematical tasks, more advice was sought and suggestions were made by Prof Ainley. In the light of her comments, necessary modifications and alterations were made and the final draft was piloted with four Turkish PhDs students majoring in

different subject areas at the University of Leicester and then translated to Turkish to be used with the actual research participants (see Appendix 6B, 6C, 6D).

Piloting

The pilot research participants were four Turkish PhDs students at the University of Leicester who were voluntarily involved in the GeoGebra learning experience in the winter term of 2011. The pilot study principally determined whether the teaching activities were appropriate for primary in-service teachers, and whether 4 hours was adequate time to assist them in getting acquainted with GeoGebra (they did not have any previous experience with GeoGebra). After the session, I took notes about my observations and subsequently revised the booklet to be used in the actual study.

The worksheets used in the exploratory phase were also piloted, to identify any shortcomings in the predefined activities for the PD model course, and to make the necessary modifications to the content and structure of the activities, without measuring their beliefs about tasks. The pilot research participants had all graduated from Turkish schools and so were familiar with the educational context. Therefore, they were able to offer useful feedback about the tasks regarding their functionality, relevance and suitability. During the pilot study, the tasks were heartily approved of and successfully carried out by the participants, and also provided an opportunity for negotiating meanings related to the pedagogical aspects of the specified activities. As a consequence of my observations I did make some small changes to the tasks for the main study. There are three main stages on the PD course; i) introductory, ii) exploratory, and iii) home-based exercises as follows:

<i>Stages</i>	<i>Content of the PD course</i>	<i>Hours</i>
Welcoming (session 1)	Welcome meeting and administration of pre-course mathematical beliefs questionnaire	1
Introductory (1-2)	Introduction to GeoGebra software application	4
Exploratory (3-8)	Working on GeoGebra-based mathematical tasks	16
Home-based	Working on particular tasks at home	-
Closing (8)	Thanks, meeting and administration of post-course mathematical beliefs questionnaire	1

Table 4.1: The content of the PD course

Introductory stage

For the study itself, as stated above, six mathematics teachers, all teaching pupils aged from 12-14 participated in eight weeks of workshops, as shown in Table 4.1. These aimed to provide them with the necessary experience to integrate mathematical software into mathematics education.

In the initial stage of the first workshop, I introduced myself to the participants and provided information about the general structure of the PD course. The introduction covered the technical activities required to help the participants obtain the basic skills needed for independent use of GeoGebra. During the introductory sessions, general information about the development and potential of DGS for teaching and learning mathematics was presented and discussed. This session, lasting 4 hours, was to help participants become familiar with GeoGebra. In order to achieve this, a booklet composed of six tasks was prepared for the participants. This booklet was given to each participant to allow them to follow the activities during the introductory stage. This includes following activities: Installation and Introduction GeoGebra, Basic Drawings, Constructing a Rectangle, Drawings, Construction and Drag Test and Equilateral Triangle Construction (For more description of these activities, see Appendix 6B). These activities helped the participants to control and use GeoGebra tools and the toolbar and this allowed them to understand their purposes. In the introductory stage, the participants also explored general features of the tools in the toolbar as a part of GeoGebra software application, and they learned how to use these tools when constructing and dragging particular geometrical figures.

Exploratory stage

The second stage of the PD course was based on six worksheets which described specific activities. Each worksheet included some activities adapted from the Turkish primary mathematics curriculum (for a detailed description these activities, see in Appendix 6C, 6D). The purpose of these activities was to place the participants in a practical context, whereby they could investigate and conjecture about GeoGebra-based mathematical tasks. The participants worked on these activities in pairs, without examples provided by me to tell them how the tasks were to be solved. When the group work was completed, they were encouraged to discuss their initial thoughts about the activities. The pair work was intended to help the teachers share their ideas about their

mathematical understanding. My role in this was only to initiate and prompt discussion or conversation; I acted as a participant observer.

Following the piloting phase, as described above, the final design of the tasks, including the rationale and a brief details for each task is presented below (For more information see in Appendix 6A):

The purpose of the first task was to enable participants to recognise how students can investigate the relationships/properties of parallel lines and polygons using GeoGebra tools, and how they can improve their understanding through using particular GeoGebra activities. It included five GeoGebra-based mathematical activities covering the following activities: angle relationships and properties of parallel lines, interior and exterior angles of polygons, the relationships between interior angles and the lengths of the sides of a triangle, and proof of the angle sum theorem.

The second task consisted of investigation into the construction of squares, regular hexagons and an exploration of circles and also a visualisation of Thales theorem. This task provided the opportunity to think about both the mathematical properties of the figure that they were to construct, and how to use the tools in GeoGebra to construct them. It involved four activities. The third task explored the parameters of linear equations and the concept of slope. Some activities in this task included an example of linking algebra and geometry with GeoGebra. A remarkable feature of GeoGebra is that it affords a dual view of objects: every expression in the algebra window corresponds to an object in the geometry window. In this task, participants can learn how to write equations, formulas and perform computations in GeoGebra. The aim of this task was to provide participants with an opportunity to see many examples of different parameters by pulling the slider with the mouse.

The fourth task consisted of an investigation of the relationship between the area and perimeter of the geometric figures. The purpose was to enable participants to recognise how students can develop an idea of the relationship between perimeter and area through GeoGebra activities, and to recognise how the examination of patterns can occur in open-ended activities. The fifth task consisted of an exploration of the Pythagorean Theorem from different points of view, using GeoGebra-based activities. The purpose was to enable the participants to recognise how GeoGebra would assist the

students in exploring mathematical ideas constructively. The participants discussed the task in terms of difference, first introducing it as a paper and pen activity and then as a GeoGebra activity.

The sixth task consisted of exploring the properties of transformations through GeoGebra-activities. The concepts of reflection, rotation and translation were introduced and explored relative to their properties and their impact on the properties of objects. In this task, participants were able to reflect, rotate and translate geometrical figures on a coordinate plane. The purpose of this task was to provide the users with a context in which the operations of geometric transformations were dynamically possible, and to enable them to recognise these transformations on the graphical window as feature of GeoGebra, so as to investigate their properties in ways not previously possible with pencil and paper.

Home-based stage

Between the group workshops the participants were required to continue working on a home based exercise in order to identify the practical results of the pair study, since they were able to apply their knowledge about GeoGebra. Here, the main aim was to provide participants with a “protected environment” wherein they would be able to examine their experiences. Then they were asked to present their solutions and findings to each other at the beginning of each workshop.

In such home based activities participants were given some responsibilities to practice their experiences so as to be ready to use those experiences in the classroom. In other words, the aim was to prepare teachers to apply their own projects with their own classes. Teachers were given an opportunity to obtain practical experience with the use of GeoGebra in a teaching context. In order to recognise the potential of teaching with GeoGebra, teachers were encouraged to work on GeoGebra based mathematical activities at home and then they were asked to articulate their solutions during workshop sessions. I and participants would then engage in some more discussion about the method for presenting solutions, in order to understand their pedagogical approaches to GeoGebra-based activities. Three activities were introduced to participants before they worked on them at home.

The first activity relates to constructing triangles and concerned the lengths of sides. The idea behind this was that teachers were given an opportunity to work on this task individually, and then provided a solution to this activity. The participants were able to perceive how the use of GeoGebra would facilitate student's learning with reference to the properties of triangles. The second activity in the home-based stage involved constructing a rolling circle. In this activity, participants were required to be able to connect the diameter of a circle to its circumference. This activity encouraged them to find the relationship between the angle and the radian measure. The last activity was based on how best to create a Pythagorean Tree (fractal). This activity led participants to think about the mathematical properties of the object that they are to construct, and also enquired as to how to use the tools in GeoGebra to construct it. These activities provided participants with an opportunity to recognise the potential of teaching with GeoGebra.

Planning and teaching approaches

I designed GeoGebra mathematical activities to exemplify how the teaching materials could be used effectively to support new practice. These activities could be used to reinforce participants' teaching in the classroom. The professional development course could then be considered as a well-designed environment in which the participants could identify an opportunity to investigate mathematical ideas and re-evaluate their conceptions about the subject matter, its pedagogy and learning by engaging with GeoGebra activities. Regarding the objectives of the actual study, my role in the PD course period could be characterised as a facilitator; this meant giving priority to the participants' freedom to articulate, explore and investigate mathematical concepts, rather than assessing solutions and conjectures.

My role initially was as a demonstrator of GeoGebra who introduced the activities and assisting the participants with their working together. I was interested in monitoring the completion of activities without attempting to intervene when they were working with activities. I provided some help by explaining and demonstrating how best to follow the worksheets and also how to use the tools in GeoGebra when they lacked confidence in the software application. However, I sometimes involved myself in their activities so as to obtain more information about their approaches and to discover why they implemented them.

As the PD progressed, I reduced my support so as to understand how competently and autonomously the participants would work in the GeoGebra-based mathematical environment. In the exploratory stage, the participants pursued the sessions through the worksheets in small groups, and I modelled for them the way I hoped they would use computers in their mathematics classrooms. After the initial sessions, I did not provide examples or explain them how the activities should be sorted out. I attempted to design a PD setting that was conducive to the participants exploring their own ideas, thinking independently, and implying their own ideas and creativity. In this regard, participants were given the responsibility to contribute to their own knowledge and skills with their own efforts. In fact, I was involved as a catalyst in the participants' discussion, and reporter of their experiences.

4.5 Sampling and Access

The issue of sampling is closely related to the generalisability concerns as discussed in the preceding section. In case studies, "nothing is more important than making a proper selection of cases" (Stake, 1994, p. 243). Purposeful sampling is therefore mainly used to select the participants. In purposeful sampling, selecting participants depending on the needs of the study is the key. The criteria for selection are as follows: a) Mathematics teachers need to have at least six years teaching experience (purposeful), b) Teachers were willing to participate in the study (volunteering), and c) All teachers were chosen from different schools in the same city (convenience).

The current study has purposive sampling because the selection criteria were that the teachers should be working in Turkish private and state primary schools when mathematics curriculum was introduced in 2005. The rationale behind the decision is that it would help to become aware of the philosophy behind curriculum change and its effects on the teachers' understanding. I intended to have at least four participants who teach in 12-14 age range from different schools in order to provide an environment where they can work both in pairs and groups. There were four teachers from the State Primary School and two teachers from the Private Primary School and in total six teachers' interactions were observed and they were interviewed before, during and after PD course. Although each teacher brought their own understanding of mathematics, its teaching and learning into PD course, common qualities among participants were investigated and discussed throughout this study. So, it was considered ideal to have at

least four teachers so as to triangulate each teacher's interactions. Both the state and the private primary schools are governed and monitored by the Ministry of National Education (see section 2.2).

I wrote a letter of application to the Education Counsellor of the Turkish Embassy to secure permission to work with primary mathematics teachers in Turkey (Appendix 1B). Some details of the proposed study were also attached to this letter of application. I have received an approval letter from MONE in order to conduct my research in Turkey. They requested details of my research results at the end of the research. The teachers were based in Kahramanmaras, a city in southeastern Turkey in the Mediterranean region. One of the most important reasons for choosing this particular city is that a colleague of mine, who was working as a teacher there, helped me contact volunteer teachers. My colleague could potentially help me to talk to these teachers about the value of taking part in the study as they had busy schedules in their schools (both private and state).

Having been granted the permission to have access to the teachers in Kahramanmaras, I paid a visit to the schools where they were working at the time of the study. First I had an appointment with the school headteachers to provide them with some details of my study. The principles kindly accepted to request mathematics teachers in their schools for their participation in my project. The teachers agreed to attend the course on Wednesday afternoons for the period of 8 weeks on a voluntary basis. The volunteer teachers and I also came to an agreement that the study would be carried out in another school's computer cluster. Although I intended to work with 7 teachers in the first place, one teacher had to withdraw due to his health problems. There were 1 female teacher and 5 male teachers in the group. More information about these participants is provided in the form of mini-profiles in section 5.1 and see also table 4.2.

Participants	Age	Education	Years of experience	Type of school	Software courses taken
Eysun (Female)	32	B.S in Maths with Teaching certificate	10	Preparator y and Private schools	Office programs
Emin (Male)	28	B.S. in mathematics education	6	Public school	Office programs Mathematica 5 and Fortran
Muslum (Male)	33	B.S in Maths with Teaching certificate	10	Private school	Office programs
Musti (Male)	29	B.S. in mathematics education	6	Public school	Logo, Cabri, Derived 6
Celal (Male)	35	B.S in Maths with Teaching certificate	11	Public school	Office programs
Asim (Male)	28	B.S. in mathematics education	6	Public school	Office programs, Mathematica 5 and Fortran

Table 4.2: The characteristics of the participating teachers

4.6 Data Collection Methods

It is important to note that “case study research is not limited to a single source of data in collecting case study data” (Yin, 2006, p.115). The idea behind this is to strengthen the evidence and findings as much as possible. Instead of relying on a single technique, it is often the case that studies into teachers’ pedagogical belief systems employ a combination of techniques for obtaining data like interviews, observations, diaries and questionnaires.

Four main different sources of data were used to examine the course participants’ initial beliefs and their beliefs on the basis of their interaction with the activities, the researcher and their peers in the PD course. Interviews were carried out and the participants were asked to write their reflections on the course activities after the sessions, and record their observations. I took also some field notes about my observation of participants’ discussion and interactions during the workshop sessions. Moreover, participants were requested to complete a questionnaire before and after the course. The point of relying upon a variety of methods to assess teachers’ beliefs was associated not only with the idea of overcoming the weaknesses of any of the methods and cross-checking results, but was also with the assumption that beliefs are reflected in

different ways in different situations. In the following, a brief description of the different methods used and relevant information about the details of the data collection are provided.

4.6.1 Interviews

An interview is a tool for particular questions to be proposed by the researcher who manages the line of questioning so as to acquire a certain response (Creswell, 2007). As Legard, Keegan, and Ward, 2003: p.139) wrote, the power of language provides access to the meanings people give to their personal and social experience. Interviewing is one of the most influential techniques employed in an effort to comprehend individual's perspective, beliefs and values. As a result of its interactive nature, interviewing has many benefits over other kinds of data gathering methods such as questionnaire (Best and Kahn, 2003).

There are a number of interview types with different terms used. Bogdan and Biklen (2007) categorise interviews as structured, semi-structured and unstructured interviewing. Data generated by structured interviews characterised by predetermined questions and fixed response categories may be perceived as impersonal, irrelevant and mechanistic (Cohen et al., 2007). This type of interviewing does not fit the scope of my exploratory, descriptive and collective-case study. Furthermore I call for the data not only to be comparable among cases but also exploratory which means there is room for informants to articulate their ideas without any impact through directive questions. Semi-structured interviews were therefore regarded to be valuable for my study as some wording of opening questions could be defined previously (Patton, 1990).

Each course participant was interviewed before the PD course, midway through the PD course, and after the final workshop session. These pre-, mid-, and post-course interviews were semi-structured, and confirmed appropriate flexibility in the arrangements to enable promising lines of development to be followed up and probed in more detail. All interviews (18) with six participants were audio recorded and transcribed, translated, and lasted at least 40 minutes.

Semi-structured interviews

Each of the interviews was of a semi-structured nature, contingent upon a group of common questions to be investigated with each informant, but enabling enough space

for probing and having the interviewees express their views in their own way (see the three interviews protocols in Appendix 4). Each interview followed three procedural steps outlined as follows: 1) Introduction aims at establishing a good relationship amongst myself and the informants and at promoting them to answer following questions; 2) Elicitation concentrates on the parts under examination; and 3) Conclusion provides the participants with the opportunity to add any further comments. The advantage of taking these steps was that detailed verbal commentary was elicited which afforded some insights into the participating teachers' beliefs and into the reasons why they responded to the PD course as they did. This was important because it was perhaps the first time for the participants to articulate their views about the subject matter, its teaching and learning.

It should be mentioned that the interviews had three main foci 1) teachers' beliefs about the subject matter, 2) what it means to do mathematics, the purpose of teaching mathematics, how a student learns mathematics, and 3) participants' experience about technology use in educational context in general, their beliefs about the function of computer in education as well as their expectations from the PD course. It should be noted, however, that while all interviews consisted of the same three parts of enquiry, the particular questions referring to these parts were not essentially the same as the underlying principles of each interview is different.

First interview

The first interview was carried out previous to the PD course to ascertain the participants' professional experience and their beliefs about mathematics, its pedagogy and learning, and computers, both in general and in the setting of mathematics education. The first-interviews were also used to assess the participants' expectations and motivations from attending the PD course. One rationale of using the first interview was to obtain positive and/or negative moments of learning mathematics as a student.

In order to understand participants' initial conceptions about the subject matter, its pedagogy and learning, and the role computer in mathematics education, they were asked some of the following interview questions:

- What is mathematics? What initially comes into your mind when you hear the word “mathematics”?

- Could you describe how did you learn mathematics in your school years? How can a student learn mathematics best regarding to your experiences?
- What are the steps you, generally, would want to follow in designing mathematics lesson, and I would like you to describe the way your mathematics classroom is organised?
- What are in your opinion the main purposes and reasons of teaching mathematics in primary school?
- Are there any differences between teaching a particular topic using pen-paper and using the computer in mathematics classroom?
- What motivated you take this course? Or what are your expectations from this course?

Since the participants' beliefs about the role of computer in mathematics were hardly addressed by the mathematical belief questionnaire, it was necessary to concentrate on the participants' beliefs about computers during first interview. These beliefs may provide an active driving force for change, if any, in their mathematical beliefs, or restrain and limit change efforts. In order to compare the participants' responses in the first interview to the second and third interviews, similar issues associated with the nature of mathematics, its teaching and learning and computer in mathematics were explored in all the interviews.

Second interview

After the fifth workshop sessions, the participants' beliefs, expectations, and perceptions about the subject matter, its pedagogy and student learning and computers in mathematics contexts were generally explored as in the first interview. The second-interview aimed at capturing first participants' responses to their involvement in the PD course and the participants' perceptions of the value of a GeoGebra-based learning environment regarding learning and teaching of mathematics. The participants were asked to describe their impressions and thoughts about the intervention strategies used in the exploratory stage. I was also interested in highlighting possible shifts in participants' beliefs by engaging them in medium-term retrospective thinking of how they handled their participation in the PD course.

As an aid to communication during the second and third interviews, a laptop with GeoGebra was prepared for the teachers. During these interviews, I sat at one side of the table with a computer, a tape-recorder, and activity worksheets used during the previous sections of the course. They were invited to demonstrate their thoughts and ideas about GeoGebra-based mathematical tasks and related episodes that came up in the discussion and described their reaction so far to the PD course. This method was an interview aid and helped me understand the ways in which they intended to use GeoGebra and its related teaching and learning tasks or activities. The conversation was usually started with the question: Was there any time, event or episode which made you feel good (positive) or bad (negative) during GeoGebra based mathematical activities?

- Did any aspects of the PD course change the overall philosophy of learning and teaching?
- What do you think about the potential relevance and application of the course activities in the classroom?
- What aspects of the GeoGebra-based PD course encourage you to use in your classroom?

In order to elicit participants' experiences in the environment and responses to the PD course by getting them to tell their stories of the course and by getting them to explain what these events; the second and third interviews were used as the primary data source.

Final interview

The purpose of conducting this interview was to promote the participants to consider the principles of learning and teaching that they had developed during the second and the third stage of the course. It was expected that this would provide the participants with an opportunity to analyse the course activities with the researcher, and to consider their mathematical beliefs. The aim was to look at how the participants reflected their own point of view about mathematics, as well as teaching and learning of mathematics with GeoGebra activities through group work.

The final interview aimed at capturing the outcomes of the GeoGebra-based PD course, namely in terms of the changes in the participants' beliefs, their degree of satisfaction with the PD course, and the perceived value of GeoGebra. Therefore, the final interview had two main focuses: (1) the teacher's general impression about and reflections on the

PD course activities. (2) Changes in the teacher's beliefs and their professional development.

In the final interview, the process of initialising the conversations was the same as in the second interview, the participants were asked to tell their good/bad incidents of the PD course in general and the sixth, seventh and eight workshop session in particular. Further questions were asked when appropriate. Some of these questions were to comprehend whether and to what extent they appreciate computer-based GeoGebra activities, and how a GeoGebra-based mathematical environment affects their views on teaching and learning mathematics and their confidence in teaching mathematical ideas with GeoGebra. Some of these questions were directly related to the participants' beliefs, expectations, as well as their mathematical beliefs. Their responses illustrated the extent to which they recognise some relationships between GeoGebra mathematics and school mathematics.

The interview was generally started with the questions: Could you define a positive and negative incident you had last workshop sessions? What can you say about it in terms of students' learning and classroom implementation from your point of view as teacher? So as to comprehend how the participants' beliefs about the nature of mathematics evolved during the course, the following question was planned: "In what ways has the PD course affected your beliefs about mathematics?"

4.6.2 Observation

Observation gives the researcher the opportunity to observe real situations rather than questioning participants about a particular issue (Patton, 1990). If enough time is spent with the participants, socially as well as in the course environment, observation can be a key tool for collecting data. By adopting an active role the researcher can develop an understanding of their patterns of interaction with one another and engagement in the course behaviour based on the evidence collected through observation.

Gold (1958) categorises the four type of observer as follows "complete participant", "participant as observer", "and observer as participant" and "complete observer" (Gold, 1958 cited in Punch, 2005). According to Jaworski (1991), a balance must be struck between these roles due to the fact that becoming a full participant could result in losing the status of observer in some situations, while it is not possible to become a participant

in other situations. One disadvantage of this technique is the difficulty associated with the recording of data. The difficulty with observing and recording may cause the loss of some data needs to be recognised. Also, being observed by the researcher may be perceived by the participants as something disquieting. By its very nature, participant observation has some ethical issues (see section 4.8): the investigation should not be carried out in a covert manner; participants should be notified of the scope and nature of the research. Alternatively, participant observation carries with it the worry that the presence of the researcher may impact the way in which the participants act. Participants may be doubtful of the researcher and unwilling to participate or be eager to please the researcher; they may interject their own impressions and biases etc. The personal rapport between researcher and participants may also influence the interaction (see limitation section 7.4). This needs to be emphasised cautiously during the fieldwork.

As the tutor of the PD course, I acted as a “participant as observer”, thus having the opportunity to observe and record their activities and reactions to these activities throughout the workshop sessions and how their mathematical views and then closely examine which of these views are modified or remained the same throughout the course. However the difficulties associated with the roles of course tutor and observer should be acknowledged. To move to a position of observer while taking the role of a course tutor was my new experience. Therefore, although during each session I was able to take some field notes, these were rather scarce. Hence, my concern was that of finding the routine of completing the field notes as soon as possible, and of reflection upon the sessions.

I used a cluster of premeditated activities by myself (see Appendix 6B, 6C) which were quite useful in framing my perspective to appreciate the participants’ engagements in the setting. I joined in discussions during pair work and whole classroom discussion sessions. This involvement took the form of monitoring and facilitating the exchange of ideas with the course participants during the activities and transcribing it soon afterwards. This approach also enabled me define the course setting and explore more broadly participants’ beliefs through their engagement with activities, and interaction with one another and the researcher.

4.6.3 Questionnaires

Questionnaire is a method of collecting information by asking people in some structured format. It can be employed both as a quantitative method or an interpretivist method conducted to advance scientific knowledge or to improve theory, in terms of the unit of analysis used (Grover, 2000). It is not surprising that questionnaires can be used for many different purposes and to answer many different kinds of research question. Questionnaires do not have to include large numbers of people but the number of participants might be small. Traditionally, questionnaires have been implemented in a group context for suitability. The investigator can provide the questionnaire to those who are present and be confident that there will be a high response rate (Denscombe, 2007). Informants can demand elucidation when they are not sure about the meaning of a question.

Even though it was planned to involve more teachers for the study, I was able to reach six teachers. Although questionnaires were employed to generate data in the research design, the data gathered through questionnaire was not entirely reported in the findings chapter of this study. I did use the questionnaire data partly in the findings, therefore, it is important to describe and rationalise the choice of questionnaires. The data derived from the self-administered questionnaire provided a measure of the participants' beliefs and of their response to the PD course which complemented the more interpretative data derived from the interviews. The questionnaire was also intended to obtain ideas about each teacher's beliefs regarding mathematics, its teaching and learning. This questionnaire data also offered results that allowed for more of a snapshot and overview between the participants' beliefs and changes in them.

Before and after the PD course, and prior to being interviewed, all the course participants completed the mathematical beliefs questionnaire. In creating the questionnaire, I sought to address beliefs about the nature of mathematics, the teaching of mathematics, predominantly those coping with what it means to do mathematics and what is significant for learners to recognize about mathematics. The questionnaire contained 36 statements had previously been utilised in former research and validated (Barkatsas & Malone, 2005; Barlow & Cates, 2006; Goos & Bennison, 2002). The mathematical beliefs questionnaire was divided into three parts (see Appendix 3). Each part comprises 12 items related to subject matter and beliefs. Half of the statements

were structured progressively (corresponding constructivist ideas), and half in a traditional format (corresponding traditional ideas). This is considered a useful means to enhance the tool with a solid theoretical basis, and to explore whether the data reflects the theoretical constructs. For example, item 18 stated that “good mathematics teaching involves class discussion in which students share ideas and negotiate meanings”, representing the former view. The latter view is reflected in item 17, which states that “good mathematics lessons progress step-by-step in a planned sequence towards the lesson objectives”. A five-point Likert-type scale was integrated with answers ranging from strongly disagree (1) to strongly agree (5). Additional comment boxes were added in each section in order to obtain more information about beliefs.

4.6.4 Participants’ Writings

Documentary data are useful in strengthening the quality of the data collected by interviews (Punch, 2009). Janesick (1999, p.522) argues that “journal writing allows participants [...] to provide an additional data set to outline, describe, and explain the exact role of the researcher in any given project”. The usefulness of writing a journal stems from the fact that the participants are offered an opportunity to write their stories without intrusion and within a less structured research environment. This means that the participants are free to write what they want or do not want.

Apart from the data collection methods mentioned in the above sections, I also asked the participants to use the journals to reflect and share their experiences, beliefs, feelings, and thoughts after each PD course sessions. These reactions were in the shape of journal-writing and founded on the experience of learning and teaching, on the roles of learners and teachers in the classroom and on intervention strategies used in the PD course (see sub-section 4.4.2). To understand their reflections on how new learning experience gained through workshop (course) sessions affected their philosophies or beliefs of learning and teaching. In order to give the participants a structure in their writing, journal entry was guided by four questions to which the researcher wished the participants to respond: Could you describe any moment, event or episode which made you feel good/positive or bad/negative during this learning experience? What do you think about my role and approaches during the task sections? What mathematical ideas did you use during the sessions? From your perspective on teaching and learning what

would you say about the activities and how can you adapt and extend them for your students?

After having completed the course sessions, participants were requested to record their reflections on their experiences in the course, and discuss ideas about the potential relevance and application of their activities in their current mathematics lessons. From a research perspective, the aim was to obtain implications from writings, such as how they regarded the PD course as being precious or contrary to the learning and teaching of mathematics, and what implications they had about the course and computer use in the Turkish educational context. In summary, observations, interviews, questionnaire and journal-writing are all tools that allow the researcher to examine the interactions between mathematics teachers' beliefs and their activities, in terms of what they say and how they expect to act (i.e. idealised practices that the teachers want to implement in their own environment).

SESSIONS	ACTIVITIES	DATA
1st Session (20 April 2011)	Welcome meeting Introduction booklet	Pre-course questionnaire First interview
2nd Session (27 April 2011)	Introduction booklet	Observation Participants' writing
3rd Session (4 May 2011)	Worksheet 1	Observation Participants' writing
4th Session (11 May 2011)	Worksheet 2	Observation Participants' writing
5th Session (18 May 2011)	Worksheet 3	Second Interview, Participants' writing and Observation
6th Session (25 May 2011)	Worksheet 4	Observation Participants' writing
7th Session (1 June 2011)	Worksheet 5	Observation Participants' writing
8th Session (8 June 2011)	Worksheet 6	Post-course questionnaire, Participants' writing, Final interview

Table 4.3: Data Collection Schedule

4.7 Data Analysis

In this section, I will outline the process of analysing the interviews/reflective writings and observational (field) notes together and then I describe how I went about analysing the questionnaire data.

4.7.1 Interviews/reflective writings and observational (field) notes

Analysis refers to a continuing process of ‘giving meaning’ to impressions about data (Stake, 1995, p.71). Qualitative data analysts tend to generate data by interpreting what they see or hear from the participants throughout the field work (Denscombe, 2007). Researchers (Bryman, 2004; Froggatt, 2001; Silverman, 2000) describe this as an “iterative” process which suggests an interrelationship between collecting and analysing data. For instance, if an emerging issue came up during the data collection, the researcher would think this through whether it is worth being followed up.

In this study, there are three types of textual data: 1) observational field notes 2) interview transcripts 3) course reflections of each participants. In the case of the current research, the subsequent steps were taken to analyse the data:

- Preparing the whole data set for analysis
- Coding
- Categorising
- Identifying broader themes

The initial step in the analysing process involves making myself familiar with each data set by re-reading the whole transcribed data and this was done in Turkish. By doing this, I became more immersed in the data. This made it possible to obtain a gradually deeper understanding of each participant’s perspective, standpoint and provided me with an overall picture and then I started to translate into English any slice of data which became substantial. While translating data, forward and backward translation approach was used with help of a Turkish colleague who was qualified in both English and Turkish languages.

Coding is the starting activity and a fundamental stage of the data analysis. In the coding phase, labels were given to units of meaning in the transcribed data including interviews, reflective writings and my observation notes. This process was conducted as

a way of reducing data into simply manageable proportions. Charmaz (1983) described ‘codes’ as a “summarizes, synthesize, sort many observations made out of the data” (p.112). An example of the coding process based on an interview transcript is that Asim’s statement that “we keep the memorising and learning in one side as a conclusion of the studies aimed at exam” was coded as ‘memorisation’ and ‘control of exam system’ see more examples in Appendix 5A and 5B. However, during the coding process, disagreement occurred in the teachers’ particular statements in the interview data. For example, one of the course participants stated that *“If we explain the abstract concepts by concretizing, the level of learning and liking the mathematics will increase. If we explain the formulas we give on the blackboard by exemplifying, they will understand and learn better. In my opinion exemplifying things is so important in this respect.”* The second coder coded the above statement as “the importance of exemplification” whereas the same statement was named by the researcher as “enjoyment”. The differences in coding stem from some words with different meanings used by the participants. When I looked at this particular statement much more closely, I noticed that this sentence contains the words of “learn better” and “understand”. The intention of this participant seems to be related to learning mathematics through exemplification, students can build mathematical ideas better and the enjoyment of mathematics will increase. If I labelled this sentence as the importance of exemplification, that would be inadequate to capture the participant’s actual intention. In order to justice to the participant’s intended meaning, we agreed on the code of enjoyment and building mathematical ideas. Another example is the participants’ particular statement in the reflective writing:

During my university years, I also did not learn mathematical ideas which I was supposed to use and teach them in the classroom. I was not taught “the proof of the sum of the angles in a triangle is 180 degrees”. Yet, it has not been asked until now. I learned this proof by working and discussing with you and my colleagues. When a student asks me why and how to prove this theorem, my answer is already ready. This became part of my knowledge.

This paragraph initially was coded as ‘the importance of collaborative learning’. However, it seemed to me that this code was a bit more general, so I thought this needed to be refined. I then thought that if I labelled this sentence as more general word, this would create a sense of confusion about coding process in this study. I attempted to overcome this difficulty in naming this sentence as “a peer interaction as a tool for better understanding”.

In the subsequent phase, that is categorising, links between codes were identified. I paid attention to the way in which I coded participants' answers to a particular question and to the way in which I coded an individual participant's answers across the entire data (Braun and Clarke, 2006). Moreover, I began to create excel sheets related to the three major areas under investigation (e.g. beliefs about the nature of mathematics) for each participant (see Appendix 5A). For each area, I used a different colour in order to gather all the same coloured chunks of texts. The summary sheets were also created and used to assemble data from each of the participants, gathered from the synthesis of the interview transcripts, my observation notes and the course reflections. Categories emerged from the data through cross-case analysis and from the researcher's previous theoretical understanding of the phenomenon under study (e.g. literature reviews) and relative to my research questions. The categorisation process led to the creation of the following broad types of beliefs regarding: a) initial beliefs about mathematics, its pedagogy and student learning and computer b) experiences with the PD course c) changes in beliefs. Finally, concerning the identification of broader themes, after the process of coding and categorising the data was completed, broader themes were tracked according to the research questions of the study.

Reporting the data analysis/findings consists of two main parts: 1) Introducing each individual participant and providing some impression about them by writing mini-profiles for each teacher, as proposed by Hoyles et al. (1991). This involves describing the distinctive characteristics of each teacher's beliefs and detecting the changes in his/her belief structures. These profiles served as background basis for the cross-case findings. The first step in the process of developing the case studies was to attempt to synthesise my observation notes, the interview transcripts, and the course reflections. During the writing of the case studies, I tried to identify some important themes in the qualitative data, and tried to elaborate them under sub-themes (see Table 4.4) throughout the case studies. 2) Identifying themes relevant to all six informants through cross-case analysis. Patton (1990) suggests that "the qualitative analyst's effort at uncovering patterns, themes, and categories is a creative process that requires making carefully considered judgements about what is really significant and meaningful in the data" (p.406). The method of constant comparison (Glaser and Strauss, 1967) was employed in the cross-case analysis in order to identify patterns within the participants as well as across all six participants. Through identification of patterns across the

participants, the summary sheets provided a rough image of beliefs change for each participant. The participants' verbal commentaries together with their answers to the beliefs questionnaire before the course started were used to figure out their interactions with the course, and simultaneously these interactions were used to comprehend their beliefs.

Views of mathematics
Pragmatic view of mathematics
Past experiences with mathematics
The role of teacher and learner
Participants' previous experiences with computers
Contextual factors
Reflections on pedagogy
Reactions on the course activities
Shifts in the beliefs
Expectations about the PD course and reactions on researcher's role

Table 4.4 Initial themes

These initial themes include overlapping themes which emerged from the case studies. In developing the cross-case analysis, more general themes had to be derived. These themes were distinguished in terms of the participants' initial beliefs and their experiences with the course and changes in their beliefs and combining some of themes such as contextual factors and the role of teacher and learner. Therefore, combining and elimination through cross-case analysis was thought to be necessary. Having this refinement through combining and elimination, I decided to elaborate the main themes which emerged from the data.

Teachers' initial beliefs	Participants' experiences	Changes in beliefs
Nature of mathematics	Participants' expectations of and reactions towards the PD course	Changes in the participants' beliefs about computer
The importance of mathematics	Reflections on the PD course activities	Changes in the participants' mathematical beliefs
Approaches to teaching and learning mathematics	Reflections on social interaction and pedagogy	
Computers in mathematics education	Linking GeoGebra with mathematics	
	Time as a key issue	

Table 4.5: Main themes

4.7.2 Analysis of Questionnaires

Questionnaires were employed to evaluate participants' beliefs about mathematics, its teaching and learning (before the course starts) and to determine changes in their beliefs (at the end of the course). Quantitative research utilise surveys, questionnaires to assemble data that is reviewed and tabulate in numbers, which enables the data to be described by the use of statistical analysis (Hittleman and Simon, 2006, p.31). This type of data analysis technique was thought to be inadequate for the nature of this research. On the other hand, participants were not able to express their beliefs in an explicit way. Therefore, mathematical beliefs items in the questionnaire were based on the literature review would help the researcher to categorise participants' beliefs in terms of constructivist and traditional views. General conclusions can be drawn from the questionnaire data. The qualitative discussions echoed views similar to the questionnaire findings, so provided some confirmation.

Since the item scores ranged from 1 (Strongly Disagree) to 5 (Strongly Agree), and there were 6 teachers who completed questionnaires both pre-course and post-course, the total score on any item could vary from 6 to 30. Goos and Bennison (2002) note that the magnitude of scores is important for indicating the degree of support for each statement and they identify scores of 24 or more, and 12 or less, as indicating general agreement and general disagreement respectively with particular statements. That is to say, they identify the mean score of 4.00 and higher as general agreement and of 2.00 and lower as general disagreement. However, I identified an average score across the teachers of 3.4 as indicating general agreement with a statement, and 2.6 as indicating general disagreement because a five-point scale contains 4 intervals and 5 categories with the ratio 4/5 being equal to 0.8 (Aydin and Tasci, 2005).

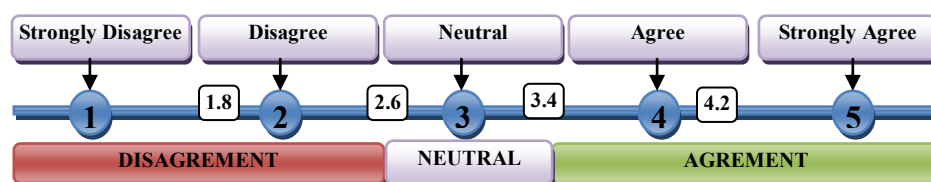


Figure 4.1: A Scale for Determining Agreement and Disagreement

4.8 Reliability and Validity

Certain steps were taken to increase the level of reliability and validity. With regards to the former, the interviews were conducted in Turkish, translation of wording might create problems about reliability. To increase the comparability of responses, similar wording in interview questions was used (Cohen et al., 2007). Where language issues arise in the translation, they were re-checked by a Turkish colleague who specialised in English language teaching. To check inter-coder reliability, the codes in some portions of the reduced data were cross-checked by the second coder with an educational research background to ensure consistency. The results of the coding and categorisation of the second coder and that of the researcher largely matched one another. However, there was also disagreement when making decision with regard to categorisation. When codes are applied to my data sets, I attempted to classify similar codes into initial categories. The teachers' particular statements in the interview data were labelled by the second coder as "pragmatic views of mathematics" whereas they were labelled by the researcher as "the importance of mathematics". The latter category does cover the former one. The importance of mathematics was therefore used while applying coded the data sets into categories.

With regards to the latter, to enhance the validity of the study, I reviewed the interview transcripts and participants' writing several times. Peer feedback (also known as peer debriefing) is the most crucial method for establishing credibility (Lincoln and Guba 1985). This technique was used to strengthen the quality of the data analysis and interpretation. Participant feedback and method triangulation were also used to make the study more valid. For the triangulation of method purposes, reflective writings were gathered to complement the comments made by the teacher in the interviews. Participant feedback was received by asking the participants to review the summarised data (Creswell, 2007; Johnson and Christensen, 2010).

4.9 Ethical Issues and Concerns

Ethical issues such as harm, consent, privacy, confidentiality and deception need to be thought about carefully throughout investigation (Punch, 1994 cited in Punch, 2005). These issues will be explained below with reference to the features of the current study.

With regard to harm, attention was paid in order not to cause any loss of self-esteem and stress (Bryman, 2008, p.118) when they are assigned a particular task during the course. Psychological and social risks were eliminated during the research process. I tried to respect personal preferences. For instance, in the first place one female participant wanted to sit and work alone during the PD course. This was quite understandable as she was the only female participant in the group. After building trust and good relationship with other participants, she agreed to study with her peers at later stages of the course.

Concerning consent, privacy and confidentiality, the personal characteristics of the participants were kept confidential so that they cannot be identified by readers of this research (Cohen et al., 2007). Keeping the participants' names and distinctive qualities their anonymity was preserved at all stages of the present study. I did not give any specific information about any individual teacher in the course to their headteachers in their own schools. The data were collected from volunteer teachers (see sampling in section 4.5) and they were informed about their right to withdraw from data collection process at any stage without stating any reason. The participants were also informed about the phases of the study, and identity of researcher, of course not with all details. Consent letters were delivered to each participant to request their participation in the study on a voluntary basis, and acknowledgements were received from each participant (English version of participation letter can be found in Appendix 2). This enabled the participants to be informed about the research project and the identity of the researcher. No attempt was made to gather data about the private life of school teachers and the participants will not face any questions or observations that are related to their privacy.

A further ethical consideration is deception (see also section 4.8: Trustworthiness), which refers to "the representation of the research as something different than what it is" (Bryman, 2008) was also addressed. A mutual understanding between the researcher and the participants about the general research purpose and the researcher's identity will be established through the signed informed consent forms to avoid deception in my project. The researcher must then think deeply about what counts as data before collecting data in order not to distort the data. For instance, some participants may say something after the interview has been completed, or may request that they make some off-the-record comments.

4.10 Chapter Summary

This chapter has presented and rationalised the theoretical perspective, research approach, research methods, and its procedures used in this study which looked for exploring participants' initial beliefs and the effects of professional development on their beliefs regarding mathematics, its teaching and learning and the use of technology. This chapter also included the rationale and design of the GeoGebra-based PD course. Descriptions of processes of coding and analysis were provided to strengthen trustworthiness and transparency. The results of the data gathered from six participants through their involvement with the PD course are introduced in the next chapter. The next chapter concentrates on the cross-case findings, and mini-profiles of six teachers respectively.

CHAPTER 5: FINDINGS

This chapter presents the findings of the study about changes in teachers' beliefs throughout a GeoGebra Professional Development course. It will focus on the participants' stated beliefs and reflections over the period of the duration of the PD course. The findings will be introduced in four main sections. In the first section, the mini-profiles of six Turkish primary mathematics teachers who took part in the GeoGebra-based course are presented. These mini-profiles introduce them as individual participants and highlight their educational background and the critical issues which result from their participation in the PD course. The last three sections address the cross-case findings of the study. It begins to depict and elucidate what seemed to be similar themes: (a) the participants' prior beliefs about mathematics, about mathematics teaching and learning, about computers (b) the participants' reactions to and engagements with the PD course activities; and (c) the changes in the participants' prior beliefs.

5.1 Six Profiles

The profiles described here are of six Turkish primary mathematics teachers. Analysing individual teachers might help readers to better understand of change process due to the nature of complexity of participants' interactions with course activities. The profiles served as background basis for the cross-case findings. In this section, I provide some impression of each of the participants as I observed them during the course and as they expressed themselves during interviews and through their reflective writings.

5.1.1 The Case of Eysun

Eysun has taught primary and secondary level mathematics for ten years. She has a bachelor's degree in mathematics and considers herself to be reasonably confident with advanced mathematics. Before working in a private school, she was a teacher in a preparatory school (aged 15-18) for seven years. Her work in this type of school, a university preparation course, is an important aspect to take into consideration in examining the case of Eysun. The characteristics of preparatory schools are quite different from those of state and private schools, and they give priority to exams consisting of multiple choice questions for those who want to enter university: teachers see their entire teaching as geared towards preparing students to be successful in the

examination. Being a teacher in a private school also might mean more subordination to both the school principle and students' parents than in teachers in the public school. Students' families have high expectations from Eysun regarding teaching practices. She had no experience of computers prior to the PD course.

At the outset, Eysun described her role as being an authority in the learning environment. She characterized the teacher's profession as "teaching students to learn mathematical ideas addressed in the curriculum" [Eysun, L92/I1]¹. She holds the idea that the teacher is concerned with transmitting mathematical knowledge and ensuring that students mastered mathematics and are able to do what the teacher has told to them in the classroom. This method of indicating to learners what to do and having them pursue this was a regular approach for her. During the PD course, frustration seemed to have originated in the conflict between the PD course's concentration on self-direction and Eysun's tendency to view learning as primarily a matter of being told what to do and pursuing that. That is, Eysun was expecting a traditional mode of lesson in which I was going to give more details about the task and give more direct answer to problems. She criticised my teaching style which was based on providing only clues and asking purposeful questions instead of giving ready-made answers. The course environment did not form a basis for Eysun to have the opportunity to use mathematics in an integrated and purposeful way and to generate new perspectives about the subject. As an educator, Eysun considered the course activities and implications as contrary to the reality of her classroom. These, in general, were not compatible with her teaching mode, which is based upon conveying knowledge within a controlled setting. However, she approved of some of the GeoGebra activities presented in the course, such as parameters of a linear equation, construction of a square activity and transformation of objects.

Eysun described herself as entirely computer illiterate. She was not confident with any particular software programme although there were computers available in her school. She expressed initial views that computers are not an essential component of learning and teaching mathematics. This was clearly illustrated by her following comments: "If a teacher teaches well with using pen and paper or expository way, then she/he does not necessarily use computer in her/his teaching" [L117/I1]. One may say that the appeal

¹ [L92/I1] refers to starting line number (L92) in participants' interview transcript within the first interview (I1).

that mathematics held for Eysun as a teacher was based on traditional paper-and-pencil exercises of an imitative nature. Afterwards, she seemed not to be aware of any potential for using computers in the mathematics classroom. However, her expectation from the PD course was to incorporate computers into her teaching as a chance to bring some fresh air to her professional life. After completion of some sessions, she wanted to use computers with her pupils but her inclination was to set up a “computer club” as a separate activity in which some students might learn to use a computer in mathematical context after the regular classroom period. As such, Eysun evidenced little realisation of the potential of computers for creating different kinds of learning environments.

After the course most of her articulated opinions about the computer maintained the same as they had been at the beginning. Specifically, Eysun thinks of the computer in learning as an extracurricular activity in order to enrich her teaching with regard to developing student learning. In general, she did not appear to have moved far away from her initial perspectives about teaching and learning mathematics. At the end, as at the beginning of the PD course, Eysun appeared to be willing to extend her beliefs and practices connected with mathematics teaching; but she knew there was little that she could do to alter the circumstance:

My role is to use mathematical ideas addressed in the curriculum to teach and is to do more practice by solving many questions. Sometimes I prefer listening students' ideas. Mostly, I have students solve a lot of problems. I do my best to prepare my students for exam. I tell lessons based on practice... [L89/I1]

Whatever I think about mathematics teaching there is a mathematics curriculum that I have to obey. I don't believe [in] the way I teach mathematics. I make that mistake myself; I want to teach maths that should not be related to what I do now but to what my thoughts are. I would like it if mathematics teaching was more investigational, but for a variety of reasons, such as time and exams I'm going to be following a truly deductive method. It is going to be like... I tell them.... [L484/I3]

5.1.2 The Case of Emin

In his late 20s, Emin teaches pupils aged 12-14. He is a primary mathematics teacher with six years of experience in a state school in an urban area. He is well-qualified mathematically, having a bachelor's degree in mathematics education. During pre-service teacher education, he took some basic computer literacy lessons for two hours a week for a year, but was not taught lessons aimed at teaching mathematics with a computer. After he became a teacher in 2005, he attended some in-service courses

which focussed on the introduction of the new curriculum objectives and the development of teaching materials for classroom use. He heard about the course from the colleagues of his school although he was personally motivated to enrol on the course. Emin seemed to hope that the PD course would offer particular information about computers, so that he could build on his computer skills in order to integrate computers into his mathematics teaching. Later, he pointed out other aspects of the course, particularly those involving new ways of learning and teaching mathematics. This is well demonstrated by his comment during the last interview:

This course has not only provided me with an invaluable experience learning different ways of teaching, and new learning models, but it has also made me aware of my deficiencies... I questioned myself about effective mathematics teaching and realised that I am inadequate when teaching... [L343/I3]

It is possible that his attendance of the PD course had brought fresh ideas about teaching and learning in general and stimulated him to question what he used to take for granted in his mathematics teaching. He came to recognise his existing approach left room for improvement, and viewed the PD course as a learning milieu where participants could engage with mathematical activities in small groups and were encouraged to articulate their ideas about these activities explicitly. This new experience led Emin to think of active involvement as an essential aspect of efficient mathematical learning, and he considered establishing a non-structured environment which encourages learners to participate in mathematical activities and provokes free discussion about problems. At the outset of the PD course, Emin expressed the view that mathematics topics might be made more interesting to the students if they included real life samples, because students have easily become discontented with mathematics when it comprises abstract concepts and ideas. He wanted to integrate real-life applications into his teaching and for his students to be able to relate the mathematics they are learning to things in their everyday life. He looked at mathematics from a utilitarian perspective which is the notion that mathematics is useful in everyday life. Seeing such relationships will help students to make sense of mathematics. Therefore, Emin's main aim in mathematics teaching was to help students to enjoy and appreciate the subject. He described his role as an entertainer.

As the course progressed, he came to see the computer as a means to encourage students to enjoy and engage in mathematical activities. Believing that it is necessary to inject

enjoyment into the teaching to break the monotony of classroom activities and thus students will not see mathematics as a scary thing anymore. He acknowledged clearly that students can learn more efficiently if they are enjoying mathematics. Emin has already held the belief that there should be an element of enjoyment in learning and teaching. Thanks to the PD course, where computer-based mathematical activities are introduced, he came to see the computer as a tool that can be used in teaching and learning mathematics. Additionally, Emin held the belief that the student and the teacher should be involved in the process of the use of GeoGebra in doing mathematical activities and the role of a teacher is to provide students with informal activities and at the same time the teacher should guide students by providing them clues. Therefore, he came to view that computers are useful to help children find things out for themselves and enjoy their learning arising from clear guidance. At the initial stage of the course, Emin had had little experience of computers in school. However, as far as mathematics teaching was concerned he participated very well in the course activities. He came to enjoy the mathematical challenges and derived pleasure from the mathematical tasks he undertook for their own sake.

I have never had fun or enjoyed myself while doing mathematics exercises in any class as a learner. I have just felt the pleasure that GeoGebra has given us...The students can find and discover rotation, reflection and translation of any figure they want in a GeoGebra environment by themselves... [L321, 370/I3]

In addition, as a result of participation in the PD course, he increased his confidence and competence in engagement with the computer and GeoGebra and was able to use these tools to explore mathematical concepts.

5.1.3 The Case of Muslum

Muslum has taught mathematics to 12-14 year olds in a private school for ten years. He had a BS degree in mathematics, pursued by a certificate of teaching. His school is well-equipped in terms of the availability of technological materials. For example, he has one laptop, an interactive whiteboard and a projector in his classroom and also computer labs available in his school. Therefore, he has some experience of using computers himself, although this is quite limited in terms of classroom use.

In considering how Muslum initially saw mathematics, he explained mathematics in terms of formulas, procedures, and calculations. In particular, Muslum mentioned that

mathematics is about, "...addition, subtraction, numbers, and algebraic expression." For him, mathematics is a subject matter that pupils undertake in a classroom, involving computation and calculations. Viewing mathematics in this respect was illustrated in what he expected from his students. His belief of mathematics was reflected on the aim of teaching in two ways; first students should develop their computational skills in order to use these skills in their everyday life; and secondly, by convincing the students that success in their future school career requires the use of computational skills throughout the national exams. He thought that the priority of mathematics teaching was test achievement and enhancing computational skills, later integrating its links with the real world. Muslum began the course with the belief that students should not be working in groups to discuss mathematical activities. He held the belief that it could be more helpful for learners to practice the activities on their own. In addition, he appeared somewhat doubtful about how students can study through pair work to find solutions to mathematical tasks without a teacher's help. As the course progressed, he articulated his opinion that how one share his/her owns thoughts and listens to the concepts of others is vital for building mathematical understanding. For him, engaging in computer based mathematical activities through group work was considered as an essential aspect of a good learning environment. Muslum initially expressed his view that traditional teaching for doing calculations and providing a visual display for mathematical concepts was more effective than the use of the computer in mathematics education. Towards the end of the PD course, he appeared to seek to understand learners' conceptions because he thought that they have their own ideas about mathematical activities. In connection with his view, Muslum considered that the nature of the feedback he obtains from pupils' mathematical activities on the computer could reinforce his knowledge about how they receive and learn, and this would serve to develop his teaching.

Musulm's engagement with the course activities appeared to have enabled him to transform his belief of the computer from being seen as a means to do calculations to considering it as an image maker which might help to fill the gaps which exist in what the students had learned in the classroom. He also began to believe that computers would enhance the process of teaching and learning and make mathematics classrooms less boring, and that both teachers and learners could benefit from computers in their teaching and learning. Considering the traditional position of many teachers towards

computers which would be a major factor in terms of integrating them into teaching and learning, Muslum was very optimistic about the implementation and dissemination of computers in mathematics education throughout the country in the near future. If the teachers have enough experience about how to use computers in mathematics education then the integration of computers in teaching mathematics will increase. Muslum is willing to present GeoGebra activities in his classroom to explore some geometric relationship in polygons.

5.1.4 The Case of Musti

Musti has six years of mathematics teaching experience at primary level. Before the PD course he was educating a class of the seven grades (aged pupils 13). He talked about disliking the subject and finding it hard as a student, and even having wanting to quit his studies, to avoid mathematics, but he was able to pass the university entrance exam to take a place in the mathematics education department. As a mathematics teacher at the beginning of his career, he was enthusiastic to find ways of using computers more effectively as part of his existing practice: “I want to incorporate it into my mathematics lesson” [L506/I3]. Musti held the belief that mathematics is present everywhere in the environment. This is illustrated by his comment during the first interview; “There is mathematics in the genetics of humans. Even the structure of our DNA is about mathematics...For example, golden ratio in mathematics appeared in the nature” [L86/I1]. He took an extreme position with regard to its presence in everyday life. He then saw mathematics as an investigation tool which helps us to make sense of the world around us. Being able to use mathematics in this way was very valuable to him.

At the outset of the PD course, he held the belief that mathematics teaching should be performed as definition, theorem, and more practice respectively. This structure puts a deductive form of mathematics to the fore. He followed this type of teaching in his classroom and believed that because low-attaining learners are often not enthusiastic to study thus lecture mode teaching is most valuable, and they are taught mathematics best through showing and practice. At the end of the PD course, Musti compared the traditional lecture mode with the teaching approach used in the course. He later described that “the course is progressed step by step and from simple to complex. There is an inductive method in it”. He derived pleasure and challenge from his personal involvement with the course. Coupled with this, he was incrementally enhancing an

understanding of a different way of teaching and learning the subject, and at the end of the course, he seemed to be aware of the pedagogical approaches in the PD course as a model in terms of teaching and learning. He believed that students came to see mathematics as meaningful and more enjoyable. Musti expressed his initial view that he did not feel the absence of computers in his teaching even though he took some lessons related to computers during his pre-service period. He entered the course with motivation of regarding how computers could be used in mathematics education. In particular, he came to believe that computers could provide him with the ability to construct geometric figures accurately and quickly which are not possible to draw them on the board. At the same time, he claimed that teaching mathematics through a computer is more difficult and more laborious than teaching through exposition, because of the technical aspects of computers. As the course progressed, he enjoyed mathematical activities on the computer and saw personal value in them, as quick and efficient, and came up with innovative ideas about mathematics. He saw his role as a mathematics teacher was to help students to develop a positive attitude, partly because he had experienced some difficulty with secondary school mathematics and been dissatisfied with the mathematics education in Turkish schools. His main complaint concerned the examination- oriented nature of the curriculum based on memorisation. For him, the reason why most students find mathematics difficult is that they do not enjoy studying mathematics in the way they do in class. He described this negative attitude towards mathematics as a psychological barrier holding students back from learning properly.

5.1.5 The Case of Celal

In his late 30s, Celal has been teaching mathematics for eleven years in state primary school (students aged 12-14). His mathematical knowledge was enriched during his undergraduate study, when he majored in mathematics. He had little experience of technology prior to the course. He generally uses technology for presenting the data and providing video tutorials as supporting material. As an experienced teacher, he was able to talk openly and was willing to share his thoughts about mathematics and the PD course.

In considering his view about learning, Celal expressed that a student can learn mathematics related topics but if she does not follow this up with information drills, the

knowledge will melt away. Celal noted, “they [students] have to jot down what the teacher says properly and then practice it in home later for a while if not, it all just goes away” [L34/I1]. This view of learning puts the learners at the receptive position, passively engaging in practice for the mastery of skills. Later, he came to the conclusion that learning mathematics through computer-based mathematical activities helps students to remember mathematical concepts easily and he came to believe that the approach adopted in the course does not usually require much student’s memorisation. Thereby, students can learn best by constructing and manipulating mathematical concepts rather than memorising. When asked to describe mathematics, he did not hold a clearly articulated belief of mathematics but largely thought of it as comprising mainly of numerically-related content. Celal noted, “I have to say that playing with numbers is what comes into my mind. In mathematics, you have to think and sort things out and give an answer” [L6/I1]. Besides that, he appeared to believe that the ideas of mathematics cannot be explained in everyday words that anyone might understand: technical mathematical language and specialist terms are needed. Therefore, it is hard for the public to understand mathematics language. Afterwards, he claimed that interaction with mathematical activities will help learners to make sense of mathematics through the use of logical thinking and they will conceptualize mathematics very easily.

When Celal first heard about the course he wondered how it could be possible for mathematics and computers to come together. In fact, his decision to attend the PD course was attributed more to the fact that it included the use of computers than the fact that the course handled mathematics. He described his reason for choosing the course: “You know everything is computers these days. It is something we should really learn. I saw this course as way to learn something about computers. I thought it would be good for students if I do something different in class” [L132/I1]. This comment also reflects his belief that the computer experience would enhance a student’s motivation towards mathematics. That is, to know how to use computers in mathematics teaching would add very little to the kind of mathematical experiences that pupils have in regular classrooms. In his final comment, Celal notes:

In general, the teacher is evaluated by the headteacher and inspectors during the school period; they are assessed in terms of their students’ exam success. When you introduce GeoGebra activities to teaching, you have to convince these people. If students are not successful in the exam, they will criticise me... [L421/I3]

Celal recognises how difficult it will be to integrate and disseminate this new approach in the school where he will be teaching. He believed the Turkish educational context is not ready for this and not flexible enough to allow teachers to spend their time in the development of computer activities, and to use group work in their class. If he attempted to do the same thing in the current system, he would run into trouble.

5.1.6 The Case of Asim

Asim is a young teacher, with six years' teaching experience. He teaches in a state primary school in an urban area. In his school, there is an only one computer lab to which he is able to gain access, and he depends on schedule of availability for computer use. He has a degree in mathematics education and seemed to enjoy mathematics and was reasonably confident with it. He had taken courses related to computer literacy and programming languages in his pre-service period.

Before entered the PD course, Asim described his actual mathematics teaching as boring, because it laid emphasis on rules and practices. He supports the idea that active involvement is central in learning and teaching processes. Believing that mathematics could not be learnt by seeing and repeating a teacher's mathematics: students should experience it themselves, and they should be actively involved in learning and putting mathematical ideas in their own words so that they can make sense of them. In addition to this, the notion of human beings discovering mathematics replicates his foundation for teaching as having pupils discover things created by God around us. Therefore, Asim gave priority to students' freedom in exploration over traditional teaching. However, he noticed a clear contradiction between his aims for teaching and those imposed on him by the constraints of the centralised system. This tension arose between the needs of his students to succeed in exams and his beliefs in the primacy of active learning as also suggested in the new curriculum. Therefore, under this condition he felt constrained to employ teaching approaches which he would not have chosen if unconstrained. In the course of time, rich mathematical discussion and engagements with course activities contributed to enhance Asim's professional growth which encompasses both a new way of thinking mathematics itself and his professional development. He seemed to have found space to give his pupils a more active role in learning and enabling them to discover mathematics. The fact that he derived great

pleasure from his participation in the PD course, and was fascinated by many of his mathematical discoveries might explain this tendency.

Asim's experiences with such non-routine mathematical activities on the PD course provoked him to reinterpret his previous experiences with mathematics. Therefore, his participation with GeoGebra based activities helped Asim extend his beliefs about the mathematics, from a somewhat absolutist-oriented beliefs to a more dynamic and creative one. He commented that: "Let alone the change in our minds, even children will be able to say that mathematics is not really something strange and to be afraid of...Mathematics will make sense... Maths is more than practising numbers on pieces of paper". By the end of the course some shifts in Asim's beliefs were noticed. In spite of his lack of confidence in his ability to interact with GeoGebra based activities, coming to know how to use the computer and using it for some period of time led Asim to reconsider his feelings: "I never thought how computer could be so useful in mathematics... I found that very exciting" [L537/I3]. During the course, he confronted a new and challenging situation, and he did not hesitate to face it. As the course progressed, he began to believe mathematics could be meaningful if students could learn with computers. In addition to this, Asim's interactions with the course appeared to have enabled him to transform the belief of the computer from seeing it as a visual display (modifying how mathematics is presented) to considering it as a tool that might be used to explore some aspects of mathematics and to allow learners to discover things for themselves.

The purpose of this section has been to offer a general picture of six Turkish primary mathematics teachers' beliefs about mathematics, its teaching and learning as well as the role of computer in mathematics education. Analysing individual teachers might help readers to better understand of change process due to the nature of complexity of participants' interactions with course activities. The following Table 5.1 represents the relationships between the ideas and beliefs of each participant. It also illustrates the dimensions that shifted as a consequence of the PD course, representing changes in the candidates' belief systems. Here, and in the subsequent mini-profiles, a schematic overview of each subject is introduced. The intention here is to illuminate the dimensions that were affected, as a means of tracking a trajectory of change in beliefs. This will serve as a foundation for the cross-case analysis in section 5.2.

	Eysun		Emin		Muslim		Musti		Celal		Asim	
Dimensions	Before	After	Before	After	Before	After	Before	After	Before	After	Before	After
<i>Personal conceptualisation of maths</i>	A school subject; Powerful expressive tool.	*	A tool for training the mind; utilitarian.	A product of interactions.	Language, content, computation and calculations.	*	Maths is present everywhere.	An investigation tool.	Unarticulated and vague.	Powerful means for making sense of relationship.	A school subject and scientific discipline; discoverable.	A more dynamic and creative one.
<i>Aims of teaching maths</i>	Exam success; Enjoyment.	*	Real world uses; Stimulating logical thinking..	To develop mathematical enjoyment.	Exam success; developing computational skills.	Real world uses.	To develop mathematical enjoyment.	Making sense of mathematics.	Exam success.	Developing logical reasoning.	Enhancing individual functions for daily life.	To access mathematical ideas.
<i>Teacher's role and student's learning</i>	Authority, rote learning. Passive receivers.	Computer makes students independent but teacher's explanation is essential.	Entertainer and no explicit ideas how to guide students.	More explicit ideas on his role as guide. Cooperative learning	Knowledge provider; one's ability and interest are important.	Treat learners more as individuals.	Lecturer; to develop a positive attitude.	Meaningful and enjoyable learning.	Memorisation and practice.	Learning based on constructing mathematical ideas.	Active involvement; students' freedom but how and why is not clear.	Facilitator; more explicit ideas on his role and active learning.
<i>Approaches to pedagogy</i>	Content-based approach. telling and repeating procedure.	Mixed perspective.	Teacher-centred; but wanted to change.	Group work, meaning making.	Traditional teaching approach. Working individually.	Active involvement; to build up mathematical understanding.	Curriculum centred, transmission model.	More explicit view of pedagogy.	Teacher centred, transmission model.	Mixed perspective.	Dissatisfied with his teaching and wanted to change.	Learner centred; to foster positive attitudes and built confidence.
<i>Personal feelings and background about computer</i>	Neutral computer illiterate.	Not confident enough but willing to use it.	Limited knowledge and lacked confidence.	Increased confidence and competence.	Limited use.	Positive; willing to use it.	Not feel absence of computers in teaching	Useful-allows maths to be done quickly.	Sceptical	Lack of confidence to integrate.	Lack of confidence.	Positive and willing to use it.
<i>The role and use of computer</i>	Motivator tool for enjoying maths and motivating students.	Extracurricular activity.	Supplementary tool.	A tool would help learner to explore mathematical ideas.	Extra (demonstration) tool; Seeing it as a means to do calculations.	An image-maker.	Concerns more about managerial aspects, computer as a time-saver.	Image maker; breaking the monotony of the everyday classroom.	Motivator tool; the computer would make students lazy in enhancing calculation skills	Supplementary tool.	Visual display and a time-saver	A pedagogical tool for exploring aspects of maths.
<i>Expectations of and reflections on the course activities</i>	Expecting a traditional mode of lesson.	The activities were isolated from the reality	*	Seeing PD as a learning milieu. Permanent learning.	*	Computer activities make maths classroom less boring.	*	Personal value in them, as quick and efficient, and came up with innovative ideas.	Expecting a traditional mode of lesson.	The activities were isolated from the reality of classroom.	*	The activities are less difficult and more productive.

Table 5.1: An overview of changes in participants' belief systems

* Indicates no change in belief system

CROSS-CASE FINDINGS

The second section explores the teachers' accounts related to their mathematical beliefs and the extent to which they were common to participants or were different from one another. This consists of three-subsections. In this first sub-section I shall confine my attention to participants' initial beliefs about: (i) mathematics; (ii) teaching and learning of mathematics; and (iii) the use of computers. The purpose of this is to present the background against which the participants' reactions to and engagements with the PD course activities are analysed. This analysis is performed in the third section. The last section investigates changes in the participants' beliefs by both comparing their beliefs at the outset of the PD course with those they held at the end, and considering their engagements with the PD course activities.

5.2 The Participants' Initial Beliefs

At the outset of the PD course the participants were interviewed and asked to complete a pre-course questionnaire (see data analysis in chapter 4). Their responses to the first interview is examined in this section with the purpose of offering information about what participants initially believed about the nature of mathematics, the teaching of mathematics, student learning and the use of computers in the context of mathematics education.

5.2.1 Nature of Mathematics

Personal conceptualisation

Before entering the PD course, the teachers were interviewed to articulate their beliefs on the nature of mathematics, but they initially hesitated before offering an answer. Not surprisingly, they found it difficult to express their ideas clearly. A few participants commented that they had never considered the question earlier. To illustrate, Musti noted that "Actually, to exactly describe the question, what is mathematics, is fairly difficult. I had never thought about this question. I think this question is a bit philosophical" [L6\I1]. Yet, eventually, the teachers offered some interesting answers.

The following comments implied that there seem to be three different views. For example, Asim seemed to consider that mathematics is seen as a subject which essentially involves a static unified body of mathematical knowledge, involving a fixed

set of procedures and formulas for computing numerical and algebraic expressions to determine fixed and objectively right answers. Another participant, Emin, characterised mathematics as pure logical thinking and as a medium for training the mind with its logic. They highlighted the mental process it entails and implied that mathematics is carried out by scientists. Thus, in their belief of mathematics is a kind of activity for development of logical thinking. The development of logic and reasoning skills in mathematics was seen as a crucial part of school mathematics. Another participant's (Eysun) remarks during the pre-course interview suggest that their descriptions of mathematics were concerned with its emotional aspects. For example, believing that mathematics is a subject which helps us to make sense of the world and that it offers an important means for expressing relationships between objects around us. This statement reflects the view that mathematics as a means to knowing the world. Here are the some participants' comments:

The first thing which comes to my mind is that mathematics is a collection of rules based on numbers...I think that mathematics consists of a combination of axioms and theorems ... [Asim, L6\I1]

Mathematics is an interpretation, an alternative way of thinking; to be able to look at both sides of the coin, a process. In general it is a journey from the unknown to the known... [Emin, L7\I1]

Most of things we do in our lives depend on cause-effect relationships. You were born, live and die. You add or subtract something and then find a result. So mathematics' philosophy is life's own... [Eysun, L14\I1]

These comments suggested that participants' beliefs seemed to be linked with the idea of the absolutist belief of mathematics which may suggest the predominance of such views in the school system (Roulet, 1998). At the same time, they had an idea of the nature of mathematics outside the school curriculum. For instance, Musti and Asim appeared to consider mathematics as a scientific discipline.

Mathematical theorems are not certain, that is, until their incorrectness will be approved. If their correctness is accepted it will be shown that it is not correct... [Musti, L18\I1]

Mathematics can be like the other sciences; where there are theories which seem to be right one day but turns out to be wrong on another day... [Asim, L12\I1]

Though expressing their initial beliefs about the nature of mathematics, Musti and Asim took a quasi-empirical perspective in which they believed that mathematical truth is not absolute but relative because in fact absolute truth is time dependent. For example,

Musti's assertion implies that whilst current knowledge seems correct, what is scientifically true today might be falsifiable in the near future as theoretical assumptions change. The underlying thinking here appeared to be that scientists are unable to discover and publish certain phenomena due to insufficient technology in that time. If they are able to examine more phenomena and understand it accurately, then mathematical knowledge will expand. Hence, mathematics is continually being modified to improve fit of what we are monitoring around us and changes itself over the course of time.

Is Mathematics an invention or a discovery of mankind?

During the first interview, I discussed with the PD course teachers whether or not mathematical ideas exist independently of human ability to discover them. It should be noted that the participants like Musti, Asim and Emin their religious beliefs were vital in their personal conceptualisation of mathematics. Their beliefs of mathematics seemed to be inseparable from their worldview. The rest of the participants thought that mathematics is a construction of human beings. These two distinct views are clearly illustrated by the following four comments:

Maths is a discovery. I think it is a discovery of existence. It is like that in every science. It even exists in nature. Human beings discover things, so they exist but we are unaware. It appears when we discover... The golden ratio of mathematics appears everywhere... There is a system and rules in nature. The person who made this system is God who created us... [Musti, L24\I1]

I believe we discovered them, as the rules were already there even before our existence. I believe God has already set certain laws in the universe and in our world; it is just a matter of finding and publishing them... This is I believe how discoveries came about... [Asim, L27\I1]

I am 100 per cent sure it is a creation. People produce it as they need.... I don't think whole mathematical knowledge has been created... [Eysun, L16\I1]

I think mankind created them. They are the results of experiments. They have either been created purely by chance or by long trial and making experimental improvements.... [Celal, L27\I1]

In the participants' accounts of the nature of mathematics, it can clearly be seen that for some of them who believe mathematics is discoverable and take a rather extreme position about the ontological argument, their religious point of view might be playing an important role in their personal conceptualisation of mathematics. For instance, Asim appeared to hold the belief that we live in a universe that is governed by fixed

mathematical laws which are already created by God, and over the decades, mathematicians have discovered and recorded systematically as mathematical theorems and rules. The underlying rationale here seems to be that scientists are discoverers rather than creators and the only creator is God. In this respect, people can do no more than discover what God has already created. Therefore, the purpose of the scientists is to find and publish mathematical realities. Within this scope, mathematics is a body of facts existing independent of human knowledge. Deductive approaches make it possible to maintain an absolutist position while mathematical knowledge expands with the discovery of new facts. For instance, Eysun appeared to view mathematics as not like other areas of science, in so much as there are no discoveries to be made. She held the belief that mathematical knowledge and theorems were invented by the minds of mathematicians and are continuously expanding its content and undergoing changes to entertain new developments as other disciplines because they need it in their lives.

It might be possible that individual teachers may express more than one perspective on a similar issue (including the nature of mathematics). The distinction can be seen in participants' remarks. Their comments reflect two different perspectives: "mathematics as a static but unified body of certain knowledge which are out there waiting to be discovered" (absolutist view). At the same time, the term "the construction of mathematics" reminds another perspective which is used by Lerman (1986) "fallibilist view" that "mathematics is dynamic and an evolving, creative human endeavour in which there is much yet to be known". This distinction is also evident in different data sources. For example, comparison between the first interview and the pre-course questionnaire revealed apparently incompatible beliefs. For example, Muslum completed pre-course questionnaire items in a way that gave the impression that he held a more rule-oriented view (absolutist view) of the nature of mathematics stressed in the first interview the idea that "student can produce a piece of mathematics for enjoyment...". This personal statement about the nature of mathematics reflects a fallibilist-oriented belief. Another example, Asim believed that mathematical knowledge was created by God, independent from humankind and at the same time believed that learners can discover mathematics. His belief included both some elements of fallibilist and absolutist-oriented beliefs about the nature of mathematics.

At the beginning of the course, the PD course participants' beliefs about the nature of mathematics can be divided into two separate spheres: first mathematics as essentially synonymous with the school subject, and second as a scientific discipline. School mathematics appears to be connected with the acceptance of the absolutist view (Lerman, 1986) that mathematics is a fixed, plausible and rule-based subject. The latter view seems to be linked with the fallibilist perspective which describes mathematics as a "social invention, its truths and concepts being relative to time and place". The participants who viewed mathematics from a religious perspective seemed to believe that mathematical truths are purely discoverable and have always been out there. To summarise participants' overall initial beliefs about mathematics, they had strongly held beliefs about the nature of mathematics and its relationships with religious and other beliefs. As we have seen in discussing whether or not mathematics is created or discovered, there are some differences between participants' beliefs about mathematics as discipline on the one hand and school mathematics on the other. It can be understood that many participants believed in a combined absolutist and fallibilist beliefs about the nature of mathematics. In particular, their beliefs appear to be partly absolutist, in that they believe that mathematics has been created by God and is therefore 'absolute', but also believe that mathematics is discovered by human beings, and therefore changes as our discoveries advance – a fallibilist perspective.

5.2.2 The Importance of Mathematics

At the outset of the PD course, I asked the participants for their opinions about the importance of mathematics and the main reasons for teaching and learning mathematics. Through analysing of participants' verbal commentary, five different purposes for teaching and learning mathematics were revealed such as *logical thinking*, *connecting to mathematics in real world*, *applications in science*, *enjoyment* and mathematics as a 'gate keeper'. For example, some thought that mathematics is being important to individuals for personal development and daily life, both mentally and socially. Another group stressed the purpose of teaching and learning was about helping students to appreciate and enjoy mathematics. The other group believed that mathematics is currently a significant subject for students at primary school in order for them to pursue further education interests and also as a foundation for other disciplines.

Developing Logical thinking

The following comments illustrated that a demand has been placed on an individual to interpret and use mathematics in everyday life. Asim held the belief that a person engaged in mathematics would assist him/her to develop reasoning skills and a sense of logic. This suggested that mathematics is the subject that trains you to have a logical mind, and improves reasoning skills. In this regard, working with mathematics is a practical activity in enhancing individual functions for daily life. Another participant, Emin, stressed that advanced mathematics is not necessary for people who may use only basic number operations in their everyday life and individuals can continue their life without knowledge of complex mathematics. Here are the two example of this view:

It would enable them to better plan and spend their money. Maths can help people to develop their analytical side and this in a way positively affects all aspects of life for that individual.... [Asim, L49/I1]

It [mathematics] isn't like bread or water. All in all a person who doesn't know any mathematics can continue his/her life. They can continue it in with addition and subtraction, when they know a little. However, considering the development of logical thinking in modern times, we can infer that mathematics offers a different, more systematic way of thinking and commenting on style... assessing people who perceive mathematics differently... [Emin, L33/I1]

These comments suggested that mathematics plays an important role in reinforcing one's logical and reasoning skills. They thought that stimulating logical thinking of their students is considered as an important part of teaching mathematics.

Real world uses

As far as its pragmatic purpose was concerned, the course participants placed the criterion of facilitating every student to develop mathematical concepts in order to apply them in daily-life applications. For instance, Emin noted that "I try to connect mathematical concepts with the daily life examples; this wasn't there before. New topics such as fractals, tessellation and probability have just come are useful and interesting for students. Their samples can be related to daily life" [L97/I1]. Such comments take an implicit utilitarian point of view of mathematics as being useful in the real world, although how and why is not explicit. In fact, the participants' accounts obviously illustrated that their principles for teaching and learning were consistent with those of the newly developed mathematics curriculum (see sub-section 2.4.1).

Applications in science

The participants also explained the fact that knowledge of mathematics is necessary both for individual and society level. They described it as also being important to a country's development and to other science. In considering this for society, mathematics is more than just a school subject. By stressing that mathematics is a science, it is helpful in the advancement of science and technology and it helps us in understanding the working of the universe. Here are some of the examples of participants' commentaries:

Considered from the point of view of science, mathematics is a prerequisite for all the sciences, although it is used variably. In summary, even if it is not everything, it is the biggest cornerstone when it is a prerequisite. It makes noteworthy contributions to the improvement of communities... [Emin, L50/I1]

Mathematics is a tool that reveals the relationship between things around us. It involves exploring a scientific truth about the order of our world... [Muslum, L58/I1]

Emin held the belief that greater demands have been placed on mathematics which is central to other disciplines in today's technology driven society, mathematics is used throughout the world as an essential tool in many fields which are based on using mathematical concepts. Mathematicians continue to make important contributions to the development of societies. In Muslum's view, if mathematics is a mode of inquiry and language of patterns, subsequently to recognize mathematics is to look into and articulate links between patterns and describing the world in which we live. So, mathematics is not only a science but also a language. It might have real importance in life and is crucial for society to understand the world and how they can change it.

Enjoyment

An interesting point about some participants' aim of teaching and learning arose from their responses in the first interview. For example, Musti believed that the purpose of teaching and learning was about helping students to appreciate and enjoy mathematics. That is, he is interested in making mathematics enjoyable for his students and this is one of the motivating reasons for carrying out hands-on activities in the classroom. These ideas reflect typical constructivist view about the primary aim of teaching and learning. The reason behind this might be because his students have negative attitude towards mathematics and students may think it is boring and difficult to understand so they have

developed negative bias about mathematics throughout their school years. This remark was from the course teachers who had similar negative experiences with mathematics at school, and therefore, considered that it is a priority for their students to enjoy mathematics. For instance, when the participants were asked about their learning experience, the fundamental importance of how their early experiences shape their perception of mathematics was revealed. Musti stated, “I don’t want my students to have the same kind of negative experiences that I had”. Considering early experiences may be helpful to make sense of the participants’ view of learning and teaching of mathematics. For instance, Eysun indicated that her feelings towards mathematics have developed as a consequence of her experiences as a teacher:

When I was in school, I didn’t use to like maths because I was not interested in mathematics. I became a maths teacher with great difficulty. After I started to study mathematics and solve problems, I began a liking to mathematics. You learn its mathematical rules, adopt them. If you find the solution to a problem, you will begin to love it... [Eysun, L44/I1]

This comment implied that although solving mathematical problems to achieve the right answer motivated Eysun to enjoy mathematics, her feelings about mathematics have evolved over time. It is interesting that she still prioritised enjoyment in this current situation even though the constraints of the examination system.

Mathematics as a gate keeper

In fact almost none of the participants had explicitly stated any specific personal reason for teaching and learning mathematics; their opinions were influenced by externally motivated pressures such as the introduction of Turkish primary mathematics curriculum and the requirements of national exams. Like other participants, Muslum expressed the same view that mathematics is an important subject which plays a crucial role in one’s chosen future career. Therefore, he described end points for his students’ desire to learn mathematics at primary school as discovering how important it is in the centralized exam. In this sense, mathematics is a necessary school subject which has to be learnt for reaching particular goals such as passing the national exam and afterwards applying for a better job. Here is the example:

It is necessary that the student complete mathematics lessons successfully in order to pass to upper class...Actually, for example, there is SSEE (Secondary School Entrance Exam) for primary school students in 8 graders class. If the student is not successful in this exam, she/he cannot transfer to a good secondary school and then she/he cannot go to a good university so consequently she

cannot have a good job. Like chains of a circle, they are all related to each other. So I think it is a must in life... [Muslum, L62/I1]

This comment suggested that being competent in mathematics gives children the opportunity to enter into a good secondary school, which subsequently impacts on their choice of university. Eysun initially believed that mathematics is seen as about being able to address the problems that come up in exams and that encouraging students' work on problems would be successful and sufficiently motivating. Mathematics as the important school subject makes up a great proportion of this exam. That is, the course teachers generally perceived mathematics to be positioned totally within an exam-oriented education system. Thus, their purposes of teaching and learning mathematics were concentrated on achievement. They really wanted their students to succeed in public exams before progression to secondary school. For example, Celal said: "my students' success must be high in exams; therefore, my goal is to attempt prepare every student to become successful in national exam (SSEE)" [L65/I1]. It is a view shared by the participants who typically made great efforts by teaching students to learn facts and to acquire the ability to manipulate numbers and solve algebraic equations.

To summarise, when participants' beliefs in the importance of mathematics is considered, it is believed that mathematics enables students developing their mental and social sides. For them, the purpose of teaching and learning was about helping students to appreciate and enjoy mathematics. It is also believed that mathematics is viewed as a gate keeper and/or as a highly important subject to get success in school and public exams for their future education and employment. As we discussed earlier, there seemed to be no relationship between past experiences and feelings about the subject, as they still enjoy working with mathematics as a teacher.

5.2.3 Approaches to Teaching and Learning Mathematics

The curriculum designed for mathematics in primary schools has been developed and is being implemented with on-going changes since 2005 by MONE as part of the reforms aiming to move from a subject-based didactic model to a learner-based constructivist one which comprises the processes of exploration, discussion, interaction and conceptualization. Students have been encouraged to engage with the new curriculum in order to construct their own knowledge by taking an active role. The former primary mathematics curriculum was designed by adapting the behaviourist theory (Bulut,

2007). The rationale of the old curriculum did not offer enough opportunities for pupils to engage in critical thinking process, including problem solving, communication, and making connections. The former curriculum placed the teacher as the centre of the teaching and learning process. In particular, the teacher was characterised as the knowledge provider who was to transfer information directly to the students without understanding (Koc, Isiksal, and Bulut, 2007). The previous curriculum seemed to have had an impact on participants' beliefs about mathematics and learning of mathematics. On-going reform movements appeared to cause interesting conflicts in participants' thinking. Asim's comments, for example, support the ideas and methods of the newly developed mathematics which emphasise the importance of a learning environment where the learners can discover, solve tasks and share and discuss their results and approaches, but he felt pressured by the demands of exams and parents, and Eysun supports the ideas and methods of the existing curriculum (which they think will work to get students through their exams).

The course participants held the belief that the school mathematics has covered many topics or areas to learn and procedures to be followed, and the teacher would ensure that pupils master knowledge of these areas and know how and when to use the rules. Technically, this approach requires telling, or providing clear, step-by step demonstrations of these procedures and students learn by watching and listening to the teachers' demonstrations and practicing them, with all students working on the same kind of task. These descriptions reflect how the teachers thought pupils learn not through active participation but through passive receipt in this process. These beliefs were evident in both participants' self-reported practices and in their responses to pre-course questionnaire items. In contrast, Emin held the belief that students learn mathematics best by 'doing and living' with help and guidance of a teacher, but they have no explicit ideas with reference to how this could or should be done. This view is compatible with a constructivist view of learning and teaching where the learner is active and the teacher's role is facilitator. On the other hand, Celal did see his role as helping students by showing them the correct ways to interpret mathematical symbols, situations and procedures in the classroom. On account of this view, pupils were seen as the passive receivers of mathematical knowledge. They characterised their teaching approach to be quite directive, and they are ready to provide all the mathematical concepts students need to succeed in the state's tests rather than eliciting mathematical

concepts from students. The shared style to teaching mathematics by most of the PD course participants appeared to be whole-class lecturing. This view is compatible with a traditional view of learning and teaching where the learner is passive and the teacher's role is knowledge provider. Here is the tension:

As a teacher, I believe that this new curriculum is more appropriate for teaching and learning mathematics... In fact it easy to provide ready- made knowledge to students. It is easy to give the formula; then doing an example and start to the new problems. For example, it will be saving the time if I give the Pythagorean Theorem only in one sentence. I think that you can solve almost fifteen examples; but if you prove this theorem you can solve only three examples.... In the new curriculum, the student goes to the way of proving, observes how it occurs; there is no information transfer... I am worried about the exam... [Asim, L108/I1]

Memorisation and Practice

At the beginning of the PD course, the participants highlighted the importance of memory and practice as an integral aspect of mathematics learning and teaching. Here is one example of this view:

Every student can be successful if they jot down what the teacher says in the classroom and then reviews it at home. I asked students to memorise formulas and use them in the exam. They will not forget and probably will get a good score in the exam. Students cannot succeed in the mathematics without memorising... [Celal, L34/I1]

Similar to other participants (e.g. Eysun and Muslum), Celal reflects a view that if students perform what they have been taught in the class immediately then they will learn better and get good results from the exams, but if they do not follow up with repetition, the information will disappear. That is, more practice and more may provide better results in the school and public exams. To learn how to follow procedures and algorithms appears to be an essential element in students' learning. This opinion appears to derive from beliefs about what works in the learning and teaching of mathematics which are shaped through teachers' experiences as a pupil in the previous system. For them, memorisation does work in the examination-oriented system so as to enable high scores in exams and progression to secondary school. Despite the fact that the mathematics curriculum has been revised in the light of constructivist learning theory, the examination-oriented system appears to attach importance to the students' memorisation of mathematical facts and their application. Therefore, teachers have been forced to teach in a traditional way so that the requirements of the exams can be met.

Although Asim appeared to be fully aware of current conditions, he was interested in looking for alternative ways of teaching and learning.

Other comments showed concerns about students' learning which focuses basically on rote learning, and teachers complained that an exam-based educational system requires students to learn ready-made mathematical knowledge that they are expected to remember and use in the exam. This type of teaching would help pupils pass an exam implemented by the teacher and to prepare themselves for the SSEE. In addition, the informants were dissatisfied with the image of the present mathematics class in which the teacher was the deliverer of knowledge and students must learn by being provided with knowledge for memorisation without understanding. In fact, both Emin's and Asim's comments suggested they believed that cooperative group work, meaning making and active participation are extremely important part of good mathematics teaching and learning. That is to say, they held the belief that meaningful mathematical activity involves students making sense of mathematical notions and constructing their own ideas, encouraging the learners to share their knowledge with others and build their own knowledge by giving them enough freedom to discover and come to understandings on their own. They were enthusiastic to use these methods in order to support their students to think, but they did not know how to achieve this. When they consider the present condition in mathematics education, they accepted that memorisation is inevitable and leaned towards accepting the status quo.

The main focus of the new curriculum is that 'every student may learn mathematics' but Muslum expressed the idea that pupils' ability and interest play an extremely important role in dealing with learning mathematics. Here is the example:

I can say that the students who have maths intelligence can apprehend mathematics better and learn formulas better and solve problems better... Actual learning is an event which depends on the student's own intelligence...Therefore, the interest in a lesson is also very important...
[Muslum, L75/I1]

This comment expresses the idea that if students have an innate ability (intelligence) for mathematics they often experience success and become high achieving students in their school. Muslum had fixed beliefs about the nature of a student's mathematical skill, which implied that some children are initially treated as 'weak', and attributed actual learning basically to stable factors, such as, 'intelligence'. According to him, it is

unrealistic to expect too much from students who have lower interests and intelligence to understand mathematics and solve problems well. This idea is compatible with a traditional view of learning and teaching.

Institutional constraints

As mentioned previously, the course participants complained about some of the key contextual factors such as the availability of materials and time, school culture and overcrowded classrooms which were impediments to implementation in their daily practice. For example, when the new curriculum was introduced, Emin was motivated to integrate group work and collaborative learning in his teaching. He stated that “the new curriculum encourages us to design teaching activities based on using group work and collaborative learning, collaboration might help students to become more independent. The implementation of these ideas in the current classroom is not possible for various factors such as crowded classrooms, lack of infrastructure of schools, examinations and time” (L1/89). In particular, time is a theme which emerged from the participants statements during the first interview. They felt that significant time was required for implementation of new curriculum principles into classroom practice. They spoke of the desire to conduct a more learner-based constructivist model of teaching, but seemed to lack time and experience to actually carry these out, fearing administration and students’ family pressures and exams. These beliefs clearly appear in the following accounts:

I can’t explain this not only to my administration but also to parents. I behave according to how I am evaluated. You have to act like that... [Eysun, L65/I1]

In fact as the new syllabus is intense, as regards subjects, giving ready-made knowledge to students, saves time... [Asim, L128/I1]

Essentially, I would like the students to actively take part in lessons. That is, I would like to spend more time on activities, and approach teaching through debates and mutual dialogues. However, students cannot play an active role in lessons because of our institutional conditions; in terms of finishing the curriculum within deadlines or in terms of exam success. If these inconveniences are removed, I could spend more time insuring that students learn. So if we were to have no trouble with finishing a subject by a deadline, I would try to make an atmosphere in which students can talk and communicate their own ideas conveniently... [Musti, L49/I1]

The new one is more interactive but gives us less time to prepare for the exams. That’s why the students do not succeed in their exams. I mean it is a good thing not to memorize; but there is limited time in the exams... [Celal, L74/I1]

These comments suggested that the current mathematics curriculum tends to limit the teachers' opportunity to take full advantage of it. It is so intense and there is not enough time to teach the whole curriculum in one year; therefore it is tempting for teachers to load students' heads with ready-made mathematical information that they are expected to remember and use in the exam, in order to save time. Their accounts also reflect the fact that they would probably always be faced with finding available time in this system. Similarly, Eysun stated that she really wants to encourage small groups of students to discuss a mathematical concept, but this was not always possible due to the national exam and time. Her willingness to meet the expectations of families and pupils' needs in exams is one of the most important aspects of her teaching. Therefore, she is eager to return to the way of teaching which she has been taught. The participants felt that the exam-oriented education system forces them to use drill and practice methods for teaching and learning mathematics.

To summarise, key pedagogical views, perceptions and beliefs about mathematics teaching and learning emerged from participants' responses to the first interview and partly pre-course questionnaire. As far as mathematics teaching and learning is concerned, a tension is apparent between how mathematics teaching and learning ought to be and how it has to be in an exam-oriented system. Institutional constraints became shared concerns of the course participants which may have an impact on their decision making about teaching and learning. In spite of a common feeling of disappointment with the current pattern of mathematics education based mainly on memorisation and practice, the participants seemed to have minimal knowledge of how to enact alternative pedagogical approaches. Furthermore, they had not explicitly articulated a clear aim for mathematics pedagogy. This study suggests that some of course participants held teachers' beliefs about teaching and learning (e.g. Eysun) at the before their participation to the course were primarily traditional in nature. The remained participants (e.g. Asim) were found to hold combining constructivist and traditional beliefs about pedagogy and learning. The link involving teachers' beliefs about teaching and learning and their beliefs about the nature of mathematics are not as straightforward as previous studies may have suggested.

5.2.4 Computers in Mathematics Education

In this sub-section, the aim is to present an image of how the participants initially understood the role of computers in mathematics education. It contains two parts: the first provides some insights about the participants' educational experience with computers, while the second illustrates the participants' conceptions of the pedagogical strategies regarding the use of computers to promote students' mathematical learning as well as the potential benefits and drawbacks of using them in mathematics classrooms. The illustrations were drawn largely from their first interview transcripts.

Participants' educational background with computers

All of the participants noted that they have already undertaken courses related to computer literacy and programming techniques during their in-service and pre-service period and had developed only their basic computer skills such as word processor, using excel, and preparing a presentation during their teacher education. In addition, both Asim and Emin reported that they had had experience with programming languages such as Mathematica 5 and FORTRAN. With the exception of these programs, they had very little familiarity with the use of computers in education. Only Musti had attended a 'Computer-based mathematics education class' which comprised some educational software such as Logo, Cabri and Derive 6 which was part of his initial teacher education programme. However, like the others, he did not have (or very limited) experience in integrating computers in his teaching. The participants had almost no knowledge of Dynamic Geometry Systems (DGS).

When I was talking to the participants about their background in relation to computers, none of the participants had been offered an opportunity to use computer-supported applications during their teaching years. The main reason behind this which all the participating teachers mentioned was the lack of appropriate computer technology equipment in their schools and lack of sufficient experience. In fact, the participants who work in private schools had considerably more access to computers than the participants who work in state schools. Generally, there are an insufficient number of computer laboratories available at state schools and thus teachers' access to computers was inadequate because the existing computers were to be used by all the teachers in school. Moreover, most of Turkish teachers have a limited view of the use of the

computer in the classroom because of poor teaching in the course they had attended. Here is an example:

I did learn how to use the Mathematica 5 program, but it was more like the teacher showing us what to do and us just replicating what he/she did. I just learned where to click, but didn't know what it was doing... it didn't help us learn about how we could use it for mathematics... [Emin, L55/I1]

This comment suggested that Emin seemed to be quite discontent with his early learning experiences. He did not have sufficient experience and realistic information about a computer in order to integrate it into his classroom. The PD course teachers believed that the current situation in Turkey connected with the use of such computers posed considerable constraints to the utilisation of computers. However, at the same time, they appeared to be hopeful about the use of computers in mathematics teaching in Turkey in the near future:

Neither do we have enough facilities nor the mentality. However, the fact that becoming a member of EU I believe things are getting better in Turkey; one day (maybe a few generations later) when the examination system and the mentality changes, we'll be up and running... [Asim, L197/I1]

The role of computers in mathematics education

The participants' initial reflections about the role of computer in mathematics education were elicited during the pre-course interview, and most of them were quite positive towards using the computer despite their lack of experiences with it in their teaching. Celal and Eysun initially felt ambivalent about the role of computer in mathematics education. "I did not believe that the two (*computer and mathematics*) can come together before I saw the course you were talking about" [Celal, L106/I1]. The remaining participants expressed the view that computer usage in education is becoming absolutely necessary for teachers who are living in a technologically oriented society. For them, there was a contemporary need to incorporate computers in Turkish mathematics classrooms. This belief also showed why most of the participants are more willing to attend the PD course. For example,

We currently live in high-tech digital age. The influence of the technological tools cannot be denied. It is necessary to benefit from technology in different aspects of education. If we use it properly, the development level of our country will increase. Therefore we need to develop and extend our selves as a teacher to be able to integrate it effectively... [Muslum, L131/I1]

In spite of limited availability of computer technology and experiences of the use of educational software, the participants shared some opinions about the possible advantages and disadvantages of computers in the teaching and learning process. For example, Celal they had little idea of what particular role the computer might play in mathematics. The general idea they shared about the computer role that it could bring a little new life to classroom environment: “it can be useful if I do something different in classroom, I may attract students’ attention” [Eysun, L139/I1]. The computer within the classroom, is seen as a good idea, and viewed just as an extra tool.

It would be more fun and more amusing to teach mathematics with the computer and it will also help students to actually understand mathematics, to be able to observe what’s occurring, and they will surmise that mathematics is not a scary thing... [Muslum, L187/I1]

The underlying his thinking here seems to be educational software as content specific, rather like a teaching device. In Emin’s view, pupils are naturally more engaged by interacting with computers than they are with traditional pencil-and-paper activities with regards to students’ motivation. Therefore, the teachers expected that using a computer in teaching mathematics will be more enjoyable and more fun. Comments suggested that if teachers are given an opportunity to design computer-related mathematical tasks that are stimulating, student’s interests towards to mathematics would rise instantly and appreciably. Here is the example:

If I am able to teach mathematics with a computer, I will have the opportunity to engage indifferent activities in class. When my students work on mathematical tasks using a computer, they will be happy about it and their appreciation of mathematics will increase... [Emin, L119/I1]

Compared to pencil-and paper activities, time was a crucial issue in terms of what the course participants thought about the use of computers. They spoke of referring to the computer as a time-saver, providing the teacher with more time to complete a lesson, enabling them to produce more accurate graphics and figures, which meant that they could allocate more time to other teaching activities. Teachers also do not need to wait for students to finish taking notes in class. As a result, they thought that computers could enable teachers to save time that is wasted on drawing on board and note taking. In contrast, Celal believes that computer-based activities are not appropriate for the Turkish mathematics lessons which are largely based on exam oriented system. The reason for this could be that computers would make students lazy in developing mathematical calculation skills. They believed that students should first improve their

abilities by doing pencil-and-paper activities, and then move onto using a computer. For instance:

Topics should be explained first before using the computer to make them more understandable... The teacher has to show the student that they are an expert in the field before using software to teach topics... On the other hand, if students use computers instead of doing more examples, they will not be able to gain calculation skills, when they take their exams... [Celal, L109/I1]

On the other hand, it is a belief shared by many participants that a computer serves as a teacher's demonstration tool and helps to offer better visual and dynamic representations of mathematical concepts. That is, they considered computers as a device that can be used for a variety purposes such as demonstration, presentation and visualisation. For instance, Muslum noted that "computer shows some the geometrical figures which we can't draw on board. It also gives the opportunity of looking at geometrical figures in a different way... and it can be good presentation equipment" [L195/I1]. For him, if students are able to observe mathematical relationships with the help of visualization, they are likely to learn mathematics better and faster. Therefore, computer-based environment is relatively active in supporting better understanding of mathematical ideas. For instance:

The drawing of geometrical figures will surely be privileged from imagination according to the seeing, hearing. The student will learn more about the permanence of a figure s/he sees; it will be learning aside from memorising... [Musti, L141/I1]

On the other hand, all participants seemed to be worried about the use of computers for mathematics. It is a view shared by them that teaching with computers would be difficult with regards to extra preparation time, extra training and managerial aspects. For instance, Musti stated that "When you are working with computer in class, you may face with some problems...while some students are studying, other may make noise at back, may deal with something else. This problem leads to lose students' attention on lesson"... [L154/I1]. This statement points to classroom management concerning students' learning while they are using the computer in the classroom. Eysun also highlighted that the pen-and-paper activities have convenience which is an important part of classroom teaching. Here is the example:

My teaching methods come to me easily. For example, while we are working with mathematical problems on the board, it is easy to show and solve problems

using a marker as it is convenient for me. Maybe I am not interested in alternative methods like searching for new adventures... [Eysun, L142\ I1].

Moreover, the school management and the curriculum do not expect teachers to use computers in their teaching. Celal suggested that if a teacher can teach mathematical concepts with traditional pen- and-paper activity, and it works this way, then why would a teacher change it? Basically, there are two distinct perspectives on the role of computers in mathematics derived from the participants' verbal commentary. Firstly, Eysun came to see computer as a motivator tool (extra tool). Within this perspective, the computer might help to develop students' motivation and interests towards learning mathematics. The second perspective highlighted computer as image maker. That is, the computer, and in particular its visual display, has the potential to transform how mathematics is introduced. From this perspective, the computer might support to improve students' understanding, and making mathematical ideas and relationship more clear and enabling concentrating on understanding.

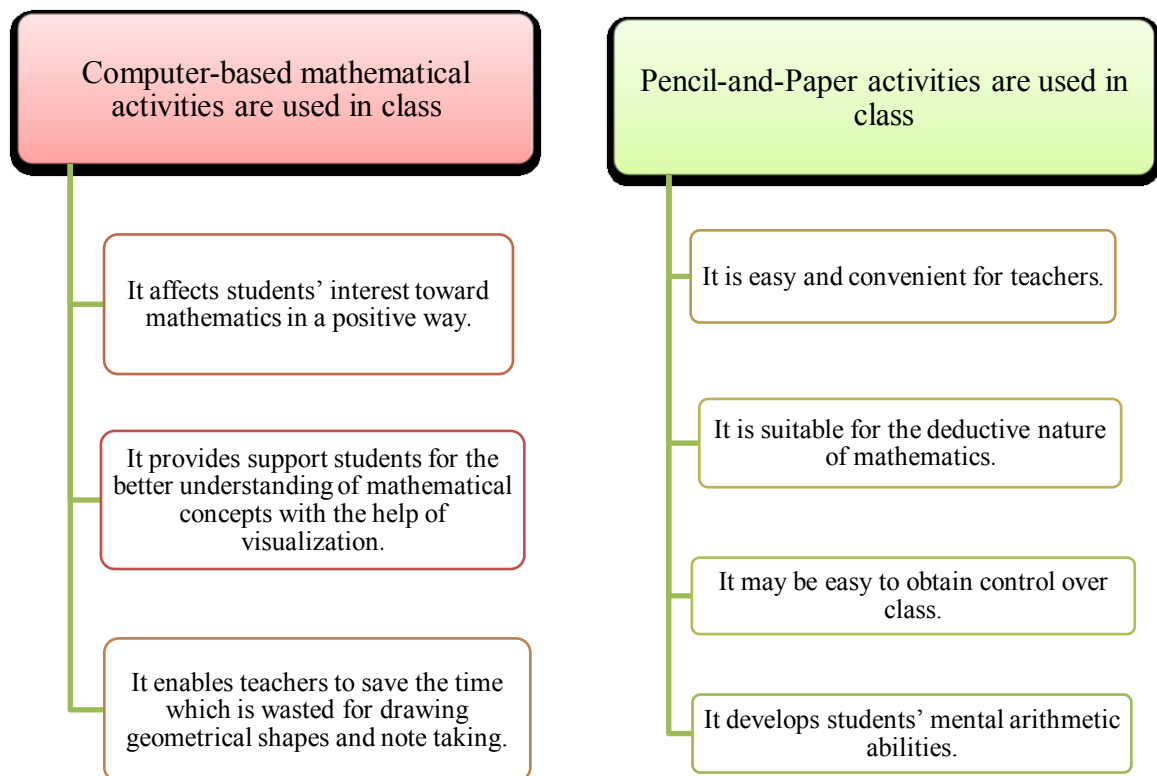


Figure 5.1: A summary of participants' initial beliefs about computer

The participants had little sense of the use of computers in mathematics education. As we saw in Table 5.1, the possibilities of seeing the computer as a catalyst for educational change are extremely complex.

5.3 Participants' Experiences with GeoGebra-Based Activities

In this section, a variety of data sources such as interviews, reflective writings and my field notes are used in order to highlight participants' experiences with GeoGebra activities which were derived from their involvement with the PD course. They shed some light on participants' views of GeoGebra-based mathematical tasks concerning mathematics, teaching and learning mathematics and the role of computers in mathematics. This section starts with the participants' expectations of the PD course and their reactions towards it so as to offer a background to the discussion of how they engaged with it. This is followed by participants' reflections on the PD course activities.

5.3.1 Participants' Expectations of and Reactions towards the PD Course

Participants' expectations of the PD course can be considered as a key issue in order to understand the unfolding of the participants' engagements with the PD course. It is unlikely that a substantial change in teachers' beliefs would occur if they did not come to the PD course with an intention to change. This sub-section elucidates the extent to which the participants made an effort in gaining ability to work with GeoGebra and the extent to which they interacted with GeoGebra-based investigational tasks during the PD course.

At the outset, the participants were interviewed to provide reasons for attending the PD course; the teachers entered this course with a variety of expectations which emerged from their participation. Initially, for Celal and Eysun, the key reason for attending the PD course was that this course offered more about learning to use the computer than coping with mathematics. These participants seemed not to be motivated to obtain new mathematical knowledge or reconstruct mathematical ideas from their involvement with GeoGebra-based course. The participating teachers explicitly expressed two main reasons about why they were willing to join this PD course. The first reason could be described as personal development which is linked with the idea that the teacher is responsible for teaching mathematics in the school; therefore they may want to enhance their knowledge and skills because of increased expectations in the new curriculum and society. For example, Asim stated that "So that I do not want to be caught off-guard and I look forward to developing myself" [L203/I2], reflecting a motivation to sustain and develop teachers' professional roles. Eysun reported that they joined the PD course due to the idea of developing student learning. Her accounts showed some sort of

willingness to improve her pedagogical knowledge and learn more about the use of computers in mathematics education to be used in their practice so as to support students' mathematical learning. The rationale underlying these kinds of reasons appeared to rely on the saying of "what more can I do for my pupils". It can be said that these two main reasons are somehow interrelated. That is, all participants referred to the same fact that they might lack pedagogical knowledge concerning the use of computer to support teaching and learning so it is needed for them to gain new knowledge, skills and challenge existing teaching routines by attending the PD course but the knowledge they acquired through course could be used for different purposes.

It should be noted that the participants who held a personal development expectation seemed more eager and committed to making more efforts to improve their skills with the computer and GeoGebra compared to those who were attending the PD course aimed at mainly supporting student learning. Throughout the course sessions, the participants in the first group would engage in the GeoGebra activities rapidly with their partner, their participation with these activities was high, and they often were able to carry out them successfully, and then to assist their peers. This difference could be attributed to the fact that some participants seemed to learn more easily in a computer-based environment.

At the time in which the first reflective writings took place, the participants' initial reflections on the PD course were gathered. Here is the one example of the participants' reactions towards the end of the third session. Eysun noted: "you did not provide us with the direct answer, I became helpless... you are the expert and you are supposed to show us these things" [4 May 2011/S3]². This suggested the influence that my intervention (i.e. the PD course) as a constructivist teacher throughout the session made on the participating teachers. In general terms, the participants did not realise what type of role they should play during the initial sessions. One of the course participants, Emin, who expected to improve their personal competence with software for its own sake, stated that: "...you are an atypical teacher. Possibly, I would not be able to deal so well with the PD course activities if a more directive method was adapted..." [4 May 2011/S3]³. This comment suggested that this participant looked for change and appeared to find what they asked for. Overtly, participants' expectations for finding

² [4 May 2011] refers to the date of the participants' reflective writings.

³ S3 refers to the date of session take place.

different ways to teach mathematics made it easier for them to interact with GeoGebra-based course activities.

During the first and second session of the PD course, the course participants had technical problems in learning and exploring the computer and GeoGebra such as installing and starting software, using toolbars, controlling the mouse, operating interactive GeoGebra pages and so on. They often asked for help from the researcher and their partners because they did not feel confident enough to deal with working on GeoGebra-based mathematical tasks within the group. It is interesting to note that both Eysun and Celal initially expected the knowledge of the use of computers directly from the researcher. These participants were surprised when I only provided them with only a clue when they asked me a question. They seemed to want more detailed explanations for what they were working on rather than experiencing with trial-and error approach, and they tended to expect an immediate answer. These participants might have felt some degree of disappointment. However, they later seemed to feel comfortable with this type of intervention which focused on activities and often emphasis on designing learning situations that require the learner to take responsibility for their learning. The participants with a personal development expectation in mind also appeared to be less worried in dealing with the difficulties and problems during the sessions in using software than the participants in the other groups. My teaching approach was intended to encourage learners to explore mathematical concepts and discuss their solution on tasks. In fact, Asim was very well aware of my role in the course as a facilitator, a guide and their role as not a passive recipient of knowledge but an active knowledge seeker. Therefore, the course participants came to see this course as a model of teaching and learning. The reason behind this could be that they enjoyed this style of learning which they had never experienced before. The evidence came from the two of the participants' reflections on my intervention:

I couldn't understand what you were trying to do at the beginning. You were standing at the back and did not intervene. You were giving us some clues, but you did not intervene with us directly. I struggled to achieve; but I found being there a strange situation. Then I understood and got used to in the end... [Eysun, L178/I2]

I believe we were closer to the ideal way of teaching in this course... You guided us by asking questions; so we caught the clues based on those questions... You didn't give the direct answer to us. You addressed open-ended questions upon the question in the group. This lead to us fully understanding the situation and

coming up with innovative ideas about solving them. Moreover I and my partner discovered different things about the activities without your help. We discovered things by our own efforts. The course was also a flexible one, thus there were a lot of different ways in which we arrived at the solutions. I felt like the students were acting like “inventors” and “innovators”... [Asim, L229/I2]

It is essential to highlight that vital part of message associated with the essence of the PD course was received by the course participants. For example, Asim deliberately used some key vocabularies in his verbal commentary such as *discovery learning*, *inventors* and *innovators* and so on. It should be noted that the participating teachers became more open to the idea of active learner and they began to consider the fact that being capable of learning mathematics and finding out things themselves were important aspects of learning. These ideas are compatible with a constructivist perspective of learning.

5.3.2 Reflections on the PD Course Activities

In this sub-section, I will discuss the reflections of the participants on the GeoGebra based activities outlined in Chapter 4 and presented in Appendix 6C, 6D. This part shows how the participants interacted with PD course activities which offered mathematical ideas; how they viewed these activities with regard to learning and teaching and how to they related them to their current teaching. The participants in general pointed out the notions below about the nature and characteristics of learning took place in their engagements with GeoGebra-based mathematical activities: a) Memorability b) Meaningfulness c) Motivation, Personal differences and Personality d) Encouraging thinking and investigation.

Memorability

This interesting concept derived from the reflections of almost all participants related to GeoGebra-based mathematical tasks. It was indicated that they believe the relations come about with help of worksheets in the GeoGebra environment will be permanent (long-time learning) due to the student’s own efforts. For example:

Students might produce mathematical ideas by using GeoGebra worksheets and they do not need to memorise information, as it were, using traditional paper-and-pencil geometry. They become more active and they could find, create and capture the relationship between those mathematical concepts which exist. This surely makes things remain permanently in the mind... [Emin, 11 May 2011/S4]

In GeoGebra dynamic environment, while previous comment suggest that the student being centralized actively and continuously in learning can enable student knowledge to

become permanent. Another participant, Asim, considered that abstract geometric relations being presented on the monitor to the student by getting concrete examples makes learning permanent. One of the statements that Muslum made is that the angle relationships and parallel lines activities could provide visual concrete experience which might help students to remember concepts. These two participants projected their feelings about these activities upon students and believed that learning abstract mathematical concepts through their concrete examples is crucial and seeing such relationships would help students to make sense of abstract concepts.

After all I think these mathematical concepts are easy to understand by the students, but I saw how we can find interference from these concepts with their concrete examples through GeoGebra activities. The fact is that these activities make abstract concepts more concrete and students can learn more about the angled relationship of parallel lines and polygons by dragging them on screen through visualisation... [Muslum, 4 May 2011/S3]

This comment implied that the concrete relations can be apprehended more easily contrary to the abstract ones; however, since traditional school mathematics mostly composes of abstract relations, it does not provide opportunity for making mathematics concrete to the students because of the immobility of objects on the board. In addition, they also came to believe that geometric objects being dynamic on the GeoGebra screen makes this abstractness concrete and this both enables learning mathematics to become easier, and provides permanency in knowledge learnt. On one hand the course participants, like Musti, who asserted that the graphical aspects of GeoGebra is going to make learning more permanent, emphasized the importance of learning by seeing; on the other hand they pointed out that students' manipulating the objects and experiencing the environment makes learning permanent. In this sense, they came to believe that GeoGebra is going to bring in substantial experiences.

The participants generally indicated that learning becomes permanent by seeing; manipulating the objects in GeoGebra and it also presents powerful opportunities to experience properties which are not seen in the traditional pen and pencil environment. For these participants, GeoGebra could be used to explore geometrical properties by dragging and sketching figures in ways beyond the scope of traditional paper-and-pencil geometry. The experiences in GeoGebra activities had played a major role in assisting their students in gaining mathematical insights about angle-length relationship but they

thinks that this important aspect does not belong to traditional paper-and-pencil geometry.

During working on task four, I and the course participants discussed the episode when dragging a rectangle with a fixed perimeter its corner, the area of the rectangle will be different. I asked Emin how you would respond to your student. He answered that:

I would ask my students to construct several rectangles of the same size if they do not understand this activity while dragging; I would help them to think about it. And then, I would leave them with this activity till they understand the point. They will understand this since a rectangle with fixed perimeter does not necessarily have a fixed area... [1 June 2011/S7]

The point is that the area might not be expected to stay the same. This comment implied offering the student with concrete examples would support the student to understand abstract ideas through GeoGebra activities. For Emin, GeoGebra as a manipulative so as to depict the situation to the student and to plan for student's construction of mathematical ideas from concrete towards abstract, as an alternative to having the student being told the correct answer. Here is a nice touch point between the participants' inclination for learning abstract mathematical concepts through concrete examples and the constructivist emphasis of the model: GeoGebra offers a means to link the two sides of this tension.

Meaningfulness

At the beginning of the course, believing that mathematical knowledge is seen as meaningless and a set of fixed rules, absolute body of laws and procedures, and knowing mathematics is viewed as having mastered them without understanding. The participants came to recognise that the GeoGebra-based environment provided opportunities for learners to make sense of mathematics which they had learned as students. This environment provides explicit support to learners for building meaningful mathematical concepts. In this sense, GeoGebra activities may help to achieve this. This perspective is compatible with a constructivist view of learning. One of the participants' reflections on this issue:

I think mathematics is about a set of meaningful relationships. It is based on this relationship and also constitutes a meaningful whole. In this respect, GeoGebra both encourages us and our students to seek these relationships and it helps them to associate with mathematical ideas based on these relationships... [Asim, L343/I2]

This comment suggested that mathematics was seen as knowledge of basic facts and relations that are interconnected with a large system of previously learnt relations and see it as necessary that students make new knowledge meaningful based on old knowledge to learn meaningfully mathematics. For example, Musti believed that learning taking place in the traditional environments can be meaningless and irrelevant and this prevents learning from taking place, in contrast GeoGebra supports meaningful learning. For example, the task (see Appendix 6C) consisted of the investigation of the relationship between area and perimeter of the geometric figures. The reflections of the participants regarding this task suggest that fact that as time went on the course did offer an alternative lens through which to model their ideas. Therefore, this open-ended activity provided an opportunity for them to learn and see for the first time a new connection between each of quadrilaterals which they had not recognised before. Perhaps the activities of the course constituted a mechanism for them to see new ways of learning and teaching. This is evident in Celal's reflection on the problem which was about the concept of the relationship between the areas of different types of quadrilaterals. Here is the example:

We cannot clarify with the students that the area of the parallelogram or the areas of the square come from the same idea. However, in this software program we see that the area of the parallelogram may be equal to the area of the square with the help of sliding a bar and dragging. Substantially, it is a profitable way of learning. We have become accustomed to every figure having a different formula to its name. But, I have never thought that all of them have a common connection... [Celal, L345/I2]

This comment suggested that working on exploratory activities on the computer allowed Celal to say things which were quite contrary to what he said about the importance of memorising in learning before the PD course. One informant, Muslum, began to view that memorising is not essential for pupils to learn mathematics and the computer as a medium of building up mathematical understanding.

It is enough to know two variables, such as base and the height in any area of quadrilateral. There is no need to make memorise many formulas. It is the most logical way to learn something by making connections. I can say about the area of the circle that it satisfied me, indeed. Student's learning needs to be improved convincingly... [Muslum, L348/I2]

Motivation, individual differences and personality

During the PD course, all the teachers put into words the view that the environment composed of GeoGebra software are going to increase students' motivation towards

mathematics. For example, one task consisted of the explorations of the properties of transformations through GeoGebra-activities. They believed this task with its GeoGebra activities would make the class enjoyable and productive and help pupils to visualise these transformations easily. Here is the example of one's reflection on this issue:

The topics I teach on the blackboard can make students feel bored. My students became bored with monotonous lesson or routines activities. I believe that it would be useful if I did something different in the classroom. For example, it is a very entertaining activity (task six) while studying in a group with GeoGebra. Students will have fun and enjoy themselves while doing this activity in my classroom. This would increase students' motivation towards mathematics... [Musti, 8 June 2011/S8]

Like Musti, Emin held the belief that teaching mathematics through GeoGebra activities break the monotony of the everyday classroom and student would become enthusiastic to learn mathematics. They thought that the interactive nature of GeoGebra could make students more familiar with mathematical concepts, when they observe transformations with shapes in different positions on the screen. They believed that students would like to see the use of different materials in the classroom and this may enable them to get more motivated. They indicated based on their experiences that students get bored of a focus only on the teacher in the classroom and instead it is necessary to arouse the interest with different things because traditional environments cannot satisfy students in terms of affective aspects.

The participating teachers thought that worksheets based on GeoGebra activities would not only increase the desires of students, but also help students to overcome the problems based on personal differences in learning. They believed that each student has a different learning style. This reality is neglected in the traditional environment and due to this negligence students have insufficient learning and this problem can be overcome with GeoGebra. They point out that traditional environment does not give students enough freedom; because of this students cannot learn properly according to their own styles. In addition, participants stressed that the GeoGebra-supported environment based on investigation and discovery may help to enhance students' confidence and learning. Here is the example of one's reflective writing:

I can say that learning mathematics can be productive through GeoGebra-based mathematical activities. In this regard, students are involved in more interaction with each other, and can benefit from each other more to develop their own learning. This would help them to motivate each other and support them to build their confidence while exploring mathematical ideas... [Emin, 18 May 2011]

The participants generally emphasized three points related to this issue:

- i) The graphical and dynamic world of GeoGebra might increase the students' desire to learn and save them from the monotony of traditional environment.
- ii) Each student learning speed and style do vary. Traditional environment neglects these personal properties. Students can learn properly according to their own style with GeoGebra-supported with worksheets.
- iii) When students find out some things in mathematics they would develop a sense of confidence and motivation.

Encouraging thinking and investigation

The participants believed that GeoGebra gives an opportunity for a discovery, thinking and finding out environment and this enables student to increase the power of thinking and gain different perspectives. For example, the second task consisted of investigation of constructing a square, regular hexagon and exploration of a circle and also visualization of Thales theorem. The last activity of the task initiated an investigation of the centre point of the circle using the given tools. All participants made a great effort to find the centre of the circle. As expected, they found this activity a difficult one, but it provides the opportunity to think about both the mathematical properties of the figure that they are to construct, and how to use the tools in GeoGebra to construct them.

I have never thought about it before with any curiosity...It was a good activity to remove the question marks in the mind. When a student asks me why and relative to what that point becomes the centre, my answer is already ready. Yet, it has not been asked until now. In my opinion my curiosity has increased with this activity, it has made my discovery feeling increased. I have realized that mathematical learning with questioning and discovery instead of accepting the ready-made mathematical knowledge is so important... [Asim, L349/I2]

This type of engagement allowed the participants to create and find or gain understanding of some mathematical principles and concepts through their own effort. Moreover, they seemed to feel that learning through discovery with using the computer is a more effective way than learning through memorising with ready-made knowledge. However, they viewed this activity in terms of trying to find a centre of circle rather than the opportunity to make conjectures.

Other participants, like Asim and Emin, complained that the traditional way of teaching mathematics results in memorising the mathematical facts, not learning. This takes the

edge off learners' sense of doing research and investigation. They believed that the activities in GeoGebra environment can evoke these feelings.

The last activity in worksheet 4, done using GeoGebra, gives the students the opportunity to discover. If a student realises and sees that the area of the quadrilateral decreases and increases in different values by using a slider, s/he can easily find the values of the biggest area and realise at the same time that a quadrilateral with the biggest area is a square. This type of activity provides an opportunity for my students to investigate and think about mathematical concepts... [Eysun, L280/I2]

This statement suggested that this activity allows for more discovery learning including more thinking and more explorations in order to find mathematical concepts. These participants thought that students can discover relationships between area and perimeter of the geometric figures through different types of representations. On the other hand, Muslum believed that when a student has developed the skills of mathematical reasoning, s/he can solve different problems on her/his own. He believed that the computer, in particular GeoGebra, has a great potential to be able to develop this logical reasoning. He came to see mathematics teaching as a kind of development of logical reasoning, believed that GeoGebra is very appropriate to reach this aim. That is to say, believing that GeoGebra can assist learners to broaden their horizon and improve their imagination because of its properties. This was evident in Emin's verbal commentary:

I think GeoGebra is open to everyone who has a different perspective, new ideas and different interpretations. For example, students can arrive at different interpretations from each task... In GeoGebra everything is in the learners' power and they can do whatever they want...The students are able to think outside the box... [Emin, L209/I2]

To summarise, the participants held the belief that mathematics is learned in a boring way and they felt that it could possibly become easier and more productive, if it was learned with GeoGebra-based activities. Though, there is a conflict between what the participants expressed about the PD course activities and what they tried to do during the work on the activities. Even the most eager participants were not able to engage with mathematical situations in meaningful way but rather they are obsessed with finding the result of the task while they were working on mathematical activities. Although participants were not familiar with open-ended exploratory activities, they displayed their enjoyment of making discoveries.

5.3.3 Reflections on Social Interaction and Pedagogy

Although new mathematics curriculum initiatives have encouraged teachers to promote collaborative learning and communication among their students and colleagues, these activities have not been sufficiently integrated into Turkish mathematics education. Almost all participants initially were not able to articulate their beliefs about collaborative learning. The common belief shared by the participants was that they tended to value mainly whole-class teaching, with all students engaged in the same kind of work as opposed to small group work which is not thought to be appropriate in current mathematics classroom due to various reasons such as crowded classrooms, examinations and time. In terms of classroom organization, the participating teachers thought students sitting in rows facing the teacher and the blackboard to be a more appropriate arrangement in the current situation. At the initial interview, the participants said that students relied heavily on teachers' help but teachers thought that collaboration might help them to more independent. Following some sessions, the participants came to realize collaborative learning from different viewpoints formed as a consequence of their interaction with GeoGebra activities as a group activity.

Even in the first session of the PD course, it became apparent that all participants displayed their enjoyment in sharing ideas with peers. This openness to others' comments about activities may be an important factor in contributing to their perspectives about mathematical pedagogy and their personal growth. During the group discussions, they were more willing to share their approaches and strategies with others. I recorded one of my reflections on this issue:

During the PD course, the participants were asked to think and share their ideas about mathematical activities within the group. It was surprising that they tended to discuss the tasks and to listen to others' ideas since this kind of professional development activity was something which they have never experienced before. It seems to me that they did not want to terminate their discussion but rather they wanted to keep talking... [Observation, 18 May 2011/S5]⁴.

This comment suggested that the course participants became more open to the idea of collaborative learning and group discussion. The participants appeared to enjoy working together and exchanging their professional ideas regarding the GeoGebra activities. As the PD course progressed, they came to recognise the possibilities for interaction and

⁴[Observation] refers to the date of the researcher's field notes.

co-operation between peers in the classroom as positive components of mathematics learning. The evidence arose from one of the participants' reflections on this issue:

No one wonders where these theorems come from and what the proof is. They just accept them as what they are. I was impressed by some of the GeoGebra activities as I have never thought about and proved them before. I and my colleagues expressed our initial thoughts about these tasks after you asked us. We have had an opportunity to discuss something. That is, the PD course allowed us to understand these theorems well through collaboration and discussion. I learned something new after this age. It had not been very crucial, but surprisingly it became rather significant. Exchanging ideas between friends about tasks increased our knowledge and motivation. This also caused us to pay more attention to this course... [Emin, 8 June 2011]

This comment suggests that this type of engagement in a course allows participants to generate or stimulate sufficient understanding about theorems further, through interaction and group work. The participant came to recognise that co-operation and discussion could function as essential motivating activities for learning mathematics. It appears that dialogue and interaction with peers increased his interest in the course. The course environment enabled the teachers to work together, supporting each other, fostering interest and promoting experientially based understanding through social interaction. These ideas are compatible with a constructivist view of learning and teaching.

The course also seemed to make it possible for the participants to recognise differences between the new environment and the traditional environment in which they often work individually. This was reflection in the social interaction that took place, as was reported in my observational field notes:

It was observed that some of the course participants were initially willing to work on GeoGebra tasks individually. When they became stuck and felt their knowledge was insufficient, they attempted to get assistance from others. Later, they were eager to test out their ideas and thoughts about particular issues with their colleagues. This may be because they wanted to formulate their own sense of mathematics, and through peer interaction, to confirm whether they had understood correctly. One of the participants' reflecting on this issue stated that "my course mates may have brought different perspectives to this situation that I may not have thought of". It should be noted that the GeoGebra activities and social interaction provided participants with great support, constituting a creative and active classroom. As the course sessions progressed, they seemed to enjoy this collaboration and interaction and give consideration to each other's ideas... [Observation, 25 May 2011/S4]

This comment illustrates that the participants' realisation of what was occurring provided a basis for how the programme shaped changes to their beliefs about mathematics pedagogy, including peer interaction. As a result of their mutual participation, they came to realise that learning could be more useful and effective when scaffolded by their peers. It appears that the course enabled the participants to change their beliefs about the role of the learner while learning mathematics. It should be noted that collaboration between the participants played a central role in improving their learning and teaching.

One of the participants, Asim, believed that communications with the groups encouraged participants to make conjectures and decisions about mathematical situations. This kind of activity in the GeoGebra-based course helped them to develop their mathematical thinking. For instance, one of his reflections on collaborative learning was that:

It was a significant experience to listen to the opinions of everyone carefully and benefit from different opinions if they exist. We tried to make as many cooperative and class debates as possible during the course. This was one of the things that led me think mathematically. We attempted to find solutions to the question which arose during discussion. It was a remarkable experience to listen to other people's opinions... [Asim, L558/I3]

The participants came to recognize that group discussion and working in a small group might provide a means for group members to deal better with mathematics. For these participants, talking and sharing mathematical ideas could be seen as a useful activity which provided an opportunity for learners to learn mathematics more effectively. The comment that "Students are studying in pairs learn more and maintain it longer" [Musti, 25 May 2011/S6] supported the notion that one of the benefits of group work is to learn mathematical content effectively and deepen mathematical understanding by examining the views and ideas of others. Some of interesting points about group work were revealed during discussion with teachers in the PD course that group work is an effective way of mathematical learning and in this way, learners could find an opportunity to express themselves and exchange their useful ideas. Asim also added that "while working in the group one can suggest alternative solutions of problems". At this point, they were giving more attention to emphasising group work as a setting that provides an opportunity to negotiate one's own findings and accelerate problem solving.

Although Eysun and Celal expressed their apprehensions about teaching through group work in current classroom, factors such as time pressures and crowded classrooms became shared concerns of these participants which may have an effect on their decision making about using collaborative work in teaching and learning. They already knew very well that the new mathematics curriculum activities and textbooks were designed in line with constructivist perspective which is open to new strategies and ideas. However, integration of such useful activities could be difficult and time consuming for teachers who wanted to follow the curriculum schedule.

Actually I heartily want to create a discussion group in my class. However, I have to pass some activities in the text book such as debating, sharing or listening to your fellow students' ideas. I should not do that. I don't defend that my behaviour is correct... [Eysun, L268/I2]

Eysun's comment illustrates that constraints exist on the time needed for teachers to complete the curriculum schedule. On the other hand, I found one participant had some doubt about learning through pair work when introducing new mathematical concepts in classroom. Here is the example of one's participants' reflection on this issue in session three:

I think pair works itself, sometimes, might not be useful for students when introducing new concepts. One discusses new concepts within the group for the first time, as not easy for students to understand; because they may not know what they are supposed to do with these concepts. It may take them a long time to explore and understand it... [Celal, 11 May 2011/S4].

This statement implies that the teacher should make necessary and sufficient interventions while those students are learning new topics in small groups. This view seems to be based on whole-class teaching approach, student should receive mathematical concepts from the teacher they are not capable of learning them by themselves. Indeed, there seemed to be some contradictions in this participant's beliefs about integration of group work in classroom.

Considering the participants' reflections on student learning, Emin was concerned with getting feedback from what students seem to learn so as to promote his teaching. This allows teachers to understand student learning. They came to recognised that students may have their own ideas about particular mathematical topics or activities and that it can be useful to ask students for feedback on their own teaching through classroom discussion. For these participants, this could provide teachers with a better understanding of their own students. This also may serve to shape their classroom

teaching activities and to develop the social interactions between students and the teacher.

While students are working with a group in a GeoGebra environment I can monitor their discussions about topics, which I have taught them in class in terms of what they are thinking about and doing. If the concept has been not clearly communicated to the students and assimilated by them, I would use different activities... [Emin, L411/I3]

However, receiving feedback from students was viewed by Eysun from a different point of view. They did not consider alternative strategies or interpretations of students. At the same time, they valued feedback as a mechanism for checking students' understanding in terms of what they understood.

5.3.4 Linking GeoGebra with Mathematics

In this sub-section, the aim is to give an image of how valuable the participants perceive GeoGebra to be in a mathematical context and the extent to which they engaged in mathematical activities throughout the course sessions. As described in sub-section 5.1.4, the course teachers initially believed that the role of the computer in mathematics was seen as entertainment rather than as a pedagogical or conceptual tool in their mathematics teaching and learning. After completion of some sessions, they may foster their competency with GeoGebra and develop a critical perspective towards the use of computers in mathematical context.

As far as the participants' reflections on the GeoGebra-based activities were concerned, almost all the participants seemed to have developed an awareness of the potential of GeoGebra in mathematics. They considered GeoGebra as a tool for providing dynamic and multiple representations of both algebraic expressions and geometrical figures that facilitate a visual learner's understanding of mathematical abstract concepts. They came to believe that visualisation and manipulation have a positive effect on mathematical understanding. For example, Muslum state that:

In this task I saw different reflections of same concepts dynamically on the computer and I observed how graphs change when different equations were typed in. GeoGebra-based activities make it easy for teachers and students to understand. They can see more examples of transformations of figures using GeoGebra... [L440/I3]

His comment suggests that the graphical aspects of GeoGebra appeared to have appealed to the course participants and encouraged them to take an interest in exploring visual effects with it, and they became interested in understanding the mathematical relationships embedded in the visual patterns. At the same time, Celal held a belief that GeoGebra may help learners to construct geometrical figures quickly and produce many examples in short time. In line with the traditional view of mathematics teaching and learning, they felt that more practice and examples could provide better understanding. This type of view seems to reveal the fact that GeoGebra-based mathematical activities provides a link between participants' preference for visual learning and the demand of the examination-oriented education system for more practice in a limited time.

The dynamic geometry environment provided by the PD course offered the means to interact with mathematical activity which appealed to the participating teachers. That is, some features of GeoGebra software such as the construction protocol, dragging tool and investigations with the slider are thought to be powerful ideas of GeoGebra. All participants were pleased with these powerful features and used them extensively throughout all the course sessions. A good example could be indicated by one's extensive use of dragging tool to explore dynamic geometrical objects:

Concepts like construction and drawing altered the reality of the structure of geometric figures not being able to cut across a drawing, their nature... We had an idea about all the equilateral triangles that we could make by dragging their edges when we construct them. It was an approach which is more encompassing and appealing in general. That's why GeoGebra showed us the reality of one figure being different from one drawing in the GeoGebra, even when only with its property. I think it is the first exciting thing that I can say... [Asim, L249/I2]

Asim's comment suggested that the participants had recognised that when creating an object by using GeoGebra in dynamic environment, the distinction between drawing and construction should be recognized clearly because these ideas would support learners in understanding the necessary characteristics of the figure. In this regard, the dragging tool in the GeoGebra environment not only allows learners to check whether the features are real or not but also to discover a variety of similar constructions and special cases. That is, learner is able to generalize mathematical concepts through using predesigned GeoGebra activities. These participants' accounts on this issue reflect the idea that this software would help them to see mathematics as a consistent system of concepts and to make connections between different parts of mathematics.

Another example described that “GeoGebra enables us to learn through trial and error. Once we constructed geometrical figures we can check and see the construction protocol where we have made a mistake and learn from it. Basically this idea increased my interest towards mathematics” [Emin, L401/I3]. This statement illustrated that dynamic geometry learning environment provides learners with getting immediate feedback about their actions that might enable them in reflecting on their conceptualization. Communication with GeoGebra provides useful feedback encouraging them to search their mistakes by looking at a construction protocol.

It is a belief shared by many participants, for example Asim, that working with the GeoGebra could be considered as a mathematical thinking activity in which one uses imagination and mathematical reasoning.

Constructing an object in GeoGebra, you have to use your imagination. When we are working on our tasks step by step; we are able to think in a mathematical way. Actually it requires thinking so much in the process of construction. When it gives you what you are expecting to see on the screen, you feel as if you are accomplishing something... [Observation, 25 May 2011/S6].

This comment suggested that these participants seemed to think of themselves as being able of thinking mathematically while they were doing GeoGebra tasks. In this respect, this helped them to build their own mathematical concepts while engaging in GeoGebra tasks. They thus expressed their self-satisfaction at being able to solve the tasks independently using mathematical thinking.

The participants’ accounts above revealed the belief that students could discover mathematical ideas and connect multiple representations of these ideas dynamically in the GeoGebra environment. In this sense, they came to see non-routine activities as an opportunity for students to discover mathematics from the GeoGebra perspective as a part of new mathematics curriculum activities. However, Musti began to view the possibility that through GeoGebra-based curricular activities; students could be encouraged to extend their prior mathematical knowledge. These participants appeared to contemplate developing similar activities for learning and teaching concepts, and they felt positive that they would be capable of doing this with GeoGebra.

To summarise, the course participants progressively began to have more focused opinions about the computer particularly about GeoGebra in mathematics education,

resulting from their learning experiences with informal GeoGebra activities through group work. They expressed different views about the potential of GeoGebra:

- ❖ It would establish a common connection with new mathematics curriculum.
- ❖ It would establish a common connection with new mathematics curriculum.
- ❖ Its activities could enhance the quality the teacher's teaching.
- ❖ It can arouse and sustain students' interest towards mathematics through exploratory activities.
- ❖ It makes abstract concepts more concrete.
- ❖ It assists learners in making associations among different domains of mathematics.
- ❖ It assists students to notice mathematics as a consistent system of concepts.
- ❖ It helps learners create their own mathematical ideas.

5.3.5 Time as a Key Issue

Through analysis of these research findings, time emerged as a key theme which was apparent in participants' reflective writing, verbal commentary and my observational field notes. Furthermore, this issue was also evident in the analysis of participants' reflections on the PD course activities and the use of GeoGebra in mathematics education. To illustrate this, they showed a tendency to attend more sessions as follow-up to become more confident in using software in a real classroom setting. They wanted to spend more time working on it, as the following excerpts suggest:

I can surely say that I need to improve myself to be able to become a good user of GeoGebra, especially in this summer time, so as to use it in the next school year, because I require more time to learn and use it in my class... [Musti, L501/I3]

We performed six tasks on the worksheets. Now I wonder how we can use GeoGebra in the other areas of the mathematics. Perhaps the time of the course can be extended in order to perform activities about other topics... [Celal, L367/I3]

There were participants who expressed difficulties in becoming, in a short period of time, a good user who needed more time to become familiar with GeoGebra. From a different viewpoint, time has been mentioned elsewhere in this thesis. Namely, Musti and Emin considered the computer as a time-saver, allowing the learner to see patterns and processes without time consuming traditional work, providing the teacher with more time to complete a mathematics lesson, enabling them to produce more accurate

geometrical figures, which meant that they could allocate more time to other teaching activities. Further, teachers do not need to wait for students to finish taking notes in class. The course participants believed that GeoGebra enables teachers to save time that are wasted on drawing on board and note taking. They provided the following account regarding the time issue:

When the slope concepts are considered, you can show five examples to teach this concept on a board in a one hour lesson, but in GeoGebra you can do five examples in five minutes. As a teacher, you can avoid wasting time. Pupils can perform and try whatever they want and they can see a lot of different examples...GeoGebra gives us the opportunity to study thousands of examples... [Muslum, L373/I2]

When I teach mathematical topics with the help of GeoGebra, I will be able to save time by using the slider. For example, if I draw three geometrical figures when I teach lines with the same slopes, I can produce many more examples for the same concept in GeoGebra... [Eysun, L270/I2]

However it lasts a long time - three hours - to demonstrate it (the slope) in the classroom. It cannot be demonstrated on the blackboard purely and there isn't enough time for the students to take notes. I should say so we can save time with the activities in GeoGebra and thanks to this GeoGebra presents us with the opportunity to see how the construction is done step by step. When the student goes home, s/he can review what s/he has just done in the classroom. The biggest advantage of GeoGebra is time. It enables us to save time... [Emin, L293/I3]

These comments suggested that the idea of computer as a time-saver for both the student and teacher can be easily drawn from the participants' comments. However, Eysun was concerned that the integration of GeoGebra into teaching will create some difficulties in planning to implement it. It is a view shared by them that extra preparation time and courses will be required for both teachers and students. They held that in order to work with computers it is necessary to spend many hours before making them work and they would have to allocate substantial time in preparing GeoGebra activities for group work. They also wanted to make sure that students' computer literacy level is suitable for applications of this kind of activities before presenting GeoGebra-based mathematical activities in classroom. Therefore, being able to use GeoGebra would oblige the teacher to allocate a reasonable amount of time in preparing students for the use of GeoGebra. They asserted that they would not be able to find convenient time for all of these in the current system. Furthermore, time became evident in considering participants' inclination to use GeoGebra in their teaching when they felt confident enough to attempt GeoGebra in their actual classroom in the primary school:

I want to use GeoGebra in my classes. But I don't know if I will completely use it. At least I want to demonstrate the activities we have done on the course to the students. For sure it is necessary to have enough computers and I need to allocate some time to it. Even if every week is not possible, I will try to use it once or twice in one month...[Celal, L338/I3]

I need more time to prepare GeoGebra activities; I should spend hours before conducting this type of lesson. I mean a single class will require many hours of planning... [Eysun, 407/I3]

However, there will be the problem. I cannot finish all the topics on time as a teacher. At the beginning, there may be difficulties when we use GeoGebra in an existing system. Later I believe that we can overcome this difficulty... [Emin, L408/I3]

These comments implied that the current mathematics curriculum tends to limit the participant's opportunity to take full advantage of the potential of the computer. The curriculum is so intense and there is not enough time to teach the whole curriculum in one year. Their comments also reflect the fact that they would probably always encounter the increasingly difficult task of finding time in this current education system. Nevertheless, Asim and Emin believed that the use of GeoGebra in Turkish primary schools does seem to be feasible.

5.4 Changes in the Participants' Beliefs

The previous section highlighted how participants initiated the move for change themselves, and continued that change during the PD course. The primary intention was to find out how participants in the PD course form their beliefs. The potential shifts in beliefs of the participants were identified using interview transcripts, participants' reflective writings and a partly post-course mathematical beliefs questionnaire. An attempt is made to bring together the data generated from participants' engagement with investigational mathematical tasks during the PD course and the extent to which conceptions and beliefs were changed, or not, within a computer-based environment. This section attempts to explore the overall changes in participants' beliefs about mathematics, learning and teaching of mathematics and computers, related to their involvement with the PD course.

5.4.1 Changes in the Participants' Beliefs about Computer

Although the nature of present study was to specifically examine school teachers' mathematical beliefs, their involvement with the PD course based on the use of dynamic

geometry would have an impact on beliefs about the computer. Actually, it turned out that changes in the participants' beliefs about computers seemed to be more pervasive than those in their beliefs about mathematics, and its teaching and learning. Changes in teachers' beliefs about mathematics, mathematics teaching and learning are not as easy to change as teachers' beliefs about computers, since these have been evolved out of short term experience. The most significant changes appeared to have taken place in following areas: (a) the participants' personal feelings towards computers; and (b) their beliefs about the role of computers in mathematics. I will now explain these two parts in turn.

Feelings towards computers

It is reasonable to expect some changes in participants' personal feelings about the computer on account of their participation in an eight-week course based on use of GeoGebra. This type of course appeared to have led to fairly prevalent changes in the participants' feelings towards computers (including enjoyment, confidence). These changes were more predominant in the course participants who had little or no experience with GeoGebra and computers. Despite the fact that some participants, Asim and Emin, lacked confidence in their ability to engage in GeoGebra based activities at the initial stage of the course, coming to know how to use the computer and using it for an extensive period of time led them to reconsider their feelings: "I never thought how computer could be so useful in mathematics"... "I dealt with course activities more easily than expected"... "I found that very exciting". These comments suggested that Asim and Emin appeared to have enjoyed engaging in such computer-based mathematical activity. An important mechanism for this re-appraisal was a sense of personal accomplishment developed out of the course activities.

Although Celal who was sceptical about the role of computer in mathematics and whose reasons for attending the PD course were the increased expectations of the new curriculum, he was enthusiastic about building his competence with GeoGebra or computers in order to use in his actual classroom. This encouraged them to engage in meaningful open-ended mathematical activities that aroused their interest and curiosity, allowing them to understand an additional dimension to the use of computers in mathematics. Yet, Eysun appeared to have disappointment and failure at the initial stage of the course. Her reflections were that "Just the thought of spending hours learning

them made me feel tired”... “I would not be able to complete some of the tasks”... “I know very little”. The important point here is that a sense of incongruity between participants’ own preference for learning and the method implemented on the course. With its stress on self-direction, the course might have presented something entirely new into these participants’ agenda as a learner and teacher. In consequence of their loss of control and progress over learning, they sometimes felt some embarrassment during group work.

The role of computers in mathematics education

Throughout the course sessions, participants’ accounts associated with the use of computers in mathematics could be categorised into three distinct views. Asim held the belief the computer as an instrument can be used to explore and find out aspects of mathematics and to allow students to discover things for their own sake. Participants in this category believed that GeoGebra-based investigational activities can play a significant role in students’ learning and students might have opportunities to review and investigate mathematical concepts by using them. Muslum believed that the computer has the potential to transform how mathematics is introduced and can be considered as an image maker to provide visual activities such as graphics, tables and diagrams so as to enhance the quality of their lessons. At the same time, GeoGebra activities might help to fill the gap which exists between what the students had learned in the classroom and what they had understood from the teacher explanations. In these respect, informal activities might support improving students’ understanding, and making mathematical ideas and relationship more clear and enabling concentration on understanding. Celal came to perceive the computer as an additional component of teaching and learning of mathematics, like a supplementary tool for their lesson in controlled ways. For them, the teacher should enable pupils to understand mathematical concepts by using several approaches, one of which is to use GeoGebra as support for an expository approach. These participants seemed not to strongly value using computer for mathematics education. For them, GeoGebra-based mathematical activities can be used after regular the class period. Within this regard, using computers in mathematics may attract students’ interest and makes them engage with the lesson.

It is important to illustrate the extent to which the PD course changed the participants’ beliefs about the role of computers in mathematics. Towards at the end of the course,

for example, Emin shifted from viewing the computer as a supplementary tool for his lessons to considering the computer as a tool that can be used to explore some aspects of mathematics and to enable learners to investigate things for themselves. This may suggest that this participant appeared more in favour of having students to explore mathematical concepts, and is more likely to create a computer-based learning environment in which students could work cooperatively and take responsibility for their own work. Asim commented that “Software like this is going to nurture students who are innovative, inventive and enthusiastic” [L406/I2]. From this perspective, pupils would have the opportunity to do mathematics rather than just focusing upon the mathematics of the curriculum. Another example indicates that Muslum initially regarded the computer as a convenient tool which can be used for demonstrations, calculation and visualisation. He appeared to show a tendency to meeting current daily demands in mathematics teaching and thought computers-based mathematical activities develop students’ calculation ability. The course offered this participant the opportunity to participate in personally meaningful mathematical activity. At a later stage of the PD course, he came to appreciate the view that the computer may make the teaching of the subject easier, interesting and attractive. In brief, the participating teachers became aware of the potential for using computer as an educational tool for the teaching and learning of mathematics as a result of attending PD course.

5.4.2 Changes in the Participants’ Mathematical Beliefs

Although the new mathematics curriculum has been developed by adopting constructivist approach, the participants’ accounts revealed that they initially held more traditional beliefs about mathematics teaching and learning. Research showed that if one’s experiences with mathematics are related to more traditional approach as a student or as a teacher over a long period time, it is more likely that he/she will develop traditional beliefs. As already mentioned, changes in the participants’ beliefs about mathematics, learning and teaching of mathematics seemed to be less substantial than in their beliefs about the computer. It is not easy to change one’s beliefs about mathematics through involvement with a short term of PD course based on the use of GeoGebra. However, in the short period of the course, rich mathematical discussion and engagements with self-directed and exploratory mathematical activities in PD course provided an opportunity to develop participants’ professional experiences which encompasses both a new way of thinking of mathematics itself and their professional

development. This was particularly true of the participants working in small groups with GeoGebra which created rich mathematical discussion environment between partners. Regarding participants' beliefs about teaching and learning, more than half of the participants became more open to accept to ideas of constructivist perspective and reported some form of shift. The following part elucidates the kind of changes which occur in the participants' beliefs about mathematics, mathematics teaching and learning.

Nature of mathematics

At the outset of the PD course, the course participants initially held a mixture of absolutist views and fallibilist views about the nature of mathematics. It can be interpreted from the findings of pre-and post-course mathematical beliefs questionnaire (see data analysis in Chapter 4); the PD course had some effect in changing teachers' beliefs on items associated with absolutist view of mathematics. After PD course, for example, Muslum was less likely to believe that doing mathematics consists mainly of using rules. This may suggest that course participants appeared to have beliefs consistent with fallibilist view on most items. However, Celal still disagreed only with the notion that "there are different forms of mathematics in different cultures around the world". In other words, he expressed the view that mathematics is universal. It can be said that the PD course had an impact on a small number of participants' beliefs about the nature of mathematics to some extent in favour of the fallibilist view.

Emin who before the PD course did express their enjoyment of mathematics, tended to express the same kind of feelings at the end of the course. Another participant Musti mentioned that "regarding that it affects me so much, I am sure that there brings out so many good results". Throughout the PD course sessions, the course participants seemed to have felt confident in coping with the GeoGebra-based mathematical activities and experienced success rather than failure. However, they had invested little time in reflecting upon the nature of mathematics. Therefore, it is not surprising to find that the relationship between participants' engagements with the PD course and their expressed beliefs of mathematics is not a core one. However, as the course progressed, Emin articulated his opinions: "new solutions can be invented by discussing with partner" and "you can come up with innovative ideas through interacting with GeoGebra activities". Underlying these comments was a new way of thinking that could be linked to a broader view of the nature of mathematics. The course participants initially believed that

mathematics consists of certain rules and procedures that students need to learn from a teacher, because they cannot invent or obtain mathematical knowledge on their own. However, later, their experience with the course helped them acquire a new sense of what it means to do mathematics. Their beliefs of mathematics became a product of interactions among people and objects. One may speculate that these expressions became evident in the problem solving perspective which sees mathematics as process of human activity and invention; This gives support to Ernest's view "continually expanding field of human inquiry, not a finished product and its result remains open to revision" (Ernest, 1989, p. 250) because students are able to explore the solutions of mathematical problems.

The participants' involvement with GeoGebra based activities helped them broaden their beliefs about the nature of mathematics, from a somewhat fixed and absolutist oriented standpoint to a more dynamic and creative one. For example, Asim commented that "...Let alone the change in our minds, even children will be able to say that mathematics is not really something strange and to be afraid of...mathematics will make sense... mathematics is more than practising numbers on pieces of paper... mathematics becomes more active subject through these activities". These accounts of mathematics are some of their course reflections point to some change in their beliefs about mathematics in that mathematics was no longer static and fixed body of knowledge. This is a new way of thinking about mathematics that is entirely different from absolutist-oriented beliefs about the nature of mathematics. And, this kind of shift appears to have been catalysed particularly by the dynamic and interactive nature of GeoGebra activities.

Apart from classroom engagements, participants' reflective writings about their experiences were gathered at the end of each session. This task motivated them to reflect on and discuss the course from professional perspectives when they met outside the class. All these engagements supported their professional development, and such shifts appear to show a natural outgrowth of the experiences they had had during both at the moment of classroom engagements and other social interactions outside the session. This type of shift is apparent in Emin's reflective writing:

When you asked us to discuss activities and problems with partner, I had never thought of mathematical ideas as discussible. I was thinking that if a mathematical problem was a simple matter if right or wrong, what was there to

discuss? Now, I believe that new conjectures can come up and new solutions can be invented through group discussions and collaboration [1 June 2011/S7].

The course teachers initially held two combined beliefs about mathematics, they started to believe that mathematics cannot be reduced to a simple matter of finding right or wrong result. They developed a new way of thinking about mathematics. For them, perhaps mathematics became fallible and less straightforward than it had been.

Beliefs about teaching and learning

Participants' engagements with open-ended mathematical activities in the PD course provided an opportunity to change their own previous experience with mathematics teaching and learning. The PD course also made it possible for the course participants to see mathematics learning and teaching from a different point of view, to think about an alternative teaching and learning model. This assisted them to engage mathematics in an active way and some were pleased with the advantages of negotiating ideas with their partners and comparing different methods.

At the beginning of the PD course, Musti and Muslum accepted the idea that memorisation and practice are important parts of mathematics teaching and learning. However, following completion of some course sessions, they began to accept the idea that learning mathematics is an active process, not passive one and the product of a student's effort to think something over in his/her own way and understand it. Their comments in the final interview suggested that this could be achieved with the help of teacher's guidance and encouragement which involve asking purposeful questions during teaching. Similarly, Emin who expressed this view suggested that students should also listen to their own voice. They started to reject the idea of memorisation and support the constructivist perspective, believing that students need to discover and are able to do mathematics to learn the concepts best. They supported the view that learning is "a process of inquiry and coming to know" (Ernest, 1989, p. 250) and utilizing computer, not a process of memorizing.

Here is how participants assessed the PD course's influence on their beliefs about mathematics learning "the approach was used in this course totally eliminates "rote learning" and leads to a more "understand the problem" and" way of looking at things". Asim was already concerned with learning mathematics in a way that was connected with the essence of the course. He derived pleasure and challenge from his personal

involvement with the PD course. Associated with this, the participating teachers were incrementally developing an understanding of a different way of teaching and learning the subject, and at the end of the course, they seemed to be conscious that the pedagogical approaches in PD course as a model in terms of teaching and learning. They seemed to have found room to give his pupils a more active role in learning. This type of the PD course seemed to have supported most participants in gaining a new understanding of mathematical learning and teaching. Here is the evidence in Asim's statement:

We always talk about student-centred education, active learning, students who can discover, make a conjecture, presumes, self-educate himself... But where are they? How can this be done? What I saw in this course... this course helps me to see this type of teaching, and will offer a great support to establish my creative and active class... [L525/I3]

After participants' experience with the PD course, some participants', for instance Celal, comments suggested that students are not capable of exploring and investigating mathematical concepts on their own without knowing how to go about the findings the solutions and teacher's explanations are necessary in student's learning. This means that the mathematics teacher should build on what the students already know which includes frequent explanations and review of topics that are related to the present task or problem. This view appears to stem from one of the pervasive traditional expectations that students cannot recreate ideas of mathematics on their own, so the teacher has no alternative but to present them. On the other hand, other came to believe that understanding mathematical ideas and concepts can be best achieved through discussion, group working and the use of GeoGebra. This type of change illustrated that these participants came to consider a mixture of two perspectives for learning mathematics: traditional view and GeoGebra that can assist mathematics learning by serving as tool for exploration and consolidation of ideas; however, they did not completely accept this idea as did previous participants. They considered GeoGebra as a catalyst for learning mathematics.

Considering the participants' responses to the questionnaire items related the role of teacher, they began to see their role as a teacher from the constructivist perspective that provides learners with tasks that promote them to wonder about and investigate mathematics. They appeared to believe that good mathematics teaching includes class discussion in which learners share concepts and negotiate meaning. They rejected the

idea of teacher's role as a teacher who simply provides knowledge to the students, but help them transform knowledge so that it becomes simpler and more powerful in solving problems and explaining mathematical relations. These participants can be labelled as a constructivist teacher. Another participant Eysun looked at her role from combining two views: a traditional and a constructivist. She appeared to believe that the teacher's role is to design active learning environment where student take on more responsibility for their own learning, and where students can use GeoGebra that might support them in mathematical knowledge construction. At the same time, the course participants were interested in monitoring the completion of activities without attempting to understand the nature of students' thinking. They also came to believe that good mathematics lesson progresses step-by-step in a planned sequence in the direction of the lesson aims. These participants form the most interesting group. Their pedagogical beliefs about mathematics teaching have indicated as both traditional and constructivist elements. This conflictual view is supported by the following reflective writing:

Teachers need to present the lesson and let their students explore. If students do not understand enough to follow the lecture, teachers will also need to use GeoGebra and computers and combine traditional methods and group work with GeoGebra... [Eysun, 25 May 2011/S6]

5.5 Summary

The PD course was designed to provide participants with a better theoretical and practical understanding of mathematics teaching and learning through engagement with computer-based mathematical activities that were consistent with the constructivist paradigm. The purpose of this chapter has been to offer a general picture of six Turkish primary mathematics teachers' beliefs about mathematics, its teaching and learning as well as the role of computer in mathematics education before interaction with GeoGebra-integrated mathematical environment and how their beliefs may have been affected as a result of their involvement with it. Participants' experiences with the course activities were highlighted in order to understand how their beliefs evolved (did not evolve) following their practice. It is recognised that some aspects of those beliefs might be changeable, some are not. It can be suggested that this type of the course seemed to have supported participants to achieve a new understanding of mathematical learning and teaching. However, it should be mentioned that the participants became aware of alternative approaches in teaching and learning but in actual classroom they

stated that they would not be able to put these beliefs into practice due to varied reasons (including institutional constraints). The evidence of this study showed that these factors might inhibit the process of change in teachers' beliefs about mathematics, its teaching and learning. The following part described some key points of changes in teachers' beliefs and these are introduced in turn.

The role of computer in mathematics education

Participants' beliefs about the use of computers in mathematics were varied, and they could be placed into three distinct views:

- ❖ The computer is viewed as an instrument that could be used to explore and investigate aspects of mathematics and to allow learners to discover things for their own sake.
- ❖ The computer has the potential to transform how mathematics is introduced and can be considered as an image maker to provide visual activities. This could provide learner to make sense of mathematical relationships.
- ❖ The computer is seen as a supplementary tool for teaching and learning of mathematics. Computer-based activities could enrich teacher's conventional mathematical lesson.

The potential of GeoGebra in mathematics

The participants articulated their beliefs about the GeoGebra on the basis of their engagements with mathematical activities. They have more concentrated thoughts about the nature and characteristics of learning during their engagements with GeoGebra-based mathematical activities. It can be summarised into four distinct views.

- ❖ The graphical and dynamic world of GeoGebra might increase and sustain students' interest towards mathematics through exploratory activities and remove from the monotony of traditional environment.
- ❖ GeoGebra activities provide an opportunity for learner to have enough freedom in order to create their own mathematical ideas.
- ❖ GeoGebra activities make abstract concepts more concrete in order to help learner to see mathematics as a consistent system of ideas.
- ❖ While working with GeoGebra activities, the student being centralised actively in learning can enable student knowledge to become permanent.

The impact of social context

The impact of social context (Ernest, 1991) became evident in this current study. The social context (including institutional factors, expectations of students, families and school culture) were impediments to implement new approaches. These contextual dynamics as a powerful set of constraints which might urge teachers to enact new models of teaching and learning mathematics. Although Turkish mathematics curriculum has been revised, teachers' stated teaching practice appeared not to reflect reform suggestions. In order to actualize recent reform movement in Turkey, the teaching practices of Turkish mathematics teachers should be compatible with new approaches to teaching and learning. This illustrated a tension between how mathematics teaching and learning ought to be and how it has to be in an autocratic oriented system. It should be noted that the PD course did offer an alternative lens through which to model participants' ideas and showed teachers the model of teaching and learning.

The role of the teacher

The PD course had an impact on teachers' beliefs about the teacher's role throughout course sessions. The course participants rejected the idea that the role of the teacher is to transfer directly ready-made mathematical knowledge to the students. They viewed their role as a facilitator who encourages learners to explore mathematical concepts and discuss their solution on tasks. They seemed to view that effective learning is a process of making sense of mathematical notions and constructing students' own ideas, encouraging them to share their knowledge with others and build their own knowledge by giving them enough freedom to discover and come to understandings on their own. Another participant Celal looked at mathematics teaching and learning from mixed perspective. He wants to play the role of clarifying knowledge to students alongside the consideration of students' need for gaps in their knowledge to be bridged.

How students learn mathematics

The evidence of this chapter implied that participant's initial beliefs about student learning were varied. A change in participants' beliefs takes place after their experience with GeoGebra activities. It can be categorised into two distinct views: First, the course participants held the belief that students can learn mathematics better on their own effort

with help of teachers who ask suitable questions and providing assistance and encouragement. They came to recognised that GeoGebra activities may facilitate this type of learning. Second, the teachers (e.g. Eysun and Celal) held two combined beliefs that when introducing new topics, it is not easy to understand for students within the group and therefore the teacher's explanation is important in students' learning. At the same time, they believed that GeoGebra make students much more independent.

In the next chapter, I will discuss the findings in relation to the literature review.

CHAPTER 6: DISCUSSION

6.1 Introduction

The purpose of this study has been to ascertain the degree to which a professional development (PD) course based on the use of dynamic geometry affected the beliefs of a group of Turkish primary school teachers. Following educational reforms in 2005, the primary mathematics curriculum was revised and constructivist approaches to learning and teaching were introduced into the Turkish educational mainstream. These changes were introduced by a top-down approach, which largely disregarded the beliefs and practices of teachers (Norton et al., 2002). However, in order for educators to be successful in curriculum reform, it is important that they attend to teachers' underlying beliefs, and work to challenge them by offering professional development programmes to underpin the theoretical foundations of that reform. Given the recognition of the importance of the teachers' role in facilitating curriculum implementation, the professional development of teachers has become a central element of educational research.

The literature review chapter (Chapter 3) revealed that teachers' beliefs about teaching tend to emerge initially from their personal experiences as students and then later through teacher education courses and the observation of other teachers, as well as in response to culture, values and engagement in professional development (Lortie, 1975). A strong link between teachers' beliefs about mathematics and their intended or enacted beliefs has been reported in the literature (Cross, 2009; Thompson, 1992; Wilson and Cooney, 2002). For example, Swan (2006) noted that any attempt to develop what mathematics teachers do in the classroom is contingent on the beliefs of those mathematics teachers and the ability to effect changes to those beliefs. This view is corroborated by Ernest (1989), who claims that "[t]eaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change" (p.249). Therefore, this study explored the possibility of effecting changes in Turkish primary teachers' mathematical beliefs through involvement in a PD course designed using GeoGebra, based on a constructivist approach.

This chapter discusses the main findings introduced in the previous chapter; in particular those associated with Turkish primary mathematics teachers' former beliefs about mathematics and their personal experience of the GeoGebra-based professional development course. As a structural basis for the discussion, this chapter examines the findings in reference to the research questions that I am aiming to answer. The chapter is divided into two sections according to the two sub questions: (i) what beliefs do Turkish primary mathematics teachers hold about the nature of mathematics, its teaching and learning before participating the PD course? And (ii) how do they change their existing beliefs while engaging in GeoGebra-based mathematical activities within the course as learners about mathematics, its teaching and learning? Each section then presents a discussion of the findings broken down into key areas as related to the literature reviewed in Chapter 3.

6.2 What beliefs do Turkish primary mathematics teachers hold about the nature of mathematics, its teaching and learning before participating in the PD course?

The particular beliefs that this study undertook to investigate were those beliefs that a group of primary mathematics teachers hold towards the nature of mathematics, its teaching and learning prior to their interaction with the PD course. This section focuses first on their beliefs about the nature of mathematics and secondly on their beliefs about the teaching and learning of mathematics.

6.2.1 Beliefs about the Nature of Mathematics

At the outset of the PD course, the course participants held strong beliefs about the nature of mathematics, even though they did not express these beliefs in an explicit way. Although there were differences between the six participants, all of them held strong beliefs that had been shaped during their own student years and also by their experiences as teachers of mathematics. This is line with findings reported by Scott (2005), who found that teachers' beliefs were mainly affected by their previous mathematics experiences and teachers, and by their graduate studies.

In the participants' accounts of the nature of mathematics in the present study, the participants' comments (e.g. Celal) indicated that they thought of mathematics as essentially synonymous with school mathematics, and then secondarily other comments (e.g. Musti) related to mathematics as a scientific discipline (see evidence in section

5.2). This dichotomy in the participants' beliefs about the nature of mathematics is entirely in accord with observations made by Beswick (2012) in her study. She observed that it is likely that teachers will hold isolated and different beliefs about mathematics as a school subject and as a discipline. Some of these beliefs (school mathematics) relate to an understanding of mathematics as being about a set of facts, rules and procedures, with a focus on objectively determining a correct and definitive answer (Ernest, 1989). For these participants, their views link to an absolutist belief of mathematics, something which is arguable predominant in the school system (Roulet, 1998). Mathematics as scientific discipline seems to be connected to a fallibilist-oriented belief, which describes mathematics as a social invention, its truths and concepts being relative to time and place (Lerman, 1986).

The results of this study imply that teachers' beliefs at the beginning of the PD course were primarily absolutist and fallibilist in nature. Muslum completed pre-course questionnaire items in a way that gave the impression that he held a more rule-oriented view (absolutist view) of the nature of mathematics stressed in the first interview the idea that "student can produce a piece of mathematics for enjoyment...". This personal statement about the nature of mathematics reflects a fallibilist-oriented belief. Another example, Asim believed that mathematical knowledge was created by God, independent from humankind and at the same time believed that learners can discover mathematics. His belief included both some elements of fallibilist and absolutist-oriented beliefs about the nature of mathematics. The results show that the participants do not necessarily directly reflect the views of either Ernest's (1989) or Lerman's (1986) classifications of perspectives on mathematics. Beswick (2005) admitted that individual teachers are unlikely to have beliefs that fall properly into a single type. At the outset of this study, I began by seeing absolutist and fallibilist-oriented beliefs about the nature of mathematics as two opposed tendencies. It is clear from the data presented in Chapter 5, the categories was reported in the literature does not completely represent teachers' beliefs about the nature of mathematics in this study.

The impact of cultural world views on beliefs about mathematics and its pedagogy

The findings reported in the previous chapter revealed interesting and unexpected responses to the question of whether mathematics is created or discovered, and the question of what precisely constitutes mathematical knowledge. One unanticipated

finding was that the participants' religious beliefs were vital in informing their personal world view, and as the research progressed it became apparent that this had consequences for their beliefs regarding mathematics and mathematics teaching and learning. They strongly asserted that their beliefs about the nature of mathematical knowledge seemed to be connected with their religious beliefs. McLeod (1992) mentioned that beliefs were inclined to grow incrementally and that cultural elements are important in facilitating their development. Half of the participants looked at mathematics from their religious perspective and those beliefs about the nature of mathematics were affected by Turkish cultural elements. For them, mathematics came from God, and so only He knows whether mathematical knowledge is absolute in nature or not.

As mentioned previously, individual teachers may express more than one perspective as to mathematics, its teaching and learning (Ernest, 1989). These beliefs could be inconsistent and contradict each other, and they may reside in different domain of the belief system (Pajares, 1992). This discrepancy can be seen in both the participants' personal remarks and the questionnaire data. However, the differences in participants' beliefs could be taken to mean that teachers are unaware of their beliefs or the influences of those beliefs on their teaching practice, or that they do not consider how their personal philosophical beliefs inform their daily pedagogical decisions. This difference could also be taken as indicative of the strengths of those stated beliefs, which as stated by Green (1971), concur with the core-peripheral dimensions of belief systems. He suggests that beliefs can be either core, which means strongly held, or peripheral, referring to beliefs which are less strongly held and more likely to be transient. From the present study, religious beliefs seemed to be core to the world views that upheld the participants' belief systems. World views (including religious beliefs) were found to take on a central role in the participants' personal conceptualisation and as such were closely associated with their beliefs about mathematics (see evidence in section 5.2). Once these beliefs are fully formed, it becomes problematic to engineer a shift without intentionally challenging them (Pajares, 1992).

Although there was a mutual relationship between the participants' religious and pedagogical beliefs, there was no way to elucidate this relationship in a simple way. Ernest (1989) argues that the beliefs of the nature of mathematics that a teacher holds

may not be consciously held views but implicit philosophies. These participants had not explicitly expressed clear pedagogical beliefs or a philosophical role for mathematics. However, religious beliefs emerged when the participants' focus was narrowed down to situations within the PD course related to learning and teaching mathematics (see section 5.2). An implication of this is the possibility that teachers' beliefs cannot be analysed adequately in isolation from their cultural context. This perspective is supported by Mansour (2009), who stated that "culture is a screen through which people view their lives and interpret the world around them. It is within this socially constituted nature of culture that beliefs play an integral role in filtering information and determining what is considered important and to be of value in the group" (p.32). Therefore, it is useful to adopt a broader, more social and cultural perspective when investigating teachers' beliefs in relation to their experiences with the PD course. This is an important issue for future researchers. The present study illustrated the importance of a possible relationship between teachers' world views (including religious beliefs) and their beliefs about mathematics, its pedagogy. While this seemed to be true in a Turkish context, it is important to remember that this might not be the case in other societies.

6.2.2 Beliefs about Teaching and Learning of Mathematics

Perceptions about the utility of mathematics

Through analysing the data, it was found that the course teachers were inclined to consider mathematics to be a very significant subject to study in terms of its applications to the real world, as a gatekeeper to find a job and as a cornerstone to the study of other sciences and the development of logical thinking. It appears reasonable to attribute such similarities to most Turkish mathematics teachers' strong traditions about the place and importance of mathematics at the individual and societal level. These perceptions seemed to have some impact on the participants' agenda as regards mathematics teaching and learning. It can be noted that some of these perceptions were consistent with objectives of the newly developed mathematics curriculum (e.g. linking mathematics applications with everyday life).

The informants in this study also considered mathematics as a subject that can and should be enjoyed. They recognised that the enjoyment of mathematics subjects increased students' motivation and their levels of interest. This echoes typical

constructivist teachers' beliefs about the aims of mathematics teaching. In fact, Stipek et al., (2001) argued that "teachers who hold the more traditional belief claimed to enjoy mathematics less and exhibited relatively less enthusiasm in their classrooms and assume extrinsic motivation" (p. 223). This argument appeared to be untrue for the participants in this study who held traditional beliefs that indicated they were moderately inclined towards enjoying mathematics as a subject. This could be suggested that there is no direct relationship between the teachers' previous learning and teaching experiences and feelings about mathematics, because they continue to develop as they work with mathematics as teachers.

At the outset, the participants of this study regarded pure logic as the main element of mathematics, and the development of logic and reasoning skills were seen as crucial aspects of mathematics as studied in school. This finding seemed to be consistent with other studies such as Pepin's (1999) study of English and French teachers who viewed mathematics as a medium of training the mind by applying logic. However, none of the activities observed in Pepin's (1999) study were designed to help students to develop logical reasoning; thus, teaching practice did not reflect beliefs.

Beliefs about approaches to and the aims of teaching and learning mathematics

There is evidence that recent reform in mathematics education in Turkey has gone some way to prompting conflicts in the participants' beliefs about teaching and learning of mathematics. Some support the ideas and methods of the newly developed mathematics which emphasise the importance of a learning environment where the learners can discover, solve tasks and share and discuss their results and approaches, and others support the ideas and methods of the existing curriculum. A possible explanation for this might be the conflict between the amalgamated influences of their own education from the time they were in school, and the new beliefs they have developed with regards to mathematics teaching and learning that stem from their experiences as teachers. This observation seems to be substantiated in this study. It might be said that the newly developed Turkish mathematics curriculum, in effect since 2005, has to some extent influenced Turkish teachers' beliefs about teaching and learning of mathematics. Stipek et al, (2001) suggest that these mixed beliefs typically emerge when a reform effort is fairly new and teachers are attempting to assimilate new teaching models to their more conventional beliefs about mathematics teaching and learning. The

participants' comments also suggested that their beliefs regarding the aims for teaching mathematics appeared to be weakly held. The teachers in this study had a tendency to articulate multiple and regularly inconsistent purposes.

The course participants in this study initially were inclined to emphasise learning mathematical rules and algorithms. They were eager to teach using the methods they had always used before and with which they felt confident; these typically involved concentrating on the more formal aspects of mathematics. They believed that memory and practice were an integral part of mathematics teaching and learning, reflecting emphasis on the achievement of good results in national examinations. The findings in the present study are consistent with those found by Foss and Kleinsasser (1996), who observed that their participants attached importance to practice and memorisation. Mewborn (2001) also concluded that most mathematics teachers adopt an explanatory teaching pattern (show and practice). In this regard, working in small groups is not a style that is prevalent, and students are not active in the learning process. This belief corresponds with Kuhs and Ball's (1986) classification of mathematics teaching, that it is "content-focused with an emphasis on performance". Participants were inclined to highlight the learning of mathematical facts and the practice of basic skills. An explanation for the agreement they expressed regarding the belief that rule-based mathematics teaching should be accepted correlates with the requirements of national examinations. Preparing students for national examinations demands the imparting of memorisation skills, instrumental teaching and practice. It can be argued that in-service and pre-service teacher education programs have been rather superficial in terms of encouraging reflection upon these issues amongst teachers and future teachers.

The factors influencing teachers' decisions about teaching

Some comments from participants corroborated the ideas and methods expressed within the new mathematics curriculum, which emphasises the importance of a learning environment in which the learners can discover, solve tasks and share and discuss their results and approaches; however, these participants also acknowledged that they felt pressured by the expectations of students and their families to teach mathematics in traditional ways. Although they were willing to teach according to the approach the new curriculum envisaged, they did not have any experience in how to implement this approach. Tension arose between teachers wanting to meet the needs of students to

succeed in their exams, but also believing in the primacy of active learning as laid out in the new curriculum.

The study also revealed some external factors which impeded the teachers from implementing their preferred way of teaching. One of the crucial factors was that teachers had experienced an explanatory mode of mathematics teaching as learners, and that they were also exposed to teacher-centred approach as student teachers in the faculties of education (Borko et al., 1992). All the teachers in Turkey follow the same curriculum and employ similar teaching plans as supplied by MONE. The educational system also places great emphasis on preparing students to pass national exams in order to obtain good results.

Ucar and Demirsoy (2010) reported similar findings in their study of Turkish teachers. Although Turkish teachers held different beliefs about mathematics teaching and learning, they were stuck in between conventional and new ways of teaching due to varied reasons. The current pattern indicated the fact that a centralised educational system and current social demands in the country have an immense impact on teachers' teaching practices and limit their freedom of action, particularly in conjunction with the country's examination system, school administration, and classroom environment (Ernest, 1989; Handal, 2003).

This study illustrated that most of course participants held teachers' beliefs about teaching and learning (e.g. Eysun) before their participation to the course were primarily traditional in nature. The remained participants (e.g. Asim) were found to hold combining constructivist and traditional beliefs about teaching and learning. Chan and Elliott (2004) highlight this may be the consequence of the amalgamation of strong cultural affects, previous experiences as a learner and as a teacher of mathematics. The relationship between participants' beliefs about mathematics pedagogy and intended teaching practices is complex and the social contexts of teaching were very influential on teachers' pedagogical decisions. This study illustrates that the relationship between teachers' beliefs about teaching and learning and their beliefs about the nature of mathematics are not as straightforward as previous studies may have suggested.

6.3 How do they change their existing beliefs while engaging in GeoGebra-based mathematical activities within the course as learners about mathematics, its teaching and learning?

This sub-section includes an exploration of how beliefs about mathematics, its teaching and learning can be associated with the participants' responses to the PD course and offers an explanation of the characteristics of their engagements with the course.

6.3.1 The Influence of the Context on Teachers' Beliefs regarding Technology Use in Mathematics

Although the intention of the present study was to specifically examine Turkish primary teachers' mathematical beliefs, their involvement with the PD course based on the use of dynamic geometry would also have an impact on beliefs about technology. In fact, learning how to use technology in mathematics was a challenging task for the course participants as they had no previous experience with it. As the PD course progressed, the participants expressed considered opinions about using the computer, particularly about GeoGebra, and they had different views about the potential for GeoGebra in teaching and learning. GeoGebra quickly gained popularity amongst all the participants. They pointed out the following ideas about the nature of the learning that took place in their interactions with GeoGebra-based tasks: memorability, meaningfulness, motivation, encouraging thinking and investigation. Most of these features were also found in the literature.

As a result of participating in the GeoGebra-based PD course, the informants developed a positive view of, and confidence about, the use of technology (in particular computers) in mathematics education, showing great enthusiasm for learning mathematics alongside technology. This was closely associated with the sense of personal achievement that emerged from the PD course. In this respect, the study produced findings which agree with the findings of the preceding work in this field (Mainali and Key, 2012; Sulaiman, 2011; Tharp et al., 1997 etc.). However, some researchers have stated that an effect on teachers' beliefs about technology use in mathematics teaching and learning cannot emerge through short-term in-service courses (Zhao et al., 2002). Teachers need more time to modify their views about technology in mathematics education in a positive manner (Thomas, Tyrrell, and Bullock, 1996). Although the course teachers do not have previous experience with using technology as

part of their own educational experience, they recognise that now technology is everywhere and this knowledge impacts on their belief systems.

The positive result regarding using technology can be explained by cultural factors and values. Since modernisation is regarded as a challenging process in developing countries, the stated aims in education have been to follow the lead of developed countries. The belief is based on the idea that following the educational trends of Western countries would assist the Turkish in becoming part of European civilisation and culture (Gok, 2006). Technology is usually considered vital for the future, so as to maintain a similar developmental level internationally. This study has revealed that the participants regarded technology as a important for their teaching career and as crucial in increasing society's development (see evidence in 5.2.4 and 5.3.1). Norton, Cooper, and McRobbie (2000) argued that cultural issues may influence mathematics teachers, affecting the school environment. It should be noted here that, although culture is a traditional force for retaining stability and impeding discernible change, under certain conditions, cultural factors and values act as an effective means for alteration.

From a cultural perspective, the concept of modernisation is important in encouraging teachers to adapt technology in their teaching so as to enhance their professional career. However, this driving force does not itself always result in the successful integration of technology, or its constructive use in the classroom. This is the reason why the aim of this research was to provide a PD opportunity as part of the methodology, giving mathematics teachers not only access to technology but also encouraging discussion of appropriate pedagogical approaches. An implication of this is the possibility that the use of technology in education cannot be considered as an issue isolated from the cultural values which affect teachers' priorities, preferences and attitudes towards technology. This is an important issue for future researchers seeking to better understand the cultural dimension in terms of the use of technology in mathematics education, and to align efforts for in depth and long-term change.

6.3.2 The Importance of Social Engagement within a Professional Learning Environment

The GeoGebra-based PD course was structured on professional development principles, one of which was that learning is best promoted by an approach founded on active involvement and self-direction. Data from this study showed the participants were able

to relate to this approach. Teachers, who experienced successes with GeoGebra, regarded their experience with the PD course as valuable and so became more receptive to its message.

One of the important features of the PD course lay in the establishment of a learning community and in encouraging the participants to work together as professionals engaged in a common objective. Interaction, co-operation and support were more extensively appreciated by most participants and sharing of ideas between individuals and among different small groups, often took place (see for evidence in 5.3.3). The sub-section 5.3.3 and 5.3.4 illustrated that both the participants' intentions to use GeoGebra with their students and their ability to connect GeoGebra with mathematics acted as catalysts for their reflection on mathematics pedagogy and social interaction. The participants recognised the importance of social engagement in relation to the teaching and learning of mathematics. This supported the promotion of the teachers' professional development with GeoGebra. Collaboration between teachers can play a central role in the improvement of their teaching; indeed, Greenfield (2005) claimed that conventional approaches of individual teachers working in isolation have not been successful, and it is apparent that there is a need for collaboration between teachers to fulfil the needs of students. Therefore, the PD course was designed for teachers to work together supporting each other, fostering interest and promoting experientially based understanding acquired through collaboration and possibly also through quality social interaction. This collaborative learning process during the PD course provided an initial touch point for the participants' priorities and preferences. During this research, it was observed that the Turkish participants appeared to appreciate breaks and used these to share what they think and know about varying topics.

The participants came to the course with the belief that engagement between teachers and students is inadequate in the mathematics classroom for various reasons. However, they began to see social engagement from different aspects as a consequence of their interaction with the GeoGebra group activities. This study suggests that learning mathematics through group activities with help of GeoGebra breaks the monotony of the everyday classroom, students would be more likely to enjoy in classroom, take the ideas of others into consideration, and therefore learn more efficiently (see evidence in sub-section 5.3.2). Therefore, the PD course provided an opportunity for participants to

see first-hand how collaboration and cooperation can be crucial in learning and in enhancing understanding of mathematics. An important outcome of the course seems to have been changing this belief that collaborative learning is unproductive within the centralised educational context.

A question could be raised here, as although changes in these beliefs appear remarkable within the PD course, what happens when the teacher is on her own, facing the complexities of daily teaching? Steele (2001) looked for an answer to this question by conducting a longitudinal study with four elementary school teachers. As a result of a teacher education program, the participants' beliefs about mathematics teaching had changed and appeared to favour a constructivist belief. This may suggest that those teachers are more likely to create a learning environment in which students could work cooperatively with their teachers and take responsibility for their own work. A follow-up study observed four teachers in their classroom practice; they did not stress the virtues of collaboration with their students and did not translate their constructivist ideas directly into their teaching. A possible explanation of this might be that participants felt pressured by their school culture, and the expectations of administrations and other teachers, to follow a traditional method of teaching in classroom (Steele, 2001). From the point of view of the present study, the connection between changing beliefs and enacted beliefs was not clear. Therefore, this data must be interpreted with caution as there was no possibility for the researcher to follow up the findings with classroom observations. The complexities of classroom culture may hinder teachers' capacity to work according to their beliefs and provide teaching that conforms to those beliefs. Although no observation was possible, a valuable touch point was found in the PD course, pointing for the first time to the possibility of interaction, co-operation and support as positive components of the classroom.

6.3.3 Dilemmas between Expectations and Reality

Although it is not the focus of this thesis to explore the relationship between teachers' beliefs and their actual practice, the social context of classrooms has a notable influence on the way in which stated beliefs are enacted. It follows, then, that constraints and opportunities generated from the social context of teaching may also serve to formulate these beliefs (Ernest, 1989). Hoyles (1992) also argues that it is pointless to separate stated and enacted beliefs since such factors (contextual) generate diverse clusters of

beliefs that are indeed enacted. Although it is impossible to comment on the extent to which the participants' in this research enacted beliefs, since they were not investigated and observed in the real classroom setting, their experiences with investigational tasks in the PD course did provide them with important opportunities to change their beliefs.

Both Asim and Emin seemed to have come to the PD course feeling motivated to change. After completion of the PD course sessions, they appeared to a certain extent to have extended their beliefs by integrating some of the pedagogical approaches to mathematics introduced in the course. In other words, they were moderately leaning towards constructivist beliefs about the teaching and learning of mathematics. However, the participants' comments indicated that they would not be able to put these beliefs into practice in their actual classroom for varied reasons (i.e. lack of time, lack of equipment, social demands). The evidence of this study shows that practical limitations are instrumental in inhibiting the process of change in teachers' teaching practice.

After gaining experience with GeoGebra-based activities, the participants themselves developed new ideas about teaching and learning mathematics. They expressed the intention to employ innovative ideas (such as mathematical investigation and discovery) in their actual classroom, but were concerned that teaching using innovative practices would create some difficulties with planning and implementation. According to the participants, the Turkish educational context is not ready for, or flexible enough to allow teachers to spend their time developing computer-based activities, or to use group work in their class. Furthermore, time was a shared concern amongst the participants; they uniformly felt that the time required to implement innovative teaching strategies in the classroom was not available to them.

The findings of this study indicated that the participants showed enthusiasm and positive attitudes towards the use of GeoGebra for teaching mathematics, even though they mentioned some difficulties in connection with existing resources and materials. Although the teachers support the ideas and methods of the new mathematics curriculum, it was felt that the PD course activities and suggestions were somewhat isolated from the reality of a centralised and examination-oriented educational system. The analysis of the participants' accounts of this issue indicated that they believed that an exploratory teaching approach, such as that adopted in the PD course, would not help students pass the existing national examinations. The literature suggests that the link

among teachers' beliefs and their teaching practice is complex in nature and reconciled by external norms (Handal, 2003; Pajares, 1992). The social context of teaching forces practicing primary school teachers to use traditional methods to teach mathematics although they may hold constructivist beliefs about mathematics teaching and learning (Handal, 2003; Perry et al., 1999). Ernest (1989) also supports this argument by saying that social context clearly hinders teacher's flexibility to make pedagogical choices. For him, the powerful impact of the social context creates a conflict in between stated and enacted beliefs in regards to teaching and learning. He cited the expectations of others as the origin of this conflict or mismatch; in particular those of students, parents, colleagues and administrators. Conflict also proceeds as a consequence of having a centralised curriculum, reliance on specific texts or curricular schemes and the system of examinations prescribed by the national educational system (Ernest, 1989). Thus, individual teachers have internalised an influential set of barriers to the delivery of new models for teaching and learning mathematics. These barriers also impact on the development of beliefs about what works and what does not in a classroom. Unquestionably, the teacher's position is powerfully influenced by a social context in which teachers are expected to make decisions quickly, in isolation, and in widely varied circumstances. The findings of the current study are consistent with the results of similar studies reported in the literature review. It is certainly the case that the nature of the Turkish education system and the curriculum profoundly affects the participants, who feel they are hindered by social demands and the centralised educational system. The participants in the current study had difficulty considering ways in which innovative ideas for teaching practice would match current social demands. Changes to practice since 2005 have been accompanied by an autocratic approach which limits individual teachers' control over their classrooms. Despite this the participants were positive about the PD course, noting that it offered a open-ended environment (social context) from which to challenge their existing beliefs without concern for the social demands and expectations of the centralised educational system. The course, in particular the home-based activities, gave the course participants the opportunity to explore the limits and constraints on their beliefs and teaching practice by giving them chance to change new beliefs free from traditional constraints. They at least were enabled to reflect constructively on the new approach. Of course, as mentioned previously, whether they would then be skilful to implement this new approach into the complexity of classroom culture is putting a question mark in the minds.

6.3.4 Changes to Teachers' Mathematical Beliefs

Hoyles (1992) asserted that all beliefs which can be reconstructed through interacting with an innovation which takes place within settings could be considered situated. As apparent from the previous chapter, the informants made changes to their beliefs following the PD course. While participants initially had mixed and multiple beliefs about mathematics, its teaching and learning, my observations and their reflections on the open-ended nature of GeoGebra showed evidence that they were leaning moderately towards fallibilist beliefs about mathematics and constructivist beliefs about pedagogy. The GeoGebra-based PD course was designed to broaden and enrich Turkish primary teachers' mathematical beliefs according to its fundamental principles. Within its boundaries, the results expressed in Chapter 5 appear to illustrate that the course provided ways of emphasising particular aspects of those beliefs. It might be said that changes to teachers' beliefs took place as a consequence of numerous interactions between their previous beliefs.

After the PD course, the participants seemed more sensitised as to their beliefs, and also were more open to re-considering and changing their mixed and multiple beliefs about mathematics learning and teaching. The findings of this study indicate that the participants' involvement in exploratory activities within the GeoGebra-based mathematical environment seemed to have led to the development of a critical perspective towards their existing teaching practices and previous learning experiences, which had been based on the explanation, practice and memorisation model. For example, the changes in the participants' beliefs about the role of the teacher in mathematics teaching were evident in their intention to use the computer and GeoGebra with their students as a part of their daily activities, applying the ideas that emerged during the PD course. They came to view their role as facilitator rather than provider or demonstrator; arguably, this was partly as a consequence of the researcher taking a facilitative role during the PD course. Through the course, the teachers seemed to have found a way to give their pupils a more active role in learning and in enabling them to discover mathematics. This view reflects the "learner-focused model" and constructivist beliefs that concentrates on the learner's own creation of mathematical knowledge through active participation in making mathematics (Kuhns and Ball, 1986). There were, however, the participants (Eysun and Celal) who experienced disillusionment with the course's approach and combine their previous beliefs and new ones. The participants'

involvement with GeoGebra based activities also helped them to broaden their beliefs about mathematics, changing from somehow seeing it as a fixed, logical and rule-based subject to a more dynamic and creative one (see sub-section 5.4.2).

Although, this result differs from a study published by Foss and Kleinsasser (1996), it is consistent with those of Sulaiman's (2011) study. He found a strong shift in most participants' beliefs in support of the use technology in general and in particular the use of Logo for their future mathematics teaching. His participants were moderately leaning towards constructivist beliefs about teaching and learning. Foss and Kleinsasser (1996) studied pre-service teachers who were involved in a methods course and engaged in experiencing learning through active problem solving and collaboration. After completion of the methods course, the pre-service teachers changed their beliefs about the nature of mathematics and what it means to teach and learn mathematics very little. In fact, they continued to hold the belief that mathematics is primarily a subject requiring rote memorisation and computational skills, not a subject of creativity or reasoning. The participants in this earlier study also continued to hold the belief that memorisation and practice were inevitable parts of mathematics teaching and learning, not accepting the effectiveness of the techniques promoted in the methods course. Foss and Kleinsasser's study highlighted the difficulty with altering teachers' beliefs about the nature of mathematics, its teaching and learning. However, it should be noted that the participants in both Foss and Kleinsasser's study and Sulaiman's study were student-teachers, and so were from very different educational contexts to those in the current research.

The participants in this study broadened their perspective of mathematics, its teaching and learning, acknowledging the value of substituting their existing learning and teaching approaches with the ideas mentioned on the PD course. These participants appeared to believe that this way of learning and teaching mathematics could be valuable in terms of improving conceptual understanding, but they cautioned that it would not fit into the centralised educational system in Turkey. This tension between developing an alternative viewpoint and unsettling prior beliefs is a manifestation of a dissonance between what was found to be the case on the PD course and the current demands made on mathematics educators. This does support the view that the PD course can act as a catalyst to breakdown fixed and conflict beliefs about mathematics,

its teaching and learning. Through professional development courses, teachers can learn to change their formerly held beliefs and perceptions, to facilitate them to engage with new experiences in new contexts.

6.4 Chapter Summary

The main purpose of this study was to explore Turkish teachers' initial beliefs about mathematics, its teaching and learning and how these beliefs may have been influenced following their participation on the PD course. The six participant teachers did not join the PD course as newcomers. They brought with them their personal experiences with mathematics as learners and as teachers, their beliefs about teaching and their understanding of the role of mathematics in educational contexts. The participating teachers held mixed beliefs about the nature of mathematics prior to participation on the PD course. This study explored the participants' world views, their previous school experiences and teaching experiences, and how these had shaped their personal philosophies regarding mathematics and its pedagogy, although this connection is difficult to clarify in an explicit way. Participants in this study did not have clear ideas about pedagogy or their philosophical position relative to mathematics. In fact, the teachers were inclined to teach using the methods with which they were familiar and felt confident using; these naturally involved focussing on the more formal aspects of mathematics. It could be argued that educational contexts promote and support the development of traditional beliefs about teaching, because of their conservative nature (Handal, 2003).

This study shows that social teaching norms are the principal factors impeding the implementation of teaching practices (Ernest, 1989). The major factor influencing Turkish teachers' decisions about teaching is the demands placed on them by the centralised educational system. Without the constraints of educational contexts, the PD course enabled the participants to challenge their fixed and conflicting beliefs about mathematics teaching and learning. In other words, this course provided an opportunity to broaden and enrich Turkish primary teachers' beliefs about mathematics teaching and learning to align them more with the PD course approaches. This may suggest that teachers' reflections on their experiences with mathematics within the PD course could be seen as an effective way to change teachers' beliefs. This study further suggests that we need to take a broader view of teachers from a more social and cultural perspective

when analysing their experiences within the PD course. However, this study does not clarify the relationship between changing beliefs and practices, since there was no follow-up to classroom observations. One cannot determine whether or not the participants were able to reflect on their new beliefs and effect changes to their teaching practice. In fact, some valuable ideas about teaching as related to mathematics were found through analysis of the participants' reflections on the PD course. It is evident that teachers need to be provided with more on-going support and interventions to embrace in depth and long-term changes in their practice (Cooney et al, 1998). Given the short period of time in which the data collection process was conducted, teachers would be invited to reflect further on the beliefs they hold so as to enact them in their classrooms.

CHAPTER 7: CONCLUSION

7.1 Introduction

Although a large volume of literature has been published examining the relationship between teachers' mathematical beliefs and their stated or enacted practice (Cross, 2009; Speer, 2005; Stipek et al., 2001; Thompson, 1992), relatively few studies have focussed on the professional development of mathematics teachers and their shifts in beliefs about their subject matter when encountering a new learning setting. In particular previous research into Turkish in-service mathematics teachers' beliefs, and how these can be affected by involvement in a short term professional development (PD) course based on the use of Dynamic Geometry Systems, does not exist. With the purpose of addressing this gap, the present study investigated teachers' initial beliefs about mathematics and its teaching and learning, and evaluated the influence on these of a PD course, both during and after the course. The PD course exposed teachers to new pedagogical practices as learners, to prompt them to confront their beliefs and, therefore, seek to change their beliefs.

In order to achieve these aims (also as stated in Chapter 1), the present study was conducted with six Turkish primary mathematics teachers, in the city of Kahramanmaraş in Turkey. They engaged in a PD course incorporating GeoGebra-based mathematical activities, involving 22 hours of workshop sessions during the spring semester from April 2011 to June 2011. Providing explanations and interpretations of the participants' experiences within the setting has enabled me to explore the changes in their beliefs in the Turkish educational context.

The subsequent sections provide a brief summary and some general conclusions drawn from the findings of this research, as reported in the preceding chapter. This chapter is comprised of the following sections: summary of the study, its implications for Turkish mathematics education, the limitations of the study, recommendations and directions for further research, and final remarks.

7.2 Summary of the Study

The focus of this study has the potential to provide a thorough understanding of learning experiences among Turkish teachers, which has significant implications for developing the quality of their teaching and learning. Overtly, I made an effort to answer a set of research questions. The main question in the present study is how has involvement in a PD course, that was designed using GeoGebra based on a constructivist approach, influenced the beliefs of Turkish mathematics teachers about mathematics and mathematics teaching and learning. This question can be subdivided as follows: 1) What beliefs do Turkish mathematics teachers in primary schools hold about the nature of mathematics, its teaching and learning before participation in the PD course? 2) How do they change their existing beliefs about the nature of mathematics, its teaching and learning, while engaging in GeoGebra-based mathematical activities within the course as learners?

I investigated and determined the teachers' initial beliefs prior to their involvement on the PD course; this was achieved by analysing their responses during a semi-structured interview and on a pre-course mathematical beliefs questionnaire. Their modified beliefs were explored during and following involvement in the PD course, through analysing their responses to a variety of data sources such as interviews, reflective writings, my field notes and the post-course questionnaire. The data gathered provided valuable remarks and insights into their modified beliefs, offering insight into the Turkish mathematics teachers' beliefs about the nature of mathematics, its teaching and learning as well as the use of GeoGebra as a pedagogical tool to foster the teaching and learning of mathematics. Some important conclusions about the complexity of the change experienced by teachers within this specific context can be drawn from the research findings.

At the outset of the PD course, the participants had little notion of how to effectively use computers in mathematics education. The general ideas they shared about the role of the computer was that it could inject new life into the traditional classroom environment, by developing students' motivation and interest in learning mathematics. The participants initially saw the course as a way to learn about using computers in their teaching. They were in part driven by the contemporary need to incorporate computers into the mathematics classrooms. The participants believed that technology would offer

a way of being modern, as this would be important for their teaching career. After following the PD course, the participants expressed considered opinions about using the computer, particularly GeoGebra, and they held different views about the potential for GeoGebra in teaching and learning (see 5.3.2 and 5.3.4). The participants were enthusiastic about building their competence with GeoGebra, as they hoped to employ it with their learners. This encouraged them to engage in meaningful open-ended mathematical activities that aroused their interest and curiosity, allowing them to comprehend an additional dimension to the use of computers in mathematics. In taking on the role of learners during the PD course, the participants had the opportunity to appreciate a potential for using computers, which extended beyond their merely being an additional tool. That is, they recognised that computers offered a valuable opportunity to enact a more learner-centred pedagogy that would fit with the constructivist approach; leading them to think deeply about more complex ways in which they could integrate computers into their classrooms. Therefore, this study illustrated substantial changes in their beliefs in favour of the use of GeoGebra in their teaching.

When questioned, it was found that the majority of the participants did not even have internalised mathematics pedagogy to guide their practice, nor did they translate their philosophical beliefs regarding the nature of mathematics into their teaching. Before conducting the study, it was anticipated that it would be possible to evaluate absolutist and fallibilist beliefs as diametric opposites. In fact, the majority of the participants were initially found to hold multiple and/or mixed beliefs, a finding that mirrored that of Beswick in 2005. Even those teachers with strong constructivist beliefs about teaching and learning, had beliefs towards mathematics that could be classified as having both absolutist and fallibilist elements. The complexity of the teachers' belief systems could not, therefore, be labelled categorically as manifesting in traditionalist or constructivist pedagogy, and neither did dividing their beliefs into either absolutist or fallibilist offer a full picture of the situation. It is clear from the data, the categories was reported in the literature does not completely represent teachers' beliefs about the nature of mathematics in this study. It should be noted that some participants' religious beliefs also seemed to be core to the world views that upheld their belief systems. Although there was a mutual relationship between the participants' religious and pedagogical beliefs, there was no way to elucidate this relationship in a simple way. Once these

beliefs are fully formed, it becomes problematic to engineer a shift without intentionally challenging them (Pajares, 1992). The findings of this study may show that participants' religious beliefs should be considered as a potential barrier to willingness to alter their absolutist-oriented beliefs about mathematics.

The study found that since its introduction in 2005, the newly developed mathematics curriculum has to some extent influenced the teachers' beliefs about teaching and learning of mathematics. These reformations appear to have gone some way to prompting conflicts and a questioning of teachers' beliefs. On the other hand, the highly centralised and examination oriented nature of the Turkish educational system were a shared concern of many of the participants, which appeared to affect their pedagogical decision making about teaching and learning. Many of the participants expressed their dissatisfaction with the emphasis on memorisation and practice in their classrooms, but had minimal knowledge of how to enact alternative pedagogical approaches. The setting for the PD course was not intended to mimic the classroom set up, rather it aimed to provide a learning environment in which it was possible for participants to change any new ideas they encountered. The relative freedom of this setting provided a mental and physical space in which the participants were able to examine their belief systems and share their discussion with me. The use of mathematical activities to explore different beliefs and stated practices was a key aspect of the PD course. As the participants learned about new ways in which to do mathematics, by utilising the scope of GeoGebra as a dynamic and interactive system, their belief systems shifted. In terms of the research itself this was a key goal; however, what emerged as an extra finding, was that both the initial beliefs (as mentioned above) and the process of addressing and changing those beliefs were considerably more complex than had been originally assumed.

In fact, most of the participants in this study experienced a broadening of their perspectives on mathematics, its teaching and learning, acknowledging the value of substituting their existing learning and teaching approaches with the ideas mentioned on the PD course. It was recognised that while some aspects of former beliefs might be subject to change, others are immutable. Notably, although most participants become aware of alternative approaches in teaching and learning, they stated that they would not be able to put these beliefs into practice in their actual classrooms for a variety of reasons (e.g. institutional factors and expectations). The evidence in this study showed

that these factors inhibit the process of change in teachers' beliefs. However, this type of the course did support participants to achieve a new understanding of mathematical learning and teaching, proving the professional learning environment can be successfully adapted to guide teachers to a vision of mathematics that is in line with recent ideas about reform, as well as helping them to develop new approaches to teaching.

7.3 Implications

The findings of the study were reported in the preceding chapter and it is evident that they have important implications for Turkish mathematics education. They covered teachers' mathematical beliefs and their change within a specific setting; as well as possible directions for further research into the professional development of Turkish teachers. The knowledge presented here in reference to primary mathematics teachers' beliefs, regarding their practice within a GeoGebra-based mathematical course environment provides valuable data for the subsequent development of Turkish mathematics education. This study also provided a taxonomy for primary teachers' mathematical beliefs, as well as the use of GeoGebra in the Turkish educational context.

The main focus of the study was on teachers' beliefs as potentially important to the effective implementation of teaching practice reforms. In order for teaching approaches, such as a constructivist pedagogy, to be implemented in the classrooms, one has to challenge and shift teachers' existing beliefs (Ernest, 1989; Swan, 2006). This means there is a need to address the pedagogical foundations and beliefs of those working within the system, if a constructivist approach is to be wholly adopted. This study demonstrated that the process of addressing and changing teachers' beliefs was significantly more complex than had been originally assumed.

The present study illustrated that participants' religious beliefs seemed to underpin their world views and belief systems. World views were found to take on a central role in the participants' personal conceptualisation, and as such were closely associated with their beliefs about mathematics (see 5.2.1). For instance, the course participants initially regarded mathematics as an absolute, fixed and created by God. They simultaneously believed that learners can discover mathematics. Green (1971) suggests that beliefs can be either peripheral, which means less strongly held, or core, which refers to beliefs

which are strongly held and more difficult to shift. Therefore, the findings of this study suggest that participants' core beliefs (religious beliefs) could be considered as a potential barrier to alter their absolutist-oriented beliefs about mathematics. However, that this does not mean individuals cannot appreciate the value of constructivist teaching methods. The fact is that participants' beliefs appear to be partly absolutist, in that they believe that mathematics has been created by God and is therefore 'absolute', but also believe that mathematics is discovered by human beings, and therefore changes as our discoveries advance – a fallibilist perspective. It is this notion of discovery which makes it possible to reconcile a belief in mathematics as a fixed creation, with a constructivist approach to pedagogy in which students are encouraged to investigate and explore. Participants in this study were found to hold a set of contradictory beliefs about the nature of mathematics. Therefore, it is evident from the data reported in Chapter 5, that the categories used in the literature do not completely represent teachers' beliefs about the nature of mathematics in this study (Lerman, 1986; Ernest, 1989).

The majority of the participants were initially found to hold traditional beliefs about the teaching and learning of mathematics. The remaining participants (e.g. Asim) were found to hold combined constructivist and traditional beliefs about teaching and learning. Moreover, they had not explicitly articulated a clear aim for mathematics pedagogy even for themselves (see evidence in 5.2.3). These beliefs typically emerge when a reform effort is fairly new and teachers are attempting to assimilate new teaching models with more conventional beliefs about the teaching and learning of mathematics (Stipek et al, 2001). The evidence of this study also revealed that the social contexts of teaching (including institutional factors, national examinations; expectations of families) were influential in affecting teachers' pedagogical decisions (Ernest, 1991). As far as mathematics teaching and learning is concerned, tension is apparent between how mathematics teaching and learning ought to be and how it has to be in an exam-oriented system. Institutional constraints became the course participants' shared concerns, which may have an impact on their subsequent decision making about teaching and learning (see evidence in 5.2.3).

Ernest (1989) argues that the beliefs that a teacher holds about the nature of mathematics may not be consciously held views, but implicit philosophies. The study findings showed that the participants did not have an internalised mathematics

pedagogy to guide their practice, nor did they translate their philosophical beliefs regarding the nature of mathematics into their teaching. However, religious beliefs emerged when the participants' focus was narrowed down to situations within the PD course, as related to learning and teaching mathematics (see section 5.2). One implication of this is the possibility that teachers' beliefs cannot be analysed adequately in isolation from their cultural context. Thus, there appears to be a clear correlation between participants' religious and pedagogical beliefs. For example, participants who held religious beliefs about the nature of mathematics still planned to integrate constructivist ideas into their teaching practice. Thus, this study suggests the relationship between participants' beliefs about mathematics and their intended teaching practices are complex, as the relationship between teachers' beliefs about teaching and learning and their beliefs about the nature of mathematics are not as straightforward as previous studies may have suggested. This finding is consistent with Liljedahl et al, (2006) and inconsistent with Chapman (2002), who suggested that beliefs are directly reflected in practice.

Most of the studies reviewed in this thesis categorised teachers' belief systems into theoretical constructs (Ernest, 1989; Lerman, 1986; Roulet, 1998). However, dividing teachers' beliefs according to these models did not offer a full picture of the situation. The study, in fact, revealed, that the complexity of teachers' beliefs does not fall neatly into the distinct categories suggested in other studies, in part due to the impact of Turkish cultural factors. The findings of the present study were consistent with Pajares (1992), and showed that teachers' beliefs included complex and messy constructs. Beliefs were inclined to grow incrementally and cultural elements were important in facilitating their development (McLeod, 1992). This implies that before attempting to comprehend the complexity of the nature of teachers' beliefs, it is essential to adopt a broader, social and cultural perspective. Therefore, further research is required in order to capture the complexity involved in the cultural dimensions that influence mathematics teaching.

This study also contributes to the limited body of research on teachers' beliefs and professional development of Turkish primary mathematics teachers in reference to a computer-integrated learning environment. One of the implications of this is that the researcher provided a tool, in the form of teachers' experiences within a GeoGebra-

based PD course environment, with which to examine the nature and development of teachers' beliefs about mathematics, its teaching and learning. The study produced information about PD in the context of those reforms currently being encouraged in Turkey. This was achieved through the representation of the participants' initial mathematical beliefs, which were largely developed in response to their own learning experiences.

A very influential ending to this research was discovery of the affect that the GeoGebra-based environment within the PD course had on the recasting of teachers' beliefs about the nature of mathematics, its teaching and learning in a technology based setting. This discovery revealed the importance of encouraging teachers to debrief about their previous belief systems as the initial phase in constructing change. The PD course then acted as a catalyst to deconstruct fixed and existing beliefs. This was accomplished in the PD course by introducing alternative ways of teaching and learning mathematics with technology (see section 5.3). Theoretically, the effectiveness of the PD course can be summarised as twofold: First, the course constituted a mechanism for participants to see new ways of learning and teaching based on the constructivist approach by participating in a professional learning environment such that they had never experienced before. They were actively engaged as learners, so were able to promote their conceptual knowledge.

Of further interest, worth noting, it was found that giving the teachers' opportunity to reflect together on activities undertaken personally during the course prompted discussions about classroom practice. The constructivist method appears specifically sufficient in itself to challenge teachers' beliefs, thereby provoking a sense of dissonance; identifying, as Appleton (1997) observed, an incomplete fit. Second, the course uses an active approach to addressing shifts in belief, by advocating teachers' involvement in hands-on-experiences, to allow for reflection, collaboration and construction of new knowledge and beliefs. Through the teachers' own experiences with mathematics in a GeoGebra-based environment they acknowledged, and moreover, believed in the value of teaching and learning mathematics through 'interaction', 'discussion', and 'sharing'.

One of the aims of the PD course was to encourage cooperation and exploration of mathematical tasks when participants are working together with their peers in a group. The researcher's role in the PD course period was that of facilitator; this meant prioritising the participants' freedom to articulate, explore and investigate mathematical concepts. Hence, placing emphasis on the value of peer collaboration, social interaction and ideas sharing to prompt teachers to attempt to use new tools and approaches. The integration of GeoGebra-based investigational tasks in the non-structured environment appeared to provide an opportunity for more purposeful teaching and learning experiences. The research illustrated that teachers' experience with mathematical activities within the course had instant and modifying influence on the beliefs concerning the nature of mathematics, its teaching and learning (see section 5.4). This finding is consistent with Liljedahl (2005).

It was observed on the PD course that in a supportive professional learning environment change is more likely, as long as the setting is sensitive to the cultural and religious values of the group of teachers. Thus, we can determine that change is most likely to occur in a PD setting that is both supportive and persuasive. In relation to the issue of barriers to implementation upon returning to the work setting, it is imperative that teachers lend continuing support and modification to the classroom settings that facilitate systematic implementation, leading to improvement.

This study provided insight into Turkish mathematics teachers' beliefs about integrating GeoGebra into their current teaching, and information on which future research can draw to examine the effectiveness of GeoGebra in the mathematics classroom to assist students' learning. It is supposed that when teachers observe positive results in their students as a consequence of their actions, their levels of personal satisfaction as an intrinsic motivation in the workplace, will improve, assisting them towards greater professional success (Guskey, 2002). Thus, it is argued that incorporating research led PD into mathematics education, and to promote educational effectiveness is beneficial in the Turkish context.

7.4 Limitations of the Study

This section emphasises three possible limitations of the study, i.e. time, and the size of the research sample and my role. Due to these limitations, the design of the study was

affected. As I was reliant on a short period of time for data collection, it was not possible to determine whether the changes in the teachers' beliefs would be lasting. There was no opportunity to follow up the beliefs they hold in classroom observation due to their busy working schedule and time limitations. There is some question over whether the teachers' involvement with the PD course could influence their beliefs and stated practices in the classroom at a duration of eight weeks after following the PD course. Time also emerged from the data as a key theme in this study. That is, all the participants mentioned difficulties in becoming a good user of GeoGebra, in such a short time period; this is necessary if the teachers are to have the confidence to integrate it with alternative approaches in their actual classroom. Hoyles, Noss and Sutherland (1991) emphasised the need for significantly longer professional development programmes. This suggestion may also be applicable for the Turkish teachers in this study, particularly as they had very little familiarity with computers in education.

Another limitation of this study was derived from the methodology. In fact, this research only consisted of a small number of teachers who volunteered to attend the GeoGebra-based PD course. Even though it was intended to involve more teachers in the study, it was only possible to reach six teachers from the different schools contacted. This may limit the generalizability of the findings presented here. However, in view of supportive results in earlier research it is possible to make some modest claims about the generalizability of the results to all educators and settings. This is because the triangulation, description and rich verification presented in the data all contribute to the validity of the analysis, making it possible for others to judge the relevance and applicability of these findings in their own contexts.

A further limitation of this study was my role in the project as both observer and leading the intervention. The fact that I was a PhD student and a novice researcher with no authorised position, this potentially created some problems from the participants' perspective about my role. The low degree of response and the fact that there were only six participants may well be a reflection of teachers' attitudes to the fact that this was a research project. This meant that it was important for me to build a good rapport with those who did participate, by spending time getting to know them socially. The fact that I was not a figure of any authority in the Turkish educational context helped me sustain

a collegial relationship with the participating-teachers. Ethical dilemmas concerning my involvement with PD course ultimately came to be less of a concern.

7.5 Recommendations and Directions for Future Research

The present study has implied that teachers' beliefs are changeable and has sketched how beliefs shifted during the course of their involvement in the PD course. Ultimately, the most important aspect of this study has been that it provided an opportunity to elucidate the surprisingly complex nature of teachers' beliefs, in its exploration of their mutability. Having acquired this realisation, it became evident that more in depth work is necessary to comprehend these issues fully. The scope of the findings in this study leads us to propose the following for further inquiry:

- A longitudinal study to investigate a model course in the school context following engagement in PD programmes is necessary to determine the long-term effects of such education. This would clarify any link, or lack thereof, between what teachers state when on the course and how they behave when back in the classroom.
- As stated, there is an interaction in play between teachers' beliefs, the educational culture of Turkey and cultural factors. To investigate this in more detail with a larger sample group would be beneficial, particularly then involving an investigation into the mutual effects on the school of teachers undertaking a new approach.
- When teachers alter their approaches through the adoption of new tools such as GeoGebra, it would be useful to ascertain their impact on students' learning needs and exam results. Thus, it is recommended that a study of the effect of GeoGebra and related new approaches be undertaken by understanding pupil outcomes.

7.6 Final Remarks

The summative conclusions and achievements within this research have shown that a substantial shift in teachers' beliefs in favour of the use of GeoGebra, as well as using constructivist ideas in their mathematics teaching can be attained through PD. It is worth remembering that successful implementation of the reform and the inclusion of technology and its tools, such as GeoGebra in mathematics education, is contingent on

teachers' beliefs. Therefore, to prompt teachers to change their beliefs about mathematics teaching and learning and the value of digital tools (e.g. GeoGebra), they must be given the opportunity to engage in new learning environment and self-reflection, similar to that of the participants in this study. I am also grateful that the study has allowed me to explore my own beliefs about mathematics, its teaching and learning, and to better understand the value of the integration of GeoGebra into the teaching context.

The Turkish Ministry of Education has over the last decade made a considerable commitment to education reform, as well as reform in mathematics education, but the fact remains that presence of technology such as GeoGebra that assists in promoting a constructivist perspective for the teaching and learning mathematics are not yet given the attention they deserve. As a result of undertaking this study, a gap between the expectations of MONE and teachers' beliefs emerged. This study illustrated that teachers do have a little knowledge and some experience of teaching mathematics with computers. In order to improve on this the changes outlined below to Turkish teacher education programs and in-service courses are recommended:

- Courses related to the use of computers in mathematics education should be made more prevalent in teacher education programs and professional development courses. GeoGebra should be introduced as a pedagogical tool to assist in a constructivist perspective for the teaching and learning of mathematics.
- Teachers should be provided first-hand experience when using technology in mathematics education. This experience will give them an opportunity to reflect on and change their own beliefs.
- An adequate professional learning environment should be designed in which teachers can engage with computer-based mathematical activities and their peers in order to reflect on their interactions and their beliefs about mathematics. Teachers' ideas and interpretations should be listened to and promoted with appropriate feedback.

Appendix 1A: Approval Letter (Turkish Version)

The Ministry of Turkish National Education approval letter and the researcher's permission letter for the field study at the Turkish primary school

T.C.
MİLLÎ EĞİTİM BAKANLIĞI
Yükseköğretim Genel Müdürlüğü

Sayı: B.08.0.YÖG.0.07.06.150.02- 4902
Konu: : Ümit KUL

14 Mart 2011

T.C.
LONDRA BÜYÜKELÇİLİĞİ
(Eğitim Müşavirliği)

1416 sayılı Kanun uyarınca yurtdışında öğrenim yapmakta olan aşağıda adı, soyadı ve durumu yazılı öğrenci hakkında alınan onayın bir örneği ekte gönderilmektedir.

Bilgilerinizi ve gereğini rica ederim.


Mehmet YAVAŞ
Bakan a.
Daire Başkanı

EKLER :

1- 11.03.2011 tarih ve 4862 sayılı onay.(2 sayfa)

Adı Soyadı	: Ümit KUL
Öğrenim Yaptığı Ülke	: İngiltere
Bursunu Veren Kurum	: Millî Eğitim Bakanlığı
Adına Öğrenim Yaptığı Üniversite/Kurum	: Kafkas Üniversitesi
Onayın Konusu	: Tez konusu üzerine çalışma yapmak ve veri toplamak üzere 15.04.2011-05.06.2011 tarihleri arasında yurtda bulunmasına, talep etmesi halinde bu çalışmasıyla ilgili olarak yurda geliş ve öğrenim yerine dönüş uçak bileti ücretinin, ayrıca yurttaki belgelendirilmiş ulaştırma giderlerinin (tren ve otobüs bileti ücreti) tamamı ile tez çalışmalarının gerektirdiği malzeme ücretlerinin en çok iki aylık yurt içi bursu kadar olan kısmının Eğitim Müşavirliğine açılan krediden karşılanması.

Appendix 1B: Request Letter (Turkish version)

07.02.2011

T.C.
MILLI EGITIM BAKANLIGI
LONDRA EGITIM MUSAVIRLIGI

Milli Egitim Bakanligi Doktora burslu ogrencisiyim. Leicester Universitesinde Matematik Egitimi alaninda doktora yapmaktayim. 20/04/2011-15/06/2011 tarihleri arasinda Turkiye de ilkogretim matematik ogretmenleriyle alan calismasi yapmayi planlamaktayim. EARGET tarafından izin alınmasini rica ediyorum. Calismamin ile ilgili detaylar dokumanlar ektedir. Gereginin yapilmasini arz ederim.

Ekler:

- Arastirmanin ilgili detayi (Amaci, kullanilan arastirma metodlari etc.)
- Anket

Adres: Flat 6, 20 Calais Hill
LEICESTER, LE1 6FF
Tel: 07879998206

Umit Kul

Appendix 2: Participant Consent Letter



Participation Consent Letter

Dear teacher,

I am a PhD student at School of Education in University of Leicester; I am conducting a research study to investigate professional development of Turkish primary mathematics teachers within a computer-integrated learning environment. Hence, I request your assistance by inviting you to participate in this current research. Your participation would help to improve and develop mathematics teaching and learning in schools of Turkish and the integrating of GeoGebra software application. This study requires your participation which is voluntary basis. There are no risks to you or to your privacy if you decide to participate to my research. But if you choose not to participate that is fine. However, your participation is important to helping me in my research. I would greatly appreciate your participation. If you have questions regarding your rights as a research participant or if a problem arises which you do not feel you can discuss with the researcher, please contact University of Leicester, or email Prof Janet Ainley: jma30@leicester.ac.uk.

This section indicates that you are giving your informed consent to participate in the research.

The Participant's consent:

I confirm that I have read this consent and understand the information provided, and do agree to participate in this study. I do understand that my participation in this study is voluntary and that I am able not to participate in the study any time by informing researcher personally or by email at uk13@le.ac.uk. I am 18 years of age or older.

I understand that I will receive a copy of this signed consent form.

Participant's Signature: _____ Date: _____

Thank you for your participation.

Umit Kul PhD Students, University of Leicester School of Education Email: uk13@le.ac.uk

Appendix 3: Mathematical Belief Questionnaire

For each item, tick the response that indicates how much you would agree with the following statements as indicated below.

Strongly Disagree Disagree Undecided Agree Strongly Agree

1 2 3 4 5

There is also a space at the end of the section that you can add your further comments.

SAMPLE:

Principle. A good education is necessary for a successful and happy life

ANSWER: 1 2 3 4 ☒ Agree 5

Section 1. Nature of mathematics

		Strongly disagree	Disagree	Undecided	Agree	Strongly agree
		1	2	3	4	5
1	In mathematics something is either right or wrong					
2	The ideas of mathematics can be explained in everyday words that anyone could understand.					
3	There are often many different ways to solve a mathematics problem					
4	Mathematics is essentially the same everywhere in the world.					
5	Doing mathematics involves creativity, thinking, and trial-and-error					
6	Technical mathematical language and special terms are needed to explain mathematics.					
7	In mathematics there are often several different ways to interpret something.					
8	Mathematics is an evolving, creative human endeavour in which there is much yet to be known.					
9	Mathematics problems can be solved in only one way.					
10	Doing mathematics consists mainly of using rules.					
11	Mathematics is a static and immutable knowledge with objective truth.					
12	In different cultures around the world there are different forms of mathematics.					

Further comments:

Section 2. Mathematics Teaching

		Strongly disagree	Disagree	Undecided	Agree	Strongly agree
		1	2	3	4	5
13	The most important component of good teaching is that teachers show students the proper procedures to answer mathematics questions.					
14	Good mathematics teachers plan so that students regularly spend time working individually to practise doing mathematics.					
15	The role of the mathematics teacher is to provide students with activities that encourage them to wonder about and explore mathematics					
16	Teachers should regularly devote time to allow students to find their own methods for solving problems					
17	Good mathematics lessons progress step-by-step in a planned sequence towards the lesson objectives.					
18	Good mathematics teaching involves class discussion in which students share ideas and negotiate meanings.					
19	Children should receive knowledge from the teacher.					
20	Good mathematics teachers only teach what is important for mathematics tests.					
21	The teacher should provide examples of problem solutions when children are doing problems					
22	Good mathematics teachers show students lots of different ways to look at the same question.					
23	Cooperative group work and class discussions are important aspects of good mathematics teaching					
24	Children learn mathematics best when they are actively involved.					

Further comments:

Section 3. Mathematics Learning

		Strongly disagree	Disagree	Undecided	Agree	Strongly agree
		1	2	3	4	5
25	Being able to memorise mathematical facts and procedures is important for mathematics learning.					
26	Students should be encouraged to build their own mathematical ideas, even if their attempts contain much trial and error.					
27	Students who have access to computers learn to depend on them and do not learn concepts and ideas properly.					
28	Students' mathematics errors often reflect their current understandings of ideas or procedures.					
29	. Students learn mathematics best if they are shown clear, precise procedures for doing things.					
30	Teachers should value periods of uncertainty, conflict, confusion or surprise when students are learning mathematics.					
31	Understanding ideas and procedures is essential in mathematics learning.					
32	Students learn mathematics by being shown the correct ways to interpret mathematical symbols, situations and procedures.					
33	Mathematics learning is enhanced if students are encouraged to use their own interpretations of ideas and their own procedures.					
34	Computers can assist mathematics learning by serving as tools for exploration and consolidation of ideas.					
35	Students' mathematical mistakes are usually caused by a lack of practice.					
36	Mathematics learning is about learning to get the right answers.					

Further comments:

Appendix 4: Interview Protocols

INTERVIEW I

1. General background information about participants.

Name? How many years have you been teaching? School? What are your qualifications?

2. This might be a difficult question. I would like to you try to answer. What mathematics really is? What initially comes into your mind when hear the word “mathematics”?

Do you think humans have “created” or “discovered” mathematics?

3. How important is mathematics to an individual and a community?

4. Could you describe how did you learn mathematics in your school years? How can a student learn mathematics best regarding to your experiences? What is the role of student while learning mathematics? Do you have positive and negative moments while learning?

5. What are the steps you, generally, would want to follow in designing mathematics lesson, and I would like you to describe the way organised your mathematics classroom today?

What are in your opinion the main purposes and reasons of teaching mathematics in primary school? What is the best way to teach mathematics? What is the role of teacher in teaching?

Did you take courses of teaching and learning methods in university?

6. Can you comment on the changes to the curriculum in 2005?

7. What is your background on computer use? What do you think about the role of computer in education? What do you think about the role of computer mathematics education?

8. Are there any differences between teaching a particular topic with using pen-paper and using computer in mathematics classroom?

9. Do you believe one day, computers might replace teachers? What major concerns do you have using the computer in your classroom?

10. There might be several reasons to come to this course. What motivated you take this course? Or what are your expectations from this course?

INTERVIEW II

1. What were your initial thoughts on GeoGebra? Did you enjoy using the computer?
 2. What criticisms and benefits about the course do you have?
 3. What can you say about my role in this course?
 4. Do you think you can teach mathematics in this way?
 5. Now I want to show you activities in introductory booklet. Could you talk about general impressions about these, for instance the most surprising one? What do you think about construction, drawing and dragging concepts?
 6. Could you describe a good and bad moment you had in one, two, three workshops session? What can you say about it in terms of students' learning and classroom implementation from your point of view as teacher?
 7. What were your thoughts on the activities in the Worksheet 1? What can you say about the activities in the Worksheet 2? What about the activities in the Worksheet 3? What do they make sense on you?
 8. What kind of opportunities does GeoGebra provide to student and teacher? What major concerns do you have about using the computer in your classroom? What is your intention to use it in your classroom? What do you think about the potential relevance and application of the course activities in the classroom?
 9. Has the course changed your thoughts in any way about the usage of technology in mathematics since the beginning of the course? Has being in a learning situation effected your perception of students' learning? Do you think the course has affected your teaching?
- Could you describe what you did when working with GeoGebra during the course?
10. What difference do you see between teaching a particular topic in pen-pencil environment and computer-based environment?

INTERVIEW III

1. Tell me how you felt throughout the course. Could you describe a good and bad moment you had in four five, six workshop sessions?

2. What can you say about it in terms of students' learning and classroom implementation from your point of view as teacher?

What were your thoughts on worksheet 4? What were your thoughts on worksheet 5? What were your thoughts on worksheet 6? What do they make sense on you? What are the mathematical ideas and concepts that you think can be explored by using GeoGebra?

4. What is your intention to use GeoGebra in your classroom? What aspects of the PD course encourage you to use it in the classroom?

5. What would be your role as teacher while engaging GeoGebra-based mathematical activities? What can the students react if you plan to use GeoGebra in the classroom?

What kind of contributions do they make use considering that you think yourself as an educator and you apply the GeoGebra program to your students?

6. Have you participated in any in-service course about using computer in mathematics education? In your opinion, can this course be model to be applied in programs in-service? What do you think as a whole?

Do you think that this course has contributed for your Professional development any?

7. What can you say about the practicability of GeoGebra on mathematics education, based on your experience at the courses you have taken?

8. As far as your teaching of mathematics is concerned, in what ways do you think you have changed your view of mathematics learning and teaching since coming on this course?

In what ways has the PD course effected your beliefs about mathematics?

9. Can you tell us about the different experiences you underwent during the course?

10. How can we improve the course and increase computer use in mathematics classroom?

Appendix 5A: An Example of Coding/Categorisation Process

appendixsd.xlsx - Microsoft Excel

	A	B	C	D	E
	Category	Sub-category	Code	Textual data	
1	The Nature Of mathematics	Personal conceptualisation	rules-oriented (school) subject	I had never thought about this question. The first thing which comes to my mind is that mathematics is a collection of rules based on numbers and consists of a combination of axioms and theorems.	
2	The Nature Of mathematics	Personal conceptualisation	open ended nature/ mathematics as a discipline	Mathematics can be like the other sciences; where there are theories which seem to be right one day but turns out to be wrong the other day. Maths is also an evolving and ever changing area thus we do not know what tomorrow holds.	
3	The importance of mathematics	Purpose of learning	Link to everyday life/ the control of exam	In class, when I was teaching trigonemtry, my students always ask me that why are we learning this ? and what is our benefit of learning this for daily life? etc. I attempted to explain establishing a connection to everyday life , sometimes not. After all, I say pupil just learn math for exam.	
4	Approaches to teaching and learning	Teaching and learning approach	memorisation/ Teacher-based teaching	The teacher was on the centre and peak, it seemed that the knowledge he gave and knew is right...You had to get the knowledge from the teacher; you had to accept without questioning where and how it came from. The memorising method only asks the student to memorise the right stuff on the exam day. After the exams are over, everybody forgets everything they learned and the teachers cannot do anything about it	
5	Approaches to teaching and learning	Social context of teaching	Institutional and social constraints	The biggest problem with the old system is the expectation of parents and sometimes the schools; as the spiral learning method requires more time to sink into the students' minds. However parents do not act very patiently and expect their children to learn as quickly as possible. This is why – sometimes, even I move to the older method. Also the exam styles are not helping as either; there are a lot of questions to be solved in a very limited time thus I have to revert to the older way of teaching.	
6	The role of computer in mathematics education	Orientation to the use of computers	Modifier tool/ timesaver/	Yes in certain areas of maths such as rotation, translation, mathematical pattern as it provides a visual way of looking at the problem. Without it, it would take a long time to even draw them, let alone understand and solve them. I had trouble in drawing the pyramid today.	
7	the role of computer in mathematics education	Expectations of technology integration in Turkey	lack of equipments/ limited experience/ modernisation	I believe students should learn how to use Computers when getting mathematical education but it concerns me whether we are up for it or not... Neither do we have enough facilities nor the mentality. However, the fact that becoming a member of EU I believe things are getting better in Turkey; one day (maybe a few generations later) when the examination system and the mentality changes, we'll be up and running	
8	Expectations about the PD course and reactions on researcher's role	The style of course/Personal impressions on PD course	ideal teaching/ active learner flexibility/ collaborative learning	I believe we were closer to the ideal way of teaching in this course. This lead to us fully understanding the situation and coming up with innovative ideas about solving them. Moreover I and my partner discovered different things about the activities without your impact... The course was also a flexible one, thus there were a lot of different ways in which we arrived at the solutions/answers. I felt like the students were acting like "inventors" and "innovators".	
9					

Ready | Asim sheet | Musti sheet | Eysun sheet | 86%

Appendix 5B: Extracts of coded data

Codes	Extracts from observational field notes and participants' writings
Willingness to share ideas	<i>During the PD course, the participants were asked to think and share their ideas about mathematical activities within the group. It was surprising that they tended to discuss the tasks and to listen to others' ideas since this kind of professional development activity was something which they have never experienced before. It seems to me that they did not want to terminate their discussion but rather they wanted to keep talking... [Observation, 18 May 2011/S5]</i>
Being positive about the course	<i>This course is somehow different from other types of course which I have so far attended. Since in this course we are in the position of a producer. That is, I and my colleague are attempting to create and construct our own knowledge. This was a kind of challenging process to investigate things by us. This would make the class enjoyable and productive... [Reflective writing, 25 May 2011/S6]</i>
The computer as a tool to visualise abstract concepts	<i>In fact, I learned these mathematical ideas when I was at primary school. However, I have got more sophisticated ideas about them now. Learning abstract concepts through GeoGebra activities makes learning easier and permanent. Students are able to see concrete examples of these abstract concepts on the monitor through visualization... [Interview, L231/I2]</i>
Contextual factors Classroom management The teacher milieu	<i>In the existing educational system the students have to learn the knowledge formulated... I am mixed up in an affair; one the one hand, I make them solve the exams with multiple choices; on the other hand I try to make them comprehend the subject. However, in this course, we used a variety ways of methods such as trial-error, group discussions. They are very useful and productive. To be honest, more experienced teachers prefer to follow traditional method mostly. It is difficult that teachers give up their regular system and teach by using the system which is more different than theirs... If least experienced teacher carries new approaches out; they find him unqualified to under control the class. And I also agree, feel that using traditional method is necessary. Since you have to follow that system because of your colleagues and some reasons in school even if you don't want. So your ambition to do some narrative things disappears. Essentially, it result from that you come to a new place and you have to comply with the school's conditions... [L386/I3]</i>
Participant's reactions The intervention strategy Discovery learning	<i>The participants entered this course with a variety of expectations. The key reason for attending the PD course was that this course offered more about learning to use the computer. My intervention often emphasis on designing learning situations that require the learner to take responsibility for their learning. One of my participants has struggled with making a relationship between sliding bar and variable at home. She had no idea about how to do it. At the end of day, she was able to deal with this problem. She stated that "I have tried to reach you, but have not found your phone number... Finally I have been able to find one. And I really enjoyed it. Although I don't have enough information about GeoGebra, I can reach where I want by discovering. It surprises me in this aspect". She later seemed to feel comfortable with this type of learning... [Observation, 1 June 2011/S7]</i>
Peer interaction as a tool for better understanding	<i>During my university years, I also did not learn mathematical ideas which I was supposed to use and teach them in the classroom. I was not taught "the proof of the sum of the angles in a triangle is 180 degrees". Yet, it has not been asked until now. I learned this proof by working and discussing with you and my colleagues. When a student asks me why and how to prove this theorem, my answer is already ready. This became part of my knowledge... [Reflective writing, 4 May 2011/S3]</i>

Appendix 6A: The Purpose of the Each Worksheet

Worksheet 1

Activity one was designed to discover the properties of parallel lines. This activity made abstract concepts more concrete so that the participants would see how their students can learn more about the angle relationship of parallel lines and polygons by dragging them around on the screen through by visualisation. The second and third activities were to examine the relationships/properties of polygons, leading to an exploration of the geometrical relationships in polygons by dragging and sketching figures in ways that extend beyond the scope of traditional paper-and-pencil geometry. Moreover, there is scope for generalisation, as the relationship between the sides and the angle of the turn is essential so as to construct a regular polygon. The fourth activity provided participants with an opportunity to gain mathematical insights into angle-length relationships and the object properties in a triangle. The participants would then conjecture about this relationship. The final activity provided participants with an opportunity to prove the sum of the angle theorem in a triangle using a slider tool; thus the participants could learn about this theorem through a concrete example, which would not have been possible using traditional paper and pen methods.

Worksheet 2

The first activity was created to explore the properties of a square. In this activity, the GeoGebra tools can be used as a compass, ruler and protractor to construct a square with a variety of characteristics. This activity provided an opportunity for learners using the drag test to check if the construction was correct. The second activity involved exploration of a regular hexagon shape. In this activity, the method for constructing a regular hexagon was found to be quite different from traditional paper-and-pencil geometry and this offered the possibility for the participants to understand the properties of the hexagon more fully. The third activity introduced a visualisation based on Thales theorem. The participants were able to perceive how the use of GeoGebra would facilitate student's learning in this context Thales theorem. The last activity in the second task initiated an investigation into the centre point of the circle using the given tools. This activity led to a discussion about the standard definition of a circle. Participants would thereby find an opportunity to make a conjecture about a centre of circle. This activity provided the opportunity to think about both the mathematical

properties of the figure that they were to construct, and how to use the tools in GeoGebra to construct them.

Worksheet 3

The first and second activities involved an examination of linear equations. By completing these activities, participants were able to explain how the variables (m and b) affect the graph of the line. These activities illustrated the concepts of observation of patterns, and the making and justification of generalisations. The third activity introduced the notion of slope on the coordinate plane. This activity guided participants to examine the influence of the equation's parameters on the line by using the sliders. The aim of this activity was to provide participants with a context in which they could explore the concept of slope with the use of GeoGebra tools to comprehend how the use of a slider would facilitate students' investigations and learning about various slopes on the coordinate plane. The last two activities were designed to examine a system of linear equations on the coordinate plane. The purpose of these activities was to provide participants with an opportunity to conjectures about finding the solution to a system of linear equations graphically.

Worksheet 4

The first activity introduced different objects constructed with two and three unit squares on a graphical view in GeoGebra, so as to discuss the notion of the conservation of area for different figures. This activity did not enable the participants to use pre-prepared applications, instead; it challenged the participants to make conjectures and then formalise them. In the second and third activity, the participants were required to investigate how to find the area of a triangle and parallelogram using a slider in order to compare the relationship between the areas of different types of quadrilaterals. The purpose was to allow the participants to see how students can make conjectures about the formula for the area of a triangle and a parallelogram. The fourth provided an opportunity for participants to express their ideas about how to find and develop the formula for the area of a circle. The purpose of the final activity was to enable the participants to recognise how students can investigate the relationship between the area and perimeter of objects through GeoGebra activities. Initially, the teachers worked on this activity using a paper and pencil so as to discriminate two concepts (area and perimeter). Participants were able to construct a graph of a rectangle with a fixed perimeter in the coordinate plane and find the dimensions with maximum area. GeoGebra encouraged the participants to illustrate this activity on the screen to clarify the relationship.

Worksheet 5

At the end of first activity, participants then investigated the relationship between the areas of the squares forming a triangle; all the activities in this task were aimed at demonstrating to the teachers, the opportunity to allow students to examine the relationships between different types of triangles accurately. Through the GeoGebra activities, the participants saw how the students could strengthen their understanding of Pythagoras Theorem and also conceptualise the relationship in more detail.

Worksheet 6

The first and second activities initiated an investigation of changing the location of objects on Cartesian coordinates, by dynamically using reflection. The aim of this activity was to provide participants with an environment in which they could discover the reflection of objects and investigate the properties of the mirror image with the original image in a dynamic context beyond the scope of traditional paper-and-pencil geometry. For example, one predesigned activity was about how to reflect an object; such as F, in the line $y=0$ or $x=0$ through GeoGebra activity. The question of which properties of the object remained the same and what properties of the object changed was asked. The third and fourth activities were designed to explore the idea of rotation in the area of transformation geometry. The participants explored the notion of the rotation of objects, and also explored how students using GeoGebra could describe an object and recognise rotations with a desirable angle and direction. The last activity introduced the idea of translation of a geometrical object in transformation geometry. This activity allowed participants to observe transformations with shapes on different positions on the screen. This could help students to see patterns and assist them with generalising.

Appendix 6B: Introduction Booklet

GeoGebra Introduction Booklet

What is GeoGebra?

GeoGebra is dynamic mathematics software that connects geometry, algebra and calculus. It is used for constructing geometric objects such as points, vectors, segments, lines, polygons, conic sections, and functions that can be changed dynamically. The most remarkable feature of GeoGebra is the dual view of objects: every expression in the algebra window corresponds to an object in the geometry window and vice versa. In the following, you will get acquainted with GeoGebra by doing five activities. You should work on them one after the other.

Activity 1: Installation and Introduction GeoGebra

Activity 2: Basic Drawings in GeoGebra

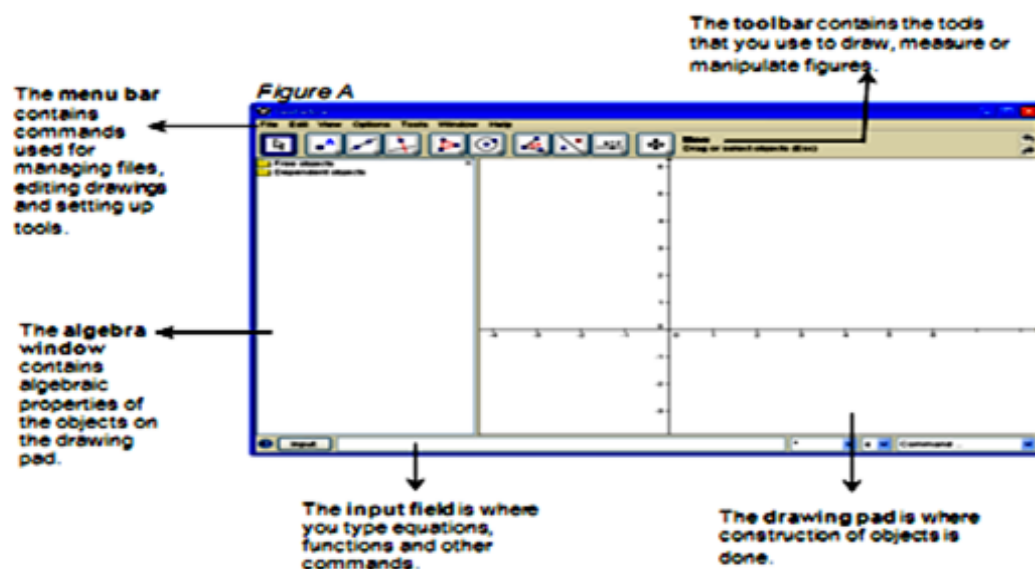
Activity 3: Constructing a Rectangle Activity

Activity 4: Drawings, Construction and Drag Test

Activity 5: Equilateral Triangle Construction

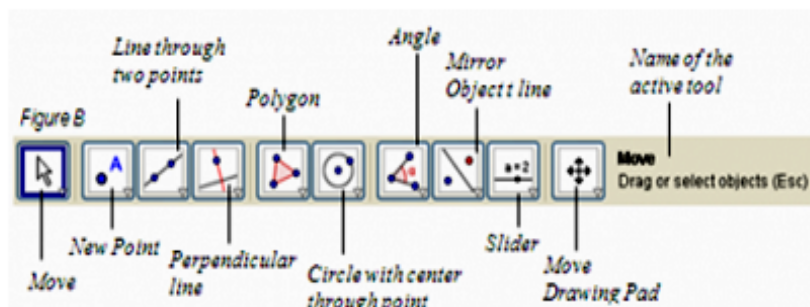
After starting GeoGebra, the window shown below appears. By means of the construction tools (modes) in the *toolbar* you can do constructions on the *drawing pad* using the mouse. At the same time the corresponding coordinates and equations are displayed in the *algebra window*. The *input text field* is used to enter coordinates, equations, commands and functions directly; these are displayed at the drawing pad immediately after pressing the enter key. Geometry and algebra side by side:

PARTS OF GEOGEBRA



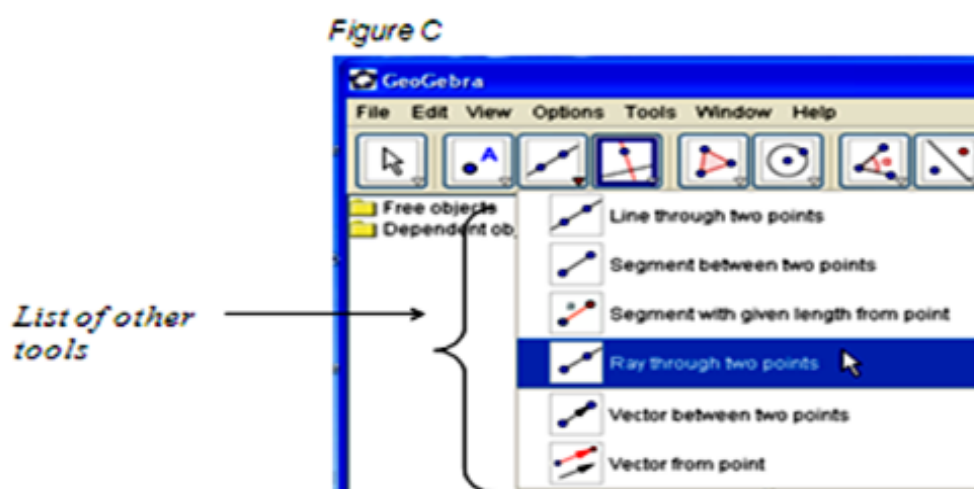
The Toolbar and the Tools

The toolbar contains the tools that are used to construct points, lines and other figures. Shown below are the default tools displayed when you open GeoGebra.



In the diagram above, the *Move tool* is highlighted by a blue border which means that it is the active tool. The name of the active tool is displayed at the right of the *Move drawing pad* tool. As long as a certain tool is active, it will construct the same drawing or perform the same task, you do not need to click it every time you construct the same object.

The icon of each tool has an *arrow (inverted triangle)* that you can click if you want to display other tools. *Figure C* displays the tools if you click the *Line through two point's* button.



Activity 1: Installing GeoGebra

Preparations

Create a new folder called GeoGebra Introduction on your desktop.

Hint: During the workshop, save all files into this folder so they are easy to find later on.

Installation WITH Internet access

Install GeoGebra Web Start

- Open your Internet browser and go to www.GeoGebra.org/webstart.

- Click on the button called *GeoGebra Web Start*.

Note: The software is automatically installed on your computer. You only need to confirm all messages that might appear with *OK* or *YES*.

Hint: Using GeoGebra Web Start has several advantages for you provided that you have an Internet connection available for the initial installation:

- You don't have to deal with different files because GeoGebra is installed automatically on your computer.
- You don't need to have special user permissions in order to use GeoGebra Web Start, which is especially useful for computer labs and laptop computers in schools.
- Once GeoGebra Web Start was installed you can use the software off-line as well.
- Provided you have Internet connection after the initial installation, GeoGebra Web Start frequently checks for available updates and installs them automatically. Thus, you are always working with the newest version of GeoGebra.

Activity 2: Basic Drawings in GeoGebra

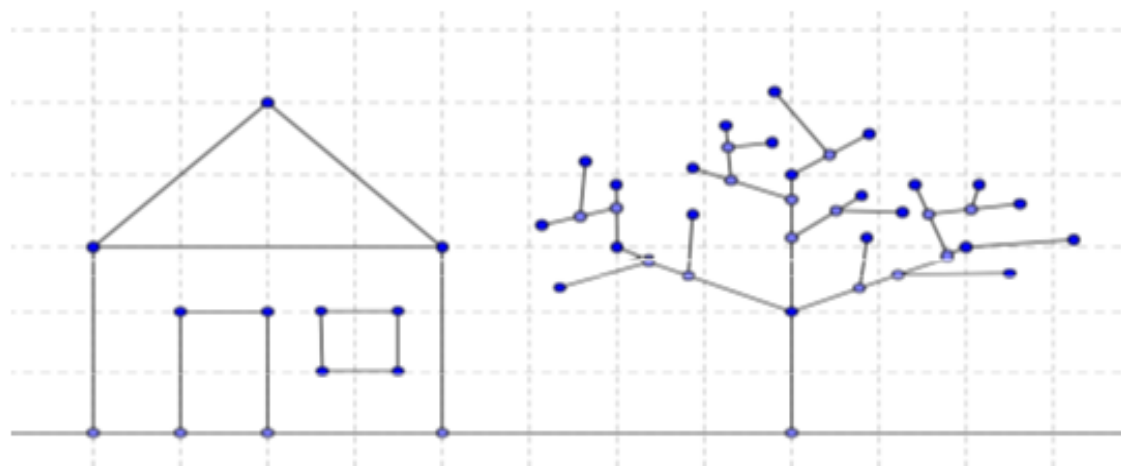
We will first see how to create simple drawings like the picture of the house shown below. The reason for doing this is to familiarise ourselves with some of the basic tools in GeoGebra

Preparations

Hide the algebra window and coordinate axes (View menu).









Show the coordinate grid (View menu).

We are now ready to begin.



Drawing a picture with GeoGebra

Use the mouse and the following selection of tools in order to draw figures on the drawing pad (e.g. square, rectangle, house, tree...).










 New Point	 Delete object
 Move	 Move Graphical view
 Line through two points	 Zoom in
 Segment between two points	 Zoom out

What to practice

- How to select an already existing object.
Hint: When the pointer hovers above an object, it highlights and the pointer changes its shape from a cross to an arrow. Clicking selects the corresponding object.
- How to create a point that lies on an object.
Hint: The point is displayed in a light blue colour. Always check if the point really lies on the object by dragging it with the mouse. How to correct mistakes step-by-step using the *Undo* and *Redo* buttons.

Activity 4: Constructing a Rectangle

Open GeoGebra. We will not need the coordinate axes. To hide the axes, click **View>Axes** from the menu bar. We also want all created objects to have labels so that we can refer to them easily. To do this, select **Options>Labelling>All New Objects** from the menu bar.

1		Create a segment AB
2		Create a perpendicular line to segment AB through point B
3		Perpendicular line to segment AB through point A
4		Now select the Move tool and drag the objects. Which of the objects can or cannot be moved? Can you explain why?
5		New point C on perpendicular line. Drag point C . What do you observe?
6		Parallel line to segment AB through point C
7		Intersection point D . Now, try dragging the objects. What do you observe?
8		Now, we hide lines b , c , and d . Select the Show/Hide Object and click the three lines. This will highlight them. To hide, select the <i>Move</i> tool or any tool other than the <i>Show/Hide</i> button.
9		Polygon $ABCD$ Hint: To close the polygon click on the first vertex again.

Appendix 6C: Worksheets

WORKSHEET 1

Activity1.1 (Angle Relationships/Properties of Parallel Lines)

We will investigate angle relationships formed by two parallel lines cut by a transversal. You

will create Figure1 with the following tools

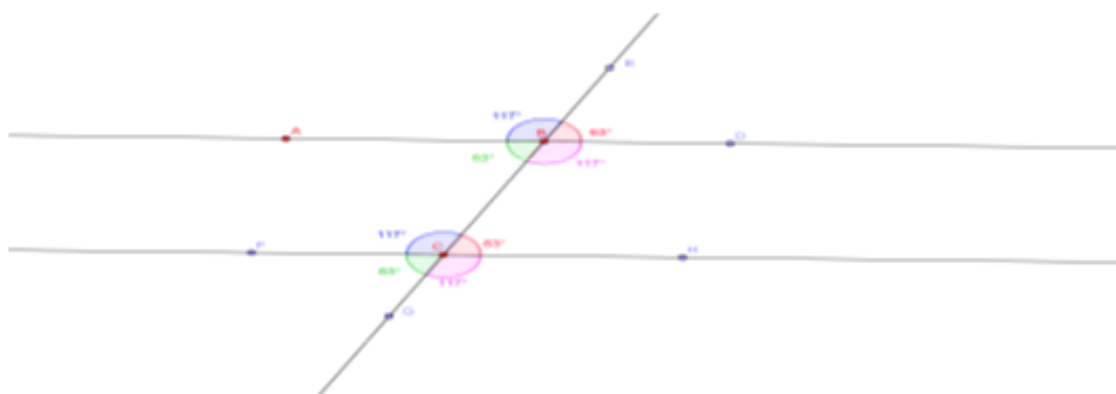


Figure 1

1. Use points A, B, and C to move the lines. What do you observe?
2. Compare different pairs of alternate interior and exterior angles. What do you notice?
3. How will students identify special pairs of angles?

Activity1.2. Interior Angles of Polygons



1. Create a triangle, quadrilateral, pentagon and hexagon.
2. “What is a diagonal?” discuss with your partner.



3. Create diagonals from a corner of all polygons.

Number of sides of the polygon	Number of Triangles created

Number of side of polygon	Number of Triangles created	Sum of the internal angles of the triangles

4. What additional pattern do you see now?
5. Could you write general procedure for regular polygons?
6. What is the correlation between the sum of interior angles of polygons and triangles formed by diagonals drawn between corners of the polygon? Discuss.
7. Could you do this activity with pencil and paper?

Activity 1.3 (Exterior Angles of Polygons)

Use this construction to investigate the sum of the exterior angles of a polygon. Open dynamic worksheet

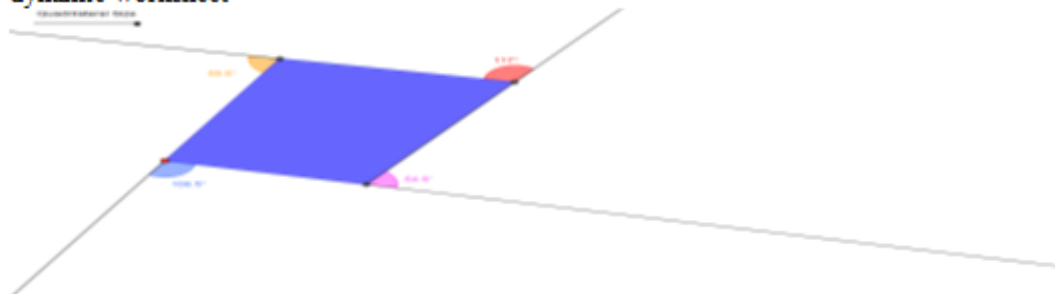


Figure: 2

1. Use the slider to resize the quadrilateral. What is the sum of the exterior angles of a quadrilateral?
2. Use the red point on the quadrilateral to change the angle measures. Now use the slider to resize the quadrilateral. Does changing the angle measures affect the sum of the exterior angles?
3. Could you prove exterior angle theorem for other polygons in this way?

Activity 1.4(Angles, Lengths and Object Properties)

We will use GeoGebra to investigate if a relationship exists between the interior angles of a triangle and the lengths of its sides. Please Create Figure 3

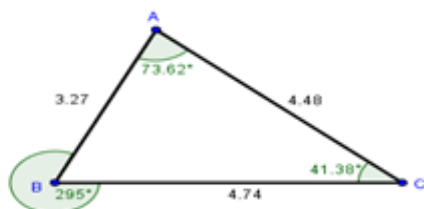


Figure: 3

Now select the **Move** tool and drag the vertices of the triangle.

1. What do you observe about the measure of the angles and the lengths of the segments?
2. If you are in the position of student, what conjecture you can make from this triangle?

Activity 1.5 Sliders and Rotation

In this activity, we will use the slider control to determine the measure of angles, and to rotate a triangle to show that is 180 degrees.

Discussion: How to prove angle sum theorem with using pen and paper

Open dynamic worksheet

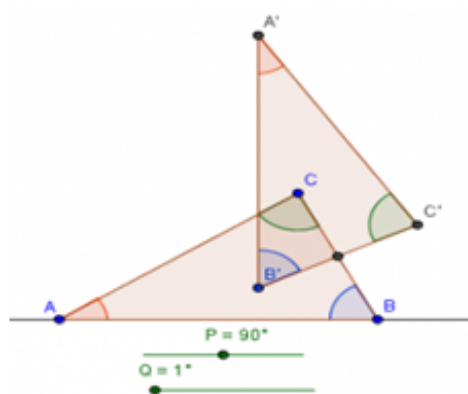


Figure: 4

1. Now move slider *P* and *Q*. What do you observe?
2. Could you prove angle sum theorem in a different way?
3. What are the differences between dynamic and static environment?











WORKSHEET 2

Activity1: Square Construction

-Open new GeoGebra file.

-Hide algebra window, input field and coordinate axes (*View menu*).

-Change the labelling setting to *New points only* (menu *Options – Labelling*).

1		Segment a = AB between points A and B
2		Perpendicular line b to segment AB through point B
3		Circle c with centre B through point A
4		Intersect circle c with perpendicular line b to get intersection point C
5		Perpendicular line d to segment AB through point A
6		Circle e with centre A through point B
7		Intersect perpendicular line d with circle e to get intersection point D
8		Create polygon ABCD
9		Hide circles and perpendicular lines
10		Use the drag test to check if your construction is correct

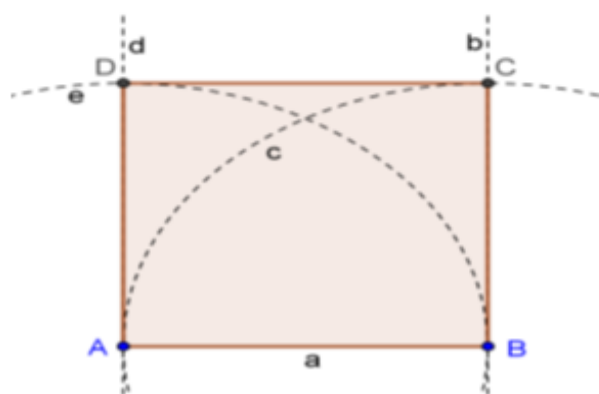








Figure: 1

Challenge: Can you come up with a different way of constructing a square?

Activity 2: Regular Hexagon Construction

For this activity you will need to use the following tools. Make sure you know how to use each tool before you begin with the actual construction of the hexagon

 Circle with centre through point	 Angle
 Intersect two objects	 Show / hide object
 Polygon	 Move

Challenge: Try to find an explanation for this construction process.

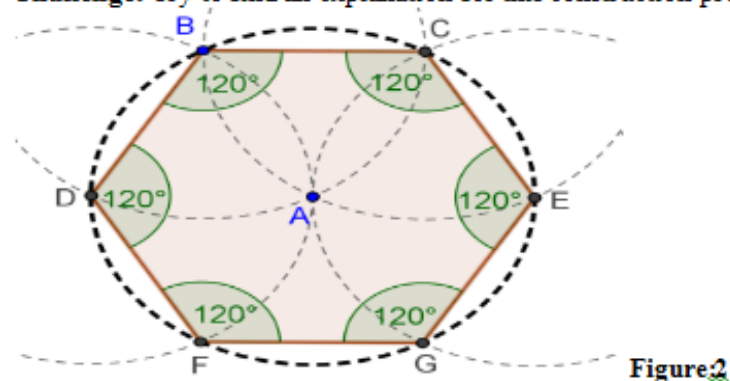








Figure 2

Activity 3: Visualization of the Thales Theorem

 Segment a = AB between points A and B	 Polygon
 Semi-circle through two points	 Angle
 New point	 Move

Below you can see a triangle whose vertex C lies on a semicircle over segment c.

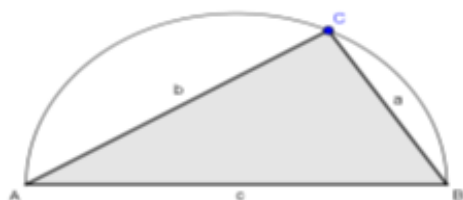


Figure: 3

1-Drag point C with the mouse along the semicircle and watch the angles of the triangle. Write down your observations on a sheet of paper.

2- Use the Angle tool to create the angles of the triangle. Check your assumptions from 1) by moving point C with the mouse (in Move mode).

3-Try to explain why angle γ is always a right angle.

Challenge: Try to make the graphical notation, the proof of this theorem

Activity4: Constructing the Centre of a Circle

Do you know how to construct the centre of a circle?

Preparations

Open a new GeoGebra file.

Hide the coordinate axes and the algebra window, but show the input field (*View* menu).

Instruction

Enter circle C 's equation: $x^2 + y^2 = 16$

Challenge: Can you find the centre point of the circle using the given tools?

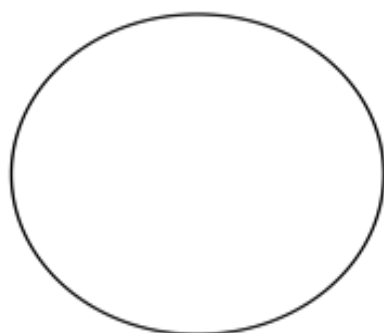
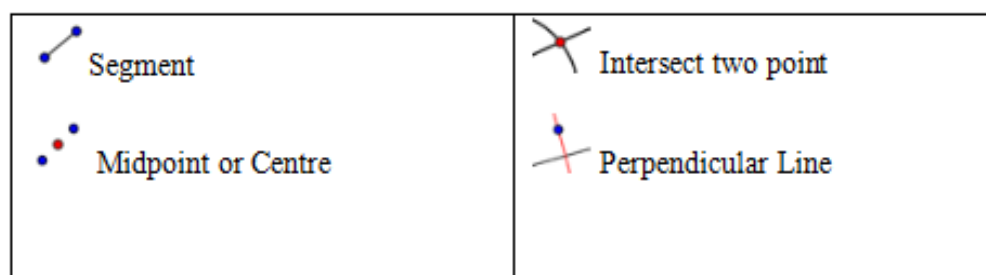


Figure: 4

WORKSHEET 3

Parameters of a Linear Equation

- Open new GeoGebra file.
- Show the algebra window, input field and coordinate axes (*View* menu).

Activity 3.1

Enter: line: $y = 0.8x + 3.2$

Tasks:

- Move the line in the algebra window using the arrow keys. Which parameter are you able to change in this way?
- Move the line in the graphics window with the mouse. Which transformation can you apply to the line in this way?

Activity 3.2

1	Delete the line created in activity 1
2	$\xrightarrow{b=2}$ Create sliders m and b using the default settings of sliders.
3	line: $y = m * x + b$ Hint: Don't forget to use an asterisk or space to indicate multiplication!

Follow the directions and use the interactive sliders for m and b to answer the questions.

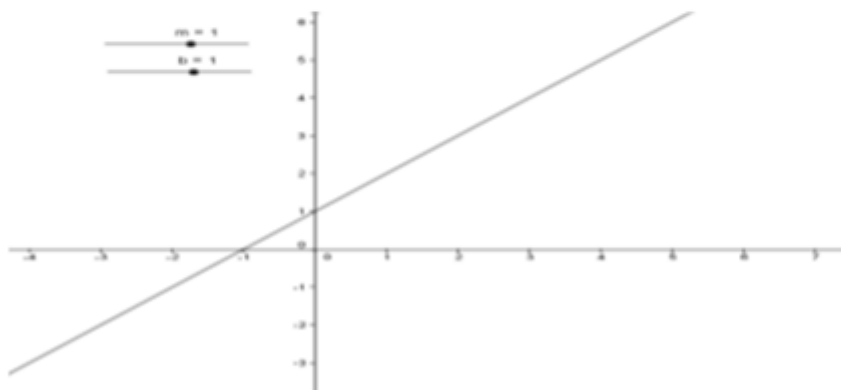







Figure: 1

The line above is created by graphing the function $y=mx+b$. By completing this exercise, you will be able to explain how the variables m and b affect the graph of the line.

1. What happens to the line when m is positive? Negative?
2. What happens to the line when m is close to zero? Close to 5 or -5?
3. What happens to the line when b increases?
4. What happens to the line when b decreases?
5. Describe how the values of m and b affect the graph of the line

Activity 3.3

1		Intersection point A between the line and the y -axis Hint: You can also use the command $A = \text{Intersect} [\text{line}, y\text{-axis}]$.
2		Point B at the origin
3		Segment between points A and B
4		Slope (triangle) of the line
5		Hide unnecessary objects

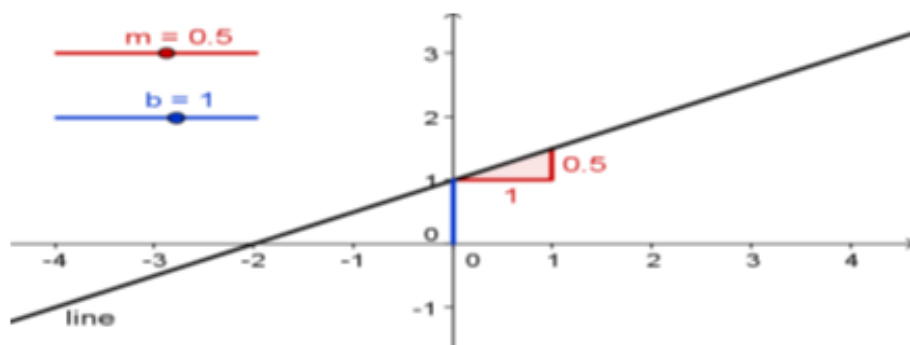


Figure:2

Task

Write down instructions for your students that guide them through examining the influence of the equation's parameters on the line by using the sliders.

Activity 3.4

Below you can see the visualization of a system of linear equations.

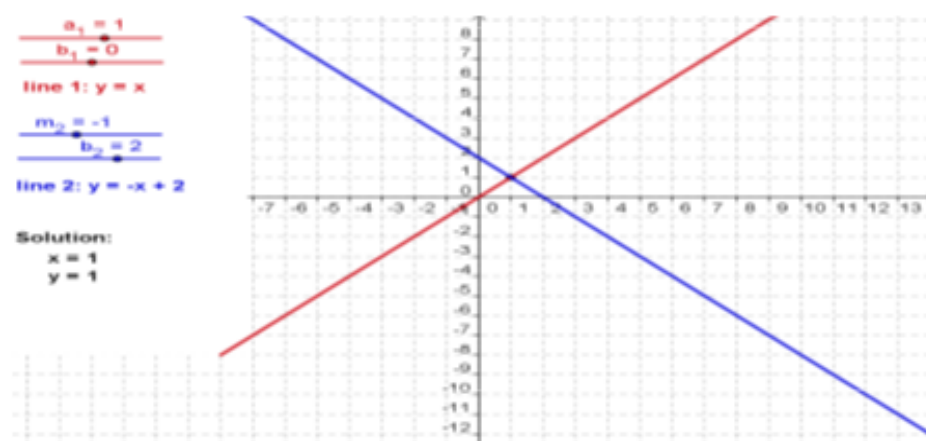


Figure:3

1. Solve the system of linear equations shown in the dynamic figure on paper. Check if your answer matches the solution displayed.
2. Use the sliders to come up with a new system of linear equations. Repeat step (1) for the new system.
3. Come up with a conjecture about finding the solution of a system of linear equations graphically.
4. What happens if the two lines are parallel? How can you tell if two linear equations are represented by parallel lines?
5. What happens if the two lines are identical? How can you tell if two linear equations are identical?

Activity 3.5

You are able to move the blue line by clicking on and moving point J. Please answer the questions to the right.

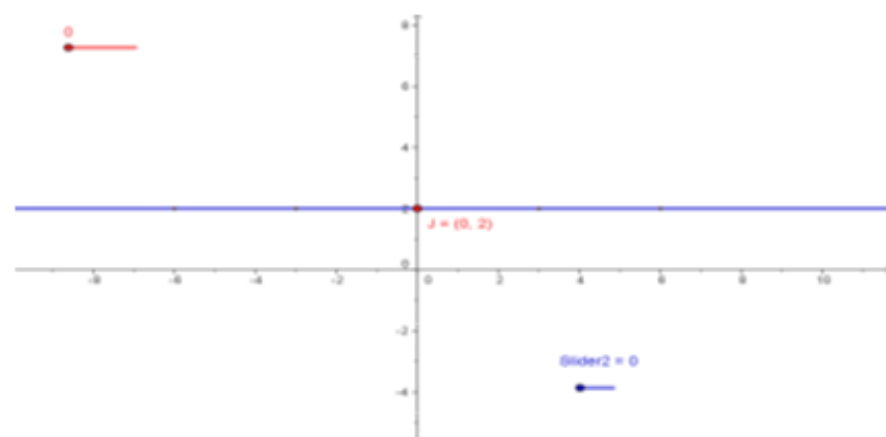


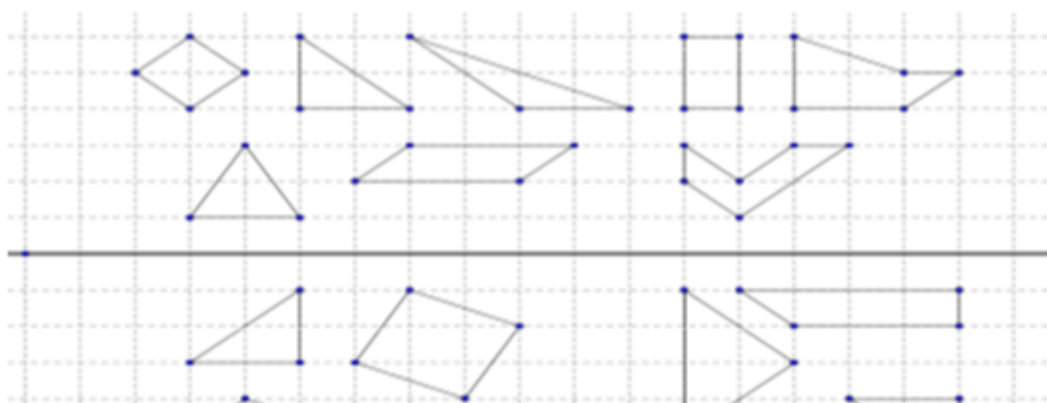
Figure: 4

1. Which axis is the blue line crossing? At what point is the blue line crossing this axis?
2. Move the red slider to 1 and write the name and coordinates of the point that appears. Do the same by moving the slider to 2, 3, and 4.
3. What do you notice about the coordinates of these points?
4. Using the answers from the above questions, make a guess as to what the equation of this line is. Move slider 2 over to check your answer.
5. Move slider 2 back to the left. Use point J to move the line along the axis. Stop at any point and write the equation of the point you stopped at along with the equation of this line. Check your answer by moving the blue slider.
6. Use any combination of two points that lie on the line to find the slope using the slope formula $(y_2 - y_1) / (x_2 - x_1)$. Make a conclusion about the equation of horizontal lines and their slopes.

What about vertical lines?

WORKSHEET 4

Activity: 4.1 Match up the shapes which have the same area and discuss what you observe



Activity 4.2: Area of a Triangle

In this worksheet you will investigate how to find the area of a triangle. Open the GeoGebra dynamic worksheet

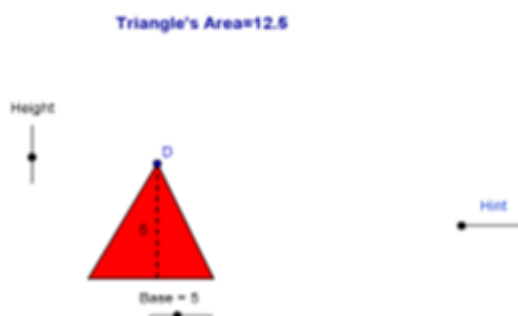


Figure:1

Investigation;

- 1) Move point D so that the triangle is a right triangle, then move the Hint slider, describe the end result.
- 2) Move point D to make an obtuse triangle and move the Hint slider, describe the end result. (Perform this task for an acute triangle as well)
- 3) What conjecture can you make about the formula for area of a triangle? Test this conjecture with 5 different base and height combinations and record your results.

Activity4.3: Area of a Parallelogram

Use the polygons to complete the investigation below.



Investigation:

- 1) Change the height of the rectangle (h) by moving point C. What happens to the height of the parallelogram?
- 2) Change the width of the rectangle by moving point L. What happens to the parallelogram?
- 3) Find the area of 3 different rectangles and compare the area of each rectangle to the area of the corresponding parallelogram.
- 4) What conjecture can you make about the formula for area of a parallelogram and why? (Use the answer to question 3 to guide you)

Activity 4.4: Area of a Circle

In this worksheet you will investigate the area of a circle.



Investigation:

- 1) Move the slider above the circle and describe what happens to the circle step by step.
- 2) Make a conjecture about the formula for the area of a circle and show how you develop this formula. (Move the slider named hint to help you)

Activity 4.5 Parameterization of Area and Length of a Rectangle

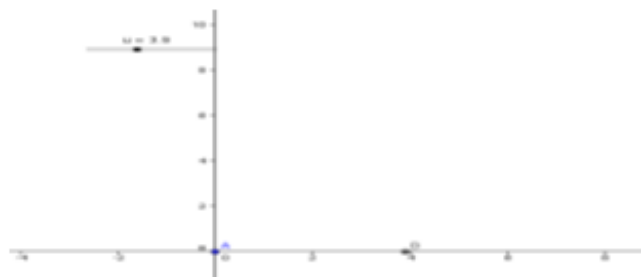
Problem: Given a rectangle with perimeter 20 units, find the dimension of the rectangle that can be formed that has the maximum area.

Let us first solve problem. We know that our perimeter is constant, so long length k , short length will be called u . or instance, if our long length is 4 units, our width will be simply $(20 - 2*4)/2$ which is equal to 1. We can get long length k in terms of by the equation $k=10-u$

Construction process

Now, create a new slider and name it u , set the interval to 0 and 10 leaving the other values as is (why we chose this range?).

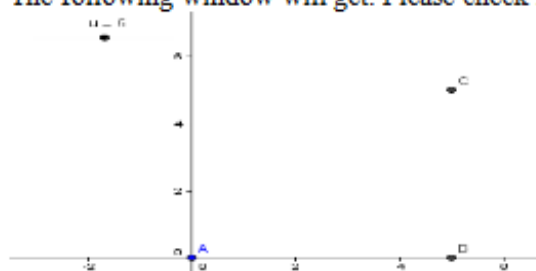
We will create points of entry by input bar, rectangle's the lower left corner $(0, 0)$ point, at the bottom right corner $(u, 0)$ point-line pairs in this way



When you change the slider, you will be able to see point B in the changes.

Now we need to construct in the upper right corner. How should be coordinates of this point?

The following window will get. Please check how the slider has creates an impact.



The C point perpendicular to the y-axis (or parallel to the x-axis) and draw the line at the intersection with the y-axis set point. This point will be the upper left corner of our rectangle. Remember to hide line.

In order to create a rectangle you should use polygon tool. Area and length so if the rectangle is automatically displayed in algebra window





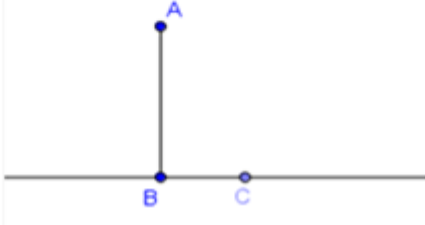


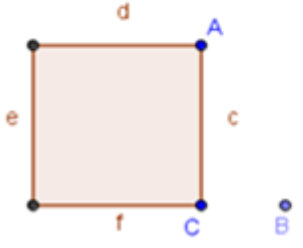



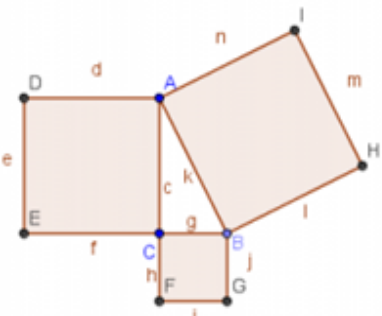
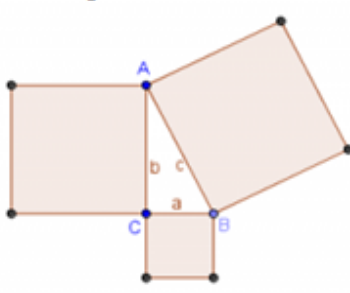
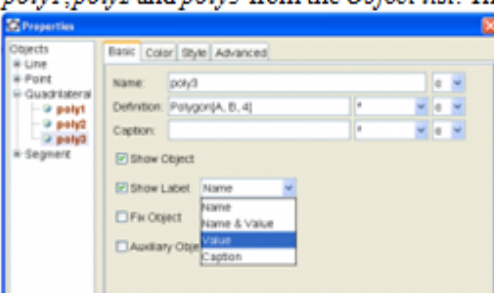

Change the shape of the rectangle by moving the slider. Observe the way the point is moving to find when the rectangle will have maximum area.

To produced point P , type $P = (u, \text{poly1})$. Note that poly1 is the area of the rectangle (see the Algebra window). Right click point P , then click check *Trace On*. This will trace the path of point P . Move point B . What do you observe? What can you say about the curve formed by the traces of point P ?

WORKSHEET 5

Activity 1(Discovering the Pythagorean Theorem) we are going to compare areas of squares formed from sides of a right triangle. To construct this figure, we first construct a right triangle, and form three squares, each of which contains one of the three sides as shown below.

	1.) Click the Segment between two points tool and click two distinct places on the drawing pad to construct segment AB .
	2.) If the labels of the points are not displayed, click the Move button, right click each point and click <i>Show label</i> from the context menu. (The <i>context menu</i> is the pop-up menu that appears when you right click an object.)
	3.) Next, we will construct a line perpendicular to segment AB and passing through point B . To do this, choose the Perpendicular line tool, click segment AB , then click point B .
	4.) Next, we create point C on the line. To do this, click the New point tool and click somewhere on the line. Your drawing should look like the figure below. Display the label of the point in case it is not shown (see no. 3)
 <p>Figure 2 - Point C on the line passing through B</p> <p>You have to be sure that C is on the line passing through B. That is, be sure that you cannot drag point C out of the line. Otherwise, you have to delete the point and create a new point C.</p>	
5.) Hide everything except the three points. To hide the line, right click the line and uncheck <i>Show Object</i> . Do this, also, to segment AB .	
6.) Next, we rename point B to point C and vice versa. To rename point B to C , right click point B , click <i>Rename</i> and then type the new name, in this case point C , in the <i>Rename text box</i> , then click the OK button. Now, rename B (or $B1$) to C .	
	7.) Next we construct a square with side AC . Click the Regular polygon tool, then click point C and click point A .
	8.) In the <i>Points text box</i> of the Regular polygon tool, type 4 since we are going to create a square. If the position of the square is displayed the wrong way (right hand side of AC) just click the Edit menu, click the Undo button and reverse the order of the clicks.
 <p>Figure 3 - Square containing side AC</p>	

	<p>9.) With the <i>Polygon</i> tool still active, click point <i>B</i> and click point <i>C</i> to create a square with side <i>BC</i>. Similarly, click point <i>A</i>, and then click point <i>B</i> to create a square with side <i>AB</i>. After step 10, your drawing should look like the one shown below.</p>  <p>Figure 4 - Squares containing sides of right triangle ABC</p>
	<p>10.) Hide the label of the sides of the squares by right-clicking them, then unchecking <i>Show label</i>.</p>
	<p>11.) Rename the sides of the rectangle as shown below.</p>  <p>Figure 5 - Triangle ABC with side lengths a, b and c.</p>
	<p>12.) Now, let us reveal the area of the three squares. Right click the interior of square with side <i>AC</i>, then click <i>Object Properties</i> from the context menu.</p>
	<p>13.) In the <i>Basic</i> tab of the <i>Object Properties</i> window, check the <i>Show Label</i> check box and choose <i>Name and Value</i> from the drop-down list box. Do this to other squares as well. You can click <i>poly1</i>, <i>poly2</i> and <i>poly3</i> from the <i>Object list</i>. These are the squares.</p>  <p>Figure 6 - Properties of squares shown in the Object Properties window</p>
	<p>14.) Move the vertices of the triangle. What do you observe about the area of the squares?</p>
	<p>15.) You may have observed that the area of the biggest square is equal to the sum of the areas of the two smaller squares. To verify this, we can put a label in the GeoGebra window displaying the areas of the three squares.</p>
	<p>16.) Suppose the side of the two smaller squares are <i>a</i> and <i>b</i>, and the side of the biggest</p>

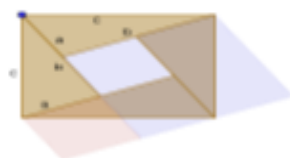
	biggest square is c , what equation can you make to express the relationship of the of the three squares?
	17.) What conjecture can you make based on your observation?

Challenge: Could you discover the Pythagorean Theorem in a different way?

Activity 2:

Open dynamic worksheet

You see here four congruent right-angled triangles with sides a , b and hypotenuse c



1. What is the area of the square built by these four triangles (including the small square in the middle)? Make a sketch of the situation and write down your solution on paper.
2. Now drag the blue point counter-clockwise as far as possible. What is the area of the small red and the big blue square? Again, make a sketch of the situation and write down your solutions on paper.
3. Do you see any connection between the areas in task (1) and (2)? Write down your conjectures.

Activity 3 Ladder against the Wall

Late at night, young Pythagoras tries to climb his 5 m long ladder up to his darling's window. The window is 4.50 m above the ground.



1. How high will he get if he places the ladder 3 m off the wall? In the drawing above, drag the point on the ground and find the solution. Sketch the solution with all its lengths (s , r , h) on paper.
2. Now, try to calculate the solution of task (1) on paper. Which lengths are given, which are sought? Do you get the same solution as in (1)?
3. At what distance of the wall should Pythagoras place his ladder in order to reach the window 4.50 meters above the ground? Find the solution again by dragging the point on the ground. Sketch the solution with all its lengths (s , r , h) on paper.
4. Now, try to calculate the solution of task (3) on paper. Which lengths are given, which are sought? Does your solution accord to your sketch made in (3)?

WORKSHEET 6

Activity 6.1

What is your opinion about the figures below?

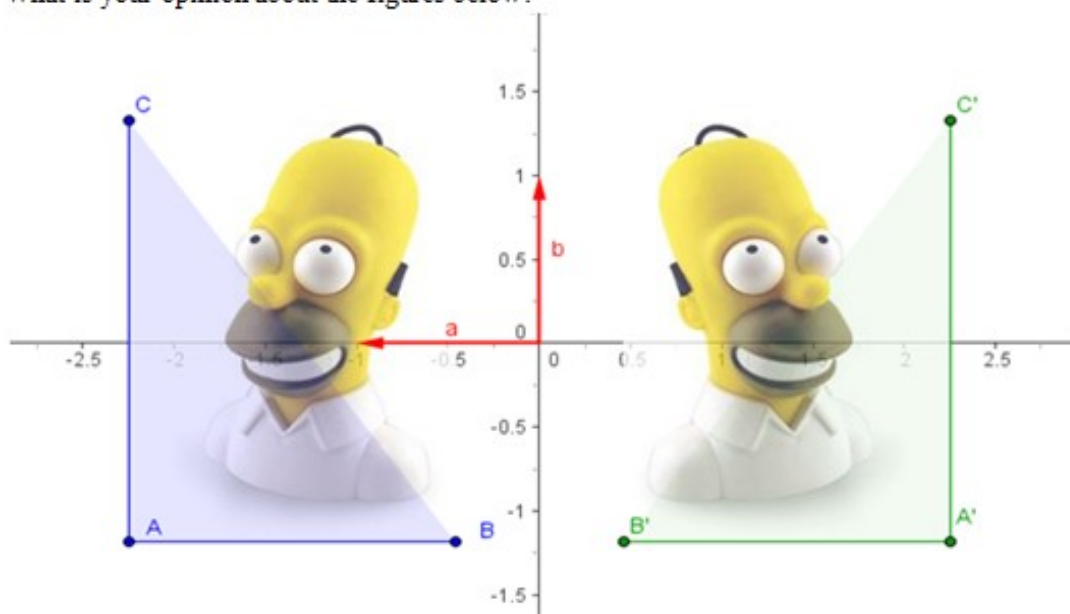


Figure:1

Activity 6.2

Find the reflection of the figure below with respect to x, y axis and origin.

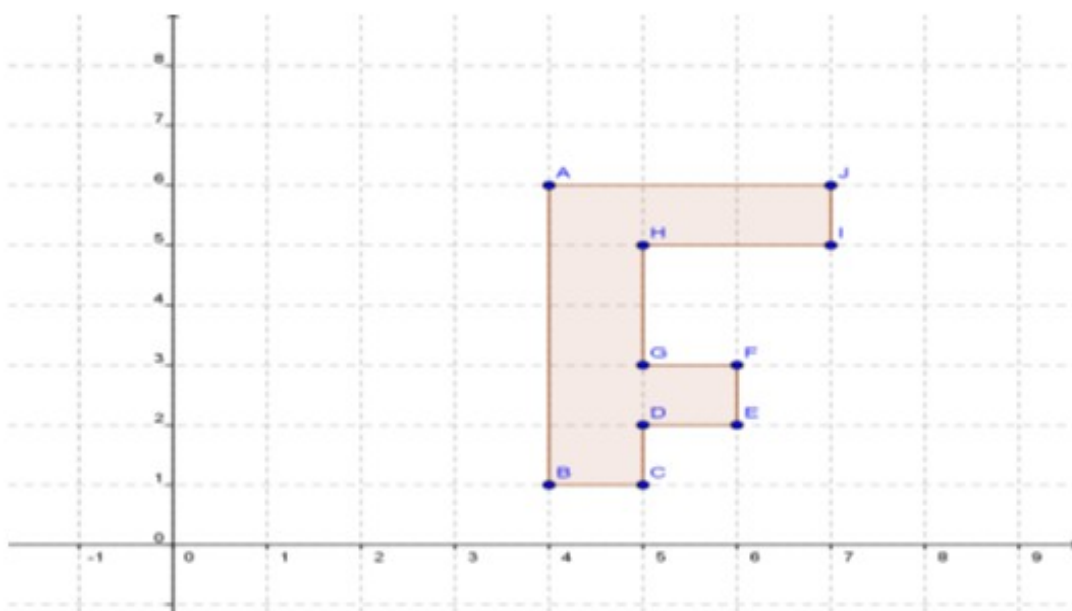
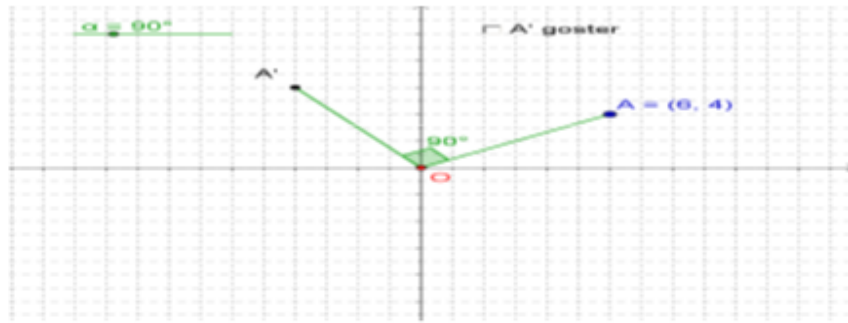


Figure:2

Activity 6.3 Find all A' points using slider.



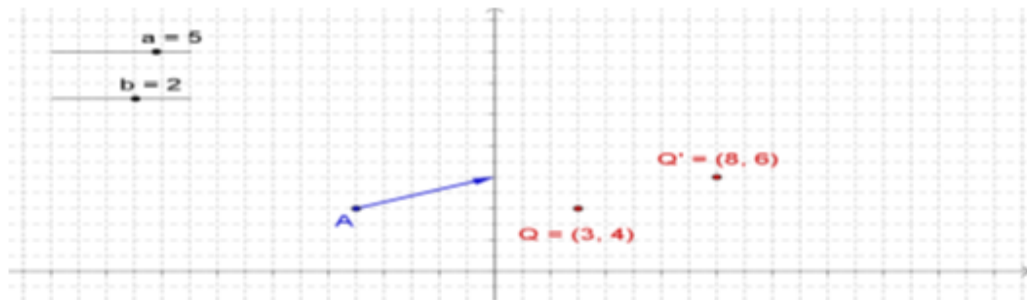
Change the A point and what do you observe?

If $\alpha = 90$ degrees, what is the value of A' ? If $\alpha = 270$ degrees, what is the value of A' ?

If $\alpha = 180$ degrees, what is the value of A' ? If $\alpha = 360$ degrees, what is the value of A' ?

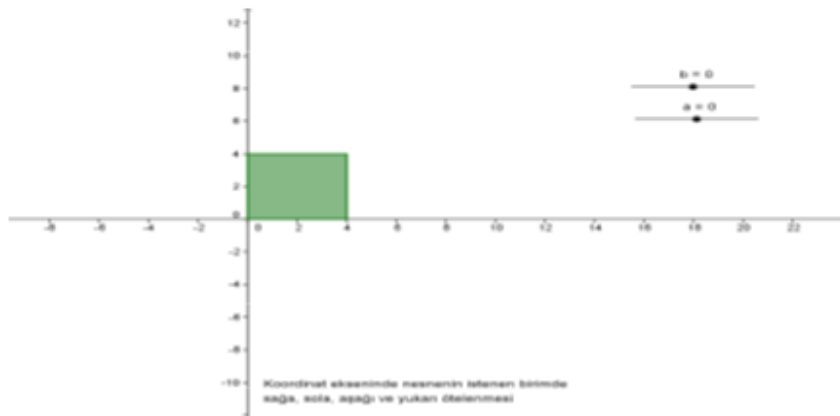
Activity 6.4

Use a and b sliders and discuss what you have observed.



Activity 6.5

Use a and b slides and discuss what you have observed

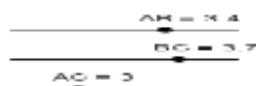


Appendix 6D: Home-based activities

Activity1 Constructing triangles given the side lengths

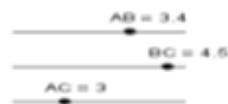
We know that we can draw a triangle in only one way if the sides are given. Let's construct a dynamic model showing how to draw this kind of triangle;

- Imagine that we will draw a triangle with A, B and C corners and to check these length of sides, let's construct slides named AB, AC and BC that change in positive values.
- Let's mark an A point in any place in the drawing area and draw an A centered circle with AB radius using "Center and radius of circle" tool.
- when radius of circle is asked, it is fine to write AB, which is the name for slide.



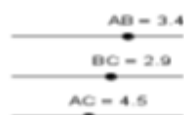
When you move the AB slide, observe that the circle gets larger and smaller.

The radius of this circle will be AB side. Mark a B point on the border of the circle and draw another circle with B centered and the radius of BC.



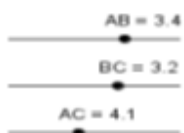
- when you determine the overlapping point of these two circles, you may think that you can construct a triangle with a C corner. However, this is a false approach. Why?

- Draw another circle with A centered and AC radius. The radius of this circle will represent AC corner.

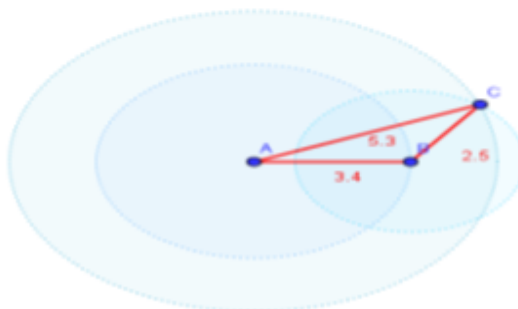
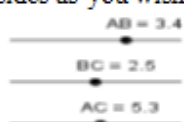


Can you tell which intersection circles will be C corner?

- Determine the overlapping corner of these two circles and construct C corner.

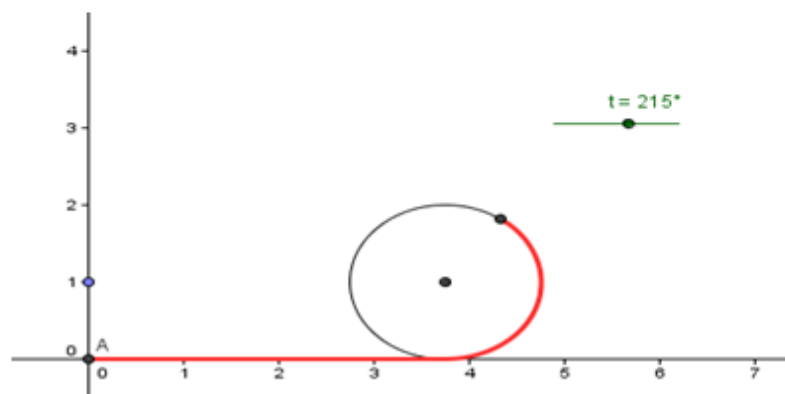


When you combined these three points, you will construct a triangle. Using slides adjust the sides as you wish. .

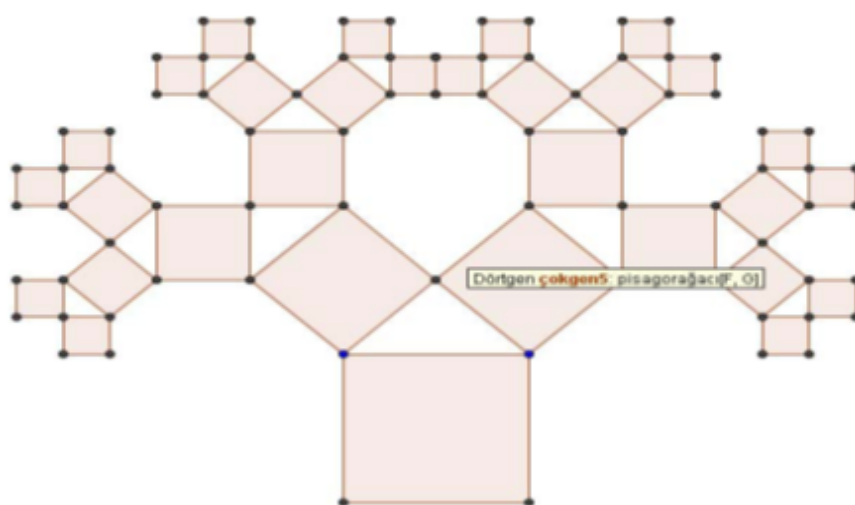


You will see that you cannot draw triangles for some side values. This is the opportunity to draw attention to what concept ?

Activity 2: The Rolling Circle



Activity 3: How to create this figure with GeoGebra(Pisagor tree)



REFERENCES

- Adams, P. (2006). Exploring social constructivism: theories and practicalities. *Education*, 34(3), 243-257.
- Aguirre, J., & Speer, N. M. (2000). Examining the relationship between beliefs and goals in teacher practice. *Journal of Mathematical Behavior*, 18(3), 327-356.
- Akkoyunlu, B. (2002). Educational Technology in Turkey: Past, Present and Future. *Educational Media International*, 39(2), 165-174.
- Aksit, N. (2007). Educational Reform in Turkey. *International Journal of Educational Development*, 27(2), 129-137.
- Al-Attas, S. M. (1990). *The Concept of Education in Islam*. Kuala Lumpur: International Institute of Islamic Thought and Civilization (ISTAC). Retrieved October 23, 2012 from <http://mef-ca.org/files/attas-text-final.pdf>.
- Alkanderi, L. (2001). *Exploring Education in Islam: Al- Ghazali's model of the master-pupil relationship applied to educational relationships within the Islamic family*. PhD Thesis, Pennsylvania State University.
- Altun, T. (2002). *Factors Influencing Teachers' Change in classroom practice due to Introduction of Information and Communications Technology (ICT) in Turkey*. Unpublished EdD Thesis, University of Nottingham.
- Andrews, P., & Hatch, G. (1999). A New Look at Secondary Teachers' Conceptions of Mathematics and its teaching. *British Educational Research Journal*, 25(2), 203-223.
- Anthony, G., & Walshaw, M. (2009). Characteristics of Effective Teaching of Mathematics: A view from the West. *Journal of Mathematics Education*, 2(2), 147-164.
- Aydin, C. H., & Tasci, D. (2005). Measuring Readiness for e-Learning: Reflections from an Emerging Country. *Educational Technology & Society*, 8(4), 244-257.
- Appleton, K. (1997). Analysis and description of students' learning during science classes using a constructivistics-based model. *Journal of Research in Science Teaching* 34(3), 303-318.
- Aysel, T. (2012). *An Exploration of the Effects of High-Stakes Examinations on the Teaching and Learning of Mathematics in Post-Primary Education in Ireland and Turkey*. PhD thesis, National University of Ireland.

REFERENCES

- Babadogan, C., & Olkun, S. (2006). Program development models and reform in Turkish primary school mathematics curriculum. *International Journal for Mathematics Teaching and Learning*. Retrieved from www.cimt.plymouth.ac.uk/journal/default.htm.
- Baki, A. (1992). Al- Khwarizmi's contributions to the science of mathematics: al kitab al jabr wa'l muqabalah. *Journal of Islamic Academy of Sciences*, 5(3), 225-228.
- Baki, A. (2000). Preparing Student Teachers to use Computers in Mathematics Classrooms through a Long-Term Pre-Service Course in Turkey. *Journal of Information Technology for Teacher Education*, 9(3), 343-462.
- Baki, A., & Celik, D. (2005). Grafik hesap makinelerinin matematik derslerine adaptasyonu ile ilgili matematik ogretmenlerinin gorusleri [Mathematics teachers' opinions about integrating graphic calculators into mathematics classrooms]. *Turkish Online Journal of Educational Technology*, 4(4), 146-162.
- Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics*, 8(1), 40-48.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it so special? *Journal of Teacher Education*, 59(5), 389-407
- Barkatsas, A. T., & Malone, J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal*, 17(2), 69-90.
- Barlow, A. T., & Cates, J. M. (2006). The Impact of Problem Posing on Elementary Teachers' Beliefs about Mathematics and Mathematics Teaching. *School Science and Mathematics*, 106(2), 64-73.
- Bassey, M. (1999). *Case Study Research in Educational Settings*. Buckingham: Open University Press.
- Becta, (2009). Primary Mathematics with ICT: A pupil's entitlement to ICT in primary mathematics. Retrieved from <http://www.bee-it.co.uk>.
- Best, J. W., & Kahn, J. V. (2003). *Research in Education* (9th ed.). Boston: Allyn and Bacon.
- Beswick, K. (2005). The Beliefs/Practice connection in broadly defined contexts. *Mathematics Education Research Journal*, 17(2), 39-68.

REFERENCES

- Beswick, K. (2011). Knowledge/beliefs and their relationship to emotion. In K. Kislenko (Ed.), *Current state of research on mathematical beliefs XVI: Proceedings of the MAVI-16 conference* (pp.43–59). Tallinn, Estonia: Institute of Mathematics and Natural Sciences, Tallinn University.
- Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics*, 79(1), 127-147.
- Bikmaz, F. (2006). New Elementary Curricula and Teachers. *Journal of Faculty of Educational Sciences*, 39(1), 97-116.
- Bishop, A. (1985). The Social Construction of Meaning: A Significant Development for Mathematics Education. *For the Learning of Mathematics*, 5(1), 24–28.
- Bodner, G., Klobuchar, M., & Geelen, D. (2001). The many forms of constructivism. *Journal of Chemical Education*, 78, 1107.
- Bogdan, R.C., & Biklen, S.K. (2007). *Qualitative research for education: An introduction to theories and methods* (5th ed.). Boston: Pearson Education.
- Booker, G. (1996). Constructing mathematical conventions formed by the abstraction and generalization of earlier ideas: The development of initial fraction ideas. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of Mathematical Learning* (pp. 381-395). Mahwah, NJ: Lawrence Erlbaum.
- Borko, H., Eisenhart, M., Brown, C., Underhill, R., Jones, D. & Agard, P. C. (1992). Learning to teach hard mathematics: Do novices and their instructors give up too easily? *Journal for Research in Mathematics Education*, 23(3), 194–222.
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101.
- Brewer, J. & Hunter, A. (1989). *Multimethod research: A synthesis of styles*. Newbury Park, CA: Sage.
- Bruner, J. (1985). Vygotsky: A historical and conceptual perspective. In J. V. Wertsch (Eds.), *Culture, communication and cognition* (pp. 21-34). Cambridge University Press.
- Bryman, A. (2004). *Social research methods* (2nd ed.). Oxford: Oxford University Press.
- Bryman, A. (2008). *Social Research methods* (3rd ed.). Oxford: Oxford University Press.

REFERENCES

- Bulut, M., & Bulut, N. (2011). Pre Service Teachers' Usage of Dynamic Mathematics Software. *The Turkish Online Journal of Educational Technology*, 10 (4), 294-299.
- Bulut, M. (2007). Curriculum reform in Turkey: A case of primary school mathematics curriculum. *Eurasia Journal of Mathematics, Science & Technology Education*, 3(3), 203-212.
- Cayir, K. (2009). Preparing Turkey for the European Union: Nationalism, national identity and 'otherness' in Turkey's new textbooks. *Journal of Intercultural Studies*, 30(1), 39-55.
- Chan, K., & Elliott, R. G. (2004). Relational analysis of personal epistemology and conceptions about teaching and learning. *Teaching and Teacher Education*, 20, 817-831.
- Chapman, O. (2002). Belief structures and inservice high school mathematics teacher growth. In G. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education* (pp. 177-194). Dordrecht: Kluwer Academic Publishing.
- Charmaz, K. (1983). The grounded theory method: An explication and interpretation. In M. R. Emerson (Eds.), *Contemporary field Research: A Collection of Readings*, 109-128. Boston: Little, Brown and Company.
- Clarke, D. M. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education*, 28(3), 278-308.
- COHE, (2010). *The Higher Education System in Turkey*. Ankara. Retrieved November 13, 2012 from <http://www.yok.gov.tr/>.
- Cohen, L., Manion L., & Morrison, K. (2007). *Research methods in education*. (6th ed.). London: Routledge.
- Confrey, J., & Kazak, S. (2006). A thirty-year reflection on constructivism in mathematics education in PME. In A. Gutiérrez and P. Boero (Eds.), *Handbook of - Research on the Psychology of Mathematics Education: Past, Present and Future* (pp. 305-345). Rotterdam: Sense Publishers.
- Cooney, T., Shealy, B., & Arvold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education*, 29(3), 306-333.

REFERENCES

- Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage.
- Creswell, J. W. (2009). *Research Design: Qualitative, quantitative and mixed methods approaches* (3rd eds.). Thousand Oaks, CA: Sage.
- Creswell, J. W., & Clark, V. L. P. (2007). *Designing and conducting mixed methods research* (1st ed.). Thousand Oaks, CA: Sage Publications.
- Cross, D. I. (2009). Alignment, cohesions, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education*, 12, 325-346.
- Crotty, M. (1998). *The Foundations of Social Research. Meaning and perspective in the research process*. London: Sage Publications Ltd.
- Cuban, L., Kilpatrick, H., & Peck, C. (2001). Paradox high access and low use of technologies in high school classrooms: Explaining an apparent. *American Educational Research Journal*, 38(4), 813-834.
- Daniels, H. (2001). *Vygotsky and Pedagogy*. London: Routledge Falmer.
- Davis, P. J., & Hersh, R. (1981). *The mathematical experience*. Boston: Birkhauser.
- Denscombe, M. (2007). *The good research guide: for small-scale social research projects*. Maidenhead, England: Open University Press.
- Diković, L. (2009). Applications GeoGebra into Teaching Some Topics of Mathematics at the College Level. *Computer Science and Information Systems*, 6 (2), 191-203.
- Donmoyer, R. (2000). Generalizability and the Single-Case Study. In R. Gomm, M. Hammersley & P. Foster (Eds.), *Case Study Method* (pp. 45-68). London: Sage Publications.
- Drier, H.S. (2001). Teaching and Learning Mathematics With Interactive Spreadsheets. *School Science and Mathematics*, 101(4), 170-179.
- Edwards, J. A., & Jones, K. (2006). Linking geometry and algebra with GeoGebra. *Mathematics Teaching, Incorporating Micromath*, 194, 28-30.
- Eggen, P., & Kauchak, D. (2007). *Educational Psychology: Windows on Classrooms* (7th ed.). New Jersey: Pearson Merrill Prentice Hall Publishing.

REFERENCES

- Ekici, G. (2005). An investigation of the attitudes of students' toward the student selection examination (SSE) through a number of variables. *Hacettepe University Journal of Education*, 28, 82–90.
- Ellis, C., &Bochner, A. (2000). Autoethnography, personal narrative, reflexivity. In N. K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (2nd ed, pp.733-768). Thousand Oaks, CA: Sage.
- Engström, L. (2004).Examples from teachers' strategies using a dynamic geometry program in upper secondary school. Paper presented at *ICME-10*. Retrieved from http://www.icmeorganisers.dk/tsg10/articulos/Lil_Engstrom.doc.
- Erez, M. M., & Yerushalmy, M. (2006). “If you can turn a rectangle into a square, you can turn a square into a rectangle . . .” Young students experience the dragging tool. *International Journal of Computers for Mathematical*, 11(3), 271-299.
- Ernest, P. (1989). The impact of Beliefs on the Teaching of Mathematics. In P. Ernest (eds.), *Mathematics Teaching: The State of the Art* (pp.249-254). London: Falmer Press.
- Ernest, P. (1991). *The philosophy of mathematics education*. London: The Falmer Press.
- Ernest, P. (1992). The nature of mathematics: Towards a social constructivist account. *Science & Education*, 1, 89-100.
- Ernest, P. (1998). *Social Constructivism as a Philosophy of Mathematics*. New York: Suny Press.
- Ertmer, P. A. (2005). Teacher pedagogical beliefs: The final frontier in our quest for technology integration? *Educational Technology Research and Development*, 53(4), 25-39.
- Forgasz, H. (2006). Factors that encourage or inhibit computer use for secondary mathematics teaching. *Journal of Computers in Mathematics and Science Teaching*, 25(1), 77–93.
- Foss, D., &Kleinsasser, R. (1996). Preservice elementary teachers' views of pedagogical and mathematical content knowledge. *Teaching and Teacher Education*, 12(4), 429-442.
- Froggatt, K.A. (2001). The analysis of qualitative data: processes and pitfalls. *Palliative Medicine*, 15(5), 433–438.

REFERENCES

- Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory: strategies for qualitative research*. Chicago, Aldine.
- Gok, F. (2006). The History and Development of Turkish Education. In M. Carlson, A. Rabo & F. Gok (eds.), *Education in 'Multicultural' Societies Turkish and Swedish Perspectives*. Istanbul: Swedish Research Institute.
- Golafshani, N., & Ross, J. A. (2006). Iranian Teachers' Professed Beliefs about Mathematics, Mathematics Teaching and Mathematics Learning. *3rd International Conference on the Teaching of Mathematics at the Undergraduate Level, Association*. Istanbul, Turkey.
- Gold, R. L. (1958). Roles in sociological field observations. *Social Forces*, 36(3), 217-223.
- Goldin, G., Rösken, B. and Törner, G. (2009). Beliefs: no longer a hidden variable in mathematical teaching and learning processes. In Maab, J. & Schlöglmann, W. (Eds.). *Beliefs and attitudes in mathematics education. New research results*. (pp. 1-18). Rotterdam: Sense Publishers.
- Goldin, G.A. (1990). Chapter 3: Epistemology, constructivism, and discovery learning in mathematics. In R.B. Davis, C.A. Maher & N. Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics. Journal for Research in Mathematics Education Monograph No: 4* (pp, 31–47). NCTM: Reston, VA.
- Goos, M., & Bennison, A. (2002). Building learning communities to support beginning teachers' use of technology. *The Annual Conference of the Australian Association for Research in Education*. Brisbane, Australia.
- Gray, D. E. (2004). *Doing Research in the Real World*. London, CA: Sage Publications.
- Green, T. F. (1971). *The activities of teaching*. New York: McGraw-Hill.
- Greenfield, W. D. (2005). Leading the teacher work group. In L.W. Hughes (Eds.), *Current Issues in School Leadership* (pp. 245-264). New Jersey: Lawrence Erlbaum.
- Grix, J. (2004). *The Foundations of Research*. London, Palgrave Macmillan.
- Grover, V. (2000). *A Tutorial on Survey Research: From Constructs to Theory*. Retrieved October 8, 2012 from <http://people.clemson.edu/~vgrover/survey/MIS-SUVY.html>
- Guba, E. G., & Lincoln, Y. S. (1989). *Fourth generation evaluation*. Newbury Park, CA: Sage.

REFERENCES

- Guba, E.G. & Lincoln, Y.S. (2005). Paradigmatic controversies, contradictions, and emerging confluences. In N.K. Denzin & Y.S. Lincoln (Eds.), *The Sage handbook of qualitative research* (3rd ed., pp. 191-215.). Thousand Oaks: Sage.
- Guskey, T. R. (1986). Staff development and the process of teacher change. *Educational Researcher*, 15(5), 5-12.
- Guskey, T. R. (2002). Professional Development and Teacher Change. *Teachers and Teaching: Theory and Practice*, 8(3/4), 381–391.
- Guven, B., Cakiroglu, U., & Akkan, Y. (2009). The gap between expectations and reality: integrating computers into mathematics classrooms. *Asia Pacific Education Review*, 10(4), 505-515.
- Halstead, J. M. (2004). An Islamic concept of education. *Comparative Education*, 40(4), 517-529.
- Handal, B., & Herrington, A. (2003). Mathematics teachers' beliefs and curriculum reform. *Mathematics Education Research Journal*, 15 (1), 59-69.
- Handal, B. (2003). Teachers' mathematical beliefs: A review. *The Mathematics Educator*, 13(2), 47–57.
- Hart, L. C. (2002). A four year follow-up study of teachers' beliefs after participating in a teacher enhancement project. In G.C. Leder, E. Pehkonen, & G. Törner, (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp.161-176). Dordrecht: Kluwer Academic Publishers.
- Healy, L., & Hoyles, C. (2001). Software Tools for Geometrical Problem Solving: Potentials and Pitfalls. *International Journal of Computers for Mathematical Learning*, 6(3), 235-256.
- Hersh, R. (1986). Some proposals for revising the philosophy of mathematics. In T. Tymoczko (Eds.), *New directions in the philosophy of mathematics* (pp.201-214). Boston: Birkhauser.
- Hills, T. (2007). Is constructivism risky? Social anxiety, classroom participation, competitive game play and constructivist preferences in teacher development. *An International Journal of Teachers' Professional Development*, 11(3), 335-352.
- Hittleman, D. R., & Simon, A. J. (2006). *Interpreting Educational Research: An introduction for consumers of research* (4th ed.). Upper Saddle River, NJ: Prentice-Hall.

REFERENCES

- Hohenwarter, M., & Preiner, J. (2007). Dynamic Mathematics with GeoGebra. *Journal for Online Mathematics and its Applications* (vol. 7). MAA, ID 1448.
- Hollingsworth, S. (1989). Prior beliefs and cognitive change in learning to teach. *American Educational Research Journal*, 26(2), 160-169.
- Hoyles, C. & Sutherland, R. (1989). *Logo Mathematics in the Classroom*. London: Routledge.
- Hoyles, C. (1992). Mathematics teaching and mathematics teachers: A meta-case study. *For the Learning of Mathematics*, 12(3), 32-44.
- Hoyles, C. and Noss, R. (1994). Dynamic geometry Environment: What's the Point? *The Mathematics Teachers*, 87(9), 716-717.
- Hoyles, C., & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Second International Handbook of Mathematics Education* (vol.1, pp. 323-349). Dordrecht: Kluwer Academic Publishers.
- Hoyles, C., Noss, R., & Sutherland, R. (1991). *Final Report of the Microworlds Project*. London: Institute of Education, University of London.
- Iqbal, M. (1932). *The Reconstruction of Religious Thought in Islam*. London: Oxford University Press.
- Isiksal, M., Koc, Y., Bulut, S., & Atay-Turhan, T. (2007). An analysis of the new elementary mathematics teacher education curriculum in Turkey. *The Mathematics Educator*, 17(2), 41-51.
- Janesick, V. (1999). A journal about journal writing as a qualitative research technique: History, issues and reflections. *Qualitative Inquiry*, 5(4), 505-524.
- Jaworski, B. (1988). Is versus seeing as: Constructivism and the mathematics classroom. In D. Pimm (Eds.), *Mathematics Teachers and Children*. London, Hodder and Stoughton.
- Jaworski, B. (1991). *Interpretations of a constructivist philosophy in mathematics teaching*. Unpublished PhD Thesis. Milton Keynes: Open University.
- Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. London: Falmer Press.

REFERENCES

- Jaworski, B. (1996). Constructivism and Teaching: The social-cultural context. Retrieved June 7, 2011 from <http://www.grout.demon.co.uk/Barbara/chreods.htm>.
- Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partnerships. *Educational Studies in Mathematics*, 54 (2/3), 249-282.
- Johnson, R. B., & Christensen, L. B. (2010). *Educational research: Quantitative, qualitative, and mixed approaches* (4th ed.). Thousand Oaks, CA: Sage.
- Joyce, B., Weil, M., & Calhoun, E. (2004). *Models of Teaching* (7th ed.). Boston: Allyn & Bacon.
- Jung, I. (2005). ICT-Pedagogy integration in teacher training: Application cases worldwide. *Educational Technology & Society*, 8(2), 94–101.
- Karakaya, S. (2004). A Comparative Study: English and Turkish Teachers' Conceptions of Their Professional Responsibility. *Educational Studies*, 30(3), 195-216.
- Karatas, I. (2011). Experiences of student mathematics-teachers in computer-based mathematics learning environment. *International Journal for Mathematics Teaching and Learning*. Retrieved from www.cimt.plymouth.ac.uk/journal/default.htm.
- Kasim, T. (2012). *Teaching and learning experiences in Malaysian higher education: A case study of a teacher education programme*. PhD thesis, Auckland University of Technology.
- Kilpatrick, J. (1987). What constructivism might be in mathematics education? In J. C. Bergeron, N. Herscovics, & C. Kieran (Eds.), *Proceedings of the Eleventh International Conference for the Psychology of Mathematics Education* (Vol. 1, pp. 3-27). Montreal.
- Koc, Y., Isiksal, M., & Bulut, S. (2007). Elementary school curriculum reform in Turkey. *International Education Journal*, 8(1), 30-39.
- Kozulin, A., & Presseisen, B. (1995). Mediated learning experience and psychological tools: Vygotsky's and Feuerstein's perspectives in a study of student learning. *Education Psychologist*, 30 (2), 67-75.
- Kucuksuleymanoglu, R. (2006). In Service Training of ELT Teachers in Turkey between 1998 - 2005. *Uludag University Educational Journal*. Retrieved May 5, 2012 from [http://home.uludag.edu.tr/users/ucmaz/PDF/egitim/htmpdf/2006-19\(2\)/M8.pdf](http://home.uludag.edu.tr/users/ucmaz/PDF/egitim/htmpdf/2006-19(2)/M8.pdf).

REFERENCES

- Kuhs, T. & Ball, D. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills and dispositions*. Retrieved March 3, 2011 from http://staff.lib.msu.edu/corby/education/Approaches_to_Teaching_Mathematics.pdf.
- Legard, R., Keegan, J., & Ward, K. (2003). In-depth interviews. In J. Ritchie & J. Lewis (Eds.), *Qualitative research practice: A guide for social research students and researchers* (pp. 138–169). Thousand Oaks, CA: Sage.
- Lerman, S. (1986). *Alternative Views of the Nature of Mathematics and their Possible Influence on the Teaching of Mathematics*. Unpublished PhD Thesis, University of London.
- Lerman, S. (1993). Can we talk about Constructivism? *Informal Proceedings of British Society for Research into Learning Mathematics Conference (BSRLM)*, 13(3), 20-23.
- Lerman, S. (2002). Situating research on mathematics teachers' beliefs and on change. In G. C. Leder, E. Pehkonen and G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 233-243). Dordrecht: Kluwer.
- Levin, T., & Wadmany, R. (2005). Changes in educational beliefs and classroom practices of teachers and students in rich technology-based classrooms. *Technology, Pedagogy, and Education*, 14(3), 281–307.
- Liljedahl, P. (2005). AHA!: The effect and affect of mathematical discovery on undergraduate mathematics students. *International Journal of Mathematical Education in Science and Technology*, 36(2-3), 219-236.
- Liljedahl, P. (2010). Noticing rapid and profound mathematics teacher change. *Journal of Mathematics Teacher Education*, 13(5), 411-423.
- Liljedahl, P., Rösken, B., & Rolka, K. (2006). Documenting changes in preservice elementary school teachers' beliefs: Attending to different aspects. Paper presented at *Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Retrieved from <http://www.pmena.org/2006/cd/PROBLEM%20SOLVING/PROBLEM%20SOLVING-0001.pdf>
- Lim, C. P. (2007). Effective integration of ICT in Singapore schools: Pedagogical and policy implications. *Educational Technology Research and Development*, 55(1), 83-116.
- Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage

REFERENCES

- Lortie, D. (1975). *Schoolteacher: A Sociological Study*. Chicago: University of Chicago Press.
- Mainali, B.R., & Key, M. B. (2012). Using dynamic geometry software GeoGebra in developing countries: A case study of impressions of mathematics teachers in Nepal. *International Journal for mathematics teaching and learning*, 1-16. Retrieved November 12, 2012 from <http://www.cimt.plymouth.ac.uk/journal/mainali.pdf>.
- Mansour, N. (2009). Science Teachers' Beliefs and Practices: Issues, Implications and Research Agenda. *International Journal of Environmental & Science Education*, 4(1), 25-48.
- McLeod, D. (1992). Research on the affect in mathematics education: A reconceptualization. In D. A. Grouws (ed.) *Handbook of Research on Mathematics Teaching and Learning*, pp. 575-596. New York: Macmillan
- Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass.
- Mewborn, D. (2001). Teachers content knowledge, teacher education, and their effects on the preparation of elementary teachers in the United States. *Mathematics Education Research Journal*, 3, 28-36.
- Mewborn, D. S. (2003). Teaching, teachers' knowledge, and their professional development. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to the Principles and Standards for School Mathematics* (pp. 45-52). Reston, VA: NCTM.
- Miles, M.B., & Huberman, A.M. (1994). *Qualitative Data Analysis*. Newbury Park, CA: Sage.
- MONE, (1995). *The Turkish Education System and Developments in Education*. Ankara. Retrieved from <http://www.meb.gov.tr/>.
- MONE, (1997, 2002). *Hizmet içi öğretmen yetistirme yönetmeliği [Turkish National Ministry Policy of In-service teachers]*. Retrieved from <http://www.meb.gov.tr/>.
- MONE, (2005a). *Introductory Handbook for Primary Classrooms' Curriculum [1-8th grade]*. Ankara. Retrieved from <http://ttkb.meb.gov.tr/>.
- MONE, (2005b). *Talim ve Terbiye Kurulu Başkanlığı, İlköğretim Matematik Dersi [6-8 Sınıflar] Öğretim Programı*. [Board of Education, Primary school mathematics curriculum 6-8th grades]. Ankara.

REFERENCES

- Nasr, S. H. (1987). *Traditional Islam in the modern world*. London and New York: Kegan Paul.
- Nasr, S. H. (1988). *İslam'da düşünce ve hayat [Islamic life and thought]*. Istanbul: İnsan Press.
- National Education Statistics, (2012). *National Education Statistics (2011-2012)*. Retrieved January 10, 2012 from <http://www.meb.gov.tr/english/indexeng.htm>.
- NCTM, (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies*, 19(4), 317–328.
- Nielsen, D. C., Barry, A. L. & Staab, P. T. (2007). Teachers' Reflections of Professional Change during a Literacy-reform Initiative. *Teaching and Teacher Education*, 24(5), 1288–1303.
- Noddings, N. (1990). Constructivism in mathematics education. In R. Davis, C. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp.7-18). Reston, VA: NCTM.
- Norton, S. J., McRobbie, C., & Cooper, T. (2002). Teachers' responses to an investigative mathematics syllabus: Their goals and practices. *Mathematics Education Research Journal*, 14(1), 37–59.
- Norton, S. J., Cooper, T., & McRobbie, C. (2000). Computer avoidance in a mathematics staff: Influences of cultural press? In M.O.J. Thomas (Eds.), *Proceedings of Technology in Mathematics Education* (pp. 211-219). Auckland, NZ.
- Oldknow, A., & Taylor, R. (2003). *Teaching Mathematics using Information and Communications Technology* (2nd ed.). London: Continuum.
- Ontario Ministry of Education, (2005). *The Ontario Curriculum Grades 1-8: Mathematics*. Retrieved November 23, 2013 from <http://www.edu.gov.on.ca/eng/>.
- Ozdas, A., Tanisli, D., Kose, N.Y., & Kilic, C. (2005). Yeni ilköğretim matematik dersi öğretim programının öğretmen görüşlerine dayalı olarak değerlendirilmesi [The evaluation of new primary school mathematics curriculum from teachers' perspective]. *Reflections on education National Congress*(pp. 239-255), 14-16 December, Ankara.
- Ozen, H. (2006). *Türkiye'de etkili matematik öğretimi için 1968-2005 yılları arasında geliştirilen ilköğretim matematik programlarının incelenmesi [Primary education*

REFERENCES

- classes the factors affect the success in mathematics education between 1968-2005*]. Master dissertation, University of Yuzuncuyil.
- Ozgun-Koca, S.A. (2000). *Using Spreadsheets in Mathematics Education*. ERIC Clearinghouse for Science, Mathematics, and Environmental Education, 1. Columbus, Ohio.
- Ozyildirim, F., Akkuş-İspir, O., Güler, V., İpek, S., & Aygün, B. (2009). Pre-service Mathematics Teachers' Views about Using Geometers' Sketchpad. *Quality of Teaching in Higher Education Conference*. Istanbul Technical University.
- Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62(1), 307–332.
- Palincsar, A. S. (1998). Social Constructivist Perspectives on Teaching and Learning. *Annual Review of Psychology*, 49, 345-75.
- Pateman, N. A., & Johnson, D. C. (1990). Curriculum and constructivism in early childhood mathematics: Sources of tension and possible resolutions. In L.P. Steffe., & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 346-356). Hillsdale, New Jersey: Lawrence Erlbaum.
- Patton, M. Q. (1990). *Qualitative evaluation and research methods* (2nd ed.). Newbury Park, CA: Sage, Publications.
- Pennington, M. C. (1995). The teacher change cycle. *TESOL Quarterly*, 29(4), 705–731.
- Pepin, B. (1999). Epistemologies, beliefs and conceptions of mathematics teaching and learning: The theory and what is manifested in mathematics teachers' work in England, France and Germany. *TNTEE Publications*, 2(1), 127-146.
- Perry, B., Howard, P., & Tracey, D. (1999). Head mathematics teachers' beliefs about the learning and teaching of mathematics. *Mathematics Education Research Journal*, 11, 39-57.
- Philippou, G., & Christou, C. (2002). A study of the mathematics teaching efficacy beliefs of primary teachers. In G.C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden Variable in Mathematics Education?* (pp. 211-231). Dordrecht: Kluwer Academic Publishers.

REFERENCES

- Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. En F. K. Lester (ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp.257-315). Charlotte, NC: NCTM.
- Phillips, D. C. (2000). An opinionated account of the constructivist landscape. In D.C. Phillips (Eds.), *Constructivism in Education* (pp. 1-18). Chicago, Illinois: The University of Chicago Press.
- Piaget, J. (1972). *The principles of genetic epistemology*. London: Routledge and Kegan Paul.
- Pierce, R., & Ball, L. (2009). Perceptions which may affect teachers' intention to use technology in secondary mathematics classes. *Educational Studies in Mathematics*, 71, 229-317.
- PISA (Organisation for Economic Co-operation and Development), (2003). Learning for Tomorrow's World First Results from PISA 2003. Retrieved November 14, 2013 from <http://www.oecd.org/dataoecd/1/60/34002216.pdf>.
- Pratt, D., & Ainley, J. (1997). The construction of meanings for geometric construction: two contrasting cases. *International Journal of Computers for Mathematical Learning*, 1(3), 293-322.
- Preiner, J. (2008). *Introducing Dynamic Mathematics Software to Mathematics Teachers: The case of GeoGebra*. PhD Thesis, University of Salzburg, Austria.
- Punch, K. F. (2009). *Introduction to research methods in education*. London: Sage Publications.
- Punch, K.F. (2005). *Introduction to Social Research: Quantitative and Qualitative Approaches* (2nd ed.). London: Sage Publications.
- Punch, M. (1994). Politics and ethics in qualitative research. In N. K. Denzin, & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 83-97). Thousand Oaks, CA: Sage.
- Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, 29(1), 4-15.
- Reynolds, D. H. (1999). What Does the Teacher Do? Constructivist Pedagogies and Prospective Teachers' Beliefs about the Role of a Teacher. *Teaching and Teacher Education*, 16, 21-32.

REFERENCES

- Richards, J. (2004). Towards reflective teaching. *The Language Teacher*, 33, 2–5.
- Richardson, V. (1990). Significant and worthwhile change in teaching practice. *Educational Researcher*, 19(7), 10-18.
- Richardson, V. (2003). Constructivist Pedagogy. *Teachers College Record*, 105(9), 1623- 1640.
- Robins, J. (2005). Beyond the bird unit. *Teacher Librarian*, 33(2), 8-19.
- Robson, C. (2011). *Real world research: a resource for users of social research methods in applied settings* (3rd ed.).Chichester: Wiley.
- Rolka, K., Rösken, B., & Liljedahl, P. (2007). The role of cognitive conflict in belief changes. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 121-128). Seoul: PME.
- Roulet, R. G. (1998). *Exemplary Mathematics teachers: Subject Conceptions and Instructional Practices*. PhD Thesis, University of Toronto.
- Ruthven, K. (2005). Expanding Current Practice in Using Dynamic Geometry to Teach about Angle Properties. *Micromath*, 21(2), 26-30.
- Ruthven, K. (2008a). The didactical challenge of technology integration in school mathematics. *Proceedings of the 17th Annual Meeting of the Mathematics Education Section of the Portuguese Society for Educational Sciences* [EIEM XVII].
- Ruthven, K. (2008b). The interpretative flexibility, instrumental evolution and institutional adoption of mathematical software in educational practice: the examples of computer algebra and dynamic geometry. *Journal of Educational Computing Research*, 39(4), 379-394.
- Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: A study of the interpretive flexibility of educational software in classroom practice. *Computer and Education*, 51(1), 297-317.
- Sangwin, C. (2007). A brief review of GeoGebra: dynamic mathematics. *MSOR Connections*, 7(2), 36.
- Scheffler, I. (1965). *Conditions of knowledge: An introduction to epistemology and education*. Glenview: Scott, Foresman and Company.

REFERENCES

- Schram, T. (2003). *Conceptualizing Qualitative Inquiry: Mindwork for Fieldwork in Education and the Social Sciences*. Upper Saddle River, NJ: Merrill/Prentice Hall.
- Scott, A. L. (2005). Pre-service teachers' experiences and the influences on their intentions for teaching primary school mathematics. *Mathematics Education Research Journal*, 17(3), 62-90.
- Shahvarani, A., & Savizi, B. (2007). Analyzing Some Iranian- High School Teachers' Beliefs on Mathematics, Mathematics Learning and Mathematics Teaching. *Journal of Environmental & Science Education*, 2(2), 54-59.
- Sigel, I. E. (1985). A conceptual analysis of beliefs. In I. E. Sigel (Eds.), *Parental belief systems: The psychological consequences for children* (pp.345-371). Hillsdale, NJ: Erlbaum.
- Silverman, D. (2000). *Doing qualitative research: A practical handbook*. Thousand Oaks, CA: Sage.
- Skemp, R.R. (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teaching*, 77, 20-26.
- Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education*, 4(1), 3-28.
- Speer, M. N. (2005). Issues of methods and theory in the study of mathematics teachers' professed and attributed beliefs. *Educational Studies in Mathematics*, 58, 361-391.
- Stake, R. E. (1994). Case studies. In N. K. Denzin, & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 236-247). Thousand Oaks, CA: Sage.
- Steele, D. F. (2001). The interfacing of preservice and inservice experiences of reform-based teaching: A longitudinal study. *Journal of Mathematics Teacher Education*, 4, 139-172.
- Steffe, L.P., Thompson, P. W., & Glasersfeld, E. V. (2000). Teaching experiment methodology: underlying principles and essential elements. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 267-306). Mahwah, NJ: Lawrence Erlbaum Associates.
- Stipek, D., Givvin, K., Salmon, J., & MacGyvers, V. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, 17(2), 213-226.

REFERENCES

- Stuyf, R. (2002). Scaffolding as a Teaching Strategy. *Adolescent Learning and Development*. Retrieved from <http://condor.admin.ccny.cuny.edu/~group4/>.
- Sulaiman, N. A. (2011). *Exploring Kuwaiti mathematics student-teachers' beliefs toward using Logo and mathematics education*. PhD Thesis, Nottingham Trend University.
- Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, 75, 58-70.
- Swan, M., & Swain, J. (2010). The impact of a professional development programme on the practices and beliefs of numeracy teachers. *Journal of Further and Higher Education*, 34(2), 165-177.
- Temiz, N. (2005). İlköğretim 4. Sınıf Matematik Dersi Yeni Öğretim Programının Yansımaları [Reflections of new 4th grade mathematics curriculum]. *XIV National Educational Science Congress*, 28-30, Denizli, Turkey.
- Temizoz, Y., & Ozgun-koca, S.A. (2008). The instructional methods that mathematics teachers' use and their perceptions on the discovery approach. *Education and Science Journal*, 33(149), 89-103.
- Tharp, M. L., Fitzsimmons, J. A., & Ayers, R. L. B. (1997). Negotiating a technological shift: Teacher perception of the implementation of graphing calculators. *The Journal of Computers in Mathematics and Science*, 16(4), 551-575.
- TIMSS (Trends in International Mathematics and Science Study), (1999). International Mathematics Report Findings from IEA's Repeat of the TIMSS at the Eighth Grade. Retrieved on 21 December, 2012, from <http://nces.ed.gov/timss/>.
- Thomas, M., Tyrrell, J., & Bullock, J. (1996). Using computers in the mathematics classroom: The role of the teacher. *Mathematics Education Research Journal*, 8(1), 38-57.
- Thompson, A. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15(2), 105-127.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of research. In D. A. Grouws (Eds.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

REFERENCES

- Toptas, V. (2006). İlkogretim Matematik dersi ogretim programinin uygulanmasinda sinif ogretmenlerinin karsilastiklari sorunlarla ilgili görüşleri (classroom teachers' views about challenges and difficulties when they utilising new primary mathematics curriculum. *National primary teacher Congress (Vol, 1. pp. 277-285)*. Ankara.
- Turan, S. (2000). John Dewey's Report of 1924 and His Recommendations on the Turkish Educational System Revisited. *History of Education*, 29(6), 543-555.
- Turner, L. J. (1999). *Use of the Logo programming language as software to support constructivist learning within a post-primary school mathematics environment: a Techne action research study*. PhD Thesis, Monash University.
- Ucar, Z., & Demirsoy, N. (2010). Tension between old and new: Mathematics teachers' beliefs and practices. *Hacettepe University Journal of Education*, 39, 321-332.
- von Glasersfeld, E. (1993). Questions and answers about radical constructivism. In K. Tobin (Eds.), *The practice of constructivism in science education* (pp. 23-38). Hillsdale, NJ: Lawrence Erlbaum Associates.
- von Glasersfeld, E. (1987). *The construction of knowledge: Contributions to conceptual semantics*. Intersystems Publications: Salinas CA.
- von Glasersfeld, E. (1996). Introduction: Aspects of constructivism. In C. Fosnot (Eds.), *Constructivism: Theory, perspectives, and practice*, (pp.3-7). New York: Teachers College Press.
- Vygotsky, L. S. (1978). *Mind in society: the development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Vygotsky, L. S. (1986). *Thought and language* (A. Kozulin, Trans.). Cambridge, MA: MIT Press.
- WanMohd, N. D. (1989). *The concept of knowledge in Islam*. Mansell, London, and N.Y.
- Wheatley, G.H. (1991). Constructivist perspectives on science and mathematics learning. *Science education*, 75 (1), 9-21.
- Williams, M. (2001). The Status of Generalization in Interpretive Research. In T. May (Eds.) *Qualitative Research: An International Guide to Issues in Practice* (pp.125-43). London: Sage.

REFERENCES

- Wilson, B.G. (1996). Introduction: What is a constructivist learning environment? In B.G. Wilson (Eds.), *Constructivist learning environments. Case studies in instructional design* (pp. 3-10). Englewood Cliffs, NJ: Educational Technology Publications.
- Wilson, M., & Cooney, T. (2002). Mathematics teacher change and development. The role of beliefs. In G. C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 127-147). Dordrecht: Kluwer Academic Publishers.
- Woolfolk, A., Hughes, M., & Walkup, V. (2008). *Psychology in education*. Harlow, UK: Longman.
- Yadav, A., & Koehler, M. (2007). The role of epistemological beliefs in preservice teachers' interpretation of video cases of early-grade literacy instruction. *Journal of Technology and Teacher Education*, 15, 335-361.
- Yildirim, A. (2003). Instructional Planning in a Centralized School System: Lessons of a Study among Primary School Teachers in Turkey. *International Review of Education*, 49(5), 525-543.
- Yin, R. K. (1994). *Case study research: Design and methods* (2nd ed.). Newbury Park, CA: Sage.
- Yin, R. K. (2006). Case study methods. In J. L. Green, G. Camilli, & P. B. Elmore (Eds.), *Handbook of Complementary Methods in Educational Research* (pp. 111-122). Mahwah, NJ: Lawrence Erlbaum Associates.
- Yin, R. K. (2009). *Case study research: Design and methods* (4th ed.). Thousand Oaks, CA: Sage.
- Zakaria, E., & Maat, S.M. (2012). Mathematics Teachers' Beliefs and Teaching Practices. *Journal of Mathematics and Statistics*, 8(2), 191-194.
- Zhao, Y., Hueyshan, T., & Mishra, P. (2001). Technology, Teaching and Learning: Whose computer is it? *Journal of Adolescent and Adult Literacy*, 44(4), 348-355.
- Zhao, Y., Pugh, K., Sheldon, S., & Byers, J. (2002). Conditions for classroom technology innovations: executive Summary. *Teachers College Record*, 104(3), 482-515.