

NUMERICAL MODELLING OF THE THERMODYNAMICS OF LAKE BAIKAL AND THE
POPULATION DYNAMICS OF THE BAIKALIAN DIATOM AULACOSEIRA BAICALENSIS

by

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A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
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Numerical Modelling of The Thermodynamics of Lake Baikal and The Population Dynamics of The Baikalian Diatom

Aulacoseira baicalensis

Karen L. Veal

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Abstract

The prediction of the past climate from diatom ratios in Lake Baikal sediments will necessitate the understanding of the relationships between the diatom life-cycles and the physical processes occurring in the lake. To this end a diatom model is coupled to a three-dimensional general circulation model of Lake Baikal for the first time. The formulation of the diatom model and its parameterizations are described. The model is able to simulate the suspension of *Aulacoseira baicalensis* organisms in the photic zone by vertical dynamical processes. Populations are able to bloom under snow-free ice but are barely maintained under snow-covered ice.

A new three-dimensional model of Lake Baikal that can simulate the lake through the Summer and Autumn has been developed. The development of the model is documented with particular emphasis placed on the numerical scheme used for the advection of momentum and temperature, the thermal forcing of the model and the parameterization of the vertical eddy viscosity and vertical eddy diffusivity coefficients. The model simulations of Lake Baikal thermodynamics during the under-ice warming period in the Spring are in good agreement with the model of Lawrence *et al.* (2002). Model simulations of the temperature fields during the rest of the year are in good agreement with available observations. The model current velocities are similar to those reported by Shimaraev *et al.* (1994) but the lack of available wind field data prevents the simulation of a realistic circulation. This work highlights the need for further field studies of Lake Baikal and its diatom populations.

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Chapter 1

Introduction

Lake Baikal is the largest freshwater lake on Earth and, with a volume of $23,615 \text{ km}^3$ (INTAS Project 99-1669 Team, 2002), holds 20% of the planet's liquid freshwater. The lake was designated a World Heritage Site by UNESCO in 1994 and supports over 1500 aquatic species with 80% of them endemic (World Heritage Convention, 1996). It is the deepest lake on Earth, with a maximum depth of 1642 m, and lies in the Baikal Rift zone, in south-eastern Siberia. The lake's bottom is 1,164 m below sea-level making it the deepest continental depression on Earth. Lake Baikal is also the Earth's oldest lake. It was formed around 25 million years ago (most of the planet's other lakes are less than 30,000 years old) and has a sediment depth of over 5 km, making it of great interest to paleo-climatologists.

With its high biodiversity, large number of endemic species, great age and size Lake Baikal is of interest to scientists of many disciplines. This work concentrates on modelling the thermodynamics of the lake and the population dynamics of one of its endemic species the diatom *A. baicalensis*. Section 1.1 describes the lake, highlighting those points that are of interest and importance to scientists and, in particular, to those modelling the lake's thermodynamics and biological systems. In section 1.2 the current state of numerical modelling of Lake Baikal is described and the objectives of this study are stated. Finally, section 1.3 outlines the structure of the rest of this thesis.

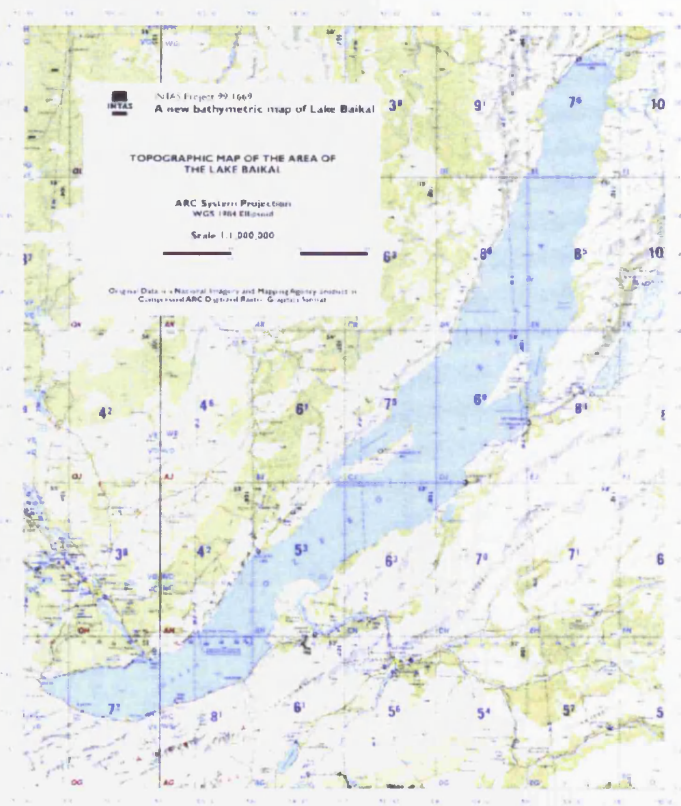
1.1 Observations of Lake Baikal

The lake was first studied by Russian scientists in the second half of the nineteenth century (Kozhova and Izmet'eva, 1998), although the earliest soundings, taken for mining purposes, took place at the end of the eighteenth century (INTAS Project 99-1669 Team, 2002). A permanent research station was established in 1920, at Bolshiye Kotye, 23 km north of the lake's only outflow, the Angara River. The Baikal

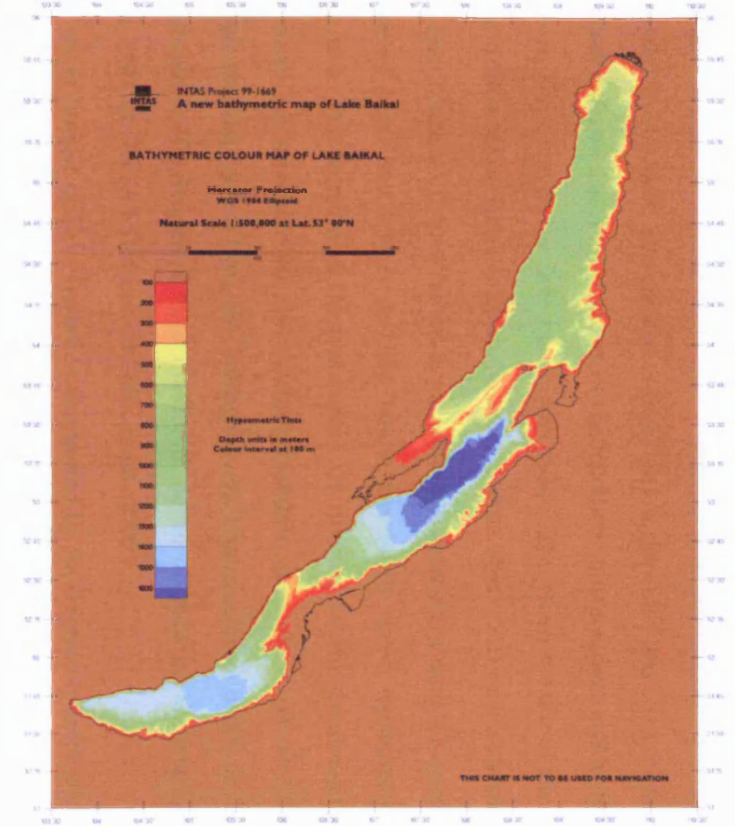
International Center for Ecological Research (BICER) was formed in 1991 by the Siberian Branch of the Russian Academy of Sciences, the Belgian Royal Institute for the Natural Sciences, the Royal Society, the Japanese Association for the Baikal International Research Programs, the Swiss Federal Institute of Technology and the University of South Carolina. An historic overview of the Russian literature on the physical limnology of Lake Baikal was published by BICER in 1994 (Shimaraev *et al.*, 1994) and the observations described in this section are taken from this review unless otherwise specified.

1.1.1 Geography and morphology

Lake Baikal is located in central Asia, in the Russian Federation and lies just to the north of Mongolia. The lake stretches in an arc between 51°28' and 55°47' latitude and 103°43' and 109°58' longitude (Figure 1.1(a)). The lake is 636 km long and has a maximum width of 81 km in the Central basin and is at its narrowest, a width of 27 km, at the Selenga delta. For most of the west coast, the lake is bordered by the Baikalskii and Primorskii mountain ranges. In the east, a coastal plain separates the shore from the Barguzinskii, Ulan-Burgasy, and Khamar-Daban mountain ranges.



(a)



(b)

Figure 1.1: Maps of Lake Baikal (INTAS Project 99-1669 Team, 2002). (a) A topographic map of the Lake Baikal region: the major river sources are the Selenga ($52^{\circ}15' \text{ N } 106^{\circ}30' \text{ E}$), the Barguzin ($53^{\circ}30' \text{ N } 109^{\circ}00' \text{ E}$), and the Upper Angara ($55^{\circ}45' \text{ N } 109^{\circ}30' \text{ E}$); the only outflow is the Angara ($51^{\circ}45' \text{ N } 105^{\circ}00' \text{ E}$). (b) A bathymetric map of Lake Baikal: the separation of the lake, by underwater ridges, into the North, Central and South Basins can be seen.

The lake lies in the Baikal Rift, a rift which is still tectonically active. The lake's cross-section is asymmetric. In the west, the coastal region, with a depth of less than 20 m, is narrow and ranges from 20 m to 200 m in width. The gradient of the lake-bed then steepens suddenly, often to angles of 60–80°. In the east, the slope of the lake-bed is less steep. The steep nature of the lake's basin sides results in less than a fifth of the lake's bottom lying at depths above 250 m (Kozhova and Izmet'seva, 1998).

The lake consists of three basins separated by subsurface ridges, (Figure 1.1(b)). These sills lie 300–400 m below the water surface so that, below this depth, direct exchange of water between the basins is prevented. The North and Central basins are divided by the Academician Ridge. The Central and South basins are separated by the Selenga Shallows, an underwater sill formed as a result of tectonic subsidence and deposition from the Selenga river. The maximum depth of the North basin is 904 m, the Central basin is the deepest with a maximum depth of 1642 m, and the South basin has a maximum depth of 1461 m (INTAS Project 99-1669 Team, 2002).

The water budget of Lake Baikal is dominated by its riverine inflows and outflow. Of the total water influx, 83% originates from river sources while just 17% comes from precipitation; similarly 81% of the outflow drains through the Angara, the only river outflow, while 19% is lost through evaporation. Over 300 rivers drain a catchment area of 540,000 km², delivering around 60 km³ a year into Lake Baikal. The average annual rate of riverine influx is 1936 m³·s⁻¹. The major river sources (see Figure 1.1(a)) are: the Upper Angara (264 m³·s⁻¹), which enters the lake on the north coast of the North Basin; the Barguzin (141 m³·s⁻¹), which flows into the northeast of the Central Basin; and the Selenga (982 m³·s⁻¹), which joins the lake at the junction of the Central and South Basins. The sole sink, the Angara, drains the lake at the north-west coast of the South Basin.

1.1.2 Climate

The Baikal region has a continental climate: the winters are long, cold and dry while the summers are short and relatively hot and wet. At Ulan-Ude, 75 km east of Lake Baikal's South Basin, the mean temperature in January, the coldest month, is –25 to –27 °C. In July, the warmest month in the Baikal region, the mean temperature at Ulan-Ude is 18 to 20 °C (Galazy, 1993). Lake Baikal is such a large body of water that it modifies the climate in regions close to the lake. Close to the shore of the South Basin the mean January air temperature is –17 to –19 °C and the mean July temperature is 14 to 15 °C (Galazy, 1993). Shimaraev *et al.* (1994) state that temperatures over the lake are 6–8 °C cooler in Summer and 10–15 °C warmer in Winter compared with air temperatures away from the lake in neighbouring regions and that the amplitude of annual temperature variation close to the lake is smaller

than that in the neighbouring areas — 30-38 °C as opposed to 41-50 °C. Temperatures peak away from the lake in July whereas next to the lake peak temperatures occur in August. This delay in reaching extremes of temperature and the reduction in annual temperature variation are typical of sea coasts.

Air temperatures over the lake vary with latitude: observations throughout the twentieth century show average Winter temperatures over the North, Central, and South Basins to be respectively -17.4 °C, -15.3 °C, and -14.5 °C; average Summer temperatures are, again respectively, 8.4 °C, 9.6 °C, and 10.5 °C.

The prevailing winds over the Baikal region are west north-westerlies (Galazy, 1993). But these are modified by the mountain ranges of the Baikal Depression so that winds that blow along or across the lake predominate. During the warm months (May-September) the winds are predominantly weak and unstable. In the Autumn and early Winter (October-December), however, there is a greater incidence of north-westerly winds and to a lesser extent north-easterly winds. Storms with wind speeds greater than 20 m s⁻¹ occur. The lake's large mass of water also affects the wind field: in the cold part of the year, the thermal effect of the water mass produces a localized low pressure area over the lake, which results in the enhancement of off-shore winds; in the warm part of the year there is a pressure maximum above the lake and on-shore winds result (Galazy, 1993; Kozhova and Izmet'seva, 1998; Shimaraev *et al.*, 1994).

The surface of Lake Baikal freezes for 4-6 months of the year. Freezing in the Autumn starts in the shallow bays during October and November. Most of the North Basin has frozen by early December and freezing of the South Basin is complete by late December or early January. The ice reaches a maximum thickness, of usually 70-90 cm, in March/April. The maximum thickness of the ice varies from year to year between 50 and 110 cm. The thickest ice is found in the North Basin due to the cooler climate in this part of the lake. The snow and ice start to melt under the influence of solar heating from late March to early April. The break-up of ice is assisted by wind action from late April to early May in first the South Basin and then the Central Basin. In the North Basin the thaw starts approximately one month later than in the South Basin. The South and Central Basins are free of ice by the middle of May and the whole lake is clear by early June. There is a large annual variation in the timing of freezing and ice-breakup and the maximum ice thickness. Long-term observations in the South Basin, at Listvenichnyi Bay, show year to year differences of up to 40 days in the timing of freezing and up to 30 days in the timing of ice break-up. Maximum ice thickness varied by up to 55 cm. These observations also reveal a long-term shortening of the period of ice-cover, especially during the last three decades of the twentieth century, which coincides with the warming of the Northern hemisphere cold season. This shortening of the period of ice-cover has implications for cold-adapted phytoplankton species such as

A. baicalensis that are endemic to Lake Baikal. The period available to these species for reproduction is also shortened and their numbers can be expected to decrease.

1.1.3 Temperature observations and dimictic turnover

In contrast to the ocean, Lake Baikal, a freshwater lake, has a low salinity (Hohmann *et al.* (1997) found ionic salinities of 94-96 mg·kg⁻¹). Freshwater is peculiar in that it reaches maximum density at a temperature above its freezing point: the expansion coefficient of water is negative at temperatures below this temperature of maximum density (T_{md}) and positive at temperatures above the T_{md} . At the lake surface, *ie.* at atmospheric pressure, the T_{md} has a value of 3.98 °C but this value decreases with increasing pressure and hence, with increasing depth. The decrease with depth is approximately linear at typical lake temperatures, at ~ 0.0021 °C per metre.

Lake Baikal is meromictic (only part of the water column overturns and mixes) and dimictic (the mixing occurs twice each year). Figure 1.2 shows the annual temperature cycle in the South Basin. The data plotted in Figure 1.2 are given by Shimaraev *et al.* (1994) and consist of averages over several data sets from different years for the South Basin. The surface temperature passes through the T_{md} twice a year: first during warming in the Spring and then again during cooling in the Autumn.

In March the lake is still covered with ice. The temperature profile exhibits a negative stratification in the upper layer. The temperature immediately under the ice is near 0 °C; the temperature then increases with depth until it reaches the T_{md} at a depth of 150-250 m. At this depth there is a 'knee' in the profile where the temperature reaches a maximum, commonly referred to as the mesothermal maximum. Below the mesothermal maximum the temperature decreases slowly with depth and the deep waters have a weak positive stratification.

In the South Basin, the change from negative to positive heat balance occurs in March and this marks the onset of the Spring warming. Solar radiation passing through the ice warms the surface water resulting in its temperature increasing towards the T_{md} . The subsequent increase in density causes it to sink but only until it meets surrounding water of a similar temperature and density. This process of free-convection results in the development of a surface mixed layer, the epilimnion. The epilimnion is separated from the deeper waters or hypolimnion by a steep temperature gradient, the thermocline.

The depth of the epilimnion increases as it continues to warm during the Spring (see Figure 1.3). Eventually, in June, the temperature and depth of the epilimnion has reached the mesothermal maximum. Subsequently convection is limited to depths where the water temperature is below the T_{md} . Once the surface temperature reaches the T_{md} free-convection stops. At this stage the temperature profile of the

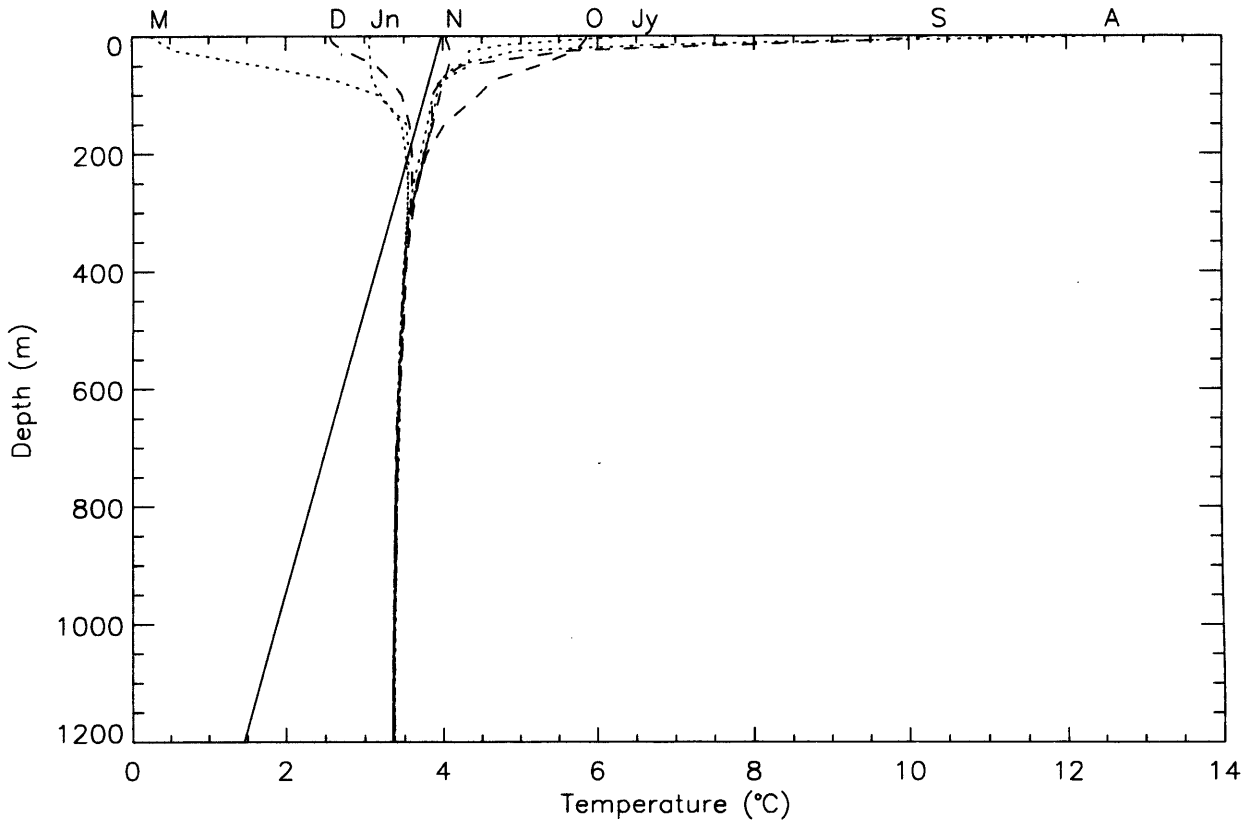


Figure 1.2: Annual temperature cycle in the South Basin of Lake Baikal. Profiles for March (M), June (Jn), July (Jy), August (A), September (S), October (O), November (N), and December (D) are plotted. Dotted lines are used for profiles during the Spring warming (March-August) and dashed lines are used for the Autumn cooling (September-December). The solid line indicates the change of T_{md} with depth. Data taken from Shimaraev *et al.* (1994).

upper 250 m resembles the profile of the T_{md} . The expansion coefficient and the stability are close to zero so that conditions favour vertical mixing.

Throughout the Summer months, continued heating sees the initiation and subsequent intensification of a positively stratified epilimnion. The epilimnion is shallow with depths limited to 5-30 m. The maximum surface temperature of the pelagic zone is reached in August with a temperature of 10-12 °C. The thermocline, at 10-40 m, has a gradient of 0.2-0.4 °C · m⁻¹. Between 60-250 m the temperature profile follows the T_{md} . The stability of the water column at these depths is weak and the vertical exchange between the upper layers and the deep zone can be considerably increased by even moderate storms.

In the Autumn, the heat flux into the lake becomes negative and the lake begins to cool. The surface waters are again increasing in density and mixing by convection occurs along with wind mixing. The

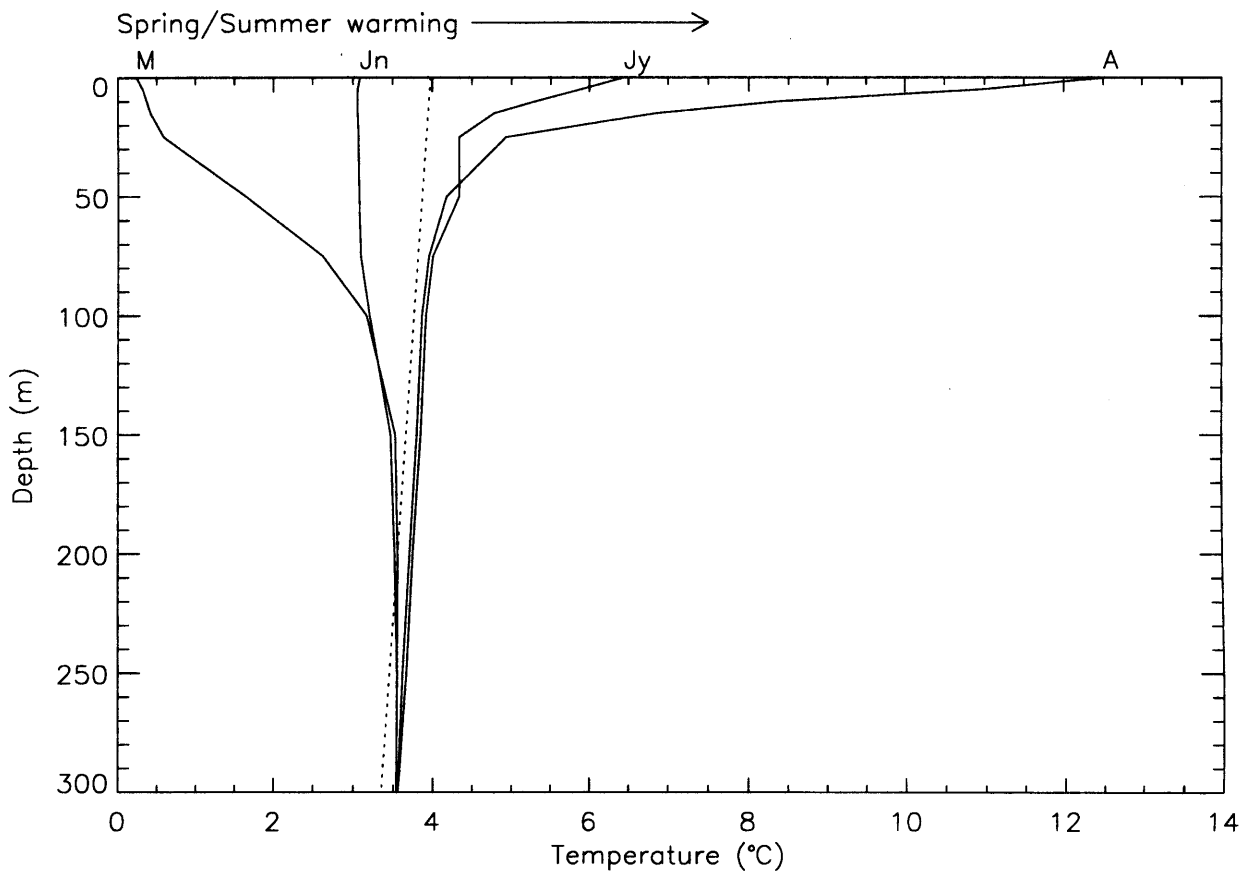


Figure 1.3: Temperature profiles in the upper 300 m of the South Basin during the Spring/Summer warming for the months of March (M), June (Jn), July (Jy), and August (A). Data taken from Shimaraev *et al.* (1994).

epilimnion deepens to 100-150 m in early November (see Figure 1.4). The water below the epilimnion warms because of vertical exchange with the upper layer.

As surface temperatures cool below the T_{md} , free-convection stops. Vertical exchange is now driven only by wind mixing. The surface continues to cool and the stability of the upper layer increases until it can no longer be overcome by the wind stress. The water column is now stable. The temperature and depth of the mesothermal maximum vary from year to year and depend on the extent of wind and convective mixing. After freezing there is a period of under-ice cooling before the onset of the next Spring warming.

The upper layer of Lake Baikal thus mixes twice a year. However, below 250 m, the hypolimnion is weakly stable and has a permanent weak stratification with a temperature gradient of $0.02\text{ }^{\circ}\text{C}/100\text{m}$. The implications of this permanent stratification are explored next.

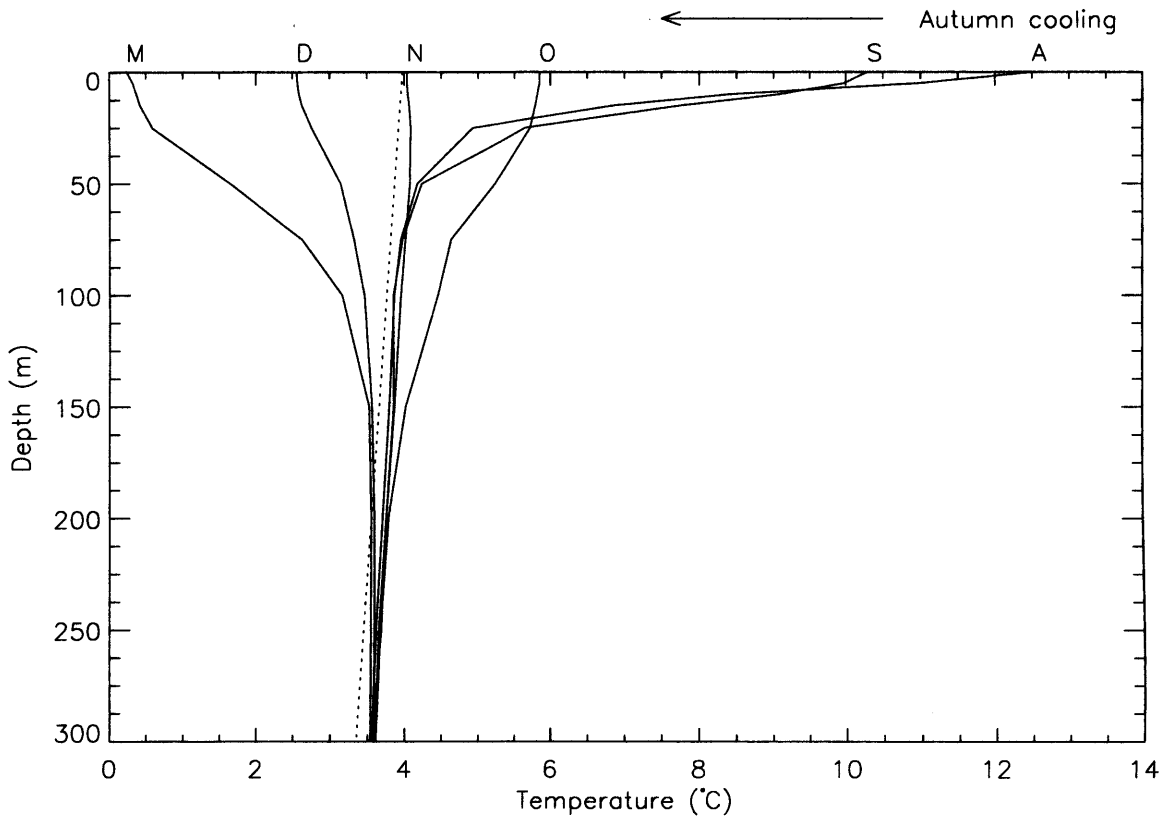


Figure 1.4: Temperature profiles in the upper 300 m of the South Basin during the Autumn cooling for the months of August (A), September (S), October (O), November (N), and December (D). A March (M) profile indicates the state of the lake at the end of the Winter. Data taken from Shimaraev *et al.* (1994).

1.1.4 Deep water renewal and thermobaric instability

Although the seasonal temperature development in Lake Baikal would appear to suggest that only the surface 250 m layer mixes, the bottom water has an oxygen concentration of over $320 \mu\text{mol} \cdot \text{kg}^{-1}$ (Weiss *et al.*, 1991) representing a saturation of over 80%, enough to support life and indicating that the bottom waters are well ventilated. During the Spring warming, water at temperatures below the T_{md} can only sink by free-convection until it meets water of the same temperature, because the warmer water below has a temperature closer to the T_{md} and is therefore denser. Colder water in the upper layer is thus prevented from freely mixing with the hypolimnion. However, if the water is forced to depths where its temperature is equal to the T_{md} (solid arrow in Figure 1.5), it is now denser than the surrounding water and can sink further, by free-convection, until it meets water of the same temperature and density (dotted arrow in Figure 1.5). This phenomenon is known as the thermobaric instability and was proposed

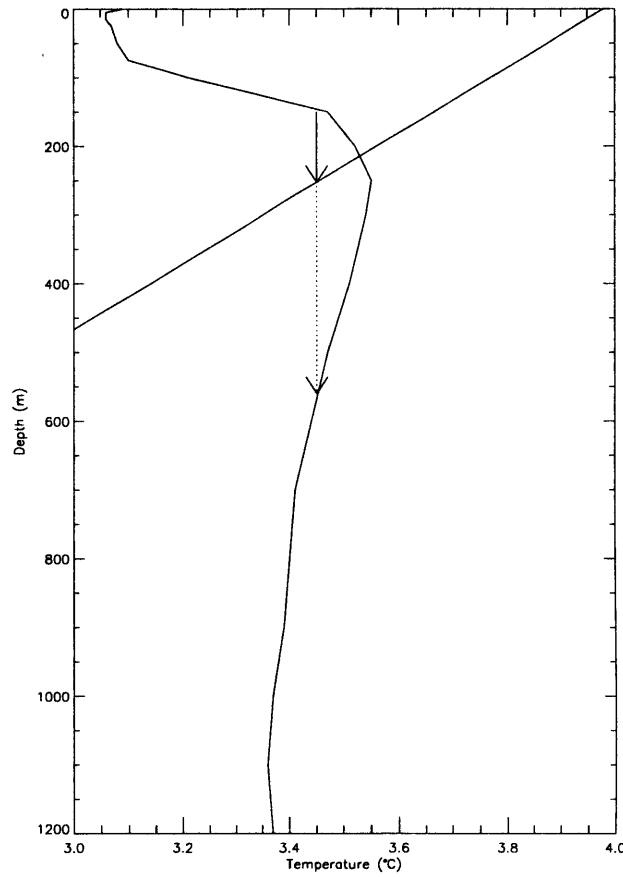


Figure 1.5: A June temperature profile from the South Basin. The thermobaric instability is illustrated. If water at 3.45 °C is forced down below the T_{md} line (solid arrow) then the water will sink by free convection (dotted arrow) to a depth of around 550 m.

as a mechanism for ventilation of the deep waters by Weiss *et al.* (1991) and Killworth *et al.* (1996). Hohmann *et al.* (1997) pointed out a problem with this argument: water temperature near the bottom of the Central Basin is around 3.1 °C requiring the water at the mesothermal maximum to be forced downward by approximately 350 m in order to cause the water at 3.1 °C to sink freely to the bottom. The authors state that thermocline displacements of more than one quarter of the required distance have rarely been measured, even after severe storms.

Shimaraev *et al.* (1993) suggested the thermal bar as a trigger for deep-water formation. In Spring, the coastal waters warm faster than the open water. This leads to a situation where the coastal water is warmer than T_{md} but the open water is cooler than T_{md} and a density front is formed. At this front, if water from the two systems mix then denser water always results, leading to a sinking jet. The authors argue a jet of cold water, originating from the open-water hypolimnion, penetrates to depths of many hundreds of metres. Peeters *et al.* (1996a) analysed the dynamics of the thermal bar and concluded

that the formation of a thermal bar can only explain vertical mixing in the top 200-300 m of the water column. This is because the T_{md} decreases with depth and the water of the jet, which forms at the thermal bar, has a temperature close to 4 °C and soon meets colder water which is denser at this depth.

Hohmann *et al.* (1997) observed that although salinity variations are very small in Lake Baikal the thermal expansion coefficient in the region of T_{md} is also small and small variations in the salinity may have a large impact on the vertical density structure. The authors found two areas where deep-water exchange takes place due to waters of different properties meeting horizontally. The first is at the Selenga Delta where relatively saline waters form a plume that sinks to the deepest part of the Central Basin in April and early May. At the Academician Ridge the small salinity difference between the North and Central Basins causes deep-water formation. However, Wuest *et al.* (2005) observed cold water intrusions in the South Basin. The cooling of bottom waters was not accompanied by an increase in ion or particle content. The authors concluded that these were not intrusions of river water and suggested that the intrusions were the result of Ekman pumping due to winds blowing parallel to the coast: winds blowing eastward, parallel to the south coast of the South Basin, produce surface currents which are turned shoreward by the Coriolis effect, the subsequent convergence at the shore produces down-welling of colder surface water.

The phenomenon of deep water renewal in Lake Baikal has been the subject of much scientific interest. The phenomenon appears to be localized and episodic but the relative importance of the proposed mechanisms remains, as yet, unclear.

1.1.5 Diatoms and their use as a tool for reconstruction of the paleoclimate

In marine and freshwater systems, the principal primary producers are the phytoplankton (the primary producers are those organisms that convert solar energy to chemical energy *i.e.* photosynthesize). The pelagic phytoplankton of Lake Baikal is particularly rich in endemic species. One of these is the diatom *Aulacoseira baicalensis*. Diatoms are single-celled plants distinguished by their intricately constructed silicate cell walls or frustules. *A. baicalensis* is non-motile (it has no means of self-propulsion) and has a sinking speed of 2.5-2.8 m/day (Gibson, 2001). It must, therefore, rely on water currents to keep it suspended in the photic zone. The photic zone is the depth of water in which light levels are sufficiently high for photosynthesis to occur and the bottom limit of the zone is usually defined as the depth to which 1% of the incident radiation reaches. In deep-water regions of Lake Baikal this can be as deep as 200 m, but under 80 cm of clear ice this reduces to 25-40 m, and under ice with a fresh covering of snow, to as little as 5 m (Kozhova and Izmet'seva, 1998). The species is well adapted to Baikal conditions.

It is able to reproduce in the conditions of low light and low temperature that are present under ice in early Spring and has a maximum growth rate at 2-3 °C (Richardson *et al.*, 2000).

Observations of the phytoplankton have necessarily been restricted to the ice-free months. Popovskaya (2000) reported the results of long-term (over 40 years) investigations into the Baikal phytoplankton. There is large inter-annual variation in phytoplankton abundance. The major bloom occurs in the Spring with a less pronounced bloom occurring in the Autumn. The Spring bloom is dominated by the diatom *Aulacoseira baicalensis*. The timing and extent of the Spring bloom vary from year to year and spatially through out the lake and is thought to be related to the timing of the ice-melting. The extent of the bloom also varies from year to year with abundant blooms occurring every 2-4 years although during the 1980s this extended to approximately every 8 years (Popovskaya, 2000). The abundance of the bloom appears to be related to the extent of the snow cover on the lake and the proportion of clear ice present during the preceding Winter. Mackay *et al.* (2003) have shown that the numbers of *A. baicalensis* valves in surface sediments are related to the extent of the snow cover on the lake in the year of deposition. Poor snow cover and much clear ice allow increased solar penetration of the water below, and this causes increased convective mixing which aids re-suspension of diatoms (Mackay *et al.*, 1998). The sensitivity of *A. baicalensis* reproduction to snow cover and the fact that another diatom, *Cyclotella minuta*, has its main growth period in September/October have led Bangs *et al.* (2000) to suggest that the relative abundance of these two diatom species in sediments may be a useful indicator of climate variability.

As has already been stated, Lake Baikal is the oldest lake on Earth and lies in the planet's deepest continental depression. It has never experienced glaciation and so, potentially, has an uninterrupted sediment record comparable to marine systems. Changes in the biogenic silica content of Lake Baikal sediments have been linked to orbital climate forcing and glacial cycles (Colman *et al.*, 1995). In turn, variability in Lake Baikal ice formation, duration and thickness are significantly related to Northern Hemisphere climate variability, notably the Scandinavian and Arctic Oscillation patterns (Todd and Mackay, 2003). Several studies have used the diatom record to reconstruct the paleoclimate over various timescales (Grachev *et al.*, 1998; Mackay *et al.*, 1998; Bangs *et al.*, 2000). However, the relationship between climate and diatom productivity is not yet understood. Laboratory and field observations suggest a number of factors may be implicated including light penetration and water temperature (Richardson *et al.*, 2000; Jewson *et al.*, 2004), ice formation, water column mixing (Huisman and Sommeijer, 2002; Ghosal *et al.*, 2000; Kelley, 1997), length of ice-free season, snow-cover (Mackay *et al.*, 2003), turbidity and nutrient cycling (Goldman *et al.*, 1996). A recent study (Mackay *et al.*, 2006) found the relative abundance of *A. baicalensis* in the diatom assemblage of sediments to be correlated with the depth of snow on the ice; the relative abundance increased with increasing snow depth, presumably due to

A. baicalensis having an increased tolerance of low irradiances over other species. The authors found the relative abundance of *C. minuta* to be increased in areas of increased ice duration *i.e.* longer ice duration in the Spring promotes high abundances of *C. minuta* the following Autumn. Diatoms are also subject to taphonomic change after death so that differences between the lake and sediment diatom assemblages may occur. The sediments may also be affected by turbidity currents resulting in alterations in the sediment diatom record. Before past climatic conditions can be quantitatively reconstructed from the diatom record, the relationships between diatom productivity and the physical processes occurring in the lake must be investigated.

1.2 Numerical Modelling of Lakes

The difficulties in making observations of such a large lake, especially one that is covered with ice for several months of the year, are obvious. Satellite observations can give high coverage of the surface but cannot provide measurements of the lake interior. Shipboard observations are confined to the ice-free period and both shipboard and buoy measurements are, of necessity, restricted in their horizontal coverage. Computational models should, therefore, be a useful tool in the investigation of the lake's thermodynamics. Computational models can also be used as a laboratory: individual processes can be isolated and varied individually which is not possible in the real lake. For instance, a model lake can be forced with different wind fields and the prevailing climatic conditions can be varied.

Modelling Lake Baikal poses a number of challenges. Lakes have highly irregular coastlines whose simulation require high model grid resolutions. The vertical grid resolution must also be high enough to resolve the temperature stratification. In the case of Lake Baikal, such a large lake requires a large model domain and, with a reasonable grid resolution, large computational resources. Most authors have attempted to solve this problem by modelling a restricted domain designed for the particular problem they were investigating. Killworth *et al.* (1996) used a longitudinal, two-dimensional, vertical section in order to investigate the possible role of wind forcing in deep water renewal. A transverse, vertical section was used by Botte and Kay (2002) to simulate the circulation produced by a strong wind. The transverse section was again used by Tsvetova (1995) and by Botte and Kay (2000) to simulate the passage of the Spring thermal bar and by Holland (2001) and Holland *et al.* (2001), to investigate the riverine thermal bar. Walker and Watts (1995) used a simple rectangular box model with small grid spacing to simulate a single plume and deep water formation. Kelley (1997) utilized a model two-dimensional eddy flow field with simple geometry to simulate a convection eddy in an ice-covered lake. The author used this model to show that convection fields under the ice in Lake Baikal are capable of

retaining *A. baicalensis* cells within the surface mixed layer long enough to enable them to reproduce. Other authors (Huisman, 1999; Huisman *et al.*, 1999; Ghosal *et al.*, 2000; Huisman and Sommeijer, 2002) have used one-dimensional models of the water column in order to investigate the effect of turbulent diffusion on plankton populations.

Transport processes in lakes depend on the interaction of surface wind fields, the lake thermodynamics, topography and seasonal thermal conditions and therefore require a three-dimensional domain for their simulation. The only three-dimensional model discussed in the literature is that of Lawrence *et al.* (2002). The authors described an n-layer model of the surface 250 m of Lake Baikal. A realistic coastline is simulated. The model runs during the period of ice-melting (middle of March to June) in 1997 and incorporates a parameterization of the surface shortwave radiation flux that simulates the attenuation of radiation by the snow and ice fields. This enables the model to simulate the differential heating that produces horizontal pressure gradients, and hence horizontal currents, while the lake is isolated by the ice from any wind forcing. This model simulates the vertical mixing field in the lake during the Spring. As this is the period when *A. baicalensis* reproduces the model is suitable for the simulation of the *A. baicalensis* population. The coupling of the general circulation model to a diatom model is described in Chapter 2 of this thesis.

Unfortunately the model of Lawrence *et al.* (2002) is not stable under wind forcing and cannot be used to simulate Lake Baikal during the ice-free period. A new model has therefore been developed. This Lake Baikal Model (LBM) runs during the ice-melting period and through the Summer and Autumn until the end of the year. The rest of this thesis documents the development of this model.

Cheng *et al.* (1976), Hodges *et al.* (2000), and Wang and Hutter (2001) have highlighted the problem of numerical diffusion which can cause the gradient of the thermocline to become less steep with time which, in turn, affects the amplitude and period of internal waves. Numerical diffusion is inherent in the finite differencing of the advection term in the momentum equations. The use of different finite difference approximations is explored in Chapter 3.

One of the difficulties in simulating geophysical fluid flows is the lack of environmental data. There is often sparse or inaccurate data concerning the forcing fields. In particular, the thermal exchange at the surface and the wind fields. Both fields are important in the vertical mixing dynamics. The wind fields also dictate the circulation pattern and local topography can produce gradients that directly drive surface gyres. The effects of different forcing fields on the LBM are described in Chapters 4, 5 and 6.

1.3 Thesis Overview

The aim of this work is to produce a documented, three-dimensional thermodynamic model of Lake Baikal that runs during the period of diatom productivity *i.e.* from the beginning of the ice-melting period until the end of the year. The model will be capable of running on a Linux workstation and will be suitable for coupling to a diatom population model so that, in the future, the dynamics of populations of *A. baicalensis* and *C. minuta* can be simulated.

Chapter 2 describes the investigation of *A. baicalensis* productivity during the period of ice-melting by coupling a model of *A. baicalensis* population dynamics to the three-dimensional general circulation model of Lawrence *et al.* (2002). The first part of the chapter describes the general circulation model and some results predicted by this thermodynamic model. Next, a diatom population dynamics model is described. The effect of various vertical dynamical processes on the diatom numbers and distribution are explored. The model prediction of the horizontal distribution of diatoms during the ice-melting period and its relationship to the ice and snow cover are described.

The general circulation model of Lawrence *et al.* (2002) is not stable under forcing by winds. In Chapter 3 a new general circulation model is described. This model is stable under wind forcing and runs during the ice-melting period, through the Summer and into the Autumn cooling period. The numerical testing of the new model is documented.

The thermal forcing of the LBM is investigated in Chapter 4. Heat exchange between the lake and its surroundings varies both temporally and spatially. Different methods of parameterizing the heat exchange processes are explored.

Chapter 5 discusses the parameterization of vertical turbulent mixing and the difficulties involved in choosing a method suitable for both under-ice conditions when the vertical current shear is very small and ice-free, wind-forced conditions when the vertical current shear is much greater.

Some results of the new thermodynamic model simulation are given in chapter 6. The new model's simulation of Lake Baikal during the ice-melting period are compared with those of the model of Lawrence *et al.* (2002). Results from the new model's simulation of the Summer and a simulated storm event are discussed.

The conclusions of this work are summarized in chapter 7. This chapter also contains suggestions for future work.

Chapter 2

Coupling A Simple Diatom Model to A General Circulation Model

In this chapter we investigate the numerical modelling of the spatial and temporal distribution of the diatom *Aulacoseira baicalensis* in Lake Baikal. The period of interest runs from the middle of March to the end of June; this period includes the melting of the snow cover and the break-up of the ice layer and coincides with the time of maximal reproduction for *A. baicalensis*.

A simple diatom model is coupled to a three-dimensional general circulation model of Lake Baikal. The diatom model consists of a tracer equation which includes a source term for diatom reproduction and a loss term for diatom mortality and predation by zooplankton. A further term is included in order to model the sinking of diatoms due to gravity. The method chosen to model the lake's thermodynamics is the three-dimensional general circulation model described by Lawrence *et al.* (2002), henceforth this model will be referred to as the LHLC model. The model runs during the period of ice-melting in the Spring and so is particularly suitable for our purpose.

The next section of this chapter describes the LHLC general circulation model and examines the development of the temperature and velocity fields calculated by the model for the Spring period. In section two the diatom model is described and special emphasis is given to the source term and the differencing of the sinking term. The results of some model runs are presented in sections three and four. Here the model is used to examine the effect of the lake dynamics calculated by the LHLC model on the diatom abundances. Finally the last section is given over to a discussion of the results.

2.1 The LHLC General Circulation Model

The LHLC model (Lawrence *et al.*, 2002) is a three-dimensional general circulation model of Lake Baikal. The model is a $N+1/2$ layer model that has been developed from Anderson and McCreary's 2 layer model of the equatorial Pacific Ocean (Anderson and McCreary, 1985). Only the surface 250 m are modelled; the deeper water is treated as an inert abyss. The model has ten layers which are allowed to vary in thickness with time. The hydrostatic approximation is made. This is justified as the vertical currents are three orders of magnitude smaller than the horizontal currents allowing the vertical acceleration to be neglected. The Boussinesq approximation is assumed so that differences in density from the mean value are neglected unless multiplied by the gravitational acceleration.

The model runs from the middle of March to the beginning of July which encompasses the period of melting and breakup of the ice. The albedo and extinction values of the lake's snow and ice cover have been derived from satellite data and these are used to calculate the spatially and temporally varying solar radiation flux passing through the lake's surface (Le Core, 1998). In the Spring, the lake is indirectly stratified with temperatures below the temperature of maximum density. Heating at the surface causes an increase in density of the surface water above the density of the water below and convection occurs. Solar heating is responsible for the vertical mixing and subsequent increase in depth of the surface mixed layer. This vertical mixing, as indicated earlier (§ 1.1.5), effects the ability of diatoms to photosynthesize so it is important that any thermodynamic model accurately reproduce the vertical mixing that exists in the real lake. We shall see that the LHLC model reproduces the development of the mixed layer well and so should, potentially, be useful in this context.

2.1.1 The model equations

The model solves the Reynolds averaged Navier-Stokes equations on a three-dimensional grid, which has dimensions 140×100 cells in the horizontal plane and 10 layers vertically. The 10 layers extend from the lake surface to a depth of 250 m and are allowed to vary in thickness with time. The equilibrium layer thickness varies monotonically from 5 m for the top layer to 50 m for the bottom layer. The vertical grid spacing was chosen in order to adequately resolve the vertical temperature profile during the period of stratification. The lake below 250 m is treated as an inert abyss. The horizontal grid cell is 0.05° longitude by 0.05° latitude which corresponds to approximately $3.3 \text{ km} \times 5.6 \text{ km}$.

The horizontal current velocity, $\mathbf{u}$, has zonal and meridional components, u and v , which are calculated from the equation for the conservation of momentum. This is formulated so as to take account of the variation in layer thickness, h , with time.

$$\frac{\partial h\mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)h\mathbf{u} = -h\frac{\nabla p}{\rho} - 2\mathbf{\Omega} \times \mathbf{u}h + \nabla \cdot (\boldsymbol{\nu} \cdot \nabla h\mathbf{u}) , \quad (2.1)$$

where, $\mathbf{\Omega}$ is the angular velocity of the earth, p is the pressure, ρ is the mean density, and $\boldsymbol{\nu}$ is the eddy viscosity.

Then, given the Boussinesq approximation, the equation for the conservation of mass describes the change in layer height with time:

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0 . \quad (2.2)$$

In order to prevent the complete emptying of a model layer a process of entrainment of water from adjacent layers is allowed. The volume of water that moves between adjacent layers is determined by the thicknesses of the relevant layers.

The development of the temperature field is described by the temperature equation:

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla)T = \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla T) + Q , \quad (2.3)$$

where T is the temperature, Q is a heat source, and $\boldsymbol{\kappa}$ is the eddy diffusion coefficient. The heat source, Q , is the energy absorbed from the solar radiation. The amount of solar radiation, $I(z)$, reaching a given depth, z , depends on the optical properties of the materials it passes through and is calculated, here, using the Beer-Lambert law:

$$I(z) = I_0 (1 - \alpha) e^{-\mu_s t_s} e^{-\mu_i t_i} e^{-\mu_w z} . \quad (2.4)$$

The surface albedo, α , and the extinction coefficients of the ice and snow, μ_i and μ_s , are inputs into the model that vary with time and surface location so that they mimic the evolution of the ice cover on the real lake (Le Core, 1998). The ice thickness, t_i , reduces with time linearly at a rate of 2/3 cm per day. The thickness of the snow cover, t_s , has a value of 2.5 cm. Lake Baikal water is known for its clarity, and its extinction coefficient, μ_w , has a typical value of 0.2 m⁻¹ (Shimaraev *et al.*, 1994).

The change in temperature due to solar radiative heating is given by

$$\frac{\partial T}{\partial t} = \frac{E}{\rho c_P h} \quad (2.5)$$

where E is the solar energy input per unit volume per second, c_P is the specific heat of water, and h is the thickness of the layer. Values for the solar radiation flux are calculated each day by linearly interpolating between whole lake, monthly averages given by Shimaraev *et al.* (1994).

A quadratic equation of state for freshwater is used to calculate the density of water at a point, $\rho(x, y, z, t)$. The density is a function of the the water temperature, $T(x, y, z, t)$ and the temperature of

maximum density, $T_{md}(z)$.

$$\rho(x, y, z, t) = \rho_0 \left[1 - \gamma \{T(x, y, z, t) - T_{md}(z)\}^2 \right]. \quad (2.6)$$

Here ρ_0 is the maximum density, $\gamma = 8 \times 10^{-6} \text{ }^\circ\text{C}^{-2}$ is an expansion coefficient and the temperature of maximum density at a depth of z (m) is $T_{md}(z) = -0.0021z + 3.98 \text{ }^\circ\text{C}$.

The pressure field is diagnosed from the temperature field using the equation of state, equation (2.6), and the hydrostatic equilibrium condition:

$$\frac{\partial p}{\partial z} = \rho \mathbf{g}, \quad (2.7)$$

where $\mathbf{g}$ is the acceleration due to gravity.

At the end of each time-step the vertical density profile is assessed. Adjacent layers are considered unstable if the density of the upper layer is greater than the density of the lower layer. If this is the case the two layers are overturned. The whole water column is treated in like manner until a stable density profile is attained. More details of this process are given in the description of the plankton model, but essentially the process acts to perturb the pressure away from the hydrostatic condition.

Finite-difference approximations for the equations are obtained using the second order leap-frog scheme (centred-in-time, centred-in-space) except for the diffusion terms where the discrete form is found using a forward-in-time, centred-in-space scheme which is also second order. The size of the time-step is limited by the Courant-Friedrich-Levy condition and is 20 s. Descriptions of the aforementioned numerical methods can be found in most basic texts on the subject; see, for example, Ferziger and Peric (2002), Press *et al.* (1992).

2.1.2 The eddy viscosity and eddy diffusion coefficients

The eddy viscosity and eddy diffusion coefficients have horizontal and vertical components. The horizontal components are constants. The value for the eddy viscosity, $\nu_h = 1.5 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$ is chosen for reasons of numerical stability and compares with that chosen by Pacanowski and Philander (1981). The horizontal component of the eddy diffusivity, $\kappa_h = 0.1 \text{ m}^2 \cdot \text{s}^{-1}$, is relatively low and allows for a reduction in horizontal turbulent mixing under ice.

Vertical turbulent mixing depends on the density profile and the method of parameterization has been chosen to model mixing under ice when the density gradient weakens and the lake is isolated from any potential wind forcing. The vertical components are, thus, functions of the vertical density gradient:

$$\nu_v = \frac{\nu_0 \kappa_{prof}}{\left(1 + \beta \left| \frac{\partial \rho}{\partial z} \right| \right)} + \nu_b. \quad (2.8)$$

$$\kappa_v = \frac{\kappa_0 \kappa_{prof}}{\left(1 + \beta \left| \frac{\partial \rho}{\partial z} \right| \right)} + \kappa_b . \quad (2.9)$$

This formulation combines elements of both fixed profile mixing and the method described by Pacanowski and Philander (1981) where the eddy viscosity and eddy diffusion coefficients are functions of the Richardson number.

$$\nu_v = \nu_b + \frac{\nu_0}{(1 + \alpha Ri)^n} , \quad (2.10)$$

where the Richardson number is

$$Ri = \frac{(\mathbf{g}/\rho_0) \partial \rho / \partial z}{(\partial u / \partial z)^2 + (\partial v / \partial z)^2} . \quad (2.11)$$

The vertical eddy viscosity, ν_v , is equal to the sum of a background value, ν_b , and a maximum eddy viscosity, ν_0 , that is modified by a term including the Richardson number. The constants, α and n , and the parameters, ν_b and ν_0 , can be adjusted to fit observations of the flow under consideration.

When the density gradient is high the water column is more stable and the turbulent mixing tends towards a background level. In (2.8) and (2.9) the background levels, ν_b and κ_b , take values of $10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ and $10^{-5} \text{ m}^2 \cdot \text{s}^{-1}$ respectively. As the density gradient decreases the coefficients tend towards a maximum value of $\nu_0 = \kappa_0 = 50 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$, modified by a fixed dimensionless profile, κ_{prof} . The fixed profile has a value of 1.0 at the surface and decreases monotonically to 0.1 in the model's bottom layer. In analogy with the method of Pacanowski and Philander (1981) the parameter, β , can be considered to provide the value of an average current shear with magnitude $|\partial u / \partial z| = \sqrt{g / \rho_0 \beta}$. It takes the value of $5 \times 10^4 \text{ m}^4 \cdot \text{kg}^{-1}$, equivalent to a current shear of order 10^{-4} s^{-1} . The values of the above parameters and the fixed profile are those used by Lawrence *et al.* (2002).

2.1.3 Boundary and initial conditions

The model temperature field is initialized with a vertical profile interpolated from data observed in the South Basin on 13th March 1997 (Jewson, 1997). The initial current field is zero.

The boundary conditions include a no-slip condition at the walls and a free-slip condition at the bottom-abyssal interface. The model does not allow for wind forcing so that there is no momentum transfer across the surface: a free-slip condition. There is no heat transfer through the walls or across the abyssal interface. The heat transfer across the surface is set equal to the solar radiation flux; while covered with ice the lake is considered insulated from all other processes that may transfer heat across the surface.

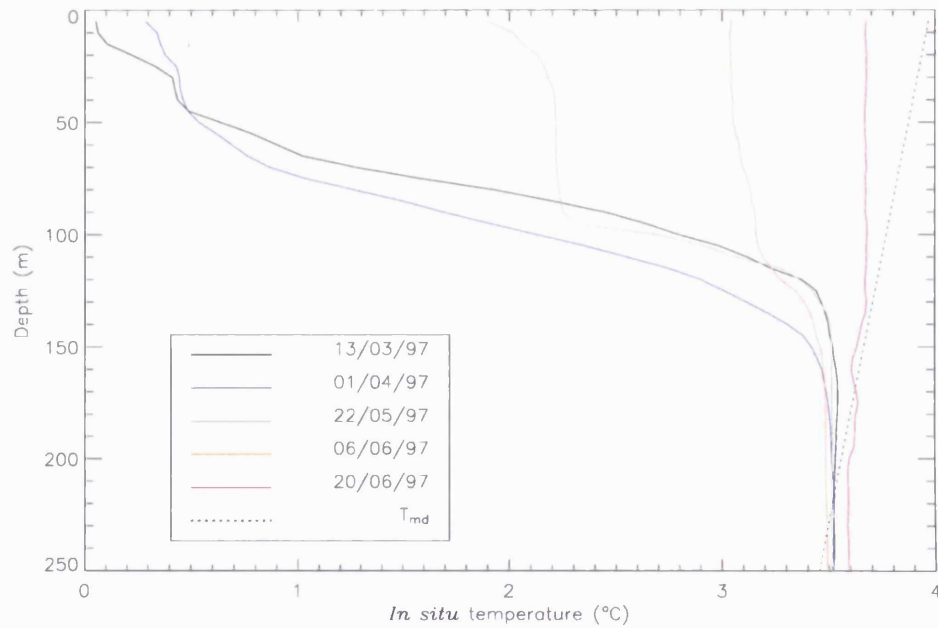


Figure 2.1: Observations of water temperature in the South Basin during the Spring, 1997. Data plotted from Jewson (1997).

2.1.4 Temperature and current fields predicted by the LHLC Model

Some *in situ* measurements of water temperature in the middle of the South Basin during the Spring of 1997, Jewson (1997), are shown in Figure 2.1. The corresponding model results are shown in Figure 2.2.

The *in situ* data chart the development from an inverse stratification to a homogeneous temperature profile and the associated deepening of the surface mixed layer. Early in the Spring the epilimnion is cold (less than 0.05 °C) and around 20-50 m deep with a steep temperature gradient down to 150 m depth where the temperature is ~ 3.5 °C. As the surface layers are warmed, they become denser and sink until they meet water of similar temperature and density. In this manner the upper mixed layer progressively deepens until by the beginning of June its temperature is ~ 3.0 °C and its depth is ~ 120 m. This progression is reproduced well in the model results (Figure 2.2) although there is less detail in the model profiles because the vertical resolution of the model is lower than the resolution of the observations. Figure 2.3 shows the state of the ice-field on 14 March 1997. Figure 2.3(a) shows the type of ice cover present on the 14 March 1997. The type of ice cover is derived from satellite data (Le Core, 1998). Figures 2.3(b) and 2.3(c) show the albedo and extinction coefficient of the particular ice-type present. Areas where the ice-cover has a high albedo and extinction coefficient will receive a lower solar radiation flux than areas with lower albedo and extinction coefficient. As a result these areas will warm

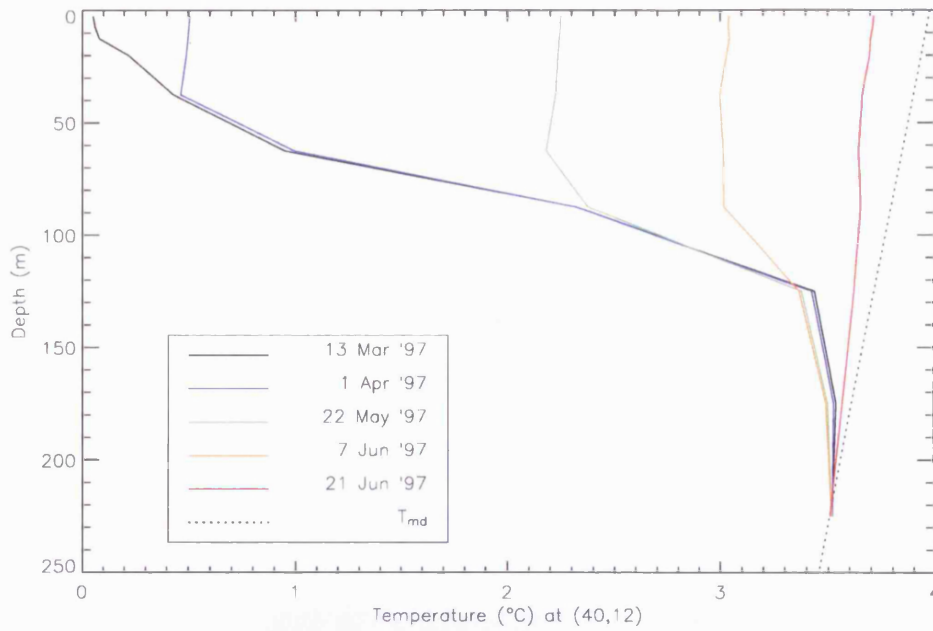


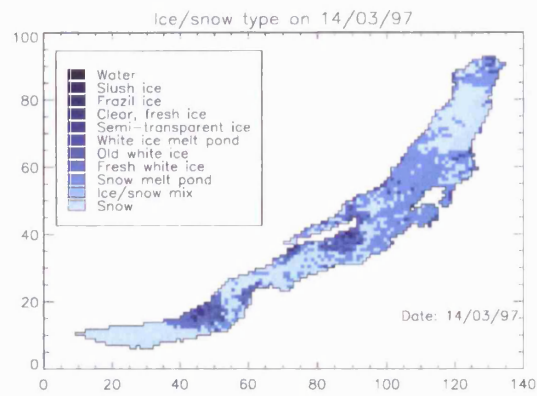
Figure 2.2: Vertical temperature profiles produced by the model for the same location and dates as the *in situ* data shown in Figure 2.1.

more slowly. Figure 2.4(a) shows the temperature and current field in the surface layer on date 14 May 1997. Comparison with Figure 2.3(a) confirms that the regions with clear ice have warmed to higher temperatures than those with snow-covered ice. The differential heating that is a consequence of the different types of ice-cover occurring on the lake results in pressure gradients that drive currents parallel to the temperature contours.

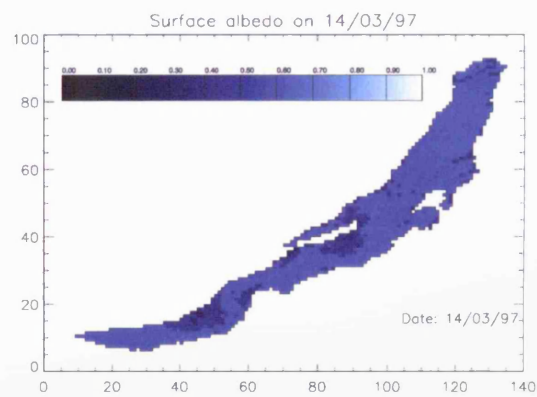
Due to the obvious difficulty in obtaining data, little is known about the current field present while the lake is covered with ice. However, the maximum current speed in the surface layer of the model of $\sim 0.02 \text{ m s}^{-1}$ is in good agreement with Zhdanov *et al.* (2001) who reported an average current velocity in the 10-50 m layer, in the South Basin, in the Spring of 1997, of 0.01-0.015 m s^{-1} . The pattern of the current field is in reasonable agreement with the mean circulation reported by Shimaraev *et al.* (1994).

The area around grid point (44,12) is covered by clear ice with a relatively low albedo and extinction coefficient whereas the area around (120,65) which is covered by snow has a higher albedo and extinction. Consequently the area at (44,12) receives a larger solar radiation flux and warms faster than the area at (120,65). Figures 2.4(b) and 2.4(c) show the temperature profiles at these two places.

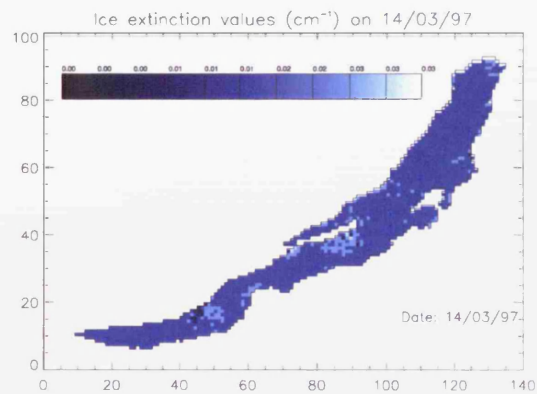
The slower warming and development of the mixed layer under snow-covered ice is demonstrated. The fact that the model reproduces the development of the mixed layer suggests that the vertical mixing is handled well by the model during the Spring. This is an important prerequisite if the model is to



(a) Type of ice or snow cover on 14 March 1997.

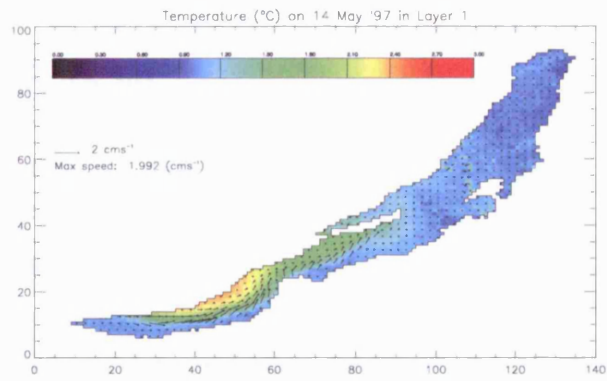


(b) Surface albedo on 14 March 1997.

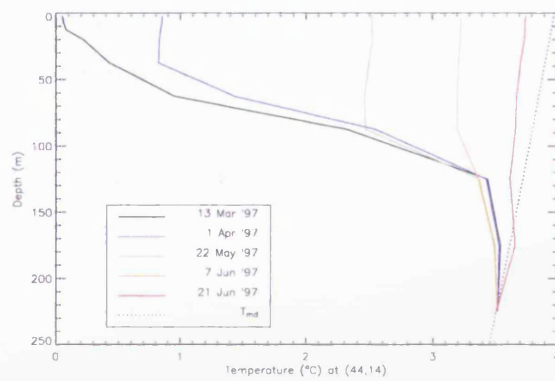


(c) Extinction coefficients for 14 March 1997.

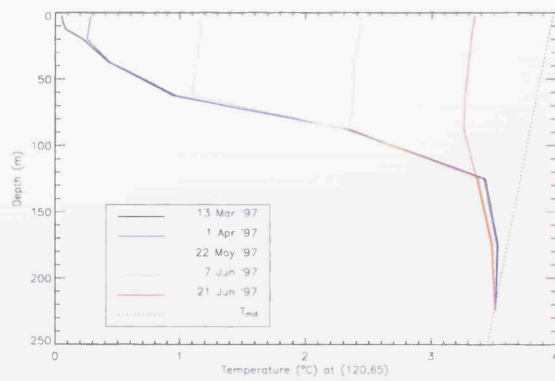
Figure 2.3: Properties of the model lake's ice and snow fields on 14 March 1997. The horizontal and vertical axes give, respectively, the x and y model grid point.



(a)



(b)



(c)

Figure 2.4: LHLC model: (a) Temperature ($^{\circ}\text{C}$) and current vectors (cm s^{-1}) in the LHLC model surface layer on 14 May 1997; model temperature profiles under (a) snow-free ice and (b) snow-covered ice.

be used to model diatom populations as these require vertical currents to stop them sinking out of the photosynthetic zone.

2.2 The Diatom Model

Commonly plankton populations are modelled with NPZ or NPZD models (see, for instance Franks (2002), Botte and Kay (2000) and Semovski (1999)). Here N, P, Z and D refer to the variables: nutrients, plankton, zooplankton and detritus. Each variable requires an additional equation. The growth of the diatom *Aulacoseira baicalensis* in the Spring is limited by the light available for photosynthesis rather than by nutrients. This diatom can, therefore, be modelled by a simple tracer equation with a light dependent source term. This requires the numerical integration of just one equation rather than the three (NPZ) or four (NPZD) of other models.

The diatom model is similar to that used by Ghosal *et al.* (2000) to investigate the effect of turbulence on the distribution of plankton in a one dimensional water column. Here we couple the diatom equation to a three-dimensional model of Lake Baikal. The density of the diatom *Aulacoseira baicalensis*, $\phi(x, y, z, t)$, obeys the partial differential equation:

$$\frac{\partial \phi}{\partial t} + (\mathbf{u} \cdot \nabla) \phi = -v_p \frac{\partial \phi}{\partial z} + \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla \phi) + S\phi. \quad (2.12)$$

The terms on the left hand side are the change in density with time and the advection of *A. baicalensis* with the fluid. The first term on the right hand side describes the sinking of *A. baicalensis* under gravity. The second term is the mixing of *A. baicalensis* due to turbulent motions of the fluid and the third term is a source-sink term representing changes due to cell division or death and predation. These terms are described below. The equation is solved on the same grid as the thermodynamic model.

2.2.1 The source-sink term

The processes of reproduction of diatoms by cell division and the loss of diatoms through mortality or predation by zooplankton are modelled as a combined source-sink term, $S\phi$. Here

$$S = P(I) - L, \quad (2.13)$$

where $P(I)$ is a specific photosynthetic growth rate which is a function of light intensity, I , and L is a specific loss rate. The specific photosynthetic growth rate is dependent only on the solar irradiation incident on the diatom. Goldman *et al.* (1996) investigated the nutrient dynamics and primary production in Lake Baikal in July 1990. They concluded that phytoplankton growth was not limited by nutrient

levels in the Spring and early Summer but that nutrient depletion may limit growth after the onset of the Summer stratification. For the period modelled here it is assumed that photosynthesis is unconstrained by nutrient levels but is strongly dependent on light levels. The functional dependence of P on I is taken from Jassby and Platt (1976), who investigated several formulations of the photosynthetic light curve for phytoplankton and found a hyperbolic tangent to be the best fit to empirical data from natural populations of marine phytoplankton.

$$P(I) = P_{max} \tanh \left(\frac{\alpha I}{P_{max}} \right), \quad (2.14)$$

where P_{max} is the specific growth rate at optimal illumination and α is the slope of the linear part of the curve. For this model the specific increase in cell number is substituted for the photosynthesis rate. Richardson *et al.* (2000) found a maximum specific growth rate of 0.15 d^{-1} and an initial slope, α , of $0.024 \text{ d}^{-1} (\mu\text{mol m}^{-2} \text{ s}^{-1})^{-1}$ for laboratory cultures of *A. baicalensis* at 2-3 °C. Jewson *et al.* (2004, unpublished) found an optimal growth rate of 0.31 d^{-1} at 3 °C. There is little known about the growth rates of *A. baicalensis* below 2-3 °C because of the difficulty in maintaining cultures at such temperatures. However, *A. baicalensis* has been shown to have growth rates that decrease with increasing temperature (Richardson *et al.*, 2000; Jewson *et al.*, 2004). The temperature experienced by diatoms during the period modelled are unlikely to exceed 3.5 °C. Due to the lack of published data concerning the temperature dependence of the growth rate of *A. baicalensis* in the range 0-3.5 °C, no temperature dependence is included in the formulation of the diatom growth rate used in this model.

This model of the P-I curve does not take into account the inhibition of photosynthesis at high light levels. Photosynthesis by *A. baicalensis* saturates at around $40 \mu\text{mol m}^{-2} \text{ s}^{-1}$, ($\sim 184 \text{ W m}^{-2}$), (Richardson *et al.*, 2000), and light levels in the lake can exceed this but in order to keep the model simple at this stage photoinhibition is ignored. A formulation that takes into account photoinhibition is described in the literature (Platt *et al.*, 1980) and could be used in future work.

The irradiances are calculated according to the Lambert-Beer law that describes the extinction of light passing through a medium:

$$I = I_0(1 - \alpha) \exp \left(-\mu_i t_i - \mu_s t_s - \mu_w z - \int_0^z \mu_p(z') dz' \right) \quad (2.15)$$

Here, I is the solar irradiation incident on a diatom, I_0 is the solar irradiation incident on the lake and α is the albedo of the lake surface, which may consist of ice, snow or water. The subscripts i , s , w and p refer to ice, snow, water and plankton; μ is the light extinction coefficient of the relevant medium and t is the thickness of the medium with the depth of water below the free surface given by z . The last term on the right hand side represents self-shading and is ignored in this model. The values of the extinction coefficients of ice, snow and water are the same as those used in the thermodynamical model.

The main predator of *A. baicalensis* is the copepod *Epischura baicalensis* which makes up 90-99 % of the Lake Baikal zooplankton. Zooplankton ingestion rates are often described in models using the Ivlev grazing formulation:

$$I = R_m (1 - e^{-\Lambda P}) . \quad (2.16)$$

The two parameters required are the maximum ingestion rate, R_m , and the rate, Λ , at which saturation is achieved with increasing plankton concentration, P . The change in plankton density due to predation is then:

$$\frac{\partial \phi}{\partial t} = -IZ , \quad (2.17)$$

where Z is the zooplankton density. The present work ignores this coupling to the zooplankton density: the specific loss, L , due to predation and mortality is constant. Though this is an over-simplification it releases us from the need either to calculate a zooplankton density field, which would require excessive computation time, or to input zooplankton density fields, the data for which are not currently available.

2.2.2 Sinking under gravity

The diatoms being modelled here are immotile; that is they have no method of self-propulsion. They are denser than the surrounding water and will, therefore, sink under gravity through a still water column. The speed of sinking will depend on the balance between the gravitational and the viscous drag forces acting on the diatom and so will vary with cell size and shape and the density of the surrounding fluid. The external surfaces of a diatom's cell wall are highly intricate and have a high degree of roughness. They can thus be expected to offer a high resistance to the flow of water past them. For this reason one can expect diatoms to sink at a terminal velocity.

From the above argument and for simplicity, we can model the process of sinking under gravity as a vertical flux of diatoms with a vertical velocity component v_p :

$$\left(\frac{\partial \phi}{\partial t} \right)_{sinking} = -v_p \frac{\partial \phi}{\partial z} \quad (2.18)$$

The sinking speed of *A. baicalensis* given by laboratory measurements is 2.5-2.8 m/day, (Gibson, 2001). In this model the sinking speed is considered to be invariant. This, of course, does not take into account any change in speed due to filament growth or change in density of the ambient water. In a freshwater lake such as Lake Baikal, any change in density due to temperature or salinity may be expected to have a negligible effect on sinking speed. The effect of filament growth, however, may be significant but is not modelled here.

As in the thermodynamic model the equation is solved by approximating the partial derivatives as finite differences. The differencing scheme used is chosen to suit the physical situation and model grid,

and must be numerically stable. The most obvious differencing scheme is the leap-frog method:

$$\phi_n^{k+1} = \phi_n^{k-1} + 2\Delta t \times - \left(\frac{v_p \phi_{n+1}^k - v_p \phi_{n-1}^k}{2\Delta z_n^k} \right). \quad (2.19)$$

In this nomenclature the superscript, k , indicates the time-step and the subscript, n , indicates the grid-spacing, in this case, along the z -axis. Both the time and space derivatives are estimated with second-order, centred differences. However, under this scheme evacuation of a layer can lead to negative plankton concentrations and unstable oscillatory behaviour can result. The transport of diatoms through sinking is in one direction only and the differencing scheme needs to reflect this.

Logically, the number of plankton sinking through a layer should not depend on the number of plankton in the layer below but only on the number sinking into the layer from above and the number sinking out of the layer. This assumes the plankton density is such that collisions do not occur. The upwind differencing scheme uses the flux through the upper and lower boundaries of the layer:

$$\phi_n^{k+1} = \phi_n^k + \Delta t \times \left[-v_p \times \frac{\phi_{n+1}^k - \phi_n^k}{\Delta z} \right]. \quad (2.20)$$

The upwind differencing scheme does not produce oscillatory behaviour or negative plankton concentrations and, though only first-order accurate, is stable as long as the Courant condition holds for the sinking velocity *i.e.* $|v_p| \leq \frac{\Delta z}{2\Delta t}$. The scheme does, however, result in numerical diffusion causing loss of amplitude. In the plankton model this causes the plankton sinking term to contain an element of diffusion and thus, as the model starts with high plankton numbers in the upper layers and no plankton in the lower layers, the apparent sinking rate is higher than wanted. This problem is exaggerated here because of the coarse nature of the grid.

The method chosen for approximating the finite differences in the sinking term of the diatom model is one first described by Hawley and Smarr (1984). The problem of numerical diffusion can be ameliorated to some extent by improving the estimate of the vertical plankton gradient used in the differencing. The method used here is their modification to the method of Wilson (1978). The sinking term can be written in terms of the flux through the upper ($n + \frac{1}{2}$) and lower ($n - \frac{1}{2}$) layer boundaries:

$$\phi_n^{k+1} = \phi_n^k + \frac{\Delta t}{\Delta z} \times \left(flux_{n-\frac{1}{2}} - flux_{n+\frac{1}{2}} \right). \quad (2.21)$$

In the upwind method described above the fluxes are:

$$flux_{n-\frac{1}{2}} = v_p \times \phi_n \text{ and } flux_{n+\frac{1}{2}} = v_p \times \phi_{n+1}. \quad (2.22)$$

In order to estimate fluxes at each interface we need a better approximation of the density distribution. First the values at the centre of adjacent layers are averaged in order to estimate the value at the

boundary. Then the gradient at the boundary is estimated using the same density values. Finally the gradient is used to extrapolate back a distance $|v_p \Delta t|$ to calculate the required density:

$$flux_{n-\frac{1}{2}} = v_p \left[\frac{1}{2} (\phi_{n-1} + \phi_n) - \frac{v_p \Delta t (\phi_n - \phi_{n-1})}{\Delta z} \right]. \quad (2.23)$$

Wilson's scheme is unstable under certain conditions. If the plankton density gradient is very steep then situations can occur where the amount removed from a layer is greater than the amount the layer contains. To prevent this Hawley and Smarr (1984) modified the scheme. A weighting system is used so that the upwind scheme, rather than the Wilson scheme, is used when the density gradient is steep and hence the potential for instability is high. The weighting is calculated from the plankton densities:

$$WT = \frac{|\phi_n - \phi_{n-1}|}{\phi_n + \phi_{n-1}}. \quad (2.24)$$

Each flux is then calculated as:

$$flux = (WT) F_1 + (1 - WT) F_2, \quad (2.25)$$

where F_1 is the upwind flux and F_2 is the Wilson Transport flux.

Finally, simpler methods are used to find finite difference approximations of the other terms in the plankton equation. The advection term is approximated using a first order upwind method. A forward-in-time-centred-in-space scheme is used for the horizontal and vertical diffusion terms. The values of the diatom diffusion coefficients are the same as those calculated for eddy diffusion. The horizontal component of the diffusion coefficient has a value of $1.0 \times 10^3 \text{ cm s}^{-1}$ everywhere. The vertical component depends on the vertical density gradient as described in § 2.1.2.

Transport of plankton by buoyancy convection of water is handled in a similar manner to the buoyancy convection of heat. The model uses a subroutine that runs at the end of every time-step. The subroutine compares the mass densities of each pair of adjacent layers in turn. If the upper layer is denser than the lower one then the two layers are mixed. The whole thickness of the thinner layer is exchanged with an equivalent mass of the thicker layer. Mixing continues until a stable density profile is obtained.

2.2.3 Boundary conditions

The boundary conditions for ϕ are those of no flux at the surface of the lake, no flux at the land-water boundary at the sides of the lake, and there is no turbulent mixing across the lower boundary but sinking across this boundary under gravity is allowed:

$$\left[v_p \phi + \kappa_v \frac{\partial \phi}{\partial z} \right]_{z=\eta} = 0, \quad (2.26)$$

$$[\nabla \cdot (\nu_h \nabla \cdot \phi)]_{\text{land-water boundary}} = 0 , \quad (2.27)$$

$$[flux]_{z=-250m} = v_p \phi , \quad (2.28)$$

where η is the free surface and in this case the gradient operator has horizontal components only.

2.2.4 Initial conditions

In view of the present paucity of published data on the under-ice distribution of *A. baicalensis* the model is initialised with a profile that has a constant value of 1, in arbitrary units, in the upper twenty-five metres and no diatoms in the layers below. Though this does not allow predictions of the absolute plankton densities, it does enable us to form a picture of the relative spatial and temporal variations in the plankton distribution.

2.3 A Diatom Model Experiment

The first experiment with the diatom model uses, in effect, a one-dimensional diatom model to examine the vertical components of the model terms and to investigate if the model can reproduce the effect of snow-cover on diatom abundance. The model is run with no horizontal diatom advection or horizontal diatom turbulent mixing. The diatoms are therefore subject only to vertical processes: their own sinking under gravity, at a speed v_p ; convection due to heating and the resulting instability of the water column; and vertical turbulent mixing. There is a light-dependent source term modelling photosynthetic reproduction but no loss term is included in these runs. The plankton concentration is thus described by the following equation:

$$\frac{\partial \phi}{\partial t} = -v_p \frac{\partial \phi}{\partial z} + \kappa_v \frac{\partial^2 \phi}{\partial z^2} + P(I) \phi + C_{mix}(\phi) , \quad (2.29)$$

where κ_v is the vertical component of the coefficient diatom turbulent diffusion and takes the same value as the vertical component of the coefficient of eddy diffusion. The terms on the right-hand side represent, respectively: sinking due to gravity, vertical mixing due to turbulent diffusion, photosynthetic reproduction and vertical mixing due to convection.

The details of the runs are shown in Table 2.1. The first four runs have no sinking term and can be thought to model a neutrally buoyant species of the plankton. The results for these runs are discussed first (§ 2.3.1 and § 2.3.2). In runs Run50a5 to Run50a8 the diatoms have a sinking speed of 2.5 m/day which is the observed sinking rate for *A. baicalensis*. These results are discussed in § 2.3.3 and § 2.3.4. Results for each type of plankter are given at two points on the lake. The results at grid cell (44,14)

Model Run	Description
Run50a1	No sinking. No vertical lake dynamics.
Run50a2	No sinking. Convection; no vertical turbulent mixing.
Run50a3	No sinking. No convection; vertical turbulent mixing.
Run50a4	No sinking. Convection; vertical turbulent mixing.
Run50a5	Sinking speed = 2.5 m/day. No vertical lake dynamics.
Run50a6	Sinking speed = 2.5 m/day. Convection; no vertical turbulent mixing.
Run50a7	Sinking, 2.5 m/day. No convection; vertical turbulent mixing.
Run50a8	Sinking, 2.5 m/day. Convection; vertical turbulent mixing.

Table 2.1: Details of the quantities varied between runs. All runs have the diatom source parameters: $P_{max} = 0.3 \text{ d}^{-1}$, $\alpha = 0.024 \text{ d}^{-1} (\mu\text{mol m}^{-2} \text{ s}^{-1})$ and $L = 0$.

illustrate the behaviour of diatoms under snow-free clear-ice, while the results at grid cell (120,65) show the behaviour of diatoms under ice which is snow-covered for the first thirty days of the run.

2.3.1 A neutrally buoyant plankton under snow-free ice

During the Spring the ice begins to melt and break up under the influence of solar heating. At grid-point (44,14) the ice is free of snow cover during the period that the model runs. As the ice melts its structure changes and its thickness reduces; consequently its albedo and light extinction coefficient changes. From day 0 to 19 the ice at this point is semi-transparent with an albedo of 0.3 and an extinction coefficient of 0.008 cm^{-1} , it then changes to white ice, albedo 0.45 and extinction coefficient 0.02 cm^{-1} . From day 26 the ice passes through stages of white-ice melt-pond, frazil ice and slush ice until at day 43 all the ice at this point on the lake has melted.

The plankton total column cell number variation with time at point (44,14) for the four runs is shown in Figure 2.5. The vertical profiles of plankton cell concentration at this snow-free point are given in Figure 2.6. Note that the x-axis scale varies between plots. Two profiles are illustrated for each run: the initial profile on day 0 and the profile for day 30.

Run50a1: Growth without vertical dynamical processes

Run50a1 has a neutrally buoyant plankton species which sees no lake dynamics and so is effectively growing in still water. The plankton cell concentration shows an exponential increase with a doubling

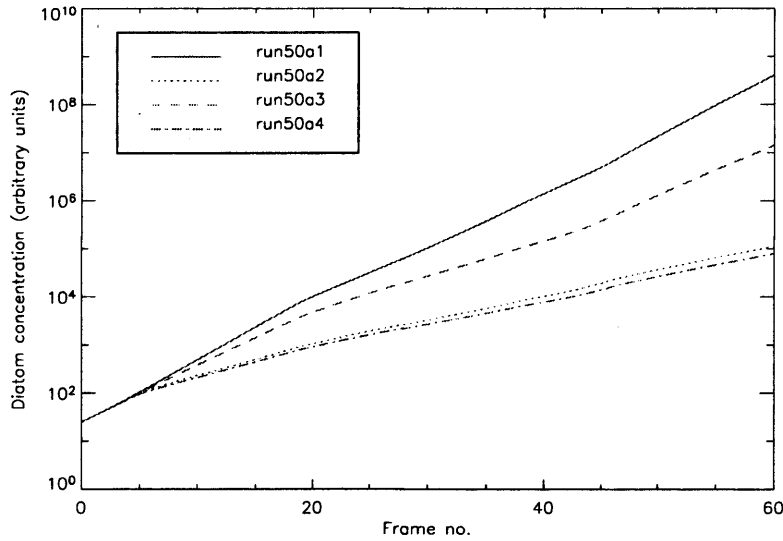


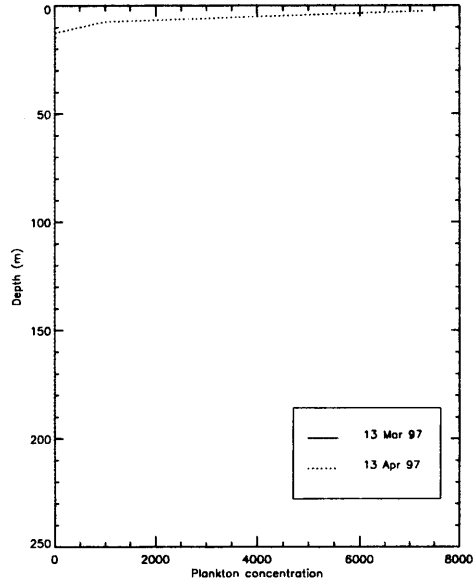
Figure 2.5: Plankton total column cell number at (44,14) under snow-free, clear ice for run50a1 to run50a4.

time that is dependent on light level. The light level is in turn dependent on depth and the absorption of light by the snow and ice cover. From day 19 the ice begins to melt and break up; the albedo reduces but the extinction coefficient increases resulting in lower light levels below the ice during this period and subsequent slower growth. After day 42 all the ice at this point has melted and the growth rate increases markedly. The plankton cell concentration doubling time under semi-transparent ice is ~ 2.26 days. This increases to nearly ~ 2.81 days after day 19 and then decreases down to ~ 2.37 days after the ice has melted.

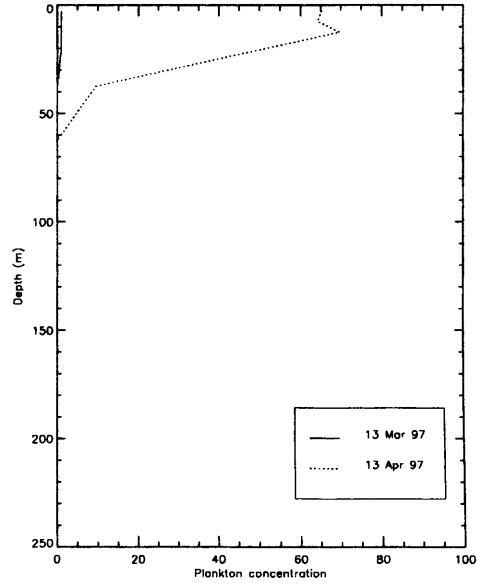
Run50a2, Run50a3 and Run50a4: Add vertical lake dynamics

The plankton cell concentration doubling time over the full 60 days is ~ 2.57 days for Run50a1 with growth only. This increases to ~ 3.34 days when vertical turbulent mixing is added (Run50a3), ~ 5.34 days when convective mixing is added (Run50a2), and ~ 5.59 days when both convective and turbulent mixing are added (Run50a4). The vertical mixing processes reduce the ability of the plankters to reproduce because the plankters are mixed down the water column into regions with lower light levels. Some plankters will be mixed up the water column but as the light levels fall off exponentially with depth the average light level experienced by the plankton cells is reduced.

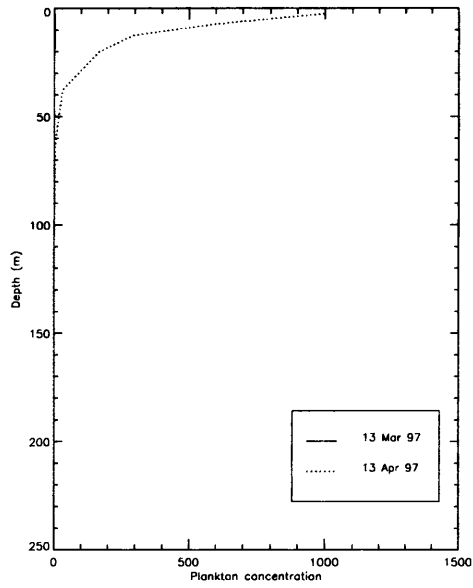
Convective mixing results in a well-mixed, almost homogeneous vertical profile of plankton cell number down to a depth of around 20 m. The presence of plankters in the 25-50 m layer by day 30 in Run50a2 shows that convective mixing has occurred down to this depth though it is more vigorous in



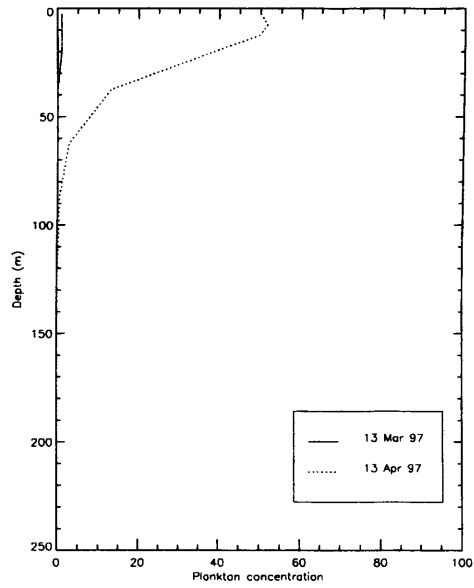
(a) Run50a1



(b) Run50a2



(c) Run50a3



(d) Run50a4

Figure 2.6: Vertical profiles of plankton cell concentration at a point under snow-free ice on day 0 (solid line) and day 29 (dotted line). Run50a1: growth only. Run50a2: convection and growth. Run50a3: vertical turbulent mixing and growth. Run50a4: convection, vertical turbulence and growth.

the upper 20 m.

Levels of turbulent mixing are low during ice-cover. Although density gradients in the upper layers are fairly low and result in a water column with limited stability, the current shear is also small. Turbulence is highest in layers with minimal density gradient, at day 30 this is the upper 50 m. Mixing of plankters by turbulent processes is lower than mixing by convection and results in a smaller reduction in plankton total column cell number (see Figure 2.5).

The effect on the plankton population is greater when turbulent and convective mixing occur together. The plankton total column cell number is reduced below the level that occurs with convective mixing alone and plankters are found at lower depths; the 50-75 m layer as opposed to the 25-50 m layer.

2.3.2 A neutrally buoyant plankton under snow-covered ice

Point (120,65) has snow cover until day 32, then ice-melt pond until day 70, except for day 38 which has snow. After day 70 the ice and snow has melted. The vertical profiles for a neutrally buoyant species under snow covered ice are shown in Figure 2.7 and the plankton total column cell number for run50a1 to Run50a4 is shown in Figure 2.8.

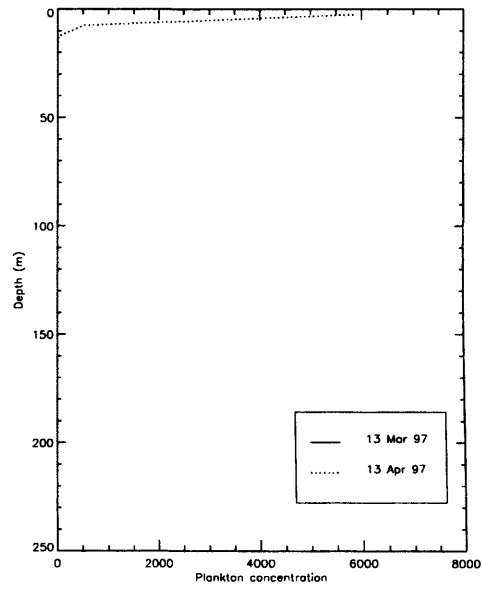
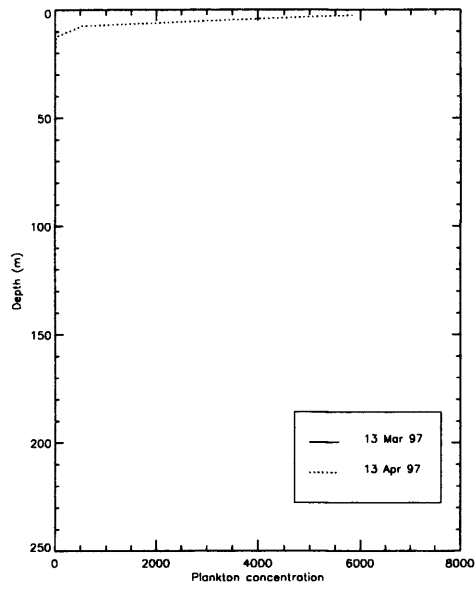
Run50a1: Growth without vertical dynamical processes

The plankton cell concentration grows at a much slower rate than under clear ice. Whereas in still water the doubling rate under snow-free ice was ~ 2.26 days, during the first 32 days of snow cover at (120,65) the doubling rate is approximately ~ 2.80 days. Growth at (120,65) accelerates when the snow clears at 32 days.

Run50a2, Run50a3 and Run50a4: Add vertical processes

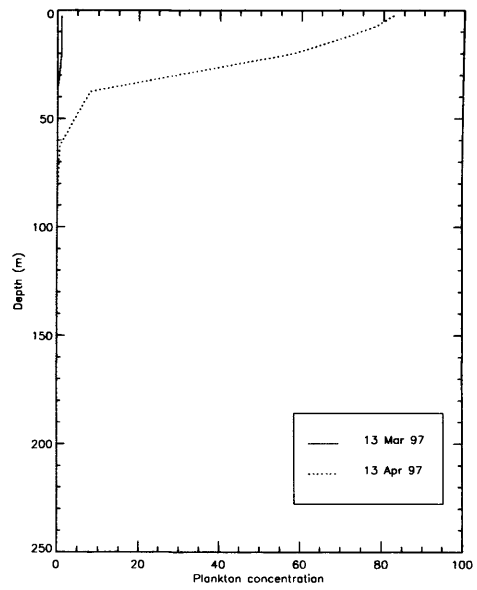
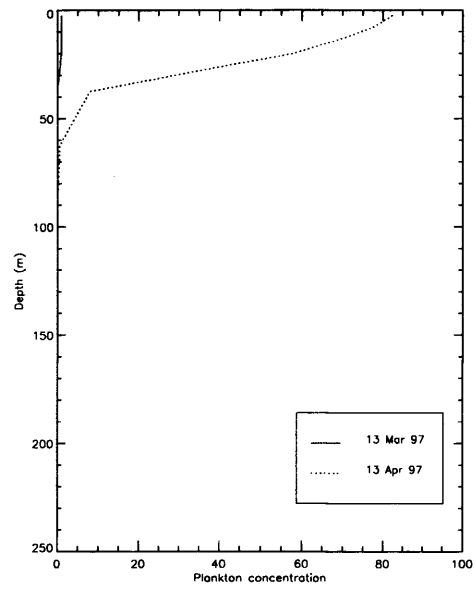
Convection has no effect until day 32. There is little heating of the water under the snow so that the density gradient is stable and no convection takes place. The vertical profiles at day 30, therefore, show no difference between Run50a1 and Run50a2, and between Run50a3 and Run50a4. After day 32 convection starts at point (120,65) and a proportion of the plankters is mixed out of the photic zone. This can be seen in the results for Run50a2: after day 32 the plankton total column cell number is lower in this run, which has convection, than in Run50a1 which does not.

Some vertical diffusion does take place under snow; although, due to the stability of the density gradient, significant turbulence is confined to the upper 25 m. Even low turbulent mixing can significantly reduce plankton populations by mixing them out of the euphotic layer and so decreasing the reproductive



(a) Run50a1

(b) Run50a2



(c) Run50a3

(d) Run50a4

Figure 2.7: Vertical profiles at day 0 and 30 for runs 50a1 to 50a4 at (120,65)

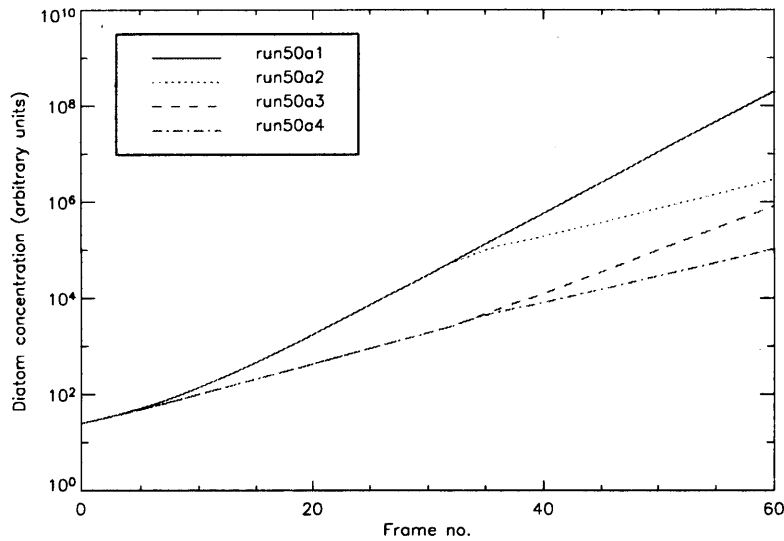


Figure 2.8: Plankton total column cell number at (120,65) under snow covered ice for run50a1 to run50a4.

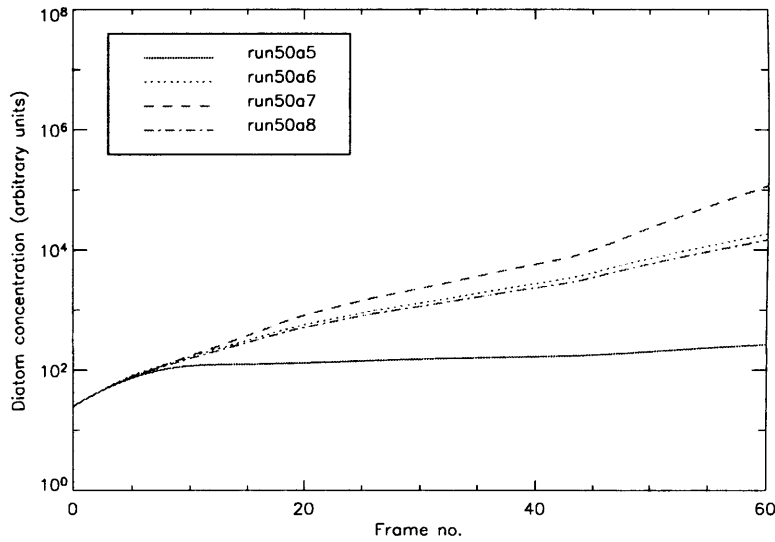
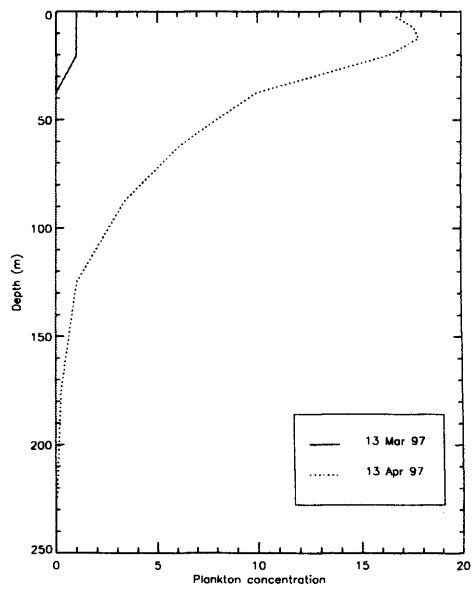
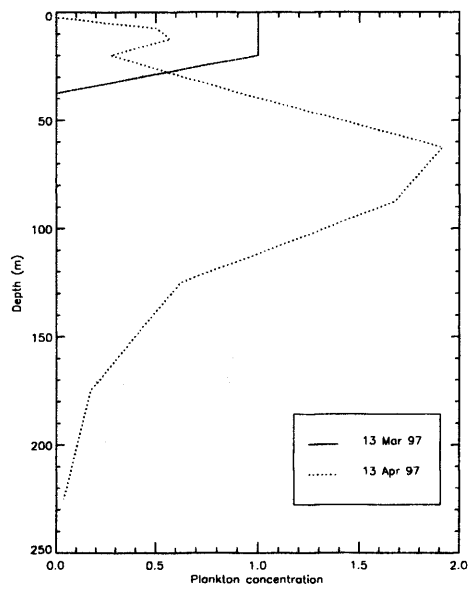


Figure 2.9: Plankton total column cell number at (44,14) under ice for run50a5 to run50a8.

rate. The vertical profile of plankton cell number for turbulent mixing under snow-covered ice is shown in Figure 2.7(c). The number of plankters in the surface layer under snow-covered ice is markedly reduced compared with the surface layer under snow-free,clear-ice (Figure 2.6(c)).

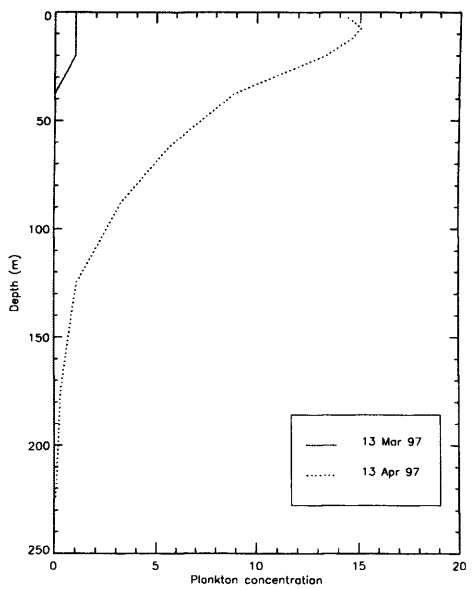
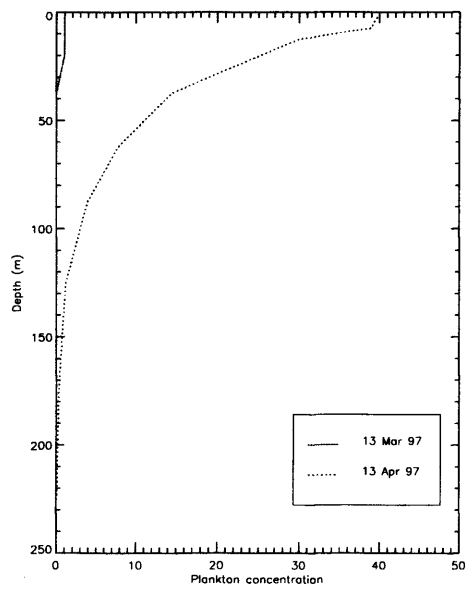
2.3.3 A sinking plankton under snow-free ice

In the runs Run50a5 to Run50a8 the plankters have a sinking speed of 2.5 m/day. Figure 2.9 shows the plankton total column cell number at this point. The vertical profiles for day 0 and day 30 under snow-free ice for the runs with sinking plankters are shown in Figure 2.10.



(a) Run50a5

(b) Run50a6



(c) Run50a7

(d) Run50a8

Figure 2.10: Vertical profiles at day 0 and 30 for runs 50a5 to 50a8 at (44,14).

Run50a5: Growth without vertical dynamical processes

In still water, there is growth while there are still plankters in the photic zone but as these sink below the photic zone the reproductive rate slows. Plankters are also lost from the water column as they sink out of the bottom layer.

When the ice begins to melt, its extinction coefficient increases and less light penetrates to the water below. The plankton growth rate is reduced and plankters sink out of the euphotic zone at a faster rate than they can be replaced by reproduction. Thus the number of cells in the upper layers reduces resulting in a vertical profile with a sub-surface maximum. At day 30, this maximum occurs at a depth of ~ 65 m (Figure 2.10(a)).

Run50a6, Run50a7 and Run50a8: Add vertical lake dynamics

In these runs, with vertical dynamical processes, growth continues because sinking plankters are mixed upwards, back into the photic zone. Both convection and turbulent mixing are seen to increase the growth rate compared with the control run. Interestingly, in the run where both convection and turbulent diffusion occur the plankton cell numbers are lower than in the runs when only one of the two processes occurs. This indicates that the two processes do not add together linearly.

2.3.4 Sinking plankton under snow-covered ice

Figures 2.11 and 2.12 show results for sinking plankters under snow-covered ice. The total column plankton is plotted in Figure 2.11 and the vertical profiles are shown in Figure 2.12.

Run50a5: Growth without vertical dynamical processes

The light levels experienced by the plankton are low because of the snow cover and because the plankters sink away from the surface. Hence, there is little growth and the amplitude of the profile is lower than under clear ice and lower than that for neutrally buoyant plankton under snow-covered ice.

Run50a6, Run50a7 and Run50a8: Add vertical lake dynamics

As convection does not start at this point until after the snow melts at day 32, the runs with convection show no difference to those without convection before this time. However, during this time turbulent mixing increases plankter numbers above the levels found in the control run. After day 32 some convection occurs and enables some slight growth but now few plankters remain in the photic zone to reproduce.

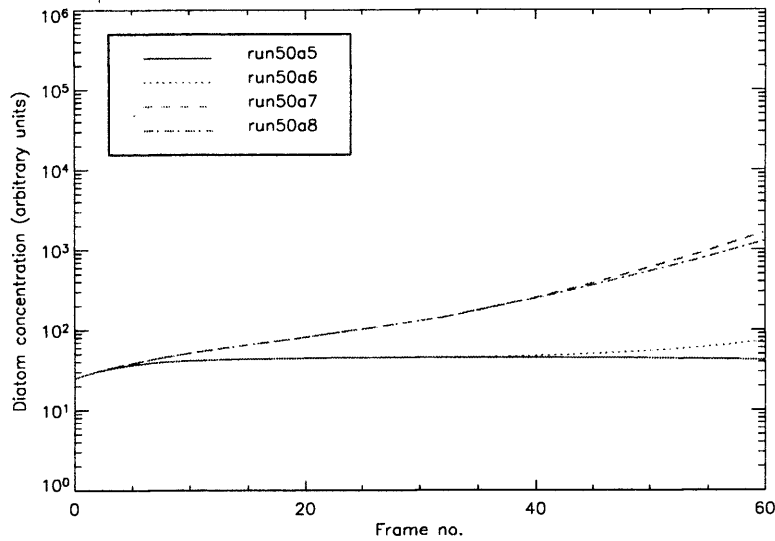


Figure 2.11: Total column plankton cell number at (120,65) under snow-covered ice for run50a5 to run50a8.

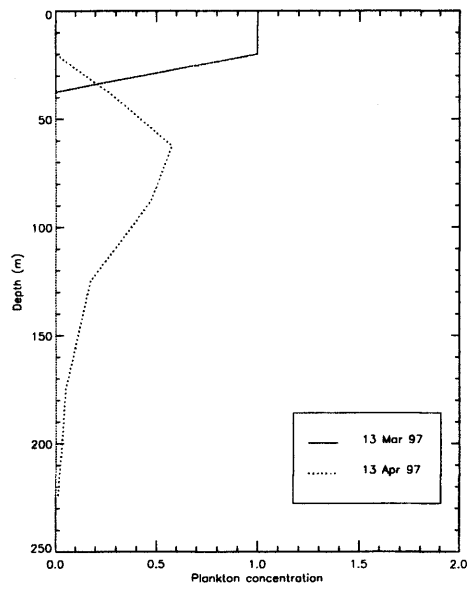
Parameter	Value
P_{max}	0.3 d^{-1}
α	$0.024 \text{ d}^{-1} (\mu\text{mol m}^{-2} \text{ s}^{-1})$
L	0.05
v_p	-2.5 m/day
κ_h	$0.1 \text{ m}^2 \cdot \text{s}^{-1}$

Table 2.2: Parameters used for diatom model run50c8ii.

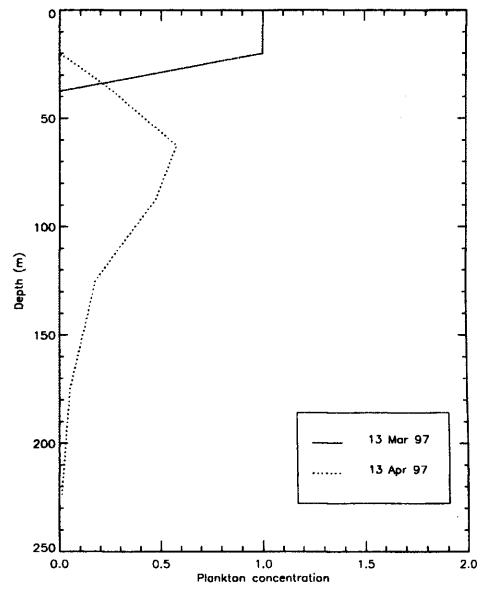
2.4 Horizontal Distribution of Diatoms

In the last section, horizontal dynamical processes were not included in the diatom model so that the various vertical processes could be isolated and examined independently. In this section the model is extended to three dimensions in order to examine the regional distribution of diatoms. The diatom model now uses the full equation given by (2.12). Although the current velocities under ice are low they are likely to play a role in carrying diatoms into and out of areas of intense vertical mixing and therefore will have an influence on bloom production. The diatom model now includes terms for horizontal advection and horizontal turbulent diffusion and entrainment in water moving between layers. The term representing diatom loss due to mortality and predation is no longer zero. The parameters used in the diatom model are given in Table 2.2

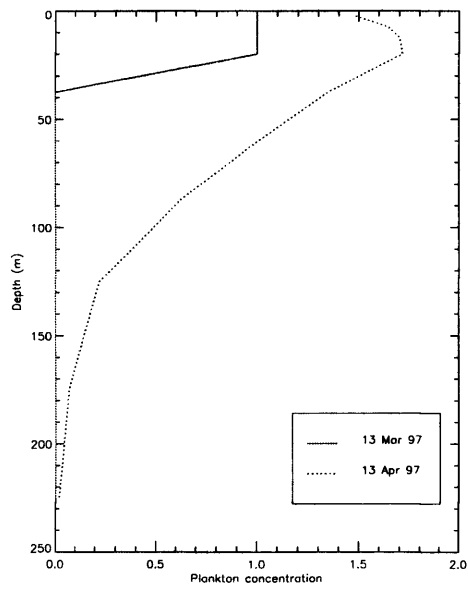
Figures 2.13 and 2.14 show maps of the diatom concentrations for each model layer. Comparison



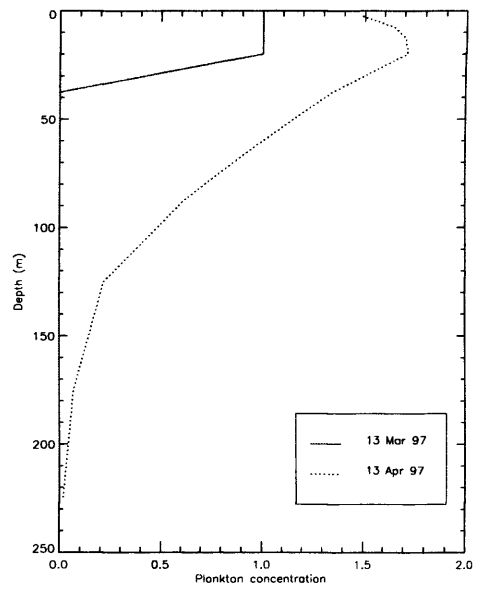
(a) Run50a5



(b) Run50a6

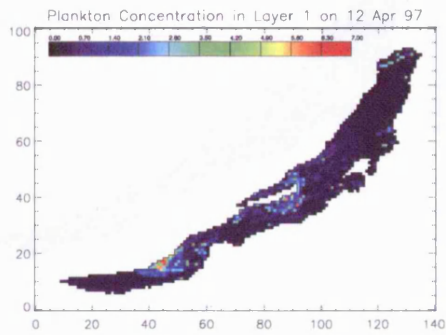


(c) Run50a7

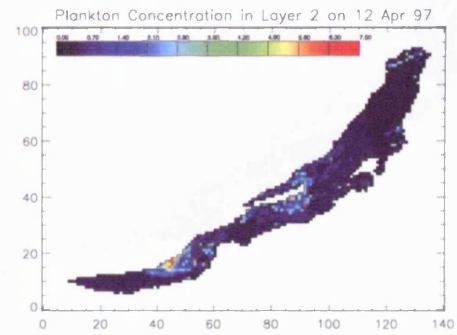


(d) Run50a8

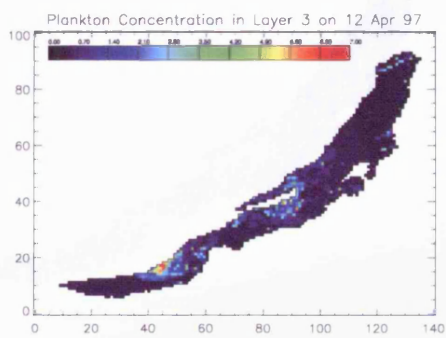
Figure 2.12: Vertical profiles at day 0 and 30 for runs 50a5 to 50a8 at (120,65).



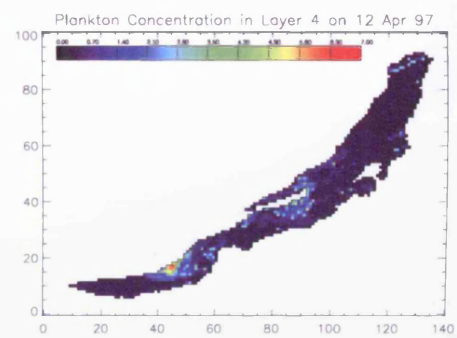
(a) layer 1



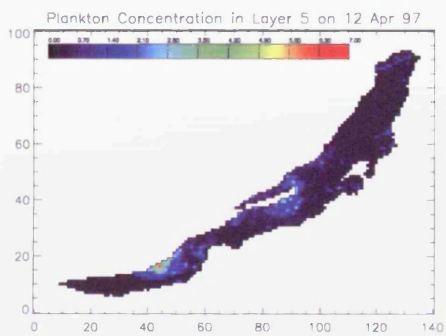
(b) layer 2



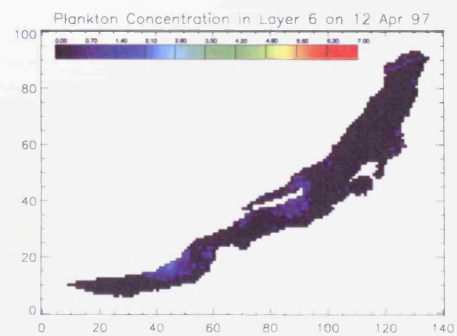
(c) layer 3



(d) layer 4



(e) layer 5



(f) layer 6

Figure 2.13: Diatom concentration in layers 1 to 6 on 12 April 1997. The colour scale runs from 0.00 to 7.00 arbitrary units in all plots.

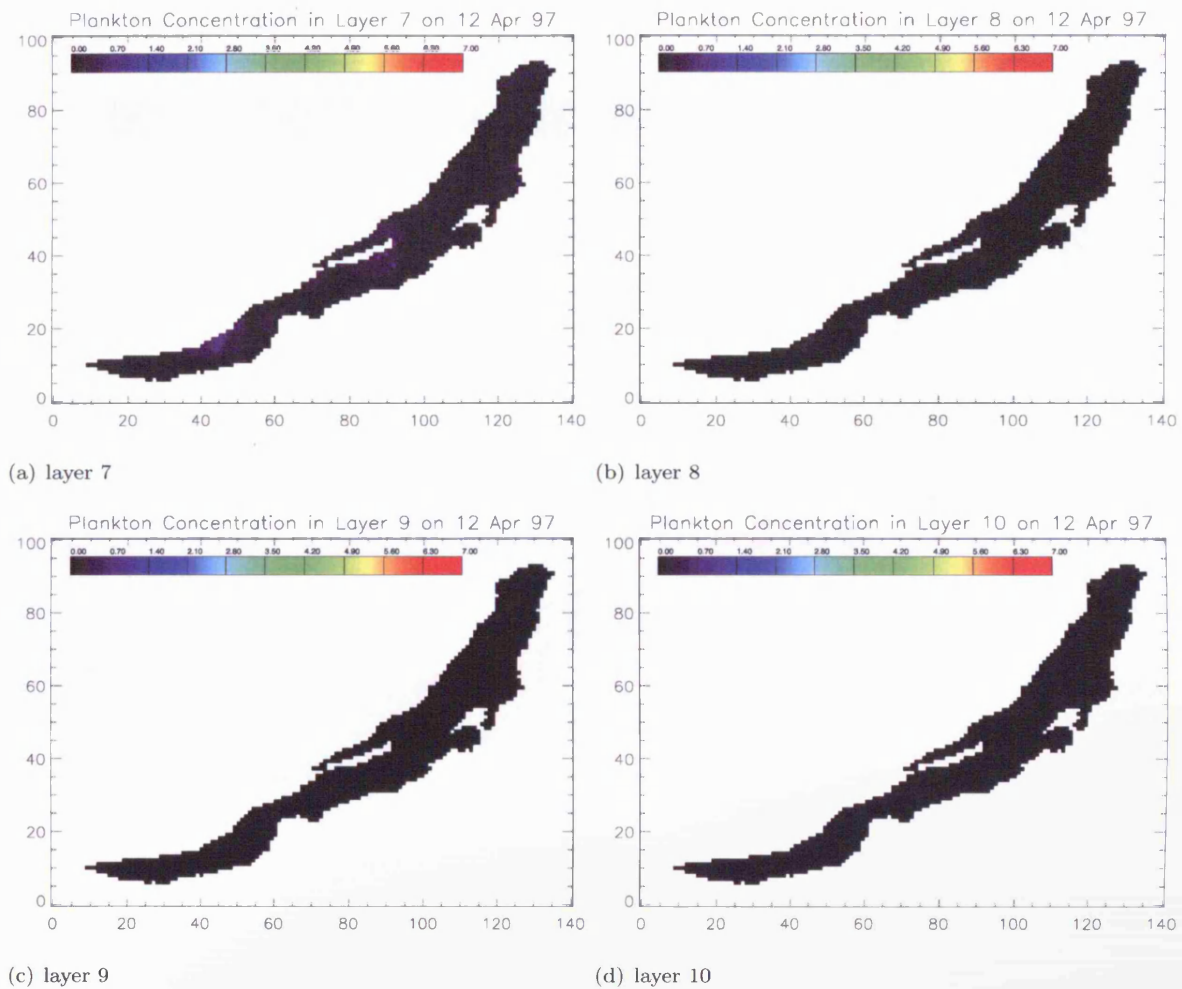


Figure 2.14: Diatom concentration in layers 7 to 10 on 12 April 1997. The colour scale runs from 0.00 to 7.00 arbitrary units in all plots.

of these maps with the map of ice types given in Figure 2.3(a) reveals the highest numbers of diatoms are found under areas of snow-free ice. Diatom numbers are highest under areas of clear ice and lowest under areas with a snow layer. Peak levels occur in layers 2 and 3 with negligible amounts of diatoms occurring in layers 8-10.

Comparison of the diatom maps with the current vectors in the surface layer (Figure 2.4(a)) shows that horizontal currents appear to have limited influence on diatoms numbers. There is no evidence to suggest that up-welling of water at areas of current divergence enhance diatom numbers; the type of ice or snow cover is more important.

A map of the total column diatom number is shown in Figure 2.15. Snow-free areas have total column diatom numbers that are nearly an order of magnitude higher than the snow-covered areas.

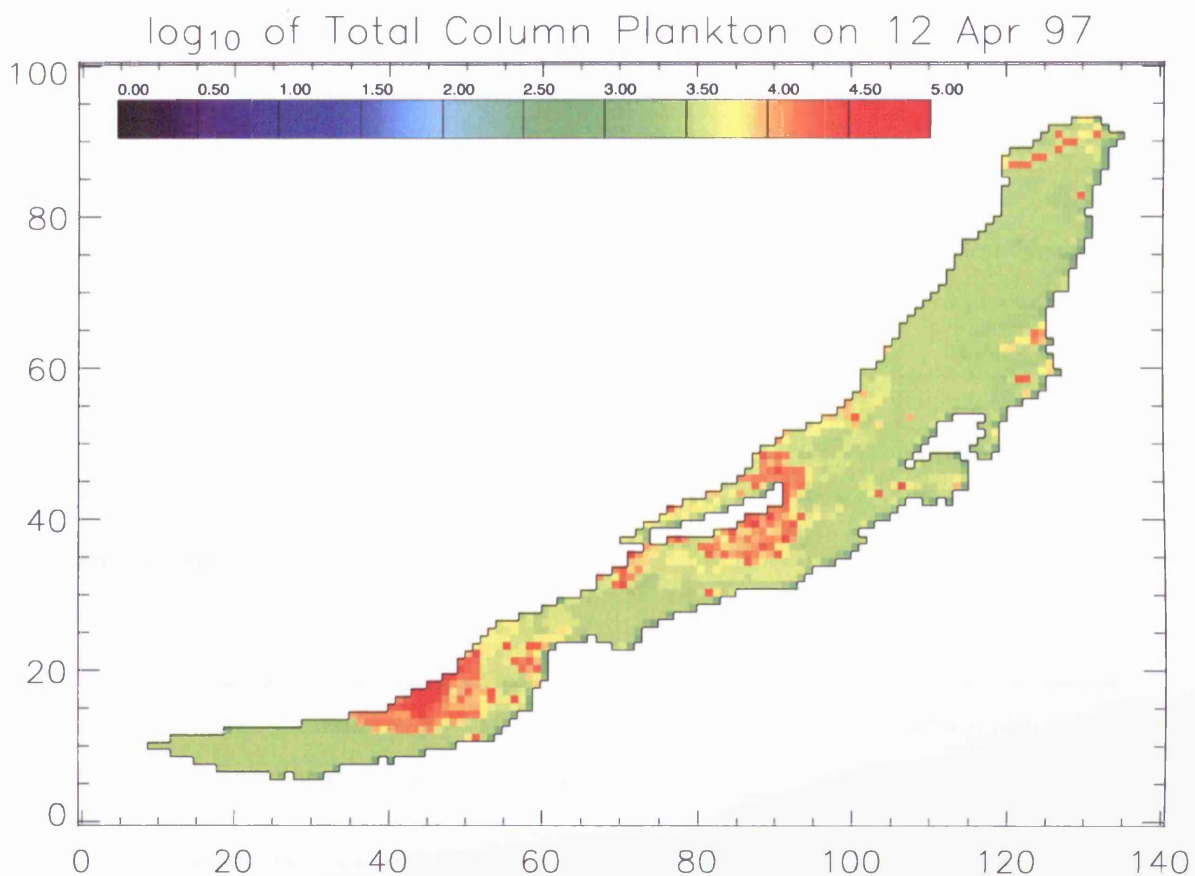


Figure 2.15: Total column plankton on 12 April 1997.

2.5 Discussion and Conclusion

A summary of the results from § 2.3 is given in Table 2.3 which shows the total column plankton for neutrally buoyant and sinking plankters on day 30. Values for runs with growth only and with vertical dynamical processes are shown at a location under snow-covered ice and under snow-free, clear ice. Under snow, growth rates are slow due to low light levels. The vertical dynamical processes appear to inhibit growth of neutrally buoyant plankton but enhance the growth of sinking plankton. The results predict that blooms of sinking plankters should occur under the snow-free, clear ice conditions present in parts of South Baikal but not under the snow-covered ice that occurred in North Baikal in the Spring of 1997.

The results concur with other models in that they show that vertical mixing is essential if sinking diatoms are to bloom (Huisman, 1999; Ghosal *et al.*, 2000) and as vertical mixing is more intense under clear, snow-free ice that is where *A. baicalensis* blooms are likely to occur (Granin *et al.*, 2001).

The model results are also in agreement with those of Kelley (1997), in that they support the

	Neutrally buoyant plankton		Sinking plankton	
	Growth only	Growth with vertical dynamical processes	Growth only	Growth with vertical dynamical processes
Clear ice	8.08×10^4	2.42×10^3	1.50×10^2	1.07×10^3
Snow-covered ice	2.27×10^4	1.62×10^3	4.53×10^1	1.24×10^2

Table 2.3: Total column plankton on day 30 for neutrally buoyant and sinking plankton with and without vertical dynamics at a point under clear ice and a point under snow-covered ice.

hypothesis that the intensity of vertical mixing under the ice in Lake Baikal is sufficient to maintain *A. baicalensis* in the euphotic zone long enough for blooms to occur, and with those of Semovski *et al.* (2000) whose model also predicted a bloom in South Baikal but not North Baikal in 1997.

Observations of *A. baicalensis* have shown that blooms occur in years of reduced snow-cover (Kozhova and Izmet'seva, 1998) and that the largest abundances occur in March-April in the layer immediately under the ice (Popovskaya, 2000; Kozhova and Izmet'seva, 1998). In March-April the vertical profiles demonstrate a surface maximum which sinks after the ice has melted and *A. baicalensis* has ceased to reproduce (Fietz *et al.*, 2005; Bondarenko *et al.*, 1996).

However, Granin (1997a) reported observations of the vertical distribution of *A. baicalensis* in March 1994 at a point under a region of snow-covered ice and at a point under a region of snow-free ice. Under the snow-free ice, the concentration of diatom cells is a factor of ten greater than the concentration under snow-covered ice. The vertical profile had a maximum at ~ 5 m under snow-covered ice and under snow-free ice the maximum was at ~ 40 m with cell concentrations dropping to near zero at 50 m.

Long-term observations indicate algal numbers in the South Basin are, on average, five times those of the North Basin (Popovskaya, 2000). This factor might be expected to increase in those years that have abundant blooms. Comparing the two points used in this study we get a factor of $\sim 8-9$ which seems reasonable and compares well with Granin (1997a).

The model reproduces the observed differences in abundances of *A. baicalensis* under snow-free and snow-covered. However, the vertical profile predicted by the model is rather broad. Numerical diffusion may result in excessive diffusion of diatoms downwards, away from the surface. Such numerical diffusion of the vertical profile may be caused by the poor vertical resolution of the model or may be implicit in the finite-difference method used for the diatom sinking term.

The model profile maximum is at a depth of ~ 5 -10 m. This agrees with the observations of Kozhova and Izmet'eva (1998) but not those of Granin (1997a) who observed a maximum at 40 m. The disparity in the observations may be due to annual variation in the timing of diatom bloom. Variation in the snow-fall or time of the break-up of the ice will result in changes in vertical mixing intensity and illumination. The population dynamics depend on the balance of growth rate, sinking rate and vertical mixing (Huisman and Sommeijer, 2002). A sub-surface maxima may form because reproduction slows to a rate below that required to replace those diatoms that sink away from the surface. Under these circumstances a thin surface bloom could be expected to sink and vertical mixing would diffuse the bloom over a greater depth range.

In the model the specific growth rate of *A. baicalensis* at optimal illumination, P_{max} , does not depend on temperature and the model does not include inhibition of photosynthesis at high light intensities. In reality the maximum growth rate for *A. baicalensis* decreases for light intensities above 31-41 $\mu\text{mol m}^{-2} \text{s}^{-1}$. The maximum growth rate also decreases as water temperature increases until at temperatures above 6-8 °C *A. baicalensis* cells cease dividing. After the ice has melted the irradiance at the water surface may reach more than 1400 $\mu\text{mol m}^{-2} \text{s}^{-1}$ and water temperatures warm quickly. *A. baicalensis* growth ceases and by July cells have sunk to lower depths Bondarenko *et al.* (1996). This inhibition of growth at higher light intensities and warmer temperatures needs to be incorporated into the model if this cessation of growth and subsequent sinking of diatoms is to be reproduced by the model. For the model to run past the period of ice break-up the maximum specific growth rate must be a function of temperature and its dependence on light intensity must be more realistic.

According to Richardson *et al.* (2000) the irradiance at which maximum growth occurs is $I_k = 0.0031 \mu\text{mol cm}^{-2} \text{s}^{-1}$. The model uses a snow thickness of 2.5 cm. The light intensity at the water surface is 0.017 $\mu\text{mols}^{-1}\text{cm}^{-2}$ for snow-covered ice and 0.0048 $\mu\text{mols}^{-1}\text{cm}^{-2}$ for clear ice. These values show that, for a snow thickness of 2.5 cm, the diatom growth rate will be maximum at the surface both under snow-free and snow-covered ice. If the snow layer is deeper then the light intensity at the surface will be reduced under snow to a level where maximum growth rates cannot be achieved. Lawrence *et al.* (2002) report that the thermodynamical model is not sensitive to the snow thickness but the diatom model is likely to be and increasing the snow thickness would increase the difference in diatom growth

rates between snow-covered and snow-free areas.

In conclusion the plankton model used here, though simple, is able, when coupled to a thermodynamic model, to show that sinking diatoms depend on the vertical mixing in the lake. Without such mixing they would sink out of the euphotic zone before they were able to reproduce.

Chapter 3

The LBM: A New 3-D

Thermodynamical Model of Lake Baikal.

Aulacoseira baicalensis blooms in the Spring under the ice and *Cyclotella minuta* numbers reach a maximum in the Autumn (Kozhova and Izmet'seva, 1998). In order to obtain the response to climatic factors of the ratio of *A. baicalensis* numbers to *C. minuta* numbers in sediments, we need to simulate the growing environment over the whole growing season; that is from February/March to November/December. The goal is, therefore, to model the thermodynamics and light-regime of the surface 250 m of the lake throughout a whole year.

Unfortunately the LHLC model is unstable under forcing by winds. Layers empty quickly causing negative layer heights and unphysical negative densities. The negative layer heights can be prevented by increasing entrainment of water from the layers below the affected level, but this caused excessive damping of currents. In order to combat this problem, the Lake Baikal Model (henceforth known as the LBM) has equations which have been re-formulated to allow calculation on a fixed grid: the layer thicknesses are no longer allowed to vary and an explicit vertical velocity component is introduced. The LHLC model assumes the lake is isolated from any wind forcing. This is clearly no longer the case once the ice has melted and so the LBM implements wind forcing in the form of a boundary condition on the momentum at the surface. This stress is assumed to be transmitted downwards through the vertical turbulent mixing processes simulated by the Reynold's momentum stresses. The introduction of the current shear into the parameterization of the vertical eddy viscosity is discussed later (Chapter 5).

Once the ice has melted the lake has lost its insulating cover and heat transfer by sensible processes (conduction and convection) and insensible processes (evaporation and condensation) can occur. The LBM attempts to simulate these processes in such a way as to maintain a realistic heat budget for the lake (see Chapter 4).

This chapter first describes the revised formulation of the model equations, the model domain, and the boundary conditions. Some of the numerical tests performed during the development of the LBM are then discussed.

3.1 The Model Equations

The LBM uses the finite difference method to solve the Reynolds-averaged Navier-Stokes equations for an incompressible fluid. In the Lake Baikal pelagic zone, the horizontal currents are three orders of magnitude greater than the vertical currents. The motion is thus almost horizontal and vertical accelerations are small enough to be neglected. This allows the pressure to be calculated as if for a static fluid and we assume the hydrostatic approximation. Within the lake, the perturbations of the density away from an average value are small. The Boussinesq approximation is therefore assumed and density perturbations are ignored except in the buoyancy term.

The temperature gradient of the lake below 250 m depth is near constant throughout the year indicating limited net heat transfer between a surface mixed layer and the relatively thermally inert, deeper lake. Since exchange between the two layers is episodic and localized (Shimaraev *et al.*, 1994), the model reproduces only the surface 250 m and treats the deeper lake as an inert abyss. The upper 250 m are represented as twenty-five layers of 10 m thickness. This is sufficient to resolve the vertical temperature structure while retaining an acceptable computation time, see § 3.2.2 for further discussion. The horizontal grid extends from 51° N to 56° N and from 103° E to 110° E. A grid of 140×100 cells (the length of each cell is equivalent to 0.05° longitude or latitude) allows resolution of the main features of Lake Baikal's three basins. Higher resolutions require shorter time-steps to maintain stability and would exceed the available computational capacity. The time-step used is 20 seconds. The grid coordinate system has the x and y axes in the zonal and meridional directions respectively and the z axis points vertically upwards with $z = 0$ the undisturbed lake surface. The equations are solved on a regular, Arakawa-type-C staggered grid. The distribution of the variables on the grid are shown in Figure 3.1.

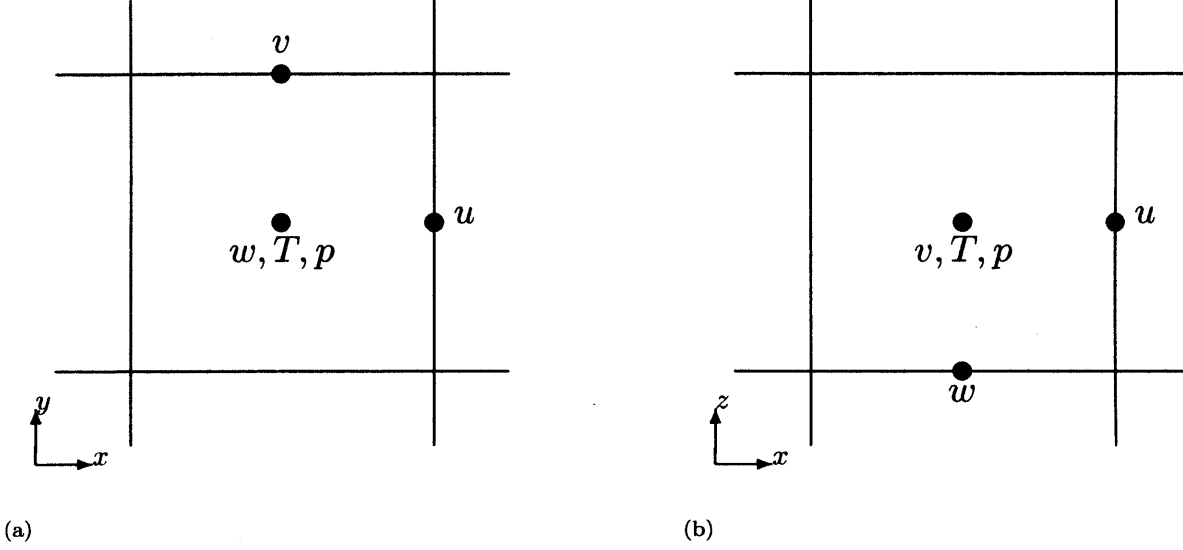


Figure 3.1: The positions of the variables in a model grid cell on the Arakawa type C grid: (a) plan view (b) side elevation. u , v , and w are the zonal, meridional and vertical components of the velocity. T is the temperature and p is the pressure.

Momentum

The horizontal momentum is predicted by the following equations:

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left(\nu_h \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu_h \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(\nu_v \frac{\partial u}{\partial z} \right) \quad (3.1)$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left(\nu_h \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu_h \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left(\nu_v \frac{\partial v}{\partial z} \right) \quad (3.2)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \quad (3.3)$$

is the substantive derivative and the Coriolis parameter, at latitude λ , is $f = 2\Omega \sin \lambda$ with Ω the Earth's angular velocity. Here the current velocity, $\mathbf{u}(u, v, w)$ has components in the zonal, meridional and vertical directions; ρ_0 is a reference density and p is the pressure.

The horizontal eddy viscosity is assumed to be constant and has a value of $\nu_h = 2 \times 10^7 \text{ cm}^2 \cdot \text{s}^{-1}$ everywhere. This value is determined by model stability criteria and is rather higher than in the real lake. The choice of value for the horizontal eddy viscosity is discussed further in § 3.2.3.

The eddy length scales are much smaller in the vertical direction and the extent of the turbulent mixing changes with depth and the seasonal changes in water column stability. The vertical component of the eddy viscosity is, therefore, not assumed to be constant. The parameterization of the vertical

eddy viscosity must allow for the different conditions under ice, when the forcing is mainly by differential heating and current velocities are small, and the conditions under wind forcing, when the lake is clear of ice and current velocities and vertical current shear is greater. Hence, the vertical eddy viscosity, ν_z is parameterized as a function of current shear and vertical density gradient. This parameterization will be discussed in more detail later (Chapter 5).

Wind forcing is implemented as a surface stress, τ . The components of this stress, τ_x and τ_y , are given by

$$\tau_x = C_d \rho_{air} |u_{wind}| u_{wind} \quad (3.4a)$$

$$\tau_y = C_d \rho_{air} |v_{wind}| v_{wind} , \quad (3.4b)$$

where $\rho_{air} = 1.2 \text{ kg} \cdot \text{m}^{-3}$ is the density of air, $C_d = 1.535 \times 10^{-3}$ is a drag coefficient and u_{wind} and v_{wind} are the zonal and meridional components of the wind velocity at 10 m above the lake surface. The components of the wind velocity are updated every hour. The wind stress is set to zero in grid boxes that are covered with ice as it is assumed that the ice-cover insulates the lake from the frictional effect of the wind.

Mass Continuity

The vertical velocity is diagnosed by the continuity equation which describes the conservation of mass for an incompressible fluid:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 . \quad (3.5)$$

Hydrostatic Equilibrium

The assumption of hydrostatic equilibrium is possible because the vertical currents are at least a factor of 10^3 smaller than those in the horizontal plane. The vertical acceleration of momentum is, therefore, neglected and a balance between the vertical pressure gradient and buoyancy is imposed:

$$\frac{\partial p}{\partial z} = \rho \mathbf{g} \quad (3.6)$$

with $\mathbf{g}$ as the acceleration due to gravity and ρ as the density.

Any static instability is eliminated by a convective adjustment: adjacent pairs of layers are compared and if deemed unstable the layers are exchanged. Stability is assessed by calculating the Brunt-Väisälä frequency, N , at the interface between the two layers.

The Brunt-Väisälä frequency is given by

$$N^2 = g \left[\alpha \left(\frac{\partial T}{\partial z} - \Gamma \right) - \beta \frac{\partial S}{\partial z} \right] \quad (3.7)$$

where

$$\alpha = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_{p,S} \quad (3.8)$$

is the volumetric isothermal expansion coefficient at constant pressure and salinity,

$$\beta = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial S} \right)_{p,T} \quad (3.9)$$

is the haline expansion coefficient and

$$\Gamma = -\frac{g\alpha}{\rho} (T_K) \quad (3.10)$$

is the adiabatic lapse rate with $T_K = T(^{\circ}\text{C}) + 273.15$. The square of the Brunt-Väisälä frequency, N^2 , compares the change in density of a parcel of fluid that is moved vertically with the change in density of the fluid surrounding the parcel as it is moved. For example, if the parcel is moved upwards and N^2 is positive then the density of the parcel increases faster than that of its surroundings and the parcel will move back towards its original position. The parcel is thus in a stable equilibrium. N is the frequency of oscillation of a particle moving with only vertical motion. If N^2 is negative then N is imaginary and the parcel is unstable and convection occurs. If $N^2 < 0$ then the layers are classified as unstable and the layers are exchanged. No mixing occurs and only heat, not momentum, is carried with the exchanged water. The adjustment routine works down the column comparing adjacent layers *i.e.* layers 1 and 2 are compared and then layers 2 and 3 and so on to the bottom of the column. The routine repeats until the whole column is stable and then moves on to the next column. The convective adjustment routine is not ideal as convection occurs within one time-step whereas in the real lake it would occur more slowly.

Equation of state

The system of equations is closed by an equation of state. Matthews and Heaney (1987) used a quadratic equation to calculate the density of freshwater, ρ , in their model of mixing in an ice-covered lake due to solar heating:

$$\rho(T) = \rho_{max} \left[1 - \gamma (T - 4)^2 \right] \quad (3.11)$$

Here, $\rho_{max} = 1000 \text{ kg} \cdot \text{m}^{-3}$ is the maximum density, $\alpha = 8 \times 10^{-6} \text{ }^{\circ}\text{C}^{-2}$ and T is temperature. They claim the density relative to the maximum density has an accuracy of 4% between 0 $^{\circ}\text{C}$ and 8 $^{\circ}\text{C}$. This is valid at the surface. At deeper levels the dependency of both the T_{md} and the maximum density on pressure must be taken into account.

At temperatures above 8 $^{\circ}\text{C}$ this equation is no longer satisfactory. For this reason, the LBM uses the equation of state for freshwater given by Chen and Millero (1986) and adopted by UNESCO. The

density, ρ , at an applied pressure, p , is given, in $\text{g}\cdot\text{cm}^{-3}$, as

$$\rho(p, S, T) = \rho_0(S, T) \left(1 - \frac{p}{K(p, S, T)} \right)^{-1}. \quad (3.12)$$

Here, ρ_0 is the density at $p = 0$ (sea level) and the pressure, p , and K are in bar.

$$\begin{aligned} \rho_0(S, T) = & 0.9998395 + 6.7914 \times 10^{-5}T - 9.0894 \times 10^{-6}T^2 + 1.0171 \times 10^{-7}T^3 \\ & - 1.2846 \times 10^{-9}T^4 + 1.1592 \times 10^{-11}T^5 - 5.0125 \times 10^{-14}T^6 \\ & + (8.181 \times 10^{-4} - 3.85 \times 10^{-6}T + 4.96 \times 10^{-8}T^2) S \end{aligned} \quad (3.13)$$

and

$$\begin{aligned} K(p, S, T) = & 19652.17 + 148.113T - 2.293T^2 + 1.256 \times 10^{-2}T^3 - 4.18 \times 10^{-5}T^4 \\ & + (3.2726 - 2.147 \times 10^{-4}T + 1.128 \times 10^{-4}T^2) p \\ & + (53.238 - 0.313T + 5.728 \times 10^{-3}p) S, \end{aligned} \quad (3.14)$$

where T is the temperature in $^{\circ}\text{C}$ and S is the salinity in $\text{g}\cdot\text{kg}^{-1}$. The authors claim the equation has a precision of better than $2 \times 10^{-6} \text{ g}\cdot\text{cm}^{-3}$ over the range typical of most freshwater lakes, including Lake Baikal: 0 – 0.6 $\text{g}\cdot\text{kg}^{-1}$ salinity, 0 – 30 $^{\circ}\text{C}$ and 0 – 180 bars.

The dependency of density on depth, at various temperatures, for the UNESCO equation and the quadratic equation are shown in Figure 3.2. The dependency of ρ_{max} on depth is included in the calculations using the quadratic equation and has been calculated according to Chen and Millero (1986). The dependency of T_{md} on depth is as given by Lawrence *et al.* (2002) and used in the LHLC model (equation (2.6)). The difference in density calculated by the two equations is less than 1.1×10^{-5} for a depth range of 0-250 m and temperature range of 0-8 $^{\circ}\text{C}$ but increases with temperatures above this range (Figure 3.3). For this reason the LBM uses (3.12) to calculate the density and assumes a salinity of zero.

Temperature

The time rate of change of temperature is calculated using the temperature equation:

$$\frac{DT}{Dt} = \frac{\partial}{\partial x} \left(\kappa_h \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\kappa_h \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\kappa_v \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial z} \left(\frac{Q}{\rho_0 c_p} \right) \quad (3.15)$$

The first three terms on the righthand side describe the diffusion of heat by turbulent processes. The eddy diffusivity, κ has horizontal and vertical components. The horizontal component has a constant

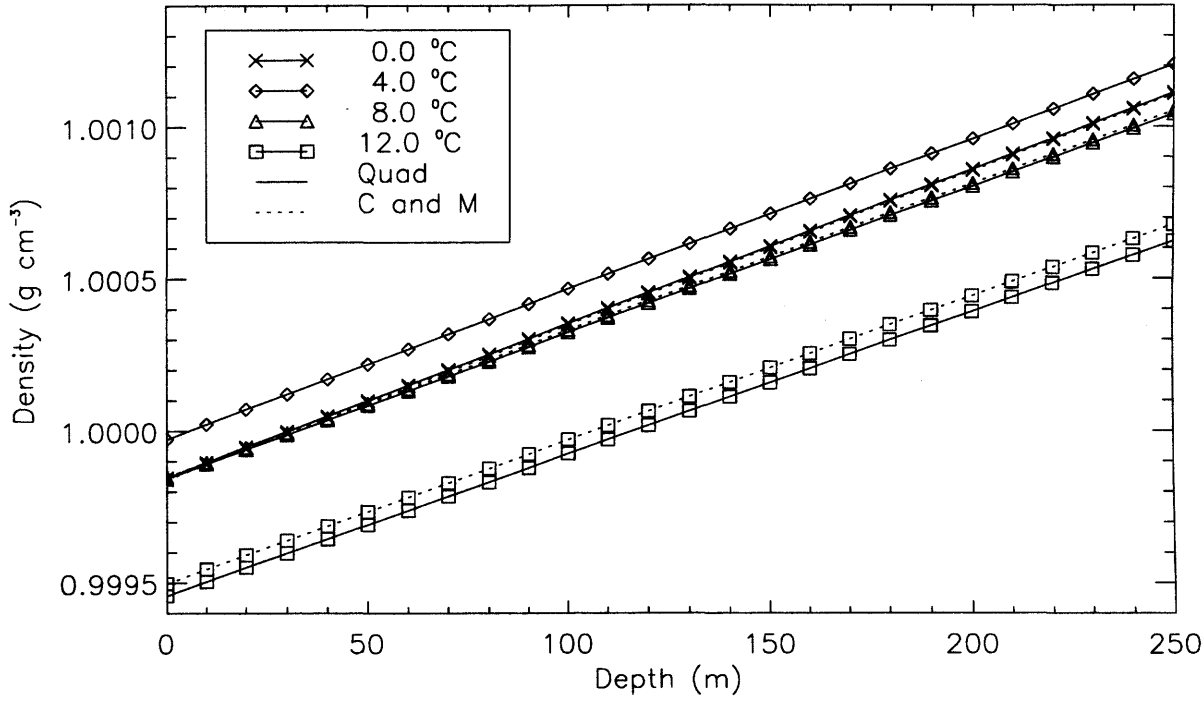


Figure 3.2: The density of freshwater as given by the equation of state of Chen and Millero (1986) (dashed lines) and Matthews and Heaney (1987) (solid lines). The densities are calculated over a depth range of 0 – 250 m for temperatures of 0, 4, 8 and 12 °C.

value of $0.1 \text{ m}^2 \cdot \text{s}^{-1}$ everywhere. This lies within the range of $0.02\text{-}0.3 \text{ m}^2 \cdot \text{s}^{-1}$ observed in several Swiss lakes by Peeters *et al.* (1996b). The parameterization of the vertical component will be discussed, with the vertical eddy viscosity, later (Chapter 5). The final term is a source-sink term describing heat transfer across the surface.

Boundary conditions

The domain is bound by solid vertical sides, a moving air/water interface at the surface, and a bottom, rigid interface between the lake and an inert abyss. At the solid boundaries $\mathbf{u} = 0$ so that the no-slip condition holds and there is no transport of momentum across the solid boundaries. There is no net flux of mass across the bottom boundary but the horizontal velocity components of the abyss can be set to a constant fraction of those in the bottom layer to allow some loss of momentum to the abyss through turbulent mixing. The vertical component of velocity is zero on the bottom giving a rigid boundary here. The vertical velocity of the upper surface is defined by the mass continuity condition:

$$w_{z=\eta} = \frac{\partial \eta}{\partial t} = \int_{z=-250 \text{ m}}^{z=\eta} \nabla \cdot \mathbf{u} \, dz \quad (3.16)$$

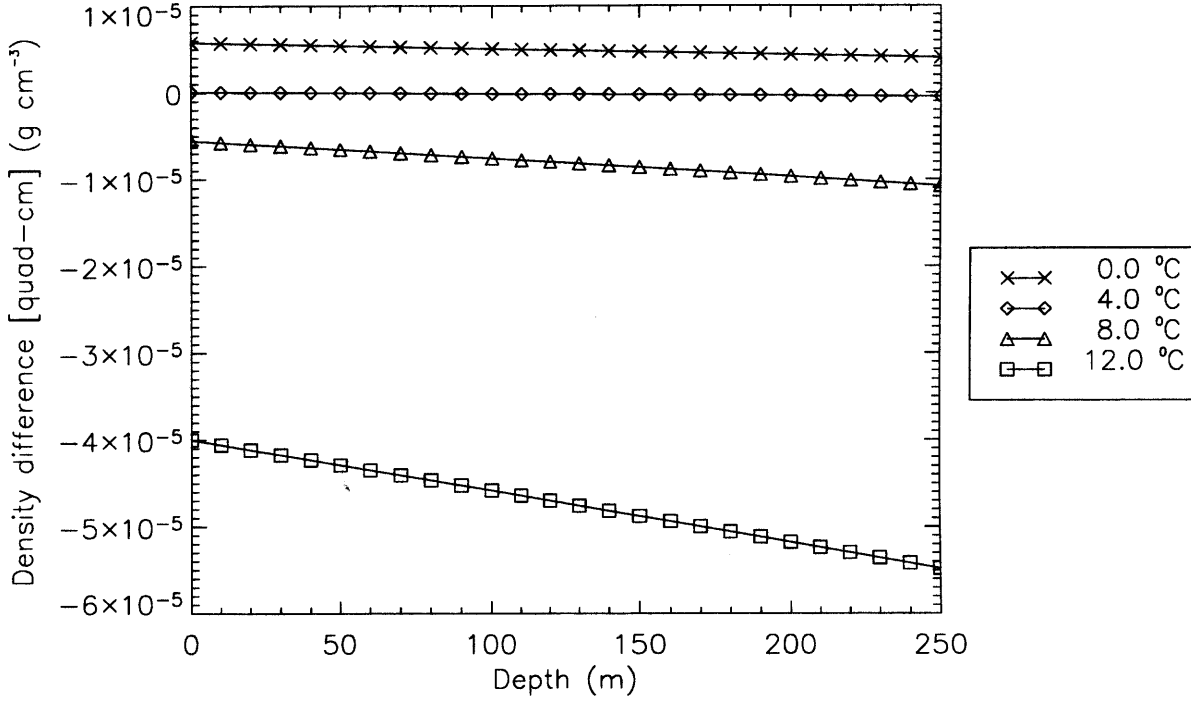


Figure 3.3: The deviation of the density calculated by the quadratic equation from that calculated by the UNESCO equation for temperatures of 0 °C, 4 °C, 8 °C and 12 °C.

where $\eta(x, y)$ is the vertical displacement of the surface from equilibrium.

At the surface the Reynold's turbulent momentum stresses are given by

$$\nu_v \frac{\partial u}{\partial z} = \frac{\tau_x}{\rho} \bigg|_{z=\eta} \quad (3.17)$$

and

$$\nu_v \frac{\partial v}{\partial z} = \frac{\tau_y}{\rho} \bigg|_{z=\eta} \quad (3.18)$$

where τ_x and τ_y are the horizontal components of the wind stress.

There is no transport of heat across the side or bottom boundaries so that

$$\frac{\partial T}{\partial x} = 0 \bigg|_{\text{east-west boundary}}, \quad \frac{\partial T}{\partial y} = 0 \bigg|_{\text{north-south boundary}} \quad (3.19)$$

and

$$\frac{\partial T}{\partial z} = 0 \bigg|_{z=-250 \text{ m}} \quad (3.20)$$

and transport of heat across the surface is parameterized in the source-sink term described later, in Chapter 4.

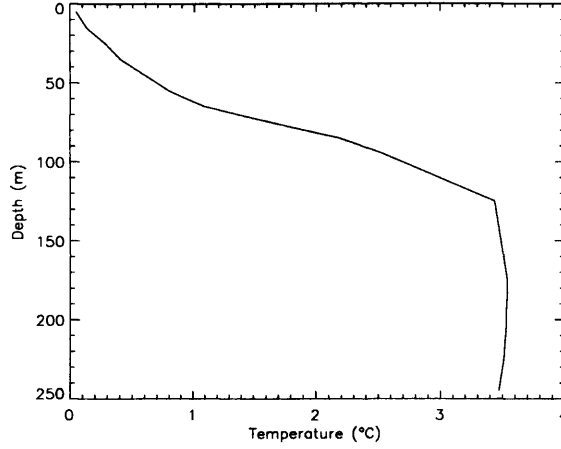


Figure 3.4: The temperature profile used to initialize the model. Data from Granin (1997b).

Initialization

When the model starts the current velocity is set to zero everywhere. The temperatures are initialized with a vertical profile observed in the South Basin in March 1997 (Granin, 1997b), which is illustrated in Figure 3.4.

3.2 Numerical Tests

The tests described in this section are aimed at choosing the best setup for the model, more specifically the most suitable discretization method for the advection terms, the best vertical grid resolution and the most suitable value for the horizontal component of the eddy viscosity. For these numerical tests we consider a simplified model lake. The test model is a rectangular lake basin with vertical sides. The horizontal dimensions are 120×20 grid boxes, with each grid cell approximately $3.33 \text{ km} \times 5.62 \text{ km}$. The lake is 250 m deep and has 25 layers of equal thickness except in the tests of vertical grid resolution (§ 3.2.2). The temperature profile consists of a 50 m surface layer at 15 °C and a lower layer, 200 m in depth, at 4 °C. This profile mimics the Summer stratification. The discontinuity at 50 m is a vigorous test of the discretization methods. The warm surface layer is thicker than the mixed layer in Lake Baikal but allows direct comparison of various layer thicknesses and vertical grid resolutions. No explicit vertical mixing is included in this test but horizontal diffusion is permitted and is actually essential for the maintenance of numerical stability. A wind blows over the lake for 12 hours at a constant magnitude equivalent to a surface stress of $0.5 \text{ N} \cdot \text{m}^{-2}$. As there is no vertical mixing in the test model the wind is implemented as a body force that decreases linearly in magnitude to zero magnitude at 50 m below the

surface. The expected response to this forcing is the formation of an internal wave.

In the case of two superimposed fluids of different densities the internal wave speed, c , can be approximated by:

$$c = \sqrt{g' \frac{h_1 h_2}{h_1 + h_2}} \quad (3.21)$$

with h_1 and h_2 the thicknesses of the two layers and ρ_1 and ρ_2 their densities (see Gill (1982) for derivation). The reduced gravity, g' , is defined as

$$g' = \frac{\rho_2 - \rho_1}{\rho_2} \quad (3.22)$$

This approximation is valid in the limit that $g'/g \rightarrow 0$. The period of the fundamental mode, T , is then given by Merian's formula:

$$T = \frac{2L}{\sqrt{g' \frac{h_1 h_2}{h_1 + h_2}}}, \quad (3.23)$$

where L is the length of the lake in the direction of travel of the wave. Merian's formula produces an estimate of approximately 14.8 days for the period of the fundamental internal wave mode in the test lake. We must remember that the model only extends to a depth of 250 m so these model runs test the differencing methods and their numerical diffusion, not the ability of the model to reproduce internal waves in the real lake. This, it cannot do unless extended to the full depth of the lake.

In the remainder of this section three groups of tests are described. The model lake is used to examine the effect of changing the discretization scheme used for the advection of temperature and velocity, secondly the result of changing the grid resolution is described and finally the optimum value for the horizontal component of the eddy viscosity is sought.

3.2.1 A comparison of three different numerical methods for the discretization of the advection terms

In the first test, we look at the effect of using different discretization methods for the advection terms in the momentum and temperature equations. In the past, ocean and lake models have commonly used the central differencing scheme (CDS) to form the discrete approximation of the advection terms and, indeed, the LHLC model uses this method. The CDS is second order and simple to implement especially on staggered grids. However, it can produce spurious oscillations in the region of steep gradients and discontinuities. In extreme situations these oscillations can lead to instabilities.

Farrow and Stevens (1994) observed anomalies in the temperature field of the Fine Resolution Antarctic Model in the region of the confluence of the Falklands and the Brazil currents when the CDS was used. Some grid points had negative temperatures and elsewhere adjacent grid points had temperatures

that differed by tens of degrees. These anomalies did not occur when the authors used a modified version of the QUICK (Quadratic Upstream Interpolation for Convective Kinematics) scheme (Leonard, 1979) to compute the advection terms.

Gerdes *et al.* (1991) described the production of an anomalous salinity maximum, which resulted from numerical dispersion of a saline front, in an experiment with the Geophysical Fluid Dynamics Laboratory ocean model. The authors compared three different treatments of the advection terms: the CDS, the upstream differencing scheme (UDS), and a flux-corrected transport (FCT) method. The UDS produced excessive vertical mixing and although the FCT method was an improvement, vertical mixing rates could not be reduced to observable values and were found to be limited by the vertical grid resolution.

Wang (2001) compared three methods, CDS, UDS and a total variational diminishing scheme (TVDS) in a model of Lake Constance. In the homogenous lake, the numerical errors resulting from the CDS were so large that in some cases barotropic motions could not be modelled by the CDS but the UDS presented no problems. In the stratified case the CDS generated oscillations in the temperature profile and caused high frequency noise to be superimposed on simulated Kelvin waves. The UDS did not produce oscillations but caused diffusion of the temperature profile and damping of Kelvin waves which resulted in reduced amplitudes and increased periods when compared with the TVDS.

The spurious oscillations produced by the CDS can be reduced by refinement of the grid, but this is computationally expensive. In the past, grids were coarse and numerical stability required high explicit diffusion which smoothed out the spurious oscillations but at some expense to the real solution. Upstream methods do not produce these oscillations but first order UDS methods have high implicit diffusion which can severely damp features such as internal waves. The TVDS method does not produce spurious oscillations and is less diffusive than the UDS and it has been shown to be suitable for the modelling of some lakes (Wang, 2001). It should be noted that Gerdes *et al.* (1991) found that vertical grid resolution was a limit on the minimum amount of diffusion that could be simulated by their model. The following tests are, therefore, designed to investigate if the model is sensitive to the method of differencing used for the advective terms.

Three methods of discretization are tested: the central difference scheme (CDS), the upstream difference scheme (UDS) and the total variation diminishing scheme (TVDS). The CDS and UDS methods are simple to implement and computationally inexpensive. They were tested to determine the extent of any problems the discretization method might produce. An alternative method can then be evaluated against these two traditional schemes. The TVDS is chosen because it is easy to implement and Wang (2001) has shown it to be superior to the CDS and UDS in their model of Lake Constance. Also, other

higher order schemes such as the QUICK method are not positive-definite and can still produce some spurious oscillations. In the rest of this section these methods are described and the results of testing the methods on a rectangular basin are given.

For simplicity, when describing the difference schemes, we shall consider only one dimension. Extension to three dimensions is straightforward. We consider the advection-diffusion equation:

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x} \left(\nu \frac{\partial \phi}{\partial x} \right) \quad (3.24)$$

Here ϕ is any transported variable, including momentum, and u is the advection velocity. The source term is ignored. This equation can be written in a discrete, flux-conservative form:

$$\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + \frac{\mathcal{F}_{i+1/2} - \mathcal{F}_{i-1/2}}{\Delta x} = \frac{\mathcal{D}_{i+1/2} - \mathcal{D}_{i-1/2}}{\Delta x}, \quad (3.25)$$

where the time-step index is indicated by the superscript, n , and the subscript, i , signifies the grid point. The advective and diffusive fluxes of ϕ on the grid-box boundaries at $x_{i\pm 1/2}$ are indicated by $\mathcal{F}_{i\pm 1/2}$ and $\mathcal{D}_{i\pm 1/2}$ respectively. The fluxes, $\mathcal{F}_{i\pm 1/2}$ and $\mathcal{D}_{i\pm 1/2}$, will take different forms depending on the discretization scheme used to define them.

In the case of an entirely diffusive equation, the leap-frog method is unconditionally unstable when used with a spatial central difference scheme. The diffusion term, therefore, is advanced using a forward Euler scheme whereas the advection term is advanced using the second-order leap-frog method. Thus the variable, ϕ , is integrated in time as

$$\phi_i^{n+1} = \phi_i^{n-1} - \frac{2\Delta t}{\Delta x} \left(\mathcal{F}_{i+1/2}^n - \mathcal{F}_{i-1/2}^n \right) + \frac{2\Delta t}{\Delta x} \left(\mathcal{D}_{i+1/2}^{n-1} - \mathcal{D}_{i-1/2}^{n-1} \right), \quad (3.26)$$

In the following description of the fluxes the time-step index has been dropped for clarity and the superscript indicates which of the staggered grids the grid index refers to. For example, if ϕ were temperature then u_i^ϕ refers to the value of u at grid point i on the temperature grid, which would be the value of u at $i - 1/2$ on the u -velocity grid.

Essentially, the following discretization methods differ in the method they use for the calculation of the fluxes at the cell boundary; this may be in their estimation of the transport velocity at this boundary or in the estimation of the value of the transported variable. In discrete numerical methods only the values of the cell averages are available so that values at the cell boundaries must be estimated from these using interpolation or averaging.

The central difference scheme

The central difference scheme is usually used for the diffusive fluxes in ocean and lake models and appears to be adequate so it will be used for the diffusion terms in all tests and only the discrete form of the

advection term will be varied. The diffusive flux is given, in the central difference method, by

$$\mathcal{D}_{i+1/2}^\phi = \frac{\nu_{i+1/2}^\phi}{\Delta x} (\phi_{i+1} - \phi_i) . \quad (3.27)$$

The central difference scheme uses a linear interpolation of adjacent cell values to obtain the value at the interface between the two cells and so is a second order method. On a uniform grid, this is just the average of the two adjacent cell values. The advective flux is, thus,

$$\mathcal{F}_{i+1/2}^\phi = u_{i+1/2}^\phi (\phi_i + \phi_{i+1}) / 2 . \quad (3.28)$$

The advection velocities at the boundaries are defined as follows: if ϕ is temperature then

$$u_{i+1/2}^\phi = (u_i + u_{i+1}) / 2 ; \quad (3.29)$$

however, if ϕ is velocity then

$$u_{i+1/2}^\phi = u_i . \quad (3.30)$$

The CDS produces numerical oscillations at regions of high gradient. These 'over shoots', if not smoothed, can lead to inaccuracies and in some cases to instability. The method is also not positive-definite and so can result in negative temperatures and unphysical negative tracer values. These properties make the CDS method unsuitable for some fluid dynamic models.

The upstream difference scheme

The UDS is a first order, positive-definite method that was introduced to avoid some of the problems of CDS. In the UDS the flux at the cell boundary is estimated using only values of the variable upstream of the boundary. This ensures the method is positive-definite and does not generate numerical oscillations. The advective flux under the first order upstream method is

$$\mathcal{F}_{i+1/2}^\phi = \begin{cases} u_{i+1/2}^\phi \phi_i & u_{i+1/2}^\phi > 0 \\ u_{i+1/2}^\phi \phi_{i+1} & u_{i+1/2}^\phi < 0 \end{cases} \quad (3.31)$$

and is implemented as

$$\mathcal{F}_{i+1/2}^\phi = u_{i+1/2}^\phi (\phi_i + \phi_{i+1}) / 2 - \left| u_{i+1/2}^\phi \right| (\phi_i - \phi_{i+1}) / 2 . \quad (3.32)$$

This scheme never produces oscillations but is numerically diffusive as its leading truncation error term resembles a diffusive flux (Ferziger and Peric, 2002).

The total variation diminishing scheme

The upstream method has implicit numerical diffusion. This can damp the real solution resulting in poor ability to simulate features such as internal waves and leads to the reduction of temperature and other tracer gradients.

The central difference scheme produces spurious oscillations in the region of high gradients due to numerical dispersion of higher modes. These oscillations can be smoothed by adding explicit numerical Laplacian diffusion but the diffusion affects the whole solution and is present everywhere not just where it is needed in the region of high gradients.

The total variation diminishing (TVD) condition requires that the sum of variations of the variable over the whole domain does not increase with time:

$$\sum_{i=0}^{N-1} |\phi_{i+1}^{n+1} - \phi_i^{n+1}| \leq \sum_{i=0}^{N-1} |\phi_{i+1}^n - \phi_i^n|. \quad (3.33)$$

Thus spurious oscillations in the value of the variable cannot develop.

The first order upstream method satisfies this condition but is highly diffusive. The aim is to find a method which fulfils the TVD condition, and so does not generate numerical oscillations in regions of high gradient, but one that has second-order accuracy in regions where the solution is smooth, so that numerical diffusion is reduced to a minimum.

The TVD condition is fulfilled by ensuring that the estimation of the value of the variable at the cell boundary is such that fluxes are limited to levels that do not produce oscillations. The UDS reconstructs the distribution of the variable in a first order piecewise fashion using the cell averaged values. The value at the boundary between two adjacent cells can take one of two values; the cell average for either of the two cells adjacent to the boundary. Which of the two values is chosen depends on the sign of the advection velocity. The TVD method also produces two values at the boundary and the sign of the advection velocity dictates which is used. However, the TVDS reconstructs the distribution of the variable in a linear, so maximum of second order, piecewise manner (see Figure 3.5). If the advection velocity is positive the value to the left of the boundary is used and is calculated as

$$\phi_{i+1/2}^L = \phi_i + \frac{1}{2} \Delta x \left. \sigma_i^\phi \right|_x. \quad (3.34)$$

The value to the right of the boundary is defined as

$$\phi_{i+1/2}^R = \phi_{i+1} - \frac{1}{2} \Delta x \left. \sigma_{i+1}^\phi \right|_x. \quad (3.35)$$

The slope limiter, $\left. \sigma_i^\phi \right|_x$, prevents the generation of numerical oscillations and is defined as

$$\left. \sigma_i^\phi \right|_x = \left. \varphi_i^\phi \right|_x (\phi_{i+1} - \phi_i) / \Delta x \quad (3.36)$$

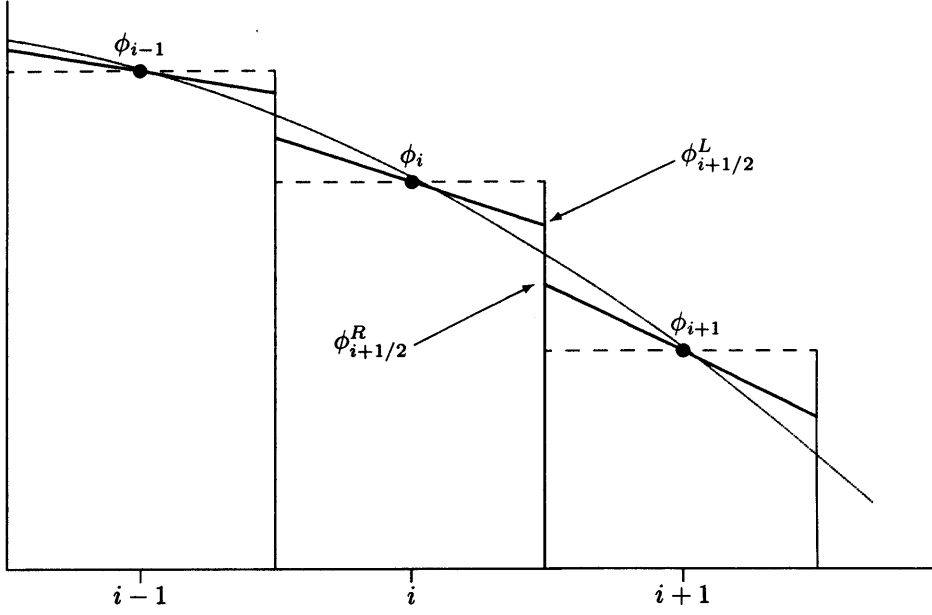


Figure 3.5: The piecewise reconstruction of the variable in the total variation diminishing scheme. The curve is the true value of the variable and the dashed lines represent the cell averaged values.

where φ is a function of θ , the ratio of the gradients in the cells adjacent to the boundary, so that

$$\varphi_i^\phi|_x = \varphi_i^\phi|_x \left(\theta_i^\phi|_x \right) \quad (3.37)$$

and

$$\theta_i^\phi|_x = \frac{\phi_i - \phi_{i-1}}{\phi_{i+1} - \phi_i} . \quad (3.38)$$

The advective fluxes are estimated as

$$\mathcal{F}_{i+1/2}^\phi = u_{i+1/2}^\phi \left(\phi_{i+1/2}^R + \phi_{i+1/2}^L \right) / 2 - \left| u_{i+1/2}^\phi \right| \left(\phi_{i+1/2}^R - \phi_{i+1/2}^L \right) / 2 . \quad (3.39)$$

If we replace $\phi_{i+1/2}^L$ and $\phi_{i+1/2}^R$ with ϕ_i and ϕ_{i+1} in (3.39) we obtain the UDS or if φ in (3.36) is not a function of θ but always has a value of one we obtain the CDS.

For the TVDS to satisfy the TVD condition and be second order, the limiter function, $\varphi(\theta)$, must lie in a certain region of $\varphi - \theta$ diagram (Sweby, 1984). The so-called Superbee limiter,

$$\varphi^{\text{Superbee}}(\theta) = \max(0, \min(1, 2\theta), \min(\theta, 2)) , \quad (3.40)$$

is defined by the upper boundary of the second-order TVD region and is the least diffusive of the second-order TVD limiters.

The total variation method introduces numerical diffusion in regions of high gradient in order to prevent the numerical oscillations. It is thus first order in regions of high gradient but reverts to a higher

order method elsewhere.

The test results

The model used in these tests has twenty-five layers of 10 m thickness hence, five model layers above the thermocline and twenty below. The horizontal eddy viscosity has a value of $2.1 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$. The effect of varying the eddy viscosity and vertical grid resolution is discussed later (§3.2.3 and §3.2.2).

The transient wind forcing sets up an internal seiche. This is a see-sawing, oscillation of the interface between the two layers of different density. Water is piled up at the east end of the lake by the westerly wind and this depresses the interface at this end. Water in the bottom layer flows to the west end of the lake causing the interface here to rise. When the wind stops the interface returns towards the equilibrium position. If the potential energy generated by the wind is sufficient an oscillation of the interface is set up. The oscillation will die away as it is damped by viscous forces. This type of oscillation is the fundamental baroclinic mode and has a wavelength equal to twice the length of the basin. A transect of the u -velocity along the lake is a half-period standing wave. The ends of the lake are nodes; the current velocities at the ends of the lake are zero. At the centre of the lake, the antinode, the horizontal current velocity attains maximum speed and oscillates between eastward and westward directions.

If the model aims to simulate no vertical mixing between the layers then the vertical temperature profile, when the lake has returned to equilibrium, should be the same as before the initial wave impulse. While the seiche exists the thermocline should retain the same gradient but just oscillate with the interface. Any widening of the thermocline will be due to numerical error, more specifically numerical diffusion implicit in the differencing method.

Figure 3.6 shows the vertical temperature profiles at grid cell (30,50) after 99 days when the model lake has returned to equilibrium. The initial profile is also shown for comparison. The CDS generates spurious oscillations in the region of the thermocline. These oscillations have led to the development of an anomalous temperature increase of over 1 °C in the surface layer, in contravention of the second law of thermodynamics. Such temperature anomalies can generate errors in the velocity field through the temperature dependency of the density field. No such oscillations are exhibited by the profile from the model using the UDS. However, numerical diffusion has led to a reduction in the temperature gradient and a consequent thickening of the thermocline from 10 m to over 50 m. It is clear that the TVDS shows the best behaviour. There are no oscillations in the profile from the TVDS model and the diffusion of the thermocline is much less than with the UDS though still more than the CDS.

The diffusion of the thermocline over time can be a problem in lake models as it results in the damping

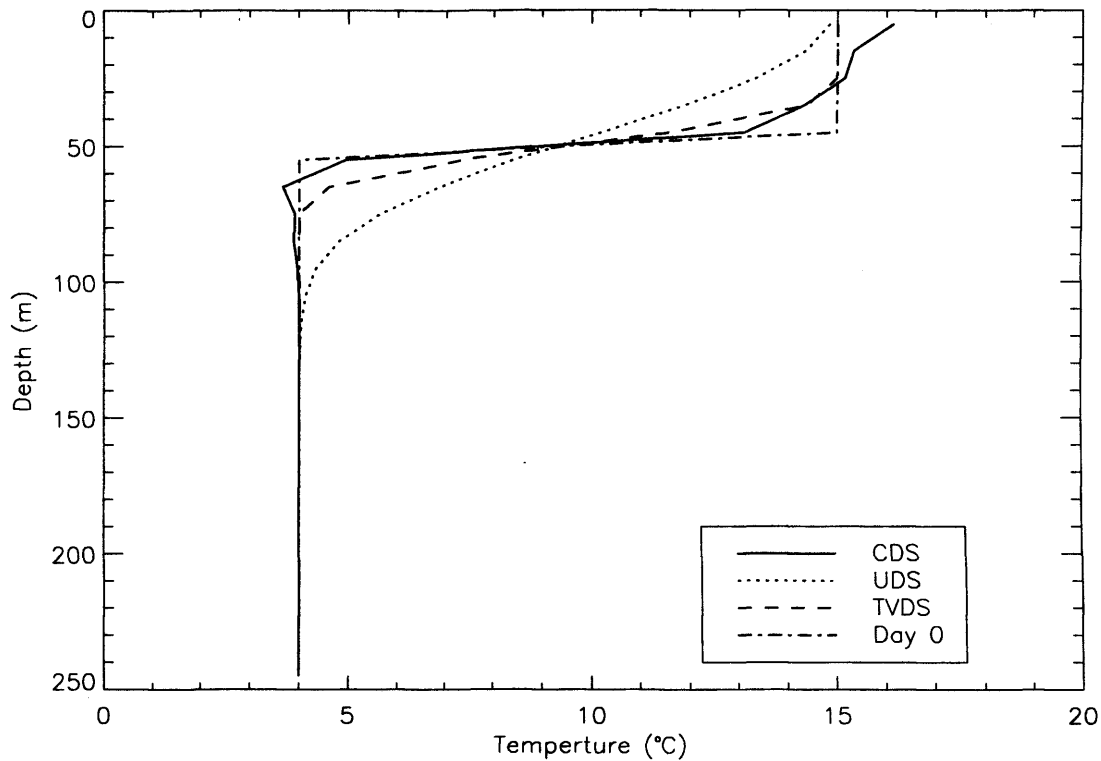


Figure 3.6: The temperature profile at day 0, and after 99 days for models with different discretization methods for the advection of temperature.

of internal waves exhibited by the lengthening of the wave period and reduction of its amplitude (Wang, 2001). This is evident in Figure 3.7, which shows a plot of the u -velocity against time at grid cell (30,50) for each of the models tested. The internal wave is manifest as an oscillation in the u -velocity. In the cases of the UDS and the TVDS the wave amplitude has greatly reduced within one period. In the CDS case the amplitude decays severely within the second period. Although the amplitude decay is most obvious with the UDS and the TVDS the lengthening of the period is more marked with the UDS than with either the TVDS or the CDS. Damping of the wave is expected because of the horizontal eddy viscosity simulated in the model. The increased extent of the damping present in the UDS and TVDS models must be due to implicit numerical diffusion.

A transect of the u -velocity along the lake in the surface layer is shown, for each scheme, in Figure 3.8. A spurious oscillation can be seen in the plot for the CDS. This may arise from differencing in regions of steep gradient in the u -velocity itself or may be produced by anomalous temperatures feeding back into the u -velocity through the pressure gradient. The oscillation is not present in the UDS or TVDS plots. The comparison of the three discretization methods has shown that the CDS does produce spurious oscillations that are not smoothed by the explicit horizontal diffusion. The UDS causes marked diffusion

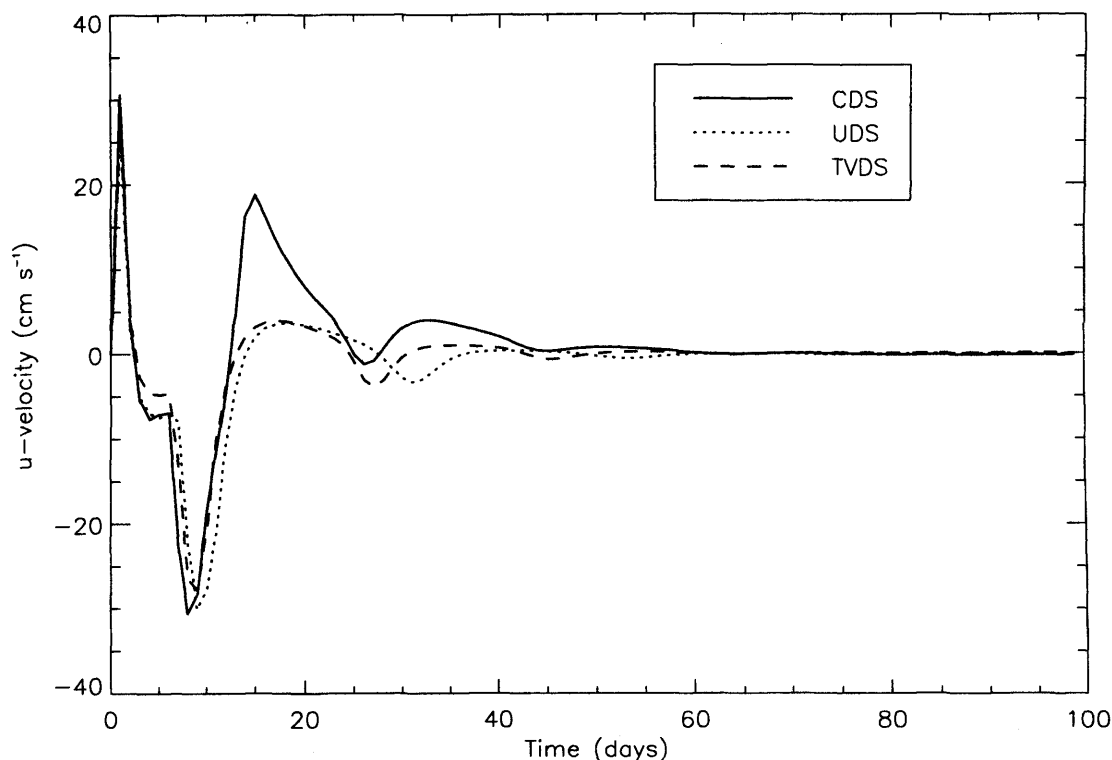
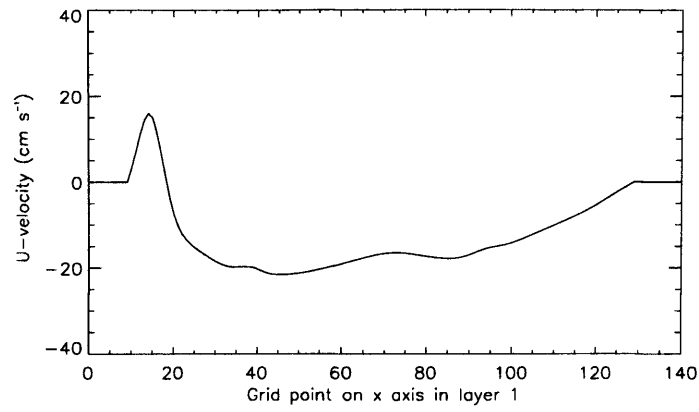


Figure 3.7: The u -velocity in top layer at grid cell (30,50) for models whose advection terms are discretized with the CDS (solid line), UDS (dotted line) and TVDS (dashed line).

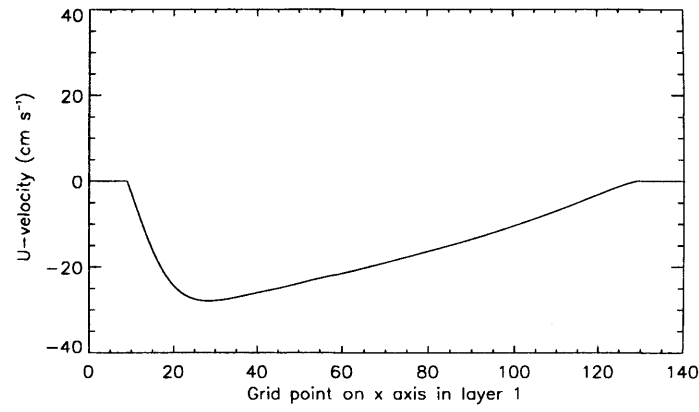
of the thermocline and damping of internal waves. This method cannot be considered suitable for this model unless very short periods of time are to be simulated. As the model is to be used to simulate at least one year a different method must be chosen. The TVDS eliminates the anomolous temperatures produced by the CDS and produces less numerical diffusion than the UDS. The results of these tests suggest that, of the three, it is the most suitable method to use for discretization of the advection terms in the LBM equations.

3.2.2 Vertical grid resolution

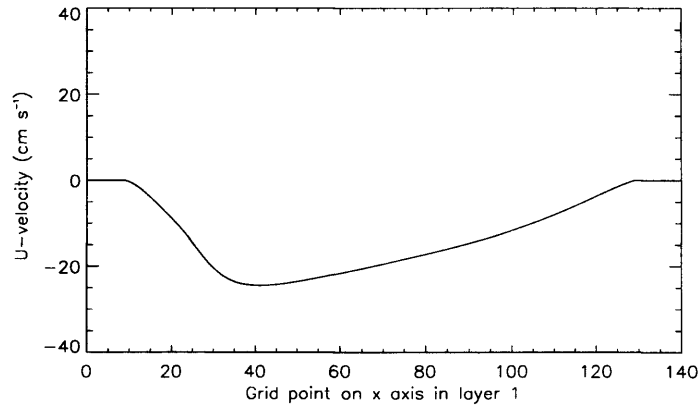
The length scales over which density and current velocity vary are much smaller in the vertical direction than in the horizontal. Hence the vertical grid resolution must be considerably higher than the horizontal grid resolution. Increasing the grid resolution reduces the error due to numerical diffusion and would therefore be expected to reduce the diffusion of the thermocline that was seen in the previous test. The next test was designed to determine the optimum vertical grid resolution within the limits imposed on the computational resources available for this study. The model set up is the same as in the previous section except that the number of layers is varied: four tests are performed with 5, 10, 25 and 50 layers.



(a) central difference scheme

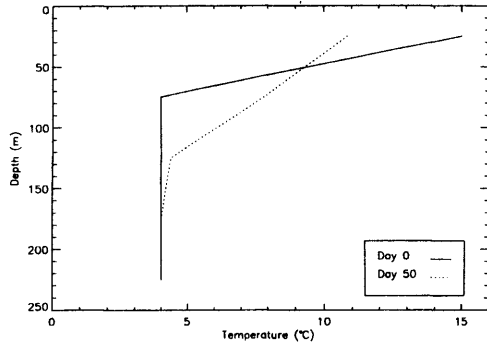


(b) upstream difference scheme

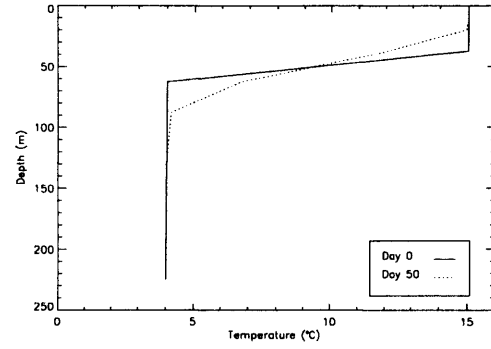


(c) total variation diminishing scheme

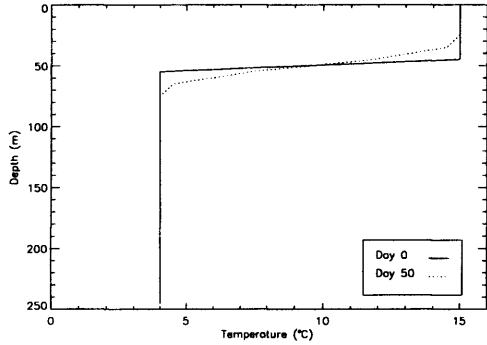
Figure 3.8: Transects of the u -velocity across the test lake after 10 days for models with different discretization methods: (a) CDS; (b) UDS; (c) TVDS.



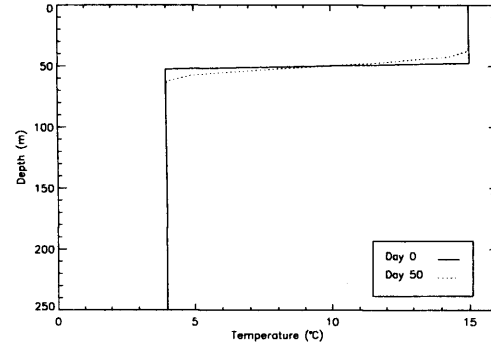
(a) 5 layers



(b) 10 layers



(c) 25 layers



(d) 50 layers

Figure 3.9: Temperature profiles after 0 and 50 days for models with 5, 10, 25 and 50 layers.

The models have layers of equal thickness except for the model with 10 layers which has layer thicknesses that increase in thickness monotonically from the surface downwards in the same manner as those in the model described in Chapter 2. All models use the TVDS for the discretization of the advection terms in the momentum and temperature equations.

Figure 3.9 shows the temperature profile after 50 days for each of the four models. The model with five layers cannot resolve the initial profile and the broadness of the thermocline increases as it becomes more diffuse. The same is true to a lesser extent for the model with 10 layers. The models with 25 and 50 layers are able to resolve the initial profile and after 15 days both exhibit some numerical diffusion of the thermocline but much less so than the other models. Increasing the vertical grid resolution reduces the numerical diffusion of the temperature profile. Figure 3.10 shows the depth of the 10 °C contour at one end of the basin with time. The internal wave can clearly be seen. The wave is severely damped in all cases. This is partly due to the horizontal diffusion required for numerical stability but the amount of damping decreases with increasing vertical resolution, suggesting some part of the damping is due to numerical error. The error due to numerical diffusion is of order $(\Delta z)^2$ whereas the computational cost increases linearly with increasing number of layers. These costs include both the processing time and

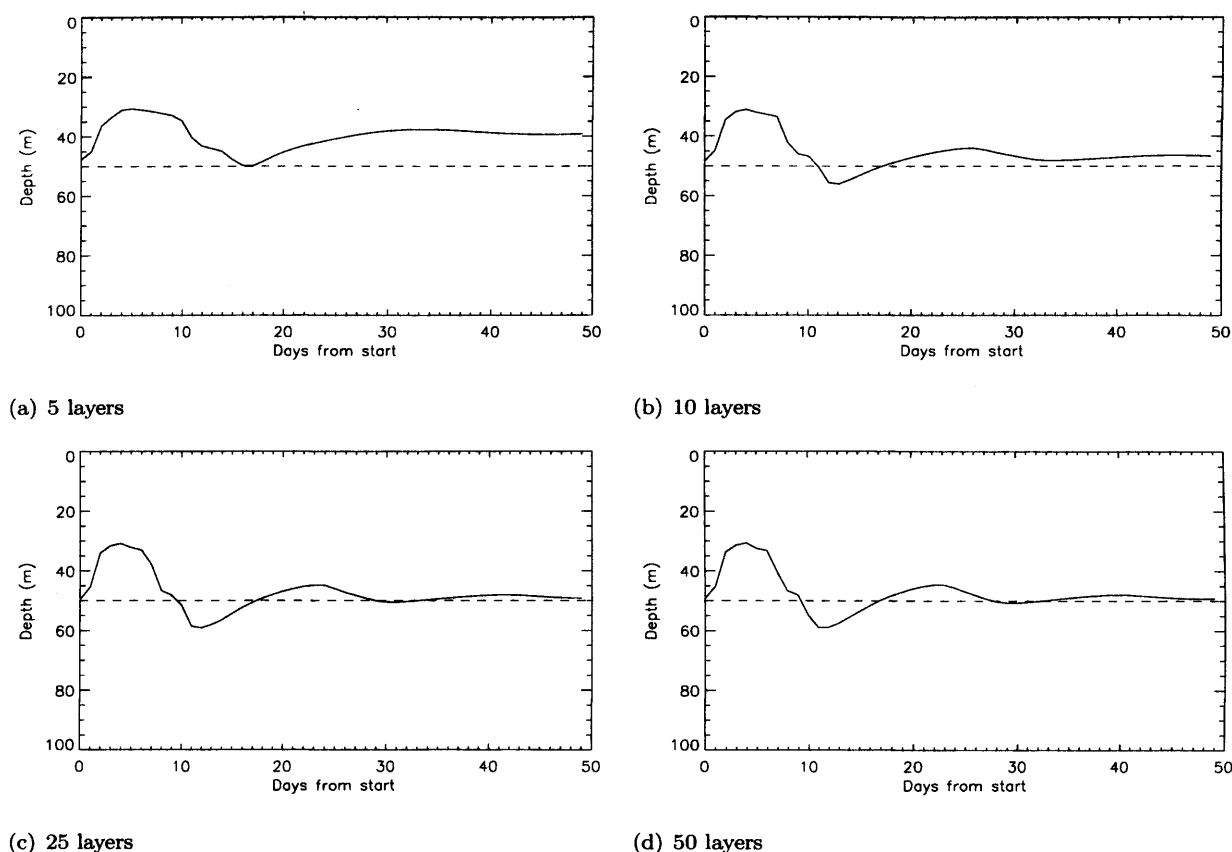


Figure 3.10: The depth of the 10 °C contour with time for models with 5, 10, 25 and 50 layers.

the increased memory requirements. There comes a point at which the cost of increasing the number of layers outweighs the benefits of error reduction. There is a small reduction in numerical diffusion when the number of layers is increased from 25 to 50 but increasing to 50 layers extends the requirement for computational resources beyond that which is available.

3.2.3 Numerical stability and the horizontal eddy viscosity coefficient

The horizontal diffusion term serves two purposes. Primarily it simulates horizontal turbulent mixing, but it is also used to combat numerical instability. The noise generated by high-order schemes in the region of high gradients, the mesh instability that is a feature of the leap-frog scheme and the non-linear instability that results from aliasing can all be smoothed by the addition of a diffusive term to the momentum equations (Philips, 1959; Mesinger and Arakawa, 1976; Press *et al.*, 1992). The value of the horizontal eddy viscosity coefficient should ideally be that which simulates the horizontal turbulent mixing but a higher value may be required for numerical stability.

Model runs were performed with varying values of the eddy viscosity coefficient. The results of runs

using the CDS and runs using the TVDS for the advection terms are shown in Figures 3.11 and 3.12. At the lowest value of eddy viscosity coefficient, $\nu_h = 1.0 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$, the model using the CDS exhibits several spurious oscillations in the u -velocity transect. These oscillations are smoothed by increasing the explicit diffusion but are not eliminated until the eddy viscosity coefficient has been increased to around $5\text{--}8 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$. Unfortunately this amount of diffusion results in considerable damping of the seiche and maximum current speed has been reduced from around 0.35 m s^{-1} to less than 0.1 m s^{-1} . The spurious oscillations are not evident in the transects for the model using the TVDS. However, there is some steepening of the wave profile in the model with $\nu_h = 1.0 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$, which is probably due to non-linear instability. This is not evident when the eddy viscosity coefficient is increased to $2.0 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$, suggesting that this is the optimum value of ν_h for this model. This value lies above the usual range of $0.1 - 10 \text{ m}^2 \cdot \text{s}^{-1}$ found in lakes (Cheng *et al.*, 1976) but is comparable with the LHLC model which has a similar horizontal grid size. This indicates that the eddy viscosity is determined by the model grid rather than the flow field which is the case with many ocean and lake models in the literature.

3.2.4 Summary

The numerical testing indicates that, of the schemes tested, the TVD method is the most suitable for the discretization of the advection terms in this model. It allows no oscillations at regions of high gradient while keeping numerical diffusion to acceptable levels. An horizontal eddy viscosity of $\nu_h = 2.0 \times 10^3 \text{ m}^2 \cdot \text{s}^{-1}$ is required to maintain numerical stability. A vertical grid of 25 layers of 10 m thickness is adequate to resolve the temperature gradient modelled in the tests while resulting in an acceptable computation time.

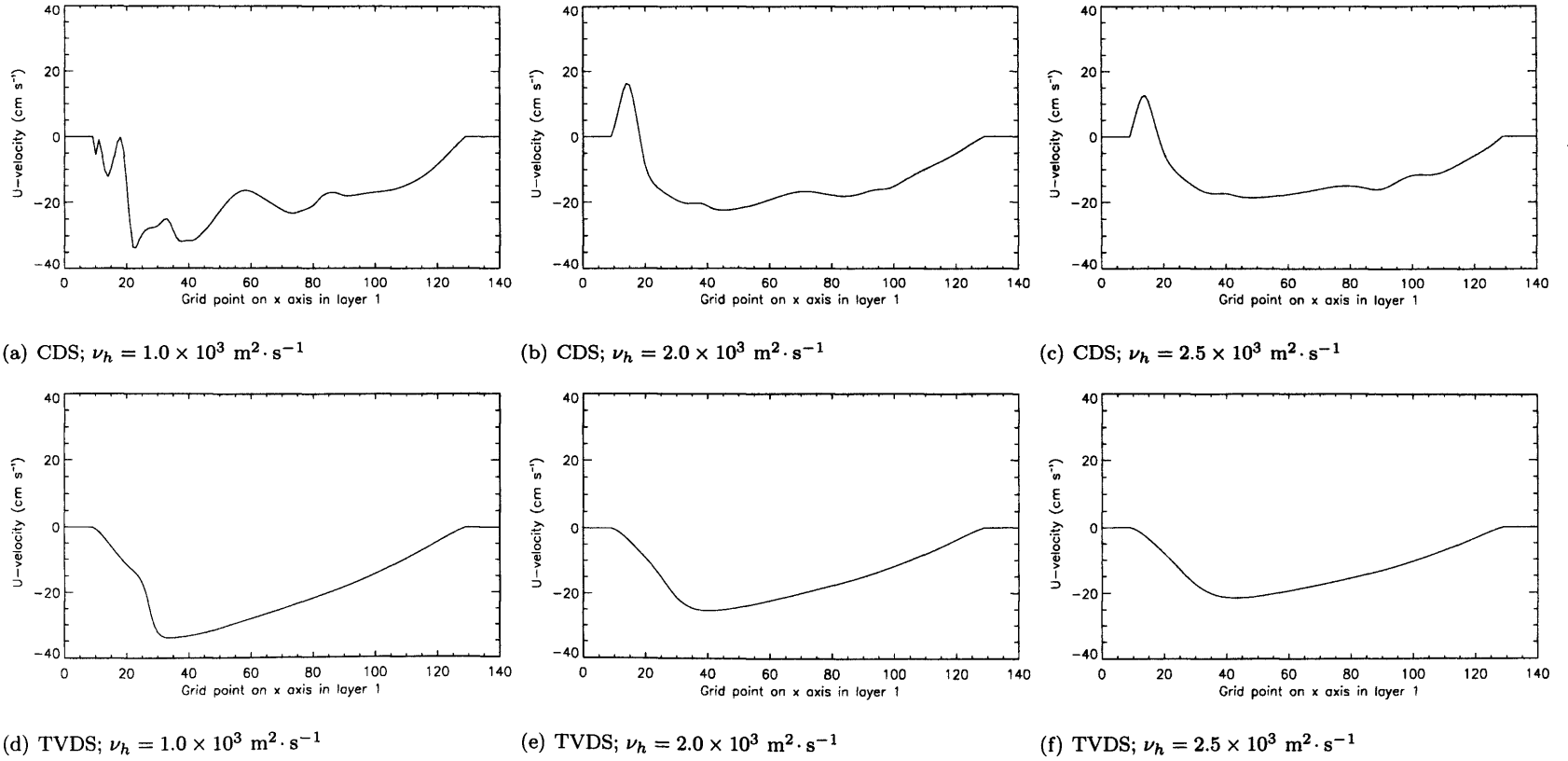


Figure 3.11: Transects of the u -velocity across the lake after 10 days for models with different horizontal viscosity coefficients: CDS = central difference scheme; TVDS = total variation diminishing scheme.

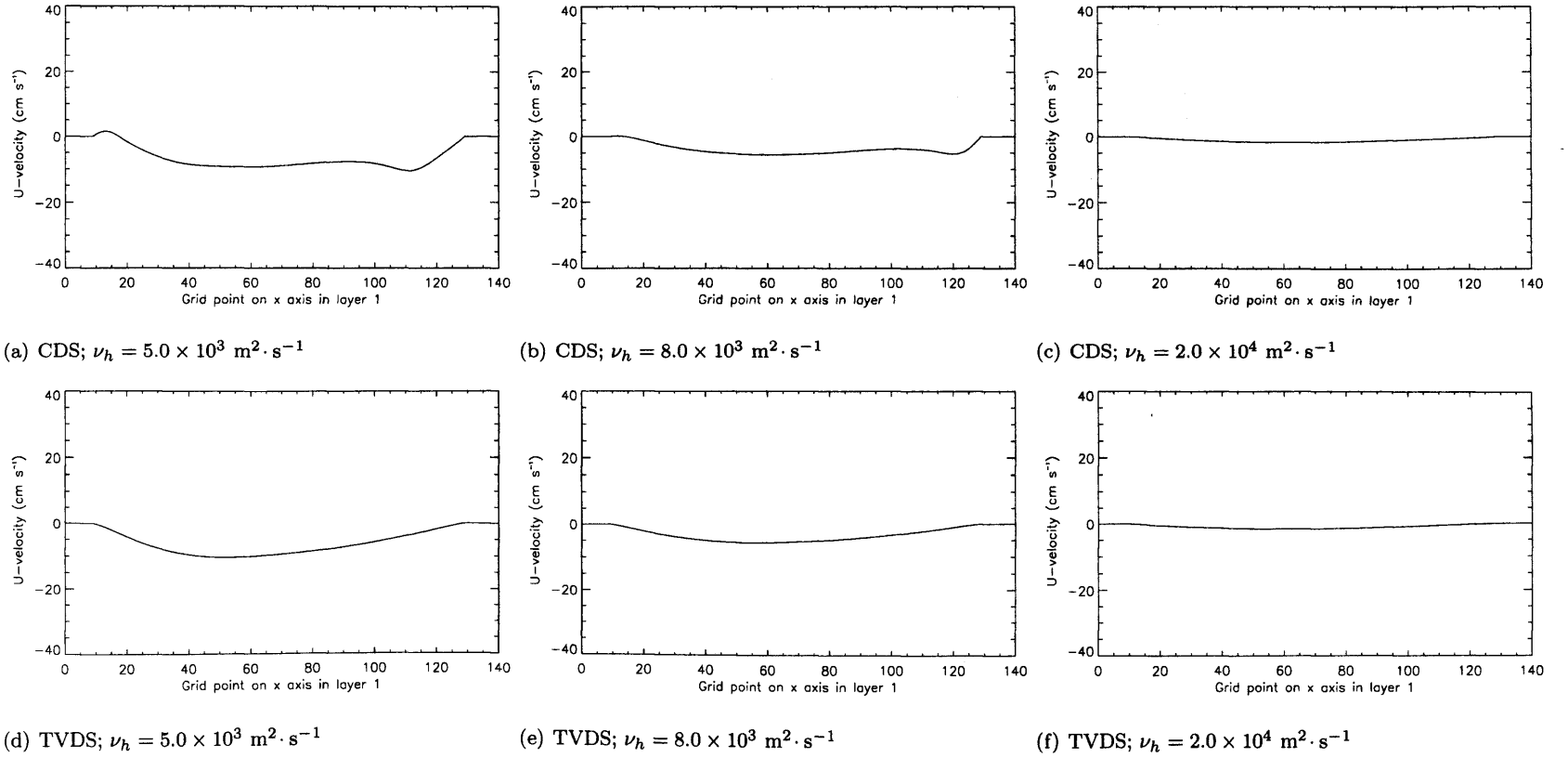


Figure 3.12: Transects of the u -velocity across the lake after 10 days for models with different horizontal viscosity coefficients: CDS = central difference scheme; TVDS = total variation diminishing scheme.

Chapter 4

The LBM: Thermal Forcing

Currents in Lake Baikal are the result of various forces acting on the lake. These forces are due to processes such as heating and cooling, the action of winds on the surface, and atmospheric pressure gradients across the lake. The frictional forcing of the lake by winds and its implementation in the LBM model are discussed in § 5.4. Here we examine the parameterization of the thermal forcing of Lake Baikal.

The growth rates of phytoplankton, including that of the diatom *Aulacoseira baicalensis*, depend on water temperature and light level. The light level experienced by a plankter will depend on the clarity of the water and on the plankter's depth. If the plankter is non-motile, as is the case with *A. baicalensis*, then it is entirely dependent on the vertical water currents for its vertical position. The model should, therefore, ideally reproduce the main features of both the water temperature and the vertical currents.

Changes in water temperature are largely brought about by the flux of heat across the lake's surface and the vertical transfer of heat by free convection and wind mixing. The rate of exchange by free convection will again depend on the heating or cooling at the surface. The annual periodic heating and cooling and the mixing which is associated with it are largely responsible for the dimictic nature of the lake and the seasonal temperature cycle that were described in § 1.1.3. Thus, it is important to choose the most suitable method of implementing the surface heat flux and to determine the sensitivity of the model to variations in this heat flux.

As well as vertical motions horizontal differences in heating rates may bring about horizontal temperature gradients and hence horizontal water movement. Spatial differences in the snow and ice cover cause spatial differences in the heating of the water below. The amount of incident radiation that is reflected at the upper surface of the ice depends on the type of ice and the presence or absence of snow. The type of ice also dictates its extinction coefficient and hence the amount of radiation that is able to

penetrate the ice-cover and heat the water underneath. These spatial differences in the heating rates of water under ice produce horizontal pressure gradients and have been proposed as a mechanism for the production of under-ice currents and the increase in current velocities at the border of areas with different thicknesses of snow cover (Zhdanov *et al.*, 2001). Adequate parameterization of the heat fluxes is essential if these under-ice currents are to be accurately simulated.

In order to model the lake's thermodynamics adequately, it is necessary that any parameterization of the lake's thermal forcing reproduces realistic temporal and spatial variation in the heat transfer across the lake's boundaries. The majority of the exchange of heat between the lake and its environment occurs in the uppermost layer of the lake. In this chapter we examine the parameterization of the surface heat fluxes. The first section describes these heat fluxes. Section 4.2 describes some observational data that will be use to compare with model predictions. The following section describes four model runs which use different parameterizations of the thermal forcing. The final section summarizes the conclusions drawn from the results of these four runs.

4.1 Processes that transfer heat between the lake and its surroundings

The inter-annual variation in the heat budget is determined by the seasonal changes in the incident solar radiation, the local climate, and the mixing of heat away from the surface into the interior of the lake. The lake absorbs radiation from the sun and atmosphere and in turn emits radiation towards the atmosphere. Heat exchange at the air-water interface also occurs by sensible and latent heat processes. The relative importance of the different heat transfer processes at Lake Baikal and some estimates of the fluxes involved are discussed below. The data and equations used are taken from Shimaraev *et al.* (1994) unless otherwise stated.

Assuming that an energy flux is positive if it results in an increase in the energy content of the lake and negative if it results in a decrease in the lake's energy content, the heat budget of a lake can be described by

$$Q = Q_S + Q_B + Q_L + Q_H + Q_I + Q_R + Q_G \quad (4.1)$$

where Q is the net heat change in the lake, Q_S is the absorbed shortwave radiation flux, Q_B is the balance of long-wave radiation, Q_L is the latent heat flux, Q_H is the sensible heat flux and Q_I is the heat of ice crystallization and melting. Of lesser importance in Lake Baikal are the heat fluxes that accompany river influx and drainage, Q_R , and heat fluxes from geothermal sources, Q_G .

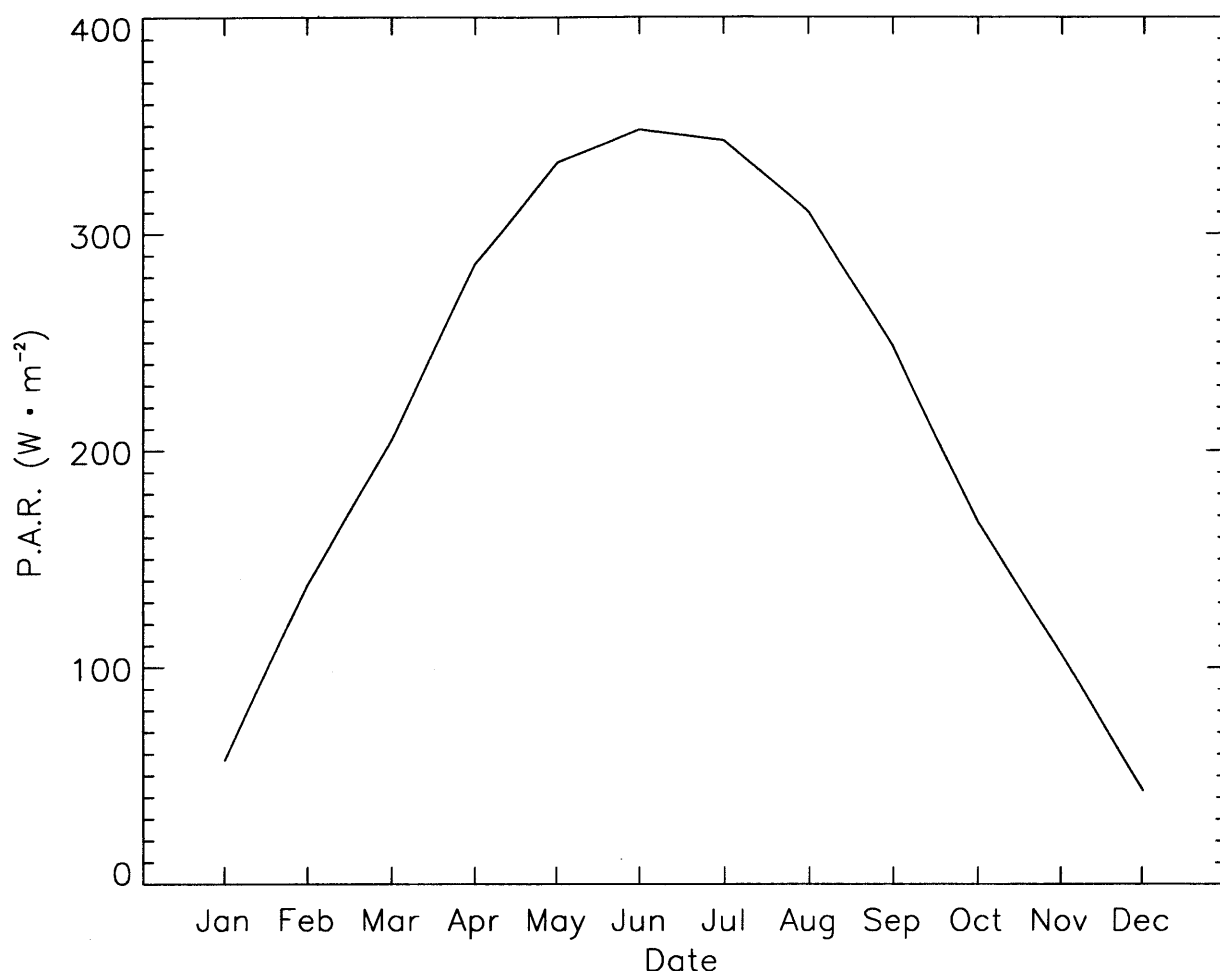


Figure 4.1: Annual variation in direct photosynthetic available radiation (P.A.R.) as observed on the 15th day of each month. Re-plotted from Shimaraev *et al.* (1994).

The annual variation in the incident solar radiation is illustrated by the monthly observations of direct photosynthetic available radiation (PAR) on clear days shown in Figure 4.1. The actual shortwave radiation incident on the lake may be reduced by cloud cover. The amount of the incident radiation that penetrates the lake rather than being reflected depends on the surface albedo. The albedo in turn depends on the nature of the surface: the roughness of the water surface or the nature of the ice or snow cover.

The annual integrated irradiance incident on the surface of the lake varies from $4050 \text{ MJ} \cdot \text{m}^{-2}$ in the North to $4600\text{-}4700 \text{ MJ} \cdot \text{m}^{-2}$ in the South. $3730 \text{ MJ} \cdot \text{m}^{-2}$ penetrates the surface over a year, and $1300 \text{ MJ} \cdot \text{m}^{-2}$ of this passes through the ice, during the period January through to May.

Long-wave radiation is emitted by both the lake, Q_W and the atmosphere, Q_A . The balance of long-wave radiation is usually calculated using an empirical formula which takes into account the temperature

of the lake surface, the temperature and humidity of the air above the surface and the frequency of cloud cover (Gill, 1982; Hutchinson, 1957, and references therein). The effective back radiation flux, $-Q_B$, for Lake Baikal can be estimated using the formula:

$$-Q_B = 0.91DT^4 (0.41 - 0.05e^{0.5}) (1 - 0.5n_t + 0.14[T_{water} - T_{air}] - 0.28) \quad (4.2)$$

where $D = 3568 \times 10^{-9} \text{ kcal} \cdot \text{cm}^{-2} \cdot \text{month}^{-1} \cdot \text{K}^{-4}$, T is the absolute air temperature, e is the vapour pressure (mb); n_t is the total frequency of cloud cover, $[T_{water} - T_{air}]$ is the difference between water and air temperature in centigrade.

The effective back radiation flux is highest in late Autumn during the months preceding ice formation. At this time the air temperature is much lower than the water temperature. The lowest fluxes occur during the Summer months (June and July) when the air temperature is higher than the water temperature. The annual average long-wave heat loss is $1820 \text{ MJ} \cdot \text{m}^{-2}$.

The annual radiation balance for the lake is $1910 \text{ MJ} \cdot \text{m}^{-2}$. The seasonal variation in the radiative balance is dominated by changes in the absorbed shortwave radiation.

The monthly heat fluxes due to various thermal and radiative processes are illustrated in Figure 4.2 which has been replotted from Shimaraev *et al.* (1994). Sensible heat transfer is the process of heat transfer by conduction and convection. During the Winter, when the lake is warmer than the atmosphere, heat is transferred to the air from the lake by the process of conduction and this heat may then be transferred away from the interface by atmospheric convection. The sensible heat flux depends then on the temperature gradient at the water surface and on the stability of atmosphere. In an unstable atmosphere, convection acts to maintain the temperature gradient at the water surface, so that in Winter heat from the lake is mixed away from the air/water interface. The sensible heat flux is positive in the Summer when the atmosphere is warmer than the lake; the average air temperature over the lake during the Summer months (June, July and August) is 9.4°C , while the average Summer temperature of the pelagic surface in August ranges from 6.03°C in the North Basin to 7.35°C in the South Basin. In Winter the sensible heat flux is negative; the average Winter air temperature over the lake is -15.8°C , while the lake surface is frozen.

The latent heat flux, heat transfer due to evaporation or condensation, depends on air humidity, temperature and wind conditions. Both sensible and latent heat transfer fluxes can be calculated with empirical formulae similar to equation (4.2). However, in Figure 4.2 the sensible and latent heat fluxes have been estimated using the heat balance equation (4.1) and Bowen ratios. The Bowen ratio is the ratio of sensible heat transfer to latent heat transfer. It can be estimated from measurements of air and water temperature, saturated vapour pressure at the interface and atmospheric vapour pressure at

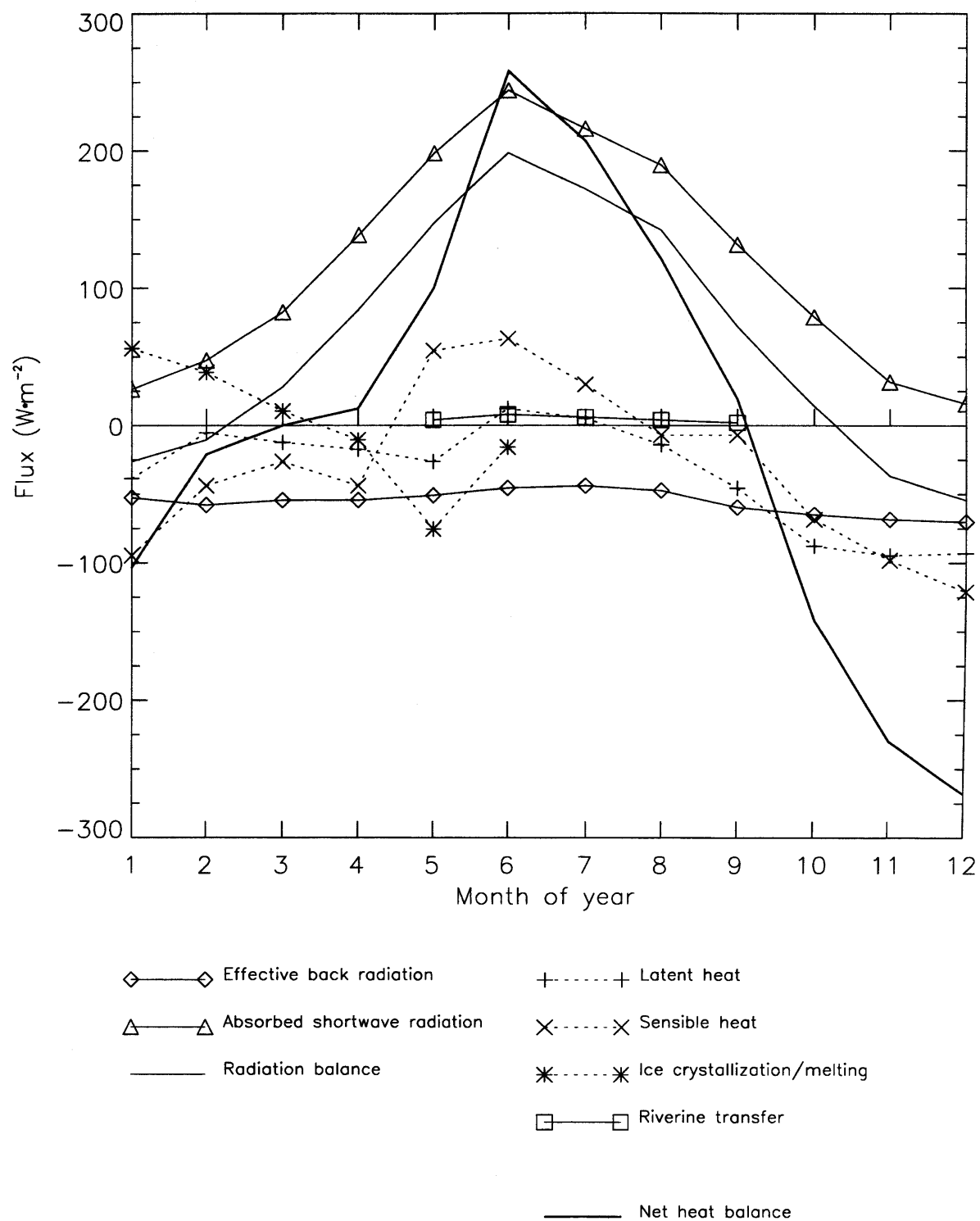


Figure 4.2: Average heat balance of Lake Baikal surface for the period 1896-1973. Re-plotted from Shimaraev *et al.* (1994).

a reference height above the water surface. The Bowen ratio, β , is then given by Lewis (1995) as

$$\beta = \frac{Q_H}{Q_L} = \frac{c_p(T_1 - T_2)P}{L(e_1 - e_2)}, \quad (4.3)$$

where C_p is the molar specific heat of air, and T_1 and T_2 are the temperature at the water surface and the air temperature at a reference height respectively. P is the atmospheric pressure, L is the molar specific latent heat of vaporization, e_1 is the saturated water vapour pressure at the temperature of the water surface, and e_2 is the water vapour pressure at the reference height.

The annual budget for latent and sensible heat transfer together is a net loss of $1940 \text{ MJ} \cdot \text{m}^{-2}$. There is a heat gain of approximately $360 \text{ MJ} \cdot \text{m}^{-2}$ during the Summer (May-July) and a heat loss of $2300 \text{ MJ} \cdot \text{m}^{-2}$ throughout the rest of the year.

Other processes contribute smaller heat fluxes to the total heat budget. The crystallization and melting of ice is associated with latent heat exchange. Ice starts to form first in the North Basin, during December and reaches maximum thickness in March. From late March to early April the ice starts to melt and breakup under the influence of solar radiation. Ice formation and break up is associated with a latent heat flux of approximately $280 \text{ MJ} \cdot \text{m}^{-2}$. The timing of ice-formation and of ice-melting, and the duration of ice-cover vary considerably from year to year under climatic variations (Shimaraev *et al.*, 1994). The net heat flux due to river inflow and drainage and precipitation per annum is $30 \text{ MJ} \cdot \text{m}^{-2}$. Lake Baikal is a rift valley lake and the rift is still active. There is a small geothermal heat flux of no more than $2\text{-}3 \text{ MJ} \cdot \text{m}^{-2}$ per annum through the deep lake bed.

4.2 Observed data

Two sets of temperature observational data will be used for comparison with the model runs. Jewson (1997) recorded temperature profiles in the South Basin on several dates between December 1996 and September 1997. This period overlaps with that of the LBM which runs from 13 March 1997 to the end of December 1997. Measurements were taken every 5 m between a depth of 5 m and the bottom. The data, henceforth referred to as the Jewson data, is plotted in Figure 4.3 for a depth range of 0-250 m. A second set of data was reported by Shimaraev *et al.* (1994) and consists of averages over several data sets from different years, for each of the three lake basins. One temperature profile is given for March and each of the months of June through December. The gap in the data between March and June and from December to March is due to the obvious difficulties in obtaining data from an ice-covered lake except when the ice is at its thickest in March. This data set has a lower sampling resolution than that of Jewson. Between 0 and 250 m temperatures are given at 0, 5, 10, 25, 50, 100, 150, 200, and 250 m.

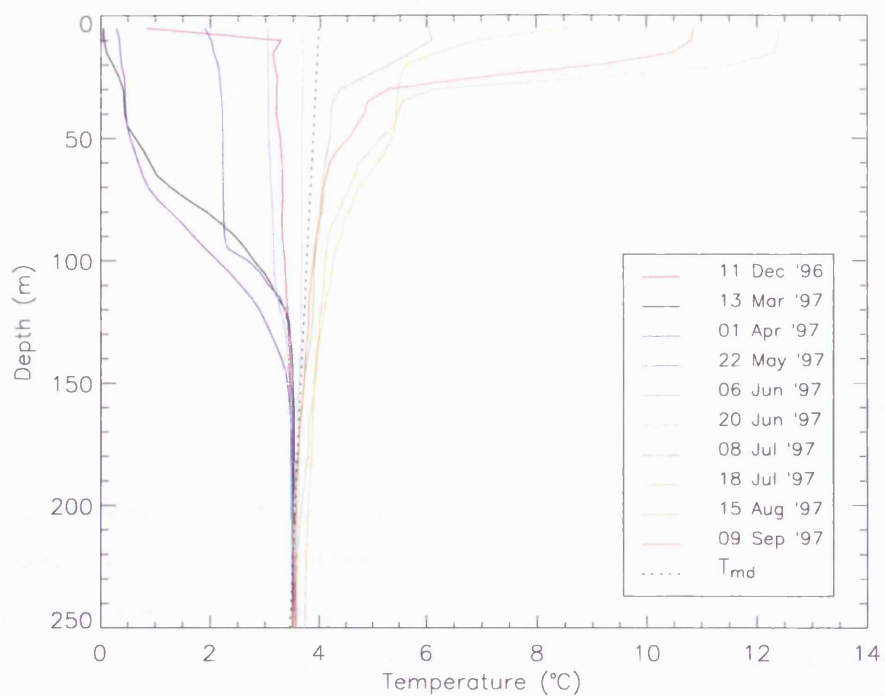


Figure 4.3: Observations of temperatures in the South Basin during 1996-97 (Jewson, 1997).

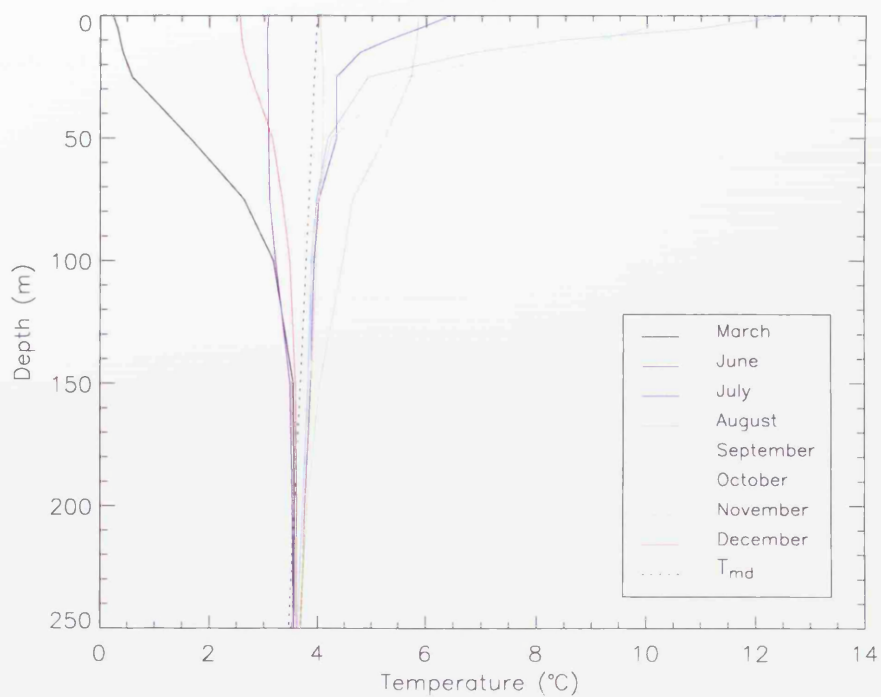


Figure 4.4: BICER data: monthly average temperatures in the South Basin, compiled from data collected over several years (Shimaraev *et al.*, 1994).

The data for the South Basin, which will be referred to as the BICER data is plotted for a depth range of 0-250 m in Figure 4.4.

Both data sets show the seasonal cycle of warming of the surface layer during the Spring and Summer with subsequent cooling during the Autumn and Winter. There are, however, some differences evident between the two data sets. The March profile has a deeper cold surface layer in the Jewson data; the temperature is below 1 °C down to ~ 65 m whereas in the BICER set the temperature exceeds 1 °C at ~ 35 m. Both data sets suggest the surface temperature warms past the T_{md} during late June or in early July. The Jewson data, however, shows the development of an epilimnion 30 m deep in the Summer of 1997 compared with 10-15 m in the averaged data. Both data sets show the Summer maximum temperature occurring in late August/early September. The BICER data shows the surface cooling past the T_{md} in November. The Jewson profile for December 1996 exhibits a narrow, cold epilimnion overlying a warmer hypolimnion; this cold layer is not evident in the BICER profile for December which displays a fairly well-mixed water column. The differences in the two data sets suggest the annual temperature cycle may exhibit considerable year-on-year differences. In conclusion, these data sets may be of value in validating model simulations but we must recognize that they contain uncertainties and that a model, which uses inputs some of which have been averaged over several years and some which are specific to one year, cannot be expected to reproduce either the averaged BICER data set or the 1997 Jewson data set.

4.3 Thermal Forcing Tests

This section describes four model runs that use different methods for modelling the external heat source. The first two use a time-varying, spatially constant surface heat flux. The other two runs implement a modification of the method of the LHLC model. The LHLC model allows for the spatially varying penetration of shortwave radiation through a spatially varying ice-field and uses the Beer-Lambert law to calculate the absorption of the radiation by the water column. A modification, used in the LBM model runs LBM-C and later, is the addition of a surface heat flux simulating the flux due to heat transfer processes other than the absorption of shortwave radiation.

4.3.1 LBM-A: A sinusoidal surface heat flux

The solar radiation incident on Lake Baikal is almost completely absorbed (99%) in an upper layer of 40m or less (Shimaraev *et al.*, 1994). It is therefore possible to consider the external heating as a surface heat flux with a positive maximum in the Summer and a negative minimum in the Winter. Killworth

et al. (1996) used a surface heat flux that varied sinusoidally with time to thermally force their two dimensional model of Lake Baikal (henceforth referred to as the Killworth model). The magnitude and phase of heating were approximations of Hutchinson's (1957) data for the Summer and Winter heat budgets of Lake Baikal. The heat flux variation with time was described by

$$Q(t) = 274 \sin(2\pi t / \text{yr}) \text{ W} \cdot \text{m}^{-2} \quad (4.4)$$

with $t = 0$ at the time of zero heat flux on 11 March. There is no spatial variation of the heat flux. The time variation of the heat flux in equation (4.4) is plotted along with the net surface heat balance described by (Shimaraev *et al.*, 1994) (also see Figure 4.2) in Figure 4.5(a). Clearly equation (4.4) fails to model the insulating effect of the ice layer (January-May) but is a good fit to the net heat balance during the ice-free months (June-December). The difference between the heat fluxes calculated with equation (4.4) and the net heat balance values is plotted in Figure 4.5(b). The sinusoidal function results in too much heat loss during the under-ice cooling period (from the beginning of January to the middle of March) and too much heat gain during the under-ice warming period (from the middle of March to the end of May).

The LBM model run LBM-A uses a similar formulation to the Killworth model. The heat flux for the middle of each month has been calculated according to equation (4.4). These values are then input to the model which calculates a value for each day by linearly interpolating between the monthly values.

The surface heat flux is implemented in the model as a boundary condition on the turbulent heat transfer. The change in temperature due to turbulent heat exchange is:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(\kappa_z \frac{\partial T}{\partial z} \right) . \quad (4.5)$$

The condition at the surface is

$$\kappa_z \frac{\partial T}{\partial z} \Big|_{z=\eta} = \frac{Q}{c_p \rho_0} , \quad (4.6)$$

where Q is the heat flux, c_p is the specific heat of water and ρ_0 is a mean density. The heat fluxes, Q , vary with time.

As in the Killworth model freezing is implemented in the following manner. At the end of each time-step if the temperature is less than zero it is set equal to zero. Ice is assumed to isolate the water below from the frictional action of the wind and so grid squares with temperatures less than or equal to 0 °C do not experience wind forcing.

The wind forcing is implemented as a surface stress in the momentum equations (see § 3.1 and § 5.4). The wind velocity field is spatially uniform however the magnitude of the wind velocity varies

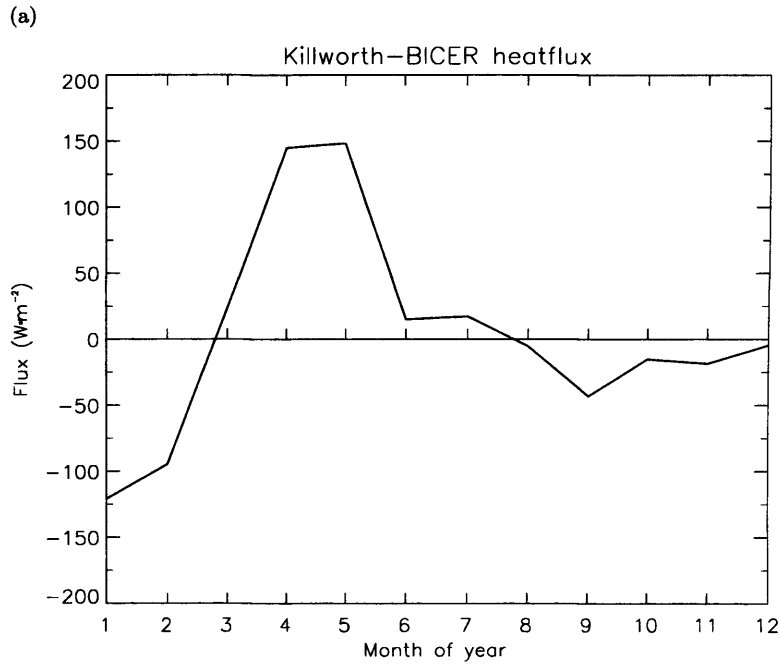
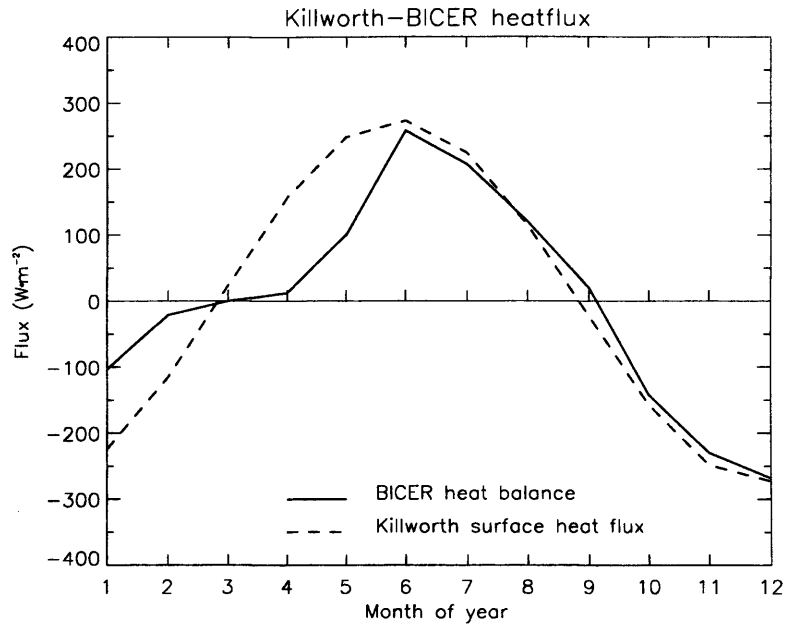


Figure 4.5: Surface heat fluxes. (a) Monthly values of the surface heat fluxes used as inputs for the LBM model run LBM-A (dashed line) and the monthly average values of the net heat balances for Lake Baikal (solid line) as given by Shimaraev *et al.* (1994) and used in model run LBM-B. (b) The difference between the fluxes shown in (a), (LBM-A) - (LBM-B)

periodically with a maximum speed of 4 m s^{-1} and a period of 3 days according to:

$$\mathbf{u}_{wind} = 4 \cos^2 [2\pi t / (3 \text{ days})] \text{ m} \cdot \text{s}^{-1} . \quad (4.7)$$

The wind direction changes randomly every 6 days in order to mimic the conditions of unstable winds that predominates for most of the year over Lake Baikal (Shimaraev *et al.*, 1994).

The initialization of the LBM model is described in § 3.1: the vertical temperature is set to the vertical temperature profile observed by Granin (1997b) on 13 March 1997 in the South Basin; the current velocity is set everywhere to zero. The model grid, domain and equations are as described in § 3.1.

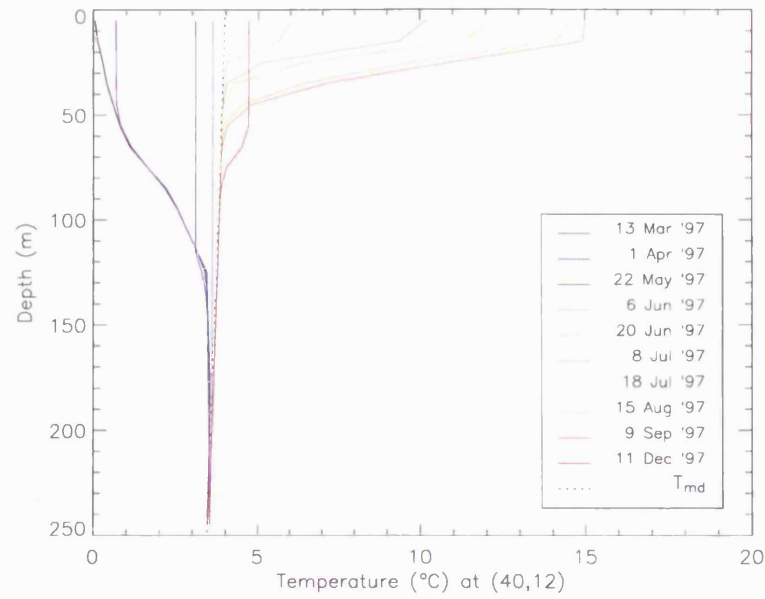
The Figures 4.6(a) and 4.6(b) show temperature profiles, for model run LBM-A, from a point in the middle of the South Basin that corresponds to the site of the observations plotted in Figures 4.3 and 4.4. The profiles in Figure 4.6(a) are from model days matching the dates of Jewson's observations and Figure 4.6(b) profiles are taken from model days corresponding to the 15th day of each month of the BICER observations. Also shown in Figure 4.6(b) are profiles for the months of April and May 1997, and January and February 1998. These are included to illustrate model behaviour during the ice-covered period when no observations are available. The dimictic seasonal cycle of warming and cooling is apparent.

The model is initialized with an observed March profile which exhibits a negative stratification with the most rapid change in temperature occurring at 60-90 m. At this time the whole lake is covered in ice and temperatures in the upper layer are near freezing. From around 10 m to 125 m there is a strong increase in temperature towards the T_{md} . Below ~ 125 m the water column has a near uniform temperature of around the T_{md} .

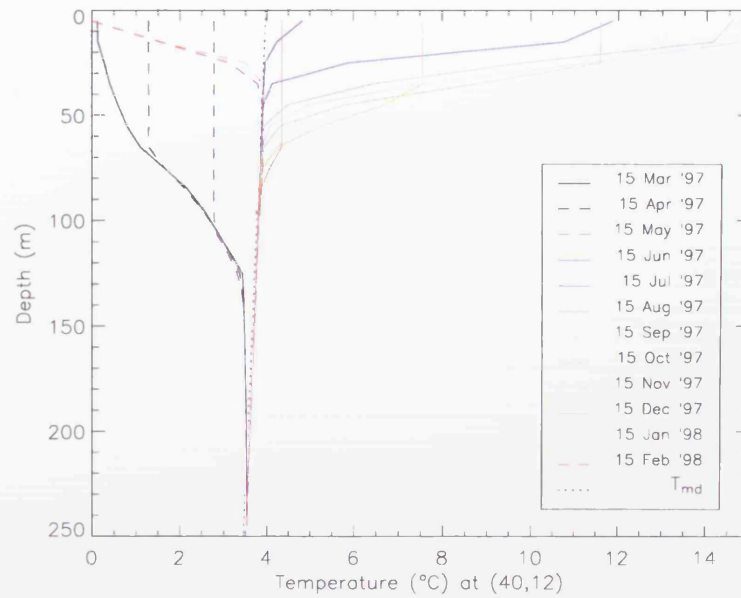
At the time of initialization the applied surface heat flux is near zero but then increases to a maximum in the middle of June. As a result, the temperature in the surface layer increases producing a positive density gradient and an unstable water column. The convective adjustment routine exchanges adjacent unstable layers thereby creating a mixed layer. This mixed layer progressively deepens as the model lake warms, until the whole water column has a uniform temperature of T_{md} .

Warming of the surface layer accelerates after its temperature has reached T_{md} . This is because there is now a stable temperature gradient and the convective adjustment routine will no longer mix heat down into lower layers. The heat is, therefore, now retained in the surface layer. This reflects the situation in the real lake where, once the surface temperature exceeds the T_{md} , a stable, positive stratification develops and free convection no longer takes place.

As heating continues during the Summer months, a warm shallow layer with a positive temperature



(a)



(b)

Figure 4.6: Temperature profiles for model run LBM-A : The heat source is a surface heat flux which varies sinusoidally with time. The profiles are located at grid point (40,12) in the South Basin. The profiles in (a) are for the same dates as the Jewson data shown in Figure 4.3. The profiles in (b) are for the 15th of each month of the BICER observations in Figure 4.4. Model profiles (dashed lines) for the months of April and May 1997, and January and February 1998 are also shown for completeness.

gradient develops. Below this the water has a uniform temperature equal to the T_{md} . These two distinct layers represent a warm epilimnion above a colder hypolimnion.

Once the surface heat flux changes sign the surface layer begins to cool and the water density here increases, again producing an unstable density gradient and the convective adjustment routine starts to mix the upper layers. During this time, model turbulent mixing, enhanced by the action of winds, may also contribute to the development of the mixed layer. As Autumn cooling progresses the mixed layer increases in depth until the whole column again has a uniform temperature of T_{md} .

When the upper layers have cooled past T_{md} , the density gradient is again stable and the convective adjustment routine no longer mixes the upper layers. An upper cold layer develops with temperatures reaching freezing point at the surface. The deeper layers remain at the T_{md} .

While the model qualitatively reproduces the seasonal cycle there are some quantitative differences between the model and the observations. The model lake warms too fast under ice. This might be expected as the model surface heat flux exceeds that of the BICER calculated heat flux during this time (see Figure 4.5(a)).

The June 15th profile (Figure 4.6(b)) shows that, in the upper layers, the model temperature has already warmed past the T_{md} ; whereas the BICER profile for June shows the water column has a temperature of 3.1 °C, colder than T_{md} , down to 80-90 m. Below this, the lake temperature increases with depth to a temperature close to the T_{md} at 150 m.

The Jewson data also indicates that, at this location in the lake, the water column warms more slowly than the model. The observations taken on 1 April 1997 suggest that the lake is still cooling at this time and that the surface heat balance has not yet become positive. The model lake, however, has a positive heat balance. The model profile shows heat has been mixed down into the lake resulting in the creation of a surface mixed layer approximately 50 m deep, with a temperature of 0.7 °C. By 22 May the model predicts this mixed layer will have a depth of 110-120 m and a temperature of 3.1 °C whereas Jewson's data for the same day reveals a mixed layer of 90-100 m depth and a temperature of 1.9-2.2 °C.

The observed data for 6 June show a mixed layer of 115-125 m with a temperature of 3.0-3.3 °C whilst the model mixed layer extends to the bottom of the model indicating that mixing has occurred throughout the water column. The column has a near uniform temperature of ~ 3.6 °C. The real lake has nearly reached this condition by 20 June when the observations show that the upper 250 m has a near uniform temperature, varying by only 0.08 °C between 3.67 °C at the surface and 3.59 °C at 250 m.

The model upper layer surface temperature reaches the T_{md} on 13 June (see Figure 4.7). Once the model temperature exceeds the T_{md} in the upper layer, a stable density gradient exists and the

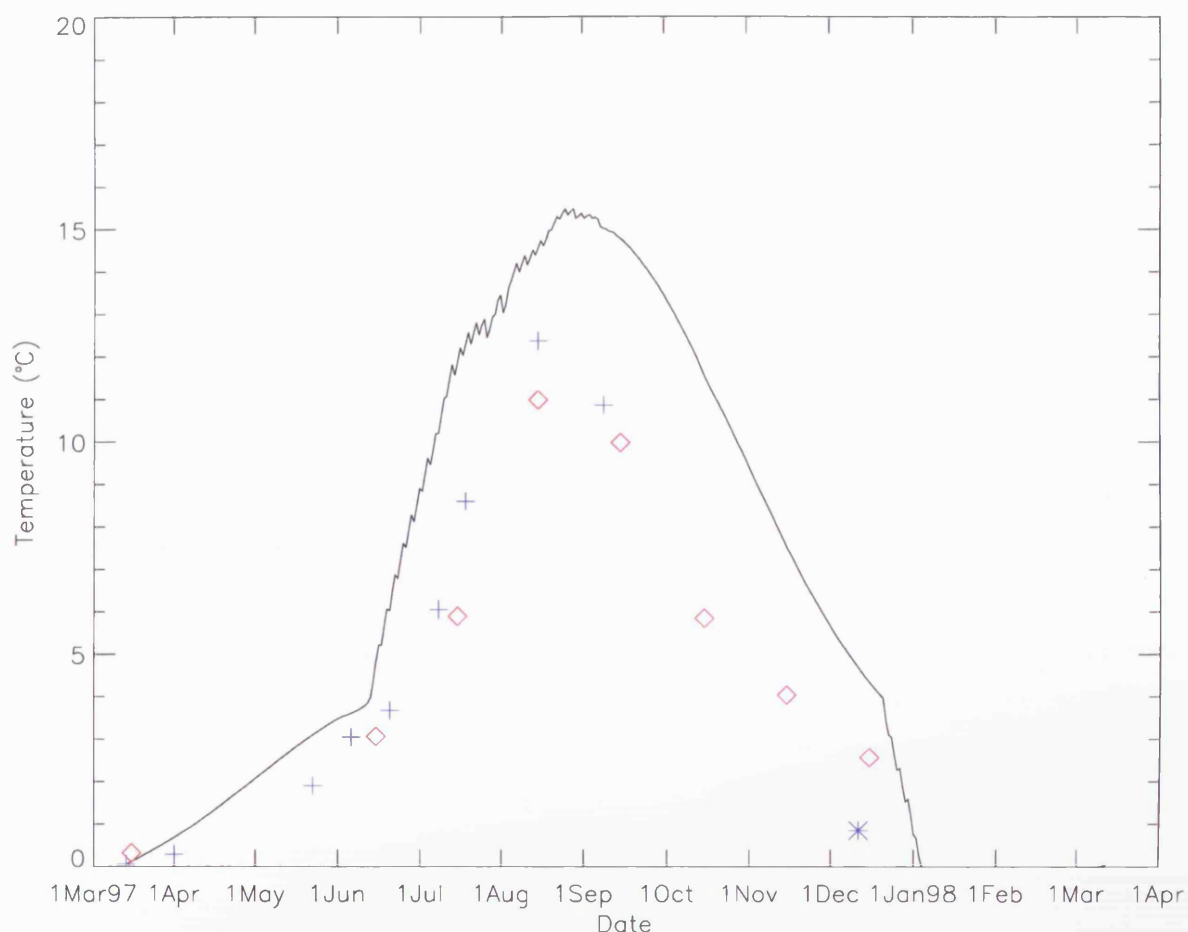


Figure 4.7: LBM-A: Temperature at the centre of the surface layer (5 m depth) at grid point (40,12): model data (solid line); BICER data (diamonds); and the Jewson data (crosses). The Jewson temperature for 11 December 1996 (asterisk) is plotted as 11 December 1997 to allow inclusion in the plot.

convective adjustment routine no longer exchanges heat between layers. Any vertical mixing of heat that now occurs will be due to the turbulent mixing term in the temperature equation.

For the 20 June, the model predicts a warm epilimnion of 20 ± 5 m deep with a temperature of 6.0°C at 5 m. Below the epilimnion the model water column has a uniform temperature equal to the T_{md} . This model profile is similar to Jewson's observations for 8 July 1997 except that the observations show temperatures at depth in excess of the T_{md} . In fact, the model predicts temperatures in the hypolimnion to be equal to the T_{md} during the whole of the Summer and Autumn period (mid-June to December). The observed lake temperatures suggest that heat is mixed down as far as the 200 m level during this time. This mixing does not occur in the model, indicating that the model vertical mixing is inadequate

for this period.

Jewson's data for 18 July 1997 show a well-mixed layer lying between the warm epilimnion, with a temperature of 8.6 °C, and the colder hypolimnion, with a temperature near T_{md} . This layer, from 20-70 m, has a temperature of ~5.5 °C. This may indicate that an episode of strong, wind-induced mixing occurred between 8 and 18 July 1997. This intermediate layer is also evident in the July profile of the BICER data suggesting that the conditions may not be peculiar to 1997 but may be normal for this time of year.

The net heat balance has a positive maximum in mid-June and decreases, through the Summer, to zero at the beginning of September. The temperature of the surface layer increases fastest during this time in both the real lake and model lake. The model surface temperature increases fastest between mid-June and late June, whereas the observations suggest that the real lake surface layer warms fastest during the second half of July. The difference in timing is due to variation in the time that the temperature of the surface layer reaches the T_{md} . Surface temperatures in the real lake are still below the T_{md} when the peak heat flux occurs, in mid-June. Heat can still be mixed downwards by convection, so reducing the rate of temperature increase in the surface layer. The model lake, however, has reached the T_{md} and so free convection cannot occur. Mixing of heat down the column is markedly reduced allowing the surface to warm more rapidly.

According to Shimaraev *et al.* (1994) maximum temperatures are reached in the lake in August. The model predicts a temperature of 14.6 °C in the surface layer for 15 August and an epilimnion thickness of 20-30 m. Jewson's observations record an epilimnion of similar thickness with a temperature of 12.4 °C for the same day. The BICER data show a narrower epilimnion, 15-20 m thick, with a temperature of 12.51 °C at the surface and 10.98 °C at 5 m. A maximum temperature of 15.5 °C is reached by the model surface layer on 28 August 1997 (see Figure 4.7).

Integrating the temperature over the depth of the water column gives an indication of the amount of heat energy in the column. This allows the net heat flux into the column in the model and that estimated for the real lake to be compared. This quantity is not affected by the vertical mixing of heat down the column and so is independent of the parameterization of the vertical diffusion coefficient. It is, however, dependent on the horizontal exchange of water with neighbouring columns. The depth integrated temperature for the model and the Jewson and BICER data sets are displayed in Figure 4.8. The overall shape of the curve for the period from March to January approximates a sine wave, reflecting the sinusoidal shape of the surface heat flux. Variation from a sinusoidal shape is most evident from mid-July to mid-October. This variation is due to the horizontal transport of heat into and out of the column, in the main due to wind-induced currents. It is this horizontal transport of heat that is responsible for

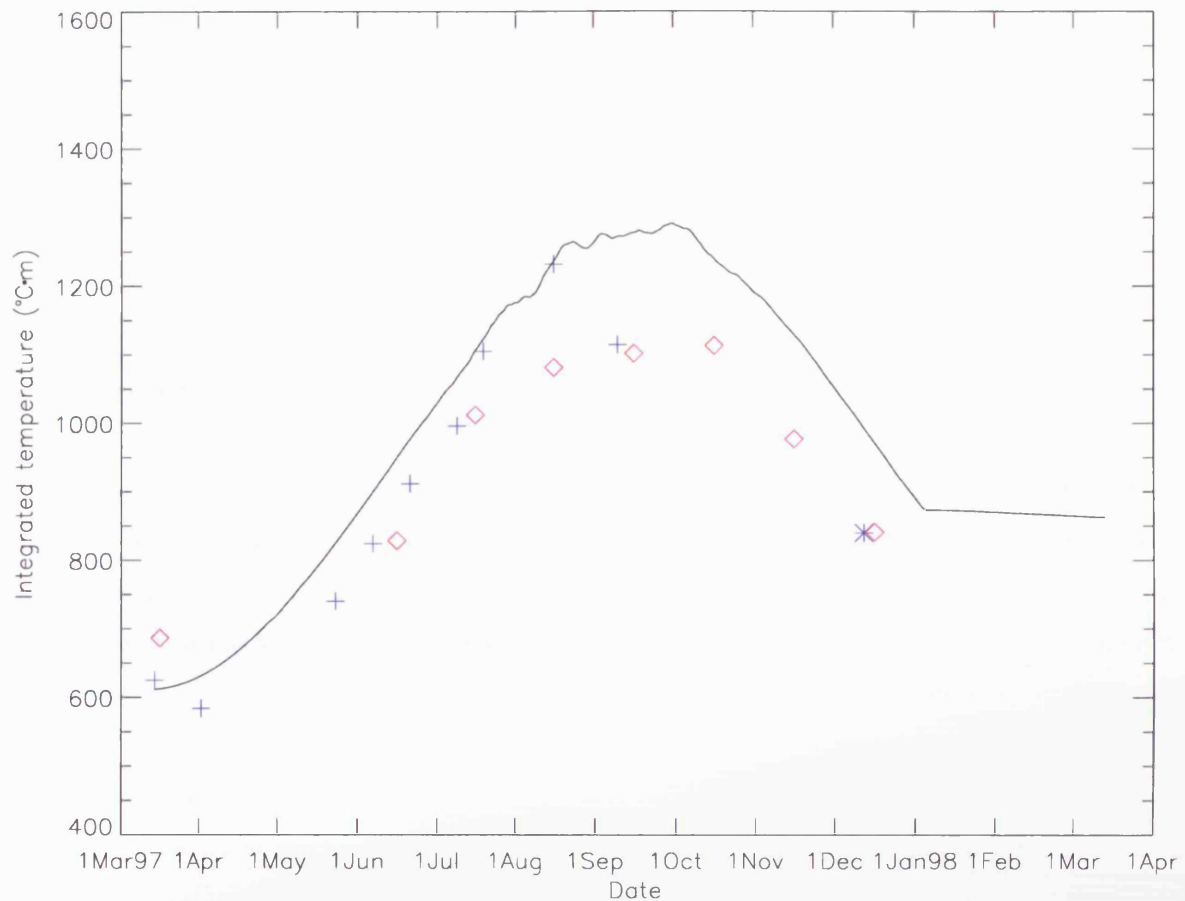


Figure 4.8: LBM-A: Depth integrated temperature at (40,12), in the South Basin: model data (solid line); Jewson data (crosses); BICER data (diamonds). Again the Jewson value for 11/12/1996 (asterisk) is plotted as 11/12/1997.

the continued warming of the water column after the surface heat flux has become negative.

For the whole of the model run, the integrated model temperature is higher than the observations would lead us to believe is the case in the real lake. This is most likely to be because the amplitude of the sinusoidal heat flux is too high, so that an excessive amount of heat is transferred into the lake during the warming phase and, indeed, Shimaraev *et al.* (1994) suggest Hutchinson's values for the Lake Baikal heat balance (the same values used by Killworth *et al.*) are too high.

Once surface temperatures start to cool free convection is again possible. Model temperature profiles for October-December (Figure 4.6(b)) show a progressively cooling and deepening mixed surface layer. The cooling of this layer lags behind the observations (see the BICER profiles in Figure 4.4) because of the excessive amplitude of the surface heat flux.

The BICER data for October records a surface temperature of 5.85 °C and the profile exhibits mixing of heat down to depths of 150-200 m. The model profile, however, has a surface layer temperature of 11.6 °C and the profile describes a distinct warm epilimnion over a cold hypolimnion. The same differences are apparent in the November profiles. The model lake still has an obvious two layer stratification whereas the observations now show a completely mixed water column with a temperature close to T_{md} . Storms with winds of speeds up to 40 m s⁻¹ occur in Autumn over Lake Baikal (Shimaraev *et al.*, 1994) and it is these storms which are able to produce strong mixing events during this time of weak stability in the water column. These storms are not implemented in this model run and this could account for the lack of mixing below 70 m.

Once the water temperature sinks below the T_{md} , free convection is no longer possible but wind-induced mixing may still occur. The BICER December profile displays evidence of mixing down to around 150 m. The model temperatures have not yet reduced below the T_{md} . When in January they have, a cold epilimnion is produced with no evidence of mixing of water with the deeper layers. Interestingly, the 11 December 1996 profile of the Jewson data shows a surface layer temperature of ~0.8 °C over a warmer hypolimnion of ~3.5 °C suggesting that in 1996 conditions had been calm enough for the formation of a narrow cold epilimnion. This may, of course, have been disrupted by subsequent storms.

The two-dimensional model of Killworth *et al.* (1996) also exhibits excessive warming in the Spring. Surface temperatures exceed the T_{md} by the time of maximal heat flux in June. However, their model does not exhibit the high surface temperature of the LBM at the time of zero heat flux in the Autumn. The LBM and the Killworth model differ in their methods of parameterization of vertical mixing. The LBM aims to model enhanced mixing in a surface layer by having a vertical eddy diffusion coefficient that depends on the Richardson number. The Killworth model does not attempt to model the mixed layer and uses a constant vertical eddy diffusion coefficient. The value used for the eddy diffusion coefficient is larger than would be the case if parameterization of a mixed layer was included. The Killworth model therefore produces a much deeper and less well-defined warm layer during the Summer. The maximum surface temperature achieved is 8.2 °C.

In summary the LBM run LBM-A reproduces the seasonal temperature cycle and dimictic turnover seen in Lake Baikal. However, model surface temperatures and the model depth integrated temperatures, in the South Basin, are nearly always greater than those observed in the real lake. This is, in part, due to the sinusoidal form of the surface heat flux, which leads to excess warming during the under-ice period, but is also caused by the model not mixing sufficient heat down to deeper layers during the Summer and Autumn. The next model run investigates the effect of reducing heat transfer during the time of ice-cover.

4.3.2 LBM-B: Surface heat flux as the empirically calculated Lake Baikal net heat balance.

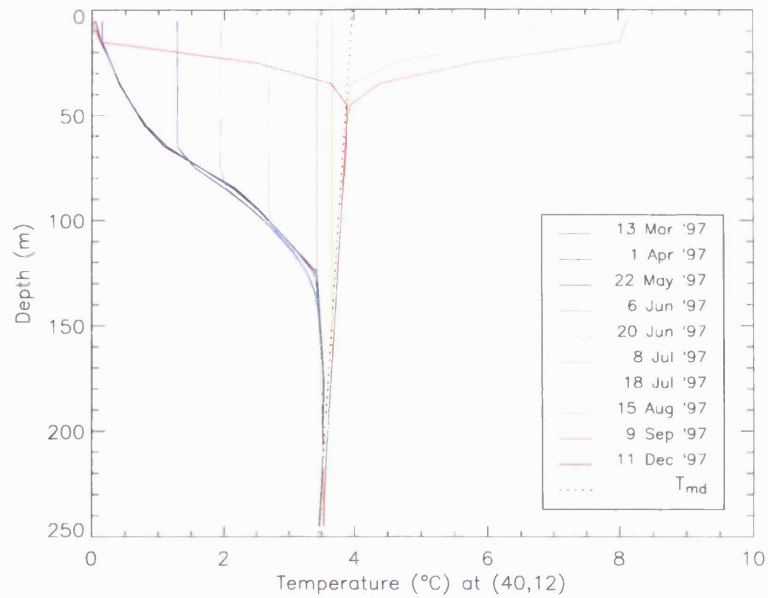
Heating in model run LBM-B is implemented in the same manner as in run LBM-A: a surface heat flux is applied as a source term in the surface model layer. The surface heat fluxes in this run are calculated using the monthly values of the net heat balance given by Shimaraev *et al.* (1994) and displayed in Figure 4.2. Daily values are obtained by linear interpolation between the monthly values. Although there is no spatial variation in this model, the heat fluxes are more realistic; their time variation reflects climatic conditions and, more noticeably, the reduction in surface heat transfer during the period of ice-cover.

The temperature profiles for run LBM-B are shown in Figure 4.9. In this run the lake appears to warm too slowly. The surface layer temperature does not warm past the T_{md} until 25 July (see Figure 4.10) whereas Jewson's observations indicate the T_{md} is reached, in the surface layer, between 20 June and 8 July in 1997. The late crossing of the T_{md} has a subsequent effect on the rest of the cycle. Delay in reaching the T_{md} results in a shortening of the period of rapid Summer warming because this process must terminate at the time of zero heat flux. The maximum temperature of 8.22 °C is achieved on 7 September, this is well short of the usual maximum of 10-12 °C. The timing of the maximum temperature is later than in run LBM-A which reaches 15.5 °C on 28 August. This is due to the difference in the time of zero heat flux which occurs on 10 September in run LBM-A and on 19 September in run LBM-B.

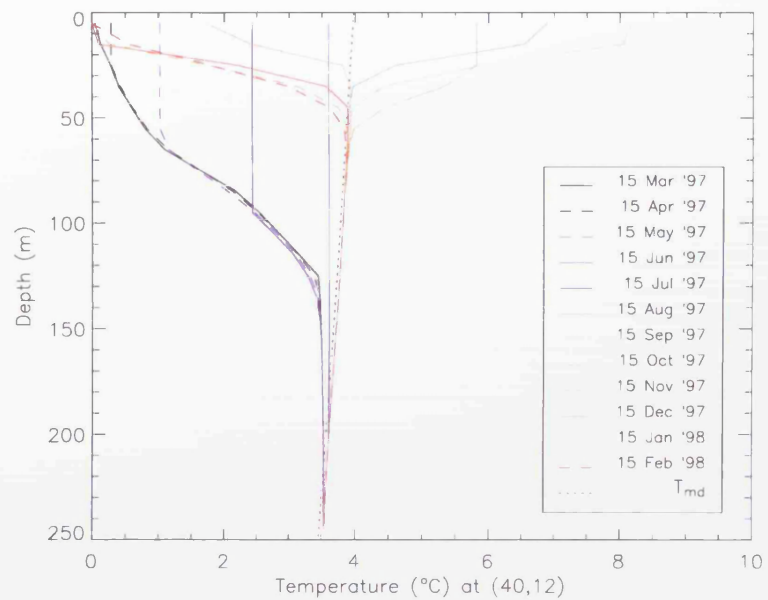
In the Autumn the surface layer cools. The surface layer temperature for 15 October (5.82 °C) is similar to that of the BICER observations (5.85 °C). The mixed layer, however, is too shallow indicating that the heat content of the water column is too low at this time. This results in the surface layer cooling more rapidly than in run LBM-A. The T_{md} is reached by 6 November, somewhat earlier than the BICER observations suggest (the BICER profile for 15 November has not yet crossed the T_{md} line) and much earlier than the model run LBM-A, which reaches T_{md} in the surface layer on 20 December. As a result freezing starts during November instead of January.

The depth integrated temperature curve (Figure 4.11) has a similar shape to that of the BICER observations. The difference between the two curves never exceeds the difference between the curves at initialization of the model, signifying that the differences between the model results and the observations arise largely because the model is initialized with a March 1997 profile and not an average March profile.

At present the model heat fluxes have no spatial variation. However, the North Basin receives less solar radiation than the South Basin due to latitudinal variation in the shortwave flux and due to climate



(a)



(b)

Figure 4.9: Temperature profiles for model run LBM-B : The heat source is a surface heat flux which varies sinusoidally with time. The profiles are located at grid point (40,12) in the South Basin. The profiles in (a) are for the same dates as the Jewson data shown in Figure 4.3. The profiles in (b) are for the 15th of each month of the BICER observations in Figure 4.4. Model profiles (dashed lines) for the months of April and May 1997, and January and February 1998 are also shown for completeness.

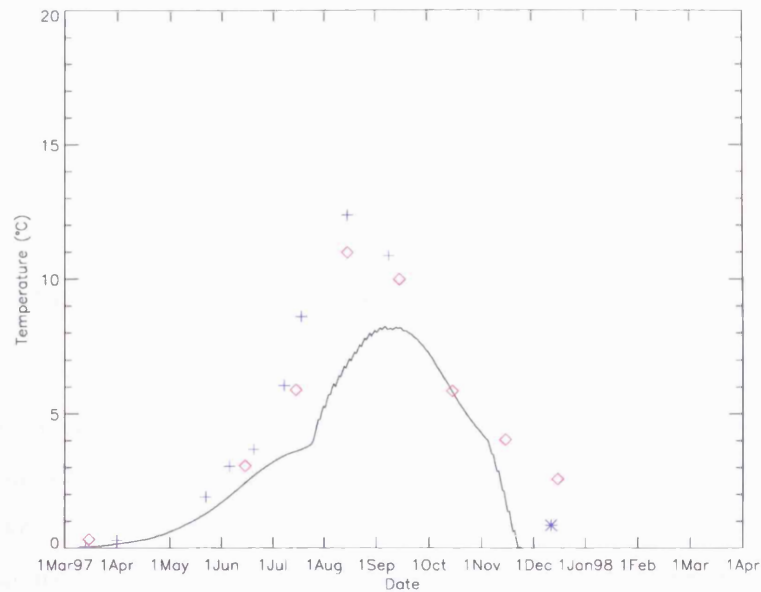


Figure 4.10: LBM-B: Temperature at the centre of the surface layer (5 m depth) at grid point (40,12): model data (solid line); BICER data (diamonds); and the Jewson data (crosses). The Jewson temperature for 11 December 1996 (asterisk) is plotted as 11 December 1997.

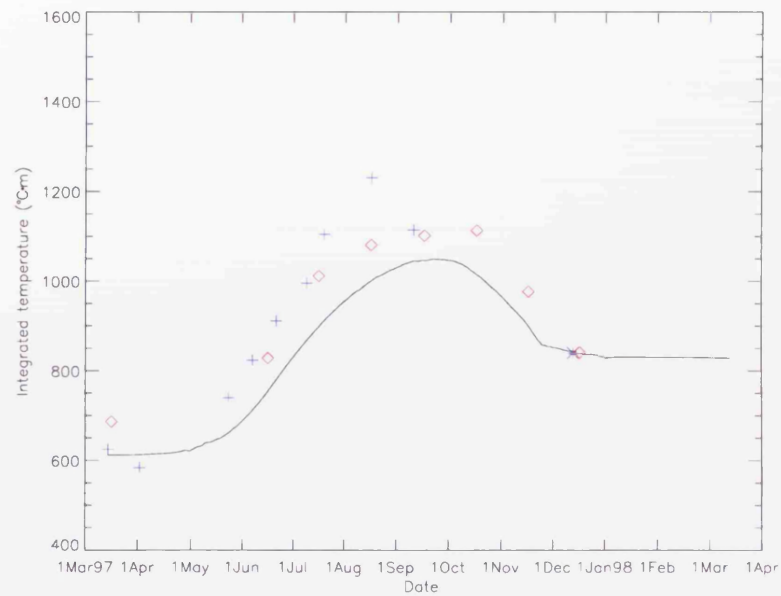


Figure 4.11: LBM-B: Depth integrated temperature at (40,12), in the South Basin: model data (solid line); Jewson data (crosses); BICER data (diamonds). Again the Jewson value for 11/12/1996 (asterisk) is plotted as 11/12/1997.

differences. Ideally these factors would be parameterized in the model heat fluxes. Currently the model is initialized with the same temperature everywhere but as the model has been shown to be sensitive to the initialization profile some spatial variation in the initialization profile may be necessary if spatial variation in the applied heat fluxes is to be attempted. The next two model runs attempt some spatial variation of the heat fluxes but only during the period of ice-cover.

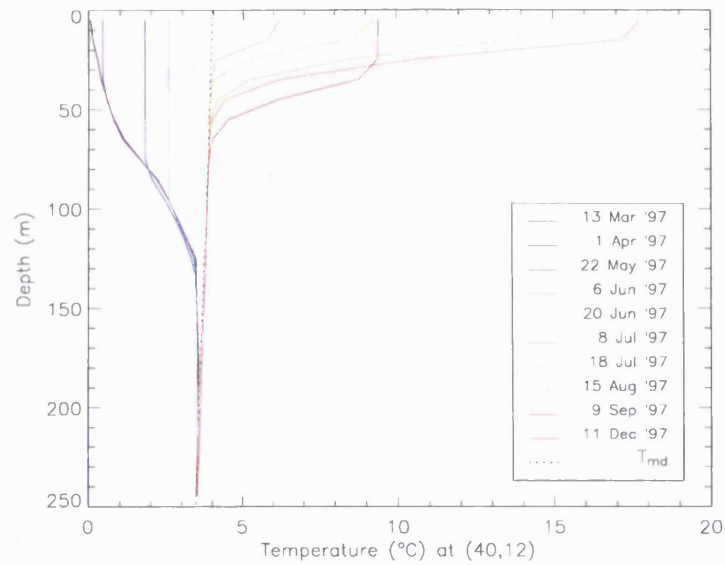
4.3.3 LBM-C: Spatial variation of the heat flux through ice.

During the period of ice-cover, spatial differences in the thickness and type of ice, and the extent of the overlying snow-cover result in spatial variations in the penetration of solar radiation through the ice to the water below. This differential heating is responsible for the production of pressure gradients and the subsequent generation of currents in the surface layer of the lake. At this time the temperature at the surface is below T_{md} and any warming results in an increase in density and sinking of the surface water. Thus heating at the surface also results in convective mixing of the upper layer.

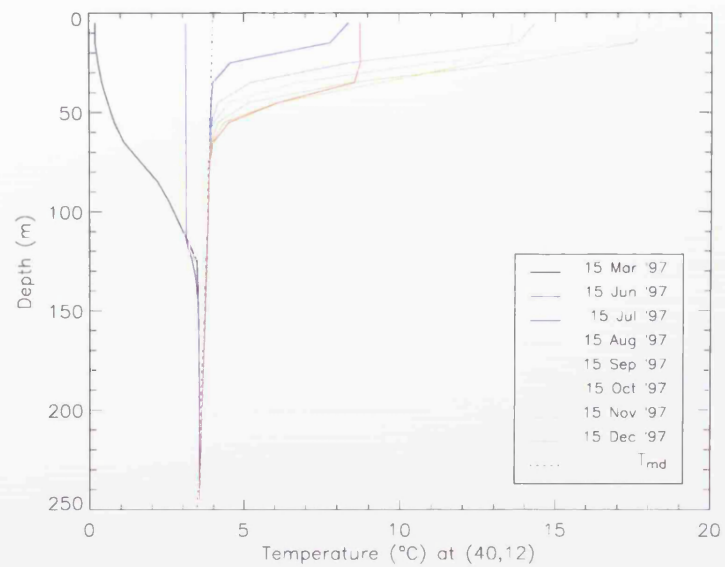
Model run LBM-C introduces spatial variation into the implementation of the external heating. The net heat flux is separated into two parts: first, the incident shortwave radiation; and secondly, heat transfer due to all other processes. The heat flux due to the incident shortwave radiation is implemented using the same method as the LHLC model (see § 2.1.1). The shortwave flux is absorbed through an ice and snow layer (when present) by the water column according to the Beer-Lambert law (2.4); the optical properties of the ice and snow layer are allowed to vary both spatially and temporally. The other heat processes are modelled as a surface heat flux, in a similar manner to runs LBM-A and LBM-B.

While ice-cover exists, the water column is considered to be insulated from all heat transfer except penetrating solar shortwave radiation. When the ice has melted, heat transfer by other processes, which have been described in § 4.1, can no longer be neglected. The values assigned to these heat fluxes are the sum of the various fluxes shown in Figure 4.2 neglecting the flux of absorbed shortwave radiation or, equivalently, the net heat balance minus the absorbed shortwave flux.

Throughout the Spring warming, the LBM, like the LHLC model, reproduces the observed temperature profiles very well (Figure 4.12). There are slight differences. The 1 April profile does not show cooling of the water column as is the case for the observed data but this is expected because the LBM net heat balance is positive at this time. The temperature in the model surface layer is 0.44°C compared with an observed temperature at 5 m of 0.289°C . Subsequent observations show that the model lake warms at a slightly slower rate than the real lake. By 22 May warming in the model has produced a mixed layer of 75 ± 5 m with a temperature of 1.79°C while observations show a warmer and deeper



(a)



(b)

Figure 4.12: Temperature profiles for model run LBM-C : The Beer-Lambert law is used to calculate the attenuation of shortwave radiation through a snow-ice layer (spatial variation is allowed) and the absorption of shortwave radiation by the water. Other heat transfer processes are modelled as a time-varying surface heat flux. The profiles are located at grid point (40,12) in the South Basin. The profiles in (a) are for the same dates as the Jewson data shown in Figure 4.3. The profiles in (b) are for the 15th of each month of the BICER observations in Figure 4.4.

mixed layer of 90-95 m with an average temperature of 2.18 °C. The profiles for 6 June show that the model mixed layer has extended down to 95 m and has a temperature of 2.55 °C; while Jewson's observations show a mixed layer of 110 m with an average temperature of 3.09 °C. At this time the model surface layer temperature is 2.55 °C and the observed temperature at 5 m is 3.04 °C.

Both the model lake and the real lake have reached homothermy, in the South Basin, by 20 June. The model water column has an average temperature of 3.43 °C with a standard deviation of 0.02 °C; the observed water column between 0 and 250 m has an average temperature of 3.64 °C with a standard deviation of 0.035 °C.

During the development of the Summer stratification the model continues to reproduce the observed surface temperatures (see Figure 4.13). On 8 July the profiles show that both the model and the observed lake have a warm epilimnion with a temperature that exceeds the T_{md} : 6.05 °C at a depth of 5 m in the observed lake; and 6.18 °C in the model surface layer. In the observed lake the depth of the epilimnion is approximately 10 m. The thermocline exists between 10 and 30 m and has a gradient of $-0.0847\text{ °C} \cdot \text{m}^{-1}$. Below 30 m the temperature gradient reduces to $\sim -0.0037\text{ °C} \cdot \text{m}^{-1}$. The model epilimnion is of comparable thickness and temperature, with a depth of 15 ± 5 m and a temperature of $\sim 5.99\text{ °C}$. The temperature then decreases to 4.08 °C at 25 ± 5 m. Below 40 m the model temperature is approximately equal to the T_{md} , so that the temperature gradient is $\sim 0.0021\text{ °C} \cdot \text{m}^{-1}$, somewhat less than in the observed lake. The model and observed profiles exhibit a notable difference; the Jewson data shows water warmer than the T_{md} present below the epilimnion. This is evidence of vertical mixing due to the action of winds, since such warm water cannot be displaced below colder water by free convection. As is the case in runs LBM-A and LBM-B, this mixing is not reproduced adequately by model run LBM-C.

Evidence of the model's inadequate mixing of sufficient heat down through the water column is present also in the profiles for 18 July and 15 August. These profiles show excessive temperatures in the model surface layer: 9.25 °C compared with the observed temperature of 8.60 °C on 18 July; and 14.4 °C, instead of 12.4 °C, on 15 August. On both these dates the model depth integrated temperature (Figure 4.14) is similar to that of the observed data, showing that while there is excessive heat in the surface layer, there is too little heat in the deeper layers.

After 15 August the model surface layer temperature and depth integrated temperature continue to increase until the beginning of October. The Jewson data suggests that in 1997 the real lake started to cool during the second half of August, with both the surface temperature and the depth integrated temperature lower on 9 September than 15 August. The BICER data, although showing the surface temperature decreasing after 15 August, shows the depth integrated temperature increasing slowly during

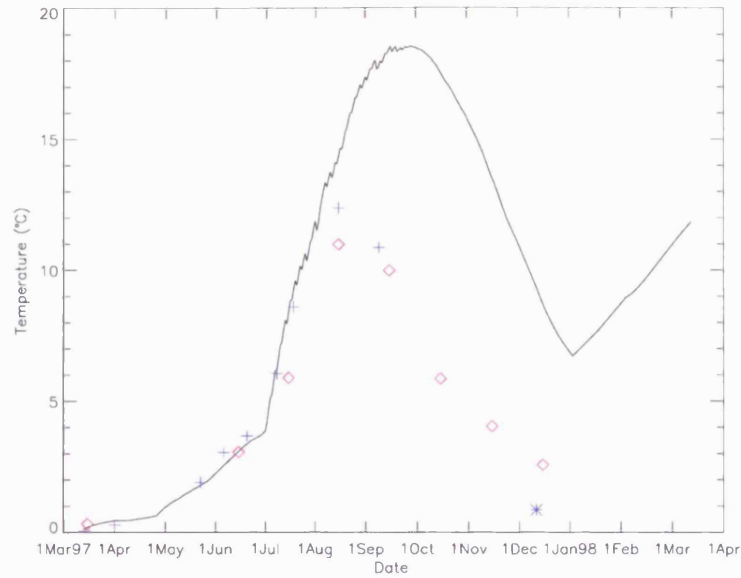


Figure 4.13: LBM-C : Temperature at the centre of the surface layer (5 m depth) at grid point (40,12): model data (solid line); BICER data (diamonds); and the Jewson data (crosses). The Jewson temperature for 11 December 1996 (asterisk) is plotted as 11 December 1997.

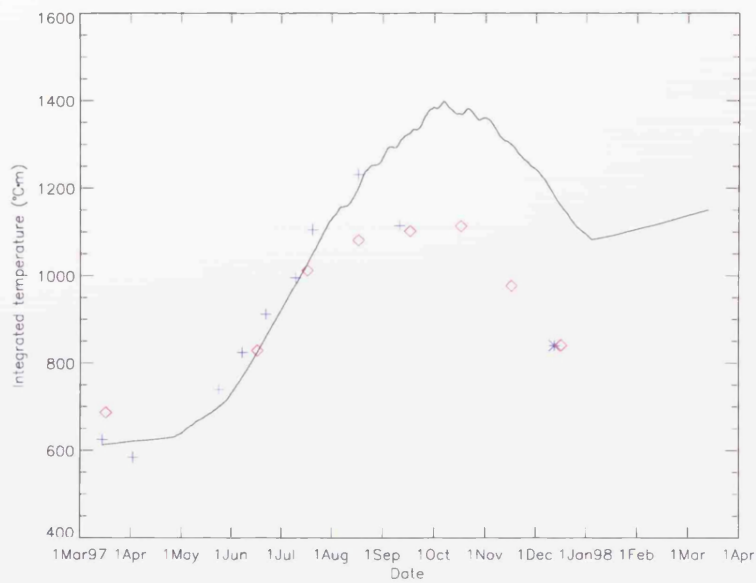


Figure 4.14: LBM-C : Depth integrated temperature at (40,12), in the South Basin: model data (solid line); Jewson data (crosses); BICER data (diamonds). The Jewson value for 11/12/1996 (asterisk) is plotted as 11/12/1997.

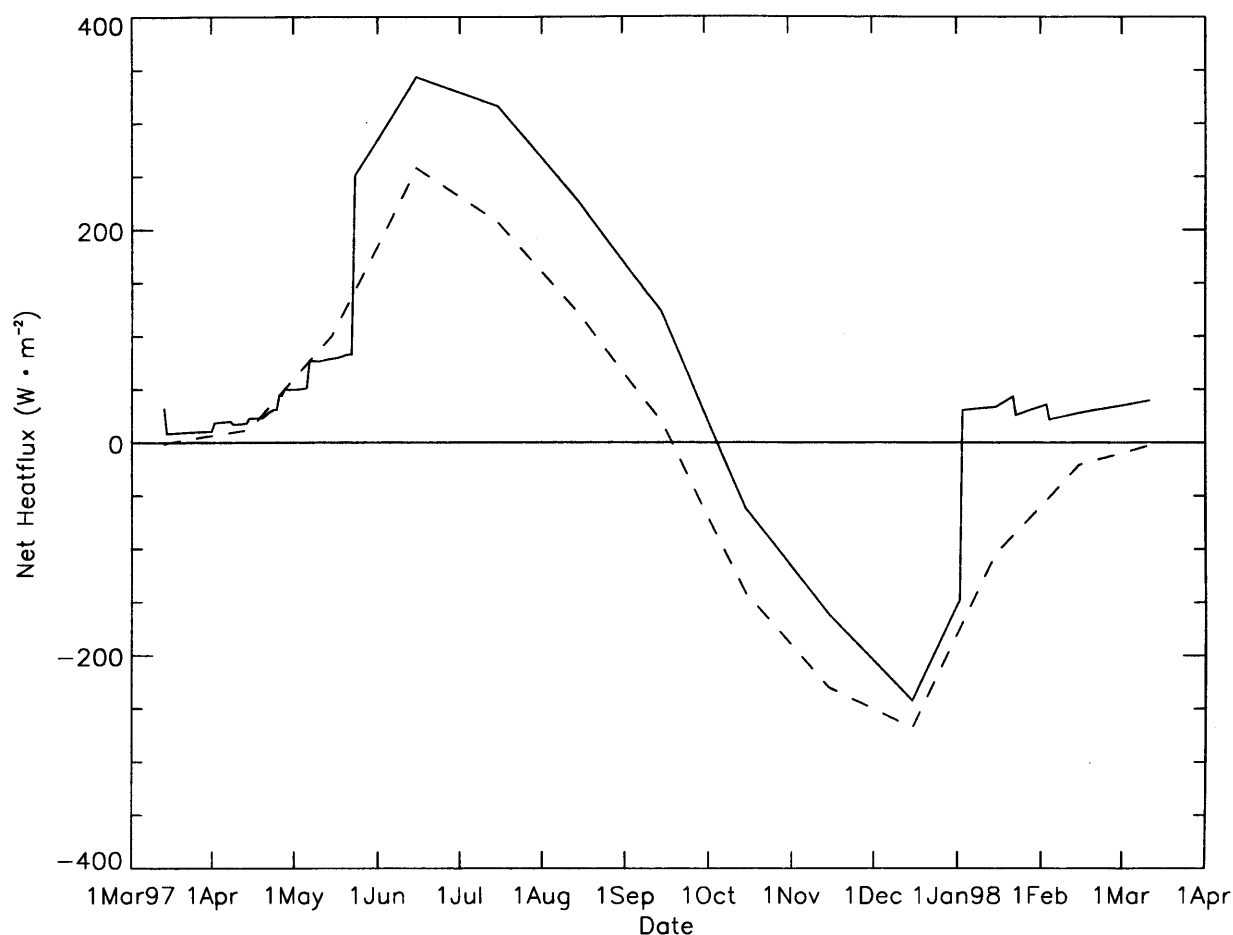


Figure 4.15: LBM-C : Daily variation in the spatially averaged net heat flux for model run LBM-C (solid line) and the BICER net heat balance (dashed line).

September before starting to cool towards the end of October. The excessive model depth integrated temperature after the middle of August shows the heat content of the lake is too high at this time. This is due to excessive heating during the late Summer and Autumn. The variation in the net flux of heat into the model lake with time is shown in Figure 4.15. The average heat flux for each day has been calculated by summing the daily temperature change for each grid box over the entire lake and converting this to a spatially-averaged energy flux. While the model heat flux reproduces the BICER net heat balance during the period of under-ice warming (13 March to 23 May), once the ice has melted the model heat flux is considerably greater than the BICER heat balance. This is largely due to the method the model uses to implement heating by solar shortwave radiation. The model uses clear sky radiation fluxes to estimate the shortwave radiation incident on the lake's surface, whereas the BICER net heat

balance is calculated using estimates of shortwave radiation absorbed by the lake. The model does not allow for attenuation of shortwave radiation by clouds. That the model accurately reproduces the shortwave fluxes during ice-cover is largely fortuitous and suggests that the ice model absorbs radiation too strongly, although the frequency of cloud cover is lowest at this time of year. The error may be in the assignment of albedos and extinction coefficients to the various ice-types or possibly in the assignment of ice-type to the satellite brightness temperatures.

The excessive shortwave radiation flux results in too much heat entering the lake during the Summer and too little heat leaving the lake during the Autumn. Although cooling of the model upper layer and convective mixing has started by the middle of October, Figure 4.13 shows that the surface layer still has a temperature of 8.75 °C in December when the upper layer of the real lake has cooled below the T_{md} . The problem with the shortwave fluxes is addressed in model run LBM-D.

4.3.4 LBM-D: Spatial variation of heat flux through ice and reduced short-wave input during Summer and Autumn.

Shimaraev *et al.* (1994) report that precipitation is maximum over the lake during the Summer season and that some areas have a second maximum in November/December. They also describe the ratio of daily sunshine duration to potential sunshine hours as varying from a maximum of 63% in February-March to a minimum of 22% in December. The actual amount of shortwave radiation reaching the lake is therefore reduced, below the clear sky radiation values, by clouds and this effect is more marked during the Summer and Autumn seasons. In model run LBM-D the clear sky radiation values are reduced in order to reflect this fact, in effect we introduce an arbitrary cloud function.

Figure 4.16(a) shows a plot of the incident shortwave radiation fluxes used in model runs LBM-C and LBM-D. The sum of the incident shortwave radiation flux and the surface heat flux due to other processes is plotted against time in Figure 4.16(b) for runs LBM-C and LBM-D. The introduction of the cloud function has reduced the net heat balance towards the BICER values in the late Summer and the Autumn. The change from positive to negative heat flux in the Autumn has been brought forward from the beginning of October to the beginning of September, so that the lake begins to cool earlier than in previous model runs.

The net heat fluxes for runs LBM-C and LBM-D are identical from initialization to 15 June so that the model temperatures are also identical up to this point. After this date the reduced heat fluxes in run LBM-D produce temperatures much closer to the observed values than in run LBM-C. Zero heat flux occurs on 8 September in run LBM-D compared with 7 October in run LBM-C. This difference and

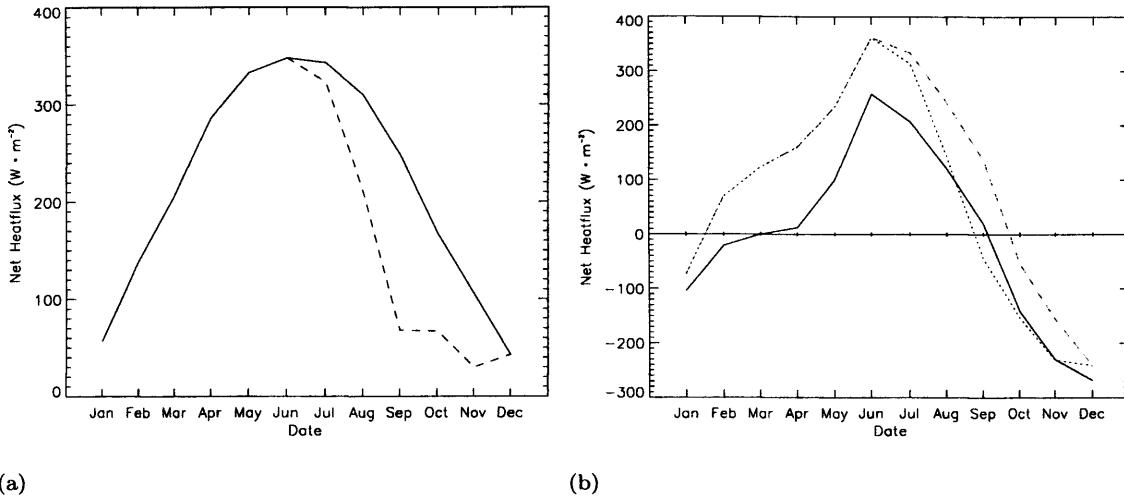


Figure 4.16: Heat fluxes for model runs LBM-C and LBM-D. (a) The incident shortwave fluxes used in model run LBM-C (solid line) and run LBM-D (dashed line). (b) The incident shortwave flux added to the surface flux due to other heat transfer processes: run LBM-C (dashed line) and run LBM-D (dotted line). The BICER net heat balance is also shown (solid line). Labels on the x-axes refer to the 15th day of the month.

the lower heat input in LBM-D result in a lower Summer maximum of 13.2 °C (18.5 °C in run LBM-C), which occurs earlier on 28 August (19 September in run LBM-C). The development of the surface layer temperature is shown in Figure 4.17.

The model depth integrated temperature (Figure 4.18) is also much nearer to the observed values. Whereas, in run, LBM-C the water column goes on gaining heat during September, in run LBM-D the heat content of the water column remains approximately the same during this time. This reduced heating and earlier cooling means that the surface layer temperature reaches T_{md} in December. Subsequently rapid cooling results in freezing occurring on 4 December.

The problem with the vertical mixing is still evident with too little heat being mixed down below the epilimnion during the Summer and Autumn (Figure 4.19). The surface temperatures remain too high during this period so that the model is still too slow to cool below the T_{md} in the Autumn. In this run the T_{md} is crossed on 4 December while the BICER data suggest this point is reached shortly after the 15 November. The low vertical mixing is also responsible for the rapid cooling towards freezing point, which observations suggest occurs at this location in early January rather than mid-December.

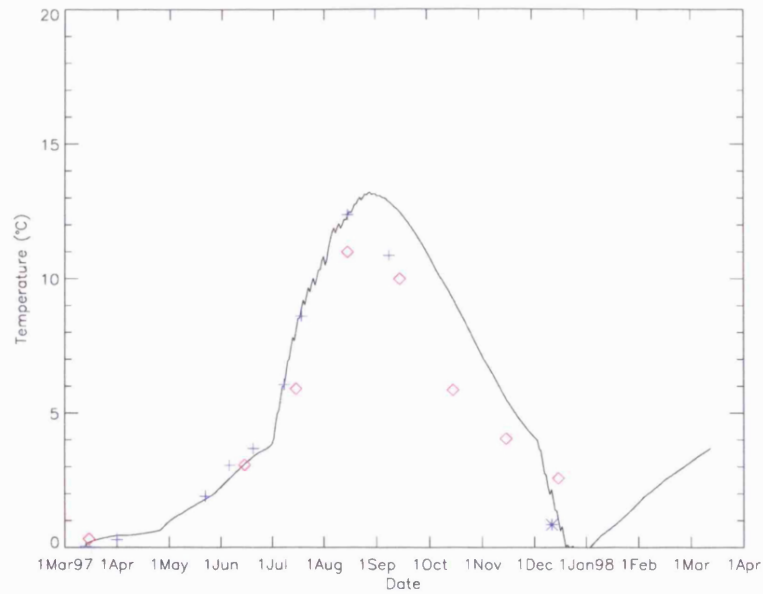


Figure 4.17: LBM-D : Temperature at the centre of the surface layer (5 m depth) at grid point (40,12): model data (solid line); BICER data (diamonds); and the Jewson data (crosses). The Jewson temperature for 11 December 1996 (asterisk) is plotted as 11 December 1997.

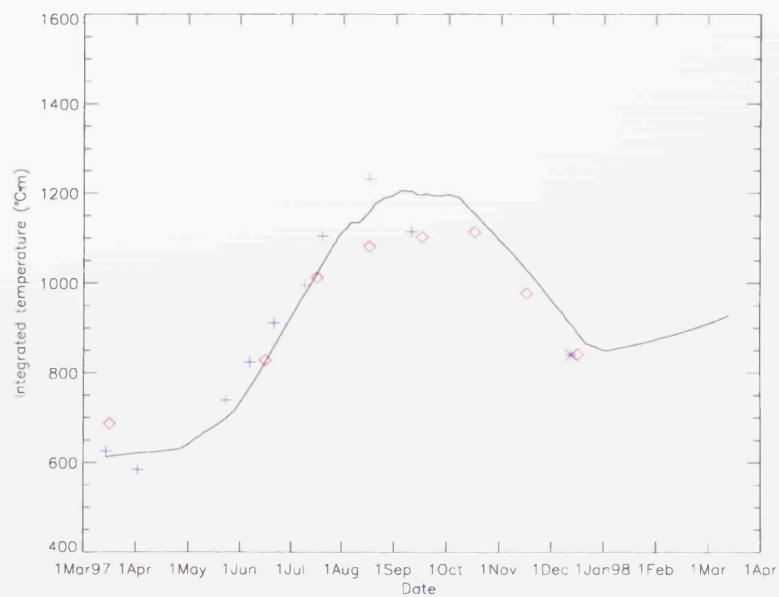
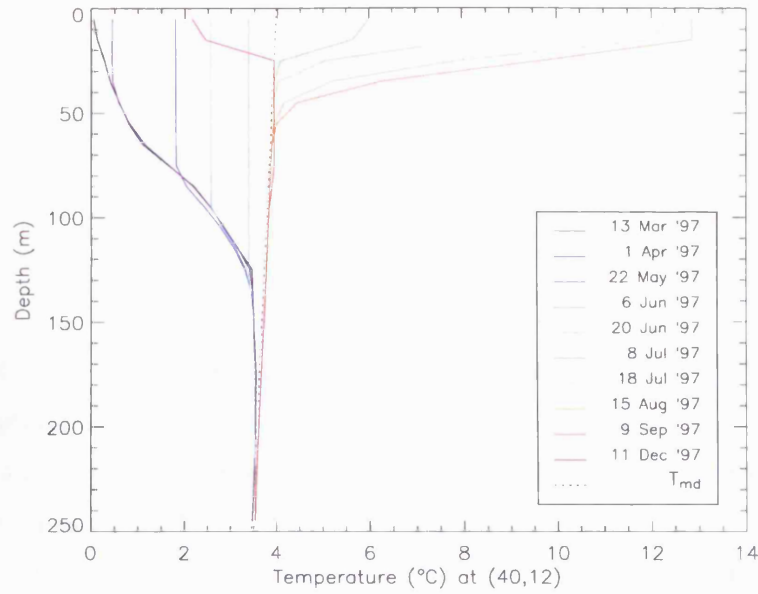
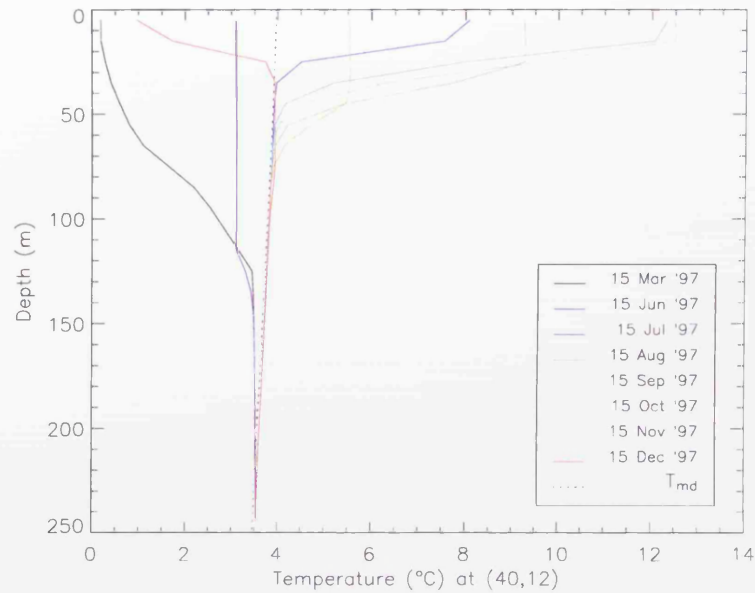


Figure 4.18: LBM-D : Depth integrated temperature at (40,12), in the South Basin: model data (solid line); Jewson data (crosses); BICER data (diamonds). The Jewson value for 11/12/1996 (asterisk) is plotted as 11/12/1997.



(a)



(b)

Figure 4.19: Temperature profiles for model run LBM-D :The heat fluxes are implemented using the same method as run LBM-C; the clear sky radiation fluxes have been reduced as shown in Figure 4.16(a). The profiles are located at grid point (40,12) in the South Basin. The profiles in (a) are for the same dates as the Jewson data shown in Figure 4.3. The profiles in (b) are for the 15th of each month of the BICER observations in Figure 4.4.

4.3.5 Conclusions

The model runs described in this section have demonstrated the sensitivity of the model to variations in the applied heat flux. The non-linearity of the equation of state of water has a large impact on the way fresh water responds to heating and cooling. This is evident in the increase in rate of change of temperature once the T_{md} is crossed, whether warming past the T_{md} in the Spring or cooling past the T_{md} in the Autumn. This is of course due to the cessation of free convection and the associated vertical transfer of heat.

Diatoms depend on the vertical mixing of water to maintain them at depths where the light level is high enough for photosynthesis to occur. The timing of the crossing of the T_{md} and the subsequent reduction in vertical mixing is therefore of vital importance to their ability to reproduce. The surface layer reaches T_{md} later in run LBM-B so that free convection persists for longer. Since *A.baicalensis* prefers cooler temperatures and free convection persists for longer, we would expect diatoms in this run to enjoy a longer growing season.

Run LBM-B uses BICER average heat fluxes but is initialized with a cooler than average, 1997 temperature profile. This probably accounts for the delay in reaching the T_{md} and emphasizes the sensitivity to initial conditions and the need to initialize the model with a profile specific to the year of study.

The LHLC ice-model allows spatial variation of the heat flux during ice-cover so that the pressure gradients that result from differential heating can be modelled. The ice-model uses an albedo that varies with ice-type and hence with space and time. It therefore requires the solar radiation flux incident on the ice surface as an input. Unfortunately such data are not readily available. The LHLC model and run LBM-C uses the clear sky radiation but this produces excessive heat fluxes during the Summer and Autumn when there is no ice-cover. The introduction of a cloud function, in run LBM-D, reduces the solar shortwave radiation flux during the Summer and Autumn and results in considerable improvement in model results.

The run LBM-D produces heat fluxes close to the empirically calculated fluxes of Shimaraev *et al.* (1994), and water temperatures close to the observations, during the period from March to December. This period encompasses both the Spring and Autumn maxima in plankton growth, suggesting the LBM may be suitable for coupling to a model of diatom population dynamics. At present, the LBM produces a poor simulation of the lake during the under-ice cooling period (January-March). This is because the method of modelling freezing by setting negative temperatures to zero introduces an unwanted positive heat flux. The model also has no method of losing heat during this period as the only heat transfer

process allowed through the ice is heating by shortwave solar radiation. In the model runs examined, so far, the model mixes insufficient heat down the water column during the Summer and Autumn. This may be due to poor parameterization of the vertical turbulent diffusion coefficient or inadequate representation of the wind forcing. These factors will be addressed in Chapter 5.

Chapter 5

The LBM: The Parameterization of Vertical Turbulent Mixing

Momentum and heat are redistributed throughout the water column by the processes of convection and turbulent mixing. In Chapter 2 we saw that these vertical mixing processes are vitally important to the diatom *A. baicalensis*. The modelling of convective overturn has been described in § 3.1. This section describes the modelling of turbulence, in particular the parameterization of vertical turbulent diffusion. First the Reynolds-averaged Navier-Stokes equations are derived and the turbulence closure problem is described. A description of the method of modelling turbulence in this study and the reasons for choosing it are given. The next sections explore the parameterization space and some results with different parameters are presented. The last section describes the model's response to a simulated storm event.

5.1 Turbulence

Most environmental fluid flows are turbulent and this is certainly true of lake flows. Turbulence is generated at the boundary through friction between the flow and the solid basin, in the lake interior through friction between flows of differing velocities, and at the surface through frictional forcing by winds. Turbulent flows contain irregular motions on different time and spatial scales. There is an overall cascade of energy from the large scale to smaller scales, with large eddies throwing off smaller eddies and these giving rise to still smaller eddies, and so on. Turbulence causes the fluid to be stirred and subsequently properties are mixed by diffusion. This mixing results in the erosion of gradients and the

dissipation of momentum as heat, so that kinetic energy is removed from the flow.

Direct numerical simulation (DNS) of turbulent flows requires the size of the grid be small enough to model all of the kinetic energy dissipation. This dissipation occurs on the smallest scales that viscosity is active. Ferziger and Peric (2002) show that the computational cost of DNS of turbulent flows scales as the Reynold's number to the power three. DNS would require multi-processor arrays not available to this study; a less computationally expensive method of approximating the turbulent flow is, therefore, required.

Motions on scales smaller than the model grid scale cannot be represented by finite difference approximations of the Navier-Stokes equations but these processes dissipate energy and must therefore be modelled somehow. Hinze (1975) noted that, "Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned." This allows us to represent the turbulent flow variables as the sum of a mean value and a turbulent fluctuation (see Hinze (1975) and other introductory texts on turbulence). So, for a turbulent flow let the variables: the components of the velocity, U_i , the pressure, p , the density, ρ , and any scalar, ϕ , be written as:

$$\begin{aligned}\tilde{U}_i &= \langle U_i \rangle + u'_i, \\ \tilde{\rho} &= \langle \rho \rangle + \rho', \\ \tilde{P} &= \langle P \rangle + p', \\ \tilde{\Phi} &= \langle \phi \rangle + \phi' .\end{aligned}\tag{5.1}$$

Here the turbulent variable, shown with a tilde, is the sum of a mean flow, in angle brackets, and a perturbation, indicated by a primed quantity. The mean flow value is the value obtained by averaging the flow over a suitable time interval. The time interval, T , must be short enough so that changes in the flow of interest are not included but long enough to average motions we wish to consider part of the turbulent flow. Then the average of, for example, a velocity component is defined as,

$$\langle U_i \rangle = \frac{1}{T} \int_{-T/2}^{+T/2} \tilde{U}_i dt .\tag{5.2}$$

In the case of the model the averaging period is the length of a time-step. The average of the random fluctuations over the period, T , is zero and the average of any mean value is just that mean value. For clarity we now drop the angle brackets and primes and represent the mean flow values by upper case characters and the perturbations by the lower case equivalent:

$$\tilde{U}_i = U_i + u_i .\tag{5.3}$$

If we assume an incompressible fluid the turbulent density, $\tilde{\rho}$, can be replaced with a constant density, ρ . The mean flow plus fluctuation are then substituted for the turbulent quantities in the Navier-Stokes equations and the equations averaged over the time interval, T . Using the properties of averaging given before, and the continuity equation we find the Reynolds-averaged Navier-Stokes equations (RANS) for an incompressible fluid. These are, neglecting body forces and sources:

$$\frac{\partial U_i}{\partial x_i} = 0, \quad (5.4)$$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left[\mu \frac{\partial U_i}{\partial x_j} - \rho \langle u_i u_j \rangle \right], \quad (5.5)$$

$$\frac{\partial \Phi}{\partial t} + U_j \frac{\partial \Phi}{\partial x_j} = +\frac{1}{\rho} \frac{\partial}{\partial x_j} \left[\mu \frac{\partial \Phi}{\partial x_j} - \rho \langle u_j \phi \rangle \right]. \quad (5.6)$$

In the above μ is the molecular dynamic viscosity and the term containing it represents dissipation due to molecular viscosity.

On comparison with the Navier-Stokes equations the Reynolds-averaged equations are seen to contain the extra terms, $-\rho \langle u_i u_j \rangle$, known as the Reynolds stresses and $-\rho \langle u_i \phi \rangle$, called the turbulent scalar flux. Now we have nine further unknowns for the three momentum equations and a further three for each scalar equation. There are now more variables than there are equations so that the equations are not closed; this is the *turbulence closure* problem.

Derivation of transport equations for the Reynolds stresses and the turbulent scalar flux produces further unknowns that require approximation (see, for instance, Ferziger and Peric (2002), Lawrence (1989)).

We choose to approximate the Reynolds stresses and turbulent scalar flux by making use of Boussinesq's concept that the turbulent stresses are analogous to the viscous stresses. We introduce the eddy viscosity, ν , and eddy diffusivity, κ . The Reynolds stresses can then be expressed as the product of an eddy viscosity and the mean velocity gradient (5.7a, 5.7b and 5.7c). Similarly the Reynolds heat fluxes are expressed as the product of an eddy diffusivity and the mean temperature gradient (5.7d).

$$\langle u' u' \rangle = -\nu_h \frac{\partial u}{\partial x}, \quad \langle u' v' \rangle = -\nu_h \frac{\partial u}{\partial y}, \quad \langle u' w' \rangle = -\nu_v \frac{\partial u}{\partial z} \quad (5.7a)$$

$$\langle v' u' \rangle = -\nu_h \frac{\partial v}{\partial x}, \quad \langle v' v' \rangle = -\nu_h \frac{\partial v}{\partial y}, \quad \langle v' w' \rangle = -\nu_v \frac{\partial v}{\partial z} \quad (5.7b)$$

$$\langle w' u' \rangle = -\nu_h \frac{\partial w}{\partial x}, \quad \langle w' v' \rangle = -\nu_h \frac{\partial w}{\partial y}, \quad \langle w' w' \rangle = -\nu_v \frac{\partial w}{\partial z} \quad (5.7c)$$

$$\langle T' u' \rangle = -\kappa_h \frac{\partial T}{\partial x}, \quad \langle T' v' \rangle = -\kappa_h \frac{\partial T}{\partial y}, \quad \langle T' w' \rangle = -\kappa_v \frac{\partial T}{\partial z} \quad (5.7d)$$

In ocean and lake flows, the length and velocity scales in the horizontal direction differ considerably

from those in the vertical direction; therefore we have assumed the Reynolds stresses to be isotropic in the horizontal plane but to differ from those in the vertical direction. For coarse grid models, the horizontal eddy coefficients are assigned constant values which are determined by considerations of numerical stability as discussed in § 3.2.3. The vertical coefficients should depend on properties of the flow, more particularly the shear stress and the density stability (Mellor and Yamada, 1974; Cheng *et al.*, 1976; Pacanowski and Philander, 1981). The next task is to determine the values of the eddy viscosity and eddy diffusivity to be used in the LBM.

5.2 Parameterization of The Vertical Components of The Eddy Coefficients

The LBM runs from Spring, when the lake is covered by ice, to the end of the year. During this time the temperature-depth profile evolves from a negative stratification through a period of homogeneity to a positive stratification and back again. While ice-cover persists, winds can have little effect but when the ice has melted the winds become the dominant forcing mechanism. If the wind stress is high enough to overcome the density-stability then mixing will occur. As this stability and the wind stress both vary with time and space the amount of mixing will also vary. Any method of parameterization of the vertical eddy coefficients must reproduce this variation.

For computational efficiency we choose to represent the eddy coefficients by algebraic expressions rather than solve further equations. We utilize a modification of the method of Pacanowski and Philander (1981) (referred to earlier in § 2.1.2) in which the expressions for the local vertical eddy coefficients are functions of the gradient Richardson number, henceforward referred to as the Richardson number. Pacanowski and Philander (1981) have

$$\nu_v = \nu_b + \frac{\nu_0}{(1 + \alpha Ri)^n}, \quad (5.8)$$

where the Richardson number is

$$Ri = \frac{(\mathbf{g}/\rho_0) \partial \rho / \partial z}{(\partial u / \partial z)^2 + (\partial v / \partial z)^2}. \quad (5.9)$$

The vertical eddy viscosity, ν_v , is equal to the sum of a background value, ν_b , and a maximum eddy viscosity, ν_0 , that is modified by a term including the Richardson number. The constants, α and n , and the parameters, ν_b and ν_0 , are fit to observations of the flow.

The Richardson number can be thought of as a ratio of the work required to move a parcel of water against a density gradient to the kinetic energy available to do that work. In regions of high density

gradient the water column is stable and a high current shear is required for turbulence to occur. On the other hand, where the density gradient is very small, low current shear may be sufficient to produce turbulence. A Richardson number below 0.25 is required for the creation of a turbulent flow (Tennekes and Lumley, 1972; Mellor and Durbin, 1975). A high Richardson number indicates a stable, laminar flow.

For high Richardson number flows the second term in the equation for the eddy viscosity (5.8) will tend to zero. So that, under these conditions, the eddy viscosity will tend to the minimum, background value, ν_b . As the Richardson number decreases towards zero, the second term in (5.8) tends towards ν_b and the eddy viscosity will tend to a maximum value of $\nu_0 + \nu_b$.

In common with the LHLC model (see § 2.1.2), we add a background average current shear, β , to the current shear in the Richardson number. Noting that the LHLC model successfully reproduces vertical mixing under ice when the current shear is very small. The inclusion of a background current shear in the expression for the Richardson number allows the LBM parameterization to tend to the LHLC parameterization (2.8) as the model-predicted current shear tends to zero. The full parameterization is given by:

$$\nu_v = \nu_b + \frac{\nu_0}{(1 + \alpha Ri)^n} \quad (5.10a)$$

$$\kappa_v = \kappa_b + \frac{\nu_0}{(1 + \alpha Ri)^n} \quad (5.10b)$$

where, in this case, the Richardson number is

$$Ri = \frac{(\mathbf{g}/\rho_0) \partial \rho / \partial z}{(\partial u / \partial z)^2 + (\partial v / \partial z)^2 + \beta^2 / \alpha} \quad (5.11)$$

Pacanowski and Philander (1981) propose $\nu_0 = (50 \text{ to } 150) \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$, $\nu_b = 1 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$, and $\kappa_b = 0.1 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ and find $\alpha = 5$ and $n = 2$ most suitable for models of the tropical oceans. The LBM run LBM-D (§ 4.3.4) has $\nu_0 = 50 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$, $\nu_b = 1 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$, and $\kappa_b = 0.1 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ and $\alpha = 1$ and $n = 1$. The average background current shear, β , is given by $g / (\rho_0 \beta^2) = 5 \times 10^5 \text{ m}^4 \cdot \text{kg}^{-1}$ so that $\beta = O(10^{-4}) \text{ s}^{-1}$ as in Lawrence *et al.* (2002). These values are so that during ice-cover the parameterization of the eddy coefficients tends to the parameterization used by Lawrence *et al.* (2002).

We now explore the use of this parameterization in the LBM and some of its consequences. Figure 5.1(a) shows the modulus of the density gradient predicted by model run LBM-D. Depth profiles at grid box (40,12) are shown for the fifteenth day of each month of the run. The position of the profiles and the dates are the same as those of the temperature profiles, for this model run, shown in Figure 4.19(b). The density of freshwater has a strong dependence on temperature and this is evident in the predicted

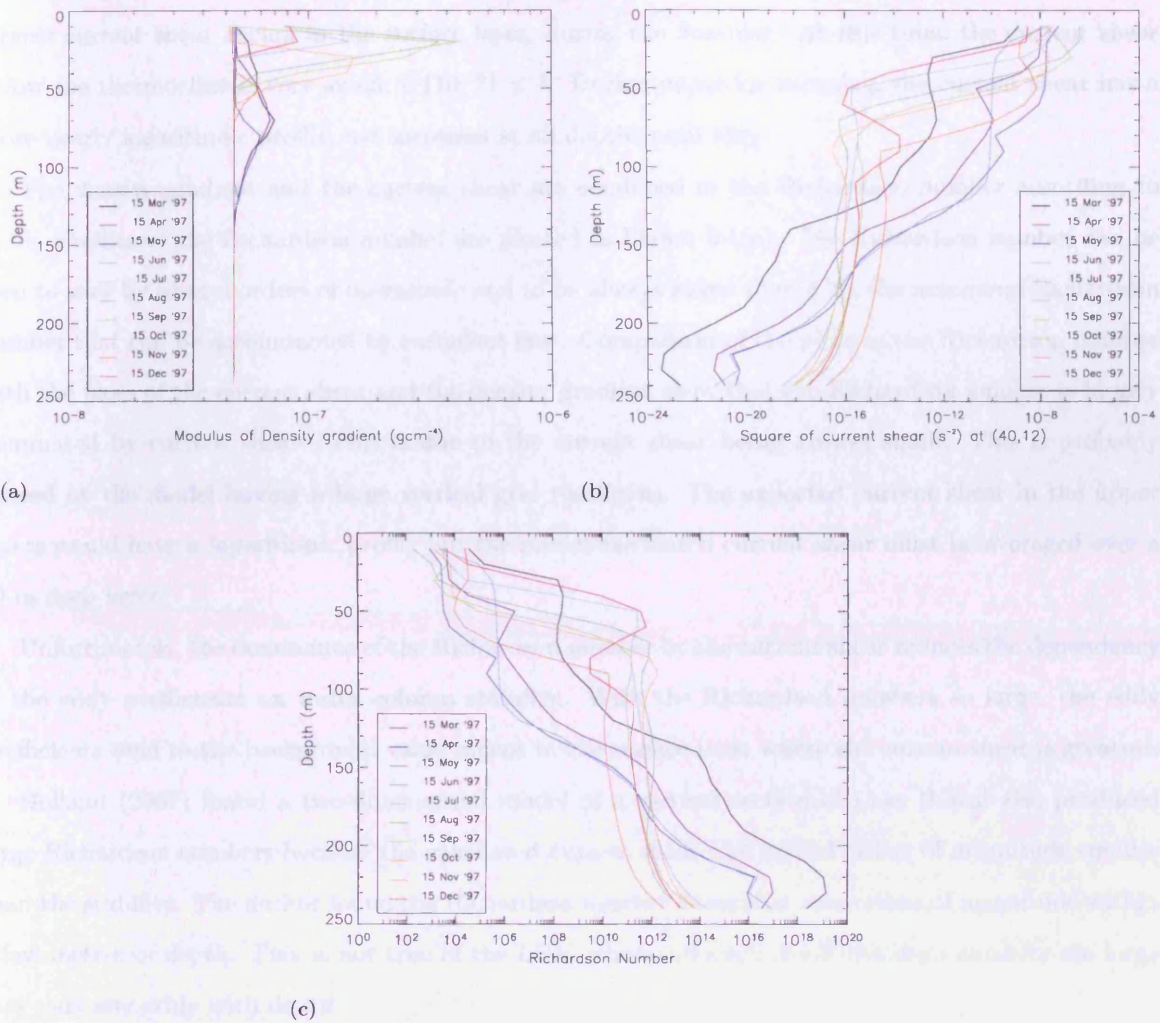


Figure 5.1: Depth profiles predicted at (40,12), by run LBM-D, on the fifteenth of each month for (a) the modulus of the density gradient, (b) the square of the current shear and (c) the Richardson number.

profiles of density gradient. Comparison of profiles of temperature and density gradient show that the depth of the thermocline coincides with the depth of the maximum density gradient. Under ice the density gradient is weak and reduces with time as near homothermy develops in the model lake. During the Summer the model predicts an increase in the density gradient at 20 m as a positive temperature stratification is established.

Figure 5.1(b) shows the square of the current shear predicted by model run LBM-D. The current shear varies through several orders of magnitude both with depth and in time. The steepest gradients in the current shear occur during the Summer stratification and coincide with the predicted thermocline. Thus, the model predicts that the thermocline acts as a barrier to downward mixing of momentum. The

largest current shear occurs in the surface layer, during the Summer. At this time, the current shear below the thermocline is very small, $O(10^{-8}) \text{ s}^{-1}$. During under-ice warming, the current shear has a more nearly logarithmic profile and increases at all depths until May.

The density gradient and the current shear are combined in the Richardson number according to (5.9). Profiles of the Richardson number are plotted in Figure 5.1(c). The Richardson number can be seen to vary by several orders of magnitude and to be always larger than 0.25, the maximum Richardson number that can be accompanied by turbulent flow. Comparison of the plots of the Richardson number with the plots of the current shear and the density gradient show that the Richardson number is largely dominated by current shear. This is due to the current shear being always small. This is probably caused by the model having a large vertical grid resolution. The expected current shear in the upper layers would have a logarithmic profile but the model-calculated current shear must be averaged over a 10 m deep layer.

Unfortunately, the dominance of the Richardson number by the current shear reduces the dependency of the eddy coefficients on water column stability. With the Richardson numbers so large, the eddy coefficients tend to the background value except in the surface layer where the current shear is greatest.

Holland (2001) found a two-dimensional model of a vertical section of Lake Baikal also produced large Richardson numbers because the calculated current shear was several orders of magnitude smaller than the stability. The author found the Richardson number fluctuated over orders of magnitude within a few metres of depth. This is not true of the LBM where although the Richardson numbers are large they vary smoothly with depth.

Holland (2001) avoided the problem by using a parameterization where the eddy coefficients were dependent on stability only and not dependent on current shear. This would not be suitable here as we require that the model predicts eddy coefficients which vary with current shear, so that the water column will respond to varying wind forcing.

Botte and Kay (2000) parameterized the eddy viscosity as the sum of two terms: one being a function of the Brunt-Väisälä frequency, and one being a function of the velocity shear. The authors' model was, however, not subject to wind forcing. With the increase in the range of values of the velocity shear that occurs with wind forcing, it is likely that the eddy coefficients will become dominated by the velocity shear dependent term. Again dependency on stability is lost and no advantage over the parameterization using the Richardson number would be gained.

The modulus of the density gradient and the current shear, predicted by LBM-D, at 10 m depth are plotted against time in Figures 5.2(a) and 5.2(b) respectively. The eddy diffusivity at 10 m depth is plotted against time in Figure 5.2(c). The density gradient at 10 m has maximum magnitude in August.

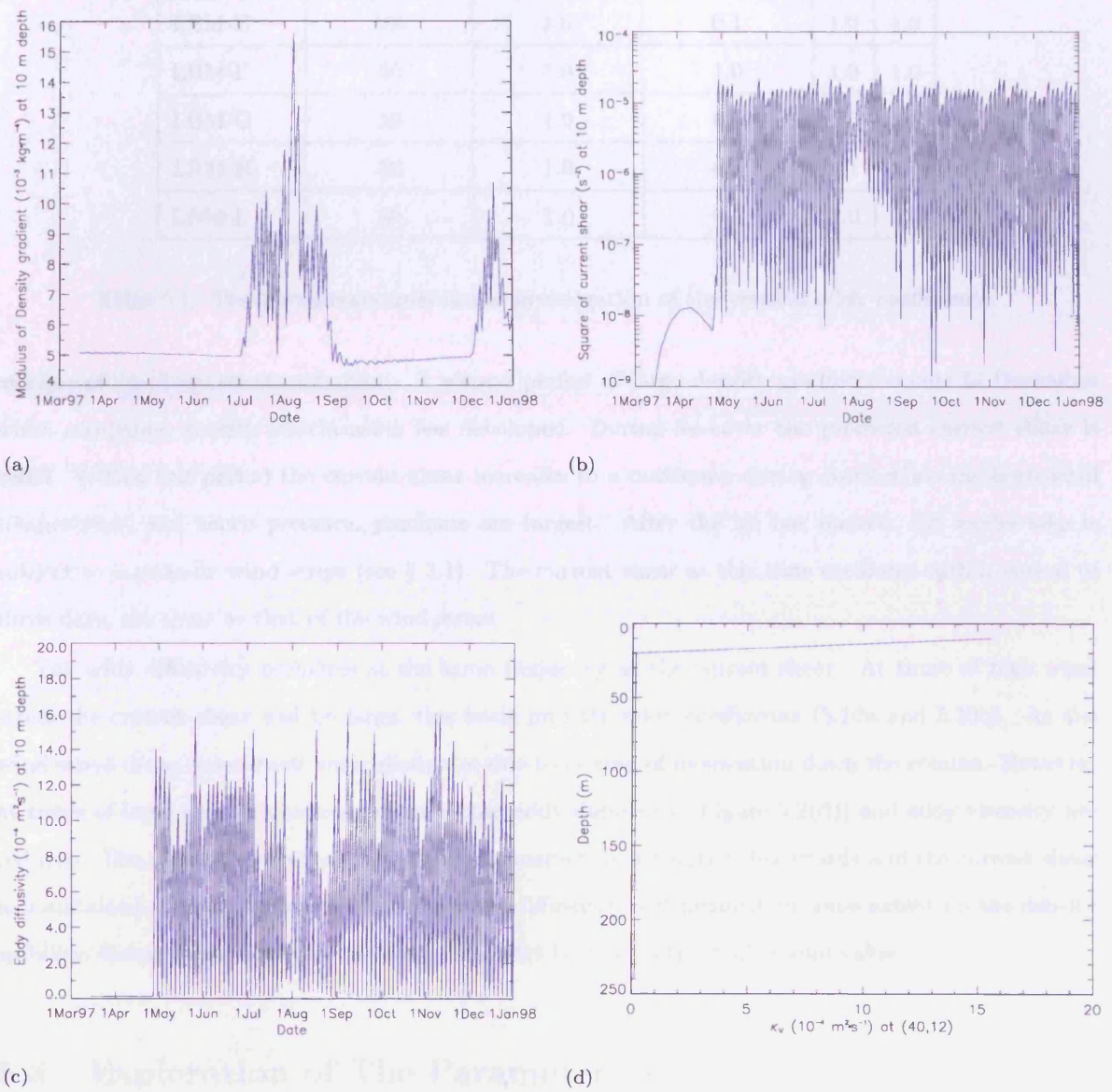


Figure 5.2: The model predicted (a) modulus of the density gradient, (b) current shear and (c) eddy diffusivity at 10 m depth for LBM-D and (d) the eddy diffusivity profile at (40,12) on 29/04/97 for LBM-D.

Model Run	ν_0	ν_b	κ_b	α	n
	$10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$	$10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$	$10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$		
LBM-D	50	1.0	0.1	1.0	1.0
LBM-E	100	1.0	0.1	1.0	1.0
LBM-F	50	1.0	1.0	1.0	1.0
LBM-G	50	1.0	0.1	0.5	1.0
LBM-H	50	1.0	0.1	0.1	1.0
LBM-I	50	1.0	0.1	1.0	0.5

Table 5.1: The parameters used in the investigation of the vertical eddy coefficients.

at time of the Summer stratification. A second period of large density gradients occurs in December when a negative density stratification has developed. During ice-cover the predicted current shear is small. Within this period the current shear increases to a maximum during April when the horizontal temperature, and hence pressure, gradients are largest. After the ice has melted, the model lake is subject to a periodic wind stress (see § 3.1). The current shear at this time oscillates with a period of three days, the same as that of the wind stress.

The eddy diffusivity oscillates at the same frequency as the current shear. At times of high wind stress the current shear will be large: this feeds into the eddy coefficients (5.10a and 5.10b). As the wind stress drops the current shear dissipates due to mixing of momentum down the column. However, at times of large density gradient the predicted eddy diffusivity (Figure 5.2(d)) and eddy viscosity are reduced. The reduced viscosity leads to less momentum being mixed downwards and the current shear is maintained. So at a depth of 10 m the eddy diffusivity is dependent to some extent on the density stability. Below 10 m, however, the eddy diffusivity is close to the background value.

5.3 Exploration of The Parameter Space

Table 5.1 gives the values assigned to the constants ν_0 , ν_b , κ_b , α and n in model runs LBM-D through LBM-I. These runs were performed in order to investigate the sensitivity of the model to the above constants and to seek out the values which would lead to the best fit of the model temperatures to the data described in § 4.2. Figure 5.3 shows the dependency of the eddy diffusivity (5.10b), on the Richardson number, as defined in (5.9), for each of the sets of constants shown in Table 5.1.

In the following discussion run LBM-D will be used as the control run for comparison with runs

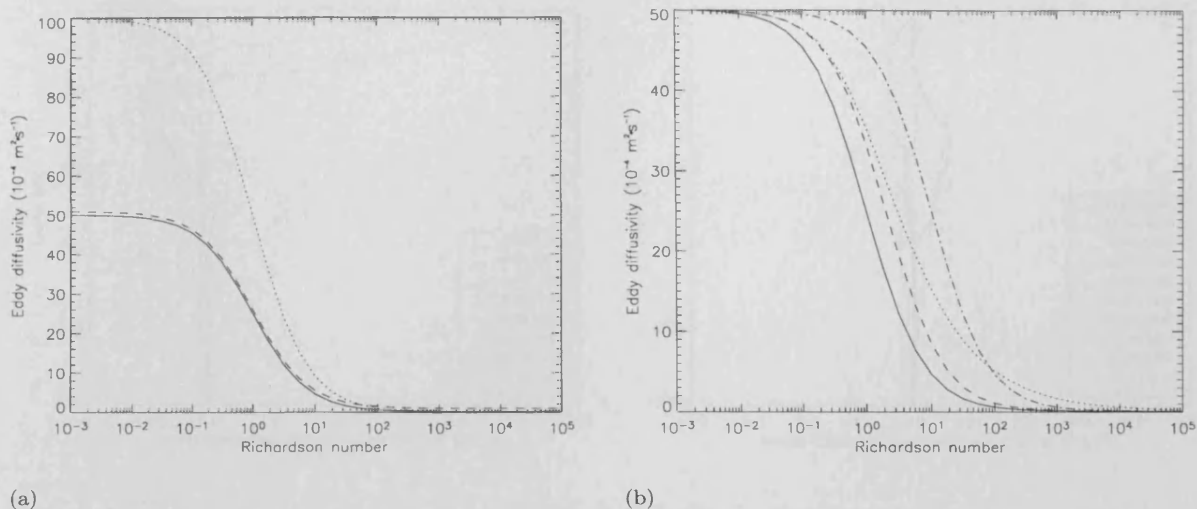


Figure 5.3: Variation of the eddy diffusivity with Richardson number for the parameters shown in Table 5.1: (a) run LBM-D (solid line), LBM-E (dotted line), LBM-F (dashed line); (b) run LBM-D (solid line), run LBM-G (dashed line), run LBM-H (dot-dashed line), run LBM-I (dotted line). Note the difference in vertical scale between (a) and (b). After Pacanowski and Philander (1981).

LBM-E to LBM-I and, for this purpose, the differences between run LBM-D temperature profiles and the observed temperature profiles for each data set are shown in Figure 5.4(a).

In run LBM-E a value of $100 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ is used for ν_0 , double that used in run LBM-D. From (5.10b) and Figure 5.3(a) it is clear that doubling ν_0 has the effect of doubling the eddy diffusivity in regions of small Richardson number where the eddy diffusivity is above the background value, κ_b . Comparison of the temperature profiles predicted by LBM-E with those predicted by LBM-D (Figure 5.5(a)) shows that the effect of increasing the value of ν_0 is small: the temperature differences between the two profiles (Figure 5.5(b)) never exceed 0.5°C . During ice-cover the profiles differ by no more than 0.01°C because at this time the Richardson number is large and the eddy diffusivity is close to the background value. Once the surface has warmed past the T_{md} the increased value of ν_0 begins to have some effect. There is an increase in the amount of heat that is mixed down away from the surface so that the model epilimnion in LBM-E is slightly cooler but also deeper and the thermocline is less steep. At depths below 80-100 m the eddy diffusivity is again close to the background value and there is little difference between the two runs at any time. During the onset of the Winter negative stratification, run LBM-E exhibits increased mixing of cooler surface water down the water column, resulting in warmer surface temperatures and a shallower thermocline than run LBM-D.

Figures 5.5(c) and 5.5(d) show comparisons of the LBM-E model profiles with the BICER and Jewson

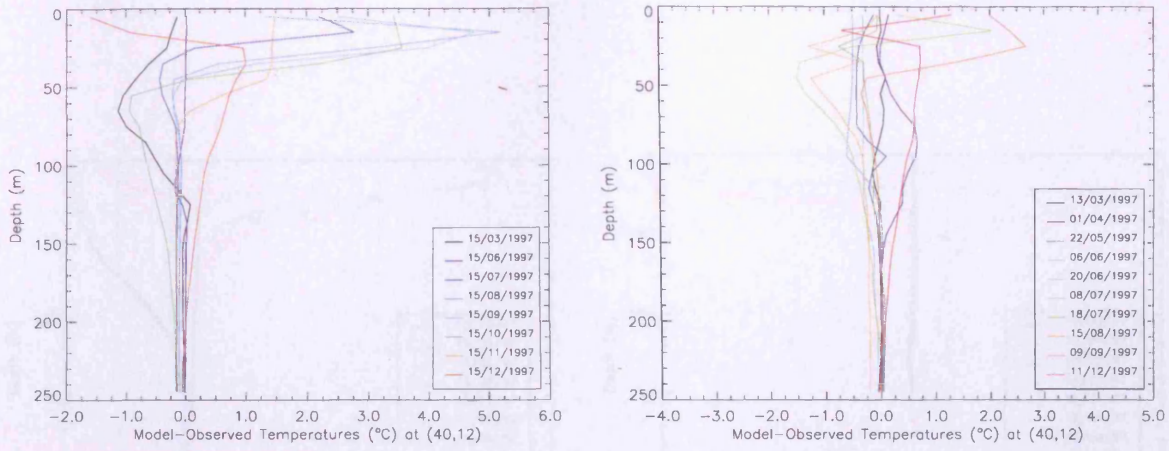
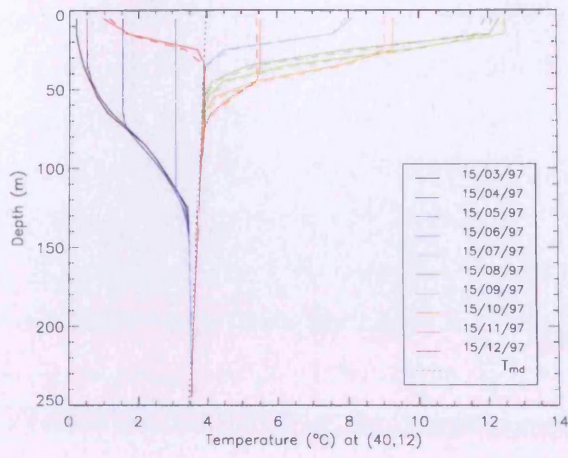


Figure 5.4: The difference between the temperature profiles predicted by the model run LBM-D, at (40,12), and the observed temperature profiles, for the South Basin, on the same days: (a) the BICER data; and (b) the Jewson data.

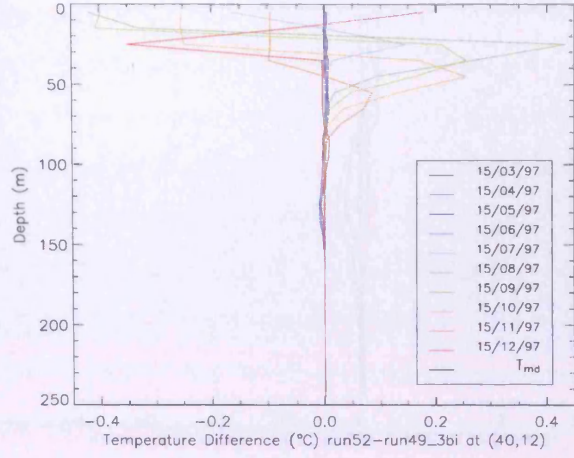
data sets. The model temperatures in run LBM-D differed from the observations most noticeably during the Summer stratification when the model surface layer was excessively warm. In run LBM-E there is some improvement as the surface layer temperatures are cooler but this is at the expense of increased warming above observed temperatures at a depth of 25 m.

Model run LBM-F has a background eddy diffusivity of $\kappa_b = 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ compared with $\kappa_b = 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}$ in run LBM-D. If the background eddy diffusivity is increased then, from (5.10b), the vertical eddy diffusivity is increased everywhere. Under ice, this results in increased mixing of surface heat downwards and faster erosion of the thermocline (Figure 5.6(a)). In the Summer the maximum surface temperature reached is lower and cooling of the surface starts earlier in the Autumn. The gradient of the Summer thermocline is shallower resulting in a broader mesolimnion. While the Summer surface temperatures are cooler than observations the November profile is closer to the observed profile (see Figures 4.4 and 4.3).

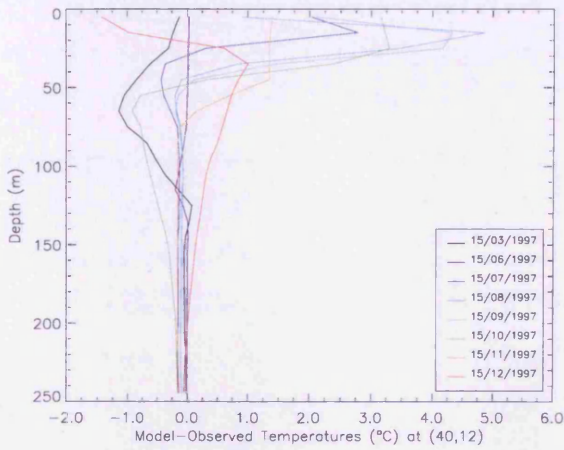
Reducing α in (5.10b) increases the Richardson number below which turbulent flow is allowed, so that the shear required to overcome a given stability is reduced. Thus, turbulent mixing is permitted over a greater part of the domain. In run LBM-D α has a value of 1.0 however in runs LBM-G and LBM-H the value of α is reduced to 0.5 and 0.1 respectively. The effect under ice is small as the eddy diffusivity tends to the background value here. However, after the onset of Summer stratification increased mixing of heat down away from surface occurs. This results in a shallower thermocline and broader mesolimnion. Also, surface temperatures during the Summer are cooler.



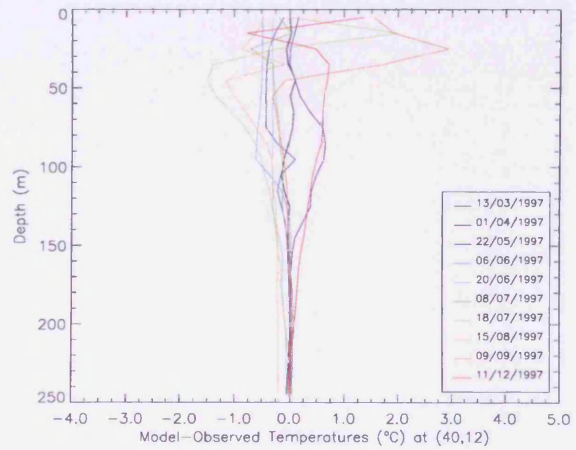
(a)



(b)



(c)



(d)

Figure 5.5: Results from model run LBM-E, which has $\nu_0 = 100$: (a) Temperature profiles for run LBM-D (solid lines) which has $\nu_0 = 50$ and for run LBM-E (dashed lines); (b) The difference between the profiles in (a), (LBM-E – LBM-D). The difference between the temperature profiles predicted by model run LBM-E and the observed temperature profiles, for the South Basin, on the same days: (c) the BICER data; and (d) the Jewson data. All plots at (40,12).

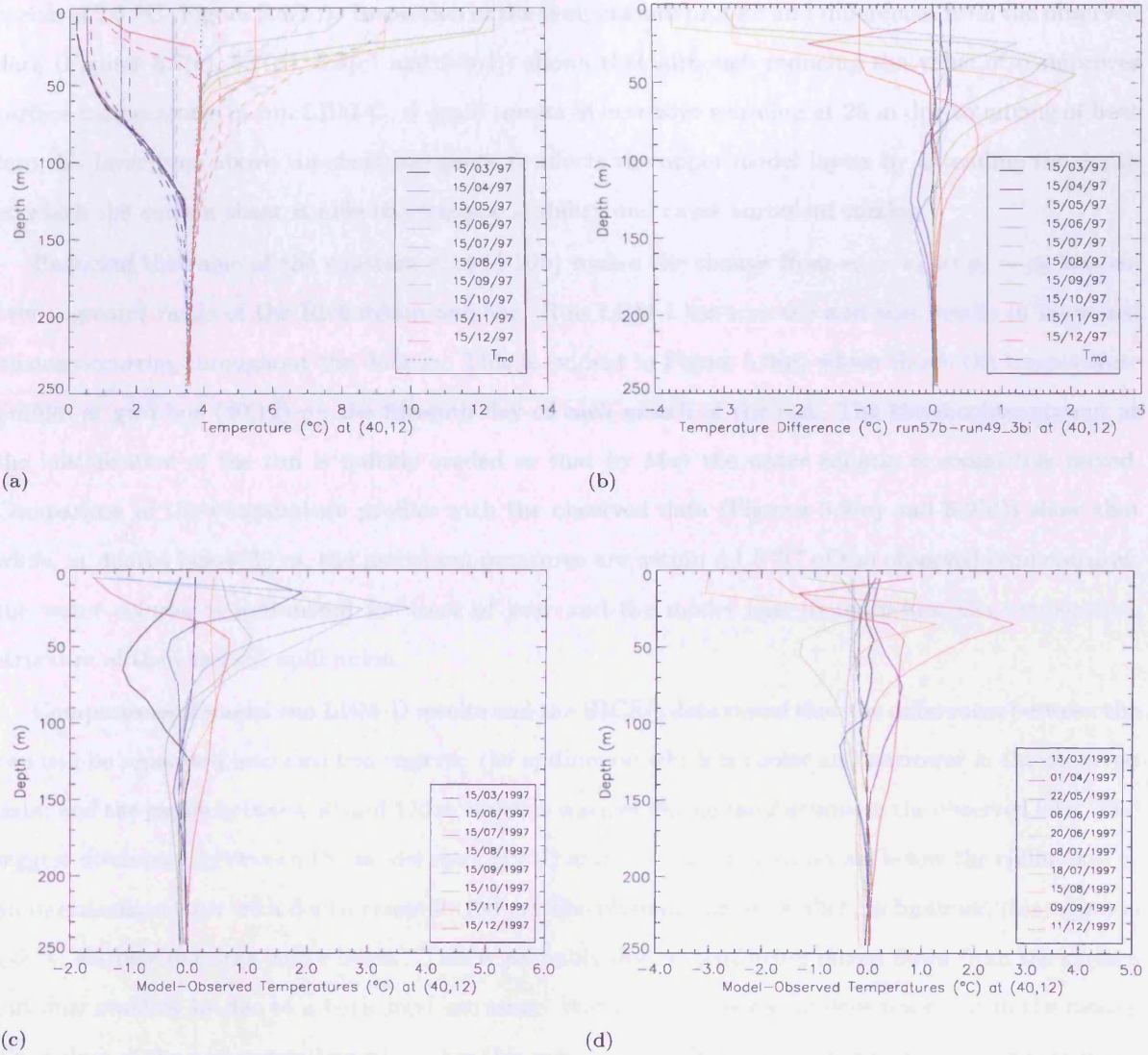


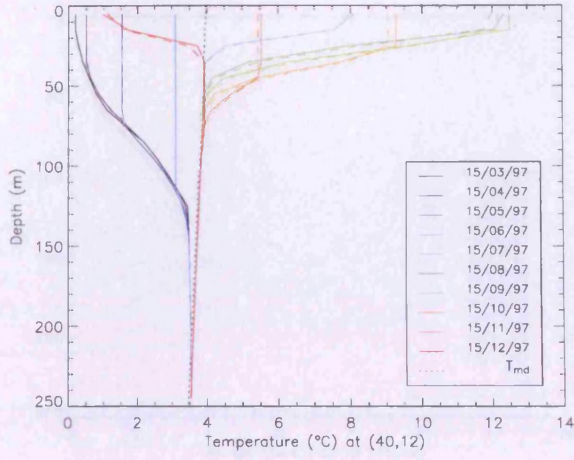
Figure 5.6: Results from model run LBM-F, which has $\kappa_b = 1.0$: (a) Temperature profiles for run LBM-D (solid lines) which has $\kappa_b = 0.1$ and for run LBM-F (dashed lines); (b) The difference between the profiles in (a), (LBM-F – LBM-D). The difference between the temperature profiles predicted by model run LBM-F and the observed temperature profiles, for the South Basin, on the same days: (c) the BICER data; and (d) the Jewson data. All plots at (40,12)

The effect of reducing α to 0.5 is small: the temperature differences between runs LBM-D and LBM-G are less than 0.3 °C (Figure 5.7(b)). However when α is reduced to 0.1, as in run LBM-H, the effect is more noticeable and the temperature differences between runs LBM-D and LBM-H reach values in excess of 2.0 °C (Figure 5.8(b)). Inspection of the temperature profiles and differences from the observed data (Figures 5.7(c), 5.7(d), 5.8(c) and 5.8(d)) shows that although reducing the value of α improves surface temperatures in run LBM-G, it again results in excessive warming at 25 m due to mixing of heat into this layer from above. In short increasing α affects the upper model layers by extending the depth at which the current shear is able to overcome stability and cause turbulent mixing.

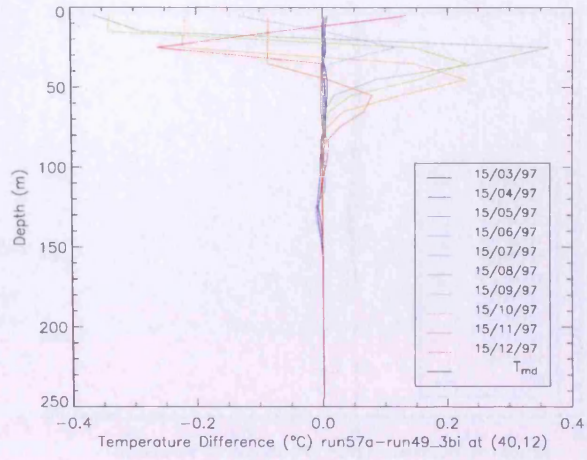
Reducing the value of the constant n in (5.10b) makes the change from $\nu_r = \nu_b$ to $\nu_r = \nu_0$ happen over a greater range of the Richardson number. Run LBM-I has $n = 0.5$ and this results in increased mixing occurring throughout the domain. This is evident in Figure 5.9(a) which shows the temperature profiles at grid box (40,12) on the fifteenth day of each month of the run. The thermocline present at the initialization of the run is quickly eroded so that by May the water column is completely mixed. Comparison of the temperature profiles with the observed data (Figures 5.9(c) and 5.9(d)) show that while, at depths below 30 m, the model temperatures are within ± 1.5 °C of the observed temperatures, the water column is well-mixed for most of year and the model fails to reproduce the temperature structure of the observed epilimnion.

Comparisons of model run LBM-D results and the BICER data reveal that the differences between the two can be separated into two regions: the epilimnion which is cooler and narrower in the observed data; and the region between 50 and 170 m which is warmer during the Autumn in the observed lake. The biggest discrepancy between the model run LBM-D and the Jewson data occurs below the epilimnion in an intermediate layer with depth range 30-100 m. The observations show that, in Summer, this region is 1-2 °C warmer than the water below. This is probably due to heat being mixed down from the surface but may possibly be due to a horizontal intrusion. Warming of this region does not occur in the model. The values of the parameters investigated in this section have, therefore, been chosen in order to make the parameterization more sensitive to the water column stability and, in doing so, increase the eddy diffusivity to a value above the background value throughout the model epilimnion and mesolimnion.

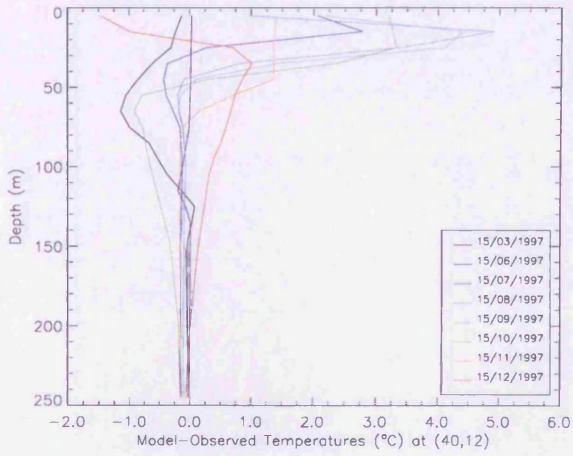
The different parameters, ν_b , ν_0 , κ_b , α , and n dominate in different regions of the domain. In the lower layers the current shear is negligible and the second term in (5.10b) tends to zero. Thus the eddy diffusivity tends to κ_b and changing the values of the parameters ν_0 , κ_b , α , and n has little effect here. The current shear is strongest in the upper layers and hence, in this region, the second term in the eddy diffusivity (5.10b) is no longer negligible and the eddy diffusivity is now dependent on ν_0 , α and n as well as κ_b . To summarize, changing the value of the background eddy diffusivity, κ_b , affects mixing in



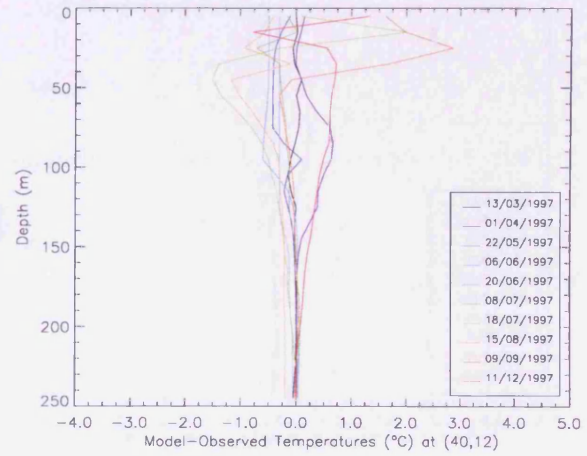
(a)



(b)

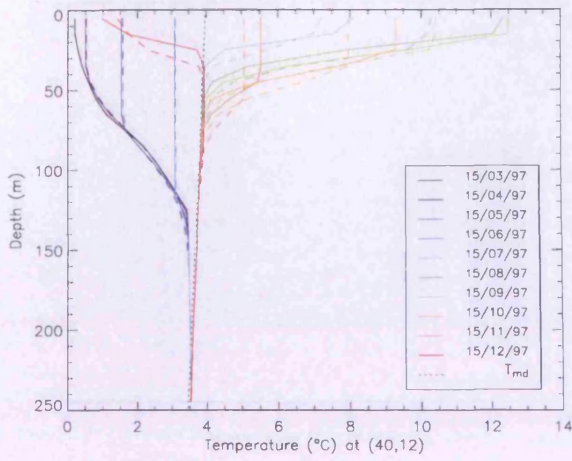


(c)

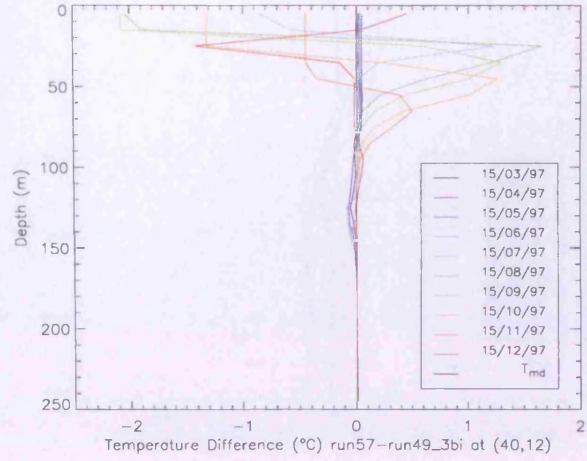


(d)

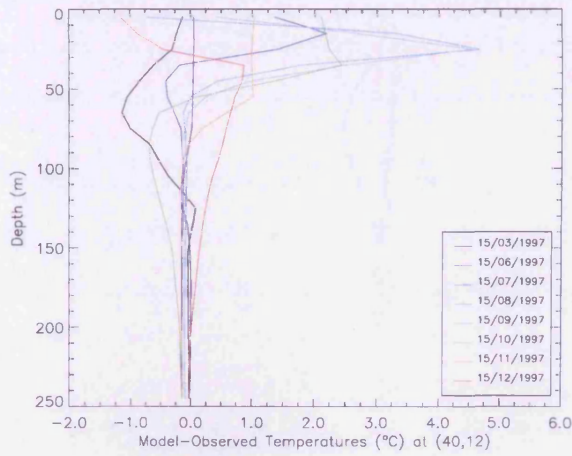
Figure 5.7: Results from model run LBM-G, which has $\alpha = 0.5$: (a) Temperature profiles for run LBM-D (solid lines) which has $\alpha = 1.0$ and for run LBM-G (dashed lines); (b) The difference between the profiles in (a), (LBM-G – LBM-D). The difference between the temperature profiles predicted by model run LBM-G and the observed temperature profiles, for the South Basin, on the same days: (c) the BICER data; and (d) the Jewson data. All plots at (40,12).



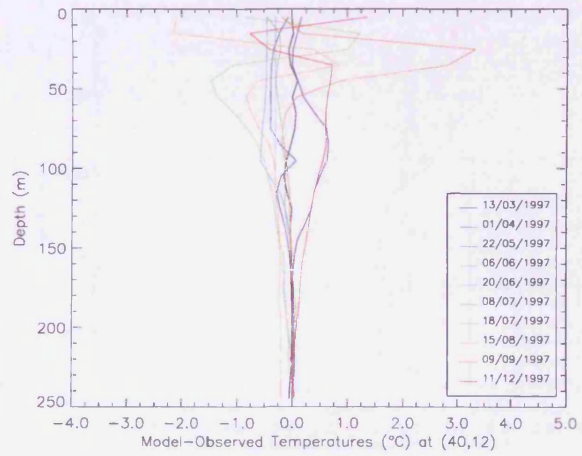
(a)



(b)

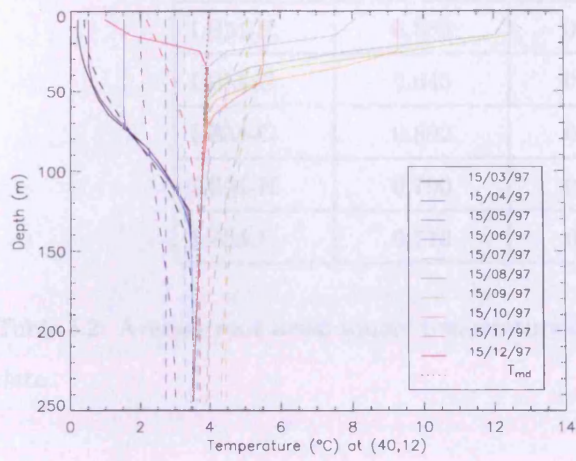


(c)

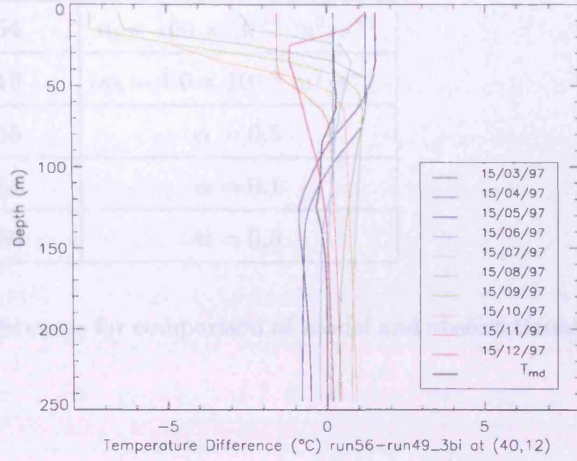


(d)

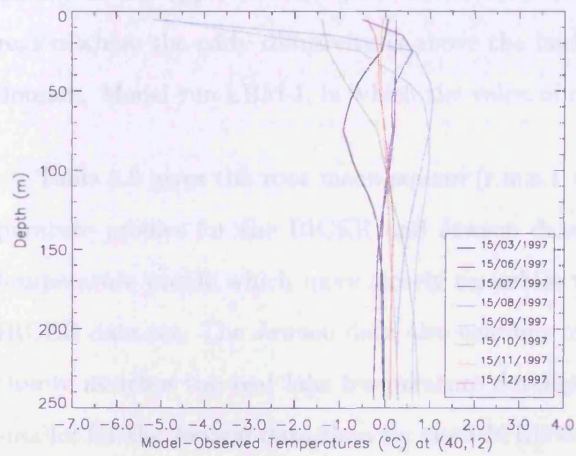
Figure 5.8: Results from model run LBM-H, which has $\alpha = 0.1$: (a) Temperature profiles for run LBM-D (solid lines) which has $\alpha = 1.0$ and for run LBM-H (dashed lines); (b) The difference between the profiles in (a), (LBM-H – LBM-D). The difference between the temperature profiles predicted by model run LBM-H and the observed temperature profiles, for the South Basin, on the same days: (c) the BICER data; and (d) the Jewson data. All plots at (40,12).



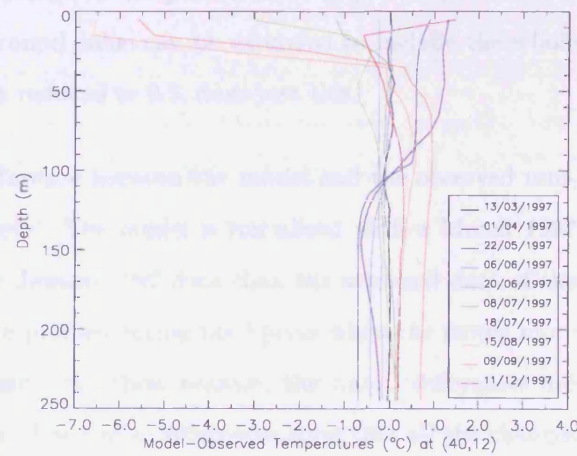
(a)



(b)



(c)



(d)

Figure 5.9: Results from model run LBM-I, which has $n = 0.5$: (a) Temperature profiles for run LBM-D (solid lines) which has $n = 1.0$ and for run LBM-I (dashed lines); (b) The difference between the profiles in (a), (LBM-I - LBM-D). The difference between the temperature profiles predicted by model run LBM-I and the observed temperature profiles, for the South Basin, on the same days: (c) the BICER data; and (d) the Jewson data. All plots at (40,12).

Model Run	BICER data	Jewson data	Model parameter changed in run
LBM-D	0.920	0.465	Control run
LBM-E	0.889	0.454	$\nu_0 = 100 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$
LBM-F	0.645	0.518	$\kappa_b = 1.0 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$
LBM-G	0.892	0.455	$\alpha = 0.5$
LBM-H	0.790	0.484	$\alpha = 0.1$
LBM-I	0.713	0.950	$n = 0.5$

Table 5.2: Average root mean square temperature differences for comparison of model and observational data.

the whole of the domain whereas, when the other parameters are altered, the effects are concentrated mainly in the upper model layers. However, if the change in the parameter is large enough then the region where the eddy diffusivity is above the background value can be extended to include the whole domain. Model run LBM-I, in which the value of n is reduced to 0.5, does just this.

Table 5.2 gives the root mean square (r.m.s.) difference between the model and the observed temperature profiles for the BICER and Jewson data sets. The model is initialized with a March 1997 temperature profile which more closely resembles the Jewson 1997 data than the averaged data of the BICER data set. The Jewson data also contains more profiles during the Spring when the model more closely matches the real lake temperature development. For these reasons, the r.m.s. differences are smaller for the Jewson data than for the BICER data. The r.m.s. differences show that all the changes made to the model parameters improve the match to the BICER data. The biggest improvement occurs when the background eddy diffusivity is increased (run LBM-F). The match to the Jewson data is improved in only two runs: LBM-E and LBM-G. A reduction in the value of α or an increase in the value of ν_0 extends mixing to deeper depths and so improves the model match with these observations.

In conclusion, the improvements to run LBM-D gained by changing the constants in (5.10b) are small and, during the Summer and Autumn, any increased mixing in the region immediately below the thermocline is accompanied by an increase in mixing in the epilimnion and the thermocline region itself. This results in a deepening of the mesolimnion with a shallower thermocline and in short the epilimnion is less well-defined.

5.3.1 Summary of parameter space exploration

Vertical mixing in the model takes place by way of two mechanisms. First, the convective adjustment algorithm maintains hydrostatic equilibrium by mixing unstable regions at the end of every time-step (see § 3.1). Thus the quantity $\mathbf{g}(\partial\rho/\partial z)$ is kept positive and negative Richardson numbers can never occur. Secondly, turbulent mixing of momentum and temperature is modelled by diffusive terms in the respective equations. The coefficients are, respectively, the eddy viscosity and eddy diffusivity. In view of the low resolution of the LBM and in the interest of computational efficiency, these coefficients are modelled by algebraic expressions and further equations are not solved.

The value of the eddy diffusivity predicted by the model is generally lower than that calculated from observations. Ravens *et al.* (2000) measured temperature microstructure and found the stratified deep water (600-250 m) to have a vertical diffusivity of $10 - 90 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ with values increasing to $O(10^{-2}) \text{ m}^2 \cdot \text{s}^{-1}$ in the upper layer. Kipfer *et al.* (2000) used inverse modelling of transient tracers and calculated a value of $4.6 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1} \pm 10\%$, in the South Basin, for the vertical diffusivity of the deep water column (below 400 m). The model eddy diffusivity tends to the background value of $0.1 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ in the lowest layer (240-250 m) and values in the surface model layer are of order $10 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$. The predicted eddy diffusivity also tends to the background value over the greater part of the domain however the model reproduces the observed temperature profiles well, suggesting that sufficient vertical mixing is occurring. This may be explained because the use of a convective adjustment routine to maintain hydrostatic equilibrium adds vertical mixing during periods of instability. As the parameterization of the eddy diffusivity in (5.10b) facilitates reasonable simulation of the temperature observations it is retained, despite the deficiencies noted here.

The constants used in the expressions for the eddy coefficients are chosen in order that the model best simulates the temperature data available. According to the r.m.s. differences the most suitable values for simulation of the Jewson data would appear to be

$$\begin{aligned}\nu_0 &= 100 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1} , \\ \kappa_b &= 0.1 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1} , \\ \alpha &= 0.5 , \\ n &= 1 ,\end{aligned}\tag{5.12}$$

although further investigation of the parameter space may enable further fine tuning of the parameters and thus better simulation of the data.

The region of the water column where the model simulation is poorest is immediately below the

epilimnion (30-100 m). The observations show this region is warmer than the model predicts. This may be due to strong mixing of heat downwards from the surface during the Summer and Autumn. Lake Baikal experiences storms with wind velocities exceeding 20 m s^{-1} , sometimes reaching up to 40 m s^{-1} in late Autumn. The resulting increased current shear will cause an increase in turbulent mixing. The next section explores how the model responds to transient strong winds and in particular the effect of such winds on vertical mixing in the model.

5.4 Vertical Mixing During A Storm Event

In order to further study the suitability of the vertical mixing parameterization, in this section the effect of a transient moderately high velocity wind stress superimposed on a low background wind stress is examined. Applying a transient, moderately high wind can be thought of as modelling a storm. The implementation of the wind stress is described in § 3.1.

The nature of the wind field over lake Baikal was reviewed by Shimaraev *et al.* (1994). The main points of their discussion are summarized here. The topography of the Baikal Depression alters the prevailing westerly winds so that the predominant winds blow along and across the depression. During the ice-free period (May-December) the winds are mainly west-north-westerlies with some longitudinal south-westerly and north-easterly winds. There is a strong seasonal pattern to the distribution of wind types. Weak unstable winds predominate from May to September. From October to December contribution from the north-westerly and north-easterly winds increases. The strongest storms are caused by north-westerly winds and have speeds in excess of 20 m s^{-1} (up to 40 m s^{-1} in the Autumn).

The results from two model runs are discussed in this section. The first has a west-northwesterly wind field, the same direction as the prevailing winds over Lake Baikal. The second aims to model the effect of storms; it has the same field but adds two episodes of strong north-westerly winds. Both model runs are forced by time-varying but spatially uniform wind stress fields.

Run LBM-J has background winds which are west-by-northwesterly in direction and have a maximum magnitude of 5 m s^{-1} . The magnitude varies as $|u_{wind}| = 5 \cos^2(2\pi t/10 \text{ days}) \text{ m s}^{-1}$ so that the magnitude peaks once every five days. Run LBM-K has background winds as above but added to this are two 'storms'. These last 24 hours and consist of a northwesterly wind with magnitude rising sinusoidally to a peak of 25 m s^{-1} after 12 hours and then falling similarly to zero m s^{-1} after 24 hours. The increase in wind speed occurs over the whole lake and is superimposed on the background wind field on 15 July and on 30 July.

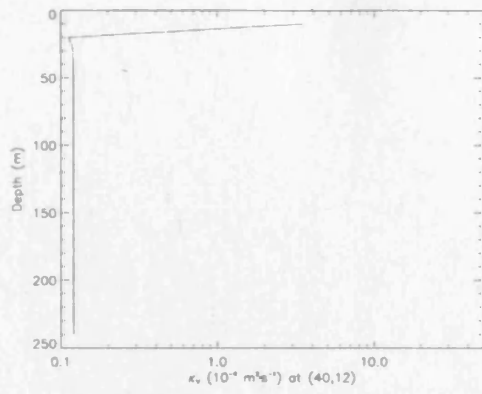
Depth profiles of the eddy diffusivity at point (40,12) are shown in Figure 5.10. The profiles show the

state of the lake at the end of the day on the date given, so for instance Figure 5.10(c) shows the eddy diffusivity 24 hours after the beginning of the storm. The background wind also peaks on 15/07/97.

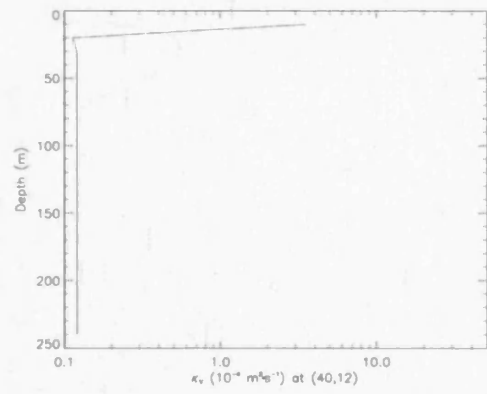
The profiles for the model with background winds only (LBM-J) show that the eddy diffusivity, at depths below 40 m, is constant with both depth and time. It takes a value of $0.120 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$, which is slightly above the background value. Above 40 m the eddy diffusivity varies with time, with peak values occurring on the days that the background wind speed reaches maximum. At 10 m the eddy diffusivity has a maximum of $25.5 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ on 15/07/97. Values of eddy diffusivity much above the background level are confined to this upper model layer. However, the eddy diffusivity profile has a different shape after a transient, but substantial, increase in wind speed. The depth profile for 15/07/97 for run LBM-K shows high values of eddy diffusivity ($[25 - 40] \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$) in the top 50 m. Below 50 m the eddy diffusivity decreases rapidly but smoothly until below 100 m the eddy diffusivity takes values similar to those for run LBM-J. The eddy diffusivity is substantially higher than in run LBM-J for the days 15-17/07/97 and evidence of the ‘storm’ can still be seen in the eddy diffusivity profile for 19/07/97 with higher values occurring down to 60 m.

The increase in the eddy diffusivity that accompanies an increase in the wind stress is, of course, due to the increase in the vertical current shear. The increased current shear feeds into the parameterization of the eddy coefficients (5.10). Profiles of the u component of velocity at (40,12) for runs LBM-J and LBM-K are shown in Figure 5.11. Although the u component of the wind velocity is always positive the u component of the current velocity takes both positive and negative values. This indicates that a seiche is occurring with some return flow occurring in the upper layers when the wind speed drops. The higher current velocities ($> 0.1 \text{ cm s}^{-1}$) are found in the upper 30 m; below 30 m the velocity component has the opposing sign and a magnitude of less than 0.1 cm s^{-1} . The current velocities in the upper layers of run LBM-K immediately after the ‘storm’ are between one and two orders of magnitude larger than the current velocities for the same day in run LBM-J. After the ‘storm’ high velocities are found in the upper 60 m indicating an increase in the depth of the wind-affected layer and a mixing of momentum to deeper depths than in run LBM-J. Thus higher wind stress leads to higher vertical current shear in the surface 60 m resulting in an increase in the eddy coefficients in this region with the subsequent mixing of momentum downwards.

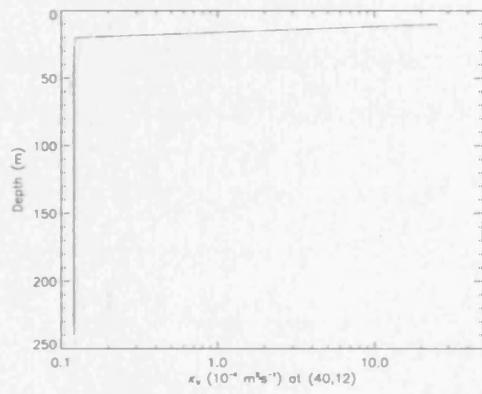
As well as momentum, the vertical mixing of heat is also enhanced by the storm event. This is evident on inspection of the temperature profiles for these runs shown in Figure 5.12(a). The profile for run LBM-K on the 18/07/97, three days after the first storm event, shows a cooler surface temperature and shallower thermocline than is the case for run LBM-J. Temperatures are above the T_{md} in the upper 80 m after the storm compared with 40 m in LBM-J when no storm occurs. This process of warming of



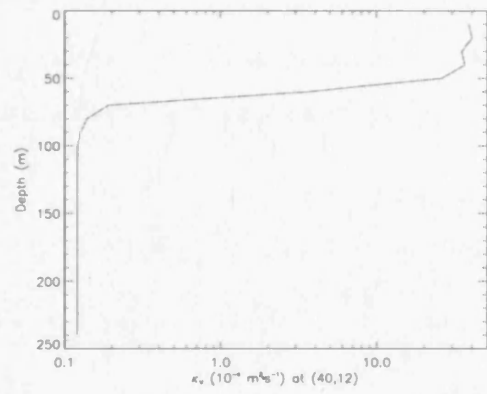
(a) LBM-J: 14/07/97



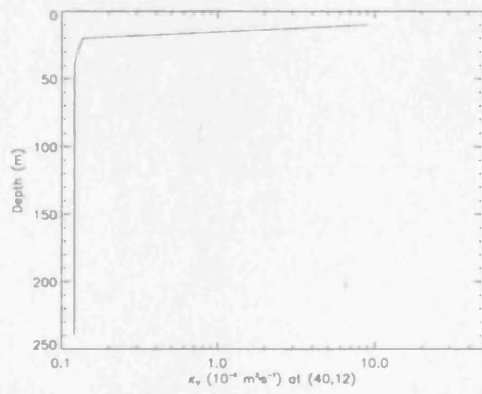
(b) LBM-K: 14/07/97



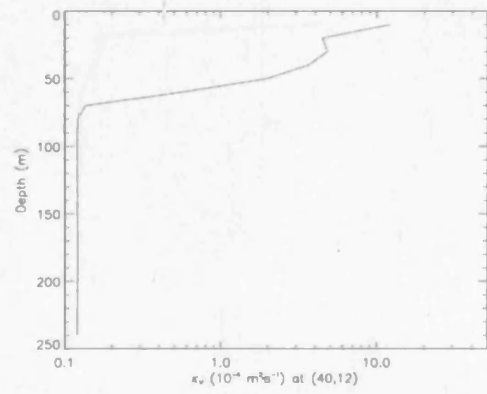
(c) LBM-J 15/07/97



(d) LBM-K 15/07/97

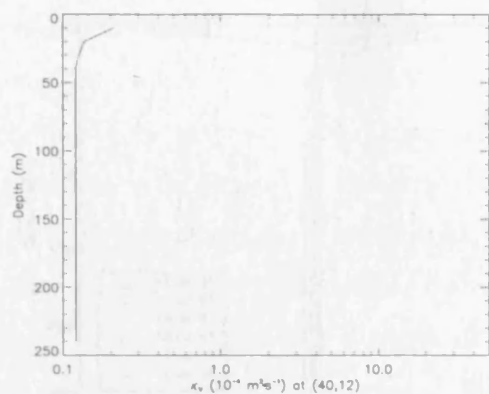


(e) LBM-J 16/07/97

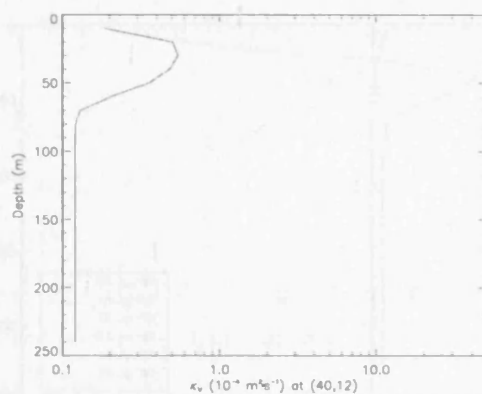


(f) LBM-K 16/07/97

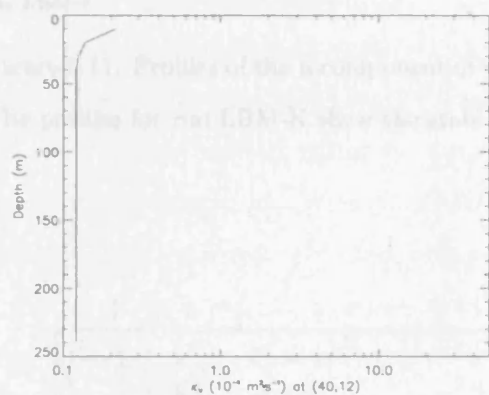
Figure 5.10: Eddy diffusivity at (40,12) for runs LBM-J and LBM-K.



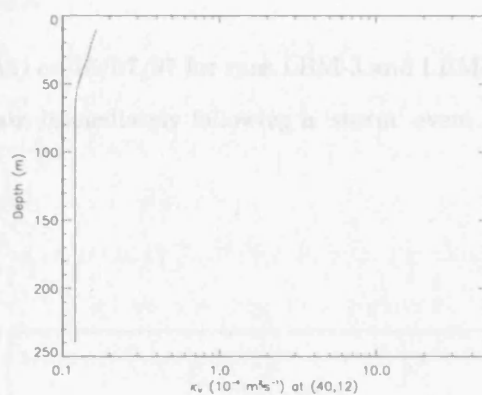
(g) LBM-J 17/07/97



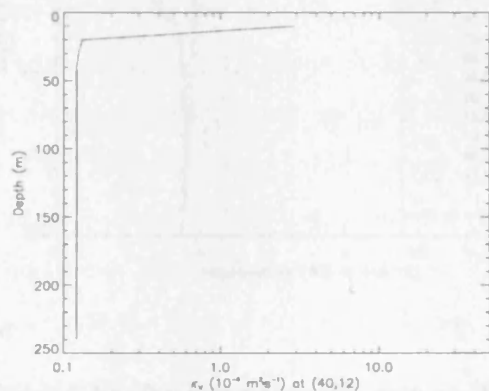
(h) LBM-K 17/07/97



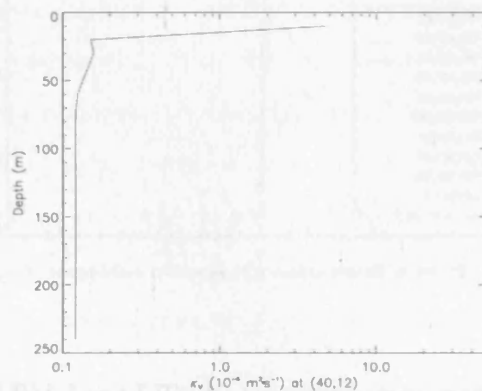
(i) LBM-J 18/07/97



(j) LBM-K 18/07/97

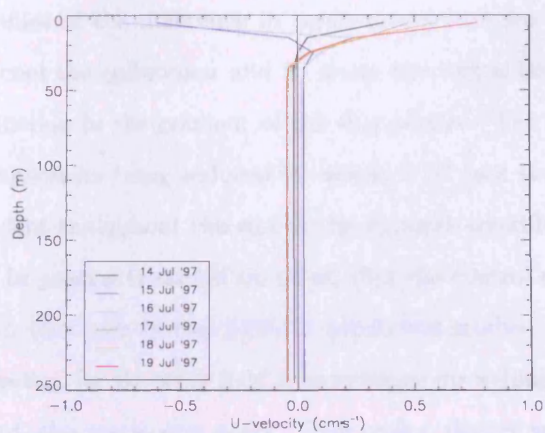


(k) LBM-J 19/07/97

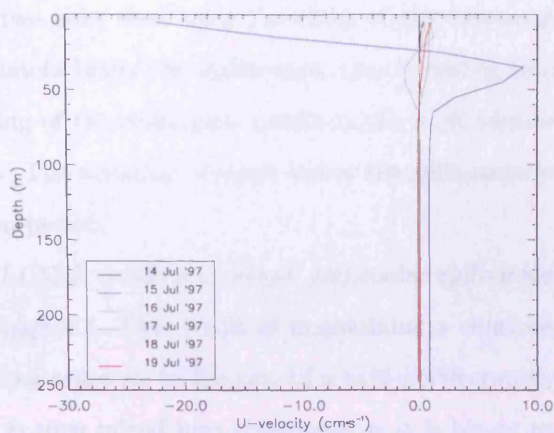


(l) LBM-K 19/07/97

Figure 5.10: Eddy diffusivity at (40,12) for runs LBM-J and LBM-K (continued).

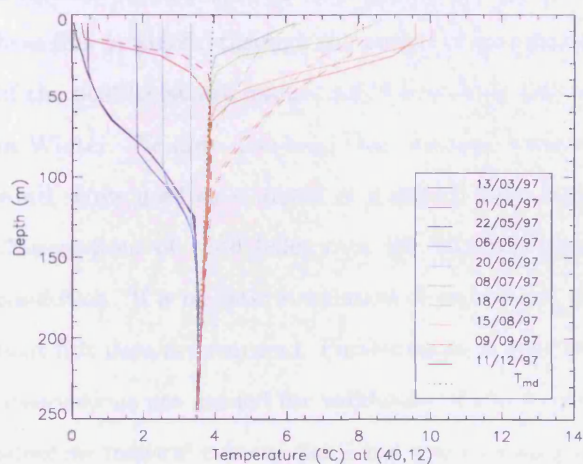


(a) LBM-J

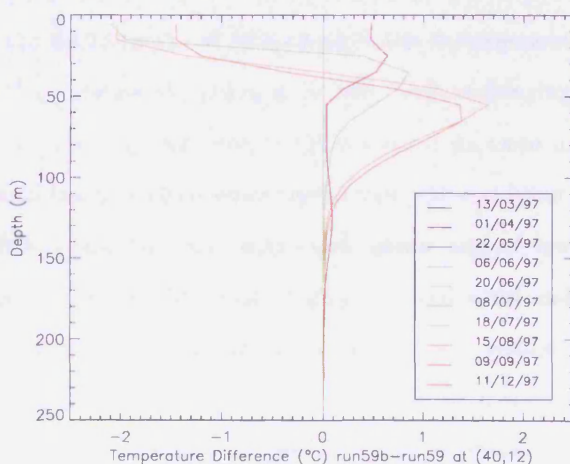


(b) LBM-K

Figure 5.11: Profiles of the u component of velocity at (40,12) on 15/07/97 for runs LBM-J and LBM-K. The profiles for run LBM-K show the state of the model lake immediately following a 'storm' event.



(a)



(b)

Figure 5.12: Temperature profile data at (40,12) for runs LBM-J and LBM-K: (a) temperature profiles on the same dates as the Jewson data for run LBM-J (solid) and run LBM-K (dashed) and (b) the differences between the profiles (LBM-K - LBM-J).

water below the epilimnion is enhanced further by the second storm on 30/07/97. The profiles for the 15/08/97 show water warmer than T_{md} at 130 m in run LBM-K compared with 60 m in run LBM-J. Profiles of the difference in temperature between the two runs show that the effect of the storms is to cool the epilimnion and to warm the region immediately below the epilimnion, thus resulting in a reduction in the gradient of the thermocline. The cooling of the epilimnion results in the peak surface temperature being reduced by almost 2 °C (not shown). The warming of water below the epilimnion is evident throughout the rest of the Summer stratification period.

In passing it should be noted that the control run, LBM-J, exhibits a deeper and cooler epilimnion than that seen in run LBM-D which was studied in Chapter 5. One result of maintaining a constant direction for the wind field is to enhance up-welling of cool water at, in the case of a west-northwesterly wind, the north-west coast. This cooler, denser water is then mixed into epilimnion as it is blown by the winds away from the coast.

The model runs examined in this section and the previous section have demonstrated the sensitivity of the model vertical turbulent mixing to the applied wind forcing. The dependence of the coastal up-welling and the turbulent vertical mixing on the properties of the wind field emphasize the need for realistic wind fields to be input to the model if the thermodynamics of the real lake are to be simulated. Further studies will require a spatially varying, as well as time-varying, wind field. The model so far reproduces a realistic development of the temperature field in the upper 250 m from the time of minimum surface heat flux in March, through the period of ice-melting, the development of homothermy, the development of the positive stratification, and the cooling past the T_{md} during the Autumn to the onset of freezing in Winter. We have also seen that the model vertical mixing responds well to the transient increase in wind stress used as a model of a storm, with mixing of warm surface water down the water column. Observations of wind fields over the lake and heat fluxes into the lake have been sparse and of low resolution. If a realistic simulation of the lake is to be performed then more highly resolved wind and heat flux data are required. Furthermore, a more complete data set of temperature and current velocity observations are needed for validation of the model. In view of the above no attempt will be made to simulate realistic current fields but the following chapter will examine the model output produced in response to the thermal forcing as discussed in § 4.3.4 and wind forcing discussed earlier in this section.

Chapter 6

Results of The LBM

It has already been noted that the lake currents and temperature distribution throughout this period are important factors in the reproduction of the diatom *A.baicalensis* which reproduces during this time. In this chapter we first examine the simulation of the lake by the LBM during the period of ice-melting and compare the LBM results to the LHLC model results. Zhdanov *et al.* (2001) found enhanced current velocities at the boundaries between areas with different degrees of snow-cover. In late March 1997 the authors measured velocities of 1-1.5 cm s⁻¹. Currents of a similar magnitude were reproduced by the LHLC model (Lawrence *et al.*, 2002) therefore good agreement between the two models would support the validity of the LBM simulation in this period. The second part of this chapter looks at the LBM simulation of Lake Baikal during the ice-free period, the months May-December: the LHLC model is unable to simulate this period. The response of the LBM to forcing, both by simulated weak, unstable winds and by storm events, is investigated.

6.1 The LBM Simulation of The Spring Warming Period.

Observations of the thermodynamics of Lake Baikal during the Spring are sparse due to the obvious difficulty, and danger, of observing beneath ice. This hinders validation of any model simulation. The data available for comparison with the model output consist of the current velocity measurements of Zhdanov *et al.* (2001), mentioned above, and the temperature data contained in the 1 April, 22 May, 6 June and 20 June profiles of the Jewson data set. References to snow and ice fields are to those of Le Core (1998), which are inferred from satellite observations and used as inputs to both the LBM and LHLC model.

The satellite observations for the 23 May 1997 show that the lake is free of ice. Thus, after the 22

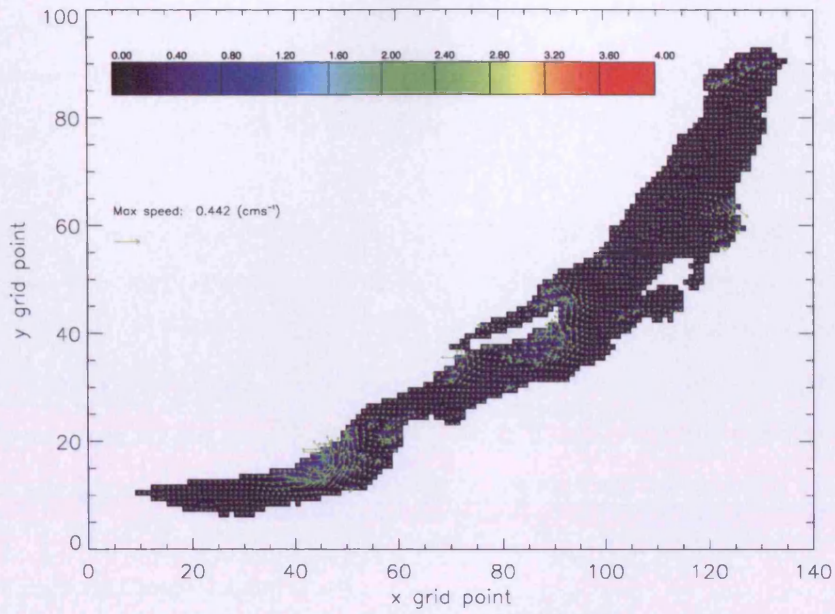
May there is no differential heating of the model lake surface and one might expect the residual currents to die away over a period of time. The LBM run LBM-M was run with no wind forcing, other parameters are as for run LBM-D, so that this process might be investigated and also to allow for a more direct comparison with the LHLC model which does not implement wind forcing. The period examined in this section will, therefore, start at the time of model initialization on 13 March 1997 and extend until the 20 June 1997, the date of the last available temperature profile before surface temperatures exceed the T_{md} .

Maps showing the water temperature and vectors of current velocity in the surface model layer, on the same dates as the Jewson temperature profiles, are given in Figure 6.1. As might be expected, these show that temperatures increase fastest under snow-free ice and where the ice is most transparent. The map of snow and ice types, derived from satellite observations, for 14 March 1997 (Figure 2.3(a)) shows an area of snow-free ice next to the north-west coast of the South Basin, adjacent to the location of the Angara outflow (around grid point [45,17]). Figure 6.1(a) shows the presence, on 1 April 1997, of an anti-clockwise circulation around this warmer area. Other areas show enhanced warming: in the Central Basin a similar, though not as distinct, anti-clockwise circulation is seen at the north coast and entrance to the Maloye More (grid point [90,47]); smaller areas in the North Basin, which lack snow-cover, show increased current velocities. The current velocities everywhere are small with the maximum velocity of 0.442 cm s^{-1} occurring in the South Basin.

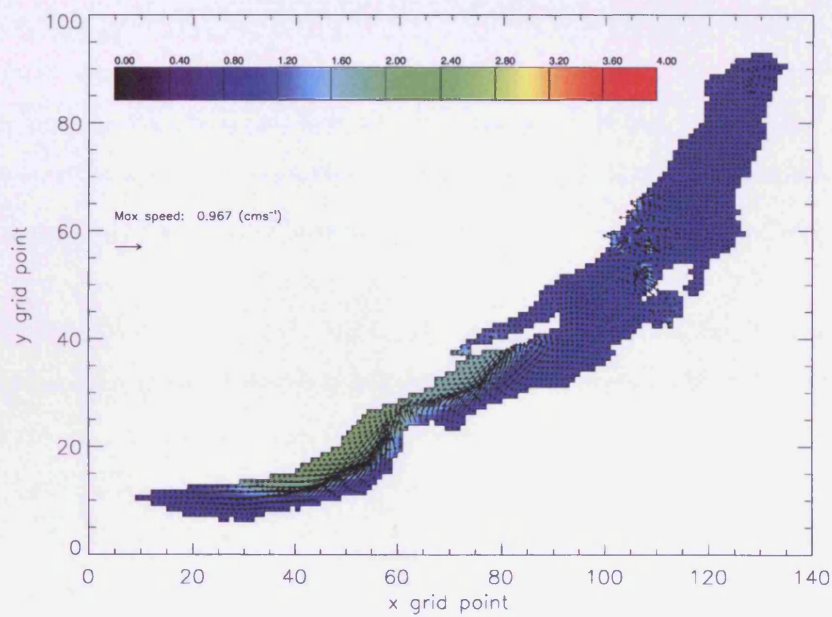
The highest current velocities are seen in the plot for 22 May 1997 (Figure 6.1(b)). By this time, the ice along the north-west coast of the South Basin has melted. The temperature difference between the warmest and the coldest areas of the lake is now more than 1.36°C . A distinct current, with speeds of up to 0.967 cm s^{-1} can be seen flowing along the edge of the ice field. Satellite data for 9 April 1997 (not shown) show a snow field in the North Basin, extending from the north-east to a line running approximately from grid point (105,62) to point (109,50); this snow field lasts until 22 April 1997. Areas of warmer water with higher current velocities can be seen where the edge of this snow field was located during April.

By the end of May, all of the ice has melted. Horizontal temperature gradients in the model lake are beginning to diminish as there is no longer a mechanism for differential heating (see Figure 6.2(a)). The currents running along the edge of the warmer areas in the South and Central basins are still evident, although their velocities are smaller. The maximum velocity is now only 0.224 cm s^{-1} . An anticlockwise circulation is established in the North Basin, at the location of the edge of the earlier snow field, but the current velocities are very small ($O(10^{-2} \text{ cm s}^{-1})$).

By 20 June 1997 (Figure 6.2(b)) the currents throughout lake have diminished still further, the

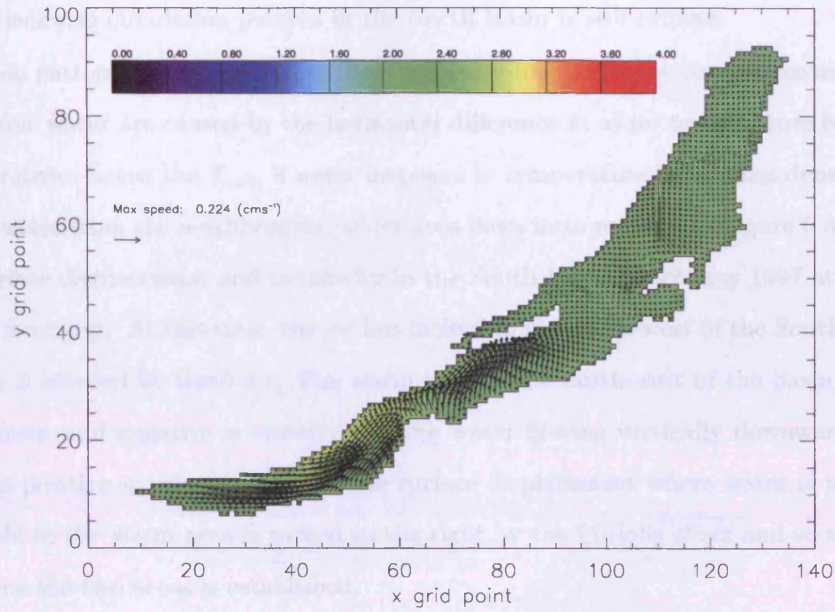


(a) 01 04 97

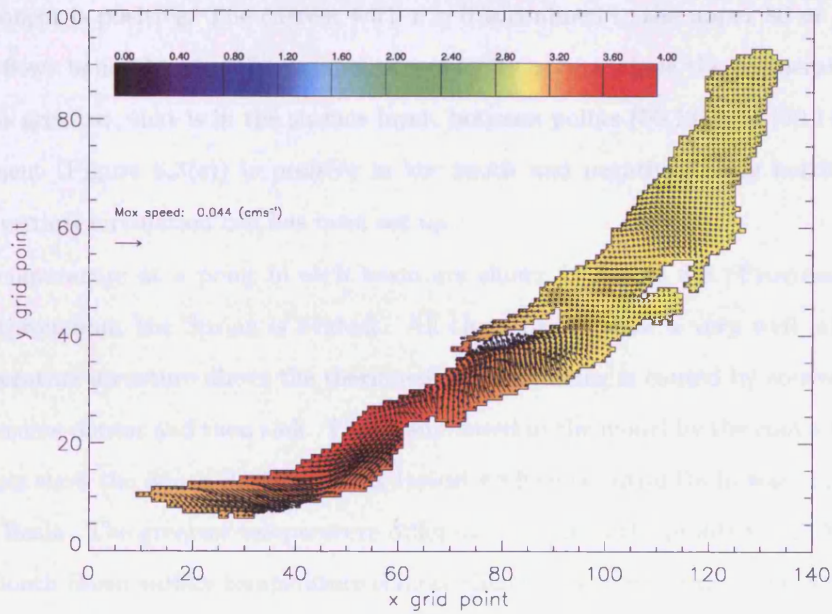


(b) 22 05 97

Figure 6.1: Maps of temperature (°C) with vectors of current velocity (cms⁻¹) for run LBM-M on (a) 1 April 1997 and (b) 22 May 1997. The magnitudes of the vectors are scaled to the field maximum, which is indicated on the left-hand side of the plot.



(a) 06 06 97



(b) 20 06 97

Figure 6.2: Maps of temperature ($^{\circ}\text{C}$) with vectors of current velocity (cms^{-1}) for run LBM-M on (a) 6 June 1997 and (b) 20 June 1997. The magnitudes of the vectors are scaled to the field maximum, which is indicated on the left-hand side of the plot.

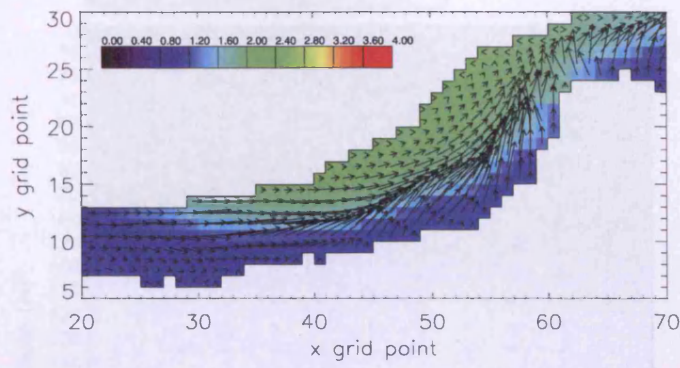
maximum velocity is now 0.044 cm s^{-1} . The currents are still strongest in the South and Central basins, though the anticlockwise circulation pattern in the North Basin is still evident.

The circulation patterns along the borders between snow-free and snow-covered ice and between ice-covered and ice-free water are caused by the horizontal difference in water temperature between the two areas. At temperatures below the T_{md} , if water increases in temperature it becomes denser. Thus warm water sinks and water from the neighbouring colder area flows in to replace it. Figure 6.3 shows maps of temperature, surface displacement and u -velocity in the South Basin on 22 May 1997 at the time when the currents are strongest. At this time, the ice has melted in the north-west of the South Basin and the rest of the basin is covered by frazil ice. The warm area in the north-west of the basin has a negative surface displacement and negative u -velocity showing water flowing vertically downwards whereas the colder area has a positive u -velocity and positive surface displacement where water is up-welling. The flow from the cold to the warm area is turned to the right by the Coriolis effect and so a flow along the boundary between the two areas is established.

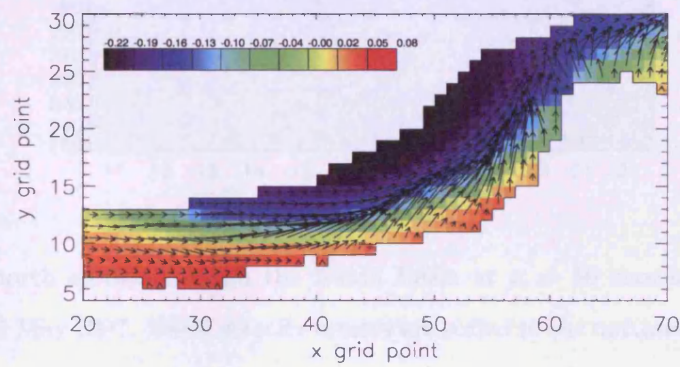
Figure 6.4 shows the temperature and v -velocity component along a vertical, south-north section through the South Basin on 22 May 1997. Along this section the surface current flows north-east so the v -velocity component is positive. The current with $v > 0$ is confined to the upper 80 m. A small return current ($v < 0$) flows beneath this. The maximum v -velocity occurs where the temperature gradient in the y -direction is greatest, that is in the surface layer, between points (50,13) and (50,14). The vertical velocity component (Figure 6.3(c)) is positive in the south and negative in the north of the section showing that a vertical circulation cell has been set up.

Profiles of temperature at a point in each basin are shown in Figure 6.5. Progressive warming of the epilimnion throughout the Spring is evident. All three basins show a very well mixed epilimnion with little temperature structure above the thermocline. The mixing is caused by convection as surface waters warm, become denser and then sink. This is simulated in the model by the convective adjustment routine. The plots show the South Basin warming fastest with the Central Basin warming slightly faster than the North Basin. The greatest temperature differences occur in the profile for 22 May 1997 which shows that the South Basin surface temperature is more than $0.7 \text{ }^{\circ}\text{C}$ warmer than the North. The mixed layer is also deeper in the South Basin having a depth of 80 m compared with 60 m in the North and Central Basins.

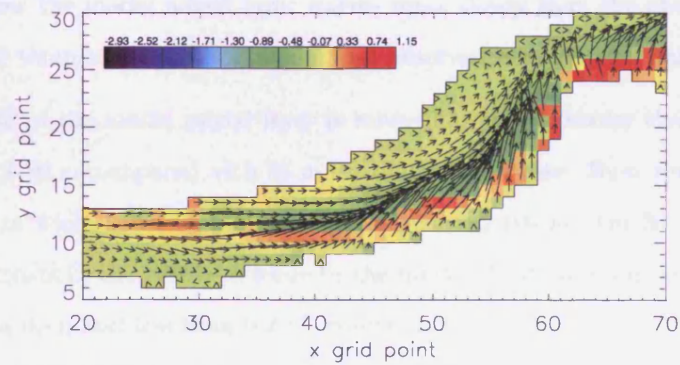
Figure 6.5(d) shows a profile of the model–observed temperature differences at grid point (40,12). The model successfully reproduces the observations to within $0.7 \text{ }^{\circ}\text{C}$. As already discussed in Chapter 4 most of the difference between the model and the observed temperatures on 1 April are due to the data suggesting the real lake is still cooling whereas the model has a positive net heat flux at this time.



(a) Temperature



(b) Surface displacement



(c) w -velocity

Figure 6.3: Maps of a) temperature ($^{\circ}\text{C}$), b) surface displacement (cm), and c) w -velocity ($10^{-3} \text{ cm s}^{-1}$) in the South Basin on 22 May 1997 for run LBM-M.

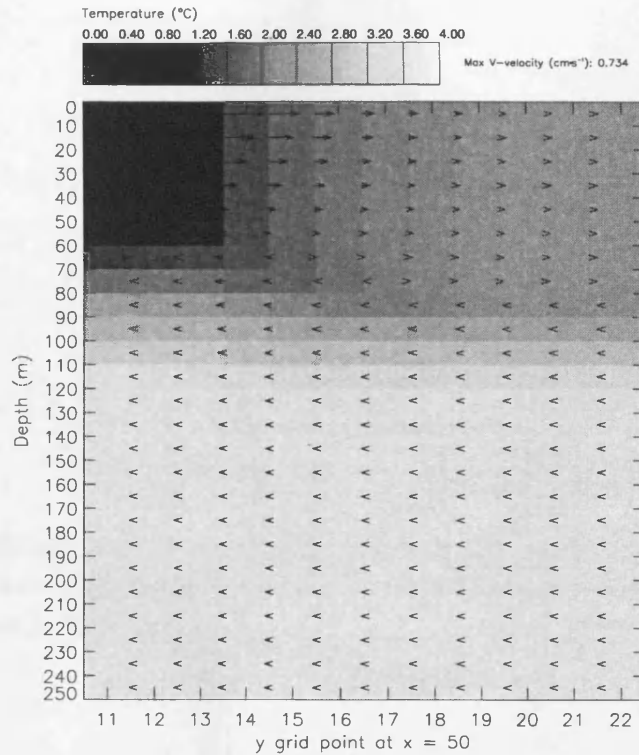
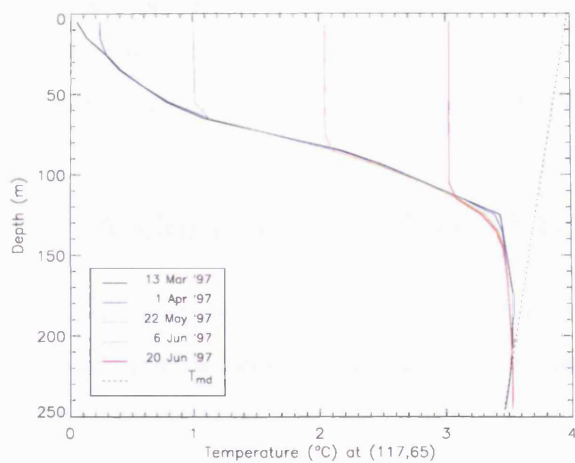


Figure 6.4: A south-north section through the South Basin at $x = 50$ showing the v -velocity and temperature ($^{\circ}\text{C}$) on 22 May 1997. The v velocity arrows are scaled to the maximum v velocity (cm s^{-1}) shown above the plot.

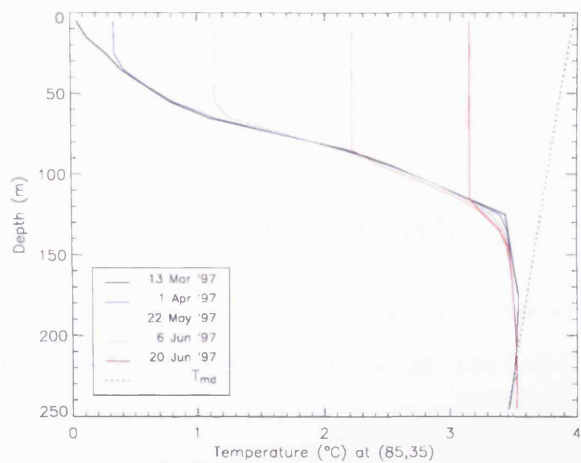
Subsequent profiles show the model mixed layer warms more slowly than the observations during May but differences between temperatures in the model and observed mixed layers reduce during June.

On 22 May the bulk of the model mixed layer is more than 0.4°C cooler than the observed mixed layer and has a depth of 80 m compared with 95 m for the observed lake. By 6 June the observed mixed layer has reached 130 m while in the model mixing has reached 100 m. On 20 June the observations indicate homothermy exists in the real lake while in the model simulation a mixed layer is still present though it is now 130 m deep and less than 0.2°C cooler than the hypolimnion. These model-observed differences though small may indicate that the model heat fluxes need further fine-tuning.

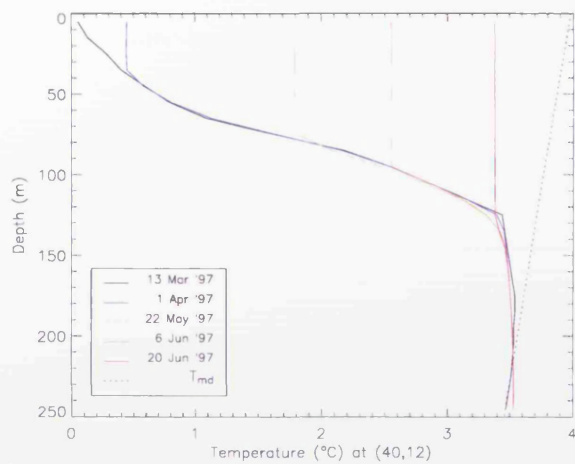
The differences in the mixed layer depth has implications for plankton production. In a deeper mixed layer plankters will experience lower light levels as they are mixed to greater depths where less light penetrates. However, this must be balanced against the tendency of some plankters such as diatoms to sink under gravity; once below the mixed layer there will be no upward current to aid re-suspension and such plankters would sink below the photic zone and thus would no longer be able to photosynthesize.



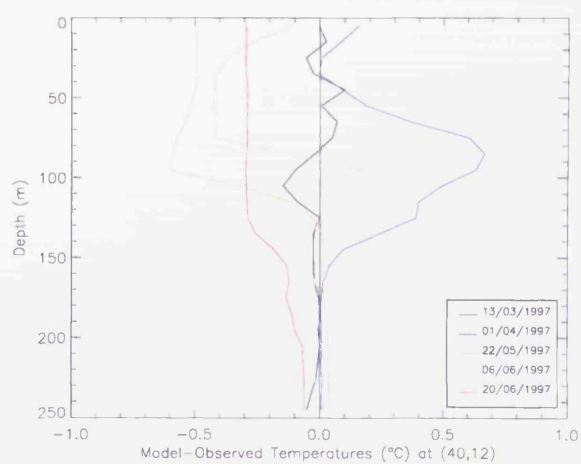
(a) North



(b) Central



(c) South



(d) South: Model-Observed

Figure 6.5: Temperature profiles ($^{\circ}\text{C}$) for run LBM-M at a) grid point (117,65) in the North Basin, b) at (85,35) in the Central Basin, c) at (40,12) in the South Basin and d) temperature differences between the model run LBM-M data at (40,12) and the Jewson data.

The preceding results show that the model is able to reproduce the enhancement of current velocity at the edge of snow fields reported by Zhdanov *et al.* (2001). The magnitude of the current velocities is of the same order as those observed by the authors. The model also predicts enhanced current velocities along the boundary between ice-covered and ice-free regions. The model simulation supports the hypothesis that differential heating due to varying ice and snow fields is a possible mechanism for the production of under-ice currents and, in this, it is in agreement with Lawrence *et al.* (2002).

6.2 A Comparison of The LBM with The LHLC Model.

In view of the limited data available for validation of the model, comparison of the LBM with another model may provide valuable information about the strengths and weaknesses of the LBM. The LHLC model is the only other model to simulate the under-ice thermodynamics of Lake Baikal and so, is the only suitable model for this purpose and will be used here. The main features of the LHLC model were described in Chapter 2.

Temperature and current vector maps, for the same dates as those of the LBM model plotted in Figure 6.1, are shown in Figure 6.6. The differences in temperature and vectors of the difference in velocity between the two models are plotted in Figure 6.7. Similar features are found in both models. Both models show a similar pattern of warming with the north-west of the South Basin warming fastest. Small areas of warm water are also present in the North Basin on 22 May in each model. Temperature profiles in the South Basin (40,12) of both models show a progressive warming and deepening of the mixed layer. However, the LHLC model exhibits faster warming of the surface layer with temperature differences between the two models being greatest in the South Basin on 22 May. The LHLC model also has areas of cooler water along the south-east coast of the North and Central basins evident on the 6 and 20 June. These are not seen in the LBM results.

Inspection of the temperature profiles (Figure 6.9(a)) shows the LHLC model mixed layer is $> 0.4^{\circ}\text{C}$ warmer than the LBM mixed layer (Figure 6.5(c)) by the 22 May. Comparison of each model with the Jewson data (Figures 6.9(b-d)) reveals a closer agreement between the LHLC model and the observed data than between the LBM and the observed data. The LBM model mixed layer warms at a slower rate than both the observed mixed layer and the LHLC simulated mixed layer. If the different vertical grid shapes of the two models are taken into consideration, the mixed layer depths are the same on the 1 April, 22 May, and 6 June. But by 21 June the LHLC model has reached homothermy at (40,12) while the LBM still has a mixed layer of 130 m depth although its temperature is only 0.2°C above the hypolimnion temperature.

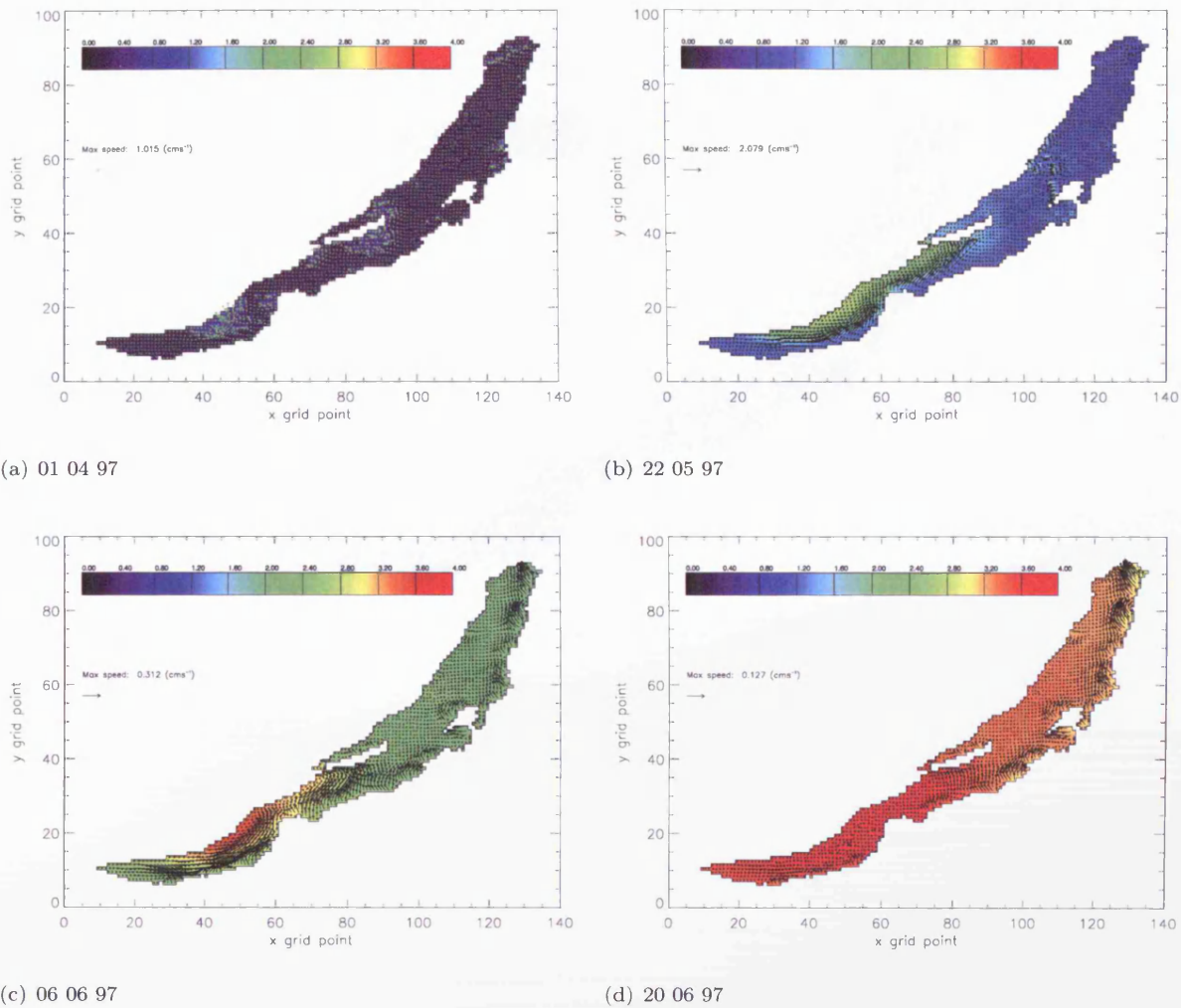
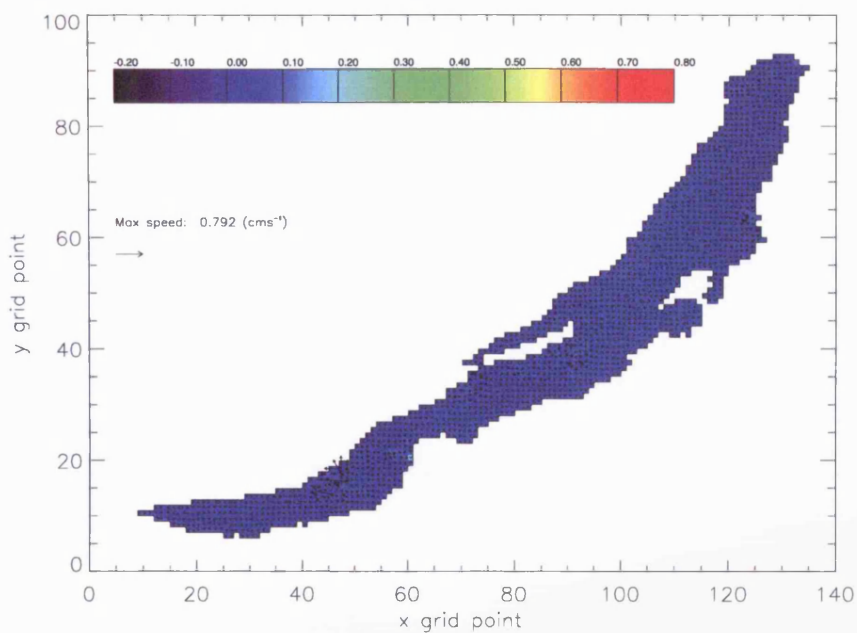
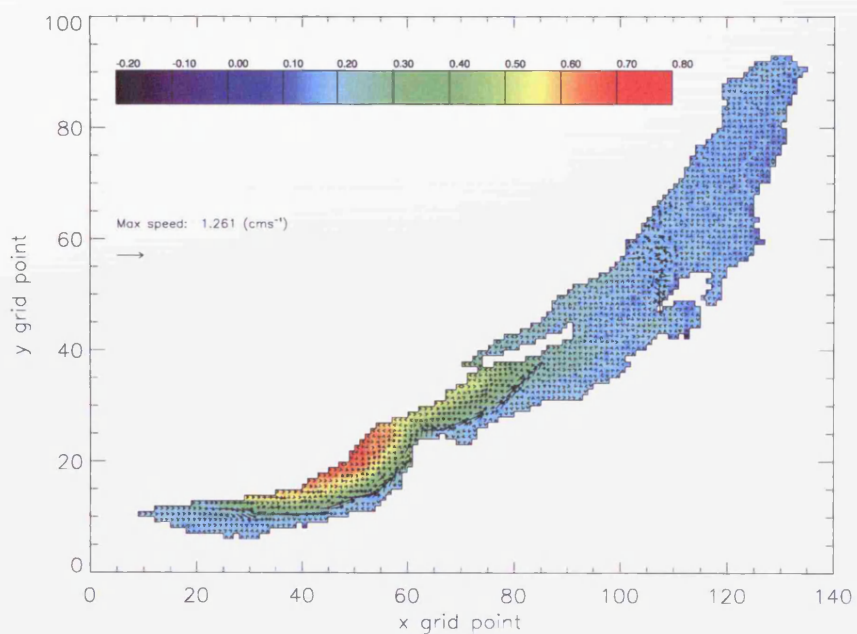


Figure 6.6: Temperatures ($^{\circ}\text{C}$) and current vectors (cms^{-1}) at 5 m for the LHLC model on (a) 1 April 1997, (b) 22 May 1997, (c) 6 June 1997 and (d) 20 June 1997.

The pattern of currents produced by LHLC model and the LBM share common features. The anticlockwise circulation around warm areas is present in both runs, both the major feature in the South Basin and also the smaller features evident in the North Basin on 22 May. Both models exhibit maximum current speeds on the 22 May when the temperature gradient across the South Basin is the greatest. The maximum speeds occur in the South Basin in both models while the current speeds in the North Basin of both runs are very small. The LHLC current velocities are generally greater than those of the LBM. Table 6.1 gives the maximum current velocity occurring on 1 April, 22 May, 6 June, and 20 June for each run and the relative differences between the two. The maximum speeds for the LHLC model are generally over twice the maximum speeds for the LBM. Profiles of the velocity vector components at (40,12) for each model are shown in Figure 6.10. Most of the profiles exhibit common

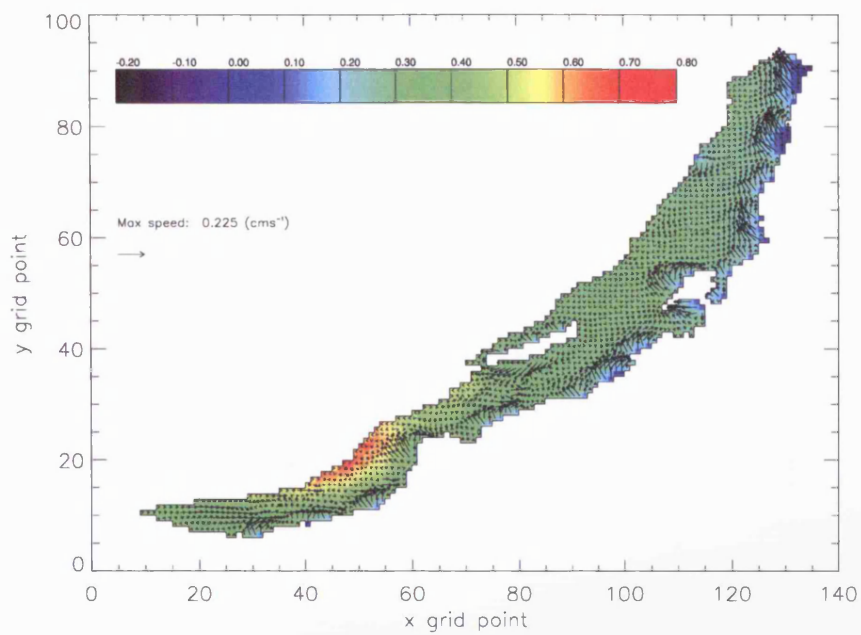


(a) 01 04 97

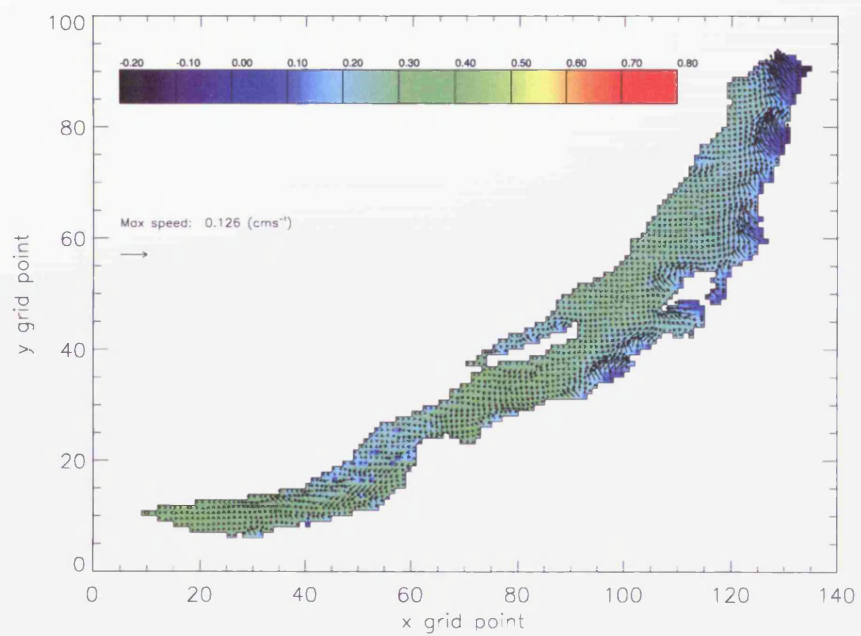


(b) 22 05 97

Figure 6.7: Temperature differences ($^{\circ}\text{C}$) and current vector differences (cms^{-1}) between LBM model run LBM-M and the LHLC model (LHLC – LBM-M) on (a) 1 April 1997 and (b) 22 May 1997.

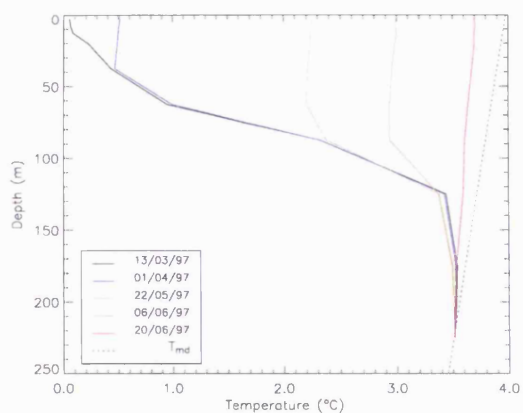


(a) 06 06 97

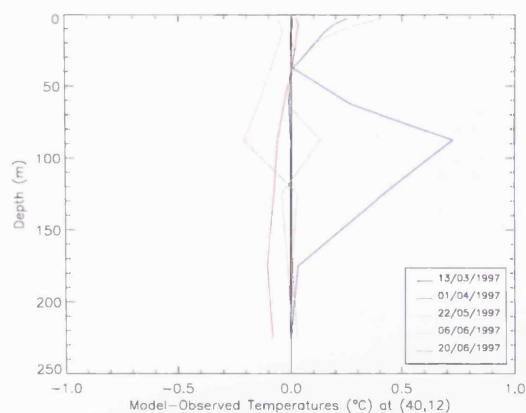


(b) 20 06 97

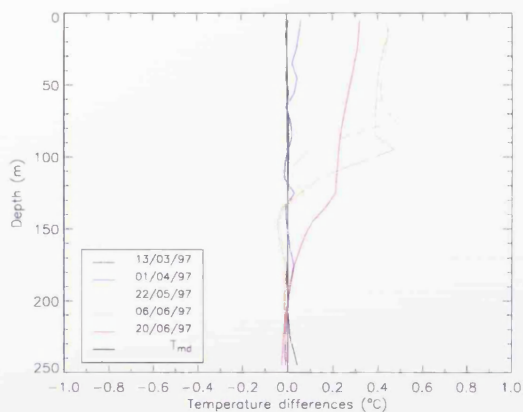
Figure 6.8: Temperature differences (°C) and current vector differences (cms⁻¹) (LHLC – LBM-M) on (a) 6 June 1997 and (b) 20 June 1997.



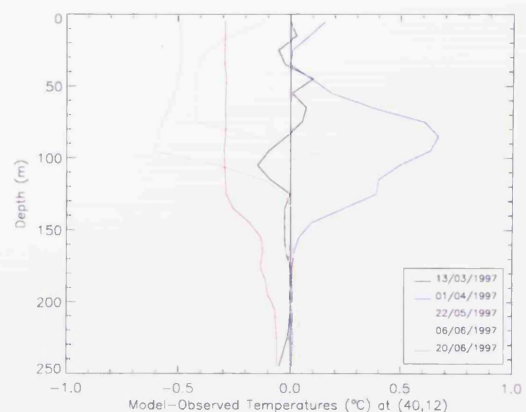
(a) LHLC model



(b) LHLC - Jewson data

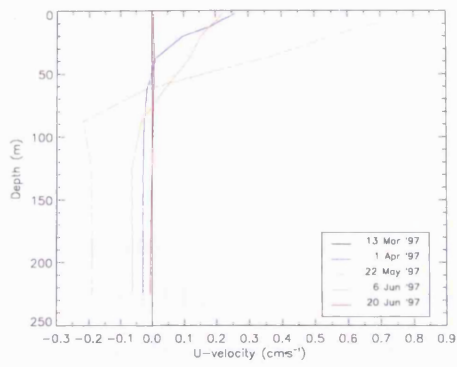


(c) LHLC - LBM-M

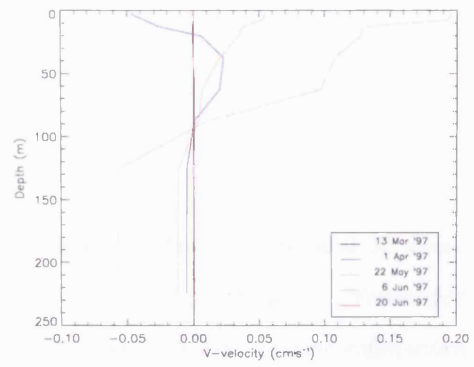


(d) LBM-M - Jewson data

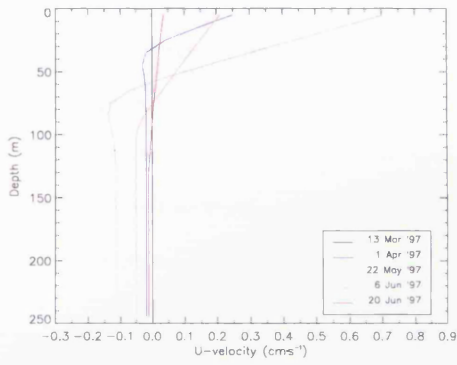
Figure 6.9: (a) Profiles of temperature in the South Basin (40,12) for the LHLC model. (b) Profiles of temperature difference between the model and the Jewson observations for the LHLC model. (c) Profiles of temperature difference between LBM model run LBM-M and the LHLC model. (d) Profiles of temperature difference between the model and the Jewson observations for the LBM model run LBM-M. All plots at (40,12) and all temperatures in centigrade.



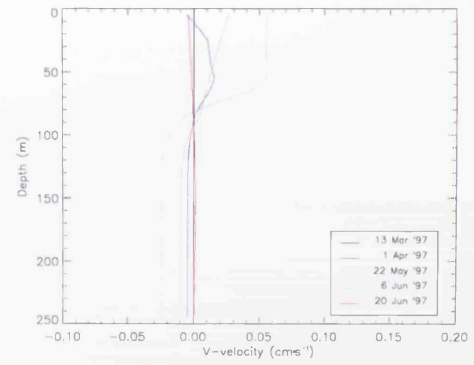
(a) LHLC



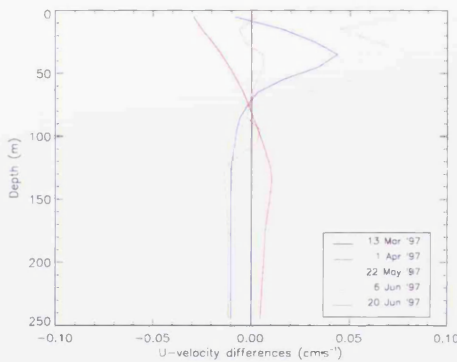
(b) LHLC



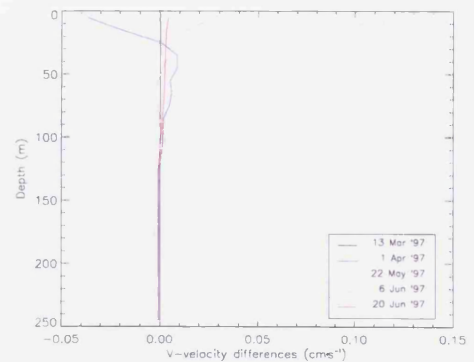
(c) LBM



(d) LBM



(e) LHLC-LBM



(f) LHLC-LBM

Figure 6.10: Profiles of (a) the u and (b) v velocity components for the LHLC model. Profiles of (c) the u and (d) v velocity components for the LBM model run LBM-M. Profiles of the difference between the LHLC model and the LBM model (e) u and (f) v velocity components (LHLC - LBM-M). All plots at (40,12). Velocities are in cms^{-1} .

Date	V_{LHLC}	V_{LBM}	$\frac{V_{LHLC}-V_{LBM}}{V_{LBM}}$
01 April 1997	1.015	0.442	1.296
22 May 1997	2.079	0.967	1.150
06 June 1997	0.312	0.224	0.393
20 June 1997	0.127	0.044	1.886

Table 6.1: The maximum current velocity at a depth of 5 m on 1 April, 22 May, 6 June and 20 June. Values for the LHLC model are the average of the velocities at 2.5 m and 7.5 m (V_{LHLC}). Values for the LBM are velocities in the uppermost model layer (V_{LBM}). Also shown are the relative differences between the two models. Velocities in centimetres per second.

features. The profiles have two distinct layers: the major current flows in the surface layer and there is a steep decrease in velocity with depth in this layer; the second, lower layer has a smaller current flowing in the opposite direction and a near uniform velocity for most of the layer. In this region of the domain the velocity components of the two models are in agreement to within 0.14 cm s^{-1} . The relative depths of the two layers change with time but are consistent between the two models.

The LHLC model and the LBM model experience different heat fluxes. As discussed in Chapter 4 the LHLC model uses a shortwave radiation flux but the LBM implements an additional surface heat flux which models other heat transfer processes and is implemented over ice-free areas. In consequence the total heat flux entering the LBM lake is lower than that entering the LHLC lake and the differences are confined to ice-free areas. This is the most probable cause of the most of the increased warming of the LHLC model. A second reason the LHLC model warms faster may be that, because of its higher grid resolution in the upper layers, it can better resolve the exponential profile of shortwave radiation absorption given by the Beer-Lambert Law. This would effect warming over the whole lake. The higher rate of heating in the LHLC model is the most likely reason for this model simulating higher current velocities than the LBM although the higher resolution in the surface layers may also be responsible. Given the differences in the two models their simulation of Lake Baikal thermodynamics during the Spring ice-melting period are remarkably similar.

6.3 The LBM Simulation During The Ice-free Period

The LBM is able to implement wind forcing and is therefore able to simulate Lake Baikal during the ice-free period. This is not true of the LHLC model which becomes numerically unstable under wind

forcing and the convective adjustment routine is unable to stabilize the model layers.

Shimaraev *et al.* (1994) state in their review that the mean monthly speeds of Baikal currents have small values, of order several centimetres per second in the upper 50 m. Speeds are high near the surface and decrease towards the thermocline with a secondary maximum before decreasing with depth to negligible values in the deep water. There is some increase in current speeds near the lake bottom. The highest current velocities occur in late Autumn when the frequency of strong winds is highest. The mean circulation exhibits a cyclonic circulation involving the whole lake and individual intra-basin cyclonic circulations. Lake Baikal also exhibits seiches. A whole basin longitudinal seiche has an inertial period of 4.55 hours.

The LBM model does not extend to the bottom of the lake and the lake bathymetry is not simulated. This, along with the poor knowledge of the wind fields experienced by the lake, means that it would be unreasonable to expect the LBM to simulate a realistic circulation. We can, however, test the response of the LBM to forcing by winds that are of similar magnitude and direction to the winds known to occur over Lake Baikal.

Temperature development in the surface layer throughout the year

A requirement of the LBM is that it simulates the seasonal development of the three-dimensional temperature field. The development of the vertical temperature profile at a point in the South Basin (grid point [40,12]) has been described in § 4.3.4. In this section we explore the model simulation of the lake temperature in a horizontal plane at a depth of 5 m, which corresponds to the midpoint of the model's uppermost layer.

Figure 6.11 shows maps of temperature in the surface model layer for run LBM-D on the 15th day of each month from March to December. The seasonal warming and cooling of the lake is evident and spatial variations in the temperature field can be discerned. The South Basin, the southern part of the Central Basin and the Maloye More, those parts of the lake that warm fastest under ice during the Spring, remain warmer than the rest of the lake throughout the year; while the south coast of the North and Central basins forms a cooler region.

The minimum, maximum, mean and standard deviation of the temperature at 5 m, on the 15th of each month of the model run are shown in Table 6.2. The maximum temperature occurs in the Maloye More in August and September and in the South Basin in other months whereas the minimum temperature occurs in the North Basin for all months except March. The mean temperatures reflect seasonal changes with temperatures starting off low in the Spring, warming through the Summer and then cooling through Autumn until freezing starts in the North Basin in December. The standard

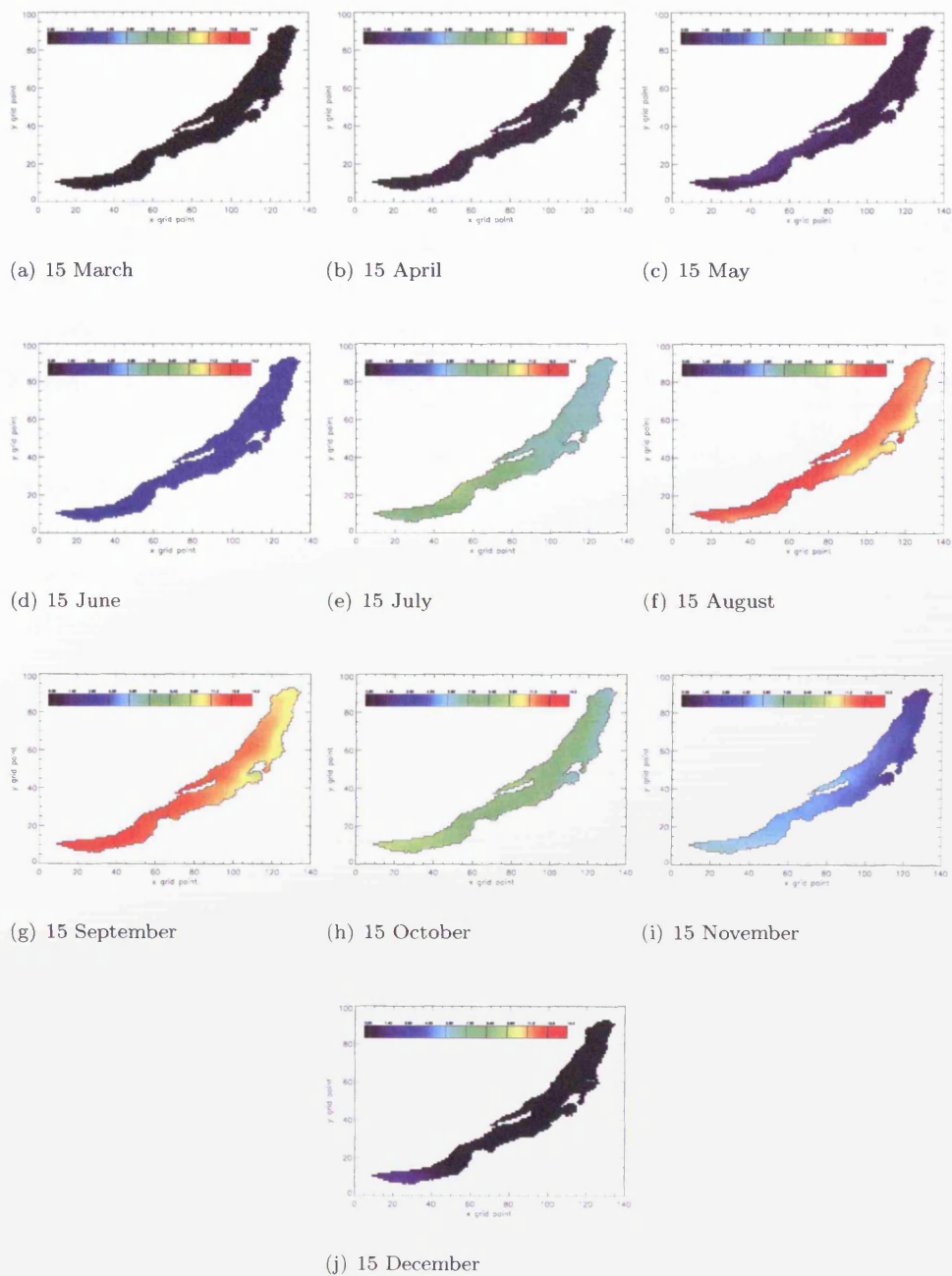


Figure 6.11: Maps of temperature in the surface layer for run LBM-D on the 15th of each month from March to December 1997. The colour bar ranges from 0.0 – 14.0 °C in intervals of 1.4 °C.

	Temperature at 5 m (°C)				
Month	Minimum		Maximum		s.d.
March	0.08	(28,10)	0.29	(43,15)	0.12
April	0.34	(123,76)	1.12	(45,17)	0.44
May	0.74	(131,86)	1.85	(45,18)	1.02
June	2.67	(130,86)	3.32	(51,23)	2.84
July	6.09	(127,70)	9.71	(49,21)	7.05
August	10.34	(114,47)	15.13	(70,37)	11.89
September	9.81	(112,48)	13.13	(70,37)	11.55
October	6.00	(134,90)	9.80	(9,10)	7.92
November	1.45	(112,42)	6.20	(18, 7)	4.39
December	0.00	(0, 0)	2.11	(26, 8)	0.18

Table 6.2: Minimum, maximum, mean and standard deviation (s.d.) of surface model layer temperatures for run LBM-D on the 15th of each month from March to December. The coordinates for the location of the minimum and maximum temperatures are given in brackets after the respective temperature. The dates are the same as for the plots in Figure 6.11. All temperatures in centigrade.

deviation gives an indication of the variation in temperature across the lake at a given time.

The standard deviation increases during the period of March through May due to spatial differences in heating through the snow/ice layer. In June, after the ice has melted and the heat flux becomes uniform over the entire lake surface, the standard deviation reduces. At this time, the depth of the mixed layer is greatest where surface temperatures are high and, because the heat flux is mixed through a greater depth, the rate of temperature increase is slowest in these regions. Thus differences in temperature across the lake are eroded.

In late June, the temperature in warmer parts of the lake (the South Basin and southern parts of the Central Basin) has exceeded the T_{md} while cooler parts still have temperatures below the T_{md} . The rate of warming in these warmer parts now accelerates and the temperature difference between those areas that are below and those that are above the T_{md} becomes large with the result that the standard deviation of the surface layer temperature on 15 July is high.

Maximum temperatures are reached towards the end of August and it is at this time that the net heat flux into the lake changes sign and the lake begins to cool. The standard deviation is again large in November. In parts of the North Basin the surface layer temperature has already cooled below the

T_{md} so that free convection has ceased and cooling of the surface has begun to accelerate. In the rest of the lake temperatures remain above the T_{md} , mixing of heat upwards into the surface layer is still occurring and so cooling is slower.

By December much of the model lake surface has frozen and has a temperature of 0 °C. Only in the South Basin does the surface temperature have values above 0 °C.

There is considerable spatial variation in the surface temperature field during the period after the ice has melted despite the heat transfer between the model lake and its environment being horizontally uniform. This variation is, in the main, due to up-welling and down-welling of water at the coasts in response to wind forcing.

The distribution of surface water temperature during the period of June-December can be compared with the contour maps of observations given by Galazy (1993) in the Baikal Atlas. The model is in good agreement with these maps however there are differences. Although both the model and Galazy have areas of warmer surface water in the north of the South Basin and south of the Central Basin, Galazy shows the surface temperature increasing towards the coast in the region of the Selenga Delta. This difference probably occurs because the LBM does not simulate either the inflow of warm water from the Selenga River or the bathymetry of the shallow coastal areas. During the Summer the coastal areas are warmer than the pelagic waters (Galazy, 1993), (Shimaraev *et al.*, 1994). The LBM cannot simulate the faster warming of shallow, coastal waters again, in part, because of the failure to simulate the bathymetry. Galazy shows regions of cool water along the south-east coast of the North and Central Basins during November and December. The LBM has cooler water in this region for most of the ice-free period. Coastal up-welling maintains cooler temperatures in this region in the LBM simulation. The amount of up-welling that occurs depends largely on the direction and magnitude of the wind forcing. Any differences between the model and the observed up-welling is probably due to the lack of realistic wind forcing.

The effect of an October storm event on the temperature and current fields simulated by the LBM.

In this section we explore the effect of a simulated storm on the model temperature and current fields. Run LBM-D has background weak, unstable winds. The magnitude is periodic with a maximum speed of 4 m s^{-1} occurring every 3 days and the direction changes every 6 days (see § 4.3.1 and Figure 6.12(a)). Run LBM-N has the same winds as LBM-D except that storms are simulated by the addition of a north-westerly wind that increases sinusoidally from 0 m s^{-1} to 20 m s^{-1} and decreases back to 0 m s^{-1} over a period of 24 hours (Figure 6.12(b)). These storms occur on the 15th and 30th of the months of

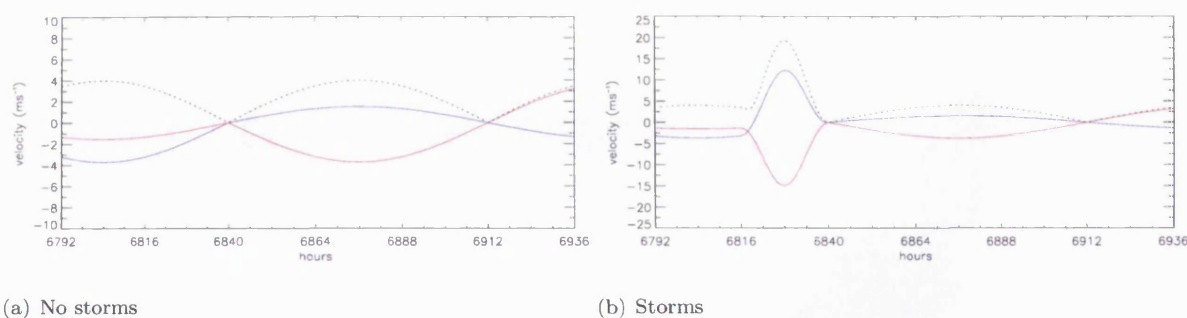


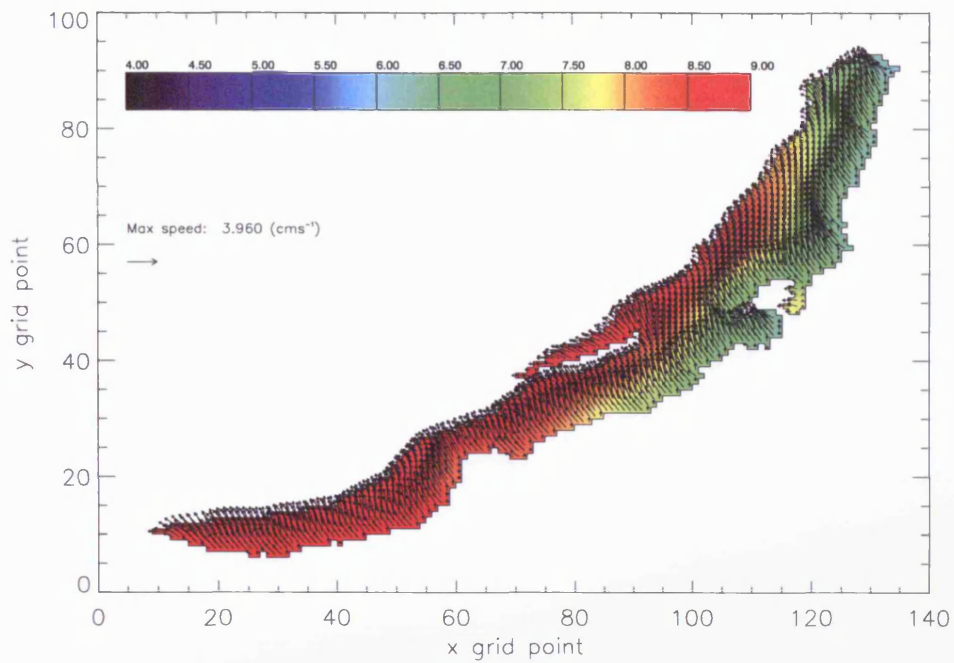
Figure 6.12: The speed (ms^{-1}) of the x (blue) and y (red) components and magnitude (black dotted) of the wind velocity for the LBM model runs (a) LBM-D and (b) LBM-N. Six days of winds are shown: 14-19 October. The x-axis labelling refers to the number of hours since the beginning of January and hour 6816 corresponds to the time of the plots of 14 October in Figure 6.13(a) and 6.16(a). Note the different scales on the vertical axes.

October, November and December.

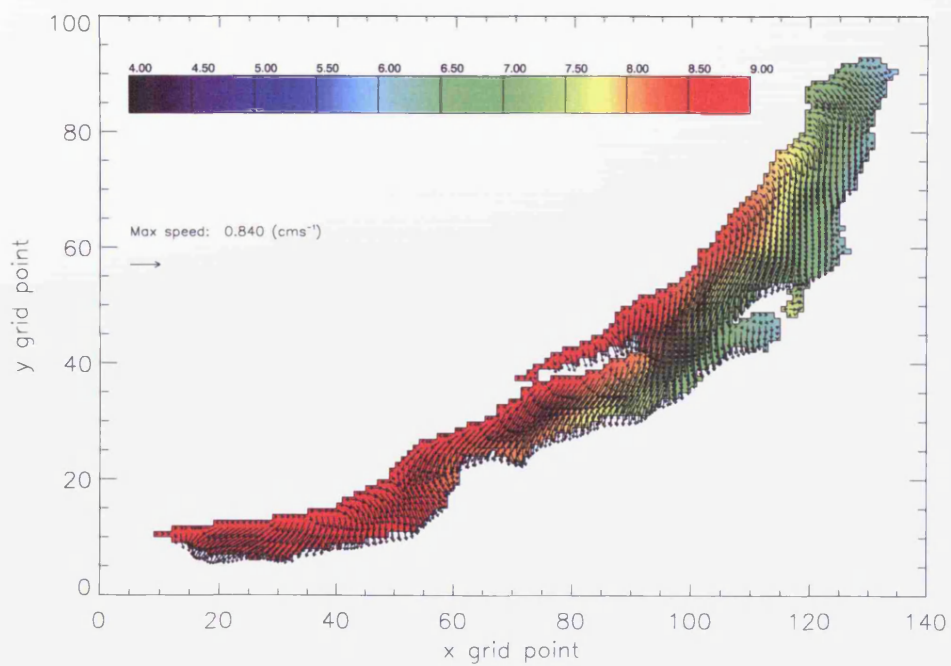
In order to evaluate the effect of a storm on the model simulation we examine the temperature and current fields on the days following the first storm, which occurs on 15 October. The temperature and current fields for run LBM-D, which has no storm, are shown in Figures 6.13 to 6.15. The variable fields plotted are those calculated at the end of each day of the period 15-19 October. Figures 6.16 to 6.18 shows plots of the temperature and current fields on the same days for run LBM-N, which has a storm on 15 October.

Examination of Figures 6.13 to 6.15 shows that the direction of the current flow is dominated by the direction of the wind; the flow towards warm areas, which is evident under ice, has been disrupted. Generally when the wind is increasing in strength the currents flow in the same direction as the wind or are turned to the right of the wind direction by the Coriolis effect (see 14 and 16,17 October) but as the wind relaxes a return flow in the opposite direction is generated (15 and 19 October). Current velocities are of the order of a few centimetres per second.

Inspection of Figures 6.16 to 6.18 shows that after a storm the current velocities on 15 October are an order of magnitude higher. The current direction on the same day is to the left of the wind direction and suggests the lake is relaxing after the storm. A north-westerly storm could be expected to pile water up along the south-east coast; as the wind relaxes this water would then flow towards the north-west and be turned towards the north-east by the Coriolis effect. On 16 October the flow is in the opposite direction (towards the south-east) suggesting the possibility of the formation of a longitudinal seiche. On 17 October the flow in the South and Central Basins is towards the south-west and the flow velocity

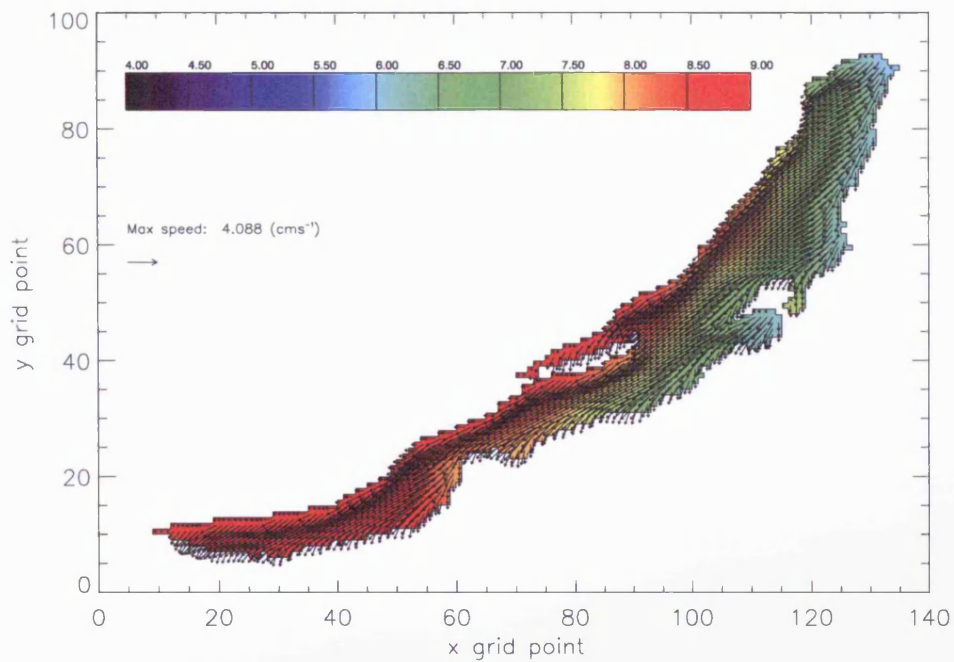


(a) 14 October, winds easterly

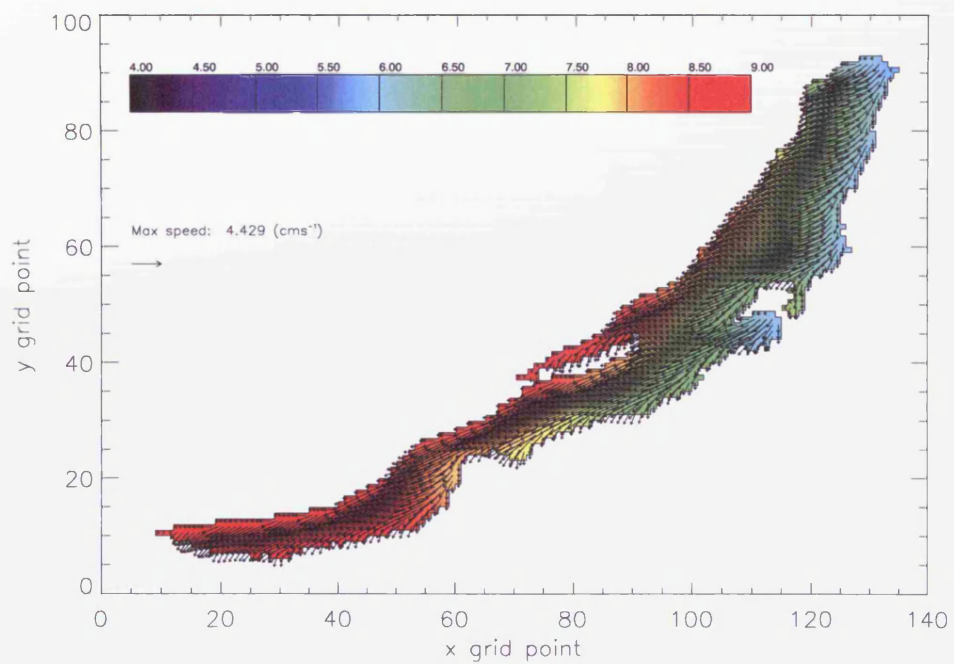


(b) 15 October, winds easterly

Figure 6.13: Temperatures ($^{\circ}\text{C}$) and current vectors (cm s^{-1}) for run LBM-D on 14 and 15 October 1997. Note that the colour bar for temperature runs from 4°C to 9°C .

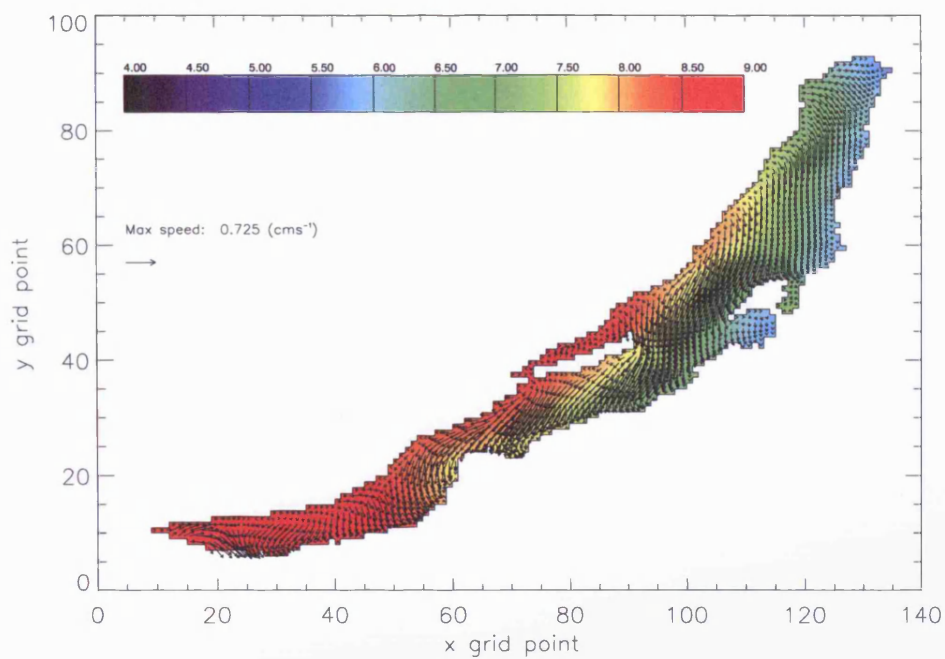


(a) 16 October, winds north-northeasterly

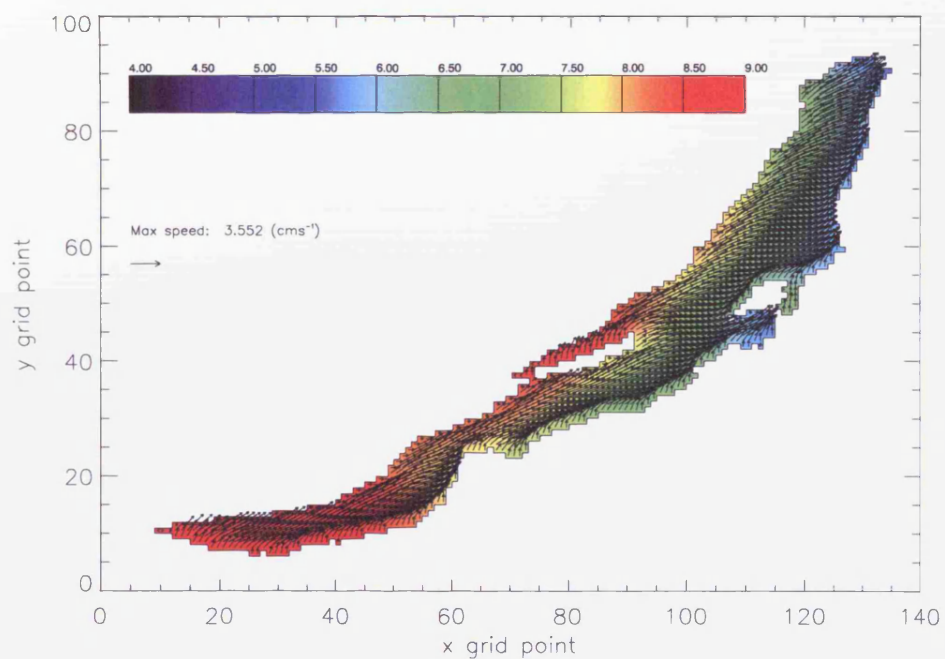


(b) 17 October, winds north-northeasterly

Figure 6.14: Temperatures ($^{\circ}\text{C}$) and current vectors (cm s^{-1}) for run LBM-D on 16 and 17 October 1997. Note that the colour bar for temperature runs from 4°C to 9°C .

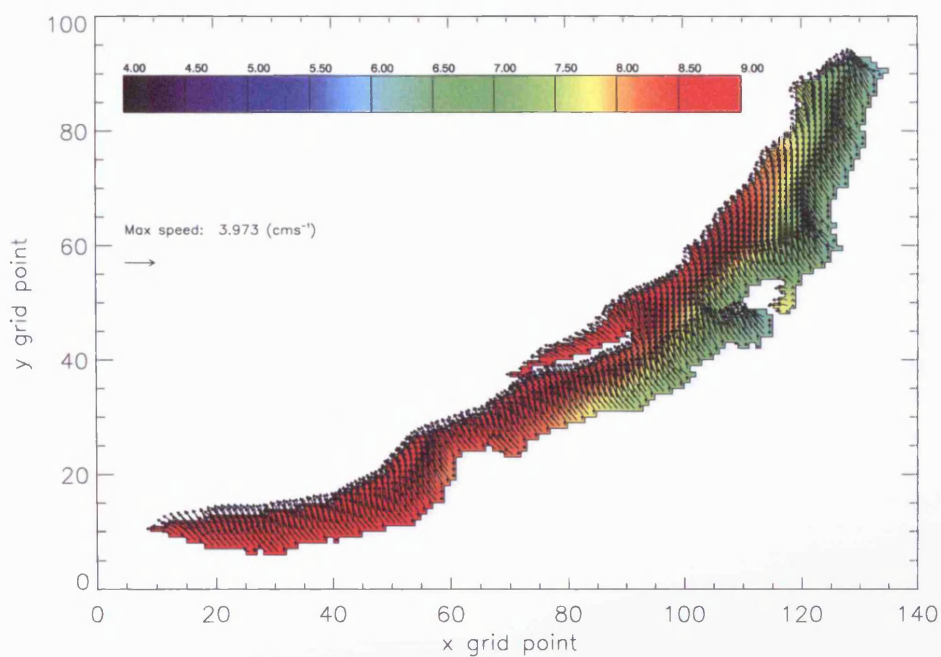


(a) 18 October, winds north-northeasterly

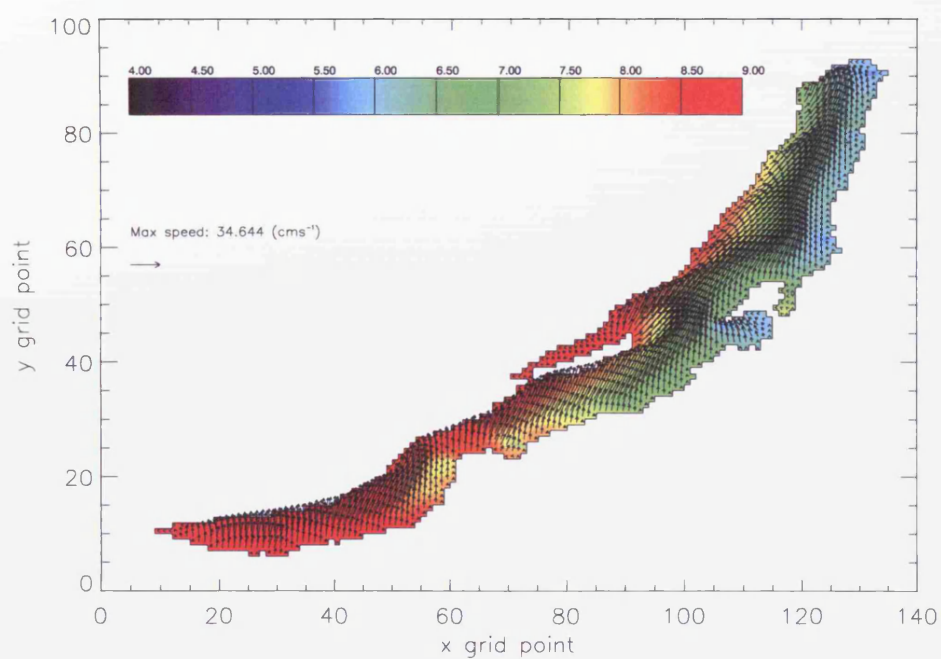


(b) 19 October, winds easterly

Figure 6.15: Temperatures ($^{\circ}\text{C}$) and current vectors (cm s^{-1}) for run LBM-D on 18 and 19 October 1997. Note that the colour bar for temperature runs from 4°C to 9°C .

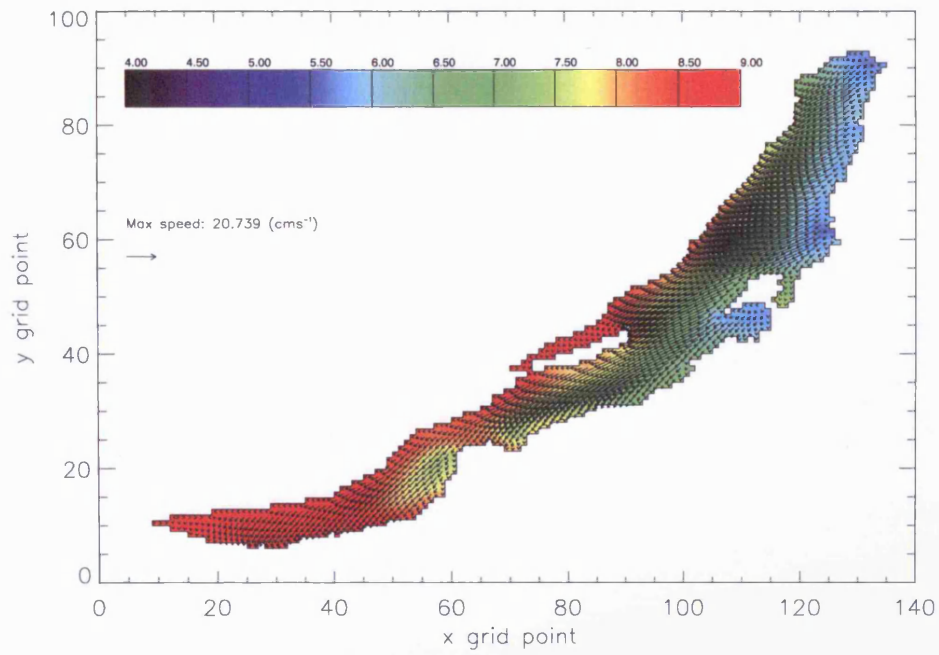


(a) 14 October

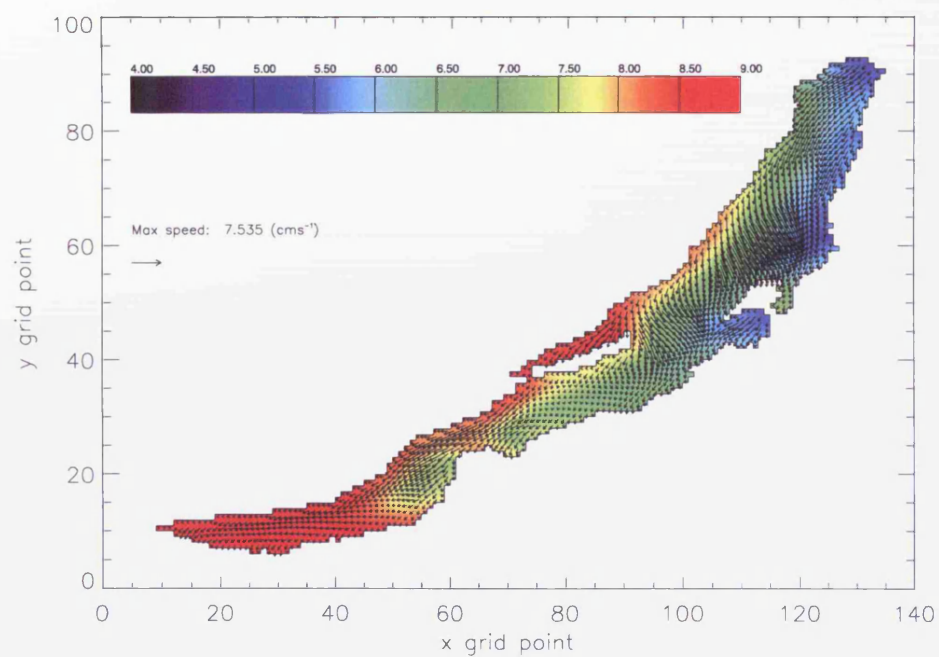


(b) 15 October

Figure 6.16: Temperatures (°C) and current vectors (cm s⁻¹) for run LBM-N on 14 and 15 October 1997. Note that the colour bar for temperature runs from 4 °C to 9 °C.

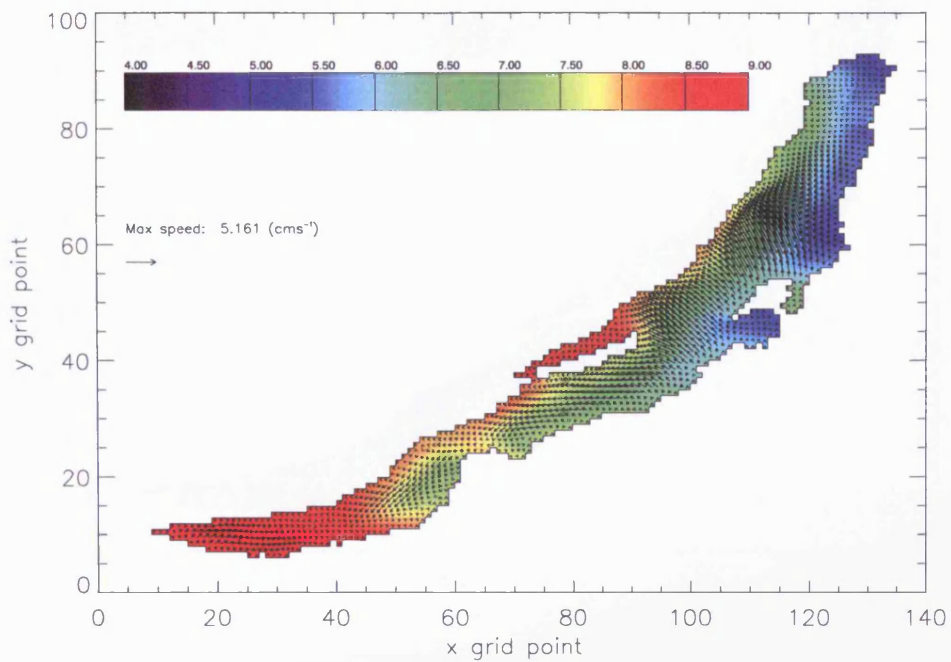


(a) 16 October

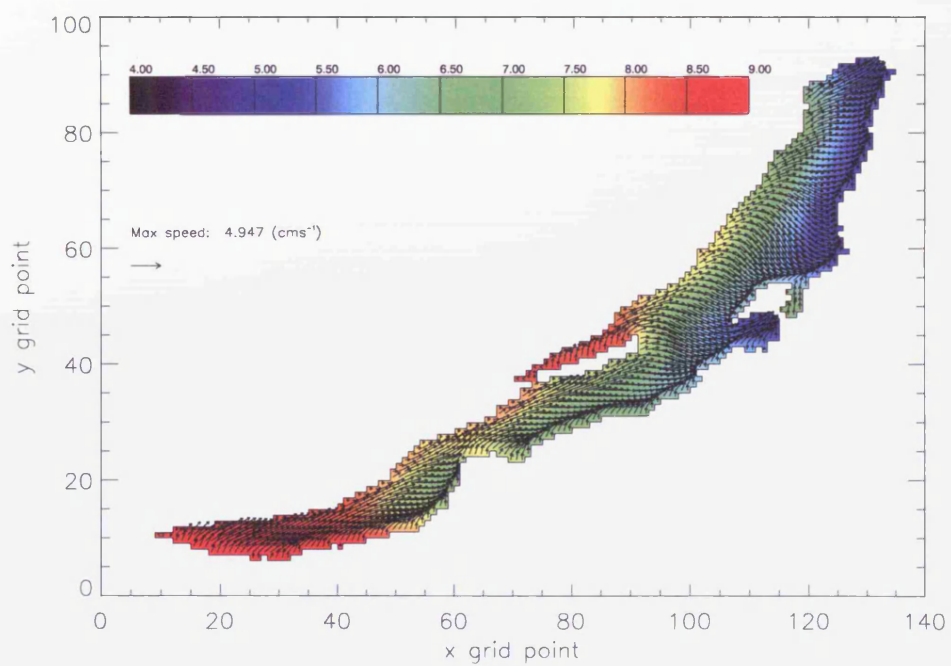


(b) 17 October

Figure 6.17: Temperatures ($^{\circ}\text{C}$) and current vectors (cm s^{-1}) for run LBM-N on 16 and 17 October 1997. Note that the colour bar for temperature runs from 4 $^{\circ}\text{C}$ to 9 $^{\circ}\text{C}$.



(a) 18 October



(b) 19 October

Figure 6.18: Temperatures ($^{\circ}\text{C}$) and current vectors (cm s^{-1}) for run LBM-N on 18 and 19 October 1997. Note that the colour bar for temperature runs from 4 $^{\circ}\text{C}$ to 9 $^{\circ}\text{C}$.

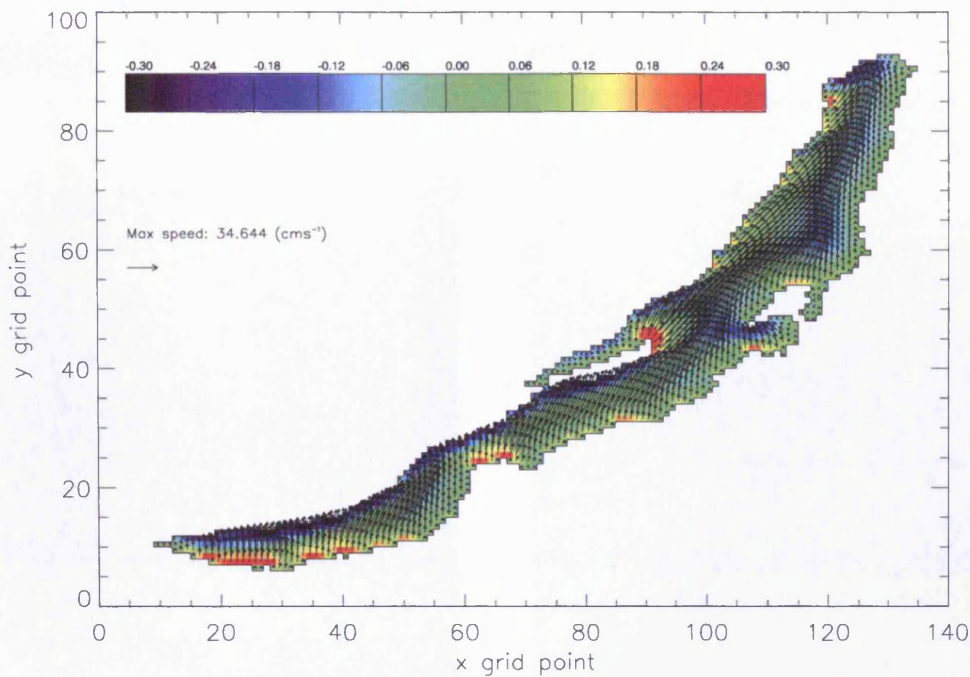
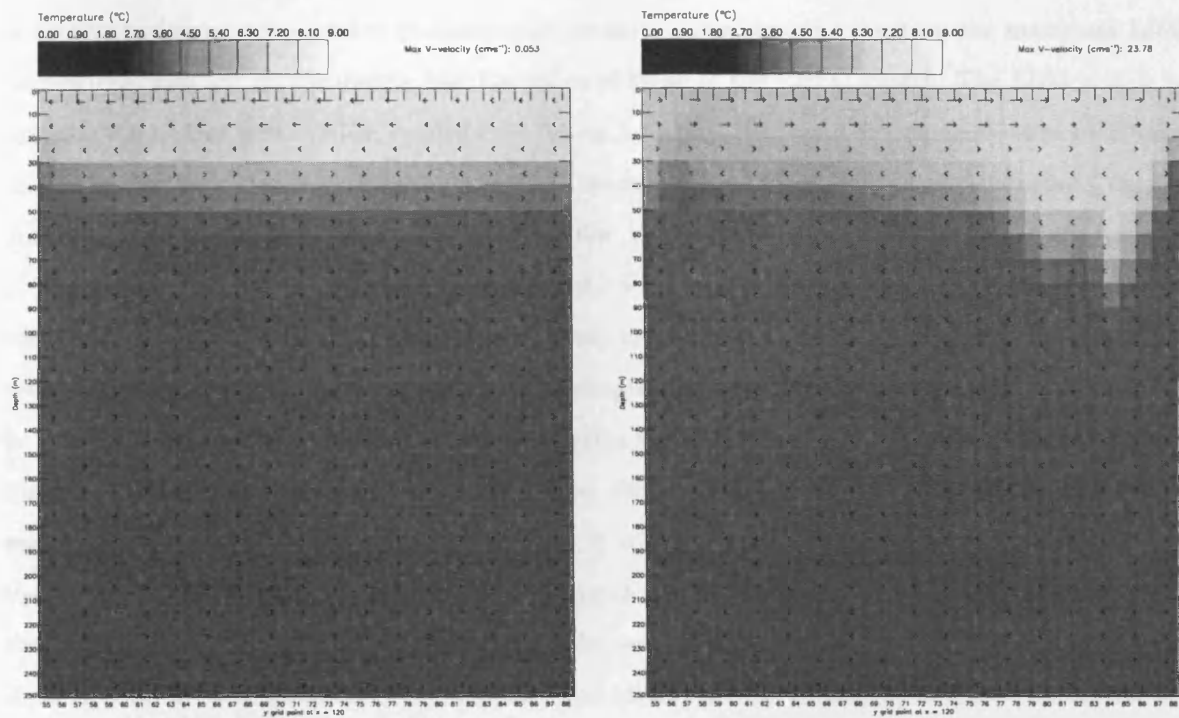


Figure 6.19: Colour map of w -velocity ($10^{-1} \text{ cm s}^{-1}$) and horizontal current vectors (cm s^{-1}) in the surface layer for run LBM-N on 15th October.

is much reduced except where the lake narrows between the two basins. The North Basin, however, exhibits some cross-basin flow in a southeastward direction. This flow is in the opposite direction on 18 October suggesting the existence of a cross-basin seiche. By 19 October current speeds are returning to similar magnitudes as those in run LBM-D without storms and the flow pattern in the South and Central Basins is also similar. However, the North Basin still has some cross-basin flow.

Figure 6.19 shows a map of the vertical velocity at 10 m on the 15 October, after the storm. Areas of up-welling are seen where the horizontal currents diverge: along the south coast of the South and Central Basins; east of the northern entrance to the Maloye More at the north of Olkhon Island; and north of the Svyatoi Nos Peninsula at (115,55). Areas of down-welling are evident along the north-west coast of the South and Central Basins, along the north coast of the North Basin, and in the centre of the North Basin where currents converge.

Down-welling of warmer surface water can be seen in transects of the temperature field shown in Figure 6.20. Transects of the temperature and v -velocity fields at $x = 120$ on 15 October for runs LBM-D and LBM-N are shown. The interface between the warm epilimnion and cooler hypolimnion is approximately horizontal in the plot for run LBM-D (Figure 6.20(a)). The plot for run LBM-N, however, shows the interface displaced downward in the northern part of the transect (Figure 6.20(b)). The



(a) run49_3bi

(b) run58

Figure 6.20: Transects at $x = 120$ showing temperature and v -velocity fields for run LBM-D and run LBM-N. Temperatures in centigrade and velocities in centimetres per second.

enhancement of down-welling of warmer surface water and up-welling of cooler water by the storm event leads to increased vertical mixing, which results in a cooler and deeper epilimnion. This is demonstrated on comparison of the model lake surface temperatures in runs LBM-D and LBM-N. Four days after the storm (Figure 6.18(b)), most of the lake surface is cooler than in the run with no storm (Figure 6.15(b)).

The results in this section have shown that the model responds realistically to the simple wind forcing fields used. The magnitude of the current velocities produced by the model are similar to those given by Shimaraev *et al.* (1994) and the model is also able to simulate the up-welling and down-welling that is expected to occur, respectively, in regions of horizontal current divergence and convergence.

6.4 Summary

The LBM is able to simulate the increased current velocities that were reported to occur at the edge of ice and snow areas by Zhdanov *et al.* (2001). The magnitudes of the currents are similar to those reported by the authors, that is of order of 1 cm s^{-1} . The LBM simulation produces similar results to the LHLC model although there are some differences. The lower net heat flux into the LBM lake results

in smaller horizontal temperature gradients that produce lower current velocities; the maximum LBM current velocities are approximately half the value of those of the LHLC model. The LBM is able to simulate the ice-free period when currents are driven by winds. Currents are generally stronger during this period and velocities depend on the velocity of the wind. The current magnitude is of order 1 cm s^{-1} during ice-cover but increases to a magnitude of a few cm s^{-1} with weak unstable winds and a few tens of cm s^{-1} with 'storm-force' winds. The amount of up-welling and down-welling exhibited by the LBM lake depends on the wind forcing. Simulated storms increase vertical mixing resulting in a cooler and deeper epilimnion. Whilst the results given in this chapter have shown that the LBM responds physically to forcing by a simplistic wind field it is not possible to reproduce a realistic Lake Baikal circulation without a realistic wind field and bathymetry. Also, since the behaviour of surface seiches and internal waves depends, in part, on the depth of the lake, it will be necessary to extend the model domain to the bottom of the lake and model a realistic bathymetry if these phenomena are to be simulated. The bathymetry data are available and extension of the model domain could be done fairly easily though some increase in the requirement for computational resources could be expected. However, the spatial and temporal resolution of available wind data are poor and the present work has identified the need for improvement in this area if the Lake Baikal circulation is to be simulated realistically.

Chapter 7

Conclusions and Suggestions for Future work

This work documents the development of a new three-dimensional model of Lake Baikal's upper 250 m and describes the coupling of a model of *A. baicalensis* population dynamics to the LHLC general circulation model. The development of the LBM is frustrated by the lack of environmental data in particular observations of surface wind fields over the lake. The purpose of this chapter is to summarize the results and conclusions of the work and to suggest ways in which the model could be developed in the future and other laboratory and field work that would be necessary for such development.

7.1 The LHLC Model

The LHLC model reproduces the vertical mixing during the period when the lake is warming under ice in the Spring. It is able to simulate the horizontal differential heating that occurs due to varying ice types and snow thickness. These properties make the LHLC particularly suitable for coupling to a diatom model for the purpose of simulating *A. baicalensis* population dynamics.

Unfortunately the LHLC model is unstable under wind forcing and so is incapable of simulating Lake Baikal thermodynamics during the ice-free period. Also, the LHLC model uses a quadratic equation of state. While Matthews and Heaney (1987) claimed this equation gives the density difference from the density at T_{md} with an error of less than 4% over the temperature range of 0-8 °C, the quadratic equation of state is less accurate at the warmer temperatures present in Lake Baikal during the Summer. The use of a quadratic equation of state also, when combined with the use of the convective adjustment

routine, prevents the model from stabilizing the layers vertically when temperatures are close to T_{md} so that the model lake is unable to warm past the T_{md} at the beginning of the Summer. Hence, whilst the LHLC model is suitable for coupling to the *A. baicalensis* model and simulating the diatom during under-ice conditions, it is not suitable for the purpose of modelling diatom populations throughout the whole year.

7.2 Modelling *A. baicalensis* Populations Under Ice

This is the first time a model of *A. baicalensis* has been coupled to a three-dimensional general circulation model of Lake Baikal. The coupled model has been used to explore the influence of the thermodynamic processes of buoyancy convection and vertical turbulent diffusion on diatom population dynamics. The model simulation of neutrally buoyant plankters indicates that vertical mixing reduces the numbers of these plankters both under snow-covered and snow-free ice. The vertical mixing processes move the plankters down the water column so that their average light exposure is reduced and hence their photosynthetic rates are slowed. Under snow-free ice, mixing occurs to a deeper level than under snow-covered ice because warming, and hence mixed layer development and deepening, is faster. The total column plankton at the points examined under snow-free and snow-covered ice are similar: under snow-free ice light levels are higher but the mixed layer is deeper, under snow-covered ice the light levels are lower but plankters are not mixed to such depths.

The analysis of the response of the plankton population to the vertical thermodynamic processes also reveals the extent to which each process occurs at different positions in the model lake. Under snow-free ice, neutrally buoyant plankter numbers are reduced more when convection is allowed than when turbulent diffusion is permitted. But under snow-covered ice, plankter numbers are unaffected when convection is allowed. This indicates that little or no convection occurs when there is snow upon the ice; here, warming is slow and mixing in the model occurs mainly by turbulent diffusion.

In the case of sinking plankters, simulating *A. baicalensis*, vertical mixing increases the number of plankters under both snow-free and snow-covered ice. Without vertical mixing the plankters sink out of the photic zone and, under snow-covered ice, are unable to maintain their numbers. The enhanced mixing that occurs under snow-free ice results in total column plankton numbers being a factor of ten higher than under snow-covered ice. The model is able to demonstrate the effect of vertical mixing processes on diatom reproduction and supports the hypothesis that the vertical mixing processes occurring in Lake Baikal during the under-ice warming period are able to maintain *A. baicalensis* organisms in the photic zone. The full three-dimensional model allows simulation of the horizontal spatial distribution of

A. baicalensis. The model predicts a negative correlation between *A. baicalensis* numbers and snow-cover and therefore, is in agreement with the observations of Granin *et al.* (2001).

One of the major obstacles to further development of the diatom model is the lack of observational data with which to initialize and validate the model. There is little published data on *A. baicalensis* numbers in the water. Until this data becomes available the model can only give relative diatom distributions rather than absolute population densities.

Further laboratory studies of the growth characteristics of *A. baicalensis* are also needed. *A. baicalensis* is peculiar, in that its growth rate decreases with increasing temperature. This property could easily be simulated by multiplying the specific growth rate (2.14) by a temperature dependent function. However, further data concerning the temperature dependency of the specific growth rate is required in order that a suitable function can be chosen.

In order to simulate populations of *C. minuta* the model must be run throughout the Summer. Irradiances during this period are high enough to cause photoinhibition of the photosynthetic rate of *A. baicalensis* and this should be included in the parameterization of the diatom's specific growth rate.

The diatom model as it stands models the light-limited growth of a diatom population. Later on in the year growth of the plankton becomes limited by nutrient availability (Goldman *et al.*, 1996). *C. minuta* blooms in the Autumn and during the Summer the picoplankton including the Cyanobacteria are in abundance. Modelling the population dynamics of *C. minuta* will require coupling further equations to the thermodynamic model. Equations for time evolution of *C. minuta* cell concentration and for nutrient concentration will be needed. Along with the need for further equations comes the need for further data for parameterization of the source-sink terms and for validation of the model; further field studies are required.

7.3 The Lake Baikal Model

For reasons given in §7.1, the LHLC model is not able to simulate the Lake Baikal warm season. A new three-dimensional general circulation model is therefore described. The main characteristics of the model are:

- The model domain includes the surface 250 m, with a realistic coastline.
- The model is able to simulate Lake Baikal from the beginning of under-ice warming to the end of December.
- The hydrostatic approximation is assumed: vertical accelerations are neglected.

- A convective adjustment routine maintains hydrostatic equilibrium.
- The Boussinesq approximation is assumed: variations from the mean density are neglected except where they affect buoyancy.
- Rotational: the horizontal components of the Coriolis acceleration are included in the equations for momentum.
- Density is approximated by the equation of state of Chen and Millero (1986), salinity is neglected, that is assumed zero.
- Sub-grid scale processes are parameterized by friction and diffusion of heat.
- Friction is parameterized by horizontal and vertical eddy viscosities.
- Diffusion of heat is parameterized by horizontal and vertical eddy diffusivities.
- Thermal forcing is modified by temporally and spatially varying ice and snow layers.
- Wind forcing is implemented as a surface stress.
- The advection of momentum and temperature are approximated using a total variation diminishing scheme (TVDS).
- The diffusion of momentum and temperature are approximated using a forward-in-time, centred-in-space scheme (CDS).

The LBM differs from the LHLC model in that: it uses a fixed model grid and includes an explicit vertical current velocity; the advection terms in the momentum and temperature equations are approximated using the TVDS; the more accurate equation of state of Chen and Millero (1986) is used to calculate the density; the convective adjustment routine determines stability by calculating the Brunt-Väisälä frequency rather than comparing layer densities; the thermal forcing is extended to heat exchange processes other than the absorption of shortwave radiation; wind forcing is implemented; the model is able to simulate the ice-free period.

The construction of finite difference approximations for the non-linear advection terms in the momentum, temperature and tracer equations can present a problem when modelling lakes. Historically, the most commonly used schemes are the central differencing scheme and the upstream differencing scheme (UDS). The CDS is known to produce numerical oscillations that produce unphysical negative concentrations of tracers and can even lead to numerical instability. Explicit diffusion has to be built into the code in order to avoid the spurious oscillations and maintain stability. The upstream method

avoids some of the problems of the CDS but at the expense of implicit diffusion. The explicit or implicit diffusion that accompanies these schemes damps internal wave processes. Numerical testing of the model shows that the TVDS is suitable for the discretization of the advection terms. This method does not produce the spurious oscillations in the variables which are seen with the CDS and the numerical diffusion, implicit in the scheme, is much less than is seen with the UDS.

The LBM model is able to simulate the thermodynamics of the upper 250 m of Lake Baikal from the beginning of the under-ice warming period in the middle of March until the beginning of freezing at the end of December. During the Spring the model predicts current flow along the edge of snow-fields and ice/water boundaries. The flow from low density (rising) towards high density (sinking) water is turned to the right by the Coriolis effect. The under-ice temperature and flow fields are in good agreement with the LHLC model. The current speeds are similar to those observed by Zhdanov *et al.* (2001). The predicted temperature profile development in the South Basin is in agreement with the observations of Jewson (1997) though the error is greater in the LBM predicted temperatures than in the LHLC model predicted temperatures and suggests some fine tuning of the thermal forcing may be required.

During the Summer and Autumn the temperature development in the South Basin predicted by the LBM is in good agreement with temperature observations made by Jewson (1997) for the same year (1997) and the model is able to predict the increased vertical mixing expected during a 'storm' event. The magnitude of the predicted currents vary from a few centimetres per second to a few tens of centimetres per second depending on the applied wind forcing. Although the magnitude of the current velocities produced by the model are similar to those given by Shimaraev *et al.* (1994), the model is unable to simulate a realistic Lake Baikal circulation. There is little published data concerning the current fields in Lake Baikal but Shimaraev *et al.* (1994) describe a cyclonic lake circulation with local gyres.

The LBM's inability to predict this circulation is probably owing to the use of a spatially uniform wind field to force the model. A more realistic wind field is required. Most global climate models fail to take into account the effect of the lake on the local climate, and hence the winds, so they are of little use as inputs to models of Lake Baikal. An alternative could be to couple the LBM to a model atmosphere or to model the effect of the local topography on the prevailing wind. Both these methods would, however, add another layer of complexity and so increase the computational resources required. If the wind fields are not to be calculated by other models then they must be obtained empirically and further observations are needed.

Prediction of a realistic circulation is also likely to require simulation of the lake's bathymetry. The LBM domain has vertical sides and, although Lake Baikal's basin is very steep, especially in the surface 250 m which forms the domain of the LBM, there are areas that are shallower with more sloping sides,

such as the Maloye More and the Selenga Delta region. Furthermore, influence of the major rivers flowing into and out of Lake Baikal will need to be included as momentum and temperature sources and sinks.

At present, the LBM prediction of the period from the start of freezing until the beginning of under-ice warming (January to the middle of March) is poor. This is, in part, owing to the model simulating freezing by setting negative temperatures to 0 °C which introduces an unphysical positive heat flux. Also, the model only allows warming by shortwave radiation of water below ice. The parameterization of the thermal forcing needs to allow conduction of heat away from the lake through the ice, perhaps at a rate that is dependent on the air temperature and the ice and snow thickness.

This thesis documents the early development of a new general circulation model of Lake Baikal. Future development of the model will require further observational data. The need for knowledge of the wind fields over the surface of the lake has been highlighted. Further current and temperature field data will also be needed for model validation. It is hoped that future field studies will yield the required data and enable further model development.

Bibliography

- D.L.T. Anderson and J.P. McCreary. Slowly propagating disturbances in a coupled ocean-atmosphere model. *Journal of Atmospheric Sciences*, 42(6):615–629, 1985.
- M. Bangs, R.W. Batterbee, R.J. Flower, D.H. Jewson, J.A. Lees, M. Sturm, E.G. Vologina, and A.W. Mackay. Climate change in Lake Baikal: diatom evidence in an area of continuous sedimentation. *International Journal of Earth Science*, 89:251–259, 2000.
- N.A. Bondarenko, N.E. Guselnikova, N.F. Logacheva, and G.V. Pomazkina. Spatial distribution of phytoplankton in Lake Baikal, Spring 1991. *Freshwater Biology*, 35(3):517–523, 1996.
- V. Botte and A. Kay. A numerical study of plankton population dynamics in a deep lake during the passage of the Spring thermal bar. *Journal of Marine Systems*, 26(3-4):367–386, 2000.
- V. Botte and A. Kay. A model of the wind-driven circulation in Lake Baikal. *Dynamics of Atmospheres and Oceans*, 35(2):131–152, 2002.
- C.T. Chen and F.J. Millero. Precise thermodynamic properties for natural waters covering only the limnological range. *Limnology and Oceanography*, 31(3):657–662, 1986.
- R. T. Cheng, T. M. Powell, and T. M. Dillon. Numerical models of wind-driven circulation in lakes. *Applied Mathematical Modelling*, 1:141–159, December 1976.
- S.M. Colman, J.A. Peck, E.B. Karabanov, S.J. Carter, J.P. Bradbury, J.W. King, and D.F. Williams. Continental climate response to orbital forcing from biogenic silica records in Lake Baikal. *Nature*, 378(6559):769–771, 1995.
- D.E. Farrow and D.P. Stevens. A new tracer advection scheme for Bryan and Cox type ocean general circulation models. *Journal of Physical Oceanography*, 25:1731–1741, 1994.
- J. H. Ferziger and M. Peric. *Computational Methods for Fluid Dynamics*. Springer, 3rd rev. edition, 2002.

- S. Fietz, G. Kobanova, L. Izmet'eva, and A. Nicklish. Regional, vertical and seasonal distribution of phytoplankton and photosynthetic pigments in Lake Baikal. *Journal of Plankton Research*, 8:793–810, August 2005.
- P.J.S. Franks. Npz models of plankton dynamics: Their construction, coupling to physics, and application. *Journal of Oceanography*, 00058(00002):379–388–, 2002.
- G.I. Galazy, editor. *Baikal Atlas*. Federal Agency for Geodesy and Cartography, Moscow, 1993. (In Russian).
- R. Gerdes, C. Koberle, and J. Willebrand. The influence of numerical advection schemes on the results of ocean general circulation models. *Climate Dynamics*, 5:211–226, 1991.
- S. Ghosal, M. Rogers, and A. Wray. The turbulent life of phytoplankton. *Centre for Turbulence Research, Proceedings of Summer Program 2000*, pages 31–45, 2000.
- C.E. Gibson. Sinking rate of *Aulacoseira baicalensis*. Personal communication, 2001.
- A. E. Gill. *Atmosphere-Ocean dynamics*, volume 30 of *International Geophysics Series*. Academic Press, 1982.
- C.R. Goldman, J.J. Elser, R.C. Richards, J.E. Reuter, J.C. Prisco, and A.L. Levin. Thermal stratification, nutrient dynamics, and phytoplankton productivity during the onset of spring phytoplankton growth in Lake Baikal, Russia. *Hydrobiologia*, 331(1-3):9–24, 1996.
- M.A. Grachev, S.S. Vorobyova, Y.V. Likhoshway, E.L. Goldberg, G.A. Ziborova, O.V. Levina, and O.M. Khlystov. A high-resolution diatom record of the palaeoclimates of east siberia for the last 2.5 my from lake baikal. *Quaternary Science Reviews*, 17(12TY - JOUR):1101–1106, 1998.
- N.G. Granin. Observations of *Aulacoseira baicalensis* in Lake Baikal in March 1997. Personal communication, 1997a.
- N.G. Granin. Temperature profile in the South Basin of Lake Baikal, 13 March 1997. Personal communication, 1997b.
- N.G. Granin, D.H. Jewson, R.Yu. Gnatovsky, L.A. Levin, A.A. Zhdanov, L.A. Gorbunova, V.V. Tsekhanovsky, L.M. Doroschenko, and N.Yu. Mogilev. Turbulent mixing under ice and the growth of diatoms in Lake Baikal. *Verhandlungen der Internationalen Vereinigung für Theoretische und Angewandte Limnologie*, 27(5):2812–2814, 2001.

- J.F. Hawley and L.L. Smarr. A numerical study of nonspherical black hole accretion. II. Finite differencing and code calibration. *The Astrophysical Journal Supplement Series*, 55:211–246, 1984.
- J.O. Hinze. *Turbulence*. McGraw-Hill, 2nd edition, 1975.
- B. R. Hodges, J. Imberger, B. Laval, and J. Appt. Modeling the hydrodynamics of stratified lakes. In *Hydroinformatics 2000 Conference*. Iowa Institute of Hydraulic Research, 2000. Pre-print.
- R. Hohmann, R. Kipfer, F. Peeters, G. Piepke, D.M. Imboden, and M.N. Shimaraev. Processes of deep-water renewal in lake baikal. *Limnology and Oceanography*, 42(5):841–855, 1997.
- P.R. Holland. *Numerical modelling of the riverine thermal bar*. PhD thesis, University of Loughborough, 2001.
- P.R. Holland, A. Kay, and V. Botte. A numerical study of the dynamics of the riverine thermal bar in a deep lake. *Environmental Fluid Mechanics*, 1(3):311–332, 2001.
- J. Huisman. Population dynamics of light-limited phytoplankton: Microcosm experiments. *Ecology*, 80(1):202–210–, 1999.
- J. Huisman and B. Sommeijer. Population dynamics of sinking phytoplankton in light-limited environments: simulation techniques and critical parameters. *Journal of Sea Research*, 48(2):83–96, 2002.
- J. Huisman, P. van Oostveen, and F.J. Weissing. Species dynamics in phytoplankton blooms: Incomplete mixing and competition for light. *Am. Nat.*, 154(1):46–68–, 1999.
- G.E. Hutchinson. *A Treatise On Limnology*, volume 1. Wiley and Sons, 1957.
- INTAS Project 99-1669 Team. *A New Bathymetric Map of Lake Baikal*. 2002. CD-ROM.
- A.D. Jassby and T. Platt. Mathematical formulation of the relationship between photosynthesis and light for phytoplankton. *Limnology and Oceanography*, 21(4):540–547, 1976.
- D.H. Jewson. Temperature profiles in the South Basin, December 1996 to September 1997. Personal communication, 1997.
- D.H. Jewson, N.G. Granin, and A. Yakushin. Affects of temperature and light on the growth of endemic planktonic diatoms from Lake Baikal. Pre-publication, 2004.
- D.E. Kelley. Convection in ice-covered lakes: effects on algal suspension. *Journal of Plankton Research*, 19(12):1859–1880, 1997.

- P.D. Killworth, E.C. Carmack, R.F. Weiss, and R. Matear. Modeling deep-water renewal in Lake Baikal. *Limnology and Oceanography*, 41(7):1521–1538, 1996.
- R. Kipfer, M. Hofer, F. Peeters, D.M. Imboden, and V.M. Domysheva. Vertical turbulent diffusion and upwelling in Lake Baikal estimated by inverse modeling of transient tracers. *Journal of Geophysical Research*, 105(C2):3451–3464, 2000.
- O.M. Kozhova and L.R. Izmet'seva. *Lake Baikal Evolution and Biodiversity*, chapter 6: Pelagic Life, pages 243–278. Biology of Inland Waters. Backhuys Publishers, 1998.
- S.P. Lawrence. Turbulence modelling in naturally convecting fluids. Technical Report CLM-R292, Culham Laboratory, United Kingdom Atomic Energy Authority, September 1989.
- S.P. Lawrence, K. Hogeboom, and H.L. Le Core. A three-dimensional general circulation model of the surface layers of Lake Baikal. *Hydrobiologia*, 487(1):95–110, 2002.
- H. L. Le Core. *Use of numerical models and satellite data to study physical processes in Lake Baikal*. PhD thesis, University of Leicester, 1998.
- B.P. Leonard. A stable and accurate convective modelling procedure based on quadratic upstream interpolation. *Computer Methods in Applied Mechanics and Engineering*, 19:59–98, 1979.
- J. M. Lewis. The story behind the Bowen ratio. *Bulletin of the American Meteorological Society*, 76(12): 2433–2443, December 1995.
- A. W. Mackay, D. B. Ryves, D. W. Morley, D. H. Jewson, and P. Riouals. Assessing the vulnerability of endemic diatom species in Lake Baikal to predicted future climate change: a multivariate approach. *Global Change Biology*, 12:2297–2315, 2006.
- A.W. Mackay, R.J. Flower, A.E. Kuzmina, L.Z. Granina, N.L. Rose, P.G. Appleby, J.F. Boyle, and R.W. Battarbee. Diatom succession trends in recent sediments from lake baikal and their relation to atmospheric pollution and to climate change. *Philosophical Transactions of the Royal Society, London. B*, 353:1011–1055, 1998.
- A.W. Mackay, R.W. Batterbee, R.J. Flower, N.G. Granin, D.H. Jewson, D.B. Ryves, and M. Sturm. Assessing the potential for developing internal diatom-based transfer functions for Lake Baikal. *Limnology and Oceanography*, 48(3):1183–1192, 2003.
- P.C. Matthews and S.I. Heaney. Solar heating and its influence on mixing in ice-covered lakes. *Freshwater Biology*, 18:135–149, 1987.

- G.L. Mellor and P.A. Durbin. The structure and dynamics of the ocean surface mixed layer. *Journal of Physical Oceanography*, 5:718–728, 1975.
- G.L. Mellor and T. Yamada. A hierarchy of turbulence closure models for planetary boundary layers. *Journal of Atmospheric Sciences*, 31:1791–1806, 1974.
- F. Mesinger and A. Arakawa. Numerical methods used in atmospheric models. GARP publications series 17, Global Atmospheric Research Program (GARP), 1976.
- R.C. Pacanowski and S.G.H. Philander. Parameterization of vertical mixing in numerical-models of tropical oceans. *Journal of Physical Oceanography*, 11(11):1443–1451, November 1981.
- F. Peeters, G. Piepke, R. Kipfer, R. Hohmann, and D.M. Imboden. Description of stability and neutrally buoyant transport in freshwater lakes. *Limnology and Oceanography*, 41(8):1711–1724, 1996a.
- F. Peeters, A. Wüest, G. Piepke, and D. M. Imboden. Horizontal mixing in lakes. *Journal of Geophysical Research*, 101(C8):18361–18375, August 1996b.
- Norman A. Philips. An example of non-linear computational instability. In *The Atmosphere and the Sea in Motion*, pages 501–504. Rockefeller Institute Press, New York, 1959.
- T. Platt, C.L. Gallegos, and W.G. Harrison. Photoinhibition of photosynthesis in natural assemblages of marine phytoplankton. *Journal of Marine Research*, 38:687–701, 1980.
- G.I. Popovskaya. Ecological monitoring of phytoplankton in Lake Baikal. *Aquatic Ecosystem Health and Management*, 3:215–225, 2000.
- W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. *Numerical Recipes in Fortran 77 : The Art of Scientific Computing*, volume 1. Cambridge University Press, 2nd ed. edition, 1992.
- T.M. Ravens, O. Kocsis, A. Wüest, and N. Granin. Small-scale turbulence and vertical mixing in Lake Baikal. *Limnology and Oceanography*, 45(1):159–173, 2000.
- T.L. Richardson, C.E. Gibson, and S.I. Heaney. Temperature, growth and seasonal succession of phytoplankton in Lake Baikal, Siberia. *Freshwater Biology*, 44:431–440, 2000.
- S.V. Semovski. The baltic sea and lake baikal underwater bio-optical fields simulation using ecodynamical model. *Ecological Modelling*, 116(2-3):149–163, 1999.
- S.V. Semovski, N.Y. Mogilev, and P.P. Sherstyankin. Lake Baikal ice: analysis of AVHRR imagery and simulation of under-ice phytoplankton bloom. *Journal of Marine Systems*, 27(1-3):117–130, 2000.

- M.N. Shimaraev, N.G. Granin, and A.A. Zhdanov. Deep ventilation of Lake Baikal waters due to Spring thermal bars. *Limnology and Oceanography*, 38(5):1068–1072, 1993.
- M.N. Shimaraev, V.I. Verbolov, N.G. Granin, and P.P. Sherstyankin. *Physical Limnology of Lake Baikal: A Review*. Baikal International Centre for Ecological Research., Irkutsk, 1994.
- P.K. Sweby. High resolution schemes using flux limiters for hyperbolic conservation laws. *SIAM Journal of Numerical Analysis*, 21:995–1011, 1984.
- H. Tennekes and J.L. Lumley. *A First Course in Turbulence*. The MIT press, 1972.
- M.C. Todd and A.W. Mackay. Large-scale climatic controls on lake baikal ice cover. *Journal of Climate*, 16:3186–3199–, 2003.
- E. A. Tsvetova. Convective currents associated with the thermal bar of Lake Baikal. In A. S. Alekseev and N. S. Bakhvalov, editors, *Advanced Mathematics: Computations and Applications*, pages 386–393. Novosibirsk Computer Centre, 1995.
- S.J. Walker and R.G. Watts. A three-dimensional numerical-model of deep ventilation in temperate lakes. *Journal of Geophysical Research*, 100(C11):22711–22731, 1995.
- Y. Wang. Comparing different numerical treatments of advection terms for wind-induced circulations in Lake Constance. In B. Straughan, R. Greve, H. Ehrentraut, and Y. Wang, editors, *Continuum Mechanics and Applications in Geophysics and the Environment*, pages 368–393. Springer, Heidelberg-New York, 2001.
- Y. Wang and K. Hutter. Comparisons of numerical methods with respect to convectively dominated problems. *International Journal For Numerical Methods in Fluids*, 37:721–745, 2001.
- R.F. Weiss, E.C. Carmack, and V.M. Koropalov. Deep-water renewal and biological production in Lake Baikal. *Nature*, 349(6311):665–669, 1991.
- J.R. Wilson. *Sources of gravitational radiation*. Cambridge University Press, 1978.
- World Heritage Convention. World Heritage Nomination - IUCN summary, Lake Baikal Basin (Russia), March 1996.
- A. Wuest, T.M. Ravens, N.G. Granin, O. Kocsis, M. Schurter, and M. Sturm. Cold intrusions in Lake Baikal: Direct observational evidence for deep-water renewal. *Limnology and Oceanography*, 50(1): 184–196, 2005.

A.A. Zhdanov, N.G. Granin, and M.N. Shimaraev. The generation mechanism of under-ice currents in Lake Baikal. *Doklady Earth Sciences*, 377(3):329–332, 2001.