Essays on Industrial Organization and Multi-market Contact

A thesis presented

by

Pak Yee Lee

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To my dearest mum

Abstract

This thesis comprises three essays on industrial organization and multi-market contact. The first chapter models leading firms in innovation markets deciding first whether to share knowledge, and then playing a market entry game. When firms are sufficiently patient, we show that the feasibility of intellectual property disclosure via licensing to outsiders or fringe firms provides a useful additional threat to entry by the punishing firm in the entry game. The opposite is true when firms are impatient; the availability of intellectual property disclosure makes coordination harder. In addition, we show that if the probability that the leading firms will be able to innovate without knowledge sharing is sufficiently high and firms are sufficiently patient, then it is also possible for the firms to enforce a knowledge sharing agreement before innovation has taken place.

The second chapter examines the incentives for predatory pricing within multi-markets. It considers an incumbent who is an uncontested monopolist in one market, but faces the threat of entry in a market for a complementary product. The paper shows that the incumbent may be able to defend its monopoly position in the complementary market even when it has a cost disadvantage and produces an inferior quality. The paper also provides conditions under which the incumbent accommodates entry.

Accommodation takes place when either the quality of the entrant's product is sufficiently high, or the entrant has a sufficiently low marginal cost. A surprising result of the analysis is that forbidding firms to price below marginal cost may reduce welfare.

The last chapter studies the incentive of a platform owner and an application developer to engage in an exclusive contract in a two-sided market setting with network externalities. The model considers two platform owners competing for advertising revenue with an application developer. We show that it can be optimal for a platform owner to allow a developer to multi-home when doing so has a strong positive effect on advertising in the two-sided market. On the other hand, if the effectiveness of the advertisement response to the number of potential viewers is at an intermediate level, then the developer is willing to accept an exclusive contract offered by a platform owner.

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[1] This chapter is a joint work with Scott Baker and Claudio Mezzetti. It has been published in the *International Journal of Economic Theory*, 7 (2011), 21-38 .

Introduction

This thesis consists of three essays in the theory of industrial organization and, more specifically, competitive behavior of a firm operating in multiple markets. The analysis of the competitive behavior of firms and its effects has a long tradition in economic theory. Competitive practices take many forms, including: undercutting rivals' prices and costs, improving the quality of its products, introducing innovations to the market and product advertising. One of the most important questions in the field of industrial organization is how firms exclude the entry of rivals through all these (anti)competitive practices and retain their dominant position in the market. However, firms usually do not produce a single product. When a firm operates in more than one market, the firm's actions in each market depend on the total profits across all markets. In this thesis, we study the firm's optimal course of action with multimarket contact and derive its welfare and efficiency implications.

"When one large conglomerate enterprise competes with another, the two are likely to encounter each other in a considerable number of markets. The multiplicity of their contacts may blunt the edge of their competition. A prospect of advantage from vigorous competition in one market may be weighed against the danger of retaliatory forays by the competitor in other markets. Each conglomerate competitor may adopt a live-and-let-live policy designed to stabilize the whole structure of the competitive relationship." (Scherer, 1980)

Following the above quote by Scherer, economists have traditionally taken the view that more collusive behavior is to be expected in the presence of multimarket contact. More recently, a growing body of literature has taken a deeper look at this issue. The focus of this literature has been on figuring out the situations in which collusive behavior is more likely and the implication for economic efficiency and antitrust and competition policy.

This thesis contains three chapters which look at different aspects of multimarket contact. The first chapter analyses information disclosure, the third predatory pricing and the fourth two-sided markets.

Intellectual Property Disclosure

In many instances, a firm must disclose its knowledge deriving from an innovation to its partner or to the public. Given the non exclusive nature of intellectual property, innovative products can be imitated by other firms getting this information. The literature on information disclosure is currently mainly concerned with how an inventor chooses between patent and trade secret protection or how a financially weak inventor should sell her innovation to a firm that may refuse to buy after having acquired partial information from the inventor.

Previous literature related to multimarket contact also points out that cross licensing is the common contract agreement and it facilitates knowledge sharing. In the first chapter, **“Intellectual Property Disclosure as ‘Threat’”**, joint work with Scott Baker and Claudio Mezzetti, we argue that a legal contract may not be necessary to enforce a knowledge sharing agreement and entry coordination. Within a multimarket setting, a joint venture agreement is easier to reach. In this chapter, we shed light on the formation of a joint venture and add to the theoretical literature on intellectual property by suggesting that firms utilize the non-exclusive nature of knowledge for enforcement of knowledge sharing agreement and coordinating market entry. In our model, the leading firms use licensing of their knowledge to outsiders, or fringe firms, as a punishment threat in order to enforce knowledge sharing agreements even the leading firms cannot enter the market with their own knowledge.

Entry Deterrence

The second chapter, **“Partner of Rival: Entry deterrence with multi-market contact”**, analyses a duopoly model where one of the firms sells two complementary goods. Up to now, the literature on entry deterrence within multiple markets in a duopolistic framework has been mainly concerned with bundling as a strategic tool for entry deterrence. In this chapter, we ask whether an incumbent always monopolizes a second market when it is allowed to price below marginal cost (predatory pricing). In the first stage, the entrant decides whether to enter one of the markets at a fixed entry cost. If it enters then nature generates the quality levels of the firms. In the third stage, firms set their prices depending on their quality levels. The incumbent has an incentive to deter entry by pricing below marginal cost in the competitive market, if the excess profit earned in the safer market (given entry deterrence) can compensate the loss in the competitive market (having another inferior firm to produce the product). The way entry can be deterred depends on the relationship between the two quality levels. If the quality levels are sufficiently close (given the costs), then entry deterrence takes place. We show that consumers may welcome predatory pricing (entry deterrence) if the cost levels of the firms are sufficiently close, the incumbent can produce at a high quality and generate more consumers surplus to the consumer than a low cost entrant produces at a lower quality. Thus the result is different from the previous literature which suggests that predatory pricing (price below marginal cost) should always be banned. With multimarket contact, the incumbent can price below marginal cost and recoup its loss from another market. Importantly, traditional antitrust analysis on predatory pricing that focuses on a single market may not be applicable as it ignores the consumer surplus generated from the other complementary market.

Two-sided Markets

In contrast to the second chapter, which analyses a model of vertical product differentiation, the third chapter deals with horizontal product differentiation in a two-sided markets framework with network externalities (e.g., social networking websites). In this chapter, **"From Nintendo to Facebook: Two-Sided Markets with exclusive contracts",** we study a model with two platforms and a potential add-on application. Positive network externalities are generated by consumers interacting with other consumers on the same platform. The platform owners and application developer's profit come from advertising. We show that it may be optimal for a platform owner to allow a developer to develop add-on applications on the rival platform. The intuition behind this result is the following. If a developer has an application on both platforms, posting an advertisement on the add-on application is more attractive than posting an advertisement on any single platforms as advertisers can reach more potential customers through the advertisement. And developer becomes a strong rival in the advertising market, however, the rival platform will also face the same strong rival in the advertisement and it will reduce its advertising price, to the advantage of the original platform that allowed multihoming by developer.

Chapter 1
Intellectual Property Disclosure as "Threat"[2]

1.1 Introduction

Once released, information is the quintessential public good. It is non-rival – my use of information does not prevent others from using that same information. It is non-exclusive. Absent some legal rights or expensive self-help, one can't easily exclude someone else from using information. The familiar argument is that intellectual property rights respond to the unique character of information. Patent, copyright, and trade secret all give innovators some ex post control over their creation. The assumption is that, without some control, the eventual appropriation of the information will stunt its development. Yet the non-rival and non-exclusive nature of information leads to another consequence under-appreciated in the academic literature or intellectual property policy debates. The same characteristics that make information-creation problematic also render the disclosure of information an effective weapon for self-policing agreements.

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[2] This chapter is a joint work with Scott Baker and Claudio Mezzetti. It has been published in the *International Journal of Economic Theory,* 7 (2011), 21-38 .

To see why, consider a tacit agreement between two firms to divide up markets. The standard story is that the threat of entry into the other firm's market can maintain the market division agreement **(**see, for example, Calem (1988) and Bulow et. al. (1985)**)**. But what if one firm has capacity to only enter one market? In that case, the capacity constrained firm lacks a credible entry threat and the market division agreement falls apart.

Intellectual property as an essential input in production alters this story. Now even though the capacity constrained firm can't enter its rival's market, any knowledge the firm has can be sold to someone else. Information licensed to one firm is never depleted. In other words, unlike physical capital, intellectual property doesn't depreciate. If the first firm who buys a license fails to innovate, that same information can be licensed to a second firm. If the second firm fails to innovate, the information can be sold to a third firm. And so on. Indeed, because information is non-rival, the leading firm can simultaneously license to a number of different fringe firms, requiring a payment only from those firms that are able to successfully use the information by innovating and entering the market. In effect, through IP disclosure a firm can guarantee that some firm will innovate, build a competing product, and enter the market of a counter-party that reneges on its promises. The fear of license-induced entry, then, provides an incentive for each firm to keep its word. Simply put, information disclosure works as a hammer to punish deviations from both express and implied agreements between firms whose business model is based on intellectual property. As a result, provided they care enough about future profits, firms holding intellectual property find enforcing agreements – whether those agreements are pro-competitive or anti-competitive– easier than firms that don't.

The power of the disclosure threat depends on two factors: (1) the degree to which knowledge can be easily transferred between firms, that is, how easy it is to learn what another firm knows; and (2) the number of fringe firms willing and able to bring a product to market if given the essential intellectual property.

Better self-enforcement works for good and ill. On the negative side of the ledger, the threat of disclosure makes it easy for firms to enforce tacit market division agreements. That is to say, there is an increased risk of collusion where firms have information that could be released upon observing a deviation from a tacit agreement. The antitrust ramifications of this point suggest care in the treatment of R&D joint ventures. Although the potential anti-competitive effects of such agreements are well-known, the literature has focused on ancillary clauses in the agreement itself, such as promises to share price information (Grossman and Shapiro (1986)), or promises to cross-license at supra-competitive rates (Shapiro (1985) and Katz and Shapiro (1985)). Our model shows that the mere presence of intellectual property at the core of the business can, under certain circumstances, facilitate collusive behavior.

On the plus side, the disclosure threat can, under some circumstances, increase the ability of firms to share knowledge in joint ventures. Firms might not need the courts (Posner, R. (2006) and Shavell (1980)) or reputational sanctions (Bernstein (1992), Posner, E. (1998), and Klein and Leffler (1981)) to generate compliance with contractual obligations to share know-how. The end result is more joint ventures, more knowledge sharing, and more new products.

The disclosure threat does not lead inevitably to this beneficial result. There are competing effects. On the one hand, when IP disclosure to fringe firms is a feasible option, sharing knowledge in a joint venture becomes more costly, because it may lead to more competition in the market following innovation. Suppose firm 1 doesn't share knowledge. Firm 2 can punish this deviation by threatening to not comply with any tacit agreement to divide up markets. But the punishment has to be credible. With licensing, it can be harder to sustain cooperation on market division. The firms have to coordinate to both (a) "not license" to the fringe and (b) "not enter" into each other's markets. When cooperation is harder, it is more likely the market division will fall apart anyway and no additional punishment will be forthcoming for a failure to share information. And, by sharing, the firm only helps their rival innovate. Combined, these two effects make knowledge sharing less attractive.

On the other hand, licensing can allow additional punishment strategies in the market entry game. The reason is that licensing allows punishment (when credible) under a wider array of circumstances. Without licensing, firms can only punish a failure to share knowledge when both firms can innovate in both markets. With licensing, firms can punish in that case as well as when one firm can innovate in only one market and the other firm can innovate in both markets. We show that knowledge sharing in a joint venture is self-enforcing provided firms care enough about future profits, and provided their ability to innovate even without knowledge sharing is sufficiently high to deter the other firms from cheating on an agreement to share knowledge.

In addition, the insights offered here provide a new rationale for the common practice of licensing technology on a non-exclusive bases to R & D joint ventures. The conventional wisdom is that firms use non-exclusive licenses because they don't want to tie up knowledge assets in the joint venture, especially if the technology might be useful for other unrelated projects. Our model shows that the non-exclusive license serves another purpose: maintaining the intellectual property disclosure threat. The non-exclusive license is equivalent to loading a gun, ready to be discharged if participants in the joint venture fail to uphold their end of the bargain. Intellectual property is disclosed in the form of non-exclusive license to maintain the knowledge sharing and entry coordination.

The model considers two R&D firms ahead of the competition in two innovation markets. The firms form a joint venture in order to share knowledge, but knowledge sharing must be self-enforcing. If possible, the firms would also like to tacitly divvy-up the two markets. That is, each firm wants to focus on developing one of the two possible innovations. As an example, consider two technology firms forming a joint venture. Each firm may or may not fully share its technology knowledge with the other firm in the joint venture. This knowledge can be the basis of a variety of potential products, from a new cell-phone to a higher-speed computer to a higher definition flat screen television. Market entry demands incorporating the technology into an innovative new product, what we denote as innovation. Each firm might be able to innovate and produce the product without access to the other firm's technology. Access, however, increases the chance of a successful innovation. The exchange of knowledge is tough to verify and, as a result, non-contractible. Under these circumstances, each firm has an incentive to withhold information from its counter-party. By withholding, a firm benefits from the other firm's knowledge, while at the same time maintaining an edge in the race to innovate.

Two mechanisms sustain both the explicit joint venture contract to share knowledge and the tacit market coordination agreement. If a firm observes its rival failing to comply with its obligations it either (1) enters and competes in the renegade's market in all the subsequent periods or (2) releases information through licensing. The first threat is a variant on the grim trigger strategy in repeated games (Friedman (1971)). Whether the threat controls deviations depends on the relationship between the gain to a one time deviation and the firm's discount rate. The more interesting second strategy – IP disclosure – is credible because it is only carried out when the punishing firm is unable to innovate on its own in the renegade's market. In that case, the punishing firm engages in sequential licensing to fringe firms until one fringe firm can innovate. The transfer of intellectual property provides fringe firms a gateway into the renegade's market.

Our paper relates to a number of literatures. First, there is the research on the strategic transfer of knowledge. Anton and Yao (1994) analyze a pro-competitive effect of information disclosure. They study the problem facing an inventor who wants to transfer knowledge in the absence of property rights. Without IP rights, contracts don't work. Any knowledge transfer will be snapped up and then the purchaser won't pay. They show that the inventor will be nonetheless able to protect his property rights by credibly threatening the buyer to disclose information to a market rival. In another set of papers, Anton and Yao (2002) and (2003) provide another justification for IP disclosure: expropriable partial disclosure can be used to credibly signal the quality of an inventor's innovation. Our model focuses on disclosures by symmetric firms, rather than private disclosures by an inventor. We show that threats of knowledge disclosure can ensure compliance with both pro-competitive and anti-competitive agreements between firms. The threat of disclosure facilitates knowledge sharing by firms, a pro-competitive effect, but it also makes it easier for firms to divide-up the innovation markets, an anti-competitive effect.

Second, our paper touches the large literature on IP licensing. Here, scholars often address the relationship between licensing and the speed of innovation (see, e.g., Katz and Shapiro (1988)). Other times, scholars are concerned with what structures the terms of the licensing agreement. For example, Gans and Stern (2000) study bargaining over the licensing terms between an incumbent and a potential entrant with a technological innovation. They find that an incumbent might, under certain conditions, invest in R &D purely to improve their position in the licensing negotiation. d'Aspremont et al. (2000) study the sharing of interim research knowledge between two firms engaged in a patent race. There, because of the nature of information, they find that the non-informed agent is able to obtain full disclosure of the informed party's knowledge, while forfeiting none of the gains from trade to the informed seller. Bhattacharya and Guriev (2006) consider two R &D firms deciding how to sell their ideas to development firms. The potential for leakage of knowledge in the patent process pushes firms toward protecting knowledge through trade secrets. Bhattacharya et al. (1992) explore two licensing contacts that ensure efficient sharing of knowledge and efficient expenditures on R &D. Like most of this literature, we focus on a special characteristics of knowledge: the ability to license the same knowledge to multiple actors. In our model, it is this characteristic that makes it easier to sustain cooperative behavior between firms.

Finally, our paper connects with the literature on multimarket contact. Bernheim and Whinston (1990) were among the first to explore the effect of multimarket contact on collusive behavior. They showed that multimarket contact may enhance the firms' ability to collude when the firms or the markets are asymmetric. We focus on symmetric firms and markets, and show that multimarket contact and the ability to disclose information via licensing to fringe firms facilitate knowledge sharing and market division.

The paper is organized as follows. Section 1.2 develops the model; in the first round two leading R&D firms decide whether to share knowledge, and in the second round they play an entry game in two potential markets. Section 2.4 studies the entry game. It shows that market coordination (each firm cornering one market) is easier to sustain if a firm can use the threat of disclosing intellectual property in the other firm's market, when it is not able to enter itself. Section 1.4 studies knowledge sharing agreements. It shows the conditions under which the threat of IP disclosure makes it easier for firms to share knowledge prior to divvying up the markets. Section 1.5 offers some concluding thoughts. All proofs are relegated to the appendix.

1.2 The Model

There are two potential innovation markets ($j \in \{A, B\}$), two leading firms ($i \in \{1, 2\}$) and n_j fringe (or start-up) firms in each market j. Each leading firm is able to introduce at most an innovation in each of the two markets, either directly, or by licensing to a fringe firm. Thus, there are four potential innovations, or products. If no firm introduces an innovation in market j, then market j does not open. If at least one firm introduces an innovation, a market stays open for an infinite number of periods. All firms have a common discount factor δ. To simplify the exposition, we assume symmetry of the two markets and the firms' payoff functions. If a firm has introduced an innovation in market j, then it obtains a payoff $V(1)$ in any period in which the other innovation is not in the market, and a payoff $V(2)$ when a second innovation has been introduced.[3]

We deliberately use the reduced form $V(\cdot)$ for the stage payoffs, in order to abstract from the firms' pricing strategies, and focus instead on their information sharing, licensing and entry strategies.[4] The two innovations in a market are substitutes, and we assume that monopoly profits are larger than total duopoly profits $V(1) > 2V(2)$.

Consider, as an example, firms introducing identical innovations into two symmetric Cournot oligopoly markets. Then our reduced form assumptions on V hold. For example, with linear demand and constant marginal cost we have $V(m) = (A-c)^2/b(m+1)^2$, where A is the vertical intercept and b is the slope of the demand function, while c is marginal cost. Our assumptions also hold if firms compete in prices, provided products are not perfectly homogeneous.

We now describe the game the firms play.

In the first stage, each leading firm decides simultaneously whether to share its knowledge about technology in the two markets with the other leading firm. We can think that the firms belong to a research joint venture, but they are able to hide

[3] It will never be the case that more than two firms enter with the same innovation. A leading firm will not want to license if it can innovate and enter, and if it licenses it will make sure that only one fringe firm will enter, so as to obtain a higher share of the market profit through the licensing fee.

[4] We also do not consider the possibility that the leading firms de facto merge by stipulating that the joint venture does the manufacturing and sells as a single entity. It is much simpler for leading firms to form an R&D joint venture than to merge.

their knowledge if they find it profitable. A leading firm's knowledge determines the probability with which it can innovate in a market. Naturally, firms with more knowledge have a greater probability of being able to innovate. To capture this idea in the simplest possible way, we assume that a leading firm can develop an innovation in a market with probability p_l using only its own knowledge, while it is able to develop the innovation with probability p_h, with $p_h > p_l$, when it also has access to the knowledge of the other leading firm. We will say that a firm with access only to its own knowledge has a low knowledge level, while it has a high knowledge level if it has access to both technologies. At the end of the first stage each leading firm learns whether it actually can innovate in each of the two markets. For simplicity, we assume that whether a firm can bring a product to market is publicly known.[5]

In the second stage, the two leading firms play a repeated entry game. In each period $t \geq 1$ the leading firms decide simultaneously whether to enter an innovation market that they have not entered before and whether to license any of their knowledge, or IP, to the fringe firms. Without any knowledge transfer from the leading firms, none of the n_j fringe firms in market j can innovate. Because knowledge transfer from a leading firm to a fringe firm might be imperfect, each fringe firm can innovate with probability $p_f \leq p_l$ when given access to a leading firm's technology.[6] If it decides to license, a leading firm issues a licensing agreements to the entire

[5] This assumption could be relaxed at the cost of complicating the analysis with little change in the main economic insights.

[6] It would seem plausible to assume that if the licensing firm has access to the other leading firm's technology, then the probability p_f^h that a fringe firm innovates after licensing is higher than the prob-

fringe. The licensing agreement stipulates that the first fringe firm that is able to innovate enters the market and pays the leading firm a fee equal to a fraction α of its stream of profits, $\frac{V(m)}{1-\delta}$, $m = 1, 2$. To keep things simple, assume that the licensing agreement also restricts entry to one fringe firm, no matter how many fringe firms can innovate. The exogenous parameter $\alpha \in [0, 1]$ measures the relative bargaining power of the leading firm. We can think of $\alpha = 0$ as the special case in which the leading firm freely and publicly discloses its knowledge. When a single leading firm licenses in a market, the probability that a fringe firm will enter is equal to the probability that at least one fringe firm will be able to innovate, which we will denote as γ_1. When both leading firms license to the fringe in a market, the probability that at least two fringe firms will be able to innovate (and hence both leading firms will be able to collect their profit shares) is denoted as γ_2. The probability that only one fringe firm will be able to innovate is $\gamma_1 - \gamma_2$; in such a case each leading firm is equally likely to be the one to license. Hence, the probability that each leading firm will receive its profit share from the only innovating fringe firm is $\frac{1}{2}(\gamma_1 - \gamma_2)$. It is natural to think that γ_1 and γ_2 are increasing function of p_f and n_j (but we need not make that assumption here).

We solve the game by backward induction, considering first the equilibrium in the repeated market entry stage and then equilibrium in the full game.

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ability p_f^l that it innovates after licensing from a leading firm with low knowledge. This assumption would complicate the notation without affecting any of the results, provided that the difference between p_f^l and p_f^h is not too large. To simplify the notation, we assume that $p_f^l = p_f^h = p_f$.

1.3 The Market Entry Stage

In this section we consider the second stage, or repeated market entry stage, of the game, after the leading firms have decided whether to share knowledge and have learned in which market they can innovate. Behavior in the second stage only depends on which leading firm can innovate in which market. Table 1 lists the possible configurations.

(1)	Firm 1 can innovate in one market; Firm 2 can innovate in the other market
(2)	Firm 1 and Firm 2 can innovate in both markets
(3)	Firm 1 can innovate in one market; Firm 2 can innovate in both markets
(4)	Firm 1 can innovate in both markets; Firm 2 can innovate in one market
(5)	Firm 1 and Firm 2 can only innovate in the same one market
(6)	At least one of the two firms cannot innovate in any markets

Table 1.1: Possible Subgame Configurations

In our setup, if each leading firm always enters any market where it can introduce an innovation, then the two firms are subject to a coordination failure. For example, if each leading firm is able to introduce an innovation in both markets, then the firms benefit from coordinating and each entering one market. Coordination may be sustained because firms have the opportunity to enter repeatedly over time.

We begin by considering the benchmark case in which the leading firms cannot disclose their IP to the fringe firms.

Absent the threat of licensing, all subgame configurations except (2) have a unique subgame perfect equilibrium in which each leading firm enters a market at time $t = 1$ if it is able to develop the innovation in that market. (Recall that a firm cannot enter a market unless it is able to develop an innovation.) In subgame (1) the leading firms coordinate trivially, because each has no alternative but to enter in the only market in which it can innovate. In the subgames (3)-(6) the firms cannot coordinate – one entering market A and the other entering market B. The reason is that the entry threat needed to maintain agreement is not credible.[7]

Consider the subgame where firm 1 can only develop in market A, while firm 2 can develop in markets A and B. Can the firms agree that firm 1 will introduce its innovation in market A and firm B will introduce its innovation in market B only? No. Firm 2 will always deviate and enter market A, too. It faces no retribution from doing so. Firm 1 can't punish firm 2's behavior because it is unable to innovate and enter market B. As we shall see, that all changes when the threat of IP disclosure is available; then market entry coordination is also possible in subgame configurations (3)-(4).

When IP disclosure is not possible, non trivial entry coordination is possible only in one subgame configuration, configuration (2). There, both firms are able to develop an innovation in both markets. In this case there are two different types of subgame perfect equilibria with no entry delay (or immediate entry).[8] In the first type

[7] In this paper, we disregard standard means of enforcing cooperation that may work even when both leading firms operate in the same market (e.g., price wars). Our focus is on how IP disclosure and licensing may help coordination.

[8] An equilibrium has no entry delay if all entry in the innovation markets takes place at $t = 1$. For our purposes, these are the most interesting and plausible equilibria and we focus on them in this paper.

of equilibrium, both firms enter both markets immediately. This type of equilibrium always exists. The strategy of each firm is to enter both markets at any time t if it did not enter the markets before. Given the opponent's strategy, each firm's strategy is sequentially rational. In the second equilibrium outcome, the focus of the next proposition, firms coordinate: One firm enters market A immediately and the other enters market B immediately. This second type of equilibrium, exists if the discount rate δ is sufficiently high. Before formalizing this result in the next proposition, define:

$$\delta_1 = \frac{V(2)}{V(1) - V(2)}.$$

Proposition 1 *Suppose IP disclosure and licensing are not possible. There exists a subgame perfect equilibrium outcome of the entry game in which each leading firm enters a different market in the following scenarios: (i) When each leading firm can develop an innovation in both markets, if $\delta \geq \delta_1$; (ii) when one leading firm can only enter market A and the other leading firm can only enter market B.*

This result is standard. If the discount factor is high (above δ_1), there exists an equilibrium of the market entry subgame (2) where firms can successfully enforce an agreement to coordinate market entry decisions. The patient firm values the one-time bump in profits from deviating on the market division agreement less than the stream of losses from competing in both markets in every future period. Enforcement of the tacit agreement is possible.

We now allow for knowledge disclosure and licensing. Licensing increases the number of subgames where the firms can coordinate their actions.

Proposition 2 *Suppose IP disclosure and licensing are possible. There exists a subgame perfect equilibrium outcome of the entry game in which each leading firm enters a different market in the following scenarios: (i) When one firm can develop an innovation in one market and the other firm can develop an innovation in both markets, if* $\delta \geq \frac{\delta}{\gamma_1}$*; (ii) when each leading firm can develop an innovation in both markets, if* $\delta \geq \delta_1$*; and (iii) when one firm can develop an innovation only in one market and the other firm can develop an innovation only in the other market, if* $\delta \geq \alpha\delta_1$.

A few remarks are worth making here. First, in market entry subgame (2), when each leading firm can develop an innovation in both markets, the condition for existence of an equilibrium with coordinated entry is the same as in the case when no licensing is possible. This is because the threat of directly entering a market is more severe than the threat of licensing to the fringe.

Second, with a sufficiently high discount factor and availability of IP disclosure, it becomes possible to coordinate entry in market entry subgames (3) and (4), corresponding to the case when one leading firm can enter both markets and the other firm only one market. Whether the equilibrium with coordination of market entry exists depends on γ_1, the chance that licensing will result in fringe entry. As

γ_1 gets smaller, the needed threshold value of δ gets bigger. A bigger γ_1 makes the entry of a fringe firm more likely, and hence the punishment of licensing more severe. In general we might think that γ_1 increases with the number of fringe firms and the probability of success of the knowledge transfer, p_f. In that situation, the power of the licensing threat to enforce entry coordination turns on the ease of knowledge transfer and the depth of the fringe.

Third, with disclosable intellectual property, entry coordination is more difficult in entry subgame (1) when in each market only one leading firm is able to innovate. This is because now each leading firm benefits from licensing its knowledge to the fringe in the market in which it cannot innovate. The leading firms now need to coordinate not to license and they must be sufficiently patient to be able to sustain such cooperation. The existence of an entry coordination equilibrium depends on the share α of the fringe profits appropriated through licensing by the leading firm. The smaller α, the smaller the temptation to license and hence the easier it is to coordinate entry.

To summarize the results of the second stage, or market entry stage, it is useful to distinguish the following three cases, depending on the degree of patience of the leading firms.

Case 1: If $\delta < \alpha\delta_1$ (the leading firms are impatient), then market entry coordination is made more difficult if information is disclosable to the fringe. The leading firms cannot coordinate not to license to the fringe when one can only innovate in market A and the other only in market B

Case 2: If $\alpha\delta_1 \leq \delta < \frac{\delta_1}{\gamma_1}$ (the leading firms are moderately patient), then the feasibility of disclosing information to the fringe has no impact on the leading firms' ability to coordinate market entry.

Case 3: If $\delta \geq \frac{\delta_1}{\gamma_1}$ (the leading firms are patient), then the ability to disclose information to the fringe makes market entry coordination more widely available to the leading firms. They can now coordinate when one can only innovate in one market and the other can innovate in both markets.

1.4 Knowledge Sharing in the First Stage

In this section, we step back to the first stage of the game and look at whether the leading firms share information. Recall that without sharing of knowledge, leading firm i can only innovate with probability p_l in each market. By sharing its knowledge a firm raises the other leading firm's probability of innovating to p_h in both markets. Thus, mutual sharing of knowledge makes innovating in each market more likely and so, everything else equal, is beneficial both to society and the leading firms. But a free riding motive is also present. Ideally, leading firm 1 would rather have firm 2 share information, but not share information itself, and vice versa.

To see the difference that IP disclosure makes, like in the preceding section we first analyze the case in which the leading firms cannot license to the fringe, and then look at the case when licensing is possible. We focus on the conditions under which the leading firms are able to share knowledge in stage one of the game, and then are able to coordinate market entry in stage 2 as described in Propositions and .

The following threshold value of p_l will be used in the next proposition, defining the equilibrium in the case in which disclosure and licensing are not possible:

$$p_l^* = p_h - \frac{p_h^3 \left[V(1) - 2V(2)\right]}{2\left[V(1) - V(2)\right]}.$$

Proposition 3 *Suppose IP disclosure and licensing are not possible. There exists a subgame perfect equilibrium in which the leading firms share knowledge in the first stage of the game if $p_l \geq p_l^*$ and $\delta \geq \delta_1$. In this equilibrium, each leading firm enters a different market in the second (market entry) stage under the following scenarios: (i) When each leading firm can develop an innovation in both markets; (ii) when one leading firm can only enter market A and the other leading firm can only enter market B.*

Information sharing can be part of an equilibrium if and only if it is coupled with coordination in the entry game. If $\delta < \delta_1$, no coordination will take place in the entry game, and thus there is no reason for a leading firm to share knowledge in the first stage (by benefiting the rival, knowledge sharing can only hurt a firm).

To sustain the knowledge sharing agreement, each leading firm credibly threatens to enter each market where it can develop an innovation if the rival firm fails to share knowledge. For this threat to serve its purpose, a leading firm must be able to innovate with sufficiently high probability, even if its rival does not share knowledge. That is to say, it must be $p_l \geq p_l^*$.[9] The restrictions on p_l makes it sufficiently likely the leading firms will end up in a market entry subgame (2) of Table 1, where both can enter both markets. Only in this subgame can meaningful entry coordination occur and, accordingly, only then can firms use threats to deviate from the coordinated scheme to punish a failure to share knowledge.

To be more precise, in subgame (1) of Table 1, firms are forced to coordinate entry, because each can only innovate in a different market. In subgames (3)-(6), the firms can't coordinate entry. If these subgames are sufficiently likely, knowledge-sharing cannot be self-enforced, no matter how patient the firms are. The chance of a firm hurting itself by sharing knowledge is simply too high. Since a coordinated equilibrium is unlikely, by sharing knowledge a firm just increases the likelihood that its rival will eventually enter more markets. The restriction $\delta \geq \delta_1$ means that, once in the subgame where both firms can enter both markets, the firms are sufficiently patient to facilitate coordination.

When $p_l < p_l^*$ or $\delta < \delta_1$, firms face a sort of prisoner's dilemma. Both firms would be better off if they could commit to share knowledge and coordinate their entries in the markets. Nevertheless, this sort of cooperation is unobtainable. In

[9] Note that $p_l^* < p_h$, since $V(1) > 2V(2)$ by assumption.

equilibrium, each firm has an incentive to take the knowledge shared by its rival, fail to return the favor, and then enter every innovation market it can.

We now consider the situation in which IP disclosure is possible, and focus on the case in which the leading firms are patient, $\delta > \delta_1/\gamma_1$. As shown in the previous section, in this case licensing gives an additional punishing tool against renegade firms, enlarging the number of subgames where cooperation can occur in the entry game. This tool can also be used to facilitate knowledge sharing in the first stage. Now if a leading firm fails to share knowledge, the rival firm can credibly threaten to license to a fringe firm in all markets in which it cannot enter. This enhances the probability a firm will experience punishment in the entry game after reneging on the knowledge-sharing agreement (punishment can be meted out in four subgames, rather than one subgame).

The availability of licensing, however, also increases the cost of sharing knowledge to a leading firm, because it allows the opponent firm to license to the fringe when unable to innovate (e.g., a firm will license in subgame (6) of Table 1). It is thus not obvious that knowledge sharing will ever take place, even when firms are patient. We will focus on this case in the next proposition and show that there is a value of p_l, the probability of innovating with low knowledge, above which sharing knowledge and then coordinating market entry can be sustained in equilibrium.

Proposition 4 *Suppose IP disclosure and licensing are possible. There is a value* $p_l^{**} < p_h$ *such that, if* $p_l \geq p_l^{**}$ *and* $\delta \geq \frac{\delta_1}{\gamma_1}$ *then there exists an equilibrium where* *firms share knowledge in the first stage and coordinate market entry (each leading firm entering a different market) in the following scenarios: (i) When one firm can develop an innovation in one market and the other firm can develop an innovation in both markets; (ii) when each leading firm can develop an innovation in both markets; and (iii) when one firm can develop an innovation in one market only and the other firm can develop an innovation only in the other market.*

Thus, a conclusion is that the feasibility of IP disclosure helps coordinating market entry and sharing knowledge between the leading firms, provided the firms are sufficiently patient and their probability of innovating even without knowledge sharing is sufficiently high. Otherwise the feasibility of IP disclosure may not help, or even hinder, knowledge sharing and entry coordination.

1.5 Concluding Remarks

The model developed in this paper demonstrates how firms can use the threat of licensing to fringe firms as a mechanism to enforce agreements to exchange knowledge and coordinate entry decisions. For some parameter configurations, the threat of knowledge disclosure deters the breach of the explicit knowledge sharing agreement and the tacit market division agreement arising out of an R&D joint venture. Some insights gained from the model follow: (1) Enforcing agreements – illegal and legal – is easier when the firms have intellectual property that can be easily released to fringe firms. (2) If technology is difficult to transfer to other firms, firms don't have any technology to transfer, or there are few firms able to innovate when given the technology, firms will have greater difficulty self-policing their agreements.

In practice, R&D knowledge sharing agreements must detail the knowledge to be shared (even if it isn't created yet). Inartful and imprecise contractual drafting can make it difficult for courts to determine "breach," especially when the contract governs ever-evolving technology. Making enforcement more problematic is the presence of judges with little technology expertise or savvy. Our model shows that enforcement concerns are potentially overstated. The threat of intellectual property disclosure to fringe firms can, under certain conditions, ensure compliance with knowledge-sharing commitments absent court intervention.

Chapter 2
Partner or Rival: Entry deterrence with multi-market contact

2.1 Introduction

Over the past three decades, high-technology consumer electronics have become ubiquitous and essential in daily life. The success of an electronics product does not rest only on the product itself, but also upon the support of complementary goods. For example, the existence of App Store helps Apple maintain its dominance over the media player market. The ability to run third party's software makes IPod not only a media player, but also a portable computer platform, and hence allows Apple to compete in the personal digital assistant market. On the other hand, the failure of Sega as a producer of home TV-game consoles illustrates the importance of complementary products. In 1994, by launching the ultra high-tech game console Saturn™, Sega, the second largest game console producer at the time, posed a difficult question to the major game developers in the market. It was very costly for developers to create a game software to fit the high hardware standard of the Saturn™. Game developers were therefore reluctant to enter the market, contributing to the final failure of the console. As complementary products are crucial to the survival of the main "complemented" product, it is important to study entry in the complementary products markets and evaluate the different strategies available to an incumbent producer of the complemented product. What strategy will the incumbent in a market use to leverage its power into a complementary market?

The usual response to this question in the literature is tying or bundling (Choi and Stefanadis [2006], Kovac [2005], Nalebuff [2004] and Whinston [1990]). According to the literature, tying provides a mechanism whereby a firm with monopoly power in one market can use the leverage provided by this power to foreclose sales in, and thereby monopolize, a second market (Whinston [1990]). An incumbent with multiple products can always bundle its products and sell at a lower price. Is tying the only mechanism for an incumbent to deter entry?

Predatory pricing can be one of the options for a firm to protect its monopoly position in its complementary market. It is the practice of a firm to sell its products or services at a low price, intending to drive rivals out of the market, or to create barriers to entry for potential rivals. Nowadays, pricing below marginal cost is regarded as predatory and it is banned by antitrust law.

We show that an incumbent who has a safe monopoly power in a market can deter entry into a complementary market without tying the two products. By lowering the price (below the rival's cost) in the (potentially) competitive complementary market, a more flexible tool, the incumbent can always deter entry and maintain its monopoly power in both markets. The idea is that a firm which is on the outside and intends to enter the complementary market faces a credible threat by the incumbent of the complemented product to lower the price below its cost. This threat of predatory pricing is credible because if the transaction volumes in the two markets are interdependent, the incumbent is able to reduce the price in the complementary market and to raise the price by the same amount in the complemented market. A multi-market structure gives the incumbent the power to transfer its profit from one market to another. Pricing below the rival's cost plays two roles: it creates new demand on the complemented good market, and squeezes the single-product entrant out of the complementary market. Unlike the case of predatory pricing in a market without complementary products, in the multi-market case entry can be deterred at an infinitesimal cost. Firm can simply undercuts its price in one market and raise its price in another market. Losses in one market then can be recouped in the other market. The incumbent firm needs not have either a cost advantage or a quality advantage, so long as (1) it has market power in one of the markets and (2) the transaction volumes in the two markets are positively correlated. Predatory pricing can be used by a firm to extend its monopoly power in one market into another competitive market.

Interestingly, accommodation can also be the optimal strategy; incumbents occasionally give up their monopoly power in one market and allow new firms to enter. In a multi-market setting, demand in one market depends on the demand in the other market and hence the quality of its complementary product. An incumbent would like to have the entrant firm produce a product which could provide strong support to the complemented market. Whether the incumbent deters or accommodates entry depends on the difference between the gain from monopolizing both markets and the gain from having the entrant as its alliance partner in one of the markets. Thus, the threat of predatory pricing is only carried out when the incumbent is unable to take advantage from the entrant. We find that if (1) the entrant's marginal cost is sufficiently low or (2) the entrant produces a sufficiently high quality product, then an incumbent accommodates entry.

We regard our results as helping to explain why in some cases low-cost firms stay out of the market in the absence of entry barrier. For example, Epson exited the market of digital range finder cameras while Leica, the leading lens maker of range finder camera, launched its own digital range finder camera afterwards. Our paper also sheds light on why Apple allows firms to develop iPhone applications which compete with its own.

While predatory pricing has always been viewed as welfare decreasing, it is not necessarily so with multi-market contact. On the negative side, predatory pricing deters competitive pricing and allows the incumbent to squeeze potential entrants out. As a result, there is a risk that the firm in the market produces at a higher cost than the potential entrants. Raising the price in the monopoly market after predatory pricing has no benefit to the consumers. On the positive side, the threat of predatory pricing could save the potential entrant's entry costs (e.g., in R&D effort), and it could stop a low cost entrant from squeezing a high-quality incumbent out of the market.

Our paper relates to a growing body of literature studying entry deterrence with multi-market contact. Of the previous work, Farrell and Katz (2000) and Kovac (2005) are closest to ours. Farrell and Katz (2000) study the innovation incentive facing an incumbent who wants to integrate the supply of the complementary product. They show that given its monopoly power in the provision of one component, the incumbent can always force its rival to charge a lower price. Hence, integration can inefficiently reduce the R&D incentive when an incumbent extracts rent from the independent firm. We extend their work by allowing consumers to have heterogeneous preferences for quality. Unlike Farrell and Katz (2000), our model focus on entry deterrence, rather than on extending monopoly power in the complementary market. We demonstrate that even with the ability to control the market of the complementary product, an incumbent may accommodate a strong entrant. Kovac (2005) constructs a model of bundling with entry deterrence. In his model, a multiproduct firm in one market bundle its products in order to prevent the inferior entrant from entering the complementary market. The paper also demonstrates that monopoly power in the first market is not required. Our paper differs in two fundamental ways. First, rather than bundling as an entry tool, we illustrate the role of predatory pricing as an effective threat. Second, we show that an inferior incumbent also has the ability to defend its monopoly position.

Another line of related research considers bundling as a deterrence tool with multi-market contact. Choi and Stefanadis (2001) and (2006) find that bundling can distort the specialization decision of entrants and reduce the entrants' incentive to invest in R&D. Salop and Scheffman (1983), and Krattenmaker and Salop (1986) demonstrate that raising the rival's cost increases the predator's profit and it can be a very effective entry barrier. Nalebuff (2004) shows that bundling in a two-sided market serves as an entry deterrence strategy as well as an effective tool for price discrimination. Carlton and Waldman (2002) show that bundling can be used to deter future entry in a dynamic model.

Finally, our paper connects with the antitrust literature. Areeda and Turner (1975) suggest that pricing below cost can be an abuse of monopoly power when an incumbent produces multiple products and advocate banning pricing below cost. Edlin (2002) further argues that pricing above cost can also be a case of predatory pricing. Evans and Noel (2005) find that pricing below cost is endemic in the video game, PC software and payment card markets, and it is not necessarily an indicator of predatory pricing with two-sided markets. Evans and Schmalensee (2005) argue that price equal to marginal cost is not the appropriate standard with multi-market contact. In this paper, we look formally at the case where predatory pricing results in a welfare loss in one market and a welfare gain in another market. Our analysis delineates when the gain is larger than the loss, and thus when predatory pricing should be allowed.

The paper is organized as follows. Section 2.2 develops the model. Section 2.3 solves the pricing subgame with multi-market contact by assuming that incumbent and entrant set the product prices simultaneously. Equilibrium is defined for three cases: (1) when the incumbent has a cost disadvantage, but a quality advantage; (2) when the incumbent has a disadvantage both in cost and quality and (3) when the incumbent has a cost disadvantage only. Section 2.4 studies the entrant's entry choice in response to the threat of predatory pricing. Section 2.5 examines antitrust policy and the welfare effects of predatory pricing. Section 2.6 concludes. Proofs are relegated to the appendix.

2.2 The Model

There are two firms $(i \in \{I, E\})$ and two product markets $(j \in \{A, B\})$. Firm I is the uncontested monopolist in market B. Initially, firm I is also an incumbent monopolist in market A and firm E is a potential entrant of market A. The two products are perfect complements. One can think of the camera body as the product in market A and the lens as the product in market B. Consumers only receive utility from consuming both products in a fixed proportion and thus there is no stand-alone value to the product in any market. Here we assume the products are consumed in a one-to-one relationship and each consumer buys at most one unit of product from each market. To simplify the exposition, assume the utility of the consumers solely depends on the quality of the product in market A. The quality of product A of firm I and of the other firm (firm E) can take two values, $q^i \in \{q_L, q_H\}$ with $q_H > q_L$.

Firm I's quality level is exogenous and publicly known. If entry takes place, then firms compete in quality as well as price.

We capture the incumbent's advantage by postulating that the potential entrant faces a fixed development cost F_E. Even though in some cases a license is essential (for example, a license has to be granted to create games for Nintendo systems), here we assume that no license is required for firm E to introduce a product in market A, and firm E is free to enter the market. Firm i has a constant marginal cost of production c_i in market A, with $q^i \geq c_i \geq 0$.

In this paper, we are particularly interested in understanding how does the nature of multi-market give the right to the incumbent to protects its market even with a cost disadvantage. Therefore, we assume the entrant, firm E has a lower marginal cost of production $(c_I > c_E)$. To focus on the entry choice in market A, and without loss of generality, we assume that firm I's marginal cost of production in market B is zero. All costs are publicly known. Equilibrium entry requires firm E to enter if and only if its expected profit exceeds the entry cost.

The timing of the game as follows: First, firm E decides whether to enter market A and then nature determines firm E's quality level in market A (q_H with probability γ and q_L with probability $1-\gamma$). Second, if E has entered, firms compete in price in market A and firm I simultaneously sets the price in market B. If firm E stays out, firm I sets prices in both markets.

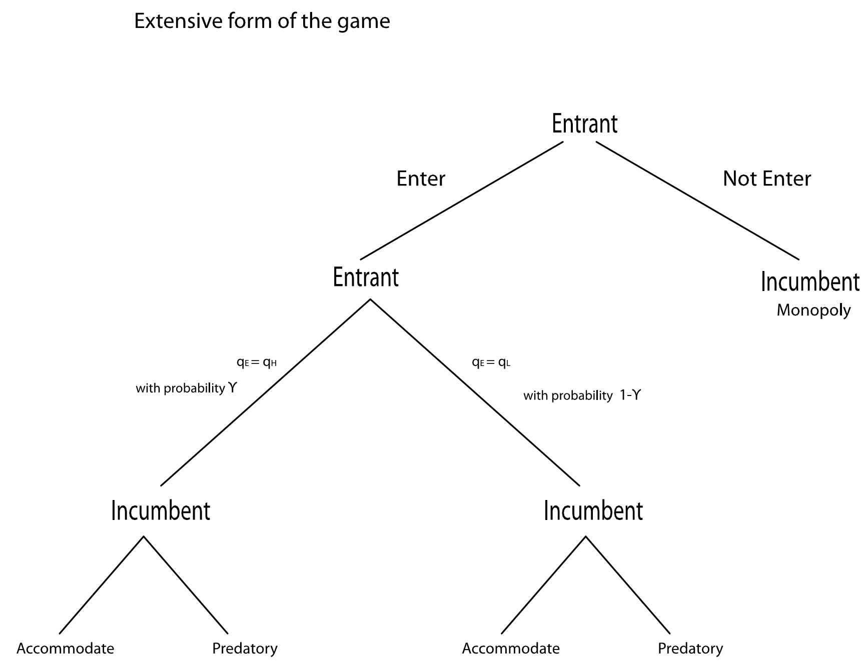

Figure 2.1: Extensive form of the game

2.2.1 The Demand Structure

A continuum of potential consumers is differentiated by a parameter θ which is assumed to be uniformly distributed on the interval $(0, 1]$. The parameter θ can be interpreted as the intensity of preference for quality of the product in market A (or the marginal utility of quality). A type θ consumer decides whether to buy the product from firm I or firm E (if firm E has entered) or not to buy in market A; the consumer also decides whether to buy the product from firm I in market B. Let k_j^θ be the purchase decision of a type θ consumer in market j, $k_A^\theta \in \{I, E, N\}$ and $k_B^\theta \in \{I, N\}$. To capture the complementarity of products in the simplest possible way, we assume that consumers only benefit from consuming both products. The utility U_θ of a consumer type θ is defined as follows:

$$U_\theta = \begin{cases} \theta q_I - p_I - p_B & \text{if } k_A^\theta = I \text{ and } k_B^\theta = I \\ \theta q_E - p_E - p_B & \text{if } k_A^\theta = E \text{ and } k_B^\theta = I \\ -p_B & \text{if } k_A^\theta = N \text{ and } k_B^\theta = I \\ -p_I & \text{if } k_A^\theta = I \text{ and } k_B^\theta = N \\ -p_E & \text{if } k_A^\theta = E \text{ and } k_B^\theta = N \\ 0 & \text{if } k_A^\theta = N \text{ and } k_B^\theta = N \end{cases}$$

where p_i is the price of the product in market A from firm i and p_B is the price of the product in market B. Each consumer buys the product combination which provides him with the highest utility and buys nothing if the utility derived from the products does not cover the total price of the products.

The quality of the product in market A plays a critical role on the demand for the product in market B. The higher the quality of the product, the higher the utility of consumers from consuming both products. Quality in market A positively influences the consumers' willingness to pay for the product in market B. Thus, both firms benefit from a high quality product in market A. If firm I is the low-quality firm and it can always sell its product at a very low price (or negative price) in market A, then firm B may be deterred from entering and introducing the high quality. In such a case firm I will extract profit solely from market B. We will allow firm I to sell the market A product at a negative price. This may be profitable because the incumbent may recoup the loss with a higher price in market B. Coupons and market B discounts after a purchase in market A are common examples of zero or negative prices that are observed in practice.

Let $\theta_{N_{IE}}$ be the consumer that, after buying in market B, is indifferent between buying from I and E in market A. It must be

$$\theta_{N_{IE}} q_I - p_I - p_B = \theta_{N_{IE}} q_E - p_E - p_B$$

and hence

$$\theta_{N_{IE}} = \frac{p_I - p_E}{q_I - q_E}.$$

Let θ_{N_I} be the consumer that is indifferent between buying from I in both markets and not buying any product. It is

$$\theta_{N_I} = \frac{p_I + p_B}{q_I}.$$

Let θ_{N_E} be the consumer that is indifferent between not buying any product and buying from E in market A (and I in market B). It is

$$\theta_{NE} = \frac{p_E + p_B}{q_E}.$$

Thus, if $q_I > q_E$, all consumers of type $\theta > \max\{\theta_{N_{IE}}, \theta_{N_I}\}$ buy both products from firm I, while all consumers of types $\theta < \theta_{N_{IE}}$ and $\theta > \theta_{N_E}$ buy from firm E in market A. Thus, the demand for good B is $Q_B = \max\{0, 1 - \max\{\theta_{N_{IE}}, \theta_{N_I}\}\} + \max\{0, \theta_{N_{IE}} - \theta_{NE}\}$.

If, on the other hand, $q_I < q_E$ then all consumer types $\theta < \theta_{N_{IE}}$ and $\theta > \theta_{N_I}$ buy both products from firm I, while all consumers $\theta > \max\{\theta_{N_{IE}}, \theta_{N_E}\}$ buy from firm E in market A. Thus, the demand for good B is $Q_B = \max\{0, \theta_{N_{IE}} - \theta_{N_I}\} +$

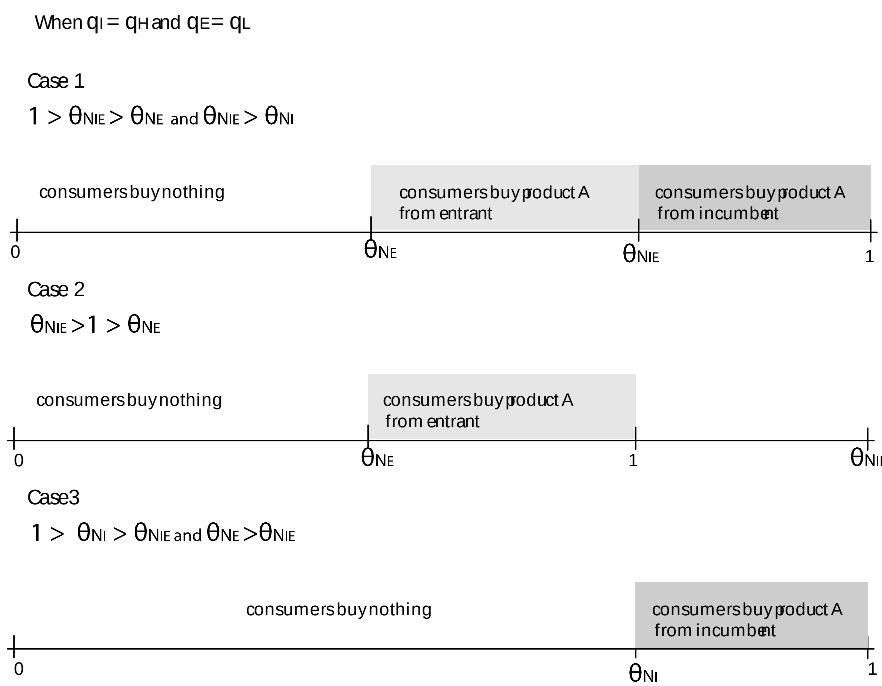

Figure 2.2: All possible demand configurations of good B when $q_I = q_H$ and $q_E = q_L$.

$\max\{0, 1 - \max\{\theta_{N_{IE}}, \theta_{NE}\}\}$.[10] The following graphs show all the possible demand configurations for good B.

We solve the game by backward induction, considering first the equilibrium in the pricing game and then the entry decision of firm E.

2.3 Equilibrium in the Pricing Subgame

In this section, we consider the case in which firm E has entered market A. There are three possible cases: (1) the incumbent offers a high-quality product and the entrant enters with low-quality, i.e. $q_I = q_H > q_E = q_L$, (2) the incumbent offers a low-

[10] $\theta_{N_{IE}} > \theta_{N_E}$ implies $\theta_{N_{IE}} > \theta_{N_I}$

quality product and the entrant enters with high-quality, i.e. $q_I = q_L < q_E = q_H$ and (3) the incumbent and the entrant offer the same quality, i.e. $q_I = q_E = q_L$ or $q_I = q_E = q_H$. In the following subgame, there exists an equilibrium which firm I always undercuts the price, however, both firm I and firm E receive a higher profit if firm I agrees to leave the market and firm E agrees to take the market when the the marginal cost of firm I is above the threshold values.[11] For the analysis that follows, we assume that if there are two possible equilibria, the Pareto dominant equilibrium prevails. In this equilibrium, firm I does not always price below the entrant's marginal cost.[12]

We begin by considering the first case, $q_I = q_H > q_E = q_L$. There are two different types of subgame perfect equilibria. In the first type of equilibrium, the entrant does not sell (i.e., it sells a zero quantity) in market A, because the incumbent's strategy is to engage in a price war after entry. In the second type of equilibrium, the incumbent accommodates entry; the incumbent gives up market A and the entrant supplies the entire market. This second type of equilibrium only exists if the entrant has a cost advantage in market A.

Before formalizing this result in the next proposition, define:

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11 $\pi_I^*(p_I^*,\, p_E^*,\, p_B^*) = \frac{(q_E - c_E)^2}{9q_E} \geq \pi_I^*(p_I^{**}, p_E^{**}, p_B^{**}) = \frac{(q_I - c_I)^2}{4q_I}$ when c_I is above the threshold values (c_1, c_2, c_3).

12 On the other hand, if the Pareto-inferior equilibrium is always played in the pricing game, then the incumbent always prices below the entrant's marginal cost and takes over the whole market, regardless of the entrant's cost level and quality type. In this case, it is optimal for the entrant to stay out of the market.

$$c_1 = q_H - \frac{2}{3}(q_L - c_E)\sqrt{\frac{q_H}{q_L}}.$$

Proposition 5 *Suppose firm E has entered market A and $q_I = q_H > q_E = q_L$. There are two possibilities: (1) If $c_I \leq c_1$, there exists a subgame perfect equilibrium outcome of the pricing game in which firm I prices below firm E's marginal cost in market A (predatory pricing equilibrium), (2) If $c_I > c_1$, then there exist a subgame perfect equilibrium outcome in which firm I sells zero quantity in market A. (accommodation equilibrium)*

Proof. See the Appendix.

A few remarks are worth making here. First, whether the incumbent engages in a price war after entry depends on the production costs of incumbent and entrant. There is a threshold level of the incumbent marginal cost, c_1, above which the incumbent accommodates and below which it deters entry.

Second, as the entrant's marginal cost c_E gets smaller, the needed threshold value of c_I gets smaller. When $q_I > q_E$, giving up market A is profitable only when the entrant produces at a sufficiently lower cost than the incumbent. In such a case, the low cost entrant with a low quality product is able to sell to a larger number of consumers than the high cost incumbent with a high quality product. By leaving market A to the entrant, the incumbent is able to induce more consumers to get on board and buy its product in market B and hence the profit of the incumbent in that market increases. On the contrary, predatory pricing is very costly to the incumbent when the entrant's cost is very low. Returning to the camera example, the incumbent who has introduced a camera body with a higher image resolution and more features is willing to exit the camera body market only if the entrant sells the camera body at a much lower price. The camera body with more features reaches fewer consumers, as some features of the camera body are not of much value to the general public and are only useful to the professional photographers. Many consumers do not value the quality difference much. If the incumbent has a high production cost for the extra camera body features, then it would find profitable to lock-in more consumers to its camera lens by letting the entrant sell a cheaper camera body with fewer features.

Third, for $c_E = 0$ there exist cost levels of the incumbent ($c_I \leq q_H - \frac{2}{3}\sqrt{q_H q_L}$) for which I deters entry of the potential entrant. In contrast, for $c_I = 0$ firm I always deter entry and the second type of equilibrium never exists. Therefore, market power in market B may prevent some lower cost entrant from entering market A.

Now we turn to the case where the entrant has a quality advantage over the product provided by the incumbent. Suppose the entrant has developed breakthrough advances in the existing imaging technology and is able to produce a camera body with advanced image technologies that make a difference to consumers. Consumers are now willing to spend more on their camera (both the lens and the camera body) with the latest development. This quality advantage increases the chances that the entrant will enter market A. Before presenting the proposition with this result, define c_2 as

$$c_2 = q_L - \frac{2\left(q_H - c_E\right)}{\left(3q_H - q_L\right)} q_L.$$

Proposition 6 *Suppose firm E has entered market A. Let $q_I = q_L < q_E = q_H$. There are two possibilities: (1) If $c_I \leq c_2$, there exists a subgame perfect equilibrium outcome of the pricing game in which firm I shares market A with firm E (predatory pricing and sharing equilibrium), (2) If $c_I > c_2$, then there exists a subgame perfect equilibrium outcome in which firm I sells zero quantity in market A. (accommodation equilibrium)*

Proof. See the Appendix.

The cost advantage gives the entrant the ability to undercut the incumbent and attract even those consumers who have little value for the quality differentials of the products. The quality advantage further strengthens the competitiveness of the entrant by increasing the consumers willingness to pay for the products in both markets. This allows the incumbent to raise the price in market B. Therefore, in order to lock more consumers in the market and generate higher profits, the incumbent always has an incentive to accommodate entry. When the incumbent has a sufficiently low marginal cost of production $c_I \leq c_2$, it has an incentive to share the market with the entrant. The incumbent with a low quality product enlarges its consumer base in market B by serving the consumers in market A who do not value quality much, while the entrant with a high quality product increase the consumers' willingness to pay on the other product. Keeping a positive market share becomes costly to the incumbent if $c_I > c_2$. In this case, letting the entrant serve the entire market is preferable. When the incumbent's cost increases, the loss from sharing the market by selling the product below cost (predatory pricing) will be higher and outweighs the gain from a larger consumer base. Referring again to our example, the release of a new generation camera body by the entrant prompts consumers to set aside more of their money for the camera body and lens. It provides room for the lens price to go higher and hence generates more profit for the incumbent through greater lens sales. If the incumbent has a sufficiently low marginal cost, then it will reach more consumers by producing the old generation camera body and sharing the market with the entrant. Otherwise, the incumbent will leave the camera body market to the entrant. Either way, a low cost entrant with higher technology is always able to enter and is viewed by the incumbent as a partner.

Note that if the entrant has zero marginal cost, $c_E = 0$ then $c_2 = (q_H - q_L)q_L/(3q_H - q_L)$; thus, there exist some low cost incumbents that share the market with the entrant. In other words, an entrant with a quality advantage and a zero production cost is not able to take over the entire complementary market if the incumbent's cost is low.

We now consider the case in which the two firms have the same quality level $q \in \{q_H, q_L\}$, that is the entrant does not add any new feature to the camera body or the new features are not compelling and do not add any value to the existing camera body. Firms then purely compete in price and cost determines firms' survival. The following threshold value of c_I will be used in the next proposition:

$$
\begin{aligned}
c_3(q) &= \frac{q+2c_E}{3} \\
&= q - \frac{2}{3}(q - c_E).
\end{aligned}
$$

Proposition 7 *Suppose firm E has entered market A. Let $q_I = q_E = q \in \{q_H, q_L\}$. (1) If $c_I \leq c_3(q)$, or equivalently $q \geq 3c_I - 2c_E$, there exists a subgame perfect equilibrium outcome of the pricing game in which firm I sets its price below firm E's marginal cost in market A (predatory pricing equilibrium), (2) If $c_I > c_3(q)$, or equivalently $q < 3c_I - 2c_E$, then there exists a subgame perfect equilibrium outcome in which firm I sells zero quantity in market A. (accommodation equilibrium)*

Proof. See the Appendix.

When incumbent and entrant have the same quality, if the quality level is sufficiently high (above $q^* = 3c_I - 2c_E$), then in the equilibrium of the pricing subgame the entrant does not sell and the incumbent sells to the entire market. On the other hand, if the quality level is below q^*, then the incumbent gives up market A and prefers not to produce in that market. The cost of taking over market A by setting p_I lower than p_E while keeping $p_I + p_B$ optimal is high when $p_E < c_I$. In such a case, it is profitable to let the entrant to serve the whole complementary market rather than to suffer a loss from serving the market by itself. This can only occur if the entrant has a sufficiently large cost advantage ($3c_I \geq 2c_E$). The incumbent free rides on the low cost entrant; the entrant charges a price lower than the incumbent's marginal cost and as a result the incumbent enjoys a larger base of consumers in market B. Note that the range of cost parameters under which the incumbent accommodates entry is larger if both firms have a low rather than a high-quality product. This can be illustrated with the telecommunication industry in the UK. Mobile phone are mainly sold by the telecommunication companies at a low price even without bundling the mobile phone with the telecommunication service. The telecommunication companies have an incentive to lower the price of the mobile phone and induce more consumers to pay for telecommunication services. Such predatory pricing also leads the mobile phone manufacturers to sell the mobile phones to the telecommunication companies rather than selling them directly to the customers. (Phones on the Nokia online website sold at a price two to three times higher than the price at the telecommunication stores.) The following graphs show the timeline of the game and the result.

Table 1 shows the equilibria in each pricing subgame.

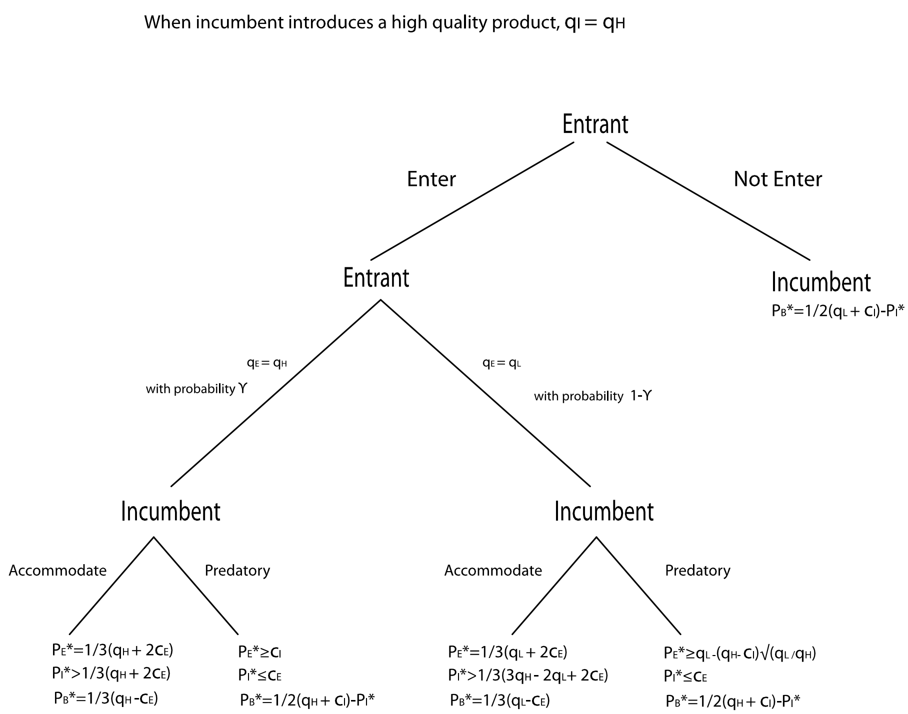

Figure 2.3: Equilibria in the game when $q_I = q_H$.

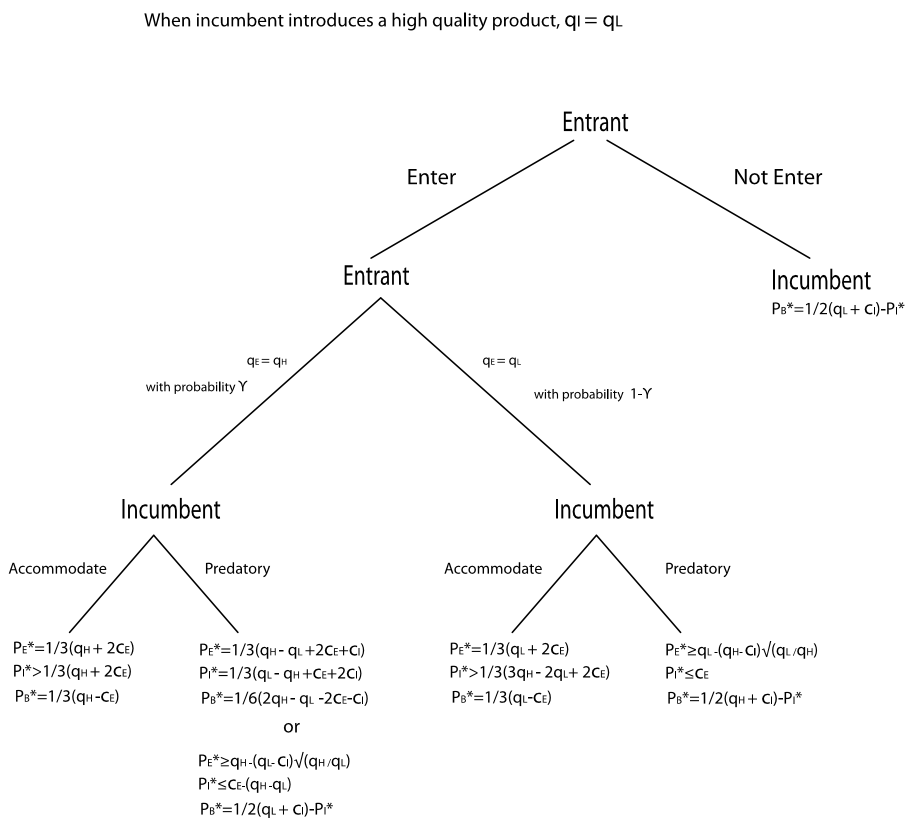

Figure 2.4: Equilibria in the game when $q_I = q_L$.

Equilibrium in the Pricing Subgame			
q_I	q_H	q_L	$q \in \{q_H, q_L\}$
q_E	q_L	q_H	$q \in \{q_H, q_L\}$
Predatory pricing equilibrium $\left(p_I^* < c_I\right)$			
Condition	Always hold		
p_E^*	$\geq q_L - (q_H - c_I)\sqrt{\frac{q_L}{q_H}}$	$\geq q_H - (q_L - c_I)\sqrt{\frac{q_H}{q_L}}$	$\geq c_I$
p_I^*	$\leq c_E$	$\leq c_E - (q_H - q_L)$	$\leq c_E$
p_B^*	$\frac{(q_H + c_I)}{2} - p_I^*$	$\frac{(q_L + c_I)}{2} - p_I^*$	$\frac{(q + c_I)}{2} - p_I^*$
Q_E^*	0	0	0
Q_I^*	$\frac{(q_H - c_I)}{2q_H}$	$\frac{(q_L - c_I)}{2q_L}$	$\frac{(q - c_I)}{2q}$
Q_B^*	$\frac{(q_H - c_I)}{2q_H}$	$\frac{(q_L - c_I)}{2q_L}$	$\frac{(q - c_I)}{2q}$
Predatory pricing and sharing equilibrium			
Condition		$c_I > q_L - \frac{2(q_H - c_E)}{(3q_H - q_L)} q_L$	
p_E^*		$\frac{(q_H - q_L + 2c_E + c_I)}{3}$	
p_I^*		$\frac{(q_L - q_H + c_E + 2c_I)}{3}$	
p_B^*		$\frac{(2q_H + q_L - 2c_E - c_I)}{6}$	
Q_E^*		$\frac{(q_H - q_L + c_I - c_E)}{3(q_H - q_L)}$	
Q_I^*		$\frac{[(q_H - q_L)(q_L - c_I) - 2(c_I q_H - c_E q_L)]}{6q_L(q_H - q_L)}$	
		$\frac{(q_L - c_I)}{2q_L}$	
Accommodation equilibrium			
Condition	$c_I \leq q_H - \frac{2(q_L - c_E)}{3}\sqrt{\frac{q_H}{q_L}}$	$c_I \leq q_L - \frac{2(q_H - c_E)}{(3q_H - q_L)} q_L$	$c_I \geq \frac{q + 2c_E}{3}$
p_E^*	$\frac{(q_L + 2c_E)}{3}$	$\frac{(q_H + 2c_E)}{3}$	$\frac{(q + 2c_E)}{3}$
p_I^*	$> \frac{(3q_H - 2q_L + 2c_E)}{3}$	$> \frac{(q_H + 2c_E)}{3}$	$> \frac{(q + 2c_E)}{3}$
p_B^*	$\frac{(q_L - c_E)}{3}$	$\frac{(q_H - c_E)}{3}$	$\frac{(q - c_E)}{3}$
Q_E^*	$\frac{(q_L - c_E)}{3q_L}$	$\frac{(q_H - c_E)}{3q_H}$	$\frac{(q - c_E)}{3q}$
Q_I^*	0	0	0
Q_B^*	$\frac{(q_L - c_E)}{3q_L}$	$\frac{(q_H - c_E)}{3q_H}$	$\frac{(q - c_E)}{3q}$

Table2.1: Equilibria in the pricing subgame.

The cost and quality differences between firms determine whether it is optimal for the incumbent to engage in predatory pricing. When the potential entrant is able to improve significantly on the incumbent's complementary product, either with a higher quality, or a lower cost product, the incumbent will profit from allowing entry.

By accommodating entry and forcing the entrant to sell at a low price in the complementary product market, the incumbent reaps the benefit in the complemented good market, where it remains a monopolist, by charging a higher price. Charging a higher price is possible because of the increase in demand that follows from the technology improvement introduced by the entrant. However, the fact that the entrant has a lower production cost or introduces a higher quality does not automatically imply that it is optimal for the incumbent to accommodate entry. Accommodation is profitable to an incumbent only if the gain from the demand increase in the complemented good market outweighs the loss from leaving all or part of the complemented good market to the entrant.

2.4 Market Entry

In this section, we analyze the entrant's incentive to enter market A. Firm E earns a positive profit in market A when firm I doesn't undercut its price after entry; there are three different cases when this happens. If it produces high quality, firm E obtains a payoff $\pi_{E_M}(q_H)$ if firm I does not sell in market A and it obtains a payoff $\pi_{E_D}(q_H)$ if it shares the market with firm I. When firm E has a low quality product, then it obtains a payoff $\pi_E(q_L)$ when firm I does not produce in market A. On the other hand, if firm I plays the predatory pricing equilibrium, firm E receives a zero payoff.

$$\pi_{E_M}(q_H) = \frac{(q_H - c_E)^2}{9q_H}.$$

$$\pi_{E_D}(q_H) = \frac{(q_H - q_L + c_I - c_E)^2}{9\,(q_H - q_L)}.$$
$$\pi_E(q_L) = \frac{(q_L - c_E)^2}{9 q_L}.$$

The next proposition describe the equilibrium in the subgame after firm E has entered market A. Define the following values. The following threshold values represent the minimum probability of being a high-quality firm required for firm E to earn a non negative expected profit regarding different cost levels.

$$\gamma^* = \frac{F_E}{\pi_{E_M}(q_H)}$$

$$\gamma^{**} = \frac{F_E}{\pi_{E_D}(q_H)}$$

$$\gamma^{***} = \frac{F_E - \pi_E(q_L)}{\pi_{E_M}(q_H) - \pi_E(q_L)}$$

Proposition 8 *Suppose predatory pricing (i.e., price below marginal cost) is possible. There exists a subgame perfect equilibrium outcome of the entry game in which firm E enters market A and firm I introduces a high quality product in the following scenarios: (i) When firm I has a marginal cost $c_I \in [c_3(q_H), c_1]$, if $\gamma \geq \gamma^*$; and (ii) when firm I has a marginal cost $c_I > c_1$, if $\gamma \geq \gamma^{***}$. There exists a subgame perfect equilibrium outcome of the entry game in which firm E enters market A and firm I introduces a low quality product in the following scenarios: (i) When firm I has a marginal cost $c_I \in [c_2, c_3(q_L)]$, if $\gamma \geq \gamma^{**}$; and (ii) when firm I has a marginal cost $c_I > c_3(q_L)$, if $\gamma \geq \gamma^{***}$.*

Proof. See the Appendix.

When the incumbent is a high-quality firm with a low cost (below c_2), it finds it profitable to charge a low price for the complementary product in market A. (The incumbent may even offer the product for free, or give a cash rebate to the consumers when they buy the complemented product in market B.) This attracts the largest consumer base in market B (the complemented product market) and since the incumbent is a monopolist in market B, it can extract consumers' surplus from that market. The entrant stays out of market A in this case. As the production cost of the incumbent increases, predatory pricing becomes more costly and the entrant becomes more helpful in boosting the demand in market B by pricing at a relatively low price in market A. Therefore, the entrant is more likely to enter market A. When the incumbent is a low-quality firm, the probability thresholds are different and the entrant is more likely to enter market A, but the general interpretation of the results is similar.

In this paper, we have assumed that the quality level of the entrant is exogenous and independent of R&D (the fixed cost in our model). However, if the strategy of improving the quality level by investing on R&D were available, the entrant would have an incentive to increase its expenditure on R&D. Therefore, the threat of predatory pricing would play two roles, it would drive a relatively high cost entrant out of the market and hence would encourage the entrant to increase the expenditure on R&D, in an attempt to build competitive and develop better quality product.

We have shown how a firm may protect its position in a complementary market by leveraging its monopoly power in the complemented market. Intuitively, a firm can also profitably use predatory pricing to extend its monopoly power from one market to another, provided the following three conditions hold: (1) the two markets are complementary; (2) the firm is in a safe market and (3) it can freely enter another market. The logic is straightforward: the firm can sell its product below cost and drive the incumbent out of the market, while increasing its profit in the safe market because of an increased demand.

2.5 Welfare Implications

So far, our analysis has focused on the impact of predatory pricing on the entrant's quality choice and on entry deterrence. Now we would like to answer our last question: If predatory pricing erects an entry barrier, should the antitrust authority prohibit it? Under which circumstances is charging a price below marginal cost welfare reducing?

Predatory pricing has an effect on welfare if and only if entry is deterred by the incumbent. Recall that the incumbent only deters entry when its cost is sufficiently low. Therefore, in this section, we only consider the case when the fixed entry cost is sufficiently small ($F_E < \pi_{E_D}(q_H)$, so that $\gamma > \gamma^{**}$) and the incumbent is not willing to leave the entire market to the entrant if predatory pricing is possible (i.e., the case when $c_I < c_3(q_L)$ if the incumbent's product is of low quality, and the case when $c_I < c_1$ if the incumbent's product is of high quality).

Define the following values.

$$c_I^*(q) = q - \frac{2\sqrt{5}}{3\sqrt{3}}\left(q_L - c_E\right)\sqrt{\frac{q_H}{q_L}} < c_1$$

$$c_I^{**}(q) = q - 2\sqrt{\frac{2q}{3}\left[\gamma\frac{5(q_H \ - c_E)^2}{18q_H} + (1-\gamma)\frac{5(q_L \ - c_E)^2}{18q_L} - F_E\right]}$$

Proposition 9 *If firm I introduces a high quality product, predatory pricing is socially optimal and $W_{pre} > W_{ban}$ in the following scenarios: (i) When firm I has a marginal cost, $c_I \in [c_3(q_H), c_I^*(q_H)]$; and (ii) when firm I has a marginal cost, $c_I \leq \min[c_I^{**}(q_H), c_3(q_H)]$. If firm I introduces a low quality product, predatory pricing is socially optimal and $W_{pre} > W_{ban}$ in the following scenarios: (i) When firm I has a marginal cost, $c_I \in [c_2, c_I^*(q_L)]$; and (ii) when firm I has a marginal cost, $c_I \leq \min[c_I^{**}(q_L), c_2]$.*

Proof. See the Appendix.

In the single market case, an incumbent may set low prices today and sacrifice current profit in order to gain a monopoly profit in the future. This practice improves consumer welfare in the short run, but in the future, supra-competitive pricing reduces welfare. Determining the welfare effect of predatory pricing is more complicated in the context of multi-markets. With uncontested market power in a safe market, the incumbent can always sacrifice profit in the complementary competitive market by pricing below cost, while raising price and profit in the safe market. Pricing low effectively serves as an entry barrier to the potential entrant. Such a practice works only if the markets are (perfect) complementary. Price cutting in the competitive market raises consumer welfare, since the incumbent does not raise the price in the safe market raise by the same amount as the price cut in the competitive market. However, this is not the end of the story. The objective of the price cutting is to deter a potential entrant from entering the competitive market. If some very low cost potential entrants are deterred by the incumbent through price cutting, consumer's welfare may be lower when pricing below marginal cost by the incumbent is allowed. Then, this type of price cutting can be regarded as "predatory" and should be banned by law.

If pricing below cost is prohibited, leaving the market is the only option for an incumbent that faces a lower cost entrant. When products are complementary and consumers value quality differently, production cost will be the sole factor deciding who stays in the market. A high quality incumbent may be driven out of the market for the complementary product by a low quality entrant. Moreover, the entrant has to pay a fixed R&D cost of entry. Thus, if the cost of production of the incumbent is only marginally higher than the cost of the entrant, a law banning pricing below marginal cost may reduce welfare. In such an instance, the antitrust authority should not intervene in the market; pricing below marginal cost should be permitted.

2.6 Concluding Remarks

Most of the literature on entry deterrence with multi-market contact has focused on bundling. On the contrary, we have argued that an incumbent with multi-market contact can successfully deter potential entry without bundling its products by using predatory pricing. We have shown that entry can be deterred at essentially zero cost in a complementary market when the incumbent is a monopolist in another "complemented" product. The incumbent needs not to have either a cost advantage or a quality advantage in the complementary market. This implies that a low cost entrant may be deterred by a weak incumbent, resulting in a welfare loss. This result explains why strong entrant firms may be unable to enter some markets, like Epson in the rangefinder market. We suggest that the antitrust authority should prohibit such predatory pricing.

However, our model also finds that a law always banning predatory pricing with multi-market contact is not socially optimal. When markets are complementary and pricing below cost has been prohibited by antitrust law, cost becomes the only factor to determine the fate of an entrant. A cost advantage gives the entrant enough power to drive the incumbent out of the market regardless of the incumbent's quality level. Taking quality into account, welfare may be lower with a low cost entrant than with a higher cost incumbent who produces a higher quality. Therefore, we suggest that the quality difference between firms should be taken into account when determining if pricing below cost should be deemed "predatory" and hence should be illegal.

Our analysis shows that, because of predatory pricing, firms never coexist when products are vertically differentiated, markets are complementary and the incumbent has a sufficiently high production cost. In future research, it would be interesting to study predatory pricing in a model of horizontally differentiated goods. We believe that predatory pricing would be less effective. An entrant can do better, and hence enter more easily, if it can horizontally differentiate its product and attract consumers that have preference for its brand. In our model, nature determines the incumbent and entrant's quality levels. It would be interesting to endogenize quality choice. In order to reduce the probability of being deterred by the incumbent, the entrant has an incentive to spend more money on R&D and improve its product quality. The higher the quality the incumbent produces, the higher the entrant's incentive to invest. It is then likely that the incumbent will produce a lower quality product than the entrant. Finally, future research could also attempt to model how predatory pricing affects the incentive to merge.

Chapter 3
From Nintendo to Facebook: Two-Sided Markets with Exclusive Contracts

3.1 Introduction

In many instances, two or more markets are closely interrelated. For example, a person will not sign a mobile phone contract if he does not have a mobile and children will not persuade their parents to buy them a Nintendo Wii if they find that the game titles are not attractive. In these cases, the good in the primary market has to be consumed with another complementary good and hence the growth of the primary market depends on the price for that complementary good. Some markets, like high-technology markets, have even stronger connections. The growth of the primary market depends on the size of the complementary market. In the case of a mobile phone, how much people are willing to pay for the mobile contract does not only depend on the contract itself, but also on the price of the mobile and how many of their friends have a mobile phone. Similarly, if a child's classmates own a Wii and they share game tips and tricks in school, that child's parents are more likely to buy him or her a Wii for their birthday present. A cross-market network effect of this kind exists in both the telecommunication market and the video game market, and these are the classic examples used in the literature on two-sided markets. Generally speaking, a two-sided (or multi-sided) market allows agents to interact through a platform or (several platforms) and each side of the market exhibits both inter-group and intra-group network externalities. The price structure of a two-sided market must be efficient to get both sides “on board”.

Over the past two decades, the video game industry has grown from a niche market to a mass market. In 2007, the worldwide video game industry revenue was estimated to have hit $41.9 billion[13] and it is now poised to overtake the music industry in the US. Recently, the significance of network externalities has been recognized and has given firms an incentive to undercut their console below cost as more players are willing to lock-in to a larger network. The well established title bases and low game console prices are credited with driving the recent dramatic growth in the video game industry. Although the Wii brings Nintendo a profit per sale of around $40 each, both Sony and Microsoft made a loss in the sale of their console.[14] Firms then recoup their losses by taking royalties from the third party game developer. Around 7% of the sales price for a third party Wii game is taken as royalties by Nintendo. In 2008, the industry faced a shock from the tremendous growth in online social gaming and we are now starting to see a new trend emerge. Social networking websites like Facebook and MySpace have become a new platform in the game industry. Dif-

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[13] According to PricewaterhouseCoopers’ Global Entertainment and Media Outlook report for 2008 http://www.empowerresearch.com/NewsLetterYears/CE_NL_Final_July2008/Gami ng.htm

[14] Sony priced its playstation 3 at $499 but with an estimated $805.85 production cost and Microsoft priced its Xbox 360 at $399 with an estimated production cost of $552.27.
http://www.pcstats.com/NewsViewArch.cfm?NewsID=48656
http://www.boston.com/business/personaltech/articles/2005/11/25/xbox_not_a_money_maker_yet/

ferent from the traditional video game industry, which charges players for the game console and game titles, consumers are free to subscribe to any social networking website, and they are allowed to play games with friends on the platform. Furthermore, no license is needed to write games (or applications) for the website and game developers are able to enter the market freely without paying the platform owner. This practice helps firms to get players and game developers “on board” easily and thereby creates a larger network.

Different from the traditional game industry, revenues of the platform owners and the developers are then generated from advertisements placed on the platform and in the games. Hence, on the one hand, the game developer works as an ally of the platform. To attract more advertisers to place advertisements on the game, the game developer has an incentive to provide better games to encourage more players to subscribe to the platform and to the game, creating a larger audience for the advertisers. In other words, the quality of the game is determined by the advertising revenue, and it is more indirectly controlled by the players as compared with the case of the video game industry, in which the players directly pay for the game title if they feel like it. On the other hand, differently from the video game industry, the game console owner receives royalties from the game developer, and the platform owner and the game developer compete for advertisements which are their only source of revenue. In this framework, the platform owner has a single strategy for profit maximization: promoting its platform to the public. However, it is common to see the platform owner allowing game developers to develop the same game for another platform. In this paper, I study how this reward system affects the incentive for the platform owner and game developer to engage in exclusive contracts. Usually, when a firm breaks out from a niche market to wider acceptance and then into the mainstream, some firms tend to apply few (if any) restrictions on collaboration in order to encourage support from other firms. Turning to the example of social networking websites, a platform may find it profitable when the developer of a popular game is willing to install the game on the platform if an exclusive contract does not exist. A well known game installed on the platform also reflects the quality or the value of the platform to the consumers. Despite this, a well-established platform like Facebook may find it profitable for a developer to support other platforms. To the developer, serving a single platform and having a smaller network can also be optimal regardless of the cost of serving the platforms. This paper investigates the reasons why this is the case. Although a great deal has been written about the advertising market and the add-on application (complementary) market, until now there has been no discussion in the economics literature of the case of a complementary product competing for advertising revenues. This paper is then, an attempt to fill this gap in the existing literature.

Traditionally, platforms make money from the services or products provided to the agents and they charge agents on both sides. In the case of social networking websites, the story is more complicated. Money is made from the advertisement and a third party product or service brings additional value to the platform. On the minus side, given the substitutability between advertising on the platform and advertising on the add-on application, a strong add-on application may reduce a platform owner's advertising revenue. On the plus side, a well developed add-on helps a platform owner to build up a stronger network for competing with the rival platform. However, a developer enjoys the maximum network size by serving both platforms, while paying the cost of having two equally strong rivals in the advertising market. Our paper shows that the platform owner's decision regarding exclusive dealing and the developer's choice on single homing depends heavily on the homing decision of the consumers, and so reflects consumer choices.

Our paper relates to the literature on the two-sided markers. Armstrong and Wright (2007), Caillaud and Jullien (2003) and Rochet and Tirole (2003) provide a theoretical discussion on the pricing structure of the two-sided markets for different governance structures. Rochet and Tirole (2006) put emphasis on the cases when agents are charged by a mix of membership fees and usage charges. Armstrong (2006) studies the equilibrium prices when one group of agents joins all platforms.

Our work is most closely related to the following papers. Gabszewicz and Wauthy (2004) model the duopoly competition between two platforms. In their model, agents are heterogeneous on both sides of the market and network effects are captured within a vertical differentiation framework. They find the existence of a multi-homing equilibrium that takes place on only one side of the markets. Instead, the network effect is homogenous to all consumers in our paper and consumers have different tastes over the platforms. We also allow consumers to multi-home on only one side, before then considering how this affects the developer's homing decision. Choi (2007) analyzes the effects of tying arrangements on the agents' decision on multi-homing and social welfare. He shows that tying arrangements would encourage more agents to engage in multi-homing and enhance social welfare. Unlike the previous literature, our model focuses on the case in which the platform owner charges one side of the market and the network externalities are only one-sided. Similar to Choi (2007), two complementary products (or two services) are provided to the consumers, however, our paper differs in two ways. First, the complementary service is provided by a third party. Second, besides having consumers subscribe to both platforms, we allow the complementary service provider (developer) to serve both platforms.

Finally, our paper also relates to the literature on advertising in two-sided markets. Anderson and Gabszewicz (2005) study the advertising market with a two-sided markets structure. They use the television market as an illustrative example. In contrast to our paper, for these authors ads are assumed to be a nuisance to viewers and platforms to be competing for advertising revenue. Instead, we argue that in the case of social networking websites, platform owners and developers can easily deliver the advertisement without annoying the consumers. Advertisement is neutral to consumers and we allow a third party to bring value to the platforms and attract advertisers, but also to compete for advertising revenue in the advertising market.

The paper is organized as follows. Section 3.2 develops the model. Sections 3.3 and 3.4 look at the cases of consumer single-homing and multi-homing, respectively. Section 3.5 studies the market equilibrium in the advertising market. Section 3.6 assumes an exclusive contract is possible for platform owners, and considers the conditions under which the developer is willing to accept an exclusive contract. Section 3.7 provides the conclusions of this study; proofs are found in the appendix.

3.2 The Model

Suppose there are three groups of agents: advertisers, consumers and a developer. Different from much of the existing literature which assume each agent of one group only values the number of agents from another group that subscribes to the same platform, here I consider a market where all the agents value the number of agents from their own group and they pay nothing for interacting with them. In the context of the social networking websites example, like a number of two-sided markets, such as telecommunication and the Internet, the utility level of the agent depends on the number of agents in his group. The value of many social games depend on the number of friends you have in the network; the number of friends limits the utility level that the consumers can obtain from the application, as well as the social networking websites. Generally speaking, the more people subscribe to the network, the more likely you are to have your friends in that network, and the more fun you will have in the network and in the game. On the other hand, the advertisements on the websites or in the application are usually delivered in ways that are least annoying to the consumers and consumers can easily skip or ignore the advertisement on the websites unless they find them useful. Here, the nuisance cost of the advertisement is negligible.[15]

There are two symmetric platforms, $i = 1, 2$ and agents interact with each other through the platforms. Consumers can subscribe to either platform 1, platform 2 or both platforms. The timing of the game is as follow: In the first stage, the developer can develop a game for a single platform (single-homing) or for more than one platform (multi-homing). Without loss of generality, we assume that if a developer decides to single-home, they will only develop the application for platform 1. In the second stage, consumers simultaneously decide which platform to subscribe[16]. Note that consumers are free to subscribe to any platform. In the third stage, given the number of consumers on the platforms and in the application, platform owners and developer simultaneously set the prices for advertisers to reach their potential consumers. In the forth stage, after observing the prices and the number of consumers on the platform and in the application, advertisers decide the media for advertising their products, they can place an advertisement either on the platform or in the application. In this model, the developer will only single-home if doing so reduces the competi-

15 Advertisment may carry information or provide value, however, given the reason for consumers to join the social network websites is to keep contact with their friends, we ignore the benefit which the consumer receives from the advertisment.

16 Here, we assume consumers have the choice to decide which platform to join and in the social network case, consumers are free to register an account with different network.

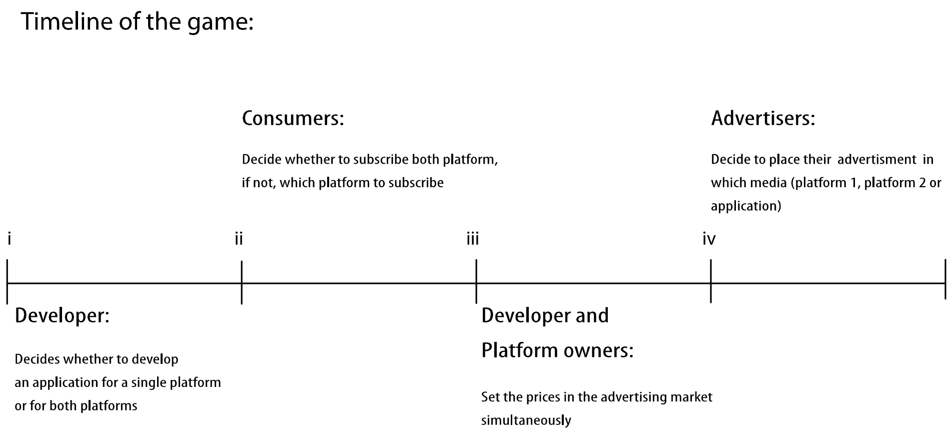

Figure 3.1: Timeline of the game

tiveness of the platforms in the advertising market which generates more profit from advertisers. We solve the model by backward induction.

3.2.1 Consumers

Formally, a continuum of potential consumers is differentiated by a parameter x which is assumed to be uniformly distributed on the interval $[0, 1]$. The parameter x can be interpreted as the intensity of preference for the ideal platform. We make the standard assumption that platforms are located at the extreme points of the interval $[0, 1]$. The consumer incurs a transportation cost $Tx \geq 0$ (disutility cost) from not subscribing to the ideal platform if she subscribes to platform 1 and $T(1-x)$ if she subscribes to platform 2.

The measure of consumers who subscribe to the platform i exclusively is denoted N_i and the numbers of consumers who enjoy the application on platform i exclusively is denoted n_i, while the number who subscribe to both platform (multi-homing) and who also enjoys the single application are denoted N and n respectively. Supposing the application is on both platforms, the utility of a consumer located at x is defined in the following way: if the platform attracts $N_i + N$ consumers and $n_i + n$ consumers enjoy the application on the platform, the utilities of subscribing to platform 1 and 2 are respectively

$$U_1(x) = V + B(N_1 + N) + b(n_1 + n) - Tx,$$

$$U_2(x) = V + B(N_2 + N) + b(n_2 + n) - T(1 - x)$$

where V is the fixed benefit the consumer obtains from subscribing to the platform, B is the network benefit the consumer obtains by subscribing to the platform which allows her to interact with other consumers on the platform and b is the network benefit the consumer obtains by interacting with other consumers in the application. When a consumer subscribes to both platforms, she obtains utility

$$U_{12}(x) = V + B(N_1 + N_2 + N) + b(n_1 + n_2 + n) - T$$

The focus of this paper is the complementary role of the application, assuming that consumers have the same preference for using the application. The proportion of consumers who play the game on the platforms does not change the analysis fundamentally, so for the sake of simplicity, we assume that everyone on the platform enjoys the application; that is, $N_i = n_i$ and $N = n$. Here, I examine the case in which the network effect of the game is limited by platforms. In other words, a consumer who uses the application on one platform does not bring extra utility to the consumer who uses the application on the other platform.

We assume V is large enough so that the market is fully-covered; all consumers wish to subscribe to at least one of the platforms. There exists a unique location at which consumers are indifferent between platform 1 and platform 2 and is given by:

$$x = \frac{1}{2} + \frac{(B+b)(N_1 - N_2)}{2T}$$

Finally, the platform i's profit, Π_i is simply equal to

$$\Pi_i = (P_i - c)\, A_i$$

where c is the costs of providing the service to each advertiser and to each consumer, A_i is the number of advertisements on the platform.

3.2.2 Developer

In this model, we assume that there is a single developer that develops applications for the platforms and that the developer decide whether to develop a game for a single platform (single-homing) or both platform (multi-homing). As we assume everyone on the platform would enjoy the application, it means that if developer multihome, all consumers would enjoy the application and hence advertisers can reach every consumer through the application. Posting an advertisement in the application is more attractive than to post an advertising on any platform, given the same price. The developer attracts consumers to use his applications in order to generate profit by allowing advertisers to post their advertisements in the application. Let a be the number of advertisements in the application, p be the price charged to the advertiser which places an advertisement in the application and c be the cost of providing the service to each advertiser. In this paper, we consider single-homing that arises for network reasons on the consumer side. Hence, the profit function of the developer from developing an application is

$$\pi_1 = (p - c)\, a$$

3.2.3 Advertisers

Each advertiser decides whether to place a single advertisement on the platform or in the application. Here, we simply model the advertising demand as a function of the expected benefit from reaching the consumers. The number of consumers who subscribe to the platform or the application plays a critical role in the demand for advertising. This is logical because the more consumers view the advertisements, the more effective is the advertising. An effective advertising shifts the demand of the product which is advertised and induces potential gain to the advertiser. Advertising fees are also assumed to be fixed and independent from the number of developers and users.

Since consumers are only differentiated by their preference for platforms, targeting advertisement is not possible in our model. Here, the advertiser's preference in advertising media is generated by the nature of the media. For example, an advertisement in the application can be more interactive and it carries more information than advertising on the platform. Different products need different types of advertising method to advertise and different media can reach different target group of consumers. In the real-world, some of the games on facebook give consumer credits to spend in the game if he or she has clicked on the advertisement. If this is the case, consumers can attain a higher utility level from the game after viewing the advertisement and consumers actively choose the level of advertising to maximize their utility level. This is the most common practice in the social networking game and such practice is a means by which developers can differentiate themselves from their competitors (the platforms).

Similar to the consumers, there is a continuum of potential advertisers that is differentiated by a parameter y which is assumed to be uniformly distributed on the unit circle. The parameter y can be interpreted as the intensity of preference for the advertising media or the ideal advertising media. An advertiser located at y decides whether to place their advertisement on the platform or in the application with a transportation cost t (disutility cost) if he does not advertise his product on the ideal advertising media. We denote the application and platforms by L_0, L_1 and L_2 and we make the standard assumption that they are equidistantly distributed on the the circle, i.e. there is a distance $d_L \equiv 1/3$ between any two advertising media. Let l_0, l_1, l_2 be their locations, with $l_0 = 0$ by definition and $l_k = kd_L$. At this point, we are not interested in the differentiation incentives and we take the equidistant location of advertising media as given.

The gross utility of any advertiser placing utility on its ideal advertising media depends on the number of the consumers who have subscribed to the advertising media. Letting N_A be the number of consumers using the application, the net utility of an advertiser located at y on the circle is defined as follows

$$u(y) = \begin{cases} wN_A - t\,|y - l_0| - p & \text{if ad is in the application} \\ w(N_1 + N) - t\,|y - l_1| - P_1 & \text{if ad is on the platform 1} \\ w(N_2 + N) - t\,|y - l_2| - P_2 & \text{if ad is on the platform 2} \\ 0 & \text{if no advertisement is placed} \end{cases}$$

where w is the expected benefit the advertiser obtains from advertising, P_i is the cost of placing an advertisement on the platform i and p is the costs of placing an advertisement in the application. We assume w is large enough that the market is fully-covered, that there exists a unique location at which advertisers are indifferent between L_0 and L_1 and that it is strictly between l_0 and l_1.[17] The location is given by:

$$y_0 = \frac{1}{6} + \frac{(P_1 - p) - w\left[(N_1 + N) - N_A\right]}{2t}$$

Similarly, advertisers indifferent to a choice between L_1 and L_2, L_2 and L_0 are respectively located at:

$$y_1 = \frac{1}{2} + \frac{(P_2 - P_1) - w(N_2 - N_1)}{2t}$$

$$y_2 = \frac{5}{6} + \frac{(p - P_2) - w\left[N_A - (N_2 + N)\right]}{2t}$$

[17] Here, we further assume $w < \frac{5t(T+B+b)}{3T}$, this ensures firms only offers positive price.

Hence, advertising demands are:

$$a = \frac{1}{3} + \frac{(P_1 - 2p + P_2) - w\,[N_1 - 2N_A + N_2 + 2N]}{2t}$$

$$A_1 = \frac{1}{3} + \frac{(P_2 - 2P_1 + p) - w\,[N_2 - 2N_1 - N + N_A]}{2t}$$

$$A_2 = \frac{1}{3} + \frac{(p - 2P_2 + P_1) - w\,[N_1 - 2N_2 - N + N_A]}{2t}$$

Note that when there are no consumers subscribed to the advertising media (the platform or the application), these cannot be located in the circle and hence advertisers never place advertisement on that platform. This makes sense because when no potential consumers view the advertisement, there is no potential gain from placing advertisement. Here, we assume other advertising media do not relocate and remain at the same location. Supposing no consumer is subscribed to platform 2, advertising demand will reduce to

$$a = \frac{1}{2} + \frac{(p - P_1) - w(N_A - N_1 - N)}{t}$$

$$A_1 = \frac{1}{2} + \frac{(P_1 - p) - w(N_1 + N - N_A)}{t}$$

3.3 Consumers single-homing

Before getting to the details of the equilibrium in the advertising market and the discussion on exclusive contracts in later sections, we consider the subscription decisions made by consumers. Similar to Armstrong and Wright (2007), we begin by making the standard assumption that the transportation cost is sufficiently high $(T > B + b)$ throughout this section; in other words, we assume that consumers are picky about platforms. The extra gain from meeting more consumers on a less preferred platform does not give value to the consumers and hence, consumers never have the incentive to multi-home. The following lemma shows that this assumption is a sufficient condition for all consumers to single-home $(N = n = 0)$ regardless of the number of platforms the developer has subscribed to. Although we allowed consumers to multi-home in the model, it is easier to analyze the implications of the developer's subscription decision taking as given the consumers' subscription decision.

Lemma 10 *Suppose $T > B + b$. All consumers single-home regardless of the subscription decision of the developer.*

Proof. Suppose not, there is at least one consumer multi-homing and the consumer who receives the lowest utility from single-homing has the most incentive to multi-home. There are three possibilities: (i) all consumers prefer subscribing to platform 1, (ii) all consumers prefer subscribing to platform 2 and (iii) some low x consumers receive higher utility from subscribing to platform 1 than subscribing to platform 2 and some high x consumers receive higher utility from subscribing to platform 2 than subscribing platform 1.

When the developer develops application to both platforms:

Suppose (i) holds and it follows that $(B + b)(N_1 - N_2) > T$, the consumer who has the most incentive to multi-home is the one who lies at $x = 1$ and the incremental benefit from multi-homing is denoted as $U_{12}(1) - U_1(1) = (B+b)N_2$. However, given $(B+b)(N_1 - N_2) > T$, the incremental benefit is always negative.

A similar proof applies to case (ii), all consumers prefer subscribing to platform 2 if and only if $(B+b)(N_2 - N_1) > T$. Consumer located at $x = 0$ has the most incentive to multi-home and receives an incremental benefit, $U_{12}(0) - U_1(0) = (B+b)N_1$ which is always negative followed by $(B+b)(N_2 - N_1) > T$.

Supposing that (iii) holds and the consumer who has the most incentive to multi-home is the consumer who is indifferent to subscribing to platform 1 and platform 2 $(U_1(x') = U_2(x'))$ and locates at $x^{'}$, where

$$x^{'} = [(B+b)(N_1 - N_2) + T]/2T$$

He or she will receive an incremental benefit $U_{12}(x') - U_1(x')$, where

$$U_{12}(x') - U_1(x') = [(B+b)(N_1 + N_2) - T]/2$$

which is always negative when $T > B + b$ and the lemma also holds.

Now, if the developer only develops an application to platform 1:

Supposing (i) holds and it follows that $B(N_1 - N_2) + b(N_1 + N) > T$, the consumer who has the most incentive to multi-home is the one who lies at $x = 1$ and the incremental benefit from multi-homing is denoted as $U_{12}(1) - U_1(1) = BN_2$ and such benefit is always negative.

Turning now to case (ii), we see that all consumers prefer subscribing platform 2 if and only if $B(N_2 - N_1) - b(N_1 + N) > T$. Consumer located at $x = 0$ has the most incentive to multi-home and receives an incremental benefit, $U_{12}(0) - U_1(0) = v + BN_1 + b(N_1 + N)$ and which is always negative followed by $T > B$ and hence the lemma also holds when $T > B + b$.

Supposing (iii) holds and the consumer who has the most incentive to multi-home is the consumer who is indifferent as to whether to subscribe to platform 1 or platform 2 $(U_1(x'') = U_2(x'')$ and locates at x'', where

$$x'' = [v + B(N_1 - N_2) + b(N_1 + N) + T]/2T$$

He or she receives an incremental benefit $U_{12}(x'') - U_1(x'')$, where

$$U_{12}(x'') - U_1(x'') = [B(N_1 + N_2) + b(N_1 + N) - T]/2$$

which is always negative when $T > B + b$. ■

3.3.1 Developer multi-homing

To find the equilibrium prices in the advertising market, the consistent demand configurations in the consumers market need to be characterized. There are four possible cases to consider: consumers single-home and the developer multi-homes, consumers and developer single-home, consumers and developer multi-home, consumers multi-home and the developer single-homes. We characterize demand configurations in each case. We first consider the case in which the developer subscribes to both platforms (multi-homing). In this setting, the number of consumers who access the application on the platforms is always equal to the mass of consumers $(N_A = 1)$.

When the developer multi-home, platforms are different for consumers and hence the demand configuration depends to a great extent on the transportation cost.

Given that no consumer multi-home, so that $N = n = 0$, if the developer multi-homes, the location of the consumer who is indifferent between the two platforms satisfies

$$V + (B + b)\, N_1 - Tx = V + (B + b)\, N_2 - T(1 - x)$$

where $N_2 = 1 - N_1$. The demand configuration is consistent provided $N_1 = x$ and $T > B + b$. The last inequality always holds by assumption when consumers prefer single-homing on one platform to multi-homing. They split between the two platforms; the equilibrium number of consumers on the platforms are

$$N_1 = \frac{1}{2} = N_2$$

This result is consistent even when consumers are able to interact with other consumers on the other platform through the application. When the developer subscribes to both platforms, if the transportation cost is sufficiently large (or consumers are not allowed to multi-home), regardless of the strength of the network effect, platforms are indifferent in general for consumers. Therefore, consumers only subscribe to the platform which they prefer most and they are equally split between two platforms. The advertising demands are

$$a = \frac{1}{3} + \frac{(P_1 - 2p + P_2) + w}{2t}$$

$$A_1 = \frac{1}{3} + \frac{2(P_2 - 2P_1 + p) - w}{4t}$$

$$A_2 = \frac{1}{3} + \frac{2(p - 2P_2 + P_1) - w}{4t}$$

3.3.2 Developer single-homing

Now suppose that the developer only subscribes to platform 1; consumers have access to the application if and only if they are on platform 1. This implies that the number of consumers of the application is equal to the number of consumers on platform 1 $(N_A = N_1)$. The location of consumers who are indifferent with regards to the two platform satisfies

$$V + (B + b)\, N_1 - Tx = V + BN_2 - T(1 - x)$$

where $N_2 = 1 - N_1$. The demand configuration is consistent provided $N_1 = x$ and $T > B + b$. The last inequality holds by assumption and it implies that the consumer located at 1 prefers to subscribe to its ideal platform (platform 2) alone, rather than to interact with all other consumers on platform 1 and through the application.

Like in the previous case, consumers split between the two platforms when they prefer single-homing on one platform than to multi-home. The equilibrium number of consumers on platform 1 is determined by the Hotelling formula

$$N_1 = \frac{T - B}{2\,(T - B) - b} = N_A$$

while $1 - N_1$ subscribe to platform 2 and

$$N_2 = \frac{T - B - b}{2(T - B) - b}$$

If the developer only subscribes to platform 1, the larger the intrinsic benefit of the application, the more value consumers attach to platform 1 when the application can generate more benefit to the consumers regardless of the size of the network. In effect, consumers find that they can attain a higher utility when they subscribe to a platform with an application. Additionally, the larger the benefit the consumer can attain from the application given the network size, the more consumers would like to subscribe to the platform with an application.

Advertising demands are

$$a = \frac{1}{3} + \frac{(P_1 - 2p + P_2)(2(T - B) - b) + wb}{2t[2(T - B) - b]}$$

$$A_1 = \frac{1}{3} + \frac{(P_2 - 2P_1 + p)(2(T - B) - b) + wb}{2t[2(T - B) - b]}$$

$$A_2 = \frac{1}{3} + \frac{(p - 2P_2 + P_1)(2(T - B) - b) - 2wb}{2t[2(T - B) - b]}$$

3.4 Consumer multi-homing

The assumption in the previous section, that the transportation cost is sufficiently large $(T > B + b)$, ensured that all consumers only single home regardless of the developer's subscription decision. On the other hand, with sufficiently low transportation cost, multi-homing becomes attractive to consumers who have no strong preference for either platform and they can enjoy being involved in a larger network by incurring a small transportation cost. When multi-homing is possible for consumers, consumers are less concerned about the subscription decision of the developer. Given a sufficiently low transportation cost, consumers can always subscribe to an additional platform with the application if the application is valuable to them. Hence, the subscription decision of the developer no longer plays an important role in the consumers' choice of platform. Now, we relax the assumption $(T > B + b)$ and consumers may single-home or multi-home depending on their strength of preference towards platforms.

3.4.1 Developer multi-homing

First, we consider the case in which the developer subscribes to both platforms. Platforms only differ in a standard Hotelling manner, the only reason for consumers to subscribe to both platforms is to interact with more consumers on the platforms and the value from multi-homing does not depend on the consumers' preference for the platforms. If interaction between consumers through the application on different platforms is not possible, the network effect of the platform will be further magnified.

Configuration 1: Consumers single-home on platform 1

Supposing that all consumers only subscribe to platform 1, the number of consumers who subscribe to platform 2 and the number of consumers multi-homing are zero. This demand configuration is consistent if and only if the following condition is satisfied for all x

$$V + (B + b)(N_1 + N) - Tx > V + (B + b)(N_2 + N) - T(1 - x)$$

provided $N_1 = 1$ and $N_2 = N = 0$. Consumers will single-home on platform 1 if $B+b+T > 0$ and $B+b > T$. The first inequality states that a consumer located at 0 prefers to subscribe to his or her ideal platform when no other consumers are on platform 2. The second inequality states that a consumer located at 1 prefers to subscribe to platform 2 if all other consumers are on platform 1. In this case, no consumer has an incentive to subscribe to platform 2 because, given that all consumers are on platform 1, no extra network benefit can be attained from subscribing to another platform and by Lemma , no consumer has then the incentive to multi-home when the second inequality holds. The number of consumers who subscribe to the application is now

$$N_A = N_1 = 1.$$

If the developer subscribes to both platforms, platforms are indifferent. When consumers have little preference about the platform, all consumers would subscribe to the same platform. This is because if consumers are more concerned with being in a large network than with responding to their preference over platforms, consumers prefer to subscribe to a large network with a high transportation cost than to subscribe to their most preferred platform with little transportation but a small number of consumers. If all consumers are using the same platform and the application is on both platform, a consumer cannot attain an extra benefit by subscribing to one more platform. Hence, all consumers single-home on one of the platforms. This can also explain the following demand configuration.

Configuration 2: Consumers single-home on platform 2

By symmetry, this configuration is consistent if and only if the following condition is satisfied for all x

$$V + v + (B + b)(N_1 + N) - Tx < V + v + (B + b)(N_2 + N) - T(1 - x)$$

provided $N_2 = 1$ and $N_1 = N = 0$. In other words, consumers will single-home on platform 2 which also requires $B + b + T > 0$ and $B + b > T$. Therefore, when these conditions are satisfied, consumers may either all single-home on platform 1 or all single-home on platform 2. This makes sense, given that the network benefit is attractive enough to all consumers and platforms are symmetric.

Configuration 3: Consumers multi-home and split between the two platforms

Supposing that consumers split between the two platforms and for any N_1, N_2 and N, it is optimal for a consumer who is located at x to multi-home if (i) $(B + b) - T \geq (B + b)(N_1 + N) - Tx$ and (ii) $(B + b) - T \geq (B + b)(N_2 + N) - T(1 - x)$ and (iii) $V + (B + b) - T \geq 0$. The first two inequalities state that the consumer prefers to multi-home than to single-home on any one of the platforms and by assumption (V is sufficiently large), the last inequality always holds, and consumers subscribe to at least one of the platforms. Solving the first two inequalities yields the equilibrium numbers of consumers on each platform, provided $N_1 = x$ and $N_2 = 1 - N_1 - N$.

$$N_1 + N = \frac{B + b}{T + B + b} = N_2 + N$$

$$N = \frac{B + b - T}{T + B + b}$$

This demand configuration is consistent if $B + b > T$ and all inequalities are satisfied. When network effects for an application are weak, i.e. consumers cannot interact through the application between platforms, the consumers are more willing to multi-home on both platforms. When consumers cannot interact with consumers between platforms even if they have subscribed to the same application, more benefit can be attained if a consumer subscribes to an additional platform and this effect will be magnified when the network effect is larger. On the other hand, if the network effect is strong, consumers have less incentive to subscribe to an additional platform as a large network effect can be generated through an application.

Configuration 4: Consumers single-home and split between the two platforms

Now, supposing $B + b < T$, no consumers are willing to multi-home, as has been proved in Lemma , and the proportion of consumers who join platform 1 is the same as the proportion of consumers who join platform 2. In other words, consumers split between the two platforms equally and

$$N_1 = \frac{1}{2} = N_2$$

3.4.2 Developer single-homing

We then turn to the case in which the developer only subscribes to platform 1. When transportation costs are sufficiently low multi-homing is possible. Consumers who prefer platform 1 over platform 2 have the incentive to multi-home only if they can interact with more consumers with one more platform. Besides the network benefit from the platform, the availability of the application also provides a rationale for consumers who prefer platform 2 to platform 1 to multi-home and interact with more consumers through the application on the platform.

Configuration 1: Consumers single-home on platform 1

The number of consumers who subscribe to platform 1 and the application is the same as the demand configuration when the developer multi-homes

$$N_A = N_1 = 1$$

This demand configuration is consistent if $b + B > T$. Given that there are more consumers on platform 1, more network benefit can be generated from both platform and application and the transportation cost can be outweighed by the network benefit. Consumers are more willing to subscribe to platform 1, than to subscribe to platform

2, as the cost from not having an ideal platform is compensated by both the network benefit from the platform and also from the application.

Configuration 2: Consumers single-home on platform 2

Now, supposing all consumers prefer platform 2 over platform 1 and when multi-homing is not possible for consumers, no consumers subscribe to platform 1 and hence the following inequality is always satisfied for all x

$$V + (B + b)\,(N_1 + N) - Tx < V + B\,(N_2 + N) - T(1 - x)$$

In this case, the consumer located at x is willing to subscribe to one more platform if the incremental benefit from multi-homing is positive. The consumer who is most likely to multi-home will be the one whose ideal platform is platform 1 and the proportion of consumers who join platform 1 is the proportion of consumers who multi-home and it is determined by the following inequalities (i) $B + b\,(N_2 + N) - T \geq B\,(N_2 + N) - T(1 - x)$ and (ii) $V + B + b\,(N_2 + N) - T \geq 0$, provided $N_2 = 1 - x$, the first inequality states that the consumer prefers to multi-home over single-homing and the last states that the consumer receives positive utility if he multi-homes. Then all consumers subscribe to platform 2 and no consumer subscribes to platform 1, given that the application benefit can only be obtained when there are consumers on platform 1. Hence, no consumer has an incentive to multi-home. The proportion of consumers who multi-home is

$$N_1 = 0 = N_A$$

$$N_2 = 1$$

This demand configuration is consistent if $B > T$.

Configuration 3: Consumers multi-home and split between the two platforms

When consumers split between the two platforms, for any $N_1,\ N_2$ and N, it is optimal for a consumer who is located at x to multi-home if (i) $B - T \geq B\left(N_1 + N\right) - Tx$, (ii) $B + b\left(N_1 + N\right) - T \geq B\left(N_2 + N\right) - T(1 - x)$, and (iii) $V + B + b\left(N_1 + N\right) - T \geq 0$. The first two inequalities state that the consumer prefers to multi-home than to single-home on any one of the platforms and by assumption (V is sufficiently large) the last inequality always holds, and consumers at least subscribe to one of the platforms. Solving the first two inequalities yields the equilibrium numbers of consumers on each platform, provided $N_1 = x$ and $N_2 = 1 - N_1 - N$.

$$N_1 + N = \frac{B(B-T)}{(B+T)\left(B-T\right)+Tb} = N_A$$

$$N_2 + N = \frac{B(B+b-T)}{(B+T)\left(B-T\right)+Tb}$$

$$N = \frac{(B-T)\left(B+b-T\right)}{(B+T)\left(B-T\right)+Tb}$$

This demand configuration is consistent if (1) $T < B - b$ and (2) $T > b$. The first inequality ensures consumers multi-homing; consumers are more interested in being in a larger network than in their ideal network. The second inequality states that consumers are more interested in being in their ideal network then in a network with the application. Therefore, when the transportation cost is sufficiently low, consumers who prefer platform 1 over platform 2 would be more willing to subscribe to platform 2 if platform 2 creates a larger network. Consumers who prefer platform 2 over platform 1 would have a smaller incentive to subscribe to platform 1 when more consumers are multi-homing. Here, the extra network benefit from application cannot drive the consumers to subscribe platform 1, given that the additional gain is outweighed by the transportation cost. However, the application benefit keeps some consumers whose ideal platform is close to platform 1 to subscribe to platform 1 only.

Configuration 4: Consumers single-home and split between the two platforms

Suppose the transportation cost is sufficiently high, consumers subscribe to the platform they like most and, by Lemma , all consumers single-home. The proportion of consumers who subscribe to platform 1 is the same as in the previous section.

$$N_1 = \frac{T - B}{2\,(T - B) - b} = N_A$$

$$N_2 = \frac{T - B - b}{2\,(T - B) - b}$$

$T > B + b$ is required for such demand configuration to be consistent.

3.5 Market Equilibrium in the Advertising Market

In the previous section, we have characterized the consistent demand configurations under different situations. Before turning to the exclusive contract discussion in the next section, we have to find the equilibrium in the advertising market given the equilibrium number of consumers in each demand configurations. As stated earlier, the main objective of this paper is to study the likelihood of the developer and platform owner entering into an exclusive contract which prohibits the developer from developing application to the rival platform (platform 2). Since we are not interested to study the equilibrium price in the advertising market if there is only a single platform (platform 2) to be the advertising media, we exclude the discussion of the pricing behavior of the single platform in the advertising market. On the other hand, if the developer stays in the market with a single platform, we assume they do not relocate around the circle and share the market given the equilibrium prices.

We consider in turn the case when a developer chooses to support two competing platforms, where platform $i = 1, 2$ attracts $N_i + N$ consumers. Each platform and the developer choose prices so as to maximize their profit from the advertising market

$$\Pi_i = (P_i - c)\, A_i$$

$$\pi = (p - c)\, a$$

Their maximum is characterized by the first order conditions:

$$\frac{\partial \Pi_i}{\partial P_i} = A_i + \frac{\partial A_i}{\partial P_i}(P_i - c) = 0$$

$$\frac{\partial \pi}{\partial p} = a + \frac{\partial a}{\partial p}(p - c) = 0$$

In particular, there are three cases: when the developer subscribes to both platforms, consumers always split between the platforms if they are interested about the platform more than the network effect (i.e. T is sufficiently large). Therefore, the equilibrium number of consumers on each platform is the same $(N_1^* = N_2^*)$ and the advertisement demands are:

$$a = \frac{1}{3} + \frac{(P_1 - 2p + P_2) - 2w\,(N_1 - 1)}{2t}$$

$$A_1 = \frac{1}{3} + \frac{(P_2 - 2P_1 + p)(T - b) - w\,(1 - N_1)}{2t}$$

$$A_2 = \frac{1}{3} + \frac{(p - 2P_2 + P_1)(T - b) - w\,(1 - N_1)}{2t}$$

The first order condition yields

$$P_1^* = \frac{5t - 3w(1 - N_1^*)}{15} + c$$

$$P_2^* = \frac{5t - 3w(1 - N_2^*)}{15} + c$$

$$p^* = \frac{5t - 6w(N_1^* - 1)}{15} + c$$

Now, when the developer subscribes to a single platform and we suppose all consumers subscribe to the application if it is available to them, the developer choose prices to maximize her profit

$$\pi = (p - c)\,a$$

In equilibrium, the number of consumers on each platform is the same $(N_1^* = N_A^*)$ and the advertisement demands are:

$$a = \frac{1}{3} + \frac{(P_1 - 2p + P_2) - w\,(N_2 - N_A)}{2t}$$
$$A_1 = \frac{1}{3} + \frac{(P_2 - 2P_1 + p)(T - b) - w\,(N_2 - N_1)}{2t}$$
$$A_2 = \frac{1}{3} + \frac{(p - 2P_2 + P_1)(T - b) - 2w\,(N_1 - N_2)}{2t}$$

and

$$P_1^* = \frac{5t - 3w(N_2^* - N_1^*)}{15} + c$$
$$P_2^* = \frac{5t - 6w(N_1^* - N_2^*)}{15} + c$$
$$p^* = \frac{5t - 3w(N_2^* - N_1^*)}{15} + c$$

On the other hand, if consumers are more interested in the network effect than their preference over the platforms, then all consumers subscribe to only a single platform. If only one platform and the developer remain in the advertising market and all consumers subscribe to the platform, then the advertising demands are

$$a = \frac{1}{2} + \frac{P_i - p}{t}$$

$$A_i = \frac{1}{2} + \frac{p - P_i}{t}$$

and hence the equilibrium prices are p^*, P_i^* respectively

$$P_i^* = \frac{t + 2c}{3}$$

$$p^* = \frac{t + 2c}{3}$$

3.6 Exclusive Contracts

$$h_1 = Bb\left(T + B + b\right) - 2T\left[\left(B + T\right)\left(B - T\right) + Tb\right].$$

Proposition 11 *Suppose multi-homing is possible for both consumers and developer. There exists a subgame perfect equilibrium outcome of the subscription game in which developer multi-home and supports both platforms in the following scenarios: (i) When $T > B + b$; (ii) when $B + b > T > B - b$ if $w > w_1$; (iii) when $B - b > T > b$, if $h_1 < 0$ and (iv) when $B - b > T$ and $T < b$, if $w > w_1$.Otherwise, the developer single-homes.*

A developer benefits when it serves both platforms for two reasons: it obtains more consumers (potential viewers for the advertisements) in its application and it makes platform 1 less competitive for advertisers. But to enjoy these benefits, the platform must incurs a cost. When the developer subscribes to both platforms, platform 2 will be as competitive as platform 1. The benefits outweigh the cost when the developer prefers two identical rivals with a larger network to a strong rival and a weak rival with a small network. This proposition describes conditions under which there are equilibria where the developer subscribes to both platforms. When the transportation cost is large $(T > B + b)$, the gain from having a large network and the gain from an application are insufficient to compensate for the loss from not having the ideal platform. Thus consumers subscribe to a single platform, their ideal platform only. When transportation costs fall, being in a larger network becomes attractive to consumers, and therefore more consumers subscribe to their less preferred platform, as well as subscribing to both platforms. Notice that the more consumers multi-home, the less the effect of the application on the platform. When consumers can multi-home, and when more and more consumers subscribe to both platforms, the difference in the number of potential viewers is reduced across the platforms. The number of extra viewers generated by the application on the platform is negligible and the platform becomes more appealing to advertisers. Apart from the low transportation cost encouraging consumers to multi-home, the subscription choice is also affected by the effectiveness of the advertisement, w. The smaller the effectiveness of an advertisement response to the number of viewers on the platform, the smaller the gain from having a larger network and the more likely the developer is to subscribe to a single platform only. When transportation cost is at an intermediate level and the network effect dominates the application effect $(h_1 < 0)$, consumers prefer a platform with a larger network over a platform with an application. Hence an application installed on the platform does not bring more consumers to the platform owner. Instead, multi-homing is more attractive to the developer with more viewers for the advertisers.

Our analysis to this point has focused on the developer side and we have assumed that the developer is free to develop an application for any platform. In practice, it is possible for platform owners to limit developers to develop applications on only one platform. For instance, an exclusive contract may be written when a license is issued to allow the developer to subscribe to the platform. Now then, we go on to study the situation in which the platform owner welcomes multi-homing.

Proposition 12 *Suppose that multi-homing is possible for both consumers and developer. There exists a subgame perfect equilibrium outcome of the subscription game in which platform* 1 *prefers developer to multi-home and supports both platforms in the following scenarios: (i) When* $T > B + \frac{3b}{2}$*; (ii) when* $B+b > T > B-b$ *if* $w < w_2$ *; and (iii) when* $B - b > T$ *and* $T < b$*, if* $w < w_2$*.Otherwise, the platform owner offers an exclusive contract.*

This proposition describes conditions under which there are equilibria in which platform 1 prefers the developer to multi-home. When the developer serves both platforms, the number of potential viewers in the application for the advertisers is the same as the consumer mass. Advertisers can reach all consumers if they post an advertisement on the application. In addition, with support from the application platform 2 becomes as competitive as platform 1, and hence reduces platform 1's competitiveness in the advertisement market. Despite this, platform 1 finds multi-homing attractive when the transportation cost is sufficiently large. In this case, consumers are concerned with their preference over platforms much more than the network size of the platform. Hence, most of the consumers subscribe to the platform which they prefer, while only very few consumers subscribe to the least preferred platform (platform 1) if the developer serves only platform 1. Moreover, when multi-homing becomes possible and attractive to the consumers, some of the consumers who are have little preference of the platform subscribe to both platform and hence, the difference of the number of consumers between the platforms is small and which reflects the competitiveness of platform 1 in the advertising market is reduced as advertiser can reach the some of the consumers from either platform. And therefore, the application then does not bring many extra consumers to platform 1 and it is likely to be as competitive as in the case where the developer multi-homes. In this case, of multihoming only increases the competitiveness of the developer and, given facing a strong rival (developer) in the advertising market, platform 2 prices its service lower to compete for more advertisements. Hence, platform 1 enjoys such a price reduction if T is sufficiently large. On the other hand, when T is small, all consumers subscribe to a single platform when the developer serves only a single platform and hence the developer is as competitive as in the multi-homing case. Then if the effectiveness of the advertisement is less responsive to the number of viewers, the loss from not having all consumers is compensated by the benefit of having another rival to compete with the developer.

Corollary 13 *Suppose that multi-homing is possible for both consumers and developer. There exists a subgame perfect equilibrium outcome of the subscription game in which the developer accepts the exclusive contract from the platform owner: (i) when $B + b > T > B - b$ if $w_1 > w > w_2$; (ii) when $B - b > T > b$, if $h_1 \geq 0$ and (iii) when $B - b > T$ and $T < b$, if $w_1 > w > w_2$.*

If it is costless to enforce exclusivity, platform 1 writes an exclusive contract with the developer, and the developer is willing to accept the offer for intermediate values of the the effectiveness of the advertisement to the number of viewers. This effectiveness must be large enough so that platform 1 can easily compensate the loss from a weak alliance (platform 2) and compete with the developer in the advertisement market due to the benefit of having a larger network. At the same time, this advertisement effectiveness must not be so sensitive to make the developer incur a great loss from not attracting all consumers and having instead a weak platform 2 as its rival. In particular, there is a case in which more consumers subscribe to the platform without the application (platform 2). This makes sense when the network effect of the platform is sufficiently large. Only a small group of consumers are willing to subscribe to their ideal platform which contains the application, rather than enjoying a larger network. It is natural for the developer to serve a single platform only if the network effect B is sufficiently small so that $h_1 \geq 0$. In this case, the loss of network size from serving only a single platform is low.

3.7 Concluding Remarks

Over the past 10 years, a significant volume of literature has contributed to our understanding of the pricing structure in two-sided markets. Many of the media industries which are supported by advertising revenue, such as newspapers, magazine, television and internet, are classic examples of two-sided markets. Yet, to date, these studies had only explored the platforms competing in the advertising market.

This article studies exclusive contract decisions relating to platforms in two-sided markets with add-on on the platforms. When add-on applications create network effect and bring more consumers to the platform, but also compete with the platforms for advertising revenue, we describe the conditions under which both platform owner and developer are interested in exclusive contracts. We show that whether add-on applications bring more consumers to the platform depends on the size of network effect from the platform relative to the size of the network effect from the add-on application. We also find that, different from the traditional game console industry, the platform owner may welcome the developer to serve both platforms. In the video game industry, the platform owner (the game console) does not compete with the game developer for revenue. Instead, they are almost purely in a cooperative situation. On the contrary, in the social networking industry the developer and platform owner compete for advertising revenue. Therefore, multi-homing is more likely to occur in the social networking industry.

Here, we simply assume that consumers are advertisement neutral; the number of advertisement does not give value nor generates nuisance cost to the consumers on the platform. We draw attention to the issue of exclusive contracts. A natural next step for future research would be to allow consumers to value advertisements. Moreover, advertising in the application can be different from advertising on the platform. In the real world, consumers can increase the value of the application by clicking on the advertisement and thus earning tokens for the application. Different advertising systems can also be discussed, such as ‘Pay per Click’ a common practice used on websites where the advertiser only pays when their advertisement is clicked on by the user.

Appendix A
Appendix to Chapter 1

Proof of Proposition 1

In scenario (i) both firms can enter both markets. Let's say that they coordinate so that firm 1 enters market A, while firm 2 enters market B. Strategies that support this equilibrium are as follows. At time $t = 1$, firm 1 enters market A. At time $t > 1$, firm 1 stays in market A and enters market B if and only if firm 2 has entered market A in a previous period. Firm 2 follows a similar strategy, entering market B at $t = 1$. Discounted continuation equilibrium payoffs are $V(1)/(1-\delta)$ for both firms. If firm 1 deviates and enters both markets in the first stage (this is the best possible deviation), then it obtains a discounted continuation payoff equal to $[V(1) + V(2)] + 2V(2)\delta/(1-\delta)$. This deviation is not profitable if $\delta \geq \delta_1$.

In scenario (ii) a firm cannot be punished for entering the only market in which it can innovate, and it cannot profit from licensing. ■

Proof of Proposition 2

Consider each scenario in turn. In scenario (i), one firm can enter one market and the other firm can enter both markets. Without loss of generality, consider the case where firm 1 can enter market A only and firm 2 can enter both markets. The following strategy supports the equilibrium where firm 1 enters market A only and firm 2 enters market B only. For firm 1: Do not license to the fringe unless firm 2 has entered market A in the previous period. For firm 2: Remain in market B only unless firm 1 has licensed and induced fringe entry into market B in the previous period. Each firm's discounted payoff from this strategy is $V(1)/(1-\delta)$. Firm 1's best deviation is to license in market B immediately. This deviation results in a payoff of $V(1) + \gamma_1 \frac{\alpha V(2)}{1-\delta} + \frac{\delta V(2)}{1-\delta}$. This deviation is unprofitable if $\delta \geq \alpha\gamma_1\delta_1$. Firm 2's best deviation is to enter firm 1's market immediately, provoking licensing by firm 1 in the following period. This deviation results in a payoff of $V(1) + V(2) + \frac{\delta(1+\gamma_1)V(2)}{1-\delta} + \frac{\delta(1-\gamma_1)V(1)}{1-\delta}$ and it is unprofitable if $\delta \geq \frac{\delta_1}{\gamma_1}$; clearly, it is $\frac{\delta_1}{\gamma_1} > \alpha\gamma_1\delta_1$.

In scenario (ii), both firms can enter both markets. By the same argument as in proposition , trigger strategies support the equilibrium where each firm enters a different market. The punishment upon observing a deviation is the leading firm's entry into the other market. This is clearly a better punishment strategy than licensing because the punishing firm need not split the proceeds with the fringe firm. A value $\delta \geq \delta_1$ ensures that this equilibrium exists.

In scenario (iii), each firm can enter a different market. The following strategy ensures that neither firm licenses to the fringe: Do not license unless the rival firm has licensed in the previous period. Each firm's discounted payoff from this strategy is $V(1)/(1-\delta)$. The best deviation for both firms is to immediately license, resulting in a payoff of $V(1) + \gamma_1 \frac{\alpha V(2)}{1-\delta} + \gamma_1 \frac{\delta V(2)}{1-\delta} + (1-\gamma_1)\frac{\delta V(1)}{1-\delta}$. This deviation is unprofitable if $\delta \geq \alpha\delta_1$. ■

Proof of Proposition 3

By Proposition , if $\delta \geq \delta_1$ it is a (continuation) equilibrium to coordinate entry in subgame (2) of stage 2 (see Table 1), when both leading firms can innovate in both market. It is also a continuation equilibrium not to coordinate. (Recall that in all other subgames of stage 2 the leading firms enter all markets in which they can innovate.) Suppose the strategy of each leading firm prescribes to share information in the first stage, and then to follow the continuation equilibrium of coordinating entry in subgame (2) when both firms shared knowledge in the first stage and otherwise to follow the no-coordination continuation equilibrium. This strategy gives the firm a discounted continuation equilibrium payoff U^E, where

$$\begin{aligned} U^E(1-\delta) &= p_h^2\left\{p_h^2 V(1) + 2p_h(1-p_h)V(2)\right\} \\ &\quad +2p_h(1-p_h)\left\{p_h^2\left[V(1)+V(2)\right] + p_h(1-p_h)V(1) + p_h(1-p_h)V(2)\right\} \\ &\quad +(1-p_h)^2\left\{2p_h^2 V(1) + 2p_h(1-p_h)V(1)\right\} \\ &= \left[p_h^4 - 2p_h^2 + 2p_h\right]V(1) - 2\left[p_h^4 - p_h^2\right]V(2) \end{aligned}$$

Failing to share knowledge in stage 1 yields the payoff U^D, where

$$\begin{aligned} U^D(1-\delta) &= p_l^2\left\{2p_h^2 V(2) + 2p_h(1-p_h)V(2)\right\} \\ &\quad +2p_l(1-p_l)\{p_h^2\left[V(1)+V(2)\right] + p_h(1-p_h)V(1) + p_h(1-p_h)V(2)\} \\ &\quad +(1-p_l)^2\{2p_h^2 V(1) + 2p_h(1-p_h)V(1)\} \\ &= 2p_h\left(1-p_l\right)V(1) + 2p_l p_h V(2). \end{aligned}$$

Simple algebra shows that $U^D \leq U^E$ if and only if $p_l \geq p_l^*$, where

$$p_l^* = \frac{(2p_h^2 - p_h^4)V(1) - 2(p_h^2 - p_h^4)V(2)}{2p_h\left[V(1) - V(2)\right]} = p_h - \frac{p_h^3\left[V(1) - 2V(2)\right]}{2\left[V(1) - V(2)\right]}.$$

This concludes the proof. ■

Proof of Proposition 4

The following is an equilibrium strategy for firm 1 (firm 2's equilibrium strategy is similar).

In the first stage of the game: share knowledge with firm 2. In the entry game:

- If firm 1 cannot innovate in either market: license in both markets.

- If firm 2, the other leading firm, cannot innovate in either market: enter any market in which can innovate, license in any market in which cannot innovate.

- If firm 1 can innovate in only one market and firm 2 can innovate in the other market: do not license if firm 2 shared in the first stage and has not entered firm 1's market; otherwise license.

- If firm 1 can innovate in both markets and firm 2 can innovate in at least one market, say market B: enter market A only, unless firm 2 licenses in that market, enters that market itself, or fails to share knowledge; otherwise enter both markets.

- If firm 1 can innovate in one market and firm 2 can innovate in that same one market only: enter that market and license to the fringe in the other market.

From Proposition 2, we know that if $\delta \geq \frac{\delta_1}{\gamma_1}$, then firms are able to coordinate market entry in subgames (1)-(4) of Table 1. It only remains to show that sharing knowledge is an equilibrium in the first stage of the game.

Suppose firm 2 follows the equilibrium strategy. If firm 1 also follow the equilibrium strategy and shares knowledge, its payoff is

$$U^E = p_h^2 U^E(2) + 2p_h(1-p_h)U^E(1) + (1-p_h)^2 U^E(0)$$

where $U^E(i)$ is firm 1's payoff when firm 2 is able to innovate in i markets. It is:

$$(1-\delta)U^E(2) = \left\{p_h^2 V(1) + 2p_h(1-p_h)V(1) + (1-p_h)^2 2\alpha\gamma_1 V(2)\right\}$$

$$(1-\delta)U^E(1) = \left\{\begin{array}{l} p_h^2 V(1) \\ +p_h(1-p_h)V(1) + p_h(1-p_h)\left[V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha(\gamma_1-\gamma_2)V(1)\right] \\ +(1-p_h)^2\left[\alpha\gamma_1 V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha(\gamma_1-\gamma_2)V(1)\right] \end{array}\right\}$$

$$(1-\delta)U^E(0) = \left\{\begin{array}{l} p_h^2 2\left[\gamma_1 V(2) + (1-\gamma_1)V(1)\right] \\ +2p_h(1-p_h)\left[\gamma_1 V(2) + (1-\gamma_1)V(1) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha(\gamma_1-\gamma_2)V(1)\right] \\ +(1-p_h)^2 2\left[\alpha\gamma_2 V(2) + \frac{1}{2}\alpha(\gamma_1-\gamma_2)V(1)\right] \end{array}\right\}$$

If firm 2 follows the equilibrium strategy, firm 1's payoff from withholding knowledge is

$$U^D = p_l^2 U^D(2) + 2p_l(1-p_l)U^D(1) + (1-p_l)^2 U^D(0)$$

where, as before, $U^D(i)$ is firm 1's payoff when firm 2 is able to innovate in i markets. It is:

$$(1-\delta)U^D(2) = \left\{p_h^2 2V(2) + 2p_h(1-p_h)[V(2) + \alpha\gamma_1 V(2)] + (1-p_h)^2 2\alpha\gamma_1 V(2)\right\}$$

$$(1-\delta)U^D(1) = \left\{\begin{array}{l} p_h^2\left[V(2) + \gamma_1 V(2) + [1-\gamma_1]V(1)\right] \\ +p_h(1-p_h)\left[V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\gamma_1 V(2)\right] \\ +p_h(1-p_h)\left[(1-\gamma_1)V(1) + \alpha\gamma_1 V(2)\right] \\ +[(1-p_h)^2\left[\alpha\gamma_1 V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \end{array}\right\}$$

$$(1-\delta)U^D(0) = \left\{ \begin{array}{l} p_h^2 2\left[\gamma_1 V(2) + (1-\gamma_1)V(1)\right] \\ +2p_h(1-p_h)\left[\gamma_1 V(2) + (1-\gamma_1)V(1) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \\ +(1-p_h)^2 2\left[\alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \end{array} \right\}$$

Note that $U^D(1) > U^D(2)$, since

$$\begin{aligned} 0 &< \left\{ \begin{array}{l} p_h^2\left[V(2) + \gamma_1 V(2) + \left[1-\gamma_1\right]V(1)\right] \\ +p_h(1-p_h)\left[V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \\ +p_h(1-p_h)\left[\gamma_1 V(2) + \left(1-\gamma_1\right)V(1) + \alpha\gamma_1 V(2)\right] \\ +[(1-p_h)^2\left[\alpha\gamma_1 V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \end{array} \right\} \\ &\quad -\left\{p_h^2 2V(2) + 2p_h(1-p_h)[V(2) + \alpha\gamma_1 V(2)] + (1-p_h)^2 2\alpha\gamma_1 V(2)\right\} \\ &= \left\{ \begin{array}{l} p_h^2\left(1-\gamma_1\right)\left(V(1)-V(2)\right) \\ +p_h(1-p_h)\left[-V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \\ +p_h(1-p_h)\left[\gamma_1 V(2) + \left(1-\gamma_1\right)V(1) - \alpha\gamma_1 V(2)\right] \\ +[(1-p_h)^2\left[-\alpha\gamma_1 V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \end{array} \right\} \\ &= \left\{ \begin{array}{l} p_h^2\left(1-\gamma_1\right)\left(V(1)-V(2)\right) \\ +p_h(1-p_h)\left[\left(1-\gamma_1\right)\left(V(1)-V(2)\right) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)\left(V(1)-2V(2)\right)\right] \\ +[(1-p_h)^2\frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)\left(V(1)-2V(2)\right) \end{array} \right\} \end{aligned}$$

Furthermore, $U^D(0) > U^D(1)$, since

$$\begin{aligned} 0 &< \left\{ \begin{array}{l} p_h^2 2\left[\gamma_1 V(2) + (1-\gamma_1)V(1)\right] \\ +2p_h(1-p_h)\left[\gamma_1 V(2) + (1-\gamma_1)V(1) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \\ +(1-p_h)^2 2\left[\alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \end{array} \right\} \\ &\quad -\left\{ \begin{array}{l} p_h^2\left[V(2) + \gamma_1 V(2) + \left[1-\gamma_1\right]V(1)\right] \\ +p_h(1-p_h)\left[V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \\ +p_h(1-p_h)\left[\gamma_1 V(2) + \left(1-\gamma_1\right)V(1) + \alpha\gamma_1 V(2)\right] \\ +[(1-p_h)^2\left[\alpha\gamma_1 V(2) + \alpha\gamma_2 V(2) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)V(1)\right] \end{array} \right\} \\ &= \left\{ \begin{array}{l} p_h^2(1-\gamma_1)\left[V(1)-V(2)\right] \\ +p_h(1-p_h)\left[\left(1-\gamma_1\right)\left(V(1)-V(2)\right) + \frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)\left(V(1)-2V(2)\right)\right] \\ +(1-p_h)^2\frac{1}{2}\alpha\left(\gamma_1-\gamma_2\right)\left(V(1)-2V(2)\right) \end{array} \right\} \end{aligned}$$

Define $\Phi(p_l) = U^E - U^D$ and note that Φ is increasing in p_l

$$\begin{aligned}
\frac{d\Phi}{dp_l} &= -2p_l U^D(2) - (2-4p_l)U^D(1) + 2(1-p_l)U^D(0) \\
&= 2p_l \left[U^D(1) - U^D(2)\right] + 2(1-p_l)\left[U^D(0) - U^D(1)\right] \\
&> 0
\end{aligned}$$

Moreover, when $p_l = p_h$ it is $\Phi(p_h) > 0$. To see this, note that

$$\Phi(p_h) = p_h^2 \left[U^E(2) - U^D(2)\right] + 2p_h(1-p_h)\left[U^E(1) - U^D(1)\right] + (1-p_h)^2\left[U^E(0) - U^D(0)\right]$$

hence $(1-\delta)\Phi(p_h)$ is equal to

$$\begin{aligned}
& p_h^2 \left\{p_h^2 \left[V(1) - 2V(2)\right] + 2p_h(1-p_h)\left[V(1) - (1+\alpha\gamma_1)V(2)\right]\right\} \\
& +2p_h(1-p_h)\left\{p_h^2\left[\gamma_1 V(1) - (1+\gamma_1)V(2)\right] + p_h(1-p_h)\left[\gamma_1\left(V(1) - V(2)\right) - \alpha\gamma_1 V(2)\right]\right\} \\
= \; & p_h^4 \left[V(1) - 2V(2)\right] + 2p_h^3(1-p_h)\left[V(1) - (1+\alpha\gamma_1)V(2)\right] \\
& +2p_h^3(1-p_h)\left[\gamma_1 V(1) - (1+\gamma_1)V(2)\right] + 2p_h^2(1-p_h)^2\left[\gamma_1\left(V(1) - V(2)\right) - \alpha\gamma_1 V(2)\right] \\
= \; & p_h^4 \left[V(1) - 2V(2)\right] + 2p_h^3(1-p_h)\left[(1+\gamma_1)\left(V(1) - 2V(2)\right) + \gamma_1(1-\alpha)V(2)\right] \\
& +2p_h^2(1-p_h)^2\gamma_1\left[V(1) - (1+\alpha)V(2)\right] \\
> \; & 0
\end{aligned}$$

It follows that there exists $p_l^{**} < p_h$ such that, for all $p_l \geq p_l^{**}$ it is $\Phi \geq 0$ and hence sharing knowledge is an equilibrium strategy. ■

Appendix B
Appendix to Chapter 2

Proof of Proposition 5

1. If $1 > \theta_{N_{IE}} > \theta_{N_E}$ and $\theta_{N_{IE}} > \theta_{N_I}$, then $Q_B = 1 - \theta_{N_E}$, $Q_{A_I} = 1 - \theta_{N_{IE}}$ and $Q_{A_E} = \theta_{N_{IE}} - \theta_{N_E}$Suppose $1 > \theta_{N_{IE}} > \theta_{N_E}$ and $\theta_{N_{IE}} > \theta_{N_I}$ holds, firms maximizes their profits by choosing the optimal price in the markets, $p_{I,}$ $p_E$and p_B and their profit functions are given as follow:

$$\begin{aligned} \pi_I(q_H) &= (p_I - c_I)\,(1 - \theta_{N_{IE}}) + p_B\,(1 - \theta_{N_E}) \\ &= (p_I - c_I)\left(1 - \frac{p_I - p_E}{q_H - q_L}\right) + p_B\left(1 - \frac{p_E + p_B}{q_L}\right). \end{aligned} \tag{B.1}$$

$$\begin{aligned} \pi_E(q_L) &= (p_E - c_E)\,(\theta_{N_{IE}} - \theta_{N_E}) \\ &= (p_E - c_E)\left(\frac{p_I - p_E}{q_H - q_L} - \frac{p_E + p_B}{q_L}\right). \end{aligned} \tag{B.2}$$

Maximizing equation B.1 and B.2 yield the optimal price in the markets:

$$p_I^* = \frac{3q_L(q_H - q_L) + c_I(q_H + q_L) - 2q_H(c_I + c_E)}{6q_H}.$$

$$p_E^* = \frac{2c_I q_H + c_E q_L}{3q_H}.$$

$$p_B^* = \frac{3q_L q_H - c_I q_L - 2c_E q_H}{6q_H}.$$

This gives $\theta_{N_{IE}} > \theta_{N_E}$ to exist if and only if $\theta_{N_{IE}} < \theta_{N_I}$, which violates the assumption. Thus, this is not the equilibrium.

2. If $\theta_{N_{IE}} > 1$ and $1 > \theta_{N_E}$, then $Q_B = 1 - \theta_{N_E}$, $Q_{A_I} = 0$ and $Q_{A_E} = 1 - \theta_{N_E}$

   When $\theta_{N_{IE}} > 1$ and $1 > \theta_{N_E}$ holds, firms maximizes their profits by choosing the optimal price in the markets, p_E and p_B and their profit functions are given as follow:

$$\begin{aligned}\pi_I(q_H) &= p_B\,(1-\theta_{N_E}) \\ &= p_B\left(1-\frac{p_E+p_B}{q_L}\right).\end{aligned}$$

$$\begin{aligned}\pi_E(q_L) &= (p_E-c_E)\,(1-\theta_{N_E}) \\ & (p_E-c_E)\left(1-\frac{p_E+p_B}{q_L}\right).\end{aligned}$$

Maximizing $\pi_{I_1}(q_H)$and $\pi_{E_1}(q_L)$ and yield the optimal price in the markets:

$$p_E^* = \frac{q_L + c_E - p_B^*}{2} = \frac{q_L + 2c_E}{3}.$$

$$p_B^* = \frac{q_L - p_E^*}{2} = \frac{q_L - c_E}{3}.$$

and this gives firms the profit $\pi_I^*(q_H)$ and $\pi_E^*(q_H)$, where

$$\pi_I^*(q_H) = \pi_E^*(q_L) = \frac{(q_L - p_E^*)^2}{4q_L} = \frac{(q_L - c_E)^2}{9q_L}.$$

For $\theta_{N_{IE}} > 1$ to hold, $p_I^* > (q_H - q_L) + p_E^*$

3. If $1 > \theta_{N_I} > \theta_{N_{IE}}$ and $\theta_{N_E} > \theta_{N_{IE}}$, then $Q_B = 1 - \theta_{N_I}$, $Q_{A_I} = 1 - \theta_{N_I}$ and $Q_{A_E} = 0$

   Consider configuration 3. For $Q_{A_E} = 0$, firm I must sells its product in market A at a price, $p_I^* \leq c_E$ (that is the minimum price for firm E to make a non negative profit). In this case, firm I maximizes its profit by choosing the optimal price in market B, p_B and its profit function is given as follow:

$$\begin{aligned} \pi_I(q_H) &= (p_I + p_B - c_I)(1 - \theta_{N_I}) \\ &= (p_I + p_B - c_I)\left(1 - \frac{p_I + p_B}{q_H}\right). \end{aligned} \tag{B.3}$$

Maximizing equation B.3 yields the optimal price in market B:

$$p_I^* + p_B^* = \frac{q_H + c_I}{2}.$$

and it gives firm I an optimal profit $\pi_I^*(q_H)$, where

$$\pi_I^*(q_H) = \frac{(q_H - c_I)^2}{4q_H}.$$

In this case, $1 > \theta_{N_I} > \theta_{N_{IE}}$ and $\theta_{N_E} > \theta_{N_{IE}}$ holds.

Now there are two sets of equilibrium candidates (configuration 2 and 3). We first consider the first set of equilibrium candidates (i.e. configuration 3), In this configuration, Firm I undercuts its price to firm E's marginal cost in market A and profit for firm I is $(q_H - c_I)^2 / 4q_H$. If firm I deviates and exits market A (equivalently setting its price above p_E^* in market A and this is the best possible deviation), then firm E chooses its monopoly price in the market to maximizes its profit and firm I obtains a profit equal to $\left(q_L - p_E^*\right)^2 / 4q_L$. This deviation is profitable if and only if $\left(q_L - p_E^*\right)^2 / 4q_L > \left(q_H - c_I\right)^2 / 4q_H$. In other words, firm I has no profitable deviation if $p_E^* \geq q_L - \left(q_H - c_I\right)\sqrt{q_L/q_H}$. Given $p_I^* \leq c_E$, entrant has no profitable deviation in this configuration.

We next consider the second set of equilibrium candidates (configuration 2). Firm I leaves market A for firm E and each firm receives a profit for$\left(q_L - c_E\right)^2 / 9q_L$. If firm I deviates and undercuts its price in market A (that is setting its price below c_E) in market A and this is the best possible deviation, then firm I chooses its price in market B as a best response to p_E^* and firm I obtains a profit equal to $\left(q_H - c_I\right)^2 / 4q_H$. This deviation is not profitable if $c_I > c_1$. There is no profitable deviation for firm E.

We now establish that there are two equilibria. If $c_I \leq c_1$, then there is only an equilibrium exists, where

$$\begin{aligned} p_I^* &\leq c_E, \\ p_E^* &\geq q_L - \left(q_H - c_I\right)\sqrt{q_L/q_H} \text{ and} \\ p_I^* + p_B^* &= (q_H + c_I)/2 \end{aligned}$$

On the other hand, if $c_I > c_1$, there are two subgame perfect equilibria exist, where

$$\begin{aligned} p_I^* &\leq c_E, \\ p_E^* &\geq q_L - \left(q_H - c_I\right)\sqrt{q_L/q_H} \text{ and} \\ p_I^* + p_B^* &= (q_H + c_I)/2 \end{aligned}$$

$$p_I^{**} > (3q_H - 2q_L + 2c_E)/3,$$
$$p_E^{**} = (q_L + 2c_E)/3 \text{ and}$$
$$p_B^{**} = (q_L - c_E)/3$$

We assume only the second type of equilibrium will be played. This makes sense. Both firm I and firm E receive a higher profit if firm I agrees to leave the market and firm E agrees to take the market. The second type of equilibrium is Pareto optimal given that $\pi_I^*(p_I^*, p_E^*, p_B^*) = \frac{(q_L-c_E)^2}{9q_L} \geq \pi_I^*(p_I^{**}, p_E^{**}, p_B^{**}) = \frac{(q_H-c_I)^2}{4q_H}$ when $c_I > c_1$. ■

Proof of Proposition 6

Consider firm E introduces a product with better quality, $q_E > q_I$. If consumers have bought product from firm B, all consumers of type $\theta > \max\{\theta_{M_{IE}}, \theta_{M_E}\}$.buys the product from firm E in market A satisfied the following conditions:

1 The utility one gets from the product of firm E is higher than that from the product of firm I, i.e. $\theta_{M_{IE}} = \frac{p_E - p_I}{(q_E - q_I)}$

2 The utility one gets from the product of firm E is higher than the utility from buying nothing, i.e. $\theta_{M_E} = \frac{p_E + p_B}{q_E}$

Hence for all consumers of type $\theta < \theta_{M_{IE}}$ and $\theta > \theta_{M_I}$ buys the product from firm I in market A, where $\theta_{M_I} = \frac{p_I + p_B}{q_I}$[18]. We consider the optimal prices in each of the possible demand functions for product in market B which listed below:

[18] $\theta_{M_{IE}} > \theta_{M_E}$ implies $\theta_{M_{IE}} > \theta_{M_I}$

$$Q_B = \begin{cases} 1-\theta_{M_I} & \text{if } 1 \geq \theta_{M_{IE}} > \theta_{M_E} \text{ and } \theta_{M_{IE}} > \theta_{M_I} \\ 1-\theta_{M_I} & \text{if } \theta_{M_{IE}} \geq 1 \text{ and } 1 > \theta_{M_I} \\ 1-\theta_{M_E} & \text{if } 1 > \theta_{M_E} > \theta_{M_{IE}} \text{ and } \theta_{M_I} > \theta_{M_{IE}} \\ 0 & \text{otherwise} \end{cases}$$

1. If $1 \geq \theta_{M_{IE}} > \theta_{M_E}$ and $\theta_{M_{IE}} > \theta_{M_I}$, then $Q_B = 1-\theta_{M_I}$, $Q_{A_I} = \theta_{M_{IE}} - \theta_{M_I}$ and $Q_{A_E} = 1 - \theta_{M_{IE}}$

   Suppose $1 > \theta_{M_{IE}} > \theta_{M_E}$ and $\theta_{M_{IE}} > \theta_{M_I}$ holds, firms maximize their profits by choosing the optimal price in the markets, $p_{I,}$ $p_E$and p_B and their profit functions are given as follow:

$$\begin{aligned} \pi_I(q_L) &= (p_I - c_I)\left(\theta_{M_{IE}} - \theta_{M_I}\right) + p_B\left(1-\theta_{M_I}\right) \\ &= (p_I - c_I)\left(\frac{p_E - p_I}{q_H - q_L} - \frac{p_I + p_B}{q_L}\right) + p_B\left(1 - \frac{p_I + p_B}{q_L}\right). \end{aligned} \tag{B.4}$$

$$\begin{aligned} \pi_E(q_H) &= (p_E - c_E)\left(1-\theta_{M_{IE}}\right) \\ & (p_E - c_E)\left(1 - \frac{p_E - p_I}{q_H - q_L}\right). \end{aligned} \tag{B.5}$$

Maximizing equation B.4 and B.5 yield the optimal price in the markets:

$$\begin{aligned} p_I^* &= \frac{q_L - q_H + c_I + p_E^*}{2} \\ &= \frac{q_L - q_H + c_E + 2c_I}{3}. \end{aligned}$$

$$
\begin{aligned}
p_E^* &= \frac{q_H - q_L + c_E + p_I^*}{2} \\
&= \frac{q_H - q_L + 2c_E + c_I}{3}.
\end{aligned}
$$

$$
\begin{aligned}
p_B^* &= \frac{q_H - p_E^*}{2} \\
&= \frac{2q_H + q_L - 2c_E - c_I}{6}.
\end{aligned}
$$

and this gives firms the profit $\pi_I^*(q_L)$ and $\pi_E^*(q_H)$, where

$$
\begin{aligned}
\pi_I^*(q_L) &= \frac{(q_L - c_I)^2}{4q_L} + \frac{(q_H - q_L + c_I - p_E^*)^2}{4(q_H - q_L)} \\
&= \frac{(q_L - c_I)^2}{4q_L} + \frac{(q_H - q_L + c_I - c_E)^2}{9(q_H - q_L)}
\end{aligned}
$$

$$
\begin{aligned}
\pi_E^*(q_H) &= \frac{(q_H - q_L - c_E + p_I^*)^2}{4(q_H - q_L)} \\
&= \frac{(q_H - q_L + c_I - c_E)^2}{9(q_H - q_L)}.
\end{aligned}
$$

$\theta_{M_{IE}} > \theta_{M_E}$ and $\theta_{M_{IE}} > \theta_{M_I}$ only hold if and only if the following condition is satisfied:

$$
(q_H - q_L)(q_L - c_I) - 2(c_I q_H - c_E q_L) > 0
$$

or, equivalently, if and only if

$$
c_I < q_L - \frac{2(q_H - c_E)q_L}{(3q_H - q_L)} = c_2
$$

2. If $\theta_{M_{IE}} \geq 1$ and $1 > \theta_{M_I}$, then $Q_B = 1 - \theta_{M_I}$, $Q_{A_I} = 1 - \theta_{M_I}$ and $Q_{A_E} = 0$

   Consider both configuration 3. For $Q_{A_E} = 0$, firm I must sells its product in market A at a price, $p_I < p_E - (q_H - q_L) = c_E - (q_H - q_L)$ (all consumers are indifferent between buying from firm E and firm I at this price and firm E makes a non negative profit) and it implies $\theta_{M_{IE}} = 1$ and firm I maximizes its profit by choosing the optimal price in market B, p_B and its profit function is given as follow:

$$\begin{aligned} \pi_I(q_L) &= (p_I + p_B - c_I)\,(1 - \theta_{M_I}) \\ &= (p_I + p_B - c_I)\left(1 - \frac{p_I + p_B}{q_L}\right) \end{aligned} \tag{B.6}$$

   Maximizing equation B.6 yields the optimal price in market B:

$$p_I^* + p_B^* = \frac{q_L + c_I}{2}.$$

   and it gives firm I an optimal profit $\pi_I^*(q_H)$, where

$$\pi_I^*(q_L) = \frac{(q_L - c_I)^2}{4q_L}.$$

   In this case, $\theta_{M_{IE}} \geq 1$ and $1 > \theta_{M_I}$ holds.

3. If $1 > \theta_{M_E} > \theta_{M_{IE}}$ and $\theta_{M_I} > \theta_{M_{IE}}$, then $Q_B = 1 - \theta_{M_E}$, $Q_{A_I} = 0$ and $Q_{A_E} = 1 - \theta_{M_E}$

   When $1 > \theta_{M_E} > \theta_{M_{IE}}$ and $\theta_{M_I} > \theta_{M_{IE}}$ holds, firms maximizes their profits by choosing the optimal price in the markets, p_E and p_B and their profit functions are given as follow:

$$\begin{aligned}\pi_I(q_L) &= p_B\left(1-\theta_{M_E}\right) \\ &= p_B\left(1-\frac{p_E+p_B}{q_H}\right).\end{aligned}$$

$$\begin{aligned}\pi_E(q_H) &= \left(p_E-c_E\right)\left(1-\theta_{M_E}\right) \\ & \left(p_E-c_E\right)\left(1-\frac{p_E+p_B}{q_H}\right).\end{aligned}$$

Maximizing $\pi_I(q_L)$and $\pi_E(q_H)$ and yield the optimal price in the markets:

$$\begin{aligned}p_E^* &= \frac{q_H+c_E-p_B^*}{2} \\ &= \frac{q_H+2c_E}{3}.\end{aligned}$$

$$\begin{aligned}p_B^* &= \frac{q_H-p_E^*}{2} \\ &= \frac{q_H-c_E}{3}.\end{aligned}$$

and this gives firms the profit $\pi_I^*(q_L)$ and $\pi_E^*(q_H)$, where

$$\pi_I^*(q_L)=\pi_E^*(q_H)=\frac{\left(q_H-p_E^*\right)^2}{4q_H}=\frac{\left(q_H-c_E\right)^2}{9q_H}.$$

Suppose $c_I < c_2$.Now there are three sets of equilibrium candidates. We first consider the first set of equilibrium candidates (i.e. configuration 1). In this configuration, firms share the markets. Firm I and firm E then receive a profit $\left(q_L-c_I\right)^2/4q_L+$

$(q_H - q_L + c_I - c_E)^2/9(q_H - q_L)$ and $(q_H - q_L + c_I - c_E)^2/9(q_H - q_L)$ respectively. If firm I deviates and undercuts its price below $c_E - (q_H - q_L)$ in market A, it then receives a profit $(q_L - c_I)^2/4q_L$. This deviation is never profitable for firm I. Another possible deviation for firm I is to exit market A (equivalently setting its price above p_E^* in market A and this is one of the best possible deviation), it is possible and firm E chooses its monopoly price in the market to maximizes its profit and firm I obtains a maximum profit equal to $(q_H - p_E^*)^2/4q_H = (q_H - c_E)^2/(3q_H - q_L)$ by setting p_I^* as a best response to p_E^*.This deviation is never profitable given that the $(q_H - q_L)(q_L - c_I) - 2(c_I q_H - c_E q_L) > 0$. There is no profitable deviation for firm E.

We next consider the second set of equilibrium candidates (configuration 2), In this configuration, Firm I undercuts its price to firm E's marginal cost in market A and profit for firm I is $(q_L - c_I)^2/4q_L$. If firm I deviates and exits market A (equivalently setting its price above p_E^* in market A and this is one of the best possible deviation), then firm E chooses its monopoly price in the market to maximizes its profit and firm I obtains a profit equal to $(q_H - p_E^*)^2/4q_H$. This deviation is profitable if and only if $(q_H - p_E^*)^2/4q_H > (q_L - c_I)^2/4q_L$. In other words, firm I does not undercut its price if $p_E^* \geq q_H - (q_L - c_I)\sqrt{q_H/q_L}$. One of the possible deviations for firm I is to sells its product at a price, $(q_L - q_H + c_I + p_E^*)/2$ and receives an extra profit $(q_H - q_L + c_I - p_E^*)^2/4(q_H - q_L)$. Such deviation is not profitable if $p_E^* \geq q_H - q_L + c_I$. Now, we consider the possible deviations for firm E. Given $p_I^* \leq c_E - (q_H - q_L)$, firm E has no profitable deviation in this configuration.

We now consider the last set of equilibrium candidates (configuration 3). Firm I leaves market A for firm E and each firm receives a profit for $(q_H - c_E)^2/9q_H$. If firm I deviates and undercuts its price in market A (that is setting its price below $c_E - (q_H - q_L)$) in market A and this is the best possible deviation, then firm I chooses its price in market B as a best response to p_E^* and firm I obtains a profit equal to $(q_L - c_I)^2/4q_L$. This deviation is not profitable if $c_I > c_4$, where

$$c_4 = q_L - \frac{2}{3}(q_H - c_E)\sqrt{\frac{q_L}{q_H}} < c_2.$$

Now, we consider another possible deviation, which is firm I sets its price equal to $(q_L - q_H + c_I + p_E^*)/2$ and share the market with firm E. Firm I then receives a profit, $(q_L - c_I)^2/9q_L$. Given that the best response for the entrant in configuration 2 and 3 are the same, it is possible for firm I to deviate and this deviation is always profitable when $c_I \geq c_2$ holds. There is no profitable deviation for firm E.

First, we suppose $c_I \geq c_2 > c_4$, in this case we have two equilibria in the subgame, where

$$\begin{aligned} p_I^* &\leq c_E - (q_H - q_L), \\ p_E^* &\geq q_H - (q_L - c_I)\sqrt{q_H/q_L} \text{ and} \\ p_I^* + p_B^* &= (q_L + c_I)/2 \end{aligned}$$

$$\begin{aligned} p_I^{**} &> (q_H + 2c_E)/3, \\ p_E^{**} &= (q_H + 2c_E)/3 \text{ and} \\ p_B^{**} &= (q_H - c_E)/3 \end{aligned}$$

On the other hand, if $c_I < c_2$, we also have two subgame perfect equilibria in the subgame, where

$$\begin{aligned} p_I^* &\leq c_E - (q_H - q_L), \\ p_E^* &\geq q_H - (q_L - c_I)\sqrt{q_H/q_L} \text{ and} \\ p_I^* + p_B^* &= (q_L + c_I)/2 \end{aligned}$$

$$\begin{aligned} p_I^{***} &= (q_L - q_H + c_E + 2c_I)/3, \\ p_E^{***} &= (q_H - q_L + 2c_E + c_I)/3 \text{ and} \\ p_B^{***} &= (2q_H + q_L - 2c_E - c_I)/6 \end{aligned}$$

We assume only the second type of equilibrium will be played. This makes sense. Both firm I and firm E receive a higher profit if firm I agrees to leave the market and firm E agrees to take the market. The second type of equilibrium is Pareto optimal given that $\pi_I^*(p_I^*, p_E^*, p_B^*) \leq \pi_I^*(p_I^{**}, p_E^{**}, p_B^{**})$ when $c_I \geq c_2$ and $\pi_I^*(p_I^*, p_E^*, p_B^*) \leq \pi_I^*(p_I^{***}, p_E^{***}, p_B^{***})$ when $c_I < c_2$. This concludes the proof. ■

Proof of Proposition 7

Suppose firm E has entered market A in the previous stage and if both firms offer the same quality level, firms merely compete in price at this stage and consumers only buy from the firm with the lowest price. There are three different possible cases. We first consider the case in which firm I sets $p_I < p_E$. Strategies that support this equilibrium are as follows. At time $t = 1$, firm I charges at a price p_I in market A where $p_I < p_E$. Given $p_I^* < p_E$, consumer only buys from firm I and hence firm E makes no sales. In this equilibrium, firm E can deviates and undercuts its price to its cost, c_E. Therefore, firm I always undercuts its price and sets $p_I^* \leq c_E$. In this case, firm E cannot make any profit if it further undercuts its cost. Therefore, firm E always exit when $p_I^* \leq c_E$ and firm I keeps its monopoly position in market A. Now firm I acts as a monopoly with the constraint $p_I^* \leq c_E$, it maximizes its profit by choosing the optimal price in market B, p_B and its profit function is given as follow:

$$\pi_{I_1}(q) = (p_I + p_B - c_I)\left(1 - \frac{p_I + p_B}{q}\right). \qquad \text{(B.7)}$$

Maximizing equation B.7 yields the optimal price in market B:

$$p_I^* + p_B^* = \frac{q + c_I}{2} = \frac{c_I - 2c_E}{2}. \qquad \text{(B.8)}$$

and it gives firm I a optimal profit $\pi_I^*(q)$, where

$$\begin{aligned} \pi_{I_1}^*(q) &= (p_I^* + p_B^* - c_I)\left(1 - \frac{p_I^* + p_B^*}{q}\right) \qquad \text{(B.9)} \\ &= \left(\frac{q + c_I}{2} - c_I\right)\left(\frac{q - c_I}{2q}\right) \\ &= \frac{(q - c_I)^2}{4q}. \end{aligned}$$

We next consider the case in which firm I sets $p_I^* = p_E$, here we assume firms share the market equally. Therefore, firms maximize profits by choosing the optimal price in its market. Their profit functions are given as follow:

$$\begin{aligned}\pi_{I_2}(q) &= (p_I - c_I)\{\frac{1}{2}\left(1 - \frac{p_I + p_B}{q}\right)\} + p_B\left(1 - \frac{p_I + p_B}{q}\right) && \text{(B.10)}\\ &= (p_E - c_I)\{\frac{1}{2}\left(1 - \frac{p_E + p_B}{q}\right)\} + p_B\left(1 - \frac{p_E + p_B}{q}\right). && \text{(B.11)}\end{aligned}$$

$$\pi_{E_2}(q) = (p_E - c_E)\{\frac{1}{2}\left(1 - \frac{p_E + p_B}{q}\right)\}. \tag{B.12}$$

Maximizing equation B.10 and B.12 yield the optimal prices:

$$\begin{aligned}p_E^* &= \frac{q + c_E - p_B^*}{2}\\ &= \frac{2q - c_I + 4c_E}{5}.\end{aligned}$$

$$\begin{aligned}p_B^* &= \frac{2q + c_I - 3p_E^*}{4}\\ &= \frac{q + 2c_I - 3c_E}{5}.\end{aligned}$$

and it gives firms the optimal profits $\pi_{I_2}^*(q)$ and $\pi_{E_2}^*(q)$, where

$$\pi^*_{I_2}(q) = (p^*_I - c_I)\{\frac{1}{2}\left(1 - \frac{p^*_E + p^*_B}{q}\right)\} + p^*_B\left(1 - \frac{p^*_E + p^*_B}{q}\right) \tag{B.13}$$
$$= \frac{(2q - c_I - p^*_E)^2}{16q}$$
$$= \frac{(2q - c_I - c_E)^2}{25q}.$$

$$\pi^*_{E_2}(q) = (p^*_E - c_E)\left\{\frac{1}{2}\left(1 - \frac{p^*_E + p^*_B}{q}\right)\right\} \tag{B.14}$$
$$= \left(\frac{p^*_E - c_E}{2}\right)\left(\frac{2q - c_I - p^*_E}{2q}\right)$$
$$= \frac{(2q - c_I - c_E)^2}{25q}.$$

Now consider the case in which firm I sets $p_I > p_E$, that is firm I leaves the market after firm E's entry. Firm E then acts as a monopoly and firms maximize profits by choosing the optimal price in its market. Their profit functions are given as follow:

$$\pi_{I_3}(q) = (p_B)\left(1 - \frac{p_E + p_B}{q}\right). \tag{B.15}$$

$$\pi_{E_3}(q_H) = (p_E - c_E)\left(1 - \frac{p_E + p_B}{q_H}\right). \tag{B.16}$$

Maximizing equation B.7 and B.16 yield the optimal prices:

$$
\begin{aligned}
p_E^* &= \frac{q + c_E - p_B^*}{2} \\
&\frac{q + 2c_E}{3}.
\end{aligned}
$$

$$
\begin{aligned}
p_B^* &= \frac{q - p_E^*}{2} \\
&\frac{q - c_E}{3}.
\end{aligned}
$$

and the optimal profits are $\pi_I^*(q)$ and $\pi_E^*(q)$, where

$$\pi_{I_3}^*(q) = \frac{\left(q - p_E^*\right)^2}{4q} = \frac{\left(q - c_E\right)^2}{9q}.$$

$$\pi_{E_3}^*(q) = \frac{\left(q - c_E\right)^2}{9q}.$$

Firm E never deviates in the any set of the above equilibrium candidates, as it always makes non-positive profit if it deviates.

Now, consider the following profitable deviations for firm I in the first set of equilibrium candidate. Suppose firm I deviates from the first set of equilibrium candidates and leaves the market, firm I then receives a maximum profit, $\left(q - p_E^*\right)^2 / 4q$. It is possible for firm I to deviate by choosing p_B^* as a best response to $p_E^* = (q + c_E - p_B^*)/2$ and such deviation is not profitable if $p_E^* \geq c_I$. Another possible deviations for firm I is to set its price equal to p_E^* and chooses p_B^* as a best response to $p_E^* = (q + c_E - p_B^*)/2$.

This deviation gives firm I a profit, $\left(2q - c_I - p_E^*\right)^2/16q$. This deviation is not profitable if $p_E^* \geq c_I$.

We next consider the second set of equilibrium candidates. Firm I can deviates and undercuts its price to $p_I^* \leq p_E^*$. Such deviation is profitable if and only if $\pi_{I_1}^*(q) \geq \pi_{I_2}^*(q)$ and and simple algebra shows that $\pi_{I_1}^*(q) \geq \pi_{I_2}^*(q)$ if and only if $c_I \leq c_3$ where

$$c_3 = \frac{q + 2c_E}{3}$$

Another possible deviation for firm I is to leave the market for the entrant. It is profitable if and only if $\pi_{I_3}^*(q) \geq \pi_{I_2}^*(q)$ and it it is equivalent to $c_I > c_3$. Therefore, if $c_I \leq c_3$, firm I always deviates from the second set of equilibrium candidates and undercuts its price. On the other hand, if $c_I > c_3$, firm I always deviates and leaves the market. Thus, setting exactly the same price as the entrant's is always not a best response of firm I.

We then consider the last set of equilibrium candidates, firm I deviates and undercuts its price below c_E. This deviation is profitable if and only if $\pi_{I_1}^*(q) \geq \pi_{I_3}^*(q)$ and simple algebra shows that $\pi_{I_1}^*(q) \geq \pi_{I_3}^*(q)$ if and only if $c_I \leq c_3$. Another possible deviation for firm I is to set $p_I^* = p_E^*$. Such deviation is profitable if and only if $\pi_{I_2}^*(q) \geq \pi_{I_3}^*(q)$ and it is equivalent to $c_I \leq c_3$.

Therefore, in summary, we have the following equilibria. If $c_I \leq c_3$, then there is only an equilibrium exists, where

$$\begin{aligned} p_I^* &\leq c_E, \\ p_E^* &\geq c_I \text{ and} \\ p_I^* + p_B^* &= (q + c_I)/2 \end{aligned}$$

On the other hand, if $c_I > c_3$, there are two subgame perfect equilibria exist, where

$$\begin{aligned} p_I^* &\leq c_E, \\ p_E^* &\geq c_I \text{ and} \\ p_I^* + p_B^* &= (q + c_I)/2 \end{aligned}$$

$$\begin{aligned} p_I^{**} &> p_E^{**}, \\ p_E^{**} &= (q + 2c_E)/3 \text{ and} \\ p_B^{**} &= (q - c_E)/3 \end{aligned}$$

We assume only the second type of equilibrium will be played. This makes sense. Both firm I and firm E receive a higher profit if firm I agrees to leave the market and firm E agrees to take the market. The second type of equilibrium is Pareto optimal given that $\pi_I^*(p_I^*, p_E^*, p_B^*) \geq \pi_I^*(p_I^{**}, p_E^{**}, p_B^{**})$ when $c_I > c_3$. ■

Proof of Proposition 8

Suppose firm E has entered market A, and predatory pricing is possible; that is, an incumbent can price below marginal cost and squeeze the entrant out of the market. Consider the case in which firm I introduces a high-quality product.

Suppose firm I has a low marginal cost, $0 \leq c_I < c_3(q_H)$. Firm I always undercuts its price and does not accommodate entry regardless of the quality level of firm E and obtains an expected payoff, π_E^e

$$\pi_E^e = -F_E.$$

Hence it is always optimal for firm E to stay out of the market.

Now suppose firm I has a marginal cost, $c_I \in [c_3(q_H), c_1]$. Firm I only undercuts its price below its marginal cost and squeeze firm E out of the market if firm E is a low-quality type and firm I finds it profitable to leave the market to firm E with high-quality product. Firm E receives an expected payoff, π_E^e

$$\pi_E^e = \gamma \pi_{E_M}(q_H) - F_E.$$

It is profitable for firm E to enter the market if $\gamma \geq \frac{F_E}{\pi_{E_M}(q_H)}$

If c_I increases and lies above c_1, entering market A is a dominant strategy for firm E, according to proposition and , firm I always welcomes any firm E to take over its market and firm E receives an expected payoff, π_E^e

$$\pi_E^e = \gamma \pi_{E_M}(q_H) + (1-\gamma)\pi_{E_M}(q_L) - F_E.$$

Firm E finds it profitable to enter the market if and only if $\gamma > \frac{F_E - \pi_E(q_L)}{\pi_{E_M}(q_H) - \pi_E(q_L)}$

Now consider the case in which firm I introduces a low-quality product. If firm I produces at a low marginal cost (i.e. $c_I \leq c_2$), firm I always prefers to lower its price in the competitive market and deters entry regardless of the type of firm E and firm E obtains an expected payoff, π_E^e

$$\pi_E^e = -F_E.$$

if it enters the market. It is always optimal for firm E to stay out of the market. When firm I has a sufficiently low marginal (i.e., $c_I \in [c_2, c_3(q_L)]$), from Proposition and , firm I finds it profitable to share the market with the entrant if the entrant has an quality advantage over it, otherwise, firm I always undercuts its price. In this case, firm E receives an expected payoff, π_E^e

$$\pi_E^e = \gamma \pi_{E_D}(q_H) - F_E.$$

Firm E finds it profitable to enter the market if $\gamma \geq \frac{F_E}{\pi_{E_D}(q_H)}$.

When $c_I > c_3(q_L)$, firm I always accommodates entry regardless of the type of firm E. Thus, firm E obtains an expected payoff, π_E^e

$$\pi_E^e = \gamma \pi_{E_M}(q_H) + (1-\gamma)\pi_{E_M}(q_L) - F_E,$$

if it enters the market and it is profitable for it to enter the market if $\gamma \geq \frac{F_E - \pi_{E_M}(q_L)}{\pi_{E_M}(q_H) - \pi_{E_M}(q_L)}$.This concludes the proof. ■

Proof of Proposition 9

Suppose predatory pricing is not possible, the entrant always enter the market as it has a cost advantage over the incumbent and leaving the market to the entrant is the only profitable option to the incumbent. Total expected producer surplus when the incumbent leaves the market to the entrant and each firm operates alone in its own market is

$$\begin{aligned} PS_{ban} &= \gamma \int\limits_{\frac{2q_H+c_E}{3q_H}}^{1} [p_E^* + p_B^* - c_E] \ d\theta + (1-\gamma) \int\limits_{\frac{2q_L+c_E}{3q_L}}^{1} [p_E^* + p_B^* - c_E] \ d\theta - F_E \\ &= \gamma \frac{2(q_H - c_E)^2}{9q_H} + (1-\gamma)\frac{2(q_L - c_E)^2}{9q_L} - F_E. \end{aligned}$$

where p_E^* is the equilibrium price offered by the entrant in market A. When the incumbent leaves the market to the entrant, total expected consumer surplus is

$$\begin{aligned} CS_{ban} &= \gamma \int\limits_{\frac{2q_H+c_E}{3q_H}}^{1} [\theta q_H - p_E^* - p_B^*] \ d\theta + (1-\gamma) \int\limits_{\frac{2q_L+c_E}{3q_L}}^{1} [\theta q_L - p_E^* - p_B^*] \ d\theta \\ &= \gamma \frac{(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{(q_L - c_E)^2}{18q_L}. \end{aligned}$$

and total welfare is

$$\begin{aligned} W_{ban} &= \gamma \int\limits_{\frac{2q_H+c_E}{3q_H}}^{1} [\theta q_H - c_E] \ d\theta + (1-\gamma) \int\limits_{\frac{2q_L+c_E}{3q_L}}^{1} [\theta q_L - c_E] \ d\theta - F_E. \\ &= \gamma \frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{5(q_L - c_E)^2}{18q_L} - F_E. \end{aligned}$$

Given predatory pricing is possible, the probability for entrant to have high quality is sufficiently high $\gamma \geq \gamma^{**}$ (i.e. $F_E \leq \gamma \pi_{E_D}(q_H)$) and the incumbent introduces a high quality product, the incumbent takes the entire market if the entrant introduces a low quality product and the incumbent leaves the market to the entrant if the entrant introduces a high quality product. Total expected producer surplus when $c_I \in [c_3(q_H), c_1]$ is

$$\begin{aligned} PS_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [p_E^* + p_B^* - c_E] \ d\theta + (1-\gamma) \int_{\frac{q_H + c_I}{2q_H}}^{1} [p_I^* + p_B^* - c_I] \ d\theta - F_E. \\ &= \gamma \frac{2(q_H - c_E)^2}{9q_H} + (1-\gamma)\frac{(q_H - c_I)^2}{4q_H} - F_E. \end{aligned}$$

where p_I^*, p_E^* and p_B^* are the equilibrium price in each market. Total expected consumer surplus with predatory pricing

$$\begin{aligned} CS_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [\theta q_H - p_E^* - p_B^*] \ d\theta + (1-\gamma) \int_{\frac{q_H + c_I}{2q_H}}^{1} [\theta q_H - p_I^* - p_B^*] \ d\theta \\ &= \gamma \frac{(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{(q_H - c_I)^2}{8q_H}. \end{aligned}$$

and total welfare is

$$\begin{aligned} W_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [\theta q_H - c_E] \ d\theta + (1-\gamma) \int_{\frac{q_H + c_I}{2q_H}}^{1} [\theta q_H - c_I] \ d\theta - F_E. \\ &= \gamma \frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{3(q_H - c_I)^2}{8q_H} - F_E. \end{aligned}$$

Simple algebra shows that $W_{pre} \leq W_{ban}$ if and only if $c_I \geq c_I^*$, where

$$c_I^*(q_H) = q_H - \frac{2\sqrt{5}}{3\sqrt{3}}(q_L - c_E)\sqrt{\frac{q_H}{q_L}} < c_1$$

Incumbent undercuts the price and deters the entry regardless of the type of the entrant when $c_I \leq c_3(q_H)$ and total expected producer surplus is

$$PS_{pre} = \int_{\frac{q_H+c_I}{2q_H}}^{1} [p_I^* + p_B^* - c_I] \ d\theta$$
$$= \frac{(q_H - c_I)^2}{4q_H}$$

Total expected consumer surplus

$$CS_{pre} = \int_{\frac{q_H+c_I}{2q_H}}^{1} [\theta q_H - p_I^* - p_B^*] \ d\theta$$
$$= \frac{(q_H - c_I)^2}{8q_H}.$$

and total welfare with only the incumbent in both markets is

$$W_{pre} = \int_{\frac{q_H+c_I}{2q_H}}^{1} [\theta q_H - c_I] \ d\theta$$
$$= \frac{3(q_H - c_I)^2}{8q_H}.$$

Simple algebra shows that $W_{pre} \leq W_{ban}$ if and only if $c_I \geq c_I^{**}$, where

$$c_I^{**}(q_H) = q_H - 2\sqrt{\frac{2q_H}{3}\left[\gamma\frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{5(q_L - c_E)^2}{18q_L} - F_E\right]}$$

Given predatory pricing is possible, the probability for entrant to have high quality is sufficiently high $\gamma \geq \gamma^{**}$ (i.e. $F_E \leq \gamma\pi_{E_D}(q_H)$) and the incumbent introduces a low quality product, the incumbent takes the entire market if the entrant introduces a low quality product and the incumbent leaves the market to the entrant if the entrant introduces a high quality product. Total expected producer surplus when $c_I \in [c_{2,}$ $c_3(q_L)]$ is

$$\begin{aligned} PS_{pre} &= \gamma \int_{\frac{2q_H+c_E}{3q_H}}^{1} [p_E^* + p_B^* - c_E] \; d\theta + (1-\gamma) \int_{\frac{q_L+c_I}{2q_L}}^{1} [p_I^* + p_B^* - c_I] \; d\theta - F_E. \\ &= \gamma \frac{2(q_H - c_E)^2}{9q_H} + (1-\gamma)\frac{(q_L - c_I)^2}{4q_L} - F_E. \end{aligned}$$

where p_I^*, p_E^* and p_B^* are the equilibrium price in each market. Total expected consumer surplus with predatory pricing

$$\begin{aligned} CS_{pre} &= \gamma \int_{\frac{2q_H+c_E}{3q_H}}^{1} [\theta q_H - p_E^* - p_B^*] \; d\theta + (1-\gamma) \int_{\frac{q_L+c_I}{2q_L}}^{1} [\theta q_L - p_I^* - p_B^*] \; d\theta. \\ &= \gamma \frac{(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{(q_L - c_I)^2}{8q_L}. \end{aligned}$$

and total welfare is

$$\begin{aligned} W_{pre} &= \gamma \int_{\frac{2q_H+c_E}{3q_H}}^{1} [\theta q_H - c_E] \; d\theta + (1-\gamma) \int_{\frac{q_L+c_I}{2q_L}}^{1} [\theta q_L - c_I] \; d\theta - F_E. \\ &= \gamma \frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{3(q_L - c_I)^2}{8q_L} - F_E. \end{aligned}$$

Simple algebra shows that $W_{pre} \leq W_{ban}$ if and only if $c_I \geq c_I^{***}$, where

$$c_I^*(q_L) = q_L - \frac{2\sqrt{5}}{3\sqrt{3}} (q_L - c_E) \sqrt{\frac{q_H}{q_L}}.$$

Incumbent undercuts the price and deters the entry regardless of the type of the entrant when $c_I \leq c_2$ and total expected producer surplus is

$$\begin{aligned} PS_{pre} &= \int_{\frac{q_L + c_I}{2q_L}}^{1} \left[p_I^* + p_B^* - c_I\right] \, d\theta. \\ &= \frac{(q_L - c_I)^2}{4q_L}. \end{aligned}$$

Total expected consumer surplus

$$\begin{aligned} CS_{pre} &= \int_{\frac{q_L + c_I}{2q_L}}^{1} \left[\theta q_L - p_I^* - p_B^*\right] \, d\theta. \\ &= \frac{(q_L - c_I)^2}{8q_L}. \end{aligned}$$

and total welfare with only the incumbent in both markets is

$$\begin{aligned} W_{pre} &= \int_{\frac{q_L + c_I}{2q_L}}^{1} \left[\theta q_L - c_I\right] \, d\theta. \\ &= \frac{3(q_L - c_I)^2}{8q_L}. \end{aligned}$$

Simple algebra shows that $W_{pre} \leq W_{ban}$ if and only if $c_I \geq c_I^{***}$, where

$$c_I^{**}(q_L) = q_L - 2\sqrt{\frac{2q_L}{3}\left[\gamma \frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{5(q_L - c_E)^2}{18q_L} - F_E\right]}$$

This concludes the proof. ■

Appendix C
Appendix to Chapter 3

Proof of Proposition 10

Consider each scenario in turn. By Lemma , if $T > B + b$, it is an equilibrium for all consumers to single-home, when the developer subscribes to only one single platform. It is also an equilibrium when developer subscribe to both platform. (Recall that in all other cases, there exist some consumers that subscribe to both platforms). Suppose that the strategy of the developer prescribes to support both platforms and sets $p*$ in the advertising market with platforms setting their prices P_1^* and P_2^* respectively, and then to follow the equilibrium in which consumers equally split between platforms, with $N_1^* = N_2^* = \frac{1}{2}$ and $N_A^* = 1$. This strategy gives the developer an equilibrium profit π_{12}^* where

$$
\begin{aligned}
\pi_{12}^* &= \left[\frac{5t - 6w(N_1^* - 1)}{15} + c - c\right] a \\
&= t\left[\frac{1}{3} + \frac{w}{5t}\right]^2
\end{aligned}
$$

Suppose that the developer only supports platform 1. More consumers subscribe to platform 1 when an extra benefit can be generated with the application on the platform. Supporting a single platform then yields the profit π_1^* with $N_1^* = N_A^* = \frac{T-B}{2(T-B)-b}$ and $N_2^* = \frac{T-B-b}{2(T-B)-b}$, where

$$\begin{aligned}\pi_1^* &= \left[\frac{5t - 3w(N_2^* - N_1^*)}{15} + c - c\right] a \\ &= t\left[\frac{1}{3} + \frac{wb}{5t\left[2\left(T - B\right) - b\right]}\right]^2\end{aligned}$$

Simple algebra shows that $\pi_1^* \geq \pi_{12}^*$ never holds.

If now consumers are concerned more about the network effect and T lies between $B + b$ and $B - b$, supposing that the developer supports platform 1 only, having all the consumers subscribe to platform 1 only, leaves the developer an equilibrium profit π_1^{**}, where

$$\begin{aligned}\pi_1^{**} &= \left[\frac{t + 2c}{3} - c\right] a \\ &= \left[\frac{t - c}{6}\right]\end{aligned}$$

Now suppose the developer supports both platforms. Then the platforms share the consumers and the developer receives a profit π_{12}^{**}, where

$$\begin{aligned}\pi_{12}^{**} &= \left[\frac{5t - 6w(N_1^* - 1)}{15} + c - c\right] a \\ &= t\left[\frac{1}{3} + \frac{2wT}{5t\left(T + B + b\right)}\right]^2\end{aligned}$$

$\pi_1^{**} \geq \pi_{12}^{**}$ if and only if $w \leq w_1$

$$w_1 = \frac{5t\left(T + B + b\right)\left(t - c - 2\right)}{12T}$$

Now, consider the special case when $T < B - b$ and $T > b$. Suppose the developer only supports platform 1. More consumers subscribe to platform 1 when an extra benefit can be generated with the application on the platform and it receives an equilibrium profit, π_1^{***}

$$\begin{aligned}
\pi_1^{***} &= \left[\frac{5t - 3w(N_2^* - N_1^*)}{15} + c - c\right] a \\
&= t\left[\frac{1}{3} + \frac{wBb}{5t\left[(B+T)(B-T) + Tb\right]}\right]^2
\end{aligned}$$

Suppose the developer supports both platforms, it receives an equilibrium profit π_{12}^{**}. $\pi_1^{***} \geq \pi_{12}^{**}$ if and only if $h_1 \geq 0$. If b is large $(T < b < B)$, and $T < B - b$, developer finds it profitable to single home, $\pi_1^{**} \geq \pi_{12}^{**}$ if and only if $w \leq w_1$. ■

Proof of Proposition 11

Consider each scenario in turn. By Lemma 1, if $T > B + b$, all consumers single-home, regardless of the subscription decision of the developer. (In all other cases, there exists some consumers that subscribe to both platform)

Suppose the strategy of the developer prescribes to support both platforms and sets $p*$ in the advertising market, the platforms set their prices P_1^* and P_2^* respectively, in equilibrium consumers equally split between platforms, with $N_1^* = N_2^* = \frac{1}{2}$ and $N_A^* = 1$. This strategy gives platform 1 an equilibrium profit $\Pi_{1(12)}^*$ where

$$\begin{aligned}\Pi^*_{1(12)} &= \left[\frac{5t-3w(1-N_1^*)}{15}+c-c\right]A_1\\ &= t\left[\frac{1}{3}+\frac{w}{10t}\right]^2\end{aligned}$$

Suppose the developer only supports platform 1 More consumers subscribe to platform 1 when extra benefit can be generate with the application on the platform. Supporting a single platform then yields the profit $\Pi^*_{1(1)}$ with $N_1^* = N_A^* = \frac{T-B}{2(T-B)-b}$ and $N_2^* = \frac{T-B-b}{2(T-B)-b}$, where

$$\begin{aligned}\Pi^*_{1(1)} &= \left[\frac{5t-3w(N_2^*-N_1^*)}{15}+c-c\right]A_1\\ &= t\left[\frac{1}{3}+\frac{wb}{5t\left[2\left(T-B\right)-b\right]}\right]^2\end{aligned}$$

Simple algebra shows that $\Pi^*_{1(1)} \geq \Pi^*_{1(12)}$ if $T < B + \frac{3b}{2}$.

Suppose now consumers are more interested in the network effect, T lies between $B+b$ and $B-b$, and the developer supports platform 1 only. Having all the consumers subscribe to platform 1 only leaves the developer an equilibrium profit $\Pi^{**}_{1(1)}$, where

$$\begin{aligned}\Pi^{**}_{1(1)} &= \left[\frac{t+2c}{3}-c\right]A_1\\ &= \left[\frac{t-c}{6}\right]\end{aligned}$$

Now, suppose developer supports both platforms, the platforms share the consumers, and developer receives a profit $\Pi^{**}_{1(12)}$, where

$$\begin{aligned}
\Pi^{**}_{1(12)} &= \left[\frac{5t - 3w(1 - N_1^*)}{15} + c - c\right] A_1 \\
&= t\left[\frac{1}{3} - \frac{wT}{5t\,(T + B + b)}\right]^2
\end{aligned}$$

$\Pi^{**}_{1(1)} \geq \Pi^{**}_{1(12)}$ if and only if $w \geq w_2$

$$w_2 = \frac{-5t\,(T + B + b)\,(t - c - 2)}{6T} = -2w_1$$

Now, consider the special case when $T < B - b$ and $T > b$. Suppose the developer only supports platform 1, more consumers subscribe to platform 1 when an extra benefit can be generated with the application on the platform and it receives an equilibrium profit, $\Pi^{***}_{1(12)}$

$$\begin{aligned}
\Pi^{***}_{1(1)} &= \left[\frac{5t - 3w(N_2^* - N_1^*)}{15} + c - c\right] A_1 \\
&= t\left[\frac{1}{3} + \frac{wBb}{5t\left[(B + T)\,(B - T) + Tb\right]}\right]^2
\end{aligned}$$

Suppose the developer supports both platforms, it receives an equilibrium profit, $\Pi^{**}_{1(12)}$. $\Pi^{***}_{1(1)} \geq \Pi^{**}_{1(12)}$ if and only if $h_2 \geq 0$, where

$$h_2 = Bb\,(T + B + b) + T\left[(B + T)\,(B - T) + Tb\right]$$

This condition always holds.

If b is large $(T < b < B)$, and $T < B - b$, developer finds it profitable to single-home, $\Pi^{***}_{1(1)} \geq \Pi^{**}_{1(12)}$ if and only if $w \geq w_2$. ■

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