Modelling Output and Inflation Using Direct Measures of Expectations

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Abstract

This thesis provides an applied macroeconometric study of the determination of output and inflation and of stabilization policy. Its focus is on the time series analysis of output and inflation, concentrating on the use of direct measures of expectations and data obtained in real time, and modelled in multivariate settings. Direct measures of expectations are useful in this study as they allow a better understanding of the expectations formation processes as well as the role played by expectations in the output and inflation dynamics and in the conduct of monetary policy. The use of real time data is important as it focuses on information sets available when decisions are made. This study considers three empirical exercises using actual and expected output and inflation series for the UK and US for the last forty years and making use of different sources of survey data. They are presented to build an increasingly sophisticated picture of the interactions between growth, inflation and stabilization policy. The first chapter concentrates on the measurement of actual and expected inflation and output from different sources of survey data, eventually developing a multivariate framework with which to analyse expectations at different time horizons. The second chapter concentrates on the dynamics between actual and expected inflation and output, developing a simple growth model that distinguishes between the long-run trend in output and short and medium term fluctuation around that trend, with inflation assumed to be driven by a model of time-consistent monetary policy. The last chapter builds on the previous ones by considering a small macro model of the economy based on three main behavioural relationships: a Phillips curve, an IS curve and a monetary policy rule; its focus is on the role of expectations in the conduct of monetary policy.

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Contents

Introduction

At the centre of macroeconomics is the relationship between economic activity and inflation. The role of expectations is crucial in this relationship and therefore understanding expectations formation processes is at the centre of policy. This understanding has evolved over time and so has accordingly the conduct of macro policy, in particular monetary policy. There already exists a rich literature on the use of survey data for the investigation of the expectations formation mechanism and its consistency with the rational expectations hypothesis; recent literature has revived the interest on the use of survey data. However, little work has been done on the simultaneous modelling of actual and expected series and inclusion of expectations in macro models and this work aims to fill that gap.

The purpose of this thesis is to study direct measures of expectations derived from different sources of survey data and incorporate those into macro modelling in order to understand better the expectations formation process itself as well as the role played by expectations in the inflation-output tradeoff and in the conduct of monetary policy. The thesis starts off by analysing sources of expectations and providing some statistics on those measures; then it creates a more elaborated set up to analyse actual and expected series within a cointegrated VAR. This will be the subject of the next chapter. The chapters that follow are yet more elaborated frameworks. The measures of expectations that have been studied in the first chapter are now incorporated into small macro models of output and inflation. The primary goal is to understand this tradeoff and the role played by expectations in this context. Chapter 3 elaborates on the previous chapter by including a new variable,

Chapter 1
Actual and Expected Output and Inflation in the UK and US

1.1 Introduction

This chapter is concerned with the measurement of inflation and output as they actually occur and as expected. We consider US and UK datasets that highlight particular features that are more or less helpful in measuring ”actual” and ”expected” variables. In the UK data, we ignore the issue of different vintages of data and data revisions, and accordingly, we assume that the most recent (post-revision) data was known at the time decisions were made. The focus here is on direct measures of expectations which are often derived from qualitative data. The use of qualitative data requires a procedure to convert it into a time series, which raises some issues, such as the introduction of measurement errors as we shall discuss. For the US, we have direct, quantitative measures of expectations straight from the survey. This type of survey does not require a conversion procedure and our focus this time is on the modelling of actual and expected data in real-time. Special attention is made to the timing of the variables. We take into account the fact that (actual) data is released with a lag; however, perceived values of output and inflation of the current period are available together with (multi-period) forecasts over next periods. Our objective is to study those series in a multivariate framework. The focus on the UK is on the conversion procedure of qualitative data and the nature of the expectational errors. The focus on the US is on the time series properties of the time t measures of the variables on first-release and the modelling of expectations at different time horizons.

The structure of the chapter is as follows: section 2 discusses the use of direct measures of expectations, analysing the issues raised by the use of qualitative and quantitative survey data; section 3 analyses inflation and output in the UK Manufacturing Sector, with expectations data provided by the Confederation of British Industry (CBI)'s Survey of Industrial Trends, and using the conversion procedure by Carlson and Parkin (1975) and Lee (1994); section 4 analyses US output and inflation in a cointegrated VAR framework, making use of direct measures of expectations provided by the Survey of Professional Forecasters; section 5 concludes

1.2 The Use of Direct Measures of Expectations in Macroeconomics

The importance of direct measures of expectations has long been acknowledged in the literature, for allowing the study of the expectations formation process itself as expectations play such a crucial role in economics and policy decision-making. In the awake of the Phillips curve and the 1970s experience, the role of expectations was emphasised. The period of high inflation and high unemployment would be hard to reconcile with the traditional Phillips curve, and rather than dismissing it, the relevance of expectations in explaining output-inflation tradeoff was recognised and the new Augmented-Phillips curve created. The need for understanding the expectations formation process became a priority and the surge of interest on direct measures of expectations is apparent in the literature. Innumerous studies in the 1970s and 1980s analyse survey data and test for the validity of Rational Expectations Hypothesis(REH)[1] which had become an important assumption in macro models. The interest in microfoundations that originated then, made REH a common assumption of models such as Fisher (1977) or Taylor (1980). However, without the use of direct measures of expectations, the study of expectations formation processes, in particular the REH could only be carried out in the context of a particular model and therefore any test would be a joint test of REH and the economic theory assumed to underlie the model. Further, because many models rely on the REH assumption, its test is important to determine the validity of this assumption, with the main studies in the literature analysing direct measures of expectations in terms of the rationality of their formation process or the absence of it. Besides the study of expectations in itself, the inclusion of direct measures of expectations in macro models have also received attention, in particular, in more recent years. Examples include Roberts (1995, 1997), Rudebusch (2002), who analysed the relevance of expectations in the Phillips curve, and Orphanides and Williams (2002) who analysed its relevance in the context of monetary policy rules. However, due to the presence of measurement errors, there is still some skepticism with regard to the use of survey data. Some prefer to obtain forecasts through different means. For example, expectations on inflation can be inferred by looking at the term structure of the interest rate or by making use of the REH assumption applied to a Phillips curve relationship. The latter has the immediate disadvantage of reliance on an assumption that may not hold (REH) and cannot be tested in this context; the use of the terms structure of the interest rate was shown

[1] Muth (1961) introduced the concept of model consistent rational expectations.

by Ang, Bekaert and Wei (2007) to have a relatively poor performance when compared to either the Phillips curve or survey sources of expectations, with the latter having the better forecasting performance. Further, survey data is a richer dataset, providing information on a range of variables.

The use of direct measures of expectations provided by the survey data is an important feature of this thesis. Its importance in this work is twofold - we are interested in the study of expectations processes on one hand, and the role of expectations in macroeconomic models, on the other. The latter will be the subject of chapters 2 and 3. The use of direct measures of expectations from survey data is the subject of the present chapter and we shall start by discussing it.

There are several sources of direct measures of expectations from survey data. However, we can classify the survey data into two different categories: quantitative and qualitative (or tendency). The first type requires precise quantitative answers from the respondents whereas the second category involves questions about the expected direction of change of the variable, namely, the respondents are asked to predict whether the variable will ”go up”, ”stay the same”, or ”go down”. Each type of survey data raises different issues and we can easily name advantages and disadvantages for either. The quantitative surveys provide us with a readily available time series, whereas the qualitative requires a conversion method, which may therefore introduce measurement errors into the derived time series[2]. These conversion methods rely on assumptions that often cannot be tested. On the other hand, we could claim that putting an exact figure into a forecast may not be satisfactory,

[2] Although they are free from conversion errors, the quantitative survey data may include other types of measurement errors, such as sampling or aggregation errors, for example.

and it would be more sensible for the respondents to attach some probability to their responses, reflecting uncertainty surrounding their forecast.[3] Furthermore, some argue that quantitative survey is more sensitive to the phrasing of the questions and more susceptible to sampling errors. In general, survey data may be subject to the criticism that respondents may choose to give an answer that does not reflect their opinion and therefore their answer does not reflect the way they will be behaving in economic transactions. However, survey data provide direct measures of expectations to serve as a proxy for expectations and allow the analysis of expectations formation processes. Sources of surveys of expectations are readily available for many countries theses days. In the US, there are several sources, with the most used ones being the Survey of Professional Forecasters (which will be used in the present work) and the Livingston survey as quantitative surveys. Using these sources are authors such as Turnovsky (1970), Carlson(1977), Mullineaux (1978, 1980), Keane and Runkle (1990), Rudebusch (2002), Papaikonomou and Pires (2006). In the case of the UK, popular sources of survey data on expectations include Gallup Polls, used in papers such as Carlson and Parkin (1975), Holden and Peel (1977), Evans and Gulamani (1984), Batchelor and Dua (1987); and the CBI survey which was used by Wren-Lewis (1986), Pesaran (1984, 1987) and Lee (1994) for example. In this chapter we will be analysing direct measures of expectations from each type of survey. The qualitative type comes in the form of the CBI survey data for the UK Manufacturing Sector. The Survey of Professional Forecasters will be the quantitative type of survey used to analyse expectations on the US output and inflation. These measures of expectations will later be incorporated in small

[3] Some surveys do provide this information. The Survey of Professional Forecasters is an example, although in the present work we will not be making use of this additional information.

macro models for the UK and US economies. This will be the subject of chapters 2 and 3. In the next sections, we will start by analysing different sources of survey data and issues raised by each.

1.2.1 Qualitative Survey data as direct sources of expectations

In this chapter we will make use of the CBI survey, on output and price expectations in the UK manufacturing Sector. When using tendency data, a procedure to convert it into a time-series is required. Two main methods have been used in the literature: the ”probability method” first introduced by Theil (1952) and first applied by Carlson and Parkin (1975) which is also the most widely used method, and the ”probability method” introduced by Pesaran (1984). The probability method, which will be used in this empirical work, will be described in detail in section 1.3.1. However, we can briefly describe the workings of the method, and criticisms and extensions it has been subject to throughout the years.

When using tendency data, the survey responses usually fall into categories, such as " go up", "go down", "stay the same", "don’t know". The probability method assumes a certain subjective probability density function that describes the expected value of a variable for each respondent for each month ($h_t(.)$); it further assumes that for very small changes in the variable under consideration, respondents are unable to report that they expect a change on it. Theil (1952) called it the "imperceptibility range" and this is assumed to be common to all individuals, and defined symmetrically around zero. Given this imperceptibility range, the respondents will answer “up” if the expected change is above the upper limit of the range; “down” if the expected change is below the lower limit (which is the negative of the upper limit); “no change” if the expected change is within the imperceptibility range. These are the basic principles. In order to quantify the data, we need (i) the survey results, (ii) the form of the distribution across the population (iii) the value of the limits on the "imperceptibility range". These require further assumptions which have been subject to criticisms in the literature.

With regard to the ”imperceptibility range”, issues have been raised with reference to the assumption made when estimating the limits of the range. In the original work, the range limits are estimated based on the assumption that expectations are unbiased. This is unsatisfactory when the empirical work that make use of survey data intends to perform a test of unbiasedness on expectations, as emphasised by Pesaran (1984). Bennett (1984) modifies this procedure by estimating the range making use of the realizations data instead of expectations on the future. The assumption made is that the indifference interval and distribution are invariant to whether respondents are asked about the future or the past.

Seitz (1988) introduced another modification to the method regarding the range. Instead of assuming a symmetric interval around zero, he introduced an asymmetric range and time-varying parameters, based on Cooley and Prescott (1976) model of parameter variation. Seitz tested whether the threshold depends on the level of the variable under consideration, in this case, inflation. He found that when inflation is high, individuals are more sensitive to price decreases than price increases as reflected by the fact that high inflation was associated with higher upper limit and smaller lower limit (in absolute value) of the imperceptibility range. Although this would provide with further support for this modification of the model, the evidence presented was weak, as judged by the statistical significance of inflation in regressions explaining the range limits. A further criticism the probability method has been subject to is based on the fact that the form of the subjective probability has to be assumed, in particular, with most empirical work assuming a normal distribution.[4]

This latter criticism has been address by Pesaran in 1984, with an introduction of an alternative method[5] - the Regression Method. This is an alternative method of computing average expectations from individuals categorical responses, which does not require making assumptions regarding the subjective probability distribution of respondents. This method, however, requires that the survey asks respondents on past perceptions as well as on future expectations on the variable.[6] This method consists of regressing actual changes in the variable upon the proportion of firms that report a rise as well as the proportion of firms that report a fall, as provided by their past perceptions. It is then assumed that the same coefficients would hold in an equation relating future expected changes of the variable and the proportion of firms that report a rise and a fall, as provided by their forecasts for the future period. In this way a time-series on expectations is created. Pesaran (1987) provides a study, comparing four different expectational series derived from the (i) regression method with the ones derived from the probability method, making use of three different probability distributions - (ii) uniform, (iii) logistic and (iv) normal distributions. The series show great similarity between them, being smoother than the actual series, but

[4] Wren-Lewis (1986) is an exception. In his empirical work on output expectations, he used a logistic distribution.

[5] See Pesaran (1987).

[6] As we will see in section 1.3.1, the CBI provide with both questions, although this is not the case for all surveys.

nevertheless exhibiting a high degree of correlation with the actual inflation. The correlation coefficients varied between 0.90 for the regression-based measure of expected inflation rate and 0.83 for the logistic distribution; the series derived under the assumption of normal distribution had a correlation of 0.86. These results seem to suggest, that despite the different methodologies and assumptions behind the derivations of each series, they have similar properties. However, and as mentioned before, the regression method has the advantage of avoiding strong and untested assumptions regarding, for example, the probability distribution of the respondents and hence minimising the measurement errors that incur when creating a quantitative series of expectations from qualitative survey data. However, we can use the probability method and remove some of the measurement errors. This is the approach introduced by Lee (1994). This method recognises that the series derived directly from the survey include measurement errors such as sampling errors, errors related to the use of questionnaires and conversion errors and hence it should not be regarded as a perfect measure of the true expected series. Through this method, we create a new series, which we consider to be the best proxy for the true and unobservable inflation expectations series. By assuming that the conversion procedure introduces the same type of errors when converting agents' responses on the past as when converting the responses on the future, Lee is able to derive a new expectational series. This new series, with improved properties that suggest elimination of (some) measurement errors, is taken to be the ”true”, unobservable series. Further support for this procedure has been provided by Lee and Shields (2007), where the nature of the measurement errors and its impact on the properties of the derived expected series is analysed. In particular, they address two of the main criticisms of the probability method we discussed earlier - the symmetry of the scaling parameter and the assumption on the form of the probability distribution. Through a simulation exercise, they showed the impact of (i) (wrongly) assuming a symmetric range in the probability method, when in fact should be asymmetric and of (ii) assuming a uniform distribution when in reality the normal distribution should have been chosen. Failure to make the correct assumption introduces measurement errors that alter significantly the properties of the expectational errors, in particular with respect to the efficient use of information from the part of economic agents. However, Lee's purging procedure is able to eliminate the systematic component of the conversion error and produce time-series that have the same features in terms of rationality as the data generation process and expectations formation process that have been simulated. In light of these results, this is the procedure we will be using in this chapter to obtain expectational series from the CBI survey data. This procedure is explained in detail in section 1.3.1.

1.2.2 Real-time data and quantitative measures of expectations

So far, we have discussed the use of qualitative or tendency survey. As mentioned before, there is a different type of survey which is quantitative and this is the type we will turn to in this section. As explained previously, this type of survey has the advantage of being a readily available time series, not requiring a conversion procedure, avoiding issues such as the introduction of measurement errors.[7]However, with quantitative data we can also acknowledge the possibility of measurement errors, such as sampling and aggregation errors. Other

[7] See previous section.

issues, such as real-time data and data revisions are particularly interesting to analyse. In this section, we introduce a new approach to analysing expectations formation processes and model actual, real-time data and expectations based on a multivariate setting.

Most studies in the literature analyse direct measures of expectations in terms of rationality of the expectations formation process or absence of it. This is also the case for quantitative survey data and examples include Baghestani (1994), Ball and Croushore (1998), Fair and Shiller (1989), Keane and Runkle(1990) among others, who have performed Rationality tests on this type of survey. However, in most of this work, very little attention has been paid to the long run relationship between actual and expected series. One exception is the joint work of Lahiri and Chun (1989), where they test for long run unbiasedness of expectations using the residual based approach by Engle and Granger. Instead, most rationality tests have been carried out having implicit the assumption that the expectational errors are stationary. Other authors, while recognising that the expected and actual series are non-stationary processes they restrict their analysis to the use of models that involve only first differences. The work of inter alia, Engle and Granger (1987) and Johansen (1988) has illustrated that in the presence of cointegration such models are misspecified as they ignore the cointegration relationship(s) between the levels of the actual and expected series.

With regard to the stationarity of expectational errors, when tests are provided, they are performed in a univariate setting and include a unit root test on the expectational error, i.e., on the difference between the actual and expected value of the variable.[8] Performing

[8] See, for example, Lee (1994).

a unit root test in this fashion implies prior knowledge of the cointegrating vector. This is not an issue in this case, as economic theory tell us, that there is a one-to-one correspondence between actual and expected series. However, our modelling framework, based upon a multivariate setting has other advantages over the simplified univariate setting. Using the multivariate setting instead of the more frequent univariate approach allows us to treat all variables endogenously without loss of valuable information on contemporaneous interactions. Thus, robustness in estimation and inference is increased and dynamics can be captured more concisely. By allowing for both, long-run and short-run analysis, the multivariate cointegration framework can provide additional insights on the workings of the expectations formation processes. The number and form of the long-run relationships do not need to be pre-determined, as theoretical predictions on these issues can be explicitly expressed in terms of model parameters, thus providing sets of testable restrictions. Short-run dynamics, on the other hand can be illustrated here with the use of impulse responses, shedding light on the adjustment process towards long-run equilibrium.

Another issue that has often been overlooked in the literature is the dissimilarity between vintages of data when it comes, for instance, to performing rationality tests and quite often these tests are carried out using the most recently revised data. Some data are subject to a series of revision processes. As a result, we end up with different time series on the same variable, which may have different stochastic properties. Furthermore, the issue of data revisions goes beyond its effects on expectations and, namely, rationality tests. Data revisions are very important and several studies have shown their impact on policy decision making. Orphanides (2003a), Croushore and Evans (2000) analysed the role of data revisions in monetary policy conduct. Other authors have analysed different issues regarding data revisions. Boschen and Grossman (1982) studied whether data revisions would affect people's expectations; Patterson and Heravi (1991) analysed the cointegration properties of different vintages of data, using different components of UK GDP, to name a few. Taking into account the results from the literature, and given the dissimilarities between actual (revised) data and real time data, we address the issue of data revisions by making use of real time data, provided by the Federal Reserve Bank of Philadelphia.

In the second part of this chapter, we model simultaneously US actual output and expected output at different time horizons within the multivariate cointegration framework of Johansen (1988) and its subsequent generalisation in Pesaran, Shin and Smith (2000). We also study actual and expected inflation. We use direct measures of expectations provided by the Survey of Professional Forecasters (SPF) together with real-time data for the US economy over the period 1968Q4-2001q2. The first aim of the paper is to fill a gap in the literature and actually test whether actual and expected series are cointegrated which will give validity to most of the rationality tests performed in the literature so far. Our modelling framework allows us to perform tests for long run unbiasedness and also provides an insight on how these series should be modelled together. More specifically, we address the following questions: (i) How should actual and expected series be modelled? What is the long-run behaviour of those series? (ii) Is there long-run unbiasedness? (iii) How do the different expected series at different time horizons relate to each other in the long run? (iv) How do these series behave in the short run? Statistical inference on these issues is carried out within the context of a cointegrating VAR(p). This analysis is given in section 1.4.

1.3 UK Inflation and Output Expectations; the Use of Qualitative Survey Data

In this section, we introduce the modelling framework that we wil be using to analyse actual and expected output and inflation in the UK Manufacturing Sector. The starting point is the discussion of the source of measures of expectations: the Confederation of British Industry (CBI)' Survey of Industrial trends.

1.3.1 Quantification of the CBI (qualitative) data: the Probability Method

For this empirical work, the Confederation of British Industry (CBI)'s Survey of Industrial trends is going to be the source of the expectations in inflation and output growth in the UK Manufacturing Industries. The CBI survey was first introduced in 1958. It covers a significant proportion of firms in the manufacturing sector. It asks each firm about past actual trends (i.e., perceptions about the past) as well as future expected trends in a number of variables (average selling price of their product, their export prices, costs, output, capacity). The surveys are weighted according to the size of the firms in the industries. CBI surveys were initially conducted 3 times each year, but since 1972 they have been carried out every quarter. However, the time horizon the questions refer to has not been changed and refers to four-month periods. The questions regarding output and prices are formulated in the following manner, respectively:

”Excluding seasonal variations, what has been the trend over the PAST 4 MONTHS, and what are the expected trends for the NEXT 4 MONTHS with regard to (i) the volume of Output and (ii) the average prices at which domestic orders are booked.

As an example, here is a table referring to output taken from the CBI surveys for the British Total Manufacturing in October 1998:

Trend over Past Four Months				Expected Trend over Next Four Months			
Up	Same	Down	N/A	Up	Same	Down	N/A
13	51	36	0	10	34	56	0

As we mentioned before, we will be using the probability method to convert this qualitative data to a quantitative series. This procedure is also the most widely employed in the literature. The probability method relies on several assumptions, which will be stated below and exemplified for the case of the expected output growth. The same procedure has been used to derive the inflation series[9].

The first assumption made by this method is that each firm i bases its survey responses on a subjective probability distribution, $h_i(\Delta y_{i,t+1}|\Phi_{it})$, with $\Delta y_{i,t+1} = y_{i,t+1} - y_{it}$ representing the actual change in firm i's output between period t and period $t+1$[10]. It is defined over the future changes in its output and is conditional on Φ_{it}, the information available to firm i at time t, such that $D_t y^e_{i,t+1} = E(\Delta y_{i,t+1}|\Phi_{it})$, with $D_t y^e_{i,t+1} = {}_t y^e_{i,t+1} - y_{it}$ representing expected output growth[11]. As mentioned before, it is assumed that there exists an "imperceptibility range". i.e., an expected change will be reported only when respondents expect the variable to change more than a certain threshold value, c. Hence, a firm reports that it expects an increase in its output if $D_t y^e_{i,t+1} \geq c$ and a decrease if $D_t y^e_{i,t+1} \leq -c$; otherwise reports no change. The interval $(-c, c)$ is also called the "indifference interval", and it is assumed to remain constant across firms and over time. The subjective distributions

9 The analysis follows Pesaran (1997).

10 With the variables defined in logarithms, the change in output represents in fact output growth rate.

11 Note that we used $D_t y^e_{t+1}$ rather than the common Δ to represent a change in a variable. We are making this distinction because $D_t y^e_{t+1}$ is actually defined, not as the difference between two expected output series between t and $t+1$ (that would be, in our notation, $\Delta_t y^e_{t+1}$) , but rather as the difference between expected output in $t+1$ and *actual* output in time t.

$h_i(\Delta y_{i,t+1}|\Phi_{it})$ are independent and have the same known form across the N firms. Furthermore, we assume that an "aggregate" distribution, $h(\Delta y_{t+1}|\Omega_t)$, can be derived, where $\Omega_t = U_{t=1}^{N}\Phi_{it}$ is the union of individuals firms' information set. We can also define actual percentage change in (or growth rate of)"average" output of the firms in the industry as $\Delta y_t = \sum_{i=1}^{N}\omega_i\Delta y_{it}$, where Δy_{it} denotes the actual growth rate in the output of firm i with ω_i denoting the weight of the firm in the industry. These assumptions allows us to write the following conditions[12]:

$$prob(\Delta y_{t+1} \leq -c|\Omega_t) = H_t(-c) =_t F_{t+1}^e \tag{1.1}$$

$$prob(\Delta y_{t+1} \geq c|\Omega_t) = 1 - H_t(c) =_t R_{t+1}^e \tag{1.2}$$

where $H_t(.)$denotes the cumulative density function of $h(\Delta y_{t+1}|\Omega_t)$and ${}_tF_{t+1}^e$and ${}_tR_{t+1}^e$are the proportion of firms that at time t expected the variable to fall or rise over the period t to $t+1$ respectively.

Given that $D_t y_{t+1}^e = E(\Delta y_{t+1}|\Phi_t) =_t y_{t+1}^e - y_t$, we would be in a position to obtain the expected output growth series from (1.1) and (1.2) if we knew (i)the form of the aggregate density function and (ii) the response threshold c. As far as the first issue is concerned, we just need to make a further assumption on its form. The uniform, logistic and normal distributions have been the most important examples of $h(\Delta y_{t+1}|\Omega_t)$. For this empirical work, we have considered that $\Delta y_{t+1}$are random drawings from a normal distribution with

12 With reference to these two conditions and as stressed by Theil (1952), the probability method cannot be used if the percentage of respondents resporting a rise or a fall is equal to zero.We have not encounter this problem for the period under consideration.

mean $D_t y^e_{t+1}$ and standard deviation ${}_t\sigma^e_{t+1}$. Using (1.1) and (1.2), we get:

$$\Phi\left[\frac{-c - D_t y^e_{t+1}}{{}_t\sigma^e_{t+1}}\right] =_t F^e_{t+1} \tag{1.3}$$

$$\Phi\left[\frac{c - D_t y^e_{t+1}}{{}_t\sigma^e_{t+1}}\right] = 1 -_t R^e_{t+1} \tag{1.4}$$

with $\Phi(.)$ representing the standard normal cumulative distribution function. We can solve equations (1.3) and (1.4) for $Dy^e_{t+1}$and ${}_t\sigma^e_{t+1}$in terms of ${}_tR^e_{t+1}$, ${}_tF^e_{t+1}$and c:

$${}_tf^e_{t+1} = \Phi^{-1}({}_tF^e_{t+1}) = \frac{-c - D_t y^e_{t+1}}{{}_t\sigma^e_{t+1}} \tag{1.5}$$

$${}_tr^e_{t+1} = \Phi^{-1}(1 -_t R^e_{t+1}) = \frac{c - D_t y^e_{t+1}}{{}_t\sigma^e_{t+1}} \tag{1.6}$$

where ${}_tf^e_{t+1}$and ${}_tr^e_{t+1}$are the abscissa of the frequencies ${}_tF^e_{t+1}$and $1-_tR^e_{t+1}$of the cumulative normal distribution:

$$\Phi({}_tf^e_{t+1}) =_t F^e_{t+1} \tag{1.7}$$

$$\Phi({}_tr^e_{t+1}) = 1 -_t R^e_{t+1} \tag{1.8}$$

From (1.5) and (1.6) it follows that

$$D_t y^e_{t+1} = c._t d^e_{t+1} \tag{1.9}$$

where

$${}_td^e_{t+1} = \frac{{}_tf^e_{t+1} +_t r^e_{t+1}}{{}_tf^e_{t+1} -_t r^e_{t+1}} \tag{1.10}$$

As it was stated before, the knowledge of the cumulative distribution is not enough to derive the expectation series,$D_t y^e_{t+1}$. In addition, we need to estimate a value for c. There are four options to estimate it:

1. $\hat{c}_{cp} = \frac{\sum_{t=1}^{n} \Delta y_t}{\sum_{t=1}^{n} ({}_{t-1}d^e_t)}$
2. regressing Δy_t on d_t, with $d_t = \frac{f_t + r_t}{f_t - r_t}$
3. regressing d_t on Δy_t
4. $\hat{c}_{rm} = \frac{\sum_{t=1}^{n} \Delta y_t}{\sum_{t=1}^{n} d_t}$

Note that d_t is defined in the same way as ${}_{t-1}d^e_t$, but it makes use of the realisations data in place of the expectations data. In the CBI questionnaire, respondents are not only required to answer about their expectations about the future, but they are also asked about the past - this is what we call the "realisations" data. The proportion of firms reporting a rise or a fall in time t is denoted by R_t and F_t . The first scaling parameter,$\hat{c}_{cp}$, initially proposed by Carlson and Parkin assumes that the expectations are unbiased. The fourth scaling parameter ($\hat{c}_{rm}$) was first proposed by Bennett (1984) and uses the backward-looking series instead, assuming that the same indifference interval, the same distribution function can be used for conversion of either data, i.e., the responses referring to the future (the expectations data) as well as the responses referring to the past (the realisation data). Alternatively, we can regress either Δy_t on d_t (option 2) or d_t on Δy_t (option 3). Pesaran (1987) argues that a consistent estimate of c is likely to fall between the estimates derived from option (2) and option (3).

Testing for the Rationality of the expectations

After choosing the appropriate scaling parameter (c), the derived expectations series can be obtained making use of equation (1.9). However, following Lee (1994), this expectation series is not considered to be the true one, since we recognise the existence of measurement errors which include problems with the conversion procedure or sampling errors. An analogy can be made considering the derived realisation series, i.e., the series derived from the responses on perceptions about the past- they do not necessarily match the actual series, as provided by official statistics. As a result, a purging procedure will be applied to the series derived from the survey in an attempt to correct for these measurement errors. So far, we have defined two series on output: Δy_t denoting actual output growth and Dy^e, denoting the expected output series, as derived from the survey responses on expectations about the future of output. We will define two further series: the realisation series ($Dy_t^r),$ derived from the responses on perceptions about the past (the realisation data) and the true, purged (purged of the measurement error) and unobservable expectation series, which we will represent by Dy_t^*. As a result, we can define the following three errors:

$\Delta y_t - Dy_t^r = \varepsilon_t^{rc}$ as the realisation conversion error;

$Dy_t^* - Dy_t^e = \varepsilon_t^{ec}$as the expectations conversion error

$Dy_t - Dy_t^* = \varepsilon_t^{ex}$ as the (true) expectational error.

The first error (ε_t^{rc}) is termed by Lee as the ”realisation conversion error” since it represents the error involved in the use of the realisations survey data and represents the difference between actual series and the series derived from the realisation data (i.e., perceptions on the past); ε_t^{ec} represents the error involved when the expectations series derived directly from the survey are used instead of the true expectational series and is termed the ”expectations conversion error”; ε_t^{ex} termed the ”expectational error” and is defined by the different between actual and (true) expected series. This is therefore the error we are interested in analysing for the purpose of testing for rationality of expectations. However, we only have observation on measured unanticipated output growth

$u_t = \Delta y_t - Dy_t^e = \varepsilon_t^{ec} + \varepsilon_t^{ex}$

which involves not only the expectational error but also the expectations conversion error. Hence, a further assumption needs to be made in order to make it possible to analyse the expectational error in isolation. Lee assumes that the error involved in the realisation conversion series is of the same form as the error involved in the expectation series. The model used to describe formally the procedure aiming at testing for rationality of expectations is the following:

$$\Delta y_t - Dy_t^r = \varepsilon_t^{rc} = \tau^{rc} + \theta^{rc} h_{t-1} + \upsilon_t^{rc} \qquad (1.11)$$

$$Dy_t^* - Dy_t^e = \varepsilon_t^{ec} = \tau^{ec} + \theta^{ec} h_{t-1} + \upsilon_t^{ec} \qquad (1.12)$$

$$\Delta y_t - Dy_t^* = \varepsilon_t^{ex} = \tau^{ex} + \theta^{ex} h_{t-1} + \upsilon_t^{ex} \qquad (1.13)$$

where $Dy_t = y_t - y_{t-1}$is the actual output growth at time t; $Dy_t^r = y_t^r - y_{t-1}$is the measure of output growth in the industry derived from the survey data on the proportion of firms in the industry who realised rises or falls in output volumes in time t; $Dy_t^e = y_t^e - y_{t-1}$is the measure of expected output growth based on the survey responses about of the future of firms in the industry; $\tau_t^{rc}, \tau_t^{ec}, \tau_t^{ex}$are scalars, $\theta^{rc}, \theta^{ec}, \theta^{ex}$ are vectors of fixed coefficients and $\upsilon_t^{er}, \upsilon_t^{ec}, \upsilon_t^{ex}$are random disturbances with zero mean and h_{t-1} is a vector $(kx1)$with the information known to agents at time t. The assumption by Lee(1994) can be described using this model and it can be expressed formally as:

$\tau^{rc} = \tau^{ec}$

$\theta^{rc} = \theta^{ec}$

This assumption makes it possible to analyse the conversion error through the realisation conversion error, which can be done by comparing the actual series with realised output growth derived from the survey data, Dy_t^r. Under these assumptions, the model can be rewritten as follows:

$$\begin{pmatrix} \Delta y_t - Dy_t^r \\ \Delta y_t - Dy_t^e \end{pmatrix} = \begin{pmatrix} \varepsilon_t^{rc} \\ \varepsilon_t^{ec} + \varepsilon_t^{ex} \end{pmatrix} = \begin{pmatrix} I & h_{t-1}^{'} \\ I & h_{t-1}^{'} \end{pmatrix} \begin{pmatrix} \tau^{rc} \\ \theta^{rc} \end{pmatrix} + \begin{pmatrix} 0 & d_0^{'} \\ I & h_{t-1}^{'} \end{pmatrix} \begin{pmatrix} \tau^{ex} \\ \theta^{ex} \end{pmatrix} + \begin{pmatrix} \upsilon^{rc} \\ \upsilon^{ec} + \upsilon^{ex} \end{pmatrix}$$

where $d_0^{'}$is a (I x k) vector of zeros.

The corresponding measures of the expected output growth series can be derived using

$$\hat{Dy}_t^{*} = \Delta y_t - \varepsilon_t^{ex} = Dy_t^e + \hat{\tau}_t^{rc} + \hat{\theta}^{rc} h_{t-1} \tag{1.14}$$

where $\hat{\tau}_t^{rc}$ and $\hat{\theta}^{rc}$ are estimates of the coefficients in (1.11). Analysis will be taken on the ”purged” expectations series, $\hat{Dy}_t^{*}$. Since REH requires the expectational errors to be orthogonal to known information, to test for it we need to test the hypothesis $H_o : \theta^{ex} = 0$, i.e., to test whether h_{t-1}has explanatory power in a regression explaining the expectational error. See equation (1.13). We will not be testing for unbiasedness of expectations, as we recognise the presence of measurement errors, in particular the bias introduced by the conversion method, which relies on the assumption of unbiasedness, as explained earlier.

The purged series will be the series used in the empirical work of chapter 2, as proxies for expectations.

1.3.2 Analysis of Output and Inflation Expectations in the UK

Actual and Expected Output Growth

The empirical work of this section covers the period 1975q3-2000q4. From the survey data, we obtained two different measures of output growth expectations, which we will be calling the derived(Dy^e) series and the purged (Dy^*) series. The former is derived employing the probability method described in the previous section and assuming a normal distribution. The latter is obtained using Lee(1994)'s procedure and it is considered to be the "true" expectational series, i.e., the one that we will be using as proxy for expectational series in the system estimation of chapter 2. The actual output growth data also requires some attention. As mentioned before, despite the fact that the CBI survey is currently conducted quarterly, the time horizon refers to four-month periods. As a result, special attention has been put into the calculation of the actual data, to match these forecasting horizons. Details of the determination of actual output growth are given in the appendix at the end of this chapter.

[Table 1.1]

Table 1.1 provides estimates of the four scaling parameters defined in section 1.3.1. The chosen scaling parameter used in conjunction with ${}_td^e_{t+1}$ as described in equation (1.9) provides an expectational series, $D_ty^e_{t+1}$.[13]We will be using the scaling parameter,

[13] The same scaling parameter will be used to derive the "realisation series" through the relation ${}_tDy^r_{t+1} =$

$c_{CP} = 0.018,$ estimated using the Carlson and Parkin method. For the construction of the ”purged” (”true”) series, the relation used was the one represented in equation (1.14) and the information set, $h_{t-1},$ includes four variables: one and two-period lagged changes in actual inflation $(\Delta\pi_{t-1}, \Delta\pi_{t-2})$ and two lags of actual output growth $(\Delta y_{t-1}, \Delta y_{t-2})$. We are therefore assuming that agents have this information available to them at time t. The actual and expected output growth series are plotted in Figure 1.1, where we compare the derived series, Dy^e, and the purged expectations series, Dy^* with actual output growth, Δy.

[Tables 1.2a-1.2b]; [Figure 1.1]

Table 1.2a provides statistics related to the series mentioned above. It is clear from the figures that output growth in the UK Manufacturing Sector has experienced considerable variability. Moreover, we can notice that expected series lie below (or above) the actual series when there is a large, unexpected increase (or decrease) in output growth. However, it is clear that the ”purged” series, Δy^* capture better the variations of output than the derived series, Δy^e. The latter is during the whole period much smoother than the former. The standard deviation of actual output growth was 2.48%. The expected series, as it is apparent from the graphs, have a lower standard deviation, presenting a value of 1.17% for the ”purged” series and an even lower value for the series Dy^e (only 0.58%). The periods of larger fluctuations include the recession of the late seventies[14], the recession of the middle eighties[15] and early nineties[16], which are cases where the expected series

$c_t d_{t+1}$, where d is defined in the way described in section 1.3.1.

[14] Associated with the oil price shocks and productivity slowdown.

[15] Associated with contractionary policies trying to control the great inflation of the 1970s.

[16] Associated with the UK entry in the Exchange rate Mechanism (ERM).

reflect changes only partially and slowly over time and where the expectational errors are larger. The manufacturing output has grown at around 0.3%, reaching the greatest growth of 4.6% in the first quarter of 1988 and having a negative value of -9.6% as the minimum during the period under consideration. The maximum and minimum values of the expected series were 2.2% and -4.0% for the "purged" series and 1.4% and -1.7% for the Dy^e series, respectively. The contemporaneous correlation between actual and expected output series is rather large but not close to one. As we can see from the table, the correlation between actual and Δy^* is 0.5453, larger than the correlation between actual output and the derived series, Δy^e, which value is 0.4259.

Table 1.2b provides further information on the properties of output series. This table shows the Dickey-Fuller statistics, indicating that the expectational errors for both series are stationary, since the hypothesis of existence of unit root is rejected. For the case of the Δy^e expectation series, the rejection is based on the Akaike Information Criteria (AIC) and Schwarz Bayesian Criterion (SBC), both of which indicate a zero order for the ADF regression. In the case of the purged series, Δy^*, we reach the same conclusions. Based on the AIC and SBC, we are able to reject the null hypothesis of a unit root in the expectational errors at the 5% significance level. The DF regression include an intercept but not a trend.

The SC statistics intends to capture (first-order) autocorrelation in the expectational errors for the output growth series. This statistics are the t-values on the estimated values of the coefficient ρ in the AR(1) specification of

$$\varepsilon_t^+ = \mu + \rho\varepsilon_{t-1}^+ + \xi_t$$

where "+" is equal to "e" or "*", depending on the series. Based on Dy^*, there is no evidence of first-order autocorrelation. This conclusion can be achieved by looking at the statistical significance of the estimated parameter of the regression described in the notes to the table, which had a t-ratio of 0.06. However, in the case of the expectational errors based on the series Dy^e, the t-value associated with the estimated coefficient had a value of 3.51, which shows that the coefficient is significant at a 5% level.

The H-statistics, which intends to give an assessment of Heteroscedasticity confirms once more the strong relation between the actual series and the expectational errors, being the errors bigger when output growth change is larger. The H-statistics represent the t-values on the estimated coefficient ρ in the regression:

$$\varepsilon_t^{+2} = \mu + \rho(y_t - y_{t-1})^2 + \xi_t$$

with "+" standing for "e" or "*". This statistics is significant for both series, being larger in the series based upon Dy^e. The t-ratio associated with the estimated coefficients took the value of 31.75 for Dy^e and 16.11 for the series Dy^*. The periods where the expectation series failed more noticeably in reflecting output changes are the recession of late 1970s, early-to-mid-80s and early 90s as we can see from Figure 1.1 and as discussed before.

[Tables 1.3a-1.3c]

Tables 1.3a-1.3c report results on the ADF tests performed on the three different output growth series. In every case, the statistics chosen by AIC and SBC were in absolute value larger than the critical value of -2.89. The underlying regressions include an intercept but not a trend. Therefore, we reject the hypothesised unit root in output growth, and as a result we treat these series as stationary.

[Table 1.4]

Table 1.4 reports the statistics testing the orthogonality of the various types of expectational errors to the information available to agents at the time expectations are formed, h_{t-1} with $h_{t-1} = (\ \Delta\pi_{t-1}, \Delta\pi_{t-2}, \Delta y_{t-1}, \Delta y_{t-2})'$ in a regression of the form:

$$\Delta y_t - Dy_t^+ = \varepsilon_t^+ = \tau^+ + \theta^+ h_{t-1} + \upsilon_t^+ \tag{1.15}$$

with $+ = e$ or $*$, depending on whether we are using the expected series derived directly from the survey(Dy_t^e) or the true, purged expectational series(Dy_t^*); ε represents the difference between each of these series and the actual series, and therefore, by definition, is an expectational error.

The statistics d_1[17], represents the test of orthogonality of the expectational errors based on Dy^e.This statistics has a value of 10.93 which should be compare with the critical value of a χ^2four degrees of freedom (which takes a value of 9.49). The orthogonality is rejected, meaning that, if Dy^e was a good measure of expectations, then agents would not be making a good use of the available information. We also test for the orthogonality of the available information with the expectational error based on what we believe to be the ”true”

[17] The statisticcs d_1 and $d_2$are heteroscedaticity-consistent as defined by White (1980). As explained in Pesaran, the d statistics is as follows:

$d = \hat{\gamma}' (G'G)\hat{V}^{-1} (G'G)\hat{\gamma}'$

where G is a nx$(s+1)$ matrix of observations on $(1, h_{t-1})'$; $\hat{V} = n^{-1} \sum_{i=1}^{n} \hat{e}_t^2 (g_t g_t')$;

$\hat{e} = \varepsilon_t - \hat{\tau} - \hat{\theta}' h_{t-1}$; $g_t = (1, h_{t-1})'$; $\gamma = (\tau, \theta)'$; s is the number of variables in h_{t-1}; ε represents the different expectational errors. See Pesaran (1987), pages 238-240.

expectational series, Dy^*.In this case, there is no evidence to reject the orthogonality, and therefore, we find support for the rationality of expectations for output growth. We get a statistics of 0.45 which has an associated p-value of 0.98.

Actual and Expected Inflation

We now turn our attention to inflation.

[Table 1.5]

The expected series for the inflation in the UK manufacturing Sector were derived, just like the output series, from the CBI survey data and making use of the same scaling parameter, the Carlson and Parkin. The Carlson and Parkin scaling parameter value (0.039) meets Pesaran's consistence criteria, according to which the scaling parameter should be within the range $[0.037, 0.047]$, with the lower and upper limit defined by the scaling parameter obtained from the regression of π on d and d on π respectively. See Table 1.5. As it was explained in section *1.3.1*, we derived three different series - one was derived from the realisation data provided by the CBI; the second one was derived from the CBI expectations data; the third one, the "true" expectation series, corresponds to the derived series corrected of measurement errors. Our analysis, however, will concentrate on the last two only. Tables 1.6a-1.6b present some statistics relating the actual inflation rate to the last two expected series mentioned above. Tables 1.7a-1.7e present ADF tests on the three different inflation series and Table 1.8 presents statistics which test for the orthogonality of each of expectational errors to the information available to agents at time t.

[Table 1.6a-1.6b]; [Figure 1.2]

As we can see from the first column of table 1.6a, the quarterly inflation rate averaged 2.12% during the period 1975q3-2000q4. The average value during the same period for the series π^e and π^*were 2.07% and 2.12% respectively. The series presenting smaller variation was the purged series, with a standard deviation of 1.64%, whereas $\pi^e$was 2.11% and the actual series exhibited a standard deviation of 1.83%. The actual series are extremely volatile during the late seventies and first two years of the eighties, varying then within a smaller range. The correlation coefficients provide in the last column of table 1.6a intends to analyse how close together the series move. The correlation between actual inflation and π^e is 0.8636, which indicates a strong relation between these two series. In the case of the purged series, π^*, the correlation presents a value of 0.9153, which is larger than the previous one, denoting an even stronger relationship between actual and expected series. Even though there is not a one-to-one correspondence between the series, the correlation coefficient is very close to one and considerably larger than the values obtained for output.

The first column of table 1.6b provides Dickey-Fuller statistics and tests the null hypothesis of a unit root in the expectational errors, $\varepsilon_t^e = \pi_t - \pi_t^e$ and $\varepsilon_t^* = \pi_t - \pi_t^*$. In both series, π_t^e and π_t^* this hypothesis is rejected and hence we can conclude that expectational errors are stationary. Since expectational errors do not exhibit a trend, the relevant statistics are the values obtained when the regression testing for the unit root includes an intercept but not a time trend. The order of the ADF regression defined has to be selected. In the case of expectational errors derived from the purged series, the values reported by the SBC and AIC model selection criteria suggest a zero-order for the regression. The Dickey-Fuller statistics for this order equal -8.28, which is in absolute value above their asymptotic 95%

critical value of -2.89. The same is true if we consider a higher order. We can therefore reject the null hypothesis of a unit root in the expectational errors at 5% significance level. For the expectational error ε_t^e, the same order of ADF regression was chosen using SBC and AIC. The statistics value of -5.08 once again provides evidence to reject the null of a unit root in these expectational errors.

Another statistics provided in table 1.6b, SC, tests for the presence of autocorrelation in the expectational errors for both series, π^e and π^*. For the first series reported, the estimated t-ratio on the coefficient was 7.27, which is significant at a 5% level. In the case of the purged series, π^*, we obtained an estimated value of 1.62, which is not significant at the standard 5% level. In the case of the purged series therefore, we cannot reject the null of zero first order serial correlation[18]

The last statistics reported in Table 1.6b is the H-statistics, as defined in the previous subsection. Through this statistics we can realise once again that the larger expectational error derived from the series π^e occur when inflation presents a larger variation. Looking at Figure 1.2, we can observe that the larger changes in inflation, which occurred at the end of the 1970s were not fully picked by these expectational series. This is still the case for the ”true”expectation series, π^*, where the estimated coefficient was significant, with a t-ratio of 5.65.

[Tables 1.7a-1.7e]

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[18] Note however that the presence of serial correlation would not necessarily eliminate the possibility of rationality in the expectations in this context. This fact happens because of the overlapping forecast horizon in the CBI data. The data is collected every quarter, but the question the firms are asked to answer refers to a period of four months. There is therefore an overlapping month that, as Pesaran (1987) argues, can introduce some serial correlation even if the Rational Expectations Hypothesis is valid

Tables (1.7a) to (1.7e) represent the ADF test statistics analysing the order of integration of the three different inflation series, π, π^e and π^*. We can see that the hypothesised unit roots cannot be rejected for either series using AIC and SBC criteria. Although the evidence for actual inflation is not as strong as it was for the case of output, we will be treating the three different inflation series as I(1) variables[19]. In tables (1.7d)-(1.7e) we also report results confirming the stationarity of the variables $\Delta\pi$ and $D\pi^* = \pi^* - \pi_{-1}$. These two variables will be modelled together in the next chapter and treated, accordingly as I(0) variables.

[Table 1.8]

The tests of rationality of expectations are provided by the statistics d_1 and d_2 reported in Table 1.8, which tests the orthogonality of the expectational errors to the vector of available information, h_{t-1}, which is defined as before as $h_{t-1} = (\ \Delta\pi_{t-1},\ \Delta\pi_{t-2},$ $\Delta y_{t-1},\ \Delta y_{t-2})'$. Like for the case of output growth, d_1 represents the test of orthogonality of the expectational errors based on π^e and represent a joint test of the significance of the variables contained in h_{t-1} in the regression. In other words, it is a test of $\theta^e = 0$ in the regression:

$$\pi_t - \pi_t^e = \varepsilon_t^e = \tau^e + \theta^e h_{t-1} + \upsilon_t^e \tag{1.16}$$

The value of this statistics is 20.00 and is very significant when compared with a χ^2 statistics with four degrees of freedom (which takes a value of 9.49). Since this test evaluates the joint significance of the coefficients attached to each variable that compose the vector h_{t-1},

19 Note that we did not perform a Normality test (non-normality could affect the results) or indeed another test to confirm the order of integration of these series. In the US analysis that follows, we are more thorough and provide with further tests to determine the order of integration of the variables. However, for the purpose of the estimation that follows in chapter 2 and uses this data, we are happy to treat inflation as an I(1) process and use these results to provide with some evidence for it.

the fact that it is significant mean that at least one of the variables has explanatory power in the expectational error regression. For these series π^e we reject rationality of the expectations. We also tested for the orthogonality of the information available with the expectational errors derived from the ”purged” series, π^*. When carrying out the test of the joint significance of the explanatory variables, we are unable to reject orthogonality at 5% significance level. In fact, the statistics d_2 presented a value of 4.95, with a p-value of 0.29. As it happened for the case of the expected output growth, Dy^*we do not have evidence to reject the rationality of expectations on the inflation rate based on the purged series.

The analysis of the current section served the purpose of deriving expectational series for both, inflation and output from the qualitative CBI data and analysing its properties. By using Lee’s procedure we were able to eliminate some of the measurement errors involved in the conversion procedure and considered these purged series (that have been denoted so far by π^* and Δy^*) as the ”true” expectational series. As a result, we have chosen these series to use as a proxy for expectations in chapter 2, where we model simultaneously actual and expected output and inflation. In order to simplify the notation, from this section on, expected inflation and expected output growth will be simply denoted using the more common subscript ”e”, as π^e and Δy^e[20].

[20] Since from now on we have only one expected series for each variable and time horizon, we will be denoting the expected series by π^e and Δy^e,i.e., we will be using the superscript e which is more intuitive and the commonly used to represent expectations.

1.4 US Inflation and Output Expectations; the Use of Quantitative Survey Data

In this section we introduce the modelling framework and empirical results for the US economy. We model, in a multivariate setting, real time actual data and expectations of inflation and output at different time horizons. We start off by introducing the source of multi-period expectations - the Survey of Professional Forecasters[21].

1.4.1 The Source of measures of expectations - the Survey of Professional Forecasters

As mentioned before, together with real-time data, we use direct measures of expectations provided by the Survey of Professional Forecasters (SPF). The SPF is a quarterly source of survey data on several economic variables for the US economy done by anonymous professional forecasters and is currently provided by the Federal Reserve Bank of Philadelphia. It was established at the initiative of the president of the American Statistical Association (ASA), Geoffrey Moore, to be conducted cooperatively with this institution and the National Bureau of Economic Research (NBER). It has been carried out since 1968q4 and in 1990, the Federal Reserve Bank of Philadelphia undertook the survey, which continued being carried out in the same manner. The SPF has a very rich dataset, covering a wide range of economic variables (including GDP, GDP price inflator, interest rates, net exports, to name just a few) . The peculiarity of this source is that it involves different forecast horizons. The respondents are provided with the last quarter's preliminary figure and are

[21] The output analysis of this chapter has been extended and published in 2006 in Economics Letters, as a paper co-authored with Dimitrios Papaikonomou, entitled "Are US output expectations unbiased? A cointegrated VAR analysis in real time".

asked to put figures into their forecasts, for the current quarter, and next five quarters. Since 1981Q3 it also include annual forecasts for most variables[22]. In addition, they also provide with probability forecast data, which reflects the degree of uncertainty involved in each respondent's forecast and 10-year forecasts. The questions that are going to be relevant for this empirical work are the questions concerning GDP and the GDP implicit price deflator. The GDP data is subject to several revisions. Official figures are released fifteen days after the end of each quarter; revised figures are released 45 days after the end of each quarter and since 1974, further revised figures are released 75 days after the end of each quarter; every July a more extensive revisions is made, reaching back between 6 and 15 quarters and in addition to that, comprehensive revisions take place occasionally. During the period 1968q4-2001q2 there were five comprehensive revisions for GDP data, which took place in January 1976; December 1980; December 1991; December 1995 and August 1999.For our empirical work, we chose the 15-day vintage. The reason behind this choice has to do with the fact that this is the vintage available to the respondents at the time the questionnaire is sent out. The respondents receive the questionnaire at the end of the first month of each quarter and they are usually asked to send it back by the middle or end of the second month of each month[23]. It seems, therefore the most appropriate vintage to use, as it reflects more accurately the information set of the respondents at the time they form their forecasts. This is exactly what we mean when claiming that our modelling is in real-time.

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[22] That makes the SPF richer when compared with other US surveys. For example, the widely used Livingston survey provides with semi-annual forecasts; National Association for Business Economics (NABE) Outlook, although also professional forecasters and with quarterly frequency, it only provides annual averages for most variables; Blue Chip provides monthly data, with long-term forecats twice a year only.

[23] For example, the questionnaire for 2001Q1 was sent in January and asked to be returned by the 12th of February; the questionnaire for 1990Q1 was again sent in January and asked to be returned by the 23 of February.

A few more notes on the data. For the price series, we use the GDP/GNP deflator, as prior to 1996 the data did not include the chain-weighted GDP price index. So, the price level was constructed by taking the ratio of nominal GDP/GNP to real GDP/GNP. The comprehensive revisions to the national income and product account (NIPA) throughout the period under study included modifying the definitions of variables. The base year for real variables was changed in January 76, December 85, late November 91 and January 96. So, some of the differences across vintages incorporate base-year changes which affect real variables. Further, prior to February 92, the real output variable is GNP and GDP from then on[24].

1.4.2 Modelling framework; the Multivariate approach

The modelling framework is based upon the multivariate cointegration approach by Johansen (1988) and its generalisation in Pesaran, Shin and Smith (2000). In particular, we will be analysing the stationarity of expectational errors and unbiasedness of expectations using this framework. The use of a multivariate setting allows us to treat all variables endogenously, without losing relevant information on contemporaneous interactions, and therefore increasing robustness of estimation as compared to the univariate setting. The framework also allows for both, long-run and short-run analysis, which help providing additional insights on the workings of the expectations formation process. The long-run relationships will be tested within the model, without requiring pre-determination of the number or form of those relationships. The short-run dynamics will be analysed with the

[24] Croushore (1993) provides a detailed description of the Survey of Professional Forecasters.

use of impulse responses, helping understanding the adjustment process towards long-run equilibrium.

Definition of the Expectational Errors

We consider the expectational errors $\eta_{i,t}$, $i = 1, 2, 3, 4$, $t = 1, 2, ..., T$, given by

$$\begin{aligned}\eta_{1,t} &= {}_t x^e_t -_{t+1} x_t, \eta_{2,t} =_t x^e_{t+1} -_{t+2} x_{t+1}, \eta_{3,t} =_t x^e_{t+2} -_{t+3} x_{t+2}, \\ \eta_{4,t} &= {}_t x^e_{t+3} -_{t+4} x_{t+3}, t = 1, 2, ..., T,\end{aligned} \tag{1.17}$$

where x represents either output (y) or inflation (π) in the empirical work that follows (i.e., $x = y$ or $x = \pi$ in section (1.4.3)) and ${}_t x_{t-1}$ is the (real-time) actual x in time $t-1$, made available to the respondents of the survey at time t, while ${}_s x^e_p$ denotes the expectation formed at time s on the value of x in time p, $s \leq p$. The expectational errors in (1.17) may be written alternatively as

$$\begin{aligned}\eta_{1,t} &= {}_t x^e_t -_t x_{t-1} - \Delta({}_{t+1}x_t), \\ \eta_{2,t} &= {}_t x^e_{t+1} -_t x_{t-1} - \Delta({}_{t+1}x_t) - \Delta({}_{t+2}x_{t+1}), \\ \eta_{3,t} &= {}_t x^e_{t+2} -_t x_{t-1} - \Delta({}_{t+1}x_t) - \Delta({}_{t+2}x_{t+1}) - \Delta({}_{t+3}x_{t+2}), \\ \eta_{4,t} &= {}_t x^e_{t+3} -_t x_{t-1} - \Delta({}_{t+1}x_t) - \Delta({}_{t+2}x_{t+1}) - \Delta({}_{t+3}x_{t+2}) - \Delta({}_{t+4}x_{t+3}),\end{aligned} \tag{1.18}$$

$t = 1, 2, ..., T$ and Δ representing the first-difference of the variables, i.e., $\Delta_{t+i} x_{t+i-1} =_{t+i} x_{t+i-1} -_{t+i-1} x_{t+i-2}$ .For the m-vector $\mathbf{z}_t = [{}_t x_{t-1}, {}_t x^e_t, {}_t x^e_{t+1}, {}_t x^e_{t+2}, {}_t x^e_{t+3}]'$ the vector of expectational errors $\boldsymbol{\eta}_t = [\eta_{1,t}, \eta_{2,t}, \eta_{3,t}, \eta_{4,t}]'$ can be expressed as

$$\boldsymbol{\eta}_t = \boldsymbol{\beta}'_0 \mathbf{z}_t - \mathbf{A}(L)\Delta\mathbf{z}_t \qquad , t = 1, 2, ..., T, \tag{1.19}$$

where $\mathbf{A}(L) = A_1 L^{-1} + A_2 L^{-2} + A_3 L^{-3} + A_4 L^{-4}$ with

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and } \boldsymbol{\beta}_0' = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1.20}$$

As is apparent from expression (1.19), the statistical properties of $\boldsymbol{\eta}_t$ depend on the properties of $\mathbf{z}_t$. In the event that $\mathbf{z}_t$ is a first difference stationary process, denoted $\mathbf{z}_t \sim I(1)$, then the expectational errors can only be stationary if the linear combination $\boldsymbol{\beta}_0'\mathbf{z}_t$ is an $I(0)$ process, i.e. $\mathbf{z}_t \sim CI(1,1)$ with cointegrating matrix $\boldsymbol{\beta}_0$. In the case that $\mathbf{z}_t \sim CI(1,1)$ with cointegrating matrix $\boldsymbol{\delta} \neq \boldsymbol{\beta}_0$, or in the absence of cointegration altogether, the expectational errors $\boldsymbol{\eta}_t$ would contain stochastic trends. Such a property would cause the first and second moments of $\boldsymbol{\eta}_t$ to be time-dependent and would thus imply a very disappointing performance on the part of economic agents in anticipating future values of the variable under analysis. For economic behaviour to be consistent with the Rational Expectations Hypothesis (REH) forecast errors should also be equal to zero on average, which by (1.19) implies

$$E[\boldsymbol{\beta}'\mathbf{z}_t] = E[\mathbf{A}(L)\Delta\mathbf{z}_t], \tag{1.21}$$

where $E[.]$ denotes the expectations operator. In what follows we illustrate how the properties of the process $\boldsymbol{\beta}'\mathbf{z}_t$ and its implications on $\boldsymbol{\eta}_t$ may be investigated within a cointegrating VAR framework.

Expectational Errors in a Cointegrating VAR Framework

Provided that $\mathbf{z}_t \sim I(1)$ with linear deterministic trending behaviour it may be approximated by the following $VAR(p)$

$$\mathbf{\Phi}(L)(\mathbf{z}_t - \boldsymbol{\mu} - \boldsymbol{\gamma} t) = \mathbf{e}_t, \qquad t = 1, 2, ..., T, \tag{1.22}$$

where L is the lag operator, $\boldsymbol{\mu}$ and $\boldsymbol{\gamma}$ are m-vectors of unknown coefficients, $\mathbf{e}_t \sim IN(\mathbf{0},\Omega)$, Ω positive-definite and the matrix lag polynomial of order p, $\mathbf{\Phi}(L) \equiv \mathbf{I}_m - \sum_{i=1}^{p} \Phi_i L^i$. We can also express the lag polynomial in the form

$$\mathbf{\Phi}(L) \equiv -\Pi L + \mathbf{\Gamma}(L)(1 - L), \tag{1.23}$$

where $\Pi \equiv -(\mathbf{I}_m - \sum_{i=1}^{p} \Phi_i)$, and $\mathbf{\Gamma}(L) \equiv \mathbf{I}_m - \sum_{i=1}^{p-1} \Gamma_i L^i$ with $\Gamma_i = -\sum_{j=i+1}^{p} \Phi_j$, $i = 1, \ldots, p-1$, the $VAR(p)$ given by (1.22) may be written in VECM form as

$$\Delta\mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \Pi\mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta\mathbf{z}_{t-i} + \mathbf{e}_t, \qquad t = 1, 2, ..., T, \tag{1.24}$$

where

$$\mathbf{a}_0 = -\Pi\boldsymbol{\mu} + [\Pi + \mathbf{\Gamma}(1)]\boldsymbol{\gamma} \tag{1.25}$$

$$\mathbf{a}_1 = -\Pi\boldsymbol{\gamma} \tag{1.26}$$

The assumption $\mathbf{z}_t \sim I(1)$ requires that $0 \leq rank[\Pi] = r < m$. In the event that $\mathbf{z}_t \sim CI(1,1)$ with cointegrating matrix $\boldsymbol{\beta}$, then $r = 4$ and the Π-matrix may be written as $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ is $m \times r$, full column rank and $\boldsymbol{\beta} = \boldsymbol{\beta}_0$and is given by (1.20). This latter condition which will be a test of whether actual and expected series are (log) linearly related with coefficients 1 and -1, is a necessary condition for long-run unbiasedness.

The MA representations for $\Delta\mathbf{z}_t$, $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$ may be obtained from (1.22) as follows:

$$\mathbf{\Phi}(L)\mathbf{z}_t = \mathbf{\Phi}(L)(\boldsymbol{\mu} - \boldsymbol{\gamma}t) + \mathbf{e}_t \tag{1.27}$$

$$\mathbf{C}(L)\mathbf{\Phi}(L)\mathbf{z}_t = \mathbf{C}(L)\mathbf{\Phi}(L)(\boldsymbol{\mu} + \boldsymbol{\gamma}t) + \mathbf{C}(L)\mathbf{e}_t \tag{1.28}$$

$$\Delta\mathbf{z}_t = (1 - L)(\boldsymbol{\mu} + \boldsymbol{\gamma}t) + \mathbf{C}(L)\mathbf{e}_t \tag{1.29}$$

$$\begin{aligned} \Delta\mathbf{z}_t &= \boldsymbol{\gamma} + \mathbf{C}(L)\mathbf{e}_t, & (1.30)\\ \mathbf{z}_t &= \boldsymbol{\mu} + \boldsymbol{\gamma}t + \mathbf{C}(1)\sum_{i=1}^{t}\mathbf{e}_i + \mathbf{C}^*(L)\mathbf{e}_t, & (1.31)\\ \boldsymbol{\beta}'\mathbf{z}_t &= \boldsymbol{\beta}'\boldsymbol{\mu} + (\boldsymbol{\beta}'\boldsymbol{\gamma})t + \boldsymbol{\beta}'\mathbf{C}^*(L)\mathbf{e}_t, \qquad t = 1, 2, ..., T, & (1.32) \end{aligned}$$

where $\mathbf{z}_0 \equiv \boldsymbol{\mu} + \boldsymbol{\gamma}t + \mathbf{C}^*(L)\mathbf{e}_0$[25],

$$\begin{aligned} \mathbf{C}(L) &\equiv \sum_{i=0}^{\infty} C_i L^i = \mathbf{C}(1) + (1 - L)\mathbf{C}^*(L), \\ C_0 &= I_n,\ C_1 = \Phi_1 - I_n,\ C_i = \sum_{j=1}^{i}\Phi_j C_{i-j}, \text{ for } i > 1, & (1.33)\\ \mathbf{C}^*(L) &\equiv \sum_{i=0}^{\infty} C_i^* L^i,\ C_0^* = I_n - \mathbf{C}(1),\ C_i^* = C_{i-1}^* + C_i, \text{ for } i > 0, \end{aligned}$$

and according to *Granger's representation theorem* the cumulative effect matrix $\mathbf{C}(1)$ may be expressed as[26]

$$\mathbf{C}(1) = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}'_\perp\mathbf{\Gamma}(1)\boldsymbol{\beta}_\perp)^{-1}\boldsymbol{\alpha}'_\perp, \tag{1.34}$$

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25 Note that $C(L)\Phi(L) = (1 - L)I_m$

26 See Johansen (1995; Theorem 4.2; pp.49-52)

where $\boldsymbol{\alpha}_{\perp}$, $\boldsymbol{\beta}_{\perp}$ are $m \times (m-r)$, full column rank and are orthogonal complements of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, respectively, so that $\boldsymbol{\alpha}'\boldsymbol{\alpha}_{\perp} = \mathbf{0}$ and $\boldsymbol{\beta}'\boldsymbol{\beta}_{\perp} = \mathbf{0}$.

Combining (1.19), (1.30) and (1.32) the MA representation of the expectational errors $\boldsymbol{\eta}_t$ may be written in the form of

$$\boldsymbol{\eta}_t = \boldsymbol{\beta}'\boldsymbol{\mu} + (\boldsymbol{\beta}'\boldsymbol{\gamma})t + \boldsymbol{\beta}'\mathbf{C}^{*}(L)\mathbf{e}_t - \mathbf{A}(1)\boldsymbol{\gamma} - \mathbf{A}(L)\mathbf{C}(L)\mathbf{e}_t, \tag{1.35}$$

$t = 1, 2, ..., T$, and $\boldsymbol{\gamma}$ is the expected growth rate of $\mathbf{z}_t$ according to (1.30). This expression immediately reveals that under the assumptions discussed above $\boldsymbol{\eta}_t$ has a time-varying mean given by[27]

$$E[\boldsymbol{\eta}_t] = \boldsymbol{\beta}'\boldsymbol{\mu} + (\boldsymbol{\beta}'\boldsymbol{\gamma})t - \mathbf{A}(1)\boldsymbol{\gamma} \qquad , t = 1, 2, ..., T. \tag{1.36}$$

where $\mathbf{A}(1) = \Sigma_{i=1}^{4} A_i$ and A_i, $i = 1, 2, 3, 4$ were defined in (1.20). As mentioned earlier, the hypothesis of long-run unbiasedness takes the form of $E[\boldsymbol{\eta}_t] = \mathbf{0}$. Expression (1.36) indicates that in the current framework this is true when $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$ and $\mathbf{A}(1)\boldsymbol{\gamma} = \boldsymbol{\beta}'\boldsymbol{\mu}$. However, using the trended series $\mathbf{z}_t$ and therefore using the $VECM$ in (1.24) we may only test $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$, not allowing a joint test of both these conditions. In fact, it only provides a direct estimate of $\boldsymbol{\beta}'\boldsymbol{\gamma}$ in the form of $\mathbf{a}_1$. So, testing $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$ corresponds to testing $\mathbf{a}_1 = 0$, which is directly observed. Rejection of this restriction signifies rejection of long-run unbiasedness. However, non-rejection of $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$, i.e., the co-trending hypothesis does not immediately verify it and further requires a test of $\mathbf{A}(1)\boldsymbol{\gamma} = \boldsymbol{\beta}'\boldsymbol{\mu}$. This restriction can only be tested given $\boldsymbol{\beta}$ and co-trending. In particular, provided that $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ and for $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5]'$, co-trending is equivalent to $\boldsymbol{\gamma} = [\gamma_1, \gamma_1, \gamma_1, \gamma_1, \gamma_1]'$. In other

[27] Note that $A(L)\gamma = A(1)\gamma$

words, the individual series in $\mathbf{z}_t$ are not only driven by a common stochastic trend, but also the expectational series share the same deterministic trend with the real-time series, ${}_t x_{t-1}$. Noting that $\mathbf{a}_0$ takes the form given in (1.25), given all current assumptions and an estimate mean of real-time growth of actual *x*, $\Delta_t x_{t-1}, \hat{\gamma}_1$, the long run unbiasedness hypothesis corresponds to the following set of restrictions[28]

$$A(1)\hat{\boldsymbol{\gamma}} = (\alpha'\alpha)^{-1}\alpha'[\boldsymbol{\Gamma}(1)\hat{\boldsymbol{\gamma}} - \mathbf{a}_0] \tag{1.37}$$

The above analysis is consistent with I(1) processes with linear deterministic trending behaviour.

In case the variables included in z_t do not have a linear trend, and provided that $z_t \sim I(1)$, we will have the following $VAR(p)$

$$\boldsymbol{\Phi}(L)(\mathbf{z}_t - \boldsymbol{\mu}) = \mathbf{e}_t, \qquad t = 1, 2, ..., T, \tag{1.38}$$

Using the re-parameterisation described above, it may be written in VECM form as

$$\Delta\mathbf{z}_t = \mathbf{a}_0 + \Pi\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\Gamma_i\Delta\mathbf{z}_{t-i} + \mathbf{e}_t, \qquad t = 1, 2, ..., T, \tag{1.39}$$

where

$$\mathbf{a}_0 = -\Pi\boldsymbol{\mu} \tag{1.40}$$

The MA representations for $\Delta\mathbf{z}_t$, $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$ may be obtained from 1.39 as follows:

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[28] This expression is obtained from equation (1.25) pre-multiplied by α' and then solved out for $\beta'\mu$, and using the co-trending restriction $\beta'\gamma = 0$.

$$\mathbf{\Phi}(L)\mathbf{z}_t = \mathbf{\Phi}(L)\boldsymbol{\mu} + \mathbf{e}_t \tag{1.41}$$

$$\Delta\mathbf{z}_t = \mathbf{C}(L)\mathbf{e}_t \tag{1.42}$$

$$\mathbf{z}_t = \boldsymbol{\mu} + \mathbf{C}(1)\sum_{i=1}^{t}\mathbf{e}_i + \mathbf{C}^*(L)\mathbf{e}_t, \tag{1.43}$$

$$\boldsymbol{\beta}'\mathbf{z}_t = \boldsymbol{\beta}'\boldsymbol{\mu} + \boldsymbol{\beta}'\mathbf{C}^*(L)\mathbf{e}_t, \qquad t = 1, 2, ..., T, \tag{1.44}$$

where $\mathbf{z}_0 \equiv \boldsymbol{\mu} + \mathbf{C}^*(L)\mathbf{e}_0$,

Combining (1.19), (1.42) and (1.44) the MA representation of the expectational errors $\boldsymbol{\eta}_t$ may be written in the form of

$$\boldsymbol{\eta}_t = \boldsymbol{\beta}'\boldsymbol{\mu} + \boldsymbol{\beta}'\mathbf{C}^*(L)\mathbf{e}_t - \mathbf{A}(L)\mathbf{C}(L)\mathbf{e}_t, \tag{1.45}$$

$t = 1, 2, ..., T$, and $\boldsymbol{\gamma}$ is the expected growth rate of $\mathbf{z}_t$ according to (1.30). This implies that the expectational errors, $\boldsymbol{\eta}_t$,have a mean given by:

$$E[\boldsymbol{\eta}_t] = \boldsymbol{\beta}'\boldsymbol{\mu} \qquad , t = 1, 2, ..., T. \tag{1.46}$$

As mentioned earlier, the hypothesis of long-run unbiasedness takes the form of $E[\boldsymbol{\eta}_t] = \mathbf{0}$. Expression (1.46) indicates that in the current framework this is true when $\boldsymbol{\beta}'\boldsymbol{\mu} = \mathbf{0}$.

1.4.3 Analysis of Output and Inflation expectations in the US

Output in the US - Empirical Analysis

In this section we apply the econometric tools discussed above in order to investigate the question of stationarity of the expectational errors on US output, as well as the empirical validity of the closely related concept of unbiasedness of output expectations. We make use of real-time data on actual output and direct measures of expectations for current period's output, the one, two and-three-period ahead forecasts which are available on a quarterly basis over the period 1968q4-2001q2. Our empirical analysis is based on the models given by (1.24), augmented by the deterministic vector $\mathbf{D}_t = [d71q4_t,\, d74q4_t,\, d81q1_t,\, d96q1_t,\, d99q4_t]'$, where $d71q4_t$ takes the value of one in 1971q4 and zero otherwise and the remaining variables are similarly defined. The dummies $d71q4_t$ and $d74q4_t$ are intended to capture the slowdown in economic activity present in the data at the end of 1971 and 1974, while $d81q1_t$, $d96q1_t$ and $d99q4_t$ are controlling for the effects of the comprehensive revisions in GDP that took place in December 1980, December 1995 and August 1999.

We consider the output expectational errors $\eta_{i,t}$, given in (1.17) when $x = y$[29], where ${}_ty_{t-1}$ is the (real-time) actual output in time $t-1$, made available to the respondents of the survey at time t, while ${}_sy_p^e$ denotes the expectation formed at time s on the value of output in time p, $s \leq p$. For the m-vector $\mathbf{z}_t = [{}_ty_{t-1},\, {}_ty_t^e,\, {}_ty_{t+1}^e,\, {}_ty_{t+2}^e,\, {}_ty_{t+3}^e]'$ the vector of expectational errors can be expressed as $\boldsymbol{\eta}_t = [\eta_{1,t},\, \eta_{2,t},\, \eta_{3,t},\, \eta_{4,t}]'$.The empirical version of

[29] Our measure of output is GNP up to 1992 and after that GDP, in order to be consistent with the survey, that changed the question from GNP to GDP in 1992.

the model given by (1.24) takes the form of

$$\Delta\mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_2\mathbf{D}_t + \Pi^*\mathbf{z}_{*,t-1} + \sum_{i=1}^{p-1}\Gamma_i\Delta\mathbf{z}_{t-i} + \mathbf{e}_t, \qquad t = 1, 2, ..., T, \tag{1.47}$$

respectively, where $\mathbf{z}_t = [{}_ty_{t-1},\ {}_ty^e_t,\ {}_ty^e_{t+1},\ {}_ty^e_{t+2},\ {}_ty^e_{t+3}]'$, $\mathbf{D}_t = [d71q4_t,\ d74q4_t,\ d81q1_t,\ d96q1_t,\ d99q4_t]'$, $\Pi^* = \boldsymbol{\alpha\beta}'_*$, $\boldsymbol{\beta}'_* = [\boldsymbol{\beta}', -\boldsymbol{\beta}'\boldsymbol{\gamma}]$, $\mathbf{z}_{*,t} = [\mathbf{z}'_t, t]'$, and $\mathbf{e}_t \sim IN(\mathbf{0},\Omega)$, Ω positive-definite.

Statistical Properties of the Data

[Tables 1.9a-1.9b]

Tables 1.9a-1.9b provide some summary statistics of the data. Over the period under consideration, actual and expected output growth averaged around 0.8% per quarter, with maximum value of actual output growth being 4.8% and a minimum value of -2.7%. Similar figures are reported for each of the expected output growth series and are of similar magnitude. The standard deviation was of around 0.01. The estimated correlation between actual and each expected series varied between 0.555 and 0.1136 for the current-forecast and the three-period ahead forecast respectively. The correlation with 1 and 2-periods ahead lie between those figures, showing that longer forecast horizons are associated with weaker correlation with actual output, as can be seen from Table 1.9b. A plot of actual and expected output level as well as output growth are represented in Figures 1.3 and 1.4.

[Tables 1.10a-1.10c]

As mentioned in the previous section the econometric framework of (1.24) requires the vector $\mathbf{z}_t = [{}_ty_{t-1}, {}_ty^e_t, {}_ty^e_{t+1}, {}_ty^e_{t+2}, {}_ty^e_{t+3}]'$ to be an $I(1)$ process with linear deterministic trending behaviour. The requirement $\mathbf{z}_t \sim I(1)$ has been investigated in a univariate setting and in a model-consistent way with the use of multivariate ADF tests. The first tests are reported in Table 1.10a, where AIC and SBC both indicate an ADF regression order of 1 for all the series under consideration and for which the unit root in the series cannot be rejected. The statistics should be compare with a 95% ADF critical value of -3.44, given that a linear trend has been included. The statistics vary between –2.78 for the actual real time output level and -2.58 for the 3-period ahead forecasts. We proceeded to test for the order of integration of first-differences in the series. As Table 1.10b shows, the hypothesised unit root can be rejected. The statistics are to be compared with a -2.88 critical value, with the underlying ADF regression including an intercept and no linear trend. We also analysed the order of integration in a multivariate setting. Table 1.10c clearly indicates that stationarity is rejected for all elements of $\mathbf{z}_t$, irrespective of the number of cointegrating relations. The null hypothesis is of stationary in the form of a unitary cointegrating vector in the variable of interest and it is conditional on the number of cointegrating vectors. The statistics are to be compared with the 5% critical $\chi^2$with $(m-r+1)$[30] number of degrees of freedom of 11.07, 9.49, 7.81 and 5.99, for the cases where there is one, two, three and four cointegrating relationships respectively[31]. In each case, for any output series, the statistics exceed the critical values, showing rejection of the null hypothesis.

[Table 1.11]

The lag-length, p, was set equal to 2 after testing for significance of additional lags within an unrestricted $VAR(6)$ in the level of $\mathbf{z}_t$, as well as with the use of the *AIC*. SBC

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30 With m standing for number of endogenous variables and r denoting the number of cointegrating relationhips.

31 Details on the Multivariate ADF tests can be seen in Harris (1995), chapter 5.

criteria suggested an order of 1. Empirically, we have found that $p = 2$ is sufficiently long to remove any serial correlation.

[Tables 1.12-1.14]

Testing for Cointegration and Co-trending in the Long Run

Having found strong evidence in favour of the aforementioned assumptions on the statistical properties of $\mathbf{z}_t$, we may proceed with the estimation of the model given by (1.47). As discussed in the previous section, we expect to find four cointegrating relations among the elements of $\mathbf{z}_t$. Table 1.12 reports the cointegration rank statistics within (1.47). Both the Max Eigenvalue and Trace statistics are clearly in favour of $r = 4$. Given this result, we next turn our attention to the form of the cointegrating relations and in particular, whether they are consistent with, what we call, the long-run unbiasedness and co-trending hypotheses. The cointegrating vectors are represented in Table 1.13, where four exactly identifying restrictions have been imposed in each vector. Our choice of restrictions is motivated by our framework, in which we try to understand the relationship between actual and the expected series in output at different time horizons. As a result, we singled out the relationship between actual and each expected series in each of the four cointegrating vectors. We then tested two overidentifying restrictions. The first one takes the form of -1's in the first column of $\boldsymbol{\beta}'_*$ in (1.47), while the second one, the co-trending hypothesis takes the form $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$, which corresponds to zero elements in the last column of $\boldsymbol{\beta}'_*$. The first test is reported in Table 1.14 as R_{OV1} . We are unable to reject the hypothesis of unitary coefficient on the actual series in every cointegrating relationship. We obtained a statistics of 5.28 which is to be compared with a χ^2 with four degrees of freedom. At 5% this value is 9.49, which is larger than the statistics reported. The co-trending hypothesis, that is equivalent to the expectational errors having a time-invariant mean, is tested given R_{OV1} and is reported as R_{OV2}. This hypothesis is asymptotically rejected at a 5% significance level, but not at 1%. However, when testing the hypothesis that the trend coefficients are absent from the cointegrating relations corresponding to $\eta_{1,t}$, $\eta_{2,t}$ and $\eta_{3,t}$ alone, leaving out the fourth expectational errors, the results are more convincing. This test is denoted by R_{OV3} and the results indicate that the joint hypothesis of efficiency and co-trending appears to be consistent with the data as far as $\eta_{1,t}$, $\eta_{2,t}$ and $\eta_{3,t}$ are concerned. Regarding $\eta_{4,t}$ the evidence appear to be less conclusive, but as shown in Papaikonomou and Pires (2006), this is a result of small sample bias[32].

Testing for Unbiasedness in the Long Run

The test of Long-run Unbiasedness was explained in section 1.4.2. Noting that $\mathbf{a}_0$ takes the form given in (1.25), given all current assumptions and an estimate mean of real-time growth of actual output, $\Delta_t y_{t-1}, \hat{\gamma}_1$, the long run unbiasedness hypothesis corresponds to the following set of restrictions: $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$ and $\mathbf{A}(1)\boldsymbol{\gamma} = \boldsymbol{\beta}'\boldsymbol{\mu}$, where $\mathbf{A}(1) = \sum_{i=1}^{4} A_i$ and A_i, $i = 1, ..., 4$, are defined along with $\boldsymbol{\beta}'$ in (1.20). The estimates of $\boldsymbol{\beta}'\boldsymbol{\mu}$ are reported below Table 1.14 together with $\mathbf{A}(1)\boldsymbol{\gamma}$, with $\hat{\gamma}_1 = 0.00752$[33].The relevant Wald statistics was found to be 2.691, with a 5% critical value of 9.49, showing unbiasedness of expecta-

[32] The small sample critical value at 5% takes the value 21.17 , which means that the hypothesis cannot be rejected. Details can be seen in Papaikonomou and Pires (2006).

[33] This figure refers to the average growth rate of actual output excluding the dummy observations. The full-sample average growth is 0.008, as reported in Table 1.9a. All results remain unchanged using either estimate.

tions. However, it should be stressed that this results rest on the joint assumption $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ and $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$, for which evidence on the expectational error $\eta_{4,t}$ was less convincing.

Vector Error Correction Model and Dynamic Behaviour of Output Expectations

[Table 1.15]

In Table 1.15 we report the Error Correction Model (ECM) under the R_{OV2}. We have a reasonably good fit, as reported by the $\bar{R}^2$, which varies between 0.69 for the actual output regression and 0.41 for the three-period ahead forecast regression, which is the lowest fit. The long-run relationship between actual and current-forecast expressed in the error correction term represented by ECM1 is statistically significant in all equations except for the current-period forecast. The ECM2, denoting the long-run relationship between actual and 1-period ahead forecast is significant at 5% in two regressions, and has a p-value of 6.4% in a third one. The ECM3, relating actual and 2-period ahead forecasts in the long-run is only significant at 10% in equation for 3-period ahead, having a p-value of 7%. The fourth ECM is statistically insignificant in any regression. In terms of diagnostic tests, the model exhibit no problems in terms of serial correlation, functional form, normality or heteroskedasticity, as it can be seen from the last four rows of the Table. The statistics presented should be compared with χ^2 with 4, 1, 2 and 1 degrees of freedom for the cases of serial correlation, functional form, normality and heteroskedasticity tests respectively. The statistics are always below the respective critical values, which, at 5% take the values $\chi^2_{(1)} = 3.84$, $\chi^2_{(2)} = 5.99$, $\chi^2_{(4)} = 9.49$.

[Figure 1.7]

Figure 1.7 illustrates the dynamic response of $\mathbf{z}_t$ to a one-standard error innovation in ${}_t y_{t-1}$ using the Generalised Impulse Responses (GIRs), which in this case are exactly equivalent to the more standard Orthogonalised Impulse Responses (OIR) because ${}_t y_{t-1}$is the first entry in $\mathbf{z}_t$.[34]. The impact effect of the shock is to raise real-time output by 0.62% and output expectations between 0.668% and 0.697%. After approximately 14 quarters all variables stabilise at 0.756% higher than their pre-shock level. This illustrates the I(1) properties of the variables that cause a given shock to have a permanent effect, as well as the stationary nature of the expectational errors that eventually eliminates the gap between actual and expected output.

Inflation in the US Economy - Empirical Analysis

In this section we analyse inflation in the US economy in real time. As for the output, we make use of direct measures of expectations for current period's expected price level, the one, two and three-period ahead forecasts which once again are taken from the Survey of Professional Forecasters. The data is quarterly and refers to the period 1968q4-2001q2. Our empirical analysis is based on the models given by (1.39). A number of dummy variables have been included. These include dummies to accommodate some of the data revisions discussed earlier $(d81q1_t, d96q1_t, d99q4_t)$. The inflation series has also been subject to several once-and-for-all events which can be accommodated with dummies. The announcement in August 1971 of the end of automatic conversion between US $/gold that characterised the Bretton Woods system, justify a dummy $d71q4$; the oil prices

[34] See Pesaran and Shin (1998) for a proof of the fact that GIR=OIR when considering a shock in the first variable of the system.

increases in October 1973 following war breaking between Israel and Arab countries justify dummies $d73q3_t, d73q4_t$; the need to include other events, associated with recessions ($d78q3_t, , d81q4_t$) and peaking of inflations expectations ($d75q2_t$) are reflected by the poor diagnostic tests on Serial Correlation and Normality; we chose to eliminate these observations through dummies to improve the statistical performance of the equations, although the results are qualitatively similar, if these dummies are dropped. [35]

The inflation expectational errors $\eta_{i,t}$, $i = 1, 2, 3, 4$, $t = 1, 2, ..., T$, can be defined with reference to (1.17) when $x = \pi$, and where ${}_t\pi_{t-1}$ is the (real-time) actual inflation in time $t-1$, made available to the respondents of the survey at time t, while ${}_s\pi^e_p$ denotes the expectation formed at time s on the value of output in time p, $s \leq p$.[36]For the m-vector $\mathbf{z}_t = [{}_t\pi_{t-1}, {}_t\pi^e_t, {}_t\pi^e_{t+1}, {}_t\pi^e_{t+2}, {}_t\pi^e_{t+3}]'$ the vector of expectational errors $\boldsymbol{\eta}_t = [\eta_{1,t}, \eta_{2,t}, \eta_{3,t}, \eta_{4,t}]'$.

The empirical version of the model given by (1.39) takes the form of

$$\Delta\mathbf{z}_t = \mathbf{a}_2\mathbf{D}_t + \Pi^*\mathbf{z}_{*,t-1} + \sum_{i=1}^{p-1}\Gamma_i\Delta\mathbf{z}_{t-i} + \mathbf{e}_t \tag{1.48}$$

$t = 1, 2, ..., T$,where $\mathbf{z}_t = [{}_t\pi_{t-1}, {}_t\pi^e_t, {}_t\pi^e_{t+1}, {}_t\pi^e_{t+2}, {}_t\pi^e_{t+3}]'$, $\Pi^* = \boldsymbol{\alpha\beta}'_*$, $\boldsymbol{\beta}'_* = [\boldsymbol{\beta}', -\boldsymbol{\beta}'\boldsymbol{\mu}]$, $\mathbf{z}_{*,t} = [\mathbf{z}'_t, 1]'$, and $\mathbf{e}_t \sim IN(\mathbf{0},\Omega)$, Ω positive-definite;

[35] The model has also been estimated without any dummies. The number of cointegrating relationships remained unaltered at four and all restrictions imposed have not been rejected. However, we encountered problems related to serial correlation on the first two equations and problems with normality across all equations. Increasing the order of the VAR from three to four did not solve the serial correlation problem, leading us to try to undertand the reasons for it. Keeping in mind that the period is rather long and in particular comprehends two particular difficult decades in term of inflation and output performance, with recessions and very high inflation rates hitting the economy, we decided to scruitinize the period and try to understand outliers in the sample.

[36] The inflation rates consist of the first differences of the (natural logarithm) of price level series, details of which are provided in the data appendix.

$$\mathbf{D}_t = [d71q4_t, d73q3_t, d73q4_t, d75q2_t, d78q3_t, d81q1_t, d81q4_t, d96q1_t, d99q4_t]'.$$

Note that we have not included a trend in the cointegrating relationship. The variables included in z_t, this time, actual and expected inflation, do not show a clear linear trend, as can be seen from the plot of the variables in Figure 1.5.

Under this formulation, the expected value of the expectational errors, $\eta_t$takes the form of

$$\mathbf{E}[\boldsymbol{\eta}_t] = \boldsymbol{\beta}'\boldsymbol{\mu}\,, \qquad t = 1, 2, ..., T \tag{1.49}$$

In this case, we directly estimate $\boldsymbol{\beta}'\boldsymbol{\mu}$,and as a result, the unbiasedness condition is tested by using the condition $\boldsymbol{\beta}'\boldsymbol{\mu} = \mathbf{0}$, in which case $\mathbf{E}[\boldsymbol{\eta}_t] = \mathbf{0}$, as required.

Statistical Properties of the Data

[Tables 1.16a-1.16b]

Some summary statistics of the data are provided in Tables 1.16a-1.16b. We also plotted the series as can be seen on figures 1.5 to 1.6. We present the mean values for inflation over the sample as well as maximum, minimum values and standard deviation. The mean quarterly inflation was around 1.1% for every series, ranging from a minimum of -0.9% to a maximum of 4.1% over the period under consideration. The standard deviation for the actual real-time series was of 0.7%, lower that the 0.1% observed for the expected series. We also present in Table 1.16b the estimated correlation coefficients between the series. As we can see, the correlation are relatively high, although not close to one. In particular, the correlation between actual and each of the expected series varies between 0.6176 and 0.5306, for the current-period forecast and 3-periods-ahead forecast, respectively. One interesting aspect of it is the decrease in correlation as we increase the forecast horizon. In other words, the correlation between actual and expected inflation is higher for the current-forecast and progressively lower for the one, two and three periods ahead, suggesting that individuals would get closer to the actual series the more information available they have, justifying the result observed. This is a common feature with the output series, described in the previous section.

[Tables 1.17a-1.17d]

As it happened for output, the modelling framework is based on the assumption that $\mathbf{z}_t$ includes variables which are integrated of order one. Once again we performed standard univariate ADF tests as well as multivariate ADF tests, reported in Tables 1.17a-1.17d. Table 1.17a reports the ADF unit root tests applied to the actual and expected price levels. The ADF underlying regression includes an intercept and a trend. The AIC and SBC indicate an ADF test of order 3 for both actual and current-period price forecast, whereas for the one and two-periods ahead forecast, SBC indicate order 2 and AIC orders 3 and 4 respectively. The three-periods ahead forecast had a chosen order of 2 using both SBC and AIC. In any case the statistic is lower (in absolute value)than the 95% critical ADF value of -3.44, indicating that the unit root hypothesis cannot be rejected. We then proceed to the analysis of those variables in their first differences, i.e., the inflation rate (given that levels are in logarithms). These results are presented in table 1.17b. The ADF regression this time did not include a time trend, as it is not clear from the figure 1.5 the presence of a linear trend on either actual or any of the expected series. The results are not as clear cut as for the levels. The SBC's order selection would actually imply a rejection of the hypothesis for the two and three-periods ahead forecast. The results though are different when considering the AIC criteria. We are unable to reject the null hypothesis for all the series, as the statistics are smaller (in absolute value) than the critical value of -2.88. However, when performing the ADF tests in a model-consistent way with the use of multivariate ADF tests, we get a clear rejection of the null hypothesis, which in this case is of stationary, as it can be seen from Table 1.17d. The results are dependent upon the number of cointegrating relationships. However, for any number of cointegrating relationships and for every single series, we obtain a statistics that are always well above the 5% critical values reported in the table. Given the ambiguity of the univariate results and the renown uncertainty surrounding the order of integration of inflation, we chose to follow the results of the model-consistent multivariate test and consider all the actual and expected series as I(1) processes. The results for the second-differences in the variable price level are reported in Table 1.17c, and there is clear rejection of the unit root hypothesis.

[Table 1.18]

When considering the order of the underlying VAR, the SBC criteria indicates a order of 1, whereas the AIC criteria indicates order 4. The adjusted LR test indicates order 3. We will be using the latter lag-length for the VAR. The underlying VAR was of order 6, and included the dummy variables referred to earlier on, and an intercept. These results are reported on Table 1.18.

[Table 1.19]

Testing for Cointegration

Having established the order of integration of our $\mathbf{z}_t$vector, we may proceed with the estimation of the model described in (1.48). Table 1.19 reports cointegration rank statistics, and it suggests, as we would expect the existence of four cointegrating relationships. When testing the hypothesis of three cointegrating relationships(being this the null hypothesis) - *versus* four, there is a rejection of the null using the Max Eigenvalue and Trace Statistics. The statistics are 57.76 and 61.19, which are larger than the respective critical values of 15.87 and 20.18 respectively. However, the null of four cointegrating relationships *versus* the alternative hypothesis of five cannot be rejected by either criteria. The statistics 3.53 is smaller than the 9.16, 5% critical value.

[Tables 1.20-1.21]

Testing for Unbiasedness in the Long Run

The hypothesis of long-run expectational unbiasedness as discussed before, takes the form of

$$\mathbf{E}[\boldsymbol{\eta}_t] = \boldsymbol{\beta}'\boldsymbol{\mu} = 0, \tag{1.50}$$

The four cointegrating relationships are represented in Table 1.20, where four exactly identifying restrictions have been imposed in each cointegrating VAR. Following the same logic as for the output, we set zeros to all expected series but the one under analysis, which coefficient we set to one. The first over-identifying restriction to be tested regards the coefficient on actual real-time inflation. To find out whether there is a one-to-one correspondence between actual and expected inflation at different time horizons, we imposed a -1 coefficient in the actual inflation series of each cointegrating vector. The results denoted R_{OV1} and expressed on the second row of Table 1.21 showed a non-rejection of the null, with a statistics of 4.248, to be compared with a critical χ^2 with four degrees of freedom, which takes a value of 9.49 at 5% significance level. The last row of Table 1.21 reports the statistics for testing the joint hypothesis of "unitary coefficient" and zero intercepts in long-run relationships. The test of unbiasedness in all four expectational errors, denoted by R_{OV2} leads to a non-rejection of the null. We obtained a statistics of 5.435 to be compared with a χ^2 with eight degrees of freedom. which at a 5% significance level takes the value of 15.507. As a result, we can conclude that the expected inflation series are unbiased in the long run.

Vector Error Correction Model and Dynamic Behaviour of Inflation Expectations

[Table 1.22]

Table 1.22 provides with the VECM estimates and relevant diagnostic statistics. All equations appear to have a good fit, as can be illustrated by the adjusted $\bar{R}^2$, which take values in the range [0.502,0.672] across the four equations, although the dummies show significantly throughout and clearly contribute to the fit. The equations are a little over-parameterised, so that only a small number of individual terms have a significant effect. However, there are still some dynamics and most importantly the error correction terms show significance in the estimated model and hence have an important impact on the system as a whole. In terms of diagnostic tests, all equations exhibit no problems in terms of serial correlation, functional form, normality or heteroscedasticity, as they pass all the tests. These tests are presented in the last four rows in the Table.

[Figure 1.7]

The dynamic response of $\mathbf{z}_t$ to a one-standard error innovation in ${}_t\pi_{t-1}$ using the Generalised Impulse is illustrated in figure 1.8. Real-time inflation raises by 0.383% on impact. Current forecast inflation raises by 0.3718%, with 1, 2 and 3-periods ahead forecast raising by 0.403%, 0.430%, 0.425% respectively. All variables stabilise at approximately

0.20% higher than pre-shock level after 29 quarters. The fact that a shock has permanent rather than temporary effect on actual and expected inflation, confirms once again the I(1) properties of all series.

1.5 Conclusions

In this chapter we made use of direct measures of expectations on output and inflation from different sources of survey with the objective of (i) creating expectational series and making a direct analysis of the expectation formation process for the case of the UK and (ii) develop a real-time, multivariate framework to analyse actual and multi-period forecasts for the case of the US.

For the case of the UK Manufacturing Sector, we first needed to convert the qualitative measures of expectations into a quantitative series. We did that by using the Carlson and Parkin method together with Lee (1994)'s method to correct for measurement errors. We showed that output was an I(1) process and that expectational errors were stationary. We also showed inflation to be an I(1) process, with stationary expectational errors. When testing for rationality we found the purged series to be the best measure of expectations and rationality could not be rejected, in particular we found that agents would take into account all the information available when formulating their expectations. Inflation was less clear-cut than output in this test, but still we had no evidence to reject our hypothesis.

The analysis on the US economy illustrated how actual output and direct measures of output expectations over different time horizons and actual and expected inflation may be studied simultaneously within a multivariate cointegrating framework. There already exists a rich literature on the use of survey data for the investigation of the expectations formation mechanism and its consistency with the rational expectations hypothesis. However, very little work has been done on the simultaneous modelling of actual and expected series and most of the existing research ignores the cointegrating properties of the series. In the second part of the chapter, we attempted to address both these issues within the long-run structural VAR framework of Pesaran, Shin and Smith (2000). This framework allows us to formulate and test the hypotheses of stationarity of expectational errors and unbiasedness of expectations in the long run in an otherwise unrestricted model that maintains the advantages of Vector Autoregressions in being able to capture complex short-run dynamics, while taking into account the $I(1)$ properties of the data with its implications on cointegration. Our findings have confirmed the quite frequent assumption of stationary expectational errors. Furthermore, all expectations may reasonably be argued to be unbiased in the long run both for output and inflation. The one exception of the three-period ahead forecast on output, for which the evidence is less convincing, although work carried out by myself and Papaikonomou after this work showed that this may be due to small sample bias. [37] The impulse responses confirmed the I(1) properties of all series. Shocks to output proved to have a permanent effect on all actual and expected output series, taking around 14 quarters to stabilise at new higher level. Inflation shocks affect all actual and expected inflation series permanently as well, but taking longer, approximately 29 quarters, for the series to convergence to the new level. In the case of output, the adjustment process is faster the shorter

[37] In Papaikonomou and Pires (2006) we used boostrap methods in order to simulate finite-sample distributions and we were unable to reject the unbiasedness hypothesis for that series.

the forecast horizon, with the 3-periods ahead forecast taking longer to stabilise, whereas for the case of inflation, the series stabilise around the same quarter.

1.6 Data Appendix

1.6.1 Definition of the variables used in the UK empirical work

The data is quarterly, seasonally adjusted and extends over the period 1975q3-2000q4. The variables are defined as follows:

Δy_t represents difference between the (natural logarithm of) output level taken at the beginning of each quarter (i.e., January, April, July and October) , from the (natural logarithm of the) level four months previously, in order to match the CBI survey data, where questions refer to changes over four-months. The monthly data is taken from the UK Output of the production Industries, Total Manufacturing Industries, Seasonally Adjusted, Monthly Digest of Statistics. The actual output growth as a result, has a quarterly frequency, but refers to changes over four-months period. (just like the CBI data).

π_t represents inflation at quarter t, and is defined in the same way as output. It represents the difference between the (natural logarithm) of price level and its level four months previously. The source of the data is again the Monthly Digest of Statistics, the series on price indices of the output of manufactured products, home sales and it is seasonally adjusted.

The expected series were derived as explained in the text, having as the source, the CBI. The questions regarding output and prices correspond to questions regarding the ”Total Sample, UK”and are formulated respectively as follows:

”Excluding seasonal variations, what has been the trend Over the PAST FOUR MONTHS, and what are the expected trends for the ”NEXT FOUR MONTHS” with regard to:

8) Volume of output

12) average prices at which domestic orders are booked.

The response regarding output has been phrased as such since 1975q3; prior to that, it refers to ”value” rather than ”volume”.

With reference to the sample table provided, we have made sure that the sum of the three "up", "down" and "same" response categories equated to 100; namely, under the presence of N/A answers, we have allocated them proportionately to the other three categories.

1.6.2 Definition of the variables used in the US empirical work

The data is quarterly, seasonally adjusted and extends over the period 1968q4-2001q2. The variables are defined as follows:

y_t is (the natural logarithm of) US real GNP prior to 1992 and GDP since then to match a change in the questionnaire formulated by the Survey of Professional Forecasters. It is quarterly, seasonally adjusted, annual rate.

The five different measures in use include

- the real-time output level released 15 days after the end of each quarter (15-day vintage), and published therefore with a period lag $({}_{t+1}y_t)$, and corresponds to real-time data, provide by the Federal Reserve Bank of Philadelphia

- and the current, one, two and three-periods ahead forecast of output level, taken to be the mean of the responses obtained in the Survey of Professional Forecasters.

A typical questionnaire, say 2001q2, would be sent to the respondents having the figure for 2001q1 filled in, with instructions to "if you use these figures in preparing your forecast, please copy them in the appropriate columns; otherwise insert your own figure". They are then asked to fill in the table for the different forecast horizons.

p_t is (the natural logarithm of) US GDP price index. It is a quarterly index, seasonally adjusted. Prior to 1992, GNP implicit deflator; since then, GDP implicit deflator. The five different measures in use are defined in the same manner as for output. The inflation rates defined in the paper consist of the first differences of the (natural logarithm) of each of the price level series.

Chapter 2
Modelling UK Manufacturing Output and Inflation Using Survey Data

2.1 Introduction

The relationship between fluctuations in real activity and price inflation is central to the study of macroeconomics and there have been innumerable studies investigating the relationship empirically. Following Phillips's(1958) seminal paper, the view developed that there exists a trade-off between real activity and inflation which could be exploited by monetary authorities. This view was challenged in Friedman (1968)'s and Phelps (1968)'s exposition of the Natural Rate Hypothesis (NRH) which argued that the trade-off, if it existed, would do so only in the short-run (over which period expectational errors were possible). Over the longer term, when expectational errors were eliminated, the trade-off would disappear. With the widespread adoption of the Rational Expectations Hypothesis (REH) as an explanation of how expectations are formed through the seventies, the joint NRH-REH assumption was used to argue that monetary policy was ineffective in systematically influencing real magnitudes over any horizon. Over the last thirty years, various arguments relating to market imperfections, nominal and real rigidities, information flows, coordination failures and so on have been used to explain why monetary policy might have real effects over the short- or medium-term, but there exists now a broad consensus that there is no trade-off between inflation and output growth over the long-term. This has important implications for the conduct of monetary policy, leading to Kydland and Prescott's (1977) analysis of the time consistency problem, later developed in Barro and Gordon (1983). They showed that monetary policy set by discretion is tempted to be time-inconsistent, which translates into a suboptimal inflation outcome.

In this chapter, we describe a modelling framework for explaining the determination of output growth and inflation. The framework incorporates a simple model of long-run growth and distinguishes between the associated long-run trend in output and short- and medium-term fluctuations around the long-run growth path. This distinction provides a taxonomy of output concepts, motivating measures of 'trend' and 'natural' levels of output, for example, and helps us to distinguish between different types of shocks. The framework also incorporates a view on the determination of inflation, assumed to be driven by time-consistent monetary policy along the lines described in Kydland and Prescott (1977). The modelling framework is used to motivate a structural Vector-Autoregressive (VAR) model of output growth and price inflation. The model is written in terms of both actual and *expected* measures of output growth and price inflation on the assumption that direct measures of expectations of these concepts are available. The identifying restrictions in this analysis arise partly out of the structure provided by the economic theory and partly out of the expectations formation process. Uncertainty regarding the best model underlying the economy leads us to start with a general model and then impose some further structure by testing the validity of restrictions, which would then help us formulate the most suitable model, given the data. Those tests are made possible due to the presence of direct measures of expectations obtained from survey data.

The plan of the remainder of the chapter is as follows. Section 2 provides background to the modelling framework, making references to the literature; Section 3 discusses the modelling framework used to motivate the structural VAR model of actual and expected output growth and inflation that is described in section 4; the empirical results are presented in section 5 and section 6 provides some concluding remarks.

2.2 Background to the modelling framework

Inflation together with unemployment is at the centre of economic policy. Both inflation and unemployment carry costs. While the costs of inflation include menu costs, shoe-leather costs, inefficient allocation of resources, undesirable distributional effects; loss of production, distributional and social negative impact are among the most cited unemployment costs. However, despite the fact that both, inflation and unemployment carry costs and these are undesirable outcomes, the existence of a Phillips curve implies that the policymaker will only be able to reduce unemployment at the cost of higher inflation. Understanding the tradeoff is therefore crucial to better guide policymakers on how to deal with it and try to pick the lowest cost combination of unemployment and inflation.

In this paper, we try to explain the behaviour of inflation and output[38] in the UK Manufacturing Sector, by extending Barro and Gordon's (1983) model with the inclusion of a simple model explaining the supply side of the economy, with reference to which the concepts of natural rate and capacity output will be defined. In this section we will give

[38] Although the introduction made reference to unemployment, for being more intuitive, through Okun's law we can easily translate unemployment into output.

some background to the theories put forward to explaining inflation and how the natural rate of output has been treated in the literature and the role it plays in the conduct of monetary policy.

2.2.1 Inflation determination

One of the most important theories that has been used to explain inflation is based on the work of Kydland and Prescott (1977) and Barro and Gordon (1983), who analysed the time-inconsistency problem of optimal policy that leads to inflation. This model has received renewed interest in more recent years. Ireland (1999, 2002), Chari, Christiano and Eichenbaum (1998), Christiano and Fitzgerald (2003) are a few examples. According to the model, policymakers wish to lower unemployment below the natural rate, and therefore are tempted to exploit the short-term inflation/unemployment tradeoff by creating surprise inflation. However, rational agents, who understand the incentives facing the policymakers will get to expect a higher inflation rate, and this will translate into a higher inflation outcome with no gain in terms of output or unemployment.

To understand better the problem let us consider a central bank that minimises a loss function of the type:

$$L = \frac{1}{2}(y_t - y^*)^2 + \frac{\gamma}{2}(\pi_t - \pi^*)^2 \tag{2.51}$$

where π_t is actual inflation; π^* is the inflation target; y_t is actual output and y_t^* is the output target. We will represent the problem in terms of output instead of unemployment to relate better to our work.

The first term denotes the costs associated with deviations of output from target, with the second term denoting the loss associated with inflation not being at the target level. The quadratic formulation, among other things[39], implies that the policymaker penalizes deviations above and below the target equally, with the nonnegative coefficient γ denoting the relative weight place on deviation of inflation from the target relative to deviations of output from target. $\gamma > 1,$ implies that central bank places more weight to deviation of inflation from the target level. For simplicity, let us assume that inflation target is zero[40]. We also assume that the policymaker wishes to target a level of output above the natural rate, i.e., above the equilibrium output, i.e., $y^* > \bar{y}$.[41]For simplicity, let us assume that $y^* = \bar{y} + \alpha,$ with α nonnegative. The fact that the monetary authorities target a level of output above the natural rate can be rationalised through the presence of distortionary taxes or monopoly power that lead to a below-optimal level of output.

In addition, we consider the following formulation for the supply curve, the Lucas-surprise equation[42]:

$$y_t = \bar{y} + \lambda(\pi_t - \pi_t^e) \tag{2.52}$$

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39 The quadratic formulation also implies increasing absolute risk aversion.

40 Introducing an inflation target different from zero does not alter the qualitative predictions of the model. Note that there is considerable disagreement on what is considered to be the optimal inflation rate. While some still argue for a zero inflation rate, many countries have adopted an inflation target that is a small positive rate. Examples include the UK, with a 2% inflation target. The non-zero inflation rate is believed to introduce more flexibility into the economy (regarding for example, wage negotiations, monetary expansion) without incurring significant costs in terms of the inefficiencies that inflation introduce.

41 Or equivalently, the policymaker wishes to target an unemployment rate below the natural rate.

42 Note that it is expressed in terms of inflation, rather the price level, ie, has a "Phillips curve" formulation rather than a "supply curve" formulation.

where $\lambda > 0$, meaning that when inflation is above expected inflation, output is above the natural rate; when inflation expectation are realised, output is in equilibrium at $\bar{y}$. This model of aggregate supply can be given micro foundations based on imperfect information. With reference to the product market, we assume that producers can only observe the change in their own product price, being unable to tell whether this reflects a change in relative price in their favour or a result of a change in the aggregate price level. Under the first hypothesis, the producer should respond by changing production, increasing it to the new optimal level; under the second hypothesis, production should be left unchanged, since changes in the aggregate price level does not alter producers optimal choice. However, given uncertainty coming from lack of perfect information, the optimal, rational response is actually to assume that the change in its own product price is a reflection of the two hypothesis, and as a result, the producer is lead to increase output. This justifies the positive relation between prices and output (i.e., $\lambda > 0$). We also assume that agents are rational, having full knowledge of the policymakers objective function and the workings of the economy, which implies that in the absence of shocks, they have perfect foresight, i.e., $\pi_t = \pi_t^e$. This condition must hold in equilibrium. As pointed out by Ireland (2002), the important issue, discussed in Kydland and Prescott (1977), is how the policymaker perceives this condition, in particular whether it is considered to be a constraint, when choosing the inflation rate. Two interesting cases can be analysed. The first one, called "commitment" where the central bank pre-commits to a choice of π_t, before agents formulate their expectations, and before any economic transaction takes place, namely wage negotiations. This is a situation where the central bank takes the condition above, $\pi_t = \pi_t^e$, as a constraint. The policymaker problems becomes:

$$\min_{\pi} L \text{ subject to } \pi_t = \pi_t^e \tag{2.53}$$

Taking into account that $y^* = \bar{y} + \alpha$,we get:

$$L = \frac{1}{2}(y_t - (\bar{y} + \alpha))^2 + \frac{\gamma}{2}(\pi_t - \pi^*)^2 \tag{2.54}$$

Substituting the supply curve into the loss function, we get:

$$L = \frac{1}{2}(\lambda(\pi_t - \pi_t^e) - \alpha)^2 + \frac{\gamma}{2}(\pi_t - \pi^*)^2 \tag{2.55}$$

This loss function is minimised with respect to π_t, and subject to $\pi_t = \pi_t^e$, leading to the following first order condition:

$$\pi_t^c = 0 \tag{2.56}$$

In other words, the optimal inflation rate under commitment, π_t^c, is zero, which is exactly the inflation target. By pre-committing to a choice of π, the central bank is no longer able to surprise economic agents and exploit the short-run tradeoff between inflation and output, by trying to push output above the natural rate.

The outcome under "no commitment" or "discretion" is very different. In this case, the central bank is unable or unwilling to pre-commit, making the choice of the inflation rate after the agents have formed their expectations. In this case, the central bank's problem is reduced to an unconstrained minimisation of the above loss function, (2.55),

$$\min_{\pi} L = \frac{1}{2}(\lambda(\pi_t - \pi_t^e) - \alpha)^2 + \frac{\gamma}{2}(\pi_t - \pi^*)^2 \tag{2.57}$$

The first order condition is given by:

$$\pi_t = \frac{(\lambda/\gamma)(\lambda\pi_t^e + \alpha)}{1 + \lambda^2/\gamma} \tag{2.58}$$

In equilibrium, $\pi_t = \pi_t^e$ and the optimal inflation rate under "no commitment" , π_t^{nc}, becomes:

$$\pi_t^{nc} = (\alpha\lambda)/\gamma \tag{2.59}$$

which is non-negative, i.e., $\pi_t^{nc} = (\alpha\lambda)/\gamma \geq 0$.This outcome is the so-called inflation bias. Before expectations are formed, the monetary authority would like to try to convince people that they will choose a zero inflation rate. This intends to give the central bank better inflation-output set of tradeoffs. The problem is that the central bank, after agents formulate their expectations, have the incentive to choose a higher than initially expected inflation rate, in order to try to push output above the natural rate. This temptation to renege on announcements or promises is recognized by the rational agents, who end up expecting a higher inflation rate, which translates into a higher actual inflation, with no gains in terms of output, which remains at the natural rate. In other words, the efforts of central bank to exploit the Phillips curve lead only to a suboptimal higher rate of inflation. The crucial point of the model is that a credible commitment to low inflation leads to better outcomes than if no ability to commit exists.

The outcome under ”no commitment” also give insights on how to potentially solve the time-inconsistency problem, and minimise or eliminate the inflation bias. Different solutions have been proposed in the literature. McCallum (1995) suggests as solutions, setting the parameter α to zero, i.e., central bank would stop targeting a level of output above the natural rate. As it can be seen from (2.59), in this simplified model, this restriction is enough to eliminate the inflation bias altogether. Another solution consists of increasing γ, ie, the weight put upon inflation deviations from target in the loss function. This is the basis of Roggoff's (1985) solution of designating a conservative central bank, with tougher views upon inflation than the public. Alesina and Summers (1993), for example, show that Central bank independence helps reducing inflation. Others believe that the key is credibility[43].

The time-consistency theory, described above, falls into what Christiano and Fitzgerald (2003) called ”the institutions view” set of theories on inflation. This theory has been used to explaining inflation dynamics, in particular, the Great Inflation of the 1970s. According to this view, the ”lack of commitment” is at the root of the high rates observed then. However, this is not the only view on the matter. Some authors have criticized the time-consistency approach on the basis that, for example, the inflation bias that it generates is quantitatively small, as referred to in Primiceri (2006). Furthermore, they claim that this theory encounters problems when trying to explain the US disinflation of the 1980s, as there has not been any major institutional change in that period (DeLong(1997)). However, as argued by Coe, Durand and Stiechier(1988), credibility was a key determinant of the disinflation that followed the great inflation. A different line of argument concentrates on economic theory and the evolution of ideas. Sargent (1999) attributed the great inflation period to the (false) belief of a permanent long-run tradeoff between inflation and output. This explains the lack of tight monetary policy during the period. The disinflation, accordingly, would have been generated by a change in belief regarding the (lack of permanent)

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[43] Other solutions to the inflation bias put forward in the literature include Walsh (1995) and involves the setting of incentive contracts for central bankers.

tradeoff which prompted contractionary policy. However, Romer and Romer (2002) find that as early as the 1970s, policymakers had changed their beliefs and accepted a version of Friedman-Phelps Natural Rate Hypothesis. Orphanides (2001, 2003a, 2003b) has started a whole different line of research on the subject, by stressing the importance of real-time data and advocating the output gap mismeasurement theory. According to Orphanides the great inflation of the 70s and subsequent disinflation was due to the fact that in the 1970s policymakers were basing their policy decisions on an mismeasured potential output. Support for this theory include Cukierman and Lippi (2005) and Bullard and Eusepi (2003). Another explanation includes Clarida, Gali and Gertler (2000) and makes reference to the taylor rule. Clarida et al. (2000) assume that monetary policy could be well represented by a Taylor type of rule, where the policy instrument is the interest rate and it responds to output gap and inflation. In their empirical work, they found structural breaks in their sample, with the period in the 70s characterised by a policy rule where the Fed did not respond sufficiently to inflation, failing to deliver a stabilizing policy. [44]

Despite the competing theories, Kydland and Prescott (1977) and Barro and Gordon (1983) have received great interest in the recent empirical literature. Broadbent and Barro (1997) extend the Barro-Gordon model by considering a small macro model of the US economy, having as an objective the investigation of the behaviour of the monetary authority, who also exploits the tradeoff inflation-output derived from inflationary surprises. Ireland (1999) estimates an extended structural version of Barro and Gordon's model. He was able to successfully accept the long-run restrictions implied by the model. However,

[44] The relevance of these two last theories are discussed in detail in chapter 3.

although he obtained reasonable and significant estimates of the parameters, the short-run dynamics were not accepted against an unrestricted version of the model. According to Ireland, Barro and Gordon should be extended, as it has difficulties in capturing the persistence in inflation. Reis (2003) shows that this persistence can be well captured by introducing uncertainty about the natural rate of unemployment into the model. In his paper he finds that, like in Barro and Gordon (1983), inflation will deviate from target, but these deviations do not arise from a deliberate attempt from the monetary authority to deceive economic agents. Instead, they are the outcome of imperfect information regarding the natural rate of unemployment. This is similar to Orphanides (2003b), although he analyses the output gap uncertainty in the context of optimal interest rate rules. This is the object of another important issue of this paper - the treatment of the natural rate of unemployment.

2.2.2 The output gap and measurement issues

The natural rate of unemployment was introduced by Phelps and Friedman in 1968 and since then its understanding and definition has changed. In fact, the natural rate of unemployment or the equivalent natural level of output has been defined and treated in several different ways. Phelps (1968)[45] defined it as the ”rate at which the actual and expected price increases (or wage increases) are equal...”. Friedman (1968) defined it as ”the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies,

[45] Phelps (1968), pp 682.

the cost of gathering information about job vacancies and labor availabilities, the costs of mobility, and so on."[46] A common definition relates to the rate consistent with stable inflation, the so called Non-Accelerating Inflation Rate of Unemployment (the NAIRU). In some work, the natural rate is seen as an equilibrium within a market-clearing framework, whereas the NAIRU is seen as consistent with market imperfections that prevent market clearing and therefore consistent with involuntary unemployment in equilibrium. However, very often the concepts are used interchangeably to denote the equilibrium unemployment. Although its usefulness as an equilibrium concept is acknowledged by many, the Natural rate or NAIRU is not directly observable and therefore its estimation requires some economic and statistical procedure and assumptions. Although several methods have been developed in the literature, there is little consensus on the best way to dealing with it and as a result there is little consensus on its exact value. The work by Staiger, Stock and Watson (1997) points out the imprecision of its estimates. They define two broad procedures to estimating the natural rate: one based on a Phillips type of curve, translating into a stable relationship between output gap and inflation; the other based on univariate behaviour of output, modelling the natural rate in different ways, including treating it as a constant or a constant with changes in time, for example. Gordon (1997), for example, assumed that the natural rate followed a random walk. In recent literature, many popular estimates of the output gap encompass a detrending procedure for output, by means of a linear or quadratic trends or more elaborated detrending procedures using filters such as the Hodrick-Prescott (1997) filter[47]. Gordon (1997) also stresses the divergence between different methodolo-

[46] Friedman (1968), pp 8.

[47] The Hodrick-Prescott filter will be dicussed in chapter 3.

gies regarding the smoothness of the estimation of the trend. Other methods to obtain trend output have been used. Lee and Shields (2000a, 200b) have created an elaborated framework to measure output trend. In their paper, direct measures of expectations on output taken from the CBI survey have been incorportated into the a VAR framework to obtain a measure of a trend output by means of a Beveridge Nelson decomposition.

As stressed by these authors, the natural rate is also a series that changes over time which creates further difficulties in measuring it. Several authors have estimated considerable change in the NAIRU of several OECD countries, including the US and UK. For ex, Gordon (1997) estimated large changes in the US natural rate of unemployment; Baker, Glyn, Howell and Schmitt (2005) have a survey on the developments on the NAIRU and explanation that have received attention in the literature ranging from institutional factors to productivity shocks. These facts about the natural rate have important implications for monetary policy conduct and in particular for the outcome of inflation in the context of the time-consistency theory. Debates on the conduct of monetary policy often focus on whether unemployment is above or below the natural rate of unemployment and therefore measures and outcomes in terms of policy objectives may be strongly related to the precision with which the natural rate is estimated. The relevance of this last point is made clear in Orphanides (2003b), according to whom poor inflation outcomes in the 1970s was due to a serious mismeasurement of the output gap. The Central bank interpreted the low output as a cyclical phenomenon, and responded to it in an attempt to counteract it, failing to realised that the natural rate of unemployment had actually increased. In this paper, we will be defining a time-varying natural rate of output and a time-varying capacity of output. Both definitions and treatment of these measures will be explained in section 2.3.1.

2.3 The Modelling Framework

In this section we present a simple model of inflation and output dynamics, where agents are assumed to be rational. The supply side of the economy will be presented with reference to three different output concepts, expanding on the literature by assuming a time-varying natural level of output as in Staiger, Stock and Watson (1997) but departing from it by introducing a time-varying capacity level of output towards which the natural level tend to. This last point elaborates on Staiger, Stock and Watson (2001) and more recently Primiceri (2006). We also assume that policymakers choose their policy conduct by minimising a loss function, having in mind a target for output that exceeds the natural level, and therefore giving rise to time-consistency problems and inflation bias in line with Barro and Gordon (1983) and the empirical work of Ireland (1999).

2.3.1 The source of shocks to output levels

Following our discussion on the output gap and different definitions given to the natural rate of output, in this subsection we define the three main definitions of output that will be used in our framework.

Our model has three equations representing the supply side of the economy and a further equation defining the demand side together with rationality on inflation expectations that will be discussed in next section. We will first introduce the model algebraically and then explain each equation and the three output concepts -actual, natural level and capacity output - that we introduce along the way. The terms ”natural level of output”, ”capacity output level” together with ”output trend” and ”NAIRU level of output” have been used in the literature in many different contexts and bearing different meanings in some papers[48] or treated as synonymous, as it happens in much recent work. In this chapter they will have very specific meaning that we shall define with reference to the algebraic formulation of our model that follows.

The three equations characterising the supply side of our model are:

$$\begin{aligned} y_t &= \bar{y}_t + a(L)\pi_t - b(L)\pi_t^e \\ &= \bar{y}_t + a(L)(\pi_t - \pi_t^e) + (a(L) - b(L))\pi_t^e \end{aligned} \tag{2.60}$$

$$e(L)\bar{y}_t = c(L)\bar{y}_t^c + \varepsilon_{1t} \tag{2.61}$$

$$d(L)\bar{y}_t^c = \varepsilon_{2t} \tag{2.62}$$

where $\pi_t = p_t - p_{t-1}$, represents actual inflation at time t[49], $\pi_t^e = p_t^e - p_{t-1}$ denotes the expected inflation based on information at time $t-1$, y_t is actual output at time t, $\bar{y}_t$is the natural level of output at time t, $\bar{y}^c$is the trend or capacity level of output, and $a(L) = a_0 + a_1 L + a_2 L^2 + ...$; $b(L)$, $c(L)$, $d(L)$ and $e(L)$ are also lag polynomials defined

[48] For example Estrella and Mishkin (1998).

[49] All levels of variables are expressed in logarithms.

similarly. We also assume that $a(L)$, $b(L)$, $c(L)$ and $e(L)$ are invertible, and that $d(L)$ has a unit root in L.

The first equation corresponds to a traditional expectations-augmented Phillips curve, expressed in terms of the output gap, to make the analysis that follows more convenient and to conform with the framework described in Barro-Gordon. It can equivalently be expressed as[50]:

$$\pi_t = \alpha(y_t - \bar{y}_t) + \gamma(L)\pi_{t-1} + \delta(L)\pi_t^e \tag{2.63}$$

with

$\alpha = 1/a(0)$; $\gamma(L) = -(a(L) - a(0))L^{-1}/a(0)$; $\delta(L) = b(L)/a(0)$.

This formulation is closer in spirit to the backwards-looking Phillips curve than the forward-looking Phillips curve motivated by models of sticky prices, for example, Calvo (1983). In our model, unlike the forward-looking formulations, there is persistence in inflation, which is a feature of the data, as argued by many, including Fuhrer and Moore (1995). However, it differs from the purely backwards-looking version of the Phillips curve in that we allow for inflation to depend not only on past (actual) inflation as in this version, but also to depend on expected inflation, which extends the approach to allow issues of credi-

[50] The time-consistency problem is analysed within a natural rate framework. In particular, a Lucas' surprise supply equation is assumed to capture the tradeoff inflation-output. According to it, only current inflation surprises will deviate output from the natural level. Although this model can be observationally equivalent to a price equation of the Keynesian school, and we can therefore translate one into the other, as stressed by King and Watson (1994), they bear differences regarding the inflation-output tradeoff and costs of disinflation. Given that we wish to test whether a Lucas supply curve translates well the tradeoff, we will be using the classical direction of fit and put output on the left-hand side, as in Lucas'. Furthermore, we obtain reasonable parameters estimates which were also statistically significant, and therefore, this direction of fit seemed adequate.

bility of policymaking to have any relevance. However, it is not based on expected future inflation, as in the New Keynesian Phillips curve. Instead, inflation depends on current and past expected inflation. It is closer in this sense to models such as Mankiw and Reis (2002)'s sticky information. In their model, inflation depends on current expectations of inflation taken at different points in time. Some price setting is based on old plans and old expectations which failed to be revised in face of new information. This occurs either because of costs of acquiring new information or costs of reoptimisation. In our model, not only current expected inflation is relevant, but we also allow for the possibility of persistence in expected inflation. Our rather general specification nests different formulations of the Phillips curve, and the use of direct measures of expectations allows us to test the different specifications and find the best empirical formulation, i.e. the specification that best fits the data.

We will assume that

$$a(1) = b(1) \tag{2.64}$$

This assumption ensures that there is no long-run tradeoff between inflation and output in the model[51]. The first two output concepts introduced in (2.60-2.62), are the actual and natural level of output and these concepts are introduced and defined by this assumption. Hence, by definition, the natural level of output is the output level achieved eventually in the absence of inflation changes or price distortions. This is a broader definition of the natural rate than is incorporated by the standard Lucas Supply curve, where output deviates

[51] Note that in our model, it also implies that there is also no medium-run tradeoff, but we will keep on referring to it as the long-run tradeoff, as it is usually referred to in the literature.

from the natural level only when *unanticipated* price changes occur. The idea of price distortions considered here includes the possibility that prices as well as inflation take time to adjust and that different prices adjust at different speeds, so that even fully anticipated price or inflation changes can cause output to deviate from the natural rate over some time period, although there is long-run neutrality.

Note that the additional restriction

$$a(L) = b(L) \tag{2.65}$$

would ensure that only unanticipated inflation,

$$(\pi_t - \pi_t^e) = \varepsilon_{3t} \tag{2.66}$$

cause output to deviate from the natural rate, and if,

$$a_o = b_{o,} a_i = b_i = 0, i = 1, 2, 3, ... \tag{2.67}$$

then only contemporaneous inflation surprises cause output to deviate from $\bar{y}_t$ and we end up with the Lucas (1972, 1973) surprise equation:

$$y_t = \bar{y}_t + a_0(\pi_t - \pi_t^e) \tag{2.68}$$

Equation (2.62) defines the third output concept in our model: the capacity output level, $\bar{y}_t^c$. The underlying notion is that it represents the level of output that would be achieved if all agents' objectives were realised and all adjustments to disturbances had taken place. The permanent innovations driving $\bar{y}_t^c$ process, ε_{2t},can be thought of as productivity shocks. We assume the existence of a unit root in $d(L)$, which ensures that $\bar{y}_t^c$is difference-stationary.

Equation (2.61) characterises the adjustment process relating $\bar{y}_t$to $\bar{y}_t^c$.We assume that $c(1) = e(1)$. This assumption translates into convergence of the natural level of output $(\bar{y}_t)$ towards the trend over time, and the ε_{1t} are therefore transitory deviations of $\bar{y}_t$from $\bar{y}_t^c$. This relationship abstracts from nominal disturbances and is associated with the growth literature.

The three equation system is clearly a stylised characterisation with expression (2.60) capturing nominal shocks, expression (2.61) incorporating transitory real variations and adjustments and the trend output in expression (2.62) being driven by permanent real innovations. All permanent changes arrive through ε_{2t} and their influence on the capacity level of output. The relationship among the different concepts can therefore be summarised as follows: we take the natural level of output at time t to be that which would be achieved in the absence of any price distortions (either from unanticipated inflation or incomplete adjustment to previous price and inflation movements), but with a given state of technological development and with a given level of capital. The trend, or capacity, level of output is the level of output achievable again in the absence of price distortions and at a given stage of technological development but with the capital stock at its *desired* level given the intertemporal desire of agents, as reflected in their savings and investment behaviour.

The distinction between the actual level, the natural level and the trend level of output can be also made with reference to the time horizon over which different equilibrating processes take place. Over the short run, price distortions can cause actual output to deviate from the natural level of output. Over the medium-term, the effects of price distortions are assumed to disappear so that actual output would coincide with the natural level in the absence of further price change.[52][53] However, actual and natural output levels might deviate from the trend or capacity output level even over the medium term as the evolution of the inputs into production, and specifically the accumulation of capital inputs, evolve more slowly over time to reflect households' and firms' intertemporal decision-making.[54]

2.3.2 Output growth, monetary policy and inflation

So far, we have concentrated on the supply side and developed a simple model that will be the basis to explaining output and the inflation-output tradeoff. In this section, we extend the theoretical model described so far to incorporate a positive model of inflation.

The model and the corresponding restrictions are derived in the spirit of the theoretical literature motivated by Kydland and Prescott (1977) concerning time consistent and credible policies. In particular, we derive and impose the theoretical restrictions implied by Barro and Gordon's (1983) theory of time consistent monetary policy and the empirical work of Ireland (1999) on a model of output and inflation.

Typically, the policymaker will choose a level of inflation that is consistent with minimising a loss function. The monetary authority, which is unable to commit to a rule, chooses a planned rate of inflation (π_t^p) at the beginning of each period, but only after

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[52] This level of output corresponds to the 'equilibrium' position considered in the supply-side models of Henry and Lee (1997), for example. Here, following a shock, price-setting decisions of firms and wage-setting decisions of workers are brought back into line at unique levels of (un)employment and output following a period of adjustment. The 'equilibrium' position is defined for given labour market institutions and capital and technology inputs.

[53] This is what we meant before when saying that in our model there is no "medium-run" tradeoff.

[54] The natural and trend output levels abstract from the effect of price and inflation movements and relate to the output concepts obtained in growth models. Hence, the trend or capacity level of output relates to the balanced growth path arising from the standard growth models of Solow, while the natural level reflects the path taken by output to the balanced growth path in these models.

private agents have formed their expectations. The loss function penalizes deviations of inflation and output from target values of π_t^* and $k\bar{y_t}$ and takes the following quadratic form,

$$L_t = \frac{1}{2}(y_t - k\bar{y}_t)^2 + \frac{\gamma}{2}(\pi_t - \pi_t^*)^2 \quad (2.69)$$

where γ is a positive number, representing the weight given to fluctuations in inflation relative to output around their desired levels, and $k > 1$, denoting the desire of the monetary authority to increase output beyond the natural rate.[55] Note that if $\gamma > 1$, the monetary authority is relatively inflation averse. Given the quadratic nature of the loss function policymaker will equally penalize, deviations above and below the target values[56].

In this framework, the policymaker cannot provide a binding commitment to a policy rule. At the beginning of each time period, the agents will incorporate such information in their decisions and form their expectations about inflation (i.e., $\pi_t^e = p_t^e - p_{t-1}$).Then, after observing π_t^e, the policymaker determine a planned rate of inflation, $\pi_t^p = p_t^p - p_{t-1}$. However, because we assume that the monetary policy cannot fully control inflation, actual inflation for the period is realised with a "control error" defined by, ζ_t. If we write

$$\pi_t = \pi_t^p + \zeta_t \quad (2.70)$$

55 Note that we included the natural rate of output rather than the potential output. We believe that, given the inability of targeting potential output, the policymakers target above the shorter-horizon measure of output, the natural rate, which is also the usual measure to compare actual output with in order to understand the possibility of any inflationary pressures building up. Further, the potential output is less useful for policymakers concerned with the outlook for inflation.

56 See paper by Chadha and Schellekens (1999) for discussion on the merits of such a formulation.

where π_t^p is the level of planned prices at time t, and $\zeta_t$is iid and normally distributed with mean zero and well-defined variance σ_ζ^2, the policymakers sets π_t^p in order to minimise the expected value of the loss function in (2.69), i.e.,

$$\min_{\pi_t^p} E_{t-1}(\frac{1}{2}(y_t - k\bar{y}_t)^2 + \frac{\gamma}{2}[(\pi_t^p + \zeta_t) - \pi_t^*]^2) \quad (2.71)$$

The coefficients γ and k will be estimated by solving the policymakers minimisation problem subject to the Phillips curve. This will determine inflation in our model. Due to the fact that the empirical version of the loss function is dependent upon the structure of the supply side, the exact structure of the inflation equation will be derived later on in section 2.4.2. As a result, this section served the purpose of motivating the empirical modelling and to give the overall picture in terms of the modelling framework.

2.4 Modelling Output Growth and Price Inflation Using Survey Data

We are concerned with modelling simultaneously the process determining (the logarithm of) actual output at time t, denoted y_t, (the logarithm of) the natural level of output at time t, denoted $\overline{y}_t$, and (the logarithm of) the trend output level at time t, denoted $\overline{y}_t^c$. We will consider unobservable, the natural level and the capacity level of output. As a result, we need to transform the model in order to define relationships among the four ”observable” variables - actual and expected output and inflation. The expected series that we will be using in this empirical work have been derived in chapter 1, section (1.3.2) from the CBI qualitative survey and consist of the "purged" series as explained at the end of that section.

Although we have direct measures of expectations, in order to transform the three-equation system above (involving two unobservable variables) into equations on those four variables only, we need to assume rationality of the expectations formation process. It is important to stress that despite this assumption, the use of direct measures of expectations is still relevant and justifiable on two main accounts. First of all, it allowed us to analyse the expectations formation process itself in chapter 1. Our results from the previous chapter lead to a non-rejection of rationality of expectations and therefore gave us validity to the the REH assumption for the present work. Secondly, the use of direct measure of expectations provide us with more structure for our model based on the simple 3-equation model introduced so far and therefore, a better understanding of the economy, through these additional relationships among the variables[57]. More specifically, it allows us to analyse the direct contribution of those expectations in determining inflation and output.

2.4.1 Defining the supply side in terms of actual and expected output and inflation

In this subsection, we transform the model in order to define relationships among the ”observable” variables - actual and expected output and inflation, making use of the relationships defining the supply side and the assumption made on the lag operators as well as the Rational Expectations Hypothesis.

We start by redefining the lag-operators for convenience of the algebra. If we write $d(L)$=$\tilde{d}(L)(1-L)$, assuming that $\tilde{d}(L)$ is invertible and $\tilde{d}(0) = 1$,the repeated substitution

57 As we will see, from the 3-equation model on 2 unobservables, we are able to define three relationship, explaining the actual output and inflation determination and expected output growth.

of expression (2.62) into (2.61) gives:

$$\bar{y}_t = e(L)^{-1}c(L)[\tilde{d}(L)]^{-1}\sum_{i=0}^{\infty}\varepsilon_{2t-i} + e(L)^{-1}\varepsilon_{1t} \tag{2.72}$$

Then, using equation (2.60) and (2.72), actual output can be written as[58]:

$$y_t = e(L)^{-1}c(L)[\tilde{d}(L)]^{-1}\sum_{i=0}^{\infty}\varepsilon_{2t-i} + e(L)^{-1}\varepsilon_{1t} + a(L)\varepsilon_{3t} + (a(L) - b(L))\pi_t^e \tag{2.73}$$

with $\varepsilon_{3t} = \Delta\pi_t - D\pi_t^e$.

By taking expectations of output in (2.73), based on information at time $t-1$, to give a formulation for expected output, y_t^e, and then subtracting from itself (i.e., from 2.73), unanticipated output movements can be given as follows[59]:

$$\Delta y_t - Dy_t^e = c(0)\varepsilon_{2t} + \varepsilon_{1t} + a(0)\varepsilon_{3t} \tag{2.74}$$

Hence, measured expected output is equal to actual output except for random terms, and the deviation of expected output from actual output at time t is orthogonal to information available at time t-1. Note the relevance of the availability of direct measures of expectations to allow enable us to identify this structural relationship.

Further manipulation shows that only ε_{2t} has a permanent impact on the three measures of output. For instance, if lagged version of expression (2.73) is subtracted from itself

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58 We used the fact that $E_{t-1}[a(L)\pi_t^e] = a(L)\pi_t^e$ and $E_{t-1}[b(L)\pi_t^e] = b(L)\pi_t^e$.

59 Note that E$_{t-1}(\pi_{t-1-i}) = \pi_{t-1-i}$, with $i = 0, 1, 2, ...$ As a result, all the terms $a(L)(\pi_t - \pi_t^e)$ cancel out with the terms in $E_{t-1}[a(L)(\pi_t - \pi_t^e)]$, except for $a(0)$.Also note that we assumed that $e(0) = 1$

to give output growth, we obtain the expression,

$$
\begin{aligned}
y_t - y_{t-1} &= e(L)^{-1} c(L)[\tilde{d}(L)]^{-1}(\varepsilon_{2t}) + (1-L)e(L)^{-1}\varepsilon_{1t} - \\
&+(1-L)a(L)\varepsilon_{3t} \\
&+(1-L)(a(L)-b(L))\pi_t^e
\end{aligned}
\tag{2.75}
$$

Therefore, in the long run, when $L = 1$, we can see that, in this system, only disturbances ε_{2t} remains to influence output.

Furthermore, from this expression we can also derive an equation for expected output growth. By defining $e(L) = \tilde{e}(L) + e(0)$, with $e(0) = 1$, we can re-write the expression for output growth as:

$$
\begin{aligned}
y_t - y_{t-1} &= \tilde{e}(L)(1-L)\bar{y}_t + c(L)[\tilde{d}(L)]^{-1}\varepsilon_{2t} + (1-L)\varepsilon_{1t} \\
&+(1-L)a(L)\varepsilon_{3t} \\
&+(1-L)(a(L)-b(L))\pi_t^e
\end{aligned}
\tag{2.76}
$$

We can obtain an expression for expected output growth from equation (2.75). If we take expectations, at time t, to this expression advanced by one period, and make use of equation (2.74)we obtain:

$$\begin{aligned} y^e_{t+1} - y_t &= -(1-L)\tilde{e}(L)(y_{t+1} - a(L)\pi_{t+1} + b(L)\pi^e_{t+1}) - \varepsilon_{1t} \\ &+c(L)\tilde{d}(L)^{-1}\varepsilon_{2t+1} - c(0)\tilde{d}(L)^{-1}\varepsilon_{2t+1} \\ &+(1-L)a(L)\varepsilon_{3t+1} - a(0)\varepsilon_{3t+1} \\ &+(1-L)(a(L)-b(L))\pi^e_{t+1} \end{aligned} \tag{2.77}$$

This equation will represent one of the lines in our VAR[60].

So far, we have developed a rather general model of actual and expected output. However, for empirical purposes, we impose some structure on the general system, presented above. In particular, we impose some structure into our lag operators. By assuming that $c(L) = 1-\rho$, $e(L) = 1+\rho L$, $d(L) = 1-L$ and $a(L) = a_0 + a_1 L + ... + a_q L^q$ and $b(L) = b_0 + b_1 L + ... + b_q L^q$ we end up with the following structural model:

$$y_t = \bar{y}_t + \sum_{i=0}^{q} a_i(\pi_{t-i} - \pi^e_{t-i}) + \sum_{i=0}^{q}(a_i - b_i)\pi^e_{t-i} \tag{2.78}$$

$$= \bar{y}_t + \sum_{i=0}^{q} a_i\pi_{t-i} - \sum_{i=0}^{q} b_i\pi^e_{t-i} \tag{2.79}$$

$$\bar{y}_t = (1-\rho)\bar{y}^c_t + \rho\bar{y}_{t-1} + \varepsilon_{1t} \tag{2.80}$$

$$\bar{y}^c_t = \bar{y}^c_{t-1} + \varepsilon_{2t} \tag{2.81}$$

[60] Note that due to the way we defined the lag operators $e(L)$ and $\tilde{e}(L)$, expected output growth depends on past output growth, ie, the first term of the equation is $(1-L)\tilde{e}(L)y_{t+1} = (1-L)(e_1 L + e_2 L^2 + ...)y_{t+1}$ $= (1-L)(e_1 y_t + e_2 y_{t-1} + ...)$.

These simplifications on the lag operators can be justified with reference to the literature. Barro and Gordon (1983) also used a similar formulation which can be represented in terms of output as:

$$\bar{y}_t = (1 - \rho)y^* + \rho\bar{y}_{t-1} + \tau_t \tag{2.82}$$

where y^* is the long-run mean of the natural rate, $\bar{y}_t$ and consists of the stochastic process for the natural rate, which is assumed to evolve as an AR(1), with the error term being an iid N(0,σ_τ^2). More recently, Primiceri (2006) also assumed that the natural rate of unemployment converges towards its unconditional expectations, a constant and it therefore consists of a simplified version of our model. This, on the other hand, consisted of a generalisation of Staiger, Stock and Watson (2001), where $\rho = 1$. Reis (2003) had also formulated a similar model for the natural rate, although he interprets the y^* (or $\bar{y}_t^c$ in our model) in the previous equations as a time-varying observable component of the natural rate.

The model described by (2.78)-(2.81) provides the following first two equations of the VAR on observable variables. The first line, which can be derived from (2.74) is given by[61]:

$$y_t - y_t^e = (1 - \rho)\varepsilon_{2t} + \varepsilon_{1t} + a_o\varepsilon_{3t} \tag{2.83}$$

We can obtain an expression for expected output growth from equation (2.77) by substituting out the lag operators[62] and using equation (2.83)[63]:

---

61 Note that $y_t - y_t^e = \Delta y_t - Dy_t^e$.

62 Note that $\tilde{e}(L) = \rho L$.

63 See appendix.

$$
\begin{aligned}
Dy_{t+1}^{e} &= (\rho-1)\Delta y_t + Dy_t^e + (1-\rho)\varepsilon_{2t} \\
&+a_0(D\pi_{t+1}^e + \varepsilon_{3t}) \\
&+\rho[\sum_{i=0}^{q+1}[(a_i - b_{i-1})\Delta\pi_{t-i} - (b_i - b_{i-1})D\pi_{t-i}^e \\
&+\sum_{i=0}^{q}[(a_{i+1} - b_i)\Delta\pi_{t-i} + (b_{i-1} - b_i)D\pi_{t-i+1}^e]
\end{aligned}
\tag{2.84}
$$

with $a_i = b_i = 0$, $i \neq 0, 1, 2$.This consists of a second line.

The remaining two lines of the model consist of equations defining the determination of inflation and will be discussed in the next subsection.

2.4.2 Inflation determination

In this subsection we concentrate on the modelling of inflation which will be represented in the last two lines of the VAR model. These include:

$$
\Delta\pi_t = D\pi_t^e + \varepsilon_{3t} \tag{2.85}
$$

as the third line and representing the imposition of rational expectations hypothesis that we were unable to reject in the previous chapter[64].

The fourth line, characterising expected inflation, will be represented in two alternative ways. The first is in its most general form and reflects relatively unrestricted relationship between output growth, expected output growth, inflation and expected inflation,

---

[64] Note that we are making use of the results obtained from chapter 1. In particular, we are taking the created purged series as the expected series. We are also making use of the result in terms of rationality of the expectations on inflaton and output.

where the only restrictions imposed in the model are to ensure that each of these series are stationary. We will be calling this line "unrestricted" fourth line, which can be represented as:

$$D\pi^e_{t+1} = \sum_{i=0}^{p} f_i Dy^e_{t+1-i} + \sum_{i=0}^{p} g_i \Delta y_{t-i} + \sum_{i=0}^{p} h_i \Delta\pi_{t-i} + \sum_{i=0}^{p-1} l_i D\pi^e_{t-i} \tag{2.86}$$

with p representing the order of the VAR. The second version is derived from the minimisation problem described in (2.71). As explained in section 2.3.2, we will be assuming that the outcome for inflation is determined by the minimisation of a loss function by the monetary authority. Depending on the underlying structure of the economy, we may make different inferences on the ultimate inflationary bias that we observe in the data. We will start the minimisation with the general Phillips curve represented by equation (2.78).

Recalling the minimisation problem explained in section 2.3.2

$$\min_{\pi^p_t} E_{t-1}(\frac{1}{2}(y_t - k\bar{y}_t)^2 + \frac{\gamma}{2}[(\pi^p_t + \zeta_t) - \pi^*_t]^2) \tag{2.87}$$

Using equation (2.78), the minimisation problem becomes

$$\min_{\pi^p_t} E_{t-1}L_t = (\frac{1}{2}(\bar{y}_t + a_0(\pi^p_t + \zeta_t) + \sum_{i=1}^{q} a_i\pi_{t-i} - \sum_{i=0}^{q} b_i\pi^e_{t-i} - k\bar{y}_t)^2 + \frac{\gamma}{2}[(\pi_t - \pi^*_t)^2] \tag{2.88}$$

The first order condition can be represented as:

$$a_0E_{t-1}[\bar{y}_t + a_0(\pi^p_t + \zeta_t) + \sum_{i=1}^{q} a_i\pi_{t-i} - \sum_{i=0}^{q} b_i\pi^e_{t-i}) - k\bar{y}_t] + \gamma E_{t-1}(\pi_t - \pi^*_t) = 0 \tag{2.89}$$

Given that in equilibrium, $\pi_t^e = \pi_t^p$, and using $E_{t-1}(\zeta_t) = 0$ and $E_{t-1}(\pi_{t-1-i}) = \pi_{t-1-i}$, with i=0,1,2...we obtain the following expression for π_t^e,

$$\pi_t^e = \frac{\gamma}{\gamma - a_0 b_0 + a_0^2}\pi_t^* + \frac{a_0(k-1)}{\gamma - a_0 b_0 + a_0^2}E_{t-1}(\bar{y}_t) - \frac{a_0}{\gamma - a_0 b_0 + a_0^2}\left[\sum_{i=1}^{q} a_i \pi_{t-i} - \sum_{i=1}^{q} b_i \pi_{t-i}^e\right] \tag{2.90}$$

In equilibrium, $\pi_t^e = \pi_t^p$,with $\pi_t = \pi_t^p + \zeta_t$;from the rationality of inflation expectations expressed in line 3, $\pi_t = \pi_t^e + \varepsilon_{3t}$, and as a result, $\zeta_t = \varepsilon_{3t}$.Taking this into account, we can write:

$$\pi_t = \frac{\gamma}{\gamma - a_0 b_0 + a_0^2}\pi_t^* + \frac{a_0(k-1)}{\gamma - a_0 b_0 + a_0^2}E_{t-1}(\bar{y}_t) - \frac{a_0}{\gamma - a_0 b_0 + a_0^2}\left[\sum_{i=1}^{q} a_i \pi_{t-i} - \sum_{i=1}^{q} b_i \pi_{t-i}^e\right] + \varepsilon_{3t} \tag{2.91}$$

If we forward this expression one period and take expectations at time t, E_t and subtract the previous expression from it, we get[65]:

$$\begin{aligned} D\pi_{t+1}^e \quad = \quad & \frac{\gamma}{\gamma - a_0 b_0 + a_0^2}E_t(\Delta\pi_{t+1}^*) + \frac{a_0(k-1)}{\gamma - a_0 b_0 + a_0^2}[E_t(\bar{y}_{t+1}) - E_{t-1}(\bar{y}_t)] \\ & - \frac{a_0}{\gamma - a_0 b_0 + a_0^2}\left[\sum_{i=1}^{q} a_i \Delta\pi_{t-i+1} - \sum_{i=1}^{q} b_i \Delta\pi_{t-i+1}^e\right] - \varepsilon_{3t} \end{aligned} \tag{2.92}$$

The expression for $[E_t(\bar{y}_{t+1}) - E_{t-1}(\bar{y}_t)]$ will be defined as a function of the ”observable” variables, by using equation (2.79):

$$E_{t-1}(\bar{y}_t) = E_{t-1}[y_t - \sum_{i=0}^{q} a_i \pi_{t-i} + \sum_{i=0}^{q} b_i \pi_{t-i}^e] \tag{2.93}$$

Forwarding this expression by one period:

---

65 Note that $E_t(\varepsilon_{3t+1}) = 0$.

$$E_t(\bar{y}_{t+1}) = E_t[y_{t+1} - \sum_{i=0}^{q} a_i \pi_{t-i+1} + \sum_{i=0}^{q} b_i \pi^e_{t-i+1}] \tag{2.94}$$

Therefore:

$$E_t(\bar{y}_{t+1}) - E_{t-1}(\bar{y}_t) = Dy^e_{t+1} + (y_t - y^e_t) - (a_0 - b_0)\Delta\pi^e_{t+1} - \sum_{i=1}^{q} a_i \Delta\pi_{t-i+1} + \sum_{i=1}^{q} b_i \Delta\pi^e_{t-i+1}] \tag{2.95}$$

Using equation (2.92), we get:

$$\begin{aligned} D\pi^e_{t+1} &= \frac{\gamma}{\gamma - a_0 b_0 + a_0^2} E_t(\Delta\pi^*_{t+1}) - \varepsilon_{3t} \\ &+ \frac{a_0(k-1)}{\gamma - a_0 b_0 + a_0^2}[Dy^e_{t+1} + (y_t - y^e_t) - (a_0 - b_0)\Delta\pi^e_{t+1} \\ &- \sum_{i=1}^{q} a_i \Delta\pi_{t-i+1} + \sum_{i=1}^{q} b_i \Delta\pi^e_{t-i+1}] \\ &- \frac{a_0}{\gamma - a_0 b_0 + a_0^2}\left[\sum_{i=1}^{q} a_i \Delta\pi_{t-i+1} - \sum_{i=1}^{q} b_i \Delta\pi^e_{t-i+1}\right] \end{aligned} \tag{2.96}$$

This equation can be further simplified by collecting the terms in $D\pi^e_{t+1}$, as follows[66]:

$$\begin{aligned} D\pi^e_{t+1} &= \frac{\gamma}{\gamma + a_0 k(a_0 - b_0)} E_t(\Delta\pi^*_{t+1}) - \varepsilon_{3t} \\ &+ \frac{a_0(k-1)}{\gamma + a_0 k(a_0 - b_0)}[Dy^e_{t+1} + (y_t - y^e_t)] \\ &- \frac{a_0 k}{\gamma + a_0 k(a_0 - b_0)}. \\ &. \left[a_1 \Delta\pi_t + (a_2 - b_1)\Delta\pi_{t-1} - b_2 \Delta\pi_{t-2} - b_1 D\pi^e_t + (b_1 - b_2) D\pi^e_{t-1} + b_2 D\pi^e_{t-2}\right] \\ &+ \varepsilon_{4t} \end{aligned} \tag{2.97}$$

[66] Note that $\Delta\pi^e_t = \pi^e_t - \pi^e_{t-1}$ which is different from $D\pi^e_t = \pi^e_t - \pi_{t-1}$. In our model, we model $D\pi^e_t$;we also make use of the result $\Delta\pi^e_{t+1-i} = D\pi^e_{t+1-i} + \Delta\pi_{t-i} - D\pi^e_{t-i}$ for $i = 0, 1, 2...$

In the empirical work we do not model target inflation and as a result, the first term of this equation will be included as part of the intercept term[67]. We also added an error term, ε_{4t} for estimation purposes. This the most general formulation, which nests the formulation for the other Phillips curves that will be analysed in this chapter[68]. For example, in case our supply side is represented by a Lucas surprise curve, i.e., $a_0 = b_0 \neq 0$ and $a_i = b_i = 0$ for $i \neq 0$, the expected change in inflation is reduced to:

$$D\pi^e_{t+1} = \Delta\pi^*_t + \frac{a_0(k-1)}{\gamma}[Dy^e_{t+1} + (y_t - y^e_t)] + \varepsilon_{4t} \tag{2.98}$$

We can express it using line 1, as follows:

$$D\pi^e_{t+1} = -\frac{a_o(1-k)}{\gamma}Dy^e_{t+1} - \frac{a_o(1-k)(1-\rho)}{\gamma}\varepsilon_{2t} - \frac{a_o(1-k)}{\gamma}\varepsilon_{1t} - (\frac{a_o^2(1-k)}{\gamma}+1)\varepsilon_{3t} + \varepsilon_{4t} \tag{2.99}$$

This will be therefore the restricted fourth line.[69] Note that all these equation are expressed in terms of variables that are observed only and hence can be estimated. The three other lines of the model can be represented as:

$$y_t - y^e_t = (1-\rho)\varepsilon_{2t} + \varepsilon_{1t} + a_o\varepsilon_{3t} \tag{2.100}$$

---

[67] Note that although we do not include intercepts in our model to make the algebra simpler, every equation contains an intercept and therefore we estimated the model as such.

[68] Note that this is a simplified exercise, without a dynamic optimisation in place.

[69] Note that we also excluded the term $\Delta\pi^*_t$, since we are showing the empirical formulation of the line we estimate.

as line 1

$$
\begin{aligned}
Dy^e_{t+1} &= (\rho-1)\Delta y_t + Dy^e_t + (1-\rho)\varepsilon_{2t} \\
&+a_0(D\pi^e_{t+1} + \varepsilon_{3t}) \\
&+\rho[\sum_{i=0}^{q+1}[(a_i - b_{i-1})\Delta\pi_{t-i} - (b_i - b_{i-1})D\pi^e_{t-i} \\
&+\sum_{i=0}^{q}[(a_{i+1} - b_i)\Delta\pi_{t-i} + (b_{i-1} - b_i)D\pi^e_{t-i+1}]
\end{aligned}
\tag{2.101}
$$

as line 2, and with $a_i = b_i = 0$, $i \neq 0, 1, 2$.

$$
\Delta\pi_t = D\pi^e_t + \varepsilon_{3t} \tag{2.102}
$$

as line 3.

The fourth line under the unrestricted model, takes the form:

$$
D\pi^e_{t+1} = \sum_{i=0}^{q+2} f_i Dy^e_{t+1-i} + \sum_{i=0}^{q+2} g_i \Delta y_{t-i} + \sum_{i=0}^{q+2} h_i \Delta\pi_{t-i} + \sum_{i=0}^{q+1} l_i D\pi^e_{t-i} \tag{2.103}
$$

with q representing the order of the lag operator in (2.78); under the restricted model, the fourth line takes the form 2.99. The model has been estimated for the period 1975q3-2000q4 using Maximum Likelihood. Details are given in the next subsection.

2.4.3 The four-variable VAR and estimation procedure

In this section, we show that the process given in the system of equations (2.100) - (2.103) can be represented by a four variable structural VAR, where it is assumed that output and inflation are first-difference stationary, and that inflation and output expectational errors are stationary. Its matrix representation is as follows:

$$A_0 z_t = c + A_1 z_{t-1} + A_2 z_{t-2} + ... A_p z_{t-p} + P\varepsilon_t \tag{2.104}$$

where p denoted the order of the VAR and

$z_t = (\Delta y_t, D_t y^e_{t+1}, \Delta\pi_t, D\pi^e_{t+1})'$

$\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t})'$

A_s, $s = 1, 2, ..., p$ and P are 4x4 matrices of coefficients

$c = (c_1, c_2, c_3, c_4)'$ is a vector of intercepts

The equivalent reduced form representation is as follows:

$$z_t = k + B_1 z_{t-1} + B_2 z_{t-2} + ... B_p z_{t-p} + u_t \tag{2.105}$$

where $k = A_0^{-1} c$

$B_s = A_0^{-1} A_s$ for $s = 1, 2, .., p$

$u_t = A_0^{-1} P\varepsilon_t$

The system can be characterised in four lines of the VAR model which will include cross-equation restrictions as well as structure on the variance-covariance matrix. The structural shocks, ε_t, are assumed to be uncorrelated, serially uncorrelated, with zero mean and constant variance. The (reduced-form) shocks have the same features, but they are correlated with each other.

$$\text{Var}(\varepsilon) = \varepsilon\varepsilon' = \mathbf{D} = \begin{bmatrix} \sigma^2_{\varepsilon_1} & 0 & 0 & 0 \\ 0 & \sigma^2_{\varepsilon_2} & 0 & 0 \\ 0 & 0 & \sigma^2_{\varepsilon_3} & 0 \\ 0 & 0 & 0 & \sigma^2_{\varepsilon_4} \end{bmatrix}$$

which, and defining $V = \mathbf{P}\varepsilon$, it follows that:

$$Var(V) = VV^{'} = (\mathbf{P}\varepsilon)(\mathbf{P}\varepsilon)^{'} = \mathbf{P}\varepsilon\varepsilon^{'}\mathbf{P'} = \mathbf{PDP}^{'} \quad (2.106)$$

which, in turn, determines the covariance matrix for u as follows:

Var$(u) = \mathrm{u}u^{'} = (A_0^{-1}V)(A_0^{-1}V)^{'} = A_0^{-1}Var(V)(A_0^{-1})^{'} = \Omega$

The reduced form of the model can be written in a more compact manner as:

$$\mathbf{Z} = \Pi X + \mathbf{U} \quad (2.107)$$

The system will be estimated using maximum likelihood. The (log) likelihood function can be represented as[70]:

$$\mathcal{L}(\mathbf{B}, \mathbf{\Omega}) = -\frac{KT}{2}\log(2\pi) + \frac{T}{2}\log\det(\Omega^{-1}) - \frac{1}{2}\sum_{t=1}^{T}(\mathbf{Z}_t - \mathbf{\Pi X}_t)'\mathbf{\Omega}^{-1}(\mathbf{Z}_t - \mathbf{\Pi X}_t) \quad (2.108)$$

Due to the presence of the matrices of contemporaneous relationships, A_0 and P, it is not possible to identify the model, unless we impose some restrictions. To determine how many restrictions are required to exactly identify the model, note that matrices A_0 and P contain 4^2 elements each, which amounts to 32[71] extra parameters when compared to the reduced form. However, the covariance matrixes have different forms and that needs to be taken into account. Namely, the structural error terms have a diagonal covariance matrix, and therefore contains 4 estimable variances, whereas the covariance of the reduced-form error terms, u, is symmetric and therefore contains in total 10 different elements[72]. Taking

70 Hamilton (1994).det denotes the deteminant.

71 In general, would be $2n^2$, with n representing the number of endogenous variables.

72 In general, a symmetric matrix contains $\frac{n^2+n}{2}$ elements, with n standing for the number of endogenous variables.

these two set of findings, the difference in parameters between the two formulations is $(2n^2 + n) - (\frac{n^2+n}{2}) = \frac{3n^2+n}{2} = 26$. Furthermore, we also usually assume to have ones in the main diagonal of each matrix, i.e., we impose some normalisations in A_0 and P, on each variable and error term, which amounts to 8 restrictions. Taking it all into account, we are left with $(\frac{3n^2-3n}{2}) = 18$ further restrictions that are required to be able to identify the structural parameters. It is easy to see that our model is overidentified, i.e., our framework contains more restrictions that required for exact-identification[73]. For the model represented above under (2.100)-(2.103), the A_0 matrix, capturing the contemporaneous relationships among the variables is given by:

$$\begin{bmatrix} 1 & 0 & a_0 & 0 \\ -(\rho-1) & 1 & -\rho a_0 - (a_1 - b_0) & -a_0 \\ 0 & 0 & 1 & 0 \\ g_0 & f_0 & h_0 & 1 \end{bmatrix}$$

and matrix P, containing the contemporaneous response of the variables to the disturbances, is given by:

$$\begin{bmatrix} 1 & 1-\rho & 0 & 0 \\ 0 & 1-\rho & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We estimated the VAR model using Maximum likelihood. The method choice is justified by the presence of cross-equation restrictions. The fact that matrix P is not an identity matrix further justifies the use of multivariate methods of estimation. We also have an over-identified system. It is easy to see just by looking at our matrix formulation that more than 26 restrictions have been imposed. We have assumed that economic theory dictates more restrictions than the required to exactly identify the model and that has been

[73] For example, having a look at line 3, that imposes rationality on inflation expectations, there are over 18 zero-restrictions on this line alone.

our starting point.[74] The fact that the model is over-identified, with different regressors across equations. means that single estimation methods would be inefficient, as important information (i.e., restrictions imposed the model) would not be taken into account. The standard error are derived from the inverse of the Hessian matrix.[75]

2.5 Empirical results

[Tables 2.1, 2.3]

We started off by using a rather general model with equation (2.78) and through a specification search, we end up with $q = 2$, leading to a VAR of order 4 ($p = 4$). The test used consisted of a likelihood ratio test. We started off by using a Phillips curve with an associated q=4 and estimated this more general model, which assumed a VAR order of 6. By imposing zero restrictions on the coefficients associated with the variables implied by an order higher than 4, we obtained a LR test of 12.79, with an associated p-value of 0.384[76]. The resulting model can be summarised on the following four lines:

$$y_t - y_t^e = (1-\rho)\varepsilon_{2t} + \varepsilon_{1t} + a_o\varepsilon_{3t} \qquad (2.109)$$

[74] In the next chapter we expand our dicussion of structural over-identified models. The model for the US economy presented in the next chapter is also over-identified. In that case, we are more thorough and include a test of the ”economy-based” over-identified restrictions, which we do not include in the present work.

[75] The likelihood function is maximised using a procedure in Gauss that uses by default the Davidon-Fletcher-Powell algorithm, which is a numerical procedure that is within the class of modified Newton-Raphson methods.

[76] Note that in a VAR of order 6, our model has 12 more coefficients to estimate; four associated with the supply curve (i.e., coefficients on actual and expected past inflation), and eight more coefficients on the 4th line. The associated chi-square has 12 degrees of freedom, which at 5% takes the value of 21.026.

as line 1

$$\begin{aligned} Dy^e_{t+1} &= (\rho-1)\Delta y_t + Dy^e_t + (1-\rho)\varepsilon_{2t} \\ &+a_0(D\pi^e_{t+1}+\varepsilon_{3t}) \\ &+\rho[\sum_{i=0}^{3}[(a_i-b_{i-1})\Delta\pi_{t-i}-(b_i-b_{i-1})D\pi^e_{t-i} \\ &+\sum_{i=0}^{2}[(a_{i+1}-b_i)\Delta\pi_{t-i}+(b_{i-1}-b_i)D\pi^e_{t-i+1}] \end{aligned} \tag{2.110}$$

as line 2, and with $a_i = b_i = 0$, $i \neq 0, 1, 2$.

$$\Delta\pi_t = D\pi^e_t + \varepsilon_{3t} \tag{2.111}$$

as line 3

$$D\pi^e_{t+1} = \sum_{i=0}^{4} f_i Dy^e_{t+1-i} + \sum_{i=0}^{4} g_i \Delta y_{t-i} + \sum_{i=0}^{4} h_i \Delta\pi_{t-i} + \sum_{i=0}^{3} l_i D\pi^e_{t-i} \tag{2.112}$$

as line 4.

This model nests several other models that we will be testing for, given the uncertainty surrounding the representation of the Phillips curve. We will be testing (i) for price homogeneity (*Model B1*), (ii) whether only unanticipated inflation deviates output from the natural rate (*Model B2*); (iii) whether only current unanticipated inflation causes output deviations and therefore whether the economy is well represented by a Lucas surprise equation (*Model B3*). The test procedure that we adopt is as follows: we first estimate the more general model (*Model A*). We then re-estimate the model imposing the first constraint that will determine Model B1. We then proceed to impose further restrictions. The models

A to B3 nest on each other; so, we do progressively impose further restrictions on the previous model. The tests were performed using maximum likelihood ratio tests. Twice the difference between the (log) likelihoods for the unconstrained ($\mathcal{L}^u$) and constrained ($\mathcal{L}^c$) is asymptotically distributed as a Chi-square with degrees of freedom equal to the number of restrictions imposed on the coefficients (k), i.e., $2(\mathcal{L}^u - \mathcal{L}^c) \sim \chi^2_{(k)}$. The tests reported in this section include an unrestricted 4th line as represented above by equation 2.112.

The first restriction we imposed on Model A was that of long-run neutrality, i.e., $a(1) = b(1)$. In particular, that $b_2 = \sum_{i=0}^{2} a_i - \sum_{i=0}^{1} b_i$. This restriction (the null hypothesis) could not be rejected, as we obtained a likelihood ratio statistics of 1.401, which is smaller than the 5% confidence one-degree of freedom chi-square of 3.84.We can therefore infer that there is neutrality in the long run, in the sense that inflation is unable to deviate actual output from its natural level as we expected from our model. The model is represented in Table (2.3) and the supply side can be summarised in the following equations:[77]

$$y_t = \bar{y}_t + 0.15(\pi_t - \pi_t^e) + 0.28(\pi_{t-1} - \pi_{t-1}^e) + 0.17(\pi_{t-2} - \pi_{t-2}^e) \quad (2.113)$$

$$-0.12\pi_t^e + 0.03\pi_{t-1}^e + 0.09\pi_{t-2}^e \quad (2.114)$$

$$\bar{y}_t = 0.57\bar{y}_t^c + 0.43\bar{y}_{t-1} + \varepsilon_{1t} \quad (2.115)$$

[77] In the table we represent the model, having eliminated the statistically insignificant coefficients on our general fourth line. We eliminated five coefficients. The LR test associated with these five zero restrictions had a value of 4.459, which is to be compared with a $\chi^2_{(5)}$. Its value of 11.07 implies a non-rejection of the null and therefore gives validity to the exlusion of those coefficients.

$$\bar{y}_t^c = \bar{y}_{t-1}^c + \varepsilon_{2t} \tag{2.116}$$

As we can see from the table, the estimation shows a good fit, with many significant coefficients. We also obtained a value for ρ that is smaller than one, but positive, as we would expect. Its value of 0.43 is statistically significant, and shows a relatively slow adjustment on the part of the natural rate towards capacity. Current and past inflation surprises have a positive and significant effect upon output. When actual inflation goes above expected inflation and therefore there is surprise inflation, output increases beyond the natural level. In this general model, further to the surprise element of inflation, current and past expected inflation also have a significant effect upon output. This effect seems to have features of persistent, in that past surprises as well as past expected inflation are still influencing output. However, the effect of actual and expected inflation does not have a lasting effect on output. As suggested by the fact that the restriction $a(1) = b(1)$ could not be rejected, output ends up at the natural rate of output. Overall, the results seem sensible. The assumption of no long-run tradeoff, which we base our supply side model on, is not rejected; the coefficient that translates the speed the natural rate converges towards potential output is significant and between zero and one, as we assumed it to be; the coefficients on actual and past surprise inflation are positive and significant, as predicted from economic theory with an extended version of Lucas surprise curve. In other words, the results so far conform with either the assumptions made when developing our small model of output and/or results predicted from economic theories we are basing it upon.

Given uncertainty regarding the true model of the economy and the best way of representing the Phillips curve, we perform some tests on the supply side of the economy. The fact that we make use of direct measures of expectations allows us to have a rather general representation of the Phillips curve that nests well known specifications, which are micro-founded and have been referred and used in the literature. One specification that is nested on our model B1 above is a Phillips curve where only unexpected inflation, either current or past have the ability of deviating output from its natural rate. This is an extension of a Lucas (1972) supply curve that allows for inflation surprise shocks to persist over time. In order to test for that, we impose on the previous model the following restriction: $a_i = b_i$, with $i = 0, 1, 2$. The resulting Phillips curve took the form:

$$y_t = \bar{y}_t + \underset{(1.270)}{0.1109}(\pi_t - \pi_t^e) + \underset{(1.600)}{0.1342}(\pi_{t-1} - \pi_{t-1}^e) + \underset{(1.692)}{0.0914}(\pi_{t-2} - \pi_{t-2}^e) \qquad (2.117)$$

The coefficients, although of the expected sign, are not significant at the 5% conventional level, having p-values of 0.204, 0.1096 and 0.0906. The latter, referring to the surprise inflation lagged two periods turned out to be significant at 10% level. We performed a likelihood ratio test of the restrictions $a_i = b_i$, on our previous model. We obtained a value of 9.36 for the likelihood ratio, which is rejected at the 5% confidence interval. In this case, we imposed two restrictions on the previous model, and therefore the statistics follows a chi-square with two degrees of freedom, which takes the value 5.991. In our system estimation of this model, the coefficient ρ was significant and once again between zero and one (its value of 0.4304 had an associated t-ratio of 21.214) and the fourth line on expected inflation and similarly to the previous case, had several significant coefficients. Lastly, we tested whether only current unanticipated inflation would deviate output from the natural rate, i.e., we imposed the restriction that $a_i = b_i = 0$, for $i \neq 0$. Starting from Model B1, we reject the hypothesis. The likelihood ratio test of 12.32 is higher than the four-degree chi-square distribution which should be compared to. Therefore, our data rejects a Lucas surprise supply curve type.

A summary of the results of the test carried out is represented in Table 2.1.

Our results seem to suggest a well documented fact that there is persistence in inflation. As a result, a simple model such as the Lucas curve, with output deviating from the natural rate due to current inflation surprises only, does not fit the data well. However, the type of persistence observed is not of a ”surprise” type, i.e., inflation shocks of a surprise type, although relevant in explaining output, are unable to account for all the movement in output. Therefore, attempting to account for inflation persistence by adding past surprise inflation terms to a Lucas representation is not adequate. Extensions should be more complex than that to be able to account for the output-inflation dynamics.

Testing Inflation determination

[Tables 2.2, 2.4]

So far, we have estimated our model with a 4th line that tries to understand the dynamics of expected changes in inflation, in an unrestricted form. In fact, we just made expected inflation dependent upon current and past values of all the variables in the VAR. As we have seen before, although we obtained several statistically significant coefficients on that line, we were not able to give it an interesting interpretation and instead, we concentrated on the output equations. In this subsection, we wish to impose some structure on that line and test the restrictions underlying it. As explained in section 2.4.2, we will be assuming that the outcome for inflation is determined by the minimisation of a loss function by the monetary authority. Depending on the underlying structure of the economy, we may make different inferences on the ultimate inflationary bias that we observe in the data. We will start the minimisation with a rather general model and derive the expression for inflation and since the supply models nest on each other, by imposing simple restrictions, we arrive to the expression for inflation for each of the other less general models. We will be using the results from estimations from the previous subsections, with an unrestricted fourth line and then estimate the system again with the restrictions derived from the simple optimisation exercise done in section 2.4.2 imposed on the fourth line. The tests will be carried out using likelihood ratio test, as before.

Table 2.2 reports the likelihood ratio tests on the different model formulations. We compared each of the unrestricted models analysed before and imposed the corresponding fourth line, derived from imposing that structure associated with the different Phillips curve representations in the loss function, as explained before. The hypothesis is rejected in every single model. Given that our previous analysis seemed to have showed that Model B1 may describe well the economy, we present the estimates for that model with the restricted 4th line (model B1T) in Table (2.4). Model B1T has some interesting features. The significance of current and past inflation on output gap gets reduced, when compared to the unrestricted version. The parameter determining the speed of convergence of the natural rate towards capacity is similar to the previous one, and very significant once again. The parameter k, that determines the value for the output target, showing how much above the natural rate the policymakers are trying to get at, is, as expected, bigger than one, showing that in fact policymakers do try to push output above the natural rate and it is statistically significant. However, the coefficient that determines the weight put on inflation relative to output is below unity, indicating that output received more weight in the optimisation problem, and therefore, it is a priority over inflation. This is also a result obtained by Ireland (1999), with more weight put into unemployment in that case than inflation. However, the coefficient is not statistically significant.

However, looking at table 2.2 that provides the likelihood ratio tests of the restrictions derived from the time-consistency exercise upon each of the models analysed before (models B1 to B3), the results suggest that the time-consistency model of determination of inflation is rejected. This might be due to the fact that we have a very simple model and does not account for all the dynamics that can be found in the data. We can also explain it by that fact that, for example, the γ parameter that reflects the relative importance of output and inflation in social welfare policymakers' preferences for inflation compared to output is not a constant (as we had assumed it) and it has changed over the sample period. In fact the stability of the model over the sample may be put under question. Throughout the period 1975q3 to 2000q4, important changes have been made with regard to policy, from monetary growth targeting, to exchange rate targeting to inflation targeting and as a result, the period could be scrutinized to try to incorporate policy regime changes. Furthermore, our simple optimisation exercise is of a static nature. We assume that the Central Bank, each period tries to minimise the loss function after agents have formed their expectations.

However, except for model B3/B3T, where output depends upon current inflation surprises, due to the presence of persistence in inflation and expected inflation in the other models, the outcome today influence the outcome tomorrow and therefore, the Central Bank should not have this myopic type of behaviour which is reflected in our optimisation exercise. Instead, a fully dynamic optimisation exercise would be more adequate and this may lead to different conclusions.

2.6 Conclusion

In this chapter we made use of direct measures of expectations on output and inflation with the objective of incorporating them in a model of actual inflation and output growth to help us understanding inflation and output determination and the inflation-output tradeoff. We used those expected and actual series together in a structural VAR which we estimated and tested a few model restrictions.

In our model, we could not reject the hypothesis of long run neutrality, in the sense that inflation is unable to deviate actual output from its natural level as we expected from our theoretical model. Our empirical model therefore, supports a vertical long-run Phillips curve. Inflation affects output only in the short-run. However, our model shows that inflation surprises alone are not enough in explaining the tradeoff, which suggest the importance of separating the influences of actual inflation and expected inflation on output. In particular, the hypothesis that only current unanticipated movements in inflation affected output was rejected, and therefore past unanticipated changes and past actual and expected prices have a real effect, causing deviation of the output from its natural level. This result should not come as a surprise. In fact, evidence has accumulated against the Lucas-type Phillips curve used in the seminal work of Barro and Gordon. For example, Christiano, Eichenbaum and Evans (2001) find that surprise inflation is not the best way of thinking of the way monetary policy translates into real effects.

The time-consistency model of determination of inflation was rejected. However, this does not imply a rejection of the ideas behind the Barro-Gordon model. In our framework, the rejection of the time-consistency model of determination of inflation can have different explanations. It might be due to the fact that we have a very simple model that does not account for all the dynamics that can be found in the data, or that the γ parameter that reflects the relative importance of output and inflation in the policymakers' preferences for inflation compared to output is not a constant (as we had assumed it) but it has changed over the sample period. Related to this is also a simplifying assumption made in relation to the target inflation rate. In our empirical work, we assumed that the inflation target remained the same throughout the period under consideration. This is a simplifying assumption, similar to the commonly used assumption of zero-inflation target. These assumptions can be easily criticized, especially considering the period under analysis, which includes different regimes in terms of conduct of monetary policy and inflation outcomes. Throughout the period 1975q3 to 2000q4, important changes have been made with regard to policy, from monetary growth targeting, to exchange rate targeting to inflation targeting. The seventies had seen the UK inflation reaching double digits after the two oil price shocks of 1973 and 1979, other commodity price shocks and developments in the labour market. Disinflationary policies followed, initially characterised by income policies, seen then as the best way to control inflation. Monetary growth targeting gained prominence since 1976. However, as argued by Nelson and Nikolov (2002), the high inflation of the 1970s was the result of policymakers not attributing inflation to monetary factors, and instead using price and wages controls[78]; different regimes followed, including the participation of the UK in the Exchange Rate Mechanism from 1990 to 1992. It was then that the inflation targeting framework which is still in practice today was implemented. In the context of Taylor type of rule, Nelson (2000) showed that the conduct of monetary policy has differed, in particular, it has been better conducted since the introduction of inflation targeting[79].This seems to suggest that one should model inflation targeting differently and a scrutiny of the sample to detect these effects would be a good extension of the model presented here. Furthermore, in our model, we pursue a very simple optimisation exercise, whereas given the persistence in inflation present in the data, a fully dynamic optimisation would be more appropriate.

As mentioned before, the rejection of the restrictions imposed by the time consistency model does not undermine the Barro-Gordon model. The model has shaped the way monetary policy started being conducted or should at least be conducted. The credibility issues are at the top of the agenda and only credible policies would bring about the necessary disinflation and at lower costs in terms of output. One attempt at gaining credibility through importing good reputation was done from 1987 to 1992, when the UK started shadowing the Deutsche Mark until formally joining the Exchange rate Mechanism in 1990

[78] The authors called this hypothesis ”Monetary Policy Neglect”. According to it, the disinflation that followed in the 1980s was a result of a change in monetary conduct, and taking inflation as a monetary phenomenon.

[79] Discussion of Taylor type of rules will be done in the next chapter. Explanations of the concept of ”stabilising” policy will be made then.

until 1992. After the unsuccessful Exchange Rate Mechanism a different route at achieving good credibility in the conduct of monetary policy was tried, this time successfully: inflation targeting which was introduced in 1992. Following Rogoff(1985)'s recommendation, in 1997, the Bank of England gained operational independence. In here lies another suggestion to eliminate inflation bias: leave monetary policy to the conduct of a conservative authority. This exposition suggests that investigating the presence of structural breaks and regime switching may be a good idea. Further, it also suggests, in our opinion, that credibility issues and the best way of gaining credibility has been recognised as crucial in implementing successful monetary policy characterised by stable inflation and therefore, Barro-Gordon's model has great merit.

Another criticism to our model is the fact that our Phillips curve (model B1), does not have microfoundations. Although other known approaches, for example a Lucas type, would have strong foundations which can be formulated in terms of imperfect information of economic agents, they do not fit the data well. A next step would be to try to motivate a Phillips curve of our type. Note that the fact that our Phillips curve includes actual (current and past) inflation together with (current and past) expected inflation, separates it from a backwards-looking Phillips curve. This, in turn, has very important implications. For example, we would not have been able to analyse the issue of credibility and time-consistency should we have a purely backwards-looking Phillips curve, with only actual, past inflation terms included in it. The fact that expectations play a role means that there is room for credibility to be of relevance. Policymakers announcements will affect expectations and therefore agents behaviour, and if credible, they may decrease disinflation costs.

Further, the models used in this chapter imply that inflation leads output, more specifically, they imply that the monetary policy affect output through its effects upon inflation. However, the empirical evidence for most countries is that output leads inflation, in the sense that monetary policy is able to affect output through it influence upon aggregate demand before inflation is affected. See, for example, Kiley (1996). Given this evidence, this is exactly the approach that will be followed in the next chapter.

Chapter 3
An Empirical Analysis of Monetary Policy and Macroeconomic Dynamics Using Survey Data

3.1 Introduction

In recent years, the empirical investigation of the effects of monetary policy has received considerable attention. This work aims to examine the decision-making processes which underlie the conduct of monetary policy and to identify the effects of monetary policy on other macroeconomic magnitudes. In particular, the empirical work aims to see whether policymaking can be reasonably described by ”Taylor-type” rules, following Taylor’s (1993) work. Having identified monetary policy innovations, many studies have considered their role in macroeconomic dynamics, often based on small estimated models or small calibrated models of the main macroeconomic variables. Examples include Clarida, Gali and Gertler (2000), Fuhrer and Madigan (1997), Haldane and Batini (1999), Ball (1999), Rudebusch and Svensson (1999a), Rotemberg and Woodford (1999). In much of this work, the nature of the expectations formation process is very important in providing the structural interpretation required to identify the monetary policy rule and in the choice of estimation technique. Usually, the assumption made in the literature is that expectations are formed rationally. But this means, of course that the analysis and results obtained are based on an assumption that many find unappealing. It would be interesting to examine the same sort of policy rules, within a small macro model but without making any assumptions on the expectations formation process. This is only possible by making use of direct measures of expectations.

One of the chapter's main contribution lies in its use of survey data on expectations. A widely used source of direct measures of expectations is the previously provided by ASA-NBER survey and now provided by the Federal Reserve Bank of Philadelphia, the Survey of Professional Forecasters (SPF). The survey constitutes a very reliable and continuous dataset, which provides forecasts over multiple horizons. The survey data has been used by many authors, including Fair and Shiller (1989), Zarnowitz (1979), Keane and Runkle (1990), Jeong and Maddala (1996). In this work, we consider a small macro model with which we analyse the dynamics of five variables, namely, actual price inflation, actual output gap and the interest rate along with direct measures of expectations on price inflation and output. The use of direct measures of expectations allows us to test the rationality hypothesis (or indeed any hypothesized expectation formation process), enabling us to capture the dynamic relationship between the actual and expected variables, and to investigate more explicitly the role of expected variables in the setting of the interest rate by the monetary authorities. The economic model behind the structural VAR is based on three behavioural relations: a Phillips curve, an IS curve and a monetary policy rule. The Phillips curve captures the tradeoff between inflation and output along the lines of Ball (1999). The IS curve, capturing the demand relationship between real interest rate and output, is in line with Judd and Rudebusch (1998). To capture the conduct of Monetary Policy we have a monetary policy rule, which consists of a specification of a Taylor type of rule.

The empirical work of the paper uses quarterly US real-time data over the period 1981q3-2001q2 and covers three issues: the use of SPF data and the test of the rationality of expectations; the estimation of the five-variable system and some tests to assess the robustness of the results. As we will see, the analysis of the results from the estimation of the VAR, provides further empirical support for simple rules such as the Taylor rule. This is because movements of the interest rate are well captured by the estimated rule, when the dynamics for actual and expected output growth and price inflation are appropriately taken into account. Due to model uncertainty, the system was estimated under different assumptions regarding the different behavioural relationships involved in the small macro model. However, the Taylor rule estimates are robust across these models. We are also able to contribute towards recent discussion over the meaning of the significant lagged inflation in explaining inflation dynamics through a Phillips curve representation.

The layout of this chapter is as follows. In Section 2, we motivate the behavioural model with some theoretical background. The modelling framework, motivating our five-variable VAR analysis is described in Section 3. In Section 4, we provide with our data analysis, rationality tests and estimates of the five-variable system. Section 5 concludes.

3.2 Motivating a theory-based behavioural model

In the previous chapter we estimated a simple model of actual and expected output and inflation. The main aim of it was the test of restrictions on the supply side, which would help understanding better the inflation-output trade-off. In that model of the economy we made the simplifying assumption that inflation was directly controlled by using monetary instruments, without modelling them. In other words, in the previous chapter, we abstracted from monetary transmission mechanism issues and how the monetary authority acts in its attempt to influence inflation and output. In this chapter we extend our analysis to include the interest rate as the policy instrument that will be used to influence inflation and output. The inclusion of the interest rate enables a better understanding of the aggregate demand curve, allowing an estimation of an IS curve. The IS curve and monetary policy rule together with a Phillips curve constitute the three main behavioural relationships in our small macro model for the US economy. The expectations formation process constitute another important component of our model that imposes further structure to it.

In this section, we provide some background to the three behavioural relationships underlying our VAR analysis. We aim to have an empirical theory-based small macro model. The behavioural relationships that we will be estimating include an IS curve, a Phillips curve and a monetary policy rule of a Taylor-type. In this section, we review the theoretical background to those relationships, starting with the monetary policy rules.

3.2.1 Monetary Policy Rules

Since Taylor (1993), many authors have estimated monetary policy rules, where the interest rate responds to inflation and output. The original formulation can be represented as:

$$i_t = \pi_t + \gamma(y_t - \bar{y}_t) + \beta(\pi_t - \pi^*) + r^* \tag{3.118}$$

where i_t is the short-term nominal interest rate, π_t is the inflation rate, $(y_t - \bar{y}_t)$ is the output gap, π^* is the inflation target and r^* the equilibrium real interest . The coefficient on inflation is $(1+\beta)$ and the intercept is $r^* - \beta\pi^*$.

Alternatively, this rule can be written in terms of the real interest rate as follows:

$$i_t - \pi_t = (r^* - \beta\pi^*) + \gamma(y_t - \bar{y}_t) + \beta\pi_t \tag{3.119}$$

This equation can have different interpretations. It can be interpreted as a reaction function, whereby the monetary authority sets the interest rates in response to events in the economy. Studies that use this interpretation often aim to identify patterns in the behaviour of the monetary authority, in particular identifying systematic relationship between the interest rate, inflation and output and/or identifying and describing differences in the conduct of monetary policy across different periods. Examples include Clarida, Gali and Gertler (2000) and Orphanides (2004). Alternatively, this rule can be interpreted as a guideline for the monetary policy, serving as a normative guide to interest rate decisions. In Taylor's (1993) seminal paper, he suggested parameter values of γ =0.5, $\beta = 0.5$, with $\pi^* = 2$ and r^*=2. Taylor's suggestions are based on empirical observation. Other papers, though, have provided normative guides with theoretical foundations, derived through optimisation exercises, finding structural parameters on output and inflation. These models derive optimal

Taylor rules, specifying restrictions on the coefficients some of which are at odds with Taylor's original formulation. See, for example, Ball (1999) and Woodford (2001) that discuss optimality of Taylor rules.

The magnitude of the coefficients in (3.118) carry important information. Taylor stresses the relevance of having $\gamma > 0$ and $1+\beta > 0$ for stabilization purposes. The interest rate should increase when output is above potential and it should be raised enough such that the real interest rate is increased whenever inflation is above target. If the parameters meet these magnitudes, the rule is said to be stabilising. Taylor (1999b)[80] himself, analysed the historical conduct of monetary policy since 1879, dividing it into different periods, resulting in different values for the coefficients. Considering two of the subperiods, 1960q1 to 1979q4, where $\gamma = 0.252$ and $(1+\beta) = 0.813$; and the period 1987q1-1997q3, with $\gamma = 0.765$ and $(1+\beta) = 1.533$. The estimates show that for both sub-periods, the coefficients on output gap and inflation are positive. However, for the period between 1960q1 and 1979q4, the coefficient on inflation, although positive is smaller than one, indicating that the real interest rate fell when inflation rose, and therefore monetary authority had failed to having a stabilising role[81]. Ball (1999) derives a Taylor rule from a structural model that includes a backwards-looking Phillips curve, IS curve and a monetary authority that minimises a quadratic loss function on deviations of inflation and output from targets. He finds that Taylor's proposed parameters are not optimal; in particular he finds that the response of

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80 In Taylor(1999a).

81 Orphanides (1999) provided with a difference interpretation for these results with reference to missmeasurements in the output gap estimates.

the interest rate to output is rather low, i.e., central banks should be advised to act more aggressively when output deviates from potential.

Taylor's original formulation of the monetary rule has been subject to several modifications. Different versions of the Taylor rule have been proposed in the literature, based both on theoretical as well as empirical grounds. A very cited example includes Clarida, Gali and Gertler (2000), where the authors proposed a forward-looking formulation of the monetary reaction function of the form:

$$i_t^* = \alpha({}_t\pi_{t+k}^e - \pi^*) + \gamma({}_ty_{t+q}^e -_t \bar{y}_{t+q}^e) + i^* \tag{3.120}$$

which can be written as:

$$i_t^* =_t \pi_{t+k}^e + \beta({}_t\pi_{t+k}^e - \pi^*) + \gamma({}_ty_{t+q}^e -_t \bar{y}_{t+q}^e) + r^* \tag{3.121}$$

where $\beta = \alpha - 1$, ${}_t\pi_{t+k}^e$ is the expected inflation rate at time $t + k$, based on information at time t, expressed in annual rates; $\pi^*$is the inflation target and $({}_ty_{t+q}^e -_t \bar{y}_{t+q}^e)$ is the expected average output gap between t and t+q; i* is the desired nominal rate when inflation and output are at the target and by construction and $r^* = i^* - \pi^*$. This formulation has theoretical foundations. As explained in Svensson (1997) and Clarida, Gali and Gertler (1999), it may be obtained from the optimisation problem of a monetary authority that faces a quadratic function in deviations of output and inflation from target, in the presence of nominal price inertia. It is also appealing from the empirical side, as it also behaves well. Clarida, Gali and Gertler (2000) find this formulation more realistic than the backwards looking Taylor type of rule. However, they also stress the fact that if future inflation can be forecasted using lagged inflation and output gap, then it is reduced to the Taylor rule.

The monetary rule above can also be written in terms of real interest rate as follows:

$$r_t^* = r^* + \beta({}_t\pi_{t+k}^e - \pi^*) + \gamma({}_t y_{t+q}^e -_t \bar{y}_{t+q}^e) \tag{3.122}$$

Using $\beta = (\alpha - 1)$

$$r_t^* = r^* + (\alpha - 1)({}_t\pi_{t+k}^e - \pi^*) + \gamma({}_t y_{t+q}^e -_t \bar{y}_{t+q}^e) \tag{3.123}$$

where $r_t^* = i_t^* -_t \pi_{t+k}^e$and $r^* = i^* - \pi^*$ is the long-run equilibrium real interest rate. The real rate is assumed to be stationary and determined by non-monetary factors in the long-run, and as a result, r*is a constant and independent of monetary policy. Similarly to the original formulation, if $\alpha > 1,$ the rule is said to be stabilising, in that, real interest rate will rise when expected inflation is above target, which depresses economic activity through the negative effect on aggregate expenditure; $\alpha \leq 1$ are likely to be destabilizing, or at least reflects an accommodating monetary policy. In terms of $\gamma,$ if $\gamma > 0$, monetary policy is stabilising; otherwise, destabilizing.

Another important modification of the original rule that tried to make it closer to the way monetary policy is conducted in practice is elaborated in Clarida, Gali and Gertler (1999). The original rule assumes that the Central Bank adjusts interest rate to target immediately, whereas in reality, there is tendency to smooth interest rates, i.e., to have a partial adjustment

$$i_t = \rho i_{t-1} + (1 - \rho) i_t^* \tag{3.124}$$

where i_t is the actual nominal interest rate and i_t^* is the target defined previously. The parameter ρ indicates the degree of interest rate smoothing. Although it does not have theoretical foundations, the interest rate smoothing from the part of the monetary authority has been justified with reference to the fear of disruption of the financial markets or loss of credibility from abrupt changes in policy.

The timing of the variables has also received great attention. In Taylor's original formulation, the policymaker changes the interest rate in response to current output and current inflation. However, the Taylor rule should reflect the information the monetary authority has at the time decisions are made, i.e., time t. Given that data becomes available with a lag, a modification of the Taylor in the literature includes lags of both inflation and output. Nelson (2000), for example, uses this formulation applied to the UK economy.

Another issue that has been put forward by Orphanides (2003b) relates to the use of revised data. Most empirical studies make use of the latest vintage of data, i.e., a revised dataset which was not available to policymakers at the time the decisions were made. As a result, analysis of policy rules may lead to the wrong historical descriptions of the conduct of policy or provide wrong recommendations. One of the main issues regards the estimates of potential output. It is well known that estimates of potential output carry a great degree of uncertainty. However, the problem becomes even more apparent when we acknowledge the presence of data revisions. Orphanides finds that , for example in the great inflation decade of the 1970s, potential output in real-time has greatly overstated potential output when compared with recent estimates making use of revised data. When estimating the Taylor rule using real-time data, and therefore real-time output gap, the policy appears to be stabilising, with the interest rates raised enough to increase the real interest rates[82]. Therefore, attempts to characterise the monetary conduct of the 1970s using the most re-

[82] With reference to the cofficients above, $\alpha > 1$, or equivalently, $\beta > 0$.

cent, revised data are misleading - instead, the use of real-time data allows us to get back in time and see through the eyes of the policymakers of the time.

The rules described so far are considered simple rules,i.e., they involve very few variables in a simple linear formulation. There is also debate over the relative merits of simple as opposed to complex rules. Rudebusch and Svensson (1999b) derive an optimal rule and show that simple rules perform almost as well. Levin, Wieland and Williams (1999) analyse robustness of simple rules versus optimal rules, finding that the latter ones are less robust. This is due to the fact that optimal rules are model-specific and as a result they do not perform well when tested in a different model of the economy. As these authors emphasise, due to uncertainty surrounding the true structure of the economy, it is adequate to try to formulate monetary policy rules that work well across different structural models. In this chapter, we also concentrate on simple types of rules.

3.2.2 The Phillips curve and the IS equation

The Phillips curve

The other two behavioural relationships of interest include the Phillips curve, analysing the inflation and output tradeoff and the IS curve, showing the relationship between interest rate and output. In this section, we will try to motivate those relationships theoretically and give an overview for the different formulations found in the literature. with regard to the Phillips curve, there has been renewed interest on New Keynesian models of price-setting, with a considerable amount of empirical work evaluating its validity and testing the relative importance of the forward-looking component in the hybrid version of it. To analyse the controversy, we may start with a hybrid formulation, as suggested in Rudebusch (2002), of the form:

$$\pi_t = a\ {}_t\pi^e_{t+1} + (1-a)\pi_{t-1} + b(y_t - \bar{y}_t) + \varepsilon_t \tag{3.125}$$

with $0 \leq a \leq 1$ and $b > 0$.There is divergence across theoretical work involving the value of a, which measures the degree of inflation inertia[83]. The treatment of the expectational variable also diverges from study to study.

Regarding the value of a, for example Ball(1999) considers $a = 0$. In this case, the Phillips curve above becomes:

$$\pi_t = \pi_{t-1} + b(y_t - \bar{y}_t) + \varepsilon_t \tag{3.126}$$

which has a backwards-looking formulation. Current inflation equals last period's inflation adjusted by current excess demand, in the form of output gap and cost-push shock ε_t. The model predicts that when the monetary authority pursues contractionary policies that brings output below the natural rate, resulting in a negative output gap, inflation will be falling over time[84].

At the other extreme, McCallum(1999), for example considers $a = 1$, with the resulting forward-looking formulation:

$$\pi_t = \pi^e_{t+1} + b(y_t - \bar{y}_t) + \varepsilon_t \tag{3.127}$$

Representations like this one, where inflation depends on the output gap and expected future inflation are the so called New Keynesian Phillips curves. One of the advantages of

[83] At the empirical level, a debate over the relative importance of the forward-looking component of the value a has raised recent debate. See issue 52 of the Journal of Monetary Economics, 2005.

[84] A negative demand shock would have the same effect of decreasing inflation.

the New Keynesian Phillips curve is the presence of strong theoretical foundations, including costs of adjustment and overlapping price and wage contracts, in line with theoretical models such as Calvo (1983)'s staggered price model, for example. Previous models, such as Lucas (1972) had relied on the assumption of price flexibility, which failed to be a good approximation of the empirical observation of the price series. These models address this issue by including some stickiness in prices; however, a Phillips curve representation such as (3.127) implicitly assumes complete flexibility of inflation - there is no lag dependence on inflation.[85] [86] Furthermore, the relationship between inflation and economic activity is reversed, i.e., contrary to intuition, a negative output gap would lead to an expected increase in inflation. In other words, a contractionary demand policy would increase expected inflation. An explanation that may justify this unintuitive result involves considering inflation as a jump variable - output below potential would made inflation to jump down and then start to rise to its new equilibrium (which would be a lower one) from below. Ball (1994) does not find evidence for such behaviour for the US inflation, where he shows that disinflationary episodes are slow processes that come at a cost in terms of loss of output. This lead several authors into developing microfounded models that would include some inflation persistence and making the hybrid formulation a more attractive one.

Theoretically,the hybrid formulation can be interpreted in different ways. Gali and Gertler (1999) develop a theoretical formulation of a hybrid Phillips curve based on the

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85 Note that starting with Phillipc curve representation $\pi_t = \pi^e_{t+1} + b(y_t - \overline{y}_t) + \varepsilon_t$, by iterating forward, we obtain $\pi_t = E_t \sum\limits_{i=0}^{\infty} [b(y_{t+i} - \overline{y}_{t+i}) + \varepsilon_{t+i}]$, where inflation depends on current and expected future variables only.

86 See Fuhrer and Moore (1995).

Calvo (1983)'s staggered price model. The starting point in Calvo's model is the assumption that each firm will keep price fixed until it receives a random signal to change its price and they will do so with a certain probability, which is independent of the time the price has been fixed for. Gali and Gertler (1999)'s extension involves making a differentiation within the firms. Out of those firms that will be changing price, only a fraction $(1-\gamma)$ will do so optimally; the remaining behave in a backwards-looking fashion, using a rule of thumb. The behaviour of this latter group is what gives the Phillips a hybrid formulation, with inflation depending upon past inflation. A different interpretation has been given by Fuhrer and Moore (1995). They altered the Taylor (1979, 1980)'s staggered-contracts model by assuming that agents care about relative real wages. Taylor's main assumption is the fact that wage contracts are not made at the same time, and assuming wages are set for two periods, only half of them are negotiated each period, which implies inertia in wages and prices. Fuhrer and Moore further assume that the agents when negotiating their contracts will take into account the real wages achieved by other contracts with which the current contract overlaps. In particular, they will be having as a reference the real wage achieved in the previous period and this becomes the crucial feature of this model, that turns a purely forward-looking Phillips curve into a hybrid one. The agents then estimate an average of this index over the current and next period (i.e., the two-period life of the contract) and expect their own real wage contract to equal this expected average except for business cycle effects in the form of the output gap. This setting, where agents care about real wages in the way defined by Fuhrer and Moore (1995), with past real wage playing a role in current wage settings, transforms Taylor's purely forward-looking inflation equation into a hybrid one[87].

Although many authors support a hybrid Phillips curve as a good representation of inflation dynamics, disagreement remains about the relative importance of the forward-looking *vs* backward-looking components. Gali and Gertler (1999) estimate the Phillips curve using the marginal cost (rather than output gap) and find that the forward-looking component of the Phillips curve is very important and outweighs the backwards looking component, as their GMM estimates find a larger coefficient on the former. This is in contradiction with Fuhrer (1997). He considers a number of different formulations for inflation, trying to understand the importance of expected inflation in determining current inflation and he finds that empirically, expected inflation does not have a significant role in explaining inflation dynamics.[88]Gali and Gertler results have been criticized on the basis of lack of robustness, in particular the dependence of the use of marginal cost as the driving variable. Lindé (2005) using full information maximum likelihood method was unable to confirm the prominence of the forward-looking component. Rudd and Whelan (2005) confirm Lindé in finding a limited role for the forward-looking terms in explaining inflation. However, Gali, Gertler and López-Salido (2005) reply to the criticisms, confirming their belief in the adequacy of the hybrid Phillips curve in explaining the behaviour of inflation.

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87 Note that this result has been criticised by Holden and Driscoll (2001), who believe that agents should care about current real wage (W_{t-1}/P_t) of other wokers rather than past real wage(W_{t-1}/P_{t-1}) and in this case, the inflation persistence disappears. However, we could still defend Fuhrer and Moore assumption on the basis that the workers in the previous aspired to the real wage (W_{t-1}/P_{t-1}), which may be very different from the real wage they are currently experiencing, W_{t-1}/P_t due to expectational errors on the price level; therefore, the benchmark should be what they aspired to, rather than what they are currently experiencing.

88 Using this evidence, we opt for a backward looking version of our model which also seems to better fit the data, as will be discussing later in the chapter.

In their view, the poor results associated with estimations of New Keynesian Phillips curve are based on the use of output gap as this measure is a poor approximation to the real marginal cost, which determines the price set by firms. Furthermore, other studies confirm their findings including Sbordone(2005) who also finds a relatively more important role for forward-looking behaviour.

Although the New Keynesian (purely forward-looking) Phillips curve has the attractiveness of microfoundations, as Gali and Gertler put it ”reconciling the new Phillips curve with the data, has not proved to be a simple task”[89]. Ball (1999) chooses to have a backwards-looking formulation on the basis that despite having strong theoretical foundations, the forward-looking models fail to fit the data. In particular, as referred to by Fuhrer and Moore (1995), they fail to produce the inertia in inflation that appears in the data. Rudd and Whelan (2005) also state that empirically, lagged inflation plays an important role in explaining inflation and puts the question of whether these reflect backward-looking expectations formation processes or they constitute a proxy for rational-forward-looking expectations, as in the New Keynesian Phillips curve. In other words, there is no consensus over the interpretation of a Phillips curve such as equation (3.126). It does characterise well the US inflation, but is it because agents form their expectations in a backward-looking manner or is it because agents are forward-looking and future inflation is correlated with past inflation? Put differently, do agents form their expectations according to adaptive expectations or do past inflation terms reflect inertia in the economy and are therefore consistent with rational, forward-looking agents? Although the representation becomes identical, they rep-

89 Gali and Gertler (1999), pp 201.

resent different dynamics and different conclusions regarding costs of disinflation. In this chapter, we try to contribute towards answering these questions.

The IS curve

On the demand side, we have two relationships. The first one, described in the previous section is a monetary rule, where the interest rate is the policy instrument. The second, which will be discussed in the section, include a demand equation in the form of an IS curve.

The IS curve relates the output to the real interest rate. The link between these two variables is aggregate expenditure, which depends negatively on the real interest rate, leading to a negative relationship between output and interest rate. Similarly to what we did with the Phillips curve, we can start by introducing a hybrid formulation of the IS, popular within the New Keynesian framework, as follows:

$$y_t - \overline{y}_t = d_0({}_t y^e_{t+1} -_t \overline{y}^e_{t+1}) + (1 - d_0)(y_{t-1} - \overline{y}_{t-1}) - d_1 r_t + v_t \qquad (3.128)$$

where r_t is the real interest rate and v_t is a demand shock[90]and $0 \leq d_0 \leq 1; d_1 > 0.$

With $d_0 = 1$, we obtain a purely forward-looking version of the above equation, which can be derived from an optimisation model with a representative agent, acquiring in this manner micro foundations, the lack of which in the traditional IS-LM model represent one of its main criticisms. Several authors, such as Woodford (1999), McCallum and Nelson (1999), accordingly have expressed the IS relationship as:

[90] This represents a demand shock other than interest change, which will affect aggregate expenditure. Examples include increased confidence about the future.

$$y_t - \overline{y}_t = ({}_t y^e_{t+1} -_t \overline{y}^e_{t+1}) - d_1 r_t + v_t \tag{3.129}$$

with $r_t = i_t - \pi^e_{t+1}$.

This formulation can be derived from an optimization problem facing a representative consumer, after deriving a log-linearisation of a consumption Euler equation, and applying the equilibrium condition in the goods market. The main difference between this formulation and the traditional IS curve relates to the output gap. Higher future output gap increases current output because consumers wish to smooth their consumption throughout their lifetimes, and therefore, expectations of higher income leads them to revise their consumption decision upwards, and as a result they consume more today. The negative effect of the real interest rate on output reflects intertemporal substitution of consumption.

The more traditional formulation can be expressed as

$$y_t - \overline{y}_t = -d_1 r_t + v_t \tag{3.130}$$

The negative effect impacts negatively on output because higher interest rate tends to depress spending by households and firms. Recognising the existence of inertia and lags, the IS curve has been represented as variations of the following dynamic equation:

$$y_t - \overline{y}_t = d_0(y_{t-1} - \overline{y}_{t-1}) - d_1 r_{t-1} + v_t \tag{3.131}$$

The presence of lags reflect, on one hand the fact that demand does not respond instantly to changes in interest and output has some inertia. Those can be explained at a micro level, by information delays and habit formation in consumers' preferences[91].

[91] Judd and Rudebusch (1998) have a similar representation, with an additional lag on output.

Comparing the traditional formulation with the forward-looking formulation, a few comments can be made. The persistence in output can explain the presence of lagged output in the IS formulation[92]; more generally, the presence of lagged variables helps capturing the dynamics in the data and they tend to have a good fit; however, these formulations do not have a theoretical foundation[93]. As noted by McCallum and Nelson (1999), the addition of a term regarding expected output is required in order to have a micro-founded IS curve, with optimising agents. As a result, current demand depends on expected future output and therefore changes in income thought to be permanent will have larger effect on spending than temporary changes. This effect is not present in the backward-looking formulation, unless expected future output is proxied by the lagged variable. The acknowledgement of costs of adjustment and habit formation has led to debate on the exact specification of IS that would translate those elements of inertia, with authors such as Svensson (1999) using the hybrid formulation, as in (3.128).

3.3 The modelling framework

3.3.1 The VAR formulation

In our model we estimate a system of five equations , which was transformed into a five-variable VAR of actual inflation, actual output gap, expected inflation, expected output gap and interest rate. It can be represented in a (structural) VAR formulation, with the following

92 King (2000).

93 Walsh (2000), chapter 10.

matrix format[94]:

$$A_o z_t = c + A_1 z_{t-1} + A_2 z_{t-2} + ... + A_p z_{t-p} + \varepsilon_t \tag{3.132}$$

with p denoting the order of the VAR and where

$z_t = (\pi_{t,t}\, \pi^e_{t+1}, y_t - \overline{y}_{t,t}\, y^e_{t+1} -_t \overline{y}^e_{t+1}, i_t)'$

$\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t}, \varepsilon_{5t})'$

A_o is a $(5x5)$ matrix that captures the contemporaneous feedback between elements of z

A_s is a $(5x5)$ matrix whose element in row i and column j is given by A_{ij}.

$c = (c_1, c_2, c_3, c_4, c_5)'$ is a vector of intercepts

This structural representation is equivalent to the following reduced form:

$$z_t = k + B_1 z_{t-1} + B_2 z_{t-2} + ... + B_p z_{t-p} + u_t \tag{3.133}$$

where $k = A_o^{-1} c$

$B_s = A_0^{-1} A_s$ for $s = 1, 2, ..., p$

$u_t = A_0^{-1} \varepsilon_t$

Alternatively, the model can be represented in a more compact way as follows:

$$\mathbf{\Gamma Z} = \mathbf{BX} + \mathbf{V} \tag{3.134}$$

where Z is a KxT matrix of T observations on K endogenous variables, X is a LxT matrix of T observations on L exogenous variables and Γ is a KxK matrix of the coefficients of the endogenous variables on the exogenous, B is a KxL matrix of coefficients and V is a

[94] Hamilton, page 326.

KxT matrix of unobserved residuals, assumed to be serially uncorrelated and uncorrelated with each other, i.e.:

$Var(\mathbf{V}) = E(\mathbf{VV'}) = \mathbf{D}$,with D being a diagonal matrix.

The reduced form of the model can be written as:

$$\mathbf{Z} = \Pi\mathbf{X} + \mathbf{U} \tag{3.135}$$

with $\Pi = \mathbf{\Gamma^{-1}B}$ and $\mathbf{U} = \mathbf{\Gamma}^{-1}\mathbf{V}$. The variance-covariance matrix for U is given by:

$$\mathbf{Var}(\mathbf{U}) = \mathbf{E}(\mathbf{UU'}) = (\mathbf{\Gamma^{-1}V})(\mathbf{\Gamma^{-1}V})' = \mathbf{\Gamma^{-1}VV'}(\mathbf{\Gamma^{-1}})' \tag{3.136}$$

$$= \mathbf{\Gamma^{-1}}Var(V)(\mathbf{\Gamma^{-1}})' = \mathbf{\Gamma^{-1}D}(\mathbf{\Gamma^{-1}})' = \mathbf{\Omega} \tag{3.137}$$

The system will be estimated using maximum likelihood. The function can be represented as[95]:

$$\mathcal{L}(\mathbf{B},\mathbf{\Omega}) = -\frac{KT}{2}\log(2\pi) + \frac{T}{2}\log\det(\Omega^{-1}) - \frac{1}{2}\sum_{t=1}^{T}(\mathbf{Z}_t - \mathbf{\Pi X}_t)'\mathbf{\Omega}^{-1}(\mathbf{Z}_t - \mathbf{\Pi X}_t) \tag{3.138}$$

With reference to the structural VAR, the (log) likelihood function becomes[96]:

$$\mathcal{L}(\mathbf{B},\mathbf{\Gamma},\mathbf{D}) = -\frac{KT}{2}\log(2\pi) + T\log|\det(\Gamma)| - \frac{T}{2}\log\det(\mathbf{D}) - \frac{1}{2}\sum_{t=1}^{T}(\mathbf{\Gamma Z}_t - \mathbf{BX}_t)'\mathbf{D}^{-1}(\mathbf{\Gamma Z}_t - \mathbf{BX}_t) \tag{3.139}$$

[97].

We will be imposing a very specific structure to our model, which is based on three main behavioural relationships (namely a Phillips curve, an IS representation and a mon-

[95] Hamilton (1994).

[96] Hamilton (1994).

[97] Harvey (1981).
Note that det stands for deterninant and $|.|$ denotes the modulus.

etary policy rule) and on the expectations formation process, which are also important behavioural relationships themselves.

3.3.2 The behavioural relationships

The first relationship to be analysed is the Phillips curve. The motivation for this relationship has been covered in section 3.2.2. However, we reviewed the theoretical foundations for the Phillips curve with reference to annual leads and lags. As discussed in Rudebusch (2002), when using quarterly data, those leads and lags become longer. This observation is reflected in the specification of the Phillips curve, as follows:

$$\pi_t = a_{\pi^e t}\pi^e_{t+1} + a_\pi(L)\pi_{t-1} + a_y(L)(y_{t-1} - \bar{y}_{t-1}) \tag{3.140}$$

where y_t is the actual output level, $\overline{y}_t$ is the natural rate of output, π_t is the actual inflation rate and ${}_t\pi^e_{t+1}$ represents inflation expectations for period t+1, taken at time t. $a_\pi(L) = a_{\pi 0} + a_{\pi 1}L + a_{\pi 2}L^2 + ...$ is a lag operator. $a_y(L)$ is also a lag operator with similar formulation.[98] The Phillips curve is given a general formulation that accommodates the various specifications described in that section, although we recognise the presence of lags in the transmission mechanism and therefore allow at least one period for output to affect inflation.

The economy is further represented by the following equation:

$$(y_t - \overline{y}_t) = b_{y^e}({}_t y^e_{t+1} -_t \overline{y}^e_{t+1}) + b_y(L)(y_{t-1} - \overline{y}_{t-1}) + b_i(L)(i_{t-1} -_{t-1} \pi^e_t) \tag{3.141}$$

[98] In this chapter we refrain from having the small three-equation model of the supply side with the three different concepts of output. Instead, we will conform with the rest of the literature (at least the great majority) and summarise the supply side into one single Phillips curve equation. This will allow an easier comparison with the literature, as our model is closer to them.

where i_t is the nominal interest rate. The equation represents aggregate demand through an IS curve, relating the output gap, defined as the difference between actual and natural rate of output with the ex-ante real interest rate. The lag operators are defined in a similar fashion to the $a_\pi(L)$ in the Phillips curve. This representation accommodates the different specifications discussed in the previous section, with longer lags to reflect the quarterly frequency of the data.

Finally, the policy rule, represented by:

$$i_t^* = d_{\pi t+k-1}\pi_{t+k}^e + d_y({}_{t+k-1}y_{t+k}^e - {}_{t+k-1}\overline{y}_{t+k}^e) \tag{3.142}$$

$$i_t = d_i i_{t-1} + (1 - d_i)i_t^* \tag{3.143}$$

where i_t^* is the interest rate target for quarter t, and k represents the "target" horizon relative to the quarter interest rates are decided. If k=+1, we get a forward-looking specification, where we assume that the policymakers look into the future when deciding upon the setting of the interest rate; with k=0, we assume that policymakers formulate policy based on information set dated at time t-1, as reflected by the public's expectations, assuming a more backwards-looking approach to the setting of the interest rates. The responsiveness of policy to expected inflation and output gap are captured by the parameters d_π and d_y. Acknowledging the relevance of the interest rate smoothing discussed in the previous section, equation (3.143) has been added. The inclusion of lagged interest rates intends to allow for the possibility that the interest rate adjusts gradually to achieve the rate recommended by the rule. This equation is showing that the actual interest rate moves in line with the target interest rate. The policy rule treats the interest rate as the instrument of monetary policy.

We have talked through the three main economic relationships. Our system has five variables and as a result, two further relationships are needed. The two extra relationships come from the use of direct measures of expectations. By making use of the survey data on expectations for inflation and output, we are able to test the expectations formation process. From this information, we will find two further relationships that define the lines on expected inflation and output. Since these results are dependent upon the test of expectation formation process, the exact formulation of these lines is left for section 3.4.2, where the specific model is introduced.

We will be representing these five equations in a five variable VAR of actual inflation, π_t,expected inflation, ${}_t\pi^e_{t+1}$,actual output gap,$y_t - \bar{y}_t$, expected output gap, ${}_ty^e_{t+1} - {}_t\overline{y}^e_{t+1}$,and interest rate, i_t. [99]For the expected variables we use direct measures of one-period ahead expectations provided by the Survey of Professional Forecasters.

3.3.3 Identification issues and estimation procedure

To identify the model is necessary to impose some identifying restrictions which include some structure into our VAR model. We will be having some zero-restrictions, i.e., we will be setting some elements of B, Γ and the covariance matrix to zero. We also fixed the diagonal of the matrix , Γ to one as a "normalisation". Under the structural representation, the number of coefficients in matrices A_i and c to be estimated consist of $K(L + (K - 1))$, with the covariance-matrix consisting of K under the case where D is diagonal[100]. Under

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[99] Note that our estimation include an intercept term for every line of the VAR, despite the fact that we do not include them while setting up the model to keep the formulation as simple as possible.

[100] Otherwise, it would be $\frac{1}{2}K(K+1)$.

the reduced form, the coefficients on matrices B_i amount to KxL, whereas the covariance matrix has $\frac{1}{2}K(K+1)$ elements to be estimated. The difference in parameters represents the amount of restrictions necessary to identify the structural parameters. In our model, we have 5 endogenous variables ($K = 5$) and $L = 26$ (which includes 5 lags of the 5 endogenous variables and the constant term). As a result, there are 145 A_i parameters and 5 constant terms to be estimated together with 5 elements in the variance-covariance matrix under the structural form when D is diagonal. Under the reduced form, the covariance matrix remains with 15 elements to be estimated, whereas there are only 130 parameters in the vector k and matrices B$_i, i = 1, ..., 4$. Therefore, the number of identifying restrictions required is 10[101]. It is easy to see that the structure that we impose on the VAR represents more than 10 restrictions and as a result, our model is over-identified. Some of those restrictions are simply imposed; some will be tested. The restrictions that we will be imposed are based on economic theory and have underlying them the behavioural relationships associated with Phillips curve, IS curve and Taylor rule as well as the rationality of the expectations.

The system has been estimated using maximum likelihood[102] and the standards errors for the coefficients have been calculated from the inverse of the Hessian matrix. When there are no restriction on the VAR model, OLS estimation of each equation separately, leads to the same results as maximum likelihood estimates, i.e. there is no gain in estimating the model as a system. However, this is not the case. The fact that the model is overidentified

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101 In the case where D is not diagonal, the number of identifying restriction rises to 20.

102 Hamilton, pp318, note that if there were no restrictions on the VAR, maximum likelihood estimation would be achieved by OLS equation.

justifies our estimation choice. Furthermore, the presence of cross-equation restrictions calls for a system estimation, as individual, single equation methods would lose this information. As stated in Hamilton (1994)[103], full information maximum likelihood typically produces the most efficient estimates, when compared with for example, the 2-stage least squares and instrumental variables.[104]

3.4 Data Analysis and Estimation results

3.4.1 Actual and expected output, potential output and inflation and interest rates in the US, 1981q3-2001q2

In this section, we analyse direct observations on expected output gap and inflation together with the actual figures for interest rates, inflation and output gap for the US economy over the period 1981q3-2001q2 using the quarterly data provided by the Survey of Professional Forecasters. The empirical work makes use of real-time data to reflect better the information available at each period in time. The current perceptions on the variables in real-time, will be used as a proxy for (real-time) actual data.[105] In our model, rather than analysing the model vintage by vintage, we have a real-time analysis across vintages. Using the current perceptions on the variables in each period of time as a proxy for our actual data means

103 On page 250.

104 The likelihood function is maximised using the Davidon-Fletcher-Powell algorithm.

105 With reference to chapter 1, the proxies for the actual data are the outlook for each variable as perceived during that quarter, ie, ${}_t x_t$. For simplicity of analysis, we will be omitting the subscript and denote the variable by x_t; y_t and p_t are based on the outlook for output and prices for quarter t as perceived during the quarter, ie, with reference to chapter 1, $y_t =_t y_t$and $p_t =_t p_t$. This follows Orphanides (2001). For simplification, we will denote them as simply as y_t and p_t .

that we are effectively using data that reflects the information available back then, when expectations and macroeconomic decisions were actually made, as opposed to using ex-post, revised data unavailable at the time.

The variables are as follows: y_t is the log of GDP; p_t is the log of GDP deflator; i_t is the federal funds rate, annualized and expressed as a fraction; π_t is the quarterly inflation rate expressed in annual rates, i.e., defined as $4(p_t - p_{t-1})$; p^e_{t+1} and y^e_{t+1}represent the expectations taken at time t for the price and output levels respectively taken from the Survey of Professional forecasters. They refer to the one-period ahead forecast, with expected inflation defined at an annual rate, i.e., $\pi^e_{t+1} = 4({}_t p^e_{t+1} - p_t)$.Actual and expected output gap are defined as $y_t - \overline{y}_t$ and $y^e_{t+1} - \overline{y}^e_{t+1}$; $\overline{y}_t$ and $\overline{y}^e_{t+1}$denote actual and expected potential output and they are computed using a Hodrick-Prescott filter applied to the actual and one-period ahead expected series respectively.[106]

Hodrick and Prescott (1997)'s framework is based on the notion that any time series can be decomposed as the sum of a cyclical and a growth component, with the latter one varying smoothly over time. The Hodrick-Prescott filter minimises the sum of squared deviations between the trend and the actual series and the squared sum of changes in the growth rate; the latter is penalized by a parameter λ[107]. This parameter regulates the degree of smoothness in the trend. For a value of $\lambda = 0$, the filter delivers a trend that tracks the movements of the series being filtered, i.e., of the original series; as λ approaches infinity

[106] Other measures of potential output were also used to assess the robustness of the formulation, given uncertainty regarding estimation of the output gap. They include fitting a linear trend; and a linear and quadratic trend to the output data. In each situation, the output gap would be defined as the residual form the OLS regression having y_t or ${}_t y^e_{t+1}$as regressors for the case of $y_t - \overline{y}_t$ and $y^e_{t+1} - \overline{y}^e_{t+1}$respectively.

[107] The problem can be formalised as $\underset{\{g_t\}^T_{t=-1}}{Min} \{\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2\}$, where c_t reflects the cyclical component and g_t the growth component.

the filter gets closer to delivering a linear time trend. We estimated the growth component using the conventional (suggested by Hodrick and Prescott for quarterly data) $\lambda = 1600$. This will be our baseline measure for the output gap.

[Table 3.1; Figures 3.1 and 3.2]

Table 3.1 provides details of the properties of the actual and expected output series, inflation and interest rates. The series for actual and expected output gap are plotted in Figure 3.1. As the Table and Figure make clear, the two series move closely together. From the table we can see that the correlation coefficient between the two series is very close to one (0.83) indicating a very strong positive association between the two series. The actual output gap averaged a value of -0.0012 during this period and the expected series a value of -0.0011, meaning that on average, throughout the period, output was slightly below potential. The standard deviations are 0.0141 and 0.0138 respectively. Table 3.1 also provides the Dickey-Fuller statistics test for the order of integration of output variables. For both, the actual and expected output gap series, the hypothesis of a unit root can be rejected, as a result those series will be treated as stationary. The Dickey-Fuller statistics presented a value of -3.71 and -3.64 for the actual and expected series respectively, both of which are smaller than the relevant critical value of -2.89.

The expected series for the inflation in the US economy given by the Survey of professional forecasters are also analysed. As we can see there seems to be a downward trend on inflation. Recalling from chapter 1, the actual series were extremely volatile during the seventies and first years of the eighties, varying then within a much smaller range. It is also interesting to note that the expected series are smoother than the actual series. See Figure 3.2. From Table 3.1 we can see that the (annualized) inflation averaged 2.77% during this period and the expected series averaged 1.98%. The correlation coefficient provided intends to analyse the degree of the relationship between the actual and expectation series. Its value was 0.61 denoting a positive relation between these two series, which is clearly smaller than the one observed for the output gap. Actual and expected inflation varied between -2.92% and 10.6% and 1.4% and 8.32% respectively. The last column of the table provides the Augmented Dickey Fuller statistics and tests the null hypothesis that there is a unit root in the actual and expected inflation. The statistics values are -8.58 and -4.89 respectively, which are to be compared with the critical value of -3.46. We reject the hypothesis, and so we will be treating the inflation rate as a I(0) variable[108]

Statistics on the interest rates can be seen in the last row of the table. The nominal interest rates averaged 6.7% over the period under analysis, and showed a good positive correlation with one-period ahead expected inflation (π^e_{t+1}), which took the value of 0.69. The actual interest rates will be treated as a stationary variable. As it can be seen from the statistics suggested by the Schwarz Bayesian Criterion, the unit root hypothesis is rejected

[Table 3.2]

As explained earlier, the use of survey data allows a test of the expectations formation process. In this case, we are going to perform a rationality test on the expectational series. In particular, we are testing for the orthogonality of the expectational error to information which is know at the time expectations are formed. Similarly to chapter 1, we

[108] Note that the evidence favouring the stationarity of the series is not strong and it is a well known property of inflation. However, during the period under consideration we do find some support in the form of Schwarz Bayesian Criterion and we will follow it up, to conform with most of the literature referred to in this chapter.

created expectational errors for inflation and the output gap and estimated the following OLS regression:

$$\varepsilon_i = \alpha_0 + \alpha_1 h_{t-1} + u_t \tag{3.144}$$

with $i = \pi$ or y and $\varepsilon_\pi = \pi_t - \pi_t^e; \varepsilon_y = (y_t - \overline{y}_t) - (y_t^e - \overline{y}_t^e)$. The information set, h_{t-1}, include two lags of each variable, i.e., $h_{t-1} = (\pi_{t-1}, \pi_{t-1}^e, (y_{t-1} - \overline{y}_{t-1}), (y_{t-1}^e - \overline{y}_{t-1}^e), i_{t-1}, \pi_{t-2}, \pi_{t-2}^e, (y_{t-2} - \overline{y}_{t-2}), (y_{t-2}^e - \overline{y}_{t-2}^e), i_{t-2})'$.

The rationality test consists of testing $\alpha_1 = 0$[109].The results are presented in Table 3.2. We obtained a Likelihood ratio statistics of 18.098 (p-value of 0.053) which is to be compared with a χ^2 with 10 degrees of freedom, which takes the value of 18.31 at a 5% confidence. As a result, we cannot reject the hypothesised rationality on the output gap.

We also perform a rationality test on the expected inflation series. In this case, the likelihood ratio test took a value of 11.55 which is below the χ^2_{10} statistics. [110]This provides us with evidence not to reject the hypothesised rationality of the expectations on inflation. The relevance of these finding in the context of the current controversy of the Phillips curve structural interpretation and the role of expectations will be analysed in the next section.

3.4.2 Estimating the five-equation model

[109] We are using an efficiency test, ie, testing whether agents had made good use of the information available to them, which by assumptions is our h_{t-1}.

[110] The F version of the test took the value of 5.99, which has an associated p-value of 0.017 and therefore, the null hypothesis could not be rejected.

The baseline model

In this section we present the estimated results for the baseline model. The intention is to consider the functioning of the macroeconomy within a strict economic framework: the ”baseline economic model”. The economic framework asserts IS and PC relationships, accommodating no contemporaneous or forward-looking dynamics. The economic framework also asserts rationality in expectation formation processes. This gives very strict structure to A_0 in which the behavioural IS and PC relations are entirely backwards-looking and the Taylor rule is the only source of forward-looking behaviour. There remains considerable potential dynamic flexibility in the baseline model, however, and it remains of interest to test whether the behavioural model conforms to the structures suggested by economic theory. For example, it is still interesting to consider dynamics of the interest rate equation to see if it reflects the Taylor principle in terms of predicted coefficients; interesting to consider dynamics of output inflation to see whether it matches predictions or assess how strong is the link interest-rate-output and so on.[111] We undertake a relatively mechanical specification search on the dynamics of the baseline economic model to investigate these issues as we will explain in more detail below. This model turned out to fit better the data and as a result, was the one chosen to be analysed in more detail. As we will see, the model performs well in terms of predictions based on empirical and theoretical grounds, when compared with others with forward-looking components. However, we also

[111] However, although useful to concentrate on the behavioural relationships, the strictness of the model selection that we will be using renders the modelling framework of little use to analysis such as impulse responses, which, despite being a very interesting exercise, is not the aim of the present work.

provide with some robustness tests, including different model formulations estimations and different measures for the output gap.

Following the modelling framework described in section 3.3, we started off with a rather general model, which involved a VAR of order 5, as suggested by the Akaike Information Criterion and Adjusted LR test.[112]See Table 3.3. [113]This determined the length of the lag operators. Taking as an example $a_\pi(L)$, it assumed the form[114]:

$$a_\pi(L) = a_{\pi 0} + a_{\pi 1}L + a_{\pi 2}L^2 + a_{\pi 3}L^3 + a_{\pi 4}L^4 = \sum_{j=0}^{4} a_{\pi j}L^j \tag{3.145}$$

The five lines of this general structural VAR model consist of:

$$\pi_t = \sum_{j=0}^{4} a_{\pi j}L^j \pi_{t-1} + \sum_{j=0}^{4} a_{y_j}L^j(y_{t-1} - \overline{y}_{t-1}) + \varepsilon_{1t} \tag{3.146}$$

as line 1, representing the Phillips curve

$$\pi^e_{t+1} = \sum_{j=0}^{4} a_{\pi_j}L^j \pi_t + \sum_{j=0}^{4} a_{y_j}L^j(y_t - \overline{y}_t) + \varepsilon_{2t} \tag{3.147}$$

as line 2, with an expression for expected inflation[115]

$$(y_t - \overline{y}_t) = \sum_{j=0}^{4} b_{y_j}L^j(y_{t-1} - \overline{y}_{t-1}) - \sum_{j=0}^{4} b_{i_j}L^j(i_{t-1} - \pi^e_t) + \varepsilon_{3t} \tag{3.148}$$

as line 3, representing the IS curve,

$$(y^e_{t+1} - \overline{y}^e_{t+1}) = \sum_{j=0}^{4} b_{y_j}L^j(y_t - \overline{y}_t) - \sum_{j=0}^{4} b_{i_j}L^j(i_t - \pi^e_{t+1}) + \varepsilon_{4t} \tag{3.149}$$

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112 The selection was made in the context of a VAR of order 6. The SBC criterion suggested a VAR of order 1.

113 On Table 3.4 we also show diagnostic tests for the VAR of order 5; in particular, the results show no problems in terms of serial correlation.

114 The other lag operators assumed similar forms.

115 This line is derived fom the Phillips curve by forwarding it one period and taking REH at time t as explained below.

as line 4, with an expression for expected output gap derived from the IS curve[116]

$$i_t = d_i i_{t-1} + (1 - d_i)(d_\pi \pi^e_{t+1} + d_y(y^e_{t+1} - \overline{y}^e_{t+1}) + \varepsilon_{5t} \tag{3.150}$$

as line 5 and representing the Taylor rule.

The error terms, ε_i are assumed to be $iid \sim N(0, \sigma_i^2)$, $i = 1, 2, 3, 4, 5$.The shocks ε_{2t} and ε_{4t} are measurement errors as it will be explained below; ε_{1t} is a supply shock; ε_{3t} is a demand shock; the error ε_{5t} captures monetary policy shocks.

Lines 2 and 4 require some more explanations. In the previous section we tested the expectations formation processes underlying the survey we are using. The finding that expectations are formed rationally allows us to impose further restrictions in our model and obtain well-defined expressions for lines 2 and 4 on expected inflation and expected output gap respectively. In particular, line 4 is derived from the IS relationship making use of the Rational Expectations Hypothesis(REH). By forwarding line 3 one period we obtain:

$$(y_{t+1} - \overline{y}_{t+1}) = \sum_{j=0}^{4} b_{y_j} L^j (y_t - \overline{y}_t) - \sum_{j=0}^{4} b_{i_j} L^j (i_t - \pi^e_{t+1}) + \varepsilon_{3t+1} \tag{3.151}$$

By taking expectations at time t, we obtain:

$$E_t(y_{t+1} - \overline{y}_{t+1}) = \sum_{j=0}^{4} b_{y_j} L^j (y_t - \overline{y}_t) - \sum_{j=0}^{4} b_{i_j} L^j (i_t - \pi^e_{t+1}) \tag{3.152}$$

where $E_t(\varepsilon_{3t+1}) = 0$.Furthermore, we assume that the output expectations taken from the survey (${}_t y^e_{t+1}$) are not the true expectational series ($E_t(y_{t+1})$), containing a measurement error, ε_{4t}, such that[117]:

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116 Similarly to line 2, this expression is obtained by forwarding the expression for the IS curve one period and taking expectations at time t.

117 This is similar to the conversion error in chapter 1.

$$({}_t y^e_{t+1} -_t \overline{y}^e_{t+1}) = E_t(y_{t+1} - \overline{y}_{t+1}) + \varepsilon_{4t} \tag{3.153}$$

Similarly, line 2 can also be obtained from the backwards looking Phillips curve represented in line 1, by forwarding it one period and taking expectations at time t.[118]. Similarly, the error term can be interpreted as a mismeasurement on inflation expectations, such that:

$${}_t\pi^e_{t+1} = E_t(\pi_{t+1}) + \varepsilon_{2t} \tag{3.154}$$

As explained in section 3.3, these 5 equations were estimated simultaneously in a VAR framework. With reference to the structural VAR formulation in (3.132), the matrix A_0 representing the contemporaneous relationship between the variables takes the form:

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -a_{\pi 0} & 1 & -a_{y0} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -b_{i0} & -b_{0y} & 1 & b_{i0} \\ 0 & -(1-d_i)d_\pi & 0 & -(1-d_i)d_y & 1 \end{bmatrix}$$

This model is represented in Table 3.5. As we can see from the table, there are many significant coefficients, but there are several insignificant parameters and the estimation feels a little overparameterised. As explained at the beginning of this section, our aim is to estimate and understand the relationships between variables through a simple model of easily recognizable and standardized equations as seen in the literature. Through a specification search procedure, we ignored the specifications where the coefficients were insignif-

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118 Alternatively, we can think of this formulation in line with Judd and Rudebusch (1998). Their formulation of the Phiilips curve took the form:

$\pi_{t+1} = a_0\pi_t + a_1\pi_{t-1} + a_2\pi_{t-2} + a_3\pi_{t-3}$
$+a_4\pi_{t-4} + a_5(y_{t-1} - \bar{y}_{t-1}) + \varepsilon_{4t+1}$.

In our model, we use ${}_t\pi^e_{t+1}$ as a proxy for π_{t+1}, which turns the error term into a t-dated error term in the regression . Given that we have not rejected the Rational Expectations Hypothesis on inflation, it seemed like a good approximation.

icant or had the wrong sign. After estimating the model described above, we performed a Likelihood ratio test, imposing zero-restrictions on those (insignificant or wrong-signed) coefficients. This include the following restrictions in lines 1 and 2, and 3 and 4, respectively:

$$a_{y0} = a_{y2} = a_{y3} = a_{y4} = 0 \tag{3.155}$$

$$b_{y1} = b_{y2} = b_{y3} = b_{y4} = 0; b_{i0} = b_{i2} = b_{i3} = b_{i4} = 0 \tag{3.156}$$

The log-likelihood value for the more general model took the value of 1279.06($\mathcal{L}^u$), whereas the model with the above restriction imposed on it obtained a likelihood value of 1267.84($\mathcal{L}^r$). The likelihood ratio (LR) test calculated as $2(\mathcal{L}^u - \mathcal{L}^r)$ is to be compared with a $\chi^2_{(12)}$ which takes the value 21.026 at 5% and 24.725 at 1%. Our LR test took the value of 22.44, which has an associated p-value of 0.033. We opted to work with this model and therefore this is the model whose results are reported on the first column of Table 3.6 and we call "baseline model". The VAR five lines consist of:

$$\pi_t = a_{\pi 0}\pi_{t-1} + a_{\pi 1}\pi_{t-2} + a_{\pi 2}\pi_{t-3} + a_{\pi 3}\pi_{t-4} + a_{\pi 4}\pi_{t-5} + a_{y1}(y_{t-2} - \overline{y}_{t-2}) + \varepsilon_{1t} \tag{3.157}$$

$${}_t\pi^e_{t+1} = a_{\pi 0}\pi_t + a_{\pi 1}\pi_{t-1} + a_{\pi 2}\pi_{t-2} + a_{\pi 3}\pi_{t-3} + a_{\pi 4}\pi_{t-4} + a_{y1}(y_{t-1} - \overline{y}_{t-1}) + \varepsilon_{2t} \tag{3.158}$$

$$(y_t - \overline{y}_t) = b_{y0}(y_{t-1} - \overline{y}_{t-1}) - b_{i1}(i_{t-2} -_{t-1} \pi^e_{t-1}) + \varepsilon_{3t} \tag{3.159}$$

$$({}_t y^e_{t+1} -_t \overline{y}^e_{t+1}) = b_{y0}(y_t - \overline{y}_t) - b_{i1}(i_{t-1} -_{t-1} \pi^e_t) + \varepsilon_{4t} \tag{3.160}$$

$$i_t = d_i i_{t-1} + (1 - d_i)(d_\pi \pi^e_{t+1} + d_y(y^e_{t+1} - \overline{y}^e_{t+1}) + \varepsilon_{5t} \tag{3.161}$$

[Table 3.6]

The Phillips curve

$$\pi_t = \underset{(3.050)}{0.125}\pi_{t-1} + \underset{(3.138)}{0.124}\pi_{t-2} + \underset{(3.951)}{0.149}\pi_{t-3} + \underset{(3.527)}{0.130}\pi_{t-4} + \underset{(4.114)}{0.150}\pi_{t-5} + \underset{(2.702)}{0.145}(y_{t-2} - \overline{y}_{t-2}) + \varepsilon_{1t} \tag{3.162}$$

[119]

Starting with the Phillips curve, the results, as explained before are based on a backwards looking version[120] and the estimated structural parameters are represented in the first six rows of Table 3.6. The results seem consistent with both theoretical predictions as well as empirical findings by different authors. Furthermore, all coefficients on lagged inflation are positive and so is the coefficient on the output gap. Inflation in period t will be higher, the higher inflation has been over the past five quarters, denoting inertia in inflation adjustment, a feature that has been well documented in the literature[121]. The output gap can be interpreted as capturing the excess demand which leads to inflationary pressure. Moreover, the output gap enters the Phillips curve with a two-period lag. This intends to capture lags in the response of inflation to output gap changes, in line with the work of Bernanke

119 The figures in brakets are t-ratios. The system is represented in Table 3.6.

120 A Phillips curve with a forward-looking component was also estimated and results are presented as one of the robustness tests.

121 See, for example, Fuhrer and Moore (1995).

and Mihov (1998) and Christiano, Eichenbaum and Evans (1996), for example. When output is above potential, this will overheat the economy. This excess demand will lead to inflationary pressures that, in our model, will come through with a gap of two-periods.

An important point that we would like to stress is the fact that our formulation of the Phillips is of a backwards-looking type. Inflation dynamics are well captured by lagged terms of inflation. As discussed in section 3.2.2 this is a well known empirical observation, although there is dispute over its meaning. Rudd and Whelan (2005) stressed exactly that in the context of the empirical relevance of hybrid Phillips curve such as the one derived by Gali and Gertler (1999) and Gali, Gertler and López-Salido (2005), and the relevance of future inflation in determining current inflation. As referred in their paper, the lagged terms either represent backward-looking expectational processes, such as adaptive expectations or they may be a proxy for rational forward-looking expectations. Our results lead to a rejection of the first hypothesis. By making use of direct measures of expectations, we were able carry out a Rationality test on inflation expectations. Being unable to reject this hypothesis, i.e., having rationality on inflation expectations allows us to reject the option that lagged inflation represent some form of adaptive expectation formation process; instead, we should interpret those terms as denoting the presence of inflation inertia or persistence in the economy and therefore consistent with rational agents.

The IS curve

$$(y_t - \overline{y}_t) = \underset{(14.592)}{0.789}(y_{t-1} - \overline{y}_{t-1}) - \underset{(2.738)}{0.114}(i_{t-2} -_{t-1} \pi^e_{t-1}) + \varepsilon_{3t} \tag{3.163}$$

Let us now consider the IS curve formulation. The b parameters in Table 3.6, first column, are the relevant ones for this analysis. We obtained a very significant coefficient on the lagged output gap and a positive relation between the actual and past output gap which reflect the relevance of costs of adjustment and habit formation explained in the previous section. The coefficient on the real interest rates has the predicted sign and is significant, with a t-ratio of 2.738. An increase in the real interest rate will decreased output gap through its negative effect upon aggregate demand. The gap between change in interest rate and observed change in output reflects the empirical finding that demand does not respond instantaneously to changes in interest rates. According to our model, and in line with other empirical findings, there are two-periods gap in the monetary transmission mechanism. If we also take into account the Phillips curve formulation, the interest rates affect output after half a year and inflation after a year, i.e., it takes double the time for the effects of the interest rates changes to come through inflation.

The Taylor rule

Finally, we have the Taylor rule formulation[122].

$$i_t^* = \underset{(4.796)}{1.412}\pi_{t+1}^e + \underset{(3.584)}{1.224}(y_{t+1}^e - \overline{y}_{t+1}^e) \tag{3.164}$$

$$i_t = \underset{(16.534)}{0.786}\, i_{t-1} + (1 - 0.786)i_t^* + \varepsilon_{5t} \tag{3.165}$$

Or alternatively, the estimated equation that combines the two above:

$$i_t = (1 - 0.786)[1.412\pi_{t+1}^e + 1.224(y_{t+1}^e - \overline{y}_{t+1}^e)] + 0.786 i_{t-1} + \varepsilon_{5t}. \tag{3.166}$$

[122] See the d parameters in Table 3.6.

We used the forward-looking version of the rule, extended to allow for interest rate smoothing and to incorporate some dynamics. These coefficients are all significant and of magnitudes similar to several empirical work. In particular, these coefficients are both positive and the coefficient on inflation is larger than one. This is in line with the coefficients obtained by Taylor in 1993. In there, Taylor found a coefficient of 1.5 on inflation and 0.5 on output gap. This seems to suggest the conduct of monetary policy through this period has been in accordance to the Taylor's principle. The coefficient on the expected inflation for next period turned out to be significant and larger than one. This denotes the expected stabilising role of Monetary Policy, in that not only nominal interest rates are raised when there are inflationary expectations, but they are raised enough to increase the real interest. The real interest rate, as seen from the IS, is the relevant variable determining aggregate demand. An increase in the real interest rate will decrease demand and therefore output (in our model with a couple of periods gap; see the IS specification). Depressing the economy in this manner will alleviate or eliminate the pressure on inflation. The stabilising role of monetary policy coming through the coefficient on inflation was also present in many empirical studies. See for example, Clarida, Gali and Gertler (2000) for a sub-period of our sample. The future output gap turned out to have a significant positive impact on the interest rate. This again goes in line with the findings of most empirical work using a Taylor type of rule. Once again, the monetary authority will raise nominal interest rates when output gap is positive, i.e., when output is above the potential output. The coefficient on the output gap is larger in magnitude than Taylor's initial finding and more in line with Ball (1999)'s recommendation in terms of efficiency of rules; he also finds that it reflects better the behaviour of the Fed[123]. It is also in line with findings by Orphanides (2001), where he also uses real-time data, where the coefficient on the output gap varied between 0.51 and 1.16, depending on vintage and proxy used for trend output. The positive sign on this coefficient once again denotes stabilising effects in that the increase in interest rate (which will be raised enough to increase real interest rate) will decrease aggregate demand and bring output down, closer to potential.

Some robustness tests

Different output gap measures

To assess the robustness of the results, and given the well documented uncertainty surrounding the output gap measure[124], we estimated the system using alternative detrending procedures. Following Taylor (1993), we used a linear trend of output (Model T1) and following Clarida, Gertler and Gali (2000), we used a linear and quadratic trend (Model T2) as proxies of potential output. The output gap is measured by the residuals from a regression of output, y_t, on a constant and a linear trend under model T1; under T2 the regression also includes a quadratic trend.

The results for models T1 and T2 are reported on Table (3.6), columns 3-4. The results tend to be similar to each other, with a few exceptions that we will be analysing shortly.

[Table 3.6]

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[123] He finds that the actual interest rate movements did not follow the interest rate movements implied by a Taylor rule with a 0.5 coefficient on the output gap. The biggest deviations happened during the boom of the 1980s and the recession of the 1990s, where the Fed had a more aggressive response than implied by the 0.5-coefficient rule.

[124] See Orphanides (1999).

In the Phillips curve formulation, the parameters on lagged inflation are significant and positive across all different measures of output gap. The differences come through the coefficient on output gap. Although positive, under models T1 and T2, the output gap is statistically insignificant at the conventional 5% confidence level. The associated p-value under models T1 and T2 are 28.5% and 15.3% respectively. As a result, these measures do not capture as well the behavioural relationships behind the Phillips curve.

With regard to the IS curve, the same IS specification was used on the different output gap measures described above. The coefficient on the real interest rate has the predicted sign in all estimations and it is statistically significant in all specifications, although the p-value associated with this coefficient under the quadratic trend is 9.2%, i.e., it is only significant at 10% level. The coefficient on one period lagged output gap is very significant in all regressions.

It is a desirable feature of a monetary rule, that it is unaltered under different model specifications, specially given the uncertainty surrounding the "right" (or true) model for the economy. The robustness of the Taylor rule was therefore tested, by checking whether the relationship between variables changes when having different output gaps estimates underlying it. For all formulations, it performed very similarly to the baseline model. Namely, the coefficients on inflation were all significant and of similar magnitude to the ones above. The coefficients on the output gap, although positive and significant (and therefore denoting a stabilising role from that end), have a smaller magnitude, of less than half the original one, being more in line with Taylor's original formulation of the rule. Note that Taylor used a linear trend to obtain the output gap measures. Our results seem to suggest that the results are sensitive to the choice of detrending procedure. When giving a quantitative significance to the parameters , the choice of the proxy for the output gap is very important. However, if the main objective is to characterise the conduct of policy, namely whether it has been stabilising or not, the conclusion are invariant to the choice of output gap measures.

Alternative Taylor rule formulations

With reference to the Taylor rule, we started by analysing the relevance of the interest rate smoothing term. We tested for the relevance of this term, by estimating the system again , imposing the restriction $d_i = 0$. The 5th line was then estimated as follows:

$$i_t = d_\pi \pi^e_{t+1} + d_y(y^e_{t+1} - \overline{y}^e_{t+1}) + \varepsilon_{5t} \tag{3.167}$$

We performed a likelihood ratio test,which took the value of 20.5. This value is higher than $\chi^2_{(1)} = 3.841$, and as result we reject the null hypothesis $d_i = 0$,which reinforces the importance of the interest smoothing term.

As another robustness test, we estimated alternative target horizons for the Taylor. With reference to the formulation:

$$i_t = (1 - d_i)[d_{\pi t+k-1}\pi^e_{t+k} + d_y({}_{t+k-1}y^e_{t+k} -{}_{t+k-1}\overline{y}^e_{t+k})] + d_i i_{t-1} + \varepsilon_{5t} \tag{3.168}$$

where k reflects the ”target” horizon relative to the quarter at which the interest rates are decided by the Fed. Our formulation so far had a forward-looking nature, with $k = +1$. In other words, we have assumed that monetary policy looks into the future when deciding upon the setting of the interest rate. We have also estimated a Taylor rule within our model with $k = 0$, which may reflect better the information available to the Central Bank when setting the interest rate at time t. The estimates under both, k=+1 and k=0 are as follows:

$$i_t = (1 - 0.786)[\underset{(4.796)}{1.412}\pi^e_{t+1} + \underset{(3.584)}{1.224}(y^e_{t+1} - \overline{y}^e_{t+1})] + \underset{(16.534)}{0.786}\, i_{t-1} \tag{3.169}$$

$$i_t = (1 - 0.771)[\underset{(4.094)}{1.171}\pi^e_t + \underset{(3.101)}{0.860}(y^e_t - \overline{y}^e_t)] + \underset{(13.642)}{0.775}\, i_{t-1} \tag{3.170}$$

The common features of these formulations include the larger-than-one coefficient on inflation and positive coefficient on the output gap. However, the influence on the output gap decreases as the horizon becomes less forward and more backwards. In terms of inflation, the magnitude of the coefficient increased as the horizon becomes more forward and less backwards. The fit of the model is also improved with a forward-looking version of the Taylor rule. These last two patterns/findings are in accordance with Orphanides (2001). However, we can say that the results are not very dissimilar, and the way monetary policy would be characterise through it in a similar way, still having a stabilising role throughout this period.

A Hybrid formulation of the economy

The results reported so far refer to what we called the ”baseline model” for the US economy, which has a formulation with an important backwards-looking component, reflecting different sources of inertia in the economy and giving a good fit to the data, as the empirical results show. Given that the VAR order selection criteria did not give unanimous suggestion, we chose to go with the Schwarz Bayesian Criterion that suggested a VAR of order one, avoiding this way the danger of over-parameterisation and exploiting the two order suggestions from the different criteria[125]. The unrestricted order 1 VAR is represented on Table 3.7 and as we can see there are no problems with serial correlation.

We impose the following hybrid structure to the economy:

$$\pi_t = a_0 \pi^e_{t+1} + (1 - a_0)\pi_{t-1} + a_{y0}(y_t - \overline{y}_t) + \varepsilon_{2t} \tag{3.171}$$

is a hybrid Phillips curve;

$$(y_t - \overline{y}_t) = b_{y0}(y^e_{t+1} - \overline{y}^e_{t+1}) + (1 - b_{y0})(y_{t-1} - \overline{y}_{t-1}) - b_{i0}(i_t - \pi^e_{t+1}) + \varepsilon_{4t} \tag{3.172}$$

represents a hybrid IS curve;

$$i_t = d_i + (1 - d_i)(d_\pi \pi^e_{t+1} + d_y(y^e_{t+1} - \overline{y}^e_{t+1})) + \varepsilon_{5t} \tag{3.173}$$

represents a forward-looking Taylor rule.

The behavioural relationships above were imposed in lines 2, 4 and 5. Lines 1 and 3 are represented by

$$y_t - \overline{y}_t = f_{y0}(y_{t-1} - \overline{y}_{t-1}) + f_{y^e 0}(y^e_t - \overline{y}^e_t) + f_{\pi 0}\pi_{t-1} + f_{\pi^e 0}\pi^e_t + f_{i0} i_{t-1} + \varepsilon_{3t}$$

$$\pi_t = g_{y0}(y_{t-1} - \overline{y}_{t-1}) + g_{y^e 0}(y^e_t - \overline{y}^e_t) + g_{\pi 0}\pi_{t-1} + g_{\pi^e 0}\pi^e_t + g_{i0} i_{t-1} + \varepsilon_{1t} \tag{3.174}$$

and will be used to test rationality of expectations on output and inflation.

Multivariate Rationality test

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[125] We have also estimated our previous model extended to include a forward-looking component, similarly to the model described below. The results are quantitatively similar. In particular, adding expected future inflation into the backward-looking formulation did not change the significance of the lagged terms, which continued to be statistically significance; the coefficient on future expected inflation was above one.

This framework allows a multivariate rationality test. After estimating the lines above, we imposed the restrictions consistent with REH. In particular, we impose zero restrictions on all the parameters of the lag operators except for the following unitary restrictions:

$$f_{y^e 0} = 1 \tag{3.175}$$

$$g_{\pi^e 0} = 1 \tag{3.176}$$

To see that these restrictions imply Rational Expectations, we note that imposing these restrictions lead to the following equations:

$$\pi_t = \pi_t^e + \varepsilon_{1t} \tag{3.177}$$

$$y_t - \overline{y}_t = y_t^e - \overline{y}_t^e + \varepsilon_{3t} \tag{3.178}$$

whereby actual output gap and actual inflation equal the measured expected output gap and expected inflation respectively except for a normally distributed error term, ε_{it} (with $i = 1, 3)$, with zero mean and standard deviations $\sigma_{\varepsilon i}$. In other words, if agents are rational, they will take into account all the information available to them when they form their expectations, leading to expectational errors (in this case $\varepsilon_{1t} = \pi_t - \pi_t^e$ and $\varepsilon_{3t} = (y_t - \overline{y}_t) - (y_t^e - \overline{y}_t^e))$ that are orthogonal to the information set. The information set is assumed to consist of actual output gap $(y - \overline{y})$, expected output gap$(y^e - \overline{y}^e)$, inflation π, expected inflation π^e,and interest rate i,dated at time $t - 1$ and earlier, as suggested by equations 3.174. Given that we are working with a VAR of order one, we restricted our information set in that manner. The rationality test consists of a Likelihood ratio test. The test takes the form $LR = 2(\mathcal{L}^u - \mathcal{L}^r) \sim \chi^2_{(j)}$, with $\mathcal{L}^u$ and $\mathcal{L}^r$being the maximum likelihood values of estimation under unrestricted and restricted model respectively, which follows a χ^2 distribution with j degrees of freedom, where j is the number of restrictions imposed in the (unrestricted) model. The number of restrictions amounts to ten, consisting of the zero-restrictions on the coefficients of the information set and the two unitary coefficients on the current output gap and current inflation expectations as explained previously. The LR took the value of 18.642, which has a p-value of 0.045. The χ^2 with 10 degrees of freedom takes the value of 18.307, and we found it close enough as an hypothesis to keep, given the possibility of small sample bias. As a result, we end up with the two lines representing REH on inflation and output as lines 1 and 3, as in (3.177) and(3.178) respectively[126].

Maximum Likelihood Estimation results

[Table 3.8]

Regarding the Phillips curve, the coefficient on future expected inflation is significant, but larger than one. This is at odds with the theoretical models that provide micro-foundations for this hybrid formulation. The coefficient on the output gap is negative and statistically insignificant. The wrong sign on the output gap is a well documented feature in empirical work with a forward-looking version of a Phillips curve when using quarterly data[127].These two estimates makes it hard to reconcile our findings with the New Keynesian Phillips curve, being more in line with a backwards-looking, traditional Phillips curve.

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[126] Note that we are allowing expectations to be biased. Since we estimate the model with interccepts, the fact that expectations may not be unbaised, does not have implications for their treatment in the model.

[127] See Gali and Gertler (2005).

This was the reason behind the choice of the backwards-looking representation of the economy. With regard to the IS curve, although the forward looking component in the form of future output gap is very significant and very close to one (and therefore leaving a much smaller weight for past output gap), the coefficient on the interest rate, despite having the right sign is statistically insignificant. Note the parallel with the Phillips curve and coefficient on the output gap. The Taylor rule, though, continues showing robustness and the results are invariant to changes in specification of the behavioural relationships representing the economy. In particular, very significant coefficients were obtained and with similar magnitudes to the ones estimated under the baseline model. However, the fact that the output gap in the Phillips curve and the real interest rate in the IS curve are statistically insignificant put down our structural model given that the two behavioural relationships are based upon those two elements. As pointed out in Smith and Wickens (2006), the New Keynesian model works through interest rate affecting output, which affects inflation. For monetary policy to be effective, both links must be strong and this is clearly not the case in the hybrid formulation of the economy.

Another issue, we would like to stress is the fact that the structural model is rejected when compared with an unrestricted VAR of order one[128]. This is also true when we impose a more general structure that translates the three behavioural relationships, as follows:

$$\pi_t = a_{\pi 0}\pi^e_{t+1} + a_{\pi 1}\pi_{t-1} + a_{y0}(y_t - \overline{y}_t) + a_{y1}(y_{t-1} - \overline{y}_{t-1}) + \varepsilon_{2t} \qquad (3.179)$$

[128] The associated LR test took the value of 136.1.

$$(y_t - \overline{y}_t) = b_{y0}(y^e_{t+1} - \overline{y}^e_{t+1}) + b_{y1}(y_{t-1} - \overline{y}_{t-1}) - b_{i0}(i_t - \pi^e_{t+1}) - b_{i1}(i_{t-1} - \pi^e_t) + \varepsilon_{4t} \quad (3.180)$$

$$i_t = d_i + (1 - d_i)(d_\pi \pi^e_{t+1} + d_y(y^e_{t+1} - \overline{y}^e_{t+1}) + \varepsilon_{5t} \quad (3.181)$$

adding these two unrestricted lines for actual inflation and output depending upon past values:

$$y_t - \overline{y}_t = f_{y0}(y_{t-1} - \overline{y}_{t-1}) + f_{y^e0}(y^e_t - \overline{y}^e_t) + f_{\pi 0}\pi_{t-1} + f_{\pi^e 0 t-1}\pi^e_t + f_{i0} i_{t-1} + \varepsilon_{3t} \quad (3.182)$$

$$\pi_t = g_{y0}(y_{t-1} - \overline{y}_{t-1}) + g_{y^e0}(y^e_t - \overline{y}^e_t) + g_{\pi 0}\pi_{t-1} + g_{\pi^e 0}\pi^e_t + g_{i0} i_{t-1} + \varepsilon_{1t} \quad (3.183)$$

This structural representation is still rejected when compared with an unrestricted VAR of order 1. An unrestricted VAR would have a Log-likelihood function value of 1430.1($\mathcal{L}^u$) whereas this more general model assumed the value of 1402.1($\mathcal{L}^r$). A likelihood ratio (LR) test would be preformed as LR=$2(\mathcal{L}^u - \mathcal{L}^r)$ which took the value of 56. This is to be compare with a $\chi^2_{(14)}$[129] which assumes the values of 23.68 and 29.14 at 5% and 1% respectively. This represents a strong rejection of the restrictions imposed by the structural

[129] The 14 restrictions include the zero restrictions on the covariance matrix as well as the zero restrictions implicit in this representation.

VAR. This is also the case under the baseline model described in the previous section[130]. One reason for that may be the presence of structural breaks.

3.5 Conclusion

In this chapter we proposed to analyse macroeconomic stability and evaluate Monetary Policy in the US economy, within the context of an estimated small macro model. The model considered a price equation, represented by a Phillips curve; we also added a standard IS curve; to analyse the conduct of monetary policy, we used a Taylor-type of rule. Since Taylor (1993) several authors have considered this type of rule to explain the movements of the interest rate in several countries and have shown that such simple formulation where interest rates respond to output gap and inflation fits the data well. Authors such as Clarida et al (2000) and Ball (1999) have also consider a Taylor rule in the context of a small macro model. This paper departs from those empirical works as far as the estimation procedure, the treatment of expectations and the use of real-time data are concerned and on these three aspects lie the main contributions of this work. Most of the empirical work is based on calibrated models where typically the authors assume rational expectations. We used direct measures of expectations and analyse the dynamics between those and the actual levels in the context of a five variable VAR of actual and expected inflation, output and interest rate variables. We analysed the expectational series, testing for the Rationality of those expec-

[130] In the case of the backwards-looking model, the result was more expected due to the high order of VAR (order 5) and subsequent possible over-parametrization of the unrestricted version of the model. In this case, the more general model described previously lead to a log-likelihood of 1292.4, with the unrestricted VAR of order 5 leading to a value of 1573.2. The associated likelihhod ratio test would take the value of 561.558, which is to be compared with a $\chi^2_{(108)}$, which takes a value of 133.26 at 5%. This represents an overwhelming rejection of the model, similarly to rejection of the ARMA representation in Ireland (1999).

tations, rather than simply assuming it. Further, the analysis of this chapter is based on real-time data. This, together with the multivariate treatment of expectations, is another feature that differentiates this paper from the literature mentioned above. Authors such as Rudebusch (2002) and Orphanides (2003) have used direct measures of expectations in behavioural relationships such as Phillips curve and Taylor rule. However the novelty of this chapter is the inclusion of those measures in a small macro model, and the multivariate treatment of the system. By using the current perception for each quarter, we reflect better the information available to agents at the time (i) expectations were formed and (ii) policy decisions were made. The direct measures of expectations together with the real-time data for output, inflation and interest rates, as captured by the perceptions made by economic agents on each variable during the current period, were then incorporated into our small macro model. The inclusion of those measures of expectations enable us to investigate more explicitly the role played by the expectations in the setting of the interest rate by the Monetary authorities and helps in the fitting of the model to the data. We showed that our results are in line with the recent econometric policy evaluation where a small macro model together with Taylor type of rules has been considered. In particular, we found no evidence to favour purely forward-looking models or even hybrid formulations, showing that backward-looking models, which intend to capture rigidities and inertia present in the economy, fitted well the data. The estimation results of the hybrid formulation were hard to interpret and reconcile with the underlying microfoundations. The monetary policy authorities, during the period under consideration, had a stabilising role, with the interest rates responding positively to deviations of output from potential and raising when inflationary pressure are felt, and by enough to increase the real interest rate and therefore having the desired dampening effect on aggregate demand, to alleviate such pressures. The robustness of the Taylor rule was confirmed, surviving different measures of output gap and different model formulations, showing once again the usefulness of simple rules when analysing the conduct of monetary policy. The use of direct of expectations also allowed us to contribute to the recent debate on Phillips curve specifications and the meaning of the relevance of lagged inflation terms in explaining inflation dynamics. The fact that we tested the rationality of expectations on inflation and we were unable to reject the hypothesised rational expectations, we ruled out a possible interpretation related to adaptive expectations from the part of agents. Instead, we were able to confirm that the lagged terms in our formulation and description of the inflation dynamics, represent inflation inertia and are therefore consistent with rationality. This provides further support for micro-founded models that use rational expectations when trying to incorporate lagged terms in New Keynesian Phillips type of curve, as opposed to models that rely on adaptive expectation and rules of thumb to obtain the persistence observed in the data. For example, our model finds further support for theoretical models to follow the work of Furhrer and Moore (1995), which derives the observed persistence in the data through considering Taylor's model of overlapping contracts but assuming that agents care about relative real wages, while keeping the rationality of expectations. This is in contrast with, for example, Gali and Gertler (1999), who base their hybrid formulation of the Phillips curve on Calvo' model of staggered prices, with some agents following a rule of thumb when determining their pricing or even Roberts (1995, 1997) where assumes that some agents form their expectations adaptively.

Conclusions

In this work, we aimed to investigate output and inflation dynamics within a multivariate setting and making use of direct measure of expectations.

The empirical work focused on the UK and US economies, using a qualitative source of expectations in the former, and a quantitative source in the latter. The issues raised with each type of survey were discussed. For the US economy, we also made use of real-time data, recognising the impact of data revisions for policy-making analysis. We explored different issues along the way, starting by analysing the expectations formation processes and then incorporating the direct measures of expectations in macro models, allowing an analysis of the output-inflation tradeoff and the role played by expectations in the conduct of monetary policy.

In chapter 1 we concentrated our analysis on the dynamics between actual and expected inflation and actual and expected output in the UK and US. For the UK, we used data for the Manufacturing Sector, getting measures of expectations from the Confederation of British Industry (CBI)'s Survey. This is a qualitative source of expectations, requiring a procedure to convert the responses into a time-series. We used Lee (1994)'s procedure. We devoted our attention to the properties of the expectations *vs* actual data. We tested the expectations formation process for both inflation and output and were unable to reject rationality, finding that agents made an efficient use of the available information when forming their expectations. For the US, we explored issues of data revision, which were not accounted for before. Furthermore, we made use of a different type of survey - quantitative in the form of the Survey of Professional Forecasters. We aimed to understand better the relationship between real-time data and expectations at different time horizons, namely by analysing their short-run dynamics in the form of impulse responses and the long-run properties. Regarding the latter, we were particularly interested in testing for long-run unbiasedness. The tests were carried out in a multivariate setting, this time in the form of a cointegration approach by Johansen (1988) and Pesaran, Shin and Smith (2000). We were unable to reject the hypothesis of stationarity of expectational errors on both inflation and output. Furthermore, expectational errors were also found to be unbiased. The I(1) properties of output levels and inflation rates were confirmed by the impulse responses, as we observed shocks to having a permanent effect on all series.

In chapter 2 we developed a modelling framework to allow the analysis of actual and expected inflation and output. We used the data analysed in the previous chapter for the UK Manufacturing Sector and made use of its properties and results obtained previously, namely the rationality of expectations. The framework included a supply side of the economy, and an inflation equation, assumed to be driven by a time-consistent model of monetary policy, in line with Barro and Gordon (1983) and Ireland (1999). We tried to motivate our model theoretically, by developing a small growth model. By making use of direct measures of expectations and the Rational Expectations Hypothesis, we were able to define our model in a four-variable structural VAR of observables. Due to the uncertainty surrounding the ”true” model of the economy, we tested a few hypothesis on the representation of the supply side of the economy. We found that the economy could be well represented within a natural rate framework, showing long-run neutrality. The hypothesis that only current inflation surprises (in line with the Lucas's supply curve), were able to deviate output from the natural level was rejected. Instead, we found the supply side to have great persistence in inflation, with past actual and past expected inflation playing an important role, although the persistence was not well captured by past surprise inflation. Inflation was not successfully represented by a time-consistent model of monetary policy, as this hypothesis was rejected by the data. The rejection of the time-consistency is also a feature in Ireland (1999)'s work.

Chapter 3 addressed the important issues of data revisions and made use of real time data to analyse inflation and output dynamics and the conduct of monetary policy, assumed to be well represented by a Taylor-type of rule, with the interest rate taken as the policy instrument. We considered three main behavioural relationships, namely a Phillips curve, an IS curve and a monetary policy rule. We motivated those relationships with reference to economic theory and we provided a multivariate modelling framework of actual and expected inflation, output and interest rate. This framework allowed a more elaborated discussion regarding policymaking. We also addressed issues such as uncertainty of output gaps estimation and uncertainty regarding the best way to represent the economy. The use of direct measures of expectations enable us to test different hypothesis regarding the Phillips curve and to analyse the role of expectations in the conduct of monetary policy. We found inflation dynamics to be well captured by backwards-looking formulations of the Phillips curves. The presence of lagged inflation terms denoted, not some form of adaptive expectations, as suggested by some, but rather they denoted the presence of inertia, consistent with rational agents. This conclusion is reached by the non-rejection of a rationality test performed on the expected inflation series. Furthermore, we found that the conduct of monetary policy is well represented by a Taylor type of monetary rule, which showed robustness across different models and measures of the output gap, and confirming successful stabilization policies conducted by the Fed since the 1980s.

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