Imperial College London

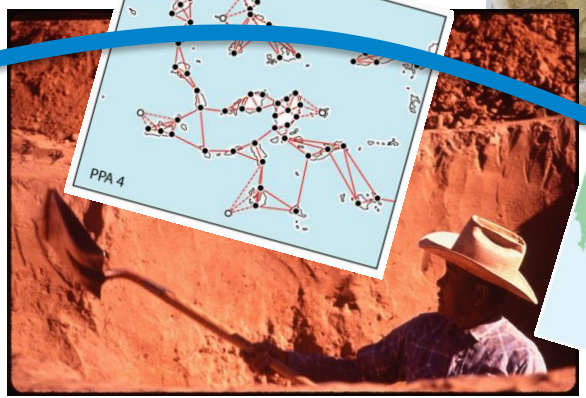
Complexity Science

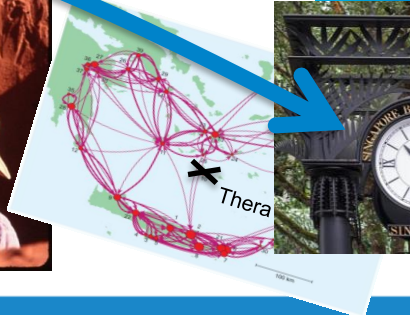
Extracting and analysing networks from spatio-temporal data,
Satellite of CCS Singapore
2/10/2019

Slides: http://bit.ly/2mMc8Kx

Spatio-Temporal Modelling for Archaeology

Tim Evans





- Introduction
  - Archaeological Data
  - Motivation
  - Modelling Choices
- Zones of Control Models
- Proximal Point Analysis
- Comparisons Using Models
- Missing Sites Examples
- Uncertainty in Modelling

Condensed Matter Physics

Particle Physics

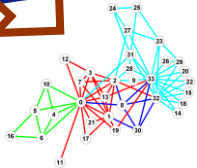
Statistical Physics

Complexity

Science

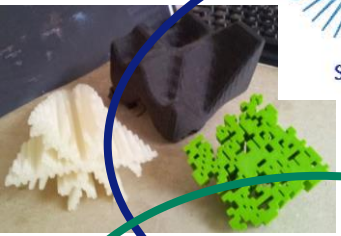
Archaeology

Spatial Networks

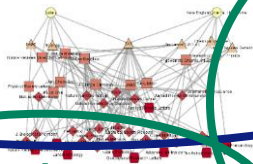


Constraint

Complexity & Networks

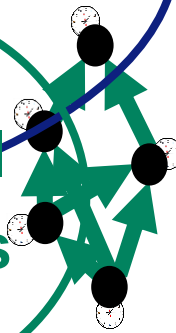


Evolution of Complex Systems



Innovation, Bibliometrics

Temporal Networks



SoCA Lab,
Data Science Institute

Archaeological Data

- Excavations necessarily partial
  - Unknown sites
  - Sites destroyed
  - Sites unreachable e.g. under modern site
- Large amounts of material
  - Documented artefacts only a tiny fraction
  - Representative items not all items
  - No universal parameters or standards
- Biased sample
  - Materials biodegrade
  - Social biases
- Paper Records
  - tiny fraction published

Lots of data! NOT Big Data

Small/Dirty Data!

Archaeological Data on Exchange

Exchange is hard to measure

- Most artefacts locally produced,
  - exchange a small fraction of that small fraction
  - ~10% of artefacts imported at Akrotiri
- Exchange difficult to identify by source
  - Analysis Expensive
  - Not always conclusive
- Direct Evidence Rare
  - Texts have biases

Uluburun Shipwreck 1300BC

Site-Site Interactions

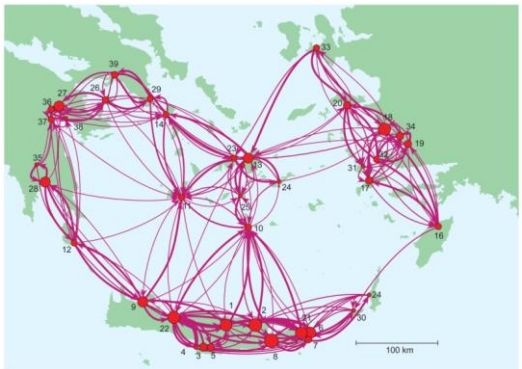
- Archaeology can be "SITE CENTRIC"
  - Regional and global interactions hard to consider

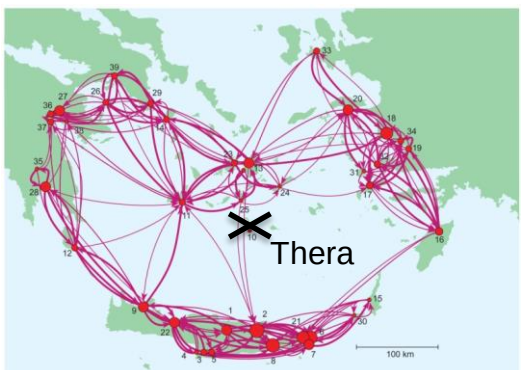
- Models can emphasise interactions
  - Mesoscopic picture
  - Macroscopic effects

What did archaeological models ever do for us?

- Importance of Sites
  - Network centrality
  - Location of missing sites
- Regions of Influence
  - Political
  - Economic
- Comparisons
  - Null models
  - Evolution in time
  - Different effect of social, geographical, economic factors

$\Rightarrow$ Proof or Narrative ?





[Knappett, TSE, Rivers, Antiquity 2012]

What is a Site?

Issues of scale:-

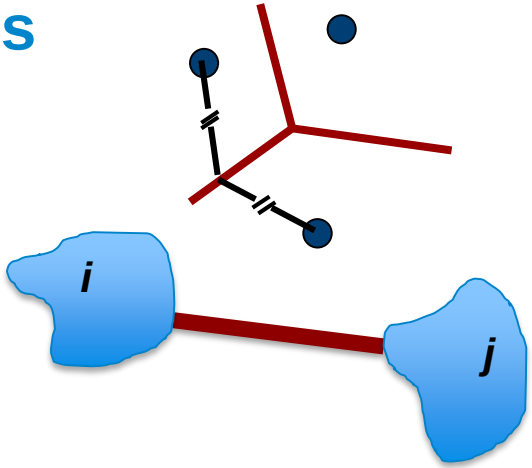
- Any Settlement
- Large Settlement
- Important Building e.g. shrine, fortification
- Region e.g. Valley defined by geography – GIS methods

The scale of a settlement is not always obvious

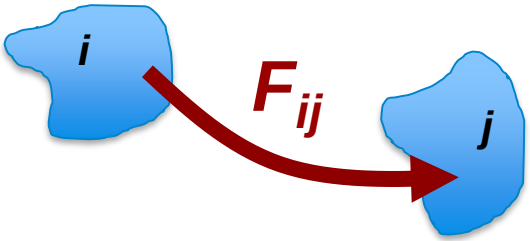
- Missing sites
- Sites now underwater
- Size of sites beneath modern cities

Interactions between sites

- Boundaries between sites
- Model binary interactions
  ⇒ SPATIAL NETWORKS
- Model flows
  ⇒ SPATIAL INTERACTION MODELS



Could be exchange of:-
- Bulk Goods
- Elite goods
- Social Exchange
- Political Exchange



Different Spaces

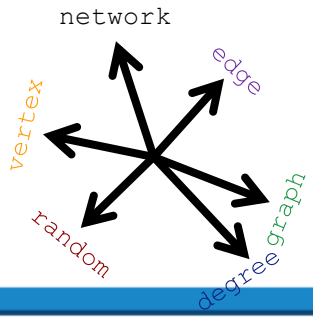
Looking at relationships between "nodes" embedded in space

Focus here on two-dimensional geographical space

- Euclidean distance
- Cost of trip
- Ranked distance, intervening opportunities

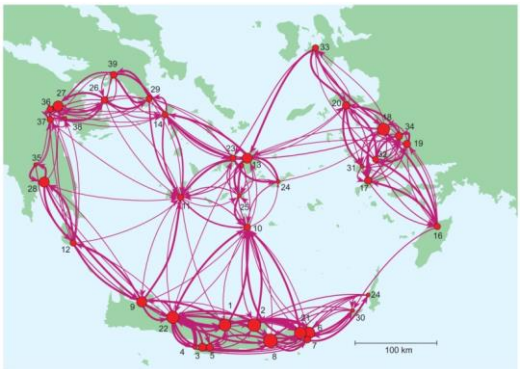
Can apply to artefact spaces

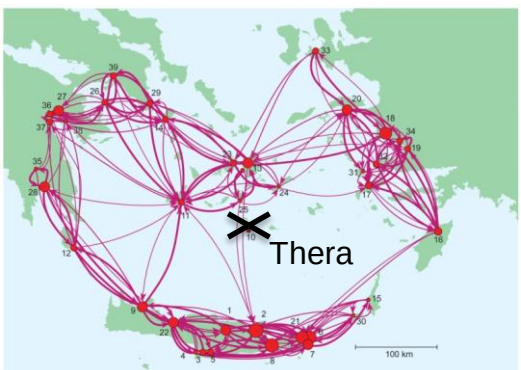
- Artefact count
- Word frequency



What did Simple Spatial Interaction Models ever do for us?

- Test Basic Principles
  - e.g. is city attractiveness a non-linear function of population? [Bettencourt at al. 2007]
- Look at general properties
  How does space effect system
- Comparisons, Null Models
  - e.g. Spatial Clustering [Expert at al. 2011]
- General Predictions
  - e.g. destruction of Thera only weakens Aegean networks





[Knappett, TSE, Rivers, Antiquity 2012]

Proof by Example

Will illustrate some of these ideas by highlighting a few examples.

- Personal Favourites
- Several other examples of similar types of analysis
- Many other types of analysis not covered here

For more examples see

- [Brughmans 2010]
- “The Connected Past”
- Proceedings of CAA

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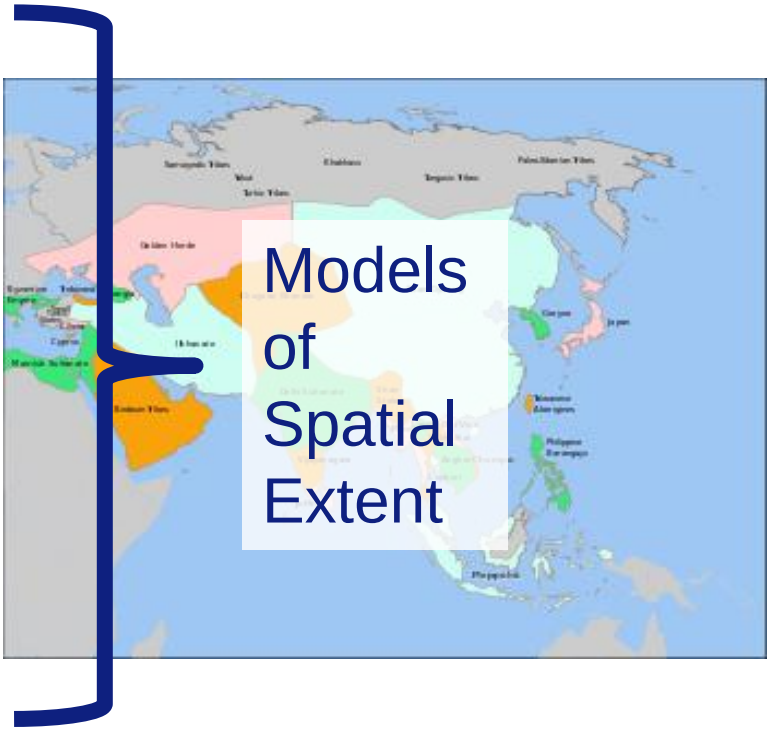
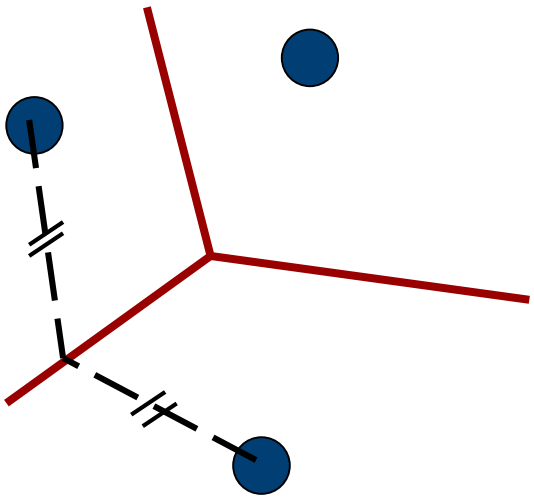
Zone of Control models

- Thiessen Polygons (Voronoi Diagrams)
  - equal site sizes
- XTent model
  [Renfrew and Level 1979]
  - Thiessen with variable site sizes
- Retail Gravity model
  [Rihll & Wilson 1987,1991]



Theissen Polygons (Voronoi Tesselation)

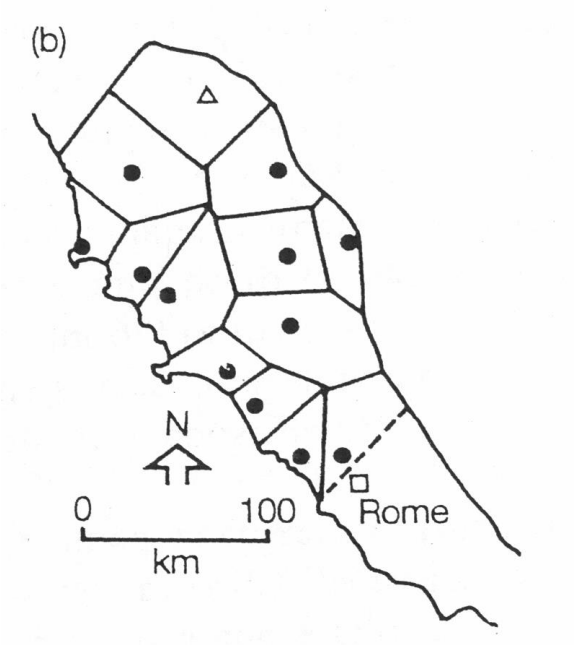
- Boundaries = Midpoint between nearest sites
- All sites equal



Theissen Polygon Example

- 12 Etrurian Cities

[Renfrew 1975]

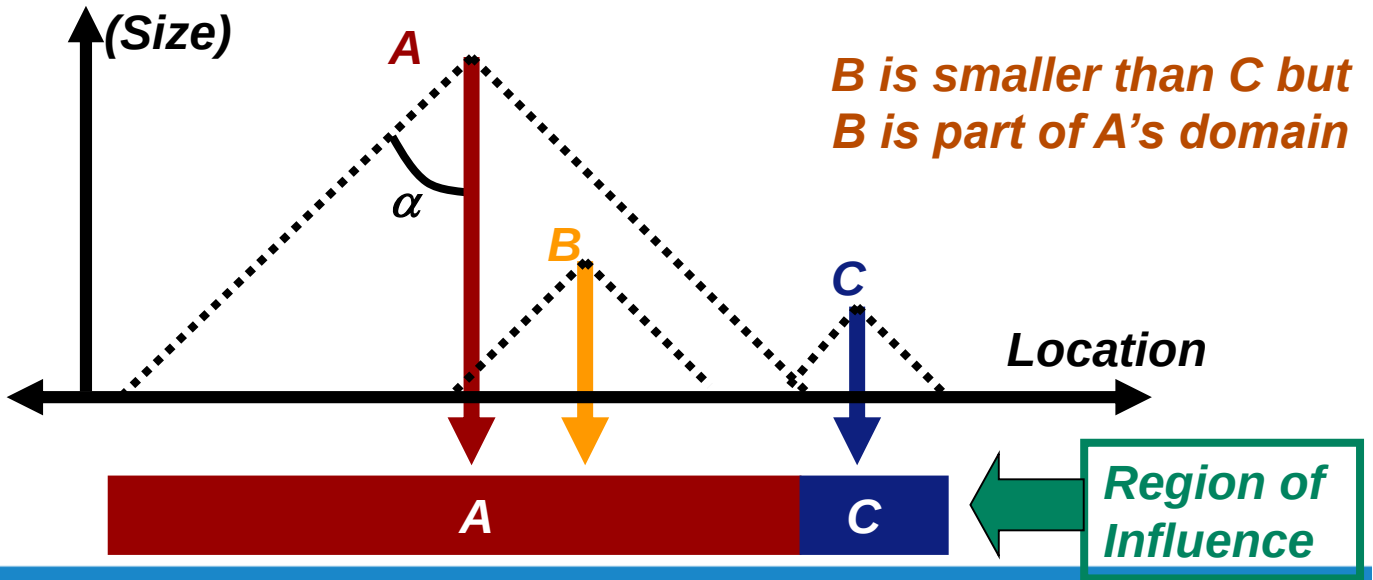


[Renfrew 1975]

[Renfrew & Level, 1979]

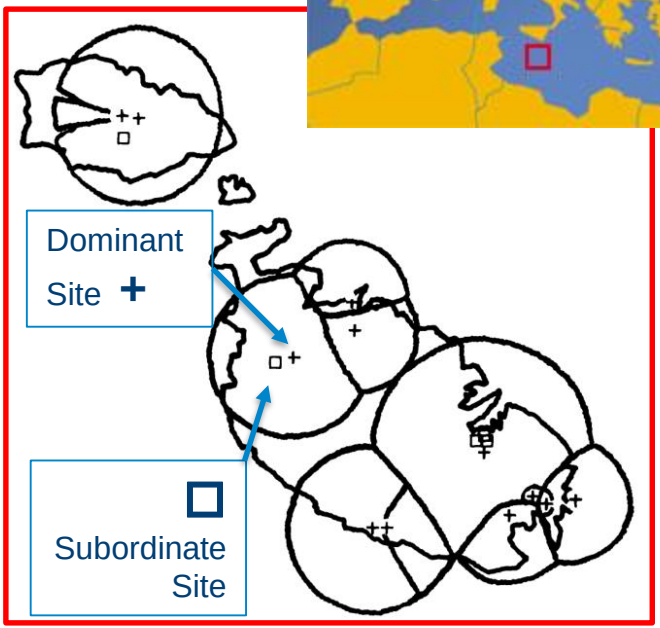
XTent Model

- Thiessen polygons for unequal size sites
- One Parameter



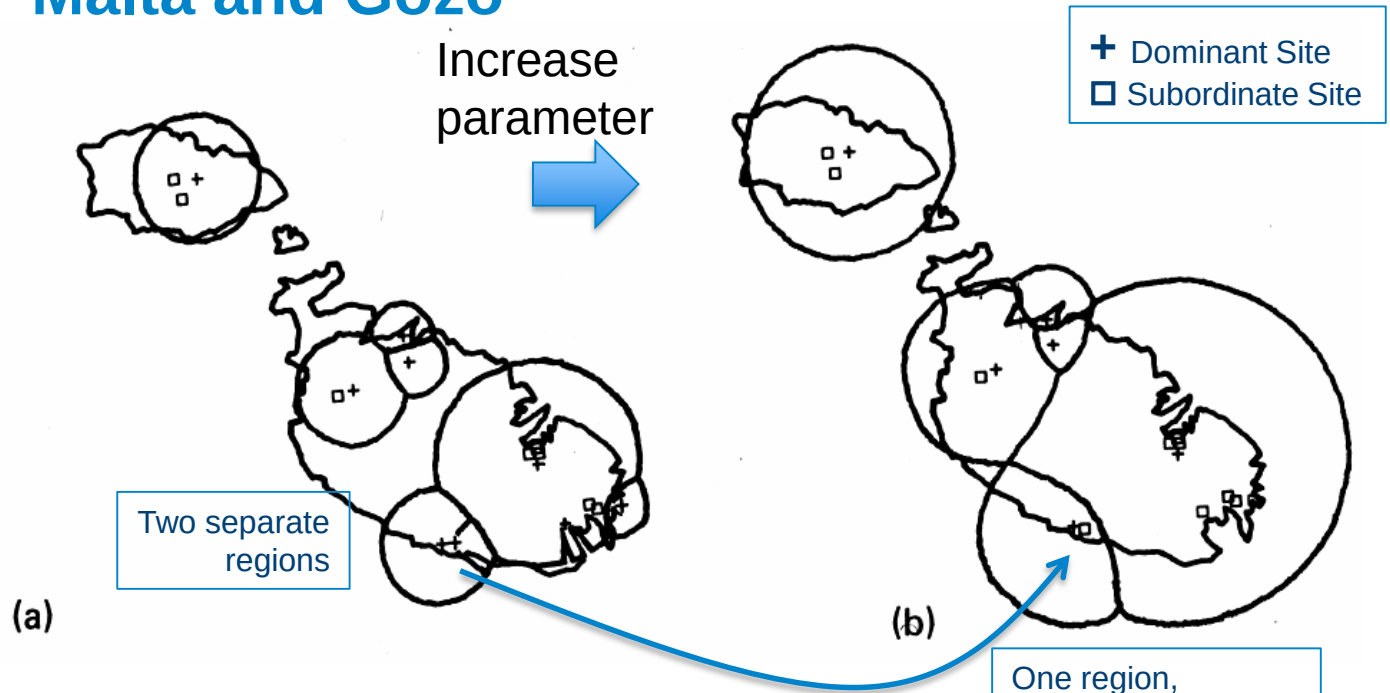
Political organization in prehistoric Malta and Gozo

- Area of Temple
  = Size of Site
- XTENT model with one parameter
- Gives “zones of control” for each site



[Renfrew and Level, 1979; Renfrew 1981]

Political organization in prehistoric Malta and Gozo

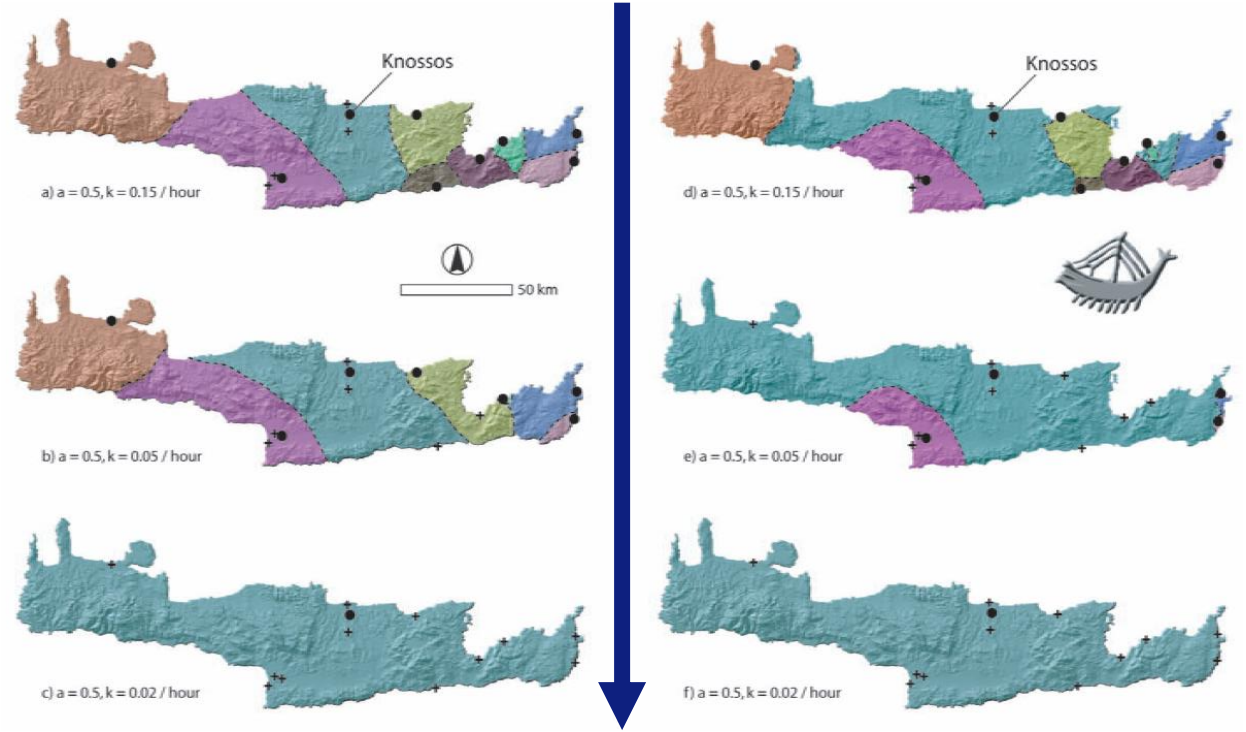


XTENT model parameter controls size of regions

[Renfrew & Level, 1979]

Xtent Model Neopalatial Crete
(~1750BC - ~1500BC)

[Bevan 2010]



On foot

By sea or land

Increasing distance

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Network Models

Long history:

from social science network analysis to anthropology

and then archaeology

for example

- Cappadocia [Tobler & Weinberg 1971]
- Solomon Islands Melanesia [Terrell 1976, 1977]
- Oceania [Hage & Harray 1991]
- Aegean [Broodbank 1993, 2000]
- etc, etc.

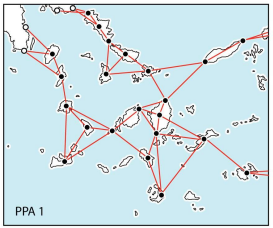
e.g. see Brughmans 2010 for a review

PPA – Proximal Point Analysis

Also known as RANDOM PROXIMITY GRAPHS or K-NEAREST NEIGHBOUR GRAPHS (k-NNG)

- Simple Network Model with one integer parameter
- Each node connects to its ***k*** nearest neighbours.

PPA - Proximal Point Analysis

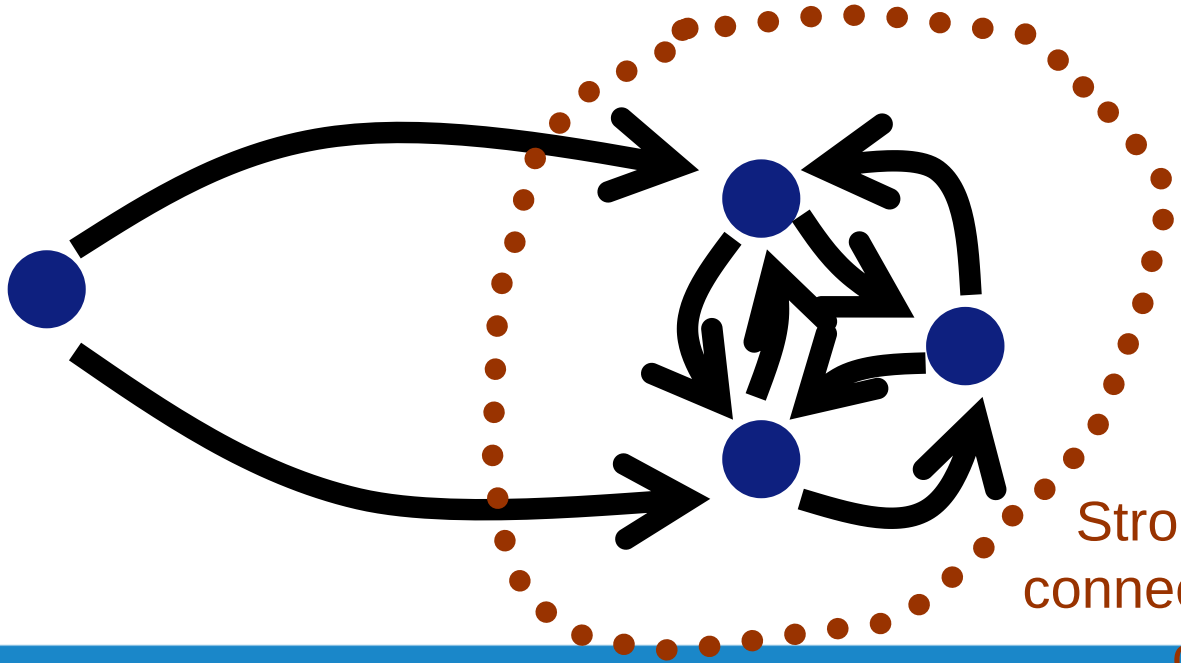


- Equal sized sites or size ignored
- Sites connect to ***k*** nearest neighbours
- Analyse graph
  - Often without directions on edges
  - Sometimes only local measures used *e.g. Degree*
  - Sometimes global measures used
    e.g. ranking, centrality, betweenness

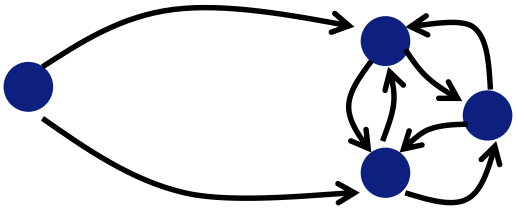
Examples: Terrell 1977; Irwin 1983; Hage & Harary 1991;
Broodbank 2000; Collar 2007

PPA Example

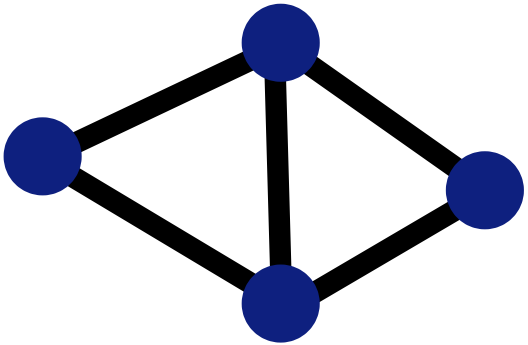
Connect each site to its ***k=2*** nearest neighbours



PPA Example



Ignore direction



- All edges equal
- Network now simply connected

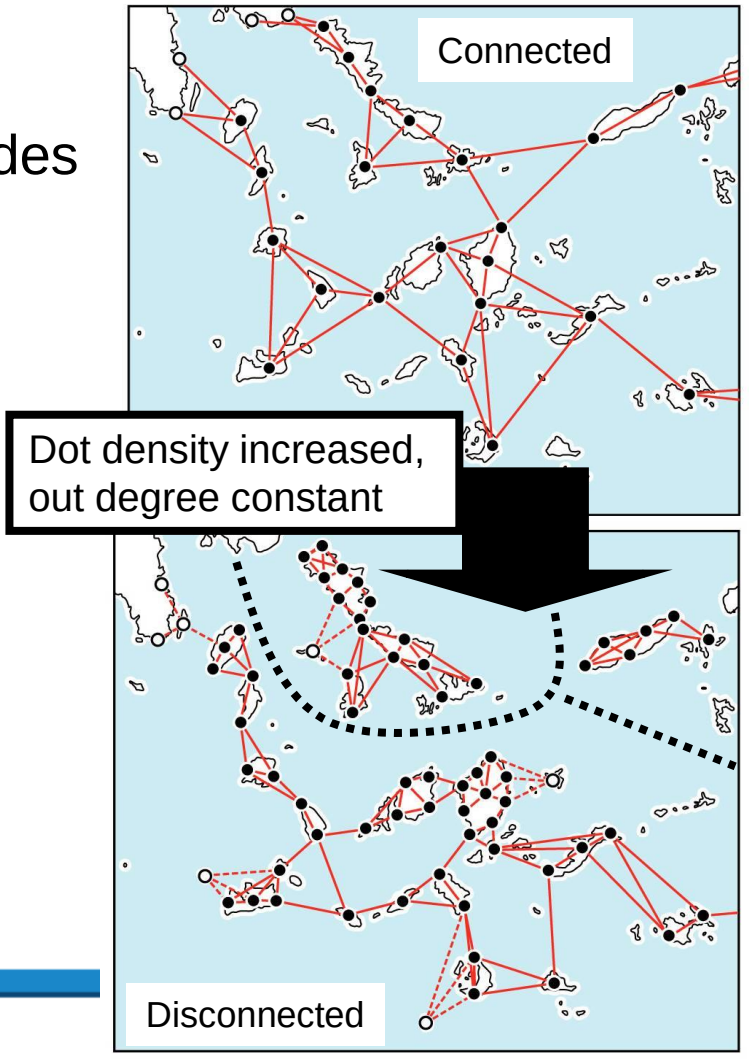
Broodbank PPA

- Early Bronze Age Cyclades
- Population = # vertices

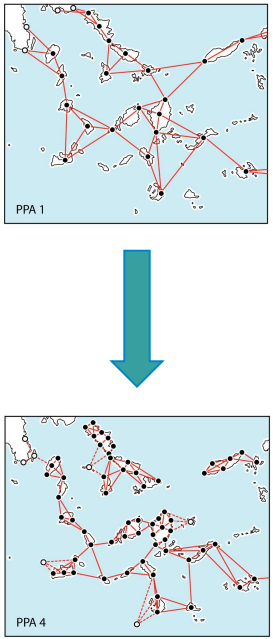
⇒Low density = connected graph

⇒High density = disconnected graph, clusters on large islands

[fig 75, p239, Broodbank 2000]



Broodbank PPA (2)



- EBA Cyclades (Early Bronze Age)
  - Settlements similar size
  - rowing ~ 10km daily

⇒ PPA appropriate

- A simple model suitable for an era with limited data?
- Is more detailed analysis inappropriate?

  e.g. use inherent directionality of edges

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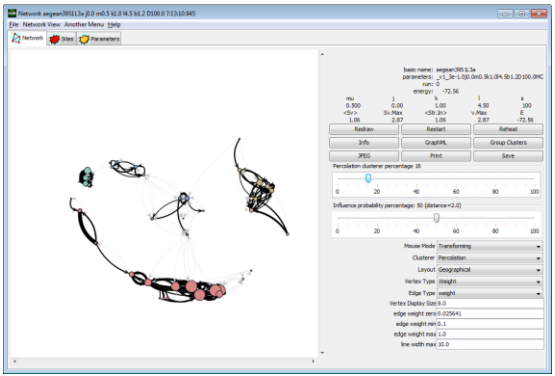
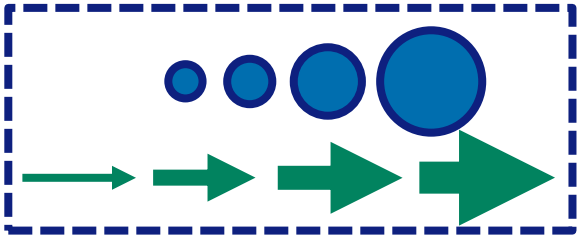
Comparisons: before and after

Model built in response to earlier network studies – **`Ariadne`** [Evans, Knappett & Rivers, 2004 onwards]

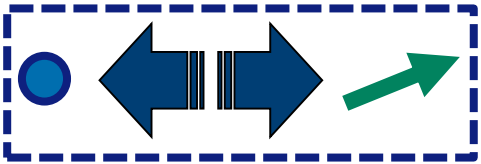


Features of `ariadne`

- Both vertices and edges of variable size

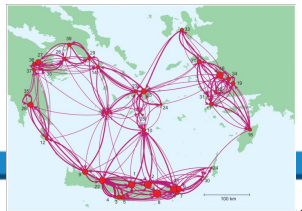


- Values of both are interlinked



- Cost/Benefit balance

- Not a fixed single solution good but never perfect



Cost/Benefit Analysis

`Energy', resources

Isolated sites have optimal size $v_i = 0.5$

Interactions (trade) bring benefits

Increasing 'population' has a cost

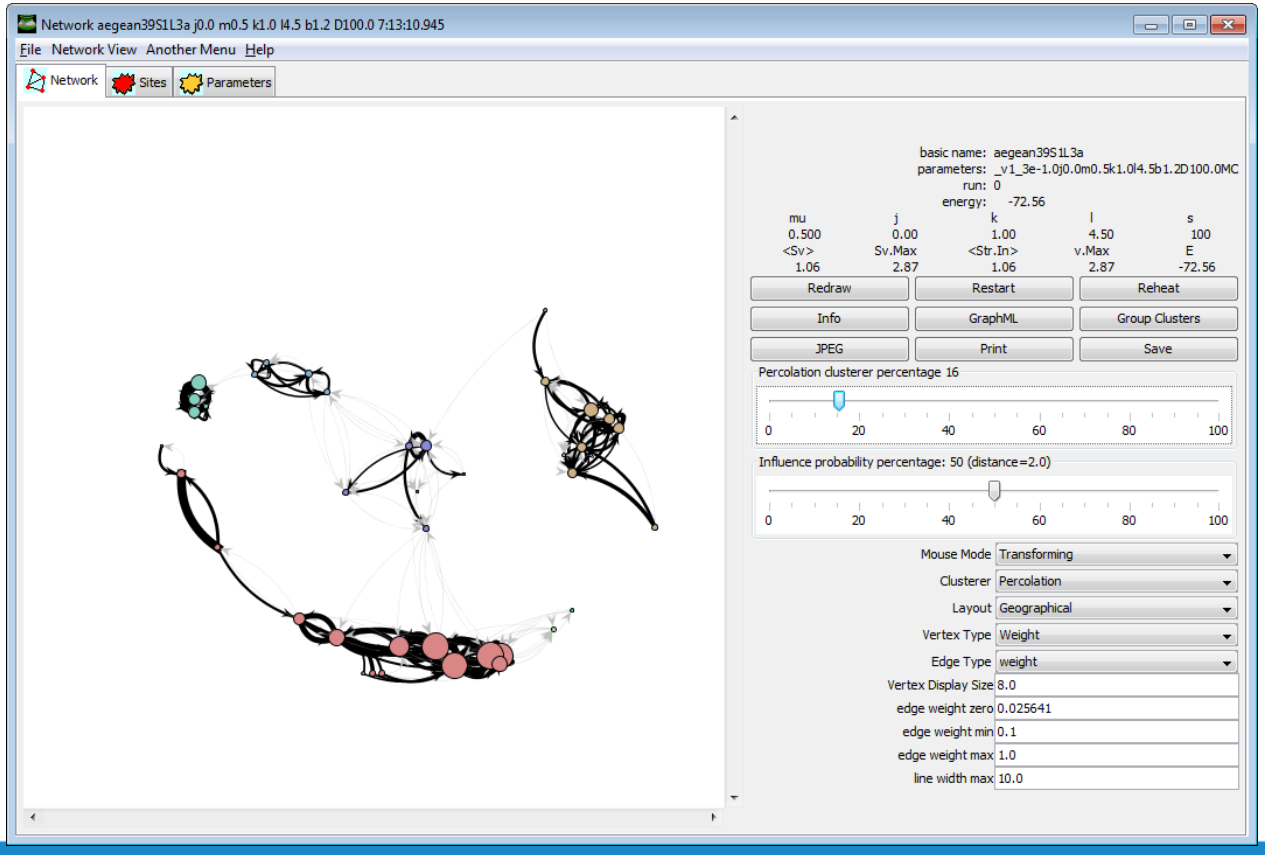
Each trade link has a cost

$$H = -\kappa \sum_i 4 S_i v_i (1 - v_i) - \lambda \sum_{i,j} (S_i v_i).e_{ij} V(d_{ij} / D).(S_j v_j) + j \sum_i S_i v_i + \mu \sum_{i,j} S_i v_i e_{ij}$$

$$0 \le \sum_j e_{ij} \le 1$$

$$0 \le v_i$$

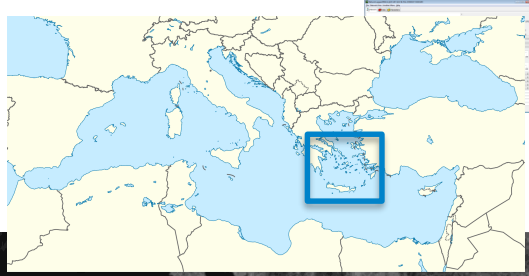
ariadne

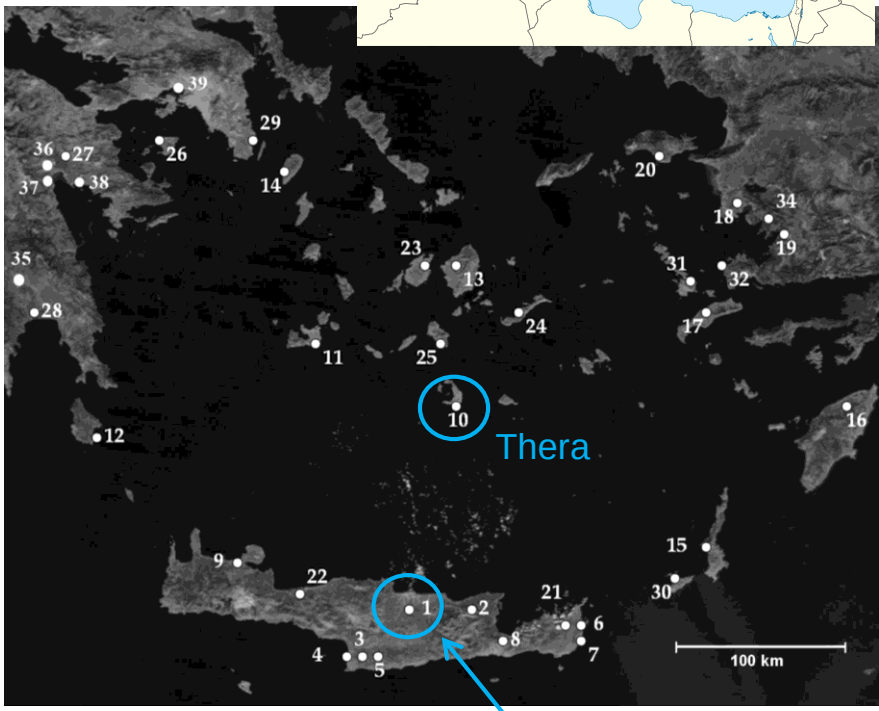

Network aegean39S1L3a j0.0 m0.5 k1.0 l4.5 b1.2 D100.0 7:13:10.945
File Network View Another Menu Help
Network
Sites
Parameters
basic name: aegean39S1L3a
parameters: _v1_3e-1.0j0.0m0.5k1.0l4.5b1.2D100.0MC
run: 0
energy: -72.56
mu j k l s
0.500 0.00 1.00 4.50 100
<Sv> Sv.Max <Str.In> v.Max E
1.06 2.87 1.06 2.87 -72.56
Redraw
Restart
Reheat
Info
GraphML
Group Clusters
JPEG
Print
Save
Percolation clusterer percentage 16
0 20 40 60 80 100
Influence probability percentage: 50 (distance=2.0)
0 20 40 60 80 100
Mouse Mode Transforming
Clusterer Percolation
Layout Geographical
Vertex Type Weight
Edge Type weight
Vertex Display Size 8.0
edge weight zero 0.025641
edge weight min 0.1
edge weight max 1.0
line width max 10.0

Focus: Minoan Aegean

Focus: Minoan Aegean

- c.2000BC distinct Minoan culture starts
  (sail replaces oar)
- c.1500BC Minoan dominance ends
  (50yr after Thera)
- Physically largely self contained
  (Egypt?)





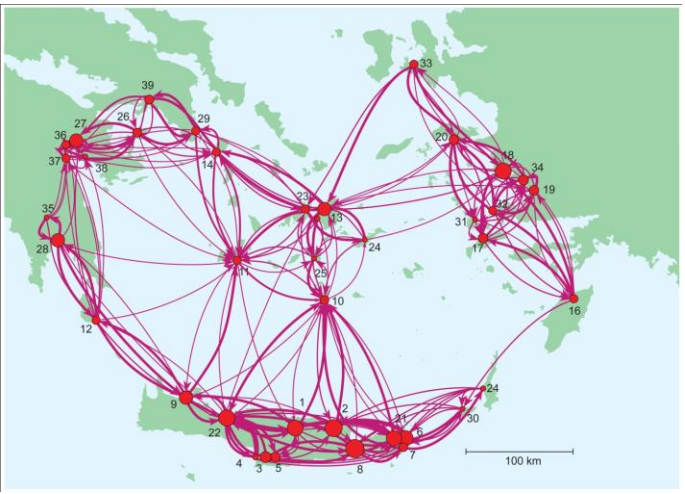
How do we use these models?

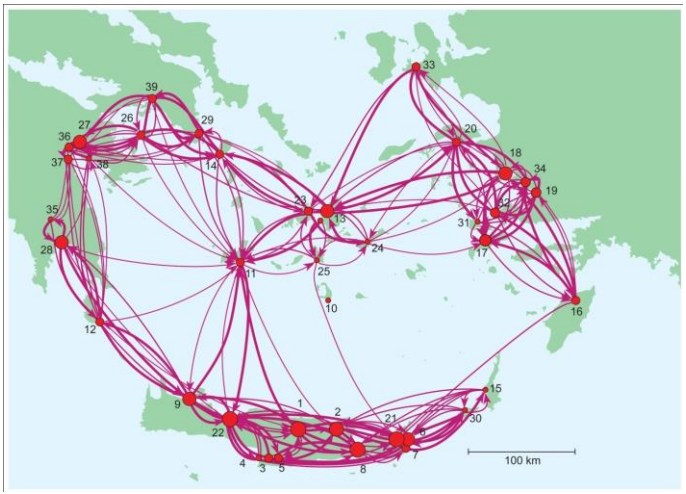
- Can not assign real values to parameters
  → make **COMPARISONS** between different values
  - e.g. vary one parameter, hold rest fixed.

- For any given set of (reasonable) values:
  a) can analyse intrinsic network measures
  e.g. degree of vertices
  b) can perform further `games' to analyse properties
  e.g. diffusion, apply cultural transmission models, ABM on this substrate.

[Knappett et al, Antiquity, 2011]

Before and After the Eruption





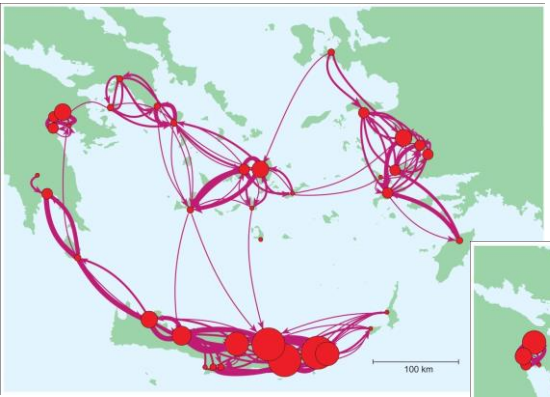
- Total population largely unchanged
- Total interaction largely unchanged

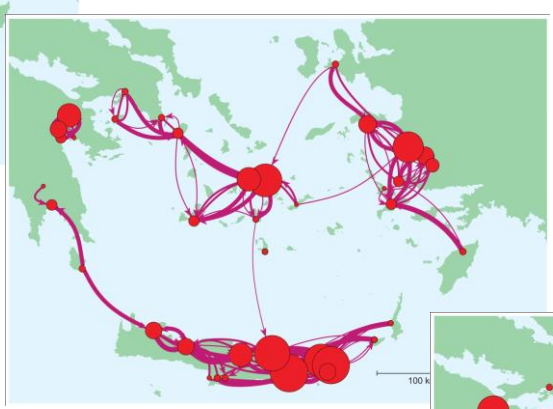
For same parameter values

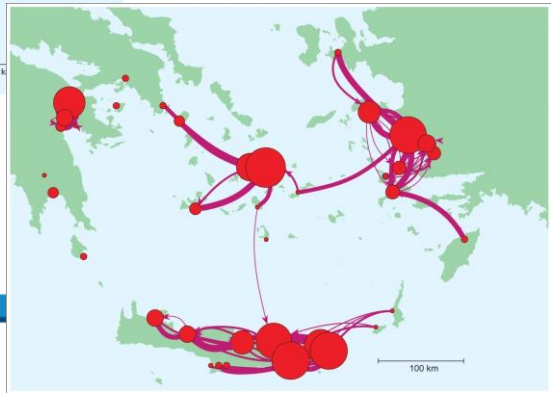
$\lambda = 4.0$, $\kappa = 1.0$, $\mu = 0.1$, $j = -2.0$, $D = 110$km

[Knappett et al, Antiquity, 2011]

Increasing Interaction Cost post Eruption







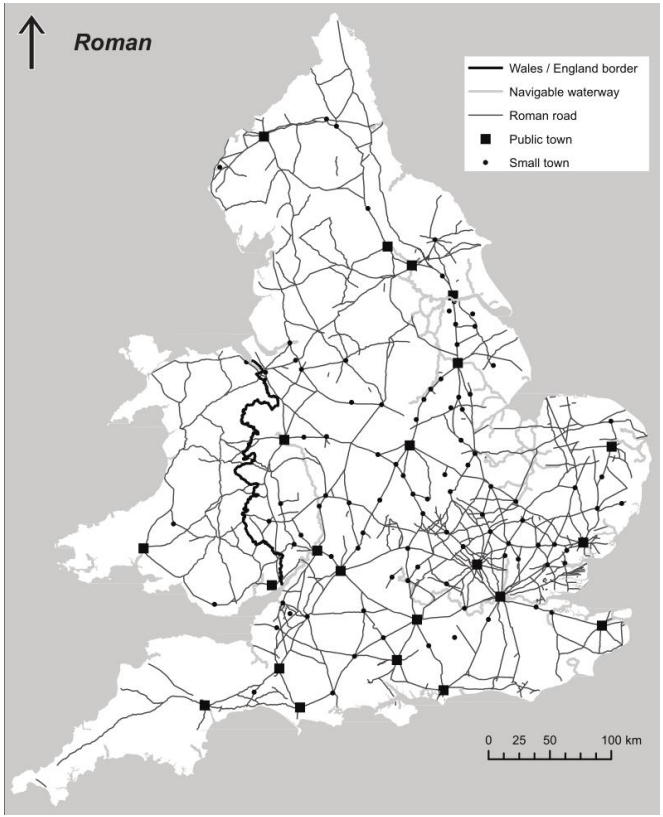
μ increasing

- Fewer but stronger links
- Shorter distances

Comparisons: The changing role of rivers

Study also illustrates the use of network centrality measures. [Brookes & Huynh 2018]

Roman Road Network

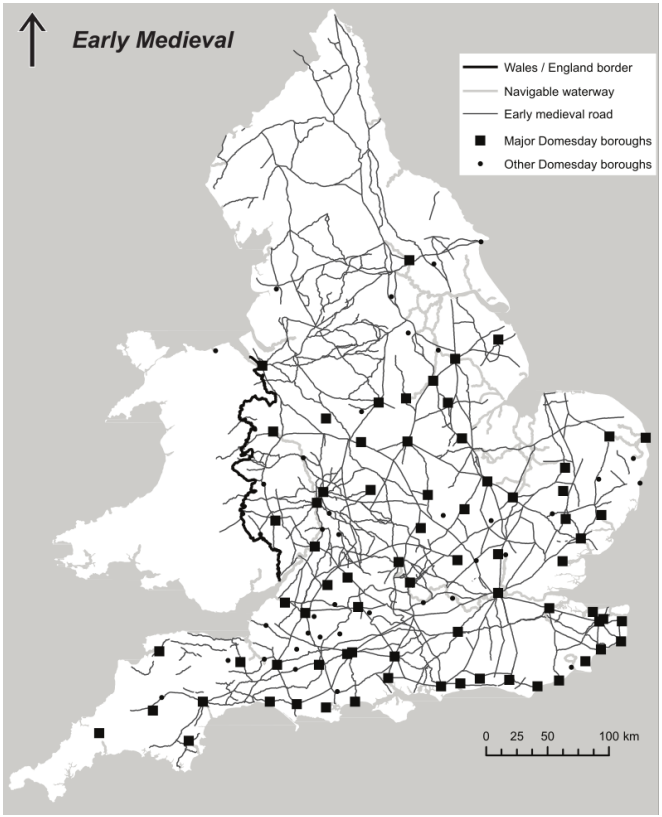


- Roman roads well understood
- Roman Settlements harder
  - 22 ‘public towns’
    civitas (capitals), colonia and municipia
  - 95 smaller centres
    ‘undefended settlements’, ‘minor towns’, ‘minor defended sites’, specialised ‘religious’ and ‘industrial’ sites
- Navigable waterways

Neil Huynh CCS Monday
[Brookes & Huynh 2018]

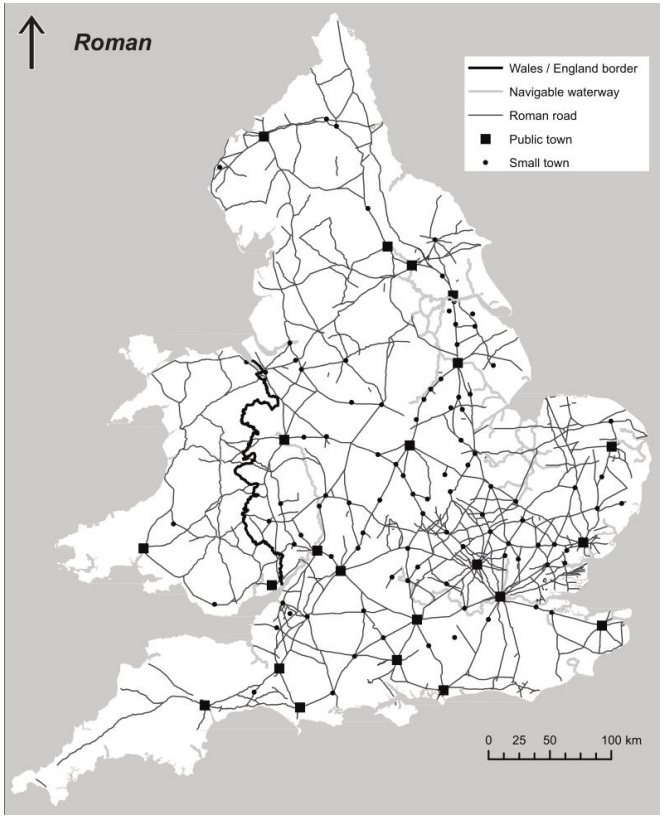
Early Medieval Period ~ 1100 CE

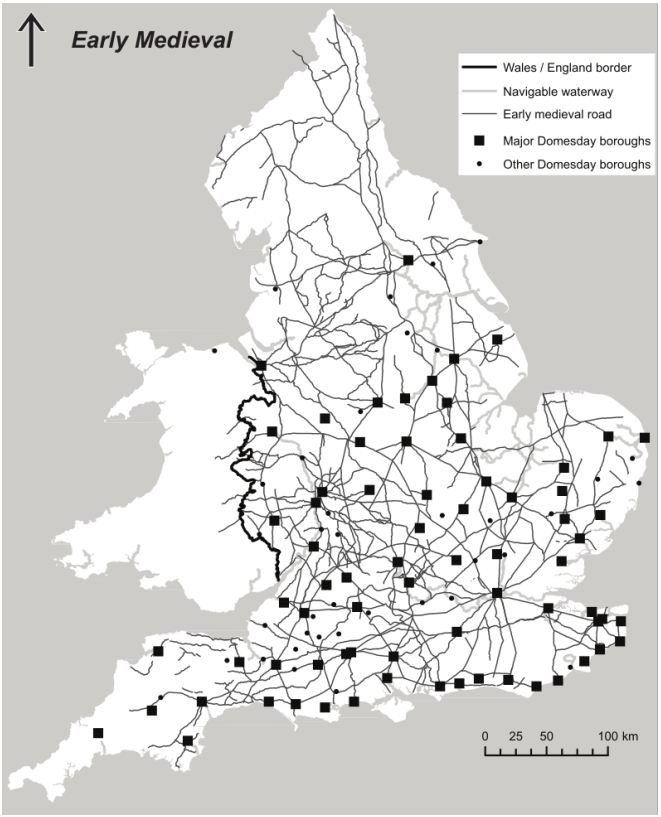
- Roads reconstructed
  [Cole 2013; Leverhulme Project 2016]
  - Some changes from Roman network
- Settlements Domesday Survey 1086CE
  - 112 settlements with 'burgesses' or townsmen
- Navigable waterways



PageRank · Brookes & Huynh

Neil Huynh CCS Monday
[Brookes & Huynh 2018]

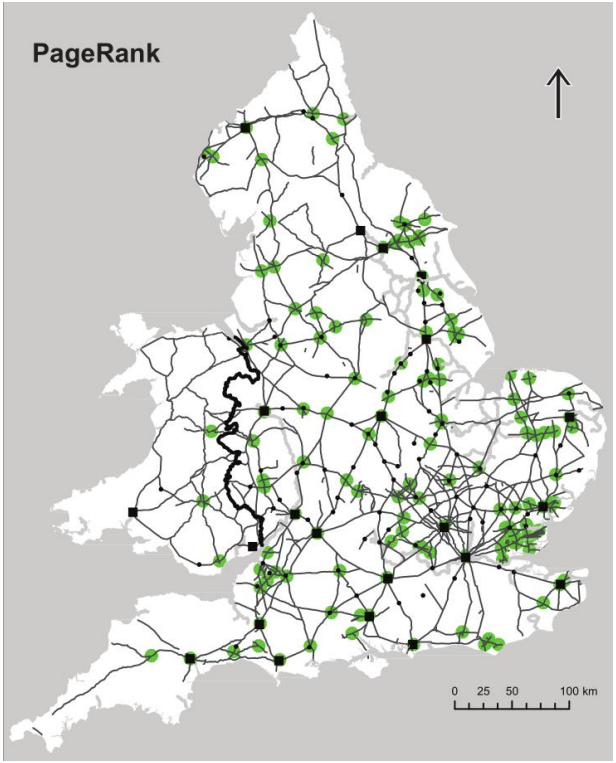




Neil Huynh CCS Monday
[Brookes & Huynh 2018]

PageRank

- PageRank more stable than other centrality measures
- Road connections laid out between these centres were especially important for military and administration
- Navigable waterways in connecting places appears to have been less important in Roman era



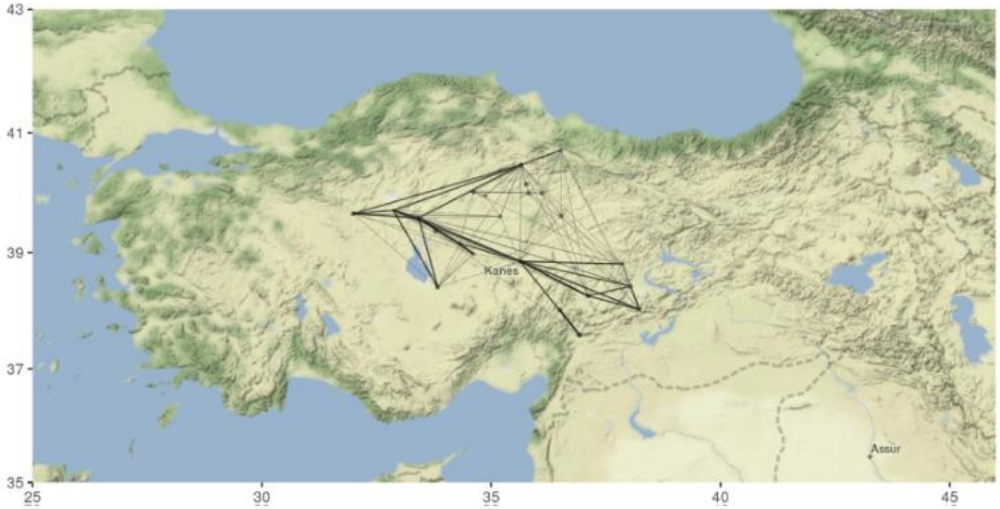
- Introduction
  - Archaeological Data
  - Motivation
  - Modelling Choices
- Zones of Control Models
- Proximal Point Analysis
- Comparisons Using Models
- Missing Sites Examples
- Uncertainty in Modelling

Missing Sites

For example

- Assyrian [Barjamovic et al, 2019]
- Minoan Crete, [Palliou & Bevan 2016]

Missing Sites

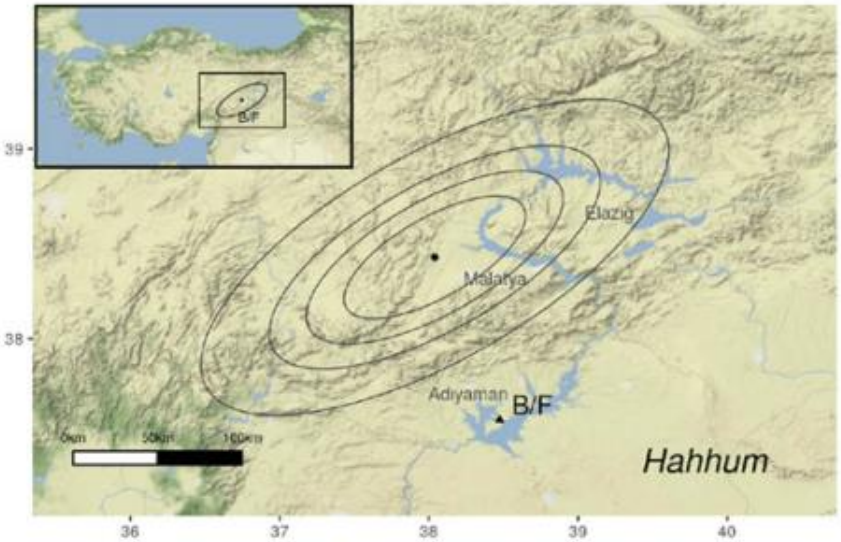


Assyrian Cities, Bronze age in Anatolia, Turkey

- 15 known sites, 10 unknown
- 198 Tablets with 227 itineraries

[Barjamovic et al, 2019]

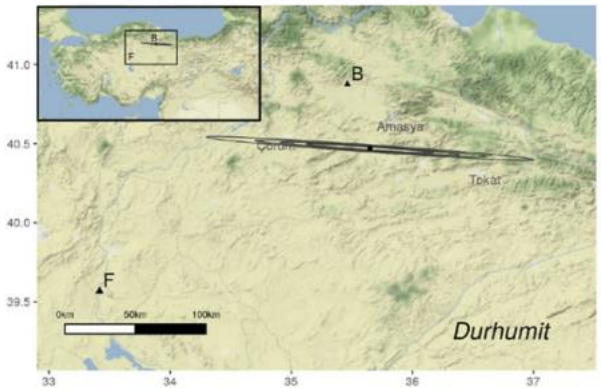
Missing Sites

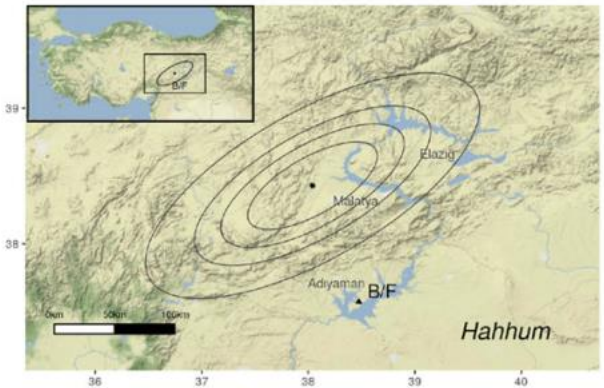


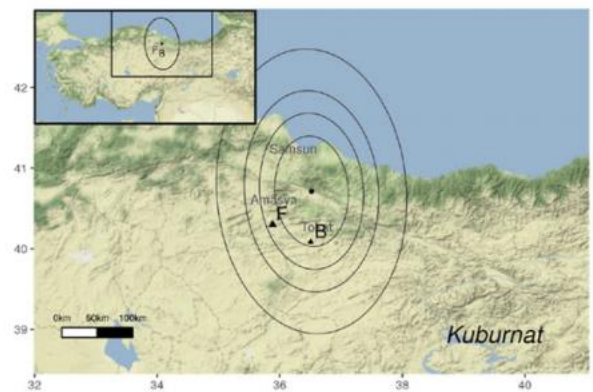
- Assign locations to missing sites
- Flows modelled with form of the gravity model
  - Interesting economic theory derivation
- Match to data on shipment counts
- NLLS estimators to optimise missing site locations
- Includes uncertainties (typical error 0.5º ~50km)

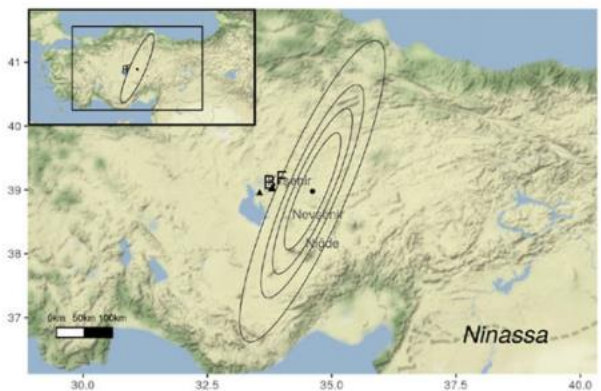
[Barjamovic et al, 2019]

Missing Sites









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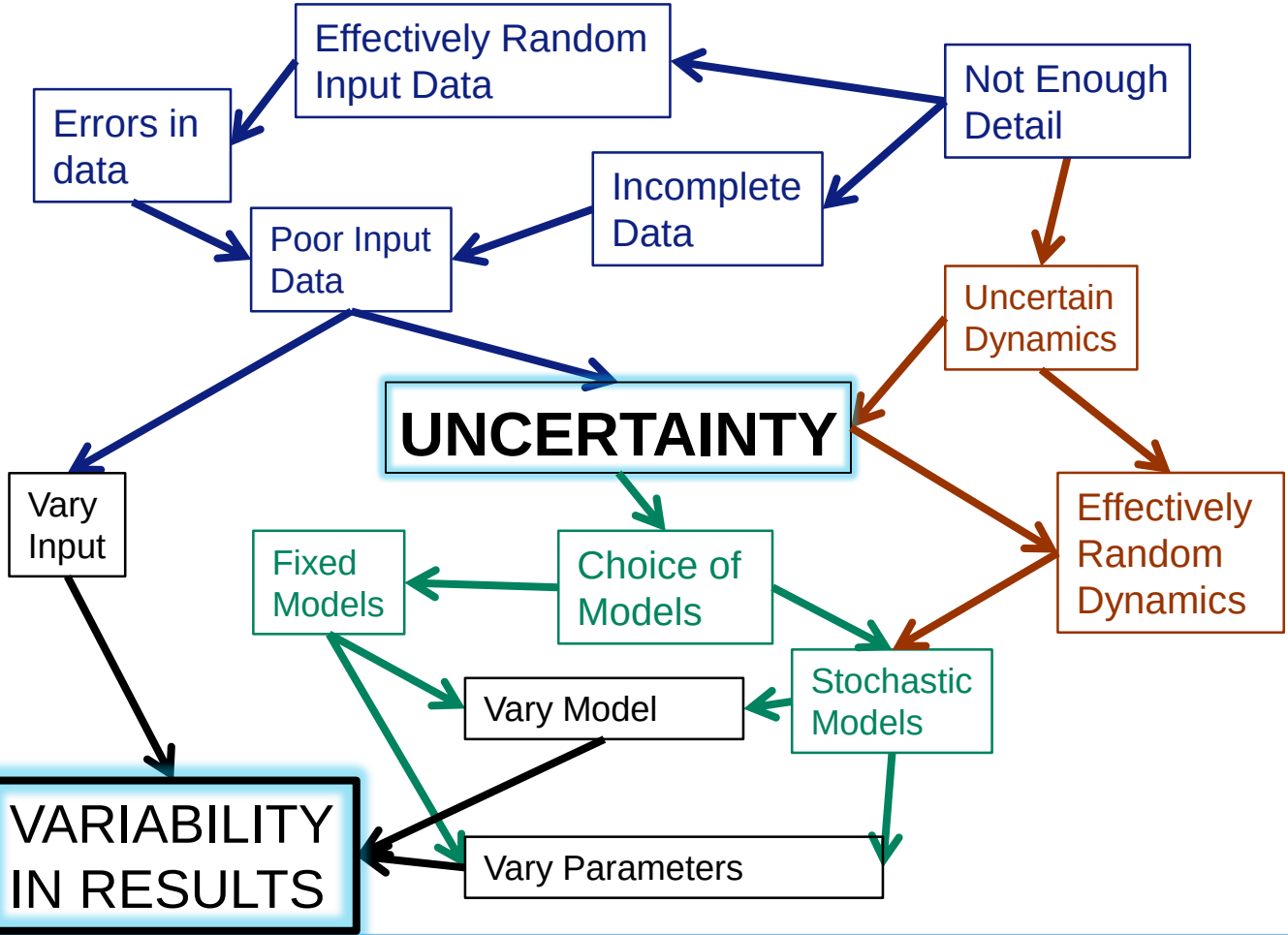
Should Thebes exist?

Example of how one can add include when modelling.

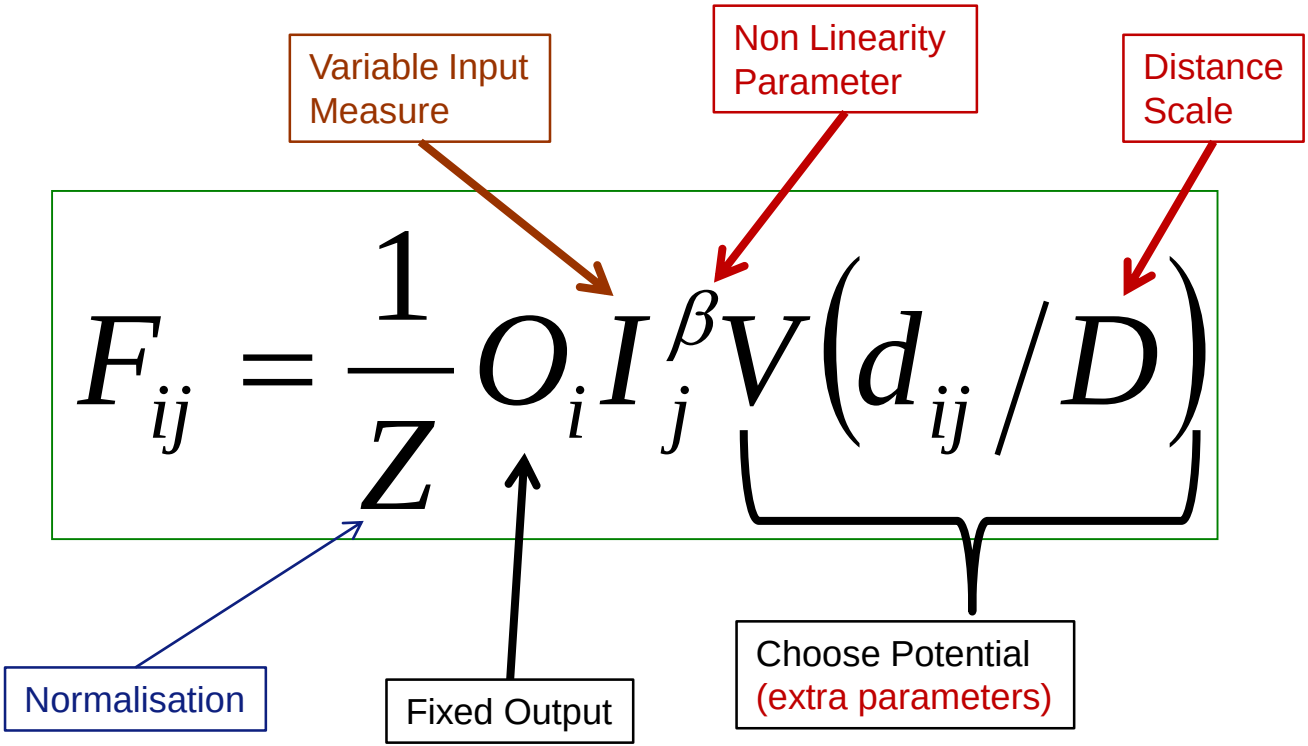
- Early study Rihll & Wilson, 1987, 1991
- Later study Evans & Rivers 2017

Thebes is one of the leading cities in classical Greece C5th BCE


Effectively Random Input Data
Errors in data
Not Enough Detail
Incomplete Data
Poor Input Data
Uncertain Dynamics
UNCERTAINTY
Vary Input
Fixed Models
Choice of Models
Effectively Random Dynamics
Stochastic Models
Vary Model
VARIABILITY IN RESULTS
Vary Parameters

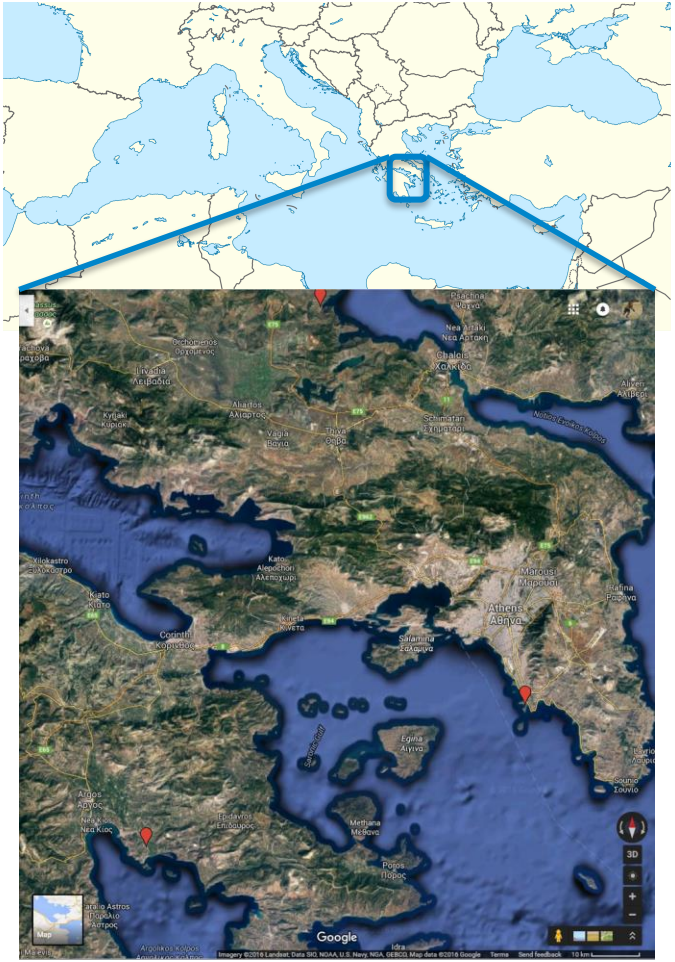
Retail Gravity Model

[Harris & Wilson, 1978]



Data

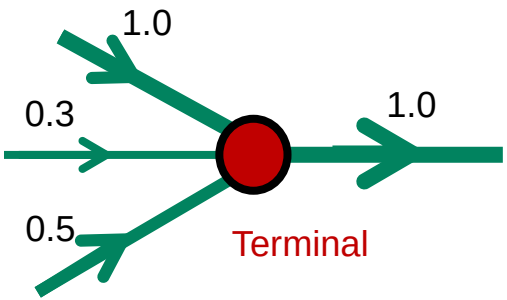
- Same data set as **RW** Rihll and Wilson 1987, 1991 (**RW87** & **RW91**)
- Late Geometric Period
- Part of central southern mainland Greece
  - Boeotia, Attica, Isthmus, Argolid
- RW aim to look at rise of unequal cities from equal sized settlements
  - Thebes, Athens, Corinth, Argos

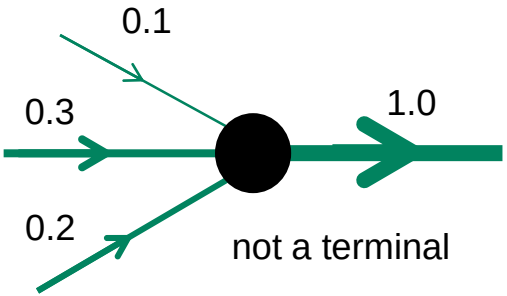


[Nystuen & Dacey 1961,
Rihll & Wilson 1991]

Sparsification using Terminals

- Rihll & Wilson define **Terminal Sites**





Terminal sites satisfy

Total Flow In > Largest Single flow out

More people owe you than you owe others

Terminals Dominant

Rihll & Wilson identify terminal sites as dominant sites emerging from equal sized villages

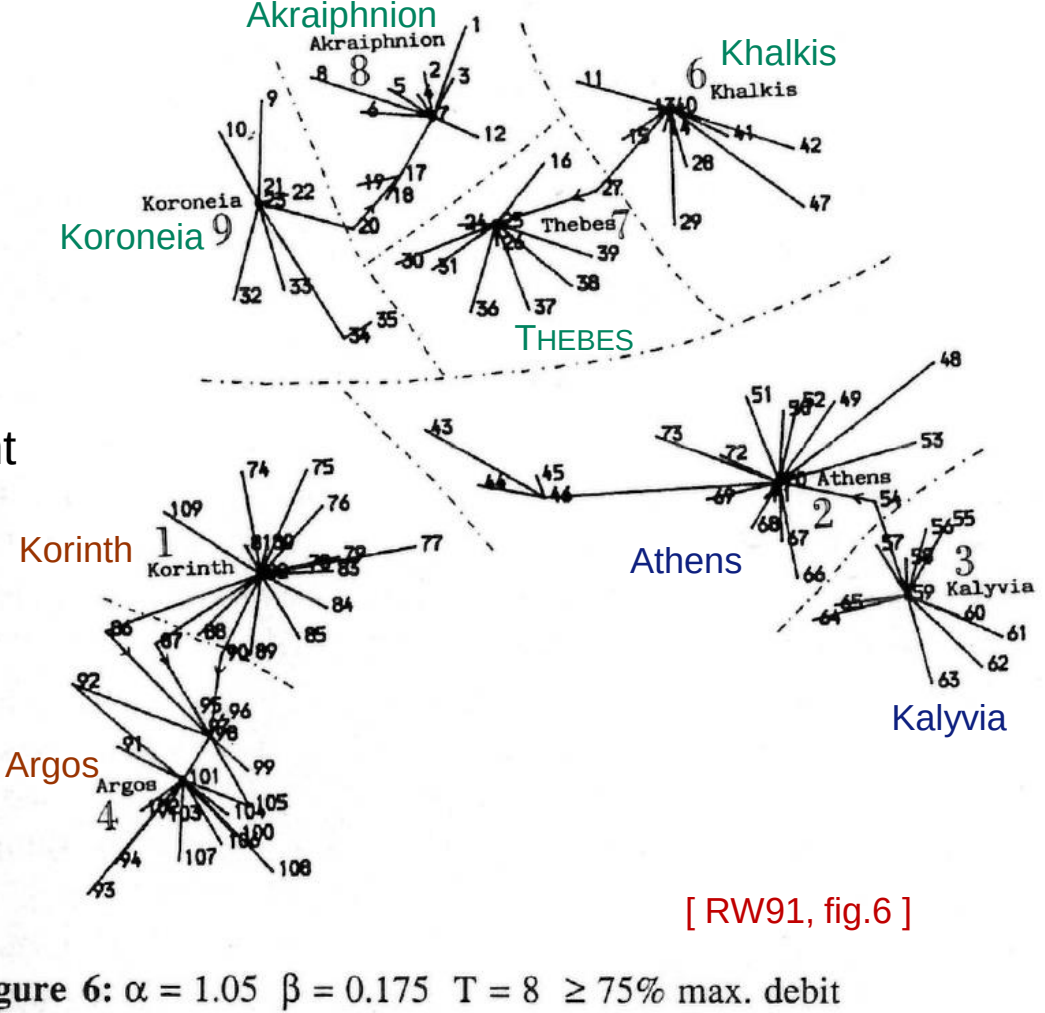


Figure 6: $\alpha = 1.05$ $\beta = 0.175$ $T = 8$ $\geq 75\%$ max. debit

Uncertainty

- Rihll & Wilson looked at several parameter values but typically found similar results
  - Thebes and Athens always identified
  - Korinth and Argos or close neighbours often identified
  - Other centres also found in addition

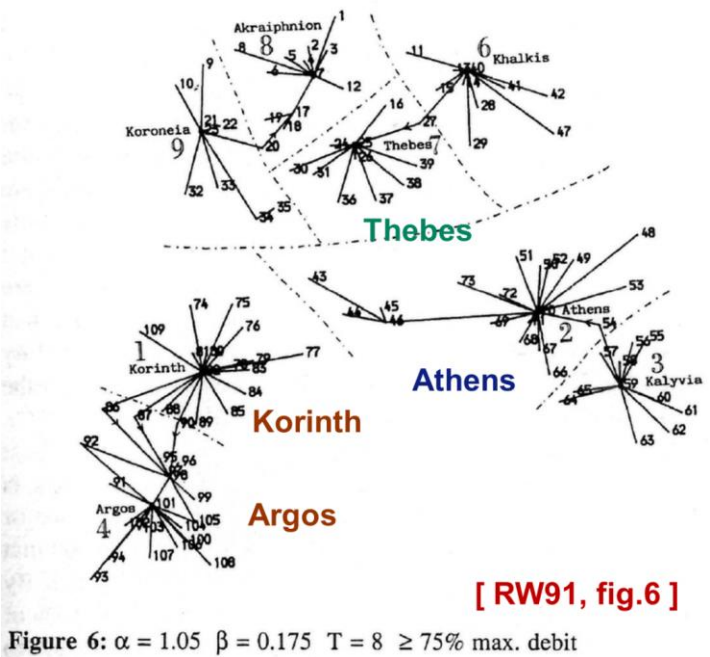


Figure 6: $\alpha = 1.05$ $\beta = 0.175$ $T = 8$ $\geq 75\%$ max. debit

Uncertainty in Results

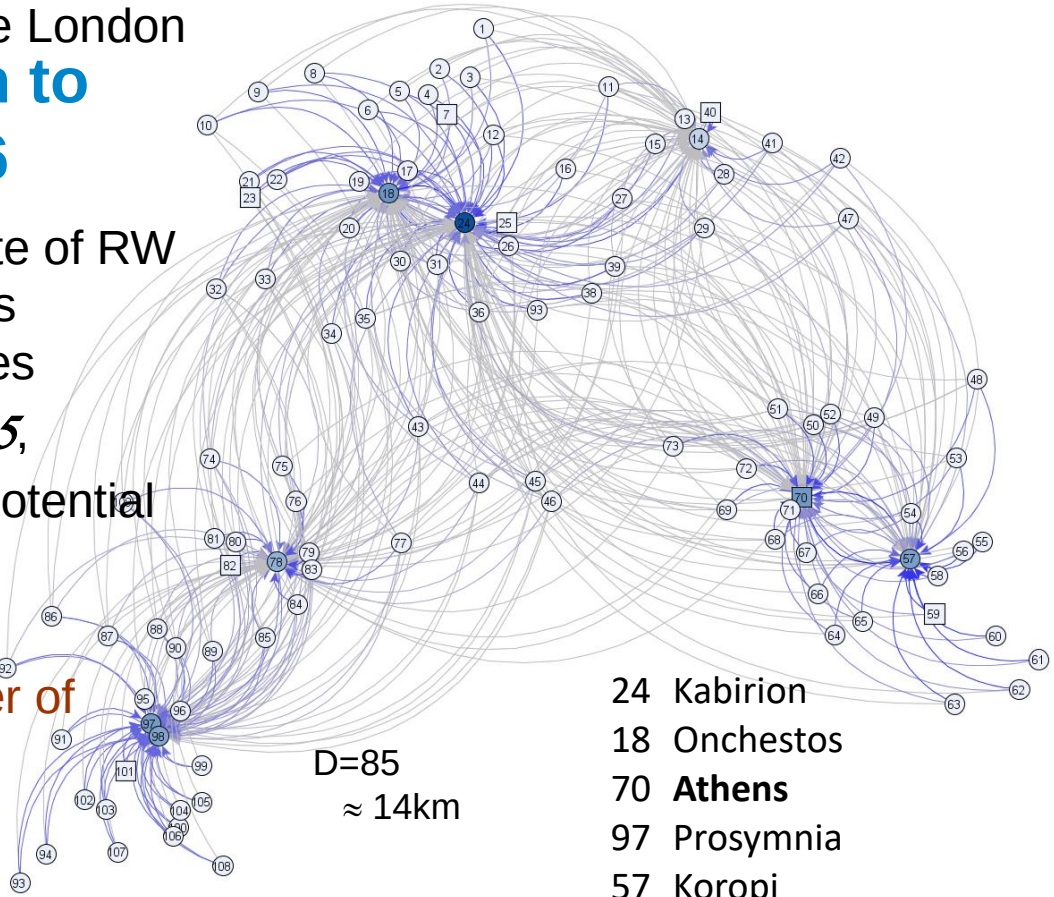
- Remarkably stable key results
- Only qualitative assessment of uncertainty
- Only one form of distance potential used
  pure exponential $\mathbf{exp}\,(-\boldsymbol{d_{ij}/D})$
- No uncertainty in site locations of distances
- Unverifiable
  - no comprehensive list of site locations and distances used

until now

Best match to RW91 Fig 6

- Best estimate of RW site locations and distances
- Same $\beta=1.05$,
- Same **exp** potential
- vary distance ***D***

⇒ same number of terminals ***8***

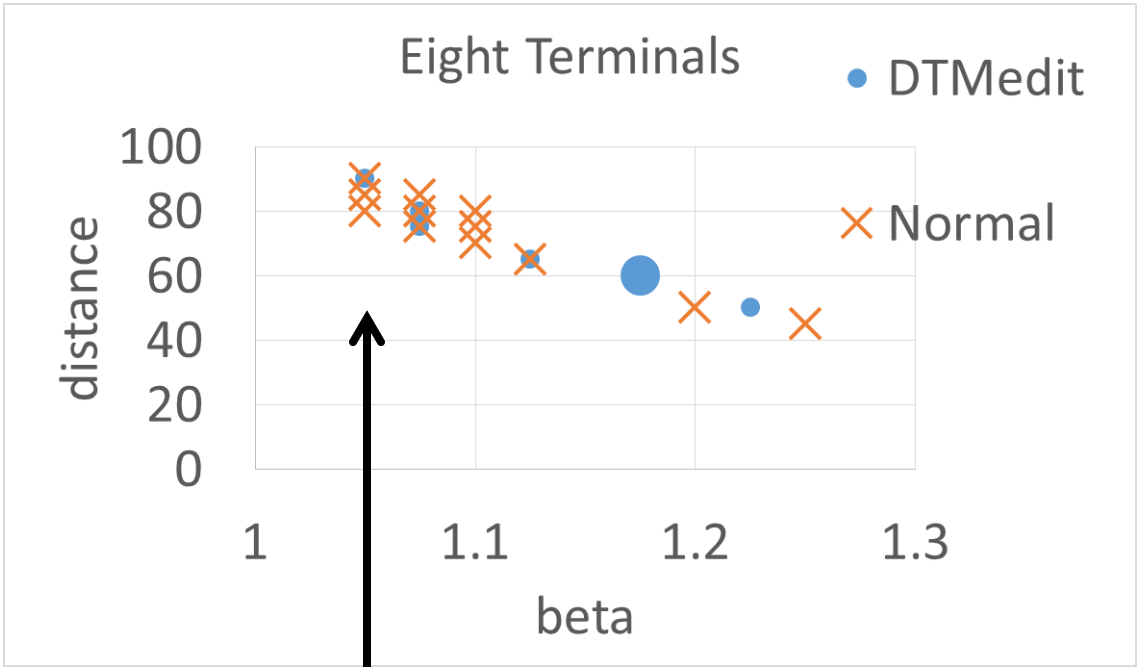


D=85
≈ 14km

24 Kabirion
18 Onchestos
70 **Athens**
97 Prosymnia
57 Koropi
98 Argive_Heraion
78 Kromna
14 Aulis

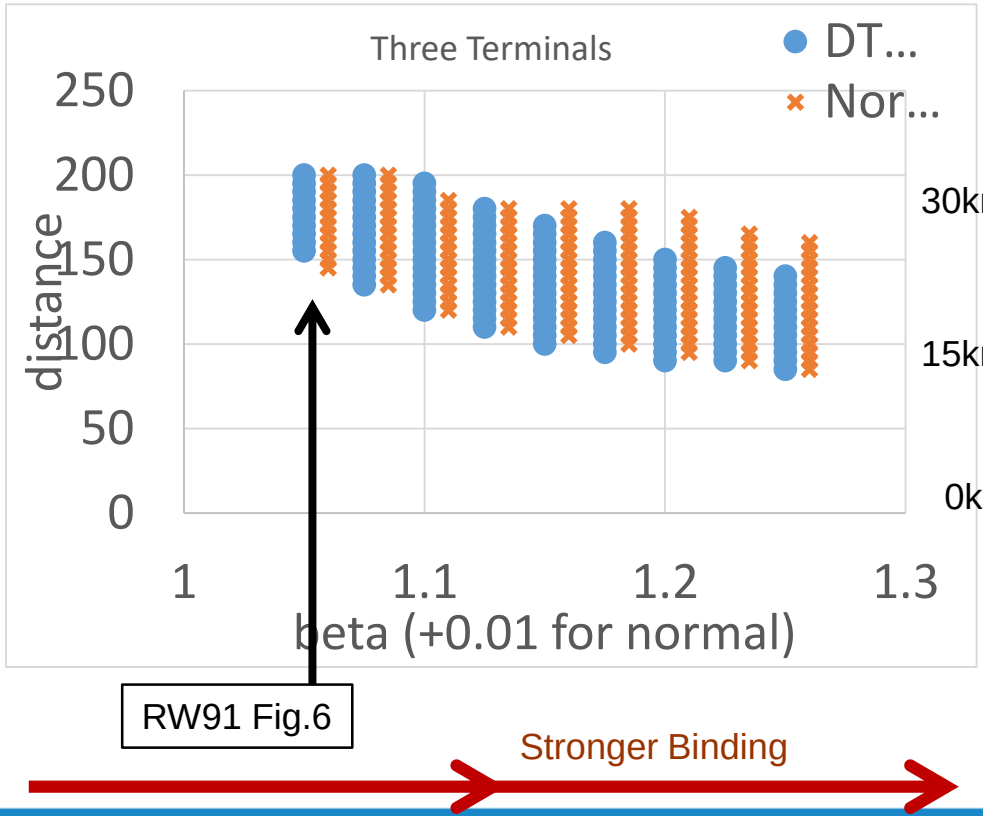
Large terminal number regions

Number of Terminals sensitive to parameter values



Large terminal number regions

- Only 3 terminals stable
- DTMedit distance scales slightly larger



8 Terminals – in order of 'hierarchy'

DTMedit D90	Normal D90	Normal D85	Normal D80	RW91 Fig 6
Potniai 26	Kabirion	Kabirion	Kabirion	Thebes (25)
Medeon 17	Athens	Onchestos	Onchestos	Akraiphnion (7), Koroneia (23)
Berbati 96	Prosymnia	Athens	Athens	Athens
Koropi 57	Koropi	Prosymnia	Koropi 57	Kalyvia 59
Athens 70	Argive Heraion	Koropi	Prosymnia 97	Argos (101)
Korinth 82	Onchestos	Argive Heraion	Argive Heraion 98	Argos (101)
Argive Heraion 98	Kromna	Kromna	Kromna 78	Korinth (82)
Mykalessos 15	Aulis	Aulis	Aulis 14	Khalkis (40)

8 Terminals – ordered by location

DTMedit D90	Normal D80, D85, D90	RW91 Fig 6	Wider Location
Mykalessos 15	Aulis 14	Khalkis 40	Euboea environs
Potniai 26	Kabirion 24	Thebes 25	Neighbourhood of Thebes
Medeon 17	Onchestos 9	Akraiphnion 7 Koroneia 23	Northern Boeotia
Athens 70	Athens 70	Athens	Athens
Koropi 57	Koropi 57	Kalyvia 59	S.Attica
Korinth 82	Kromna 78	Korinth 82	Neighbourhood of Corinth
Berbati 96	Prosymnia 97	Argos (101)	Neighbourhood of Argos
Argive Heraion 98	Argive Heraion 98		

Largely consistent on scale of about 10km

8 Terminals

Now we have ‘error bars’

Ranges of uncertainty

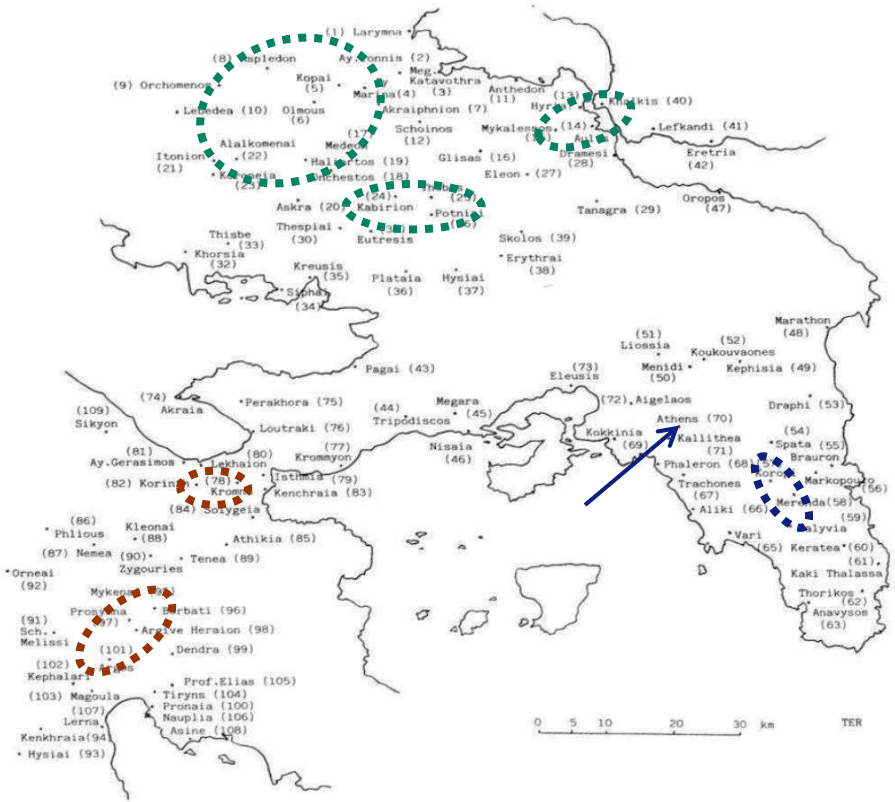


FIGURE 1

3 Terminal Sites – Exponential Potential

Distance Data	Distance D	Terminal Sites		
		Boeotia	Attica	Isthmus/ Argolid
Normal	150	24 Kabirion	70 Athens	96 Berbati
Normal	155, 160, 165, 170, 175, 180	31 Eutresis	70 Athens	96 Berbati
Normal	185	31 Eutresis	70 Athens	89 Tenea
Normal	190,200	36 Plataia	70 Athens	89 Tenea
DTMedit	155,160,170, 175	26 Potniai	71 Kallithea	96 Berbati
DTMedit	180,190,200	26 Potniai	71 Kallithea	89 Teneai

Exponential potential

3 & 8 Terminals

Ranges of uncertainty

Grey =
8 Terminals

Coloured =
3 Terminals

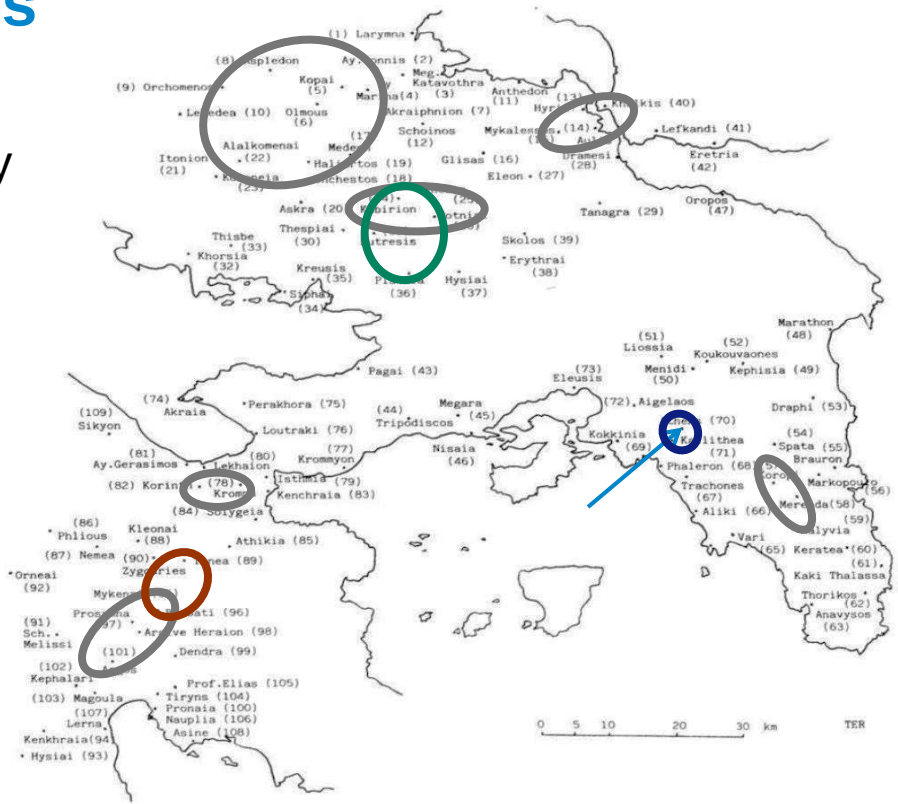


FIGURE 1

Summary

Tim Evans
http://netplexity.org

- Uncertainty should be reflected in results
- Should see “errors” – variability measures
  - Vary initial conditions ✓
  - Stochastic dynamics × (but `ariadne/ERG`)
  - Vary parameters ✓
  - Different Models ✓
- Illustration with Rihll and Wilson data (1987, 1991)

Slides: http://dx.doi.org/10.6084/m9.figshare.3840249

- General Approach to Modelling in Archaeology
- Geography and Zones of Control
- Geography and Interactions
- Our Model - **`ariadne`**
- Summary

Summary

- Use of modelling networks is now increasing in archaeology
- Many models very simple
- Role of geography relatively easy to study
- Comparing against finds much harder
- **Many options remain to be explored**

All my work done with
Carl Knappett (Toronto)
Ray Rivers (Imperial)

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