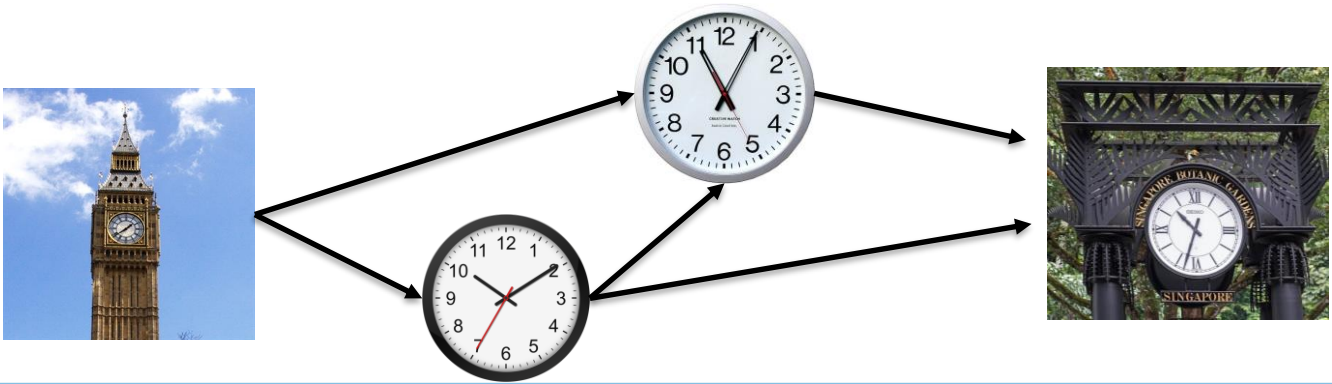
Complexity Science

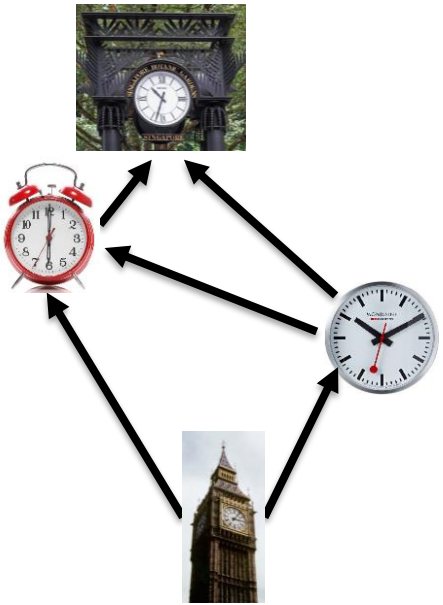
Making Communities Show Respect for Order

Tim Evans

Work with Vaiva Vasiliauskaite



Directed Acyclic Graphs = DAG

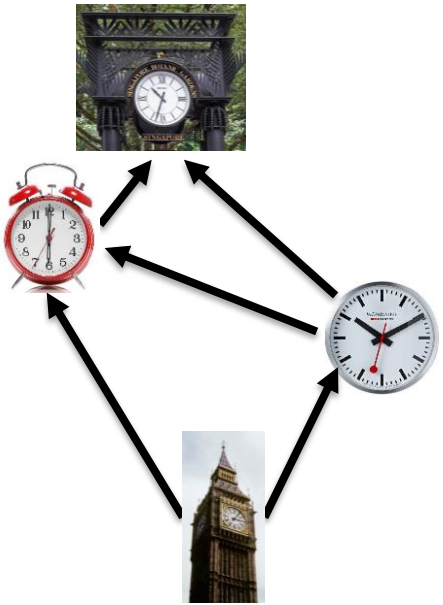


- A directed network with no cycles
- Defines a PARTIAL ORDER of nodes
- Order constrains direction of edges

e.g. TEMPORAL VERTEX NETWORK

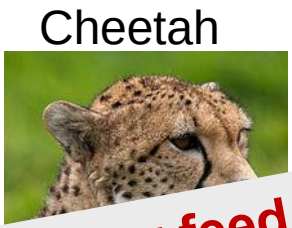
- vertices assigned a time
- edges respect the ARROW-OF-TIME

Directed Acyclic Graphs = DAG Examples



- Citation networks
  - Papers
  - Patents
  - Court judgements
  - Blogs
- Task scheduling
- Food webs
- Cryptocurrency Transactions
- Programme Dependencies
- Causal set approach to quantum gravity

Similar nodes in DAGs are not connected



predator

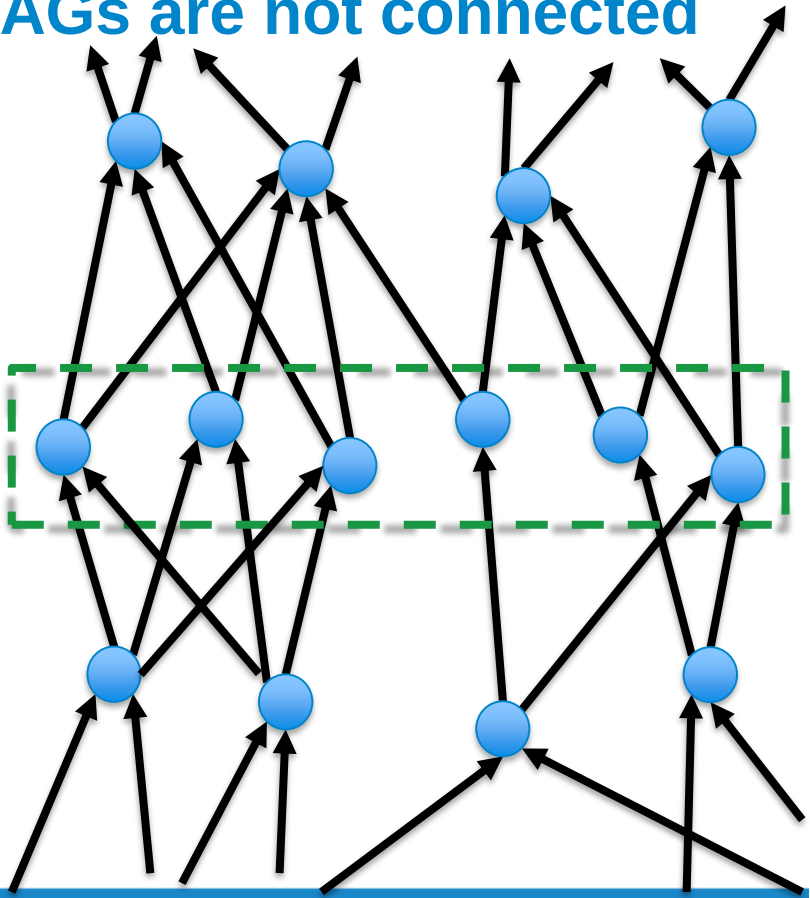
prey

Similar species DO NOT feed on each other
Similar species DO feed on similar prey

Similar nodes in DAGs are not connected

Similar nodes form an **ANTICHAIN**

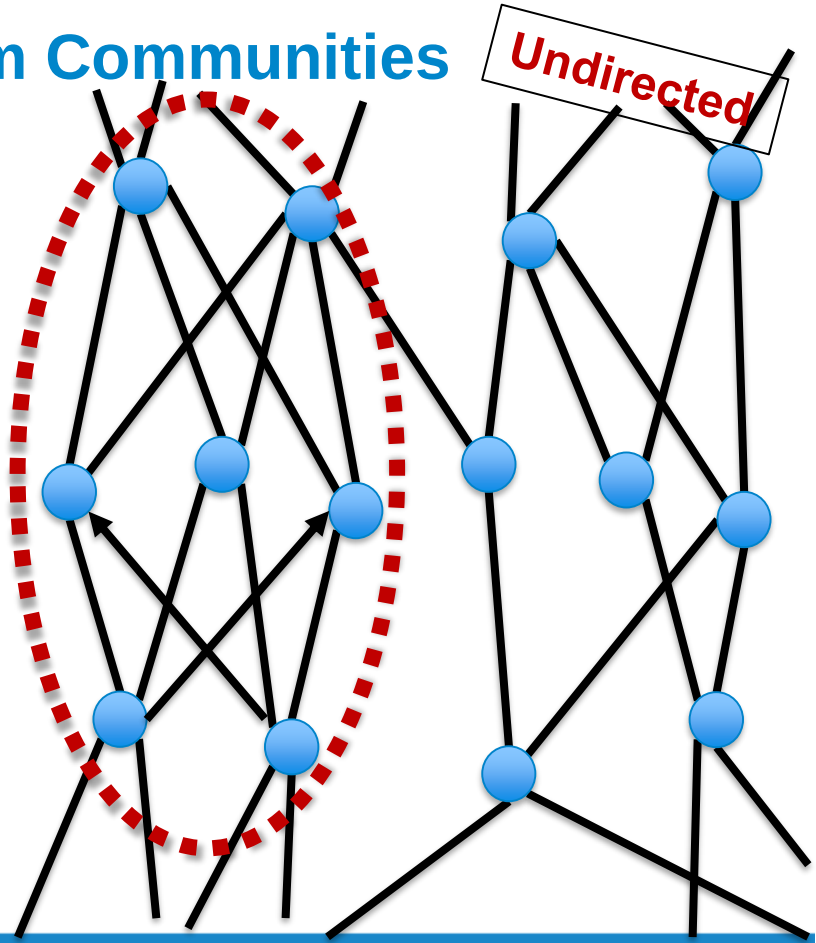
An **ANTICHAIN** is a set of nodes with **no path** between any of the nodes



Similar nodes form Communities

A COMMUNITY is a set of **similar** nodes

Usually a set of nodes with more connections within the community than outside

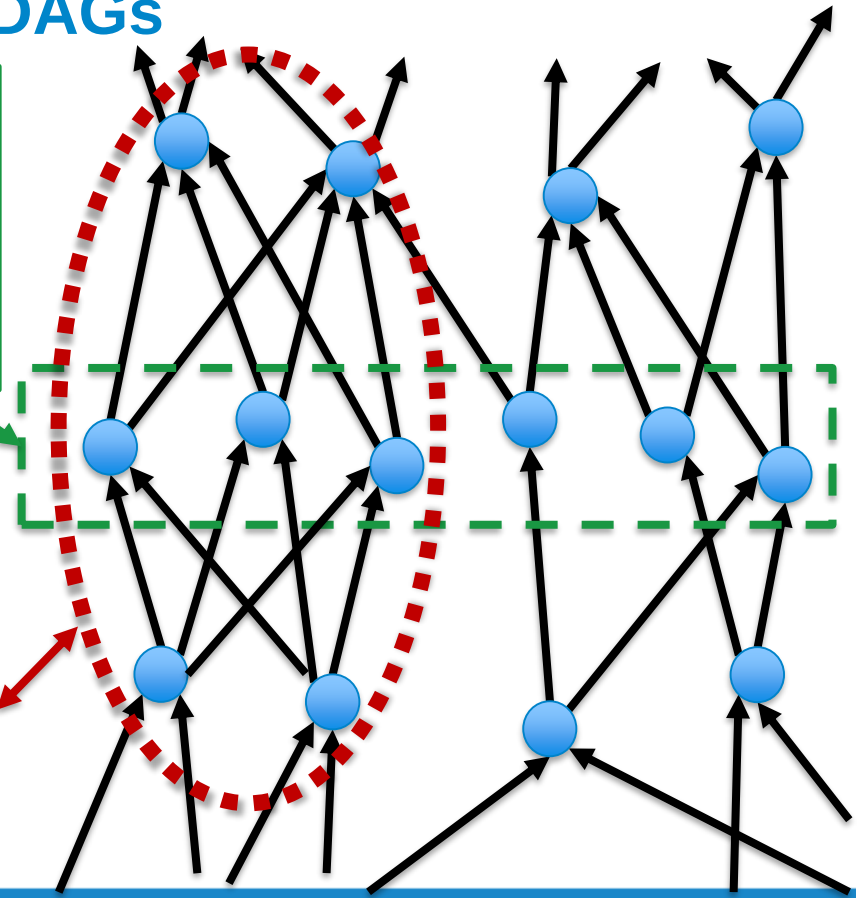


Communities in DAGs

Similar nodes form an **ANTICHAIN** of disconnected nodes in a DAG

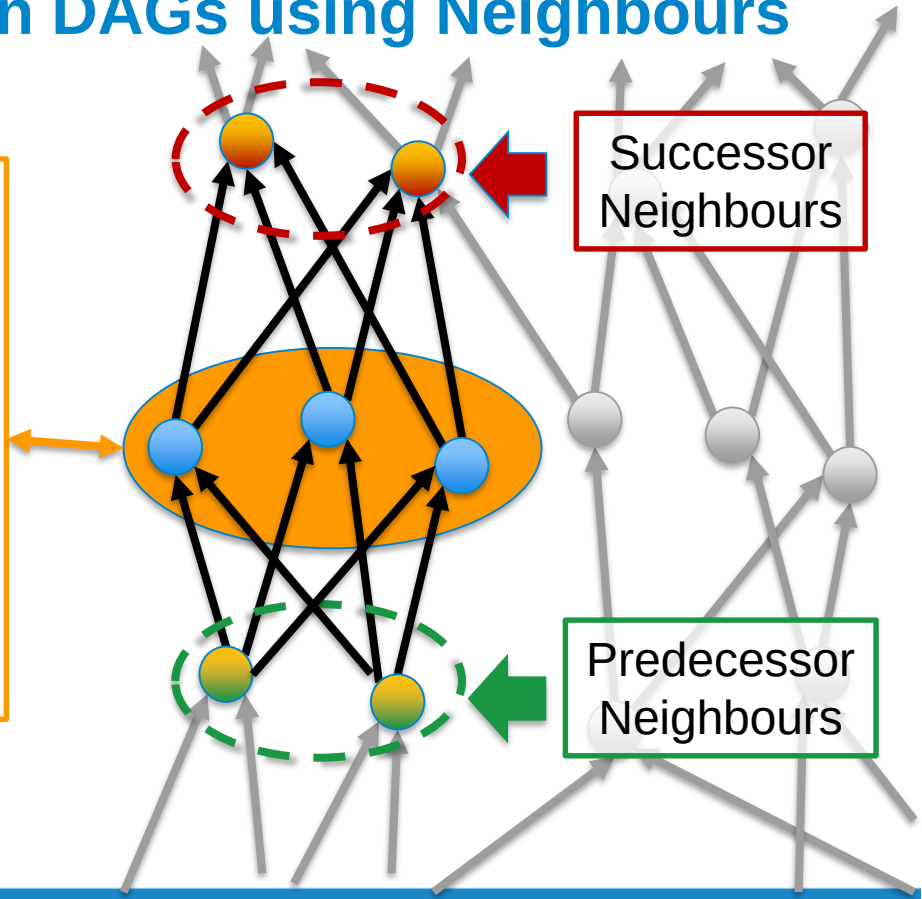
Tension

A **COMMUNITY** is a set of tightly connected nodes



Communities in DAGs using Neighbours

The disconnected nodes of an antichain are similar **if** they have similar neighbours



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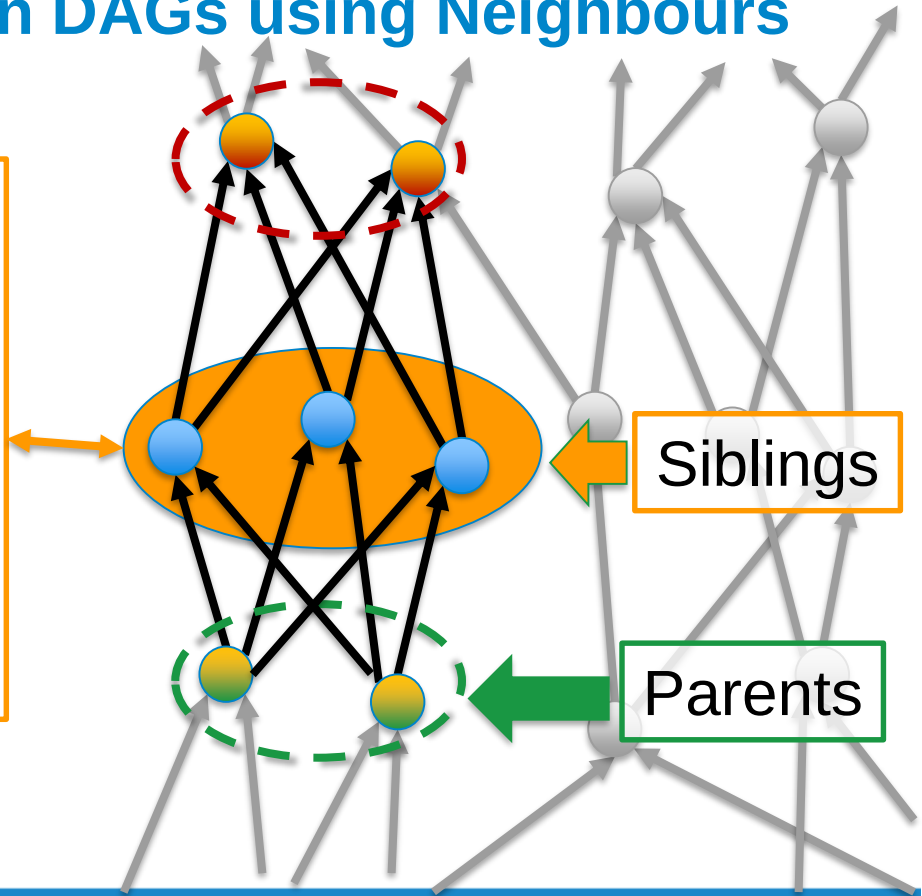


Siblinarity

- $\mathcal{A}$ = Antichain = Community,
  set of vertices with no path between any pair

- $\mathfrak{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \} =$ Partition of nodes into antichains

- $S(\mathfrak{A}) =$ SIBLINARITY measures the quality of a partition of nodes into antichains

  For each pair of nodes in the same antichain:-
  - add a measure of their similarity
  - subtract expected value of similarity in a null model

$$S(\mathfrak{A}) = \sum_{\mathcal{A} \in \mathfrak{A}} \sum_{n,m \in \mathcal{A}} \left(\mathrm{sim}(n,m) - \mathrm{sim}_{\mathrm{null}}(n,m) \right)$$

Siblinarity

A is adjacency matrix of DAG

SIBLINARITY quality function

$$T = A^{\mathrm{T}} A$$

Degree

$$\kappa_n := \sum_m T_{nm}$$

$$S(\mathfrak{A}) = \sum_{\mathcal{A}\in\mathfrak{A}} \sum_{n\in\mathcal{A}} \sum_{m\in\mathcal{A}} \left(T_{nm} - \frac{\kappa_n \kappa_m}{W} \right)$$

Total Weight

$$W = \sum_{n,m} T_{nm}$$

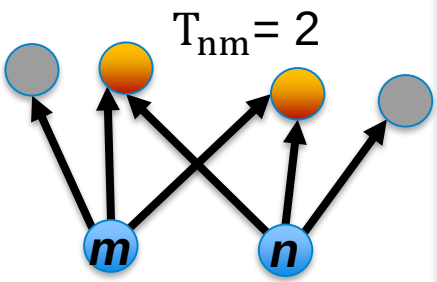
Antichain

Partition

$$\mathfrak{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \}$$

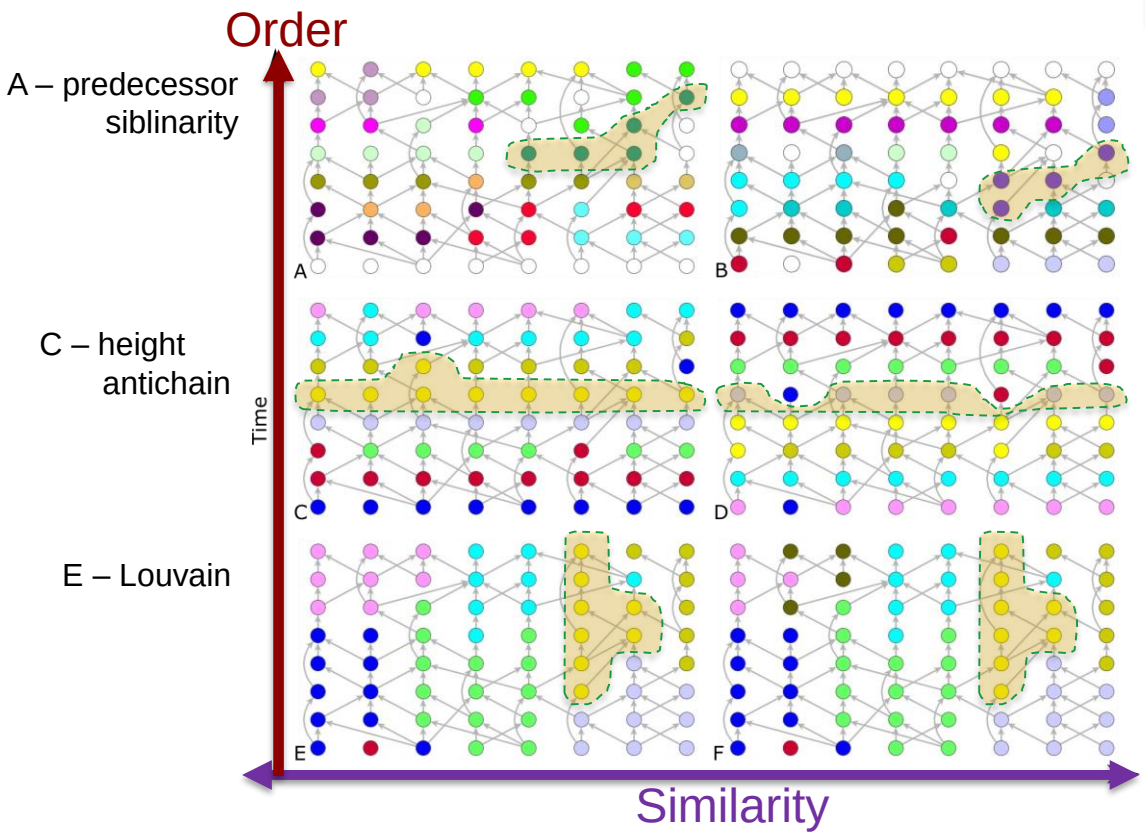
For example

$\mathrm{T_{nm}}$ = number of common successors



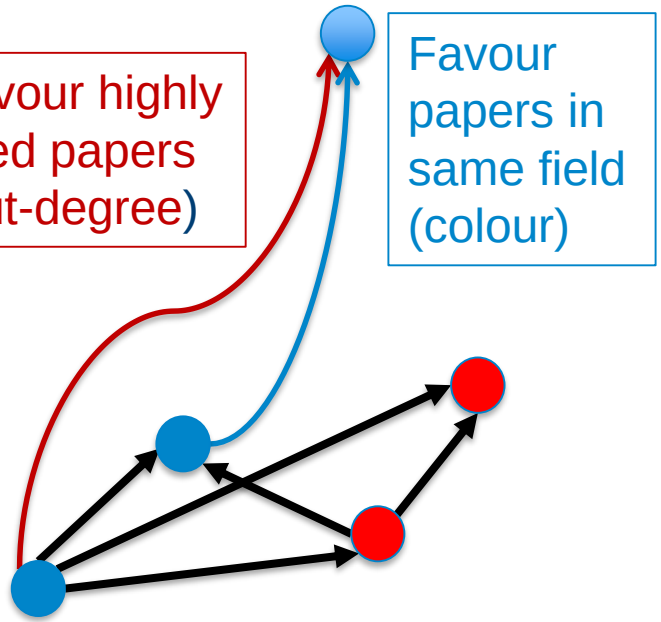
One DAG – Different Communities

= one example community



Price Model (1965) with Fields

- Models a citation network = DAG
- Directed version of BA model
- New paper assigned random field ***f***
- Cites older papers based on
  - preferential attachment
  - Prefer papers from own field ***f*** with probability ***p***.



Diversity in Communities

High D = All fields appear equally

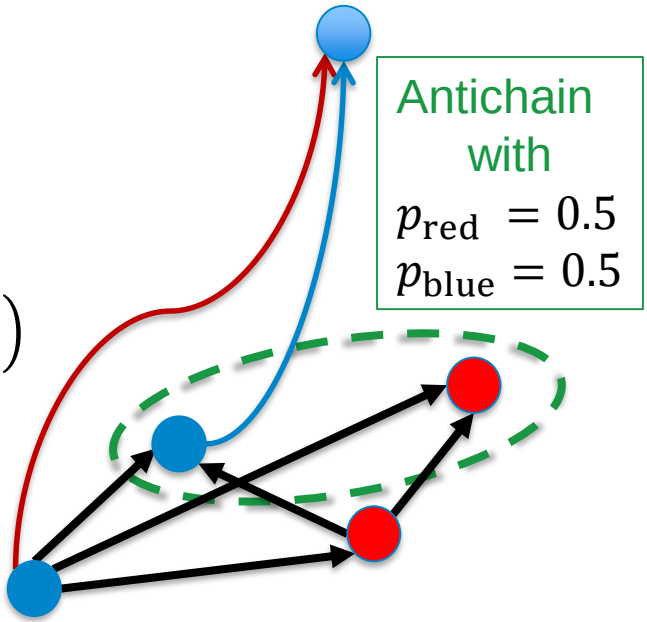
Measure diversity of fields in antichains (= communities) with

$$D(\mathfrak{A}) = \frac{1}{|\mathfrak{A}|} \sum_{\mathcal{A} \in \mathfrak{A}} \exp\big(S(\mathcal{A})\big)$$

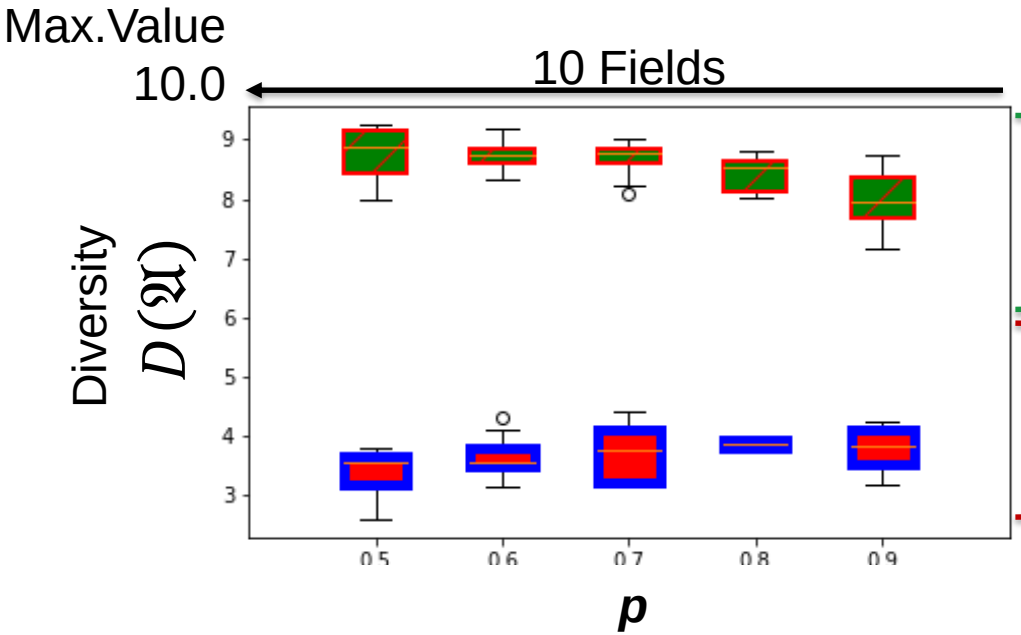
$$S(\mathcal{A}) = -\sum_{f} p_f(\mathcal{A}) \log\left(p_f(\mathcal{A})\right)$$

Sum over all fields

$p_f(\mathcal{A})$ = Fraction of nodes of field *f* in $\mathcal{A}$



Diversity of Communities in Modified Price model



Height partitions have diverse fields

Siblinarity communities are more similar

One Siblinarity Community of Papers in arXiv repository hep-th section

U-Duality and Central Charges in Various Dimensions Revisited

Enhancement of Supersymmetry Near 5d **BLACK HOLE** Horizon

Extremality Versus Supersymmetry in Stringy **BLACK HOLES**

Microscopic Entropy of $N = 2$ Extremal **BLACK HOLES**

Dipole Moments of **BLACK HOLES** and String States

Static $N = 2$ **BLACK HOLES** For Quadratic Prepotentials

Electrically Charged **BLACK-HOLES** for the Heterotic String Compactified

Vertical versus Diagonal Dimensional Reduction for p-branes

Four Dimensional **BLACK HOLES** and Strings with Rescaled Tension

Supersymmetric dyonic **BLACK HOLES** of IIA string on Six Torus

The Complete Form of $N = 2$ Supergravity and its Place in the General

Wrapped Supermembrane

Antichain Partition based on common citers

Conclusions

arXiv:1909.11818

- The **ORDER** in a Directed Acyclic Graph must be considered in any network measurement.
- To find communities in DAGs we use **ANTICHAINS**
- We use neighbour overlap to find similar nodes in antichains

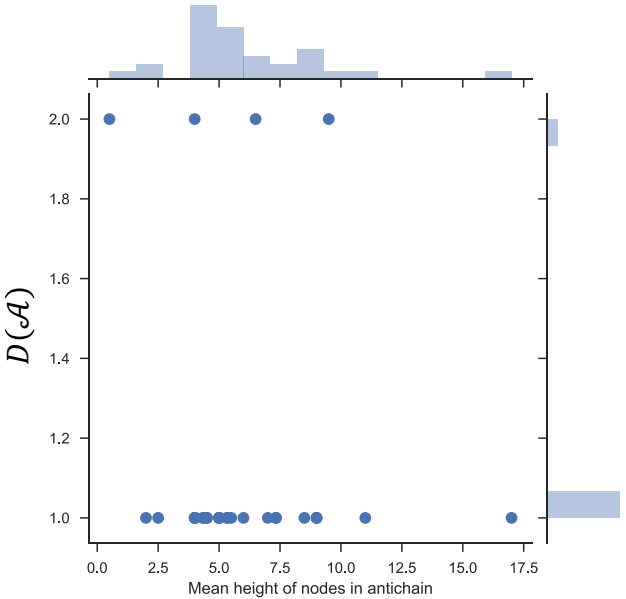
Work with Vaiva VASILIAUSKAITE

Extra Slides

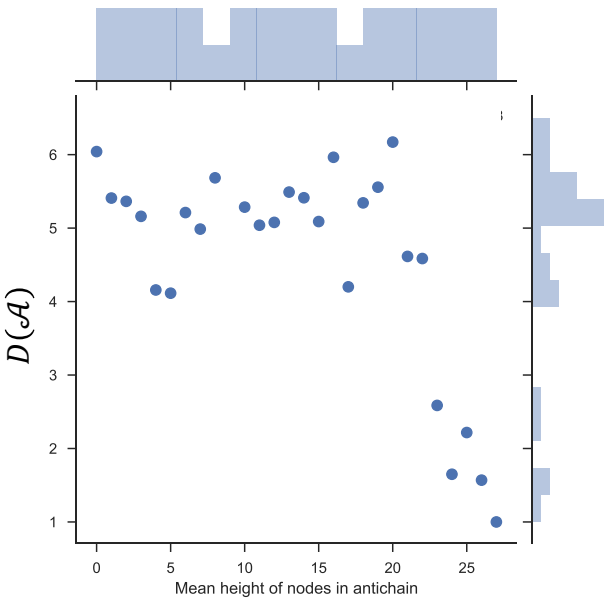
$$D(\mathcal{A}) = \exp\left(-\sum_f p_f \log p_f\right)$$

Diversity in Cora citation network

Siblinarity communities



Height antichains

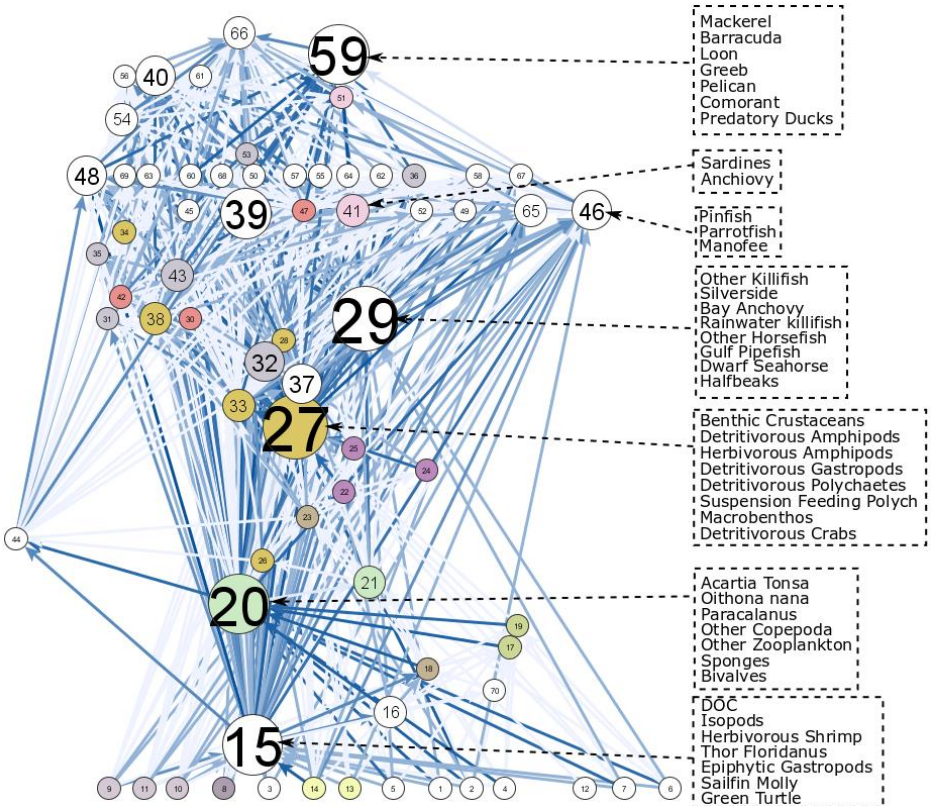


Diversity of topic labels *f* of papers in siblinarity communities (based on common citers) is smaller than the diversity in height antichains

Florida Bay Food Web

Nodes = Antichain Communities

An induced graph is a coarser version of the initial food web and can help understand more clearly the flows of energy in the system



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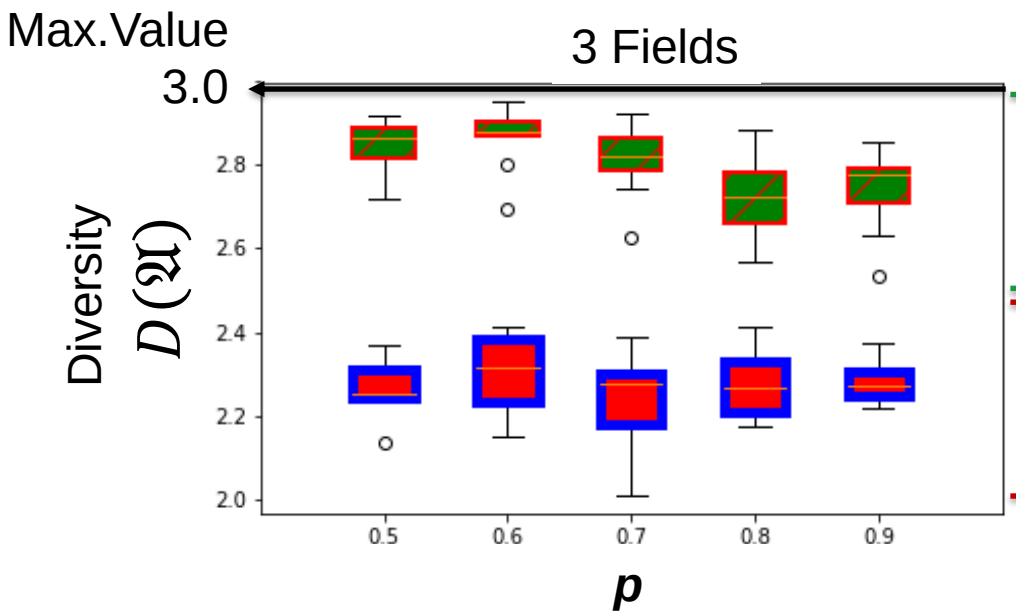

Height partitions have diverse fields

Siblinarity communities are more similar

Two antichains of papers from High Energy Physics Th
arXiv repository

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Modular invariance of **string** theory on AdS_3

Quantum Coherent **String** States in AdS_3 and $SL(2,R)$ WZ

Vertex Operators in $AdS_3 \times S^3$ Background with NS_NS Fl
Berkovits-Vafa-Witten Variables

Superstring Theory on $AdS_3 \times G/H$ and Boundary $N =$
Superconformal Symmetry

Constructing Classical and Quantum Superconformal Algeb
Boundary of AdS_3

String Theory on AdS_3 as Discrete Light-Cone Liouville T

Correlation functions for M^N/S_N orbifolds

Conformal symmetry of **superstrings** on $AdS_3 \times S^3 \times T$
system

Conformal Blocks and Correlators in WZNW Model. I. Genu

Topological **String** on $AdS_3 \times N$

Boundary Fluctuations of AdS **String**

based on common citers)