Estimative of gravity-gradient tensor components via fast iterative equivalent-layer technique

Larissa S. Piauilino*

Fillipe C. L. Siqueira

Vanderlei C. Oliveira Jr

Valeria C. F. Barbosa

Observatório Nacional, Brazil

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Agenda

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Agenda

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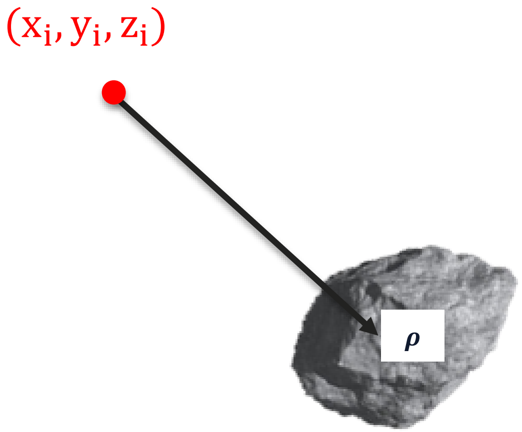
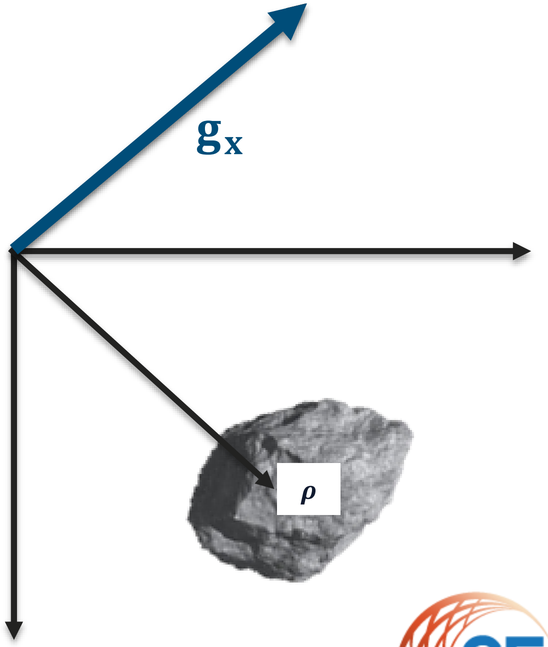
- Gravitational attraction produced by a 3D gravity source.



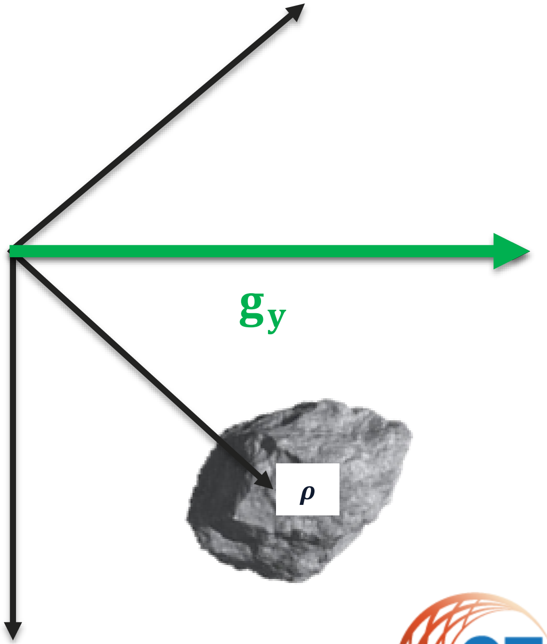
- Gravitational attraction produced by a 3D gravity source.
  Can be decomposed in three components:

$$\begin{bmatrix} g_x \\ \\ \\ \end{bmatrix}$$



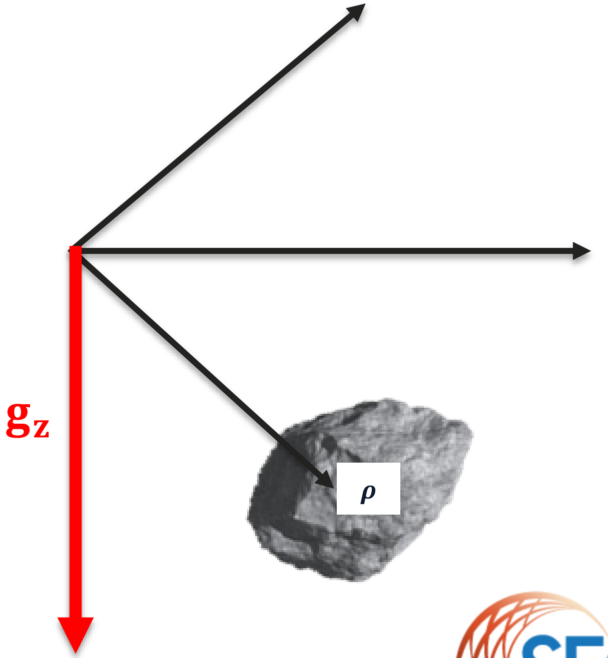
- Gravitational attraction produced by a 3D gravity source.
  Can be decomposed in three component:

$$\begin{bmatrix} g_y \end{bmatrix}$$



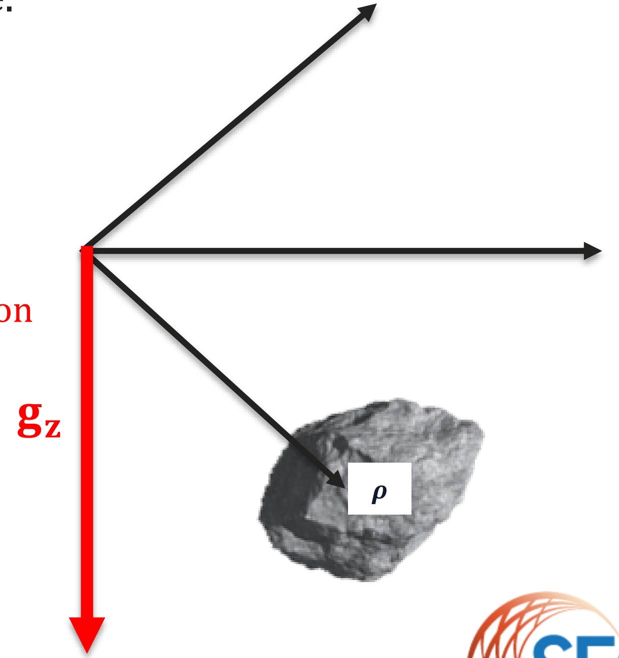
- Gravitational attraction produced by a 3D gravity source.
  Can be decomposed in three component:

$$\begin{bmatrix} \\ \\ g_z \end{bmatrix}$$



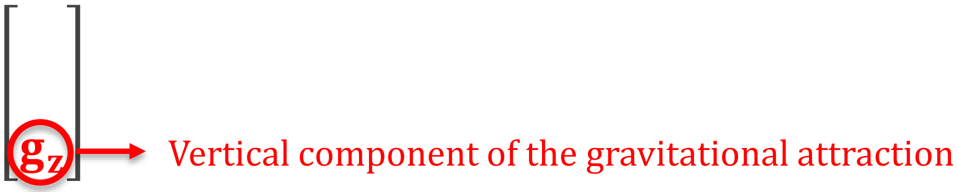
- Gravitational attraction produced by a 3D gravity source.
  Can be decomposed in three component:

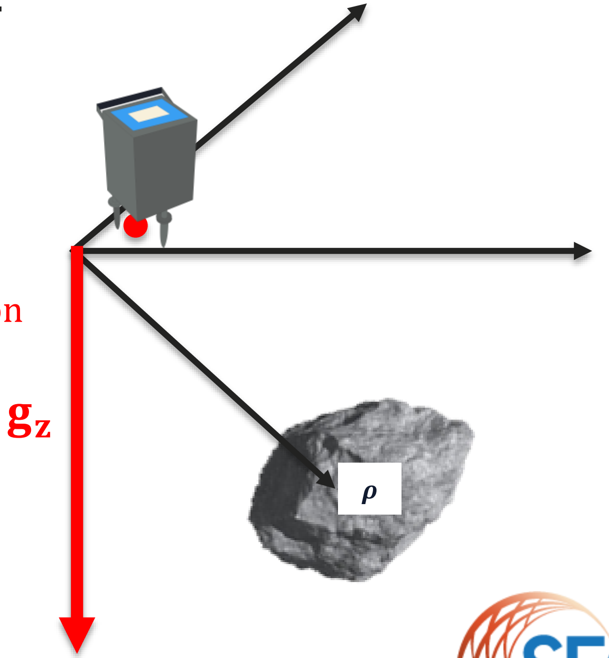
$$\begin{bmatrix} \\ \\ g_z \end{bmatrix} \rightarrow$$ Vertical component of the gravitational attraction



- Gravitational attraction produced by a 3D gravity source.

  Can be decomposed in three component:

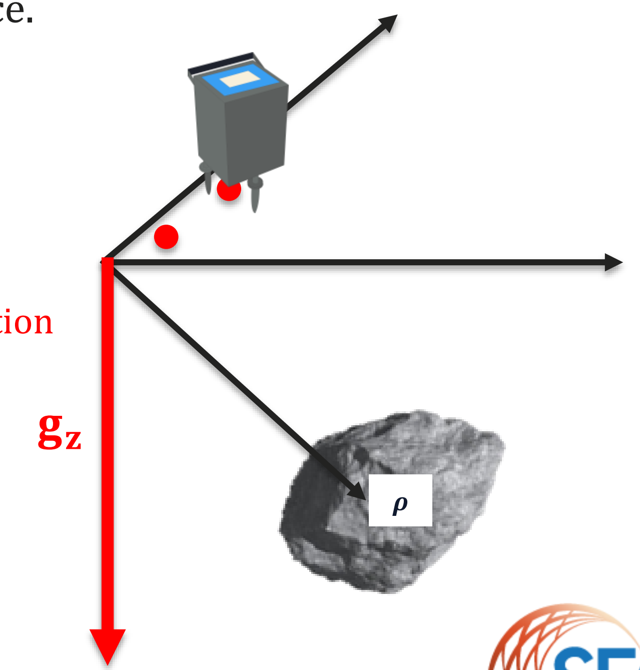




- Gravitational attraction produced by a 3D gravity source.

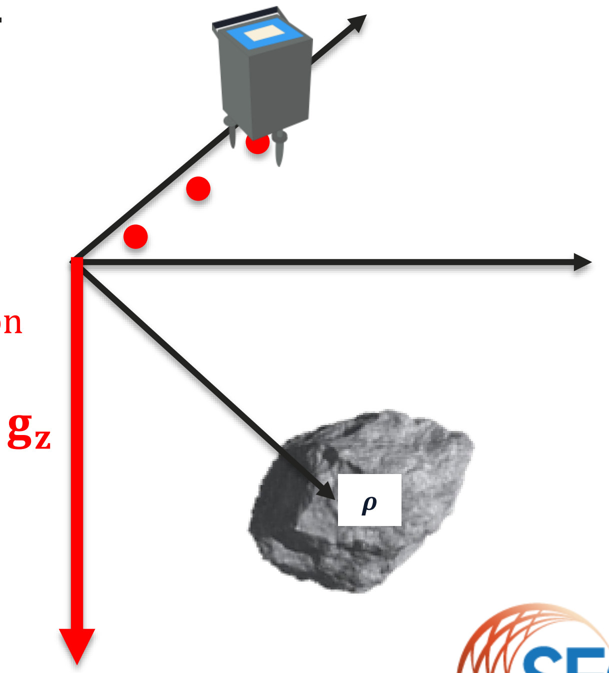
  Can be decomposed in three component:

$$\begin{bmatrix} \\ \\ g_z \end{bmatrix} \rightarrow$$ Vertical component of the gravitational attraction



- Gravitational attraction produced by a 3D gravity source.
  Can be decomposed in three component:

$$\begin{bmatrix} \\ \\ g_z \end{bmatrix}$$ → Vertical component of the gravitational attraction



- Gravitational attraction produced by a 3D gravity source.
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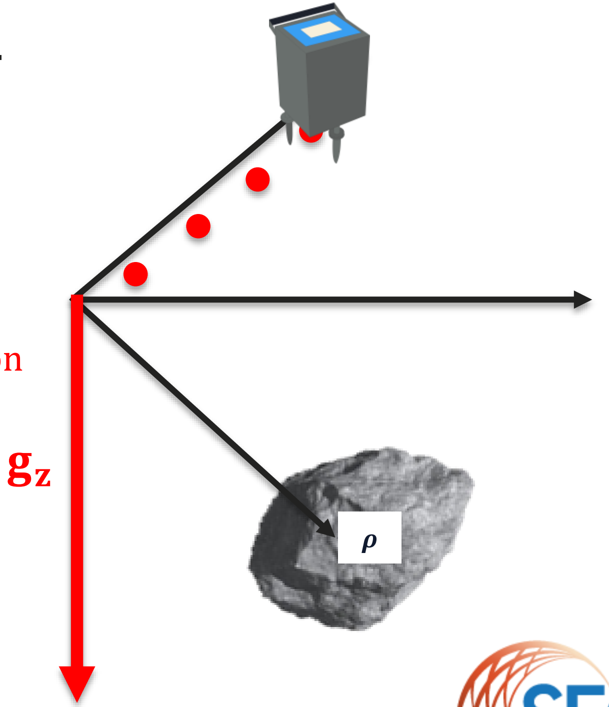
$\begin{bmatrix} \\ \\ g_z \end{bmatrix}$ → Vertical component of the gravitational attraction



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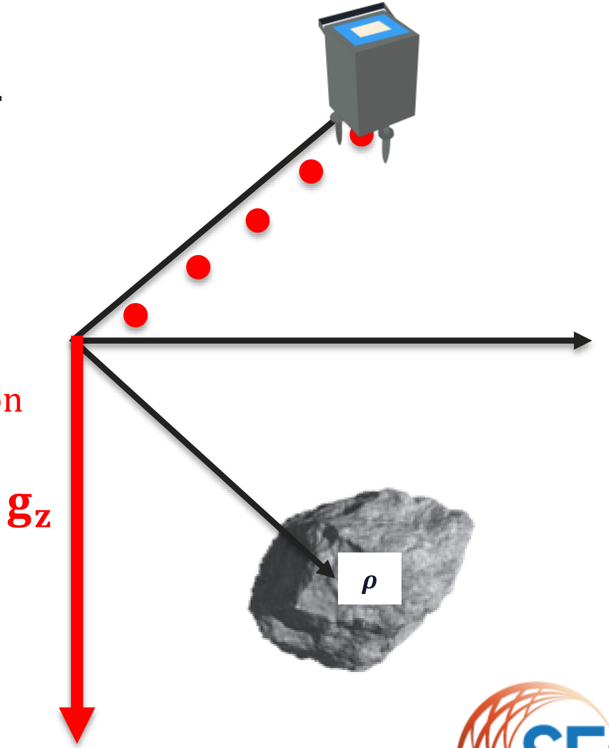

- Gravitational attraction produced by a 3D gravity source.
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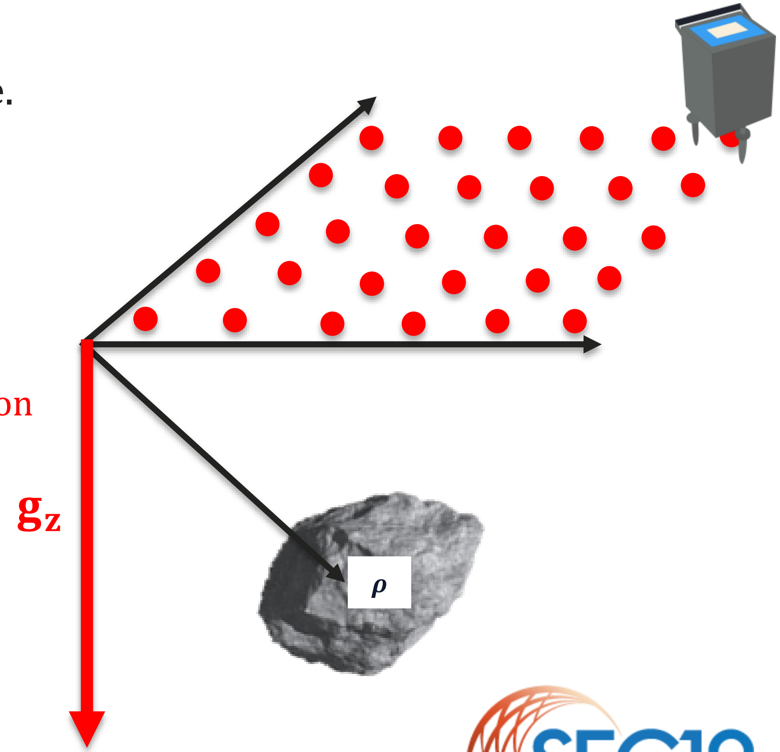
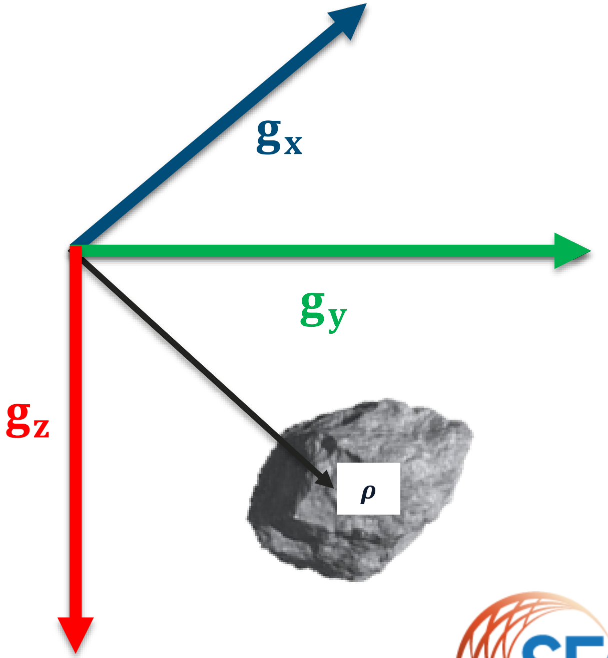


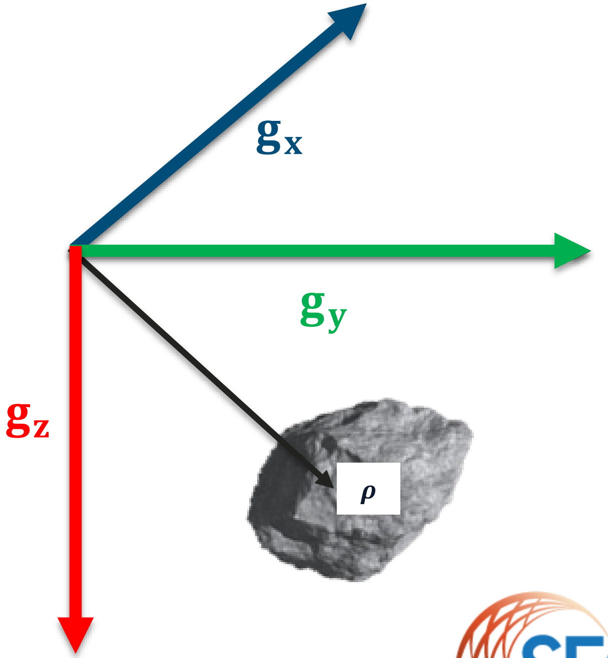
- Gravitational attraction produced by a 3D gravity source.
  Can be decomposed in three component:

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$



Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

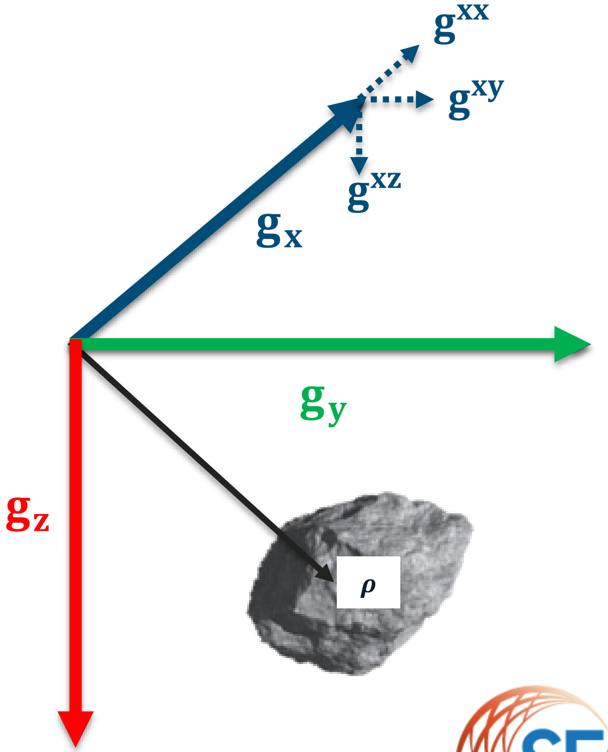


Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

Gravity-gradient tensor

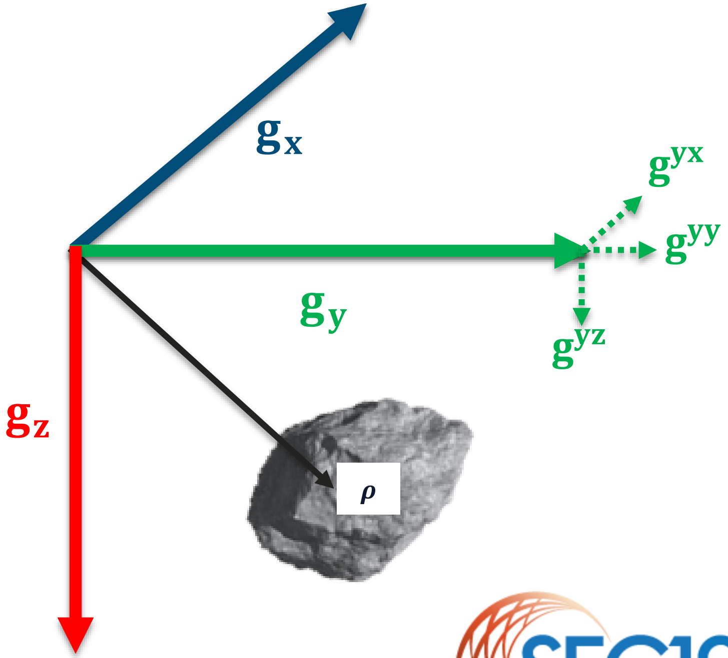
$$\longrightarrow \begin{bmatrix} g^{xx} & g^{xy} & g^{xz} \\ & & \\ & & \end{bmatrix}$$



Gravity vector

Gravity-gradient tensor

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} \longrightarrow \begin{bmatrix} & & \\ g^{yx} & g^{yy} & g^{yz} \\ & & \end{bmatrix}$$



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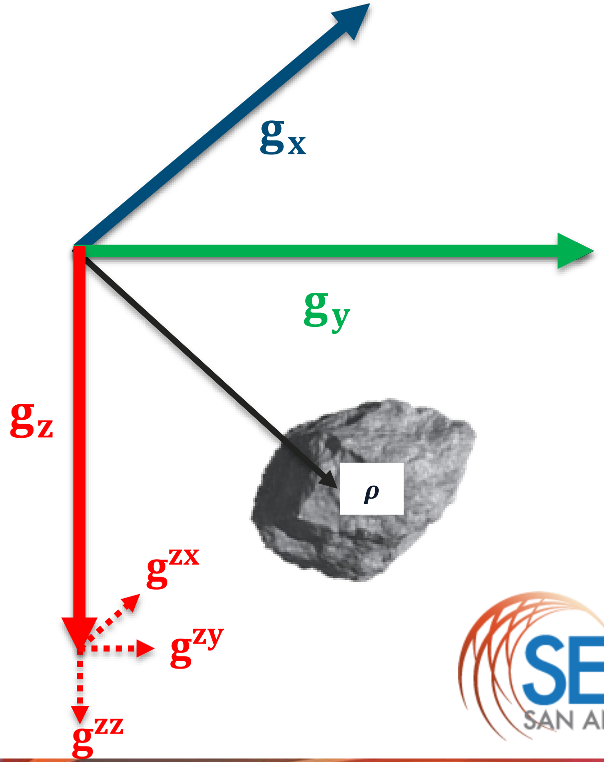
Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

$\longrightarrow$

Gravity-gradient tensor

$$\begin{bmatrix} & & \\ & & \\ g^{zx} & g^{zy} & g^{zz} \end{bmatrix}$$

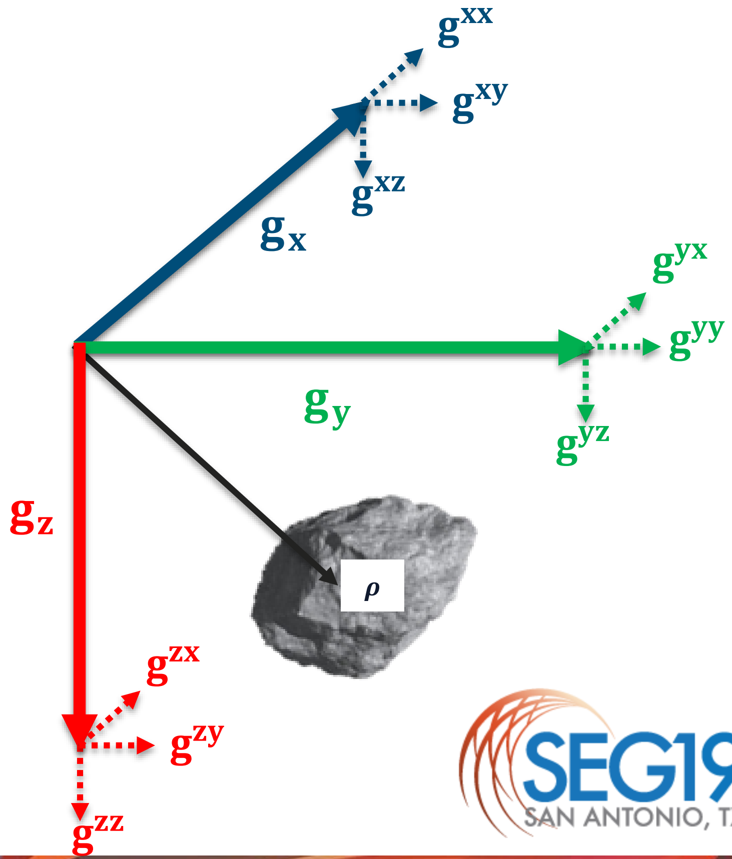


Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

Gravity-gradient tensor

$$\begin{bmatrix} g^{xx} & g^{xy} & g^{xz} \\ g^{yx} & g^{yy} & g^{yz} \\ g^{zx} & g^{zy} & g^{zz} \end{bmatrix}$$



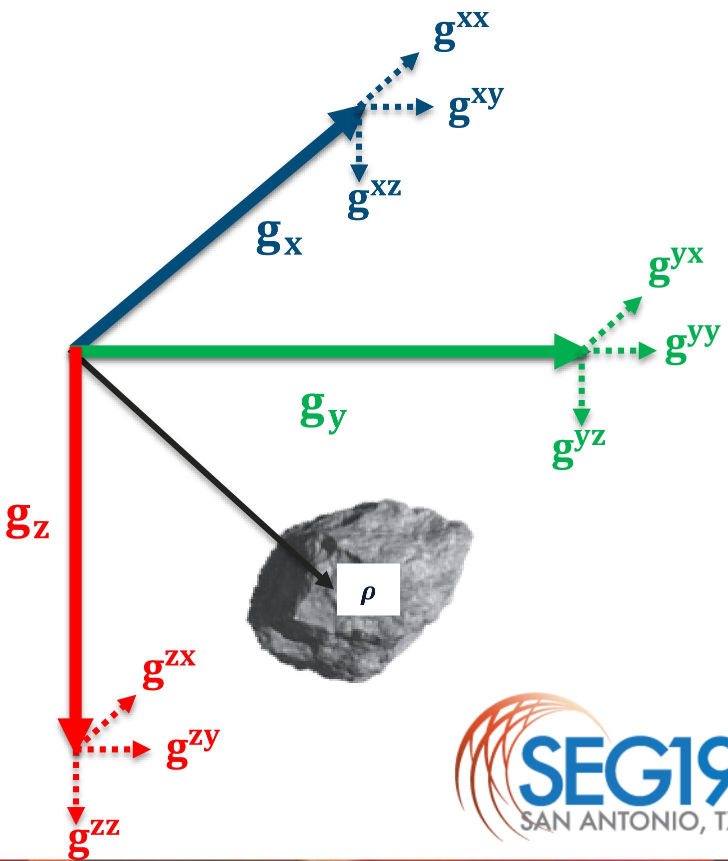
Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

Gravity-gradient tensor

$$\begin{bmatrix} g^{xx} & g^{xy} & g^{xz} \\ g^{yx} & g^{yy} & g^{yz} \\ g^{zx} & g^{zy} & g^{zz} \end{bmatrix}$$

Symmetric matrix !



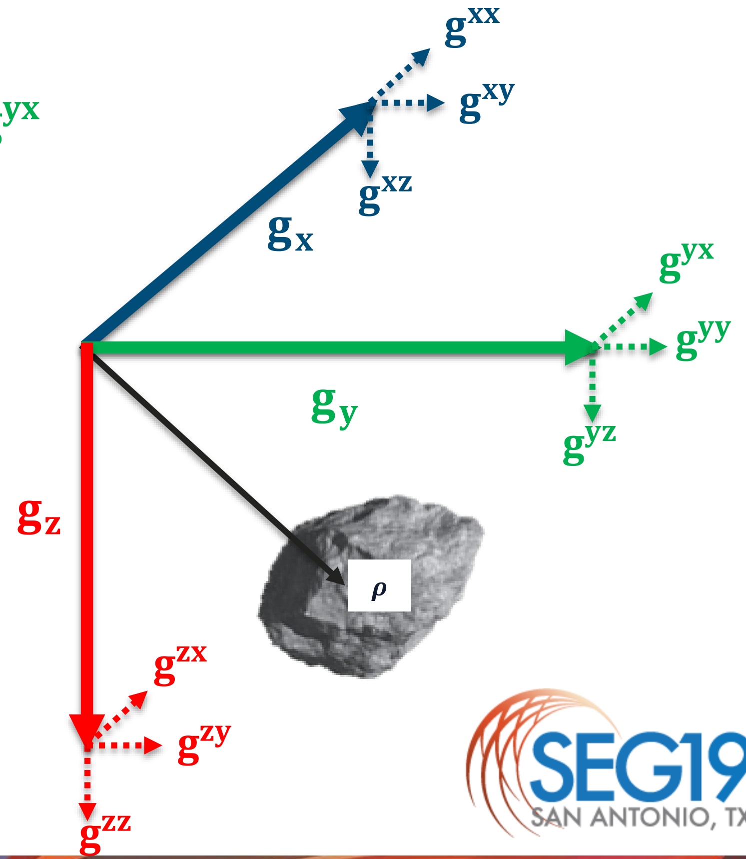
Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

Gravity-gradient tensor

$$\begin{bmatrix} g^{xx} & g^{xy} & g^{xz} \\ g^{yx} & g^{yy} & g^{yz} \\ g^{zx} & g^{zy} & g^{zz} \end{bmatrix} \quad \left\{ g^{xy} = g^{yx} \right.$$

Symmetric matrix !



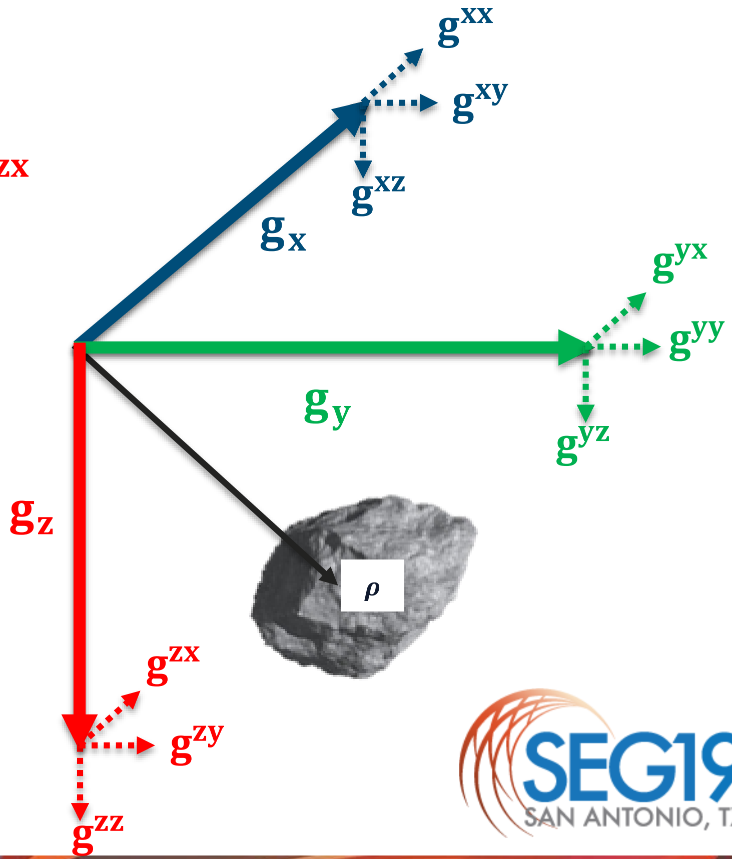
Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

Gravity-gradient tensor

$$\begin{bmatrix} g^{xx} & g^{xy} & g^{xz} \\ g^{yx} & g^{yy} & g^{yz} \\ g^{zx} & g^{zy} & g^{zz} \end{bmatrix} \left\{ g^{xz} = g^{zx} \right.$$

Symmetric matrix !



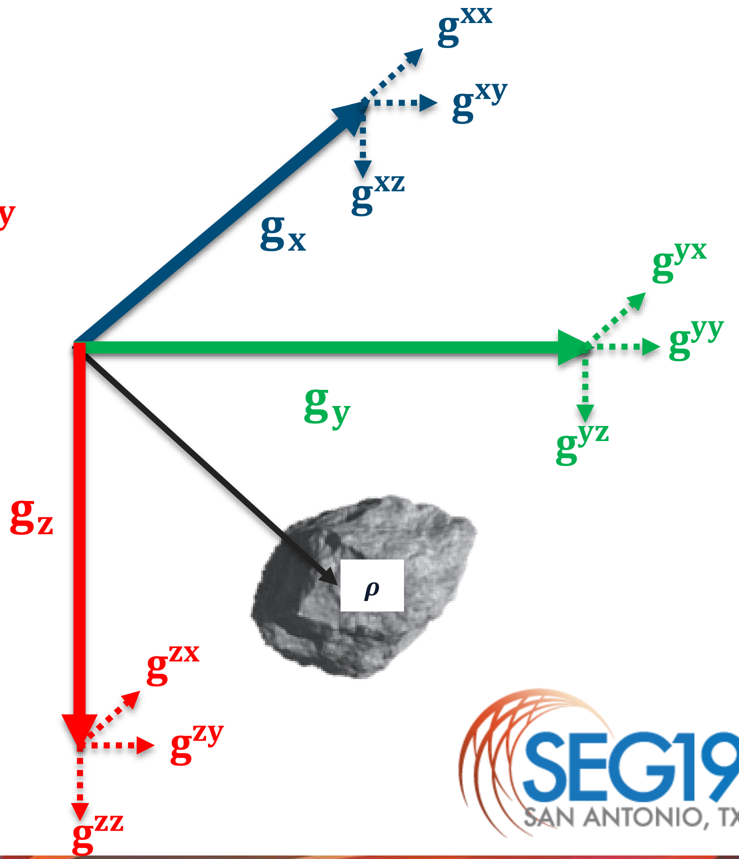
Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

Gravity-gradient tensor

$$\begin{bmatrix} g^{xx} & g^{xy} & g^{xz} \\ g^{yx} & g^{yy} & g^{yz} \\ g^{zx} & g^{zy} & g^{zz} \end{bmatrix} \quad \left\{ g^{yz} = g^{zy} \right.$$

Symmetric matrix !



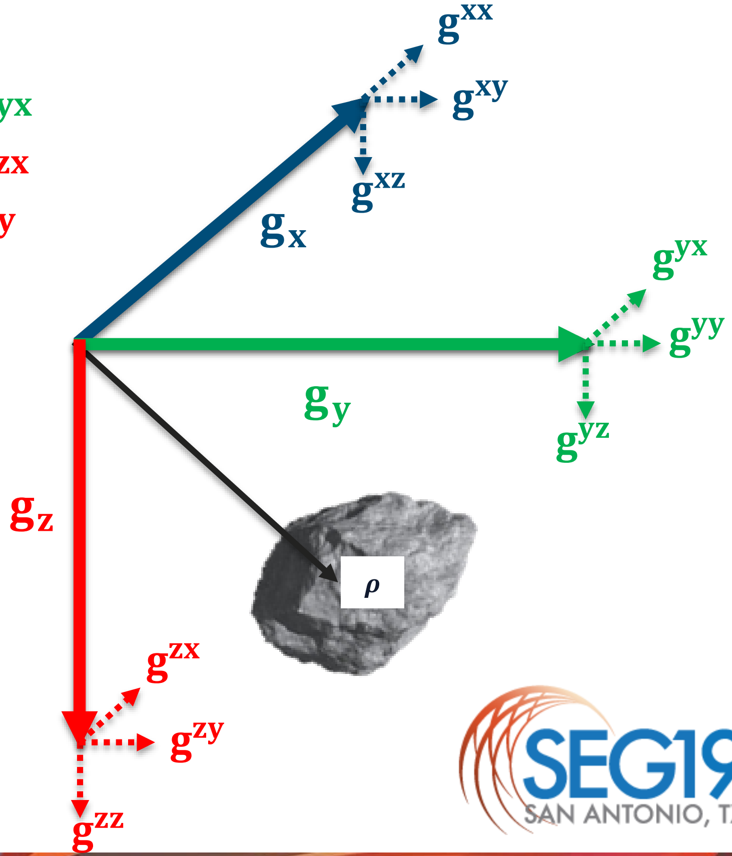
Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

Gravity-gradient tensor

$$\begin{bmatrix} g^{xx} & g^{xy} & g^{xz} \\ g^{yx} & g^{yy} & g^{yz} \\ g^{zx} & g^{zy} & g^{zz} \end{bmatrix} \qquad \begin{cases} g^{xy} = g^{yx} \\ g^{xz} = g^{zx} \\ g^{yz} = g^{zy} \end{cases}$$

Symmetric matrix !

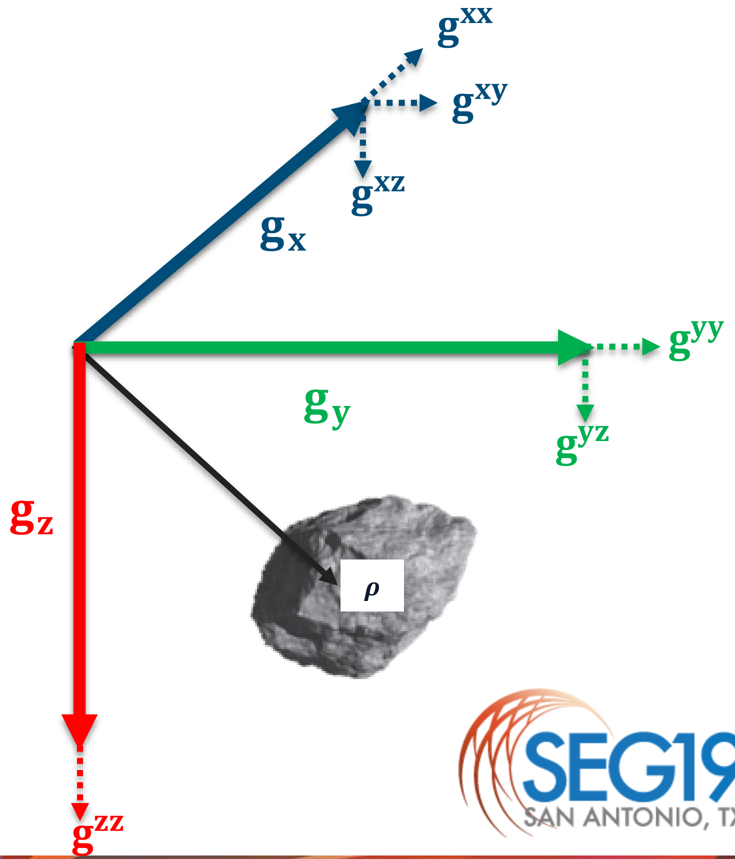


Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

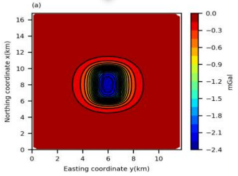
Gravity-gradient tensor

$$\begin{bmatrix} g^{xx} & g^{xy} & g^{xz} \\ & g^{yy} & g^{yz} \\ & & g^{zz} \end{bmatrix}$$



Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$



Gravity-gradient tensor

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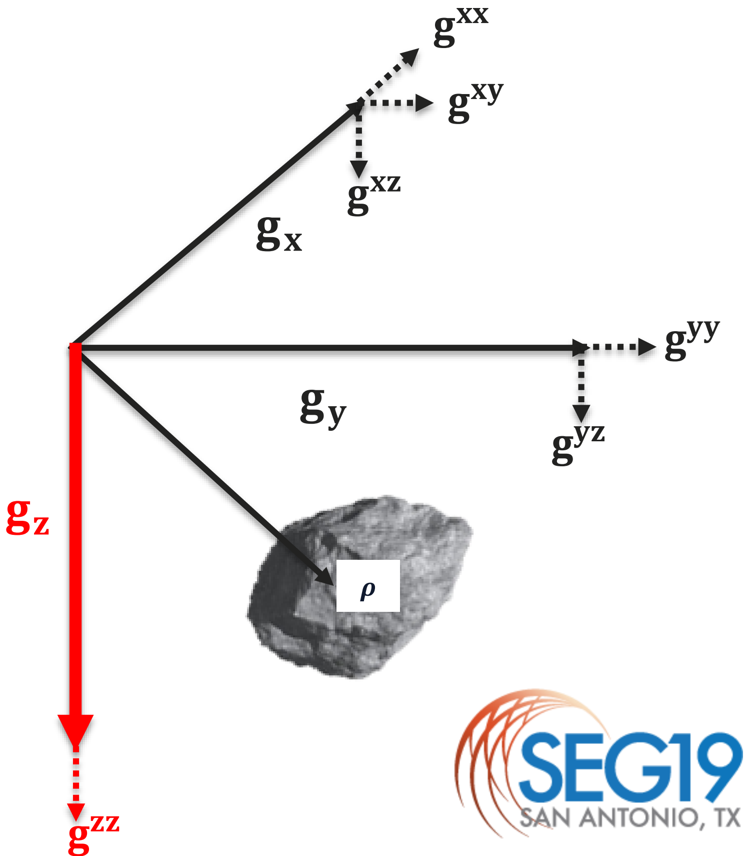

Gravity vector

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}$$

Gravity-gradient tensor

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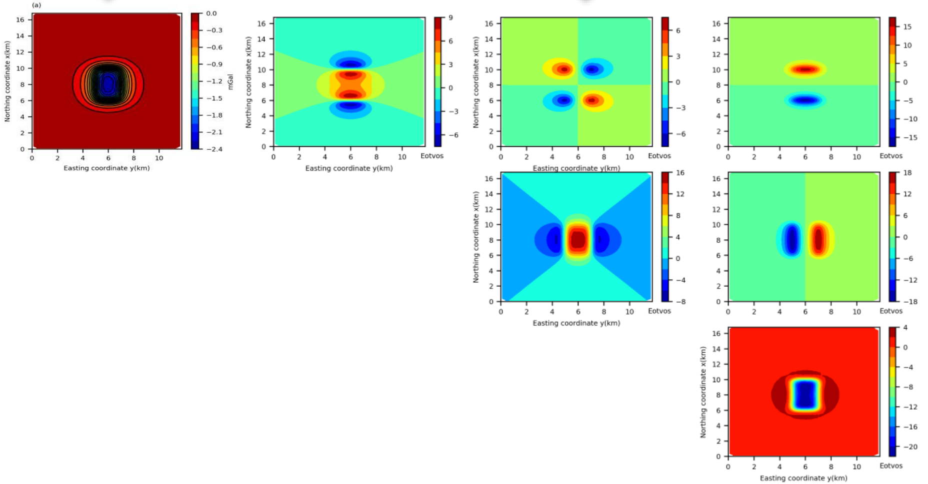

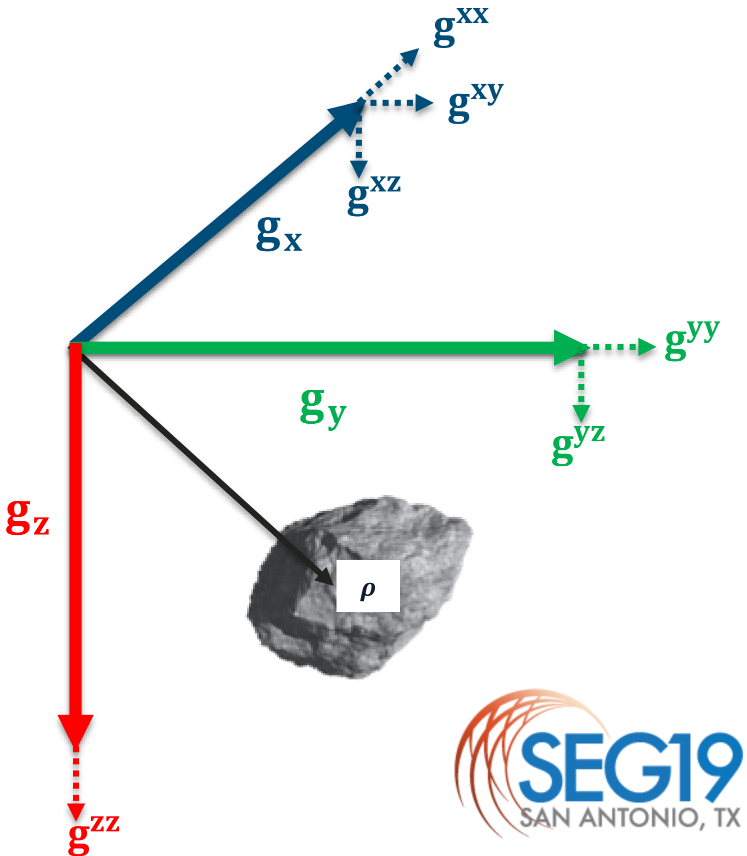

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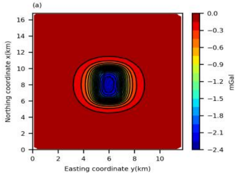
$\mathbf{g}_z$–component

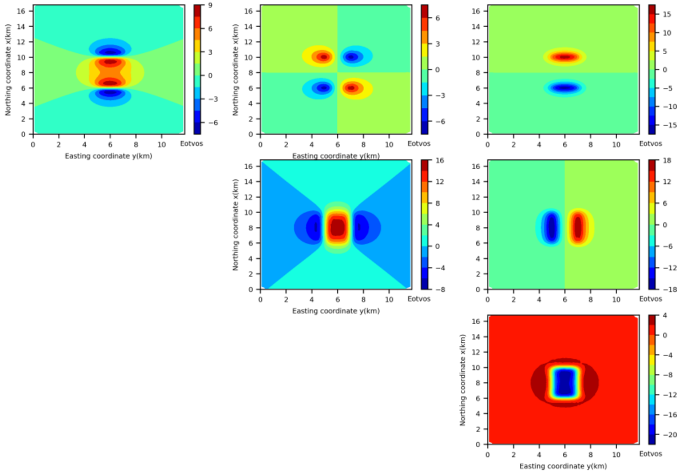
$$\begin{bmatrix} \mathbf{g}_x \\ \mathbf{g}_y \\ \mathbf{g}_z \end{bmatrix}$$



- The $\mathbf{g}_z$– component has historically been used because of the ease of interpretation and the low-cost of measurement;
- Qualitative interpretation; e.g.: Horizontal delimitation of the source.
- Quantitative interpretation; e.g.: Inversion.

Gravity-gradient tensor

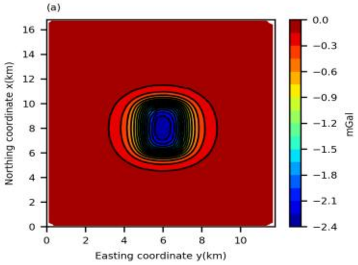
$$
\begin{bmatrix}
g^{xx} & g^{xy} & g^{xz} \\
 & g^{yy} & g^{yz} \\
 & & g^{zz}
\end{bmatrix}
$$



- Since the great improvement in the acquisition of accurate gravity-gradient data, these data have increasingly been used in geophysical prospecting (mining and hydrocarbon explorations; e.g., Zhdanov et al. 2004; Uieda and Barbosa, 2012; Martinez et al., 2013; and Carlos et al., 2014).
- Qualitative interpretation; e.g.: Horizontal delimitation of the source.
- Quantitative interpretation; e.g.: Inversion.

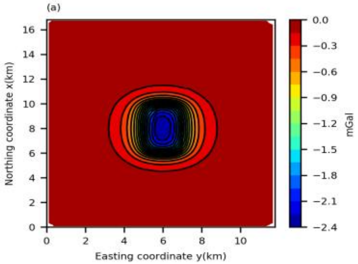
Objective

$\mathbf{g_z}$ -component



Objective

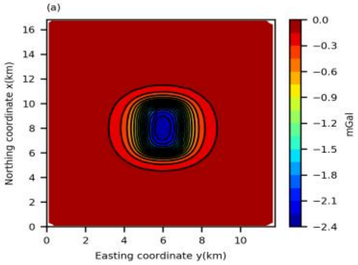
g_z -component



transforming

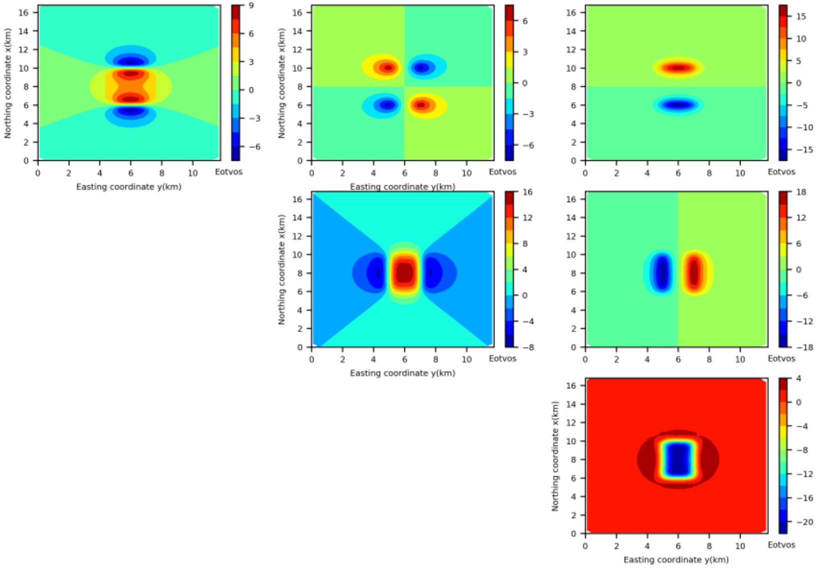
Objective

$\mathbf{g_z}$ -component



transforming

gravity gradient tensor components



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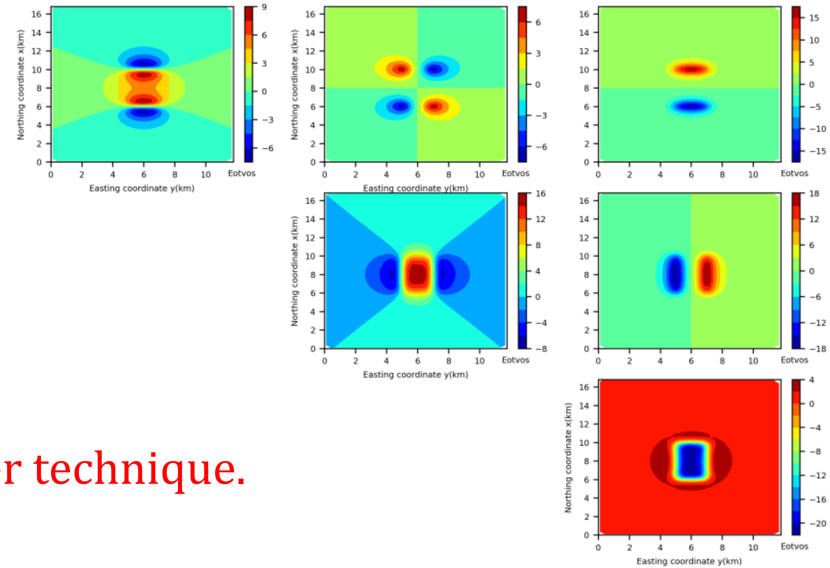
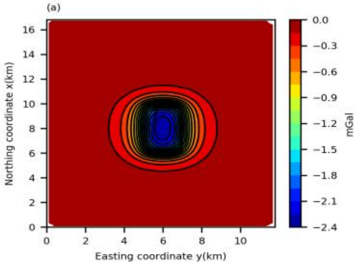
Objective

gravity gradient tensor components



g_z -component



transforming

...using the fast equivalent-layer technique.

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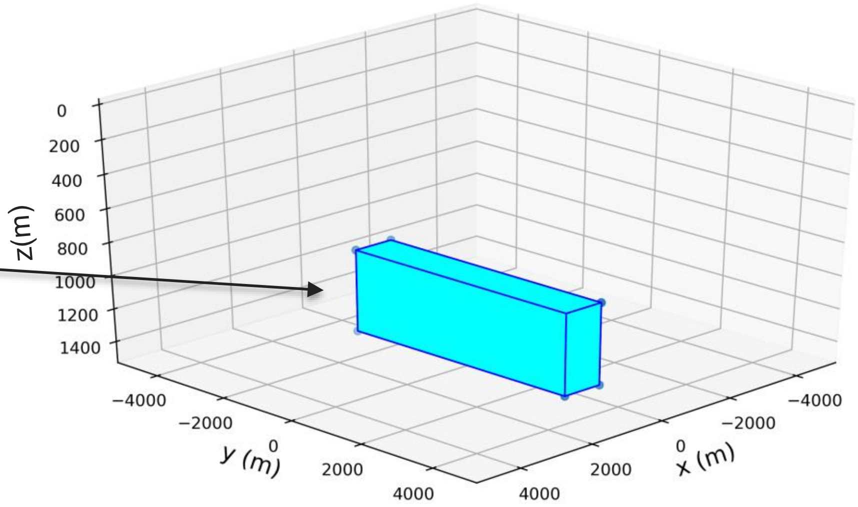
Agenda

1. Introduction
2. The classical equivalent layer
3. The fast equivalent layer
4. Applications to synthetic data
5. Application to real data
6. Conclusions
7. Acknowledgments

3D source with a density contrast

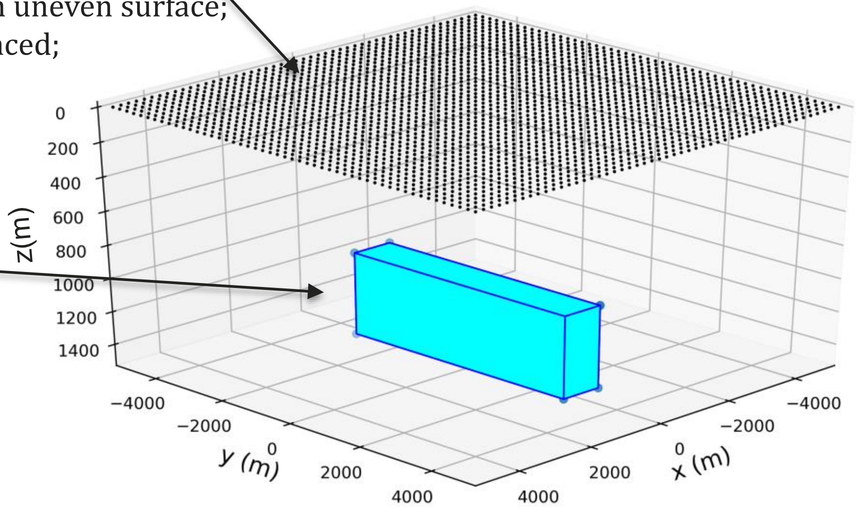


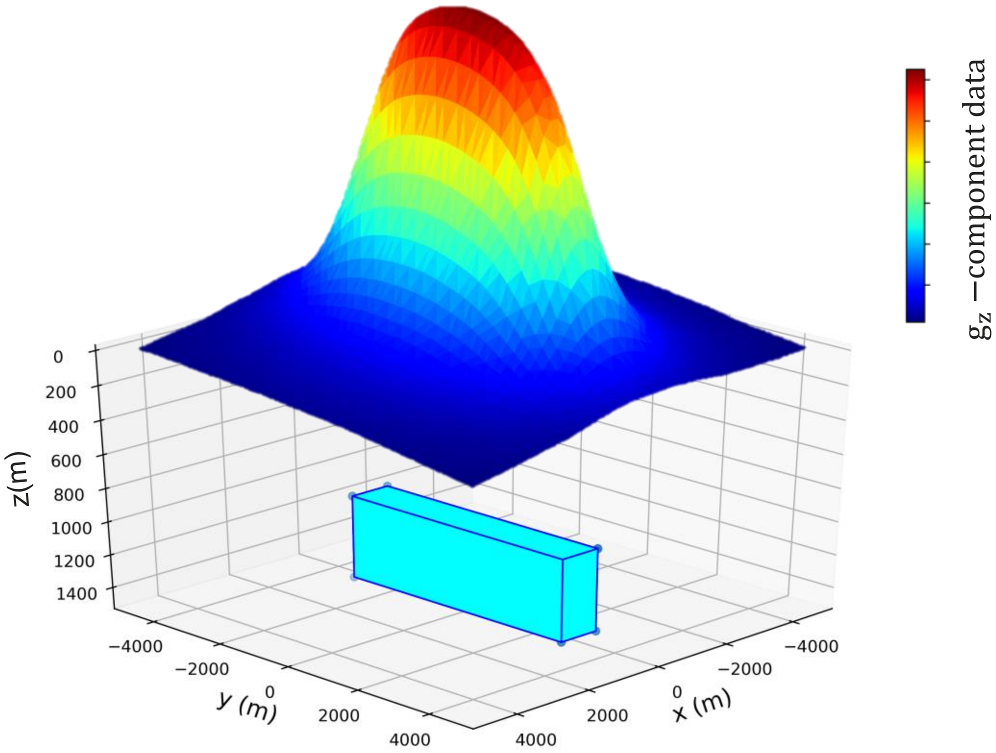
N observed g_z −component

may be: - acquired on an uneven surface;
- irregularly spaced;

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}$$

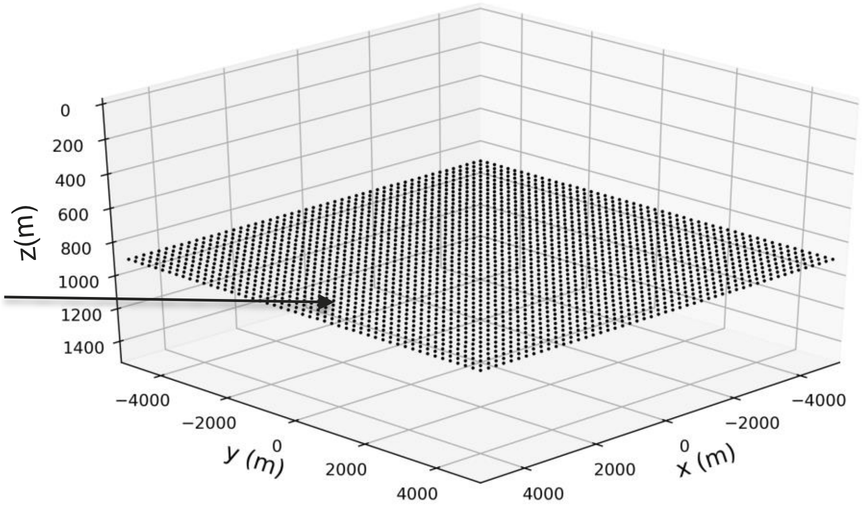
3D source with a density contrast



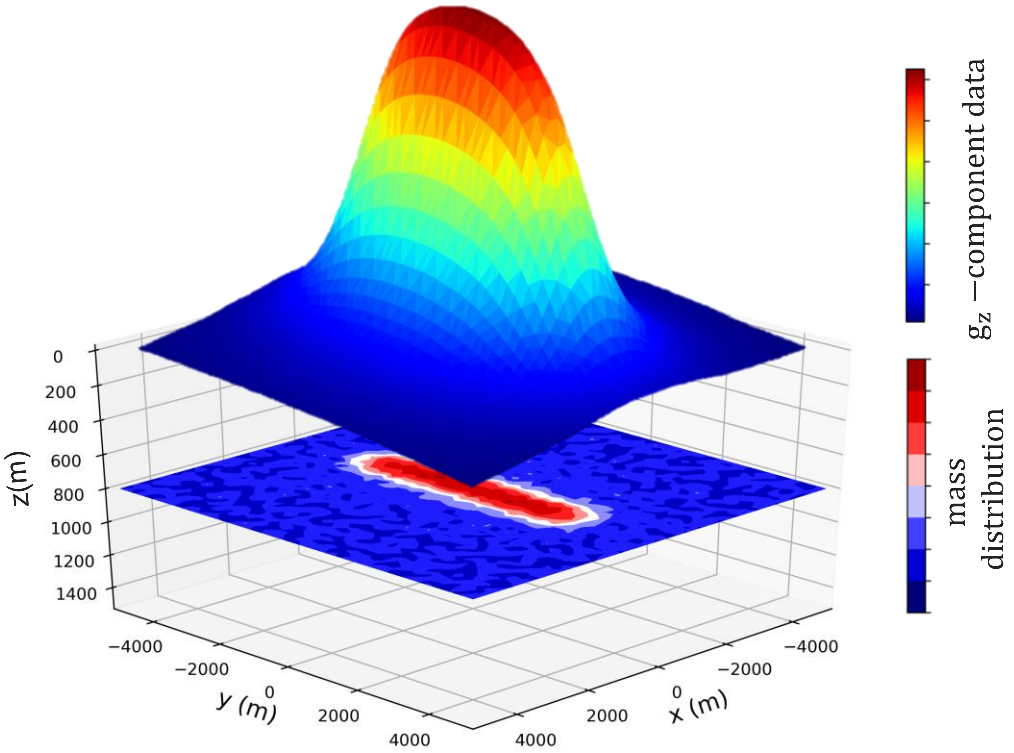


$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}$$

M equivalent sources (point masses)



$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}$$



$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}$$

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}$$

Forward modeling

$$\mathbf{g} = \mathbf{Am}$$

g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

Forward modeling

$$\mathbf{g} = \mathbf{A}\mathbf{m}$$

N x M matrix of the Green's function

Forward modeling

$$\mathbf{g} = \mathbf{A}\mathbf{m}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

Forward modeling

$$\mathbf{g} = \mathbf{A}\mathbf{m}$$

g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

Forward modeling

$$\mathbf{g} = \mathbf{A}\mathbf{m}$$

The $\hat{\mathbf{m}}$ vector can be estimated by using the zeroth-order Tikhonov regularization (Tikhonov and Arsenin, 1977):

$$\hat{\mathbf{m}} = (\mathbf{A}^{T}\mathbf{A} + \mu\mathbf{I})^{-1}\mathbf{A}^{T}\mathbf{g}^{o}$$

N > M

g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

Forward modeling

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g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

Forward modeling

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g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

N > M

Forward modeling

g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

$$\mathbf{g = Am}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

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$$\hat{\mathbf{m}} = (\mathbf{A}^T\mathbf{A} + \mu\mathbf{I})^{-1}\mathbf{A}^T\mathbf{g}^o$$

μ is the regularizing parameter

Forward modeling

$$\mathbf{g} = \mathbf{A}\mathbf{m}$$

The $\hat{\mathbf{m}}$ vector can be estimated by using the zeroth-order Tikhonov regularization (Tikhonov and Arsenin, 1977):

$$\hat{\mathbf{m}} = (\mathbf{A}^{\intercal}\mathbf{A} + \mu\mathbf{I})^{-1}\mathbf{A}^{\intercal}\mathbf{g}^{o}$$

μ is the regularizing parameter

I is an identity matrix of order *N*

g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

Forward modeling

$$\mathbf{g} = \mathbf{A}\mathbf{m}$$

g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

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$$\hat{\mathbf{m}} = (\mathbf{A}^T\mathbf{A} + \mu\mathbf{I})^{-1}\mathbf{A}^T\mathbf{g}^o$$

What is the problem with this estimate?

Forward modeling

g_z-component data vector

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}_{Nx1}$$

$$\mathbf{g} = \mathbf{Am}$$

Parameter vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix}_{Mx1}$$

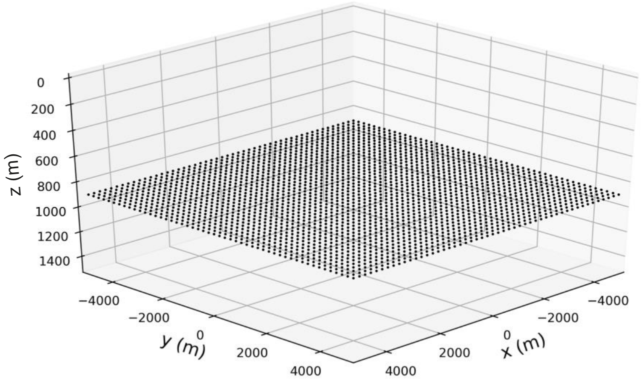
The $\hat{\mathbf{m}}$ vector can be estimated by using the zeroth-order Tikhonov regularization (Tikhonov and Arsenin, 1977):

$$\hat{\mathbf{m}} = \underbrace{(\mathbf{A}^{\intercal}\mathbf{A} + \mu\mathbf{I})}_{M \ x \ M}{}^{-1}\mathbf{A}^{\intercal}\mathbf{g}^{o}$$

What is the problem with this estimate? Computationally costly!

Agenda

- The iterative equivalent-layer proposed by Siqueira et al. (2017) is grounded on the excess of mass and on the positive correlation between the $\mathbf{g_z}$-component data and the masses on the equivalent layer.



M equivalent sources

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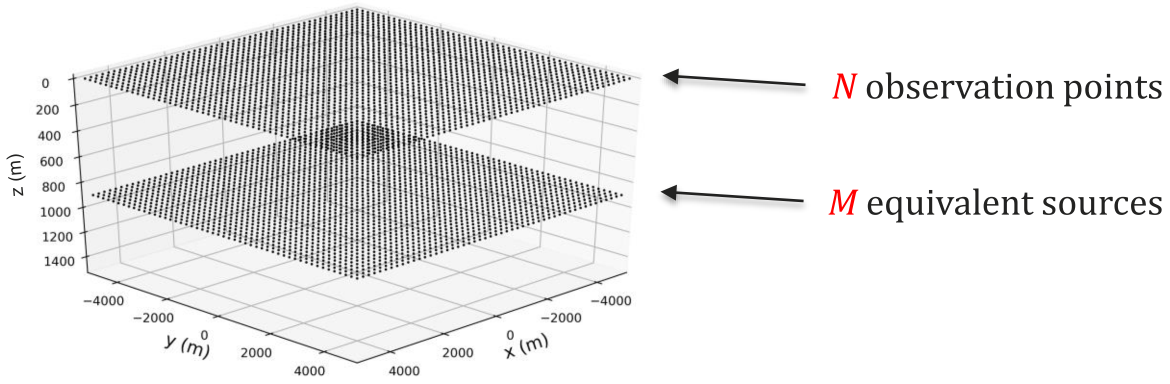

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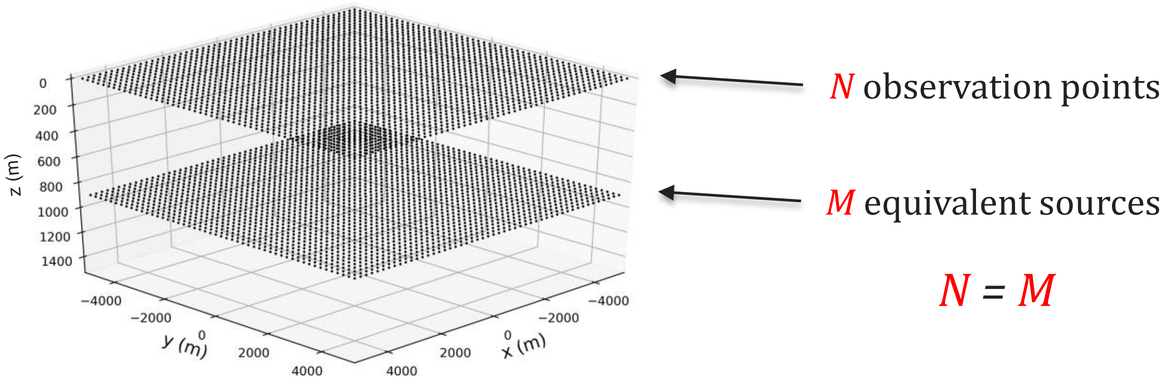

$$N = M$$

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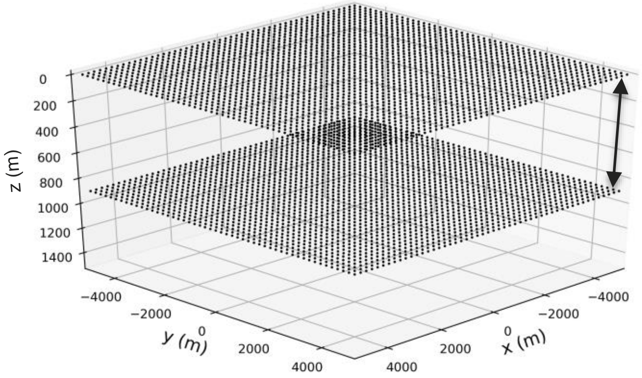
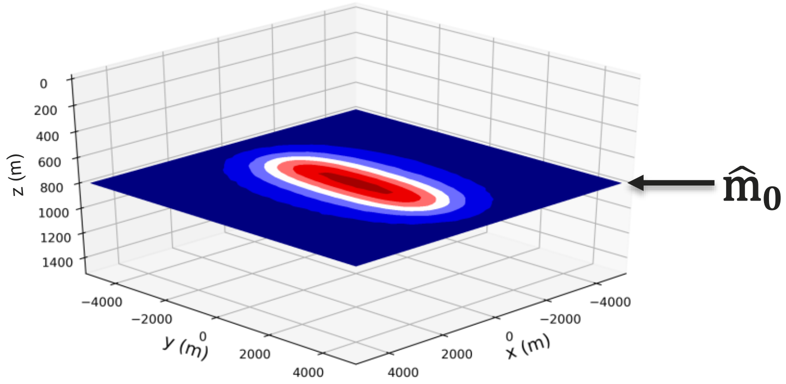


each equivalent source is located directly below each observation point

- The iterative equivalent-layer proposed by Siqueira et al. (2017) is grounded on the excess of mass and on the positive correlation between the $\mathbf{g_z}$-component data and the masses on the equivalent layer.

- The Gauss-Newton's method is used for estimating a mass distribution on the equivalent layer.



The initial aproximation:

$$\mathbf{m}_0 = \tilde{\mathbf{A}}^{-1}\mathbf{g}^o$$

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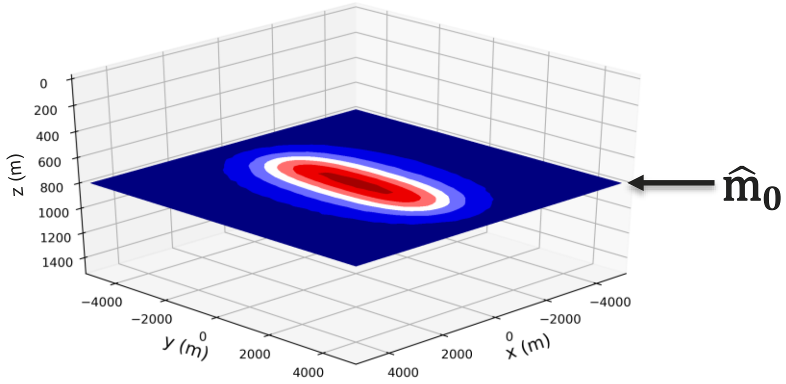

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$$\mathbf{m}_0 = \tilde{\mathbf{A}}^{-1}\mathbf{g}^o$$

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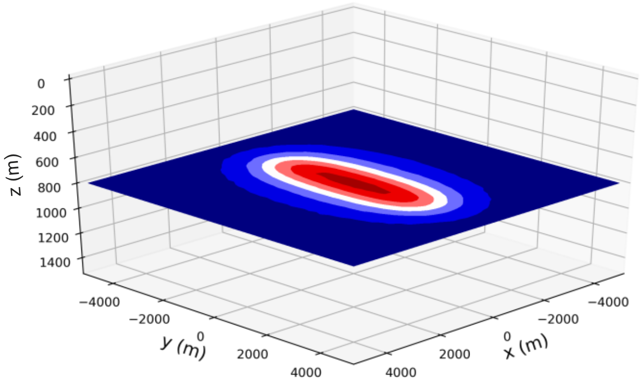

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γ is Newton's gravitational constant

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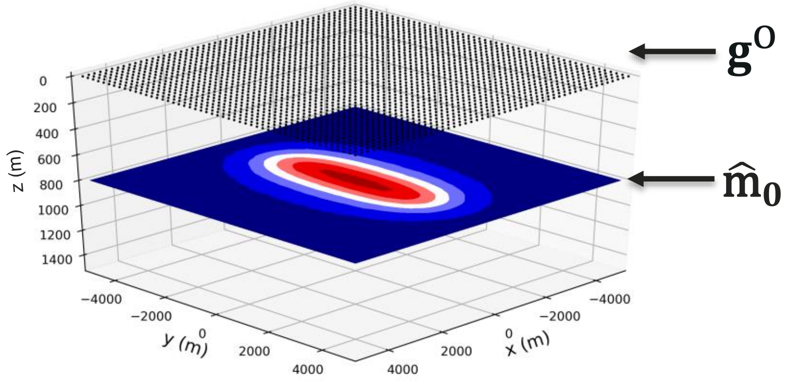

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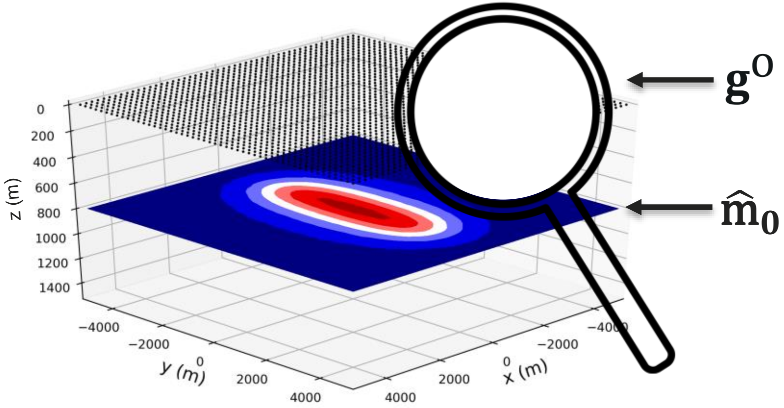

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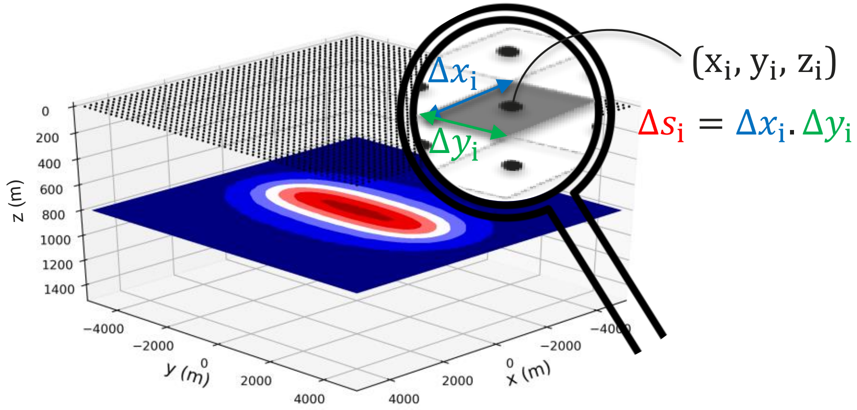

whose Δs_i is the horizontal area located at depth z_i and centered at the horizontal coordinates (x_i, y_i) of the *i*th $\mathbf{g_z}$-component data .

The initial aproximation:

$$\mathbf{m}_0 = \tilde{\mathbf{A}}^{-1}\mathbf{g}^o$$

$$\tilde{\mathbf{A}} = 2\pi\gamma\mathbf{\Delta S}^{-1}$$

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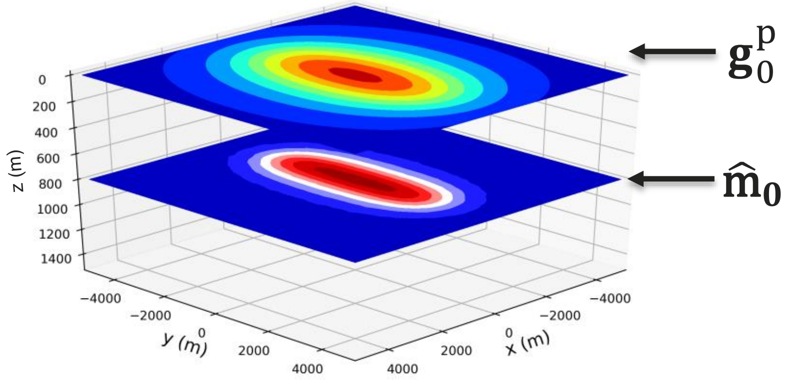
Iteration 0

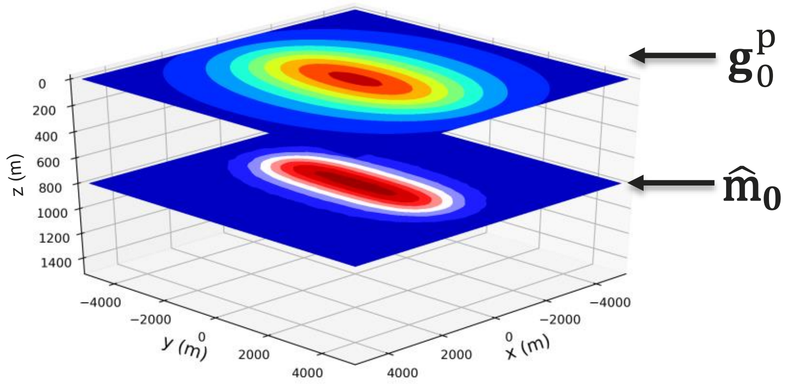
$$\mathbf{m}_0 = \tilde{\mathbf{A}}^{-1}\mathbf{g}^o$$



Iteration 0

$$\mathbf{m}_0 = \tilde{\mathbf{A}}^{-1}\mathbf{g}^o$$

$$\mathbf{r}_0 = \mathbf{g}^o - \mathbf{A}\mathbf{m}_0$$



Iteration 0

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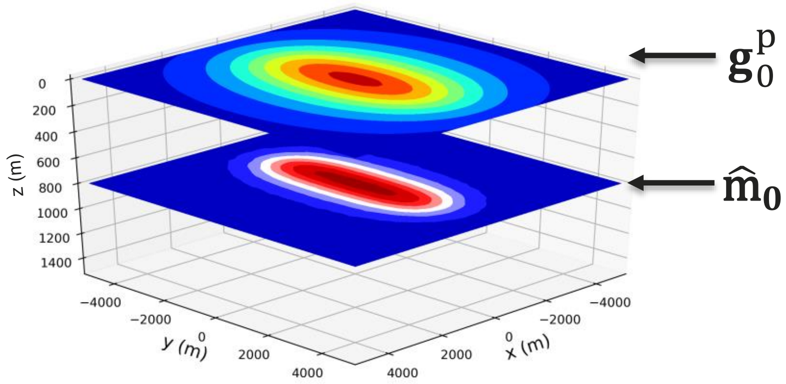
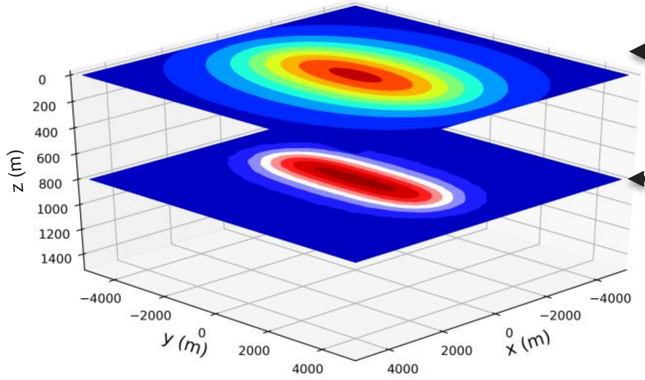


$$\mathbf{r}_0 = \mathbf{g}^o - \mathbf{A}\mathbf{m}_0$$

the predicted $\mathbf{g_z}$−component data ($\mathbf{g}^p$) at the 0 iteration

Iteration 0



$\mathbf{g}_0^p$

$\hat{\mathbf{m}}_0$

$$\mathbf{m}_0 = \tilde{\mathbf{A}}^{-1}\mathbf{g}^o$$

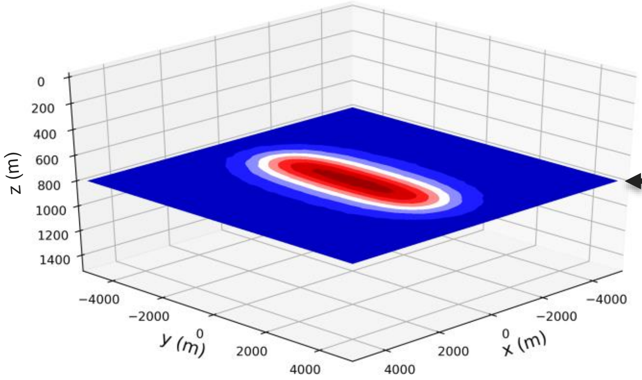
$$\mathbf{r}_0 = \mathbf{g}^o - \mathbf{A}\mathbf{m}_0$$

the predicted $\mathbf{g_z}$−component data ($\mathbf{g}^p$) at the 0 iteration

$$\Delta\hat{\mathbf{m}}_0 = \tilde{\mathbf{A}}^{-1}\mathbf{r}_0$$

The excess mass contraint

1^{st} Iteration

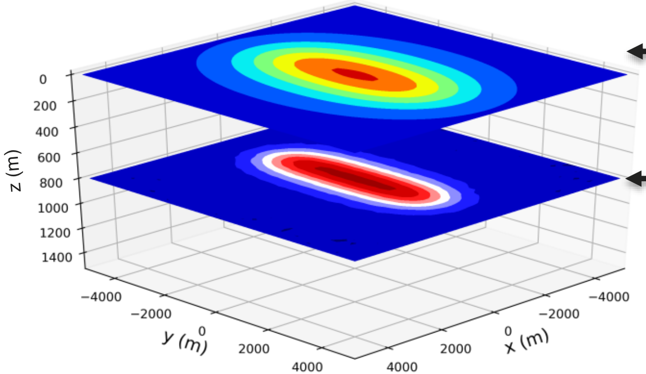


$\hat{\mathbf{m}}_1$

$$\hat{\mathbf{m}}_1 = \hat{\mathbf{m}}_0 + \Delta\hat{\mathbf{m}}_0$$

the mass distribution updated at the 1^{st} iteration

1^{st} Iteration



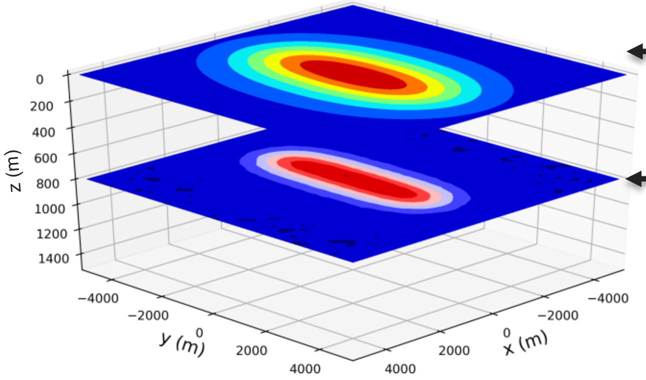
$\mathbf{g}_1^p$

$\hat{\mathbf{m}}_1$

$$\hat{\mathbf{m}}_1 = \hat{\mathbf{m}}_0 + \Delta\hat{\mathbf{m}}_0$$

the mass distribution updated at the 1^{st} iteration

2^{nd} Iteration



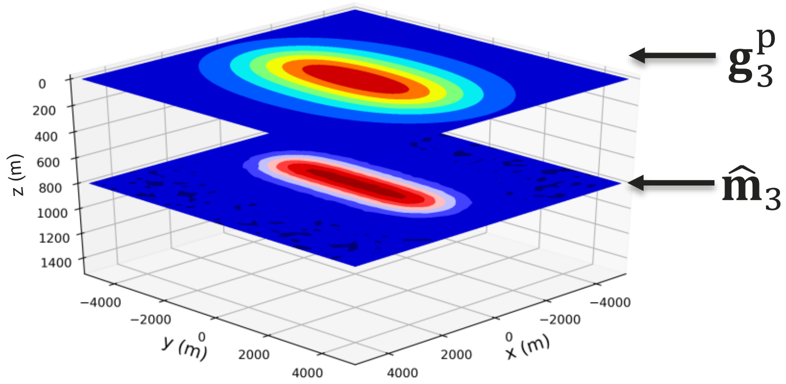
$\mathbf{g}_2^p$

$\hat{\mathbf{m}}_2$

$$\hat{\mathbf{m}}_2 = \hat{\mathbf{m}}_1 + \Delta\hat{\mathbf{m}}_1$$

the mass distribution updated at the 2^{nd} iteration

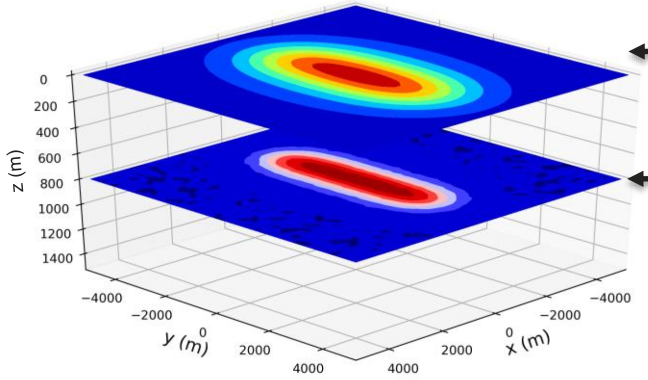
3^{rd} Iteration



$$\hat{\mathbf{m}}_3 = \hat{\mathbf{m}}_2 + \Delta\hat{\mathbf{m}}_2$$

the mass distribution updated at the 3^{rd} iteration

4^{th} Iteration



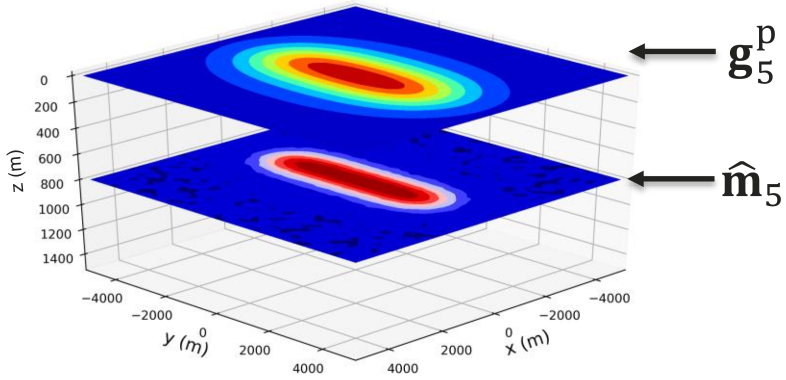
$\mathbf{g}_4^p$

$\hat{\mathbf{m}}_4$

$$\hat{\mathbf{m}}_4 = \hat{\mathbf{m}}_3 + \Delta\hat{\mathbf{m}}_3$$

the mass distribution updated at the 4^{th} iteration

5^{th} Iteration ....



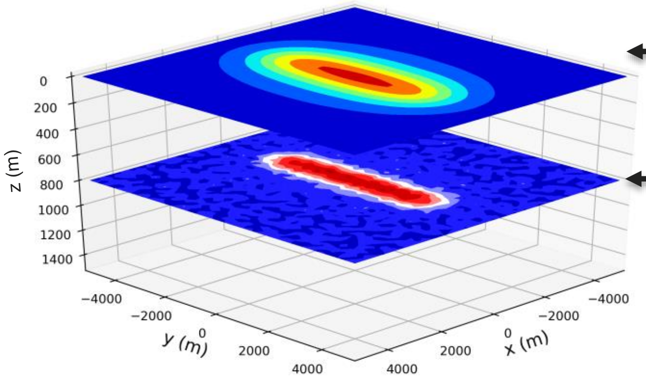
$$\hat{m}_5 = \hat{m}_4 + \Delta\hat{m}_4$$

the mass distribution updated at the 5^{th} iteration

...

... 30th Iteration

...



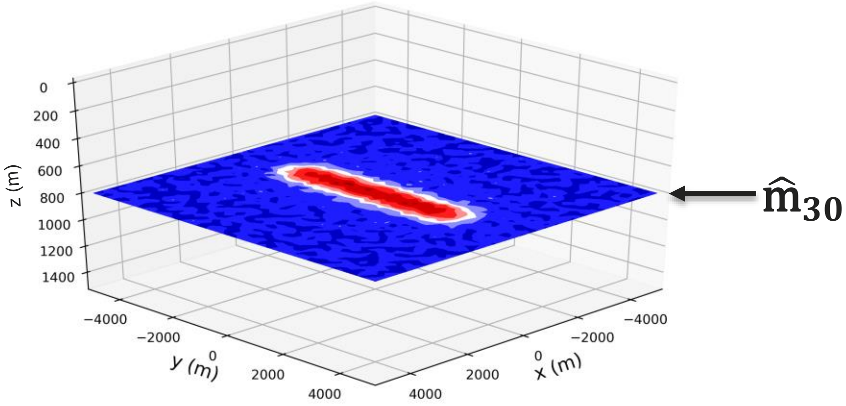
$\mathbf{g}_{30}^{p}$

$\hat{\mathbf{m}}_{30}$

$$\hat{\mathbf{m}}_{30} = \hat{\mathbf{m}}_{29} + \Delta\hat{\mathbf{m}}_{29}$$

the mass distribution updated at the 30th iteration

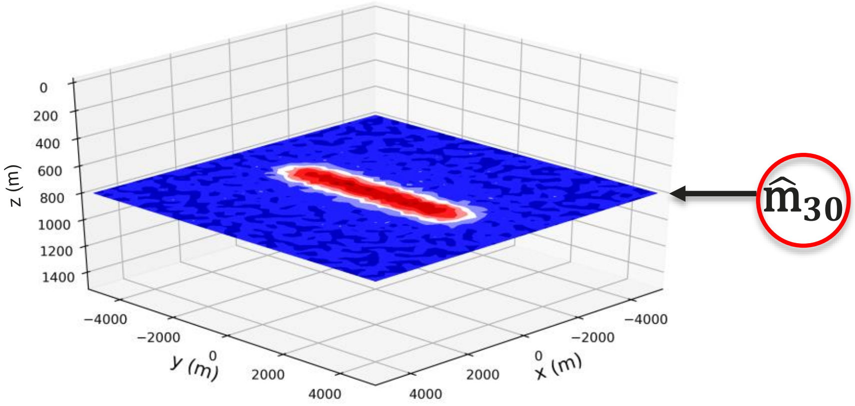
Calculating the gravity-gradient data



N-dimensional vector $\mathbf{g}^{\alpha\beta}$ that contains the $g^{\alpha\beta}$-component of the gravity-gradient tensor:

$$\mathbf{g}^{\alpha\beta} = \mathbf{T}^{\alpha\beta}\hat{\mathbf{m}}$$

Calculating the gravity-gradient data



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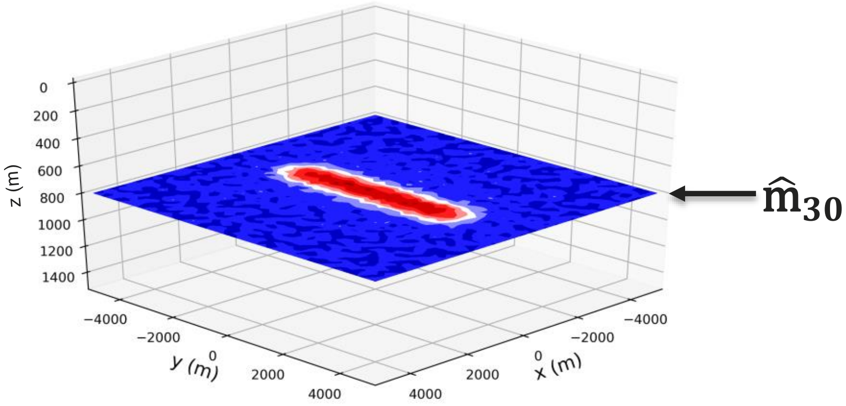

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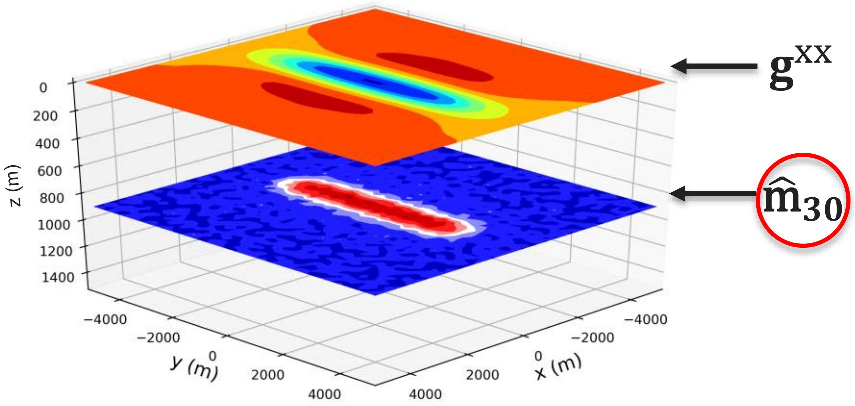
$$\mathbf{g}^{\alpha\beta} = \mathbf{T}^{\alpha\beta}\hat{\mathbf{m}}$$

$$T_{ij}^{\alpha\beta} = \begin{cases} \dfrac{3\left(\alpha_i - \alpha_j'\right)}{r^5} - \dfrac{1}{r^3} & \text{if} \quad \alpha = \beta \\ \dfrac{3\left(\alpha_i - \alpha_j'\right)\left(\beta_i - \beta_j'\right)}{r^5} & \text{if} \quad \alpha \neq \beta \end{cases}$$

$$\alpha_i, \beta_i = x_i, y_i, z_i \qquad \alpha_j', \beta_j' = x', y', z_0$$

$$\Big\{ r = \left[(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_0)^2\right]^{1/2}$$

Calculating the gravity-gradient data



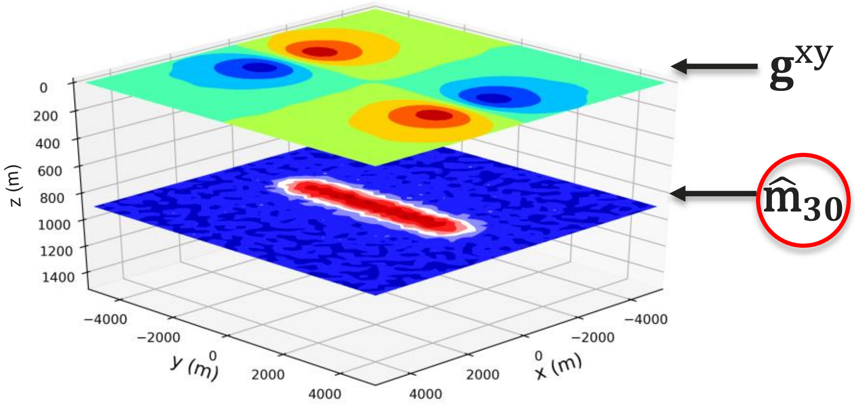
N-dimensional vector $\mathbf{g}^{xx}$ that contains the g^{xx}-component of the gravity-gradient tensor:

$$\mathbf{g}^{xx} = \mathbf{T}^{xx}\hat{\mathbf{m}}$$

$$T_{ij}^{xx} = \frac{3\left(x_i - x_j'\right)}{r^5} - \frac{1}{r^3}$$

$$\left\{ r = \left[(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_0)^2\right]^{1/2} \right.$$

Calculating the gravity-gradient data



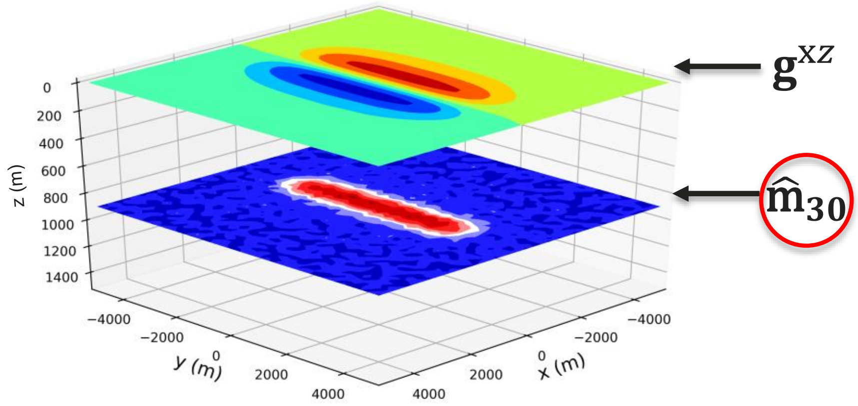
N-dimensional vector $\mathbf{g}^{xy}$ that contains the g^{xy}-component of the gravity-gradient tensor:

$$\mathbf{g}^{xy} = \mathbf{T}^{xy}\hat{\mathbf{m}}$$

$$T_{ij}^{xy} = \frac{3\left(x_i - x_j'\right)\left(y_i - y_j'\right)}{r^5}$$

$$\left\{ r = \left[(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_0)^2\right]^{1/2} \right.$$

Calculating the gravity-gradient data



N-dimensional vector $\mathbf{g}^{xz}$ that contains the g^{xz}-component of the gravity-gradient tensor:

$$\mathbf{g}^{xz} = \mathbf{T}^{xz}\hat{\mathbf{m}}$$

$$T^{xz}_{ij} = \frac{3\left(x_i - x_j'\right)\left(z_i - z_j'\right)}{r^5}$$

$$\left\{ r = \left[(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_0)^2\right]^{1/2} \right.$$

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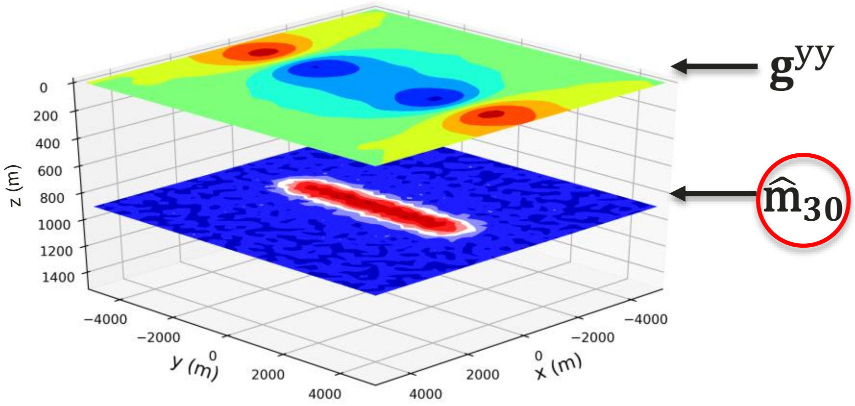
Calculating the gravity-gradient data

N-dimensional vector $\mathbf{g}^{yy}$ that contains the g^{yy}-component of the gravity-gradient tensor:

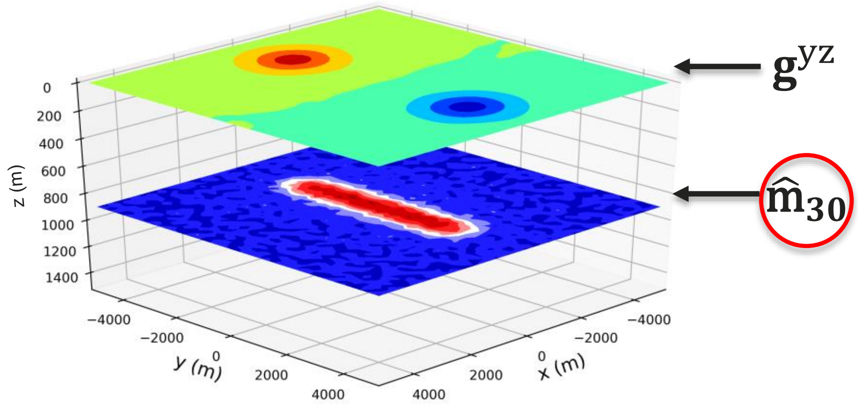
$$\mathbf{g}^{yy} = \mathbf{T}^{yy}\,\hat{\mathbf{m}}$$

$$T_{ij}^{yy} = \frac{3\left(y_i - y_j'\right)}{r^5} - \frac{1}{r^3}$$

$$\left\{ r = \left[(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_0)^2\right]^{1/2} \right.$$



Calculating the gravity-gradient data



N-dimensional vector $\mathbf{g}^{yz}$ that contains the g^{yz}-component of the gravity-gradient tensor:

$$\mathbf{g}^{yz} = \mathbf{T}^{yz}\hat{\mathbf{m}}$$

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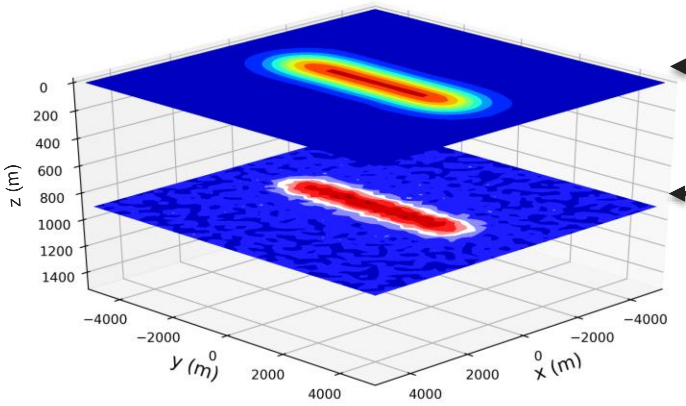
$$\left\{ r = \left[(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_0)^2\right]^{1/2} \right.$$

Calculating the gravity-gradient data



N-dimensional vector $\mathbf{g}^{zz}$ that contains the g^{zz}-component of the gravity-gradient tensor:

$$\mathbf{g}^{zz} = \mathbf{T}^{zz}\hat{\mathbf{m}}$$

$$T_{ij}^{zz} = \frac{3\left(z_i - z_j'\right)}{r^5} - \frac{1}{r^3}$$

$$\left\{ r = \left[(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_0)^2\right]^{1/2} \right.$$

Agenda

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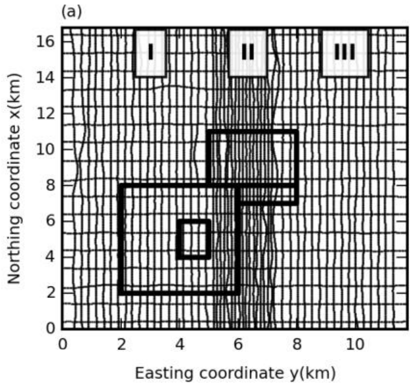
1st synthetic test: g_z-component data without a regional trend

- Flight lines and horizontal projection of the 3D sources



1st synthetic test: g_z-component data without a regional trend

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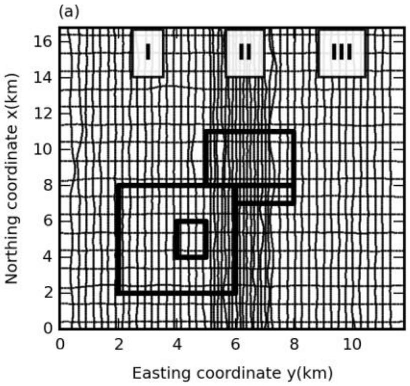
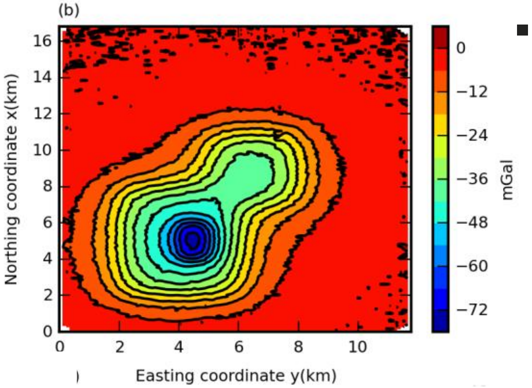
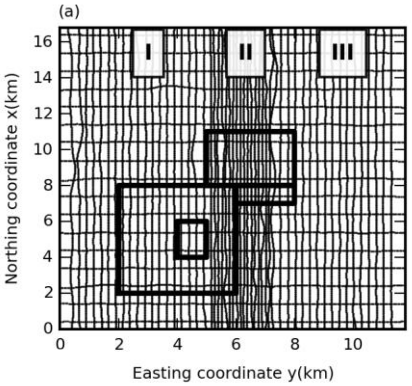


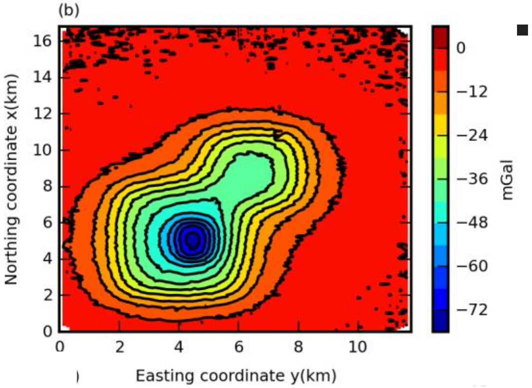


- Simulated g_z -component data

1st synthetic test: g_z-component data without a regional trend

- Flight lines and horizontal projection of the 3D sources



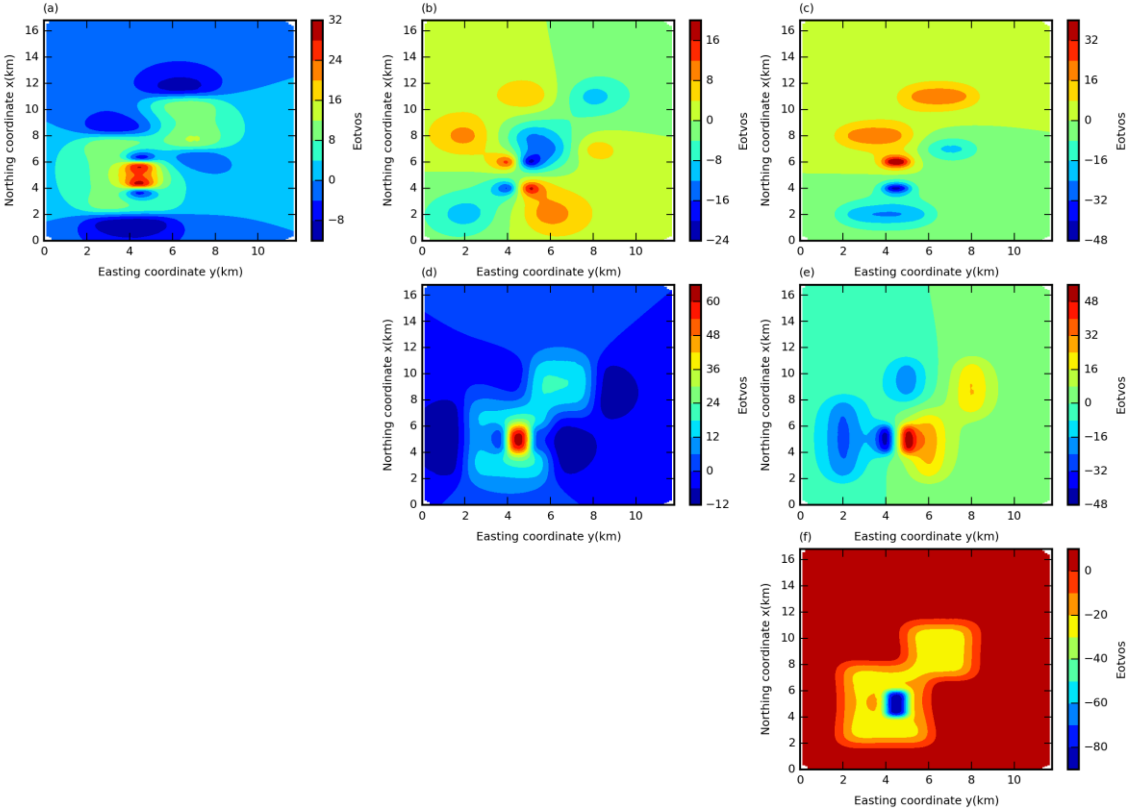


- Simulated g_z -component data

21.095 observation points: the number of flops (floating-points operations) required to estimate the mass distribution is approximately 173.37 times less than the number of flops required by the classical approach.

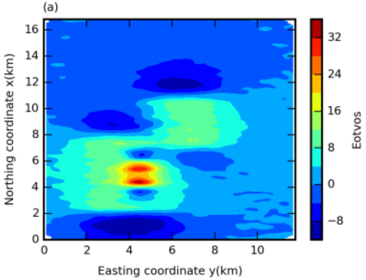
1^{st} synthetic test: g_z-component data without a regional trend

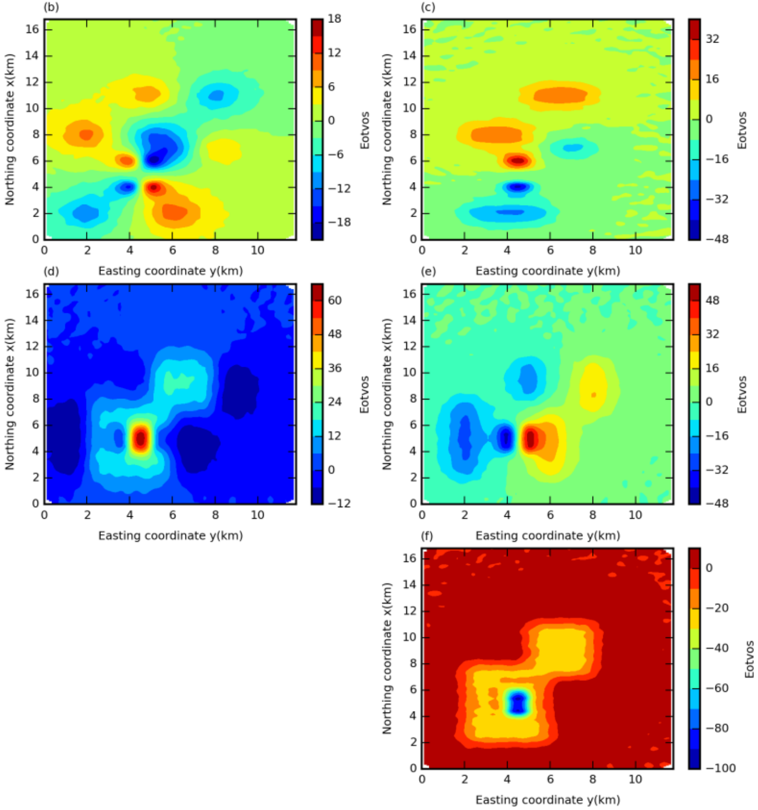
- True gravity-gradient data



1^{st} synthetic test: g_z-component data without a regional trend

- Predicted gravity-gradient data

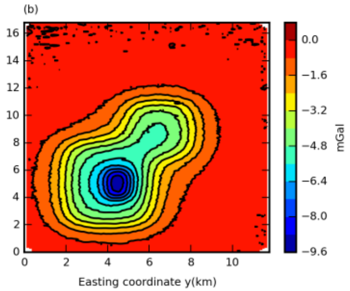




$z_j = 400\text{m}$

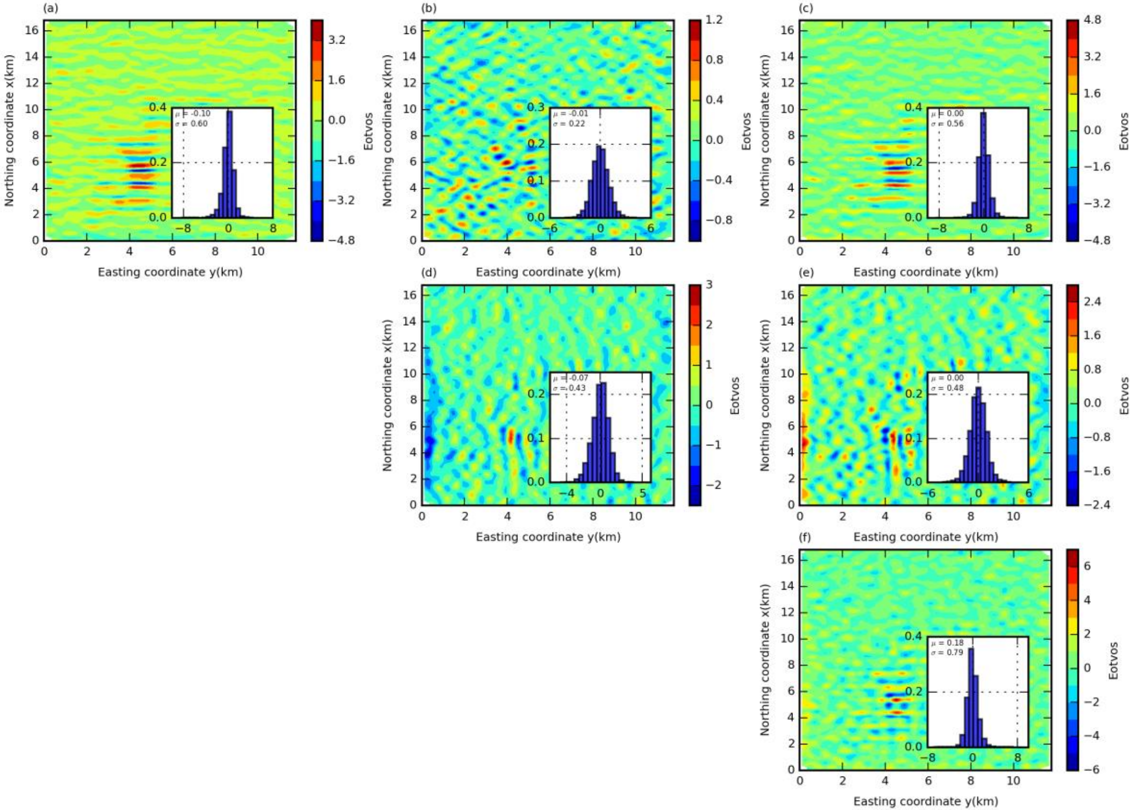
30 iterations

g_z-component data



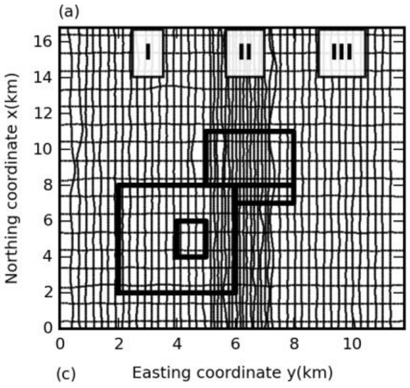
1^{st} synthetic test: g_z-component data without a regional trend

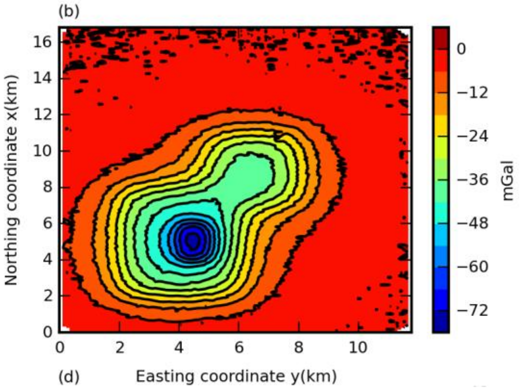
- Residuals



2nd synthetic test: g_z-component data with a regional trend

- Flight lines (simulating the real data) and horizontal projection of the 3D sources

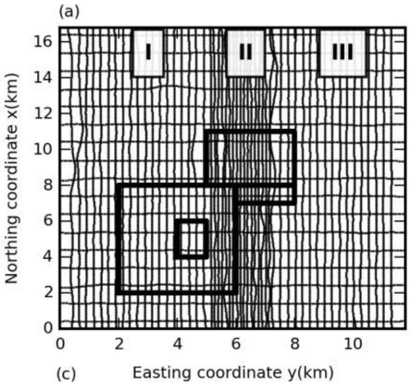


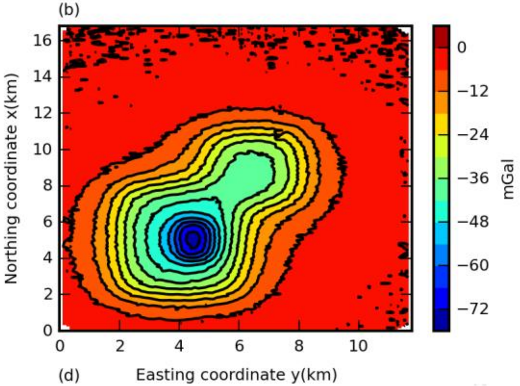


- Simulated g_z - component data

2^{nd} synthetic test: g_z-component data with a regional trend

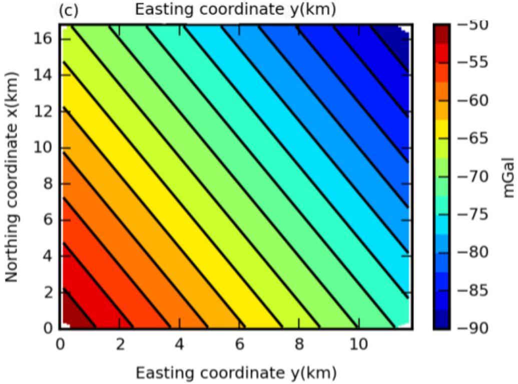
- Flight lines (simulating the real data) and horizontal projection of the 3D sources





- Simulated g_z - component data

- Regional trend simulated by a first-order polynomial



- Total g_z - component data

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2nd synthetic test: g_z-component data with a regional trend

- Flight lines (simulating the real data) and horizontal projection of the 3D sources
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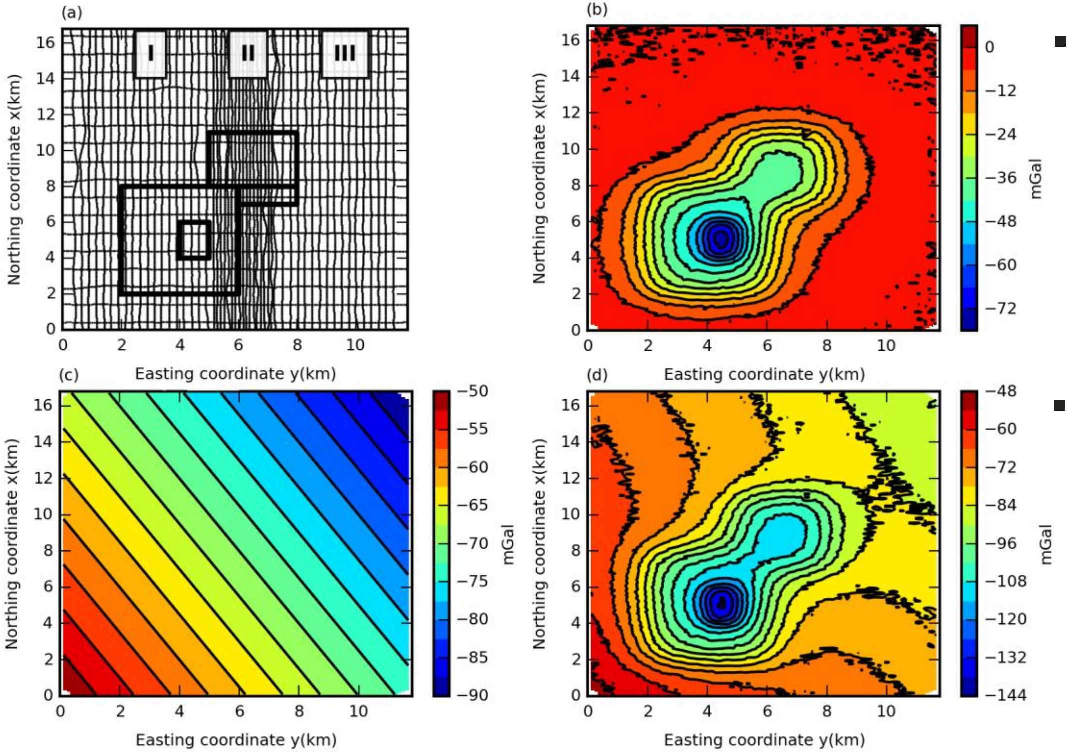
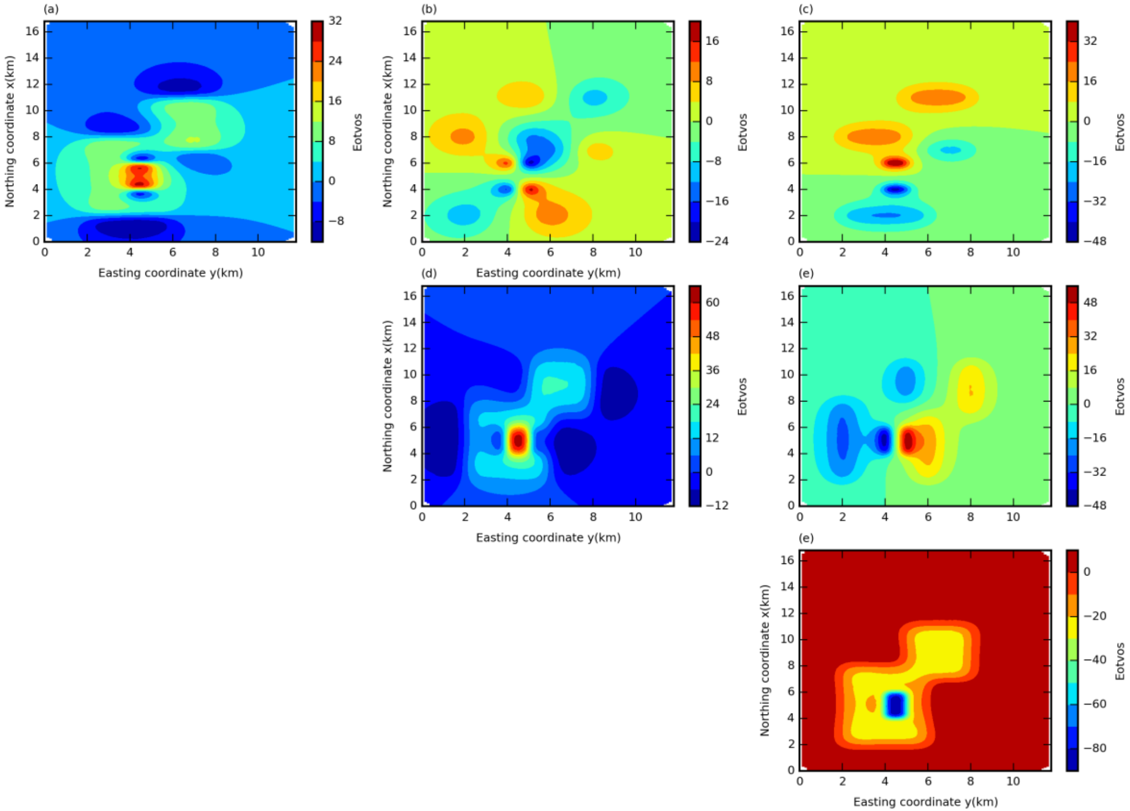


- Simulated g_z - component data
- Total g_z - component data

2^{nd} synthetic test: g_z-component data with a regional trend

- True gravity-gradient data

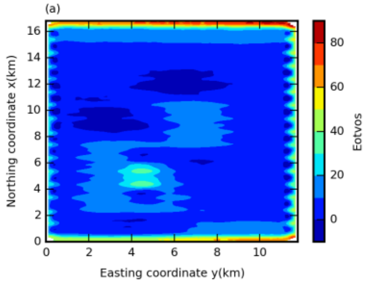


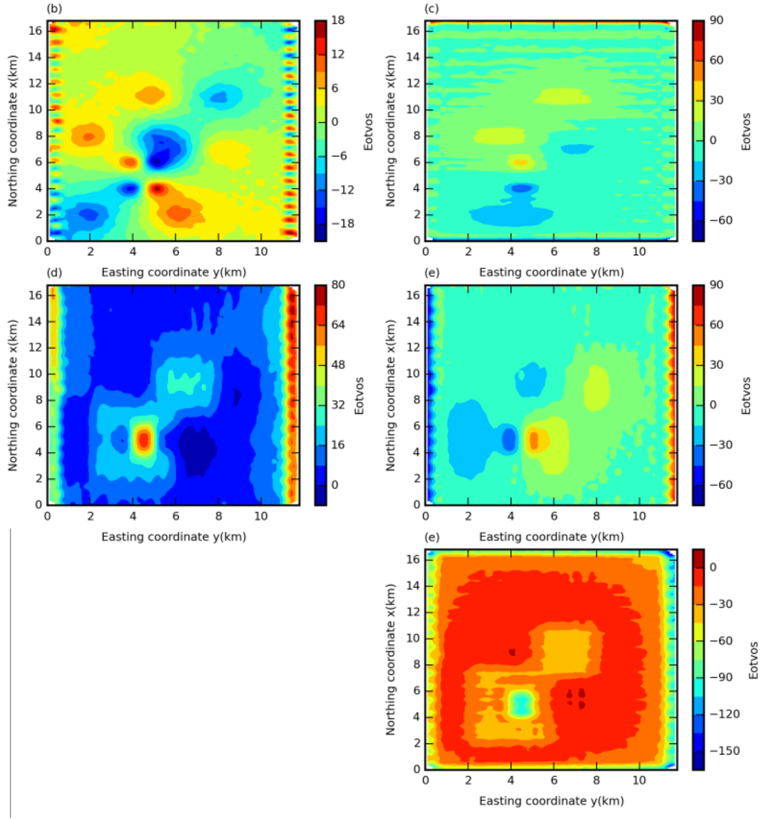
2nd synthetic test: g_z-component data with a regional trend

- Predicted gravity-gradient data

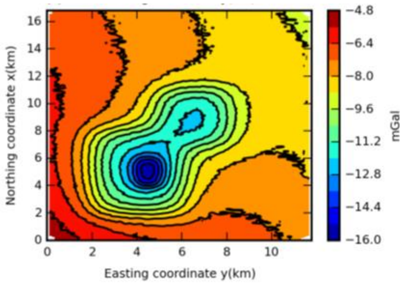
$z_j = 400\text{m}$

30 iterations



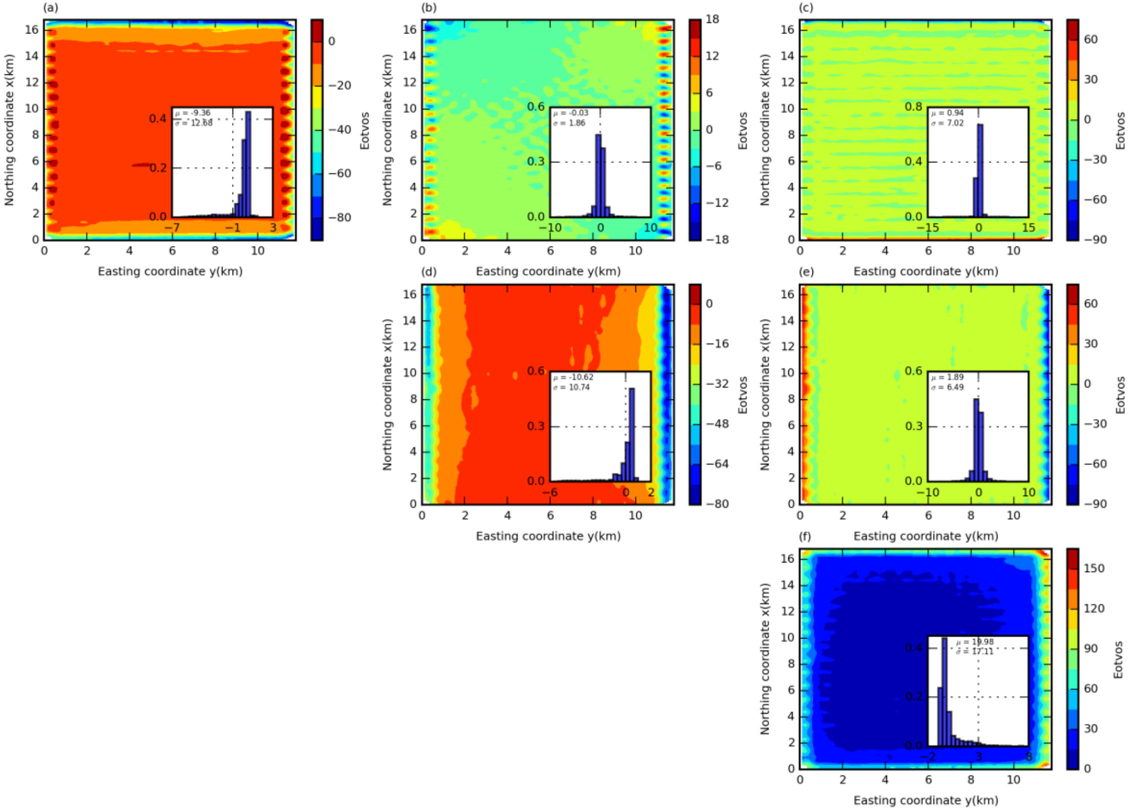


Total g_z-component data



2nd synthetic test: g_z-component data with a regional trend

- Residuals



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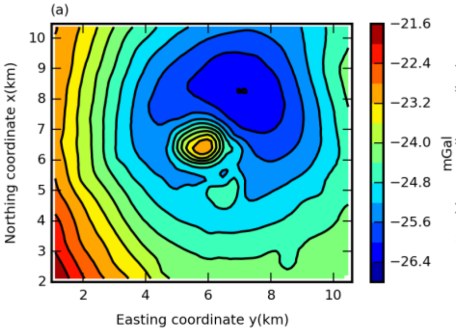
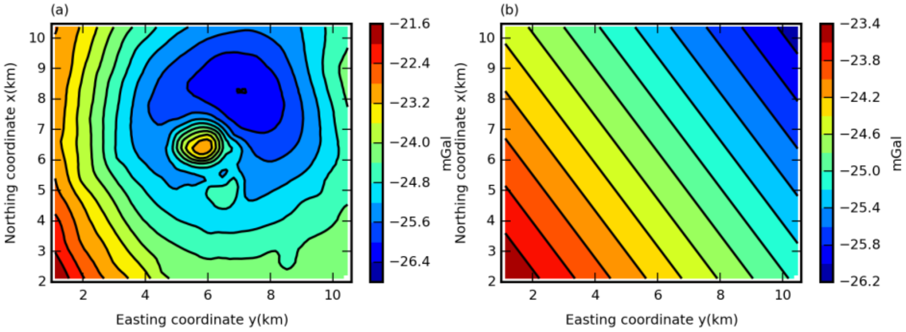
Vinton salt dome, Louisiana, USA

- g_z-component data



Vinton salt dome, Louisiana, USA

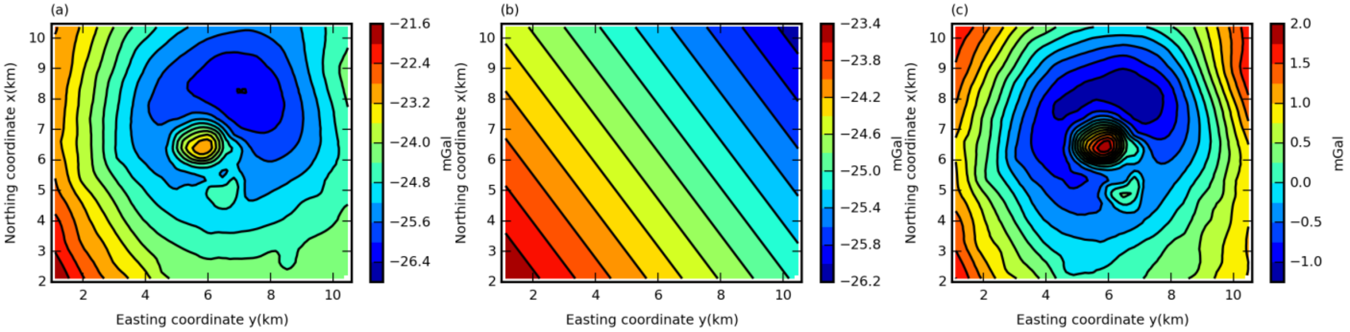
- g_z-component data
- Regional trend removed



Beltrão et al. (1991): regional-residual separation method.

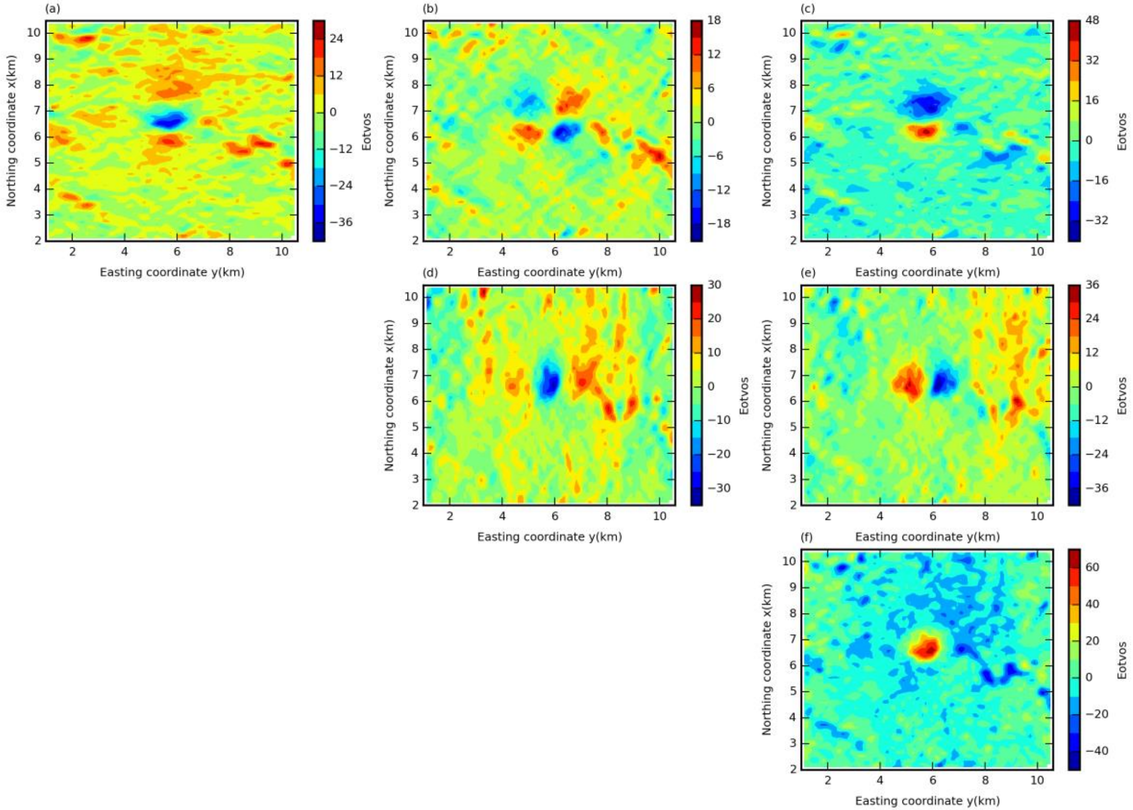
Vinton salt dome, Louisiana, USA

- $\mathbf{g_z}$-component data
- Regional trend removed
- Residual $\mathbf{g_z}$-component data



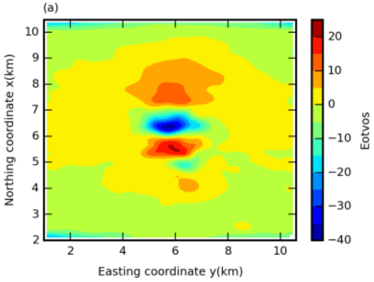
Vinton salt dome, Louisiana, USA

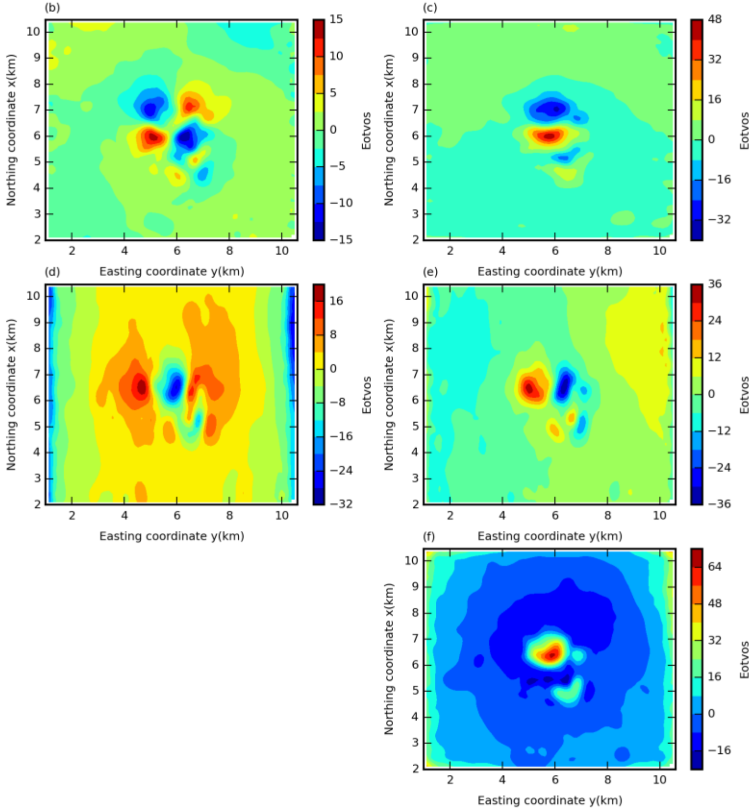
- Observed gravity-gradient data



Vinton salt dome, Louisiana, USA

- Predicted gravity-gradient data



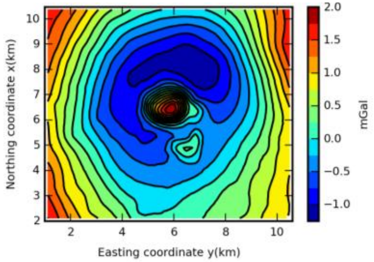


$z_j = 400\text{m}$

30 iterations

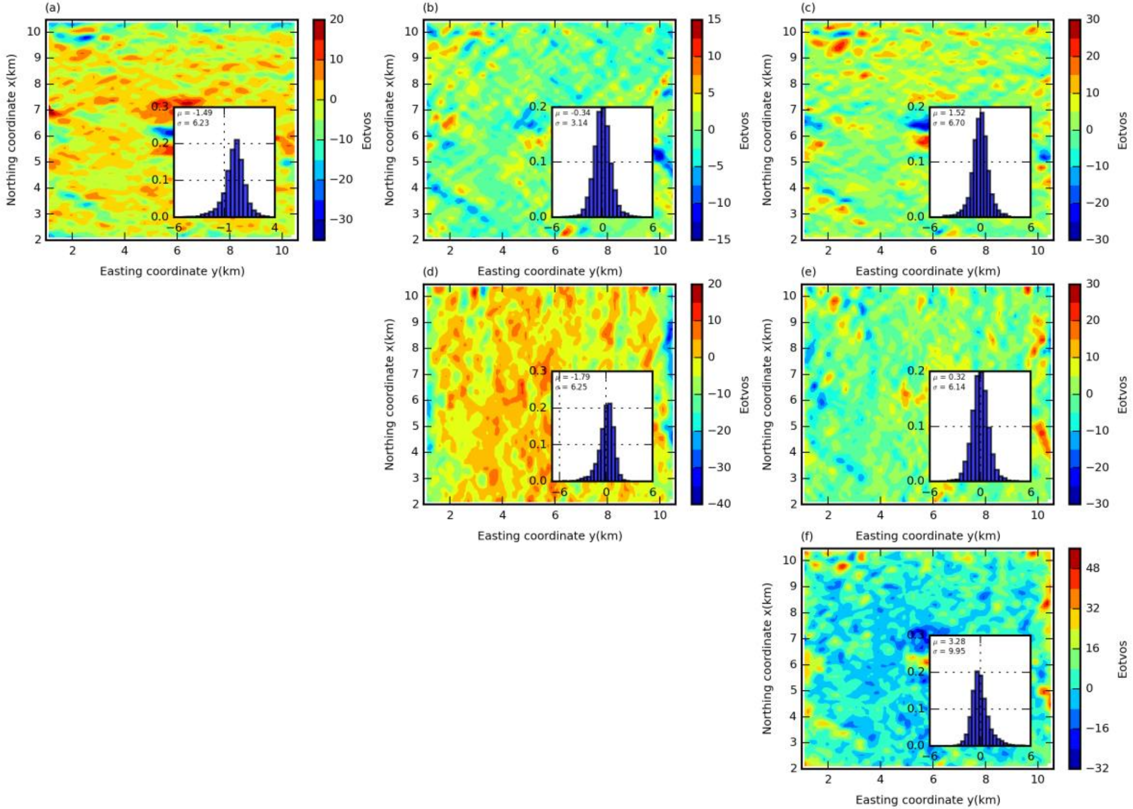
Residual

$\mathbf{g_z}$-component data



Vinton salt dome, Louisiana, USA

- Residuals



Agenda

- We have used a fast iterative equivalent-layer technique for calculating gravity-gradient data from $\mathbf{g}_z$-component data.

- This method uses the excess of mass and the positive correlation between the observed $\mathbf{g}_z$-component and the masses on the equivalent layer.

- The computational efficiency of the method relies heavily on the fast estimation of the mass distribution on the equivalent layer without requiring matrix multiplications and the solution of linear systems.

- Applications to synthetic and real data show the ability of the method to calculate the gravity-gradient tensor from large data set when a regional data is removed. The presence of a regional data may result in errors in the calculation of the components.

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1 Introduction

2 The classical equivalent layer

3 The fast equivalent layer

4 Application to synthetic data

5 Applications to real data

6 Conclusions

7 Acknowledgments

Support, scholarships, fellowships and dataset:

PINGA
Grupo de problemas inversos em geofísica
A research group for inverse problems in **geophysics**
pinga-lab.org

fatiando a terra
modeling and inversion | fatiando.org

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CAPES

CNPq
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