Equivalent-layer technique for estimating magnetization direction

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Overview

- Total-field anomaly and magnetization direction;
- Equivalent layer and the positivity;
- Forward and inverse problems;
- Numerical simulations;
- Field data application;
- Conclusion.



















Magnetized rocks + North Fo Magnetic field can be measured over the surface Depends on the magnetization direction of the magnetic source

Mathematically...

$$\Delta T = \hat{\mathbf{F}}_0^T \mathbf{B}$$

Total-field anomaly





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Total-field anomaly

 $\Delta T = \gamma m \hat{\mathbf{F}}_0^T \mathbf{H} \, \hat{\mathbf{h}}$





Mathematically...

$$\Delta T = \hat{\mathbf{F}}_0^T \mathbf{B}$$

Total-field anomaly

 $\Delta T = \bigcap m \hat{\mathbf{F}}_0^T \mathbf{H} \hat{\mathbf{h}}$ Constant with vacuum permeability





Mathematically...

$$\Delta T = \hat{\mathbf{F}}_0^T \mathbf{B}$$

Total-field anomaly

 $\Delta T = \gamma \hat{\boldsymbol{m}} \hat{\boldsymbol{F}}_0^T \boldsymbol{H} \hat{\boldsymbol{h}}$

Magnetization intensity







How to estimate the magnetization direction?



1. Methods that presume some geometry for the source

Set of prisms or spherical geometries (e.g.,Battacharyya,1996; Emilia e Massey, 1974; Medeiros e Silva, 1995; Parker et al, 1987; Kubota e Uchiyama, 2005; Oliveira Jr et al, 2015) 2. Methods that not presume any information about the source

Analysis of the magnetic moment of sources (e.g., Tontini e Pedersen, 2008)

Correlating of potential quantities (e.g.,Roest e Pilkington,1993; Dannemiller e Li, 2006; Gerovska, 2009; Li et al, 2017; Zhang et al, 2018)

Succesive RTP (Fedi, 1994)



Total-field anomaly



Here we use the **Equivalent-layer technique**



An interesting feature!



We can retrieve the data generated by a 3D source using a 2D physical-property distribution



Total-field anomaly (layer) Total-field anomaly (source) F $\Delta T(x_i, y_i, z_i)$ +North p(x',y',z_c) Z 7 $+\infty +\infty$ $\Delta T_i = \int \int p(x', y', z_c) [\gamma \hat{\mathbf{F}}_0^T \mathbf{M} \, \hat{\mathbf{m}}(\mathbf{q})] dx' dy'$ $\Delta T = \gamma m \hat{\mathbf{F}}_0^T \mathbf{H} \, \hat{\mathbf{h}}$ $-\infty -\infty$









The magnetic-moment distribution is all-positive over the layer.







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This feature was explored by Pedersen (1991) and Li et al (2014).





$\hat{\mathbf{m}}(\mathbf{q}) = \hat{\mathbf{h}}$

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Lima and Weiss (2007) and Baratchart et al (2013) pointed out that uniqueness can be achieved by imposing an unidirectional solution.









How does it work in practice?



3D magnetization distribution



3D magnetization distribution





3D magnetization distribution



Observed data vector



Equivalent layer





Equivalent layer



The equivalent layer can be approximated by a set of **equivalent sources** that, in this case, are **dipoles with unit volume**



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Equivalent layer Fo Х **m**(**q**) Z_c P_i VΖ The **magnetic moment** of the jth dipole, j = 1, ..., M.






















The procedure for estimating magnetization direction





















How to solve this inverse problem?



We propose a nested algorithm that solves the problem in two steps



Given an initial guess **q**₀ for magnetization direction

We propose a nested algorithm that solves the problem in two steps

1
$$(\mathbf{G}_{p}^{k^{T}}\mathbf{G}_{p}^{k} + \mu f_{0}^{k}\mathbf{I})\mathbf{p}^{*^{k}} = \mathbf{G}_{p}^{k^{T}}\boldsymbol{\Delta}\mathbf{T}^{o}$$

$$2 \left(\mathbf{G}_{q}^{k^{T}} \mathbf{G}_{q}^{k} + \lambda \mathbf{I} \right) \mathbf{\Delta q}^{*^{k}} = \mathbf{G}_{q}^{k^{T}} \left[\mathbf{\Delta T}^{o} - \mathbf{\Delta T}(\mathbf{s}) \right]$$



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 Nonnegative (NNLS)
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2
$$(\mathbf{G}_{q}^{k^{T}}\mathbf{G}_{q}^{k}+\lambda\mathbf{I})\mathbf{\Delta q}^{*^{k}} = \mathbf{G}_{q}^{k^{T}}[\mathbf{\Delta T}^{O}-\mathbf{\Delta T(s)}]$$

Levenberg-Marquardt
method



An overview of the algorithm





Numerical simulations

1. Unidirectional magnetization sources

2. Unidirectional with shallow-seated source

3. Shallow-seated source with different direction



*(top, bottom) in meters



A model composed by five magnetic sources :



*(top, bottom) in meters



*(top, bottom) in meters



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For the inversion





M = 1225 equivalent sources (same number of observations)



For the inversion



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Depth of z = 1150 m



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L-curve for choosing the regularization parameter proposed by Hansen (1992).



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The initial guess of $\mathbf{q}_0 = (-10^\circ, -10^\circ)$



1. Unidirectional magnetization sources





The magnetization direction for all sources:

(I,D) = (-25°,30°)



(I,D) = (-25°,30°) (I,D) = (-28.6°,30.8°)

Data Fitting



(I,D) = (-25°,30°) (I,D) = (-28.6°,30.8°)




Magnetic moment distribution

Magnetic moment distribution



 $(I,D) = (-25^{\circ}, 30^{\circ})$ $(I,D) = (-28.6^{\circ}, 30.8^{\circ})$

Note the positive magnetic moment distribution!







These results show that the all-positive magnetic-moment distribution and the estimated magnetization direction produce an acceptable data fitting





2. Unidirectional with shallow-seated source









(I,D) = (-25°,30°) (I,D) = (-28.7°,31.7°)

Data Fitting



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Magnetic moment distribution





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(I,D) = (-25°,30°) (I,D) = (-28.7°,31.7°)





Despite the large residual located above the shallow-seated source, we consider that the methodology produced a reliable result because the estimated magnetization direction is very close to the corresponding true magnetization direction.





3. Shallow-seated source with different direction





Now the shallow-seated source has a different magnetization direction

 $(I,D) = (20^{\circ},-30^{\circ})$





Now the shallow-seated source has a different magnetization direction

 $(I,D) = (20^{\circ}, -30^{\circ})$

while the other sources:

(I,D) = (-25°,30°)



(I,D) = (-25°,30°) (I,D) = (-30.4°,27.6°)

Data Fitting



(I,D) = (-25°,30°) (I,D) = (-30.4°,27.6°)

Data Fitting



Magnetic moment distribution

1e9





(I,D) = (-25°,30°) (I,D) = (-30.4°,27.6°)

Note the positive magnetic moment distribution!









Despite the slight difference from the true magnetization direction, the estimated magnetic-moment distribution produces an acceptable data fit, with the exception of the small area exactly above the smallseated prism.





Field data application





Goiás Alkaline Province (GAP): Complex of Montes Claros de Goiás





)	Goiás Alkaline Province (GAP): Complex of Montes Claros de Goiás
	This area was flown with an aeromagnetic survey
лТ	The flight height of 100 m
D	A total of N=1787 observations
C	(I ₀ , D ₀) = -19°,-18°
C	

SAN ANTONIO, TX



For the inversion

M = 1787 equivalent sources (same number of observations) Depth of z = 840 m L-curve for choosing the Г regularization parameter proposed by Hansen (1992). The initial guess of $\mathbf{q}_0 = (-70^\circ, 50^\circ)$



Data Fitting

(I,D) = (-50.2°,34.9°)



Data Fitting

(I,D) = (-50.2°,34.9°)



Magnetic moment distribution

Magnetic moment distribution 1e10



(I,D) = (-50.2°,34.9°)

Note the positive magnetic moment distribution!









We check the quality of the estimated magnetic-moment distribution and estimated magnetization direction by computing the reduction-topole of the observed total-field anomaly!





The reduction-to-pole



We can note predominantly positive values over the study area. For this reason, we consider the estimated magnetization direction satisfactory



The reduction-to-pole



From these results, we can also conclude that the all-positive magnetic moment distribution and the estimated magnetization direction produce an acceptable data fitting. We can also conclude that these intrusions present a strong remanent magnetization component.



Conclusions



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- We propose a nested algorithm for solving the inverse problem in two steps;
- This method can be applied to estimate the total magnetization direction of multiple sources;
 - We show that the positive magnetic-moment distribution can auxiliate the magnetization estimation;
 - After synthetic tests, we conclude that the method is capable to retrieve the magnetization direction of magnetic sources;
 - The field data application confirms that our method can be a reliable tool for interpreting geological scenarios
 - However, despite the reliable magnetization estimate, we cannot infer if shallow sources has the same direction of the final solution.



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Thank you!

