On "On Recursion"

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Recursion in linguistics by Watumull et al. (2014) is compared with recursion in mathematics by Turing (1937).

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§1 On the shoulders of giants

¶1 · In the abstract of "On Recursion", a paper by Watumull et al. (2014), we can read that "the concept of recursion as articulated in the context of linguistic analysis has been perennially confused." So their "essay is an attempt to bring conceptual clarity to such discussions [in linguistics on recursion] as well as to future empirical investigations". And they do it by examining "the standard mathematical definition of recursion as understood by Gödel and Turing".

¶2 · I have done the same, but what I have found, which is entirely due to Turing (1937), is not what Watumull et al. (2014) are saying. After comparing recursion in mathematics by Turing (1937) with recursion in linguistics by Watumull et al. (2014), I propose to follow Turing in linguistics, because recursion in mathematics is a deep and far-reaching concept that has an exact meaning, and having an exact meaning is the best way to prevent confusion.

¶3 · In addition, linguistics does not need to deviate from mathematics on recursion, because we can use mathematical recursion to discriminate human language from other animal communication systems.

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Any comments on it to papa@ramoncasares.com are welcome.

§2 Recursion in mathematics

¶1 · Turing wrote in 1937: "The purpose of the present paper [Turing 1937] is to show that the computable (Turing 1936) functions introduced by the author are identical with the λ-definable (Church 1935) functions of Church and the general recursive (Kleene 1935) functions due to Herbrand and Gödel and developed by Kleene. It is shown that every λ-definable function is computable and that every computable function is general recursive. [...] If these results are taken in conjunction with an already available (Kleene 1936) proof that every general recursive function is λ-definable we shall have the required equivalence of computability with λ-definability" (page 153). The diagram can help to see Turing's (1937) plan to use Kleene's (1936) proof.

$$\begin{array}{ccc} \text{recursive function} & \Rightarrow & \lambda\text{-definable function} \\ \Uparrow & & \Downarrow \\ & \text{computable function} & \end{array}$$

¶2 · Kleene (1936) had already shown the equivalence of λ-definability with recursion "by proving that all recursive functions, in a wide sense of the term recursive, due to Herbrand and Gödel, are λ-definable; and conversely, all λ-definable functions of the type in question are recursive" (page 343).

¶3 · We can express the identity of computable with recursive functions proved by Turing (1937), with the help of Kleene (1936), as a mathematical theorem:

every recursive function is computable

and

every computable function is recursive.

§3 Recursion in linguistics by Watumull et al.

¶1 · Watumull et al. (2014) state that a recursive function has to fulfill three requirements: "(i) computability, (ii) definition by induction, and (iii) mathematical induction." Therefore, for them, every recursive function is computable, because of the first requirement, but not every computable function is recursive, because of the other two requirements. This means that they are not using mathematical recursion, because their result contradicts Turing's theorem.

¶2 · Of course, in front of this fact, they can argue that they are defining 'linguistic recursion', which is more stringent than mathematical recursion. The extra-mathematical requirements for recursion, which are (ii) definition by induction to generate hierarchical structures and (iii) mathematical induction to generate items beyond any bound, are used by Watumull et al. (2014) to argue that recursion is unique to our species (see subsection "Evolution" in page 5), as proposed by Hauser, Chomsky, and Fitch (2002), because then other species computable functions are not recursive under their constricted definition of recursion. But I would not recommend to persist in walking on this slippery slope for the following three reasons.

¶3 · Firstly, because this is why recursion, that has an exact meaning in mathematics, "has been perennially confused" in linguistics, as Watumull et al. (2014) admit. Ironically, that very paper, by "amending" mathematical recursion to "clarify" what was the meaning of recursion that was used in a previous paper by Hauser, Chomsky, and Fitch (2002), only adds confusion to confusion. That is, as long as there is not an agreement on linguistic recursion, so every linguist can keep using their own recursion to push their own interests, recursion was, is, and will be confusing in linguistics.
¶4 · Secondly, because if linguistics does not use mathematical recursion, but its own linguistic recursion, then linguistics cannot take advantage of the mathematical results on recursion. And mathematical recursion is a deep and far-reaching concept.

§4 Human language and mathematical recursion

¶1 · However, human language is quite different from other animal communication systems and this difference relies on mathematical recursion.
¶2 · Using the hierarchy of languages presented in Casares (CH), human language is Turing complete, while all other species languages are asyntactic. By definition, a language is Turing complete if and only if any computable function can be expressed and calculated in it. And, as computable functions are equivalent to recursive functions, see §2, then mathematically what qualifies a language as Turing complete or not is whether or not any recursive function can be expressed and calculated in the language.
¶3 · And then thirdly, because mathematical recursion is enough to discriminate human language from other species communication systems.

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