Supplementary Material

Taming Chimeras in Networks by Multiplexing Delays

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1 Exploring $D_x - E_y$ space for Undelayed Case:

We use the variance to measure the synchronization in the layers of the multiplex network with undelayed inter-layer links. The variance which is a common measure for synchronization can be defined as

$$\sigma^2 = \langle \frac{1}{N} \sum_{i=1}^{N} \{x_i - \bar{x}\}^2 \rangle_T \tag{1}$$

where $\bar{x} = \frac{1}{N}\sum_{k=1}^{N} x_k$ depicts the mean.

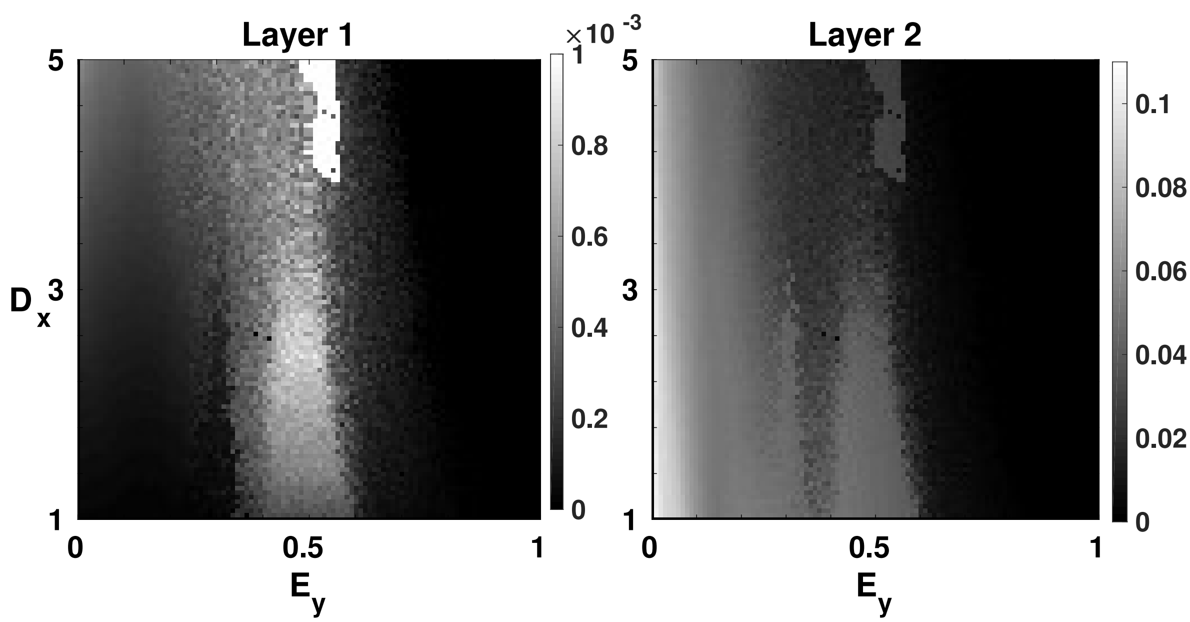

Figure 1: (Color Online) Phase Diagram of variance (σ^2) in D_x - E_y space for (a) layer 1 and (b) layer 2 of the multiplex network. with undelayed inter-layers edges. Notice that the colorbar scales are different in both layers. The parameters are $\varepsilon = 0.9$,$r = 0.32$, $N = 100$ in each layer; σ^2 is averaged over 1000 time steps.

Fig. 1 depicts a complete synchronization for layer 1 regardless of the D_x or E_y value due to the high coupling strength (ε). Layer 2 display a varied region of synchronization due to E_y which decrease of the effective coupling strength in layer 2.

2 Exploring $D_x - E_y$ space for Delayed Case:

To identify the chimera states in the parameter space, we define correlation measure of normalized probability distribution function $g(|\ \bar{D}\ |)$ as follows [2]

$$g_0(t) = \int_0^\delta g(|\ \bar{D}(t)\ |)d(|\ \bar{D}(t)\ |), \tag{2}$$

where δ is a small positive threshold and $|\bar{D}(t)|$ is Laplacian distance measure with its components $d_i(t)$ defined as $d_i(t) = |(z_{i+1}(t) - z_i(t)) - (z_i(t) - z_{i-1}(t))|$. The correlation measure $g_0(t)$ essentially represents fractional size of coherence with $g_0 = 0$ representing complete incoherent state whereas $g_0 = 1$ depicting coherent state. Anyvalue in range $0 < g_0 < 1$ ideally represents a chimera state.

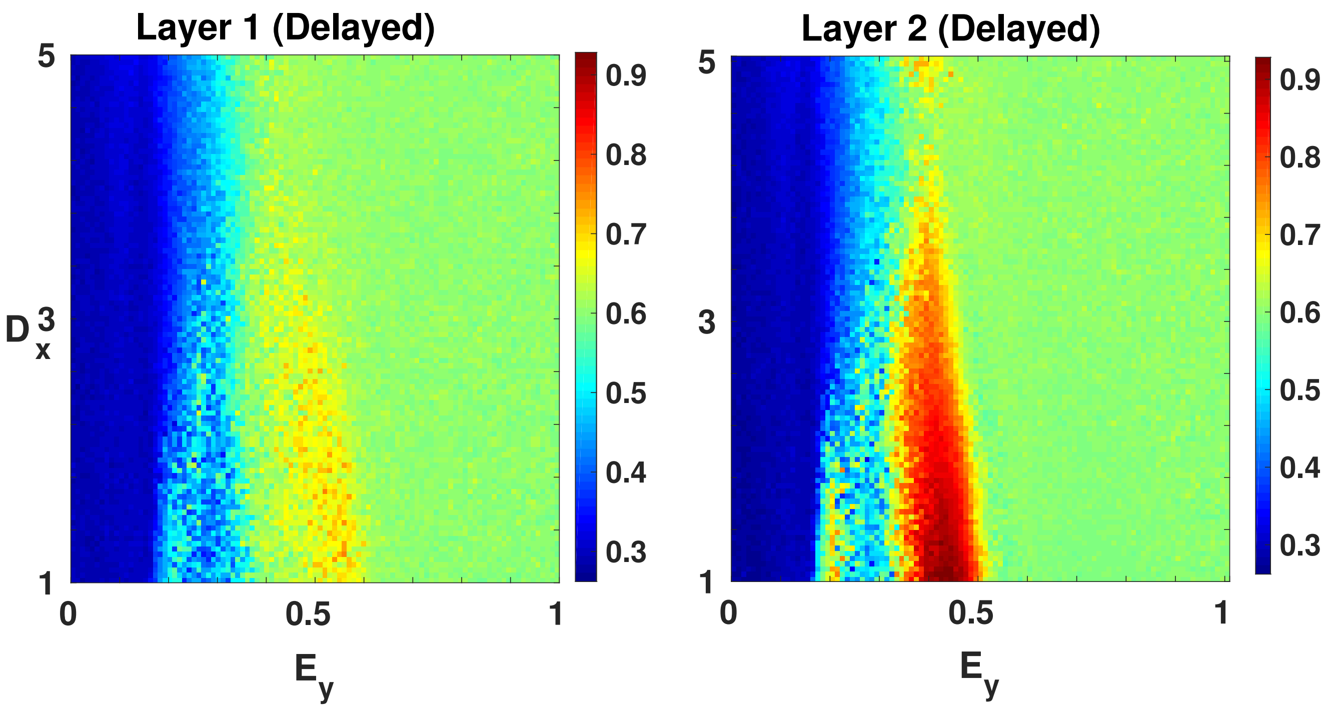

Figure 2: (Color Online) Phase diagram of normalized correlation measure g_0 in whole D_x - E_y space for (a) layer 1 and (b) layer 2 of the multiplex network. Half of the inter-layers edges are heterogeneously delayed and the delay values are chosen from a uniform random distribution with $\tau_{max} = 20$. Other parameters are same as Fig. 1.

References

[1] ATAY F. M., JOST J. and WENDE A. *Phys. Rev. Lett.* **92(14)** (2004) 144101.

[2] KEMETH F.P. *et al.*, *Chaos* **26** (2016) 094815