Median bias: sampling distributions

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Contents

Bias is defined as the distance between the mean of the sampling distribution (here estimated using a Monte-Carlo simulation) and the population value. In this notebook, we look in more detail at the shape of the sampling distribution, which was ignored by Miller (1988), and we consider other aspects of the distributions:

- median: typical sample value;

- standard deviation (SD): measure of spread;
- 50% highest density interval (HDI): spread and location of the bulk of the observations;
- P(sample < population value): another measure of bias, sensitive to the asymmetry of the sampling distribution.

```
# dependencies
library(ggplot2)
library(tibble)
library(tidyr)
library(cowplot)
library(retimes)
library(HDInterval)
source("./functions/akerd.txt")
source("./functions/skew.txt")
```

```
sessionInfo()
```

```
## R version 3.4.4 (2018-03-15)
## Platform: x86_64-apple-darwin15.6.0 (64-bit)
## Running under: OS X El Capitan 10.11.6
##
## Matrix products: default
## BLAS: /Library/Frameworks/R.framework/Versions/3.4/Resources/lib/libRblas.0.dylib
## LAPACK: /Library/Frameworks/R.framework/Versions/3.4/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_GB.UTF-8/en_GB.UTF-8/en_GB.UTF-8/C/en_GB.UTF-8/en_GB.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] HDInterval_0.1.3 retimes_0.1-2    cowplot_0.9.2    tidyr_0.8.0
## [5] tibble_1.4.2     ggplot2_3.0.0
##
## loaded via a namespace (and not attached):
##  [1] Rcpp_0.12.17     bindr_0.1.1      knitr_1.20       magrittr_1.5
##  [5] tidyselect_0.2.4 munsell_0.4.3    colorspace_1.3-2 R6_2.2.2
##  [9] rlang_0.2.1      stringr_1.2.0    plyr_1.8.4       dplyr_0.7.6
## [13] tools_3.4.4      grid_3.4.4       gtable_0.2.0     withr_2.1.2
## [17] htmltools_0.3.6  yaml_2.1.16      lazyeval_0.2.1   rprojroot_1.3-1
## [21] digest_0.6.15    assertthat_0.2.0 bindrcpp_0.2.2   purrr_0.2.5
## [25] glue_1.2.0       evaluate_0.10.1  rmarkdown_1.9    stringi_1.1.6
## [29] compiler_3.4.4   pillar_1.2.1     scales_0.5.0     backports_1.1.2
## [33] pkgconfig_2.0.1
```

Define ex-Gaussian parameters

```
load('./data/miller_exg_param.RData')
```

Save KDE

Save kernel density estimates of the sampling distributions of the mean and the median for each of the 12 distributions and each sample size.

```
load('./data/sim_miller1988.RData')

# save kernel density estimates
x <- seq(300, 900, 1)
kde.m <- array(NA, dim = c(length(x), nP, length(nvec)))
kde.md <- array(NA, dim = c(length(x), nP, length(nvec)))
kde.md.bc <- array(NA, dim = c(length(x), nP, length(nvec)))
for(S in 1:length(nvec)){
  for(P in 1:nP){
    kde.m[,P,S] <- akerd(sim.m[,P,S], pts = x, pyhat = TRUE, plotit = FALSE)
    kde.md[,P,S] <- akerd(sim.md[,P,S], pts = x, pyhat = TRUE, plotit = FALSE)
    kde.md.bc[,P,S] <- akerd(sim.md.bc[,P,S], pts = x, pyhat = TRUE, plotit = FALSE)
  }
}

save(
  kde.m,
  kde.md,
  kde.md.bc,
  nvec,
  nsim,
  x,
  file='./data/sim_miller1988_kde.RData')
```

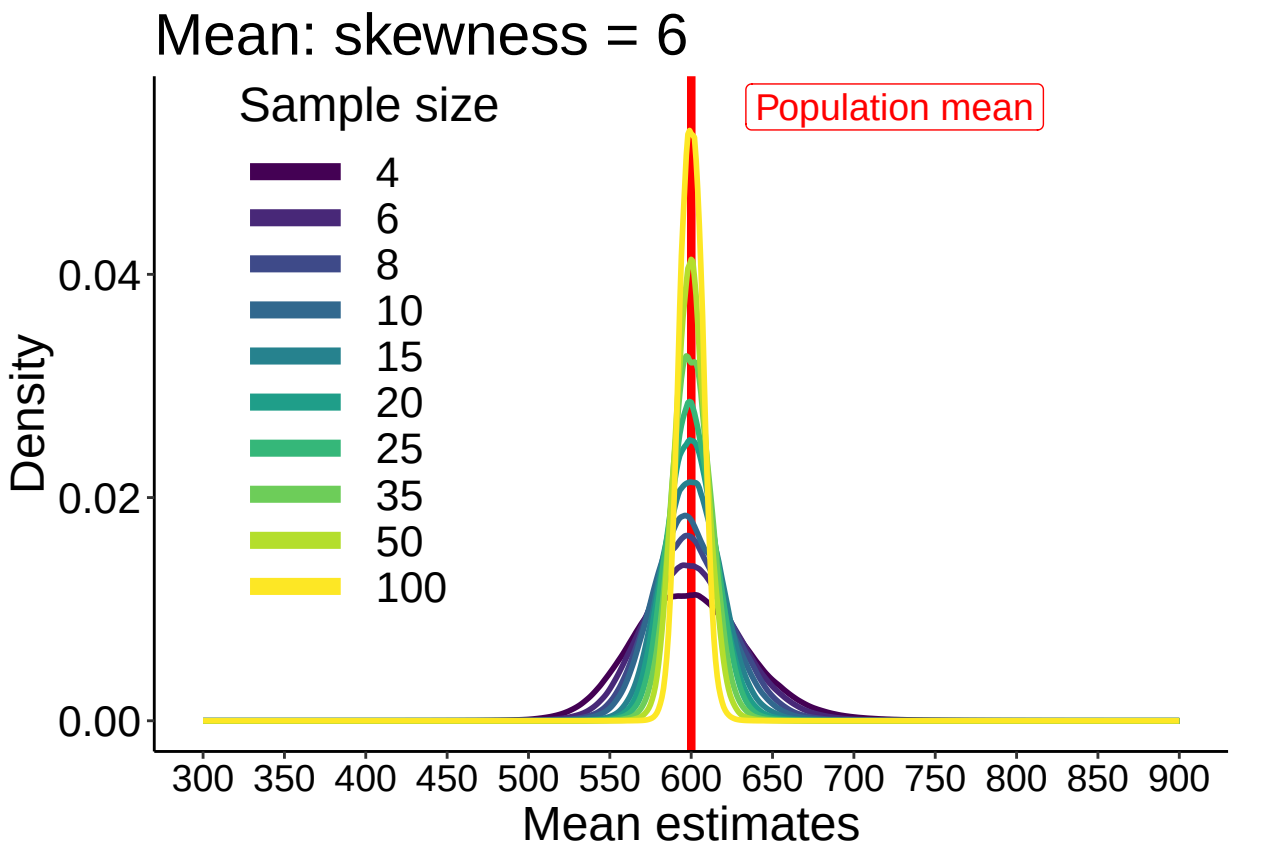
Sampling distribution of the mean

Least skewed distribution

```
load('./data/sim_miller1988.RData')
load('./data/sim_miller1988_kde.RData')
P <- 12 # least skewed distribution
df <- tibble(sd = rep(x, length(nvec)),
             kde = as.vector(kde.m[,P,]),
             `Sample size` = factor(rep(nvec, each = length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
  geom_vline(xintercept=pop.m[P], linetype=1, colour = "red", size = 1.5) +
          geom_line(aes(colour = `Sample size`), size=1) +
       scale_colour_viridis_d(direction=1) +
          theme(plot.title = element_text(size=22),
                axis.title.x = element_text(size = 18),
                axis.text.x = element_text(size = 14, colour="black"),
                axis.text.y = element_text(size = 16, colour="black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = c(0.2,0.6),
```

```
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) +
  scale_x_continuous(breaks = seq(300, 900, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Mean estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.m[P]+125, kde=0.055),
             label = "Population mean", angle = 90, colour="red", size=5) +
  ggtitle(paste0("Mean: skewness = ",round(pop.m[P] - pop.md[P])))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```



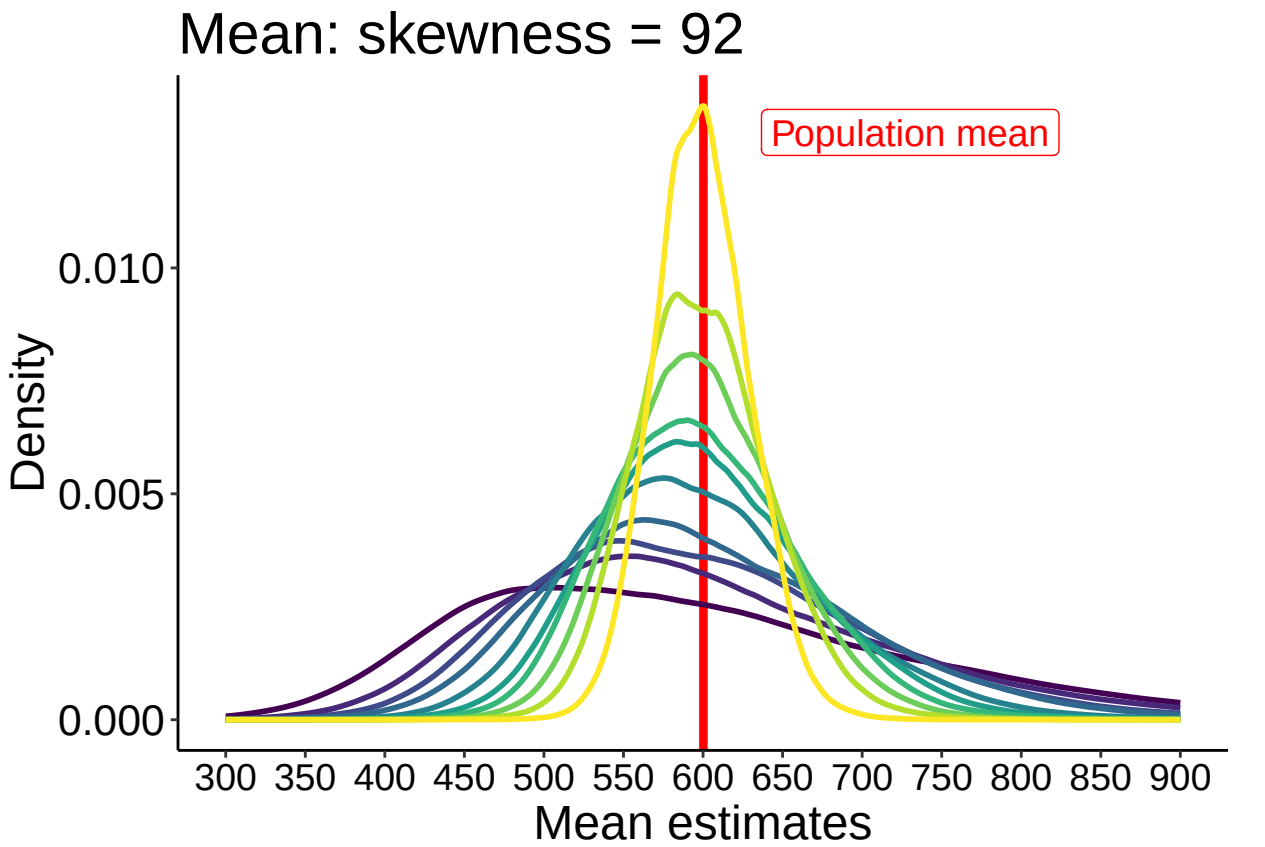

```
p.m.P12 <- p
```

Most skewed distribution

```
P <- 1 # most skewed distribution
df <- tibble(sd = rep(x, length(nvec)),
             kde = as.vector(kde.m[,P,]),
             `Sample size` = factor(rep(nvec, each = length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
      geom_vline(xintercept=pop.m[P], linetype=1, colour = "red", size = 1.5) +
```

```
      geom_line(aes(colour = `Sample size`), size=1) +
       scale_colour_viridis_d(direction=1) +
          theme(plot.title = element_text(size=22),
                axis.title.x = element_text(size = 18),
                axis.text.x = element_text(size = 14, colour="black"),
                axis.text.y = element_text(size = 16, colour="black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = "none", #c(0.2,0.55),
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) + #grey90
  scale_x_continuous(breaks = seq(100, 1000, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Mean estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.m[P]+130, kde=0.013),
             label = "Population mean", angle = 90, colour="red", size=5) +
  ggtitle(paste0("Mean: skewness = ",round(pop.m[P] - pop.md[P])))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```




```
p.m.P1 <- p
```

Highest density intervals

We use HDI to represent the location of the bulk of the sample means and medians. HDI are computed using a function from John K. Kruschke's Doing bayesian data analysis book. The function is also available in the supplementary information of this article.

Compute highest density intervals

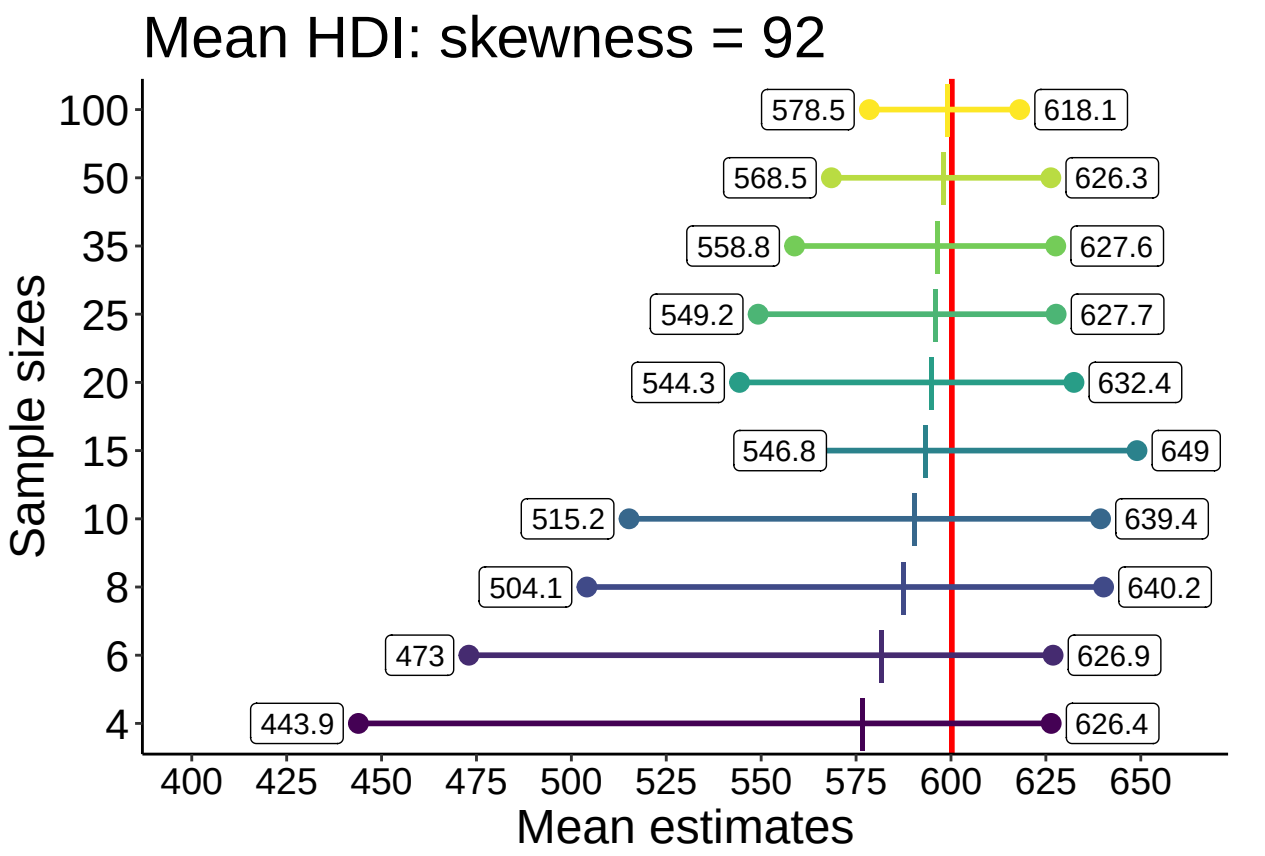
```
hdi.res <- matrix(0, nrow=length(nvec), ncol=2)
for(N in 1:length(nvec)){
hdi.res[N,] <- hdi(sim.m[,P,N], credMass=0.50)
}
hdi.res <- round(hdi.res, digits = 1)
# m.dist <- hdi.res[,2]-hdi.res[,1]
# 181.5 152.3 136.8 125.0 100.1  87.0  81.1  67.1  55.6  39.7
# md.dist <- hdi.res[,2]-hdi.res[,1]
# 166.8 141.8 124.6 116.9  98.8  85.8  77.4  65.6  55.4  39.9
dist.md <- apply(sim.m[,P,], 2, median) # median of sampling distribution
```

Illustrate results

```
df <- tibble(x = as.vector(hdi.res),
             y = rep(seq(1,length(nvec)),2),
             label = as.character(as.vector(hdi.res)))
df.seg <- tibble(x = hdi.res[,1],
                 y = seq(1,length(nvec)),
                 xend = hdi.res[,2],
                 yend = seq(1,length(nvec)))
# df.label1 <- tibble(x = hdi.res[,1],
#               y = seq(1,length(nvec)),
#               label = as.character(hdi.res[,1]))
df.label1a <- tibble(x = hdi.res[1:5,1],
             y = seq(1,5),
             label = as.character(hdi.res[1:5,1]))
df.label1b <- tibble(x = hdi.res[6:10,1],
             y = seq(6,10),
             label = as.character(hdi.res[6:10,1]))
df.label2 <- tibble(x = hdi.res[,2],
             y = seq(1,length(nvec)),
             label = as.character(hdi.res[,2]))
df.md <- tibble(x = dist.md,
                y = seq(1,length(nvec)))

p <- ggplot(df, aes(x=x, y=y)) + theme_classic() +
  geom_vline(xintercept = pop.m[P], colour="red", size=1) +
  geom_point(size=3, aes(colour=y)) +
  geom_segment(data=df.seg, size=1, aes(x=x, xend=xend, y=y, yend=yend, colour=y)) +
  scale_colour_viridis_c(direction=1) +
  scale_y_continuous(breaks = seq(1,length(nvec)), labels = as.character(nvec)) +
  geom_point(data=df.md, shape=124, size=7, aes(colour=y)) +
  # geom_label(data=df.label1, aes(label=label), hjust = "inward", nudge_x = -3) +
  geom_label(data=df.label1a, aes(label=label), hjust = "outward", nudge_x = -4) +
  geom_label(data=df.label1b, aes(label=label), hjust = "inward", nudge_x = -4) +
```

```
  geom_label(data=df.label2, aes(label=label), hjust = "outward", nudge_x = 4) +
  scale_x_continuous(breaks = seq(400, 650, 25)) +
  coord_cartesian(xlim=c(400, 660)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 14, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "none",
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Mean estimates", y = "Sample sizes") +
  ggtitle(paste0("Mean HDI: skewness = ",round(pop.m[P] - pop.md[P])))
p
```



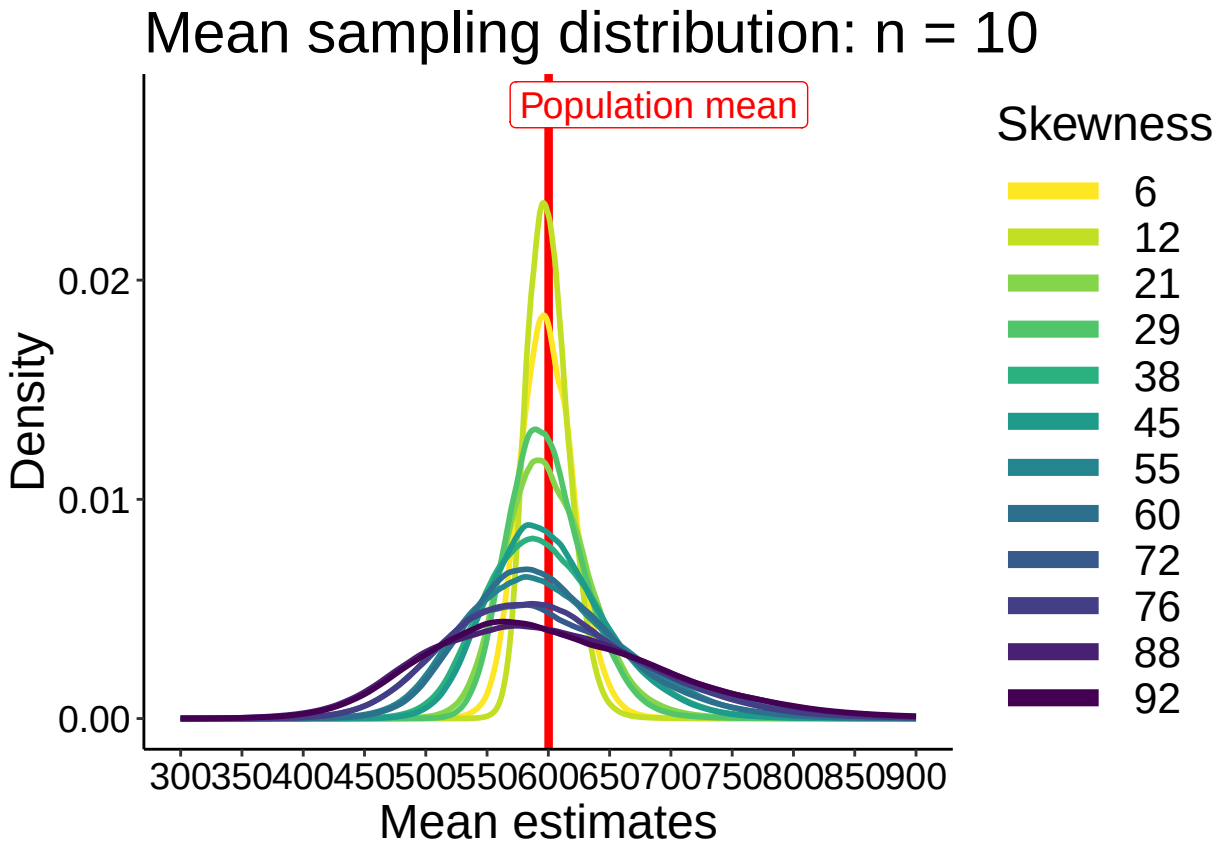

```
p.m.P1.hdi <- p
# save figure
# ggsave(filename = 'figure_m_diff_size_hdi.jpg',width=10,height=6) #path=pathname
```

For small sample sizes, the 50% HDI is offset to the left of the population mean, and so is the median of the sampling distribution. This means that the typical sample mean tends to under-estimate the population mean - that is to say, the mean sampling distribution is median biased. This offset reduces with increasing sample size, but is still present even for n=100.

n=10 - illustrate skewness effect

```
N <- 4 # n=10
df <- tibble(sd = rep(x, nP),
             kde = as.vector(kde.m[,,N]),
             `Skewness` = factor(rep(round(pop.m-pop.md), each = length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
  geom_vline(xintercept=pop.m[P], linetype=1, colour = "red", size = 1.5) +
          geom_line(aes(colour = `Skewness`), size=1) +
       scale_colour_viridis_d(direction=-1) +
          theme(plot.title = element_text(size = 22),
                axis.title.x = element_text(size = 18),
                axis.text = element_text(size = 14, colour = "black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = "right",#c(0.15,0.55),
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) +
  scale_x_continuous(breaks = seq(100, 1000, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Mean estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.m[P]+90, kde=0.028), # same mean for all distributions
             label = "Population mean", angle = 90, colour="red", size=5) +
  ggtitle(paste0("Mean sampling distribution: n = ",nvec[N]))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```




```
p.m.N10 <- p
```

Highest density intervals

Compute highest density intervals

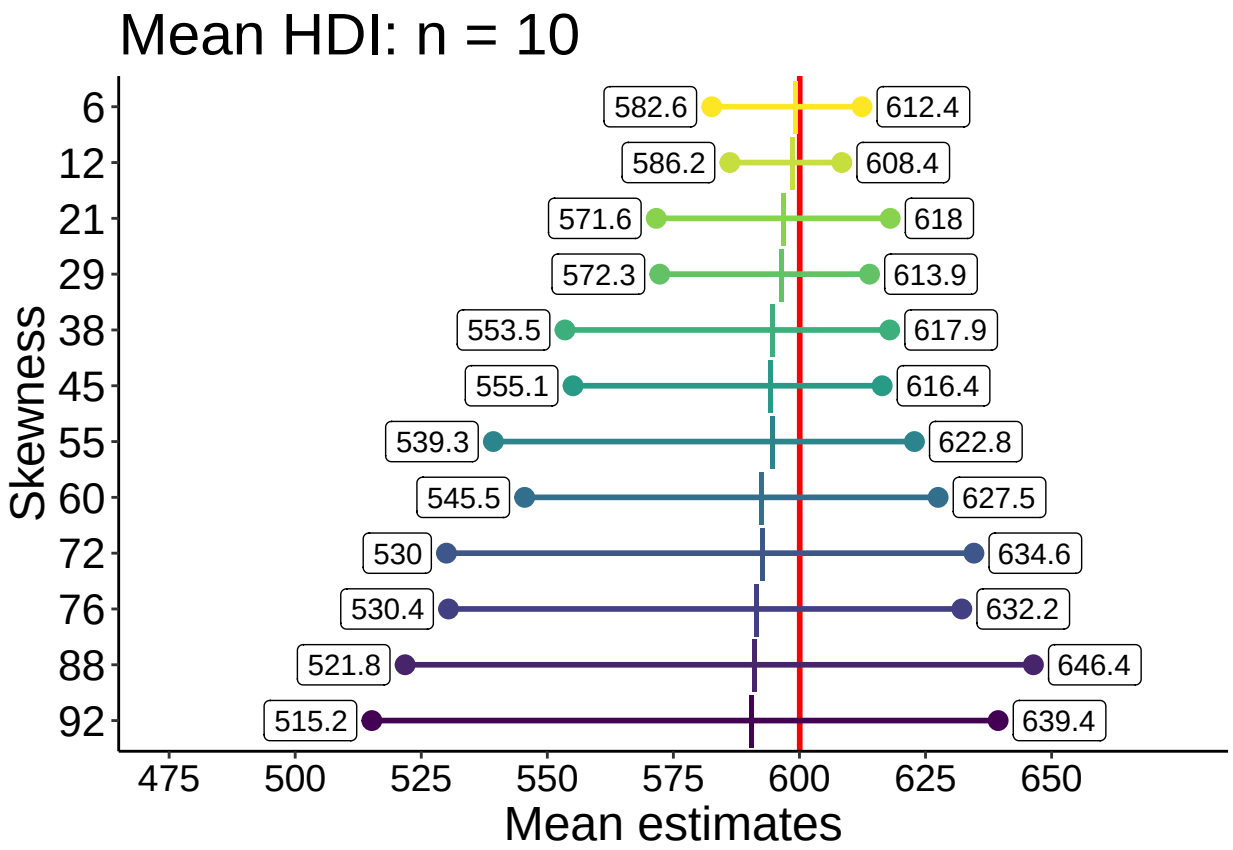
```
hdi.res <- matrix(0, nrow=nP, ncol=2)
for(P in 1:nP){
hdi.res[P,] <- hdi(sim.m[,P,N], credMass=0.50)
}
hdi.res <- round(hdi.res, digits = 1)
dist.md <- apply(sim.m[,,N], 2, median) # median of sampling distribution
```

Illustrate results

```
df <- tibble(x = as.vector(hdi.res),
             y = rep(seq(1,nP),2),
             label = as.character(as.vector(hdi.res)))
df.seg <- tibble(x = hdi.res[,1],
                 y = seq(1,nP),
                 xend = hdi.res[,2],
                 yend = seq(1,nP))
df.label1 <- tibble(x = hdi.res[,1],
             y = seq(1,nP),
             label = as.character(hdi.res[,1]))
df.label1a <- tibble(x = hdi.res[1:10,1],
```

```
              y = seq(1,10),
              label = as.character(hdi.res[1:10,1]))
df.label1b <- tibble(x = hdi.res[11:12,1],
              y = seq(11,12),
              label = as.character(hdi.res[11:12,1]))
df.label2 <- tibble(x = hdi.res[,2],
              y = seq(1,nP),
              label = as.character(hdi.res[,2]))
df.md <- tibble(x = dist.md,
                y = seq(1,nP))

p <- ggplot(df, aes(x=x, y=y)) + theme_classic() +
  geom_vline(xintercept = pop.m[P], colour="red", size=1) +
  geom_point(size=3, aes(colour=y)) +
  geom_segment(data=df.seg, size=1, aes(x=x, xend=xend, y=y, yend=yend, colour=y)) +
  scale_colour_viridis_c(direction=1) +
  scale_y_continuous(breaks = seq(1,nP), labels = as.character(round(pop.m-pop.md, digits=0))) +
  geom_point(data=df.md, shape=124, size=7, aes(colour=y)) +
  # geom_label(data=df.label1, aes(label=label), hjust = "outward", nudge_x = -3) +
  geom_label(data=df.label1a, aes(label=label), hjust = "outward", nudge_x = -3) +
  geom_label(data=df.label1b, aes(label=label), hjust = "inward", nudge_x = -3) +
  geom_label(data=df.label2, aes(label=label), hjust = "outward", nudge_x = 3) +
  scale_x_continuous(breaks = seq(400, 650, 25)) +
  coord_cartesian(xlim=c(475, 675)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 14, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "none",
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Mean estimates", y = "Skewness") +
  ggtitle(paste0("Mean HDI: n = ",nvec[N]))
p
```



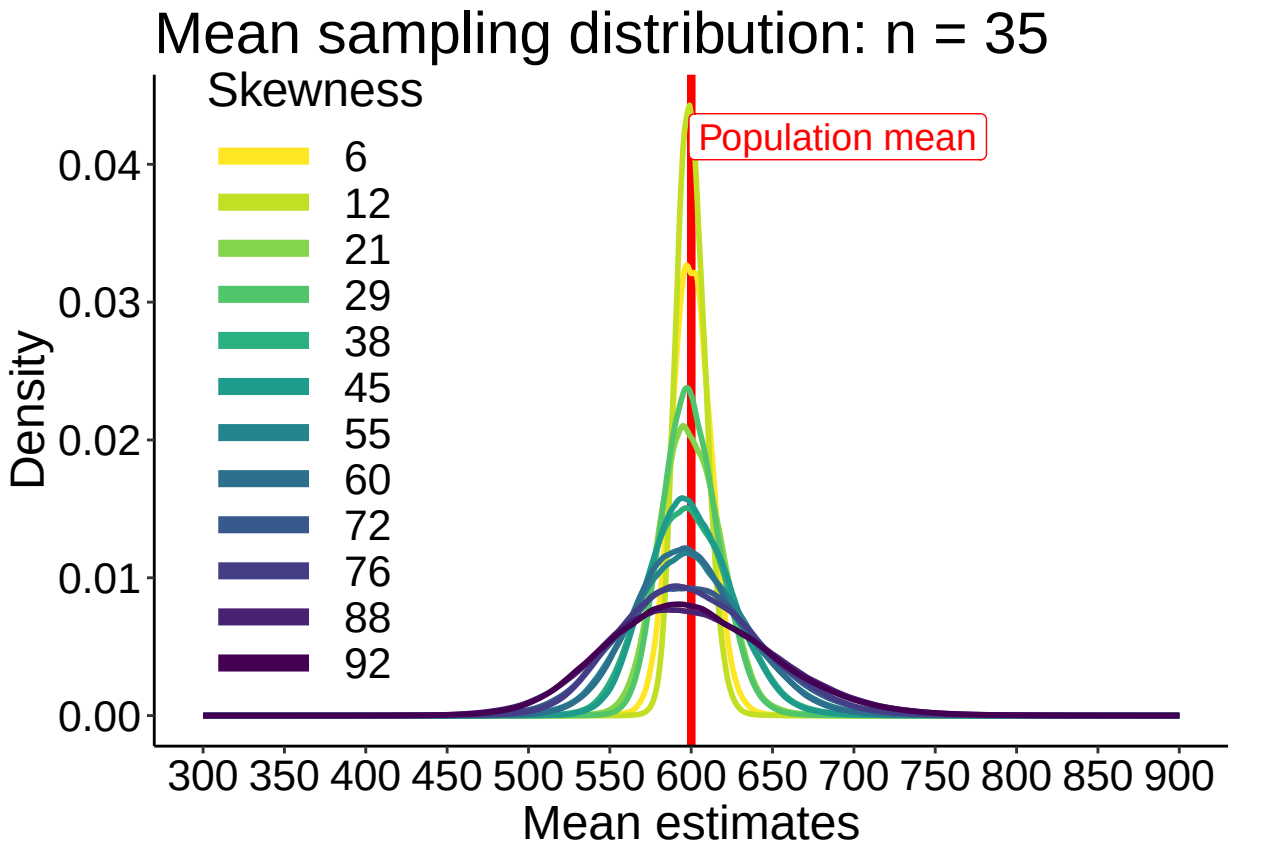

```
p.m.N10.hdi <- p
# save figure
# ggsave(filename = 'figure_m_diff_size_hdi.jpg',width=10,height=6) #path=pathname
```

n=35 - illustrate skewness effect

```
N <- 8 # n=35
df <- tibble(sd = rep(x, nP),
             kde = as.vector(kde.m[,,N]),
             `Skewness` = factor(rep(round(pop.m-pop.md), each = length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
  geom_vline(xintercept=pop.m[P], linetype=1, colour = "red", size = 1.5) +
          geom_line(aes(colour = `Skewness`), size=1) +
       scale_colour_viridis_d(direction=-1) +
          theme(plot.title = element_text(size = 22),
                axis.title.x = element_text(size = 18),
                axis.text = element_text(size = 16, colour = "black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = c(0.15,0.55),
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) +
  scale_x_continuous(breaks = seq(100, 1000, 50)) +
```

```
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Mean estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.m[P]+90, kde=0.042),
             label = "Population mean", angle = 90, colour="red", size=5) +
  ggtitle(paste0("Mean sampling distribution: n = ",nvec[N]))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```




```
p.m.N35 <- p
```

All mean HDI

Compute HDIs for all skewness levels.

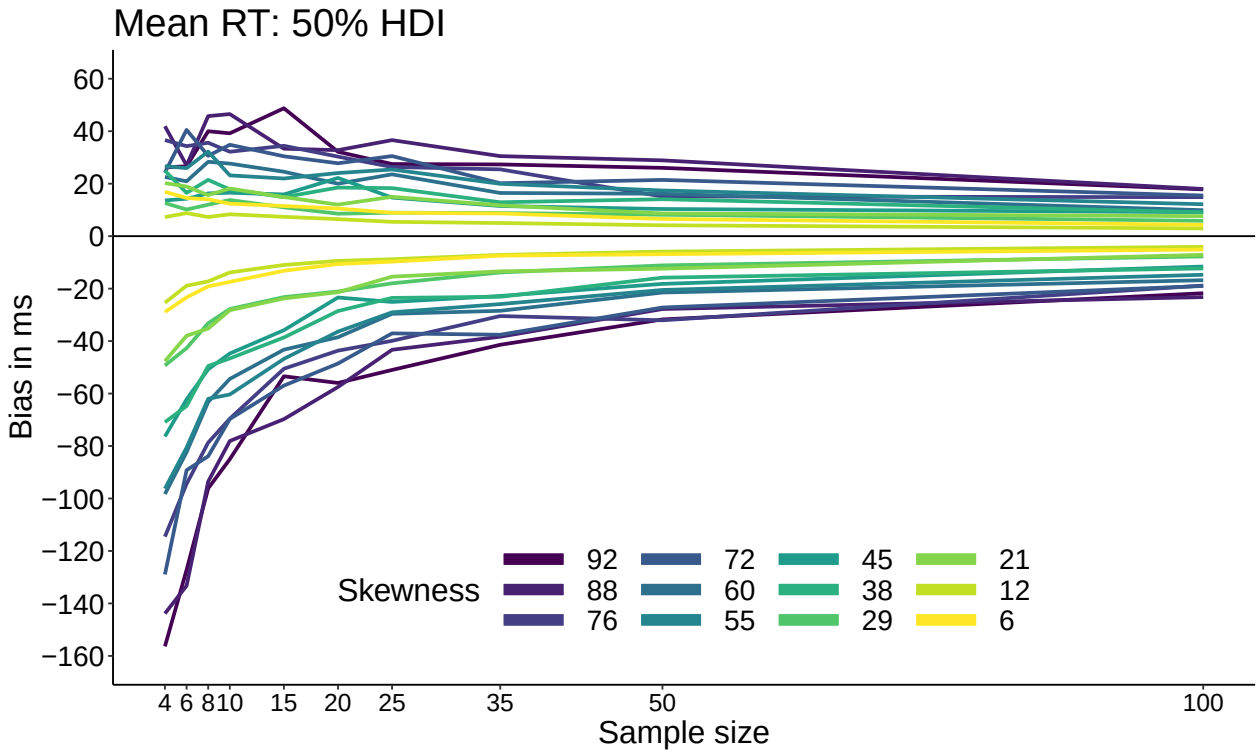
```
# 50% HDI of mean bias
hdi.m <- array(NA, dim = c(2, nP, length(nvec)))
for(iter.n in 1:length(nvec)){
  for(P in 1:nP){
      hdi.m[, P, iter.n] <- hdi(sim.m[, P, iter.n]-pop.m[P], credMass=0.50)
  }
}
```

Illustrate HDIs for all skewness levels.

```
# df <- tibble(`Bias`=c(as.vector(hdi.m[1,,]),as.vector(hdi.m[2,,])),
#              `Size`=rep(rep(nvec,each=nP),2),
#              `Skewness`=rep(rep(round(pop.m - pop.md),length(nvec)),2),
#              `Side`=c(rep("lower",length(nvec)*nP), rep("upper",length(nvec)*nP)))
df1 <- tibble(`Bias`=as.vector(hdi.m[1,,]),
              `Size`=rep(nvec,each=nP),
              `Skewness`=rep(round(pop.m - pop.md),length(nvec)))
df2 <- tibble(`Bias`=as.vector(hdi.m[2,,]),
              `Size`=rep(nvec,each=nP),
              `Skewness`=rep(round(pop.m - pop.md),length(nvec)))

df1$Skewness <- as.character(df1$Skewness)
df1$Skewness <- factor(df1$Skewness, levels=unique(df1$Skewness))
df2$Skewness <- as.character(df2$Skewness)
df2$Skewness <- factor(df2$Skewness, levels=unique(df2$Skewness))

# make plot
p <- ggplot(df1, aes(x=Size, y=Bias)) + theme_classic() +
  geom_line(aes(colour = Skewness), size = 1) + #linetype = Side
  geom_line(data=df2, aes(colour = Skewness), size = 1) +
  geom_abline(intercept=0, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
 scale_y_continuous(breaks=seq(-160,60,20)) +
  coord_cartesian(ylim=c(-160,60)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 14, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = c(0.5,0.15),
        legend.direction = "horizontal",
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "Bias in ms") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Mean RT: 50% HDI")
p
```




```
p.m.hdi <- p
# save figure
# ggsave(filename='./figures/figure_miller_bias_m_hdi.pdf',width=10,height=6)
```

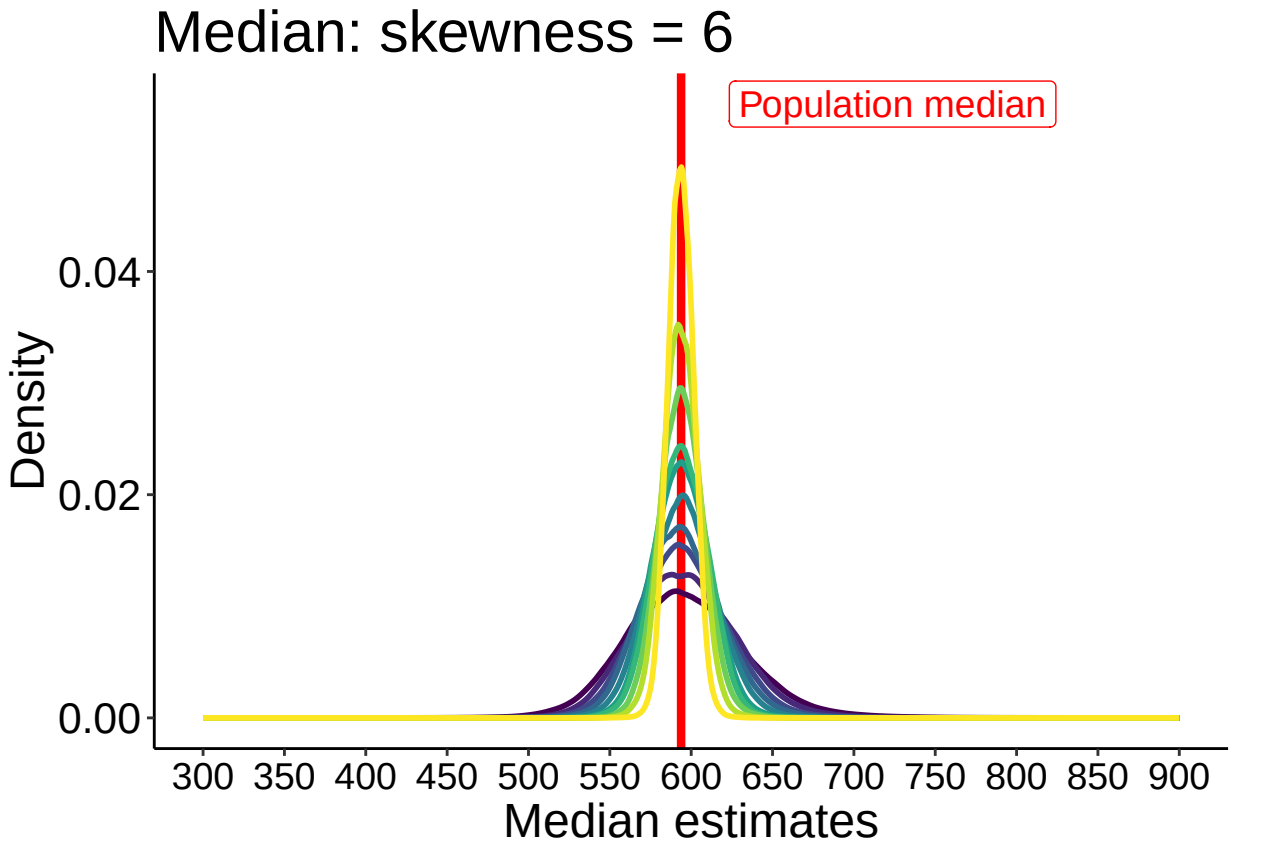
Sampling distribution of the median

Least skewed distribution

```
# load('./data/sim_miller1988_sampdist.RData')
P <- 12 # least skewed distribution
df <- tibble(sd = rep(x, length(nvec)),
             kde = as.vector(kde.md[,P,]),
             `Sample size` = factor(rep(nvec, each = length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
  geom_vline(xintercept=pop.md[P], linetype=1, colour = "red", size = 1.5) +
          geom_line(aes(colour = `Sample size`), size=1) +
       scale_colour_viridis_d(direction=1) +
          theme(plot.title = element_text(size=22),
                axis.title.x = element_text(size = 18),
                axis.text.x = element_text(size = 14, colour="black"),
                axis.text.y = element_text(size = 16, colour="black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = "none",#c(0.15,0.6),
                legend.text=element_text(size=16),
```

```
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) +
  scale_x_continuous(breaks = seq(300, 900, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Median estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.md[P]+130, kde=0.055),
             label = "Population median", angle = 90, colour="red", size=5) +
  ggtitle(paste0("Median: skewness = ",round(pop.m[P] - pop.md[P])))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```



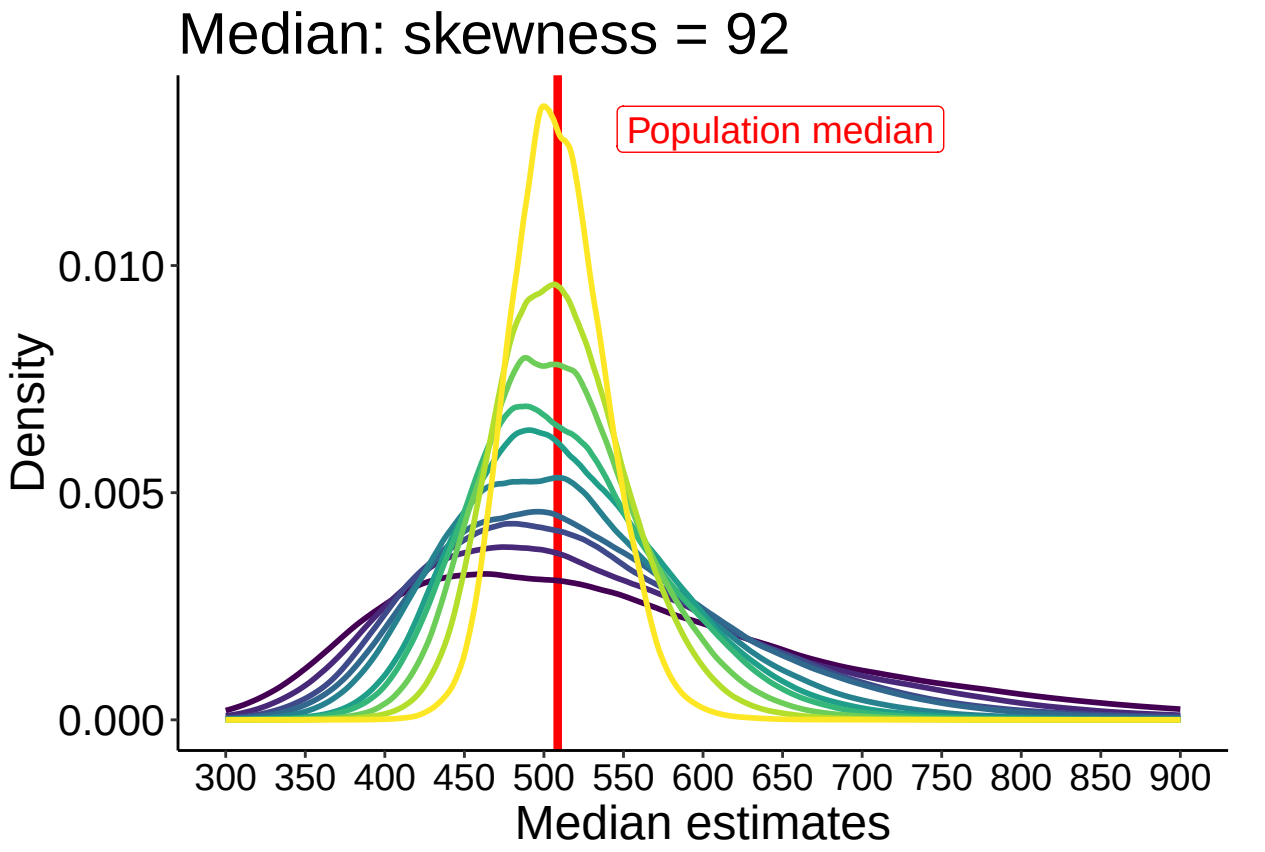

```
p.md.P12 <- p
```

Most skewed distribution

```
P <- 1 # most skewed distribution
df <- tibble(sd = rep(x, length(nvec)),
             kde = as.vector(kde.md[,P,]),
             `Sample size` = factor(rep(nvec, each = length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
      geom_vline(xintercept=pop.md[P], linetype=1, colour = "red", size = 1.5) +
      geom_line(aes(colour = `Sample size`), size=1) +
```

```
      scale_colour_viridis_d(direction=1) +
         theme(plot.title = element_text(size=22),
               axis.title.x = element_text(size = 18),
               axis.text.x = element_text(size = 14, colour="black"),
               axis.text.y = element_text(size = 16, colour="black"),
               axis.title.y = element_text(size = 18),
               legend.key.width = unit(1.5,"cm"),
               legend.position = "none", #c(0.2,0.55),
               legend.text=element_text(size=16),
               legend.title=element_text(size=18),
                panel.background = element_rect(fill="white")) + #grey90
  scale_x_continuous(breaks = seq(100, 1000, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Median estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.md[P]+140, kde=0.013),
             label = "Population median", angle = 90, colour="red", size=5) +
  ggtitle(paste0("Median: skewness = ",round(pop.m[P] - pop.md[P])))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```




```
p.md.P1 <- p
```

Highest density intervals

Compute highest density intervals

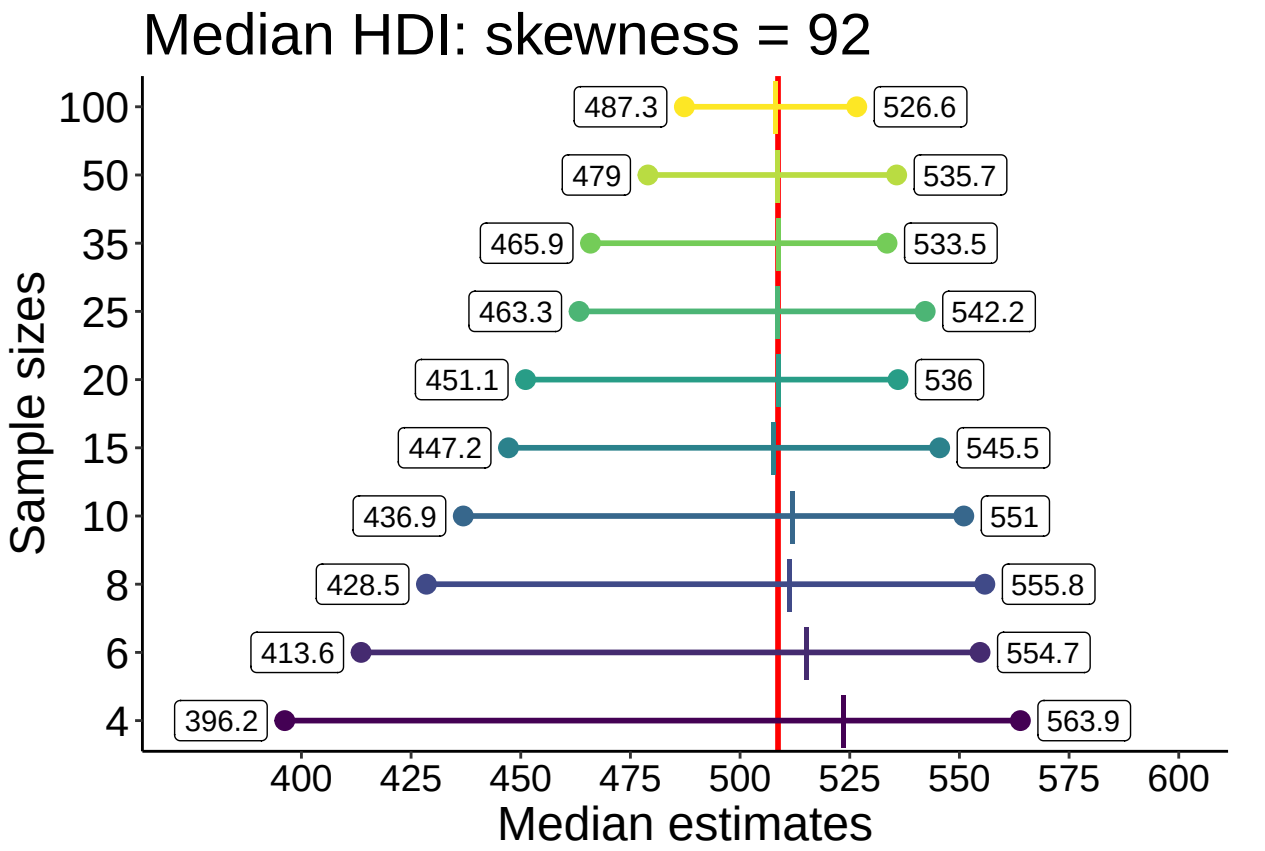
```
hdi.res <- matrix(0, nrow=length(nvec), ncol=2)
for(N in 1:length(nvec)){
hdi.res[N,] <- hdi(sim.md[,P,N], credMass=0.50)
}
hdi.res <- round(hdi.res, digits = 1)
# md.dist <- hdi.res[,2]-hdi.res[,1]
# 166.8 141.8 124.6 116.9  98.8  85.8  77.4  65.6  55.4  39.9
dist.md <- apply(sim.md[,P,], 2, median) # median of sampling distribution
```

Illustrate results

```
df <- tibble(x = as.vector(hdi.res),
             y = rep(seq(1,length(nvec)),2),
             label = as.character(as.vector(hdi.res)))
df.seg <- tibble(x = hdi.res[,1],
                 y = seq(1,length(nvec)),
                 xend = hdi.res[,2],
                 yend = seq(1,length(nvec)))
df.label1 <- tibble(x = hdi.res[,1],
             y = seq(1,length(nvec)),
             label = as.character(hdi.res[,1]))
df.label1a <- tibble(x = hdi.res[1:4,1],
             y = seq(1,4),
             label = as.character(hdi.res[1:4,1]))
df.label1b <- tibble(x = hdi.res[5:10,1],
             y = seq(5,10),
             label = as.character(hdi.res[5:10,1]))
df.label2 <- tibble(x = hdi.res[,2],
             y = seq(1,length(nvec)),
             label = as.character(hdi.res[,2]))
df.md <- tibble(x = dist.md,
                y = seq(1,length(nvec)))

p <- ggplot(df, aes(x=x, y=y)) + theme_classic() +
  geom_vline(xintercept = pop.md[P], colour="red", size=1) +
  geom_point(size=3, aes(colour=y)) +
  geom_segment(data=df.seg, size=1, aes(x=x, xend=xend, y=y, yend=yend, colour=y)) +
  scale_color_viridis_c(direction=1) +
  scale_y_continuous(breaks = seq(1,length(nvec)), labels = as.character(nvec)) +
  geom_point(data=df.md, shape=124, size=7, aes(colour=y)) +
  geom_label(data=df.label1, aes(label=label), hjust = "outward", nudge_x = -4) +
  # geom_label(data=df.label1a, aes(label=label), hjust = "outward", nudge_x = -3) +
  # geom_label(data=df.label1b, aes(label=label), hjust = "inward", nudge_x = -3) +
  geom_label(data=df.label2, aes(label=label), hjust = "outward", nudge_x = 4) +
  scale_x_continuous(breaks = seq(400, 650, 25)) +
  coord_cartesian(xlim=c(375, 600)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 14, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "none",
```

```
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Median estimates", y = "Sample sizes") +
  ggtitle(paste0("Median HDI: skewness = ",round(pop.m[P] - pop.md[P])))
p
```




```
p.md.P1.hdi <- p
# save figure
# ggsave(filename = 'figure_m_diff_size_hdi.jpg',width=10,height=6) #path=pathname
```

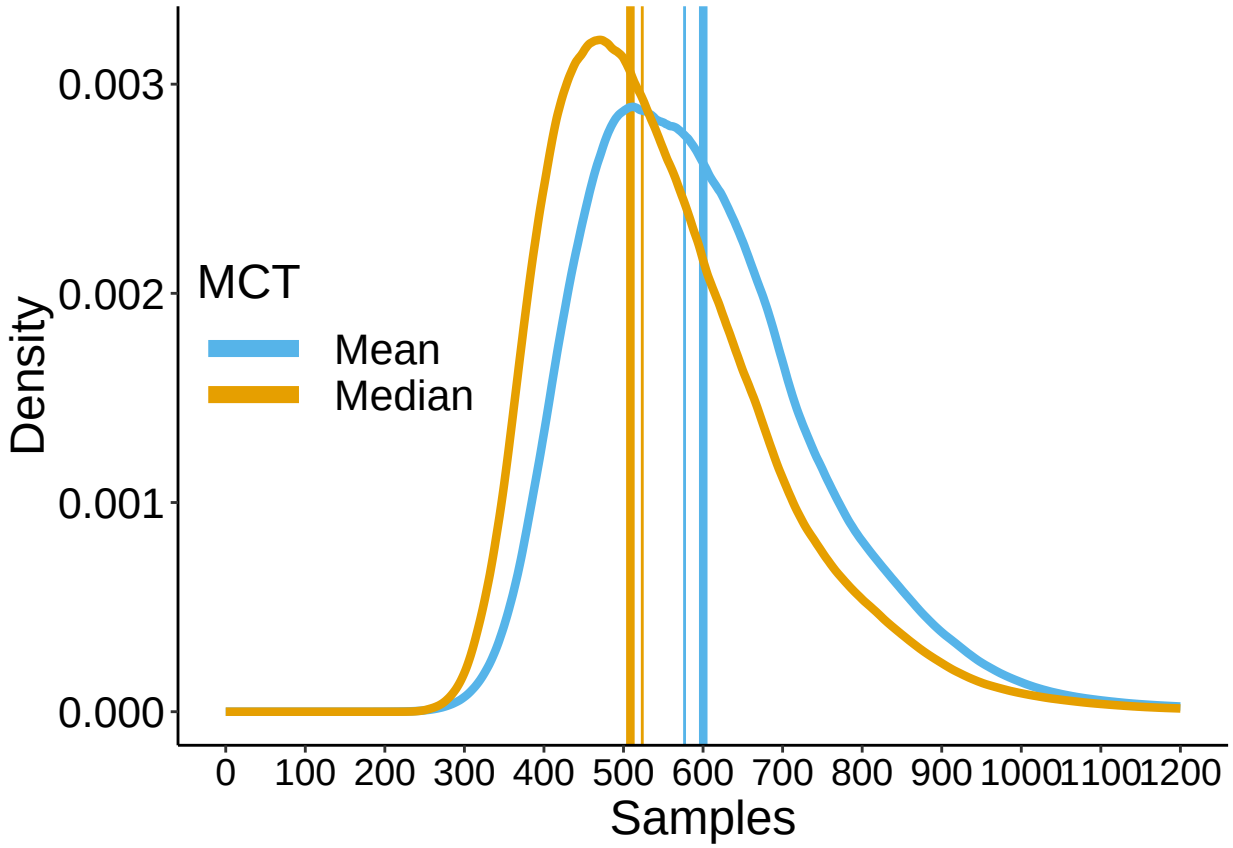
With small sample sizes, there is a discrepancy between the 50% HDI, which is shifted to the left of the population median, and the median of the sampling distribution, which is shifted to the right of the population median. This contrasts with the results for the mean, and can be explained by differences in the shapes of the sampling distributions, in particular the larger skewness and kurtosis of the median sampling distribution compared to that of the mean (see next figure). The offset between 50% HDI and the population reduces quickly with increasing sample size. For n=10, the median bias is already very small. From n=15, the median sample distribution is not median bias, which means that the typical sample median is not biased.

Compare mean to median for n=4

```
P <- 1 # most skewed distribution
S <- 1 # n=4
# save kernel density estimates
x <- seq(0, 1200, 1)
kde.m1 <- akerd(sim.m[,P,S], pts = x, pyhat = TRUE, plotit = FALSE)
kde.md1 <- akerd(sim.md[,P,S], pts = x, pyhat = TRUE, plotit = FALSE)
```

```r
df <- tibble(sd = rep(x, 2),
             kde = c(kde.md1, kde.m1),
             MCT = c(rep("Median",length(x)), rep("Mean",length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
      geom_vline(xintercept=pop.md[P], linetype=1, colour = "#E69F00", size = 1.5) +
      geom_vline(xintercept=pop.m[P], linetype=1, colour = "#56B4E9", size = 1.5) +
      geom_vline(xintercept=median(sim.md[,P,S]), linetype=1, colour = "#E69F00", size = 0.5) +
      geom_vline(xintercept=median(sim.m[,P,S]), linetype=1, colour = "#56B4E9", size = 0.5) +
      geom_line(aes(colour = `MCT`), size=1.5) +
      scale_color_manual(values=c("#56B4E9", "#E69F00")) +
                theme(plot.title = element_text(size=22),
                axis.title.x = element_text(size = 18),
                axis.text.x = element_text(size = 14, colour="black"),
                axis.text.y = element_text(size = 16, colour="black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = c(0.15,0.55),
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                panel.background = element_rect(fill="white")) + #grey90
  scale_x_continuous(breaks = seq(0, 1200, 100)) +
  coord_cartesian(xlim = c(0, 1200)) +
  labs(x = "Samples", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3)))
  # geom_label(data=tibble(sd=pop.md[P]+140, kde=0.013),
  #            label = "Population median", angle = 90, colour="red", size=5) +
  # ggtitle(paste0("Median: skewness = ",round(pop.m[P] - pop.md[P])))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```



Thick lines show the population values.
Thin lines show the medians of the sampling distributions. The sample mean is more median biased than the sample median.

Median sampling distribution is more skewed and kurtotic than the mean's

```
skew(sim.md[,P,S])
```

```
## [1] 1.092176
```

```
skew(sim.m[,P,S])
```

```
## [1] 0.9580953
```

```
kurt(sim.md[,P,S])
```

```
## [1] 4.56367
```

```
kurt(sim.m[,P,S])
```

```
## [1] 4.349415
```

Summary figure: effect of sample size at skewness 6 and 92

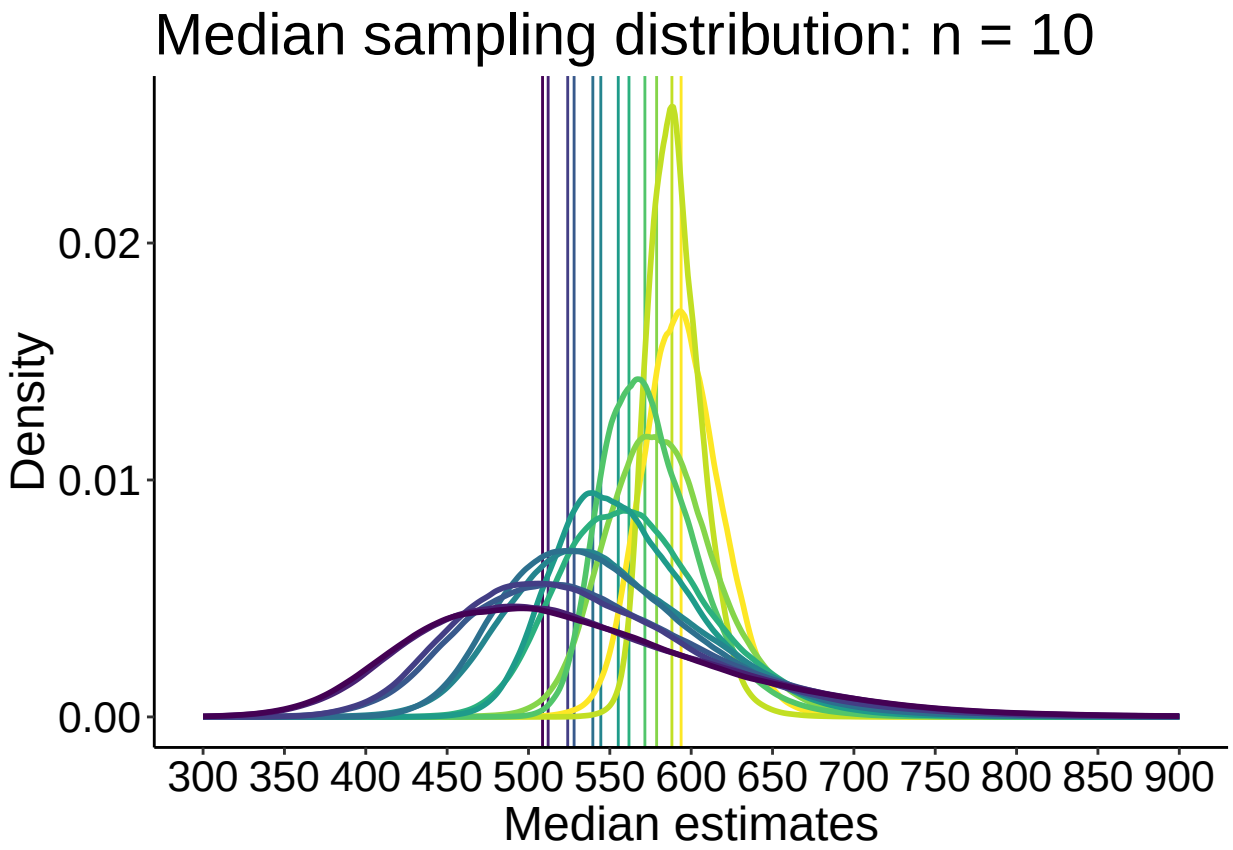
```
# combine panels into one figure
cowplot::plot_grid(p.m.P12, p.md.P12,
                   p.m.P1, p.md.P1,
```

```
                    p.m.P1.hdi, p.md.P1.hdi,
                           labels = c("A", "D", "B","E", "C", "F"),
                           ncol = 2,
                           nrow = 3,
                           rel_widths = c(1, 1, 1, 1, 1, 1),
                           label_size = 20,
                           hjust = -0.5,
                           scale=.95,
                           align = "h")
# save figure
ggsave(filename='./figures/figure_samp_dist_summary.pdf',width=12,height=15) #path=pathname
```

n=10 - illustrate skewness effect

```
load('./data/sim_miller1988.RData')
load('./data/sim_miller1988_kde.RData')
N <- 4 # n=10
df <- tibble(sd = rep(x, nP),
             kde = as.vector(kde.md[,,N]),
             `Skewness` = factor(rep(round(pop.m-pop.md), each = length(x))))
df.pop <- tibble(pop = pop.md,
                 Skewness = factor(round(pop.m-pop.md)))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
  geom_vline(data=df.pop, aes(xintercept=pop, colour = `Skewness`), linetype=1, size = 0.5) +
          geom_line(aes(colour = `Skewness`), size=1) +
       scale_colour_viridis_d(direction=-1) +
          theme(plot.title = element_text(size = 22),
                axis.title.x = element_text(size = 18),
                axis.text = element_text(size = 16, colour = "black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = "none",#c(0.15,0.55),
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) +
  scale_x_continuous(breaks = seq(100, 1000, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Median estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  # geom_label(data=tibble(sd=pop.md+90, kde=0.028),
  #            label = "Population median", angle = 90, colour="red", size=5) +
  ggtitle(paste0("Median sampling distribution: n = ",nvec[N]))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```




```
p.md.N10 <- p
```

Highest density intervals

Compute highest density intervals

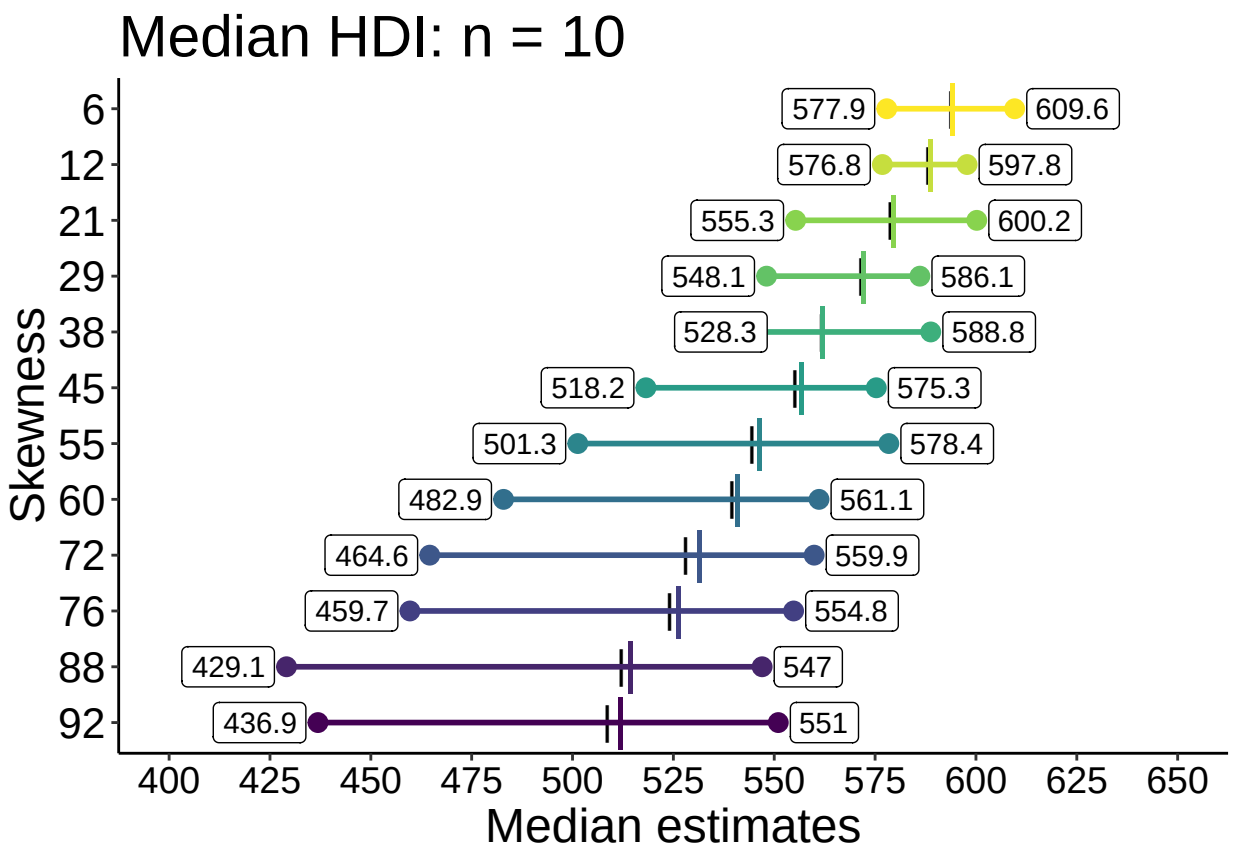
```
hdi.res <- matrix(0, nrow=nP, ncol=2)
for(P in 1:nP){
hdi.res[P,] <- hdi(sim.md[,P,N], credMass=0.50)
}
hdi.res <- round(hdi.res, digits = 1)
dist.md <- apply(sim.md[,,N], 2, median) # median of sampling distribution
```

Illustrate results

```
df <- tibble(x = as.vector(hdi.res),
             y = rep(seq(1,nP),2),
             label = as.character(as.vector(hdi.res)))
df.seg <- tibble(x = hdi.res[,1],
                 y = seq(1,nP),
                 xend = hdi.res[,2],
                 yend = seq(1,nP))
df.label1 <- tibble(x = hdi.res[,1],
             y = seq(1,nP),
             label = as.character(hdi.res[,1]))
df.label1a <- tibble(x = hdi.res[1:8,1],
```

```
              y = seq(1,8),
              label = as.character(hdi.res[1:8,1]))
df.label1b <- tibble(x = hdi.res[9:12,1],
              y = seq(9,12),
              label = as.character(hdi.res[9:12,1]))
df.label2 <- tibble(x = hdi.res[,2],
              y = seq(1,nP),
              label = as.character(hdi.res[,2]))
df.md <- tibble(x = dist.md,
                y = seq(1,nP))
df.pop <- tibble(x = pop.md,
                 y = seq(1,nP))

p <- ggplot(df, aes(x=x, y=y)) + theme_classic() +
  # geom_vline(xintercept = pop.m, colour="red", size=1) +
  geom_point(data=df.pop, shape=124, size=5, colour="black") +
  geom_point(size=3, aes(colour=y)) +
  geom_segment(data=df.seg, size=1, aes(x=x, xend=xend, y=y, yend=yend, colour=y)) +
  scale_color_viridis_c(direction=1) +
  scale_y_continuous(breaks = seq(1,nP), labels = as.character(round(pop.m-pop.md, digits=0))) +
  geom_point(data=df.md, shape=124, size=7, aes(colour=y)) +
  # geom_label(data=df.label1, aes(label=label), hjust = "outward", nudge_x = -3) +
  geom_label(data=df.label1a, aes(label=label), hjust = "outward", nudge_x = -3) +
  geom_label(data=df.label1b, aes(label=label), hjust = "inward", nudge_x = -3) +
  geom_label(data=df.label2, aes(label=label), hjust = "outward", nudge_x = 3) +
  scale_x_continuous(breaks = seq(400, 650, 25)) +
  coord_cartesian(xlim=c(400, 650)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 14, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "none",
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Median estimates", y = "Skewness") +
  ggtitle(paste0("Median HDI: n = ",nvec[N]))
p
```




```
p.md.N10.hdi <- p
# save figure
# ggsave(filename = 'figure_m_diff_size_hdi.jpg',width=10,height=6) #path=pathname
```

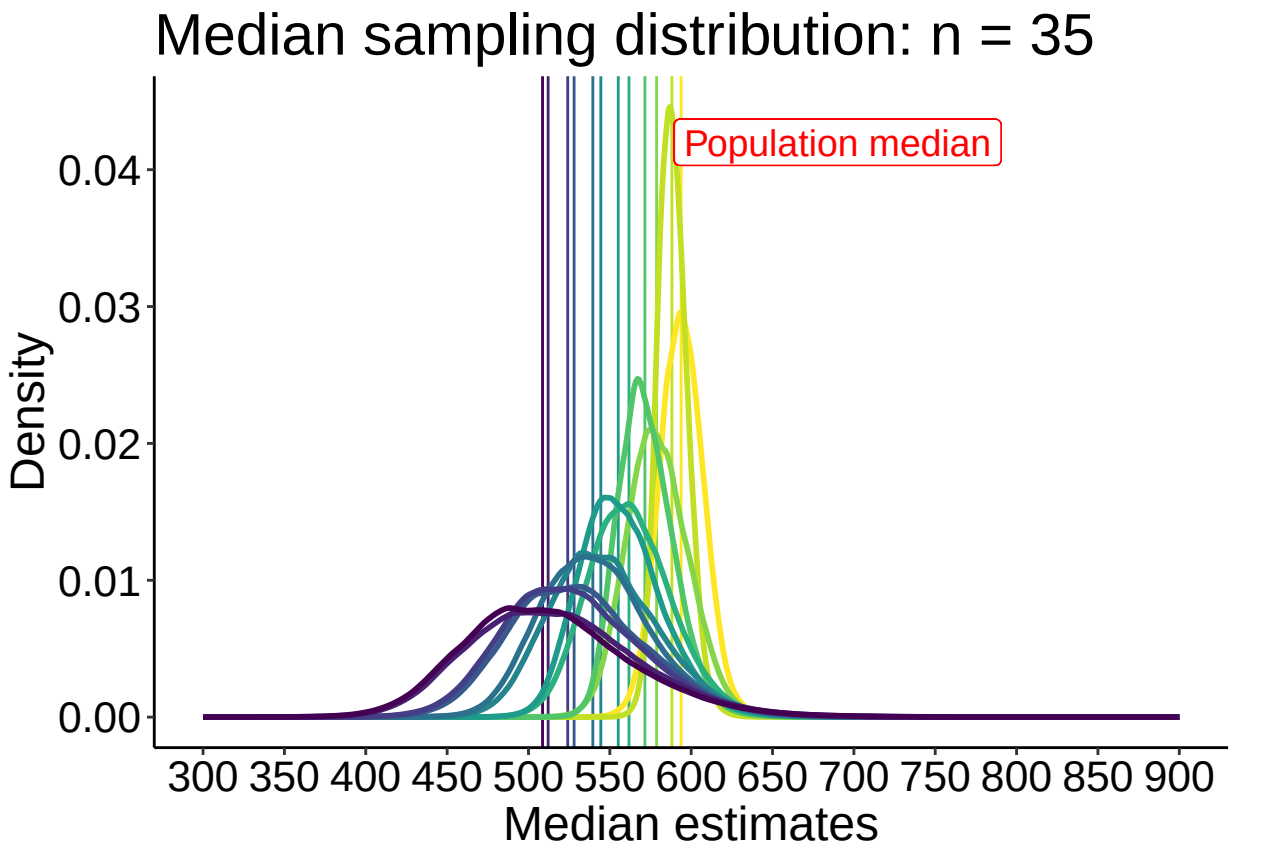
n=35 - illustrate skewness effect

```
N <- 8 # n=35
df <- tibble(sd = rep(x, nP),
             kde = as.vector(kde.md[,,N]),
             `Skewness` = factor(rep(round(pop.m-pop.md), each = length(x))))

df.pop <- tibble(pop = pop.md,
                 Skewness = factor(round(pop.m-pop.md)))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
  # geom_vline(xintercept=pop.m, linetype=1, colour = "red", size = 1.5) +
  geom_vline(data=df.pop, aes(xintercept=pop, colour = `Skewness`), linetype=1, size = 0.5) +
         geom_line(aes(colour = `Skewness`), size=1) +
       scale_colour_viridis_d(direction=-1) +
         theme(plot.title = element_text(size = 22),
               axis.title.x = element_text(size = 18),
               axis.text = element_text(size = 16, colour = "black"),
               axis.title.y = element_text(size = 18),
               legend.key.width = unit(1.5,"cm"),
               legend.position = "none",#c(0.15,0.55),
```

```
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) +
  scale_x_continuous(breaks = seq(100, 1000, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Median estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.m+90, kde=0.042),
             label = "Population median", angle = 90, colour="red", size=5) +
  ggtitle(paste0("Median sampling distribution: n = ",nvec[N]))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```




```
p.md.N35 <- p
```

All median HDI

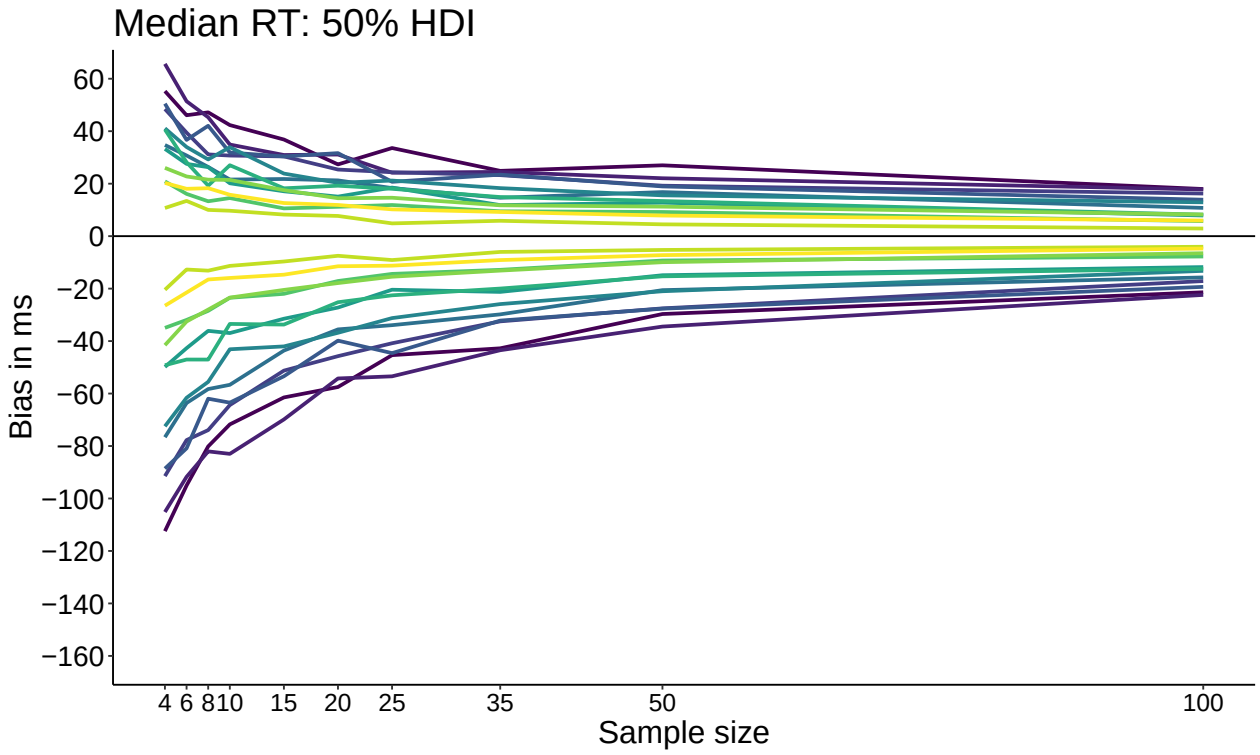
Illustrate the HDIs for all skewness levels.

```
# 50% HDI of mean bias
hdi.md <- array(NA, dim = c(2, nP, length(nvec)))
for(iter.n in 1:length(nvec)){
  for(P in 1:nP){
      hdi.md[, P, iter.n] <- hdi(sim.md[, P, iter.n]-pop.md[P], credMass=0.50)
  }
}
```

```r
df1 <- tibble(`Bias`=as.vector(hdi.md[1,,]),
              `Size`=rep(nvec,each=nP),
              `Skewness`=rep(round(pop.m - pop.md),length(nvec)))
df2 <- tibble(`Bias`=as.vector(hdi.md[2,,]),
              `Size`=rep(nvec,each=nP),
              `Skewness`=rep(round(pop.m - pop.md),length(nvec)))

df1$Skewness <- as.character(df1$Skewness)
df1$Skewness <- factor(df1$Skewness, levels=unique(df1$Skewness))
df2$Skewness <- as.character(df2$Skewness)
df2$Skewness <- factor(df2$Skewness, levels=unique(df2$Skewness))

# make plot
p <- ggplot(df1, aes(x=Size, y=Bias)) + theme_classic() +
  geom_line(aes(colour = Skewness), size = 1) + #linetype = Side
  geom_line(data=df2, aes(colour = Skewness), size = 1) +
  geom_abline(intercept=0, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  scale_y_continuous(breaks=seq(-160,60,20)) +
  coord_cartesian(ylim=c(-160,60)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 14, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "none",#c(0.5,0.15),
        legend.direction = "horizontal",
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "Bias in ms") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Median RT: 50% HDI")
p
```




```
p.md.hdi <- p
# save figure
# ggsave(filename=paste0('figure_miller_bias_md_hdi.pdf'),width=10,height=6)
```

Summary figure: all HDI for mean and median

```
# combine panels into one figure
cowplot::plot_grid(p.m.hdi, p.md.hdi,
                           labels = c("A", "B"),
                           ncol = 1,
                           nrow = 2,
                           rel_widths = c(1, 1),
                           label_size = 20,
                           hjust = -1.5,
                           scale=.95,
                           align = "h")
# save figure
# ggsave(filename='./figures/figure_samp_dist_hdi_summary.pdf',width=18,height=6) # 2 rows
ggsave(filename='./figures/figure_samp_dist_hdi_summary.pdf',width=10,height=10)
```

Standard deviation results

Compute the SD of the Monte-Carlo samples.

Compute SD

```
sd.m <- apply(sim.m, c(2,3), sd)
sd.md <- apply(sim.md, c(2,3), sd)
```

Illustrate SD results

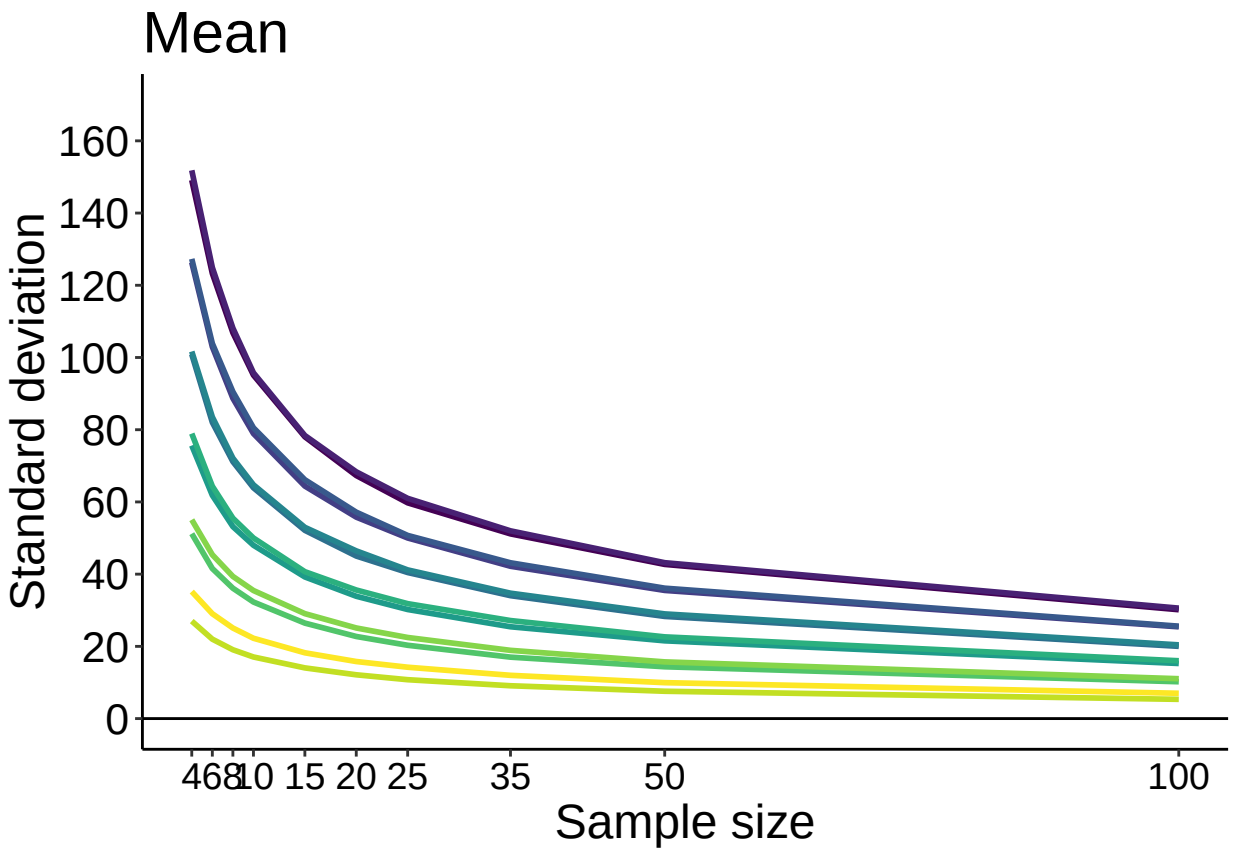
We illustrate SD as a function of skewness and sample size.

MEAN:

```
df <- tibble(`Bias`=as.vector(sd.m),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  scale_y_continuous(limits=c(0,170), breaks=seq(0,170,20)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 14, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "blank",#c(0.85,0.65),
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "Standard deviation") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Mean")
p
```



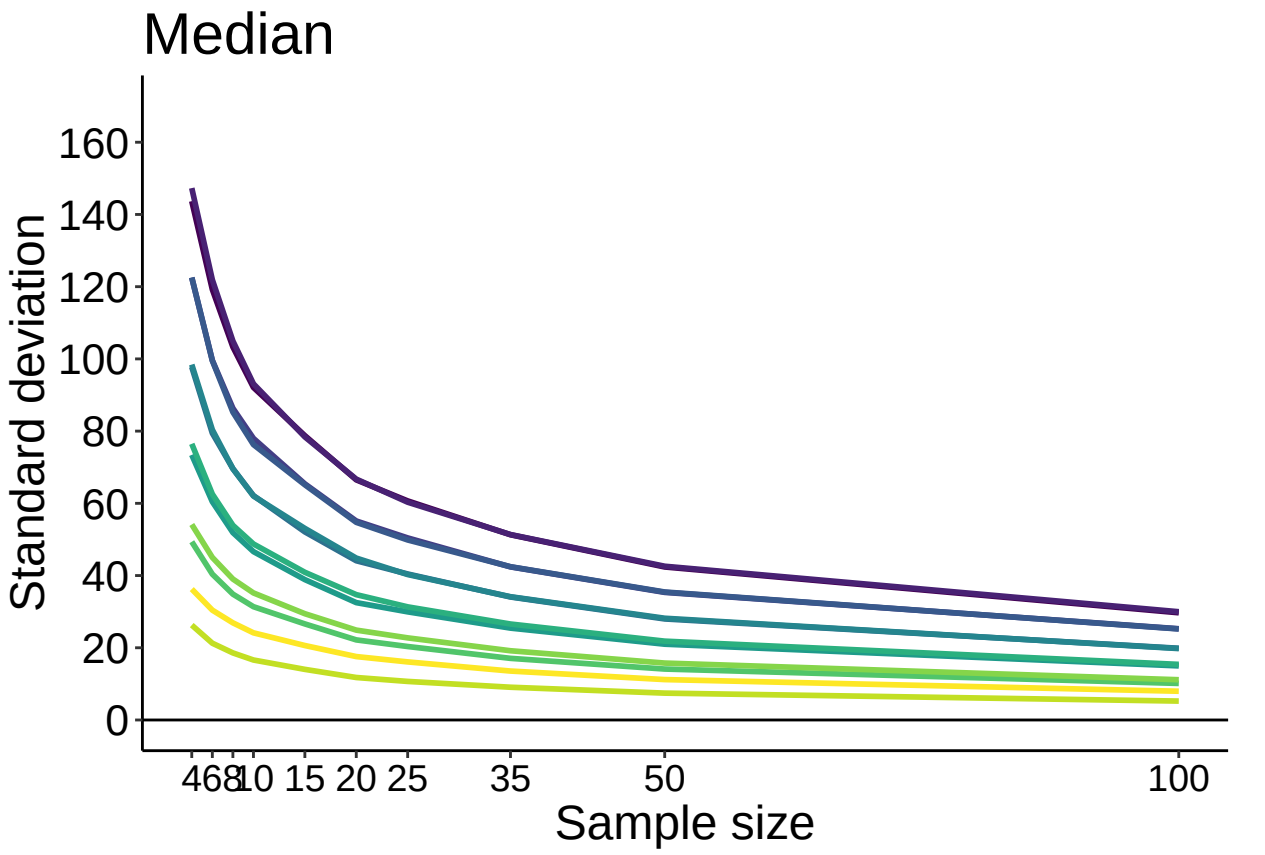
MEDIAN:

```
df <- tibble(`Bias`=as.vector(sd.md),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  scale_y_continuous(limits=c(0,170), breaks=seq(0,170,20)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 14, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "blank",#c(0.85,0.65),
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "Standard deviation") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
```

```
  ggtitle("Median")
p
```



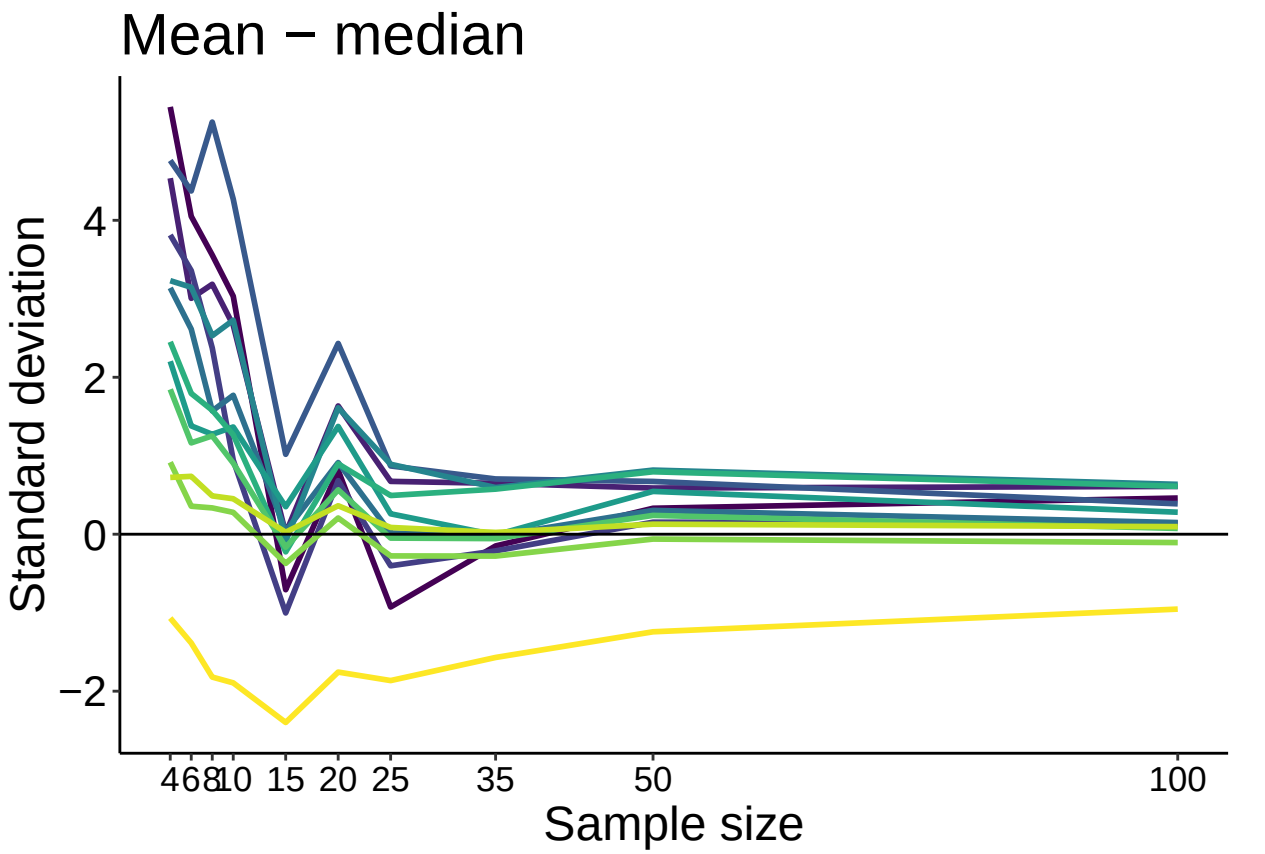
MEAN minus MEDIAN

```
df <- tibble(`Bias`=as.vector(sd.m - sd.md),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  # scale_y_continuous(limits=c(0,170), breaks=seq(0,170,20)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 13, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "blank",#c(0.85,0.65),
        legend.text=element_text(size=16),
```

```
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "Standard deviation") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Mean - median")
p
```



Mean SD tends to be larger than median SD across conditions, especially for small sample sizes. The notable exception is for the distribution with least skewness (yellow): as we get closer to a normal distribution, the median SE tends to over-estimate the mean SE, which reduces power of tests based on the median (Wilcox & Rousselet, 2018).

P(MC > pop) results

Probability that a Monte-Carlo estimates is inferior to the population value.

Compute P(MC > pop)

```
ppop.m <- matrix(0, nP, length(nvec))
ppop.md <- matrix(0, nP, length(nvec))
for(P in 1:nP){
ppop.m[P,] <- apply(sim.m[,P,] >= pop.m[P], 2, mean)
ppop.md[P,] <- apply(sim.md[,P,] >= pop.md[P], 2, mean)
}
```

Illustrate P(sample > population) results

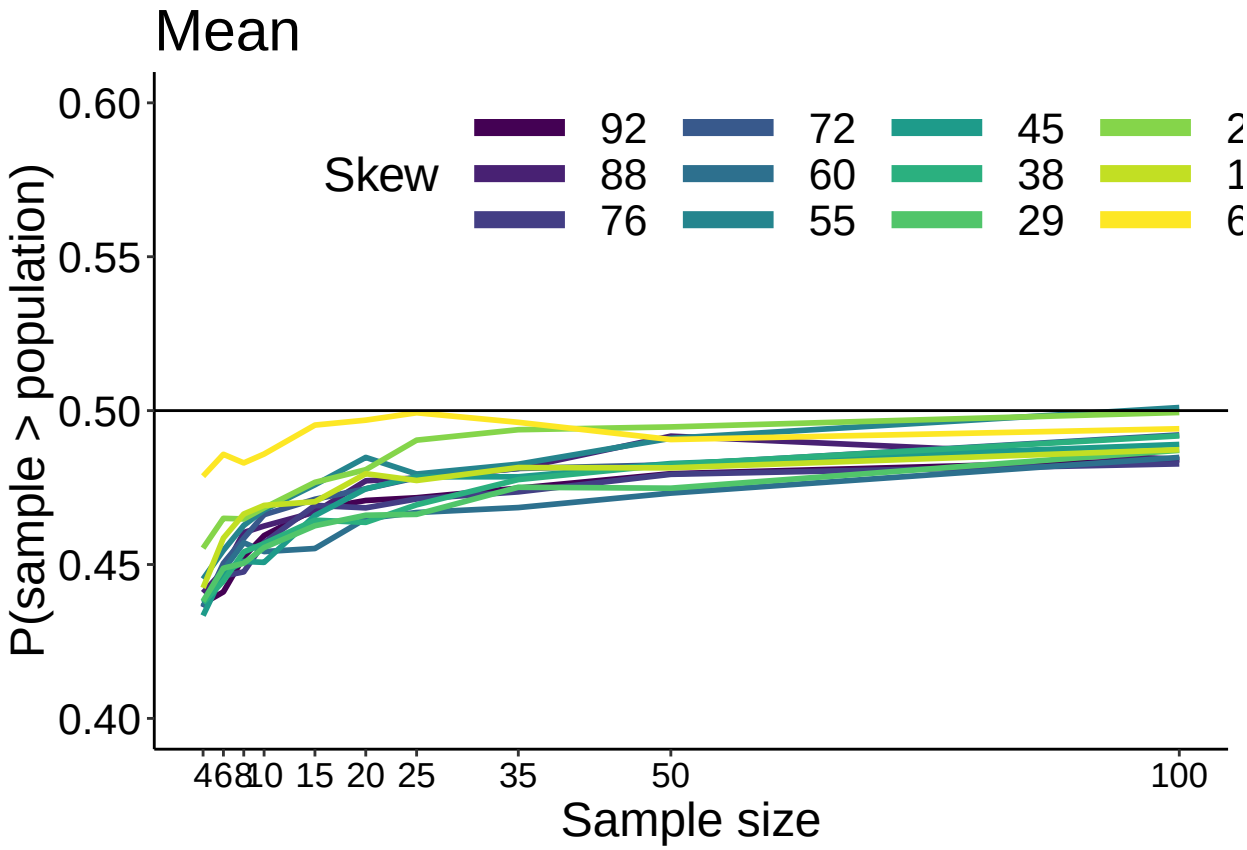
We illustrate P(sample > population), the probability that a sample mean or median is larger than the population value, as a function of skewness and sample size.

MEAN

```
df <- tibble(`Bias`=as.vector(ppop.m),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0.5, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  scale_y_continuous(limits=c(0.4,0.6), breaks=seq(0.4,0.6,0.05)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 13, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = c(0.6,0.85),
        legend.direction = "horizontal",
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "P(sample > population)") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Mean")
p
```




```
psp.m <- p
```

The distribution of probabilities is not centered at 0.50, as expected from the illustrations of the sampling distributions. The typical sample mean tends to under-estimate the population mean for all skewness levels and even with large sample sizes.

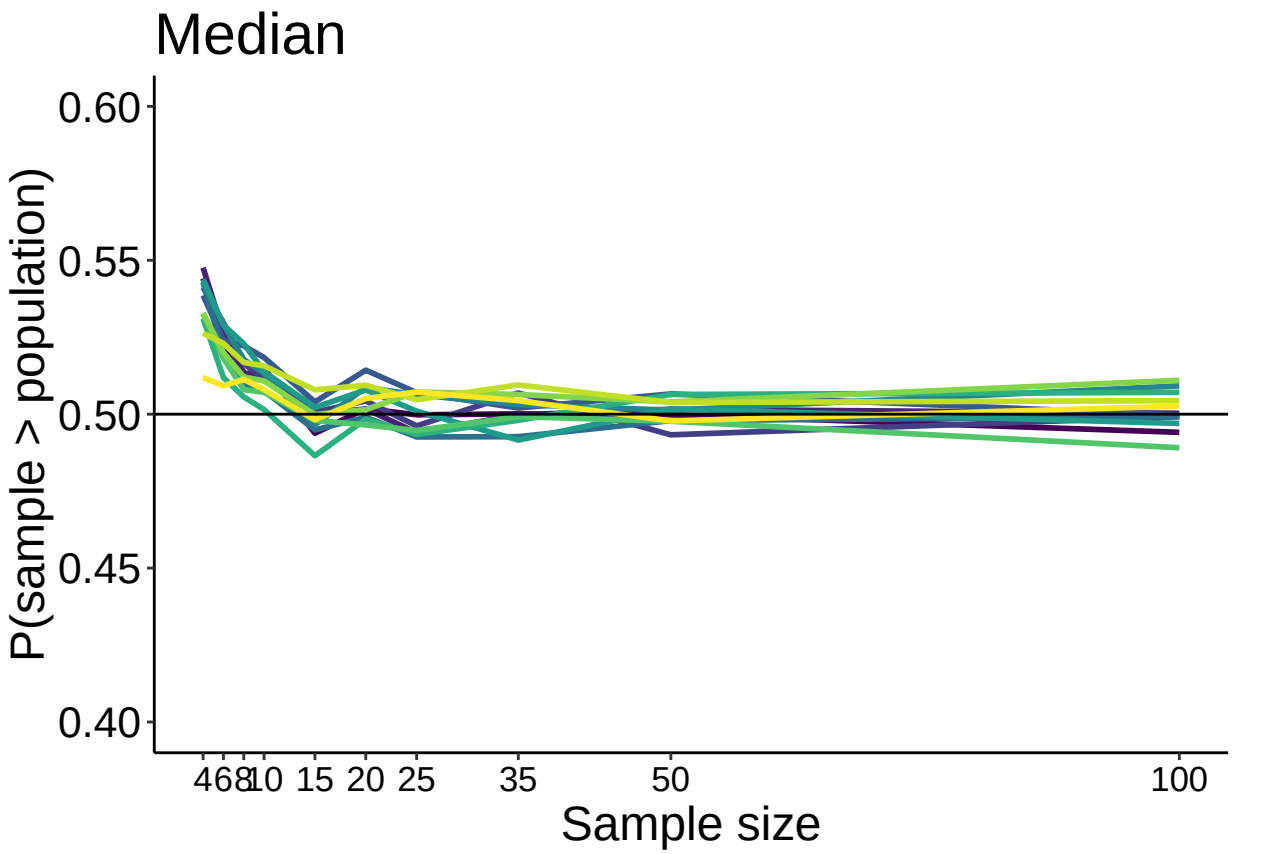
MEDIAN

```
df <- tibble(`Bias`=as.vector(ppop.md),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0.5, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  scale_y_continuous(limits=c(0.4,0.6), breaks=seq(0.4,0.6,0.05)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 13, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
```

```
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "blank",#c(0.85,0.65),
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "P(sample > population)") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Median")
p
```




```
psp.md <- p
```

The sample median tends to over-estimate the population median for all skewness levels for small sample sizes. The offset reduces with increasing sample size, and faster than it does for the mean.

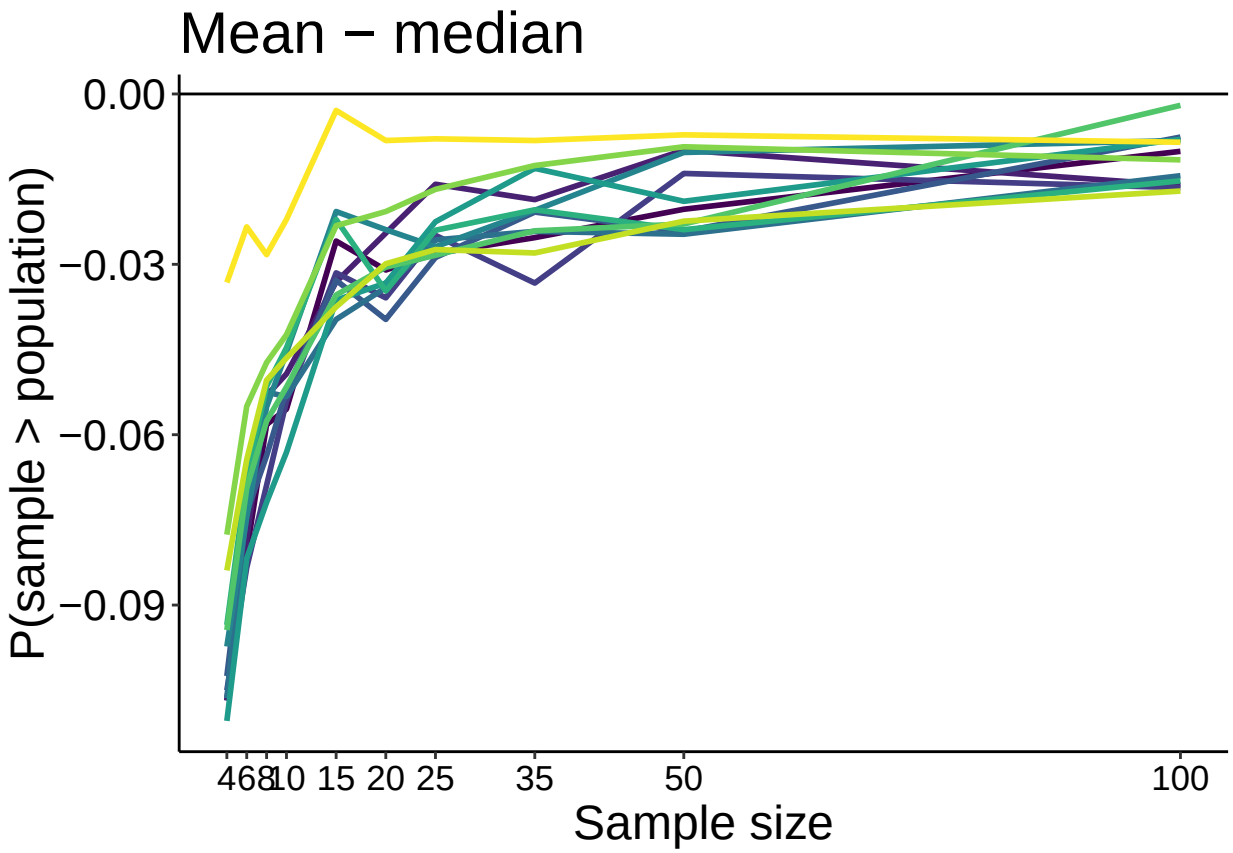
MEAN - MEDIAN

```
df <- tibble(`Bias`=as.vector(ppop.m - ppop.md),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
```

```
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  # scale_y_continuous(limits=c(0,170), breaks=seq(0,170,20)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 13, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "blank",#c(0.85,0.65),
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "P(sample > population)") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Mean - median")
p
```



P(MC <= pop +/- 10) results

Probability that a Monte-Carlo estimates is within 10 ms of the population value.

Compute P(MC <= pop +/- 10)

```
ppop10.m <- matrix(0, nP, length(nvec))
ppop10.md <- matrix(0, nP, length(nvec))
for(P in 1:nP){
ppop10.m[P,] <- apply(abs(sim.m[,P,]-pop.m[P])<=10, 2, mean)
ppop10.md[P,] <- apply(abs(sim.md[,P,]-pop.md[P])<=10, 2, mean)
}
```

Illustrate P(sample <= pop +/- 10) results

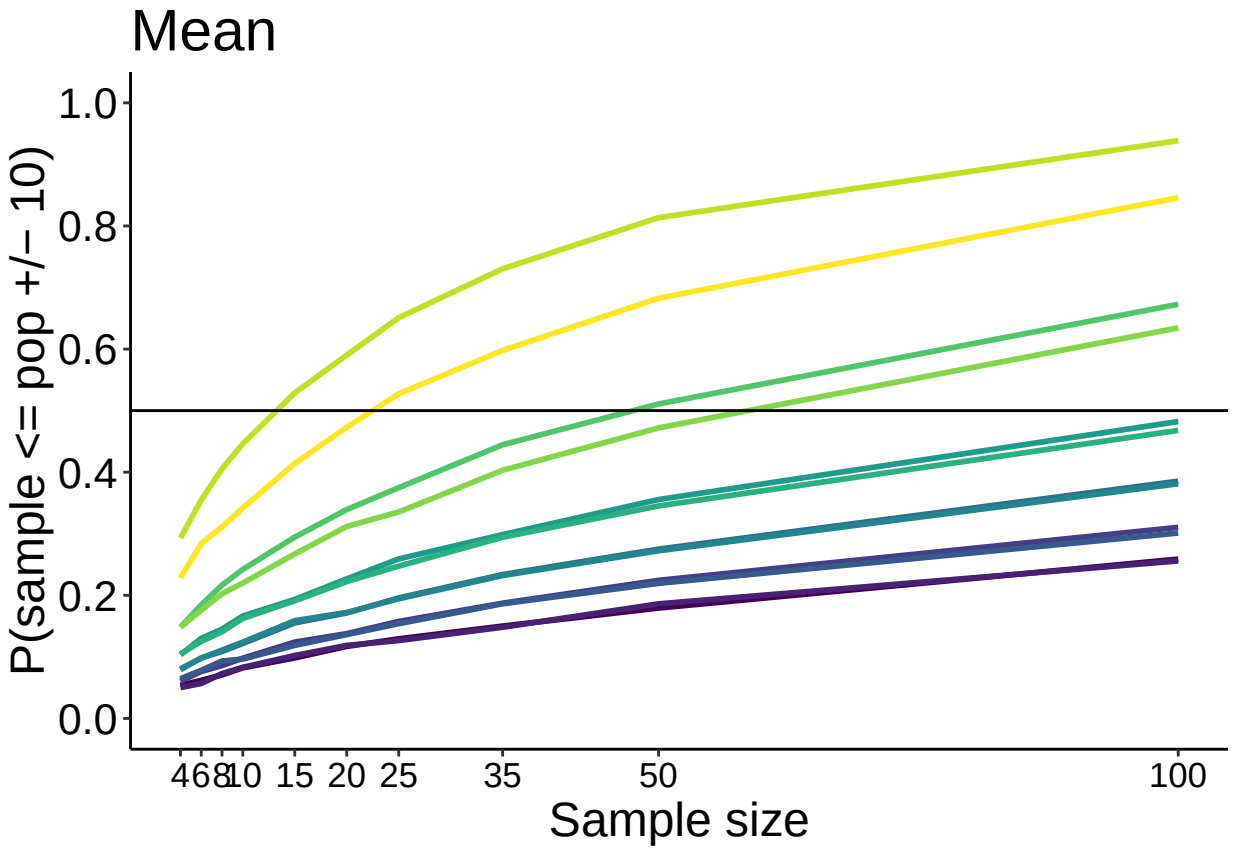
We illustrate P(sample <= pop +/- 10), the probability that a sample mean or median is within 10 ms of the population value, as a function of skewness and sample size.

MEAN

```
df <- tibble(`Bias`=as.vector(ppop10.m),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0.5, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  scale_y_continuous(limits=c(0,1), breaks=seq(0,1,0.2)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 13, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "none",#c(0.6,0.85),
        legend.direction = "horizontal",
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "P(sample <= pop +/- 10)") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Mean")
p
```




```
# psp.m <- p
```

Long run measurement precision increases with sample size and with lower skewness.

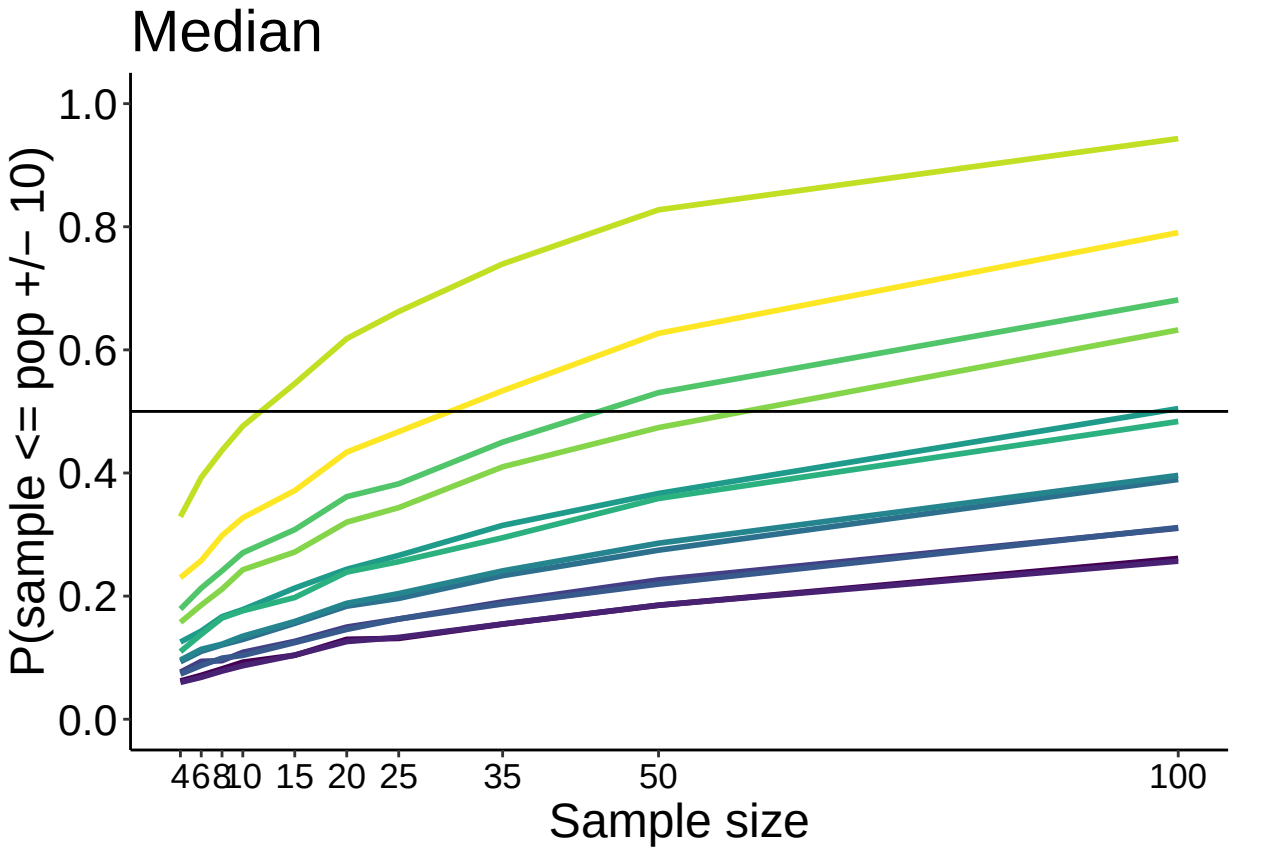
MEDIAN

```
df <- tibble(`Bias`=as.vector(ppop10.md),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0.5, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
  scale_y_continuous(limits=c(0,1), breaks=seq(0,1,0.2)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 13, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
```

```
        legend.position = "blank",#c(0.85,0.65),
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "P(sample <= pop +/- 10)") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Median")
p
```




```
# psp.md <- p
```

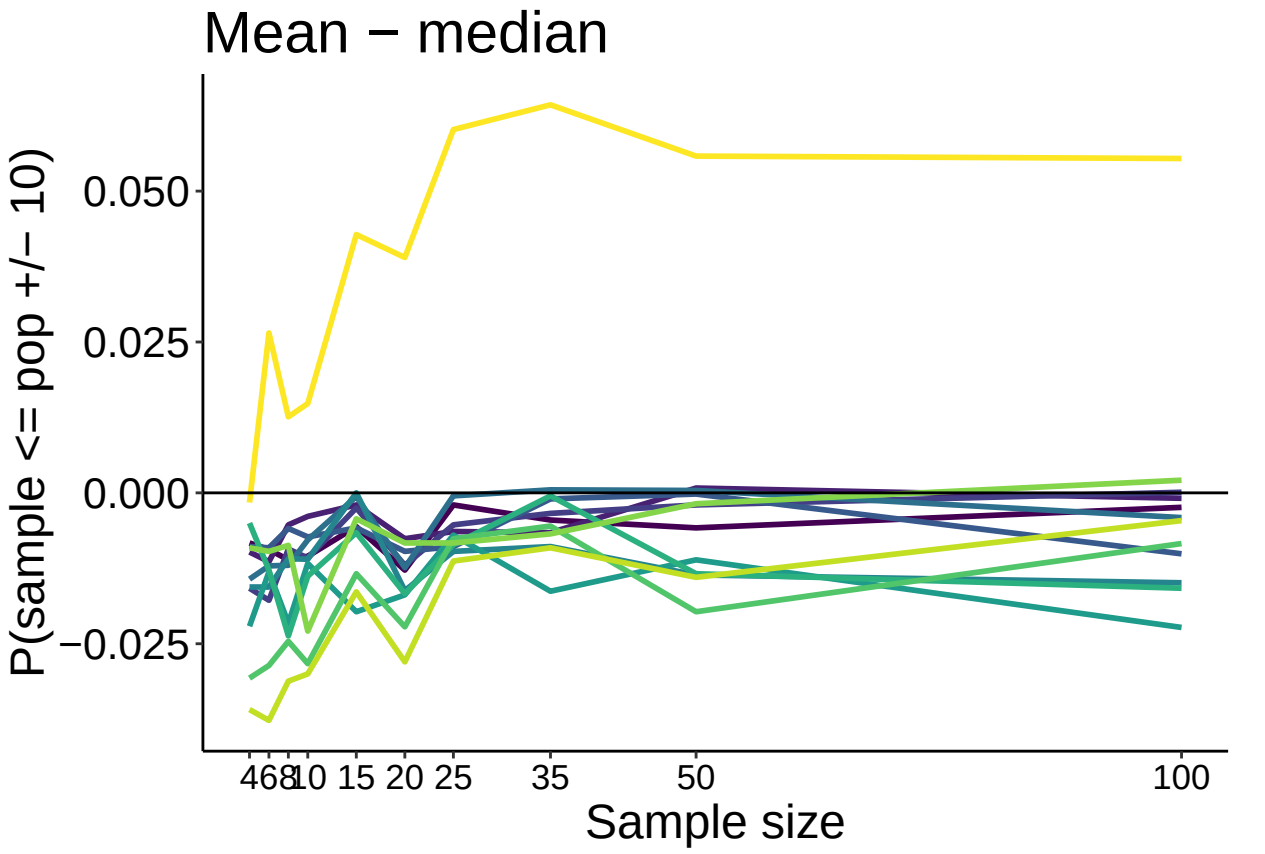
MEAN - MEDIAN

```
df <- tibble(`Bias`=as.vector(ppop10.m - ppop10.md),
             `Size`=rep(nvec,each=nP),
             `Skew`=rep(round(pop.m - pop.md),length(nvec)))

df$Skew <- as.character(df$Skew)
df$Skew <- factor(df$Skew, levels=unique(df$Skew))

# make plot
p <- ggplot(df, aes(x=Size, y=Bias), group=Skew) + theme_classic() +
  geom_line(aes(colour = Skew), size = 1) +
  geom_abline(intercept=0, slope=0, colour="black") +
  scale_colour_viridis_d() +
  scale_x_continuous(breaks=nvec) +
```

```
  # scale_y_continuous(limits=c(0,170), breaks=seq(0,170,20)) +
  theme(plot.title = element_text(size=22),
        axis.title.x = element_text(size = 18),
        axis.text.x = element_text(size = 13, colour="black"),
        axis.text.y = element_text(size = 16, colour="black"),
        axis.title.y = element_text(size = 18),
        legend.key.width = unit(1.5,"cm"),
        legend.position = "blank",#c(0.85,0.65),
        legend.text=element_text(size=16),
        legend.title=element_text(size=18)) +
  labs(x = "Sample size", y = "P(sample <= pop +/- 10)") +
  guides(colour = guide_legend(override.aes = list(size=3))) + # make thicker legend lines
  ggtitle("Mean - median")
p
```



The sample median tends to be closer to the popluation than the mean, except for the least skewed distribution.

Sampling distribution of the median after bias correction

Quantiles of sampling distributions

```
load('./data/sim_miller1988.RData')
S <- 1 # n = 4
P <- 12 # least skewed
```

```
round(quantile(sim.md[,P,S]))
```

```
##   0%  25%  50%  75% 100%
##  480  572  595  619  788
```

```
round(quantile(sim.md.bc[,P,S]))
```

```
##   0%  25%  50%  75% 100%
##  460  568  593  620  811
```

```
P <- 1 # most skewed
round(quantile(sim.md[,P,S]))
```

```
##   0%  25%  50%  75% 100%
##  302  444  524  628 1316
```

```
round(quantile(sim.md.bc[,P,S]))
```

```
##   0%  25%  50%  75% 100%
##  -34  406  494  607 1420
```

```
S <- 6 # n = 20
P <- 12 # least skewed
round(quantile(sim.md[,P,S]))
```

```
##   0%  25%  50%  75% 100%
##  516  582  594  606  678
```

```
round(quantile(sim.md.bc[,P,S]))
```

```
##   0%  25%  50%  75% 100%
##  499  580  594  607  688
```

```
P <- 1 # most skewed
round(quantile(sim.md[,P,S]))
```

```
##   0%  25%  50%  75% 100%
##  351  468  509  556  902
```

```
round(quantile(sim.md.bc[,P,S]))
```

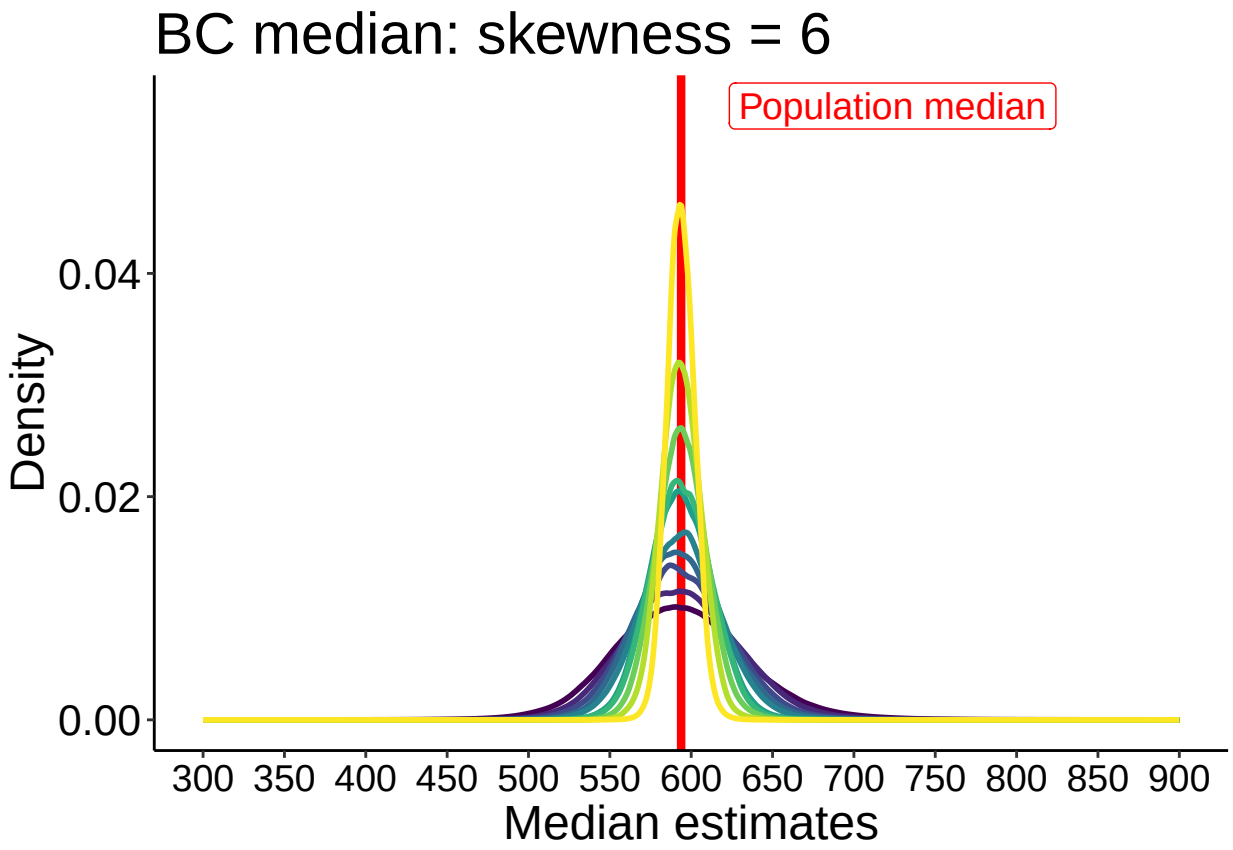
```
##   0%  25%  50%  75% 100%
##  306  454  500  554  914
```

Least skewed distribution

```
P <- 12 # least skewed distribution
df <- tibble(sd = rep(x, length(nvec)),
             kde = as.vector(kde.md.bc[,P,]),
             `Sample size` = factor(rep(nvec, each = length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
  geom_vline(xintercept=pop.md[P], linetype=1, colour = "red", size = 1.5) +
          geom_line(aes(colour = `Sample size`), size=1) +
       scale_colour_viridis_d(direction=1) +
          theme(plot.title = element_text(size=22),
                axis.title.x = element_text(size = 18),
```

```
                axis.text.x = element_text(size = 14, colour="black"),
                axis.text.y = element_text(size = 16, colour="black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = "none",#c(0.15,0.6),
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) +
  scale_x_continuous(breaks = seq(300, 900, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Median estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.md[P]+130, kde=0.055),
             label = "Population median", angle = 90, colour="red", size=5) +
  ggtitle(paste0("BC median: skewness = ",round(pop.m[P] - pop.md[P])))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```



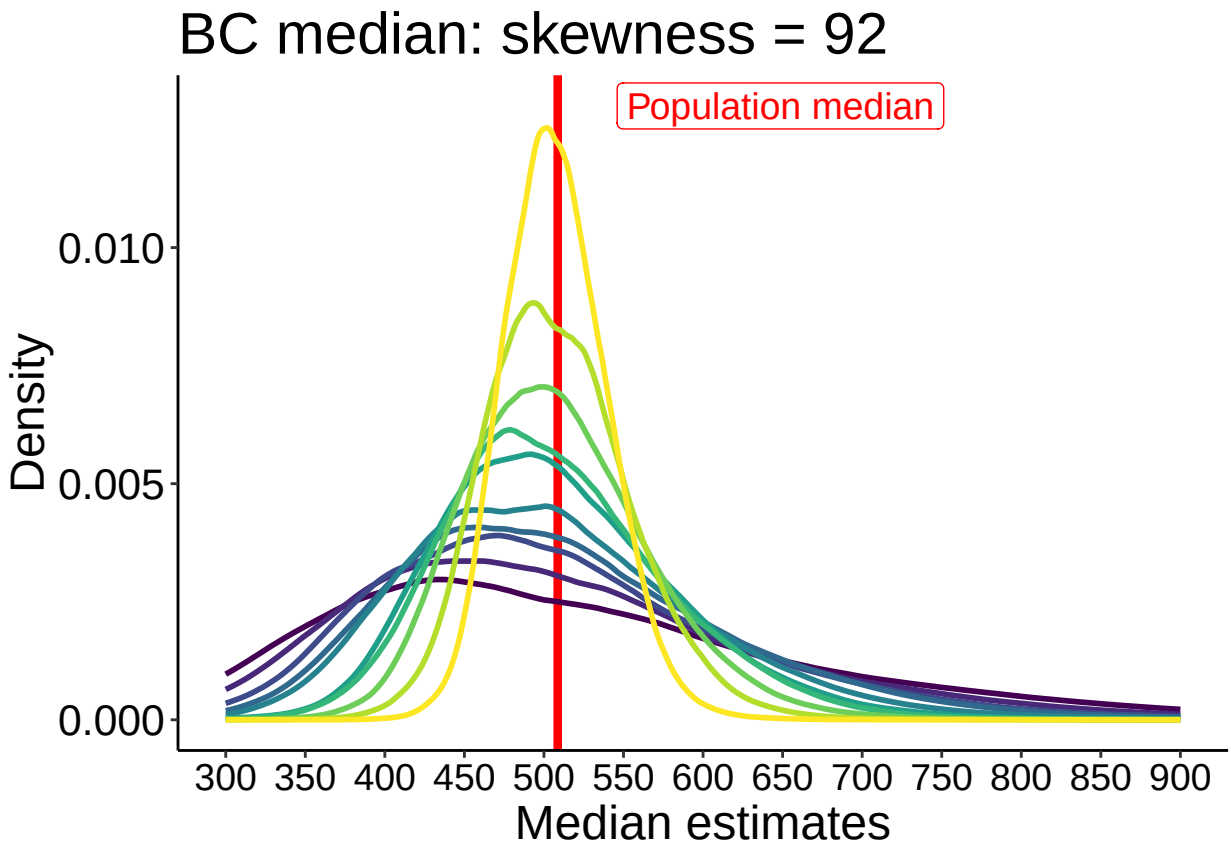

```
# p.md.P12 <- p
```

Most skewed distribution

```
P <- 1 # most skewed distribution
df <- tibble(sd = rep(x, length(nvec)),
             kde = as.vector(kde.md.bc[,P,]),
```

```
            `Sample size` = factor(rep(nvec, each = length(x))))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
      geom_vline(xintercept=pop.md[P], linetype=1, colour = "red", size = 1.5) +
      geom_line(aes(colour = `Sample size`), size=1) +
       scale_colour_viridis_d(direction=1) +
          theme(plot.title = element_text(size=22),
                axis.title.x = element_text(size = 18),
                axis.text.x = element_text(size = 14, colour="black"),
                axis.text.y = element_text(size = 16, colour="black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                legend.position = "none", #c(0.2,0.55),
                legend.text=element_text(size=16),
                legend.title=element_text(size=18),
                 panel.background = element_rect(fill="white")) + #grey90
  scale_x_continuous(breaks = seq(100, 1000, 50)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Median estimates", y = "Density") +
  guides(colour = guide_legend(override.aes = list(size=3))) +
  geom_label(data=tibble(sd=pop.md[P]+140, kde=0.013),
             label = "Population median", angle = 90, colour="red", size=5) +
  ggtitle(paste0("BC median: skewness = ",round(pop.m[P] - pop.md[P])))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```



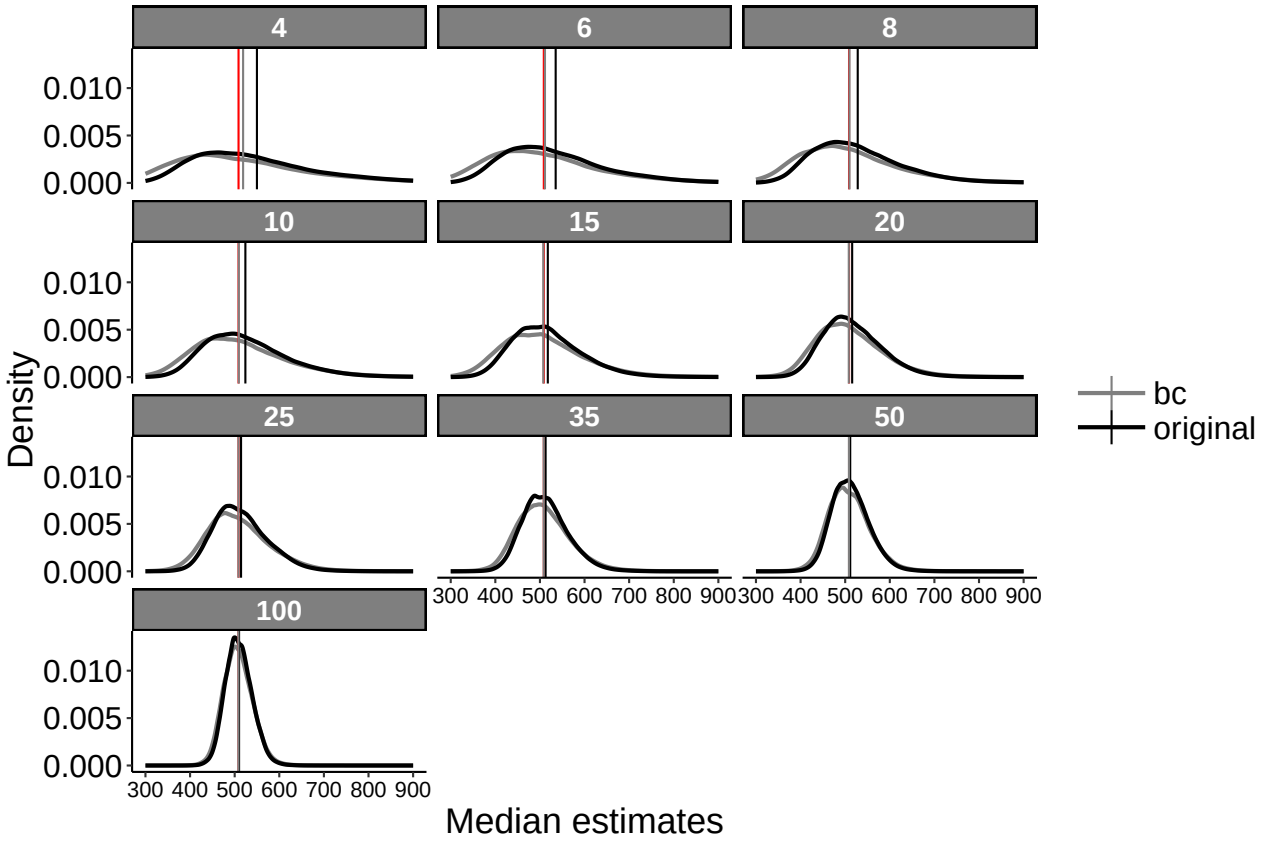
```r
# p.md.P1 <- p
```

Most skewed distribution: compare before and after bias correction

```r
P <- 1 # most skewed distribution
df <- tibble(sd = rep(rep(x, length(nvec)),2),
             kde = c(as.vector(kde.md[,P,]),as.vector(kde.md.bc[,P,])),
             samplesize = factor(rep(rep(nvec, each = length(x)),2)),
             bc = factor(c(rep("original",length(nvec)*length(x)),rep("bc",length(nvec)*length(x)))))

ori.mean <- apply(sim.md[,P,],2,mean)
bc.mean <- apply(sim.md.bc[,P,],2,mean)
df2 <- tibble(md = c(ori.mean, bc.mean),
              bc = factor(c(rep("original", length(ori.mean)),rep("bc", length(bc.mean)))),
              samplesize = factor(c(nvec,nvec)))

# make plot
p <- ggplot(df, aes(x=sd, y=kde)) + theme_classic() +
          geom_line(aes(colour = bc), size=1) +
          scale_color_manual(values=c("grey50","black")) +
  # population median
  geom_vline(xintercept=pop.md[P], linetype=1, colour = "red", size = 0.5) +
  # sample median before and after bias correction
  geom_vline(data = df2, aes(xintercept=md, colour=bc), size = 0.5) +
       # scale_colour_viridis_d(direction=1) +
          theme(plot.title = element_text(size=22),
                axis.title.x = element_text(size = 18),
                axis.text.x = element_text(size = 11, colour="black"),
                axis.text.y = element_text(size = 16, colour="black"),
                axis.title.y = element_text(size = 18),
                legend.key.width = unit(1.5,"cm"),
                # legend.position = "none",#c(0.15,0.6),
                legend.text = element_text(size=16),
                legend.title = element_blank(),
                strip.text = element_text(size=14, face="bold", colour="white"),
                strip.background = element_rect(colour="black", fill="grey50"),
                panel.background = element_rect(fill="white")) +
  scale_x_continuous(breaks = seq(300, 900, 100)) +
  coord_cartesian(xlim = c(300, 900)) +
  labs(x = "Median estimates", y = "Density") +
  facet_wrap( ~ samplesize, ncol=3)
  # guides(colour = guide_legend(override.aes = list(size=3))) +
  # geom_label(data=tibble(sd=pop.md[P]+130, kde=0.055),
  #            label = "Population median", angle = 90, colour="red", size=5) +
  # ggtitle(paste0("BC median: skewness = ",round(pop.m[P] - pop.md[P])))
  # annotate("text", x = pop.sd-10, y = 0.006, label = "Population SD", angle=90, size=5, colour="red")
p
```




```
# p.md.P12 <- p
```

References

Miller, J. (1988) A warning about median reaction time. J Exp Psychol Hum Percept Perform, 14, 539-543.

Wilcox, R.R. & Rousselet, G.A. (2018) A Guide to Robust Statistical Methods in Neuroscience. Curr Protoc Neurosci, 82, 8 42 41-48 42 30.