FAST 3D INVERSION OF THE MAGNETIC VECTOR AMPLITUDE PRODUCED BY A BASEMENT RELIEF USING GAUSS-LEGENDRE QUADRATURE

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INTRODUCTION

The magnetic components produced by a unique prism (Bhattacharyya, 1964) is computationally expensive because it involves a series of loops and trigonometric functions. The scenario is more complicated when the forward modelling of prisms is used in a nonlinear magnetic inversion than in a linear inversion because the forward modelling is calculated at every iteration. Another bottleneck in the magnetic inversion is the knowledge about the magnetization vector. In this work, we present a regularized nonlinear magnetic inversion for depth-to-basement estimate by inverting the amplitude of the magnetic anomaly vector (amplitude data). To overcome the high computational cost of forward modelling, we approximate the $x-$, $y-$, and $z-$ components of the magnetic vector produced by a prism by the numerical integral produced by an assemble of dipoles along the vertical axis of the prism. What is more, the amplitude data inversion is weakly dependent on the magnetization vector direction.

METHOD

Forward Model: Sedimentary basin consisting of nonmagnetic sediments and magnetic basement surface parametrized as a grid of M 3D vertical prisms juxtaposed in the horizontal directions, where the prisms' tops represent the depths to the magnetic basement. The centers of the prisms are directly below each observation point and the depths to the bottoms of all prisms are at the same depth Z_2.

$$|\mathbf{B}|(x_i, y_i, z_i) = \sum_{j=1}^{M} \sqrt{B_{x_j}^2 + B_{y_j}^2 + B_{z_j}^2}, \qquad i = 1, \dots, N, \tag{1}$$

where N is the number of observations, B_{x_j}, B_{y_j} and B_{z_j} are the $x-, y-$, and $z-$components of the anomalous magnetic vector produced by the jth prism calculated at the ith observation point

Approximate the components of the magnetic vector produced by the jth prism by the ones produced by a dipole field integrated along the z −axis of the *jth* prism, i.e.:

$$|\mathbf{B}|(x_i, y_i, z_i) \cong \sum_{j=1}^{M} \sqrt{\left(\int_{p_j}^{Z_2} \varnothing_x^{ij}(z'_\ell)dz'\right)^2 + \left(\int_{p_j}^{Z_2} \varnothing_y^{ij}(z'_\ell)dz'\right)^2 + \left(\int_{p_j}^{Z_2} \varnothing_z^{ij}(z'_\ell)dz'\right)^2} \quad i = 1, \dots, N \tag{2}$$

where $\varnothing_x^{ij}(z'_\ell), \varnothing_y^{ij}(z'_\ell)$ and $\varnothing_z^{ij}(z'_\ell)$ are the $x-$, $y-$, and $z-$components of the magnetic vector produced by a dipole along the z −axis of the prism and calculated at the ith observation point.

The integrals in equation 2 can be solved numerically using Gauss-Legendre quadrature (GLQ) method (Mathews and Fink, 2004). The GLQ method consists in approximating the integrals in equation (2) by the weighted sum of the calculated $\varnothing_{x,y,z}^{ij}(z'_k)$ at some specific points (nodes) around de z −axis of the jth prism.

Inverse Problem: Minimize the objective function:

$$\delta(\mathbf{p}) = \|\mathbf{d}^o - \mathbf{f}(\mathbf{p})\|^2 + \lambda\left\|\overline{\overline{\mathbf{R}}}\mathbf{p}\right\|^2, \tag{3}$$

where d^o and $f(p)$ are, respectively, the observed and calculated amplitude data $\overline{\overline{\mathbf{R}}}$ is a first-order finite-difference matrix and λ is the regularization parameter.

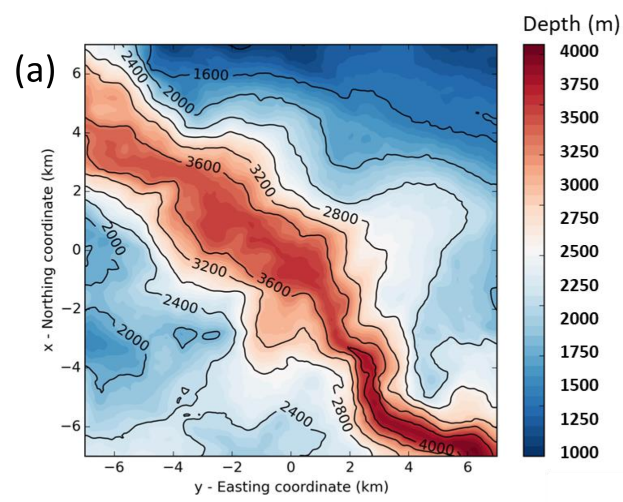
Using Gauss-Newton Method:

$$\Delta\mathbf{p}^{k+1} = \left(\overline{\overline{\mathbf{A}}}_k^T\overline{\overline{\mathbf{A}}}_k + \lambda\overline{\overline{\mathbf{R}}}^T\overline{\overline{\mathbf{R}}}\right)^{-1}\left(\overline{\overline{\mathbf{A}}}_k^T\boldsymbol{\varepsilon}^k - \lambda\overline{\overline{\mathbf{R}}}^T\overline{\overline{\mathbf{R}}}\mathbf{p}^k\right), \tag{4}$$

where $\boldsymbol{\varepsilon}^k$ is the difference between observed and calculated amplitude data at the kth iteration and $\overline{\overline{\mathbf{A}}}_k$ the Jacobian or sensitivity matrix whose a_{ij} element is the partial derivative of $\mathbf{f}(\mathbf{p}^k)$ with respect to the jth parameter calculated at the ith observation point.

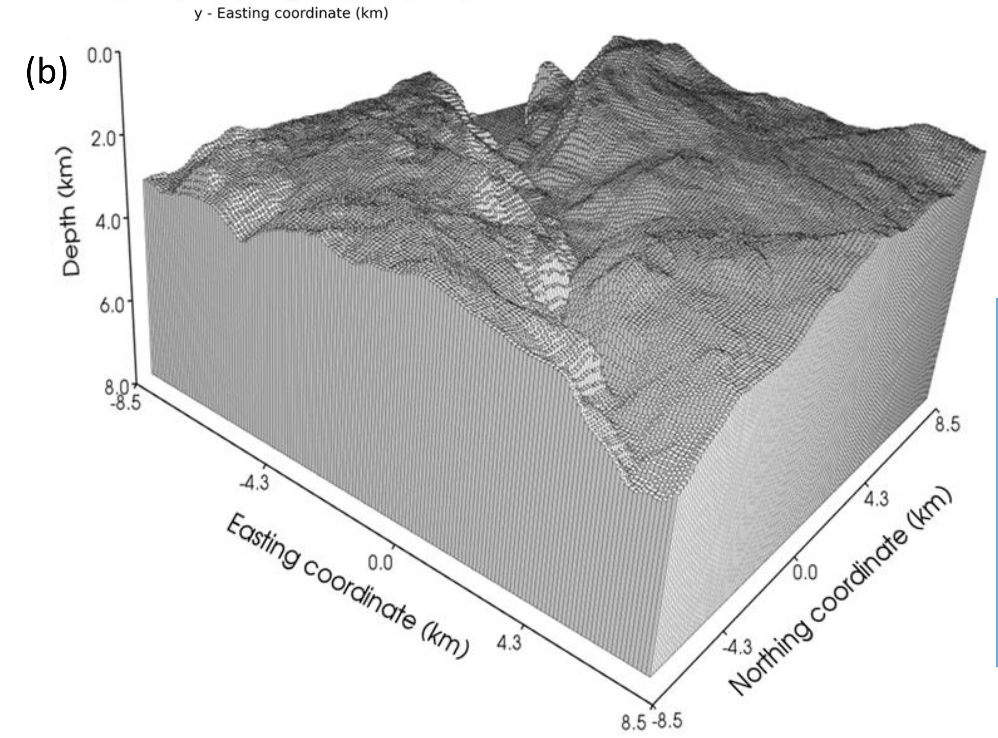
RESULTS

(a)

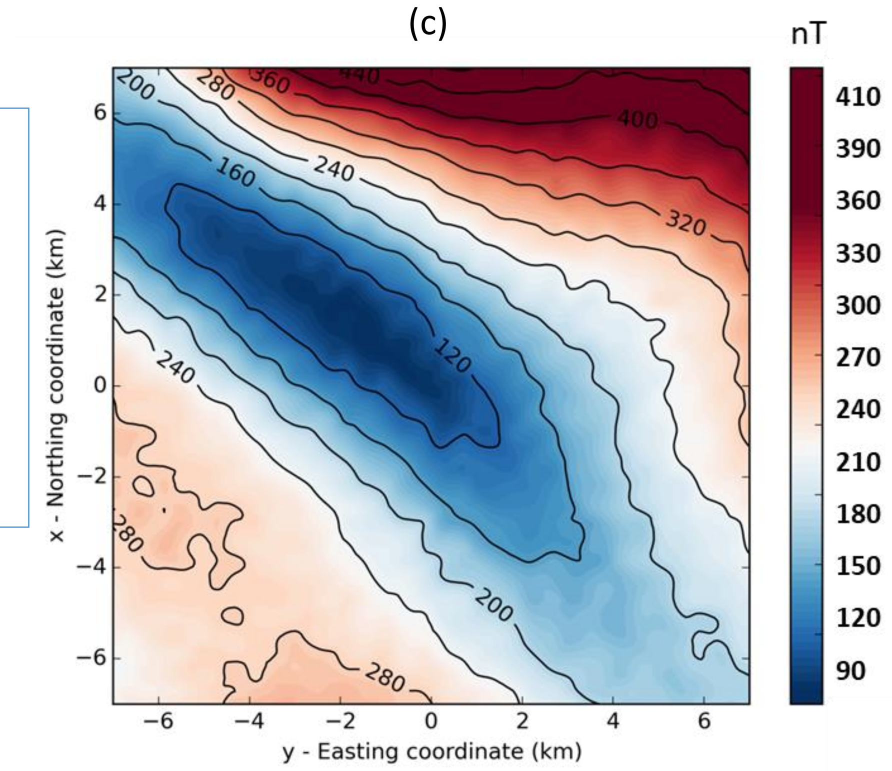


- Magnetic basement overlain by nonmagnetic sediments.
- Uniformly magnetization with a magnetization vector:
  - Intensity of 2.0 A/m
  - Inclination of −20°
  - Declination of 34°.

(b)



(c)



Calculated amplitude of the magnetic vector amplitude:

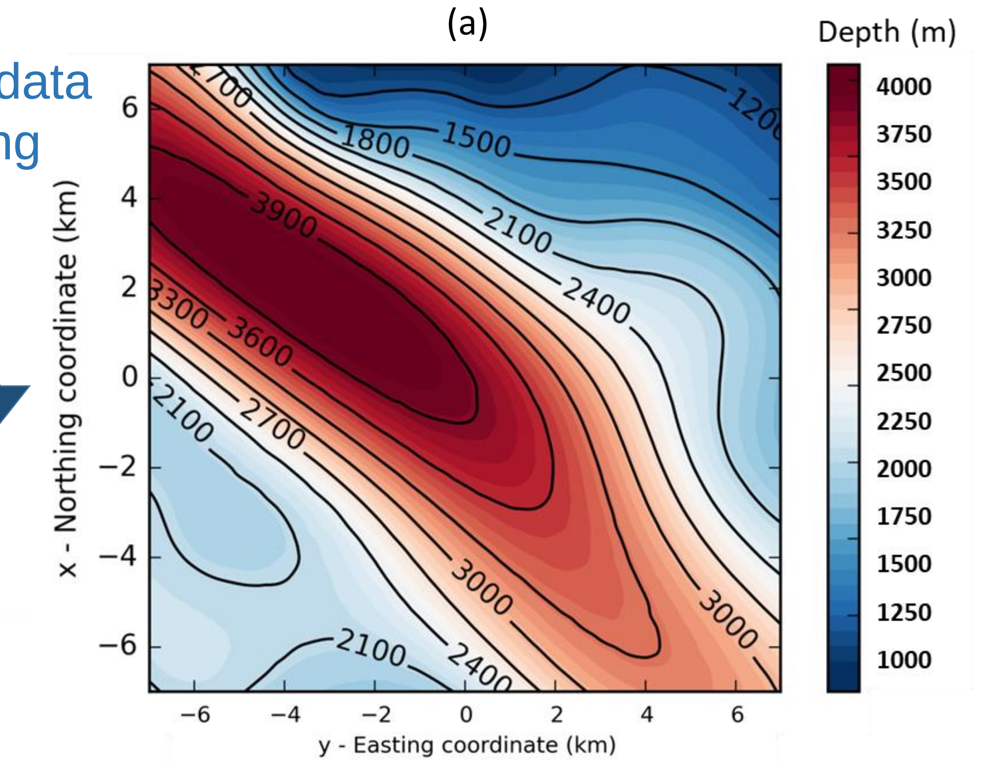
- a regular grid of 130 × 130
- grid spacing of 160 m
- 150 m height
- Corrupted with pseudorandom zero-mean Gaussian noise with a standard deviation of 10 nT.

Figure 1 – Synthetic test - Simulated magnetic basement relief show in map view (a) and in a 3D view (b) of a sedimentary basin consists of non-magnetic sedimentary rocks. (c) Noise-corrupted amplitude of the anomalous magnetic vector produced by the magnetic basement relief shown in a and b.

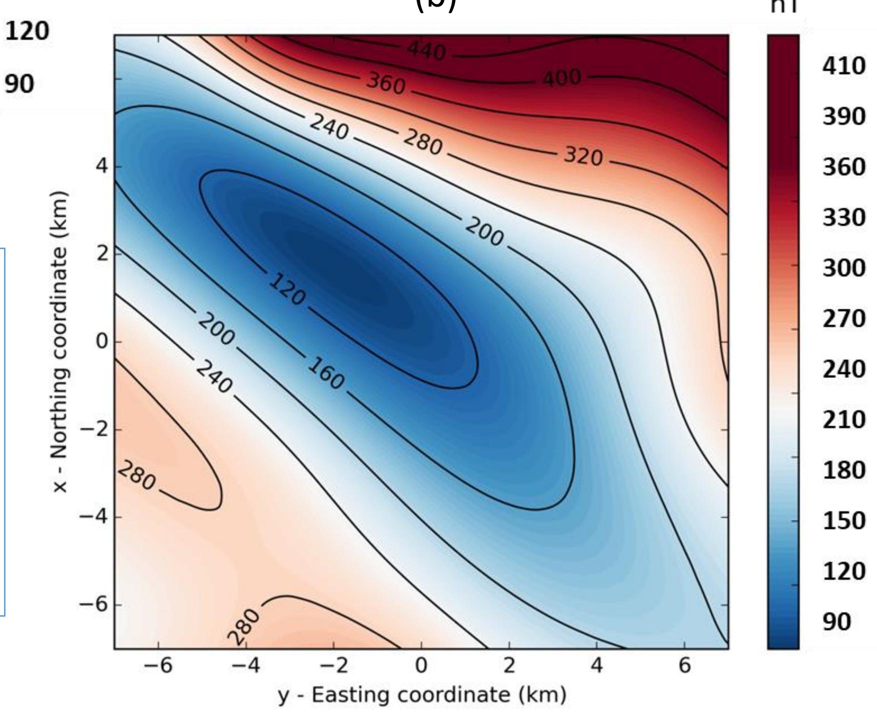
Inversion of the magnetic amplitude data using quadrature Forward Modelling

Using a wrong magnetization vector direction:
- Inclination of 30°
- Declination of −10°

(a)



(b)



Estimated basement relief smoother than the simulated basement:

i) the observed amplitude data do not reflect such steep variations and
ii) first-order Tikhonov regularization

(c)

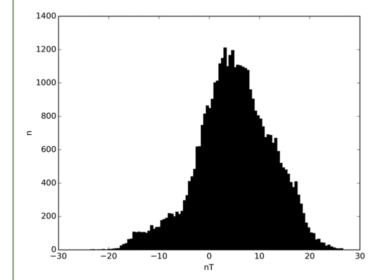


Figure 2 – (a) Calculated amplitude of the magnetic vector produced by (b) the estimated basement relief. (c) Histogram of the residuals (difference between the observed and predicted amplitude data).

CONCLUSIONS

- Fast-regularized inversion of the amplitude of the magnetic anomaly vector (amplitude data) for depth-to-basement estimate based on an efficient way to compute amplitude data produced by an arbitrary interface separating nonmagnetic sediments from a magnetic basement.
- Because the amplitude data is weakly dependent on the magnetization direction, the presence of remanent magnetization does not affect the solution.
- New forward model to calculate the magnetic components are simplified by a 1D integration taken with respect to the z-axis of a prism (prism thickness) and then multiplied by the horizontal area of the prism. The 1D integration is solved using the Gauss-Legendre quadrature using dipoles.

4x Faster than using the analytical solution to calculate the magnetic vector anomaly produced by a prism

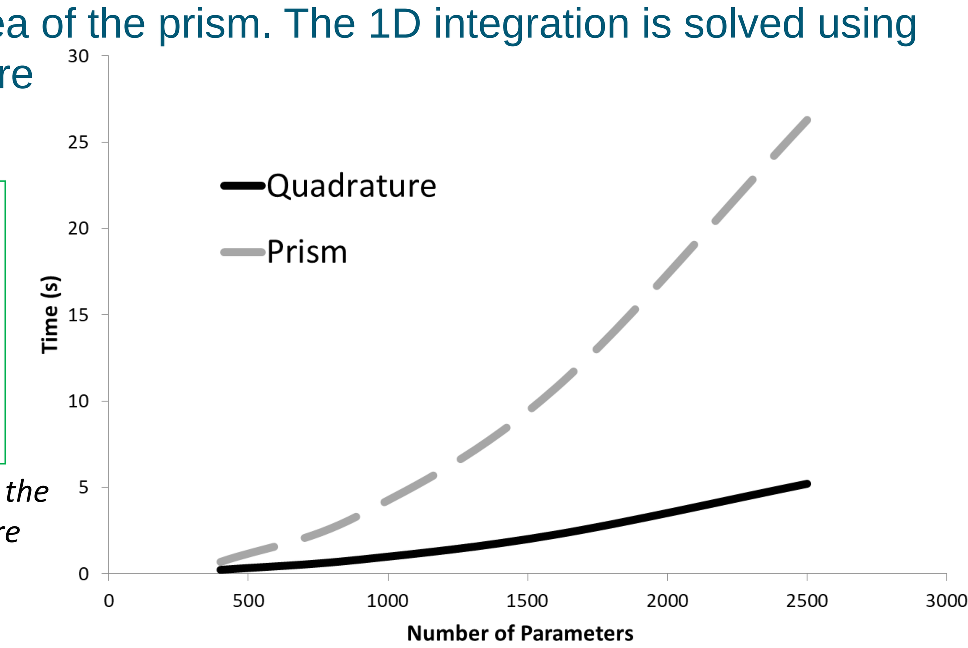


Figure 2 – Computational time consuming of the forward modeling using prism and quadrature integral versus the number of parameters.

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