Online Supplement for "Location, inventory and testing decisions in closed-loop supply chains: a multimedia company" by Zhi-Hai Zhang, Gemma Berenguer, and Xiaoyong Pan

1. Proof of Proposition 1

In order to start reformulating ($\mathscr{P}$) to an equivalent CQMIP we linearize the absolute deviation in (10) by introducing the auxiliary variable Λ_s, which is equivalent to the following program

$$\begin{aligned} min \quad & \Gamma_s - \sum_{s' \in S} \omega_{s'} \Gamma_{s'} + 2\Lambda_s, \\ s.t. \quad & \Gamma_s - \sum_{s' \in S} \omega_{s'} \Gamma_{s'} + \Lambda_s \geq 0. \end{aligned}$$

We also linearize constraint (11) by introducing two auxiliary decision variables:

$$\bar{V}_{ijk}^{F(R)} = \begin{cases} 1, & \text{if new (returned) products are processed at (re)manufacturer } k \text{ and} \\ & \text{shipped to (from) retailer } i \text{ through DC } j, \\ 0, & \text{otherwise.} \end{cases},$$

In particular, the nonlinear term $Y_{ij}^F V_{jk}^F$ and $Y_{ij}^R V_{jk}^R$ in $\widetilde{WI}_j^F(D_j^F, Q_{js}^F)$, $\widetilde{WI}_j^R(D_{js}^R, Q_{js}^R)$, $\widetilde{SS}_j(\mathbf{Y_j^F}, L_j)$, and $\mathbf{R}(\mathbf{Y_j^R}, Q_{js}^R)$ are substituted by $\bar{V}_{ijk}^F$ and $\bar{V}_{ijk}^R$, respectively.

$$\begin{aligned} \widetilde{WI}_j^F(D_j^F, Q_{js}^F) &= \frac{F_j^F}{Q_{js}^F}\chi\sum_{i\in I}\mu_i^F Y_{ij}^F + \frac{\beta}{Q_{js}^F}\sum_{k\in K}\sum_{i\in I}\bar{g}_{jk}^F\chi\mu_i^F\bar{V}_{ijk}^F + \beta\sum_{k\in K}\sum_{i\in I}\bar{a}_{jk}^F\chi\mu_i^F\bar{V}_{ijk}^F + \frac{\theta h}{2}Q_{js}^F, \\ \widetilde{WI}_j^R(D_{js}^R, Q_{js}^R) &= \frac{F_{js}^R}{Q_{js}^R}\chi\sum_{i\in I}\mu_{is}^R Y_{ij}^R + \frac{\beta}{Q_{js}^R}\left(\sum_{k\in K}\sum_{i\in I}\bar{g}_{jk}^R\chi\mu_{is}^R\bar{V}_{ijk}^R\right) + \beta\sum_{k\in K}\sum_{i\in I}\bar{a}_{jk}^R\chi\mu_{is}^R\bar{V}_{ijk}^R + \frac{\theta h}{2}Q_{js}^R, \\ \widetilde{SS}_j(\mathbf{Y_j^F}, L_j) &= z_\alpha\sqrt{\sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\,(\sigma_i^F)^2\,\bar{V}_{ijk}^F}, \\ R(\mathbf{Y_j^R}, Q_{js}^R) &= \gamma U\left(\frac{\chi Q_{js}^R}{2} + \frac{1}{k}\sum_{i\in I}d_{ij}\chi\mu_{is}^R Y_{ij}^R + \frac{1}{k}\sum_{k\in K}\sum_{i\in I}\bar{a}_{jk}^R\chi\mu_{is}^R\bar{V}_{ijk}^R\right). \end{aligned}$$

Additionally, the corresponding constraints (A.2)$\sim$(A.4) are added to the model so the bilinear terms $Y_{ij}^F V_{jk}^F$ and $Y_{ij}^R V_{jk}^R$ can be substituted by $\bar{V}_{ijk}^F$ and $\bar{V}_{ijk}^R$, respectively.

Three sets of auxiliary decision variables Δ_{js}^F, Δ_{js}^R and Ω_j are introduced, which satisfy the following inequalities:

$$\frac{\theta h}{2}\Delta_{js}^F \geq \frac{F_j^F}{Q_{js}^F}\chi\sum_{i\in I}\mu_i^F Y_{ij}^F + \frac{\beta}{Q_{js}^F}\sum_{k\in K}\sum_{i\in I}\bar{g}_{jk}^F\chi\mu_i^F\bar{V}_{ijk}^F + \frac{\theta h}{2}Q_{js}^F,$$

$$
\begin{aligned}
\frac{\theta h + W\gamma U\chi}{2}\Delta_{js}^{R} &\geq \frac{F_j^R}{Q_{js}^R}\chi\sum_{i\in I}\mu_{is}^R Y_{ij}^R + \frac{\beta}{Q_{js}^R}\sum_{k\in K}\sum_{i\in I}\bar{g}_{jk}^R\chi\mu_{is}^R\bar{V}_{ijk}^R + \frac{\theta h + W\gamma U\chi}{2}Q_{js}^R, \\
\Omega_j &\geq \sqrt{\sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\left(\sigma_i^F\right)^2\bar{V}_{ijk}^F}.
\end{aligned}
$$

Then, the linearized constraint (A.5) can be obtained. Note that $Y_{ij}^2 = Y_{ij}$ and $\bar{V}_{ijk}^2 = \bar{V}_{ijk}$ if Y_{ij} and $\bar{V}_{ijk}$ are binary variables, so the above inequalities can be reformulated as the conic mixed-integer constraints (A.7)$\sim$(A.9).

Furthermore, chance constraints (9), (10), and (11) should be remodeled as conic quadratic constraints as follows. Constraint (A.10) can be derived from constraint (9) by following steps in Appendix C of Ozsen et al. (2008). To show Constraint (A.11), let $\mathbb{D}_k^F = \sum_{j\in J}\tilde{D}_j^F V_{jk}^F$ and it follows a normal distribution with mean $\sum_{j\in J}\sum_{i\in I}\chi\mu_i^F\bar{V}_{ijk}^F$ and variance $\sum_{j\in J}\sum_{i\in I}\chi\left(\sigma_i^F\right)^2\bar{V}_{ijk}^F$. From $Pr\left\{\sum_{j\in J}\tilde{D}_j^F V_{jk}^F \geq C_k^F\right\} \leq \rho^F$ in (13) we have that

$$
\begin{aligned}
Pr\left\{\frac{\mathbb{D}_k^F - \sum_{j\in J}\sum_{i\in I}\chi\mu_i^F\bar{V}_{ijk}^F}{\sqrt{\sum_{j\in J}\sum_{i\in I}\chi\left(\sigma_i^F\right)^2\bar{V}_{ijk}^F}} \leq \frac{C_k^F - \sum_{j\in J}\sum_{i\in I}\chi\mu_i^F\bar{V}_{ijk}^F}{\sqrt{\sum_{j\in J}\sum_{i\in I}\chi\left(\sigma_i^F\right)^2\bar{V}_{ijk}^F}}\right\} &\geq 1-\rho^F, \\
\frac{C_k^F - \sum_{j\in J}\sum_{i\in I}\chi\mu_i^F\bar{V}_{ijk}^F}{\sqrt{\sum_{j\in J}\sum_{i\in I}\chi\left(\sigma_i^F\right)^2\bar{V}_{ijk}^F}} &\geq z_{1-\rho^F}.
\end{aligned}
$$

Furthermore, an auxiliary decision variable $\bar{\Omega}_k^F$ is introduced, which satisfies constraint (A.12), $\left(\bar{\Omega}_k^F\right)^2 \geq \sum_{j\in J}\sum_{i\in I}\chi\left(\sigma_i^F\right)^2\left(\bar{V}_{ijk}^F\right)^2$. Correspondingly, Constraint (10) can be formulated as $\sum_{j\in J}\sum_{i\in I}\chi\mu_i^F\bar{V}_{ijk}^F + z_{1-\rho^F}\bar{\Omega}_k^F \leq C_k^F$. Constraints (A.13) and (A.14) can be shown in the same way. □

2. Solution method based on the Sample Average Approximation approach

We suggest a more general setting for our problem in which we relax the normal distribution of demand of new and returned products. If we do so, we cannot linearize the chance constraints as done in Proposition 1 and then the model is non-convex, which cannot be solved by our proposed method. To solve this issue, we next present a scenario-based approach to linearize the chance constraint and to be able to reformulate the problem as a sample average approximation (SAA) model.

In order to formulate the problem as a SAA model that considers demand and return uncertainty, we define scenario $e \in E$ associate with random demand and return instead of the return rate of returned products. π_e is the probability of scenario e. Under scenario e, the quantities of new and returned products at retailer i are μ_{ie}^F and μ_{ie}^R, respectively, where $\mu_{ie}^R = \phi(s)\mu_{ie}^R$ with probability ω_s, $s \in S$. The mean and standard deviation of new products at retailer i are $\hat{\mu}_i^F$ and $\hat{\sigma}_i^F$. Then, the safety stock of new products at DC j is $\hat{z}_\alpha \sqrt{\sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\left(\hat{\sigma}_i^F\right)^2\left(\bar{V}_{ijk}^F\right)^2}$. $\hat{z}_\alpha$ is a safety factor, which can be obtained by a convex approximation method proposed by Bonami & Lejeune (2008). The reorder point at DC j (r_j) is $\sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\hat{\mu}_i^F + \hat{z}_\alpha\sqrt{\sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\left(\hat{\sigma}_i^F\right)^2\left(\bar{V}_{ijk}^F\right)^2}$.

$$Pr\left\{Q_{js}^F + Q_{js}^R + r_j - \tilde{D}_{j,L}^F \geq C_j\right\} \leq \bar{\rho},$$

$$Pr\left(Q_{js}^F + Q_{js}^R + z_\alpha\Omega_j + \sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\left(\hat{\mu}_i^F - \mu_{is}^F\right)Y_{ij}^F \geq C_j\right) \leq \bar{\rho}, \forall j \in J.$$

Then, the SAA model is formulated as follows.

$$\begin{aligned}
\min_{X,Y} Z \quad = \quad & \sum_{k\in K} p_k^F Z_k^F + \sum_{j\in J} f_j^F X_j^F + \sum_{k\in K} p_k^R Z_k^R + \sum_{j\in J} f_j^R X_j^R - \sum_{j\in J}\hat{s}_j^C X_j^C \\
& + \sum_{e\in E}\pi_e\mathit{\Gamma}_e + \lambda\sum_{e\in E}\pi_e\left[\mathit{\Gamma}_e - \sum_{e'\in E}\pi_{e'}\mathit{\Gamma}_{e'} + 2\mathit{\Lambda}_e\right], && (1)\\
s.t \quad & (A.2) \sim (A.4), \\
& Y_{ij}^F \leq X_j^F, Y_{ij}^R \leq X_j^R, \forall i \in I, \forall j \in J, && (2)\\
& X_j^C \leq X_j^F, X_j^C \leq X_j^R, \forall j \in J, && (3)\\
& \sum_{k\in K} V_{jk}^F = X_j^F, \sum_{k\in K} V_{jk}^R = X_j^R, \forall j \in J, && (4)\\
& V_{jk}^F \leq Z_k^F, V_{jk}^R \leq Z_k^R, \forall j \in J, \forall k \in K, && (5)\\
& Z_k^R \leq \sum_{j\in J} V_{jk}^F, \forall k \in K, && (6)\\
& \mathit{\Gamma}_e \geq \sum_{j\in J}\left\{\sum_{i\in I}\beta\chi d_{ij}\mu_{ie}^F Y_{ij}^F + \theta h\hat{z}_\alpha\Omega_j + \beta\sum_{k\in K}\sum_{i\in I}\bar{a}_{jk}^F\chi\mu_{ie}^F\bar{V}_{ijk}^F + \frac{\theta h}{2}\Delta_{je}^F\right\} \\
& \qquad + \sum_{j\in J}\left\{\sum_{i\in I}\beta\chi d_{ij}\mu_{ie}^R Y_{ij}^R + \beta\sum_{k\in K}\sum_{i\in I}\bar{a}_{jk}^R\chi\mu_{ie}^R\bar{V}_{ijk}^R\right. \\
& \qquad \left. + W\gamma U\left(\frac{1}{\kappa}\sum_{i\in I} d_{ij}\chi\mu_{ie}^R Y_{ij}^R + \frac{1}{\kappa}\sum_{k\in K}\sum_{i\in I}\bar{a}_{jk}^R\chi\mu_{ie}^R\bar{V}_{ijk}^R\right) + \frac{\theta h + W\gamma U\chi}{2}\Delta_{je}^R\right\}, \\
& \hfill \forall e \in E, && (7)\\
& \mathit{\Gamma}_e - \sum_{e'\in E}\omega_{e'}\mathit{\Gamma}_{e'} + \mathit{\Lambda}_e \geq 0, \forall e \in E, && (8)\\
& \frac{\theta h}{4}\left(\Delta_{je}^F + Q_{je}^F\right)^2 \geq F_j^F\chi\sum_{i\in I}\mu_{ie}^F\left(Y_{ij}^F\right)^2 + \beta\sum_{k\in K}\sum_{i\in I}\bar{g}_{jk}^F\chi\mu_{ie}^F\left(\bar{V}_{ijk}^F\right)^2 \\
& \qquad + \frac{3\theta h}{4}\left(Q_{je}^F\right)^2 + \frac{\theta h}{4}\left(\Delta_{je}^F\right)^2, \forall j \in J, e \in E, && (9)\\
& \frac{\theta h + W\gamma U\chi}{4}\left(\Delta_{je}^R + Q_{je}^R\right)^2 \geq F_j^R\chi\sum_{i\in I}\mu_{ie}^R\left(Y_{ij}^R\right)^2 + \beta\sum_{k\in K}\sum_{i\in I}\bar{g}_{jk}^R\chi\mu_{ie}^R\left(\bar{V}_{ijk}^R\right)^2
\end{aligned}$$

$$+\frac{3\left(\theta h+W\gamma U\chi\right)}{4}\left(Q_{je}^{R}\right)^{2}+\frac{\theta h+W\gamma U\chi}{4}\left(\Delta_{je}^{R}\right)^{2},\forall j\in J,e\in E, \tag{10}$$

$$\Omega_j^2\geq\sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\left(\hat{\sigma}_i^F\right)^2\left(\bar{V}_{ijk}^F\right)^2,\forall j\in J, \tag{11}$$

$$\sum_{j\in J}\sum_{i\in I}\chi\mu_{ie}^F\bar{V}_{ijk}^F\geq\sum_{j\in J}\sum_{i\in I}\chi\mu_{ie}^R\bar{V}_{ijk}^R,\forall k\in K,e\in E, \tag{12}$$

$$\sum_{e\in E}\pi_e\mathrm{II}\left(Q_{je}^F+Q_{je}^R+z_\alpha\Omega_j+\sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\left(\hat{\mu}_i^F-\mu_{ie}^F\right)Y_{ij}^F\geq C_j\right)\leq\bar{\rho},\forall j\in J, \tag{13}$$

$$\sum_{e\in E}\pi_e\mathrm{II}\left(\sum_{j\in J}\sum_{i\in I}\chi\mu_{ie}^FY_{ij}^FV_{jk}^F\geq C_k^F\right)\leq\rho^F,\forall j\in J, \tag{14}$$

$$\sum_{e\in E}\pi_e\mathrm{II}\left(\sum_{j\in J}\sum_{i\in I}\chi\mu_{ie}^RY_{ij}^RV_{jk}^R\geq C_k^R\right)\leq\rho^R,\forall j\in J, \tag{15}$$

$$\Delta_{je}^F,\Delta_{je}^R,\Omega_j,Q_{je}^F,Q_{je}^R,\mathit{\Gamma}_e,\mathit{\Lambda}_e\geq 0,\forall j\in J,k\in K,e\in E, \tag{16}$$

$$X_j^F,X_j^R,X_j^C,Y_{ij}^F,Y_{ij}^R,Z_k^F,Z_k^R,V_{jk}^F,V_{jk}^R,\bar{V}_{ijk}^F,\bar{V}_{ijk}^R\in\{0,1\},\forall i\in I,\forall j\in J,\forall k\in K. \tag{17}$$

where $\mathrm{II}(f(x)\leq t)$ is 1 if $f(x)\leq t$ and 0 otherwise. Constraints (13), (14), and (15) are associated with chance constraints (9), (10), and (11) in the paper, respectively.

Furthermore, constraint (13) is linearized by introducing binary variable δ_s and it is equivalent to the following two constraints.

$$\begin{cases} Q_{je}^F+Q_{je}^R+\hat{z}_\alpha\Omega_j+\sum_{k\in K}\sum_{i\in I}\bar{L}_{jk}\left(\hat{\mu}_i^F-\mu_{ie}^F\right)Y_{ij}^F\geq\delta_eC_j+(\delta_e-1)M, \quad \forall e\in E,\\ \sum_{e\in E}\pi_e\delta_e\leq\bar{\rho}, \end{cases}$$

where M is a sufficient larger and positive number. Similarly, constraints (14) and (15) can be linearized. The SAA model is still a second-order cone program. Following the solution approaches of the SAA model proposed by Ahmed et al. (2008), Luedtke & Ahmed (2008), Pagnoncelli et al. (2009), we can solve the model. Moreover, several reformulation techniques such as valid inequalities (e.g., star inequalities (Ahmed et al. 2008), quantile inequalities (Xie & Ahmed 2017)) and solution algorithms such as branch and cut decomposition algorithm (Luedtke 2014, Ahmed et al. 2017), scenario decomposition algorithm (Ahmed et al. 2017) and cutting plane approaches (Xie & Ahmed 2017) can be employed to solve the program efficiently. This is beyond the scope of the paper and it is one of the our future research.

3. Case study additional results

		Cost components					MFs		DCs		
Model	τ_i	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
integrated	0.5	440862124	389753192	51106470	8404	10876	1, 2	1, 2	3	7	3
sequential	0.5	758797361	707809959	50981664	4944	10679	0, 1, 2	1, 2	6	10	2
integrated	0.7	443692362	390179095	53510279	8404	11429	1, 2	1, 2	3	5	3
sequential	0.7	761122652	707836888	53279683	4944	11027	0, 1, 2	1, 2	6	6	2
integrated	1	446320306	390284705	56044991	8404	12156	1, 2	1, 2	3	3	3
sequential	1	765610492	707865200	57740158	4944	10101	0, 1, 2	1, 2	6	2	2

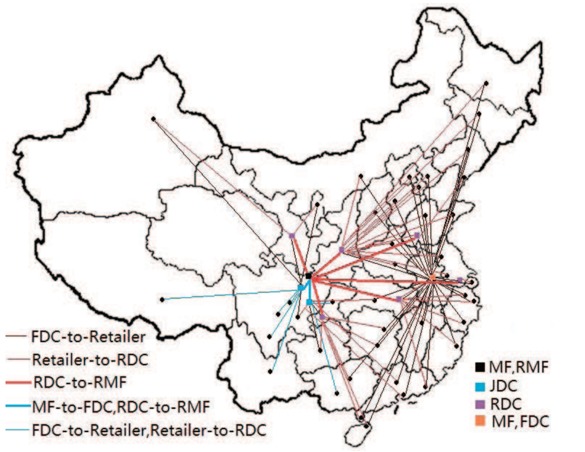
Table S1: Costs and location results under the integrated and sequential robust approach with unit shipping cost ratios $(MF-to-DC)/(DC-to-retailer) = 1$. The sequential robust approach is based on the current forward supply chain and using the robust model to find the optimal reverse supply chain.

		Cost components					MFs		DCs		
Testing	τ	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
MF	1	762167044	707865551	54297083	4944	9413	0,1,2	1,2	6	2	2
DC	0.9	762884063	707858779	55018895	4944	11443	0,1,2	1,2	6	4	2
DC	0.8	762089777	707848468	54235134	4944	11122	0,1,2	1,2	6	4	2
DC	0.7	761122652	707836888	53279683	4944	11027	0,1,2	1,2	6	6	2
DC	0.6	760053857	707825735	52222447	4944	10620	0,1,2	1,2	6	6	2
DC	0.5	758797361	707809959	50981664	4944	10679	0,1,2	1,2	6	10	2

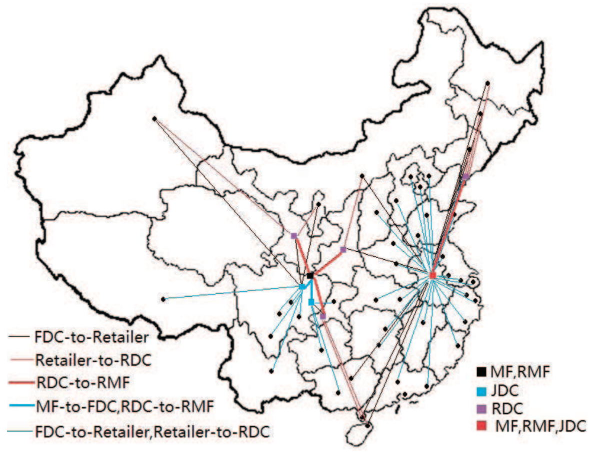
Table S2: Costs and location results under DC and MF testing and varying τ. The incremental testing cost at the DCs is assumed to be $a_j^{test,R} = 1$. These experiments are based on the current forward supply chain and using the robust model

4. Model with single return rate

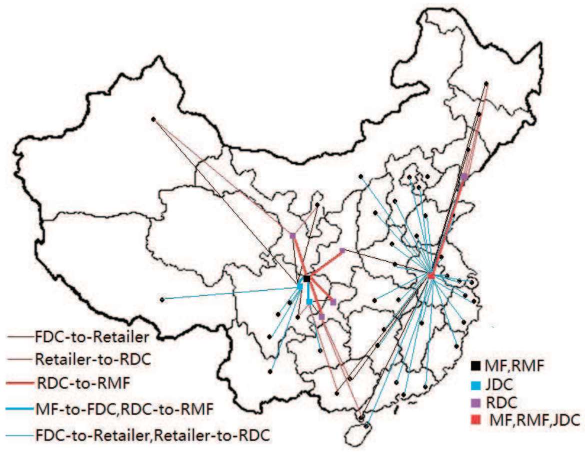
$$
\begin{aligned}
\underset{X,Y}{min}\, Z \quad &= \quad \sum_{k\in K} p_k^F Z_k^F + \sum_{j\in J} f_j^F X_j^F + \sum_{k\in K} p_k^R Z_k^R + \sum_{j\in J} f_j^R X_j^R - \sum_{j\in J} \hat{s}_j^C X_j^C + \mathit{\Gamma}, \\
S.t. \quad & (3) \sim (7), (A.2), (A.3), (A.4), \\
& \mathit{\Gamma} \geq \sum_{j\in J} \left\{ \sum_{i\in I} \beta\chi d_{ij}\mu_i^F Y_{ij}^F + \theta h z_\alpha \Omega_j + \beta \sum_{k\in K}\sum_{i\in I} \bar{a}_{jk}^F \chi \mu_i^F \bar{V}_{ijk}^F + \frac{\theta h}{2}\Delta_j^F \right\} \\
& \quad + \sum_{j\in J} \left\{ \sum_{i\in I} \beta\chi d_{ij}\mu_i^R Y_{ij}^R + \beta\sum_{k\in K}\sum_{i\in I} \bar{a}_{jk}^R \chi\mu_i^R \bar{V}_{ijk}^R \right. \\
& \quad \left. + W\gamma U \left(\frac{1}{\kappa}\sum_{i\in I} d_{ij}\chi\mu_i^R Y_{ij}^R + \frac{1}{\kappa}\sum_{k\in K}\sum_{i\in I} \bar{a}_{jk}^R \chi\mu_i^R \bar{V}_{ijk}^R \right) + \frac{\theta h + W\gamma U\chi}{2}\Delta_j^R \right\}, \\
& \frac{\theta h}{4}\left(\Delta_j^F + Q_j^F\right)^2 \geq F_j^F \chi \sum_{i\in I} \mu_i^F \left(Y_{ij}^F\right)^2 + \beta\sum_{k\in K}\sum_{i\in I} \bar{g}_{jk}^F \chi\mu_i^F \left(\bar{V}_{ijk}^F\right)^2 \\
& \quad + \frac{3\theta h}{4}\left(Q_j^F\right)^2 + \frac{\theta h}{4}\left(\Delta_j^F\right)^2, \forall j \in J, \\
& \frac{\theta h + W\gamma U\chi}{4}\left(\Delta_j^R + Q_j^R\right)^2 \geq F_j^R \chi \sum_{i\in I} \mu_i^R \left(Y_{ij}^R\right)^2 + \beta\sum_{k\in K}\sum_{i\in I} \bar{g}_{jk}^R \chi\mu_i^R \left(\bar{V}_{ijk}^R\right)^2 \\
& \quad + \frac{3\left(\theta h + W\gamma U\chi\right)}{4}\left(Q_j^R\right)^2 + \frac{\theta h + W\gamma U\chi}{4}\left(\Delta_j^R\right)^2, \forall j \in J,
\end{aligned}
$$



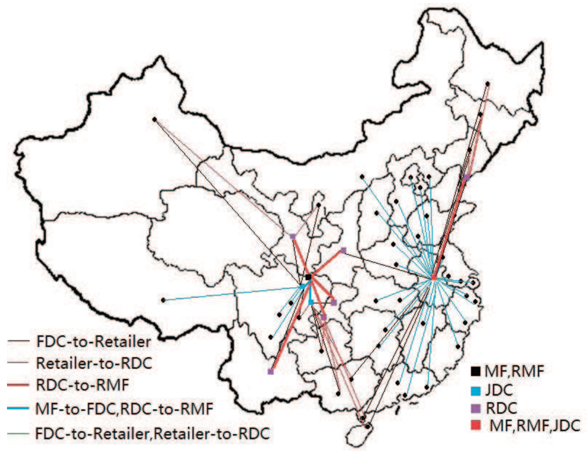
(a) Return ratio=0.1, total cost=CNY412541869, the numbers of MF, RMF, FDC, RDC, and JDC are 2, 1, 3, 8, 2, respectively



(b) Return ratio=0.3, total cost=CNY438408837, the numbers of MF, RMF, FDC, RDC, and JDC are 2, 2, 3, 7, 3, respectively



(c) Return ratio=0.5, total cost=CNY463447967, the numbers of MF, RMF, FDC, RDC, and JDC are 2, 2, 3, 8, 3, respectively



(d) Return ratio=0.7, total cost=CNY488181622, the numbers of MF, RMF, FDC, RDC, and JDC are 2, 2, 3, 9, 3, respectively

Figure S1: Closed-loop supply chain network obtained by the integrated approach. MF= manufacturing location, RMF= remanufacturing location, FDC= forward DC, JDC= joint DC, and RDC= reverse DC

$$\Omega_j^2 \geq \sum_{k \in K} \sum_{i \in I} \bar{L}_{jk} \left(\sigma_i^F\right)^2 \left(\bar{V}_{ijk}^F\right)^2, \forall j \in J,$$

$$Q_j^F + (z_\alpha - z_{\bar{\rho}})\Omega_j + Q_j^R \leq C_j, \forall j \in J,$$

$$\sum_{j \in J} \sum_{i \in I} \chi \mu_i^F \bar{V}_{ijk}^F + z_{1-\rho^F} \bar{\Omega}_k^F \leq C_k^F, \forall k \in K,$$

$$\left(\bar{\Omega}_k^F\right)^2 - \sum_{j \in J} \sum_{i \in I} \chi \left(\sigma_i^F\right)^2 \left(\bar{V}_{ijk}^F\right)^2 \geq 0, \forall k \in K,$$

$$\sum_{j \in J} \sum_{i \in I} \chi \mu_i^R \bar{V}_{ijk}^R + z_{1-\rho^R} \bar{\Omega}_k^R \leq C_k^R, \forall k \in K,$$

$$\left(\bar{\Omega}_k^R\right)^2 - \sum_{j \in J} \sum_{i \in I} \chi \left(\sigma_i^R\right)^2 \left(\bar{V}_{ijk}^R\right)^2 \geq 0, \forall k \in K,$$

$$\sum_{j \in J} \sum_{i \in I} \chi \mu_i^F \bar{V}_{ijk}^F \geq \sum_{j \in J} \sum_{i \in I} \chi \mu_i^R \bar{V}_{ijk}^R, \forall k \in K,$$

$$\Delta_j^F, \Delta_j^R, \Omega_j, \bar{\Omega}_k^F, \bar{\Omega}_k^R, Q_j^F, Q_j^R, \Gamma \geq 0, \forall j \in J, k \in K,$$

$$X_j^F, X_j^R, X_j^C, Y_{ij}^F, Y_{ij}^R, Z_k^F, Z_k^R, V_{jk}^F, V_{jk}^R, \bar{V}_{ijk}^F, \bar{V}_{ijk}^R \in \{0,1\}, \forall i \in I, \forall j \in J, \forall k \in K.$$

5. Computational performance

In this supplement we design and report on the computational performance of the model proposed in order to assert whether the solution CPU times and optimality gaps are reasonable given different numerical experiments.

The performance of the model without adding valid inequalities by varying the coefficients associated with the transportation and inventory costs (θ and β) are evaluated firstly. Five random instance are generated by adding noise to the mean demand, standard deviation, and fixed costs by multiplying these values with $(1+\epsilon)$ where ϵ is drawn from a uniform [-0.1, 0.1]. These instances are solved and the average of the five instance per row is reported for different parameter values in the columns labeled by "CPLEX" in Tables S3 and S4. CPU times and gaps are reported under different settings. We observe that most of the instances can find the optimal solutions (gap $\leq 0.01\%$ in CPLEX) within the time limits when $cap_{49} = cap_{88} = 1000$ and $\xi = 0.8$ (Table S3). More instances cannot find the optimal solutions within the established time limits when decreasing the capacities of the MFs and DCs to $cap_{49} = cap_{88} = 800$ and $\xi = 0.4$ (Table S4). To show the computational benefits of adding these valid inequalities, we resolve these instances after adding valid inequalities (extremal extended polymatroid inequalities) to the model. The last columns in the tables list the number of valid inequalities that are added to the root node of the branch and bound tree in CPLEX. Compared with the model without adding the valid inequalities, the model with valid inequalities can either find the optimal solutions within the time limits or improve the solutions' quality. Therefore, for these popular data sets, the solution approach is efficient and the usage of the valid cuts improves performance significantly.

			CPLEX		CPLEX+CUTS		
#DCs	θ	β	CPU (s)	GAP (%)	CPU (s)	GAP (%)	CUTS
49	0.1	0.001	10012.16	0.01	3701.57	0.01	17
49	0.1	0.002	9438.17	0.01	805.40	0.01	13
49	0.1	0.003	2991.44	0.01	693.68	0.01	8
49	0.1	0.004	699.53	0.01	483.44	0.01	6
49	0.1	0.005	525.93	0.01	513.84	0.01	2
49	0.2	0.002	3998.60	0.01	668.73	0.01	14
49	0.5	0.005	735.95	0.01	615.75	0.01	6
49	1.0	0.005	794.79	0.01	470.63	0.01	7
49	2.0	0.005	677.87	0.01	595.83	0.01	7
49	5.0	0.005	923.32	0.01	769.13	0.01	13
88	0.1	0.001	14403.60	1.48	11234.15	0.01	31
88	0.1	0.002	8067.17	0.01	5795.65	0.01	37
88	0.1	0.003	7069.66	0.01	2739.60	0.01	32
88	0.1	0.004	14404.33	0.15	7817.02	0.01	36
88	0.1	0.005	-	-	14404.26	0.03	34
88	0.2	0.002	6746.54	0.01	6394.62	0.01	57
88	0.5	0.005	3800.75	0.01	3598.29	0.01	40
88	1.0	0.005	14404.98	0.03	12630.39	0.01	54
88	2.0	0.005	14404.14	0.53	6436.92	0.01	77
88	5.0	0.005	14404.69	0.45	14404.79	0.15	168

Table S3: The comparison between the instances with and without the valid equalities, time limits=14400s, $|K|$=5, cap_{49}=cap_{88}=1000, W=1, $\xi = 0.8$. '-' means that the instances can not find any feasible solutions within the time limits

			CPLEX		CPLEX+CUTS		
#DCs	θ	β	CPU (s)	GAP (%)	CPU (s)	GAP (%)	CUTS
49	0.1	0.001	14400.70	16.76	14400.62	4.02	35
49	0.1	0.002	14400.70	2.20	9297.30	0.01	35
49	0.1	0.003	14401.70	1.79	11051.27	0.01	10
49	0.1	0.004	3353.48	0.01	1293.72	0.01	12
49	0.1	0.005	2892.10	0.01	2073.52	0.01	10
49	0.2	0.002	14405.33	0.51	14403.64	0.36	41
49	0.5	0.005	1418.40	0.01	1191.77	0.01	13
49	1.0	0.005	1307.51	0.01	1271.56	0.01	14
49	2.0	0.005	3057.71	0.01	1562.56	0.01	15
49	5.0	0.005	14402.70	0.79	1517.42	0.01	24
88	0.1	0.001	14403.45	28.34	14403.32	22.58	36
88	0.1	0.002	14403.21	14.53	14403.17	9.43	37
88	0.1	0.003	11340.24	0.01	10616.91	0.01	30
88	0.1	0.004	14326.22	0.01	13577.96	0.01	38
88	0.1	0.005	5810.32	0.01	4741.15	0.01	23
88	0.2	0.002	14404.07	16.26	14404.01	0.07	50
88	0.5	0.005	6206.27	0.01	4044.56	0.01	42
88	1.0	0.005	8905.55	0.01	7051.76	0.01	53
88	2.0	0.005	14404.35	3.42	14404.66	2.04	80
88	5.0	0.005	14404.03	10.96	14404.78	4.24	163

Table S4: The comparison between the instances with and without the valid equalities, time limits=14400s, $|K|$=5, cap_{49}=cap_{88}=800, W=1, $\xi = 0.4$

τ_i	Cost components					MFs		DCs		
	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
0.10	152615992	140351778	12246600	1985	19599	0,1,2	1	7	9	5
0.20	152626262	140342475	12259653	1922	26056	0,1,2	1	8	9	5
0.30	152643529	140341809	12271527	1484	31677	0,1,2	1	8	9	4
0.40	167649984	140341310	27276565	1776	33885	0,1,2	2	7	9	4
0.50	167667805	140345968	27289419	3198	35616	0,1,2	2	6	7	4
0.60	167682794	140348385	27299782	2847	37474	0,1,2	2	9	4	4
0.70	167685630	140343686	27305683	1756	38017	0,1,2	2	5	4	1
0.80	167689237	140343250	27309214	2199	38972	0,1,2	2	8	4	3
0.90	167694464	140343707	27315388	1756	37125	0,1,2	2	8	2	1
1	177666810	140344630	37298810	4944	28314	0,1,2	1,2	7	2	2
1	175511537	140342956	35145098	3188	26671	0,1,2	1,2	8	2	1

Table S5: Costs and location results under different τ when testing and disposition decisions at the DC level ($a_j^{test,R} = 1$). The solution with testing at the MFs is represented by $a_j^{test,R} = 0$ and $\tau_i = 1$ in the last row (with Table 5)

6. Experimental results associated with Changhong data sets and the single scenario model

The additional experiments are generated based on the data from Changhong Electric Co. Ltd and using the same parameter settings described in Section 6 when required. Experimental results shown in Tables S5∼S8 correspond to those shown in Tables 5 to 8, respectively.

		Cost components					MFs			DCs	
Shipment	Ratio	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
indirect	0.1	175381808	140228203	35116177	1881	39309	0,1,2	1,2	13	6	4
direct	0.1	180185101	140221184	39944931	0	18986	0,1,2	1,2	14	0	0
indirect	0.2	175431631	140272349	35128113	3140	34309	0,1,2	1,2	13	5	4
direct	0.2	180235894	140271977	39944931	0	18986	0,1,2	1,2	12	0	0
indirect	0.3	175499156	140333002	35137990	4634	32798	0,1,2	1,2	12	5	5
direct	0.3	180281737	140317820	39944931	0	18986	0,1,2	1,2	12	0	0
indirect	0.4	175511537	140342956	35145098	3188	26671	0,1,2	1,2	8	2	1
direct	0.4	180312456	140348539	39944931	0	18986	0,1,2	1,2	6	0	0
indirect	0.5	175529121	140359158	35148758	4944	26149	0,1,2	1,2	3	2	2
direct	0.5	180318088	140354171	39944931	0	18986	0,1,2	1,2	4	0	0
indirect	0.6	175531023	140359361	35150498	4944	26108	0,1,2	1,2	3	2	2
direct	0.6	180327509	140363592	39944931	0	18986	0,1,2	1,2	4	0	0
indirect	0.7	175544548	140370217	35153216	4944	26059	0,1,2	1,2	3	2	2
direct	0.7	180326618	140362701	39944931	0	18986	0,1,2	1,2	3	0	0
indirect	0.8	175549632	140373815	35154703	4944	26058	0,1,2	1,2	3	2	2
direct	0.8	180329437	140365520	39944931	0	18986	0,1,2	1,2	3	0	0
indirect	0.9	175556039	140378576	35156283	4944	26124	0,1,2	1,2	4	2	2
direct	0.9	180340607	140376690	39944931	0	18986	0,1,2	1,2	3	0	0
indirect	1	175552544	140373608	35157752	4944	26128	0,1,2	1,2	3	2	2
direct	1	180368501	140404584	39944931	0	18986	0,1,2	1,2	3	0	0

Table S6: Costs and location results under different ratios when indirect/direct shipments of returned products (with Table 6)

	Cost components					MFs			DCs	
γ (%)	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
0.01	175488156	140343899	35146182	4944	3019	0,1,2	1,2	7	3	2
0.10	175511537	140342956	35145098	3188	26671	0,1,2	1,2	8	2	1
0.30	175568074	140344991	35147324	4944	80703	0,1,2	1,2	6	2	2
0.50	175613269	140340740	35147556	4943	129916	0,1,2	1,2	8	2	2
0.70	175670088	140345523	35147920	4943	181588	0,1,2	1,2	9	2	2
0.90	175722247	140343122	35148414	4944	235655	0,1,2	1,2	5	2	2

Table S7: Costs and location results under different γ (with Table 7)

7. Experimental results associated with Daskin (1995) data sets and the multiple-scenario (robust) model

The additional experimental results associated with Daskin (1995) data sets and the robust model are reported in the section. In particular, we use the same table of rates of returned products and associated probabilities as the Changhong data set. The rest of parameter values are the as the ones used in Section 6. Accordingly, we show results related to measuring the impact of the fraction of accepted returned products (τ), indirect/direct shipments, and marginal value of time of returned products (γ). The results are reported in Tables S9, S10, and S11, respectively. Compared to the results reported in Tables 5, 6, and 7, which are obtained from the single scenario model, we draw the same conclusions obtained in Sections 6.2, 6.3 and 6.4. This implies that the conclusions are independent of the different scenarios related to different rates of returned product.

Rate	Cost components					MFs		DCs		
	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
0.2	165449546	140341594	25093194	1756	16514	0,1,2	2	7	1	1
0.4	175520042	140344145	35153383	4944	27458	0,1,2	1,2	8	2	2
0.6	185702315	140347383	45298004	4187	61115	0,1,2	0,1	7	2	2
0.8	200663024	140347541	60271737	4944	48690	0,1,2	0,2	8	2	2
1	210760762	140347863	70349796	7025	70128	0,1,2	0,1,2	8	4	4

Table S8: Costs and location results under different rates of returned products. Note: $\mu_i^R = return\ rate \times \mu_i^F$, $\left(\sigma_i^R\right)^2 = (return\ rate)^2 \times \left(\sigma_i^F\right)^2$ and $C_k^R = 1.0 \times \chi \sum_{i \in I} \mu_i^F \times 0.4$ (with Table 8)

τ_i	Cost components					MFs		DCs		
	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
0.1	108716	87802	20675	1074	1314	2,3,4	4	17	9	9
0.2	110370	87844	21766	1074	1835	2,3,4	4	18	9	9
0.3	112122	87839	23222	1206	2268	2,3,4	4	18	10	10
0.4	113681	87846	24669	1206	2373	2,3,4	4	19	10	10
0.5	115233	87848	26114	1206	2479	2,3,4	4	18	10	10
0.6	118163	87839	28854	1058	2528	2,3,4	3,4	18	9	9
0.7	118768	87834	29122	900	2713	2,3,4	3,4	18	8	8
0.8	119490	87825	29762	921	2825	2,3,4	3,4	18	8	8
0.9	120031	87829	29966	809	3046	2,3,4	3,4	18	7	7
1.0	120679	87833	30402	809	3254	2,3,4	3,4	18	7	7
1.0	115415	87831	25146	809	3249	2,3,4	3,4	18	7	7

Table S9: Costs and location results under different τ when testing and disposition decisions at the DC level ($a_j^{test,R} = 1$). The solution with testing at the MFs is represented by $a_j^{test,R} = 0$ and $\tau_i = 1$ in the last row (with Table 5)

8. References

Ahmed, S., Luedtke, J., Song, Y., & Xie, W. (2017). Nonanticipative duality, relaxations, and formulations for chance-constrained stochastic programs. *Mathematical Programming*, *162*, 51–81.

Ahmed, S., Shapiro, A., & Stewart, H. M. (2008). Solving chance-constrained stochastic programs via sampling and integer programming. In *State-of-the-Art Decision-Making Tools in the Information-Intensive Age* (pp. 261–269). INFORMS.

Bonami, P., & Lejeune, M. (2008). An exact solution approach for portfolio optimization problems under stochastic and integer constraints. *Operations Research*, *57*, 650–670.

Daskin, M. (1995). *Network and discrete location: models, algorithms, and applications*. Wiley-Interscience series in discrete mathematics and optimization. Wiley.

Luedtke, J. (2014). A branch-and-cut decomposition algorithm for solving chance-constrained mathematical programs with finite support. *Mathematical Programming*, *146*, 219–244.

Luedtke, J., & Ahmed, S. (2008). A sample approximation approach for optimization with probabilistic constraints. *Siam Journal on Optimization*, *19*, 674–699.

Pagnoncelli, B. K., Ahmed, S., & Shapiro, A. (2009). Sample average approximation method for chance

		Cost components					MFs			DCs	
Shipment	Ratio	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
indirect	0.1	82291	57869	21999	1078	3502	2,3,4	3,4	23	9	9
direct	0.1	87341	57748	26451	0	3143	2,3,4	3,4	23	0	0
indirect	0.2	93710	68170	23059	921	3402	2,3,4	3,4	22	8	8
direct	0.2	97734	68142	26451	0	3143	2,3,4	3,4	22	0	0
indirect	0.3	104782	78118	24326	921	3259	2,3,4	3,4	20	8	8
direct	0.3	107688	78096	26451	0	3143	2,3,4	3,4	20	0	0
indirect	0.4	115415	87831	25146	809	3249	2,3,4	3,4	18	7	7
direct	0.4	117330	87738	26451	0	3143	2,3,4	3,4	17	0	0
indirect	0.5	119328	91084	25575	539	3210	1,2,3,4	3,4	14	5	5
direct	0.5	120527	90935	26451	0	3143	1,2,3,4	3,4	13	0	0
indirect	0.6	122756	93631	26480	539	3185	1,2,3,4	3,4	12	5	5
direct	0.6	123040	93448	26451	0	3143	1,2,3,4	3,4	12	0	0
indirect	0.7	125255	95493	26977	421	3207	1,2,3,4	3,4	9	4	4
direct	0.7	124902	95309	26451	0	3143	1,2,3,4	3,4	9	0	0
indirect	0.8	127391	96889	27765	421	3159	1,2,3,4	3,4	7	4	4
direct	0.8	126128	96536	26451	0	3143	1,2,3,4	3,4	6	0	0
indirect	0.9	127850	96974	27949	225	3153	1,2,3,4	3,4	4	2	2
direct	0.9	126323	96731	26451	0	3143	1,2,3,4	3,4	4	0	0
indirect	1	127848	96973	27948	225	3153	1,2,3,4	3,4	4	2	2
direct	1	126323	96731	26451	0	3143	1,2,3,4	3,4	4	0	0

Table S10: Costs and location results under different ratios when indirect/direct shipments of returned products (with Table 6)

	Cost components					MFs			DCs	
γ (%)	Cost_T	Cost_F	Cost_R	Cost_S	Cost_L	F	R	#F	#R	#C
0.01	112425	87840	25068	809	327	2,3,4	3,4	18	7	7
0.10	115415	87831	25146	809	3249	2,3,4	3,4	18	7	7
0.30	121978	87822	25186	809	9779	2,3,4	3,4	18	7	7
0.50	128300	87824	25279	700	15898	2,3,4	3,4	18	6	6
0.70	134728	87811	25360	700	22257	2,3,4	3,4	18	6	6
0.90	142739	87832	32118	806	23596	1,2,3,4	2,3,4	18	6	6

Table S11: Costs and location results under different γ (with Table 7)

constrained programming: Theory and applications. *Journal of Optimization Theory & Applications*, *142*, 399–416.

Xie, W., & Ahmed, S. (2017). On quantile cuts and their closure for chance constrained optimization problems. *Mathematical Programming*, *1*, 1–26.