The Dynamics of a Holling-Tanner Predator-Prey Model with Allee Effect

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Original Holling-Tanner Model [1]

$$\frac{dx}{dt} = rx\left(1-\frac{x}{K}\right) - \frac{qxy}{x+a}$$

$$\frac{dy}{dt} = sy\left(1-\frac{y}{nx}\right)$$

$$\Omega = \{(x,y)\in\mathbb{R}^2, x>0, y\geq 0\}$$
$$(r,K,q,a,s,n)\in\mathbb{R}^6_+$$

$$rx\left(1-\frac{x}{K}\right) \Rightarrow rx\left(1-\frac{x}{K}\right)(x-m)$$

Nondimensionalisation & Desingularisation [2]

$$\varphi(u,v,\tau) = \left(Ku, nKv, \frac{u\left(u+\frac{a}{K}\right)}{Kr}\right) = (x,y,t)$$

The system is topologically equivalent to system

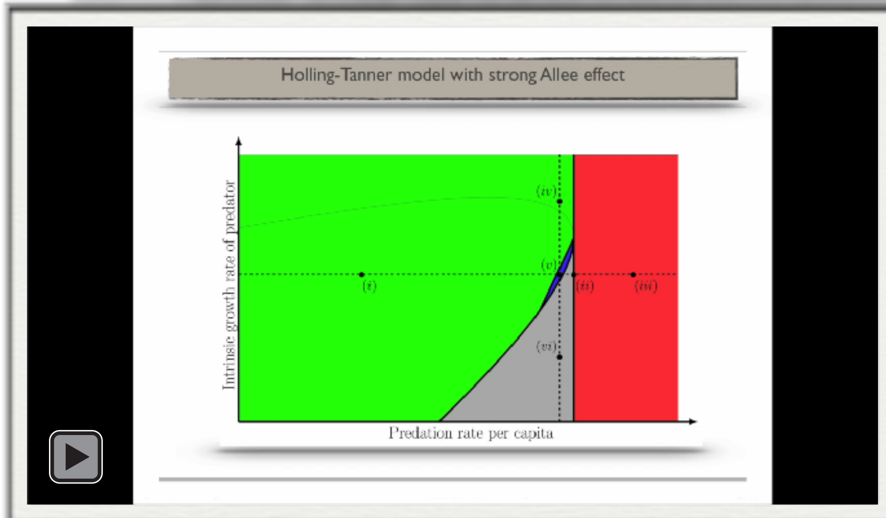
$$\frac{du}{d\tau} = u^2\left((u+A)(1-u)(u-M) - Qv\right),$$
$$\frac{dv}{d\tau} = Sv(u+A)(u-v).$$

$$A := \frac{a}{K} < 1, \quad S := \frac{s}{rK}, \quad Q := \frac{nq}{rK} \quad \text{and} \quad M := \frac{m}{K}$$

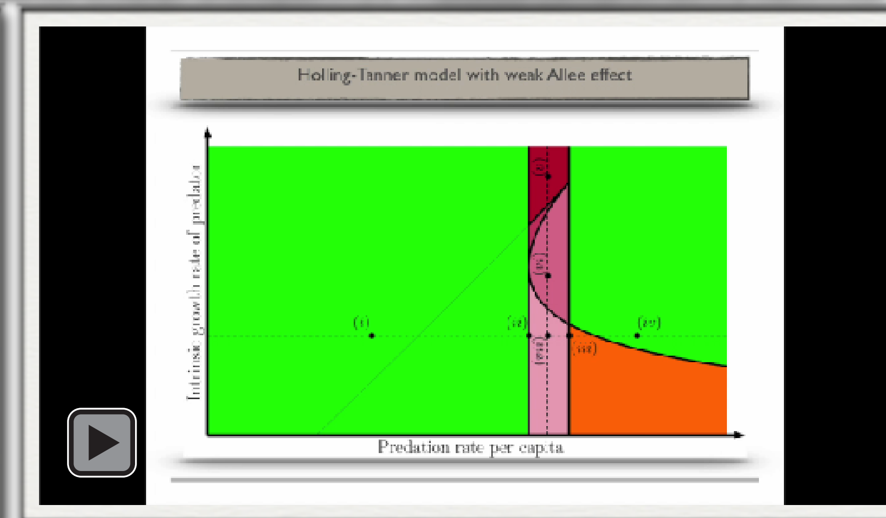
$$\Phi = \{(u,v),\ 0\leq u\leq 1,\ 0\leq v\leq 1\}$$

This set is an invariant region

Simulations of Strong and Weak Allee Effect Using System Parameters [3,4]



http://doi.org/10.6084/m9.figshare.7284530



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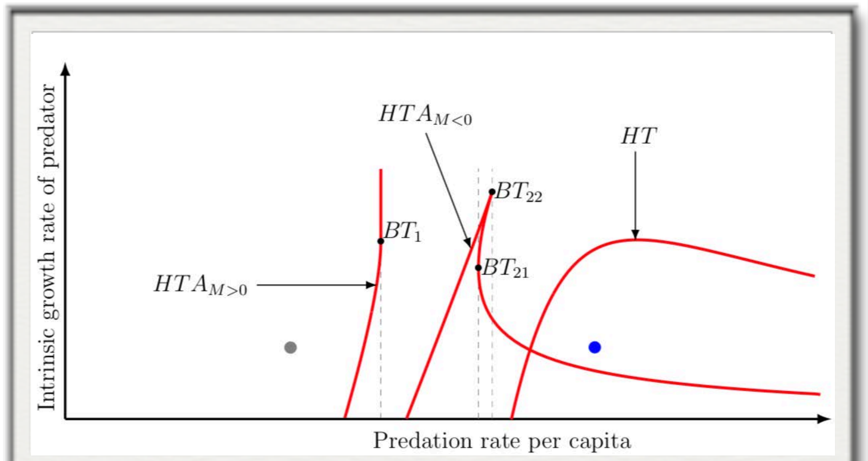
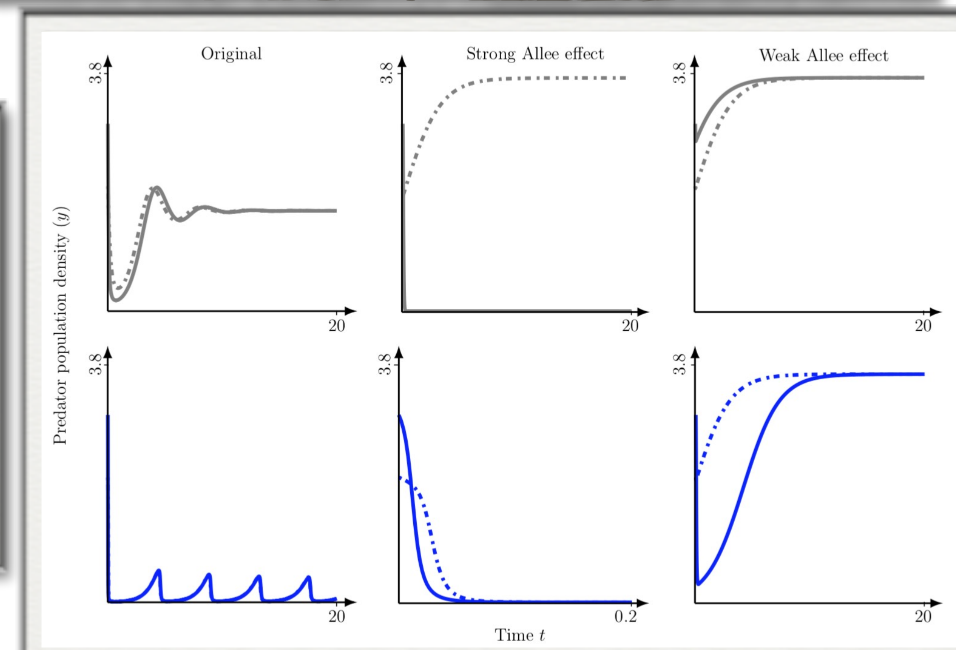
Comparing the Dynamics





Future Work

- Considering alternative food sources for the predator modifying the environmental carrying capacity.
- Pattern formation (Turing and travelling waves) in spatial spatio-temporal extended Holling-Tanner models.

References

[1] E. Saez and E. Gonzalez-Olivares, Dynamics on a predator–prey model, SIAM Journal on Applied Mathematics, 59 (1999), pp. 1867–1878.

[2] C. Arancibia-Ibarra, J. Flores, G. Pettet, and P. van Heijster, A Holling-Tanner predator-prey model with strong Allee effect, ArXiv e-prints, (2018), https://arxiv.org/abs/1809.05854.

[3] I. Hanski, P. Turchin, E. Korpimaki, and H. Henttonen, Population oscillations of boreal rodents: regulation by mustelid predators leads to chaos, Nature, 364 (1993), p. 232.

[4] P. Turchin and I. Hanski, An empirically based model for latitudinal gradient in vole population dynamics, The American Naturalist, 149 (1997), pp. 842–874.