

1. The problem

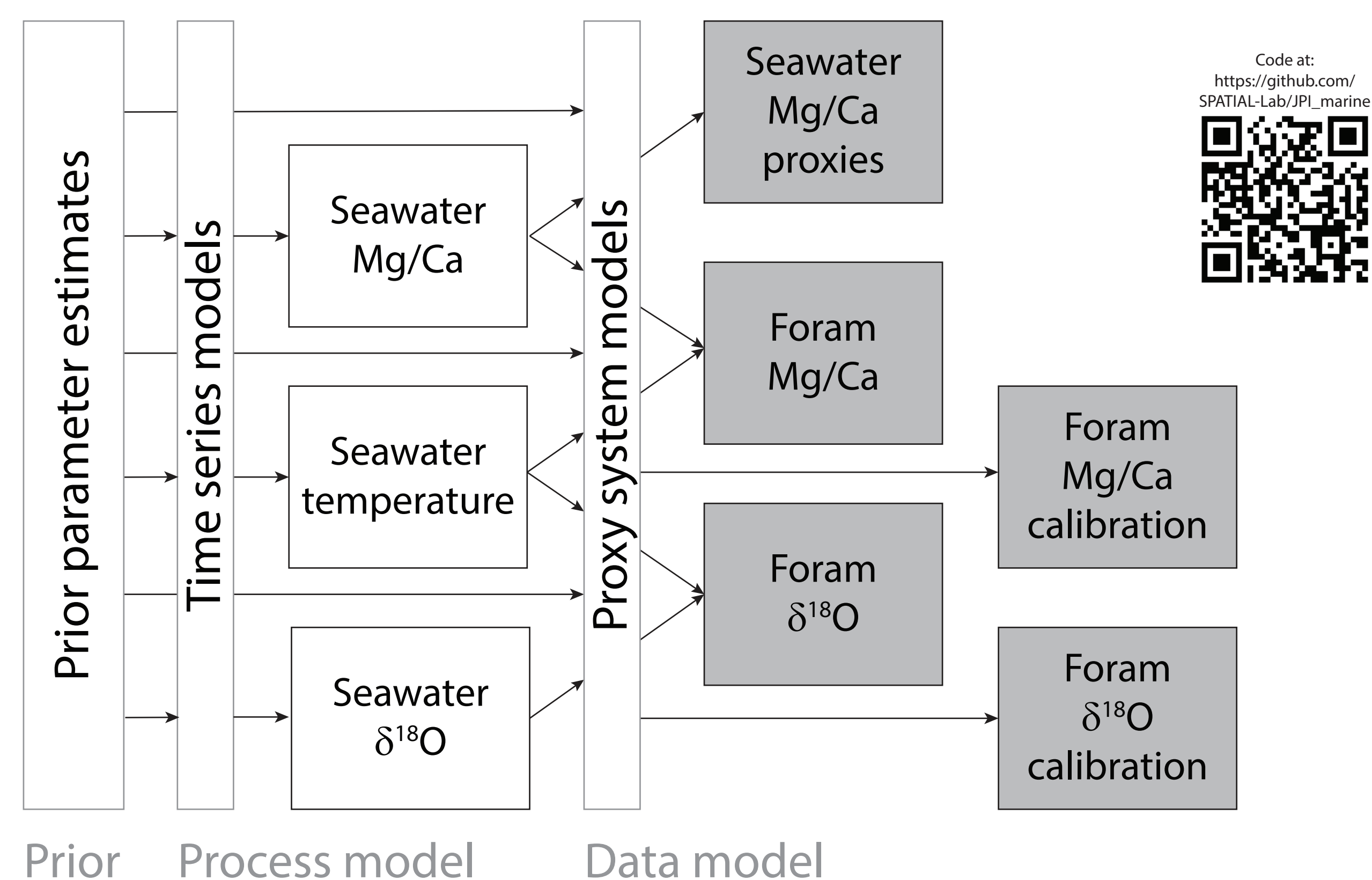
- *All proxies are imperfect
 - We want to reconstruct a particular variable
 - But we get a signal impacted by multiple factors
- *We'd like to
 - Minimize bias from these factors
 - Quantify the uncertainty imparted
 - Maximize the information gained

2. The idea

- *Combine multiple proxies
 - Benefit from complimentary information
- *Leverage proxy system models
 - Represent multiple influences on the proxy record
- *Link proxies to paleo-environmental time series
 - Gain information from temporal autocorrelation

3. The implementation

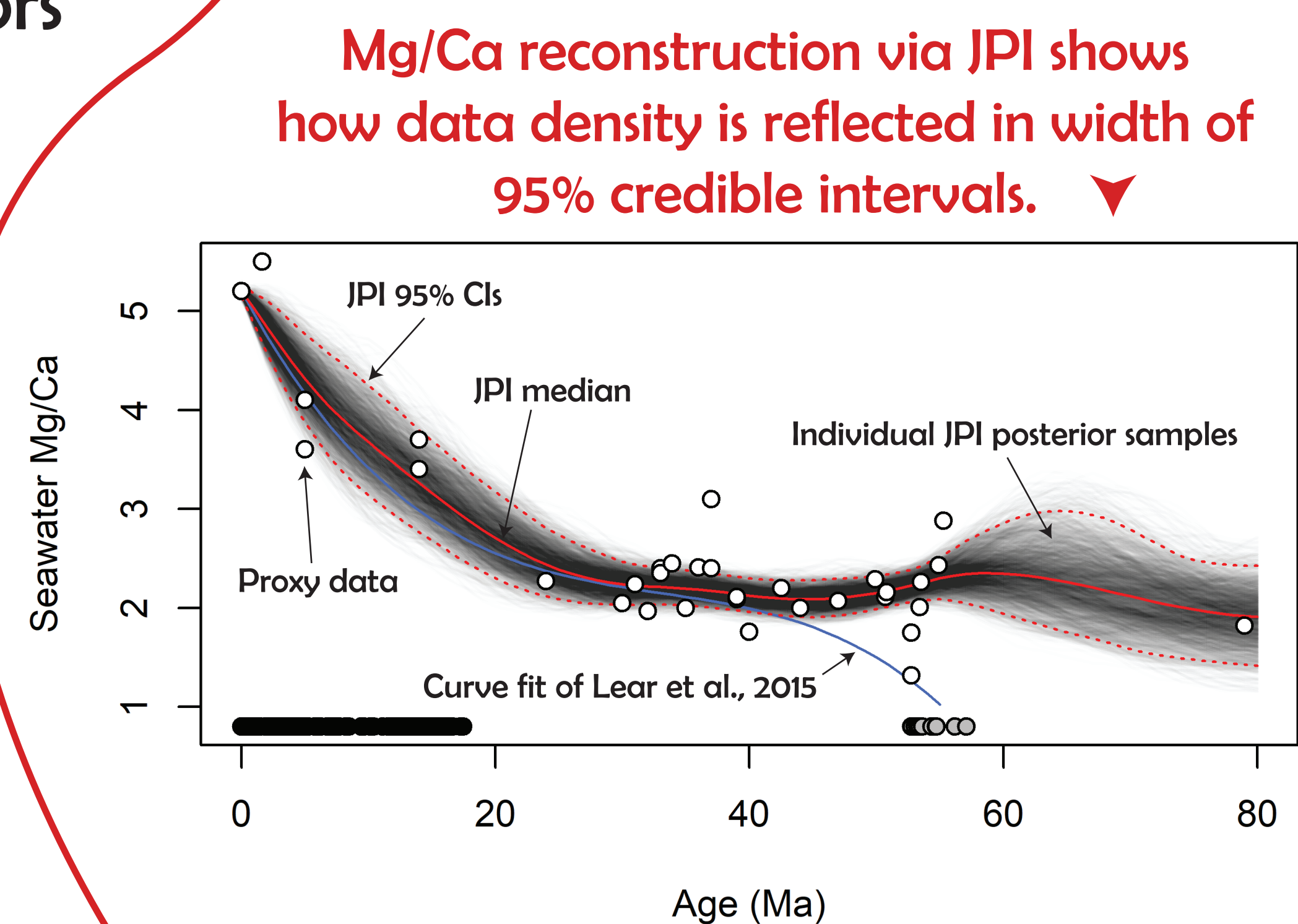
- *Hierarchical forward model for proxy records
 - Environmental time series (correlated random walk)
 - Proxy model (statistical or process-based)
 - Data model (for calibration +/- paleo proxy data)
- *Bayesian inversion using MCMC
- *Example system: benthic foraminiferal Mg/Ca + $\delta^{18}\text{O}$



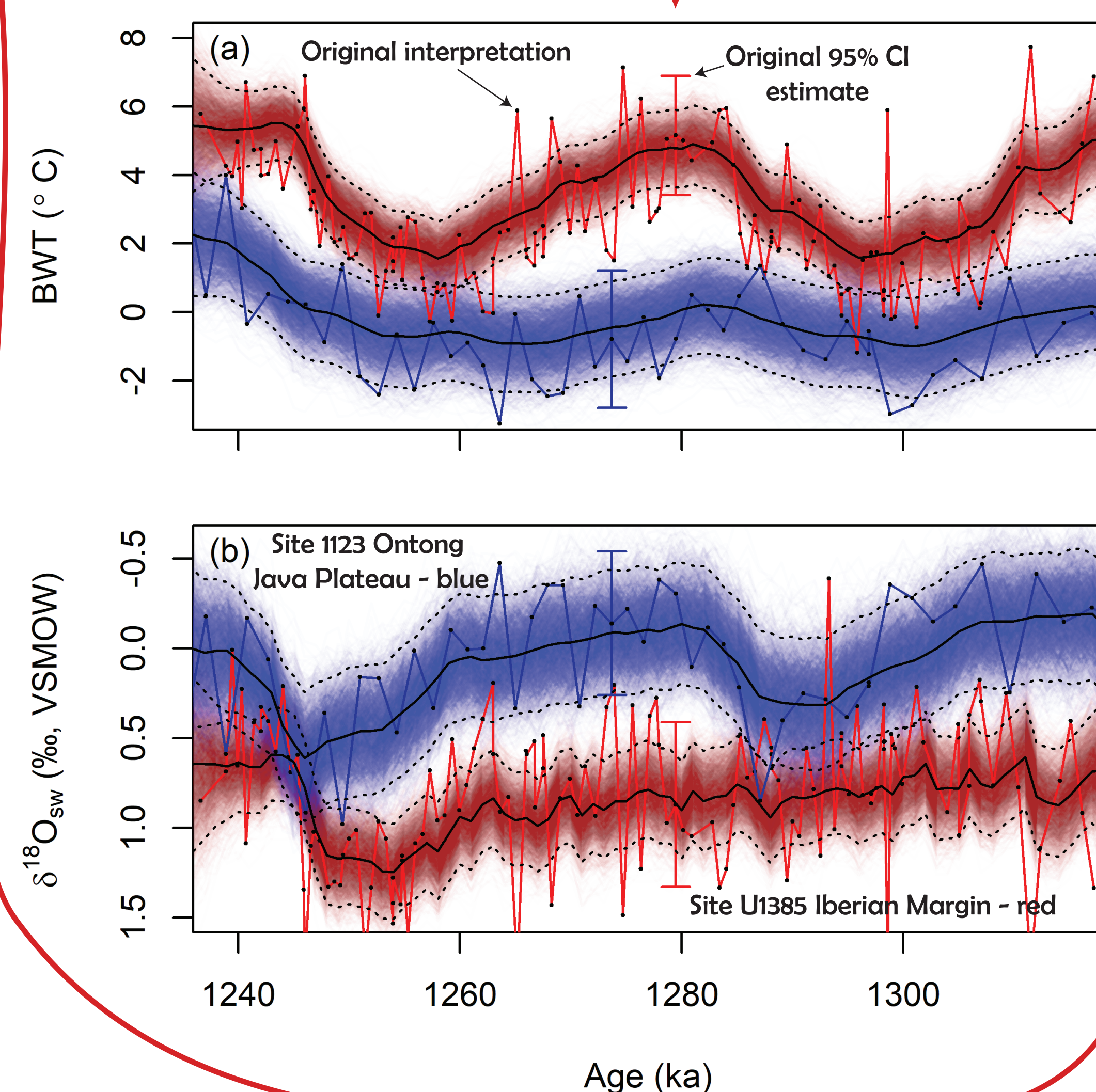
Schematic representation of the JPI Bayesian hierarchical model for the coupled foraminiferal Mg/Ca and $\delta^{18}\text{O}$ proxy systems. The process model describes paleoenvironmental conditions using a correlated random walk. The data model consists of transfer functions linking proxy and proxy calibration observations to the environmental state.



4. Paleoenvironmental reconstructions with comprehensive uncertainty assessment

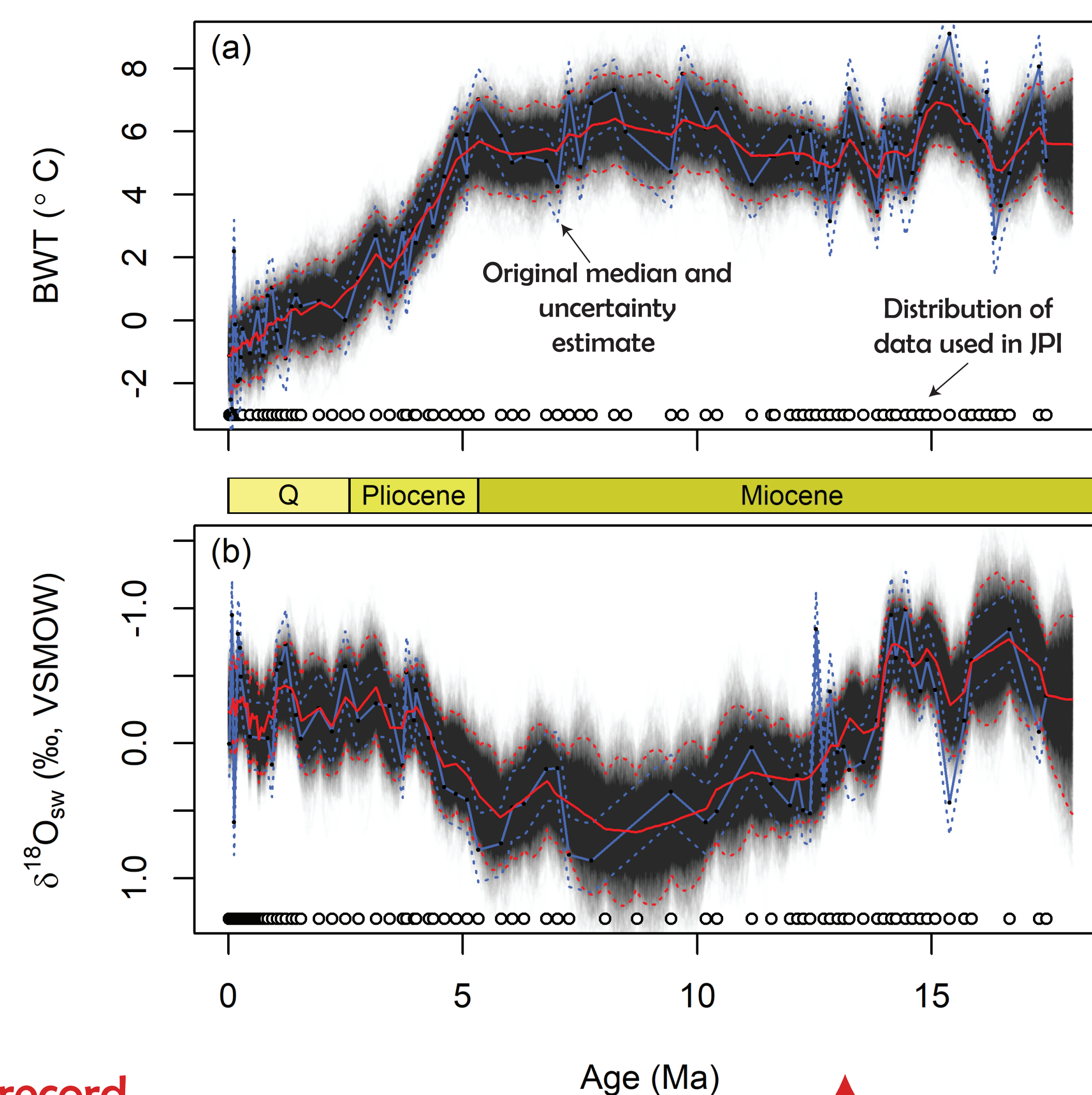


JPI applied simultaneously to 2 Pleistocene records shows narrower 95% CIs than original error propagation, especially for densely-sampled Site U1385 record.



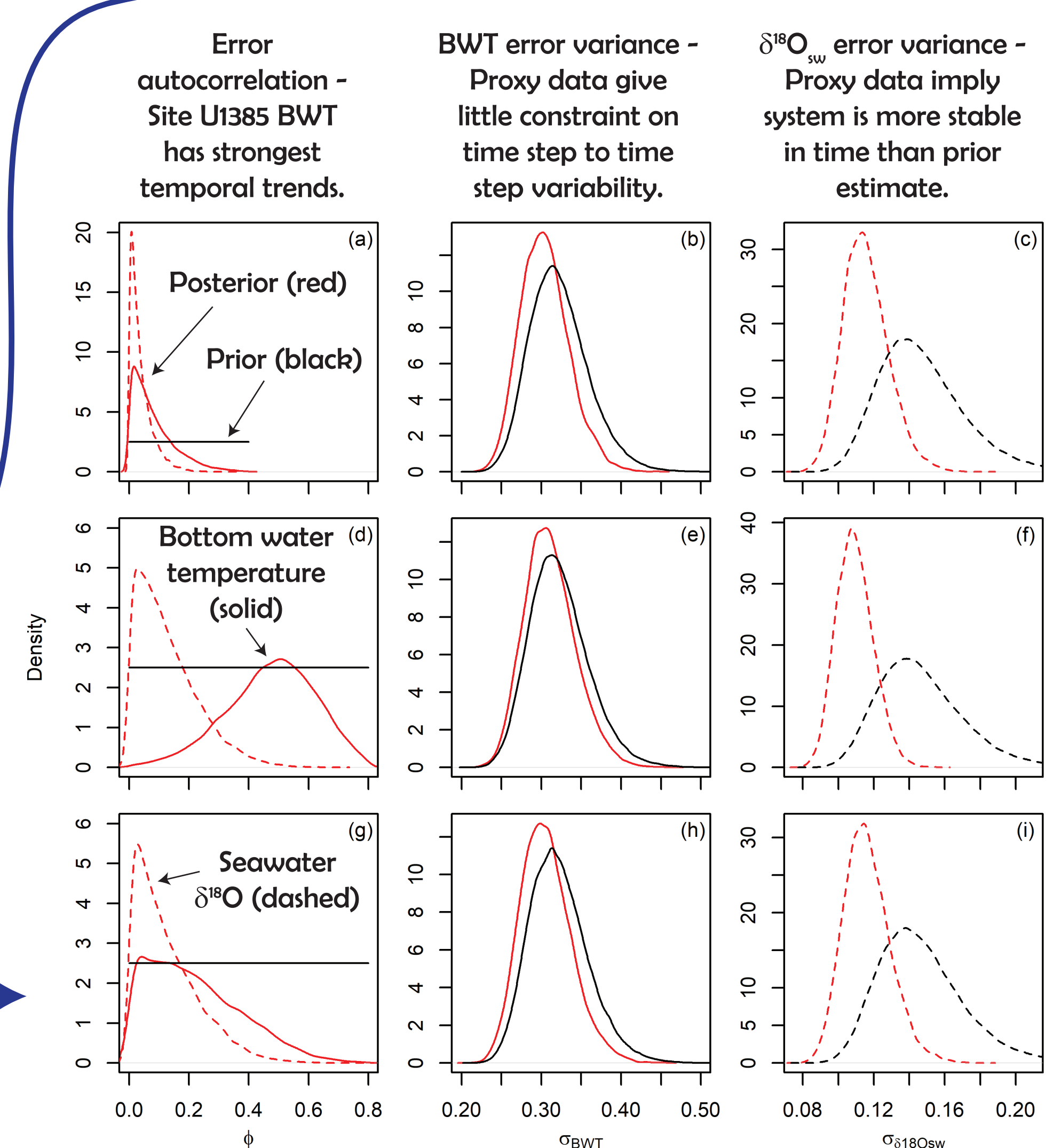
6. Paleoenvironmental inference

JPI posterior estimates of time series parameters embed information about paleoenvironmental systems properties.

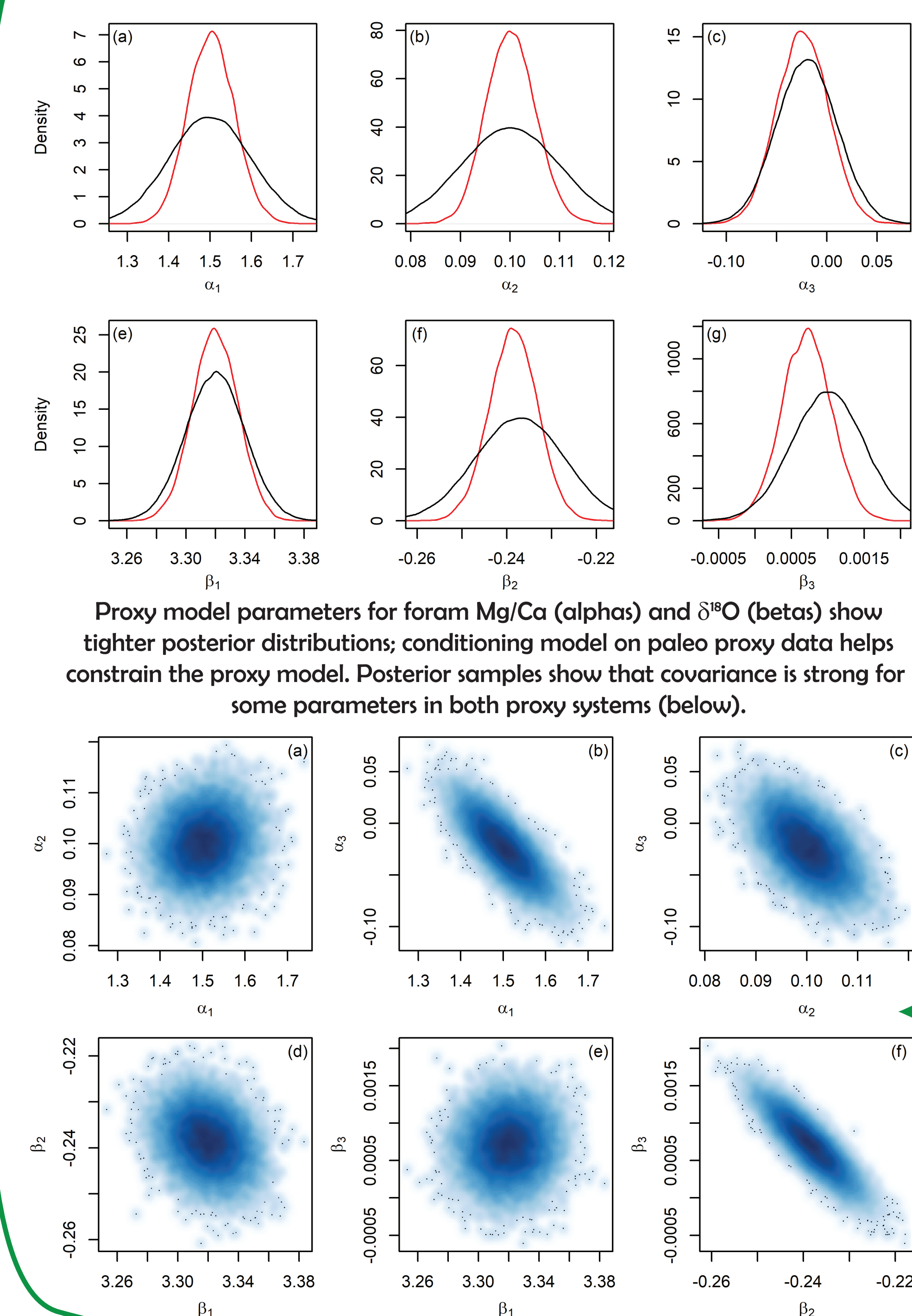


JPI for Site 806 data gives continuous reconstruction, despite uneven sampling of proxy records, and shows how uncertainty accumulates between sampling points.

Data sources:
Site 806: Lear, C. H., et al. (2015), *Paleoceanography* 30(11): 1437-1454. Lear, C. H., et al. (2003), *Earth and Planetary Science Letters* 210(3-4): 425-436. Site U1385: Birner, B., et al. (2016), *Paleoceanography* 31(10): 203-217. Site 1123: Elderfield, H., et al. (2012), *Science* 337(6095): 704. Seawater Mg/Ca: Coggan, R. M., et al. (2010), *Science* 327: 1114-1117. Dickson, J. A. D. (2002), *Science* 298(5596): 1222. Evans, D., et al. (2018), *Proceedings of the National Academy of Sciences* 115(6): 1174. Horita, J., et al. (2002), *Geochimica et Cosmochimica Acta* 66(21): 3733-3756. Louvenstein, T. K., et al. (2009), *Science* 294: 1086-1088. Calibration: Elderfield, H., et al. (2010), *Quaternary Science Reviews* 29(1): 160-169. Lear, C. H., et al. (2015), *Paleoceanography* 30(11): 1437-1454. Marchitto, T. M., et al. (2014), *Geochimica et Cosmochimica Acta* 130: 1-11. Software: Plummer, M. (2018), <https://CRAN.R-project.org/package=jags>. Su, Y.-S., and M. Vojima (2015), <https://CRAN.R-project.org/package=R2jags>.



5. Proxy calibration inference

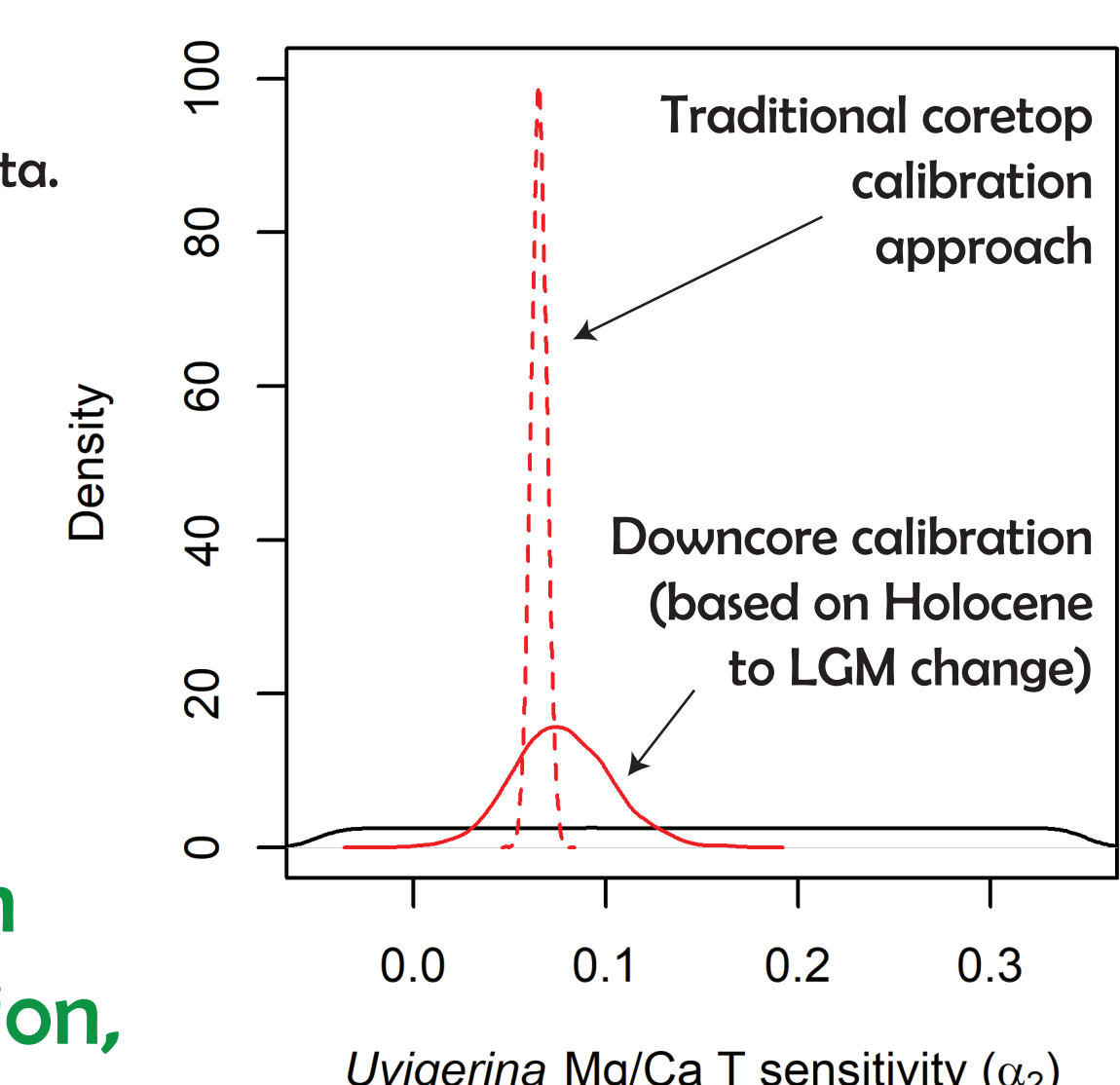


Proxy model parameters for foram Mg/Ca (alphas) and $\delta^{18}\text{O}$ (betas) show tighter posterior distributions; conditioning model on paleo proxy data helps constrain the proxy model. Posterior samples show that covariance is strong for some parameters in both proxy systems (below).

JPI posterior distributions demonstrate how both calibration and paleo-proxy data constrain proxy calibration. Here posteriors for Site 806 JPI show more precise estimates of most parameters than obtained by fitting proxy model to calibration data only (prior).

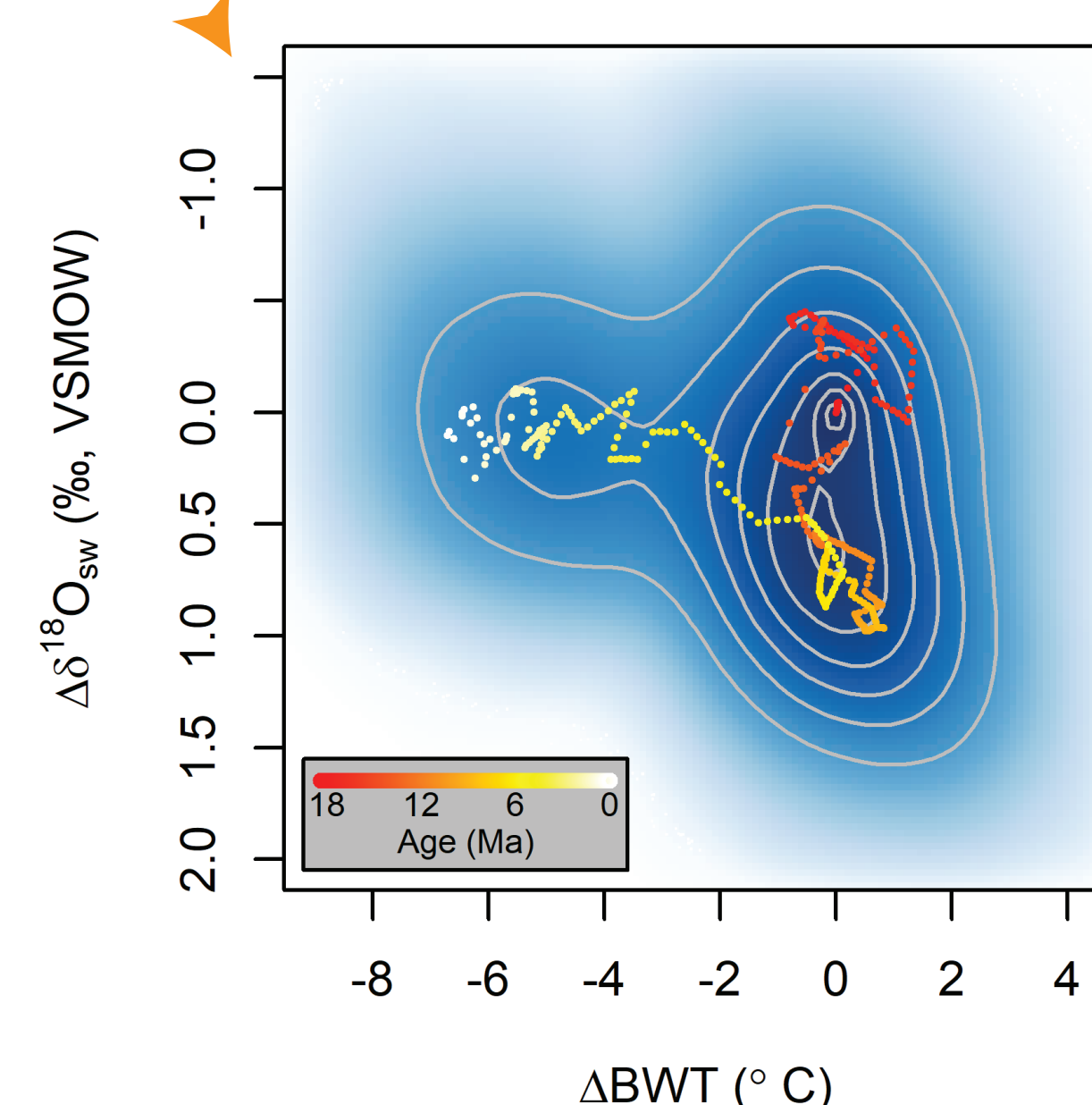
Proxy variance is especially tightly constrained by paleo data.

Parameter covariance is reflected in JPI posterior samples. This is often ignored in error propagation, which can inflate estimates of uncertainty.



▲ Sidenote - JPI using Elderfield 2010 downcore & coretop Mg/Ca calibration methods - former adds little info.

Posterior time series from Site 806 describe multivariate environmental "space" occupied and pathways of change. Three semi-stable states are suggested.



Individual samples from JPI posterior represent self-consistent realizations of the paleoenvironmental system. Statistical tests should be based on within-sample comparisons, e.g., is BWT different from modern at Site 806 (a) or is seawater $\delta^{18}\text{O}$ different at sites U1385 and 1123 (b).

7. Derivative analyses

