**Supplemental data**

Title: An inverse method for estimating air volume fraction of sea foam from emissivity data.

Authors: En-Bo Wei, Hong-Xiu Gao, Shu-Bo Liu, Guang-Yan Li, and Le Gao

**Part A:** The spectral representation of coated particle composites

Let the potential , where is the polarization potential induced by the composites. Substituting  into Equation (5), we have

. (A1)

Equation (A1) is a Poisson’s equation. The boundary conditions of Equation (A1) are given as follows:  and  for  and , where  is very large. The solution of Equation (A1) can be expressed as Green function .

, (A2)

where the composite region  is enclosed by the curved surface , and  is the value of function  on the curved surface boundary . The  is the normal derivative of Green function on the curved surface . The operator  is related to variable . Because  is zero on the curved surface (e.g. ) and the Green function satisfies with on the boundary  of the composite region, we have

 (A3)

Then, the potential is given as,

 (A4)

To obtain the solution of Equation (A4), we define an integral-differential operator , which satisfies

, (A5)

with an inner product defined as

, (A6)

where “” denotes the conjugate function. With Equations (A5) and (A6), we express the potential  in Equation (A4) as:

, (A7)

where, . Note that we can demonstrate that when the function  is real function, the operatoris a Hermitian operator, which satisfies .

For the linear Hermitian operator with the real function , we assume that there is a complete series of orthogonal eigenfunctions  corresponding to the real eigenvalue  so that  (Dong et al. 2005；Zhang and Cherkaev, 2009). Then we can expand the potential solution  of Equation (A7) in terms of eigenfunctions.

, (A8)

where  are unknown constants. Substituting Equation (A8) into Equation (A7), we have

, (A9)

where . The unknown coefficients are determined, . Then the solution of Equation (A8) is obtained.

Using Equations (A8) and (A9) in Equation (4) with its boundary condition, the EDC is formulated as,

 (A10)

where. We obtain from Equation (A10) the following expression for EDC:

  . (A11)

where we introduce the spectral representation for the summation term. Function  is analytical everywhere except for a finite number simple pole on the semi-close segment (Bergman 1978). The summation of residues has the following expression,

, (A12)

which is related to the characteristic function .

**Part B**: The spectral representation of coated spherical foam composites

The EDC of the seawater-coated spherical composite can be derived from Equation (7) of Wei (2013) given as,

, (B1)

. ,and are the dielectric constants of the spherical core, host medium and spherical shell, respectively. and are the inner and outer radii of coated spheres, respectively. and are the volume fractions of the core and spherical shell materials, respectively.

We let  and , where  and  are the dielectric constants of air and seawater and (i.e. the volume fraction of seawater). Equation (B1) is the EDC of composite media of the seawater-coated spheres embedded in the air, where , . As an approximation, we let , where is the volume fraction of air. Using, we have and .

Let , Equation (B1) is rewritten as,

 (B2)

Using  and substituting into Equation (B2) , we get

 (B3)

where , ,,. It is noted that the spectral coefficients  and  (）satisfy Equation (19).