Supplemental Materials

Contents

1 Conditional Posterior Distributions

With a multivariate normal prior for $\boldsymbol{\lambda}_0$, $p\left(\boldsymbol{\lambda}_0\right) = MN\left(\boldsymbol{\lambda}_0^{pr}, \boldsymbol{V}_\lambda^{pr}\right)$, the conditional posterior for $\boldsymbol{\lambda}_0$ is

$$\begin{aligned}
& p\left(\boldsymbol{\lambda}_0 | \boldsymbol{V}_\lambda^{(m)}, \boldsymbol{\lambda}_i^{(m)}, i = 1, 2, \ldots N_s\right) \\
\propto & \prod_{i=1}^{N_s} p\left(\boldsymbol{\lambda}_i \middle| \boldsymbol{\lambda}_0, \boldsymbol{V}_\lambda^{(m)}\right) p\left(\boldsymbol{\lambda}_0\right) \\
= & MN\left(\boldsymbol{\lambda}_0^{pt}, \boldsymbol{V}_\lambda^{pt}\right)
\end{aligned}$$

where $\boldsymbol{\lambda}_0^{pt} = \boldsymbol{V}_\lambda^{pt}\left[\left(\boldsymbol{V}_\lambda^{(m)}\right)^{-1} \sum_{i=1}^{N_s} \boldsymbol{\lambda}_i^{(m)} + \left(\boldsymbol{V}_\lambda^{pr}\right)^{-1} \boldsymbol{\lambda}_0^{pr}\right]$ and $\boldsymbol{V}_\lambda^{pt} = \left[N_s \left(\boldsymbol{V}_\lambda^{(m)}\right)^{-1} + \left(\boldsymbol{V}_\lambda^{pr}\right)^{-1}\right]^{-1}$.

With a Wishart prior for $\boldsymbol{V}_\lambda$, $p\left(\boldsymbol{V}_\lambda\right) = W\left(df_\lambda^{pr}, \boldsymbol{V}_\lambda^{pr}\right)$, the conditional posterior for $\boldsymbol{V}_\lambda$ is

$$\begin{aligned}
& p\left(\boldsymbol{V}_\lambda | \boldsymbol{\lambda}_0^{(m+1)}, \boldsymbol{\lambda}_i^{(m)}, i = 1, 2, \ldots N_s\right) \\
\propto & \prod_{i=1}^{N_s} p\left(\boldsymbol{\lambda}_i \middle| \boldsymbol{\lambda}_0^{(m+1)}, \boldsymbol{V}_\lambda\right) p\left(\boldsymbol{V}_\lambda\right) \\
\propto & \left|\boldsymbol{V}_\lambda\right|^{\frac{df_\lambda^{pr} - d_\lambda - 2}{2}} \exp\left\{-\frac{1}{2} tr\left(\left(\boldsymbol{V}_\lambda^{pr}\right)^{-1} \boldsymbol{V}_\lambda + \sum_{i=1}^{N_s}\left(\boldsymbol{\lambda}_i^{(m)} - \boldsymbol{\lambda}_0^{(m+1)}\right)\left(\boldsymbol{\lambda}_i^{(m)} - \boldsymbol{\lambda}_0^{(m+1)}\right)' \boldsymbol{V}_\lambda^{-1}\right)\right\}
\end{aligned}$$

where d_λ is the number of rows of $\boldsymbol{V}_\lambda$. The Metropolis-Hastings sampling can be used to sample this kernel of the posterior. The conditional posterior distributions for $\boldsymbol{\phi}_0$ and $\boldsymbol{V}_\phi$ are similar to those of $\boldsymbol{\lambda}_0$ and $\boldsymbol{V}_\lambda$ respectively.

The conditional posterior distribution of $\boldsymbol{\lambda}_i$ is

$$\begin{aligned}
& p\left(\boldsymbol{\lambda}_i | \boldsymbol{\lambda}_0^{(m+1)}, \boldsymbol{\phi}_i^{(m)}, \boldsymbol{V}_\lambda^{(m+1)}, \boldsymbol{V}_{ri}\left(\bar{\boldsymbol{r}}\right), \boldsymbol{r}_i\right) \\
\propto & p\left(\boldsymbol{r}_i \middle| \boldsymbol{\lambda}_i, \boldsymbol{\phi}_i^{(m)}, \boldsymbol{V}_{ri}\left(\bar{\boldsymbol{r}}\right)\right) p\left(\boldsymbol{\lambda}_i \middle| \boldsymbol{\lambda}_0^{(m+1)}, \boldsymbol{V}_\lambda^{(m+1)}\right) \\
= & MN\left(\boldsymbol{\rho}_i\left(\boldsymbol{\lambda}_i, \boldsymbol{\phi}_i^{(m)}\right), \boldsymbol{V}_{ri}\left(\bar{\boldsymbol{r}}\right)\right) \times MN\left(\boldsymbol{\lambda}_0^{(m+1)}, \boldsymbol{V}_\lambda^{(m+1)}\right)
\end{aligned}$$

where $\boldsymbol{\rho}_i\left(\boldsymbol{\lambda}_i, \boldsymbol{\phi}_i^{(m)}\right) = vech\left(\boldsymbol{\Lambda}_i \boldsymbol{\Phi}_i^{(m)} \boldsymbol{\Lambda}_i'\right)$.

The conditional posterior distribution of $\boldsymbol{\phi}_i$ is

$$\begin{aligned}
& p\left(\boldsymbol{\phi}_i | \boldsymbol{\phi}_0^{(m+1)}, \boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{V}_\lambda^{(m+1)}, \boldsymbol{V}_{ri}\left(\bar{\boldsymbol{r}}\right), \boldsymbol{r}_i\right) \\
\propto & p\left(\boldsymbol{r}_i | \boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{\phi}_i, \boldsymbol{V}_{ri}\left(\bar{\boldsymbol{r}}\right)\right) p\left(\boldsymbol{\phi}_i | \boldsymbol{\phi}_0^{(m+1)}, \boldsymbol{V}_\phi^{(m+1)}\right) \\
= & MN\left(\boldsymbol{\rho}_i\left(\boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{\phi}_i\right), \boldsymbol{V}_{ri}\left(\bar{\boldsymbol{r}}\right)\right) \times MN\left(\boldsymbol{\phi}_0^{(m+1)}, \boldsymbol{V}_\phi^{(m+1)}\right)
\end{aligned}$$

Strictly speaking, the conditional posterior distribution of ϕ_{ij} can be specified explicitly if a specific CFA model is given. The resulting posterior distribution would be a normal distribution. However, we cannot find a generic formula for all CFA models and thus we cannot express the equation in the matrix form.

By assuming MCAR missingness mechanism, the conditional posterior of missing correlations of study i is

$$\begin{aligned}
& p\left(\boldsymbol{r}_{miss,i} | \boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{\phi}_i^{(m+1)}, \boldsymbol{V}_{ri}\left(\bar{\boldsymbol{r}}\right), \boldsymbol{r}_{obs,i}\right) \\
= & MN\left(\boldsymbol{\mu}_{r,miss}^{pt}, \boldsymbol{V}_{r,miss}^{pt}\right)
\end{aligned}$$

where

$$\boldsymbol{\mu}_{r,miss}^{pt} = \boldsymbol{\rho}_{miss,i}\left(\boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{\phi}_i^{(m+1)}\right) + \boldsymbol{V}_{ri,miss\times obs}\left(\bar{\boldsymbol{r}}\right)\left[\boldsymbol{V}_{ri,obs}\left(\bar{\boldsymbol{r}}\right)\right]^{-1}\left[\boldsymbol{r}_{obs,i} - \boldsymbol{\rho}_{obs,i}\left(\boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{\phi}_i^{(m+1)}\right)\right]$$

and

$$\boldsymbol{V}_{r,miss}^{pt} = \boldsymbol{V}_{ri,miss}\left(\bar{\boldsymbol{r}}\right) - \boldsymbol{V}_{ri,miss\times obs}\left(\bar{\boldsymbol{r}}\right)\left[\boldsymbol{V}_{ri,obs}\left(\bar{\boldsymbol{r}}\right)\right]^{-1} \boldsymbol{V}_{ri,obs\times miss}\left(\bar{\boldsymbol{r}}\right).$$

Here, $\boldsymbol{\rho}_i\left(\boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{\phi}_i^{(m+1)}\right) = \left[\begin{array}{c} \boldsymbol{\rho}_{miss,i}\left(\boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{\phi}_i^{(m+1)}\right) \\ \boldsymbol{\rho}_{obs,i}\left(\boldsymbol{\lambda}_i^{(m+1)}, \boldsymbol{\phi}_i^{(m+1)}\right) \end{array}\right]$ and $\boldsymbol{V}_{ri}\left(\bar{\boldsymbol{r}}\right) = \left[\begin{array}{cc} \boldsymbol{V}_{ri,miss}\left(\bar{\boldsymbol{r}}\right) & \boldsymbol{V}_{ri,miss\times obs}\left(\bar{\boldsymbol{r}}\right) \\ \boldsymbol{V}_{ri,obs\times miss}\left(\bar{\boldsymbol{r}}\right) & \boldsymbol{V}_{ri,obs}\left(\bar{\boldsymbol{r}}\right) \end{array}\right]$.

2 Supplemental Simulation Results

2.1 The Homogeneous Case

Table 1: Results for Condition With $NS = 20$ and $\bar{N} = 100$ When Both Models M1 and M2 are Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	EBIAS	ESD	ASD	95% CR	Est.	EBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.297	-0.936	0.0370	0.0380	0.970	0.294	-1.953	0.0369	0.0471	0.990
$\lambda_{0,11}$	.700	0.695	-0.765	0.0331	0.0359	0.980	0.691	-1.252	0.0331	0.0424	0.990
$\lambda_{0,21}$	.600	0.601	0.080	0.0293	0.0332	0.960	0.599	-0.232	0.0294	0.0402	1.000
$\lambda_{0,31}$	.500	0.498	-0.420	0.0292	0.0313	0.980	0.497	-0.695	0.0293	0.0387	0.990
$\lambda_{0,42}$	.700	0.697	-0.500	0.0338	0.0357	0.950	0.693	-0.947	0.0338	0.0421	0.969
$\lambda_{0,52}$	.600	0.600	-0.046	0.0305	0.0330	0.960	0.597	-0.449	0.0303	0.0400	0.990
$\lambda_{0,62}$	.500	0.500	0.026	0.0309	0.0310	0.960	0.499	-0.248	0.0312	0.0384	0.990
V_ϕ	.000	-	-	-	-	-	0.010	1.020	0.0051	0.0137	-
$V_{\lambda,11}$	.000	-	-	-	-	-	0.005	0.534	0.0034	0.0086	-
$V_{\lambda,21}$	.000	-	-	-	-	-	0.005	0.531	0.0027	0.0084	-
$V_{\lambda,31}$	.000	-	-	-	-	-	0.006	0.562	0.0029	0.0085	-
$V_{\lambda,42}$	.000	-	-	-	-	-	0.005	0.503	0.0020	0.0084	-
$V_{\lambda,52}$	.000	-	-	-	-	-	0.005	0.516	0.0027	0.0084	-
$V_{\lambda,62}$	.000	-	-	-	-	-	0.006	0.556	0.0038	0.0084	-
Convergence rate				1.000					0.970		
% PP p favoring M1						0.825					
PP p 5th percentile				0.132					0.132		
PP p rejection rate				0.010					0.021		
% DIC favoring M1						0.876					

Note: Pop. = population values; Est. = average of estimates across replications; EBIAS = estimation bias (in percentage); ESD = empirical standard deviation; ASD = average posterior standard deviation; 95% CR = 95% coverage rates; PP p = posterior predictive probability; DIC = deviance information criterion. Underlined italic numbers indicate substantial bias and bold numbers indicate coverage rates lower than 0.90.

Table 2: Results for Condition With $NS = 20$ and $\bar{N} = 200$ When Both Models M1 and M2 are Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	EBIAS	ESD	ASD	95% CR	Est.	EBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.297	-1.012	0.0260	0.0270	0.950	0.296	-1.219	0.0250	0.0340	0.990
$\lambda_{0,11}$	.700	0.699	-0.101	0.0210	0.0250	0.980	0.698	-0.308	0.0210	0.0290	0.990
$\lambda_{0,21}$	.600	0.599	-0.142	0.0220	0.0230	0.990	0.599	-0.245	0.0210	0.0280	0.990
$\lambda_{0,31}$	.500	0.499	-0.126	0.0180	0.0220	0.970	0.499	-0.191	0.0180	0.0270	0.990
$\lambda_{0,42}$	.700	0.700	0.003	0.0230	0.0250	0.970	0.698	-0.322	0.0230	0.0300	0.990
$\lambda_{0,52}$	.600	0.598	-0.316	0.0200	0.0230	0.970	0.598	-0.366	0.0200	0.0280	0.990
$\lambda_{0,62}$	.500	0.496	-0.802	0.0190	0.0220	0.990	0.495	-0.969	0.0190	0.0270	1.000
V_{ϕ}	.000	-	-	-	-	-	0.006	0.558	0.0030	0.0070	-
$V_{\lambda,11}$	.000	-	-	-	-	-	0.002	0.248	0.0010	0.0040	-
$V_{\lambda,21}$	.000	-	-	-	-	-	0.003	0.261	0.0010	0.0040	-
$V_{\lambda,31}$	.000	-	-	-	-	-	0.003	0.264	0.0010	0.0040	-
$V_{\lambda,42}$	.000	-	-	-	-	-	0.003	0.274	0.0010	0.0040	-
$V_{\lambda,52}$	.000	-	-	-	-	-	0.003	0.254	0.0010	0.0040	-
$V_{\lambda,62}$	.000	-	-	-	-	-	0.003	0.289	0.0020	0.0040	-
Convergence rate				1.000					0.980		
% PP p favoring M1						0.980					
PP p 5th percentile				0.167					0.156		
PP p rejection rate				0.000					0.000		
% DIC favoring M1						0.786					

Note: Pop. = population values; Est. = average of estimates across replications; EBIAS = estimation bias (in percentage); ESD = empirical standard deviation; ASD = average posterior standard deviation; 95% CR = 95% coverage rates; PP p = posterior predictive probability; DIC = deviance information criterion. Underlined italic numbers indicate substantial bias and bold numbers indicate coverage rates lower than 0.90.

Table 3: Results for Condition With $NS = 50$ and $\bar{N} = 100$ When Both Models M1 and M2 are Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	EBIAS	ESD	ASD	95% CR	Est.	EBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.298	-0.687	0.0210	0.0240	0.960	0.297	-0.967	0.0210	0.0260	1.000
$\lambda_{0,11}$	.700	0.698	-0.351	0.0200	0.0220	0.960	0.696	-0.599	0.0200	0.0240	0.978
$\lambda_{0,21}$	.600	0.598	-0.343	0.0180	0.0200	0.980	0.598	-0.287	0.0180	0.0220	0.989
$\lambda_{0,31}$	.500	0.499	-0.162	0.0150	0.0190	0.990	0.499	-0.131	0.0150	0.0210	1.000
$\lambda_{0,42}$	.700	0.695	-0.677	0.0190	0.0220	0.980	0.694	-0.924	0.0190	0.0240	0.978
$\lambda_{0,52}$	.600	0.600	-0.070	0.0190	0.0200	0.940	0.599	-0.109	0.0200	0.0220	0.957
$\lambda_{0,62}$	.500	0.498	-0.430	0.0190	0.0190	0.970	0.498	-0.331	0.0190	0.0210	0.978
V_{ϕ}	.000	-	-	-	-	-	0.005	0.473	0.0020	0.0050	-
$V_{\lambda,11}$	.000	-	-	-	-	-	0.002	0.244	0.0010	0.0030	-
$V_{\lambda,21}$	.000	-	-	-	-	-	0.002	0.215	0.0010	0.0030	-
$V_{\lambda,31}$	.000	-	-	-	-	-	0.003	0.263	0.0010	0.0030	-
$V_{\lambda,42}$	.000	-	-	-	-	-	0.002	0.246	0.0010	0.0030	-
$V_{\lambda,52}$	.000	-	-	-	-	-	0.002	0.228	0.0010	0.0030	-
$V_{\lambda,62}$	.000	-	-	-	-	-	0.002	0.247	0.0020	0.0030	-
Convergence rate				1.000					0.930		
% PP p favoring M1						1.000					
PP p 5th percentile				0.159					0.114		
PP p rejection rate				0.000					0.000		
% DIC favoring M1						0.086					

Note: Pop. = population values; Est. = average of estimates across replications; EBIAS = estimation bias (in percentage); ESD = empirical standard deviation; ASD = average posterior standard deviation; 95% CR = 95% coverage rates; PP p = posterior predictive probability; DIC = deviance information criterion. Underlined italic numbers indicate substantial bias and bold numbers indicate coverage rates lower than 0.90.

2.2 The Heterogeneous Case

Table 4: Results for Condition With $NS = 20$ and $\bar{N} = 100$ When Model M2 is Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	EBIAS	ESD	ASD	95% CR	Est.	EBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.306	1.951	0.0640	0.0410	**0.770**	0.305	1.781	0.0650	0.0710	0.970
$\lambda_{0,11}$	.700	0.695	-0.700	0.0510	0.0440	0.920	0.690	-1.380	0.0480	0.0500	0.960
$\lambda_{0,21}$	.600	0.592	-1.318	0.0520	0.0390	**0.860**	0.592	-1.346	0.0510	0.0560	0.980
$\lambda_{0,31}$	.500	0.496	-0.892	0.0590	0.0360	**0.770**	0.494	-1.123	0.0570	0.0660	0.990
$\lambda_{0,42}$	.700	0.703	0.424	0.0430	0.0440	0.940	0.698	-0.271	0.0400	0.0510	0.990
$\lambda_{0,52}$	.600	0.588	-2.003	0.0520	0.0390	**0.860**	0.590	-1.679	0.0480	0.0560	0.960
$\lambda_{0,62}$	.500	0.490	-1.931	0.0540	0.0360	**0.800**	0.491	-1.722	0.0560	0.0660	0.980
V_{ϕ}	.050	-	-	-	-	-	0.058	*16.664*	0.0240	0.0400	0.990
$V_{\lambda,11}$	.010	-	-	-	-	-	0.014	*42.298*	0.0080	0.0160	0.990
$V_{\lambda,21}$	.020	-	-	-	-	-	0.026	*29.146*	0.0130	0.0230	0.990
$V_{\lambda,31}$	.040	-	-	-	-	-	0.048	*19.392*	0.0240	0.0340	0.960
$V_{\lambda,42}$	.010	-	-	-	-	-	0.015	*50.916*	0.0090	0.0170	0.980
$V_{\lambda,52}$	.020	-	-	-	-	-	0.026	*30.104*	0.0130	0.0230	0.990
$V_{\lambda,62}$	.040	-	-	-	-	-	0.047	*16.864*	0.0220	0.0340	0.949
Convergence rate				1.000					.990		
% PP p favoring M2						.939					
PP p 5th percentile				0.076					0.189		
PP p rejection rate				0.212					0.000		
% DIC favoring M2						1.000					

Note: Pop. = population values; Est. = average of estimates across replications; EBIAS = estimation bias (in percentage); ESD = empirical standard deviation; ASD = average posterior standard deviation; 95% CR = 95% coverage rates; PP p = posterior predictive probability; DIC = deviance information criterion. Underlined italic numbers indicate substantial bias and bold numbers indicate coverage rates lower than 0.90.

Table 5: Results for Condition With $NS = 20$ and $\bar{N} = 200$ When Model M2 is Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	EBIAS	ESD	ASD	95% CR	Est.	EBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.294	-2.074	0.0620	0.0340	**0.740**	0.294	-1.996	0.0600	0.0660	0.970
$\lambda_{0,11}$	.700	0.696	-0.503	0.0400	0.0410	0.940	0.693	-0.957	0.0370	0.0410	0.929
$\lambda_{0,21}$	.600	0.595	-0.856	0.0500	0.0370	**0.840**	0.596	-0.655	0.0470	0.0480	0.939
$\lambda_{0,31}$	.500	0.488	-2.447	0.0560	0.0330	**0.720**	0.490	-1.913	0.0550	0.0610	0.960
$\lambda_{0,42}$	.700	0.699	-0.184	0.0420	0.0410	0.940	0.697	-0.440	0.0370	0.0420	0.960
$\lambda_{0,52}$	.600	0.596	-0.734	0.0450	0.0360	**0.830**	0.597	-0.450	0.0440	0.0500	0.960
$\lambda_{0,62}$	.500	0.499	-0.114	0.0630	0.0330	**0.690**	0.500	0.069	0.0590	0.0620	0.960
V_ϕ	.050	-	-	-	-	-	0.061	<u>*21.146*</u>	0.0240	0.0350	0.960
$V_{\lambda,11}$	.010	-	-	-	-	-	0.013	<u>*29.877*</u>	0.0080	0.0120	0.939
$V_{\lambda,21}$	.020	-	-	-	-	-	0.023	<u>*16.675*</u>	0.0110	0.0170	1.000
$V_{\lambda,31}$	.040	-	-	-	-	-	0.045	<u>*12.623*</u>	0.0180	0.0290	0.980
$V_{\lambda,42}$	.010	-	-	-	-	-	0.013	<u>*31.254*</u>	0.0090	0.0130	0.960
$V_{\lambda,52}$	.020	-	-	-	-	-	0.026	<u>*28.439*</u>	0.0110	0.0200	0.960
$V_{\lambda,62}$	.040	-	-	-	-	-	0.046	<u>*14.162*</u>	0.0180	0.0300	0.980
Convergence rate				1.000					0.990		
% PP p favoring M2						1.000					
PP p 5th percentile				0.038					0.221		
PP p rejection rate				0.455					0.000		
% DIC favoring M2						1.000					

Note: Pop. = population values; Est. = average of estimates across replications; EBIAS = estimation bias (in percentage); ESD = empirical standard deviation; ASD = average posterior standard deviation; 95% CR = 95% coverage rates; PP p = posterior predictive probability; DIC = deviance information criterion. Underlined italic numbers indicate substantial bias and bold numbers indicate coverage rates lower than 0.90.

Table 6: Results for Condition With $NS = 50$ and $\bar{N} = 100$ When Model M2 is Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	EBIAS	ESD	ASD	95% CR	Est.	EBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.301	0.313	0.0440	0.0260	**0.800**	0.297	-0.894	0.0420	0.0420	0.918
$\lambda_{0,11}$	.700	0.696	-0.603	0.0260	0.0280	0.950	0.697	-0.439	0.0240	0.0280	0.949
$\lambda_{0,21}$	.600	0.598	-0.391	0.0320	0.0250	**0.860**	0.598	-0.349	0.0280	0.0310	0.969
$\lambda_{0,31}$	.500	0.495	-1.097	0.0380	0.0230	**0.760**	0.493	-1.489	0.0370	0.0380	0.939
$\lambda_{0,42}$	.700	0.703	0.377	0.0250	0.0280	0.980	0.701	0.180	0.0250	0.0270	0.959
$\lambda_{0,52}$	.600	0.597	-0.452	0.0300	0.0250	0.900	0.596	-0.639	0.0280	0.0310	0.949
$\lambda_{0,62}$	.500	0.501	0.250	0.0360	0.0230	**0.820**	0.499	-0.272	0.0370	0.0390	0.969
V_ϕ	.050	-	-	-	-	-	0.053	6.625	0.0160	0.0190	0.949
$V_{\lambda,11}$	.010	-	-	-	-	-	0.010	-4.986	0.0060	0.0070	0.949
$V_{\lambda,21}$	.020	-	-	-	-	-	0.020	1.855	0.0070	0.0090	0.949
$V_{\lambda,31}$	.040	-	-	-	-	-	0.040	-0.556	0.0130	0.0140	0.959
$V_{\lambda,42}$	.010	-	-	-	-	-	0.009	-6.617	0.0060	0.0060	0.949
$V_{\lambda,52}$	.020	-	-	-	-	-	0.020	-0.606	0.0080	0.0090	0.949
$V_{\lambda,62}$	.040	-	-	-	-	-	0.043	6.251	0.0130	0.0150	0.959
Convergence rate				1.000					0.980		
% PP p favoring M2						1.000					
PP p 5th percentile				0.030					0.172		
PP p rejection rate				0.806					0.000		
% DIC favoring M2						1.000					

Note: Pop. = population values; Est. = average of estimates across replications; EBIAS = estimation bias (in percentage); ESD = empirical standard deviation; ASD = average posterior standard deviation; 95% CR = 95% coverage rates; PP p = posterior predictive probability; DIC = deviance information criterion. Underlined italic numbers indicate substantial bias and bold numbers indicate coverage rates lower than 0.90.

3 Simulation Results Without $\boldsymbol{\omega}_i$

3.1 The Homogeneous Case

Table 7: Results for Condition With $N_s = 20$ and $\bar{N} = 100$ When Both Models M1 and M2 are Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	RBIAS	ESD	ASD	95% CR	Est.	RBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.302	0.522	0.0340	0.0360	0.980	0.299	-0.233	0.0330	0.0460	1.000
$\lambda_{0,11}$	.700	0.694	-0.799	0.0310	0.0310	0.940	0.693	-1.060	0.0310	0.0390	1.000
$\lambda_{0,21}$	.600	0.601	0.161	0.0290	0.0290	0.940	0.599	-0.224	0.0280	0.0370	1.000
$\lambda_{0,31}$	.500	0.502	0.418	0.0300	0.0280	0.940	0.499	-0.110	0.0300	0.0370	0.979
$\lambda_{0,42}$	.700	0.697	-0.377	0.0330	0.0310	0.960	0.695	-0.704	0.0340	0.0390	0.969
$\lambda_{0,52}$	.600	0.603	0.527	0.0300	0.0290	0.930	0.602	0.321	0.0290	0.0370	0.959
$\lambda_{0,62}$	.500	0.495	-0.920	0.0310	0.0280	0.950	0.494	-1.198	0.0310	0.0360	0.979
V_ϕ	.000	-	-	-	-	-	0.010	1.009	0.0060	0.0130	-
$V_{\lambda,11}$	.000	-	-	-	-	-	0.006	0.631	0.0040	0.0090	-
$V_{\lambda,21}$	.000	-	-	-	-	-	0.006	0.578	0.0030	0.0080	-
$V_{\lambda,31}$	.000	-	-	-	-	-	0.006	0.606	0.0030	0.0080	-
$V_{\lambda,42}$	.000	-	-	-	-	-	0.006	0.596	0.0040	0.0090	-
$V_{\lambda,52}$	.000	-	-	-	-	-	0.005	0.549	0.0030	0.0080	-
$V_{\lambda,62}$	.000	-	-	-	-	-	0.006	0.617	0.0040	0.0080	-
Convergence rate				1.000					0.970		
% PP p favoring M1						0.464					
PP p 5th percentile				0.136					0.184		
PP p rejection rate				0.031					0.000		
% DIC favoring M2						-					

Note: Pop. = population values; Est. = estimate averages; RBIAS = relative bias; ESD = empirical standard deviation; ASD = average posterior standard deviation; 95% CR = 95% coverage rates; PP p = posterior predictive probability; DIC = deviance information criterion. Underlined italic numbers indicate substantial bias and bold numbers indicate coverage rates below .90.

Table 8: Results for Condition With $N_s = 20$ and $\bar{N} = 200$ When Both Models M1 and M2 are Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	RBIAS	ESD	ASD	95% CR	Est.	RBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.302	0.528	0.0270	0.0260	0.950	0.301	0.273	0.0270	0.0330	1.000
$\lambda_{0,11}$	.700	0.700	-0.034	0.0220	0.0220	0.960	0.699	-0.176	0.0220	0.0280	0.990
$\lambda_{0,21}$	.600	0.598	-0.397	0.0190	0.0200	0.950	0.597	-0.552	0.0190	0.0260	0.990
$\lambda_{0,31}$	.500	0.498	-0.481	0.0170	0.0200	0.990	0.496	-0.746	0.0180	0.0260	0.990
$\lambda_{0,42}$	.700	0.698	-0.243	0.0240	0.0220	0.920	0.697	-0.396	0.0250	0.0270	0.980
$\lambda_{0,52}$	.600	0.594	-0.956	0.0180	0.0210	0.960	0.594	-0.992	0.0180	0.0260	0.990
$\lambda_{0,62}$	.500	0.499	-0.191	0.0170	0.0200	0.980	0.498	-0.444	0.0170	0.0260	1.000
V_ϕ	.000	-	-	-	-	-	0.006	0.563	0.0030	0.0070	-
$V_{\lambda,11}$	.000	-	-	-	-	-	0.003	0.307	0.0020	0.0040	-
$V_{\lambda,21}$	.000	-	-	-	-	-	0.003	0.272	0.0010	0.0040	-
$V_{\lambda,31}$	.000	-	-	-	-	-	0.003	0.288	0.0010	0.0040	-
$V_{\lambda,42}$	.000	-	-	-	-	-	0.003	0.299	0.0020	0.0040	-
$V_{\lambda,52}$	.000	-	-	-	-	-	0.003	0.275	0.0010	0.0040	-
$V_{\lambda,62}$	.000	-	-	-	-	-	0.003	0.326	0.0020	0.0040	-
Convergence rate				1.000					0.980		
% PP p favoring M1						0.622					
PP p 5th percentile				0.078					0.109		
PP p rejection rate				0.082					0.041		
% DIC favoring M2						-					

Note: The same notation is used as in Table 7.

Table 9: Results for Condition With $N_s = 50$ and $\bar{N} = 100$ When Both Models M1 and M2 are Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	RBIAS	ESD	ASD	95% CR	Est.	RBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.298	-0.773	0.0250	0.0230	0.930	0.298	-0.677	0.0260	0.0260	0.957
$\lambda_{0,11}$	.700	0.698	-0.341	0.0200	0.0190	0.940	0.696	-0.549	0.0200	0.0210	0.978
$\lambda_{0,21}$	.600	0.600	0.035	0.0180	0.0180	0.940	0.600	-0.056	0.0170	0.0210	0.978
$\lambda_{0,31}$	.500	0.500	-0.077	0.0200	0.0180	0.920	0.500	-0.082	0.0200	0.0200	0.957
$\lambda_{0,42}$	.700	0.697	-0.360	0.0200	0.0190	0.910	0.696	-0.590	0.0200	0.0220	0.968
$\lambda_{0,52}$	.600	0.599	-0.114	0.0190	0.0180	0.940	0.598	-0.301	0.0190	0.0210	0.957
$\lambda_{0,62}$	.500	0.498	-0.367	0.0210	0.0180	**0.860**	0.497	-0.532	0.0210	0.0200	0.925
V_ϕ	.000	-	-	-	-	-	0.006	0.562	0.0030	0.0050	-
$V_{\lambda,11}$	.000	-	-	-	-	-	0.002	0.247	0.0010	0.0030	-
$V_{\lambda,21}$	.000	-	-	-	-	-	0.003	0.274	0.0020	0.0030	-
$V_{\lambda,31}$	.000	-	-	-	-	-	0.003	0.268	0.0020	0.0030	-
$V_{\lambda,42}$	.000	-	-	-	-	-	0.003	0.291	0.0020	0.0030	-
$V_{\lambda,52}$	.000	-	-	-	-	-	0.003	0.266	0.0010	0.0030	-
$V_{\lambda,62}$	.000	-	-	-	-	-	0.003	0.318	0.0020	0.0030	-
Convergence rate				1.000					0.930		
% PP p favoring M2						0.559					
PP p 5th percentile				0.047					0.103		
PP p rejection rate				0.043					0.054		
% DIC favoring M2						-					

Note: The same notation is used as in Table 7.

Table 10: Results for Condition With $N_s = 50$ and $\bar{N} = 200$ When Both Models M1 and M2 are Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	RBIAS	ESD	ASD	95% CR	Est.	RBIAS	ESD	ASD	95% CR
ϕ_0	.300	0.300	0.018	0.0170	0.0160	0.940	0.300	-0.102	0.0180	0.0180	0.955
$\lambda_{0,11}$	.700	0.700	-0.055	0.0130	0.0140	0.980	0.699	-0.103	0.0130	0.0150	0.978
$\lambda_{0,21}$	.600	0.600	0.040	0.0140	0.0130	0.940	0.600	-0.038	0.0140	0.0140	0.989
$\lambda_{0,31}$	.500	0.499	-0.171	0.0110	0.0120	0.970	0.499	-0.198	0.0110	0.0140	1.000
$\lambda_{0,42}$	.700	0.700	0.033	0.0130	0.0140	0.980	0.700	0.009	0.0130	0.0150	0.978
$\lambda_{0,52}$	.600	0.599	-0.122	0.0120	0.0130	0.960	0.598	-0.261	0.0120	0.0150	0.989
$\lambda_{0,62}$	.500	0.497	-0.504	0.0110	0.0130	0.960	0.497	-0.523	0.0120	0.0140	0.989
V_ϕ	.000	-	-	-	-	-	0.003	0.283	0.0010	0.0030	-
$V_{\lambda,11}$	.000	-	-	-	-	-	0.001	0.126	0.0010	0.0010	-
$V_{\lambda,21}$	.000	-	-	-	-	-	0.001	0.128	0.0010	0.0010	-
$V_{\lambda,31}$	.000	-	-	-	-	-	0.002	0.155	0.0010	0.0010	-
$V_{\lambda,42}$	.000	-	-	-	-	-	0.001	0.128	0.0010	0.0010	-
$V_{\lambda,52}$	.000	-	-	-	-	-	0.001	0.125	0.0010	0.0010	-
$V_{\lambda,62}$	.000	-	-	-	-	-	0.001	0.144	0.0010	0.0010	-
Convergence rate				1.000					**0.890**		
% PP p favoring M2						0.674					
PP p 5th percentile				0.085					0.077		
PP p rejection rate				0.067					0.124		
% DIC favoring M2						-					

Note: The same notation is used as in Table 7.

3.2 The Heterogeneous Case

Table 11: Results for Condition With $N_s = 20$ and $\bar{N} = 100$ When Model M2 is Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	RBIAS	ESE	ASE	95% CR	Est.	RBIAS	ESE	ASE	95% CR
ϕ_0	.300	0.295	-1.507	0.0590	0.0360	**0.710**	0.294	-1.951	0.0560	0.0720	0.979
$\lambda_{0,11}$	.700	0.702	0.224	0.0510	0.0310	**0.720**	0.700	0.003	0.0490	0.0490	0.968
$\lambda_{0,21}$	.600	0.595	-0.809	0.0460	0.0290	**0.760**	0.594	-0.923	0.0440	0.0530	0.957
$\lambda_{0,31}$	.500	0.492	-1.641	0.0600	0.0280	**0.620**	0.486	-2.790	0.0580	0.0640	0.968
$\lambda_{0,42}$	.700	0.699	-0.120	0.0430	0.0310	**0.840**	0.695	-0.673	0.0410	0.0490	0.957
$\lambda_{0,52}$	.600	0.597	-0.464	0.0510	0.0290	**0.750**	0.600	-0.052	0.0460	0.0540	0.979
$\lambda_{0,62}$	.500	0.492	-1.550	0.0610	0.0280	**0.600**	0.494	-1.208	0.0600	0.0640	0.968
V_ϕ	.050	-	-	-	-	-	0.065	*30.639*	0.0330	0.0420	0.947
$V_{\lambda,11}$	.010	-	-	-	-	-	0.017	*70.713*	0.0100	0.0170	0.926
$V_{\lambda,21}$	.020	-	-	-	-	-	0.026	*31.721*	0.0140	0.0220	0.968
$V_{\lambda,31}$	.040	-	-	-	-	-	0.047	*16.251*	0.0210	0.0320	0.968
$V_{\lambda,42}$	.010	-	-	-	-	-	0.017	*73.303*	0.0100	0.0170	0.947
$V_{\lambda,52}$	.020	-	-	-	-	-	0.028	*39.683*	0.0150	0.0220	0.947
$V_{\lambda,62}$	.040	-	-	-	-	-	0.046	*14.714*	0.0200	0.0320	0.968
Convergence rate				1.000					0.940		
% PP p favoring M2						1.000					
PP p 5th percentile				0.000					0.227		
PP p rejection rate				1.000					0.011		
% DIC favoring M2						-					

Note: The same notation is used as in Table 7.

Table 12: Results for Condition With $N_s = 20$ and $\bar{N} = 200$ When Model M2 is Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	RBIAS	ESE	ASE	95% CR	Est.	RBIAS	ESE	ASE	95% CR
ϕ_0	.300	0.291	-3.061	0.0600	0.0260	**0.560**	0.286	-4.701	0.0560	0.0680	0.979
$\lambda_{0,11}$	.700	0.695	-0.742	0.0370	0.0220	**0.740**	0.696	-0.595	0.0360	0.0390	0.958
$\lambda_{0,21}$	.600	0.596	-0.604	0.0430	0.0210	**0.630**	0.594	-0.938	0.0420	0.0480	0.947
$\lambda_{0,31}$	.500	0.492	-1.699	0.0540	0.0200	**0.520**	0.491	-1.873	0.0540	0.0600	0.937
$\lambda_{0,42}$	.700	0.699	-0.137	0.0390	0.0220	**0.680**	0.699	-0.161	0.0390	0.0410	0.979
$\lambda_{0,52}$	.600	0.596	-0.615	0.0390	0.0210	**0.690**	0.594	-1.038	0.0370	0.0480	1.000
$\lambda_{0,62}$	.500	0.494	-1.274	0.0550	0.0200	**0.600**	0.492	-1.613	0.0540	0.0620	0.958
V_ϕ	.050	-	-	-	-	-	0.066	*32.458*	0.0230	0.0370	0.968
$V_{\lambda,11}$	.010	-	-	-	-	-	0.013	*34.424*	0.0080	0.0120	0.937
$V_{\lambda,21}$	.020	-	-	-	-	-	0.026	*31.599*	0.0120	0.0180	0.947
$V_{\lambda,31}$	.040	-	-	-	-	-	0.047	*16.673*	0.0190	0.0290	0.968
$V_{\lambda,42}$	.010	-	-	-	-	-	0.015	*46.347*	0.0080	0.0130	0.926
$V_{\lambda,52}$	.020	-	-	-	-	-	0.025	*26.468*	0.0100	0.0180	0.979
$V_{\lambda,62}$	.040	-	-	-	-	-	0.049	*23.222*	0.0200	0.0300	0.937
Convergence rate				1.000					0.950		
% PP p favoring M2						1.000					
PP p 5th percentile				0.000					0.245		
PP p rejection rate				1.000					0.000		
% DIC favoring M2						-					

Note: The same notation is used as in Table 7.

Table 13: Results for Condition With $N_s = 50$ and $\bar{N} = 100$ When Model M2 is Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	RBIAS	ESE	ASE	95% CR	Est.	RBIAS	ESE	ASE	95% CR
ϕ_0	.300	0.292	-2.729	0.0390	0.0230	**0.770**	0.290	-3.369	0.0380	0.0430	0.939
$\lambda_{0,11}$	.700	0.693	-0.932	0.0240	0.0200	**0.890**	0.694	-0.849	0.0230	0.0270	0.970
$\lambda_{0,21}$	.600	0.602	0.310	0.0300	0.0180	**0.760**	0.601	0.144	0.0290	0.0310	0.970
$\lambda_{0,31}$	.500	0.497	-0.655	0.0370	0.0180	**0.660**	0.496	-0.829	0.0360	0.0370	0.949
$\lambda_{0,42}$	.700	0.697	-0.376	0.0290	0.0200	**0.800**	0.699	-0.165	0.0280	0.0270	0.939
$\lambda_{0,52}$	.600	0.598	-0.347	0.0290	0.0190	**0.790**	0.595	-0.761	0.0290	0.0310	0.949
$\lambda_{0,62}$	.500	0.492	-1.511	0.0330	0.0180	**0.700**	0.492	-1.606	0.0330	0.0380	0.980
V_ϕ	.050	-	-	-	-	-	0.057	*14.325*	0.0170	0.0190	0.939
$V_{\lambda,11}$	.010	-	-	-	-	-	0.011	*14.536*	0.0050	0.0070	0.970
$V_{\lambda,21}$	.020	-	-	-	-	-	0.023	*14.065*	0.0080	0.0090	0.929
$V_{\lambda,31}$	.040	-	-	-	-	-	0.041	3.502	0.0120	0.0140	0.980
$V_{\lambda,42}$	.010	-	-	-	-	-	0.012	*17.856*	0.0060	0.0070	0.949
$V_{\lambda,52}$	.020	-	-	-	-	-	0.023	*12.597*	0.0090	0.0090	0.919
$V_{\lambda,62}$	.040	-	-	-	-	-	0.043	6.677	0.0120	0.0140	0.960
Convergence rate				1.000					0.990		
% PP p favors M2						1.000					
PP p 5th percentile				0.000					.234		
PP p rejection rate				1.000					0.000		
% DIC favors M2						-					

Note: The same notation is used as in Table 7.

Table 14: Results for Condition With $N_s = 50$ and $\bar{N} = 200$ When Model M2 is Correct.

		M1 (Homogeneous)					M2 (Heterogeneous)				
	Pop.	Est.	RBIAS	ESE	ASE	95% CR	Est.	RBIAS	ESE	ASE	95% CR
ϕ_0	.300	0.298	-0.804	0.0400	0.0160	**0.580**	0.296	-1.461	0.0380	0.0380	0.949
$\lambda_{0,11}$	.700	0.699	-0.101	0.0250	0.0140	**0.700**	0.698	-0.252	0.0220	0.0230	0.959
$\lambda_{0,21}$	.600	0.600	0.007	0.0270	0.0130	**0.700**	0.600	-0.009	0.0260	0.0280	0.939
$\lambda_{0,31}$	.500	0.492	-1.682	0.0390	0.0120	**0.420**	0.491	-1.887	0.0390	0.0350	0.939
$\lambda_{0,42}$	.700	0.701	0.164	0.0270	0.0140	**0.640**	0.700	-0.068	0.0250	0.0230	0.929
$\lambda_{0,52}$	.600	0.601	0.166	0.0290	0.0130	**0.600**	0.602	0.383	0.0270	0.0280	0.969
$\lambda_{0,62}$	.500	0.493	-1.495	0.0320	0.0120	**0.500**	0.492	-1.510	0.0290	0.0340	0.959
V_ϕ	.050						0.053	6.509	0.0140	0.0150	0.980
$V_{\lambda,11}$	.010	-	-	-	-	-	0.011	*13.576*	0.0050	0.0050	**0.898**
$V_{\lambda,21}$	.020	-	-	-	-	-	0.022	9.214	0.0060	0.0070	0.949
$V_{\lambda,31}$	.040	-	-	-	-	-	0.041	3.596	0.0110	0.0120	0.949
$V_{\lambda,42}$	.010	-	-	-	-	-	0.011	9.069	0.0050	0.0050	0.949
$V_{\lambda,52}$	.020	-	-	-	-	-	0.022	9.297	0.0060	0.0070	0.949
$V_{\lambda,62}$	.040	-	-	-	-	-	0.039	-2.142	0.0100	0.0120	0.959
Convergence rate				1.000					0.980		
% PP p favoring M2							1.000				
PP p 5th percentile				.000					.228		
PP p rejection rate				1.000					0.010		
% DIC favoring M2							-				

Note: The same notation is used as in Table 7.

4 Impact of $\boldsymbol{\omega}_i$ on Computation Time

Table 15: The impact of adding $\boldsymbol{\omega}_i$ into the model in terms of computation time.

	Homogeneous							
	NS=20,N=100		NS=20,N=200		NS=50,N=100		NS=50,N=200	
	M1	M2	M1	M2	M1	M2	M1	M2
With $\boldsymbol{\omega}_i$	149.89	202.95	145.63	235.84	639.30	947.05	610.48	955.83
Without $\boldsymbol{\omega}_i$	204.15	315.32	194.17	301.14	653.22	1314.26	636.84	1075.95
	Heterogeneous							
	M1	M2	M1	M2	M1	M2	M1	M2
With $\boldsymbol{\omega}_i$	154.00	237.79	153.00	206.53	649.88	830.72	644.51	957.08
Without $\boldsymbol{\omega}_i$	194.35	337.92	192.85	333.78	690.50	1183.86	701.70	1140.00

Note: Computation time was the amount of time (in seconds) for a central processing unit to run the first 60,000 iterations. Analysis was performed on a Macbook Pro with an Intel i7 processor (3.1 GHz) with 16.00 GB DDR3.

5 R Code Implementing FIMASEM to Analyze Personality Data

```
# Modified based on the supplemental materials of Yu, Downes, Carter, and O'Boyle (2018)
# The heterogeneity problem in meta-analytic structural equation modeling (MASEM) revisited
# A reply to Cheung
# SRMR is computed using a self-developed function because R function fitMeasuresMx
# is not available now

library(metaSEM)
require('matrixcalc')
library(OpenMx)
library(Matrix)
library(MASS)
```

```
mySRMR <- function(oC,mC){
            p = nrow(oC)
            return(sqrt(sum((oC-mC)^2)/p/(p+1)))
}

wd = 'D:/Research/MASEM/FIMASEM'
setwd(wd)

# data
data = read.table('MIMM.dat', header = TRUE)
vR = as.matrix(data[1:83,-1])
missfew.id = which(apply(vR,1,function(x) sum(is.na(x))<35)==TRUE)
vR = vR[missfew.id,]
Ni = data[missfew.id,1]
N = sum(Ni)
k = nrow(vR)
reps <- 10000
NpS = 1/mean(1/Ni)
#NpS = N/k

#Reformat the data for TSSEM input
B5.names <- c('N','A','C','E','O')
varnames <- c(paste(B5.names,'s',sep=''),paste(B5.names,'p',sep=''))
cormats <- list()
for (i in 1:k){
  temp <- matrix(1,nrow=10,ncol=10)
  temp[lower.tri(temp)] <- as.numeric(vR[i,])
  temp2 <- t(temp)
  temp[upper.tri(temp)] <- temp2[upper.tri(temp2)]
  colnames(temp) <-  rownames(temp) <- varnames
  cormats[[i]] <- temp
}
#cormats <- cormats[which(sapply(cormats,is.positive.definite))][1:k]


######################################
# conduct multivariate FIMASEM
######################################
varnames <- c(paste(B5.names,'s',sep=''),paste(B5.names,'p',sep=''))
step.one <- tssem1(cormats,Ni,method="REM",RE.type="Diag") #run TSSEM Step 1
rho.mult <- diag(1,nrow=length(varnames),ncol=length(varnames))
rho.mult[lower.tri(rho.mult)] <- (coef(step.one,select="fixed"))
temp <- t(rho.mult)
rho.mult[upper.tri(rho.mult)] <- temp[upper.tri(temp)]
sigma.mult <- diag(coef(step.one,select="random"))
#the sigma matrix from this analysis will have Tau^2 along the diagonal and zeros in the off-diagonal.
#There are a number of possible approaches to this sigma matrix; we think this is most appropriate.
dimnames(rho.mult) <- list(varnames,varnames)
matrices.mult <- rCorPop(rho.mult,sigma.mult,corr=T,k=reps,nonPD.pop="nearPD")


######################################
#          CTOM5
```

```
########################################
varnames <- c(paste(B5.names,'s',sep=''),paste(B5.names,'p',sep=''),B5.names,'Self','Peer')
A.values = rbind(cbind(matrix(0,5,10),diag(0.6,5),rep(0.6,5),rep(0,5)),
          cbind(matrix(0,5,10),diag(0.6,5),rep(0,5),rep(0.6,5)),matrix(0,7,17))
A.lbound = rbind(cbind(matrix(0,5,15),c(0,rep(-1,4)),rep(0,5)),
          cbind(matrix(0,5,15),rep(0,5),c(0,rep(-1,4))),matrix(0,7,17))
A.ubound = rbind(cbind(matrix(0,5,10),diag(1,5),rep(1,5),rep(0,5)),
          cbind(matrix(0,5,10),diag(1,5),rep(0,5),rep(1,5)),matrix(0,7,17))
Atmp = matrix(NA,5,5)
diag(Atmp) <- paste('L',1:5,sep = '')
A.labels = rbind(cbind(matrix(NA,5,10),Atmp,paste('LM',1:5,sep=''),rep(NA,5)),
          cbind(matrix(NA,5,10),Atmp,rep(NA,5),paste('LM',6:10,sep='')),matrix(NA,7,17))
A.free = A.values!=0
A <- mxMatrix(type = 'Full',free = A.free,values = A.values,labels = A.labels,
          lbound = A.lbound,ubound = A.ubound,name="A") # First order factor loadings
dimnames(A) <- list(varnames,varnames)

Stmp = matrix(.1,5,5)
diag(Stmp) = 1
S.values = rbind(cbind(diag(.1,10),matrix(0,10,7)),cbind(matrix(0,5,10),Stmp,matrix(0,5,2)),
          cbind(matrix(0,2,15),diag(1,2)))
Stmp = matrix(NA,10,10)
diag(Stmp) = paste('s2e',1:10,sep='')
Stmp2 = matrix(c(
NA,'rNA','rNC','rNE','rNO',
'rNA',NA,'rAC','rAE','rAO',
'rNC','rAC',NA,'rCE','rCO',
'rNE','rAE','rCE',NA,'rEO',
'rNO','rAO','rCO','rEO',NA),5,5)
S.labels = rbind(cbind(Stmp,matrix(NA,10,7)),cbind(matrix(NA,5,10),Stmp2,matrix(NA,5,2)),matrix(,2,17))
S.lbound = rbind(cbind(diag(-1,10),matrix(0,10,7)),cbind(matrix(0,5,10),matrix(-1,5,5),matrix(0,5,2)),
          cbind(matrix(0,2,15),diag(-1,2)))
S.ubound = rbind(cbind(diag(1,10),matrix(0,10,7)),cbind(matrix(0,5,10),matrix(1,5,5),matrix(0,5,2)),
          cbind(matrix(0,2,15),diag(1,2)))
S.free = S.values!=0; diag(S.free[11:17,11:17]) = FALSE
S <- mxMatrix(type = 'Full',free = S.free,values = S.values,labels = S.labels,
          lbound = S.lbound,ubound = S.ubound,name="S") # First order factor loadings
dimnames(S) <- list(varnames,varnames)
mF = cbind(diag(1,10),matrix(0,10,7))
matrF <- create.mxMatrix(c(mF),nrow=10,ncol=17,name="F")
exp     <- mxExpectationRAM("A","S","F", dimnames=varnames )

# Run SEM on those random matrices using the same technique we use for V&O FIMASEM
coefs.fits.multivariate.FIMASEM <- as.data.frame(t(sapply(1:reps,function(i) {
  openmxmodel <- mxModel("temp",mxData(matrices.mult[[i]],type="cov",numObs = NpS),
  matrA = A,matrS = S,matrF=matrF,exp=exp,funML=mxFitFunctionML());
  openmxfit <- mxRun(openmxmodel,silent=T);
  if (openmxfit$output$status[[1]] == 6) {openmxfit <- mxRun(openmxfit,silent=T);}
  modelsummary <- summary(openmxfit);
  coefs <- coef(openmxfit)[c(1:15,26:35)]
  coef.names <- names(coefs)
   mC <- openmxfit$expCov$result
  oC <- matrices.mult[[i]]
  output <- c(coefs,mySRMR(oC,mC),modelsummary$CFI,openmxfit$output$status[[1]]);
```

```r
  names(output) <- c(names(coefs),'SRMR','CFI','openMxStatus')
  output
}))) #returns a dataframe of SEM parameter estimates (i.e., fit indices and path coefficients)

del.id = which(coefs.fits.multivariate.FIMASEM[,28]>0)
print('########################################')
print('## MULTIVARIATE FIMASEM RESULTS')
print('########################################')
print('means')
print(sapply(coefs.fits.multivariate.FIMASEM,function(x){mean(x[-del.id])}))#MEANS
print('sd')
print(sapply(coefs.fits.multivariate.FIMASEM,function(x){sd(x[-del.id])}))#SD

print('% SRMR < .10')
print(sum(coefs.fits.multivariate.FIMASEM$SRMR[-del.id] < .1)/(reps-length(del.id)))
print('% CFI > .90')
print(sum(coefs.fits.multivariate.FIMASEM$CFI[-del.id] > .90)/(reps-length(del.id)))# %cfi > .90


########################################
#        2SF
########################################
varnames <- c(paste(B5.names,'s',sep=''),paste(B5.names,'p',sep=''))
L.values = rbind(cbind(diag(.6,5),rep(0.6,5),rep(0,5)),cbind(diag(.6,5),rep(0,5),rep(0.6,5)))
L.lbound = rbind(cbind(diag(0,5),c(0,rep(-1,4)),rep(0,5)),cbind(diag(0,5),rep(0,5),c(0,rep(-1,4))))
L.ubound = rbind(cbind(diag(1,5),rep(1,5),rep(0,5)),cbind(diag(1,5),rep(0,5),rep(1,5)))
Ltmp = matrix(NA,5,5)
diag(Ltmp) <- paste('L',1:5,sep = '')
L.labels = rbind(cbind(Ltmp,paste('LM',1:5,sep=''),rep(NA,5)),
        cbind(Ltmp,rep(NA,5),paste('LM',6:10,sep='')))
L.free = L.values!=0
L <- mxMatrix(type = 'Full',free = L.free,values = L.values,labels = L.labels,
        lbound = L.lbound,ubound = L.ubound,name="L") # First order factor loadings

U1.values = diag(0.1,10)
U1.lbound = diag(0,10)
U1.ubound = diag(1,10)
U1.labels = matrix(NA,10,10)
diag(U1.labels) = paste('s2e',1:10,sep='')
U1.free = U1.values!=0
U1 <- mxMatrix(type = 'Diag',free = U1.free,values = U1.values,labels = U1.labels,
        lbound = U1.lbound,ubound = U1.ubound, name="U1") # First order uniquenesses

LL.values = matrix(c(rep(0.6,3),rep(0,7),rep(0.6,2),rep(0,2)),7,2)
LL.lbound = matrix(c(0,rep(-1,2),rep(0,11)),7,2)
LL.ubound = matrix(c(rep(1,3),rep(0,7),rep(1,2),rep(0,2)),7,2)
LL.labels = matrix(c(paste('LL',1:3,sep=''),rep(NA,7),
        rep('LL4',2),rep(NA,2)),7,2)
LL.free = LL.values!=0
LL <- mxMatrix(type = 'Full', free = LL.free,values = LL.values,labels = LL.labels,
        lbound = LL.lbound,ubound = LL.ubound,name="LL") # Second order factor loadings
Phi2 <- mxMatrix(type = 'Symm',nrow = 2,ncol = 2,free = c(FALSE,TRUE,FALSE),values = c(1,.3,1),
        labels = c(NA,'rAB',NA),lbound = rep(-1,3),ubound = rep(1,3),name = 'Phi2')
Im <- mxMatrix(type = "Iden", nrow = 7, ncol = 7, name = "Im")
```

```r
U2 <- mxAlgebra(diag2vec(Im - LL %*%Phi2%*%t(LL)), name = "U2")
Phi <- mxAlgebra(LL%*%Phi2 %*%t(LL) + vec2diag(U2),name = 'Phi')

ecov <- mxAlgebra(L%*%Phi%*%t(L) + U1 , name="expCov")
expectation <- mxExpectationNormal(cov="expCov",dimnames = varnames)

# Run SEM on those random matrices using the same technique we use for V&O FIMASEM
coefs.fits.multivariate.FIMASEM <- as.data.frame(t(sapply(1:reps,function(i) {
  openmxmodel <- mxModel("temp",mxData(matrices.mult[[i]],type="cov",numObs = NpS),
  L,LL,Phi2,U1,Phi,Im,U2,ecov, expectation,
  funML=mxFitFunctionML());
  openmxfit <- mxRun(openmxmodel,silent=T);
  if (openmxfit$output$status[[1]] == 6) {openmxfit <- mxRun(openmxfit,silent=T);}
  modelsummary <- summary(openmxfit);
  coefs <- coef(openmxfit)[c(1:20)]
  coef.names <- names(coefs)
  mC <- openmxfit$expCov$result
  oC <- matrices.mult[[i]]
  output <- c(coefs,mySRMR(oC,mC),modelsummary$CFI,openmxfit$output$status[[1]]);
  names(output) <- c(names(coefs),'SRMR','CFI','openMxStatus')
  output
}))) #returns a dataframe of SEM parameter estimates (i.e., fit indices and path coefficients)

del.id = which(coefs.fits.multivariate.FIMASEM[,23]>0)
print('##############################################')
print('## MULTIVARIATE FIMASEM RESULTS')
print('##############################################')
print('means')
print(sapply(coefs.fits.multivariate.FIMASEM,function(x){mean(x[-del.id])}))#MEANS
print('sd')
print(sapply(coefs.fits.multivariate.FIMASEM,function(x){sd(x[-del.id])}))#SD

print('% SRMR < .10')
print(sum(coefs.fits.multivariate.FIMASEM$SRMR[-del.id] < .1)/(reps-length(del.id)))
print('% CFI > .90')
print(sum(coefs.fits.multivariate.FIMASEM$CFI[-del.id] > .90)/(reps-length(del.id)))# %cfi > .90


########################################
#          2SFG
########################################
varnames <- c(paste(B5.names,'s',sep=''),paste(B5.names,'p',sep=''))
L.values = rbind(cbind(diag(.6,5),rep(0.6,5),rep(0,5)),cbind(diag(.6,5),rep(0,5),rep(0.6,5)))
L.lbound = rbind(cbind(diag(0,5),c(0,rep(-1,4)),rep(0,5)),cbind(diag(0,5),rep(0,5),c(0,rep(-1,4))))
L.ubound = rbind(cbind(diag(1,5),rep(1,5),rep(0,5)),cbind(diag(1,5),rep(0,5),rep(1,5)))
Ltmp = matrix(NA,5,5)
diag(Ltmp) <- paste('L',1:5,sep = '')
L.labels = rbind(cbind(Ltmp,paste('LM',1:5,sep=''),rep(NA,5)),
          cbind(Ltmp,rep(NA,5),paste('LM',6:10,sep='')))
L.free = L.values!=0
L <- mxMatrix(type = 'Full',free = L.free,values = L.values,labels = L.labels,
          lbound = L.lbound,ubound = L.ubound,name="L") # First order factor loadings

U1.values = diag(0.1,10)
```

```
U1.lbound = diag(0,10)
U1.ubound = diag(1,10)
U1.labels = matrix(NA,10,10)
diag(U1.labels) = paste('s2e',1:10,sep='')
U1.free = U1.values!=0
U1 <- mxMatrix(type = 'Diag',free = U1.free,values = U1.values,labels = U1.labels,
         lbound = U1.lbound,ubound = U1.ubound,name="U1") # First order uniquenesses

LL.values = matrix(c(rep(0.6,3),rep(0,7),rep(0.6,2),rep(0,2),rep(0.6,5),rep(0,2)),7,3)
LL.lbound = matrix(c(0,rep(-1,2),rep(0,11),0,rep(-1,4),rep(0,2)),7,3)
LL.ubound = matrix(c(rep(1,3),rep(0,7),rep(1,2),rep(0,2),rep(1,5),rep(0,2)),7,3)
LL.labels = matrix(c(paste('LL',1:3,sep=''),rep(NA,7),rep('LL4',2),
         rep(NA,2),paste('LLG',1:5,sep=''),rep(NA,2)),7,3)
LL.free = LL.values!=0
LL <- mxMatrix(type = 'Full', free = LL.free,values = LL.values,labels = LL.labels,
         lbound = LL.lbound,ubound = LL.ubound,name="LL") # Second order factor loadings
Im <- mxMatrix(type = "Iden", nrow = 7, ncol = 7, name = "Im")
U2 <- mxAlgebra(diag2vec(Im - LL %*% t(LL)), name = "U2")
Phi <- mxAlgebra(LL %*%t(LL) + vec2diag(U2),name = 'Phi')

ecov <- mxAlgebra(L %*%  Phi %*% t(L) + U1 , name="expCov")
expectation <- mxExpectationNormal(cov="expCov",dimnames = varnames)

# Run SEM on those random matrices using the same technique we use for V&O FIMASEM
coefs.fits.multivariate.FIMASEM <- as.data.frame(t(sapply(1:reps,function(i) {
  openmxmodel <- mxModel("temp",mxData(matrices.mult[[i]],type="cov",numObs = NpS),
  L,LL,U1,Phi,Im,U2,ecov, expectation,
  funML=mxFitFunctionML());
  openmxfit <- mxRun(openmxmodel,silent=T);
  if (openmxfit$output$status[[1]] == 6) {openmxfit <- mxRun(openmxfit,silent=T)}
  modelsummary <- summary(openmxfit);
  coefs <- coef(openmxfit)[c(1:24)]
  coef.names <- names(coefs)
  mC <- openmxfit$expCov$result
  oC <- matrices.mult[[i]]
  output <- c(coefs,mySRMR(oC,mC),modelsummary$CFI,openmxfit$output$status[[1]]);
  names(output) <- c(names(coefs),'SRMR','CFI','openMxStatus')
  output
}))) #returns a dataframe of SEM parameter estimates (i.e., fit indices and path coefficients)

del.id = which(coefs.fits.multivariate.FIMASEM[,27]>0)
print('####################################')
print('## MULTIVARIATE FIMASEM RESULTS')
print('####################################')
print('means')
print(sapply(coefs.fits.multivariate.FIMASEM,function(x){mean(x[-del.id])}))#MEANS
print('sd')
print(sapply(coefs.fits.multivariate.FIMASEM,function(x){sd(x[-del.id])}))#SD

print('% SRMR < .10')
print(sum(coefs.fits.multivariate.FIMASEM$SRMR[-del.id] < .1)/(reps-length(del.id)))
print('% CFI > .90')
print(sum(coefs.fits.multivariate.FIMASEM$CFI[-del.id] > .90)/(reps-length(del.id)))# %cfi > .90
```