**Supplementary Material D**

for the paper “On theDevelopment of Instabilities in an Annulus and a Shell Composed of a Poro-Hyperelastic Material”

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**Appendix D.**

In this Appendix, we present the procedure for numerical solution of differential equations (3.15) and (3.22) using MATLAB software. The time-dependent differential equation (3.22) can be discretized in time using implicit backward Euler scheme as suggested by Sun [92]

, (D.1)

where the subscripts  and  signify the time step number and  is the time step magnitude. The solution at time step  is assumed known and the solution at time step  is sought. Equation (D.1) is now recast into the form suitable for solving 1D boundary-value problem using MATLAB built-in function bvp4c. Namely, differential equation of the second order (D.1) is represented as a system of two differential equations of the first order. In this system, one of the unknown function is the fluid pressure  and the second unknown function is the pressure gradient . Thus, we can obtain

 (D.2)

In the system of equations (D.2), if a quantity is missing a time step number, then the time step  is implied.

Next, we represent differential equation of equilibrium (3.15) as a system of two differential equations of the first order. In order to achieve that, two more unknown functions are introduced: the radial displacement  and the displacement gradient . The stretches  and can be easily expressed in terms of the functions  and . Then, we can obtain from (3.15)

 (D.3)

where

|  |  |
| --- | --- |
| $$\begin{matrix}f=\frac{2}{D\_{1}}λ\_{1}\frac{1}{R}\left(λ\_{1}-λ\_{2}\right)+ \\\frac{1}{R}\left(λ\_{1}-λ\_{2}\right)\frac{2μ}{3α}λ\_{1}^{-\frac{α}{3}-1 }λ\_{2}^{-\frac{α}{3}-2}\left(-2\left(\frac{α}{3}+1\right)λ\_{1}^{α}-\left(\frac{2α}{3}-1\right)λ\_{2}^{α}+\frac{α}{3}+1\right)+\\\begin{matrix}\frac{1}{R}\frac{2μ}{α}λ\_{1}^{-\frac{α}{3} }λ\_{2}^{-\frac{α}{3}-2}\left(λ\_{1}^{α}-λ\_{2}^{α}\right)-y\_{2};\\g=\frac{2}{D\_{1}}λ\_{2}+\frac{2μ}{3α}λ\_{1}^{-\frac{α}{3}-2 }λ\_{2}^{-\frac{α}{3}-1}\left(2\left(\frac{2α}{3}-1\right)λ\_{1}^{α}+\left(\frac{α}{3}+1\right)λ\_{2}^{α}+\frac{α}{3}+1\right)\end{matrix}\end{matrix}$$ | (D.4) |

As before, the time step  is implied for all quantities with missing time step number. The systems of equations (D.2) and (D.3) are now solved simultaneously. The term , present in (D.2), is set equal to  according to the second equation of (D.3).

Boundary conditions (3.17) and (3.23) also need to be supplied to the solver bvp4c. At the inner surface , we have

|  |  |
| --- | --- |
| $$\left\{\begin{matrix}y\_{1}-P\_{A}=0\\\frac{2μ}{3α}λ\_{1}^{-\frac{α}{3}-1}λ\_{2}^{-\frac{α}{3}-1}\left(2λ\_{1}^{α}-λ\_{2}^{α}-1\right)+\frac{2}{D\_{1}}\left(λ\_{1}λ\_{2}-1\right)-y\_{1}+P\_{A}=0\end{matrix}\right.$$ | (D.5) |

At the outer surface , we have

|  |  |
| --- | --- |
| $$\left\{\begin{matrix}y\_{2}=0\\\frac{2μ}{3α}λ\_{1}^{-\frac{α}{3}-1}λ\_{2}^{-\frac{α}{3}-1}\left(2λ\_{1}^{α}-λ\_{2}^{α}-1\right)+\frac{2}{D\_{1}}\left(λ\_{1}λ\_{2}-1\right)-y\_{1}=0\end{matrix}\right.$$ | (D.6) |

One of the advantages of using this solution procedure is the absence of undesirable pressure oscillations (numerical instability) caused by steep fluid pressure distribution near the inner surface  in the short-term. The oscillations in the fluid pressure distribution will be present in the finite element solution once linear elements for displacement and pressure fields are used. But the disadvantage of the current MATLAB procedure is that there is no automatic time step selection. In contrast, the ABAQUS finite element software enables the user to take advantage of automatic time step selection.