**Supplemental File**

**Table 1.** Different considered scenarios when $k=2, 3, 5$, and 10. In this table $(x\_{y})$ means that $x$ is repeated $y$ times and $x-y$ means $x$, $x+1, …, y$.

|  |  |  |  |
| --- | --- | --- | --- |
| $$k$$ | **Scenarios** | $$n$$ | $$σ^{2}$$ |
| $$k=2$$ | 1-6 | $$(6, 6)$$ | $$\left(0.1\_{2}\right), \left(0.5\_{2}\right), \left(1\_{2}\right), \left(1, 2.5\right), \left(1, 0.1\right), (1, 0.5)$$ |
| 7-12 | (6, 10) | $$\left(0.1\_{2}\right), \left(0.5\_{2}\right), \left(1\_{2}\right), \left(1, 2.5\right), \left(1, 0.1\right), (1, 0.5)$$ |
| 13-18 | (10, 6) | $$\left(0.1\_{2}\right), \left(0.5\_{2}\right), \left(1\_{2}\right), \left(1, 2.5\right), \left(1, 0.1\right), (1, 0.5)$$ |
| 19-24 | ($10\_{2}$) | $$\left(0.1\_{2}\right), \left(0.5\_{2}\right), \left(1\_{2}\right), \left(1, 2.5\right), \left(1, 0.1\right), (1, 0.5)$$ |
| 25-30 | ($20\_{2}$) | $$\left(0.1\_{2}\right), \left(0.5\_{2}\right), \left(1\_{2}\right), \left(1, 2.5\right), \left(1, 0.1\right), (1, 0.5)$$ |
| 31-36 | (10, 20) | $$\left(0.1\_{2}\right), \left(0.5\_{2}\right), \left(1\_{2}\right), \left(1, 2.5\right), \left(1, 0.1\right), (1, 0.5)$$ |
| $$k=3$$ | 1-6 | $$(6\_{3})$$ | $$\left(0.1\_{3}\right), \left(0.5\_{3}\right), \left(1\_{3}\right), \left(2.5, 0.5, 2.5\right), \left(0.1, 0.5, 1\right), (0.1, 0.5, 2)$$ |
| 7-12 | (6, 10, 6) | $$\left(0.1\_{3}\right), \left(0.5\_{3}\right), \left(1\_{3}\right), \left(2.5, 0.5, 2.5\right), \left(0.1, 0.5, 1\right), (0.1, 0.5, 2)$$ |
| 13-18 | (10, 6, 10) | $$\left(0.1\_{3}\right), \left(0.5\_{3}\right), \left(1\_{3}\right), \left(2.5, 0.5, 2.5\right), \left(0.1, 0.5, 1\right), (0.1, 0.5, 2)$$ |
| 19-24 | $$(10\_{3})$$ | $$\left(0.1\_{3}\right), \left(0.5\_{3}\right), \left(1\_{3}\right), \left(2.5, 0.5, 2.5\right), \left(0.1, 0.5, 1\right), (0.1, 0.5, 2)$$ |
| 25-30 | $$(20\_{3})$$ | $$\left(0.1\_{3}\right), \left(0.5\_{3}\right), \left(1\_{3}\right), \left(2.5, 0.5, 2.5\right), \left(0.1, 0.5, 1\right), (0.1, 0.5, 2)$$ |
| 31-36 | (10, 50, 10) | $$\left(0.1\_{3}\right), \left(0.5\_{3}\right), \left(1\_{3}\right), \left(2.5, 0.5, 2.5\right), \left(0.1, 0.5, 1\right), (0.1, 0.5, 2)$$ |
| $$k=5$$ | 1-6 | $$(6\_{5})$$ | $$\left(0.1\_{5}\right), \left(0.5\_{5}\right), \left(1\_{5}\right), \left(0.5, 2.5\_{3}, 0.5\right), \left(0.1, 1\_{3}, 0.1\right), (1, 0.1\_{3},1)$$ |
| 7-12 | (6, $10\_{3}$, 6) | $$\left(0.1\_{5}\right), \left(0.5\_{5}\right), \left(1\_{5}\right), \left(0.5, 2.5\_{3}, 0.5\right), \left(0.1, 1\_{3}, 0.1\right), (1, 0.1\_{3},1)$$ |
| 13-18 | (10, $6\_{3}$, 10) | $$\left(0.1\_{5}\right), \left(0.5\_{5}\right), \left(1\_{5}\right), \left(0.5, 2.5\_{3}, 0.5\right), \left(0.1, 1\_{3}, 0.1\right), (1, 0.1\_{3},1)$$ |
| 19-24 | $$(10\_{5})$$ | $$\left(0.1\_{5}\right), \left(0.5\_{5}\right), \left(1\_{5}\right), \left(0.5, 2.5\_{3}, 0.5\right), \left(0.1, 1\_{3}, 0.1\right), (1, 0.1\_{3},1)$$ |
| 25-30 | $$(20\_{5})$$ | $$\left(0.1\_{5}\right), \left(0.5\_{5}\right), \left(1\_{5}\right), \left(0.5, 2.5\_{3}, 0.5\right), \left(0.1, 1\_{3}, 0.1\right), (1, 0.1\_{3},1)$$ |
| 31-36 | (10,$20\_{3}$,10) | $$\left(0.1\_{5}\right), \left(0.5\_{5}\right), \left(1\_{5}\right), \left(0.5, 2.5\_{3}, 0.5\right), \left(0.1, 1\_{3}, 0.1\right), (1, 0.1\_{3},1)$$ |
| $$k=10$$ | 1-8 | $$(6\_{10})$$ | $\left(0.1\_{10}\right), \left(0.5\_{10}\right), \left(1\_{10}\right), \left(1\_{3}, 0.5\_{4}, 1\_{3}\right), \left(0.5\_{3}, 1\_{4}, 0.5\_{3}\right),$ $\left(2.5\_{3}, 0.5\_{4}, 2.5\_{3}\right), \left(0.5\_{3}, 2.5\_{4}, 0.5\_{3}\right), (1\_{3},2\_{4},1\_{3})$ |
| 9-16 | ($6\_{5}$, $10\_{5}$) | $\left(0.1\_{10}\right), \left(0.5\_{10}\right), \left(1\_{10}\right), \left(1\_{3}, 0.5\_{4}, 1\_{3}\right), \left(0.5\_{3}, 1\_{4}, 0.5\_{3}\right),$ $\left(2.5\_{3}, 0.5\_{4}, 2.5\_{3}\right), \left(0.5\_{3}, 2.5\_{4}, 0.5\_{3}\right), (1\_{3},2\_{4},1\_{3})$ |
| 17-24 | $$(10\_{10})$$ | $\left(0.1\_{10}\right), \left(0.5\_{10}\right), \left(1\_{10}\right), \left(1\_{3}, 0.5\_{4}, 1\_{3}\right), \left(0.5\_{3}, 1\_{4}, 0.5\_{3}\right),$ $\left(2.5\_{3}, 0.5\_{4}, 2.5\_{3}\right), \left(0.5\_{3}, 2.5\_{4}, 0.5\_{3}\right), (1\_{3},2\_{4},1\_{3})$ |
| 25-32 | $$(10\_{5}, 20\_{5})$$ | $\left(0.1\_{10}\right), \left(0.5\_{10}\right), \left(1\_{10}\right), \left(1\_{3}, 0.5\_{4}, 1\_{3}\right), \left(0.5\_{3}, 1\_{4}, 0.5\_{3}\right),$ $\left(2.5\_{3}, 0.5\_{4}, 2.5\_{3}\right), \left(0.5\_{3}, 2.5\_{4}, 0.5\_{3}\right), (1\_{3},2\_{4},1\_{3})$ |
| 33-40 | $$(50\_{10})$$ | $\left(0.1\_{10}\right), \left(0.5\_{10}\right), \left(1\_{10}\right), \left(1\_{3}, 0.5\_{4}, 1\_{3}\right), \left(0.5\_{3}, 1\_{4}, 0.5\_{3}\right),$ $\left(2.5\_{3}, 0.5\_{4}, 2.5\_{3}\right), \left(0.5\_{3}, 2.5\_{4}, 0.5\_{3}\right), (1\_{3},2\_{4},1\_{3})$ |

Note: Each scenario corresponds to a value of$σ^{2}$.

**Table 2.** The results of simulation study for the method proposed by Lin and Wang (2013). The considered scenarios are shown in Table 1.

|  |  |  |
| --- | --- | --- |
|  | **Coverage Probability** | **Average Length** |
| **Scenarios** | $k=$**2** | $k=$**3** | $k=$**5** | $k=$**10** |  | $k=$**2** | $k=$**3** | $k=$**5** | $k=$**10** |
| **1** | 0.932 | 0.919 | 0.901 | 0.877 |  | 0.464 | 0.362 | 0.272 | 0.186 |
| **2** | 0.942 | 0.929 | 0.902 | 0.866 |  | 1.319 | 1.025 | 0.759 | 0.515 |
| **3** | 0.942 | 0.923 | 0.890 | 0.842 |  | 2.213 | 1.716 | 1.262 | 0.855 |
| **4** | 0.945 | 0.921 | 0.905 | 0.855 |  | 2.943 | 2.557 | 1.190 | 0.661 |
| **5** | 0.951 | 0.935 | 0.915 | 0.864 |  | 0.690 | 0.621 | 0.438 | 0.602 |
| **6** | 0.942 | 0.934 | 0.912 | 0.864 |  | 1.625 | 0.633 | 0.355 | 0.795 |
| **7** | 0.934 | 0.927 | 0.914 | 0.869 |  | 0.369 | 0.311 | 0.213 | 0.661 |
| **8** | 0.949 | 0.929 | 0.916 | 0.831 |  | 1.002 | 0.855 | 0.566 | 1.001 |
| **9** | 0.943 | 0.928 | 0.913 | 0.907 |  | 1.642 | 1.404 | 0.922 | 0.152 |
| **10** | 0.942 | 0.939 | 0.907 | 0.893 |  | 2.520 | 1.795 | 1.082 | 0.406 |
| **11** | 0.947 | 0.934 | 0.918 | 0.875 |  | 0.460 | 0.568 | 0.412 | 0.658 |
| **12** | 0.947 | 0.936 | 0.927 | 0.885 |  | 1.126 | 0.580 | 0.248 | 0.515 |
| **13** | 0.933 | 0.933 | 0.909 | 0.889 |  | 0.370 | 0.276 | 0.229 | 0.470 |
| **14** | 0.942 | 0.932 | 0.917 | 0.891 |  | 0.998 | 0.736 | 0.617 | 0.618 |
| **15** | 0.949 | 0.935 | 0.912 | 0.894 |  | 1.645 | 1.201 | 1.014 | 0.516 |
| **16** | 0.954 | 0.920 | 0.928 | 0.873 |  | 1.902 | 2.020 | 0.792 | 0.773 |
| **17** | 0.947 | 0.940 | 0.928 | 0.916 |  | 0.659 | 0.432 | 0.307 | 0.133 |
| **18** | 0.945 | 0.943 | 0.916 | 0.899 |  | 1.346 | 0.440 | 0.347 | 0.343 |
| **19** | 0.946 | 0.930 | 0.921 | 0.894 |  | 0.314 | 0.250 | 0.190 | 0.551 |
| **20** | 0.946 | 0.932 | 0.926 | 0.905 |  | 0.827 | 0.656 | 0.496 | 0.434 |
| **21** | 0.949 | 0.934 | 0.920 | 0.906 |  | 1.342 | 1.061 | 0.801 | 0.396 |
| **22** | 0.949 | 0.932 | 0.931 | 0.907 |  | 1.734 | 1.540 | 0.753 | 0.518 |
| **23** | 0.945 | 0.945 | 0.935 | 0.907 |  | 0.449 | 0.410 | 0.297 | 0.434 |
| **24** | 0.947 | 0.943 | 0.930 | 0.883 |  | 0.998 | 0.421 | 0.245 | 0.643 |
| **25** | 0.940 | 0.937 | 0.930 | 0.929 |  | 0.246 | 0.156 | 0.146 | 0.106 |
| **26** | 0.950 | 0.942 | 0.935 | 0.925 |  | 0.627 | 0.390 | 0.370 | 0.268 |
| **27** | 0.948 | 0.941 | 0.929 | 0.915 |  | 0.996 | 0.616 | 0.587 | 0.426 |
| **28** | 0.947 | 0.949 | 0.932 | 0.921 |  | 1.475 | 0.447 | 0.680 | 0.336 |
| **29** | 0.952 | 0.944 | 0.937 | 0.921 |  | 0.297 | 0.310 | 0.280 | 0.308 |
| **30** | 0.946 | 0.947 | 0.935 | 0.918 |  | 0.699 | 0.313 | 0.167 | 0.400 |
| **31** | 0.943 | 0.940 | 0.936 | 0.925 |  | 0.208 | 0.169 | 0.130 | 0.336 |
| **32** | 0.944 | 0.944 | 0.935 | 0.911 |  | 0.524 | 0.424 | 0.324 | 0.494 |
| **33** | 0.947 | 0.943 | 0.937 | 0.950 |  | 0.830 | 0.670 | 0.512 | 0.057 |
| **34** | 0.951 | 0.941 | 0.941 | 0.942 |  | 1.063 | 0.953 | 0.484 | 0.140 |
| **35** | 0.953 | 0.946 | 0.939 | 0.944 |  | 0.292 | 0.269 | 0.198 | 0.219 |
| **36** | 0.951 | 0.948 | 0.939 | 0.945 |  | 0.631 | 0.274 | 0.165 | 0.175 |
| **37** |  |  |  | 0.946 |  |  |  |  | 0.160 |
| **38** |  |  |  | 0.942 |  |  |  |  | 0.206 |
| **39** |  |  |  | 0.946 |  |  |  |  | 0.175 |
| **40** |  |  |  | 0.941 |  |  |  |  | 0.253 |

**The related R code for the method proposed by Lin and Wang (2013)**

N = 10000

MM = 10000

n = c(10,10,10,10,10)

Var = c(1,0.1,0.1,0.1,1) \* 1

k = length(n)

Nu0 = nu1 = 0

Nu = rep(nu1, k)

Mu = Nu - Var / 2

alpha = 0.05

#===============================Power Simulations===========================

L1 = L2 = array(0, MM)

Rt = array(0, N)

count1 = count2 = 0

M = matrix(rep(n, N), byrow=T, ncol=k, nrow=N)

M1 = M - 1

for(i in 1:MM){

 X1 = rnorm(k, Mu, sqrt(Var / n))

 S2 = Var \* rchisq(k, n - 1) / (n - 1)

 VrG = (n-1)\*S2/(n\*(n-3))+(n-1)^2\*S2^2/(2\*(n-5)\*(n-3)^2)

 W = sum(1 / VrG)

 VrR = 1 / W #Variance of the R

 W = 1 / (VrG \* W)

 MUG = X1 + (n-1)\*S2/(2\*(n-3))

 MUR = sum(MUG \* W) #Mean of the R

 S2N = matrix(rep(S2, N), byrow=T, ncol=k, nrow=N)

 X1N = matrix(rep(X1, N), byrow=T, ncol=k, nrow=N)

 Sms = matrix(rep(sqrt(S2 / n), N), byrow=T, ncol=k, nrow=N)

 T = matrix(rt(N \* k, n - 1), byrow=T, nrow=N, ncol=k)

 U = matrix(rchisq(N \* k, n - 1), byrow=T, nrow=N, ncol=k)

 RS2 = M1 \* S2N / U

 RMU = X1N - T \* sqrt(S2N / M)

 RG = RMU + RS2 / 2

 R = RG %\*% W

 RTilda = (R - MUR) / sqrt(VrR)

 for(j in 1:N) Rt[j] = mean(abs(RTilda-RTilda[j]))

 Qa = quantile(Rt, prob=1 - alpha, names=F)

 g = function(eta0) mean(abs(eta0 - RTilda)) - Qa

 Upper = sqrt(VrR) \* uniroot(g,lower=0,upper=10)$root + MUR

 Lower = sqrt(VrR) \* uniroot(g,lower=-10,upper=0)$root + MUR

 if(Nu0 >= Lower & Nu0 <= Upper) count1 =count1 + 1

 L1[i] = Upper - Lower

}

#--------------------coverage probability------------------------------------

count1 / MM #Lin and Wang 2013

mean(L1)