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Accurate Independent Domination in Graphs

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Abstract: A dominating set D of a graph $G = (V, E)$ is an *independent dominating set*, if the induced subgraph $\langle D\rangle$ has no edges. An independent dominating set D of G is an *accurate independent dominating set* if $V - D$ has no independent dominating set of cardinality $|D|$. The *accurate independent domination number* $i_a(G)$ of G is the minimum cardinality of an accurate independent dominating set of G. In this paper, we initiate a study of this new parameter and obtain some results concerning this parameter.

Key Words: Domination, independent domination number, accurate independent domination number, Smarandache H-dominating set.

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§1. Introduction

All graphs considered here are finite, nontrivial, undirected with no loops and multiple edges. For graph theoretic terminology we refer to Harary [1].

Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$. Let $\Delta(G)(\delta(G))$ denote the *maximum (minimum)* degree and $\lceil x\rceil(\lfloor x\rfloor)$ the *least (greatest)* integer greater(less) than or equal to x. The *neighborhood* of a vertex u is the set $N(u)$ consisting of all vertices v which are adjacent with u. The *closed neighborhood* is $N[u] = N(u) \cup \{u\}$. A set of vertices in G is *independent* if no two of them are adjacent. The largest number of vertices in such a set is called the *vertex independence number* of G and is denoted by $\beta_o(G)$. For any set S of vertices of G, the *induced subgraph* $\langle S\rangle$ is maximal subgraph of G with vertex set S.

The *corona* of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2. A *wounded spider* is the graph formed by subdividing at most $n-1$ of the edges of a star $K_{1,n}$ for $n \geq 0$. Let $\Omega(G)$ be the set of all pendant vertices of G, that is the set of vertices of degree 1. A vertex v is called a support vertex if v is neighbor of a pendant vertex and $d_G(v) > 1$. Denote by $X(G)$ the set of all support vertices in G, $M(G)$ be the set

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of vertices which are adjacent to support vertex and $J(G)$ be the set of vertices which are not adjacent to a support vertex. The diameter $diam(G)$ of a connected graph G is the maximum distance between two vertices of G, that is $diam(G) = max_{u,v\in V(G)}\ d_G(u,v)$. A set $B \subseteq V$ is a *2-packing* if for each pair of vertices $u, v \in B, N_G[u] \cap N_G[v] = \phi$

A *proper coloring* of a graph $G = (V(G), E(G))$ is a function from the vertices of the graph to a set of *colors* such that any two adjacent vertices have different colors. The chromatic number $\chi(G)$ is the minimum number of colors needed in a proper coloring of a graph. A *dominator coloring* of a graph G is a proper coloring in which each vertex of the graph dominates every vertex of some color class. The *dominator chromatic number* $\chi_d(G)$ is the minimum number of color classes in a dominator coloring of a graph G. This concept was introduced by R. Gera at.al [3].

A set D of vertices in a graph $G = (V, E)$ is a *dominating set* of G, if every vertex in $V - D$ is adjacent to some vertex in D. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a dominating set. For a comprehensive survey of domination in graphs, see [4, 5, 7].

Generally, if $\langle D\rangle \simeq H$, such a dominating set D is called a *Smarandache H-dominating set*. A dominating set D of a graph $G = (V, E)$ is an *independent dominating set*, if the induced subgraph $\langle D\rangle$ has no edges, i.e., a Smarandache H-dominating set with $E(H) = \emptyset$. The *independent domination number* $i(G)$ is the minimum cardinality of an independent dominating set.

A dominating set D of $G = (V, E)$ is an *accurate dominating set* if $V - D$ has no dominating set of cardinality $|D|$. The *accurate domination number* $\gamma_a(G)$ of G is the minimum cardinality of an accurate dominating set. This concept was introduced by Kulli and Kattimani [6, 9].

An independent dominating set D of G is an *accurate independent dominating set* if $V - D$ has no independent dominating set of cardinality $|D|$. The *accurate independent domination number* $i_a(G)$ of G is the minimum cardinality of an accurate independent dominating set of G. This concept was introduced by Kulli [8].

For example, we consider the graph G in Figure 1. The accurate independent dominating sets are $\{1, 2, 6, 7\}$ and $\{1, 3, 6, 7\}$. Therefore $i_a(G) = 4$.

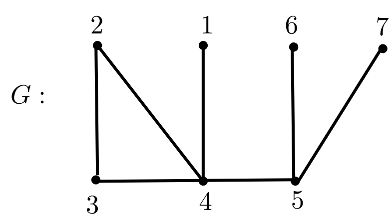


Figure 1

§2. Results

Observation 2.1

1. Every accurate independent dominating set is independent and dominating. Hence it is a minimal dominating set.

2. Every minimal accurate independent dominating set is a maximal independent dominating set.

Proposition 2.1 *For any nontrivial connected graph G, $\gamma(G) \leq i_a(G)$.*

Proof Clearly, every accurate independent dominating set of G is a dominating set of G. Thus result holds. □

Proposition 2.2 *If G contains an isolated vertex, then every accurate dominating set is an accurate independent dominating set.*

Now we obtain the exact values of $i_a(G)$ for some standard class of graphs.

Proposition 2.3 *For graphs P_p, W_p and $K_{m,n}$, there are*

(1) $i_a(P_p) = \lceil p/3 \rceil$ *if* $p \geq 3$;
(2) $i_a(W_p) = 1$ *if* $p \geq 5$;
(3) $i_a(K_{m,n}) = m$ *for* $1 \leq m < n$.

Theorem 2.1 *For any graph G, $i_a(G) \leq p - \gamma(G)$.*

Proof Let D be a minimal dominating set of G. Then there exist at least one accurate independent dominating set in $(V - D)$ and by proposition 2.1,

$$i_a(G) \leq |V| - |D| \leq p - \gamma(G).$$

Notice that the path P_4 achieves this bound. □

Theorem 2.2 *For any graph G,*

$$\lceil p/ \vartriangle +1 \rceil \leq i_a(G) \leq \lfloor p \vartriangle / \vartriangle +1 \rfloor$$

and these bounds are sharp.

Proof It is known that $p/ \vartriangle +1 \leq \gamma(G)$ and by proposition 2.1, we see that the lower bound holds. By Theorem 2.1,

$$\begin{aligned} i_a(G) &\leq p - \gamma(G), \\ &\leq p - p/ \vartriangle +1 \\ &\leq p \vartriangle / \vartriangle +1. \end{aligned}$$

Notice that the path $P_p, p \geq 3$ achieves the lower bound. This completes the proof. □

Proposition 2.4 *If $G = K_{m_1, m_2, m_3, \cdots, m_r}, r \geq 3$, then*

$$i_a(G) = m_1 \quad if \quad m_1 < m_2 < m_3 \cdots < m_r.$$

Theorem 2.3 *For any graph G without isolated vertices* $\gamma_a(G) \leq i_a(G)$ *if* $G \neq K_{m_1,m_2,m_3,\cdots,m_r}, r \geq 3$. *Furthermore, the equality holds if* $G = P_p(p \neq 4, p \geq 3), W_p(p \geq 5)$ *or* $K_{m,n}$ *for* $1 \leq m < n$.

Proof Since we have $\gamma(G) \leq \gamma_a(G)$ and by Proposition 2.1,$\gamma_a(G) \leq i_a(G)$.

Let $\gamma_a(G) \leq i_a(G)$. If $G = K_{m_1,m_2,m_3,\cdots,m_r}$, $r \geq 3$ then by Proposition 2.4, $i_a(G) = m_1$ if $m_1 < m_2 < m_3 \cdots < m_r$ and also accurate domination number is $\lfloor p/2 \rfloor + 1$ i.e., $\gamma_a(G) = \lfloor p/2 \rfloor + 1 > m_1 = i_a(G)$, a contradiction. □

Corollary 2.1 *For any graph G,* $i_a(G) = \gamma(G)$ *if* $diam(G) = 2$.

Proposition 2.5 *For any graph G without isolated vertices* $i(G) \leq i_a(G)$. *Furthermore, the equality holds if* $G = P_p$ $(p \geq 3)$, W_p $(p \geq 5)$ *or* $K_{m,n}$ *for* $1 \leq m < n$.

Proof Every accurate independent dominating set is a independent dominating set. Thus result holds. □

Definition 2.1 *The double star* $S_{n,m}$ *is the graph obtained by joining the centers of two stars* $K_{1,n}$ *and* $K_{1,m}$ *with an edge.*

Proposition 2.6 *For any graph G,* $i_a(G) \leq \beta_o(G)$. *Furthermore, the equality holds if* $G = S_{n,m}$.

Proof Since every minimal accurate independent dominating set is an maximal independent dominating set. Thus result holds. □

Theorem 2.4 *For any graph G,* $i_a(G) \leq p - \alpha_0(G)$.

Proof Let S be a vertex cover of G. Then $V - S$ is an accurate independent dominating set. Then $i_a(G) \leq |V - S| \leq p - \alpha_0(G)$. □

Corollary 2.2 *Fr any graph G,* $i_a(G) \leq p - \beta_0(G) + 2$.

Theorem 2.5 *If G is any nontrivial connected graph containing exactly one vertex of degree* $\triangle\ (G) = p - 1$, *then* $\gamma(G) = i_a(G) = 1$.

Proof Let G be any nontrivial connected graph containing exactly one vertex v of degree $deg(v) = p - 1$. Let D be a minimal dominating set of G containing vertex of degree $deg(v) = p = 1$. Then D is a minimum dominating set of G i.e.,

$$|D| = \gamma(G) = 1. \tag{1}$$

Also $V - D$ has no dominating set of same cardinality $|D|$. Therefore,

$$|D| = i_a(G). \tag{2}$$

Hence, by (1) and (2) $\gamma(G) = i_a(G) = 1$. □

Theorem 2.6 *If G is a connected graph with p vertices then $i_a(G) = p/2$ if and only if $G = H \circ K_1$,where H is any nontrivial connected graph.*

Proof Let D be any minimal accurate independent dominating set with $|D| = p/2$. If $G \neq H \circ K_1$ then there exist at least one vertex $v_i \in V(G)$ which is neither a pendant vertex nor a support vertex. Then there exist a minimal accurate independent dominating set D' containing v_i such that

$$|D'| \leq |D| - \{v_i\} \leq p/2 - \{v_i\} \leq p/2 - 1,$$

which is a contradiction to minimality of D.

Conversely, let l be the set of all pendant vertices in $G = H \circ K_1$ such that $|l| = p/2$. If $G = H \circ K_1$, then there exist a minimal accurate independent dominating set $D \subseteq V(G)$ containing all pendant vertices of G. Hence $|D| = |l| = p/2$. □

Now we characterize the trees for which $i_a(T) = p - \Delta(T)$.

Theorem 2.7 *For any tree T, $i_a(T) = p - \Delta(T)$ if and only if T is a wounded spider and $T \neq K_1,\ K_{1,1}$.*

Proof Suppose T is wounded spider. Then it is easy to verify that
$i_a(T) = p - \Delta(T)$.

Conversely, suppose T is a tree with $i_a(T) = p - \Delta(T)$. Let v be a vertex of maximum degree $\Delta(T)$ and u be a vertex in $N(v)$ which has degree 1. If $T - N[v] = \phi$ then T is the star $K_{1,n}, n \geq 2$. Thus T is a double wounded spider. Assume now there is at least one vertex in $T - N[v]$. Let S be a maximal independent set of $\langle T - N[v]\rangle$. Then either $S \cup \{v\}$ or $S \cup \{u\}$ is an accurate independent dominating set of T. Thus $p = i_a(T) + \Delta(T) \leq |S| + 1 + \Delta(T) \leq p$. This implies that $V - N(v)$ is an accurate independent dominating set. Furthermore, $N(v)$ is also an accurate independent dominating set.

The connectivity of T implies that each vertex in $V - N[v]$ must be adjacent to at least one vertex in $N(v)$. Moreover if any vertex in $V - N[v]$ is adjacent to two or more vertices in $N(v)$, then a cycle is formed. Hence each vertex in $V - N[v]$ is adjacent to exactly one vertex in $N(v)$. To show that $\Delta(T) + 1$ vertices are necessary to dominate T, there must be at least one vertex in $N(v)$ which are not adjacent to any vertex in $V - N[v]$ and each vertex in $N(v)$ has either 0 or 1 neighbors in $V - N[v]$. Thus T is a wounded spider. □

Proposition 2.7 *If G is a path P_p, $p \geq 3$ then $\gamma(P_p) = i_a(P_p)$.*

We characterize the class of trees with equal domination and accurate independent domination number in the next section.

§3. Characterization of (γ, i_a)-Trees

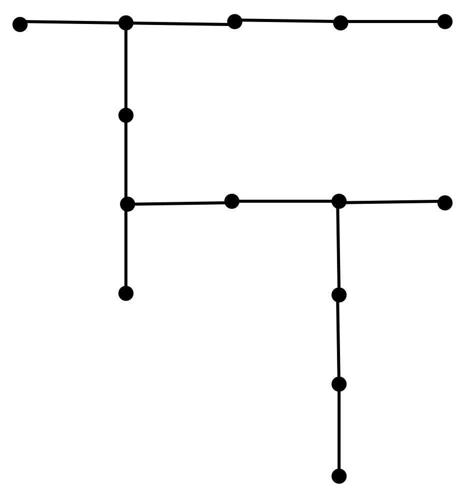
For any graph theoretical parameter λ *and* μ, we define G to be (λ, μ)-graph if $\lambda(G) =$

$\mu(G)$. Here we provide a constructive characterization of (γ, i_a)-trees.

To characterize (γ, i_a)-trees we introduce family τ_1 of trees $T = T_k$ that can be obtained as follows. If k is a positive integer, then T_{k+1} can be obtained recursively from T_k by the following operation.

Operation O Attach a path P_3(x,y,z) and an edge mx, where m is a support vertex of a tree T.

$$\tau = \{T/\text{obtained from } P_5 \text{ by finite sequence of operations of } O\}$$



Tree T belonging to family τ_1

Observation 3.1 If $T \in \tau$, then

1. $i_a(T) = \lceil p + 1/3 \rceil$;
2. $X(T)$ is a minimal dominating set as well as a minimal accurate independent dominating set of T;
3. $\langle V - D \rangle$ is totally disconnected.

Corollary 3.1 *If tree T with $p \geq 5$ belongs to the family τ then $\gamma(T) = |X(T)|$ and $i_a(T) = |X(T)|$.*

Lemma 3.1 *If a tree T belongs to the family τ then T is a $(\gamma, i_a) - tree$.*

Proof If $T = P_p,\ p \geq 3$ then from proposition 2.7 T is a $(\gamma, i_a) - tree$. Now if $T = P_p,\ p \geq 3$ then we proceed by induction on the number of operations $n(T)$ required to construct the tree T. If $n(T) = 0$ then $T \in P_5$ by proposition 2.7 T is a (γ, i_a)-tree.

Assume now that T is a tree belonging to the family τ with $n(T) = k$, for some positive integer k and each tree $T' \in \tau$ with $n(T') < k$ and with $V(T') \geq 5$ is a (γ, i_a)-tree in which $X(T')$ is a minimal accurate independent dominating set of T'. Then T can be obtained from a tree T' belonging to τ by operation O where $m \in V(T') - (M(T') - \Omega(T'))$ and we add path (x, y, z) and the edge mx. Then z is a pendant vertex in T and y is a support vertex and $x \in M(T)$. Thus $S(T) = X(T') \cup \{y\}$ is a minimal accurate independent dominating set of T. Therefore $i_a(T) \geq |X(T)| = |X(T')| + 1$. Hence we conclude that $i_a(T) = i_a(T') + 1$. By the induction hypothesis and by observation 3.1(2) $i_a(T') = \gamma(T') = |X(T')|$. In this way $i_a(T) = |X(T)|$ and in particular $i_a(T) = \gamma(T)$. □

Lemma 3.2 *If T is a $(\gamma, i_a) - tree$, then T belongs to the family τ.*

Proof If T is a path $P_p,\ p \geq 3$ then by proposition 2.7 T is a $(\gamma, i_a) - tree$. It is easy to verify that the statement is true for all trees T with diameter less than or equal to 4. Hence we may assume that $diam(T) \geq 4$. Let T be rooted at a support vertex m of a longest path P. Let P be a $m - z$ path and let y be the neighbor of z. Further, let x be a vertex belongs to $M(T)$. Let T be a (γ, i_a)-tree. Now we proceed by induction on number of vertices $|V(T)|$ of a (γ, i_a)-tree. Let T be a (γ, i_a)-tree and assume that the result holds good for all trees on $V(T) - 1$ vertices. By observation 3.1(2) since T is (γ, i_a)-tree it contains minimal accurate independent dominating set D that contains all support vertices of a tree. In particular $\{m, y\} \subset D$ and the vertices x and z are independent in $\langle V - D\rangle$.

Let $T' = T - (x, y, z)$. Then $D - \{y\}$is dominating set of T' and so $\gamma(T') \leq \gamma(T) - 1$. Any dominating set can be extended to a minimal accurate independent dominating set of T by adding to it the vertices (x, y, z) and so $i_a(T) \leq i_a(T') + 1$. Hence, $i_a(T') \leq \gamma(T') \leq \gamma(T) + 1 \leq i_a(T) - 1 \leq i_a(T')$. Consequently, we must have equality throughout this inequality chain. In particular $i_a(T') = \gamma(T')$ and $i_a(T) = i_a(T') + 1$. By inductive hypothesis any minimal accurate independent dominating set of a tree T' can be extended to minimal accurate independent dominating set of a tree T by operation O. Thus $T \in \tau$. □

As an immediate consequence of lemmas 3.1 and 3.2, we have the following characterization of trees with equal domination and accurate independent domination number.

Theorem 3.1 *Let T be a tree. Then $i_a(T) = \gamma(T)$ if and only if $T \in \tau$.*

§4. Accurate Independent Domination of Some Graph Families

In this section accurate independent domination of *fan graph*,*double fan graph*, *helm graph* and *gear graph* are considered. We also obtain the corresponding relation between other dominating parameters and dominator coloring of the above graph families.

Definition 4.1 *A fan graph, denoted by F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex.*

Observation 4.1 Let F_n be a fan. Then,

1. F_n is a planar undirected graph with $2n + 1$ vertices and $3n$ edges;
2. F_n has exactly one vertex with $\Delta(F_n) = p - 1$;
3. $Diam(F_n) = 2$.

Theorem 4.1([2]) *For a fan graph* $F_n, n \geq 2,$ $\chi_d(F_n) = 3$.

Proposition 4.1 *For a fan graph* $F_n, n \geq 2,$ $i_a(F_n) = 1$.

Proof By Observation 4.1(2) and Theorem 2.5 result holds. □

Proposition 4.2 *For a fan graph* $F_n, n \geq 2,$ $i_a(F_n) < \chi_d(F_n)$.

Proof By Proposition 4.1 and Theorem 4.1, we know that $\chi_d(F_n) = 3$. This implies that $i_a(F_n) < \chi_d(F_n)$. □

Definition 4.2 *A double fan graph, denoted by* $F_{2,n}$ *isomorphic to* $P_n + 2K_1$.

Observation 4.2

1. $F_{2,n}$ is a planar undirected graph with $(n+2)$ vertices and $(3n-1)$ edges;
2. $Diam(G) = 2$.

Theorem 4.2([2]) *For a double fan graph* $F_{2,n}, n \geq 2,$ $\chi_d(F_{2,n}) = 3$.

Theorem 4.3 *For a double fan graph* $F_{2,n},$ $n \geq 2$, $i_a(F_{2,2}) = 2$, $i_a(F_{2,3}) = 1$, $i_a(F_{2,5}) = 3$ *and* $i_a(F_{2,n}) = 2$ if $n \geq 7$.

Proof Our proof is divided into cases following.

Case 1. If $n = 2$ *and* $n \geq 7$, then $F_{2,n},$ $n \geq 2$ has only one accurate independent dominating set D of $|D| = 2$. Hence, $i_a(F_{2,n}) = 2$.

Case 2. If $n = 3,$ $then F_{2,3}$ has exactly one vertex of $\Delta(G) = p - 1$. Then by Theorem 2.5, $i_a(F_{2,n}) = 1$.

Case 3. If n=5 and D be a independent dominating set of G with $|D| = 2$, then $(V - D)$ also has an independent dominating set of cardinality 2. Hence D is not accurate.

Let D_1 be a independent dominating set with $|D_1| = 3$, then $V - D_1$ has no independent dominating set of cardinality 3. Then D_1 is accurate. Hence, $i_a(F_{2,n}) = 3$.

Case 4. If n=4 and 6, there does not exist accurate independent dominating set. □

Proposition 4.3 *For a double fan graph* $F_{2,n}$, $n \geq 7$,

$$\gamma(F_{2,n}) = i(F_{2,n}) = \gamma_a(F_{2,n}) = i_a(F_{2,n}) = 2$$

.

Proof Let $F_{2,n},$ $n \geq 7$ be a Double fan graph. Then $2k_1$ forms a minimal dominating set of $F_{2,n}$ such that $\gamma(F_{2,n}) = 2$. Since this dominating set is independent and in $(V - D)$ there is no independent dominating set of cardinality 2 it is both independent and accurate independent dominating set. Also it is accurate dominating set. Hence,

$$\gamma(F_{2,n}) = i(F_{2,n}) = \gamma_a(F_{2,n}) = i_a(F_{2,n}) = 2.$$ □

Proposition 4.4 *For Double fan graph $F_{2,n}$, $n \geq 7$*

$$i_a(F_{2,n}) \leq \chi_d(F_{2,n}).$$

Proof The proof follows by Theorems 4.2 and 4.3. □

Definition 4.3([1]) *For $n \geq 4$,the wheel W_n is defined to be the graph $W_n = C_{n-1} + K_1$. Also it is defined as $W_{1,n} = C_n + K_1$.*

Definition 4.4 *A helm H_n is the graph obtained from $W_{1,n}$ by attaching a pendant edge at each vertex of the n-cycle.*

Observation 4.3 A helm H_n is a planar undirected graph with (2n+1) vertices and 3n edges.

Theorem 4.4([2]) *For Helm graph H_n , $n \geq 3, \chi_d(H_n) = n + 1$.*

Proposition 4.5 *For a helm graph H_n, $n \geq 3$, $i_a(H_n) = n$.*

Proof Let H_n , $n \geq 3$ be a helm graph. Then there exist a minimal independent dominating set D with $|D| = n$ and $(V - D)$ has no independent dominating set of cardinality n. Hence D is accurate. Therefore $i_a(H_n) = n$. □

Proposition 4.6 *For a helm graph H_n , $n \geq 3$*

$$\gamma(H_n) = i(H_n) = \gamma_a(H_n) = i_a(H_n) = n.$$

Proposition 4.7 *For a helm graph H_n , $n \geq 3$*

$$i_a(H_n) = \chi_d(H_n) - 1.$$

Proof Applying Proposition 4.5 , $i_a(H_n) = n = n + 1 - 1 = \chi_d(H_n) - 1$ by Theorem 4.4, $\chi_d(H_n) = n + 1$. Hence the proof. □

Definition 4.5 *A gear graph G_n also known as a bipartite wheel graph, is a wheel graph $W_{1,n}$ with a vertex added between each pair of adjacent vertices of the outer cycle.*

Observation 4.4 *A gear graph G_n is a planar undirected graph with $2n + 1$ vertices and $3n$ edges.*

Theorem 4.5([2]) *For a gear graph G_n , $n \geq 3$,*

$$\chi_d(G_n) = \lceil 2n/3 \rceil + 2.$$

Theorem 4.6 *For a gear graph G_n , $n \geq 3$, $i_a(G_n) = n$.*

Proof It is clear from the definition of gear graph G_n is obtained from wheel graph $W_{1,n}$ with a vertex added between each pair of adjacent vertices of the outer cycle of wheel graph $W_{1,n}$. These n vertices forms an independent dominating set in G_n such that $(V - D)$ has no independent dominating set of cardinality n. Therefore, the set D with cardinality n is accurate independent dominating set of G_n. Therefore $i_a(G_n) = n$. □

Corollary 4.1 *For any gear graph G_n, $n \geq 3, \gamma(G_n) = i(G_n) = n - 1$.*

Proposition 4.8 *For a gear graph G_n , $n \geq 3$,*

$$i_a(G_n) = \gamma_a(G_n).$$

Proposition 4.9 *For a graph G_n , $n \geq 3$*

$$i_a(G_n) = \gamma(G_n) + 1 = i(G_n) + 1.$$

Proof Applying Theorem 4.6 and Corollary 4.1, we know that $i_a(G_n) = n = n - 1 + 1 = \gamma(G_n) + 1 = i(G_n) + 1$. □

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