A Voltage Control Scheme for Generation-Dominated Networks to Maximise Power Export

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***Abstract*—This letter describes a voltage control scheme which maximises the exported active power from generation-dominated networks. The proposed scheme exploits recent theoretical advances which rigorously characterise the origins of power losses in electrical networks. In situations where load-driven losses can be disregarded, these theoretical advances simplify the calculations and data inputs required by an export-maximizing optimal control scheme.**

I. Introduction

Whether in onshore or offshore contexts, it is increasingly common to construct expansive electrical networks for the harvesting of renewable energy [1]. In the absence of substantial loads on these networks, power quality concerns are subordinate and it becomes viable to exploit the voltage control capabilities of individual renewable generators to minimize network active power losses, and thus to maximise the aggregate export of clean power to the wider system. Extant approaches to loss minimization typically involve formulating the full AC power flow equations and solving to an optimal reactive power dispatch [2]. The present work demonstrates that a simplified, more lightweight formulation of the loss-minimization problem is possible in generation-dominated networks. A novel result is also provided on how to monitor generator outputs in such networks.

II. Methodology

1) Characterising network power losses.: To begin, the $\boldsymbol{Y}_{\text{bus}}$ matrix is reordered per [3], such that the m generator buses and n load buses are grouped together:

$$\begin{bmatrix}\boldsymbol{i}_G\\ \boldsymbol{i}_L\end{bmatrix} = \begin{bmatrix}\boldsymbol{Y}_{GG} & \boldsymbol{Y}_{GL}\\ \boldsymbol{Y}_{LG} & \boldsymbol{Y}_{LL}\end{bmatrix}\begin{bmatrix}\boldsymbol{v}_G\\ \boldsymbol{v}_L\end{bmatrix} \tag{1}$$

Work by Abdelkader [4] has shown that the total prevailing power loss within a power system is equal to:

$$v_G^T\boldsymbol{Y}^*_{GGM}v_G^* + v_G^T(\boldsymbol{Y}^*_{GL}\boldsymbol{Z}^*_{LL} - \boldsymbol{Y}^T_{LG}\boldsymbol{Z}^T_{LL})i_L^* + i_L^T\boldsymbol{Z}^T_{LL}i_L^* \tag{2}$$

Where $\boldsymbol{Z}_{LL} = \boldsymbol{Y}_{LL}^{-1}$ and $\boldsymbol{Y}_{GGM} = \boldsymbol{Y}_{GG} - \boldsymbol{Y}_{GL}\boldsymbol{Z}_{LL}\boldsymbol{Y}_{LG}$.

Abdelkader's loss characterisation is seen to consist of three distinct components. The first of these, the *circulating current loss* is directly controllable by the system operator, as it depends on the vector $\boldsymbol{v}_G$, itself dependant on each generator's active power output and voltage magnitude setpoint. The central term, the *mismatch loss*, turns out to be negligible for typical networks [4]. The final term, the *load current* loss, cannot be directly affected by the system operator as it solely depends on i_L, but in any case is insignificant in generation-dominated networks. Recent work [6] has used Abdelkader's characterisation to articulate a closed-form equation for the generator dispatch that nullifies the circulating current term and brings losses to their theoretical minimum by equalising generator complex voltages. The present control scheme builds on this insight, but instead uses reactive power resources to *manage* the circulating currents to maximise the export of active power.

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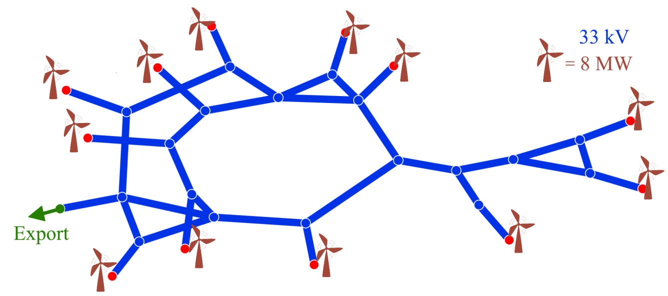

Fig. 1: A diagram of the offshore energy harvesting network, drawn using techniques in [5]

2) Characterising power injections.: While equation (2) shows the terms which sum to give the scalar of total losses, in [7] this paradigm is extended to likewise attribute individual branch current flows to the same three distinct causes. In brief, one component of branch current flows is affected only by load currents i_L, whereas a distinct component, the *circulating current*, exists because of heterogeneous generator voltages, $\boldsymbol{v}_G$. In the absence of meaningful load-serving currents i_L in a generation-dominated network, each generator's output must therefore principally consist of a circulating current component, the formula for which is given in [7] as:

$$\boldsymbol{i}^{Circ} = \boldsymbol{Y}_{GGM}\boldsymbol{v}_G \tag{3}$$

Multiplying element-by-element with the relevant connection point voltages gives the vector of circulating active power injected by each generator:

$$\boldsymbol{p}^{Circ} = \Re(\boldsymbol{v}_G \circ \boldsymbol{i}^*_{Circ}) \tag{4}$$

As this is the only power component injected by generators in this type of network, we note that monitoring active power outputs merely requires observability of the complex vector of generator voltages, v_G, and access to the static Y_{GGM} matrix. The vector v_G can readily be observed with phasor measurement units (PMU).

TABLE I:
CONTROL SCHEME PERFORMANCE

	Convergence	Mean Runtime	Total Exports
Baseline	100%	-	2926 MWh
ACOPF	97%	0.07 s	2949 MWh
Proposed	100%	1.3 s	2951 MWh

TABLE II:
OPTIMIZATION FORMULATION SIZE

	Optimization Variables	Inequality Constraints	Equality Constraints
ACOPF	88	70	62
Proposed	52	39	39

3) Novel optimal voltage control scheme.: Using the foregoing insights, it is possible to optimally select the controlled voltage magnitudes $|v_G|$ to maximise the network's export of active power, which is equivalent to minimising total losses. This is achieved by maximising the power absorbed at the slack generator, as described using equation (4):

$$max(\boldsymbol{p}_{Slack}^{Circ}) \tag{5}$$

The generator voltage magnitude setpoints $|v_G|$ are the sole control variables. The optimal voltage profile must however maintain the active power injections of the non-slack generators at their initial values $\boldsymbol{P}_{init}$ (these can be inferred from equation (4) using PMU measurements)

$$p_{Nonslack}^{Circ} = \boldsymbol{P}_{init} \tag{6}$$

Constraint (6) ensures that, while generator voltage magnitudes may be controlled freely, the resulting complex voltage vector must maintain the power injection profile of the non-slack generators. Additionally, the controlled voltages must respect magnitude limits:

$$v^- \leq |\boldsymbol{v}_G| \leq v^+ \tag{7}$$

Taken together, equations (3), (4), (5), (6) and (7) are sufficient to model the export maximization problem. This allows an optimal voltage magnitude schedule to be calculated for all generators using just PMU data.

III. CASE STUDY

To build a notional energy harvesting network, loosely corresponding to an offshore wind farm network, the 33 kV distribution circuits were extracted from the `nesta_case30_ieee` system [8]. Nodal loadings were reduced to 5% of their original levels, for a total load of just 5.24 MW, and these minor loads correspond to equipment overheads. Twelve 8 MW turbines were added at various points in this network, as illustrated in figure 1, and these were assumed to enjoy the flexible voltage control capabilities typical of modern renewable generators [9]. The resulting total installed generation capacity, of 96 MW, is comparable to the original total loading in the network of 104.7 MW: this modified network is dominated by generation rather than by loads. Voltage limits were imposed as v^- = 0.95 and v^+ = 1.05.

One hundred network scenarios were created: in each, individual turbine wind speeds were sampled from a Rayleigh distribution with scale parameter $= 7ms^{-1}$ and these wind speeds were mapped to MW outputs using a power curve [10] with a cut-in speed of $3ms^{-1}$, a rated speed of $12ms^{-1}$ and a cut-out speed of $30ms^{-1}$. Each scenario notionally corresponds to one hour of operation.

The optimization was implemented in MATLAB [11] using MATPOWER [12], the YALMIP toolbox [13] and the IPOPT solver [14]. This case study, as well as calculated results and corresponding code, is available online at [15].

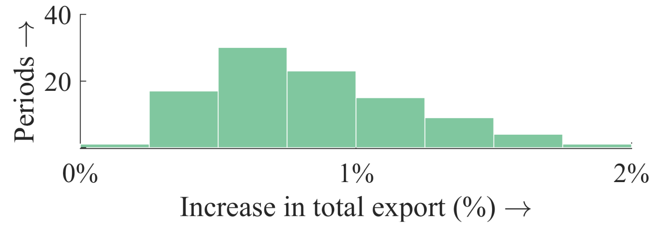

Fig. 2: A histogram showing the increase in wind power export, compared against the uncontrolled case

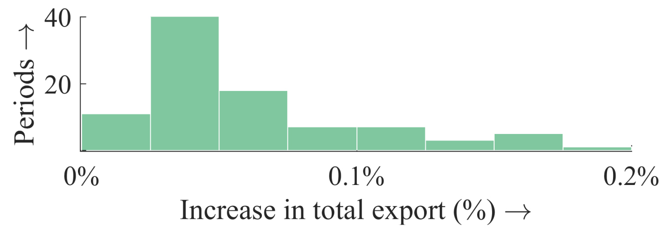

Fig. 3: A histogram showing the increase in wind power export, compared against the ACOPF in [11]

IV. RESULTS

Three power flow calculations were performed for each of the one hundred scenarios: first, a baseline case with all generator voltages set equal to $1pu$, then with generator voltage setpoints optimally selected by a conventional loss-minimising ACOPF implementation [12], and finally with the generator voltage setpoints given by the proposed optimization. Note that the proposed scheme inferred the required $\boldsymbol{P}_{init}$ parameters from only the voltage angles found by the initial power flow calculations, to reflect the technique's sole reliance on PMU data.

1) Results overview.: An overview of the results is provided in Table I. It can be seen that both active control schemes outperformed the uncontrolled baseline case, with the proposed scheme achieving slightly more energy exported over the hundred scenario hours. Also notable is that the proposed scheme achieved convergence in all periods, whereas the standard ACOPF failed to converge in three cases. However, the standard ACOPF was substantially faster on average.

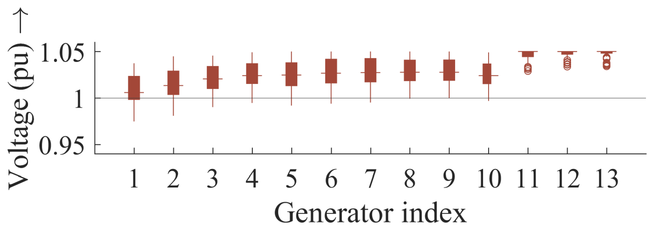

Fig. 4: A boxplot showing the distribution of controlled voltages for each PV bus over the hundred periods under the proposed regime

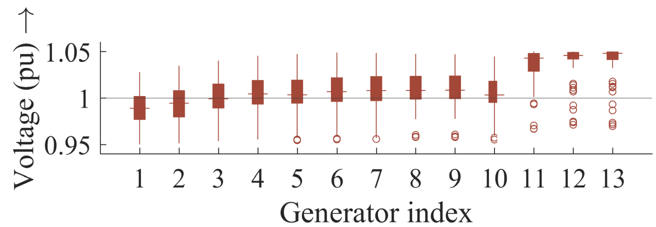

Fig. 5: A boxplot showing the distribution of controlled voltages for each PV bus over the hundred periods under the ACOPF regime

The summary in Table II shows that the proposed formulation results in a more compact optimization problem, and given the achieved results, it appears plausible that the tractability benefits of this reduction in problem size outweigh the minor inaccuracies introduced by the simplifying assumptions. Likewise, the more compact formulation likely aids convergence.

2) Percentage improvement over baseline case.: As shown in Figure 2, across every single period the proposed optimization scheme successfully reduced losses and increased the export of energy from the network, typically by around one percent, as compared against the baseline case. Net of internal loads, the available wind power ranged between 10 and 57 MW over these scenarios, so the percentage increase in exported power is meaningful. Not only do these results demonstrate the efficacy of the novel voltage control scheme, they likewise validate the use of (4) for power injection monitoring using PMU data.

3) Percentage improvement over ACOPF *case.:* The incremental improvement of the proposed scheme over the ACOPF case is shown in figure 3. The proposed scheme increased exports in every scenario, albeit modestly. Not only does the novel scheme permit a lightweight formulation, it appears to consistently outperform an established optimization implementation which enjoys full network observability.

4) Controlled voltage profiles.: The boxplots in figures 4 and 5 offer some insights into how this improved performance is realised. The distributions in figures 4 indicate that the generator voltages were typically controlled to be noticeably higher under the proposed scheme. These generally higher voltage profiles seemingly result in a slightly more optimal reduction in losses. Although the percentage increase in exports is modest, even a marginal improvement in the capacity factor of a wind harvesting network can affect its economic viability.

V. Conclusions

The presented results have demonstrated the viability of a lightweight optimal voltage control scheme, which needs only PMU data, to maximise harvested power from generation dominated networks. The proposed scheme exploits new theoretical findings which characterise the origins of power losses in electrical networks, and this characterisation permits a more compact problem formulation with reduced data requirements. As the control scheme requires only PMU data and has demonstrated superior convergence performance, it seems well-suited for real time control of energy harvesting networks.

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