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# FINITE ELEMENT INVESTIGATION OF MULTI-STORY POST-TENSIONED ROCKING FRAMES

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# ABSTRACT

Modern code-conforming buildings have a high probability of surviving major seismic events without collapse, hence minimizing the number of casualties. Nevertheless, the risk of substantial post-earthquake economic losses remains high, as a consequence of inadequate damage prevention guidelines in current earthquake design codes. Rocking post-tensioned moment-resisting frames present a viable damage-free structural solution, with a nominal increase in building costs compared with conventional buildings. This structural system comprises of: (i) unbonded posttensioned strands to provide overturning resistance and self-centering capability, and (ii) opening joints at the column-foundation and beam-column interfaces designed to rock during a seismic event. Rocking frames with various forms of supplemental damping have been previously examined numerically, adopting different finite element frameworks. However, there is a shortage of numerical studies studying the non-linear dynamic response of pure rocking multi-story posttensioned moment frames, exclusive of supplementary energy-dissipation elements and devices. Hence, it is critical to develop modelling procedures for multiple stories which adequately capture the full range of their nonlinear dynamic behavior due to the joint rocking mechanism, and investigate the resulting response. Numerical studies are presented herein, including static and dynamic analyses of three- to nine-story building models. The proposed modelling methods are shown to effectively predict the non-linear response of multi-story rocking frames over a wide range of forcing frequencies and amplitudes. It is further concluded that the structural response is influenced by both sub-harmonic resonances and beam-column interactions.

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Modern code-conforming buildings have a high probability of surviving major seismic events without collapse, hence minimizing the number of casualties. Nevertheless, the risk of substantial post-earthquake economic losses remains high, as a consequence of inadequate damage prevention guidelines in current earthquake design codes. Rocking post-tensioned moment-resisting frames present a viable damage-free structural solution, with a nominal increase in building costs compared with conventional buildings. This structural system comprises of: (i) unbonded post-tensioned strands to provide overturning resistance and self-centering capability, and (ii) opening joints at the column-foundation and beam-column interfaces designed to rock during a seismic event. Rocking frames with various forms of supplemental damping have been previously examined numerically, adopting different finite element frameworks. However, there is a shortage of numerical studies studying the non-linear dynamic response of pure rocking multi-story post-tensioned moment frames, exclusive of supplementary energy-dissipation elements and devices. Hence, it is critical to develop modelling procedures for multiple stories which adequately capture the full range of their nonlinear dynamic behavior due to the joint rocking mechanism, and investigate the resulting response. Numerical studies are presented herein, including static and dynamic analyses of three- to nine-story building models. The proposed modelling methods are shown to effectively predict the non-linear response of multi-story rocking frames over a wide range of forcing frequencies and amplitudes. It is further concluded that the structural response is influenced by both sub-harmonic resonances and beam-column interactions.

# Introduction

Major earthquake events regularly devastate communities across the globe, claiming lives, and causing widespread destruction of property, as well as disruption of businesses and social systems. Structures are conventionally designed to avoid collapse and accept a level of damage by means of ductile inelastic response and plastic energy dissipation. For this purpose, controlled yielding of the main structural elements is permitted at designated locations. Hence, modern design codes are successful in minimizing the number of associated casualties. However, they do not explicitly address damage prevention which leads to permanent structural deformation and astounding economic losses.

The lingering social and economic impacts of earthquakes have propelled cutting-edge research into earthquake resilient structures. Damage avoidance self-centering systems are a highly sustainable approach to seismic design which enables structures to remain operational after earthquake events. Self-centering rocking frames, are types of damage-avoidance systems with opening joints at the base of the columns and beam-column connections. They utilize unbonded post-tensioning tendons to restore a structure to its initial state of zero residual displacements, in the aftermath of a seismic event. Energy dissipation is typically provided by supplemental means rather than through the yielding of structural members, which means that the inelastic structural

deformations under lateral forces can be reduced or even eliminated.

Traditional capacity design approaches have been previously applied to the design of selfcentering structures, wherein, the design forces are calculated using lateral force distributions. These distributions are predominantly based on the first mode response combined with some correction for higher modes. However, an amplification of peak seismic force demands has been observed in rocking systems, which can be attributed to the higher-mode effect. The potential contribution from higher-modes may limit the economic viability of these systems. This means that a structural design may be uneconomical if higher force demands are predicted, or unsafe if designed according to current approaches [1].

There is abundant research on various aspects of the non-linear behavior of multi-story rocking moment frames. However, the vast majority of these studies include supplementary energy-dissipation devices, elements or mechanisms. By contrast, numerical studies associated with studying the pure non-linear dynamic response of multi-story rocking moment frames, independent of external energy-dissipation, are limited. Similarly, higher-mode effects on rocking structures have been formerly investigated, and demonstrated to significantly increase member design forces. Nevertheless, the assessment of the level and nature of higher mode contribution specific to rocking moment-resisting frames remains unexplored and requires extensive insight. The following paper numerically examines the non-linear behavior of multi-story rocking building models. The conclusions drawn can further lead to contributions towards feasible design approaches, and strategies to mitigate the observed effects.

# **Literature Review**

The motivation for study of the rocking phenomenon can be traced back to the father of modern seismology, Robert Mallet [2], who explored the possibility of estimating seismic intensities by deriving numerical formulations for the rocking response of blocks. This concept was adapted into the pioneering works of Milne and Perry [3 - 5], where the principle of conservation of energy was used to propose empirical formulae for a rigid rocking block. Investigations concerning rocking structures diverged from their earlier purpose, when it was observed that various slender inverted pendulum type structures survived major seismic events with only minor damages, illustrating the potential economies of utilizing the rocking mechanism for damage resistant design [6 - 8]. Thereafter, the first feasibility studies of building structures comprising steel frames with base-uplifting columns were carried out, and it was observed that the rocking motion resulted in significant story displacements [9]. The possibility of utilizing unbonded post-tensioning tendons in pre-cast concrete connections for seismic resilience and stability was further introduced [10], providing compelling evidence of the benefits of building systems featuring rocking connections.

The promising results from earlier studies prompted the substantial PRESSS (Precast Seismic Structural Systems) program [11], which explored the advantages of non-emulative connections for precast concrete frame and wall structures for use in high-risk seismic areas. The program culminated with the test of a 60%-scale, five-story two-bay building which incorporated: a hybrid coupled wall to provide lateral force resistance in one direction, moment-resisting frames with and without unbonded tendons in the other direction, and components exhibiting rocking, at beam-column or wall-foundation interfaces. The structure withstood events up to 50% greater than the design level event with minimal cosmetic damage [12]. It was also observed that the base

shear in the wall rocking system was amplified relative to the design values, due to the higher mode effect [13, 14]. It was concluded that the self-centering frame system was a robust and efficient structural system, and that it had the ability to accommodate large deformations with minimal residual drifts without losing its load carrying capacity.

The concepts developed during the PRESSS project captured the interests of numerous researchers who conducted detailed investigations to illustrate the reliability of integrating selfcentering rocking connections in steel structures. Beam-column moment connection with wideflange members were experimentally tested by Garlock et al. [15, 16]. The seismic behavior of a 6-story, 4-bay post-tensioned steel moment-resisting frame was also examined numerically, with energy-dissipation provided by sacrificial bolted top and seat angles in the connection [16]. It was concluded that post-tensioning in steel moment-resisting rocking connections provides sufficient strength, stiffness, and drift capacity under seismic loading. The non-linear dynamic behavior of single-story rocking post-tensioned moment frames exclusive of supplemental energy-dissipation components was explored in [17 - 19] using numerical, analytical and physical modelling. A quarter-scale physical model representing a single bay and story from the PRESSS building was adopted in these studies. The Discrete Element (DE) software UDEC [20], was used to capture the fundamental nonlinear mechanics of this physical model [19]. Low-order empirical models were also presented using energy considerations to represent the equivalent single-degree-of-freedom (SDOF) response of post-tensioned rocking frames [17]. These analytical formulations were validated using the experimental results and it was demonstrated that the equation of motion for a two-dimensional, post-tensioned frame is dynamically equivalent to a single tied rocking block on an elastic foundation. From the foregoing, it follows that if a self-centering rocking frame is dynamically equivalent to a tied rocking block, its dynamic response is first-mode dominant. Nevertheless, it should be noted that the analytical study was limited to the investigation of a single-story system and the response of multi-story structures under pure rocking require further examination to evaluate the contribution from higher-modes.

Concentrically-braced steel rocking frames were numerically investigated by Sause et al. [21] and it was determined that the base overturning moment was dominated by the first mode, while the base shear was influenced by the higher modes. An in-depth assessment of higher mode effects in rocking self-centering steel-braced frames was further carried out by Wiebe et al. [1, 22]. It was demonstrated that the peak roof displacement was likely to be dominated by the first-mode behavior, while most force-based response quantities would be dominated by the second mode of response. For example, the base shear force was expected to increase with earthquake intensity due to the amplification of contributions from the higher modes. It was noted that although the base-rocking mechanism minimizes the overturning moment, it does not limit the base and story shears [23]. Nevertheless, these studies were concerned with base-rocking systems, and did not include additional opening joints at the beam-column interfaces. The majority of the subsequent research has focused on shear walls, and steel braced frame structural systems. Also, a considerable amount of literature has been devoted to the study of individual joint assemblages. Hence, there is a shortage of understanding concerning overall structural assemblies under pure rocking motion, independent of supplemental energy-dissipation mechanisms, devices, and elements. Likewise, limited insight is available for the higher mode effects in moment-resisting steel frames featuring gap-opening mechanisms at both the beam-column and column-foundation interfaces.

#### **Description of Proposed Structures**

Three-, six- and nine-story single-bay planar steel frames with moment-resisting rocking mechanisms at the beam-column and column-foundation connections are studied herein. The aspect ratios and elevations for each of the structural models are illustrated in Fig. 1. Hollow steel square sections were used for the frame members, with identical sections for beams and columns. The inertial mass applied on the frames was two tons per bay. The higher story models were designed based on the single-story physical model described in [17]. In this preceding study, the detailed design of the single-story benchmark structure was carried out in accordance with design guidelines presented as a part of the PRESSS research programme [24]. The sections sizes and post-tensioning tendons for the multi-story models were chosen using an iterative analysis-design technique. The design and analysis of frames were carried out simultaneously in an iterative pattern, incrementing the section sizes and checking for member forces and dynamic behavior, such that all sections remain elastic for the duration of the highest forcing amplitude.

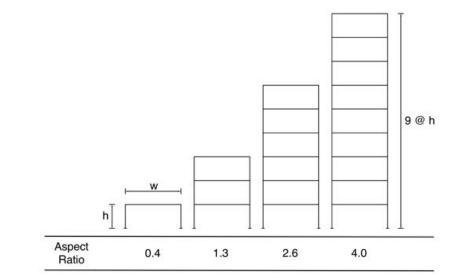


Figure 1. Elevations and Aspect Ratios of the 1-, 3-, 6-, and 9-Story models.

# **Numerical Modelling Strategies**

A series of planar models with two-degrees-of-freedom per node were constructed in OpenSees [25]. The Lobatto integration scheme introduced in [26] was applied with stiffness distributed along the member widths using 10 linear springs (Table 1 and Fig. 2). All degrees of freedom were constrained at the foundation level to simulate a fixed base beneath the rocking interface. The base opening joint was horizontally restrained to simulate the friction support in the X-direction. Vertical displacement restraints were added for the zero-length elements between the beam and column to facilitate shear transfer. This was achieved by using the equal degree-of-freedom constraint (master-slave nodes) for adjacent nodes between the beam-end rigid links simulating the gap opening. Nodal masses were lumped at the top of each column element and defined in the horizontal and vertical degrees of freedom. The columns and tendons were modelled as continuous linear elements along the frame heights. Corotational geometric transformation [25] was applied for columns and beams to account for geometric nonlinearity, and linear transformation was used for the rigid links. The post-tensioning tendons were modelled using Corotational Truss elements

with a uniaxial material (Steel02 material in OpenSees) and an initial stress to simulate the pretension force. The post-tensioning tendons were anchored at the frame centerline. The frame and post-tensioning elements were modelled with their respective material elastic moduli and yield strengths. Elastic beam-column elements were used for modelling the beams, columns, and rigid links.

 Table 1.
 Position of gap elements starting at center of contact element, and weight multipliers for individual stiffnesses of the respective element

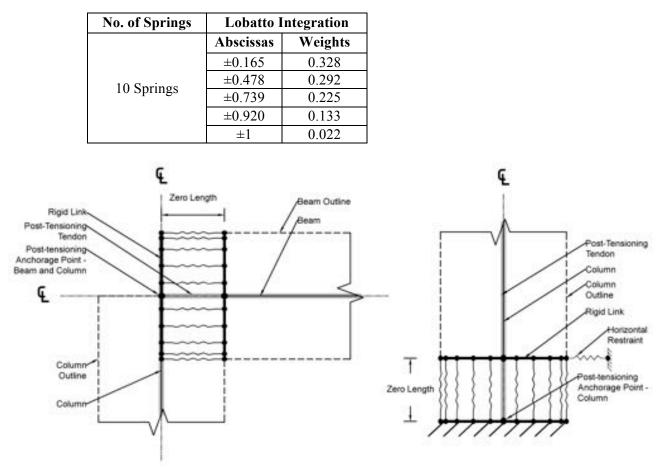


Figure 2. Beam-column connection (Left); Column-foundation connection detail (Right).

Elastic Perfectly Plastic material was defined for the gap element, with no stiffness in tension. The stiffness properties of each spring element was calculated using Eq. 1 and 2 below:

$$K_{spring,i} = \frac{EA}{2L_i} * w_i \tag{1}$$

$$Fs_{spring,i} = \frac{L_i A}{2} * w_i \tag{2}$$

where, E, A and  $F_y$  is the Modulus of Elasticity, Cross-Sectional Area, and Yield Strength of the connecting member;  $w_i$  is the weight of each spring assigned in accordance with the Lobatto integration scheme;  $L_i$  is the influence length, with  $L_i$ , column selected to be one-third of the story

height and  $L_{i, beam}$  to be approximately one-sixth of the beam length. These values were further calibrated using an iterative process.

Gravity loads were applied as nodal loads on columns in order to decrease processing time for convergence. The static lateral loads during pushover were also applied at the nodes assuming linear first-mode response. A displacement control strategy was used to perform the pushover analyses. Furthermore, resonance response curves were generated using a sine-dwell (discrete sine-sweep) input ground motion, where the frequency was incremented by discrete amounts, giving the structures time to reach a steady-state response. MATLAB [27] was used to generate input harmonic base-acceleration histories based on the frequency range, time increment and sampling rate. Subsequently, an initial stiffness proportional damping of 5% was specified.

#### **Results and Discussion**

# **Static Analysis**

The static analysis results for the multi-story models are illustrated in Fig. 3. The pushover response was normalized against the weight and height of the structures. Sensitivity analyses for structural parameters such as the influence length, member sizes, pretension force, and area of post-tensioning tendons, were carried out for all the models. The effects of modifying some of these characteristics for the six-story numerical model are also presented in Fig. 3. It was observed that altering the initial pretension force of the columns did not have a significant impact on the static response, whereas increasing the initial pretension force of the beam by a factor of 1.4 resulted in a 25% higher ultimate load. Additionally, doubling the cross-sectional area of the post-tensioning tendons in the columns, resulted in a response with negative post-elastic stiffness. This led to the counter-intuitive conclusion that a greater size for the post-tensioning tendons does not ensure a structure with greater strength and stability, and an increase in the initial force and/or area of post-tensioning tendons can have a detrimental impact on the response of rocking systems [28].

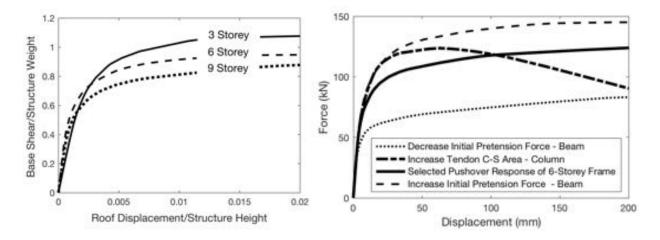


Figure 3. Pushover response for numerical models (Left); Pushover response illustrating the impact of altering the properties of post-tensioning tendons (Right).

# **Dynamic Analysis**

A series of dynamic analyses were performed for low, medium, and high forcing amplitudes. The resonance response plots for the 9-story model obtained from increasing sine-dwell (sine-sweep) input motions, are presented in Fig. 4 in terms of displacement and acceleration. In contrast to the displacement-based resonance curves, multiple peaks were observed in the acceleration-based plots at the second story level (Peaks A, B, and C in Fig. 4). The peak at point A demonstrates the importance of sub-harmonic effects on the numerical response. Points A, B, and C correspond to frequencies of 1.27, 2.02, and 2.49 Hz, with B representing the fundamental frequency of the structure. Fig. 5 shows the maximum (Point B) displacements, accelerations, shears and moments along the structural height. Response values associated with Points C are further illustrated in Fig. 6, for high amplitude forcing.

The resonance response graphs were further utilized to identify the resonant frequencies of the building models. A backbone curve was first constructed by connecting the maximum response coordinates (highest inflection points) in the displacement-based resonance response plots, for a range of forcing amplitudes. For nonlinear systems, a feature of backbone curves is that the end of the curves (lowest amplitude response) approaches the fundamental frequency of the system [17]. The resonant frequency was further used to generate sinusoidal loading and perform harmonic analysis for all the numerical models. The results were compared with the respective static pushover responses. Typical plots obtained for the 9-story model are illustrated in Fig. 4. Minor deviations in the cyclic response were observed at intersections of the static and plastic branches of the backbone (pushover) curve. These can be attributed to an influence from sub-harmonic resonances. Moreover, the graphs also demonstrate that the structural system does not possess significant energy-dissipation (damping) capability innately.

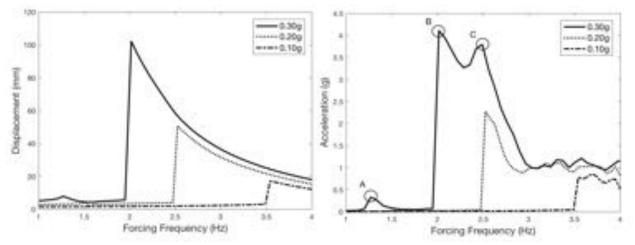


Figure 4. 9-story resonance curves showing response displacements (left) at roof level and accelerations (Right) at Story 2 (max) level corresponding to forcing frequencies for low (0.1g), medium (0.2g), and high (0.3g) amplitude excitation.

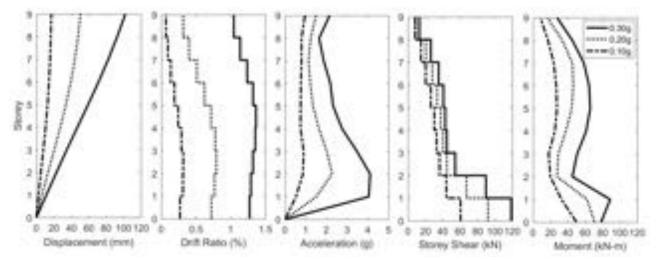


Figure 5. Maximum (Point B, Fig. 5) values of roof displacements, drift ratios, roof accelerations, column shears, and moments for 9-Story numerical model, to low (0.10g), medium (0.20g), and high (0.30g) amplitude excitation.

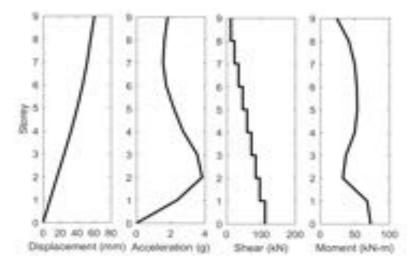


Figure 6. Values corresponding to Point C (Fig. 5) in 9-Story Numerical Model for a 0.30g (high) amplitude forcing.

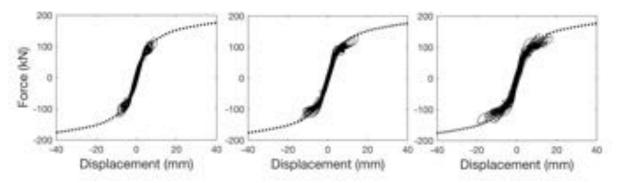


Figure 7. 9-story frame hysteresis curves in response to cyclic (sinusoidal) loading for amplitudes 0.10g (left), 0.20g (center), and 0.30g (right).

The results obtained show that the displacements and shear forces are governed by the first mode of response. At lower amplitude forcing, the bending moment and acceleration plots for the 3- and 6-story models were qualitatively similar to the primary mode response of a conventional fixed/pinned based moment frame. Moreover, in the bending moment plots for the resonant frequency, a peak is observed at the first story level for all models, where the moment magnitude is greater than the value at ground level. This can be attributed to the complex beam interactions at resonance, when the structure is subjected to high amplitude forcing. It can also be noted that for the 9-story model, the peak is not observed for Points A and C (Fig. 7). In the case of the 3-story model, moments due to beam interaction effects govern. Subsequently, it is evident that the frequency response functions for the multi-story frames experience a contribution from sub-harmonic resonances, and the structural response quantities do not presently indicate a significant influence from higher-mode effects.

## Conclusions

Multi-story frame models, comprising three-, six- and nine-stories were developed within the finite element framework of OpenSees. Quasi-static and dynamic analyses were performed. The results obtained demonstrate that the displacement quantities are governed by the first mode of response, whereas the force-based quantities were impacted by beam-column interaction effects. Moreover, typically around the first story, the beam-column interactions lead to a peak in the moment magnitudes of the columns. The bending moment values due to this effect are important and can govern the design of the joint. Subsequently, the structural models need to be further evaluated using a series of earthquake ground motion records. The beam interaction effects will also be explored extensively in subsequent studies.

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