PORTFOLIO SELECTION: A HEURISTIC APPROACH*

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I. INTRODUCTION

THE PROBLEM of selecting a portfolio can be divided into two components: (1) the analysis of individual securities and (2) the selection of a portfolio or group of securities based on the previous analysis. Up to now, the majority of writers have focused on the first part of the problem and have developed several, well-accepted methods of analysis.¹ Little attention has been paid to the second phase of the problem. It is to this second part of the portfolio selection process that this paper is principally devoted.

Recently a normative approach to portfolio selection for a particular kind of investor has been proposed by Markowitz.² He defines a decision problem (in this case the selection of a set of securities), assumes a decision function, and observes the behavior which the system generates when inputs are varied. In his analysis, Markowitz shows that, for given securities, a rational investor can determine the "efficient" set.³ To obtain an optimal portfolio from the efficient set, additional assumptions are required: namely, a Markowitz investor must choose that combination of mean and variance which provides maximum utility. But, whatever form the

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1. B. Graham and D. Dodd, *Security Analysis* (3d ed.; New York: McGraw-Hill Book Co., Inc., 1951), is an example of one of the more comprehensive works in this area.

2. H. Markowitz, *Portfolio Selection* (New York: John Wiley & Sons, 1959).

3. *Ibid.* Portfolios which provide the maximum return for a given variance are "efficient."

decision function takes, it must be such as to make its mathematical representation tractable and soluble.

A positive theory of portfolio selection does not yet exist. Such a theory must describe and predict the investment behavior of individuals under uncertainty. Whether one constructs a positive theory or compares the results of a normative theory with existing procedures, knowledge of actual behavior is a prerequisite. Since neither a theory nor an adequate description of the selection process is available, the aim of this paper is partially to fill both gaps.

The focus of our study is the investment of trust funds held by banks. We view this process as a problem in decision-making. A heuristic⁴ model, written as a computer program, simulates the procedures used to assign accounts to a common trust fund or to select particular portfolios. The analysis is based on the operations at a medium-sized national bank (with trust assets approximately equal to the average for all national banks). The decision-maker of our problem is the trust investment officer;⁵ our simulation asks the computer program to select a portfolio based on information available to the investment officer at the time his decision is made.

This approach is related to the traditional literature of financial analysis and portfolio selection. Like the traditional approach, it is based on rules of thumb (heuristics) which guide the decision-maker from the original input of information about the client, the securities markets, and the economy to the choice of particular portfolios. But, unlike that approach, the rules must be completely specified, unambiguous, and capable of being refuted by empirical tests. When the rules for processing information (or heuristics) yield results consistent with those obtained by human subjects, the model is said to have "simulated" the decision process; the set of heuristics (or simulation model) has "predicted" the behavior of the subject.⁶

Even if the model fails to predict, simulation provides valuable information about the decision process in the form of a step-by-step record of the procedures used. Sources of error can frequently be identified and eliminated. In this way, the model, through a

4. Heuristics are important, as they often lead us quickly to solutions which we would otherwise reach much more expensively by analytic techniques. For a more extensive discussion of heuristic programs see—H. A. Simon and A. Newell, "What Have Computers To Do with Management?" (RAND Publication P-1708 [May 21, 1959]).

5. It should be noted carefully that our results reflect the behavior of one investor and hence may not describe the general case.

6. For a more complete discussion of the theory and technique of simulation see the forthcoming symposium on simulation in the December, 1960, *American Economic Review*.

series of successive tests, can be designed to approximate the behavior of the subject or subjects.

Simulation accommodates both the inductive and the deductive approach. One may simulate the processes used by a number of individuals and attempt to generalize the results. Alternatively, one can construct a model and test it against a wide variety of observed behavior. The latter approach has been used in this paper.

Simulation need not involve a computer, just as addition does not require an adding machine. The advantage of the computer is its ability to solve complex problems more accurately. Moreover, the computer permits the addition to the simulation program of as many mechanisms as are interesting and important, subject only to the speed and storage capacity of the computer. Thus, by using the computer, one can move farther away from an assumed decision function and focus on the actual operations performed by a decision-maker.

The next section briefly describes some recent developments in the theory of human problem-solving. Section III discusses the application of problem-solving to the trust investment decision. We then describe the computer model which selects the assets to be held in particular accounts. The results of some of the tests of the model are shown in the appendix.

II. COMPUTERS AND THE THEORY OF HUMAN PROBLEM-SOLVING

Recent interest in the theory of human problem-solving⁷ has focused on the computer programming of mental processes. Most of this work has been directed toward developing an understanding of the operations performed in thinking; some recent work has focused on the application of these techniques to industrial or business problems.⁸

Basic to these studies is the assumption that thinking processes

7. This section is largely based on A. Newell and H. A. Simon, *The Simulation of Human Thought* (RAND Corporation, June, 1959).

8. For examples, see the following: L. A. Hiller, Jr., and L. M. Isaacson, *Experimental Music* (New York: McGraw-Hill Book Co., Inc., 1959); A. Newell, J. C. Shaw, and H. A. Simon, "Empirical Explorations of the Logic Theory Machine," *Proceedings of the Western Joint Computer Conference*, February 26-28, 1957, pp. 218-30; A. Newell, J. C. Shaw, and H. A. Simon, "Chess-playing Programs and the Problem of Complexity," *IBM Journal of Research and Development*, No. 2 (October, 1958), pp. 320-35; E. A. Feigenbaum, "An Information Processing Theory of Verbal Learning" (unpublished Ph.D. thesis, Carnegie Institute of Technology, 1959); J. Feldman, "An Analysis of Predictive Behavior in a Two-Choice Situation" (unpublished Ph.D. thesis, Carnegie Institute of Technology, 1959); and F. M. Tonge, "A Heuristic Program for Assembly Line Balancing" (unpublished Ph.D. thesis, Carnegie Institute of Technology, 1959).

can be isolated as well as identified and that they can be represented by a series of straightforward mechanical operations. This is not to say that thought processes are simple and easy to represent but rather that they can be broken down into their elemental parts, which, in turn, consist of collections of simple mechanisms. These operations are written as a set of statements and rules which, when coded in computer language, become a computer program. The program is tested by running it on a digital computer, and, as in the more familiar case of mathematical theory, the logical consequences of the initial conditions are derived by performing the operations according to the specified rules.

In an actual simulation the derived computer statements are compared with the output of human subjects who have verbally reported (in detail) their thought processes and decisions. If the humans and the computer use similar processes, the computer is said to have successfully simulated the behavior of the humans. Moreover, if this occurs, the computer program is sufficient to account for the "observed" behavior.

It must be remembered, however, that computers are neither necessary nor sufficient devices for building heuristic models. A human can replace the computer and perform each operation as directed by the program. But humans are inefficient at this task and are usually replaced by digital computers.

One particular characteristic of computers, called "transfer" or "branching," is essential to the study of problem-solving and information-processing. Conditional transfer operations permit a program to choose between alternatives and/or follow strategies.

As a hypothetical example, in a problem-solving context, consider the following translation problem:⁹ The computer is supplied with a Russian-language dictionary, a program, and a Russian story. The program specifies that symbols representing Russian words be read and that the corresponding English words be printed out. The program finds a word in the Russian story. It is instructed to *search* through a list of commonly used words (or dictionary), until it finds symbols identical with the symbols it is using to represent the Russian word. The conditional transfer operation specifies that (1) if the symbols are identical, replace the Russian symbols with the corresponding English symbols and transfer to the next word (set of symbols) in the story, then repeat the process

9. While this is not the process followed by most translation programs, it is illustrative of the economies inherent in conditional transfer operations.

for the next Russian word in the *story*, but (2) if the symbols are not identical, transfer to the next Russian word in the *dictionary*.

Three points are worth emphasizing. First, the program is iterative, i.e., it uses its operations repetitively to process different pieces of information or to solve quite separate problems. During the processing, it sorts information, retaining those parts which are useful, discarding the irrelevant. Second, the program is capable of modifying the "dictionary" or lists. Frequently used "words" may be separated, to narrow future search activity in the interests of economizing time. Third, the hypothetical program described above is general. Any type of list could replace the Russian dictionary as an input without necessitating modification of the search-compare-transfer operations.

While the general processes which the computer follows remain unchanged, each successful simulation must recognize the constraints which arise within the context of the particular problem. These constraints restrict the program to those processes that are consistent with the operations performed by humans engaged in similar tasks. In the translation example, a constraint might call for initiating search by looking at the first letter of the word; in searching a list of Treasury notes, the computer might first consider their yields or maturities.

In our work, a list of common stocks becomes the basic list of the problem—i.e., the dictionary. The goals of the client and the amount of money to be invested represent the Russian story of our example. And conditional transfer operations allow the program to follow the strategies of portfolio selection.

When the constraints which arise in the choice of portfolios are imposed on the general theory of human problem-solving, a theory of portfolio selection emerges. The following section describes the constraints and the resulting theory.

III. SIMULATION OF THE TRUST INVESTMENT PROCESS

An investor is confronted with a large assortment of information which he may use in making decisions. There is a wide variety of data, past and current, on the operation of firms and the market valuation of their stocks. There are many published predictions about the present and future state of the general economy, the stock market, and particular industries and firms. There are legal restrictions and the desires of clients to be considered when an investor acts in an agency of fiduciary capacity. These factors, when evalu-

ated and combined with an investment policy, ultimately result in a decision to buy specific quantities of particular stocks and bonds.

An investor choosing a portfolio is processing information: he sorts the useful from the irrelevant and decides which parts of the total information flow are most important. As we have seen, the theory of human problem-solving was built to handle problems of this type. The postulates of the theory particularly relevant for our purposes are that the following exist:

(1) A control system consisting of a number of *memories*, which contain symbolized information and are interconnected by various ordering relations. . . .

(2) A number of *primitive information processes*, which operate on the information in the memories. . . .

(3) A perfectly definite set of rules for combining these processes into whole programs of processing. From a program it is possible to deduce unequivocally what externally observable behavior will be generated.¹⁰

In the portfolio selection problem, these postulates consist of (1) The memory, i.e., lists of industries each of which has associated a sublist of companies. The memory also contains lists of information associated with the individual companies.¹¹ (2) Search procedures for selecting a portfolio from the information stored in the memory. These function in a manner similar to the traditional clerk who prepares lists of stocks suitable for current investment by scanning a master list. (3) A set of rules or criteria which guide the decision-making processes by stipulating when and how each primitive process is to be used. The set of rules constitutes the processing program for an individual investor. It might be compared with the heuristics of the traditional "expert," but, as noted, there is an important difference—the program must be unambiguous.

Like any problem-solving program, the simulation of the portfolio selection process relies principally on this set of basic operating rules. The rules are specified in advance and may be modified by the outcome of specific decisions. In particular, the record of past successes, failures, and the processes involved in each are stored in memory. The program modifies its behavior by eliminating

10. A. Newell, J. C. Shaw, and H. A. Simon, "Elements of a Theory of Human Problem Solving," *Psychological Review*, LXV (1958), 151 ff.

11. Investors categorize companies by industry. Not all investors may associate identical companies with a given industry, but the process of classification by industry remains invariant as the primary basis for listing companies in the memory. The information associated with each company also varies among investors, but each has a list of attributes and values stored in memory (e.g., growth rate, dividend rate, price, price/earnings ratio, expected earnings, expected yields, etc.).

such unsuccessful procedures. In this sense it learns from its past experience.¹²

In common with other heuristic programs, the process is iterative. Lists of industries and companies are searched for particular attributes; sublists are created, searched, and again divided. For example, to obtain a high-growth portfolio, the list of companies stored in memory is searched to obtain shares with the desired characteristics. Additional criteria are employed to narrow (or expand) this list. Further search and matching against desired criteria yield the specific selection of stocks to buy.

Like the investor it simulates, the computer stores the final result (list) for future use. When the same problem recurs, the entire search process need not be repeated. The list may be judged by present criteria, accepted, adapted to new conditions, or completely rejected. In the latter event, the computer would use a conditional transfer operation to renew search activity until a new list had been formed.

Within this general framework, the problem of constructing a model of investment behavior becomes a problem of uncovering the basic rules (operations) which lead to a decision to purchase particular securities. The following procedure was used to obtain these data: First, the trust department of a local bank was observed by attending committee meetings called to review past and future decisions. Interviews were then conducted with departmental officers to obtain a better understanding of the lines of authority. From these procedures it became apparent that the investment officer was the primary locus of all decisions relevant to the choice of portfolios.

Interviewing as a technique provided helpful background information. However, as portfolio selection has a well-developed lore, this technique failed to separate the relevant from the irrelevant criteria.

Second, the history of several accounts was examined. Naïve behavioral models were constructed to approximate the recorded behavior and to help uncover those processes which appeared to be invariant between accounts.

Third, and most important, the investment officer was asked to

12. For a complete discussion see G. P. E. Clarkson and H. A. Simon, "Micro-Simulation: The Simulation of Individual and Group Behavior," *American Economic Review*, December, 1960.

permit "protocols" to be made of his decision processes.¹³ To accustom the subject to verbalizing his procedures, the first case was based on an account with which he had dealt before. Artificiality was introduced into the description of the beneficiary and the past history of the account. Successive protocols recorded the investment officer's decision processes for accounts which arose in the course of his work. The decisions made during these problem sessions determined the particular securities which were purchased for these accounts.

From these protocols a program of the investment decision process was built. As yet, the number of protocols is insufficient to answer all the problems that are raised in writing such a program. But our experience has shown that programing focuses our attention on precisely those details for which our specific knowledge is weakest. To date, there are still large gaps in our understanding of the decision-making process, especially in the areas of goal formation¹⁴ and the association of particular industries with particular goals. Also the selection process which determines the particular company and the number of shares to be purchased has not been completely determined. However, an adequate amount of information has been gathered to program a substantial part of the portfolio selection process.

IV. THE PORTFOLIO SELECTION PROCESS

This section describes the step-by-step simulation of the trust investment process in a medium-sized bank. At present we are directly concerned with the way in which common stocks are chosen for individual portfolios. The selection of bonds and preferred stocks has not yet been explicitly considered.

The investment officer's behavior can be described by a flow chart (Fig. 1) detailing the sequential pattern followed in the decision-making process. Each of the elements in the flow chart requires a specific decision by the investment officer. Although the model operated with a basic list of eighty stocks, specification of the goal

13. A "protocol" is a transcript of the verbalized thought and actions of a subject when the subject has been instructed to think or problem-solve aloud. Thus the transcript is a record of the subject's thought processes while engaged in making a decision. Since a protocol is a detailed description of what a person does, it avoids some of the problems inherent in interview and questionnaire techniques, which ask the subject to state his reasons for behaving as he does. For further discussion see Newell, Shaw, and Simon, "Elements of a Theory of Human Problem Solving," *op. cit.*

14. E.g., the precise way in which a "growth account" differs from an "income account."

of the account (step 2) eliminates securities inconsistent with the goal and reduces the list to approximately thirty stocks.

The model was required to predict the portfolios for two accounts with different goals. That is, operations 6–9 were performed as directed by the program. The output was compared with the investment officer's recorded decisions. The results are shown in the appendix.

FIG. 1

The descriptions which follow detail the processes used. Translated into symbolic form, they become the computer program.¹⁵

Information on account and client.—There are two basic sources of information on each account: the administrative officer's interviews with the client and the written record, containing a copy of the legal instrument (often a will) setting up the trust.¹⁶ From the

15. The program was written in an information-processing language IPL-V (Newell and Simon).

16. In most cases, this contains information about the beneficiaries, the investment powers of the bank, what is to happen to the principal, what should be done with the income, etc. From these sources he also gets information on what the beneficiaries' age,

accumulated data and the subjective impressions of the administrative officer, the investment officer proceeds to step 2: formulating a concept of what the client wants the trust to do, i.e., the goal of the account. Before transforming this concept into a goal (or investment policy) for the account, the investment officer must choose between two courses of action. Conditional transfer operations direct the program to (1) invest the assets in the common trust fund (C.T.F.),¹⁷ (2) set up an individual portfolio for the account.

The bank prides itself on the "individual" investment service which it offers to its customers. Thus there are clear preferences for setting up individual accounts whenever the size of the account permits. The following rules (or procedures) guide the decision to invest the assets in the common trust fund:

a) All "legal"¹⁸ trusts are eligible for investment in C.T.F. The funds of beneficiaries who have waived legal requirements are not so invested.

b) All legal trusts which have less than \$K¹⁹ in assets are automatically placed in the C.T.F.

c) Legal trusts greater than \$K may or may not be placed in C.T.F., depending on the goals of the beneficiary. However, as noted, no account may participate for more than \$100,000. Thus, in the range between \$K and \$100,000 the decision will be determined by the goal of the account. If the client has goals consistent

marital status, number and age of dependents, place of legal residence, income tax bracket, and status and age of future beneficiaries, if any.

17. The common trust fund was established to provide a medium for the collective investment of trust funds held by the bank in a fiduciary capacity. Investments are restricted to those considered legal for investment in Pennsylvania. Under Federal Reserve Board regulations, no account may participate for more than \$100,000. Under Orphan's Court rulings, not more than 10 per cent of the fund may be invested in securities of any one corporation, with the exception of direct and guaranteed obligations of the United States government. In addition, the fund may not own more than 5 per cent of any one class of stock of any corporation or have the amount invested in common stocks exceed one-third of the total investment in the fund.

18. "Legal investment" statutes fall into two general categories: (1) those that restrict all or part of the investments to specific investments or specific classes of investments, and (2) those that limit investment in non-legal securities to a given percentage of the account or fund. The statutory limitations on investment in non-legal securities range from 30 per cent to 50 per cent of the market value (in one state, inventory value) of the fund" ("Survey of Common Trust Funds, 1958" *Federal Reserve Bulletin*, May, 1959, p. 477). Many people, when setting up the trust relation, specifically waive these investment restrictions. Thus "legal" refers to situations in which the investment officer must comply with these investment restrictions.

19. To protect the bank's anonymity, the precise dollar values are not revealed. Nationally, the average C.T.F. participation is approximately \$22,000 (*Federal Reserve Bulletin*, May, 1958, p. 537).

with expected C.T.F. results and does not have assets which permit the purchase of five common stocks in round lots, C.T.F. is indicated, and the process ends.²⁰

The investment of assets in the C.T.F. is an all-or-none decision. But all legal accounts greater than \$100,000²¹ and all accounts which are not of a fiduciary nature have their own portfolios. The minimum size for these accounts depends on the asset composition. For accounts with participation in bonds, as well as common stocks, a minimum of $\frac{1}{2}K$ is required; if the account participates only in common stocks, a minimum of \$0.4K is required. Smaller accounts are refused or placed in C.T.F. The funds of very small accounts are deposited in a savings bank.

Derive the goal.—For all portfolios not invested in C.T.F., the investment officer must formulate a goal. Data previously collected are transformed into an investment policy that approximates his perception of what the client wants. The number of possible combinations is very large. But the goal he decides on must lie somewhere along a continuum between the extremes of growth and income. The bank's records indicate that accounts are categorized into four or five classes: pure growth, growth with some income, income with some growth, and income alone.²²

Determine percentages to be invested in bonds, preferreds and commons.—The main function of the program is to select the particular common stocks to be held in any given account.

In legal trusts, the maximum amount that can be invested in common stocks is $33\frac{1}{3}$ per cent.²³ In trusts where the legal require-

20. This problem and many of those which follow clearly lend themselves to "conditional transfer" operations in computer terminology.

21. As yet, we have not programed the heuristics underlying the choice of the C.T.F. portfolio. As noted above, we are concerned here only with explaining the mechanisms underlying the decisions on common stocks for individual accounts.

22. A growth account is roughly defined as one in which the monetary value of the assets appreciates at an average rate of 10 per cent per year for a five-year period. In an account seeking current dividend and interest income, a minimum yield of 4-4½ per cent is expected. In a mixed growth and income account, a capital appreciation of 5 per cent combined with a dividend of 3-4 per cent is customary.

The goal of an account is determined from the initial data in the following manner. Data on the client: lawyer, high current income, high tax bracket, no pension on retirement, married with no children, desires security after retirement, earnings to be reinvested. The goal of growth with current income as a secondary goal is indicated for the present. Income will be emphasized after the client has retired and is in a lower income tax bracket.

23. This figure is determined by Pennsylvania state law (Act. No. 340 of 1951) and is the amount designated by the state as constituting a "prudent investment." The prudent-investment criterion limits banks, in practice, to choosing securities which, if preferred stocks, have paid dividends for 16 years and which, if commons, have had

ments are waived or do not exist, this decision is left to the investment officer. Except under unusual circumstances, such as a statement that the entire fund be placed in commons, the amount invested in commons ranges between 40 and 65 per cent.²⁴

Select industries appropriate to goal.—Despite the large overlap between industries, the investment officer associates a set of industries with each goal. These are chosen from a previously selected "preference list."²⁵ A one-to-one correspondence between goal and industry does not exist. But each goal invokes a search of the preference list (memory), which leads to the selection of a particular list of industries. The length of the list depends on the size of the account, since each industry is represented only once in any given account. Thus the association of industries with goals narrows the search for appropriate securities to a much shorter list.²⁶

Select companies.—Once an industry has been selected, the company to be chosen for participation is picked by the following series of conditional transfer operations. Companies are examined on the following criteria: (i) subject to tax in Pennsylvania, (ii) legal in Pennsylvania, (iii) current uncertainty, (iv) growth, (v) yield, (vi) expected earnings, (vii) past earnings, (viii) expected dividends, (ix) expected price-earnings ratio, (x) past price-earnings ratio, (xi) amount spent on expansion and/or research and development.²⁷ The first three criteria are used in an absolute manner to

positive earnings and have paid dividends in 12 out of 16 years. A list of securities meeting these requirements is prepared by the Pennsylvania Bankers Association.

24. It is abundantly clear from the protocols that the process involved in choosing government bonds is similar to the one described for common stocks. Under present market conditions, it appears to consist of selecting the highest yield from a table listing short-term governments. Hence it is a search procedure using established processes within a given memory.

25. The preference list is the investment officer's working list of stocks. This list of approximately 80 stocks is categorized by industry. The investment officer refers to it on every selection that he makes. The preference list is designed to cover various economic situations. Although it is re-examined every three months, few changes are made. We take the list as given.

26. The investment officer's rule of thumb seeks to spread risk by diversification. But, as Markowitz (*op. cit.*, p. 109) has shown, when the returns on securities are correlated, this may not be accomplished if the amount which the client deposited is relatively small.

Further recording of protocols is expected to specify the selection process that associates particular industries and particular goals. However, it is clear that this association depends on the characteristics of the goal and the general characteristics of the companies within each industry. Some industries contain companies which vary only slightly in their individual characteristics, e.g., banks or utilities. Others, like oils, are more heterogeneous, i.e., appear on several lists.

27. Large current expenditures on plant expansion and/or research and development will lower current dividends while raising expectations of future earning power. For

reduce the lists further:

Rule a: If the beneficiary is a resident of Pennsylvania, reject all stocks which are subject to personal property tax in Pennsylvania.

Rule b: If the trust is a legal trust, reject all stocks which do not have legal status in Pennsylvania.

Rule c: Reject further purchases of stocks in which there are "current uncertainties." The investment officer would not buy du Pont stock pending the court's decision on what they are to do with their holdings in General Motors. During the Middle East crisis of 1958, international oil companies were labeled "current uncertainties."

The next two criteria (iv and v) are used in a somewhat similar manner. If the goal is growth, all stocks which do not meet minimum growth criteria are rejected. Similarly, if income is desired, low-yielding stocks are rejected.

The rest of the criteria are used in a relative manner. A rough simulation has been achieved by matching the remaining companies on these criteria and seeing which has the most points in its favor. To do this, the program sets up a three-valued scale for each criterion (low = 1, medium = 2, high = 3) and makes binary choices by subtracting the value of a particular criterion of one company from the value of the same criterion for the other company. The result of any one comparison will be a positive, negative, or zero number. All the remaining criteria are matched in this manner, and the resulting scores are added algebraically to yield a unique value for the particular comparison. Since one company's criteria are always subtracted from the other's, a positive sign on the summation will denote that the first company is chosen; a negative sign, the second company. In the case where the sum is zero, no choice has been indicated.²⁸

companies heavily dependent on the discovery of new products, e.g., chemicals, drugs, and office equipment, the amount spent on research and development is used as an indicator of the company's intention to continue developing new and profitable products.

28. More recent protocols suggest an alternative selection routine, which lists all the companies in a preference order on the two basic criteria of growth and income. For the goal of growth or income the program would take the first company on the growth or income list and check through each of the remaining criteria to see whether it met a specified standard or not. If it did, the company would be accepted, and search in that industry would terminate. If it did not, the first company would be interchanged with the second company on the list, and the test would be repeated. Changes in suitability occur because the stored data on price, income, earnings, dividends, etc., are kept up to date.

An example will clarify this process. Assume that a portfolio of high-yield stocks is required and that the selection process has reached the point where it is starting to select stocks on the basis of attributes vi through xi. At this point the choice lies between Company A and Company B. Since we are considering only attributes vi through xi, let their values for Company A be given by the vector (3, 3, 1, 3, 3, 2) and for Company B by the vector (2, 3, 1, 3, 2, 2). As mentioned above, the selection process consists of subtracting the values of the attributes of Company B from the values of the similar attributes of Company A. In this case the result of this subtraction yields a vector whose values are given by the following six numbers: 1, 0, 0, 0, 1, 0. Since the algebraic sum of these numbers is positive, Company A is selected. If more alternatives are available, a transfer operation directs the program to match Company A against the next alternative.

Determine the size of participation.—The investment officer divides the accounts into two classes. For accounts with less than \$½K to invest in commons, his rules are as follows:

a) Given the amount to be invested and the number of participations, determine the average amount which can be invested in each company.

b) Divide this average amount by the current price of the stock to obtain the number of shares which can be purchased.²⁹

c) Since each purchase may be slightly over or under the average dollar amount to be spent, maintain a continuous count of "funds remaining" figure and not the average number.

In accounts with more than \$½K to invest in commons, a different procedure is used. Once the amount to be invested and the number of participations are determined as above, the minimum round lot is purchased for each company that is selected. Again a "funds remaining" account is kept to determine the size of the last participation.

V. CONCLUSIONS

In recent years new techniques for the study of human problem-solving have been developed. Of these, the simulation of individual

29. If this number of shares is 90 or greater, 100 shares (a round lot) are purchased; if less than 90, but greater than 10, the number is reduced to its nearest multiple of 5, and that number of shares is purchased; if less than 10 but greater than 5, 10 shares are purchased; if less than 5, 5 shares are purchased. Note, however, that, in general, this process will not lead to selection of a portfolio which is "Markowitz-efficient."

behavior is most apposite to the study of problems of choice under uncertainty. Application of this technique has been facilitated by the use of digital computers capable of storing and processing large blocks of information.

This paper proposes the use of simulation as a basis for studying portfolio selection. Clearly, the choice of securities by individuals or their agents is an application of the theory of decision-making under uncertainty. We contend that focusing on the decision-making process per se is a more appropriate technique for dealing with this problem than those which, though mathematically more elegant, either (1) lead to non-testable implications or (2) rest on probabilistic assumptions.

Building computer programs focuses attention on areas of least knowledge. Moreover, since computer statements must be operational, hypotheses advanced must clearly specify assumptions about the mechanisms at work.

Using information recorded from "protocols," we programed portions of the decision rules employed by a trust investment officer. The program was tested by two simulations, and, although such small samples are never conclusive, we believe that the results strongly indicate the potential power of the theory as a predictor. (A crude test for "goodness of fit" is shown in the appendix.)

Future work will be directed at discovering the rules that are used in the formation of goals, in the association of industries with goals, and on parts of the present program that are not yet fully defined. As programs are added, we expect to generate more of the recorded behavior. We suggest that in this way a descriptive theory of portfolio selection can be developed to serve either (1) as a predictor of investor behavior or (2) as the basis for a theory of optimal portfolio selection.

APPENDIX

RESULTS OF SIMULATION OF ABC ACCOUNT, 7/7/58

Description of Account:

1. Agency account
 2. Revocable
 3. Goal of account: high growth with little or no concern for income; fluctuations in principal not a problem
 4. Investment restrictions: not a legal trust, hence not restricted to legal list; donor stated that all assets were to be invested in common stocks
 5. Amount available for investment in common stocks: assumed to be given
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The *program* selected the following portfolio for the ABC Account: The portfolio selected by the investment officer on 7/7/58 was:

85 shs. Monsanto Chem. comm.	80 shs. Monsanto Chem. comm.
10 shs. I.B.M. comm.	10 shs. I.B.M. comm.
50 shs. Continental Oil comm.	45 shs. Continental Oil comm.
45 shs. Owens Corning comm.	50 shs. Owens Corning comm.

The funds-remaining figure was too small to generate new activity.

RESULTS OF SIMULATION OF XYZ ACCOUNT, 3/28/58

Description of Account:

1. Agency account
2. Revocable
3. Goal of account: high income and stability of income
4. Investment restrictions: not a legal trust, hence not restricted to legals
5. Amount available for investment in common stocks: assumed to be given

The *program* selected the following portfolio for the XYZ Account: The portfolio selected by the investment officer on 3/28/58 was:

100 shs. Philadelphia Elec. comm.	100 shs. Philadelphia Elec. comm.
100 shs. Equitable Gas comm.	100 shs. Equitable Gas comm.
60 shs. Socony Mobil Oil comm.	50 shs. Socony Mobil Oil comm.