Helium: investments in the future

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This article develops and implements a method for evaluating an exhaustible resource (helium) whose rate of production is governed by the rate of production of a second exhaustible resource (natural gas). We determine optimum future price and consumption paths, optimal production rates from various sources, and optimal storage policies for a number of scenarios. We conduct a sensitivity analysis to find which of several possible storage policies performs best for a variety of demand growth rates and discount rates.

1. Introduction

■ Helium conservation has emerged as a major public policy issue whose importance is likely to grow (Helium Study Committee, 1978; Department of the Interior *et al.*, 1978; U.S. General Accounting Office, 1979). By separating and storing helium today, society can insure a cheap, abundant supply in the 21st Century (Laverick, 1978; Howland and Hulm, 1974). Failing to do so could make many advanced energy generation, transmission, and storage technologies unattractive (Lipper, 1978; Laverick, 1974). Deciding how much helium to store requires answering more general questions about the allocation of resources over time, the role of government, and the potential future demand by helium dependent technologies (Lave, 1978).

□ **Background.** Helium is an exceptional substance. The Department of the Interior and other agencies (1978, p. 4) report:

If there is anything indisputable about helium, it is the unique nature of the element itself. A colorless, odorless, and tasteless gas, it is seven times lighter than air, nonflammable, and will not combine with other substances. Helium has the lowest boiling point of any substance, and it is the second best gaseous conductor of heat and electricity. It flows through a hole faster than any gas and transmits sound faster than any gas save one. Its most important properties are its inertness, low boiling point, and lightness. However, it is this combination of properties in one substance that makes helium indispensable in many uses.

Although it is the second most common element in the universe and there are large quantities in the atmosphere, it is present only in low concentrations

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(about 5 parts per million). Helium is concentrated in natural gas and other gases trapped underground. As the helium-rich natural gas is extracted and burned, the helium is released into the atmosphere.

Not all natural gas contains helium. Some gas in Kansas, Texas, and Oklahoma has more than 3000 parts per million (PPM) helium; in other natural gas the helium concentration is only a few parts per million. Although substantial quantities of natural gas have been found throughout the world, to date high concentrations of helium have been found only in the United States. In addition, small amounts of ("nondepleting") helium-rich natural gas have been found which consist primarily of nitrogen and helium; its heating value is so low that the primary value is due to helium.

Helium has been used extensively in the laboratory, for lighter-than-air craft, and for rockets. In 1977 commercial uses included welding under inert conditions (19 percent), an oxygen-helium mixture for deep sea diving (9 percent), pressurizing (16 percent), and cryogenics (33 percent) (Department of the Interior *et al.*, 1978). Aside from laboratory applications and cryogenics, few of these uses exploit the unique properties of helium.

In the early 1960s, liquid fuel rockets were being used extensively for defense and space exploration. To insure an adequate supply of helium for government agencies, the Congress assigned the Bureau of Mines responsibility for contracting for large amounts of helium. The program provided long-term contracts to several companies, and established an underground reservoir in the field which was formerly the richest source of helium. On the premise that government demand would rise exponentially, the program assumed that storage would take place in the 1960s, with helium from the reserves being needed in the 1970s. Net revenues were to repay the Treasury fully by 1980, at a price calculated to recoup the Treasury's investment and interest.

However, solid fuel rockets assumed the burden of defense, space exploration activities declined, and demand growth slowed. Private companies found they could separate helium and sell it below the government's price. Expenses continued to exceed revenues, and, when it became clear that storage in the Cliffside field was sufficient to last government agencies decades, the government cancelled its contracts.

□ **Litigation.** Two controversies have resulted in litigation. The first concerned the 1970 cancellation of the purchase agreements, which contractor companies contend was illegal. The second is a suit by the owners of the land containing the natural gas. These owners contend that they sold their natural gas, but not the helium. Thus, they are suing to be paid for the helium.

Largely as a result of the suits, the government no longer accepted helium at the Cliffside field. Either the helium was not separated from the natural gas or was separated (to upgrade the gas to pipeline quality) and then vented into the atmosphere. The thicket of litigation is yet to be resolved and promises to continue impeding solution.

Thus, the situation in the early 1970s was that roughly two billion cubic feet of helium were being separated per year, about .6 billion cubic feet were being sold, and the rest was being vented. The helium companies were suing the government for breach of contract, and the landowners were suing the helium companies and the government. At the same time, over one billion cubic feet of helium were being vented into the air.

In 1974 the Bureau of Mines changed its policies and allowed companies to store private helium in the Cliffside field for a nominal charge. Since then, venting of helium has decreased somewhat. In fiscal 1976, .88 BCF were separated and sold, .54 BCF were separated and stored, .98 BCF were separated and vented, and 4.74 BCF were dissipated in fuel gas (Helium Study Committee, 1978, p. 10).

□ **Current policy options.** Scientists feel strongly about helium for two reasons. First, helium is used in many laboratory experiments and is essential for many technological innovations. Thus, having insufficient helium would tend to stifle R&D as well as to deny society the benefits of innovation. Second, entropy inexorably increases when helium goes from a concentration of 3,000 PPM to a concentration of 5 PPM, as occurs when helium-rich natural gas is burned without extracting the helium. Since this failure to conserve helium means that we are increasing the cost of helium for future generations, some scientists suggest that the current government program is immoral. Apparently, helium ought to be separated and stored from as many sources as would keep entropy from rising.

Congress is attempting to establish a new helium policy. Recommendations abound. The principal question is: How much helium does society desire to save? It is important to recognize that there are a variety of institutional arrangements for deciding the answer and for handling whatever amount it is desired to store.

Everyone agrees that separated helium ought to be stored in the Cliffside field rather than vented. Beyond this step, there is no agreement on who should own the helium, on who should pay for putting it into storage, or on how the legal issues should be resolved.

2. Theoretical framework

■ The two most common models for natural resources are (1) the renewable model (e.g., how much fish to catch each year) and (2) the finite stock model (e.g., time-phased production from a fixed pool of oil) (Peterson and Fisher, 1977). We present a third model where availability of the resource is exogenous or, rather, where there is an upper bound on availability, with the property that whatever part of the resource is not currently used (or stored) is available only at a much higher (possibly infinite) price. This class of problems maximizes the conflict between physicists and economists; incurring costs now to minimize entropy, the solution often proposed by physicists, makes sense only if the discount rate is zero.

The new model describes a wide class of resource problems. One extreme case might be represented by the minerals lost when coal is burned; the other extreme case might be gold ore, where the tailings are carefully saved and periodically reworked, as price increases. Which and how much of the minerals in ore, coal, and sand and gravel should be extracted now lest the future cost be much higher?

The most crucial variable for our analysis is the productivity of capital invested in the economy. If the productivity of invested capital is low, then current separation and storage of helium is a much more attractive policy than if the productivity of invested capital is high. If technological change does not

keep pace with the combination of resource depletion, population growth, and exogenous adversities, *per capita* income will decline and strife may destroy capital goods (Georgescu-Roegen, 1971; Daly, 1977). Under such conditions, the discount rate could be expected to be low, or even negative. Given available evidence on long-term returns to capital (Ibbotson and Sinquefeld, 1977), we believe that a discount rate of 5 percent (after accounting for inflation) reflects the productive opportunities open to society; however, we have conducted the analysis with alternative rates of 2 percent and 10 percent.

Five sources of helium are available, ranging from helium rich natural gas to extraction from the atmosphere, as described in Section 4. All sources of supply are assumed subject to technological change which decreases extraction costs by one percent per year. The demand schedule and other assumptions are described in Section 4.

3. Private market or governmental action?

■ There are economies of scale in helium storage and a limited number of facilities suitable for storage. The Cliffside field appears to be the best available storage location. If the uniqueness of this storage facility is the only reason for government intervention in the market, then government action can be limited to providing storage services to private firms at cost with the private market's determining price, production, and the amount of storage.

Will the private economy provide adequate storage since future generations are not represented in existing markets? Baumol (1978) offers a lucid discussion of the case (which we find compelling) against the argument that the private market inadequately provides for future generations. Even if one accepts the argument that the discount rate is too high to provide for adequate helium storage, it is not at all clear that government regulation of production and pricing is appropriate. A simpler policy would be to take action to reduce the discount rate used by firms engaged in helium production by, for example, reducing the profit tax rate applied to profits earned in helium production.

With more than half a dozen domestic producers of helium, many other potential entrants, and thousands of private owners of helium-rich natural gas (including the government), the current and future markets for helium should behave competitively.

The appropriate role of government in the helium market is to (1) provide a storage facility at cost, (2) help resolve the current litigation it helped cause, and (3) if necessary, use tax policy to equate the public and private discount rates for helium storage.

4. Programming formulation

■ We have set up the problem as one of maximizing the present discounted sum of future utility, given an aggregate production function, assumed rate of technological change, known supply options for helium, and time dependent demand for helium. Let J denote the number of sources of helium indexed by increasing cost of helium extraction. We assume that helium can be extracted from each source at constant returns to scale up to an exogenously specified maximum rate per period. The objective is to find the price and storage paths and the production rates from each source that maximize discounted consumer

benefits less discounted extraction costs. The objective function is:

$$L = \sum_{t=0}^T \left\{ \left[\int_0^{x_t} P(x,t) dx - \sum_{j=1}^J C_{jt} q_{j,t} \right] (1+r)^{-t} + \mu'_t \left(\sum_{s=1}^t \sum_{j=1}^J q_{j,s} + Q_0 - \sum_{s=1}^t x_s \right) + \sum_{j=1}^J \lambda_{j,t} (\bar{q}_{j,t} - q_{j,t}) \right\}, \quad (1)$$

where r is the discount rate, $P(x,t)$ is the inverse demand function at time t , x_t is quantity consumed at time t , Q_0 is initial storage, $q_{j,t}$ is the quantity extracted from source j at time t , $C_{j,t}$ is extraction cost from source j , and $\bar{q}_{j,t}$ is the maximum amount obtainable from source j at time t . The integral over x is the area under the demand curve, the consumer's surplus measure of utility. The first set of constraints with multipliers μ'_t states that initial storage plus cumulative production at each date must exceed cumulative consumption. The second set of constraints with multipliers $\lambda_{j,t}$ state that production from source j at time t cannot exceed the maximum available from that source at date t .

The most costly source of helium is extraction from the atmosphere, and the supply from that source is unlimited. Therefore, this source will serve as the backstop, and the time horizon, T , will be chosen to be sufficiently large that the backstop comes into use. Once the backstop is in use, production from the backstop equals consumption each period into the indefinite future.

The model permits helium in excess of current demand to be extracted and stored. Helium may then be withdrawn from storage at any future time. The model is constrained to use helium from current production, rather than storage, as long as it is available. We neglect the additional costs associated with getting the gas into storage and withdrawing it, since they are small.

We derive and investigate the first-order conditions for this objective function in the Appendix. There we show that there are two types of solutions that may occur. It is possible that positive storage is optimal for all periods preceding the introduction of the backstop. For this case, the price rises at the rate of interest until the backstop technology is introduced. The second possibility is that storage is exhausted at some date preceding the introduction of the backstop. In this case, price rises at the rate of interest until storage is exhausted. At the date that storage is exhausted, a source intermediate in cost between the lowest cost source and the backstop comes into production. Demand is entirely satisfied each year by production from that source and lower cost sources for some number of years. During this period price is equal to the cost of production from the intermediate source, and hence price may fall if there is technological progress in helium extraction. Storage from this intermediate source eventually becomes desirable as demand growth and decline in output from the intermediate source necessitate the eventual introduction of higher cost sources. When storage is undertaken, price again rises at the rate of interest. Once storage is reintroduced, the previous pattern may be repeated with a higher cost intermediate source's temporarily supplying demand, or the price may rise at the rate of interest until the backstop is introduced. The second case is thus one in which the exponential rise in price is interrupted temporarily one or more times before the backstop source is introduced.

By establishing that these are the two types of solutions that may occur,

we are able to devise an efficient computational procedure for determining price and storage policies that maximize the objective function in equation (1), as is explained in the Appendix.

Five supply sources of helium are available. Sources (1)–(4) consist of helium from natural gas of varying richness (where the extraction rate of natural gas is assumed to be set exogeneously) and source (5) is helium as the primary product of liquifying air (available in unlimited quantities). With the exception of helium as the primary product from air, all of the supply sources offer fixed quantities of helium at given costs. For sources (1)–(4), if the helium is not separated at the moment of availability, it is assumed to diffuse into the atmosphere and not noticeably increase the concentration of helium.

5. Implementation and data

■ We have developed a program which computes the optimal solution. Below we present and explain our choice of data and parameters for our scenarios. Policy alternatives will be discussed in Section 6.

We have assumed a constant elasticity demand function, with a shift in elasticity at \$120/MCF. Below a price of \$120/MCF, the elasticity is $-.3$, while above that price the elasticity is $-.7$ (cf. Howland and Hulm, p. 38–40). Of greater importance than the price elasticity is the exogenous increase in demand. Detailed estimates have been made; however, they are no more than guesses. The *Interagency Helium Study* (Department of the Interior *et al.*, 1978, pp. 85–90) considers a low demand scenario (viewed as a business-as-usual scenario), an intermediate demand scenario, and a high demand scenario.¹ These three scenarios have a compound annual rate of demand growth of 2.7 percent from 1977 to 2000 and growth rates of 1.5, 3.2, and 5.4 percent, thereafter. The implied 1977 through 2030 rates are 2.1, 3.0, and 4.2 percent, respectively. The “most probable” demand scenario is said to lie between one and two, leading us to choose 2 and 3 percent as our low and high demand scenarios.

We want to emphasize the variation around these estimates. If containment fusion became commercially attractive, if superconducting transmission became attractive, or if some of the other prospective uses for very low temperatures became commercial, the demand for helium could rise more quickly in the next several decades than is shown in the high demand projection. However, if none of these uses came about, and helium were commercially attractive only because of its properties of lightness, diffusion, and inertness, the demand could be much lower than the low demand projection.

The supply options for helium appear more concrete, but still have enormous uncertainty associated with them. The helium content of much of the natural gas reserves is known; however, a large amount of natural gas is projected to exist, but is yet undiscovered (U.S. General Accounting Office, 1979). Samples of natural gas found in other countries have been found to contain only small quantities of helium.

We have assumed that natural gas is produced from reserves at a rate of drawing down the reserve by 10 percent a year. We also assume that 10 percent of undiscovered reserves are added to resources each year. The backstop tech-

¹ The *Interagency Helium Study* does not assume constant growth rates. The rates cited are compound rates computed from the demand projections in the *Study's* Table 21.

TABLE 1
ESTIMATED HELIUM RESERVES AND EXTRACTION COSTS

SOURCE	DISCOVERED (BCF)	UNDISCOVERED (BCF)	CONCENTRATION (PPM HELIUM)	EXTRACTION COST/MCF
INITIAL STORAGE	39.6	0	1×10^6	0
1	39.4	0	5300	\$7.00 ^a
2	51.1	0	5300	\$18.90 ^b
3	19.2	153.6	2400	\$41.67 ^c
4	54.0	283.9	950	\$105.30 ^d
*	11.5	60.0	100	> \$1600.00 ^e
5	UNLIMITED			\$1600.00 ^f

^aDEPARTMENT OF THE INTERIOR ET AL. (1978, P. 15).
^{b-d}HOWLAND AND HULM (1979, P. 84); QUANTITIES ARE TAKEN FROM
POTENTIAL GAS COMMITTEE (1973) AND HOWLAND AND HULM (1974).
^eTHE SOURCE DENOTED * IS MORE COSTLY THAN EXTRACTION OF HELIUM
FROM AIR AND IS INCLUDED IN THE TABLE FOR THE SAKE OF COMPLETENESS
(HOWLAND AND HULM, P. 87).
^fHOWLAND AND HULM (1974, P. 87).

nology is extracting helium from the air as a primary product. About 200,000 MCF of air must be liquified to get 1 MCF of helium. These supply options in terms of cost and total quantity are shown in Table 1.

Howland and Hulm (1974, p. 86) have suggested that a 15-percent reduction in helium extraction costs could be achieved by the turn of the century. This is apparently somewhat lower than rates experienced historically. We have adopted a rate of decline of costs of one percent per year to reflect technical improvements.

6. Simulations and results

■ Shown in Tables 2 and 3 are the optimal sources of supply and the total level of storage at each decade for the low and high demand cases under discount rates of 2, 5, and 10 percent.

All prices and dollar values in the Tables are in constant 1973 dollars. Thus, as will be seen in Table 4, the policy 1 price of \$15 in 1973 dollars is approximately \$25 when adjusted to 1978 dollars. This may be compared to the wholesale price of helium in 1978 of \$24.53.

These results dramatically illustrate the importance of the discount rate as a determinant of optimal prices and storage levels. If the productivity of capital is low, helium storage is a good investment and the optimal policy is to accumulate a very large inventory of helium over the next 40 or 50 years.

The dramatic differences among scenarios in the optimal storage levels in the early decades of the 21st century should not obscure a very clear message for policy over the next two decades. That message is that separation and storage of very high cost sources of helium will not be optimal under any scenario. Source 3, which contains roughly 2500 PPM of helium is the highest cost source in production among the scenarios. The current cost of production for that source is roughly \$42/MCF (1973 dollars). These results suggest

TABLE 2

PRICE, HIGHEST COST SOURCE PRODUCED, AND STORAGE BY DECADE FOR THE LOW DEMAND GROWTH SCENARIO*

DATE	INTEREST RATE								
	2%			5%			10%		
	PRICE (\$1973)	HIGHEST COST SOURCE	STORAGE (BCF)	PRICE (\$1973)	HIGHEST COST SOURCE	STORAGE (BCF)	PRICE (\$1973)	HIGHEST COST SOURCE	STORAGE (BCF)
1977	31.	2	40	8.	1	40	2.	0	40
1981	34.	2	72	10.	1	40	3.	0	30
1991	41.	3	128	15.	1	51	8.	1	14
2001	50.	3	173	25.	2	48	20.	2	5
2011	61.	3	187	41.	3	48	38.	3	1
2021	75.	4	190	67.	3	44	91.	4	2
2031	91.	4	187	109.	4	41	236.	4	3
2041	111.	4	174	177.	4	31	612.	4	1
2051	136.	4	154	288.	4	19	753.	5	0
2061	165.	4	132	469.	4	7	681.	5	0
2071	202.	4	108	616.	5	0	616.	5	0
2081	246.	4	82	557.	5	0	557.	5	0
2091	299.	4	54	504.	5	0	504.	5	0
2101	365.	4	24	456.	5	0	456.	5	0
2111	412.	5	0	412.	5	0	412.	5	0
*THE DECLINE IN PRICE AFTER SOURCE 5 COMES INTO USE IS A RESULT OF TECHNICAL PROGRESS IN EXTRACTING HELIUM FROM THE ATMOSPHERE.									

a policy of storing all the helium currently being extracted or of extracting all helium from helium-rich natural gas and storing it. In a decade or so, when more is known about helium supply and demand, one might want to reassess the policy.

It is, however, all too painfully obvious that society does not have a crystal ball and that decisions must be made in the face of immense uncertainty. Thus, any surprises on the supply side are likely to take the form of additional, rather than diminished, availability. Thus, we believe that the high demand case is as high as demand is likely to be and that the assumed supply function is conservative as regards the future availability of helium.

But the future is uncertain. Demand might be high or low, and events over the next decade or two might lead us to revise our estimate of the productivity of capital (and thus the discount rate). We assume that the events of the next decade and a half will resolve much of the uncertainty concerning helium supply, demand, and the discount rate. However, we must decide today what policy to pursue over the next decade or so until the uncertainties are lessened. What would be the cost of pursuing a policy which is subsequently revealed to have been in error? One method of dealing with the uncertainty is to devise policies that would be optimal under each of the future conditions that might prevail. But, since we do not know what the future will be, we must evaluate each of these policies under the other possible futures. If we are fortunate, one policy will turn out to be optimal, or close to optimal, under the range of possible futures.

Six general policies have been proposed by various groups. (1) Everyone

TABLE 3

PRICE, HIGHEST COST SOURCE PRODUCED, AND STORAGE BY DECADE FOR THE HIGH DEMAND GROWTH SCENARIO*

DATE	INTEREST RATE								
	2%			5%			10%		
	PRICE (\$1973)	HIGHEST COST SOURCE	STORAGE (BCF)	PRICE (\$1973)	HIGHEST COST SOURCE	STORAGE (BCF)	PRICE (\$1973)	HIGHEST COST SOURCE	STORAGE (BCF)
1977	46.	3	40	10.	1	40	2.	0	40
1981	51.	3	95	13.	1	50	4.	0	30
1991	62.	3	185	22.	2	59	9.	1	16
2001	76.	3	229	36.	3	61	24.	2	4
2011	93.	4	271	58.	4	70	55.	3	2
2021	113.	4	284	94.	4	70	142.	4	7
2031	138.	4	274	154.	4	60	369.	4	5
2041	168.	4	251	250.	4	42	833.	5	0
2051	204.	4	221	408.	4	22	753.	5	0
2061	249.	4	184	664.	4	2	681.	5	0
2071	304.	4	139	616.	5	0	616.	5	0
2081	370.	4	87	557.	5	0	557.	5	0
2091	451.	4	25	504.	5	0	504.	5	0
2101	456.	5	0	456.	5	0	456.	5	0
2111	412.	5	0	412.	5	0	412.	5	0

*THE DECLINE IN PRICE AFTER SOURCE 5 COMES INTO USE IS A RESULT OF TECHNICAL PROGRESS IN EXTRACTING HELIUM FROM THE ATMOSPHERE.

agrees that helium which is currently being separated should not be vented. For the first 15 years, while uncertainty is being resolved, the price would be set at current levels (about 15 1973 dollars per MCF). Policy 2 is identical to policy 1, except that the price would be set at \$25 to curtail use. Policy 3 would mean separation and storage of helium for sources 1 and 2, with costs less than \$25; the price over the next decade and a half would be \$25. Policy 4 is identical to policy 3, except that the price would be increased to \$35 to curtail use and encourage recovery. Policy 5 extends separation and storage to source 3, where natural gas has only 2400 PPM helium; the price is assumed to be set at \$35. Finally, policy 6 would involve separation and storage of all helium from natural gas where the cost of separation would be less than that of extracting helium from the atmosphere. This policy of maximum storage is assumed to have a price of \$35.

Shown in Table 4 is an exploration of the excess social cost of following each of these six policies, given that the actual demand for helium will turn out to be either high or low as the discount rate really is 2, 5, or 10 percent. If demand turns out to be low, and 5 percent is the discount rate, the optimal policy (shown in the middle column of Table 2) requires using only source 1 to produce helium for the first 15 years. Policies 3–6 involve separating and storing a great deal more helium at much greater costs. These policies increase current expenditures, but the resources taken away from other investments are not so productive as they would have been if left. Certainly it is true that storing this additional helium as in policy 3, results in delaying the time when helium would be extracted from the atmosphere by 35 years, but these current

TABLE 4

POLICY EXPLORATION FOR AN UNCERTAIN FUTURE

DISCOUNT RATE	DEMAND	POLICY					
		1	2	3	4	5	6
2%	HIGH	3.1	3.0	1.0	1.3	0.0	7.1
	LOW	1.0	0.9	0.1	0.1	0.4	11.3
5%	HIGH	0.1	0.1	0.2	0.3	1.9	13.4
	LOW	0.0	0.1	0.4	0.4	2.4	14.0
10%	HIGH	0.1	0.1	0.5	0.6	2.2	11.0
	LOW	0.1	0.1	0.6	0.6	2.3	11.1

NUMBERS IN THE TABLE REPRESENT ADDITIONAL COSTS (IN 10⁹\$ 1973) OF THE POLICY SHOWN OVER THE OPTIMAL POLICIES IN TABLES 2 AND 3.

POLICIES ARE:

1. FIX PRICE AT \$15 FOR 15 YEARS, PRODUCE SOURCES COSTING LESS THAN \$15 (I.E., SOURCE 1) FOR 15 YEARS.
2. FIX PRICE AT \$25 FOR 15 YEARS, PRODUCE SOURCE 1 ONLY FOR 15 YEARS.
3. FIX PRICE AT \$25 FOR 15 YEARS, PRODUCE SOURCES COSTING LESS THAN \$25 (I.E., SOURCES 1 AND 2) FOR 15 YEARS.
4. FIX PRICE AT \$25 FOR 15 YEARS AND PRODUCE SOURCES COSTING LESS THAN \$35 (SOURCES 1 AND 2).
5. FIX PRICE AT \$35 FOR 15 YEARS AND PRODUCE SOURCES 1, 2, AND 3 FOR 15 YEARS.
6. FIX PRICE AT \$35 FOR 15 YEARS AND PRODUCE ALL SOURCES EXCEPT FROM ATMOSPHERE.

expenditures could have been employed more productively in research and development or in building new plants and equipment.

Policy 2, which merely involves a higher price, is almost identical to policy 1 in terms of its social costs and benefits; it is slightly less beneficial, since it gives rise to too high a price and thus curtails demand needlessly. Thus Table 4 shows a zero-excess social cost for policy 1 in this case. Policy 2 has a slightly greater cost, and policies 3–6 represent still greater costs.

For low demand and discount rates above 2 percent, policy 3 involves at least a doubling of the social loss; the higher the interest rate, the greater is the social loss. Since policy 4 increases the price, it involves still greater loss. Policy 5 involves utilizing still another source for helium, with a dramatic escalation of social loss, even for the 2-percent discount rate. Finally, policy 6, which attempts to save all helium, involves huge social losses under all assumptions about the future.

7. Conclusion

■ As shown in Table 4, policy 1 dominates for discount rates higher than 2 percent, whatever the demand level. Indeed, if demand were below that of the "low" level or if additional helium were found in new discoveries of natural gas here and abroad, the costs of policies 2–6 would rise dramatically. Furthermore, for a 10-percent discount rate, the optimal policy is not to store any helium for at least 10 years, and merely to consume the helium already stored. The analysis indicates that policy is not sensitive to precise assumptions about future demand or the discount rate. As long as investment earns more than a 2 percent per year return (after accounting for inflation) and as long as the

demand for helium is not markedly greater than that assumed, policy 1, storage of helium that is currently separated, or which could be separated cheaply in existing plants, is the preferred social policy. We expect that the appropriate amount of storage would be undertaken by the private market if government were to (1) operate a storage facility at cost, (2) resolve the onerous litigation, and (3) if necessary, change tax policy so as to eliminate any gap between public and private returns to helium storage; additional government activities are likely to increase uncertainty and be counterproductive.

Appendix

The programming formulation

■ The Kuhn-Tucker conditions for the maximization of (1) are:

$$\frac{\partial L}{\partial x_t} = (1+r)^{-t}P(x_t, t) - \sum_{s=t}^T \mu'_s \leq 0, \quad x_t \frac{\partial L}{\partial x_t} = 0 \quad (\text{A1a})$$

$$\frac{\partial L}{\partial q_{j,t}} = -C_{j,t}(1+r)^{-t} + \sum_{s=t}^T \mu'_s - \lambda_{j,t} \leq 0, \quad q_{j,t} \frac{\partial L}{\partial q_{j,t}} = 0 \quad (\text{A1b})$$

$$\frac{\partial L}{\partial \mu'_t} = \sum_{s=1}^t \sum_{j=1}^J q_{j,s} + Q_0 - \sum_{s=1}^t x_s \geq 0, \quad \mu'_t \frac{\partial L}{\partial \mu'_t} = 0 \quad (\text{A1c})$$

$$\frac{\partial L}{\partial \lambda_{j,t}} = \bar{q}_{j,t} - q_{j,t} \geq 0, \quad \lambda_{j,t} \frac{\partial L}{\partial \lambda_{j,t}} = 0 \quad (\text{A1d})$$

$$x_t \geq 0, \quad q_{j,t} \geq 0, \quad \mu'_t \geq 0, \quad \lambda_{j,t} \geq 0. \quad (\text{A1e})$$

The implications of these conditions and a relatively simple computational scheme for finding a solution will now be developed. We shall assume that consumption each period is positive, $x_t > 0$, so that (A1a) holds as an equality. Let

$$\mu_t = \sum_{s=t}^T \mu'_s. \quad (\text{A2})$$

Since $\mu'_s \geq 0$, it follows from (2) that μ_t is nonincreasing over time. From (A1a),

$$P(x_t, t) = \mu_t(1+r)^t. \quad (\text{A3})$$

Since μ_t is nonincreasing, it follows that $P(x_t, t)$ cannot rise over time more rapidly than the rate of interest.

When storage is positive, it follows from (A1c) that $\mu'_t = 0$. Consider a scenario in which storage is positive for all periods preceding the introduction of the backstop and let T be the date of introduction of the backstop. Then $\mu'_t = 0$ for $t < T$ and $\mu'_T > 0$. It follows from (2) that $\mu_t = \mu'_T$ for all t . Then, for this case, (3) implies that the price increases at the rate of interest for all periods prior to the introduction of the backstop. At the date of introduction of the backstop, production from the backstop, $q_{J,T}$ is less than potential production $\bar{q}_{J,T}$, so $\lambda_{J,T} = 0$ from (A1d). Condition (A1b) is an equality for (J, T) , since $q_{J,T} > 0$. It then follows from (1a) and (1b) that:

$$P(x_t, t) = C_{J,T}(1+r)^{t-T}. \quad (\text{A4})$$

FIGURE A1

OPTIMAL PRICE PATHS UNDER DIFFERENT COST ASSUMPTIONS

Thus, for this case, the price at each period is simply the discounted unit cost of producing from the backstop source.

The case discussed above is particularly easy to compute. One begins by choosing a value for T and then computing the price path from (4). Next, compute economic rent $\lambda_{j,t}$ for each source each period from equation (A1b). Condition (A1d) implies that one produces at capacity from each source with a positive rent. Finally, compute cumulative production from all sources up to date T and cumulative consumption. Condition (A1c) should be an inequality for all periods preceding T and should become an equality at date T . If cumulative production exceeds (falls short of) cumulative consumption at date T , then T is increased (reduced) and the above procedure is repeated.

It may be that condition (A1c) is violated at some intermediate time period, even if cumulative production equals cumulative consumption at date T . Violation of condition (A1c) at some date implies negative storage at that date. This might occur, for example, if technical progress were making some source less costly to exploit over time. The discounted cost of production at some date subsequent to t would then be less than the cost of producing at date t , thereby giving rise to an incentive to consume now and produce later. Negative storage is infeasible, of course, so such a solution is inadmissible.

Violation of condition (A1c) at some date t simply means that one is attempting to defer the introduction of some intermediate technology beyond

the date at which storage plus production from less costly technologies is able to satisfy demand on the chosen price path. When this condition arises, one simply treats the technology in question as a backstop, and chooses a price path as described in the preceding paragraph to just exhaust storage at the date this technology solution is introduced.

Figure A1 illustrates the two cases described thus far. In Figure A1(a) cumulative production exceeds cumulative consumption for all periods up to the introduction of the backstop. The solid line is the optimal price path, and the dashed line is the unit cost of the highest cost technology in use at each date. In Figure A1(b) cumulative production will fall short of cumulative consumption at some intermediate date if the price path is chosen in the same way as the path for A1(a). When this occurs, it is necessary to compute the price path in more than one stage.

One first finds the date T_1 at which the cumulative production equals cumulative consumption when the price path is obtained by discounting back the cost of the intermediate technology from date T_1 . Thus, T_1 is found in the same manner as T is found in Figure A1(a). The intermediate technology satisfies demand for some time without storage. However, at some point, say T_2 , storage from this intermediate technology becomes desirable. Points T_2 and T_3 are found as follows. Choose T_3 and discount the cost of the backstop C_{J,T_3} back until the time T_2 is found at which the discounted cost of the backstop equals the cost of the intermediate technology. Points T_2 and T_3 must also satisfy the condition that cumulative production from time T_2 to T_3 must just equal cumulative consumption over that interval. Thus, the initial choice of T_3 is adjusted upward or downward until these conditions are satisfied simultaneously.

The two cases discussed above characterize the two types of solution paths which may be obtained. It may be that more than one intermediate technology will affect the solution in this manner, but the computational procedure for the optimal path in such cases is an obvious extension of the methods discussed above. The case in which an intermediate technology dominates the price path temporarily is illustrated in Figure A1(b). This illustration assumes no technical progress. With technical progress in exploiting the intermediate technology, the price would fall between T_1 and T_2 .

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