CONGESTION AND URBAN LOCATION

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Each of the functional systems within a city is highly interactive. The demand for transportation depends on the locations of residential areas, work areas, shopping areas, and recreation areas. The location and type of housing demanded depends on the quality and cost of available transportation. Other important components in the analysis include the size of city, demand for freight transportation, amount of congestion, and means for regulating congestion. Most attempts to isolate a single system for analysis, such as the demand for urban passenger transportation, have been less than complete successes because these other systems cannot be held constant; the various systems are simply too interdependent for "other things held constant" to be a very useful technique of analysis.

In an earlier paper, I made a modest beginning in an attempt to explore some of these interdependencies [8]. One model of the interaction between residential location and transportation mode was explored along with models of congestion and congestion tolls and a model of urban decentralization. This paper is the product of efforts to generalize some of these models and to include congestion in each aspect of transportation and city size. It should be stressed that even though the models become more complicated, they are pale images of reality; every effort has been made to keep them sufficiently simple that analytical results can be derived.

The basic model is more than one hundred years old [22, 4] and seems to have been rediscovered by a number of investigators, including Mohring [12] and this author. Perhaps the most complete development of this type of model is done by Alonso [1] who generalizes the problem to nonlinear functions and explicitly considers the utility function of a landowner in deriving locational patterns. In a recent paper, Beckmann [2] derives the residential location pattern from an explicit maximization of the consumer's utility function (assuming a Pareto distribution for income and a logarithmic utility function).

A SIMPLE VON THÜNEN-ELLET MODEL

Consider a city whose factories and stores are located in the city center. All land (outside the city center) is allocated to housing. Initially, it is assumed that

Any views expressed in this paper are those of the author, who is consultant to The Rand Corporation and should not be interpreted as reflecting the views of Rand or the official opinion or policy of any of its governmental or private research sponsors.

This research was supported, in part, by the United States Department of Transportation, Federal Railroad Administration, Office of High Speed Ground Transportation. A related version of this material was produced for the Northeast Corridor Transportation Project as RAND Memorandum RM-5874 DOT, *Transportation, City Size and Congestion Tolls*.

the demand for land is totally inelastic (for simplicity, all lots are of identical size) as is the demand for travel. The cost of travel (to work or to shop) is defined to include both the cash outlay and the time spent; it is a function of the distance to be traveled. Throughout this paper, it is assumed that the city center is a point (it takes up no space itself) and that travel is direct, "as the crow flies." A plausible form for the cost of travel is shown in equation (1),

$$\begin{aligned} T_i &= P(a + br) \\ &= r_0^2(a + br) \end{aligned} \quad (1)$$

where T_i is the dollar cost per day of the annual trips between the city center and a lot of unit area which is r feet from the city. Travel cost is a linear function of distance with a multiplicative factor, the city's population P , to account for congestion.¹ When the size of residential lots is constant, population is proportional to the area of the city, where the boundary is r_0 from the city center. Since all shopping and work are in the city center, it is assumed that all trips are between home and the city center.² For such a city, the cost of travel will be minimized in a circular configuration with the city center located at the geometric center of the circle.

It is further assumed that the only reason for living close to the city center is that travel cost is lower than if one lives farther out. Residents can be expected to bid for "in town" property in order to save commuting costs. The rent of any piece of land will reflect the savings in travel cost. Thus, for all land which is occupied, the sum of the daily rent and the travel cost will be constant (assuming that residents make the same number of round trips per day). "Marginal land" is land far enough from the city center so that its rent is zero. Rent falls to zero at the point where the commuting cost is so high that, even with positive rent, land nearer the city center has lower total cost (rent plus travel). Let r_0 be the distance from the city center of marginal land (that which carries a zero rent).³ The rent will be zero and the travel cost

$$T_0 = r_0^2(a + br_0).$$

Any lot r_i (which is r_i feet from the city center) will have a travel cost

¹ This formulation has congestion increase very rapidly as the city grows. It might be defended as a reasonable approximation to what would happen if other kinds of adjustment, e.g., other transportation modes and some decentralization, were not allowed to occur. If the reader finds the assumption extreme, the model is sufficiently simple that it is easy to substitute an alternative assumption.

² An equivalent set of assumptions is that all travel which is not between the city center and home occurs in a manner which is independent of the location of one's abode, i.e., local shopping centers are spread uniformly throughout the city, and one's friends live in close proximity to home. Although this set of assumptions is more general than that in text, it is not quite equivalent to that in the text if congestion is greater near the city center.

³ The distance of marginal land from the city center will vary as the number of persons demanding land varies. Higher demand means higher rent, which increases the distance from the center one can go before travel cost exceeds rent at the center.

$$T_i = r_0^2(a + br_i)$$

and a rent

$$R_i = T_0 - T_i \\ = r_0^2 b(r_0 - r_i)$$

the cost of occupying a lot at r_i is the sum of rent and travel cost as shown in equation (2).

$$T_i + R_i = r_0^2(a + br_i) + r_0^2 b(r_0 - r_i) \\ = r_0^2(a + br_0). \quad (2)$$

What is the supply curve for residential land? The cost of occupying a lot depends on the travel cost from the marginal land. The larger the city, the greater will be the rent of any lot a fixed distance from the center. If marginal land is r_i feet from the city center, the total cost of occupying a square foot of land (rent plus travel cost) will be $r_0^2(a + br_0)$. The total area of the city will be $A = \pi r_0^2$. The supply curve for residential land is given in equation (3).

$$L = T + R = \left[a + b \left(\frac{A}{\pi} \right)^{1/2} \right] \frac{A}{\pi} \quad (3)$$

where the total cost of occupying a square foot of land (the "standard" size lot) is

FIGURE 1. THE SUPPLY CURVE FOR RESIDENCE IN THE CITY

a function of the area of the city, as shown in Figure 1. (This formulation holds not only for a round city but for any shape which is the sum of equal radius pie-shaped wedges.)

A Measure of Benefit in the Model

The total cost of residence in the city is easily calculated in this model. Each square foot of land carries a total daily cost of $r_0^2(a + br_0)$. There is a total of πr_0^2 square feet of land in the city, and so the total cost of occupying land is given in equation (4).

$$L_0 = r_0^2(a + br_0)\pi r_0^2. \quad (4)$$

Note that the total travel cost is

$$T = \int_0^{r_0} T_i 2\pi r dr = \int_0^{r_0} r_0^2(a + br) 2\pi r dr = \left(a + \frac{2}{3}br_0\right)\pi r_0^4,$$

and the total expenditure on rent is

$$R = \int_0^{r_0} R_i 2\pi r dr = \int_0^{r_0} br_0^2(r_0 - r) 2\pi r dr = \frac{b\pi r_0^5}{3}.$$

If transportation were free and instantaneous, all rents would be zero in this model. Rents exist only because transportation is costly (in terms of both money and time). If a way could be found to improve transportation, its cost would decline and all residents would benefit.⁴ Rents might also fall, but this drop cannot be added to the decrease in transportation cost. Rent changes are purely a transfer; changes in travel cost are changes in the use of scarce resources. In this model, ignoring distributional considerations, the correct measure of benefit from a transportation improvement is the drop (both in terms of time and money) in transportation cost.

There are two sorts of changes that might improve transportation. The first is a fall in a , the fixed cost of travel. This "cost of getting ready to travel" must be borne by everyone, and everyone would benefit equally from a reduction in a . The effect of a change in a is shown in the first part of equation (5).

$$\begin{aligned} \frac{\partial T}{\partial a} &= \pi r_0^4 \\ \frac{\partial T}{\partial b} &= \pi r_0^5/3. \end{aligned} \quad (5)$$

In this case, there will be no change in rents. The second change that would be a benefit involves a fall in b , the cost per foot traveled. Such a fall might come from expanding highways or synchronizing traffic lights. Those living farthest from the city center would obtain the greatest reduction in transport cost from a fall in b , while those living in the center would obtain a relatively small immediate

⁴ Except absentee landlords, whose rental income would fall.

gain. However, since central property now has a smaller advantage vis-a-vis transportation, its rent must fall relative to marginal land. This will continue until total cost (travel cost plus rent) is again the same throughout the city. This change in the structure of rents due to competition will cause a transfer from landlords to tenants so that all tenants eventually benefit equally while landlords lose a uniform portion of their rents.

If the assumptions of this model were relaxed slightly so that the demand for land had some positive price elasticity, a further consequence would occur. One might expect people to occupy more land than previously, since its price had fallen as a result of the fall in b . As a consequence, the town would expand and transportation expenditures would rise, though possibly not back to the former level. This increase in land occupied indicates the existence of a further benefit. This result is important for the measurement of benefit. When the demand for land increases, the fall in transportation cost (calculated on the existing quantity of transport services) is no longer a precise measure of the benefit of a technological change.

Immigration

What is the effect of migration into or out of the city? Immigrants will require their own lots and so the area of the city must expand. Suppose the migrants locate around the periphery, between r_0 and r_1 . Their travel cost will be higher than that at r_0 . Consequently, they will be willing to bid some positive rent for the previously marginal land at r_0 . If the distance of marginal land from the city center increases from r_0 to r_1 , $\pi(r_1^2 - r_0^2)$ additional square feet of land will be added to the city. If the immigrants occupy n square feet of land, $r_1 = (n/\pi + r_0^2)^{1/2}$.

Before the migrants arrived, at r_0 the rent was zero and the total travel cost was $(a + (2/3)br_0)\pi r_0^4$. With migration, both rent and travel costs are higher. Rent (per square foot) now rises by $b(r_1^2 - r_0^2) + br_1(r_1^2 - r_0^2)$, and travel cost per day per person increases by $(r_1^2 - r_0^2)(a + br_1)$. The cost per day to everyone of the immigration is shown in equations (6)–(8).

$$\text{Rent increase} \quad \frac{b}{3} \pi (r_1^2 - r_0^2), \quad (6)$$

$$\text{Travel cost increase} \quad a\pi(r_1^2 - r_0^2) + \frac{2}{3} b\pi(r_1^2 - r_0^2), \quad (7)$$

$$\text{Total increase} \quad a\pi(r_1^2 - r_0^2) + b\pi(r_1^2 - r_0^2). \quad (8)$$

The immigrants pay only part of these increases; current residents must pay the rest. The external cost of immigration is the increased cost which is not borne by the immigrant; these costs are shown in equations (9)–(11). These equations are obtained by multiplying the rent and travel cost increases per square foot by the number of square feet occupied by the original residents.

Rent externality

$$\int_0^{r_0} [b(r_1^3 - r_0^3) + br(r_0^2 - r_1^2)] 2\pi r dr = \frac{b}{3} \pi r_0^3 (3r_1^3 - 2r_0r_1^2 - r_0^3). \quad (9)$$

Travel cost externality

$$\int_0^{r_0} (r_1^2 - r_0^2)(a + br) 2\pi r dr = \frac{1}{3} \pi r_0^3 (r_1^2 - r_0^2)(3a + 2br_0). \quad (10)$$

Rent plus travel externality

$$r_0^3 \pi [a(r_1^2 - r_0^2) + b(r_1^3 - r_0^3)]. \quad (11)$$

To optimize the allocation of resources, technological externalities must be taken into account in deciding to immigrate or emigrate. However, before rushing off to place a tax on immigration and a subsidy on emigration, the costs in equations (9) and (10) must be examined carefully. Who receives the increased rent shown in equation (9)? This rent is a transfer payment from renters to owners; changes in this sort of rent are a pecuniary externality and need not be considered in optimizing resource allocation (increased rent does not mean that any additional scarce resources are being used).⁵ The increase in travel cost does represent a technological externality, that is, additional resources are required to make the same trip after the city has expanded since the migrants increase congestion for all travelers. A larger, more populous city means that more cars will be on the road at any given time; unless the highway network is expanded, congestion increases and all travel becomes more expensive.

In this model, there is a technological externality *only* because of the increase in congestion. If there were no congestion, there would be no travel cost externality, and the increase in rent would be a pecuniary externality; no taxes or subsidies would have to be imposed to optimize the allocation of resources. Thus, congestion leads to a qualitative change in this simple model. The existence of congestion means that immigration will lead to a misallocation of resources unless congestion tolls and subsidies are imposed.⁶

Thus, a per day tax equal to the amount shown in equation (10) ought to be imposed on immigrants.⁷ Similarly, any current resident should be offered a bribe

⁵ The fact that the increased rent is a pecuniary externality can best be seen by supposing that each person owns his house and land. When a migrant enters, the value of every person's house and lot rises, but this increase is of no concern. If someone residing at r_1 were to decide to capitalize on the increased value of his house, he would sell it to the immigrant and move to the no-rent periphery, which is now at r_1 . He would find that the increase in the value of his property due to immigration is just large enough to offset the increase in travel costs due to the fact that he must now travel from r_1 instead of r_0 .

⁶ The model can be altered so that the immigration leads to a technological externality without congestion; see the formulation in [8].

⁷ The reason for taxing migration should be clearly understood. The tax is not levied because expanding a city results in a higher cost to all residents. Rather, the tax is required because a migrant does not face the marginal cost of migration in deciding to enter or leave. If I decide to

of this amount to leave the city. (Note that all beneficial effects of greater city size are neglected, such as possible economies of agglomeration.)

Everyone should face a per day tax of the magnitude given in equation (10). This amount is the penalty he is imposing on all other people by living in the city and thereby adding to highway congestion. If everyone were to pay this tax, the city would collect a substantial amount of revenue. Such a tax might provide sufficient revenue to meet the city's needs. Indeed, the tax could be great enough so that part of it would have to be returned to the taxpayers as a subsidy. On equity grounds, one could ask that the subsidy be in proportion to the amount paid. However, one would have to be careful that the purpose of the tax was not destroyed by the subsidy.

Even a cursory look at urban transportation problems makes it evident that there is a substantial cost to people concentrating in an area to form a city. Since the trend to concentrate in urban areas is continuing, one might infer that there are some offsetting economies of agglomeration.⁸ Thus, when an immigrant moves in, current residents experience some of the benefits from the economies of agglomeration, and the immigrant experiences the rest. At the same time, current residents experience some of the costs in the form of both pecuniary and technological externalities while the immigrant also experiences some of the costs. A major reason for the disarray in our cities is the fact that important technological externalities are not accounted for. It would only be by the purest chance that external costs and benefits of immigration were remotely equal. But even if they were identical, urban resources would be allocated badly. To achieve an efficient allocation of resources, taxes and subsidies for technological externalities must be properly levied. The tax in this model is a tax on congestion designed to internalize the increase in travel cost that one resident imposes on all other travelers. Relaxing the assumptions of the model a bit, the effect of the tax is to encourage people to share rides and travel at off-peak times. A subsidy to encourage immigration and internalize economies of agglomeration is a subsidy for participation in the city; it has little to do with how many people ride in a car or when people travel. To achieve efficient resource allocation, both tax and subsidy must be levied separately so as to encourage both immigration and less congestion on the highways.

The Demand for Property

So far, it has been assumed that all lots are the same size. Thus, R is the rent for a "standard" lot and T is the transportation cost. If each individual is to be enter this city, current residents will have to pay an amount whose sum is equal to equation (10); if a current resident decided to leave, all other residents would receive a total benefit equal to equation (10). In order to have each individual face the true marginal cost of migration, immigrants must pay a tax and emigrants receive a subsidy equal to equation (10).

⁸ Another possibility is that, from a social standpoint, it is not optimal to have large cities continue to grow. Perhaps, if all technological externalities were accounted for, large cities would cease to grow.

allowed to choose the size of his lot, the travel cost function, $T_i = r_i^2(a + br_i)$ will have to be modified. The radius of the city r_0 is in the function as a surrogate for the number of cars on the highway. The reader should not be misled into believing that congestion is a function of city size (number of cars on the highway and highway capacity held constant). City size is relevant in that the larger the city, *ceteris paribus*, the longer one must bear each level of congestion. However, this fact does not result in a technological externality. As shown by Strotz [19], the fact that one must bear a given amount of congestion for a longer time when lot sizes increase is completely captured in the rental price of land; no additional tax or subsidy is required to optimize the allocation of resources.

As shown previously [8], allowing the size of lots to vary will result in central lots being smaller than average and lots on the periphery larger than average. At the same time, the structure of land rents will change so that the rental price (per square foot) is higher for central land and lower for peripheral land than that calculated in the model sketched here. Thus, in comparison with a city where all lots are of identical size, a city whose lots are free to vary will have lot sizes increase as one moves away from the city center and will have a rent gradient that falls more rapidly as one moves away from the city center.

A TWO-MODE MODEL

So far, transportation has been assumed to cost the same amount for everyone. An interesting variation occurs if this cost varies from one individual to another, as when transportation is assumed to be time-consuming, and the value of time is not the same for all residents. Let w_r be the value of time to the "rich" and w_p be the value of time to the "poor." Suppose travel time is a linear function of distance, $t_i = \alpha + \beta r_i$; initially, there is no congestion in the model. The time cost of travel is $w_r(\alpha + \beta r)$ for the rich and $w_p(\alpha + \beta r)$ for the poor. The total cost of transportation is $w_r(\alpha + \beta r) + a + br$ for the rich and $w_p(\alpha + \beta r) + a + br$ for the poor. Since it costs the rich more to travel a given distance (the value of their time is greater), land near the center has a relatively high value to them. Whatever land at r_i is worth to the rich, a lot at $r_i + 1$ is worth $w_r\beta + b$ less for the rich and $w_p\beta + b$ less for the poor; these expressions determine the rent gradients of the rich and poor, respectively.

Since travel is relatively more costly for the rich, they will bid away land near the city center until they have satisfied their demand.⁹ The poor will live around them in a ring, the width of which is determined by their demand. To determine the rent structure, note that land at r_0 has a zero rent and a travel cost of $w_p(\alpha + \beta r_0) + a + br_0$. The sum of rent and travel cost for the poor must always equal this expression. The rent function for the poor is $w_p(\alpha + \beta r_0) + a + br_0 - [w_p(\alpha + \beta r_i) + a + br_i]$; that is, the savings in travel cost over that incurred at r_0 . At the

⁹ Demand for lots is assumed to be price inelastic in this model, with the rich demanding a total area a_r and the poor a total area a_p .

inner edge of the poor ring (r_e), the rent is $w_p\beta(r_0 - r_e) + b(r_0 - r_e)$. This r_e is marginal land for the rich; however, its rent is not zero to them. In order to obtain the inner circle, they must bid it away from the poor. At r_e , they must bid as much as the poor, and their rent gradient must be steeper than $w_p\beta + b$. Knowing that rent at r_0 is $w_p\beta(r_0 - r_e) + b(r_0 - r_e)$ and that the travel cost is $w_r(\alpha + \beta r_e) + a + br_e$, it is apparent that the total cost of occupying a lot for the rich is $w_p\beta(r_0 - r_e) + b(r_0 - r_e) + W_r(\alpha + \beta r_e) + a + br_e$. Thus, the cost of any lot for the rich will be

$$w_p\beta(r_0 - r_e) + b(r_0 - r_e) + w_r\beta(r_e - r_i) + b(r_e - r_i).$$

This argument is illustrated in Figure 2.

FIGURE 2. OPTIMAL LOCATION WITH TWO VALUES OF TIME

The horizontal axis measures the distance of a lot from the city center; the vertical axis is in dollars. For any r_i , the vertical distance to the rent gradient measures the rent for a lot r_i from the city center (note that the rent at r_0 is zero). The total cost of occupying a lot is measured by the height of a rectangle. Thus, the travel cost at r_i is the vertical distance from the rent gradient to the top of the

rectangle. At r_0 , the rent of rich and poor are equal and so the two rent gradients intersect. Note finally that the travel cost at $r_i = 0$ equals $w_r\alpha + a$.

Suppose now that there are two modes but only one valuation of time. One mode is fast and costly; the other is cheap but time consuming. The "costs" of the two modes are the sum of time and cash costs. For the fast mode, the costs are $w(\alpha_f + \beta_f r)$ and $a_f + b_f r$; for the slow mode, the costs are $w(\alpha_s + \beta_s r)$ and $a_s + b_s r$. One mode might always dominate or the slow mode might be cheaper to use for a short distance, although it is cheaper to use the fast mode for greater distances.

As an illustration, suppose that the fast mode costs $42 + 3r$, and the slow mode costs $2 + 13r$. As shown in Figure 3, the slow mode will be taken for distances

FIGURE 3. TRANSPORTATION COST WITH TWO MODES

up to 4, and beyond that the fast mode will be taken. The rent of a lot at r_i is (assuming $r_0 \geq 4$)

$$3(r_0 - r_i) \quad r_i \geq 4$$

$$3(r_0 - 4) + 13(4 - r_i) \quad r_i \leq 4.$$

The total cost of occupying a square foot of residential land is $42 + 3r_0$ throughout the city, depending only on the cost of the mode that dominates at the edge r_0 .

Finally, one might suppose the existence of two modes and two valuations of time. The costs of the two modes will differ for rich and poor. The cost of the fast mode "auto" is $w_r(\alpha_f + \beta_f r) + a_f + b_f r$ for the rich and $w_p(\alpha_f + \beta_f r) + a_f + b_f r$ for the poor. The cost of the slow mode "walking" is $w_r(\alpha_s + \beta_s r) + a_s + b_s r$ for the rich and $w_p(\alpha_s + \beta_s r) + a_s + b_s r$ for the poor. It is not very interesting if

one mode is cheaper for both rich and poor. Suppose $w_r > w_p$; $w\alpha_s < a_s < w\alpha_f < a_f$ for both rich and poor; and $w\beta_f r < b_f r < w\beta_s r < b_s r$ for both rich and poor. In other words, travel time is worth more to the rich than to the poor, the fixed cost of travel (the cost of getting ready to travel) is higher for the fast mode than for the slow mode, and the marginal cost of travel (the cost of going an extra foot) is more expensive on the slow mode than on the fast one. Under these conditions, as is shown in Figure 4, there will be two crossover points: at r_a , the rich will change from walking to auto, and, at r_b , the poor will change.

FIGURE 4. TRANSPORTATION COST WITH TWO MODES AND TWO VALUES OF TIME

This case is best analyzed via a particular illustration. Let the cost of the fast mode be $w(10 + 2r) + 32 + r$, and the cost of the slow mode is $w(1 + 11r) + 1 + 2r$. The fast mode has a relatively high fixed cost ($32 + 10w$ as compared to $1 + w$), since it is both time consuming and expensive to warm up an automobile and take it out of the garage. However, the fast mode has a low variable cost ($r + 2rw$ as compared to $2r + 11rw$), since it is both faster and cheaper per mile than walking. Thus, for short distances, one is willing to walk, since the cost of getting ready to walk is small; for longer distances, the high setup cost of driving is worthwhile since it is more than counterbalanced by the low variable cost of driving.

Now suppose that the wage rate of the rich is $w_r = 2$, and the wage rate of the poor is $w_p = 1$. Then the cost of the fast mode is $52 + 5r$ to the rich and $42 + 3r$ to the poor; the cost of the slow mode is $3 + 24r$ to the rich and $2 + 13r$ to the poor. It is now easy to calculate the points where rich and poor elect to switch modes. For distances greater than $2^{11/10}$, the rich will always take the fast mode;

for shorter distances, they will always take the slow mode. For distances greater than 4, the poor will always take the fast mode; for shorter distances, they will always take the slow mode.

Given the demand for property by rich and poor, all locations, rents, and transportation costs are determined. (Remember that each individual wishes to minimize the total cost of occupying a lot, that is, the sum of rent and travel costs.) There are four possible locational patterns for cities of our model; these are shown in Figure 5. Pattern I corresponds to the locational patterns shown in Figure 2, since both rich and poor walk; the rich live in the inner circle, and the poor live in

FIGURE 5. COMPLETE LOCATIONAL PATTERNS FOR TWO-MODE MODEL.

the ring surrounding them. As the demand for land of rich and poor increases, a Pattern I city will become a Pattern II city, and some of the rich will begin to live in a ring surrounding the ring of the poor. Note that the rich who live in the outer ring use the fast mode (auto) while everyone else walks. Those rich using the fast mode have a comparative advantage in travel and so live toward the edge of the city. As demand of rich and poor grows, the city will assume the shape of a Pattern III city. As shown in Figure 5, all of the rich now live in an outer ring, and the poor have taken over the whole of the central circle. If the city continues to grow, additional poor will locate in a ring beyond the rich. The poor who locate in the outermost ring use the fast mode; since the value of the time is lower than that of the rich, they locate beyond the rich. This situation is represented by Pattern IV.

To make these notions more precise, let us call the boundary of the inner circle r_0 , the boundary of the ring between poor and rich r_s (the "suburbs"), and the boundary between "suburbs" and "rural" area, r_c . Then the rent functions for a Pattern I city are given in equation (12).

$$\begin{aligned} 13(r_0 - r) - r_s &\leq r \leq r_0 \\ 13(r_0 - r_s) + 24(r_s - r) &= 0 \leq r \leq r_c. \end{aligned} \quad (12)$$

To be precise, for this pattern, r_c must be less than or equal to $2^{11/10}$, the point at which the rich begin to use the fast mode, and, in addition, the area demanded by the poor must be small enough so that it is not to the advantage of any rich to move to r_0 and drive. This means that the total cost of occupying a lot at $r = 0$ must be less than or equal to the total cost of occupying space at r_0 for a rich man. Thus, $13(r_0 - r_s) + 24r_s + 3 \leq 52 + 5r_0$. This implies that if $r_s = 2^{11/10}$, there are no poor in the city ($r_c = r_0$); if $r_s = 1^{6/11}$, $r_0 = 4$ and the poor occupy an area of 13.61π ; if $r_s = 0$, $r_0 = 6^{1/8}$ and the poor occupy an area of 37.5π . In order for a city to assume Pattern I, r_s and r_0 must satisfy the inequality $11r_s + 8r_0 \leq 49$.

For a Pattern II city, the rent functions are shown in equation (13).

$$\begin{aligned} 5(r_0 - r) - r_s &\leq r \leq r_0 \\ 5(r_0 - r_s) + 13(r_s - r) - r_c &\leq r \leq r_s \\ 5(r_0 - r_s) + 13(r_s - r_c) + 24(r_c - r) &= 0 \leq r \leq r_c. \end{aligned} \quad (13)$$

Note that the total cost of a lot (to the rich) must be the same in the suburbs as it is in the inner circle. Thus, $52 + 5r_0 = 3 + 24r_s + 13(r_s - r_c) + 5(r_0 - r_s)$ or $49 = 11r_s + 8r_c$. Thus, when $r_c = 0$, $r_s = 6^{1/8}$; when $r_c = 2^{11/10}$, $r_s = 2^{11/10}$. In the former case, the poor occupy an area of 37.5π , and there are no rich. In the latter case, there are no poor, and the rich occupy an area of 6.65π . In order for a city to assume Pattern II, r_s and r_c must satisfy the equation $49 = 11r_s + 8r_c$.

When the demand for land of the poor grows enough, the city fits Pattern III; the rich occupy the central area, the poor surround them out to the suburbs, which

are occupied by the rich, and beyond the suburbs are the rural poor. The rent functions are shown in equation (14).

$$\begin{aligned}
 3(r_0 - r) \quad & r_r \leq r \leq r_0 \\
 3(r_0 - r_r) + 5(r_r - r) \quad & r_s \leq r \leq r_r \\
 3(r_0 - r_r) + 5(r_r - r_s) + 13(r_s - r) \quad & r_0 \leq r \leq r_s \\
 3(r_0 - r_r) + 5(r_r - r_s) + 13(r_s - r_0) + 24(r_s - r) \quad & 0 \leq r \leq r_0.
 \end{aligned} \tag{14}$$

The structure of a Pattern III city is derived by noting that the cost of residence for the rich must be the same whether they live in suburbs or central city, and the cost of residence for the poor must be the same whether they live in the city or surrounding rural area. These constraints give rise to the simultaneous equations shown in equation (15).

$$\begin{aligned}
 49 - 11r_r &= 8r_s \\
 40 - 8r_s &= 2r_r.
 \end{aligned} \tag{15}$$

One example of a Pattern III city is $r_0 = 29/11$, $r_s = 3$, $r_r = 8$, and $r_0 = n$ where n can be any number greater than 8.

A further increase in demand will change a Pattern III city into a Pattern IV city by forcing all of the rich out of the inner circle into the suburbs. The rent functions are shown in equation (16).

$$\begin{aligned}
 3(r_0 - r) \quad & r_r \leq r \leq r_0 \\
 3(r_0 - r_r) + 5(r_r - r) \quad & r_s \leq r \leq r_r \\
 3(r_0 - r_r) + 5(r_r - r_s) + 13(r_s - r) \quad & 0 \leq r \leq r_s.
 \end{aligned} \tag{16}$$

The requirement that the cost of residence for the poor be the same in the rural area as in the suburbs means that $40 = 8r_s + 2r_r$. An example of a Pattern IV city is $r_s = 2.5$, $r_r = 10$, $r_0 = n$ where n is any number greater than 10.

Pattern I might be identified as a small city, Pattern II as a moderate sized city, and Patterns III and IV as large cities. These identifications are not precise in that they depend on the relative demands of rich and poor for land, as made precise in the conditions above (for example, $r_0 < (49/19)$ must be a Pattern I city, and $(49/19) \leq r_0 \leq (49/8)$ may be a Pattern I city). These identifications also depend on the time and cost characteristics of the two modes remaining unchanged as city size increases. This assumption seems doubtful in that congestion might impede the fast mode more than the slow one.

Without belaboring this model excessively, one might add a congestion factor to see its effect on rents and urban location. If both fast and slow modes become equally congested, the underlying structure is held constant, and all of the previous results will be reproduced in substance, even though the exact numbers will change

slightly. The more interesting case occurs when one mode, presumably the fast one, is subject to congestion while the slow mode is congestion free. Suppose that the time and cost of travel on the fast mode depends on the number of people in the city; that is, all commuters, not just those using the fast mode, contribute to fast mode congestion. For example, if the slow mode were walking or bicycling, the number of pedestrians or cyclists would slow the average speed of automobiles. Assuming that lot size is constant, the travel cost of the fast mode would become $(52 + 5r)r_0^2/100$ for the rich and $(42 + 3r)r_0^2/100$ for the poor; the cost of the slow mode is unchanged. The point where one switches from slow to fast mode now becomes a function of population (r_0) as shown in Table 1.

TABLE 1
CROSSOVER POINTS WITH CONGESTION

City size: r_0	$\sqrt{30}$	$\sqrt{40}$	$\sqrt{100}$	$\sqrt{200}$
Rich crossover*	.56	.81	2.58	7.22
Poor crossover [†]	.88	1.25	4.00	11.71
* The crossover point is	$.52r_0^2 - 3$ $24 - .05r_0^2$			
[†] The crossover point is	$.42r_0^2 - 2$ $13 - .03r_0^2$			

Congestion does not affect the fact that there are four possible city location patterns. The rent functions for Patterns I-IV are shown, respectively, in equations (17)-(20). Thus, congestion does not complicate the formulation in the sense of changing location patterns or the basic rent structure.

$$\begin{aligned} 13(r_u - r) \quad r_e \leq r \leq r_0 \\ 13(r_u - r_e) + 24(r_e - r) \quad 0 \leq r \leq r_e, \end{aligned} \quad (17)$$

$$\frac{r_0^2}{20}(r_0 - r) \quad r_e \leq r \leq r_0$$

$$\frac{r_0^2}{20}(r_u - r_e) + 13(r_e - r) \quad r_e \leq r \leq r_e$$

$$\frac{r_0^2}{20}(r_0 - r_e) + 13(r_e - r_e) + 24(r_e - r) \quad 0 \leq r \leq r_e. \quad (18)$$

$$.03r_0^2(r_u - r) \quad r_e \leq r \leq r_0$$

$$.03r_0^2(r_0 - r_e) + .05(r_e - r) \quad r_e \leq r \leq r_e$$

$$.03r_0^2(r_0 - r_e) + .05(r_e - r_e) + 13(r_e - r) \quad r_e \leq r \leq r_e$$

$$.03r_0^2(r_u - r_e) + .05(r_e - r_e) + 13(r_e - r_e) + 24(r_e - r) \quad 0 \leq r \leq r_e. \quad (19)$$

$$\begin{aligned}
 &.03r_u^2(r_u - r) \quad r_e \leq r \leq r_u \\
 &.03r_u^2(r_u - r_e) + .05r_e^2(r_e - r) \quad r_u \leq r \leq r_e \\
 &.03r_u^2(r - r_e) + .05r_e^2(r_e - r) + 13(r_u - r) \quad 0 \leq r \leq r_e. \quad (20)
 \end{aligned}$$

This simple model is suggestive of one aspect of the urban location decision. Of course, the model is terribly simple, and reasoning from it to the world is tenuous. However, one might note that cities have grown considerably in the last two decades. In these two decades, many cities have switched from Pattern I to Pattern II; that is, they have developed suburbs for the middle class. It is probable that these high income people were not fleeing the central city, but rather were taking advantage of the automobile as a better way of commuting. If one desired to stop the "flight to the suburbs" of high income residents, one should stop spending money to improve auto transportation and rather spend the money on improving public transportation. One would be foolish to predict that transportation time and cost alone determine urban location. However, one would also be foolish to deny that they have an effect. The better is commuting by auto, both in time and cost, the more people can be expected to move out to the suburbs. Not that it pays the rich to move out first (as city size expands). On the other hand, the better is the public transportation system (both in terms of time and cost), the more poor people will leave the city center. Therefore, it would seem unreasonable for cities to be concerned about the growing percentage of urban poor in the city and the falling tax revenues caused by high income citizens moving to the suburbs and, at the same time, spend large sums to improve auto transportation to the suburbs.¹⁰

SUMMARY AND CONCLUSION

This paper has outlined an exploration of the interactions between the nature of transportation, rent patterns, and location patterns. Within these simple models, changes have been explored (such as that resulting from congestion or that stemming from an improvement in transportation technology) along with the implications for social welfare. The primary advantage of such a simple model is the ease with which alternative assumptions can be explored. Its simplicity allows analytical solutions to be derived, which means that a benefit-cost analysis of improving urban transportation is easily accomplished within the model. The model is sufficiently complicated that complex patterns of location and rents emerge, when two modes and two values of time are introduced. High and low income residents cluster into patterns suggestive of those prevailing in contemporary cities.

Congestion, which slows the pace of urban transportation and increases its

¹⁰ Another bit of casual empiricism is that New York seems to be a Pattern III city and Los Angeles a Pattern IV city. Good public transportation combined with automobile congestion in New York has tended to keep some rich in the city center. In Los Angeles, a car is regarded as a necessity; thus there is no advantage to living in the city center (for the rich). The poor are forced to put up with poor public transportation or walk.

cost, is ubiquitous. When congestion is incorporated into these models, it has the effect of complicating locational patterns and associating a technological externality with immigration. To optimize city size, it is necessary to place a congestion tax on city residence and to offer a subsidy to current residents if they leave the city.

Some analytical models of congestion have been explored elsewhere.¹¹ Their implication is that traffic congestion leads to a marked technological externality. If the externality were to be incorporated in user's decisions properly via a congestion toll, the toll would be quite substantial and could be expected to have a marked effect on transportation decisions. The tolls are sufficiently high that one would expect a significant increase in the number of occupants per car and in the proportion of off-peak travel.

This formulation suggests a rather simple model of decentralization. A benefit occurs in the model when transportation costs are reduced. For example, if a city could be divided so that there are two city centers, transportation expenditures would fall by 30 per cent. Dividing a city into four city centers would lead to a 50 per cent reduction in transportation expenditures.

To determine optimal city size, the economies of agglomeration must be added to offset the reduced travel cost of decentralization. One simple way of doing so is to suppose that there are N commodities (or commodity groups), each of which is produced under substantial economies of scale. In dividing a city into several pieces, it is assumed that each city center will produce all of any particular group of commodities. Thus, with two city centers, each produces half of the commodity groups and ships half of its production to the other city center. Thus, the offsetting economy of agglomeration is the cost of shipping commodities to the other city center(s).

As shown in [8], as soon as it becomes profitable to decentralize the city at all, the optimal number of city centers is indefinitely large. The only factor limiting the number of centers is the assumption that, while each commodity group is produced under economies of scale, the production of any two commodity groups is independent. Thus, one city center would be engaged in the production of coal-iron-steel while another was engaged in producing research-education. The conclusions of this model seem to accord quite well with reality in that as soon as transportation technology made it profitable to decentralize a large city, the result has been a multitude of individual shopping-work centers rather than a gradual move to first two then three city centers.

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¹¹ See [8, sect. III], [6], [7], [9], [14], [15, 16], [17, 18], [19], [20, 21], and [23, 24].

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