ACCOUNTING RECOGNITION, MORAL HAZARD, AND COMMUNICATION

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Abstract: Two complementary sources of information are studied in a multi-period agency model. One is an accounting source which partially but credibly conveys the agent's private information through accounting recognition. The other is an unverified communication by the agent (i.e., a self-report). In a simple setting with no communication, alternative labor market frictions lead to alternative optimal recognition policies. When the agent is allowed to communicate his private information, accounting signals serve as a veracity check on the agent's self-report. Finally, such communication sometimes makes delaying the recognition optimal. We see contracting and confirmatory roles of accounting as its comparative advantage. As a source of information, accounting is valuable because accounting reports are credible, comprehensive, and subject to careful and professional judgment. While other information sources may be more timely in providing valuation information about an entity, audited accounting information, when used in explicit or implicit contracts, ensures the accuracy of the reports from non-accounting sources.

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Accounting Recognition, Moral Hazard, and Communication

Section I Introduction

All information systems manage their sources. The U.S. Labor Department uses elaborate rules and procedures to determine whether the price of a particular consumer good should be included in calculating the consumer price index (CPI). Judges use legal codes and their professional opinions to decide whether a piece of evidence should be heard by a jury. Likewise, accountants are selective about what can be recorded in an entity's financial records. The primary means to achieve this selectivity in accounting is through recognition rules, which dictate how and at what point in time economic events may enter accounting books. For example, according to GAAP, internally generated goodwill and some types of holding gains are not recognized in the accounting records until the corresponding assets are sold.

In the accounting policy arena, rhetoric about recognition is abundant. In its conceptual framework, the FASB prescribes four fundamental recognition criteria: definition, measurability, relevance, and reliability.¹ The conceptual statements further emphasize the tension between relevance and reliability. For instance, recording revenue before cash is received may sacrifice some information reliability. However, if "enough" uncertainty has been resolved, recognition is justified because relevant information may be conveyed in time to help users make various decisions.

The objective of this paper is to examine the economic forces that underlie the accounting recognition issue in order to better understand the comparative advantages of accounting as a source of information. Numerous studies have addressed the recognition issue. In the first half of 20th century, accounting writers stressed an economic measurement perspective (e.g., Paton [1922], Canning [1929],

¹ FASB Statement of Financial Accounting Concepts No. 5, *Recognition and Measurement in Financial Statements of Business Enterprises*, paragraph 63. Essentially, accounting recognition may occur when the economic item in question has met the definition of an accounting element and is measurable, relevant, and reliable. All four criteria are subject to the pervasive cost-benefit constraint and a materiality threshold.

and Alexander [1948]). The recognition debate was part of the larger income debate.² Contemporary authors have adopted an information content approach (e.g., Beaver [1968], Butterworth [1972], Demski [1972], and Feltham [1972]). They view accounting as a source of information as opposed to a measure of some underlying stock or flow of value. Under this approach, recognition has been studied in the terms of consumption planning (e.g., Antle and Demski [1989]) and security price behavior (e.g., Antle, Demski, and Ryan [1994] and Beaver and Ryan [1995]).

In this paper, we add two themes to the recognition debate. First, we focus on the incentive use of accounting information (e.g., to evaluate and compensate managers). Prior research has stressed the valuation use (e.g., to predict the future payoff of an entity). However, accounting measures are widely used in managerial evaluation and compensation schemes (e.g., Antle and Smith [1985], Lambert and Larcker [1987], and Sloan [1993]). In general, the information system best suited for valuation purposes may not be best suited for incentive (or stewardship) purposes (e.g., Gjesdal [1981] and Feltham and Xie [1994]). By implication, one would expect that the best recognition rule for valuation purposes may not be the best rule for stewardship purposes.

Second, we consider the interaction between accounting and non-accounting information sources. There are many non-accounting information sources concerning a typical corporate entity, such as voluntary disclosures by its managers and news stories from the financial press. Casual observation suggests information from these non-accounting sources is often more timely than the typical accounting source. When determining the optimal recognition rule, it is critical to consider other information users may already have.

We construct a multi-period agency setting in which the principal's major concern is motivating a privately informed agent. Alternative recognition rules partially convey the agent's private information at

² See AAA Committee Report [1965], Horngren [1965], and Sprouse [1965]. The broader accounting vs. economic income debate is illustrated by Paton [1922], Canning [1929], Edwards and Bell [1961], and Lee [1974].

different points in time. We then analyze the usefulness of these recognition rules. Next, a manager's self-report is introduced, playing the role of a non-accounting information source. We use this expanded setting to study how other information sources affect the use of accounting information and the choice of the optimal recognition rule.

By adopting an agency perspective, this paper adds new insights to the recognition debate. First, we provide a setting where it is best to have recognition occur in the period when the moral hazard problem is most critical rather than the period when the most uncertain event in the earning process takes place.³ Second, and more importantly, we show that when other, less credible, information sources are present, the credibility of accounting information leads to a veracity check role. Specifically, contracting on audited accounting information helps encourage a truthful self-report by the manager. While the self-report is, in equilibrium, useful in predicting future cash flows, we show it is the pending accounting signal that ensures the self-report is reliable.⁴ While feeding timely information to the security market is not the comparative advantage of accounting, the veracity check role makes accounting uniquely valuable among competing information sources.⁵ Third, the existence of an earlier self-report, coupled with this veracity check role of accounting, suggests that delaying accounting recognition may be optimal at times. Timeliness (early production and dissemination of information) is not necessarily a virtue for accounting

³ Antle and Demski [1989] considered a model of revenue recognition with moral hazard. However, in their setting, moral hazard exists only in one period (i.e., the first period). This interaction between severity of moral hazard and timing of recognition is, therefore, absent.

⁴ There have been a number of prior studies of agency models with communication. However, the emphasis has been placed upon (1) conditions under which communication is strictly valuable (e.g., Christensen [1981], Penno [1984]); (2) the trade-off between centralization and delegation of planning decisions (e.g., Melumad and Reichelstein [1987]); (3) the interaction between communication and the agent's different types of activities (e.g., directly related vs. indirectly related activities in Penno [1990a]); and (4) the function of internal audit (e.g., Penno [1990b]). In this paper, the focus is the explicit accounting structure (e.g., the timing of accounting recognition) in a multi-period agency setting.

⁵ Sundem, Dukes, and Elliott [1997] make a similar point in their monograph on the value of accounting and auditing. Auditing plays a very important role here. To be able to serve as a veracity check on other sources of information, the integrity of accounting information must be sustained.

as a source of information. Delaying an accounting report can enhance its disciplining role (through the auditing process).

Section II describes the sequence of productive activities in the basic agency model. In Section III, we expand the basic model by introducing accounting signals as additional contracting variables. Section IV further expands the model by allowing a self-report by the agent and examines the veracity check issue. Section V gives concluding comments.

Section II Organizational Setting

A stochastic technology is operated by a manager (the agent) who is hired by the owner (the principal) of the technology. This agency relationship lasts for three periods. The agent supplies two unobservable labor inputs, denoted $a_t \in A$ ($t=1, 2$), at a pecuniary personal cost of $c(a_t)$.⁶ To use the simplest model to convey our main ideas, we employ binary structures wherever possible. Each labor input can be either high or low: $A = \{H, L\}$ with $c(H) > c(L)$, and $c(L)$ set to 0. After supplying the labor input in period t , the agent privately observes a signal, denoted $z_t \in Z$ ($t=1, 2$). Each signal can be either good or bad news: $Z = \{G, B\}$. A single output, denoted $x \in X$, is realized and observed publicly at the end of the third period. The output can be zero or one: $X = \{0, 1\}$. The monetary value of output x is given by qx with $q > 0$. The principal pays I_t to the agent at the end of period t based upon the publicly available information at that time. Figure 1 summarizes the sequence of events.

We neutralize the principal's risk-sharing desire and consumption timing by assuming that the principal is risk-neutral and only cares about the end-of-the-game net cash flow. The principal's utility is given by $qx - I_1 - I_2 - I_3$.

⁶ In the third period, there is no explicit productive input provided by the agent. The model yields the same results if an unobservable and productive a_3 is admitted. Sidestepping an explicit a_3 merely simplifies the calculation.

The agent is risk-averse, with the utility function $U(I_1, I_2, I_3; a_1, a_2) = \frac{1}{r} \exp(-r(I_1 + I_2 + I_3 - c(a_1) - c(a_2)))$. The utility function exhibits constant absolute risk aversion (CARA) with the Arrow-Pratt measure $r (> 0)$. It is also multiplicatively separable over time periods. This means the agent has no income-smoothing desires and only cares about total income less total personal cost, with a zero discount rate.⁷ If the agent chooses not to participate in the agency, he receives a reservation utility of $\underline{U}$.

Let $P(x, z_1, z_2 | a_1, a_2)$ denote the joint probability of x, z_1, z_2 , given the agent's input sequence a_1, a_2 . In this section and Section III, we assume:

$$[A1] \quad P(x, z_1, z_2 | a_1, a_2) = P(x | a_1, a_2) P(z_1 | a_1) P(z_2 | a_2), \text{ for all } a_1, a_2$$

[A1] entails certain separability about the stochastic environment. In particular, given any input sequence, x, z_1 , and z_2 are conditionally independent. [A1] also implies that the agent's choice of a_1 does not affect the probability of z_2 and his choice of a_2 does not affect the probability of z_1 . The latter is natural since z_1 is realized before a_2 is chosen.

We label the agent's effort and the output such that high effort in either period produces a higher chance of success, i.e., $P(x=1 | HH) > P(x=1 | HL) > P(x=1 | LL)$ and $P(x=1 | HH) > P(x=1 | LH) > P(x=1 | LL)$.⁸ Following the agency literature, we assume there is decreasing return to effort such that the Concavity of Distribution Function Condition (CDFC) is satisfied:⁹

$$[A2] \quad \begin{aligned} P(x=1 | HL) &> \frac{1}{2} P(x=1 | LL) + \frac{1}{2} P(x=1 | HH) \\ P(x=1 | LH) &> \frac{1}{2} P(x=1 | LL) + \frac{1}{2} P(x=1 | HH) \end{aligned}$$

⁷ Presumably, one can assume banking opportunities exist and explicitly model the consumption plans for the agent. However, it would create unnecessary distractions for the model (e.g., the information set available to the banker, how the banking market works, etc.). This assumption on the agent's intertemporal tastes is a simple way of sidestepping the distractions. See Fellingham et al. [1985], Malcomson and Spinnewyn [1988] and Fudenberg et al. [1990].

⁸ We adopt the mnemonic notation HH to represent $+H, H$, and similarly for HL, LH, LL.

⁹ As shown in the proof of proposition 1, with CDFC, input sequence $+L, L$, is so unproductive that in designing the optimal labor contract, the principal can ignore the incentive compatibility constraint involving $+L, L$, once other constraints are satisfied. See Grossman and Hart [1983] for more on the CDFC assumption.

where $2 = 1/(1 + \exp(r(c(H))))$

Similarly, we label the news such that high effort produces a higher chance of good news, i.e., $P(z_1 = G | a_1 = H) > P(z_1 = G | a_1 = L)$ and $P(z_2 = G | a_2 = H) > P(z_2 = G | a_2 = L)$. Given [A1], one can “learn more about act a_i ” from output x and signal z_i than from output x alone. Formally, we say z_i is incentive informative about a_i conditional on x . Following Gjesdal [1978], we adopt the following definition of incentive informativeness:

[D1] The information source giving signal z_i is said to be incentive informative about act $a_i \in \{H, L\}$ conditional on x if $P(z_i | x, H) \dots P(z_i | x, L)$ for some z_i , and some x .

Intuitively, an information source is incentive informative about a_i if different choices of a_i produce different conditional (on x) probability of z_i . [A1] implies z_i is incentive informative about a_i conditional on x ($i = 1, 2$).¹⁰

Since the agent observes some signal before choosing a_2 , his second-period policy can be thought of as mapping $\sigma: \{\text{all possible signals available to the agent before } a_2 \text{ is chosen}\} \rightarrow \{H, L\}$. Along with his first-period act, the agent’s strategy for the entire game can be represented by (a_1, σ) . We assume q is large enough that the principal always prefers the agent to provide high effort in both periods regardless of what information might become available to either party. Thus, the preferred strategy is (H, σ^H) , where σ^H denotes the second-period policy where high effort is provided for all possible pre- a_2 signals.¹¹

Principal's Problem (Basic Model)

¹⁰ In section IV, we will relax [A1] to allow the z ’s to be informative about x . In general, we think of the z ’s as representing all possible information within the firms (e.g., internal financial reports, private information about demands and cost structures, etc.). This set of information may be informative about the future payoff (e.g., x) as well as about the managerial actions (i.e., the a ’s). In this paper, we choose to focus our attention on each of these aspect separately. While considering a more general setting where z_i has both attributes (i.e., informative about x and a at the same time) is appealing, it would be hard to tease out the different effects.

¹¹ In general, the optimal labor input is endogenous to the principal’s problem. In this paper, we neutralize the production decision in order to focus on the incentive use of accounting information.

We now introduce accounting recognition. We show that alternative labor market frictions affect the usefulness of the accounting recognition rules.

Accounting Recognition

In our setting the recognition issue centers upon when to produce verifiable information that can help the principal control the agent's actions. We consider two recognition rules: one calls for early and the other for late recognition. The early recognition rule, called R_1 , produces an accounting signal denoted $y_1 \in Y \setminus \{1, 2\}$ at the end of the first period. Since typically financial accounting numbers are publicly reported and subject to audit, we assume y_1 is verified, hence contractible. However, y_1 does not perfectly reveal z_1 and is only a noisy signal of z_1 . Further, we assume conditional on z_1 , y_1 is not informative about any other variables in the model (i.e., the agent's actions a_t and the output x).¹⁵ The late recognition rule R_2 produces accounting signal $y_2 \in Y$ at the end of the second period. We assume y_2 is a noisy signal of both z_1 and z_2 in the similar fashion. Formally, y_1 (resp. y_2) is a garbling of z_1 (resp. (z_1, z_2)).

$$[A3] \quad y_1 \text{ (} y_2 \text{) is a garbling of } z_1 \text{ (} z_1 \text{ and } z_2 \text{), and } P(y_1 = 1 | z_1 = G) \dots P(y_1 = 1 | z_1 = B), \\ P(y_2 = 1 | z_1 = G) \dots P(y_2 = 1 | z_1 = B), P(y_2 = 1 | z_2 = G) \dots P(y_2 = 1 | z_2 = B)$$

The way we model accounting recognition reflects our attempt to capture the timing aspect of producing accounting information. An accounting information system produces an accounting series (i.e., the y 's) by taking all available information within the firm (i.e., the z 's and x) as inputs and applying an set of accounting recognition rules.¹⁶ An early recognition rule produces an early signal y_1 without the knowledge of z_2 while the late rule produces the late signal y_2 with both z_1 and z_2 as inputs.

¹⁵ Here, accounting merely transmits the agent's private information with some noise but the resulting accounting number is verified, publicly available and therefore contractible. In reality, accounting systems also require some new information to be created in addition to conveying what the agent privately knows. In order to highlight the recognition aspect of accounting, we assume there is no new information generated.

¹⁶ If we think of $q_1 x$ as the lifetime cash flow to the firm and the y 's as the periodic accounting income, we can preserve the accounting articulation by introducing an "adjusting accrual" y_3 in the third period to make sure $y_1 + y_3 = q_1 x$ ($t = 1, 2$). From an information content point of view, knowing x and y_1 , y_3 does not provide any additional information and is clearly redundant in our setting.

Principal's Problem with Accounting Recognition

$$C(R_i) / \min_{I(x, y_i)} E[I(x, y_i) | H, "H"] \quad (4)$$
$$E[U(I(x, y_i); \theta | H, \mathbf{a}_i^H)] \leq E[U(I(x, y_i); \theta | \mathbf{a}_i, \mathbf{a}_i^H)] \quad (6)$$

An alternative source of information (to recognition) is to ask the agent to self-report his private information (i.e., the z 's). Typically, the principal could design compensation contracts which encourage self reporting by promising to under-utilize the information. There may be Pareto gain from such communication (e.g., Christensen [1981]). However, since the z 's are assumed to be only informative

[illegible]

about the actions (the a 's) and not informative about the stochastic environment (i.e., the z 's are assumed to be independent of x). A self-report will not be useful to the principal. We relax the independence assumption and study the communication setting formally in Section IV.

Usefulness of Accounting Recognition

The principal weakly prefers accounting recognition R_t to no recognition, i.e., $C(R_t) \# C^*$ ($t=1, 2$). Clearly, the principal can always choose (and commit) not to use the additional information generated by the accounting recognition and resort to the optimal contract in the basic model.²⁰ (The original optimal contract is feasible in the expanded program.) So in our setting, early (late) recognition is strictly valuable (relative to no recognition) if contracting on y_1 (y_2) reduces the principal's ex ante expected payment to the agent in equilibrium.

Proposition 1: Assume [A1], [A2], and [A3], then:

- (i) $P(x=1 | LH) > P(x=1 | HL)$ implies early and late recognition rules are strictly valuable to the principal; and
- (ii) $P(x=1 | LH) < P(x=1 | HL)$ implies late but not early recognition rule is strictly valuable to the principal.

When $P(x=1 | LH) > P(x=1 | HL)$, shirking in the first period (i.e., $+L$, " H ") is "less likely to be detected" than shirking in the second period (e.g., $+H$, " L "). In turn, the principal is more concerned with the agent supplying low effort in the first period than in the second period. In this case, both early and late recognition rules are valuable because both y_1 and y_2 are generally incentive informative about a_1 conditional on output x . Which is better, early or late recognition, depends upon the relative informativeness of the y 's about z_1 . Under the restrictive condition that y_2 is independent of z_1 , we have the stronger result that only early recognition is useful.

²⁰ In general, when pre-decision information (i.e., y_1 before a_2) is introduced, this commitment argument does not follow. However, it does here because y_1 is a garbling of z_1 , which is available to the agent regardless. With no recognition, the agent's second period strategy is the mapping $\sigma: Z \rightarrow A$. With recognition of y_1 , the mapping is $\sigma: Z \times Y \rightarrow A$. If the principal commits not to use y_1 , the agent can not gain by conditioning his second period act (a_2) non-trivially on y_1 .

When $P(x=1 | LH) < P(x=1 | HL)$, the principal is more concerned with the agent supplying low effort in the second period. Here only late recognition is valuable because y_2 is incentive informative about a_2 conditional on x while y_1 is not. In short, the principal's preference over R_1 and R_2 depends upon which moral hazard problem (a_1 or a_2) is more critical.²¹

Delayed output realization, among other features, contributes to the above result. In the basic model, output x is used to control the agent's labor inputs in both periods. Technically, this implies that, in the basic model, the two relevant IC constraints involving $+H$, $''^L$, and $+L$, $''^H$, are nested. With our binary structure, each constraint essentially imposes a “steepness requirement” on the incentive scheme. As a result, only the steeper of the two requirements is in effect, leaving the other IC constraint inactive (or not binding).²² Naturally, if the IC constraint involving $+L$, $''^H$, is inactive, information about a_1 is useless. This is in contrast to repeated moral hazard models (e.g., Lambert [1983], Rogerson [1985], Radner [1985], Malcomson and Spinnewyn [1988], and Fudenberg et al. [1990]) where periodic output is observed between the agent's labor inputs, and long-term effects are typically neutralized. Therefore, the issue of nested IC constraints is not present.²³

²¹ Earlier studies of informativeness criterion in agency settings (e.g., Holmström [1979]) and Kim [1995]) worked with a single IC constraint. Here, there are two IC constraints that may bind. A signal's informativeness about the agent's act may not guarantee its usefulness in contracting because the IC constraint with respect to that act may not bind.

²² There is an issue of redundant constraints here. If $P(x=1 | HL) = P(x=1 | LH)$, the two IC constraints are identical and one is clearly redundant. Technically, it causes an indeterminacy of the Lagrange multipliers associated with the two IC constraints. If this is the case, the rank condition in the Arrow-Hurwicz-Uzawa theorem is not satisfied (Takayama [1974]). This rank condition is a sufficient condition for the validity of characterizing the solution using the Kuhn-Tucker conditions. To avoid complicating the matter, we simply assume $P(x=1 | LH) \dots P(x=1 | HL)$ to satisfy the rank condition.

In the general case of more than two input choices or a continuum of inputs, desirable labor input choice (i.e., which act to induce) becomes a non-trivial issue. Interactions between input choice and incentive design become relevant. In this paper our focus is on incentive issues, so we purposely fixed desirable action choice.

²³ In repeated moral hazard models cited, the agent's action in each period only affects the output of that particular period (i.e., no long-run effects). Typically, each period's IC constraint binds individually.

The interaction among nested IC constraints is also present in Demski [1994]. In his single-period-multiple-task setting, certain IC constraint may “free ride” other IC constraints. The interaction “between the problem of motivating input for the task *pe se* and the problem of coordinating supply of inputs across the array of tasks” (Demski [1994], p. 577) may result in “good” performance measures driving out “bad” measures or “bad” driving out “good” measures. In our multi-period setting, a similar force is in play.

Discussion

The key idea in this section is that the optimal recognition rule is affected by the nature of labor market frictions. “When to recognize” depends upon which of the two labor inputs poses a more critical incentive problem.

The notion of critical event has played an important role in recognition debates since Myers [1959] first introduced such a concept. Take revenue recognition as an example. Usually, some critical event, such as a transfer of merchandise, must occur to trigger revenue recognition. Most of the literature treats uncertainties associated with the major events in the earning process as the focus of the recognition issue (e.g., Johnson and Storey [1983]); the control aspect of these events is not at center stage.

This paper stresses moral hazard concerns in the recognition debate. When control is a viable concern, the optimal time to produce information about managerial actions is not when the most uncertain event in the earning process has occurred, but when the critical labor input appears. To illustrate, suppose $P(x=1 | LH) < P(x=1 | HL)$, we can infer that first-period labor input a_1 is marginally more productive than second-period input a_2 . If we treat labor inputs as purely random events (i.e., no control problem is present), the critical event in this earning process occurs in the first period in the sense that knowing a_1 leaves less uncertainty about future cash flow x . However, in the presence of moral hazard, Proposition 1 tells us that information about a_1 is useless while information about a_2 is valuable.

Therefore, the critical event occurs in the second period. In short, the critical event in valuation settings can be different from that in agency settings.²⁴

Section IV Accounting Recognition and Veracity Check

In this section, we consider a setting where the agent's private signals are informative about the realization of output x . Communicating such information (e.g., a self-report on z_t by the agent) can reduce agency costs (e.g., Christensen [1981], Penno [1984], and Melumad and Reichelstein [1987]). We study the role of accounting recognition in such a communication regime. In the previous section, accounting signals were used to control the agent's labor inputs. In this section, they are used to control the agent's self-report as well.

When signal z_t is informative about the output x , we say it is valuation informative. Following Gjesdal [1978], we adopt the following definition:²⁵

[D2] The information source giving signal z_t is said to be valuation informative about future cash flow x conditional on signal T in the presence of input sequence a_1, a_2 , if $P(z_t | x \neq 0, T, a_1, a_2) \dots P(z_t | x = 0, T, a_1, a_2)$ for some $x \neq 0$, some z_t , and some T .

In our context, the conditioning information source T may refer to other z_t or the accounting signal y_t . Intuitively, z_t is valuation informative about x if z is not independent of x , T , for a given input sequence. [A1] clearly precludes z_t from being valuation informative about x because it assumes

²⁴ We use a numerical example to elaborate. Let $P(x=1 | HH)$, $P(x=1 | HL)$, $P(x=1 | LH)$, and $P(x=1 | LL)$ be 0.8, 0.7, 0.6, and 0.3 respectively. Suppose the labor inputs are purely random (1/4 probability for all four combinations). The prior probability $P(S)$ is .6 $(= (.8 + .7 + .6 + .3)/4)$. If $a_1 = H$, the posterior $P(x=1 | a_1 = H) = (.8 + .7)/2 = .75$; and similarly if $a_1 = L$, $P(x=1 | a_1 = L) = .45$. Changes from prior are $\pm .15$. On the other hand, knowing a_2 only changes the prior by $\pm .10$. Therefore, a_1 can be thought of as "the most uncertain event" in the earning process because knowing the realization of a_1 changes the posterior probability the most. However, if the agent's acts are not random but subject to moral hazard, by Proposition 1 we know (since $P(x=1 | LH) < P(x=1 | HL)$), the useful information is about a_2 , not a_1 .

²⁵ Notice in our context, consumption decisions are neutralized. Therefore, valuation informativeness is defined without reference to a decision context and is purely from a probabilistic point of view.

independence among x , z_1 , and z_2 for all input sequences. We replace [A1] with two assumptions. First, we assume:

$$\begin{aligned} \text{[A4]} \quad & P(z_t | a_1, a_2) = P(z_t) \text{ for all } a_1, a_2, t=1, 2; \text{ and} \\ & P(z_1, z_2 | x, a_1, a_2) = P(z_1 | x, a_1, a_2) P(z_2 | x, a_1, a_2) \end{aligned}$$

The agent's labor input choice does not affect the probability of z_t and private signals z_1 and z_2 are conditionally independent.²⁶ In addition to [A4], we assume for all z_t :

$$\begin{aligned} \text{[A5]} \quad & P(x | z_t = G, a_1, a_2) \dots P(x | z_t = B, a_1, a_2) \text{ when } a_1, a_2 = +H, H; \\ & P(x | z_t, a_1, a_2) = P(x | a_1, a_2) \text{ when } a_1, a_2 = +H, L, +L, H, +L, L, \end{aligned}$$

Under [A5], z_1 and z_2 are valuation informative about x only when the agent provides high effort in both periods. When the agent supplies low effort in either period, x , z_1 , and z_2 are mutually independent (in fact z_1 and z_2 reduce to pure noise).²⁷ One can interpret [A5] as a complementarity effect among the two factors of production (i.e., the agent's efforts and the information content of the signals). If the agent works hard, he is likely to receive forward-looking information. If he shirks, his information is not valuation informative about x .²⁸

²⁶ This assumption is part of our experimental design, which isolates the valuation informativeness (and therefore the veracity check role) of accounting information. As noted earlier, a self-reported z_t that is only incentive informative about a_t is not useful to the principal. If z_t is also incentive informative about a_t , then accounting signal y_t is also incentive informative about a_t when z_t is not known. It will be hard to tell whether the usefulness of y_t is attributed to its valuation informativeness (about x) or its incentive informativeness (about a_t). By making [A4], we remove the interaction between the two types of informativeness of z_t and streamline the analysis.

²⁷ [A5] simplifies the derivation of the sufficient conditions for communication to be strictly preferred (Observation 1). However, Proposition 2, the more important result, does not rely on [A5].

²⁸ We give a numerical example of such a joint probability $P(x, z_1 | a_1, a_2)$ structure. Consider the following:

	$a_1, a_2 = +H, H,$		$a_1, a_2 = +H, L,$		$a_1, a_2 = +L, H,$		$a_1, a_2 = +L, L,$	
$x =$	1	0	1	0	1	0	1	0
$z_1 = G$	.51	.09	.36	.24	.42	.18	.18	.42
$z_1 = B$	.29	.11	.24	.16	.28	.12	.12	.28

Consistent with [A4], $P(z_1 = G | a_1, a_2) = P(z_1 = G) = .6$. [A5] is also readily verified. The joint probability

Turning to the contracting problem with communication, we focus on a setting where a single message is reported by the agent immediately after he privately observes z_1 . Therefore, the agent's self-reporting strategy is a mapping $m: Z \rightarrow Z$, with $m(z_1)$ denoting the message reported when z_1 is observed. Relying on the Revelation Principle, we restrict our attention to truth-telling mechanisms.²⁹ Notice under both recognition regimes (R_1 and R_2), accounting signal is produced subsequent to the self-report $m(\cdot)$. Let m^T denote the truth-telling reporting strategy and let $E[U(I(x, y_t, m(\mathbb{h})); \theta | a_1, m, \cdot)]$ denote the agent's expected utility if he supplies input sequence $a_1, \cdot$, and adopts reporting strategy m under payment scheme $I(x, y_t, m(\mathbb{h}))$. The mechanism design program for the principal can be written as:

$$E[U(I(q; q) | H, m^T, \mathbf{h})] \leq E[U(I(q; q) | a_1, m, \mathbf{h})] \leq a_1, m, \mathbf{h} \quad (9)$$

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Demand For Veracity Check

We begin with the case in which neither y_1 nor y_2 is recognized. In this case, the agent issues a self-report on z_1 and the two parties contract on x and $m(\cdot)$.³¹

Observation 1: Assume [A2], [A4], [A5] and no recognition, communication of z_1 is strictly valuable if

(i) $P(x=1|LH) > P(x=1|HL)$; and

$$(ii) \frac{P(x=1|HH) P(x=0|LH)}{P(x=1|LH) P(x=0|HH)} < \frac{P(x=1|z_1=G, HH) P(x=0|z_1=B, HH)}{P(x=1|z_1=B, HH) P(x=0|z_1=G, HH)}$$

Intuitively, the two conditions appeal to mutual gains through communication. Under condition (i), the only binding IC constraint involves $+L$, H , absent communication. This gives the principal more flexibility in designing the optimal contract. Condition (ii) requires that the probability revision caused by z_1 is large enough to make the communication worthwhile.³² Observation 1 merely identifies a parameter region where communication is strictly valuable. The value of communication in agency settings has been established by previous studies mentioned before (Christensen [1981] etc.).

We give the following numerical example to illustrate Observation 1. Suppose, $P(z_1=G)=.6$, $P(x=1|z_1=G, HH)=.85$, $P(x=1|z_1=B, HH)=.725$, $P(x=1|LH)=.7$, $P(x=1|HL)=.6$, $P(x=1|LL)=.3$, $c(L)=0$, $c(H)=2,000$, $r=.0001$, and $\underline{U} = \exp(-r15,000)$. Without communication, the expected payment to the agent is 21,640. With a self-report on z_1 , the expected payment is 21,542.³³ Notice condition (i) and (ii) in the observation can be verified: (i) $P(x=1|LH) = .7 > P(x=1|HL) = .6$; and (ii) $(.8) (.3)/((.7)(.2)) = 1.714 < (.85) (.275)/((.725)(.15)) = 2.149$.

³¹ The agent's decision tree in this special case is:

$$\begin{array}{ccccccc} \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} & \text{I} \\ \text{a}_1, \{H, L\} & z_1 O\{G, B\} & m(z_1) O\{G, B\} & a_2, \{H, L\} & z_2 O\{G, B\} & x O\{0, 1\} & I(x, m) \end{array}$$

³² This is similar to the notion of information gap in Christensen [1981].

³³ The payment schedule for the case with no communication is $I(x=1)=24,848$; $I(x=0)=8,807$ and with communication, $I(m=G, x=1)=25,020$; $I(m=G, x=0)=8,671$; $I(m=B, x=1)=24,110$; and $I(m=B, x=0)=9,173$.

When communication is strictly valuable, accounting recognition is useful as long as y_t reveals something about the realization of z_t . In other words, there exists a strict demand for a veracity check for the earlier self-report.

Proposition 2: Assume [A2], [A3], [A4], and that communication is strictly valuable, both recognition rules (R_t , $t=1,2$) are strictly valuable to the principal.

Recall that accounting signal y_1 (resp. y_2) is at best a garbling of the agent's private signal z_1 (resp. z_1, z_2). Normally when y_t is a garbling of z_t , contracting on y_t is not useful when z_t is already used in the contract. In our setting, however, z_1 is self-reported through $m(z_1)$, and the self-reporting is subject to additional (induced) moral hazard.³⁴ In turn, contracting on accounting signal y_t helps the principal combat the moral hazard associated with the self-reporting. Should he choose to lie in his report (i.e., $m(z_1) \neq z_1$), the agent runs the risk of being "punished" by the upcoming accounting report. This disciplining role is what makes y_t valuable for the principal.

Discussion

In general, an entity's accounting report and the voluntary disclosure by its managers are both useful to its stakeholders. In our setting, both the accounting signal (y_t) and the self-report ($m(z_t)$) help the principal mitigate his contracting problem. More importantly, the two sources of information complement each other as well. The self-report has the comparative advantage of being early and having the ability to predict x . However, it lacks trustworthiness because, if not controlled, the agent has the incentive to lie to his advantage. On the other hand, the accounting signal may not be valuation informative about x conditional on a truthfully reported z_t by the agent. But it has the advantage of being a veracity check on the agent's earlier self-report because the typical accounting report is subject to audit

³⁴ The moral hazard on reporting is induced because any misreporting *pe se* does not factor into the agent's utility. The agent has no incentive to lie just for the sake of lying. However, the agent is also asked to provide unobservable, and personally costly, labor inputs, as well as the report.

and there is no (or considerably less) incentive problem associated with this source.³⁵ This is the key to understanding the usefulness of accounting information in our context.

This result has implications for the different functions served by accounting. In empirical research, especially event studies, earnings announcements fail to explain a large part of security price movement, and this is interpreted as suggesting, if not implying, accounting reports lack usefulness (e.g., Lev [1989]). In this paper, however, accounting is useful not for its expediency in providing timely valuation information to the security market, but for its ability to provide a veracity check on other, unaudited sources of information. These other sources (e.g., the manager's self-report) are more readily controlled because there is a pending, undisputable accounting report. Therefore, the noted empirical regularity does not necessarily imply the lack of usefulness of accounting information. In fact, limited reaction to the accounting report is expected in equilibrium. Timeliness (early production and dissemination of information) is not necessarily a virtue for accounting as a source of information. Delaying an accounting report can enhance the disciplining role of accounting (through the auditing process). This paper highlights credibility as the key characteristics of accounting. The principal can use accounting report readily while she must control the self-report (through additional truth-telling IC constraints). The usefulness of accounting comes from its disciplining function through labor contracting.³⁶

³⁵ The auditing process and the reputation management by the accounting professionals are outside of this model. We take the easy route of assuming that they result in no incentive problems associated with these professionals. But it is by no means implied that the process and behavior are unimportant. In fact, they are critical to the result.

The growing literature on earnings management has indicated that managers have, on the margin, some control over the accounting reports. This possibility is absent in this paper. Our current focus is the interaction between audited accounting reports and other, unaudited sources of information.

³⁶ Here the disciplining function is through formal contracting, which is a modeling convenience. In practice, the disciplining may be achieved through managerial reputation and retention, etc. The focus of this study is on the disciplining function, not the form of the disciplining function.

Communication and Optimal Recognition

Finally, we examine how the presence of the earlier communication changes the usefulness of alternative recognition rules. We have established that communication gives rise to an additional role of credible accounting information (i.e., veracity check). Now we probe further to see whether this additional role makes early or late recognition more or less attractive.

Proposition 3: *Assume [A2], [A3], [A4] and that communication is strictly valuable, there exists a parameter region with positive measure in which early recognition is strictly preferred when no communication is allowed but late recognition is strictly preferred when communication is allowed.*

We use a series of numerical examples to illustrate Proposition 3 to see how the presence of communication may prompt the principal to favor the late recognition rule (R_2). We continue with the numerical specifications from the last numerical example; in addition, suppose z_2 is such that $P(z_2 = G) = 0.5$ and $P(x = 1 | z_2 = G, HH) = 0.82$. Under R_1 , y_1 is a garbling of z_1 with $P(y_1 = 2 | z_1 = G) = .8$ and $P(y_1 = 2 | z_1 = B) = .05$. Under R_2 , $P(y_2 = 2 | z_2 = G) = .65$ and $P(y_2 = 2 | z_2 = B) = .35$. In Figure 2, we plot the four expected payments S $C(R_1)$, $C(R_2)$, $C(R_1, m)$, and $C(R_2, m)$ S for different values of conditional probability O / $P(y_2 = 2 | z_1 = G)$ with $O \in [.5, .8]$. Intuitively, higher O means y_2 conveys “more information” about z_1 .

Absent communication $C(R_2)$ decreases in O as “more information” about z_1 is available with higher O . Naturally, $C(R_1)$ is a constant as the stochastic property of y_1 does not change with O . With communication, $C(R_2, m)$ and $C(R_1, m)$ display a similar pattern. Examination of Figure 2 shows that in the parameter region where O ranges from approximately .70 to .775, communication makes delaying accounting recognition optimal (i.e., R_1 is preferred absent communication and R_2 is preferred with communication).

To understand the intuition behind this result, recall that incentive informativeness has been removed by assumption [A4]. The only benefit from producing additional information (either through accounting signals y_i and/or the self-report $m(z_i)$) is derived from early read on output x to improve the

contracting. In these examples, both z_1 and z_2 are valuation informative about x . Therefore under early recognition, accounting signal y_1 , correlated with z_1 , is informative about x through the fact z_1 is informative about x . And under late recognition, the accounting signal y_2 , correlated with both z_1 and z_2 , is informative about x through two independent “channels:” (1) the fact z_1 is informative about x ; and (2) the fact z_2 is informative about x .

In the interesting parameter region, without communication, R_1 is preferred because the principal can infer more about x from y_1 than from y_2 . With communication, under both recognition regimes, z_1 is, in equilibrium, revealed truthfully, which provides information about x . With R_1 , y_1 is used only to discipline the self report $m(z_1)$; but with R_2 , y_2 does two jobs: (1) disciplining $m(z_1)$ (y_2 is correlated with z_1); and (2) providing additional information about x since y_2 is also correlated with z_2 (and z_2 is valuation informative about x independently of z_1). With z_1 truthfully communicated, more is known about x with y_2 than with y_1 so the principal is willing to tolerate some noise in y_2 about z_1 (i.e., the disciplining role). Here, the benefit of the additional information about x out-weights the cost of noise.

The key idea shown in these numerical examples is that other information sources (e.g., the manager's self-report) interact with the accounting source. Therefore, when evaluating alternative recognition rules, one must keep in mind this interactive effect among the proposed recognition rule and other sources of information. In this instance, the presence of other information makes delaying the accounting recognition optimal.

The opposite of Proposition 3 may also be true. In other words, there exist a set of parameters that late recognition is strictly preferred when no communication is allowed but early recognition is strictly preferred when communication is allowed.³⁷ We have not considered soliciting a second self-

³⁷ Here are the specifics of the counter-example: set $P(y_1 = 2 | z_1 = G) = .7$ and $P(y_1 = 2 | z_1 = B) = .2$ and keep everything else the same the examples in the text. At $O = .56$, we have $C(R_1) = 21,359$, $C(R_2) = 21,337$, $C(R_1, m) = 21,264$, and $C(R_2, m) = 21,312$.

report (on z_2) from the manager. Since z_2 is informative about the outcome x , we expected such communication can be valuable (Christensen [1981], etc.). The demand for a later veracity check may also exist. However, introducing a second self-report may further complicate the model. Christensen and Feltham [1997] study sequential vs. simultaneous self-reports in an agency setting with two private signals and Farlee [1998] studies timely vs. delayed self-reports in similar settings.

Section V Conclusions

In accounting theory and practice, recognition issues have been controversial. We seek to enrich the debate by acknowledging the incentive use of accounting information and the interaction between accounting and other information sources. We cast a recognition choice problem in a stewardship framework and allow other information sources into the picture. We see that the optimal recognition choice depends on whether the moral hazard at the proposed recognition time is critical, not whether the most uncertainty about the earning process has been resolved. When we allow other information sources into the model, the veracity check role of accounting surfaces in our analysis. Finally, the presence of other information may call for later recognition.

In our model, the contracting and the confirmatory roles of accounting are highlighted. We see these two roles as the comparative advantage of accounting as a source of information. Accounting is valuable to the extent that it is credible, comprehensive, and subject to careful and professional judgment. Naturally, this makes accounting information a perfect candidate for contracting and confirmatory uses.³⁸ While other information sources may be more timely in providing valuation information about an entity, audited accounting information, when used in explicit or implicit contracts, helps ensure the accuracy of the reports from other sources.

³⁸ Gigler and Hemmer [1998] also emphasizes the confirmatory role of financial accounting reports.

Tractability concerns clearly limit the analysis. As an interesting extension of the study, we can allow the agent to have partial control over the accounting apparatus. This will enable us to examine the accounting structure in the more realistic setting where performance manipulations may exist.

Figure 1: Sequence of Events

	t = 1		t = 2		t = 3
	/	/	/	/	/
agent's inputs	$a_1 \in \{H, L\}$		$a_2 \in \{H, L\}$		
agent's private information		$z_1 \in \{G, B\}$		$z_2 \in \{G, B\}$	
output					$x \in \{0, 1\}$
accounting recognition rules:					
early: R_1		$y_1 \in \{1, 2\}$		n/a	
late: R_2		n/a		$y_2 \in \{1, 2\}$	
agent's net compensation		$I_1 \leq c(a_1)$		$I_2 \leq c(a_2)$	I_3
principal's net cash flow		$\leq I_1$		$\leq I_2$	$q \leq x \leq I_3$

Figure 2: Expected Payments under Alternative Recognition Rules and Communication Regimes

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APPENDIX: PROOFS

Proof of Proposition 1:

We begin with the basic model:

$$C^* / \min_{I(x)} E[I(x) | H, \theta^H] = E_x P(x | HH) I(x) \quad (1)$$

$$\text{Subject to } E[U(I(x); \theta | H, \theta^H)] \leq \underline{U} \quad (2)$$

$$E[U(I(x); \theta | H, \theta^H)] \leq E[U(I(x); \theta | a_1, \theta^H)] \quad \forall a_1, \theta^H \quad (3)$$

with $a_1 \in \{H, L\}$ and $\theta^H \in \{G, B\} \subset \{H, L\}$, the agent has eight possible strategies:

Strategy	a_1	θ^H	
		$\theta^H(G)$	$\theta^H(B)$
(i)	H	H	H
(ii)	H	L	H
(iii)	H	H	L
(iv)	H	L	L
(v)	L	H	H
(vi)	L	L	H
(vii)	L	H	L
(viii)	L	L	L

In general, there are seven IC constraints in the basic model (e.g., strategy (i) is preferred to (ii), (iii)).

We label the seven IC constraints by their off-equilibrium strategy numbers, (ii) through (viii). Given [A1] and [A2], we can collapse the seven IC constraints into two IC constraints.

To proceed, constraint (ii) requires:

$$P(G|H) E[U|G, HH] + P(B|H) E[U|B, HH] \leq P(G|H) E[U|G, HL] + P(B|H) E[U|B, HH]$$

Canceling common terms, the constraint simplifies to:

$$E[U|G, HH] \leq E[U|G, HL] \quad (AI-1)$$

[A1] implies $P(x, z_2 | z_1, a_1, a_2) = P(x | a_1, a_2) P(z_2 | a_2)$ for all z_1, z_2 , and a_1, a_2 . Therefore, constraint (ii) reduces to:

$$E[U|HH] \leq E[U|HL] \quad (AI-2)$$

$$\text{where } E[U|a_1, a_2] / G_X P(x|a_1, a_2) U(I(x); a_1, a_2) \quad (\text{AI-3})$$

Similarly, by [A1], constraints (iii) and (iv) are identical to (AI-1). Thus, constraints (ii), (iii), and (iv) can be replaced by (AI-1), which we rename the second-period IC constraint.

Similarly, [A1] simplifies constraints (v) through (viii) to these four inequalities:

$$\text{constraint (v):} \quad E[U|HH] \geq E[U|LH]$$

$$\text{constraint (vi):} \quad E[U|HH] \geq P(G|L) E[U|LL] + P(B|L) E[U|LH]$$

$$\text{constraint (vii):} \quad E[U|HH] \geq P(G|L) E[U|LH] + P(B|L) E[U|LL]$$

$$\text{constraint (viii):} \quad E[U|HH] \geq E[U|LL]$$

Clearly constraints (v) and (viii) imply constraints (vi) and (vii). Given [A2], the CDFC assumption, it is easy to verify constraint (viii) does not bind.

We rename constraint (v) the first-period IC constraint. Thus far, all but the following three constraints in the basic model have been eliminated: (1) the IR constraint, (2) the first-period IC constraint (constraint (v)), and (3) the second-period IC constraint (inequality (AI-1)).

Let λ , θ_1 , and θ_2 be the non-negative Lagrange multipliers associated with these three constraints respectively. We set up the following Lagrangian:

$$\begin{aligned} \mathcal{L} = & E[I(x)|H, \theta^H] - \lambda (E[U|HH] - E[U|LH]) \\ & - \theta_1 (E[U|HH] - E[U|LL]) \\ & - \theta_2 (E[U|HH] - E[U|HL]) \end{aligned} \quad (\text{AI-4})$$

From the first-order conditions, it is easy to verify that:

- (i) $\lambda > 0$ (Holmström and Milgrom [1987]),
- (ii) if $P(x=1|LH) > P(x=1|HL)$, then $\theta_1 > 0$ and $\theta_2 = 0$ (because $+L, H$, dominates $+H, L$, in the sense of first order stochastic dominance and $I(x=1) > I(x=0)$ in equilibrium), and
- (iii) if $P(x=1|LH) < P(x=1|HL)$, then $\theta_2 > 0$ and $\theta_1 = 0$.

Now we analyze the usefulness of R_1 . Under R_2 , y_2 is just an *ex post* monitor. From Holmström [1979] and Grossman and Hart [1983] where there is only one binding IC constraint, we know a monitor, say y_2 , is useful if the likelihood ratio associated with the constraint is a function of y_2 . This is the case

here because when $P(x=1|LH) > P(x=1|HL)$, z_1 is incentive informative about a_1 and y_2 is a garbling, but not independent, of z_1 . A parallel argument applies to the case when $P(x=1|LH) < P(x=1|HL)$.

Now consider R_1 . First, suppose $P(x=1|LH) < P(x=1|HL)$, we claim R_1 is useless. Under R_1 , the agent's second-period policy is a mapping $\sigma: Z \times Y \rightarrow A$. There are now 16 possible σ mappings. Mixed with a_1 , there are 32 possible strategies. To avoid repetition, we replace the 15 IC constraints involving $+H, \sigma, (\sigma \dots \sigma^H)$ with the following four constraints:

$$E[U|z_1, y_1, HH] \geq E[U|z_1, y_1, HL] \quad \forall z_1, y_1 \quad (\text{AI-5})$$

These four constraints imply that the agent, having chosen $a_1=H$, will choose $a_2=H$ for all possible realizations of z_1 and y_1 . If (AI-5) is satisfied, the 15 IC constraints that involve $+H, \sigma, (\sigma \dots \sigma^H)$ are also satisfied.³⁹

[A1] and the fact y_1 is a garbling of z_1 imply $P(x|z_1, y_1, a_1, a_2) = P(x|a_1, a_2)$, so (AI-5) reduces to the following two constraints:

$$E[U|y_1, HH] \geq E[U|y_1, HL] \quad \forall y_1 \quad (\text{AI-6})$$

where $E[U|y_1, a_1, a_2] = \int G_X P(x|a_1, a_2) U(I(x, y_1); \mathbb{A})$

We solve the optimization problem with only the two constraints in (AI-6) and the IR constraint, (ignoring the other 16 IC constraints involving $+L, \sigma$, for the moment). We obtain the following first-order conditions:

$$\frac{-1}{rV(I(x, y_1))k^2} = \mu + \lambda(y_1) \left(1 - \frac{P(x|HL)}{kP(x|HH)} \right) \quad \forall x, y_1 \quad (\text{AI-7})$$

³⁹ To see this, write the 15 constraints as: $E[U(\mathbb{A})|H, \sigma^H] \geq E[U(\mathbb{A})|H, \sigma] \geq \dots \geq E[U(\mathbb{A})|H, \sigma^H]$. But $E[U(\mathbb{A})|H, \sigma^H] = \int_{Z \times Y} E[U|z_1, y_1, HH]$ and $E[U(\mathbb{A})|H, \sigma] = \int_{Z \times Y} E[U|z_1, y_1, H, \sigma(z_1)]$. Given (AI-5), each of the four terms in $E[U(\mathbb{A})|H, \sigma^H]$ is greater than or equal to its counterpart in $E[U(\mathbb{A})|H, \sigma]$.

From here it is clearly that R_1 is useless if $P(x=1|LH) < P(x=1|HL)$, as strict use would needlessly impose risk on the agent. If the principal does not use y_1 , the omitted IC constraints are clearly satisfied, R_1 is indeed useless.

Now, suppose $P(x=1|LH) > P(x=1|HL)$, but let $I^*(x, y_1) = I^*(x)$, where $I^*(x)$ is the optimal contract from the basic model. We show this solution violates the optimality conditions in the expanded program. With this supposed solution, the IR constraint binds, only one IC constraint is binding by complementary slackness. The binding IC constraint must involve $+L, \theta^H$. All other 30 off-equilibrium strategies result in input sequence $+H, L$, or $+L, L$, with non-zero probability and are, therefore, dominated by $+L, \theta^H$, under incentive scheme $I^*(x)$. (CDFC is used here.) The first-order conditions evaluated using the supposed solution is:

$$\frac{-1}{rV(I^*(x, y_1))k^2} = \mu + \lambda(L, \alpha^H) \left(1 - \frac{P(x|LH) \sum_z P(y_1|z_1)P(z_1|a_1=L)}{kP(x|HH) \sum_z P(y_1|z_1)P(z_1|a_1=H)} \right) \otimes x, y_1 \quad (AI-8)$$

The right-hand-side of (AI-8) is a non-trivial function of y_1 because (i) z_1 is incentive informative about a_1 and (ii) y_1 is a garbling but not independent of z_1 . However, the left-hand-side of (AI-8) is not, under the supposed solution.

Proof of Observation 1:

When only x is contractible, the design program with communication is as follows:

$$C(m) / \text{minimum } E[I(x, m(z_1)) | H, m^T, \theta^H]$$

$$\text{Subject to } E[U(I(\theta; \theta) | H, m^T, \theta^H)] \leq \underline{U}$$

$$E[U(I(\theta; \theta) | H, m^T, \theta^H)] \leq E[U(I(\theta; \theta) | a_1, m, \theta)] \otimes a_1, m, \theta$$

The agent's strategy for the entire game is represented by $+a_1, m, \theta$. There are four possible self-reporting policies, denoted m^T, m^{GG}, m^{BB} , and m^{BG} , where m^T is the truth-telling policy, m^{GG} (resp. m^{BB}) is the policy that always reports good news (resp. bad news), and m^{BG} is the policy under which the

agent always lies. Recall there are four second-period input policies (θ). Therefore, for the entire game, the agent has 32 possible strategies. The preferred strategy is θ^H, m^T, θ^H . The design program has 31 IC constraints and one IR constraint.

We use a variation argument to prove communication is strictly useful. Let v_1^*, v_0^* be the optimal payment scheme, in utility terms, for the mechanism design program without communication. We construct a trial solution $v_{m(\theta)x^*}$ in utility terms, to the program with communication: $v_{m(\theta)x} = v_x^* \otimes x$, $m(\theta) \in Z$. Clearly, the solution is feasible. Since $P(x=1|LH) > P(x=1|HL)$, only 7 of 31 IC constraints are satisfied with equality, which correspond to the following off-equilibrium strategies:

a_1	m	θ
H	m^{GG}	θ^H
H	m^{BB}	θ^H
H	m^{BG}	θ^H
L	m^T	θ^H
L	m^{GG}	θ^H
L	m^{BB}	θ^H
L	m^{BG}	θ^H

Each of the seven strategies has $\theta = \theta^H$. All other 24 off-equilibrium strategies (i.e., $\theta^L, m, \theta^L, \theta^H, \dots, \theta^H$) result in input sequence θ^H, L , or θ^L, L , with non-zero probability and are, therefore, dominated by θ^L, m, θ^H , or θ^H, m, θ^H , under the trial solution $v_{m(\theta)x^*}$.

Along with the IR constraints, we have the following eight constraints satisfied with equality:

$$E[U] \geq v_{G1} + (1 - \alpha_G)v_{G0} + (1 - \alpha_B)v_{B1} + (1 - \alpha_B)(1 - \alpha_B)v_{B0} \quad \text{(IR)} \quad \$ 0$$

$$E[U] \geq v_{G1} + (1 - \alpha_G)v_{G0} \quad \text{(TT}_{GG}) \quad \$ 0$$

$$E[U] \geq v_{B1} + (1 - \alpha_B)v_{B0} \quad \text{(TT}_{BB}) \quad \$ 0$$

$$E[U] \geq v_{G1} + (1 - \alpha_G)v_{B0} + (1 - \alpha_B)v_{G1} + (1 - \alpha_B)(1 - \alpha_B)v_{G0} \quad \text{(TT}_{BG}) \quad \$ 0$$

$$E[U] \geq \alpha_G v_{G1} + (1 - \alpha_G)\alpha_B v_{G0} + (1 - \alpha_G)\alpha_B v_{B1} + (1 - \alpha_G)(1 - \alpha_B)\alpha_B v_{B0} \quad \text{(IC}_T) \quad \$ 0$$

$$E[U] \geq \alpha_G v_{G1} + (1 - \alpha_G)\alpha_B v_{G0} \quad \text{(IC}_{GG}) \quad \$ 0$$

$$E[U] \geq \alpha_B v_{B1} + (1 - \alpha_B)\alpha_G v_{B0} \quad \text{(IC}_{BB}) \quad \$ 0$$

$$E[U] \geq \beta \cdot \mathbb{N}k_{B1} + \beta(1 - \mathbb{N})k_{B0} + (1 - \beta) \cdot \mathbb{N}k_{G1} + (1 - \beta)(1 - \mathbb{N})k_{G0} \geq 0 \quad (IC_{BG})$$

where:

$$\beta = \frac{P(z_1 = G)}{P(z_1 = G) + \frac{P(x = 1 | z_1 = G, HH)}{P(x = 1 | z_1 = B, HH)}}; \quad \mathbb{N} = \frac{P(x = 1 | LH)}{P(x = 1 | HH)} = \frac{\beta \cdot P(z_1 = G)}{\beta \cdot P(z_1 = G) + (1 - \beta) \cdot P(z_1 = B)}$$

Rewriting the principal's objective function, in utility terms, we have:

$$E[I(x, m(\mathbb{A})) | H, m^T, \theta^H] = \beta \cdot V^{1-\eta}(v_{G1}) + \beta(1 - \mathbb{N})V^{1-\eta}(v_{G0}) + (1 - \beta) \cdot V^{1-\eta}(v_{B1}) + (1 - \beta)(1 - \mathbb{N})V^{1-\eta}(v_{B0})$$

Totally differentiating (IR), (TT_{GG}), (IC_{BB}), and the principal's objective function at the trial solution, we have:

$$\begin{aligned} \text{IR} &= \beta \cdot dv_{G1} + \beta(1 - \mathbb{N})dv_{G0} + (1 - \beta) \cdot dv_{B1} + (1 - \beta)(1 - \mathbb{N})dv_{B0} \\ \text{TT}_{GG} &= \text{IR} - \mathbb{N} \cdot dv_{G1} - (1 - \mathbb{N})dv_{G0} \\ \text{IC}_{BB} &= \text{IR} - \mathbb{N}k_{B1} - (1 - \mathbb{N})k_{B0} \\ E[I(\mathbb{A})] &= \frac{M}{V} V^{1-\eta} [\beta \cdot dv_{G1} + (1 - \beta) \cdot dv_{B1}] + \frac{M}{V} V^{1-\eta} [\beta(1 - \mathbb{N})dv_{G0} + (1 - \beta)(1 - \mathbb{N})dv_{B0}] \end{aligned}$$

For a fixed $dv_{G1} > 0$, choose dv_{G0} , dv_{B1} , and dv_{B0} so that $\text{IR} = 0$, $\text{IC}_{BB} = 0$, and $\text{TT}_{GG} = 0$.

Therefore we have system of three linear equations with three unknowns. Solving the system⁴⁰, we have:

$$\text{sign}(\text{E}[I(\mathbb{A})]) = \text{sign}(\mathbb{T}) = \text{sign} [\mathbb{N} \cdot (1 - \mathbb{N})(1 - \beta) - \mathbb{N} \cdot \beta(1 - \mathbb{N})(1 - \beta)]$$

Therefore, the expected payment is reduced, i.e., $\text{E}[I(\mathbb{A})] < 0$, if

$$\frac{\zeta(1 - \zeta')}{\zeta'(1 - \zeta)} < \frac{\zeta_G(1 - \zeta_B)}{\zeta_B(1 - \zeta_G)}$$

With dv_{G1} , dv_{G0} , dv_{B1} , and dv_{B0} chosen in such a way, we can readily verify that constraints

(TT_{BB}), (TT_{BG}), (IC_T), (IC_{GG}) and (IC_{BG}) are satisfied.

⁴⁰ One can readily verify the determinant of the coefficient matrix is not zero, so the solution to the system of three linear equations exists.

Proof of Proposition 2:

The design program with recognition and communication is as follows:

$$C(R_1, m) / \underset{I(A)}{\text{minimum}} E[I(x, y_1, m(z_1)) | H, m^T, \theta^H] \quad (7)$$

$$\text{Subject to } E[U(I(\theta; \theta) | H, m^T, \theta^H)] \leq \underline{U} \quad (8)$$

$$E[U(I(\theta; \theta) | H, m^T, \theta^H)] \leq E[U(I(\theta; \theta) | a_1, m, \theta)] \quad \forall a_1, m, \theta \quad (9)$$

Under R_2 , the agent has 32 strategies. Let $\lambda(a_1, m, \theta)$ denote the non-negative Lagrange multipliers and let μ denote the multiplier associated with the IR constraint. The first order condition can be written as:

$$\frac{-1}{rV(I(x, y_2, m(\cdot)))k^2} = \mu + \sum_{(a_1, m, \alpha) \neq (H, m^T, \alpha^H)} \lambda(a_1, m, \alpha) \Lambda(a_1, m, \alpha)$$

where $\Lambda(a_1, m, \theta)$ will be specified in the following. Specifically, the first order condition with respect to $I(x, y_2, G)$ can be expanded to:

$$\begin{aligned} \frac{-1}{rV(I(x, y_2, G))k^2} = & \mu + \sum_{(a_1, \alpha) \neq (H, \alpha^H)} \lambda(a_1, m^T, \alpha) \left(1 - \kappa(a_1, \alpha(G)) \frac{P(x|z_1 = G, a_1, \alpha(G))}{P(x|z_1 = G, HH)} \right) + \sum_{a_1, \alpha} \lambda(a_1, m^{BB}, \alpha) \\ & + \sum_{a_1, \alpha} \lambda(a_1, m^{GG}, \alpha) \left(1 - \kappa(a_1, \alpha(G)) \frac{P(x|z_1 = G, a_1, \alpha(G))}{P(x|z_1 = G, HH)} - \phi_2 \kappa(a_1, \alpha(B)) \frac{(1-\gamma)P(x|z_1 = B, a_1, \alpha(B))}{\gamma P(x|z_1 = G, HH)} \right) \\ & + \sum_{a_1, \alpha} \lambda(a_1, m^{BG}, \alpha) \left(1 - \phi_2 \kappa(a_1, \alpha(B)) \frac{(1-\gamma)P(x|z_1 = B, a_1, \alpha(B))}{\gamma P(x|z_1 = G, HH)} \right) \end{aligned} \quad (AII-9)$$

where m^{GG} , m^{BB} , and m^{BG} are defined as in the proof of Observation 1, $N_2 = P(y_2|z_1=B)/P(y_2|z_1=G)$, and $\phi(a_1, a_2) = \exp(r(c(a_1) + c(a_2) - 2c(H)))$. Similarly, the first order condition with respect to $I(x, y_2, B)$ can be expanded to:

$$\begin{aligned}
\frac{-1}{rV(I(x, y_2, B)k^2)} = & \mu + \sum_{\langle a_1, \alpha \rangle \in \{H, \alpha^H\}} \lambda(a_1, m^T, \alpha) \left(1 - \kappa(a_1, \alpha(B)) \frac{P(x|z_1 = B, a_1, \alpha(B))}{P(x|z_1 = B, HH)} \right) + \sum_{a_1, \alpha} \lambda(a_1, m^{GG}, \alpha) \\
& + \sum_{a_1, \alpha} \lambda(a_1, m^{BB}, \alpha) \left(1 - \kappa(a_1, \alpha(B)) \frac{P(x|z_1 = B, a_1, \alpha(B))}{P(x|z_1 = B, HH)} - \phi_2^{-1} \kappa(a_1, \alpha(G)) \frac{\gamma P(x|z_1 = G, a_1, \alpha(G))}{(1 - \gamma) P(x|z_1 = B, HH)} \right) \\
& + \sum_{a_1, \alpha} \lambda(a_1, m^{BG}, \alpha) \left(1 - \phi_2^{-1} \kappa(a_1, \alpha(G)) \frac{\gamma P(x|z_1 = G, a_1, \alpha(G))}{(1 - \gamma) P(x|z_1 = B, HH)} \right)
\end{aligned} \tag{AII-10}$$

We know N_2 is a non-trivial function of y_2 because y_2 is not independent of z_1 . If the principal strictly prefers communication with no recognition, then at least one truth-telling constraint is binding, i.e., at least one of the multipliers $\delta(a_1, m^{GG}, \cdot)$, $\delta(a_1, m^{BB}, \cdot)$, or $\delta(a_1, m^{BG}, \cdot)$ is positive. Thus, we have the right-hand-side of (AII-9) as a non-trivial function of y_2 (if some $\delta(a_1, m^{GG}, \cdot)$, or $\delta(a_1, m^{BG}, \cdot)$ is non-zero), or we have the right-hand-side of (AII-10) as a non-trivial function of y_2 (if some $\delta(a_1, m^{BB}, \cdot)$, or the $\delta(a_1, m^{BG}, \cdot)$ is non-zero), or both⁴¹. Therefore, y_2 is useful for contracting.

Now consider R_1 . First consider the program with no recognition. Let $\mathbf{S}$ denote the set of off-equilibrium strategies in this program, strategies denoted $a_1, m, \cdot$. In the program with R_1 , the agent chooses among strategies $\underline{a}_1, \underline{m}, \underline{\cdot}$. (We use an underline “ $\underline{\cdot}$ ” to denote the elements of strategies in the program with R_1 to avoid confusion.) Here, $\underline{\cdot}: Z \times Y \rightarrow A$. There are sixteen possible “ $\underline{\cdot}$ ” mappings. Mixed with $\underline{a}_1 \in \{H, L\}$ and $\underline{m} \in \{m^T, m^{GG}, m^{BB}, m^{BG}\}$, the agent has one hundred twenty-eight strategies.

In this expanded program, we construct an off-equilibrium strategies set, denoted $\underline{\mathbf{S}}$, in the following way. For each strategy $a_1, m, \cdot \in \mathbf{S}$, find the strategy in the program with R_1 such that $\underline{a}_1 = a_1$, $\underline{m} = m$, and $\underline{\cdot}(z_1, y_1) = \cdot(z_1)$ for all y_1 .

By construction, the agent’s second-period input is not a function of y_1 for strategies in $\underline{\mathbf{S}}$. We write $\underline{\cdot}(z_1, \cdot)$ to reflect this fact. Further, we partition $\underline{\mathbf{S}}$ into four subsets: those that use m^T , $\underline{\mathbf{S}}_T$; those that use m^{GG} , $\underline{\mathbf{S}}_{GG}$, etc.

⁴¹ Note that in the right-hand-sides of both (AII-9) and (AII-10), the coefficients on N_2^{-1} are either negative or zero.

Now we take the optimal solution in the program with no recognition to construct a trial solution to the program with R_1 by setting $I(x, y_2, m(\mathbb{N})) = I^*(x, m(\mathbb{N}))$. If all constraints involving strategies not in $\underline{S}$ are redundant, (which will be proved to be true later,) we can evaluate the first order conditions in the program with R_1 at the trial solution as follows:

$$\begin{aligned} \frac{-1}{rV(I(x, y_1, G)k^2)} = \mu + \sum_{\langle \underline{a}_1, \underline{m}, \underline{\alpha} \rangle \in \underline{\Omega}_T} \lambda(\underline{a}_1, \underline{m}^T, \underline{\alpha}) & \left(1 - \kappa(\underline{a}_1, \underline{\alpha}(G, \bullet)) \frac{P(x|z_1 = G, \underline{a}_1, \underline{\alpha}(G, \bullet))}{P(x|z_1 = G, HH)} \right) + \sum_{\langle \underline{a}_1, \underline{m}, \underline{\alpha} \rangle \in \underline{\Omega}_{BB}} \lambda(\underline{a}_1, \underline{m}^{BB}, \underline{\alpha}) \\ + \sum_{\langle \underline{a}_1, \underline{m}, \underline{\alpha} \rangle \in \underline{\Omega}_{GG}} \lambda(\underline{a}_1, \underline{m}^{GG}, \underline{\alpha}) & \left(1 - \kappa(\underline{a}_1, \underline{\alpha}(G, \bullet)) \frac{P(x|z_1 = G, \underline{a}_1, \underline{\alpha}(G, \bullet))}{P(x|z_1 = G, HH)} - \phi_1 \kappa(\underline{a}_1, \underline{\alpha}(B, \bullet)) \frac{(1 - \gamma)P(x|z_1 = B, \underline{a}_1, \underline{\alpha}(B, \bullet))}{\gamma P(x|z_1 = G, HH)} \right) \\ + \sum_{\langle \underline{a}_1, \underline{m}, \underline{\alpha} \rangle \in \underline{\Omega}_{BG}} \lambda(\underline{a}_1, \underline{m}^{BG}, \underline{\alpha}) & \left(1 - \phi_1 \kappa(\underline{a}_1, \underline{\alpha}(B, \bullet)) \frac{(1 - \gamma)P(x|z_1 = B, \underline{a}_1, \underline{\alpha}(B, \bullet))}{\gamma P(x|z_1 = G, HH)} \right) \end{aligned} \quad (AII-11)$$

and

$$\begin{aligned} \frac{-1}{rV(I(x, y_1, B)k^2)} = \mu + \sum_{\langle \underline{a}_1, \underline{m}, \underline{\alpha} \rangle \in \underline{\Omega}_T} \lambda(\underline{a}_1, \underline{m}^T, \underline{\alpha}) & \left(1 - \kappa(\underline{a}_1, \underline{\alpha}(B, \bullet)) \frac{P(x|z_1 = B, \underline{a}_1, \underline{\alpha}(B, \bullet))}{P(x|z_1 = B, HH)} \right) + \sum_{\langle \underline{a}_1, \underline{m}, \underline{\alpha} \rangle \in \underline{\Omega}_{GG}} \lambda(\underline{a}_1, \underline{m}^{GG}, \underline{\alpha}) \\ + \sum_{\langle \underline{a}_1, \underline{m}, \underline{\alpha} \rangle \in \underline{\Omega}_{BB}} \lambda(\underline{a}_1, \underline{m}^{BB}, \underline{\alpha}) & \left(1 - \kappa(\underline{a}_1, \underline{\alpha}(B, \bullet)) \frac{P(x|z_1 = B, \underline{a}_1, \underline{\alpha}(B, \bullet))}{P(x|z_1 = B, HH)} - \phi_1^{-1} \kappa(\underline{a}_1, \underline{\alpha}(G, \bullet)) \frac{\gamma P(x|z_1 = G, \underline{a}_1, \underline{\alpha}(G, \bullet))}{(1 - \gamma)P(x|z_1 = B, HH)} \right) \\ + \sum_{\langle \underline{a}_1, \underline{m}, \underline{\alpha} \rangle \in \underline{\Omega}_{BG}} \lambda(\underline{a}_1, \underline{m}^{BG}, \underline{\alpha}) & \left(1 - \phi_1^{-1} \kappa(\underline{a}_1, \underline{\alpha}(G, \bullet)) \frac{\gamma P(x|z_1 = G, \underline{a}_1, \underline{\alpha}(G, \bullet))}{(1 - \gamma)P(x|z_1 = B, HH)} \right) \end{aligned} \quad (AII-12)$$

where $N_1 = P(y_1|z_1=B)/P(y_1|z_1=G)$. We know N_1 is a non-trivial function of y_1 because y_1 is not independent of z_1 . If the principal strictly prefers communication with no recognition, then at least one truth-telling constraint is binding, i.e., at least one of the multipliers $\mathbf{8}(\underline{a}_1, \underline{m}^{GG}, \underline{\alpha})$, $\mathbf{8}(\underline{a}_1, \underline{m}^{BB}, \underline{\alpha})$, or $\mathbf{8}(\underline{a}_1, \underline{m}^{BG}, \underline{\alpha})$ is positive. So we have the right-hand-side of (AII-11) as a non-trivial function of y_1 or we have the right-hand-side of (AII-12) as a non-trivial function of y_1 , or both. Therefore, y_1 is useful.

Now we prove that under the trial payment scheme, the constraints involving strategies not in $\underline{S}$ are, in fact, redundant. Let $\underline{\underline{S}}$ denote the set of all such strategies. Partition $\underline{\underline{S}}$ into two subsets denoted $\underline{\underline{S}}^1$ and $\underline{\underline{S}}^2$. $\underline{\underline{S}}^1$ is the set of strategies in which the agent's second-period input is a function of y_1 only when his first-period private signal (z_1) is $z_1\mathbb{N}$ and it is not a function of y_1 when $z_1 = z_1\mathbb{O} \dots z_1\mathbb{N}$. On the other hand, $\underline{\underline{S}}^2$ is the set of strategies in which the agent's second-period input is a function of y_1 when $z_1 = z_1\mathbb{N}$ as well as when $z_1 = z_1\mathbb{O} \dots z_1\mathbb{N}$.

We first prove if $E[U(\mathbb{A})|H, m^T, {}^H] \geq E[U(\mathbb{A})|a_1, \underline{m}, \underline{c}]$ for $a_1, \underline{m}, \underline{c}, 0 \in \underline{S}$, then $E[U(\mathbb{A})|H, m^T, {}^H] \geq E[U(\mathbb{A})|a_1, \underline{m}, \underline{c}]$ for $a_1, \underline{m}, \underline{c}, 0 \in \underline{S}^1$ at the trial solution, i.e., constraints involving strategies in $\underline{S}^1$ are redundant. Second, we prove if constraints involving strategies in $\underline{S}^1$ are redundant, then constraints involving strategies in $\underline{S}^2$ are also redundant.

First, suppose, without loss of generality, a representative strategy $a_1, \underline{m}, \underline{c}, 0 \in \underline{S}^1$ is such that $\underline{c}(z_1N, y_1N) \dots \underline{c}(z_1N, y_10)$ for z_1N equal to, say, good news (i.e., $z_1N = G$), and $\underline{c}(B, y_1N) = \underline{c}(B, y_10) = \underline{c}(B, C)$. Evaluating the agent's expected utility of adopting this strategy, we have:

$$\begin{aligned} E[U(\mathbb{A})|a_1, \underline{m}, \underline{c}] &= G_Z P(z_1, y_1N|a_1) E[U|z_1, y_1N, a_1, \underline{m}, \underline{c}] + G_Z P(z_1, y_10|a_1) E[U|z_1, y_10, a_1, \underline{m}, \underline{c}] \\ &= P(G, y_1N|a_1) E[U|G, y_1N, a_1, \underline{m}, \underline{c}(G, y_1N)] \\ &\quad + P(B, y_1N|a_1) E[U|B, y_1N, a_1, \underline{m}, \underline{c}(B, C)] \\ &\quad + P(G, y_10|a_1) E[U|G, y_10, a_1, \underline{m}, \underline{c}(G, y_10)] \\ &\quad + P(B, y_10|a_1) E[U|B, y_10, a_1, \underline{m}, \underline{c}(B, C)] \end{aligned} \quad (AII-13)$$

Now select a strategy, denoted $a_1^\circ, \underline{m}^\circ, \underline{c}^\circ$, from $\underline{S}$ such that: $a_1^\circ = a_1, \underline{m}^\circ = \underline{m}, \underline{c}^\circ(G, C) = \underline{c}(G, y_1N)$, and $\underline{c}^\circ(B, C) = \underline{c}(B, C)$. The constraint associated with this strategy requires:

$$E[U(\mathbb{A})|H, m^T, {}^H] \geq$$

$$\begin{aligned} E[U(\mathbb{A})|a_1^\circ, \underline{m}^\circ, \underline{c}^\circ] &= P(G, y_1N|a_1) E[U|G, y_1N, a_1, \underline{m}, \underline{c}^\circ(G, C)] \\ &\quad + P(B, y_1N|a_1) E[U|B, y_1N, a_1, \underline{m}, \underline{c}^\circ(B, C)] \\ &\quad + P(G, y_10|a_1) E[U|G, y_10, a_1, \underline{m}, \underline{c}^\circ(G, C)] \\ &\quad + P(B, y_10|a_1) E[U|B, y_10, a_1, \underline{m}, \underline{c}^\circ(B, C)] \end{aligned}$$

Since $\underline{c}^\circ(G, C) = \underline{c}(G, y_1N)$, and $\underline{c}^\circ(B, C) = \underline{c}(B, C)$ by construction, the constraint can be written as:

$$E[U(\mathbb{A})|H, m^T, {}^H] \geq$$

$$\begin{aligned} E[U(\mathbb{A})|a_1^\circ, \underline{m}^\circ, \underline{c}^\circ] &= P(G, y_1N|a_1) E[U|G, y_1N, a_1, \underline{m}, \underline{c}(G, y_1N)] \\ &\quad + P(B, y_1N|a_1) E[U|B, y_1N, a_1, \underline{m}, \underline{c}(B, C)] \\ &\quad + P(G, y_10|a_1) E[U|G, y_10, a_1, \underline{m}, \underline{c}(G, y_1N)] \\ &\quad + P(B, y_10|a_1) E[U|B, y_10, a_1, \underline{m}, \underline{c}(B, C)] \end{aligned} \quad (AII-14)$$

Combining (AII-13) and (AII-14) yields:

$$\begin{aligned} E[U(\mathbb{A}) | \underline{a}_1^{\circ}, \underline{m}^{\circ}, \underline{c}^{\circ}] & \geq E[U(\mathbb{A}) | \underline{a}_1, \underline{m}, \underline{c}] \\ & = P(G, y_1 0 | \underline{a}_1) (E[U | G, y_1 0, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 \mathbb{N})] \geq E[U | G, y_1 0, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 0)]) \end{aligned} \quad (\text{AII-15})$$

Select another strategy, denoted $\underline{a}_1^{\circ\circ}, \underline{m}^{\circ\circ}, \underline{c}^{\circ\circ}$, from $\underline{S}$ such that: $\underline{a}_1^{\circ\circ} = \underline{a}_1, \underline{m}^{\circ\circ} = \underline{m}, \underline{c}^{\circ\circ}(G, \mathbb{C}) = \underline{c}(G, y_1 0)$, and $\underline{c}^{\circ\circ}(B, \mathbb{C}) = \underline{c}(B, \mathbb{C})$. In similar fashion, the constraint associated with that strategy can be written as:

$$\begin{aligned} E[U(\mathbb{A}) | H, m^T, c^H] & \geq \\ E[U(\mathbb{A}) | \underline{a}_1^{\circ\circ}, \underline{m}^{\circ\circ}, \underline{c}^{\circ\circ}] & = P(G, y_1 \mathbb{N} | \underline{a}_1) E[U | G, y_1 \mathbb{N}, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 0)] \\ & \quad + P(B, y_1 \mathbb{N} | \underline{a}_1) E[U | B, y_1 \mathbb{N}, \underline{a}_1, \underline{m}, \underline{c}(B, \mathbb{C})] \\ & \quad + P(G, y_1 0 | \underline{a}_1) E[U | G, y_1 0, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 0)] \\ & \quad + P(B, y_1 0 | \underline{a}_1) E[U | B, y_1 0, \underline{a}_1, \underline{m}, \underline{c}(B, \mathbb{C})] \end{aligned} \quad (\text{AII-16})$$

Combining (AII-13) and (AII-16) yields:

$$\begin{aligned} E[U(\mathbb{A}) | \underline{a}_1^{\circ\circ}, \underline{m}^{\circ\circ}, \underline{c}^{\circ\circ}] & \geq E[U(\mathbb{A}) | \underline{a}_1, \underline{m}, \underline{c}] \\ & = P(G, y_1 \mathbb{N} | \underline{a}_1) (E[U | G, y_1 \mathbb{N}, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 0)] \geq E[U | G, y_1 \mathbb{N}, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 \mathbb{N})]) \end{aligned} \quad (\text{AII-17})$$

Suppose $E[U(\mathbb{A}) | \underline{a}_1, \underline{m}, \underline{c}] \neq E[U(\mathbb{A}) | \underline{a}_1^{\circ\circ}, \underline{m}^{\circ\circ}, \underline{c}^{\circ\circ}]$, constraint (AII-13) is implied by constraint (AII-16) and is, therefore, redundant. If $E[U(\mathbb{A}) | \underline{a}_1, \underline{m}, \underline{c}] > E[U(\mathbb{A}) | \underline{a}_1^{\circ\circ}, \underline{m}^{\circ\circ}, \underline{c}^{\circ\circ}]$, (AII-17) implies:

$$E[U | G, y_1 \mathbb{N}, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 \mathbb{N})] > E[U | G, y_1 \mathbb{N}, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 0)]$$

But under the trial solution, $E[U | G, y_1 \mathbb{N}, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 \mathbb{N})] = E[U | G, y_1 0, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 \mathbb{N})]$ and $E[U | G, y_1 \mathbb{N}, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 0)] = E[U | G, y_1 0, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 0)]$, so we have:

$$E[U | G, y_1 0, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 \mathbb{N})] > E[U | G, y_1 0, \underline{a}_1, \underline{m}, \underline{c}(G, y_1 0)]$$

By (AII-15), this implies $E[U(\mathbb{A}) | \underline{a}_1^{\circ}, \underline{m}^{\circ}, \underline{c}^{\circ}] > E[U(\mathbb{A}) | \underline{a}_1, \underline{m}, \underline{c}]$. So constraint (AII-13) is implied by constraint (AII-14) and is, again, redundant. Therefore, constraint (AII-13) is implied by either (AII-14) or (AII-16) and can be eliminated under the trial solution. This is true for each strategy in the set $\underline{S}^1$.

A parallel argument applies to the strategies in $\underline{S}^2$ so that strategies in the set $\underline{S}^2$ can be ignored when the constraints are evaluated at the trial solution.

Proof of Proposition 3:

Consider a numerical specification of the following:

$$P(z_1 = G) = .6, P(x = 1 | z_1 = G, HH) = .85, P(x = 1 | z_1 = B, HH) = .725,$$

$$P(x = 1 | LH) = .7, P(x = 1 | HL) = .6, P(x = 1 | LL) = .3,$$

$$c(L) = 0, c(H) = 2,000, r = .0001, \text{ and } \underline{U} = \exp(-r15,000).$$

$$P(z_2 = G) = 0.5 \text{ and } P(x = 1 | z_2 = G, HH) = 0.82.$$

Under R_1 , y_1 is a garbling of z_1 with $P(y_1 = 2 | z_1 = G) = .8$ and $P(y_1 = 2 | z_1 = B) = .05$.

Under R_2 , y_2 is a garbling of z_1 and z_2 with $P(y_2 = 2 | z_2 = G) = .7$, $P(y_2 = 2 | z_2 = B) = .3$, $P(y_2 = 2 | z_1 = G) = .73$ and $P(y_2 = 2 | z_1 = B) = .5$.

With the specification above, the optimal contracts yielded the following expected payments.

$$C(R_1) = 21241 \quad C(R_1, m) = 21137 \quad C(R_2) = 21269 \quad C(R_2, m) = 21113$$

So at this point in the parameter space, the programs exhibit the property in proposition 3. That is, early recognition is preferred without communication (i.e., $C(R_1) < C(R_2)$) and late recognition is preferred with communication (i.e., $C(R_2, m) < C(R_1, m)$).

Now we argue that at the neighborhood of this point in the parameter space the property is also active using implicit function theorem and a continuity argument.

Recall the expected payments, as functions defined over the parameter $P(y_2 = 2 | z_1 = G)$, are implicit in the Kuhn-Tucker conditions. Strictly non-binding constraints can be ignored because there will exist an open ball around the point above that these constraints will continue to be strictly non-binding. For the binding constraints, the Jacobian of the implicit functions is generic non-singular. Since all functions in the implicit function are continuous, the expected payment function defined by the implicit function is also continuous. Therefore there exists a parameter region with positive measures around the above point in the parameter space that the four expected payment functions will continue to possess the property in proposition 3.