User Case Studies

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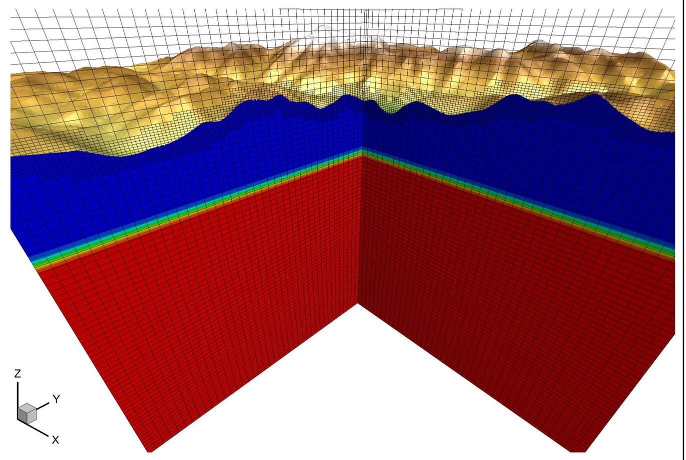
Example: The Elastic Wave Equation

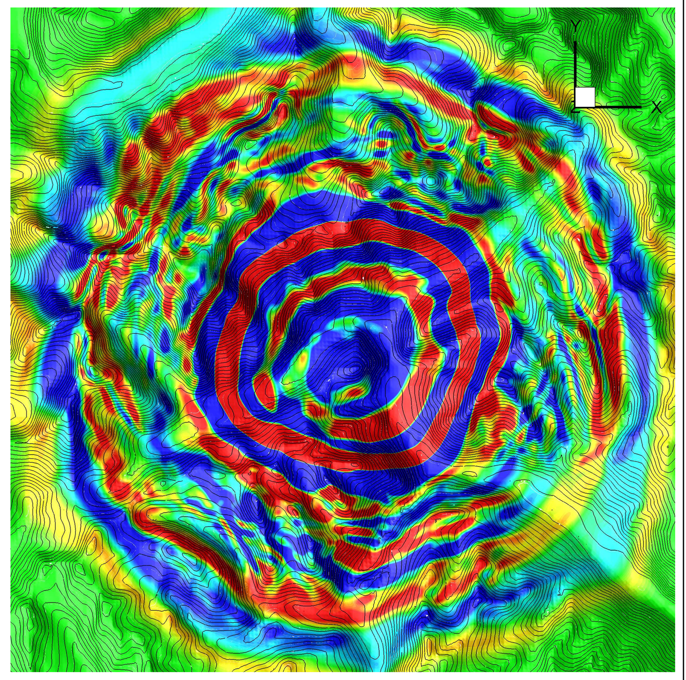
$$\frac{\partial \sigma}{\partial t} - \mathbf{E} \cdot \frac{1}{\alpha}\nabla(\alpha v) + \mathbf{E} \cdot v \otimes \nabla\alpha = 0,$$
$$\frac{\partial \alpha v}{\partial t} - \frac{\alpha}{\rho}\nabla \cdot \sigma - \frac{1}{\rho}\sigma\nabla\alpha = 0,$$

- $\mathbf{E}$ depends only on Lamé constants λ and μ
- ρ is the mass density
- $\sigma \in \mathbb{R}^d \times \mathbb{R}^d$ the stress tensor
- $\mathbf{v}$ is the velocity field

Diffuse Interface Approach:
augment the elastic wave equation with α, which identifies the location of solid medium.

1. At the boundaries the fluxes are *non-linear*
2. Physically, α represents the volume fraction of the solid medium present
3. This new approach *completely avoids* the problem of mesh generation





Use in Teaching

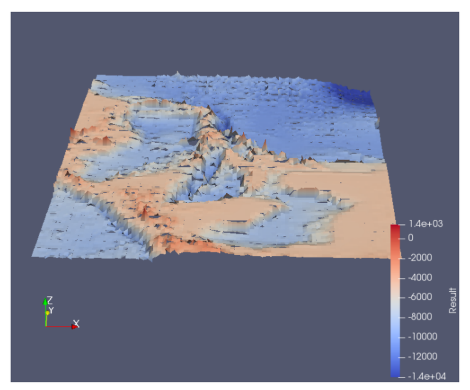
We use a stable snapshot with more complicated features disabled for teaching. Students pick up how to use the engine quickly due to the simple interface.

Ferienakademie

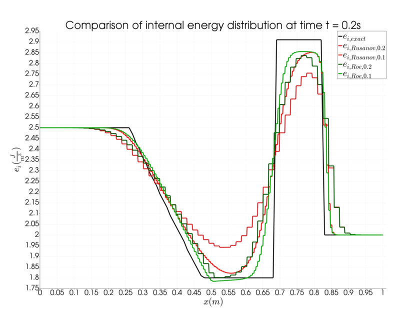
- During three half day sessions students implemented several simple test cases
- In one case this required writing a reader to read in the required data format, but was still quickly set up

Hyperbolic PDE Seminar

- Adding a new PDE system (Maxwell equations): required only 159 lines of code
- Implementing a new Riemann solver: required only 320 lines of code

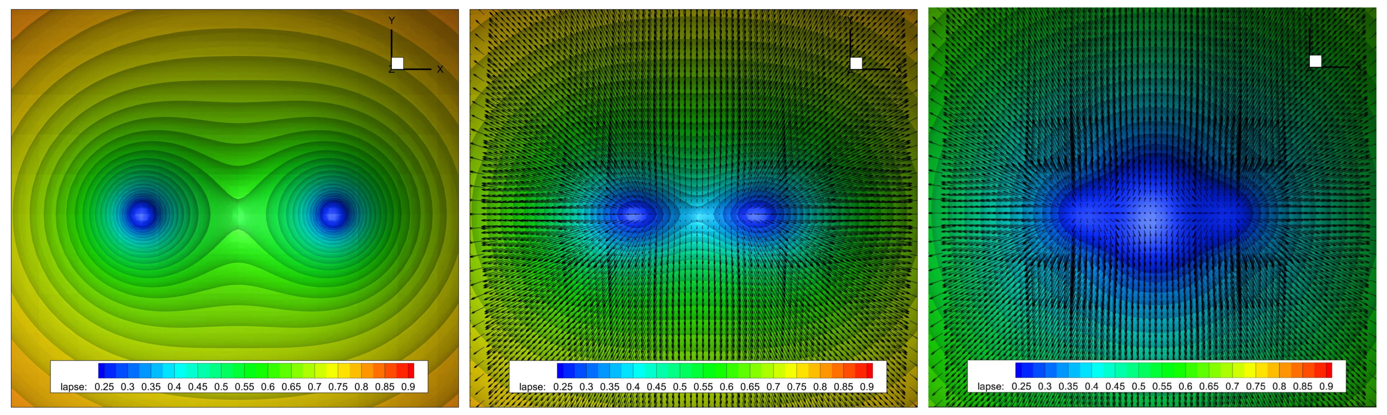


Bathymetry/topography near Japan



Comparison of Roe vs. Rusanov Riemann solver

Example: The Einstein Equations



Einstein's equations describe the theory of gravitation given by the equality of mass, energy and curved spacetime. Loosely speaking, one can refer to the spacetime of a black hole *outside* of the black hole center point as a *vacuum* solution, i.e.

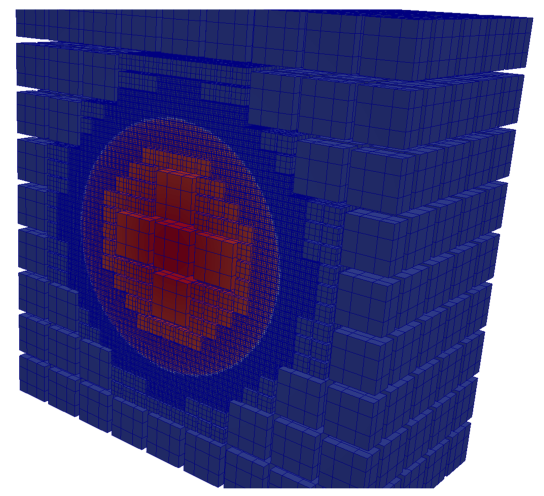
$$G_{\mu\nu} \sim GM\delta(x),$$

the Einstein tensor $G_{\mu\nu}$ has a zero source whenever the Dirac delta gives no contribution at $x > 0$.

The formulation of Einstein's equations as an initial value problem suitable for numerical integration is an ongoing challenge, and within ExaHyPE a novel first order formulation of the Constraint damping Z4 formulation of Einstein's equations (CCZ4) was developed, see [2].
In this formulation, there are 59 scalar variables which describe the Einstein tensor. The state vector is given as

$$Q = (g_{ij}, K_{ij}, \Theta, Z_i, \alpha, \beta_i, b_i, A_i, B_{ij}, D_{ijk}, K, \phi, P_k, K_0).$$

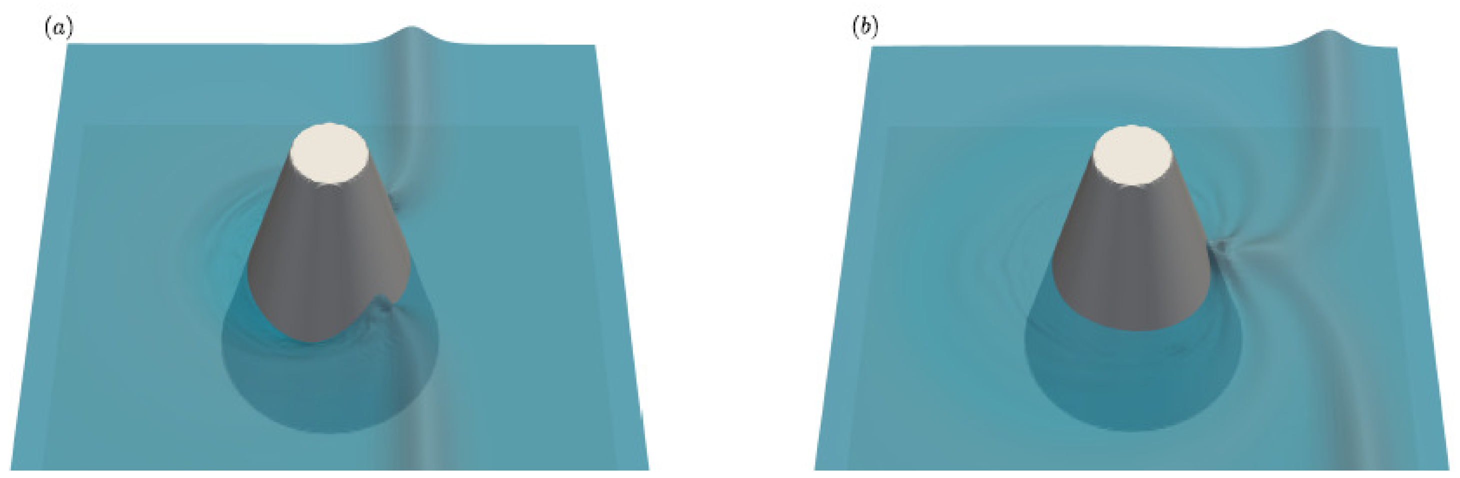
- A simple test for Einsteins equations in vacuum is the **Gauge Wave**, in which the metric takes the form of a sine wave. The gauge wave test evolves flat empty space (Minkowski spacetime) and the sine wave simply propagates.
- **Static single black holes:** A more advanced application is the evolution of a static single black holes. Here the evolution remains close to the initial conditions and violations of the constraints remain small.



One single evaluation of the CCZ4 PDE system requires 1 division, 2 exponential function evaluations, 8434 additions and 11589 multiplications, i.e. more than 20,000 double precision floating point operations

Geophysical free surface flows

- During a 3 month visit a guest with a background in applied mathematics implemented a new application within ExaHyPE.
- **Main objective:** development of ADER-DG schemes with a posteriori subcell FV limiter for nonhydrostatic free surface flows. Comparison with experimental data.
- **Preparatory work of the guest:** access to the code, getting familiar with the schemes used in the ExaHyPE engine and the essential data structures.
- **Main problem:** Standard models for nonhydrostatic shallow water flows require the solution of an additional elliptic equation for the nonhydrostatic pressure. This PDE a priori does not fit into the design concept of ExaHyPE.
  → The main work lies in the development and analysis of the PDE system.
- **Results:** The implementation of the new PDE system into the engine took only one week, including debugging.
- **Results:** The rest of the stay was used for the production of results.



3D plots of the free surface at t = 8 (left) and at t = 10 (right).

ExaHyPE User workshop

We invite existing and interested potential users alike to join us for four days to try out the ExaHyPE Engine. The goal of the workshop is to start the week with a problem in the form of a hyperbolic PDE and to go home with a working first prototype using ExaHyPE.

Date:
Monday (evening), 22 April 2019 to
Friday 26 April 2019

Location:
Grey College at Durham University

Remote access to a supercomputer will be provided for the duration of the workshop by the Leibniz Supercomputing Centre.

The workshop is free of charge.

More Information and Registration at:
`https://exahype-user-workhop.eventbrite.co.uk`

This poster is joint work with Luke Boyard, Luciano Rezzolla, Sven Köppel (Frankfurt Institute for Advanced Studies), Dominic Charrier, Tobias Weinzierl (University of Durham), Michael Dumbser, Francesco Fambri, Maurizio Tavelli (University of Trento), Alice Gabriel, Kenneth Duru (Ludwig-Maximilians-University Munich), Jean-Mathieu Gallard, Leonhard Rannabauer and Philipp Samfaß(Technical University of Munich).
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