

maha_v8.R

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Two simple R functions to compute multivariate standardized differences between two groups (Mahalanobis' D), either from raw data with function `maha` or from Cohen's d values and correlations with function `maha.summary`. The functions return uncorrected and bias-corrected values of D , exact confidence intervals, bootstrapped confidence intervals (only from raw data), heterogeneity coefficients, and a number of additional diagnostics and effect sizes. Disattenuated estimates can also be computed if desired. For more information see Del Giudice (2009, 2013, 2019).

Confidence intervals: exact confidence intervals for D are computed with Reiser's (2001) method. Especially for small D values, the equations may not be solvable; in those cases, one or both CI bounds are set to NA. For more information see Reiser (2001). The bootstrapped CIs are bias-corrected and accelerated; for details see Kelley (2005).

Additional effect sizes: additional effect sizes are calculated from D under the assumptions of multivariate normality and equality of covariance matrices. These are: the overlapping coefficient OVL ; Cohen's alternative coefficient of overlap OVL_2 calculated on the joint distribution (equivalent to $1-U_1$); the common language effect size CL (McGraw & Wong, 1992); and the probability of correct classification (PCC) based on linear discriminant analysis (assuming equal group sizes). For details and discussion see Del Giudice (2019).

Heterogeneity coefficients: the heterogeneity coefficient H_2 quantifies heterogeneity in the variables' contribution to the multivariate effect size (0 = max homogeneity; 1 = max heterogeneity). Coefficient EPV_2 represents heterogeneity as the proportion of contributing variables that would result in the same heterogeneity (equivalent proportion of variables), in a hypothetical scenario where a certain proportion of variables contribute equally to D while the remaining ones make no contribution (range: 0-1). See Del Giudice (2017, 2018).

Similarity between correlation matrices: before computing D and other effect sizes, the correlation matrices of the two groups are pooled by taking weighted averages, unless a common correlation matrix is directly provided to function `maha.summary`. If two matrices are provided to function `maha.summary` without specifying the size of the groups, the unweighted average is taken. Tucker's Congruence Coefficient (CC) can be used as an index of similarity between the two sample correlation matrices (-1 = max dissimilarity; 1 = max similarity). Values above .85 indicate fair similarity; values above .95 indicate high similarity (Lorenzo-Seva & ten Berge, 2006). The significance of the difference between correlation matrices can be tested with Steiger's test of equality (Steiger, 1980); however, significance tests can be overly sensitive to small differences when sample size is large. See Del Giudice (2019) for more details.

Bias correction: the bias-corrected distance D_u removes the upward bias in D , which is especially large when sample size is small relative to the number of variables (see Hess et al., 2007; Del Giudice, 2019). The bias-corrected version of Cohen's d is d_u (also known as Hedges' g); the approximate formula is used here (see Kelley, 2005).

Disattenuation: disattenuated estimates of D and other effect sizes can be obtained by supplying a vector of reliability coefficients (e.g., Cronbach's α). For details see Del Giudice (2019).

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Function maha

Computes the Mahalanobis distance D , confidence intervals, and various relevant indices from raw data. Can compute disattenuated estimates if desired.

```
maha(dataA, dataB, rel_values=NULL, conf.level=.95, boot.n=NULL, round.digits=3)
```

Arguments

<code>dataA, dataB</code>	raw data matrices/data frames for the two groups
<code>rel_values</code>	vector of reliability coefficients (optional: required for disattenuation)
<code>conf.level</code>	CI width (default is 95%)
<code>boot.n</code>	number of bootstrap samples (optional; recommended: 5,000-10,000)
<code>round.digits</code>	rounding digits for the output (default is 3)

Value

returns a list object containing some or all of the following:

<code>D</code>	Mahalanobis' D
<code>CI_exact_D</code>	exact CI for D (lower bound, upper bound); NA if not solvable
<code>CI_boot_D</code>	bootstrapped CI for D (lower bound, upper bound)
<code>OVL</code>	overlapping coefficient OVL (single distribution)
<code>OVL2</code>	Cohen's coefficient of overlap OVL_2 (joint distribution; equals $1-U_1$)
<code>CL</code>	common language effect size CL
<code>PCC</code>	probability of correct classification (PCC ; linear discriminant analysis)
<code>H2</code>	heterogeneity coefficient H_2
<code>EPV2</code>	EPV_2 coefficient (equivalent proportion of variables)
<code>CC_cor</code>	Tucker's Congruence Coefficient CC (similarity between correlation matrices)
<code>steiger.p</code>	p value for Steiger's test of equality of correlation matrices (null hypothesis: equality)
<code>d_values</code>	vector of Cohen's d values

Du	bias-corrected D_u
CI_boot_Du	bootstrapped CI for D_u (lower bound, upper bound)
OVLu	overlapping coefficient (single distribution) based on D_u
OVL2u	Cohen's coefficient of overlap (joint distribution) based on D_u
CLu	common language effect size based on D_u
PCCu	probability of correct classification based on D_u
du_values	vector of bias-corrected d_u values (equivalent to Hedges' g)
Dc	disattenuated D_c
OVLc	disattenuated overlapping coefficient (single distribution)
OVL2c	disattenuated Cohen's coefficient of overlap (joint distribution)
CLc	disattenuated common language effect size
PCCc	disattenuated probability of correct classification
H2c	disattenuated heterogeneity coefficient H_2
EPV2c	disattenuated EPV_2 coefficient
dc_values	vector of disattenuated Cohen's d_c values
Dcu	disattenuated, bias-corrected D_{cu}
OVLcu	disattenuated overlapping coefficient (single distribution) based on D_{cu}
OVL2cu	disattenuated Cohen's coefficient of overlap (joint distribution) based on D_{cu}
CLcu	disattenuated common language effect size based on D_{cu}
PCCcu	disattenuated probability of correct classification based on D_{cu}
dcu_values	vector of disattenuated, bias-corrected d_{cu} values (disattenuated Hedges' g)

Function maha.summary

Computes the Mahalanobis distance D , confidence intervals, and various relevant indices from summary statistics. Can compute disattenuated estimates if desired.

```
maha.summary(d_values, corA, corB=NULL, nA=NULL, nB=NULL, rel_values=NULL,
             conf.level=.95, round.digits=3)
```

Arguments

d_values	row vector of Cohen's d values
corA	either the common correlation matrix, or the correlation matrix for group A
corB	correlation matrix for group B (optional)
nA, nB	sample size of the two groups (optional: required for bias correction, exact CIs, and Steiger's test)
rel_values	vector of reliability coefficients (optional: required for disattenuation)
conf.level	CI width (default is 95%)
round.digits	rounding digits for the output (default is 3)

Value

returns a list object containing some or all of the following:

D	Mahalanobis' D
CI_exact_D	exact CI for D (lower bound, upper bound); NA if not solvable
OVL	overlapping coefficient OVL (single distribution)
OVL2	Cohen's coefficient of overlap OVL_2 (joint distribution; equals $1-U_1$)
CL	common language effect size CL
PCC	probability of correct classification (PCC ; linear discriminant analysis)
H2	heterogeneity coefficient H_2
EPV2	EPV_2 coefficient (equivalent proportion of variables)
CC_cor	Tucker's Congruence Coefficient CC (similarity between correlation matrices)
steiger.p	p value for Steiger's test of equality of correlation matrices (null hypothesis: equality)
d_values	vector of Cohen's d values
Du	bias-corrected D_u
OVLu	overlapping coefficient (single distribution) based on D_u
OVL2u	Cohen's coefficient of overlap (joint distribution) based on D_u
CLu	common language effect size based on D_u
PCCu	probability of correct classification based on D_u
du_values	vector of bias-corrected d_u values (equivalent to Hedges' g)
Dc	disattenuated D_c
OVLc	disattenuated overlapping coefficient (single distribution)
OVL2c	disattenuated Cohen's coefficient of overlap (joint distribution)
CLc	disattenuated common language effect size
PCCc	disattenuated probability of correct classification
H2c	disattenuated heterogeneity coefficient H_2
EPV2c	disattenuated EPV_2 coefficient
dc_values	vector of disattenuated Cohen's d_c values
Dcu	disattenuated, bias-corrected D_{cu}
OVLcu	disattenuated overlapping coefficient (single distribution) based on D_{cu}
OVL2cu	disattenuated Cohen's coefficient of overlap (joint distribution) based on D_{cu}
CLcu	disattenuated common language effect size based on D_{cu}
PCCcu	disattenuated probability of correct classification based on D_{cu}
dcu_values	vector of disattenuated, bias-corrected d_{cu} values (disattenuated Hedges' g)