## File S2: Differential Equations for Model

The model depicted in Figure 6a describes the following set of reactions:

$$E + \sigma^{70} \stackrel{k_{fE\sigma^{70}}}{\rightleftharpoons} E\sigma^{70} E\sigma^{70} E + \sigma^{38} \stackrel{k_{fE\sigma^{38}}}{\rightleftharpoons} E\sigma^{38} E\sigma^{38} E\sigma^{38} E\sigma^{38} Rsd + \sigma^{70} \stackrel{k_{fRsd}}{\rightleftharpoons} Rsd - \sigma^{70} Escondown end to the equation of the eq$$

These can be written as a system of differential equations, as given below:

$$\begin{aligned} \frac{d[E]}{dt} &= -k_{fE\sigma70}[E][\sigma^{70}] + k_{bE\sigma70}[E\sigma^{70}] - k_{fE\sigma38}[E][\sigma^{38}] + k_{bE\sigma38}[E\sigma^{38}] \\ & (\text{Equation S1}) \end{aligned}$$

$$\begin{aligned} - k_{fNS}[E][DNA] + k_{bNS}[EDNA] + e[E_{e70}] + e[E_{e38}] \\ \frac{d[\sigma^{70}]}{dt} &= -k_{fE\sigma70}[E][\sigma^{70}] + k_{bE\sigma70}[E\sigma^{70}] - k_{fRsd}[Rsd][\sigma^{70}] + k_{bRsd}[Rsd - \sigma^{70}] + e[E\sigma^{70}P_{70}] \\ & (\text{Equation S2}) \end{aligned}$$

$$\begin{aligned} \frac{d[\sigma^{38}]}{dt} &= -k_{fE\sigma38}[E][\sigma^{38}] + k_{bE\sigma38}[E\sigma^{38}] + c[E\sigma^{38}P_{38}] \\ \frac{d[E\sigma^{70}]}{dt} &= k_{fE\sigma70}[E][\sigma^{70}] - k_{bE\sigma70}[E\sigma^{70}] - k_{f6S}[6SRNA][E\sigma^{70}] + k_{b6S}[6SRNA - E\sigma^{70}] \\ & - k_{fNS}[E\sigma^{70}][DNA] + k_{bNS}[E\sigma^{70}DNA] - k_{fE\sigmaP}[E\sigma^{70}][P_{70}] + k_{bE\sigmaP}[E\sigma^{70}P_{70}] \\ \end{aligned}$$

$$\begin{aligned} \frac{d[E\sigma^{38}]}{dt} &= k_{fE\sigma38}[E][\sigma^{38}] - k_{bE\sigma38}[E\sigma^{38}] - k_{fNS}[E\sigma^{38}][DNA] + k_{bNS}[E\sigma^{38}DNA] \\ & - k_{fE\sigmaP}[E\sigma^{38}][P_{38}] + k_{bE\sigmaP}[E\sigma^{38}P_{38}] \end{aligned}$$

$$\begin{aligned} \end{aligned}$$

$$\frac{a_{[Rsd]}}{dt} = -k_{fRsd}[Rsd][\sigma^{70}] + k_{bRsd}[Rsd - \sigma^{70}]$$
(Equation S6)  

$$\frac{d[Rsd - \sigma^{70}]}{dt} = k_{fRsd}[Rsd][\sigma^{70}] - k_{bRsd}[Rsd - \sigma^{70}]$$
(Equation S7)

$$\begin{aligned} \frac{d[6SRNA]}{dt} &= -k_{f6S}[6SRNA][E\sigma^{70}] + k_{b6S}[6SRNA - E\sigma^{70}] & (Equation S8) \\ \frac{d[6SRNA - E\sigma^{70}]}{dt} &= k_{f6S}[6SRNA][E\sigma^{70}] - k_{b6S}[6SRNA - E\sigma^{70}] & (Equation S9) \\ \frac{d[DNA]}{dt} &= -k_{fNS}[E][DNA] + k_{bNS}[EDNA] - k_{fNS}[E\sigma^{70}][DNA] & (Equation S10) \\ &+ k_{bNS}[E\sigma^{70}DNA] - k_{fNS}[E\sigma^{38}][DNA] + k_{bNS}[E\sigma^{38}DNA] & (Equation S11) \\ \frac{d[EDNA]}{dt} &= k_{fNS}[E][DNA] - k_{bNS}[EDNA] & (Equation S11) \\ \frac{d[E\sigma^{70}DNA]}{dt} &= k_{fNS}[E][DNA] - k_{bNS}[E\sigma^{70}DNA] & (Equation S12) \\ \frac{d[E\sigma^{38}DNA]}{dt} &= k_{fDN}[E\sigma^{70}][DNA] - k_{bNS}[E\sigma^{70}DNA] & (Equation S13) \\ \frac{d[E\sigma^{38}DNA]}{dt} &= k_{fDNA}[E\sigma^{38}][DNA] - k_{bDNA}[E\sigma^{38}DNA] & (Equation S13) \\ \frac{d[E\sigma^{38}DNA]}{dt} &= k_{fE\sigma_{P}}[E\sigma^{70}][P_{70}] + k_{bE\sigma_{P}}[E\sigma^{70}P_{70}] + c[E\sigma^{70}P_{70}] & (Equation S14) \\ \frac{d[Bal}{dt} &= -k_{fE\sigma_{P}}[E\sigma^{70}][P_{70}] + k_{bE\sigma_{P}}[E\sigma^{70}P_{70}] - c[E\sigma^{70}P_{70}] & (Equation S15) \\ \frac{d[E\sigma^{38}P_{38}]}{dt} &= k_{fE\sigma_{P}}[E\sigma^{38}][P_{38}] - k_{bE\sigma_{P}}[E\sigma^{38}P_{38}] - c[E\sigma^{38}P_{38}] & (Equation S17) \\ \frac{d[Ea\sigma^{10}]}{dt} &= c[E\sigma^{70}P_{70}] - e[E_{e70}] & (Equation S18) \\ \frac{d[E_{e38}]}{dt} &= c[E\sigma^{38}P_{38}] - e[E_{e38}] & (Equation S19) \\ \end{array}$$

## Where:

 $\begin{aligned} \frac{k_{bE\sigma70}}{k_{fE\sigma70}} &= \mathbf{K}_{E\sigma70} \\ \frac{k_{bE\sigma38}}{k_{fE\sigma38}} &= \mathbf{K}_{E\sigma38} \\ \frac{k_{bRsd}}{k_{fRsd}} &= \mathbf{K}_{Rsd} \\ \frac{k_{b6S}}{k_{f6S}} &= \mathbf{K}_{6S} \\ \frac{k_{bNS}}{k_{fNS}} &= \mathbf{K}_{NS} \\ \frac{k_{bE\sigmaP} + c}{k_{fE\sigmaP}} (\approx \frac{k_{bE\sigmaP}}{k_{fE\sigmaP}}) = \mathbf{K}_{E\sigmaP} \end{aligned}$ 

Rates of transcription are given by:

$\frac{d[mRNA_{70}]}{dt} = \mathbf{e}[\mathbf{E}_{e70}]$	(Equation S20)
$\frac{d[mRNA_{38}]}{dt} = \mathbf{e}[\mathbf{E}_{e38}]$	(Equation S21)

To obtain steady state values, the left-hand sides of equations (S1) - (S19) were set to zero. We initialized values of E =  $E_{total}$ ,  $\sigma^{70} = \sigma^{70}_{total}$ ,  $\sigma^{38} = \sigma^{38}_{total}$ , Rsd = Rsd<sub>total</sub>, 6S RNA = 6SRNA<sub>total</sub>, DNA = DNA<sub>total</sub>, P<sub>70</sub> = P<sub>70total</sub>, P<sub>38</sub> = P<sub>38total</sub> and all other variables = 0. The differential equations were solved until the system reached steady state.