A Appendix [for online publication]

A.1 Dynamic Demand (extension)

A.1.1 Foster et al. (2016)

Let Z_{it} be the demand "stock" of the firm which depends on the previous firm sales. The stock evolves as $Z_{it} = (1-\delta)\,Z_{i,t-1} + p_{it}q_{it}$. We consider the nonparametric case here where $p_{it} = p(q_{it}, Z_{i,t-1}, \varepsilon^d_{it})$ with Z_{it} as specified above. Foster et al. (2016) also consider age as a state variable. For convenience let us abstract from age, as this adds nothing substantial to the analysis and we would need to keep track of one extra state variable.

By the end of the period the firm chooses the level of materials that maximizes the following objective function

$$
\begin{aligned}
V(\varepsilon^d_{it}, \omega_{it}, Z_{i,t-1}) &= \max_{M_{it}} p(q_{it}, Z_{i,t-1}, \varepsilon^d_{it})q_{it} - p^M_{it} M_{it} \\
&\quad + \rho E_{\varepsilon^d_{i,t+1}, \omega_{i,t+1}} V(\varepsilon^d_{i,t+1}, \omega_{i,t+1}, Z_{it} | \varepsilon^d_{it}, \omega_{it}, Z_{i,t-1}) \\
\text{s.t. } q_{it} &= q(K_{it}, L_{i,t}, M_{it}, \omega_{i,t}) \\
Z_{it} &= (1-\delta)\,Z_{i,t-1} + p_{it}q_{it}
\end{aligned}
$$

where $\rho < 1$ is the discount factor and we now have an extra term that includes the continuation value. Notice that if we let $y_{it} = p(q_{it}, Z_{i,t-1}, \varepsilon^d_{it})q_{it}$, and since $Z_{it} = (1-\delta)\,Z_{i,t-1} + y_{it}$, the sum of static returns and continuation value can be written as $y_{it} + \rho E_{\varepsilon^d_{i,t+1}, \omega_{i,t+1}} V(...|\varepsilon^d_{it}, \omega_{it}, Z_{i,t-1}, y_{it})$. This is a fundamental property and greatly simplifies the analysis since we can use the chain rule to first differentiate with respect to y_{it} (by construction the derivative of Z_{it} with respect to y_{it} is 1) and then we differentiate with respect to M. The first-order condition is

$$
[\sigma_{p,q} + 1]\, p(q_{it}, Z_{it}, \varepsilon^d_{it}) q_m \,(1 + \rho EV'_Z) = p^M_{it} \tag{A.1}
$$

where $EV'_Z = E_{\varepsilon^d_{i,t+1}, \omega_{i,t+1}} V'_{Z_{it}}(\varepsilon^d_{i,t+1}, \omega_{i,t+1}, Z_{it} | \varepsilon^d_{it}, \omega_{it}, Z_{i,t-1})$ is the effect sales today on the demand stock tomorrow. The main difference from before is the continuation term EV'_Z. So, the marginal returns of increasing M_{it} (LHS) consists of the increase in short-term revenues and future revenues due to the increase in the demand stock Z. Put in another way, firms are willing to produce more and receive a lower price because every unit they sell also increases future sales. We can rewrite the reduced form optimal solution as

$$M_{it}^{*} = m(\omega_{it}, L_{it}, K_{it}, Z_{i,t-1}, \varepsilon_{it}^{d})$$

In the beginning of the period the firm chooses the level of employment that maximizes the expected returns

$$\max_{L_{i,t}} E_{\varepsilon_{it}^{d}} \left[\begin{array}{c} p(q_{it}, Z_{it}, \varepsilon_{it}^{d}) q_{it} - p_{it}^{M} M_{it} + \\ \rho E_{\varepsilon_{i,t+1}^{d}, \omega_{i,t+1}} V(\varepsilon_{i,t+1}^{d}, \omega_{i,t+1}, Z_{it} | \varepsilon_{it}^{d}, \omega_{it}, Z_{i,t-1}) | \Omega_{it} \end{array} \right] - p_{it}^{L} L_{it}$$

$$\begin{aligned} \text{s.t. } q_{it} &= q(K_{it}, L_{it}, M_{it}, \omega_{it}) \\ Z_{it} &= (1-\delta) Z_{i,t-1} + p_{it} q_{it} \end{aligned}$$

which gives[8]

$$E_{\varepsilon_{it}^{d}} \left[\frac{\partial p(q_{it}, Z_{it}, \varepsilon_{it}^{d}) q_{it}}{\partial q_{it}} \frac{\partial q_{it}}{\partial L_{it}} | \Omega_{it} \right] = E_{\varepsilon_{it}^{d}} \left[\left[\sigma_{p,q} + 1 \right] p(q_{it}, Z_{it}, \varepsilon_{it}^{d}) \left(1 + \rho E V_{Z}'\right) q_{l} | \Omega_{it} \right] = p_{it}^{L} \tag{A.2}$$

Again the same dynamic element is at work. Increasing revenues also brings in future additional sales. Using the previous two we obtain the input ratio

$$\frac{p_{it}^{M} M_{it}}{p_{i,t}^{L} L_{i,t}} = \underbrace{\frac{1}{\epsilon_{m.l}}}_{\text{Supply (ETS)}} \underbrace{\frac{\left[\sigma_{p,q}(q_{it}, Z_{i,t}, \varepsilon_{it}^{d}) + 1\right] p(q_{it}, Z_{i,t}, \varepsilon_{it}^{d}) \left(1 + \rho E V_{Z}'\right) q_{l}}{E_{\varepsilon_{it}^{d}} \left[\left[\sigma_{p,q}(q_{it}, Z_{i,t}, \varepsilon_{it}^{d}) + 1\right] p(q_{it}, Z_{i,t}, \varepsilon_{it}^{d}) \left(1 + \rho E V_{Z}'\right) q_{l} | \Omega_{it} \right]}}_{\text{Demand}}$$

and again

$$\frac{p_{it}^{M} M_{it}}{p_{it}^{L} L_{it}} = \frac{M_{it}}{E_{\varepsilon^{d}} \left[\epsilon_{m.l} M_{it} | \Omega_{it} \right]}$$

A.1.2 Gourio and Rudanko, 2014

We now consider the extension to the dynamic demand model formulated by Gourio and Rudanko (2014) and show how the analysis is similar in a model with endogenous

[8] From equation (A.1), $\frac{\partial p(q_{it}, Z_{it}, \varepsilon_{it}^{d}) q_{it}}{\partial q_{it}} \frac{\partial q_{it}}{\partial M_{it}} \left(1 + \rho E V_{Z}'\right) = p_{it}^{M}$. The total derivative is $E_{\varepsilon^{d}} \left[\frac{\partial p(q_{it}, Z_{it}, \varepsilon_{it}^{d}) q_{it} \left(1 + \rho E V_{Z}'\right) - p_{it}^{M} M_{it}^{*}}{\partial L_{it}} \right] - p_{it}^{L} = E_{\varepsilon^{d}} \left[\frac{\partial p(.) q_{it}}{\partial q_{it}} \frac{\partial q_{it}}{\partial L_{it}} \left(1 + \rho E V_{Z}'\right) + \left(\frac{\partial p(.) q_{it}}{\partial q_{it}} \frac{\partial q_{it}}{\partial M_{it}} \left(1 + \rho E V_{Z}'\right) - p_{it}^{M} \right) \frac{\partial M_{it}}{\partial L_{it}} \right] - p_{it}^{L}$ (by the envelope theorem). Replacing the above we obtain equation (A.2).

dynamic demand considerations. Contrary to Foster et al. (2016), this model avoids dynamic pricing decisions. Instead, dynamic demand considerations are addressed with sales force effort decisions. Since sales force is an expense and does not build up, this allows us to split the dynamic considerations about future demand (stock building) from the optimal input allocations (labor and materials). Consider the previous model but let us assume that prices are given, as in Gourio and Rudanko (2014). Let us also ignore capital decisions to simplify the analysis, as everything holds if we add the extra state variable, capital. In this case the production function is

$$q_{it} = q_0 L_{it}^{\beta} M_{it}^{\gamma} \omega_{it}$$

where comparing to the previous model, $q_0 = K_{it}^{\alpha}$, so we are assuming that capital is fixed. We adapt the specification in Gourio and Rudanko (2014) so that the number of units sold is given by

$$q_{it} = n_{it} + s_{it}\eta(\frac{b_{it}}{s_{it}})$$

where n_{it} is the number of consumers carried from the last period, b_{it} is the number of buyers in the market (given), and s is the number of sellers (sales force). The function $\eta\left(\frac{b}{s}\right) = \xi\left(\frac{b}{s}\right)^{\phi}$ is the (Mortensen-Pissarides) matching function with $\phi \in (0,1)$. In this sense, $s\eta(.)$ is the number of new customers attracted when there are b buyers and s sellers. Also, assume that employing these sellers generates a cost that is increasing in the number of sellers. In particular let the cost be quadratic $\kappa(s) = s^2/2$. Notice that given this structure, the number of units sold is determined by the number of customers (customer base), the number of buyers, and the number of sellers. In this way, the firms' input decision is how to best choose the intermediate inputs to minimize the cost (dual). We can show this by maximizing the firms' profit (value) function. We first need to introduce two more elements: the customer base and the potential buyers. The customer base n decays at rate δ_n. This means that in every period a fraction δ_n of the customers leave the company. That means that the customer base available for next period is $n_{i,t+1} = (1-\delta_n)\, q_{it}$. On the other hand, let the number of potential buyers in the market follow a first-order Markov process, $b_{it} = h(b_{it-1})e^{\varepsilon_{it}}$ for some smooth function $h(.)$. This introduces a shock to demand ε_{it} which is absent in the original model of Gourio and Rudanko (2014). Again, this shock is observed after L_{it} has been determined.

In the last period after the demand shock (ε_{it}) is revealed, the company wants to maximize its long-term value (note that choosing s_{it} is equivalent to choosing M_{it} since $M_{it} = \left(\frac{q_{it}}{e^{\omega_{it}}L_{it}^{\beta}}\right)^{1/\gamma} = \left(\frac{n_{it}+s_{it}\eta(b_{it}/s_{it})}{e^{\omega_{it}}L_{it}^{\beta}}\right)^{1/\gamma}$ and we have to account for the fact that increasing s_{it} will make the company sell more and also incur larger costs as it needs to purchase more intermediate inputs, M_{it})

$$\max_{s_{it}} pq_{it} - p_{it}^{M}M_{it} - \kappa(s_{it}) + \beta E_{\nu_{i,t+1},\varepsilon_{i,t+1}}\left[V(n_{i,t+1},\omega_{i,t+1},b_{i,t+1})|b_{it},n_{it},\omega_{it}\right]$$

$$\begin{aligned}
\text{s.t. } q_{it} &= L_{it}^{\beta}M_{it}^{\gamma}e^{\omega_{it}} \\
q_{it} &= n_{it} + s_{it}\eta(b_{it}/s_{it}) \\
b_{it} &= h(b_{it-1})e^{\varepsilon_{it}} \\
n_{i,t+1} &= (1-\delta_n)\,q_{it} \\
\omega_{i,t+1} &= g(\omega_{it})e^{\nu_{it+1}}
\end{aligned}$$

where $E\left[V(n_{i,t+1},\omega_{i,t+1},b_{i,t+1})|b_{it},n_{it},\omega_{it}\right]$ is the expected continuation value from a company with customer base n_{it}, productivity ω_{it} and a stock of potential buyers of b_{it}. Or equivalently

$$\max_{s_{it}}\left\{\begin{array}{c} p\left(n_{it}+\xi s_{it}^{1-\phi}b_{it}^{\phi}\right) - p_{it}^{M}\left(\frac{n_{it}+\xi s_{it}^{1-\phi}b_{it}^{\phi}}{e^{\omega_{it}}L_{it}^{\beta}}\right)^{1/\gamma} - s_{it}^{2}/2 \\ +\beta E_{\nu_{i,t+1},\varepsilon_{i,t+1}}\left[V(n_{i,t+1},\omega_{i,t+1},b_{i,t+1})|b_{it},n_{it},\omega_{it}\right] \end{array}\right\}$$

$$\begin{aligned}
\text{s.t. } b_{it} &= h(b_{it-1})e^{\varepsilon_{it}} \\
n_{i,t+1} &= (1-\delta_n)\,q_{it} \\
\omega_{i,t+1} &= g(\omega_{it})e^{\nu_{it+1}}
\end{aligned}$$

The first-order condition is

$$
\begin{aligned}
& p\left(1-\phi\right)\xi\left(\frac{b_{it}}{s_{it}}\right)^{\phi}+\beta\left(1-\delta_{n}\right)\left(1-\phi\right)\xi\left(\frac{b_{it}}{s_{it}}\right)^{\phi}E\left[V_{n}(n_{i,t+1},\omega_{i,t+1},b_{i,t+1})|b_{it},n_{it},\omega_{it}\right] \\
= \; & p_{it}^{M}\frac{1}{\gamma}\left(e^{\omega_{it}}L_{it}^{\beta}\right)^{-1/\gamma}\left(n_{it}+\xi s_{it}^{1-\phi}b_{it}^{\phi}\right)^{(1-\gamma)/\gamma}\left(1-\phi\right)\xi\left(\frac{b_{it}}{s_{it}}\right)^{\phi}+s_{it} \qquad \text{(A.3)}
\end{aligned}
$$

where $E\left[V_n\left(.\right)\right] = E\left[\frac{\partial V(.)}{\partial n'}\right]$. The first term, $p\left(1-\phi\right)\xi\left(\frac{b_{it}}{s_{it}}\right)^{\phi}$, is the marginal short-term benefit from increasing the sales force while the second two terms are the marginal cost of doing so. First the cost in terms of increasing intermediate input purchases, $p_{it}^{M}\frac{1}{\gamma}\left(e^{\omega_{it}}L_{it}^{\beta}\right)^{-1/\gamma}\left(n_{it}+\xi s_{it}^{1-\phi}b_{it}^{\phi}\right)^{(1-\gamma)/\gamma}\left(1-\phi\right)\xi\left(\frac{b_{it}}{s_{it}}\right)^{\phi}$ and the second is simply the marginal cost of more sellers, s_{it}. The final component is the marginal net benefit for the future increase in the customer base, $\beta\left(1-\delta_{n}\right)\left(1-\phi\right)\xi\left(\frac{b_{it}}{s_{it}}\right)^{\phi}E\left[V_{n}(n_{i,t+1},\omega_{i,t+1},\varepsilon_{i,t+1})\right]$.

In the beginning of the period the firms sets its labor before knowing the shock to demand,ε_{it} and maximizes the expected future value

$$
\begin{aligned}
& \max_{L_{it}} E_{\varepsilon_{it}}\begin{pmatrix} pq_{it}-p_{it}^{M}M_{it}-\kappa(s_{it}))-p_{it}^{L}L_{it} \\ +\beta E_{\nu_{i,t+1},\varepsilon_{i,t+1}}\left[V(n_{i,t+1},\omega_{i,t+1},b_{i,t+1})|b_{it},n_{it},\omega_{it}\right]\end{pmatrix} \\
\text{s.t. } q_{it} &= L_{it}^{\beta}M_{it}^{\gamma}e^{\omega_{it}} \\
q_{it} &= n_{it}+s_{it}\eta(b_{it}/s_{it}) \\
b_{it} &= b_{it-1}e^{\varepsilon_{it}} \\
n_{i,t+1} &= \left(1-\delta_{n}\right)q_{it} \\
\omega_{i,t+1} &= g(\omega_{it})e^{\nu_{it+1}}
\end{aligned}
$$

Or equivalently

$$
\begin{aligned}
& \max_{L_{it}} E_{\varepsilon_{it}}\begin{pmatrix} p\left(n_{it}+\xi s_{it}^{1-\phi}b_{it}^{\phi}\right)-p_{it}^{M}\left(\frac{n_{it}+\xi s_{it}^{1-\phi}b_{it}^{\phi}}{e^{\omega_{it}}L_{it}^{\beta}}\right)^{1/\gamma} \\ -s_{it}^{2}/2-p_{it}^{L}L_{it}+\beta E\left[V(n_{i,t+1},\omega_{i,t+1},\varepsilon_{i,t+1})\right]\end{pmatrix} \\
\text{s.t. } b_{it} &= h(b_{it-1})e^{\varepsilon_{it}} \\
n_{i,t+1} &= \left(1-\delta_{n}\right)q_{it} \\
\omega_{i,t+1} &= g(\omega_{it})e^{\nu_{it+1}}
\end{aligned}
$$

By the envelope theorem, the first-order condition becomes

$$E_{\varepsilon_{it}}\left(-p_{it}^{M}\frac{\partial M_{it}}{\partial L_{it}}-p_{it}^{L}\right)=0$$

since for the Cobb Douglas production function $\frac{\partial M}{\partial L}=-\frac{\beta}{\gamma}\frac{M}{L}$, the solution becomes

$$\frac{p_{it}^{M}M_{it}}{p_{it}^{L}L}=\frac{\gamma}{\beta}\frac{M_{it}}{E_{\varepsilon_{it}}\left(M_{it}\right)}$$

which is exactly the same solution we had before in Equation (4). The difference is that now, $\frac{M_{it}}{E_{\varepsilon_{it}}(M_{it})}$ is a more complicated expression. In particular,

$$\begin{aligned}\frac{M_{it}}{E_{\varepsilon_{it}}\left(M_{it}\right)} &= \frac{\left(\frac{n_{it}+\xi s_{it}^{1-\phi}b_{it}^{\phi}}{e^{\omega_{it}}L_{it}^{\beta}}\right)^{1/\gamma}}{E_{\varepsilon_{it}}\left[\left(\frac{n_{it}+\xi s_{it}^{1-\phi}b_{it}^{\phi}}{e^{\omega_{it}}L_{it}^{\beta}}\right)^{1/\gamma}\right]}\\ &= \frac{\left(n_{it}+\xi s_{it}^{1-\phi}b_{it-1}^{\phi}e^{\phi\varepsilon_{it}}\right)^{1/\gamma}}{E_{\varepsilon_{it}}\left[\left(n_{it}+\xi s_{it}^{1-\phi}b_{it-1}^{\phi}e^{\phi\varepsilon_{it}}\right)^{1/\gamma}\right]}\end{aligned}$$

and we see that, as before, the input ratio is a function of the demand shock (and not a function of the productivity shock) but is now also a function of the demand state variables, namely the customer base, the number of potential buyers, and the (endogenous) number of sellers. To address the endogeneity of s_{it} (since s_{it} is correlated with ε_{it}), we can use L_{it} (or K_{it} in the model extended with capital) as a valid instrument as demonstrated in the first-order condition in Equation (A.3). The previous equation can be estimated by non-linear or nonparametric IV depending on whether we can obtain a solution to $E_{\varepsilon_{it}}\left[\left(n_{it}+\xi s_{it}^{1-\phi}b_{it-1}^{\phi}e^{\phi\varepsilon_{it}}\right)^{1/\gamma}\right]$.

Call $v_n^e(b_{it},n_{it},\omega_{it})=E\left[V_n(n_{i,t+1},\omega_{i,t+1},b_{i,t+1})|b_{it},n_{it},\omega_{it}\right]$. Note that the first-order condition for s_{it} is

$$s_{it}=\left(\left(1-\phi\right)\xi b_{it}^{\phi}\left[p+\beta\left(1-\delta_n\right)v_n^e(b_{it},n_{it},\omega_{it})-\frac{1}{\gamma}\frac{p_{it}^{M}}{\left(e^{\omega_{it}}L_{it}^{\beta}\right)^{1/\gamma}}\left(n_{it}+\xi\left(\frac{b_{it}}{s_{it}}\right)^{\phi}\right)^{\frac{(1-\gamma)}{\gamma}}\right]\right)^{\frac{1}{1+\phi}}$$

which illustrates both the endogeneity of s_{it} with respect to ε_{it} (via b_{it}) and the validity of L_{it} as an instrument.

A.2 Non-Fixed Labor/temporary work (extension)

In this section we extend the previous results to the case with two types of employment: permanent and temporary. The cost of hiring an extra temporary worker equals the cost of hiring a regular worker with a "fiscal" advantage $\left((1-\tau)\, p_{it}^{L^P}\right)$ plus an extra cost that increases with the number of temporary workers and their regular cost, $p_{it}^{L^P}\eta\frac{L_{it}^T}{L_{it}^P}$. The cost of a temporary worker is less than the cost of a regular worker for small amounts but increases with the amount of temporary workers. The total cost is thus $p_{it}^{L^T}L_{it}^T = (1-\tau)\, p_{it}^{L^P}L_{it}^T + p_{it}^{L^P}\left(\eta\frac{\left(L_{it}^T\right)^2}{L_{it}^P}\right)$. The "fiscal" advantage is related with the money a company can save when it hires a temporary workers on social security, taxes and other expenses (e.g. medical insurance, etc). In our case, this rationalizes the use of temporary workers in small amounts.

Companies first maximize for permanent employment and later optimize for temporary employment and materials.

By the end of the period the firm chooses the level of materials that maximizes the following objective function

$$
\begin{aligned}
& \max_{L_{it}^T, M_{it}} p(q_{it}, Z_{it}, \varepsilon_{it}^d)q_{it} - p_{it}^M M_{it} - p_{it}^{L^T}L_{it}^T \\
\text{s.t. } q_{it} &= q(K_{it}, L_{i,t}, M_{it}, \omega_{i,t}) \\
L_{it} &= L_{it}^T + L_{it}^P \\
p_{it}^{L^T} &= p_{it}^{L^P}\left(1-\tau+\eta\frac{L_{it}^T}{L_{it}^P}\right)
\end{aligned}
$$

where p_{it}^M is the price of materials. Assume an interior solution for temporary employment, $L_{it}^T > 0$. When this constraint binds we are in the original solution where temporary employment is not possible. From the first-order conditions

$$
\left[\sigma_{p,q}+1\right] p(q_{it}, Z_{it}, \varepsilon_{it}^d)q_m = p_{it}^M \tag{A.4}
$$

$$[\sigma_{p,q}+1]\,p(q_{it},Z_{it},\varepsilon_{it}^{d})q_{l}=p_{it}^{L^{P}}\left(1-\tau+2\eta\frac{L_{it}^{T}}{L_{it}^{P}}\right) \tag{A.5}$$

where $\sigma_{p,q}<0$ is the elasticity of demand, $q_m=\frac{\partial q(.)}{\partial M}$ and $q_l=\frac{\partial q(.)}{\partial L}$. We can rewrite the reduced form optimal solutions as

$$M_{it}^{*}=m(\omega_{it},L_{it}^{P},K_{it},Z_{it},\varepsilon_{it}^{d})$$

$$\left(L_{it}^{T}\right)^{*}=l^{T}(\omega_{it},L_{it}^{P},K_{it},Z_{it},\varepsilon_{it}^{d}).$$

Putting the two equations together

$$\frac{p_{it}^{M}M_{it}}{p_{it}^{L^{P}}L_{it}}=\frac{1}{\epsilon_{m.l}}\frac{p_{it}^{L^{T}}}{p_{it}^{L^{P}}}=\frac{1}{\epsilon_{m.l}}\left(1-\tau+2\eta\frac{L_{it}^{T}}{L_{it}^{P}}\right). \tag{A.6}$$

The input ratio is now also a function of the wedge created by the labor market friction, together with the ETS.

In the beginning of the period the firm chooses the level of employment that maximizes the expected returns

$$\begin{aligned} &\max_{L_{i,t}^{P}} E_{\varepsilon^{d}}\left[p(q_{it},Z_{it},\varepsilon_{it}^{d})q_{it}-p_{it}^{M}M_{it}-p_{it}^{L^{T}}L_{it}^{T}|\Omega_{it}\right]-p_{it}^{L^{P}}L_{it}^{P}\\ \text{s.t. } q_{it} &= q(K_{it},L_{it},M_{it},\omega_{it})\\ L_{it} &= L_{it}^{T}+L_{it}^{P}\end{aligned}$$

which gives

$$E_{\varepsilon^{d}}\left[\frac{\partial p(q_{it},Z_{it},\varepsilon_{it}^{d})q_{it}}{\partial q_{it}}\frac{\partial q_{it}}{\partial L_{it}^{P}}|\Omega_{it}\right]=E_{\varepsilon^{d}}\left[[\sigma_{p,q}+1]\,p(q_{it},Z_{it},\varepsilon_{it}^{d})q_{l}|\Omega_{it}\right]=p_{it}^{L^{P}}. \tag{A.7}$$

Combining the FOCs for L_{it}^{T} and L_{it}^{P}, we obtain the following restriction

$$E_{\varepsilon^d}\left[p_{it}^{L^P}\left(1-\tau+2\eta\frac{L_{it}^T}{L_{it}^P}\right)|\Omega_{it}\right]=p_{it}^{L^P}.$$

This is because the benefit the company derives from one unit of temporary labor is identical to the benefit it derives from one unit of permanent labor, the only difference is the cost that it wants to equate. Replacing the cost of temporary workers we obtain

$$E_{\varepsilon^d}\left[\frac{L_{it}^T}{L_{it}^P}|\Omega_{it}\right]=\frac{\tau}{2\eta}$$

or we can write this as $E_{\varepsilon^d}\left[L_{it}^T|\Omega_{it}\right]=\frac{\tau}{2\eta}L_{it}^P$, so the company would like to keep the ratio of temporary to permanent work as a constant

$$\frac{L_{it}^T}{L_{it}^P}=\frac{\tau}{2\eta}\frac{L_{it}^T}{E_{\varepsilon^d}\left[L_{it}^T|\Omega_{it}\right]}. \tag{A.8}$$

The two ratios of materials to labor and temporary employment to labor are a function of $\frac{L_{it}^T}{E_{\varepsilon^d}\left[L_{it}^T|\Omega_{it}\right]}$

$$\frac{p_{it}^M M_{it}}{p_{it}^{L^P}L_{it}}=\frac{1}{\epsilon_{m.l}}\left(1-\tau+\tau\frac{L_{it}^T}{E_{\varepsilon^d}\left[L_{it}^T|\Omega_{it}\right]}\right)$$

$$\frac{p_{it}^{L^P}L_{it}^T}{p_{it}^{L^P}L_{it}^P}=\frac{\tau}{2\eta}\frac{L_{it}^T}{E_{\varepsilon^d}\left[L_{it}^T|\Omega_{it}\right]}.$$

Given that $L_{it}=L_{it}^T+L_{it}^P$, even the parametric case does not deliver an elegant analytic solution. However, we can see that the case with temporary employment delivers similar results to the case without it. The materials to wages ratio is a function the the ETS and the temporary employment to the expected temporary employment. The ratio $\frac{L_{it}^T}{E_{\varepsilon^d}\left[L_{it}^T|\Omega_{it}\right]}$ is a function of the demand shock. We can also use the ratio of temporary to permanent employment, $\frac{L_{it}^T}{L_{it}^P}$.

A.3 Higher order process (extension)

The input ratio is

$$\frac{p^M_{it}M_{it}}{p^L_{i,t}L_{i,t}} = \frac{\gamma}{\beta}\frac{\left(\varepsilon^d_{it}\right)^{\frac{1}{1-\gamma(\sigma+1)}}}{E\left[\left(\varepsilon^d_{it}\right)^{\frac{1}{1-\gamma(\sigma+1)}}|\Omega_{it}\right]}.$$

The input ratio is a sole function of the ETS and the demand component. For simplification, let the demand shock be a separable s order process,

$$\varepsilon^d_{it} = g(\zeta^d_{i,t-1}, ...\zeta^d_{i,t-s}, \nu^d_{it}) = g(\zeta^d_{i,t-1}, ...\zeta^d_{i,t-s})\nu^d_{it} \tag{A.9}$$

where $\zeta^d_{i,t} = (\varepsilon^d_{it}, \mathbf{z}_{it})$ and z is an n-dimensional vector of information known to the firm at period t. Again, write $\tilde{\nu}^d_{it} = \frac{1}{1-\gamma(\sigma+1)}\nu^d_{it}$ and normalize $E_{\nu^d_{it}}\left[\tilde{\nu}^d_{it}\right] = 0$,

$$\ln\left[\frac{p^M_{it}M_{it}}{p^L_{i,t}L_{i,t}}\right] = \ln\left(\gamma/\beta\right) + \tilde{\nu}^d_{it}.$$

As we can see from above, the restriction to a first-order Markovian process is immaterial.

A.4 Nonparametric Identification

In this section I discuss how the assumptions and observed data allow us to nonparametrically identify the distribution of demand shocks. This is instructive, to explain that identification of demand shocks does not depend on the parametrization presented in Section 2.4. Instead, identification is obtained from meaningful variation together with the economic restrictions. There is a total of four equations: the two first-order conditions, the demand function and the production function. This is the case when prices and quantities are observed. However, since prices (and quantities) are not observed we need to combine the production and demand functions into a revenue function. This delivers three equations[9].

[9] In this section, to reduce the number of variables in the notation, input prices (P^M_{it}, P^L_{it}) are included in Z_{it}.

$$
\begin{aligned}
y_{it} &= p(q_{it}, Z_{it}, \varepsilon^d_{it})q(K_{it}, L_{it}, M_{it}, \omega_{it}) \\
M_{it} &= m(\omega_{it}, L_{it}, K_{it}, Z_{it}, \varepsilon^d_{it}) \\
L_{it} &= l(\omega_{it}, K_{it}, Z_{it}, \Omega_{it})
\end{aligned}
$$

Unfortunately, this is not sufficient to identify the demand shock nor is this sufficient to identify productivity. The reason is that we have two unobservables (ω_{it}, ε^d_{it}) and the vector of information known to firm i when it chooses labor, Ω_{it}. This is potentially correlated with all the observables due to serial correlation (note Ω_{it} includes $\omega_{i,t-1}$, $\varepsilon^d_{i,t-1}$ and the other state variables) and three equations.[10] If we attempt to estimate any of the three equations individually, we would face endogeneity of potentially all the explanatory variables since $(y_{it}, M_{it}, L_{it}, K_{it}, Z_{it})$ are correlated with $(\omega_{it}, \varepsilon^d_{it}$, and $\Omega_{it})$. Note that if we allow both inputs to be fully flexible, the equation for labor $(l(.))$ becomes a function of $\varepsilon^d_{i,t}$ instead of Ω_{it}, and both components (ω_{it} and $\varepsilon^d_{i,t}$) enter all three equations. This is the reason why demand and supply shocks cannot be separately identified. Furthermore, when prices and quantities are observed, the two equations allow us to separately identify the demand from the supply conditions $p(q_{it}, Z_{it}, \varepsilon^d_{it})$ and $q(K_{it}, L_{it}, M_{it}, \omega_{it})$. This is the approach followed in Santos, Costa, and Brito (2016).

In order to make progress, let the demand shock be separable in ν^d_{it},

$$
\varepsilon^d_{it} = g(\Omega_{it})e^{\nu^d_{it}} = g_{it}e^{\nu^d_{it}}
$$

where ν^d_{it} is the "news" to demand or the unexpected changes to demand conditions. One simple case is when the demand follows a separable first-order Markov process, $\varepsilon^d_{it} = g(\varepsilon^d_{i,t-1})e^{\nu^d_{it}}$. The news term is, by assumption, serially independent. Out of all the information contained in the information set, $g(\Omega_{it})$ is now a sufficient statistic for ε^d_{it}. This single index restriction is important because it links ε^d_{it} with Ω_{it}. When the firm chooses L_{it}, it must forecast ε^d_{it}. However, given the separability assumption on the demand shock, $g_{it} = g(\Omega_{it})$ is now a sufficient statistic. In the general case, there is now

[10] Notice that if we allow adjustment costs to labor, lagged employment would enter the input demand function for labor, which would become, $l(\omega_{it}, K_{it}, Z_{it}, L_{i,t-1}, \Omega_{it})$.

a system with four equations

$$\begin{aligned}
y_{it} &= y(K_{it}, L_{it}, M_{it}, \omega_{it}, Z_{it}, \varepsilon_{it}^d) \\
M_{it} &= m(\omega_{it}, L_{it}, K_{it}, Z_{it}, \varepsilon_{it}^d) \\
L_{it} &= l(\omega_{it}, K_{it}, Z_{it}, g_{it}) \\
\varepsilon_{it}^d &= g(\Omega_{it})e^{\nu_{it}^d} = g_{it}e^{\nu_{it}^d}
\end{aligned}$$

where $y(K_{it}, L_{it}, M_{it}, \omega_{it}, Z_{it}, \varepsilon_{it}^d) = p(q(K_{it}, L_{it}, M_{it}, \omega_{it}), Z_{it}, \varepsilon_{it}^d)q(K_{it}, L_{it}, M_{it}, \omega_{it})$. In Section 2.4 we used the ratio of the amount spent on the two inputs. This was because taking the ratio eliminates ω_{it} and g_{it} from the ratio equation. The ratio is then a function of the state variables and the demand shock ν_{it}^d. The demand shock is independent from all the state variables, which guarantees identification of both the equation of interest and the shock distribution. In the general case, taking the ratio does not eliminate ω_{it} and g_{it}. Instead, we can characterize the reduced form solutions obtained from the first-order conditions. First we can eliminate ε_{it}^d by replacing with $g(\Omega_{it})e^{\nu_{it}^d}$ which leaves three equations

$$\begin{aligned}
y_{it} &= y(K_{it}, L_{it}, M_{it}, \omega_{it}, Z_{it}, g_{it}e^{\nu_{it}^d}) \\
M_{it} &= m(\omega_{it}, L_{it}, K_{it}, Z_{it}, g_{it}e^{\nu_{it}^d}) \\
L_{it} &= l(\omega_{it}, K_{it}, Z_{it}, g_{it}).
\end{aligned}$$

From the last two equations we can solve for ω_{it} and g_{it} as functions of the remaining variables $(M_{it}, L_{it}, K_{it}, Z_{it})$. We study the technical conditions for invertibility in the next subsection. Plugging these solutions into the first equation

$$\begin{aligned}
y_{it} &= y(K_{it}, L_{it}, M_{it}, Z_{it}, \omega_{it}, g(\Omega_{it})e^{\nu_{it}^d}) \\
&= y(K_{it}, L_{it}, M_{it}, Z_{it}, \omega\left(M_{it}, L_{it}, K_{it}, Z_{it}\right), g\left(M_{it}, L_{it}, K_{it}, Z_{it}\right)e^{\nu_{it}^d}). \qquad \text{(A.10)}
\end{aligned}$$

From equation (A.10) the distribution of "news" to demand term, ν_{it}^d is identified up to standard normalization and location restrictions on the nonparametric functions (see

Matzkin, 2007) as long as all the variables $(M_{it}, L_{it}, K_{it}, Z_{it})$ are independent from ν_{it}^d. That is coherent with the model for the state variables (L_{it}, K_{it}, Z_{it}). However, from the the model's assumptions, we know that M_{it} is not independent from ν_{it}^d. As long as there is serial correlation in ω_{it}, lagged values of $M_{i,t-1}$ and $(L_{i,t-1}, K_{i,t-1}, Z_{i,t-1})$ become valid instruments simply by inverting the equation $\omega_{i,t-1} = \tilde{m}^{-1}(.)$. Equation (A.10) can be estimated by nonparametric instrumental variables when the demand shock is separable (Newey and Powell, 2003). When the demand shock is nonseparable, we need to consider further restrictions to implement the nonparametric IV estimator (see for example, Chen et al., 2014 and Blundell and Matzkin, 2013).

Note that in some cases the equation for intermediate inputs can be written as $M_{it} = \tilde{m}(L_{it}, K_{it}, Z_{it}, \nu_{it}^d)$. This is the parametric case presented in Section 2.4. In this case we do not require invertibility conditions that guarantee that we can write ω_{it} and $g(\Omega_{it})$ as functions of the remaining variables $(M_{it}, L_{it}, K_{it}, Z_{it})$. In such case, we can use the equation $\tilde{m}(.)$ directly to identify the distribution of the demand shocks ν_{it}^d (again up to standard normalization and location restrictions.

A.4.1 Invertibility

We can express the previous problem of obtaining a solution for $g_{it} = g(\Omega_{it})$ and ω_{it} as an invertibility problem. Inverting the labor demand function ($g_{it} = l^{-1}(\omega_{it}, K_{it}, Z_{it}, L_{it})$, see Levinshon and Petrin (2003) for invertibility conditions), and substituting it in the materials input function, we obtain

$$\begin{aligned} M_{it} &= m(\omega_{it}, L_{it}, K_{it}, Z_{it}, l^{-1}(\omega_{it}, K_{it}, Z_{it}, L_{it})e^{\nu_{it}^d}) \\ &= \tilde{m}(\omega_{it}, L_{it}, K_{it}, Z_{it}, \nu_{it}^d). \end{aligned} \tag{A.11}$$

The next step is inverting the $\tilde{m}$ function with respect to ω_{it} to obtain

$$\omega_{it} = \tilde{m}^{-1}(M_{it}, L_{it}, K_{it}, Z_{it}, \nu_{it}^d).$$

Replacing ω_{it} back in $g_{it} = l^{-1}(\omega_{it}, K_{it}, Z_{it}, L_{it})$ we obtain the expression for g_{it}. We can thus solve for g_{it} and ω_{it} as functions of $(M_{it}, L_{it}, K_{it}, Z_{it}, \nu_{it}^d)$ and replace g_{it} and ω_{it} in the revenue Equation (A.10).

We will discuss below the case where $\tilde{m}$ is not a function of ω. In this situation we can obtain the distribution of ν_{it}^d by $M_{it} = \tilde{m}(L_{it}, K_{it}, Z_{it}, \nu_{it}^d)$ directly. This is the parametric case presented in Section 2.4.

A.4.2 Technical conditions for invertibility

Monotonicity of the first-order conditions is not sufficient to obtain invertibility. Take the system formed by the two first-order conditions in equations (1) and (2) where $\varepsilon_{it}^d = g(\Omega_{it})e^{\nu_{it}^d}$. If the determinants of **all** principal submatrices of the Jacobian

$$\begin{bmatrix} \frac{\partial[\sigma_{p,q}+1]p(q_{it},Z_{it},\varepsilon_{it}^d)q_m - p_{it}^M}{\partial \varepsilon_{it}^d}\frac{\partial \varepsilon_{it}^d}{\partial g_{it}} & \frac{\partial[\sigma_{p,q}+1]p(q_{it},Z_{it},\varepsilon_{it}^d)q_m - p_{it}^M}{\partial \omega_{it}} \\ \frac{E_{\varepsilon^d}\left[[\sigma_{p,q}+1]p(q_{it},Z_{it},\varepsilon_{it}^d)q_l\right] - p_{it}^L}{\partial g_{it}} & \frac{E_{\varepsilon^d}\left[[\sigma_{p,q}+1]p(q_{it},Z_{it},\varepsilon_{it}^d)q_l\right] - p_{it}^L}{\partial \omega_{it}} \end{bmatrix}$$

are non-vanishing, it follows by Theorem 7 in Gale and Nikaido (1965) that the system is invertible. We thus obtain a solution to this system in (g_{it}, ω_{it}) as functions of the remaining variables $\left(M_{it}, K_{it}, L_{it}, Z_{it}, \nu_{it}^d\right)$. A necessary condition to obtain a solution is,[11]

$$\frac{\frac{\partial[\sigma_{p,q}+1]p(q_{it},Z_{it},g(\Omega_{it})e^{\nu_{it}^d})q_m}{\partial \varepsilon_{it}^d}\frac{\partial \varepsilon_{it}^d}{\partial g_{it}}}{\frac{\partial \int \left[[\sigma_{p,q}+1]p(q_{it},Z_{it},g(\Omega_{it})e^{\nu_{it}^d})q_l\right] df(\nu_{it}^d)}{\partial g_{it}}} \neq \frac{\frac{\partial[\sigma_{p,q}+1]p(q_{it},Z_{it},g(\Omega_{it})e^{\nu_{it}^d})q_m}{\partial \omega_{it}}}{\frac{\partial \int \left[[\sigma_{p,q}+1]p(q_{it},Z_{it},g(\Omega_{it})e^{\nu_{it}^d})q_l\right] df(\nu_{it}^d)}{\partial \omega_{i,t}}}.$$

This condition states that we require the differential effect of the demand shock on material vs. labor (slope) to be different from same differential effect for the supply shock. This is because labor/intermediate input choices "control" for one of the shocks ($g_{i,t}$ or ω_{it}). In the nonparametric case, productivity shocks have an effect on the input ratio and the input ratio varies with both demand and supply shocks (e.g. non-Hicks neutral productivity). If the differential effects are the same for supply and demand

[11] Notice that when the determinat is zero and invertibility condition holds with equality (and we cannot solve the system explicitly), the productivity term drops from the equation for M_{it}, i.e. $M_{it} = \tilde{m}(L_{it}, K_{it}, Z_{it}, \nu_{it}^d)$ and the equation is clearly no longer invertible in ω_{it}. However, in this case the problem is actually simplified and guarantees that the demand shock is identified since we just need to nonparametrically regress M_{it} on (L_{it}, K_{it}, Z_{it}). This is the collinearity case of ACF (2006). In this case only the two FOCs and the separability condition (A.3) are required.

shocks, changes to the ratio allow us to distinguish one shock from the other. When the condition is not verified, it implies that both shocks have a symmetric effect on the two first-order conditions such that varying one of them is in fact equivalent to varying the other. The system is not invertible. However, this is equivalent to the parametric case presented in Section 2.4. It is particularly useful because we suffice with the two first-order conditions. That is, inverting the labor demand equation effectively "controls" for the two shocks ω and ε^d and

$$\begin{aligned} m_{it} &= m(\omega_{it}, L_{it}, K_{it}, Z_{it}, l^{-1}(\omega_{it}, K_{it}, Z_{it}, L_{it})e^{\nu^d_{it}}) \\ &= \tilde{m}(L_{it}, K_{it}, Z_{it}, \nu^d_{it}) \end{aligned}$$

and $\tilde{m}$ is not a function of ω. We can nonparametrically regress M_{it} against L_{it}, K_{it}, Z_{it} to identify the distribution of ν^d_{it}.

A.5 Results

A.5.1 Year-averaged estimated demand shocks and self-reported changes in market conditions, by industry.

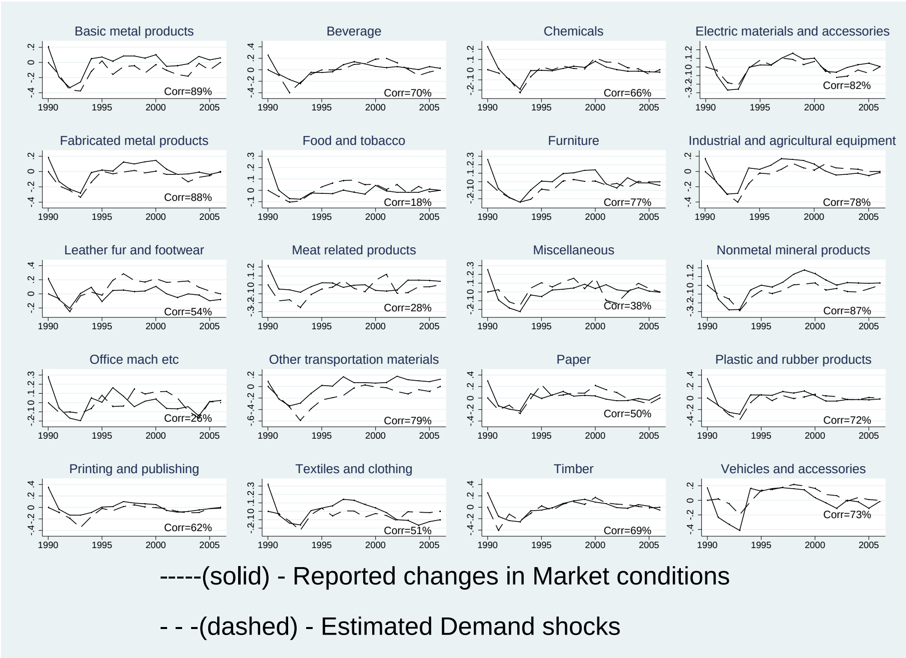


Figure A.1: Year-averaged estimated demand shocks and self-reported changes in market conditions, by industry.

A.5.2 Production function estimates

	Dep. Variable: Physical Output									
Sector	Meat	Food	Drinks	Text.	Foot.	Timber	Paper	Publish.	Chemic.	Plastics
Capital	0.024	0.035***	0.192***	0.043***	0.064***	0.126***	0.096***	0.049***	0.050***	0.011
Labor	0.167***	0.202***	0.282***	0.483***	0.202***	0.341***	0.547***	0.302***	0.203***	0.415***
Materials	0.520***	0.621***	0.172***	0.247***	0.487***	0.303***	0.297***	0.389***	0.479***	0.367***
Services	0.096***	0.072***	0.186***	0.082***	0.080***	0.078***	0.099***	0.028***	0.156***	0.120***
Constant	5.583***	3.991***	6.694***	7.753***	5.261***	6.474***	6.166***	7.258***	4.949***	6.447***
N	642	2240	461	2375	720	610	730	1235	1736	1306

	Dep. Variable: Physical Output									
Sector	Non-metal Minerals	Basic Metal	Fabricat. Metal	Equip.	Offic./data Precision	Electric.	Vehicles	Other Transp.	Furniture	Misc.
Capital	0.034*	0.025	0.035**	0.001	-0.006	-0.023*	0.092***	0.02	0.026*	0.065**
Labor	0.439***	0.265***	0.419***	0.314***	0.275***	0.437***	0.385***	0.261***	0.364***	0.425***
Materials	0.354***	0.541***	0.442***	0.524***	0.511***	0.412***	0.456***	0.599***	0.382***	0.278***
Services	0.128***	0.144***	0.075***	0.068***	-0.003	0.074***	0.019***	0.112***	0.068***	0.090***
Constant	6.208***	4.157***	5.635***	5.473***	7.037***	6.881***	5.481***	3.874***	6.683***	7.241***
N	1731	776	2402	1903	396	1729	1188	538	1186	543

Notes: Production function estimates. Regression reults with firm specific (fixed) effects.
Standard errors clustered at firm level. * $p<0.05$; ** $p<0.01$; *** $p<0.001$

Table A.1: Production function estimates, fixed effects results by industry.

A.5.3 Input regression estimates (self reported shocks to demand)

Industry	Meat	Food	Drinks	Text.	Foot.	Timber	Paper	Publish.	Chemic.	Plastics
Dep. Var:	ln(wage bill)									
Self reported demand	-0.092	0.034	0.061	0.042	-0.064	-0.064	0.038	0.001	-0.011	0.021
ln(Capital)	0.138	0.262***	0.369***	0.288***	0.236***	0.191***	0.266***	0.228***	0.371***	0.291***
TFP	0.134	0.266**	0.333**	0.188*	0.201*	0.412***	0.065	0.396***	0.162	0.190
Cons.	12.479***	10.503***	9.306***	9.770***	10.055***	11.058***	10.754***	11.003***	9.659***	10.260***
Dep. Var:	ln(materials)									
Self reported demand	0.021	0.130***	0.056	0.244***	0.120	-0.039	0.193**	0.130**	0.067	0.046
ln(Capital)	0.264**	0.270***	0.468**	0.288***	0.325**	0.183*	0.368***	0.246***	0.423***	0.274***
TFP	0.093	0.080	0.003	0.029	-0.971*	0.416	0.051	0.266**	0.159	0.008
Cons.	12.120***	11.181***	8.716***	9.706***	9.789***	12.042***	9.990***	10.720***	9.854***	11.283***
N	662	2402	477	2600	809	687	756	1350	1793	1366

Industry	Non-metal Minerals	Basic Metal	Fabricat. Metal	Equip.	Offic./data Precision	Electric.	Vehicles	Other Transp.	Furniture	Misc.
Dep. Var:	ln(wage bill)									
Self reported demand	0.073**	0.010	0.017	0.023	0.075	-0.007	0.033	0.099*	0.036	-0.001
ln(Capital)	0.427***	0.184*	0.226***	0.236***	0.446***	0.242***	0.178*	0.214*	0.346**	0.162*
TFP	0.218*	0.162*	0.193**	0.260***	0.290*	0.268**	0.295*	0.188	0.128	0.242
Cons.	8.105***	12.580***	11.037***	10.909***	8.616***	11.493***	13.014***	12.066***	8.958***	11.639***
Dep. Var:	ln(materials)									
Self reported demand	0.150***	0.096*	0.212***	0.205***	0.227*	0.197***	0.131**	0.399*	0.203***	0.342**
ln(Capital)	0.453***	0.211	0.245***	0.237***	0.510***	0.434*	0.229***	0.087	0.431***	0.038
TFP	0.025	0.252	0.120	-0.094	0.047	-0.029	0.307	0.058	0.099	-0.220
Cons.	8.100***	13.447***	11.202***	10.992***	7.920***	9.354***	13.116***	14.775***	8.139***	13.523***
N	1832	778	2530	1968	407	1786	1201	562	1269	598

Notes: OLS estimates for the regression of the wage bill (top panel) and materials (bottom panel) on self-reported demand shocks, TFP and capital stock, by industry. Year dummies included. S.e.'s clustered at the firm level. All variables in logs
* $p<0.05$; ** $p<0.01$; *** $p<0.001$

Table A.2: Input demand equations (with self reported demand shocks).

A.5.4 Demand estimates: GMM results

Industry	Meat	Food	Drinks	Text.	Foot.	Timber	Paper	Publish.	Chemic.	Plastics
Dep. Var:					Quantity					
σ	-3.529**	-1.143***	-1.896***	-4.212***	-14.69***	-5.903***	-1.818***	-2.648***	-1.015**	-2.439***
ϕ_a	0.358	-0.505***	-0.314***	-2.450***	0.0487	-2.158***	-0.610***	-1.698***	-1.467***	-1.192***
ϕ_z	-0.285	1.085***	1.068***	2.420***	0.321	1.731***	0.991***	1.656***	1.522***	1.075***
g_1	1.004***	0.854***	0.867***	0.996***	1.046***	0.969***	0.852***	1.008***	0.991***	0.940***
N	557	1,994	389	2,150	666	545	626	1,109	1,497	1,114

Industry	Non-metal Minerals	Basic Metal	Fabricat. Metal	Equip.	Offic./data Precision	Electric.	Vehicles	Other Transp.	Furniture	Misc.
Dep. Var:					Quantity					
σ	-3.240***	-2.235***	-1.720***	-1.037***	-0.432**	-0.973***	-13.87***	-0.499	-14.64***	-1.240***
ϕ_a	-1.868***	-1.048***	-1.111***	-1.077***	-1.320***	-1.082***	-0.642***	-0.350*	-1.048***	-0.547***
ϕ_z	1.794***	1.117***	1.116***	1.146***	1.088***	1.055***	0.842***	0.998***	1.268***	1.145***
g_1	0.990***	0.956***	0.941***	0.950***	0.943***	0.953***	1.032***	0.740***	1.037***	0.865***
N	1,481	649	2,055	1,624	328	1,503	985	471	1,054	506

Notes: GMM estimates of the demand model. Instrument set includes current and lagged TFP, age and capital stock and lagged output, prices and demand stock. Year dummies included. S.e.'s clustered at the firm level. * $p<0.05$; ** $p<0.01$; *** $p<0.001$

Table A.3: Demand estimates, IV results per industry.

A.5.5 Correlation of the demand shock estimated via demand function with the self reported demand change and the demand shock estimated via input ratio

Dep. Variable: Estimated Demand Shocks (via input ratio)										
Sector	Meat	Food	Drinks	Text.	Foot.	Timber	Paper	Publish.	Chemic.	Plastics
Panel A										
$\hat{\nu^d}$	0.439***	0.512***	0.256	0.307***	0.123***	0.440***	0.260*	0.422***	0.335**	0.618**
Panel B										
$\hat{\nu^d}$	0.008	-0.007	0.064	0.063*	0.06	-0.028	0.037	0.03	0.007	-0.065
Self-reported mkt.	0.438***	0.514***	0.241	0.290***	0.116***	0.446***	0.256*	0.415***	0.400***	0.635**
Observations	683	2473	511	2472	754	663	705	1305	1734	1248

Sector	Non-metal Minerals	Basic Metal	Fabricat. Metal	Equip.	Offic./data Precision	Electric.	Vehicles	Other Transp.	Furniture	Misc.
Panel A										
$\hat{\nu^d}$	0.264**	0.384***	0.456***	0.525***	0.507***	0.539***	0.100***	0.352***	0.100***	0.644***
Panel B										
$\hat{\nu^d}$	0.015	-0.01	0.034	0.024	0.013	0.039	0.044*	0.118	0.097***	0.049
Self-reported mkt.	0.260**	0.386***	0.443***	0.517***	0.504***	0.528***	0.096***	0.344***	0.093***	0.620***
Observations	1709	770	2326	1866	386	1712	1135	537	1221	563

Notes: Results for linear regression of the demand shock (via input ratio) on the demand shock (via demand function residual), by industry. Each panel reports separate results. Standard errors clustered at the firm level. * $p<0.1$; ** $p<0.05$; *** $p<0.01$

Table A.4: Correlation of the demand shock estimated via demand function with the self reported demand change and the demand shock estimated via input ratio.

A.5.6 Input regression estimates (estimated demand residuals)

Industry	Meat	Food	Drinks	Text.	Foot.	Timber	Paper	Publish.	Chemic.	Plastics
Dep. Var:	ln(wage bill)									
Demand residual	0.096	0.195*	0.211	0.226**	0.056	0.073	-0.061	0.070	0.294***	0.008
Capital	0.620***	0.693***	0.812***	0.574***	0.431***	0.501***	0.689***	0.528***	0.703***	0.657***
TFP	1.070***	0.380**	0.640*	0.480***	0.468**	0.815**	-0.148	1.040***	0.262	0.546***
Cons.	5.172***	4.002***	1.959	5.833***	7.665***	6.305***	4.117***	6.506***	4.307***	4.684***
Dep. Var:	ln(materials)									
Demand residual	0.553***	0.774***	0.612	0.680***	0.206*	0.279**	0.207	0.719***	0.569***	0.708***
Capital	0.688***	0.903***	0.600***	0.690***	0.686***	0.645***	0.765***	0.626***	0.743***	0.773***
TFP	0.517	0.205	1.735***	1.576***	-0.582	0.969	0.260	0.936***	0.212	0.672
Cons.	5.783*	1.846***	6.548***	4.575***	5.150***	5.417***	3.989***	5.170***	4.688***	3.614***
N	607	2177	433	2364	725	590	674	1209	1627	1193

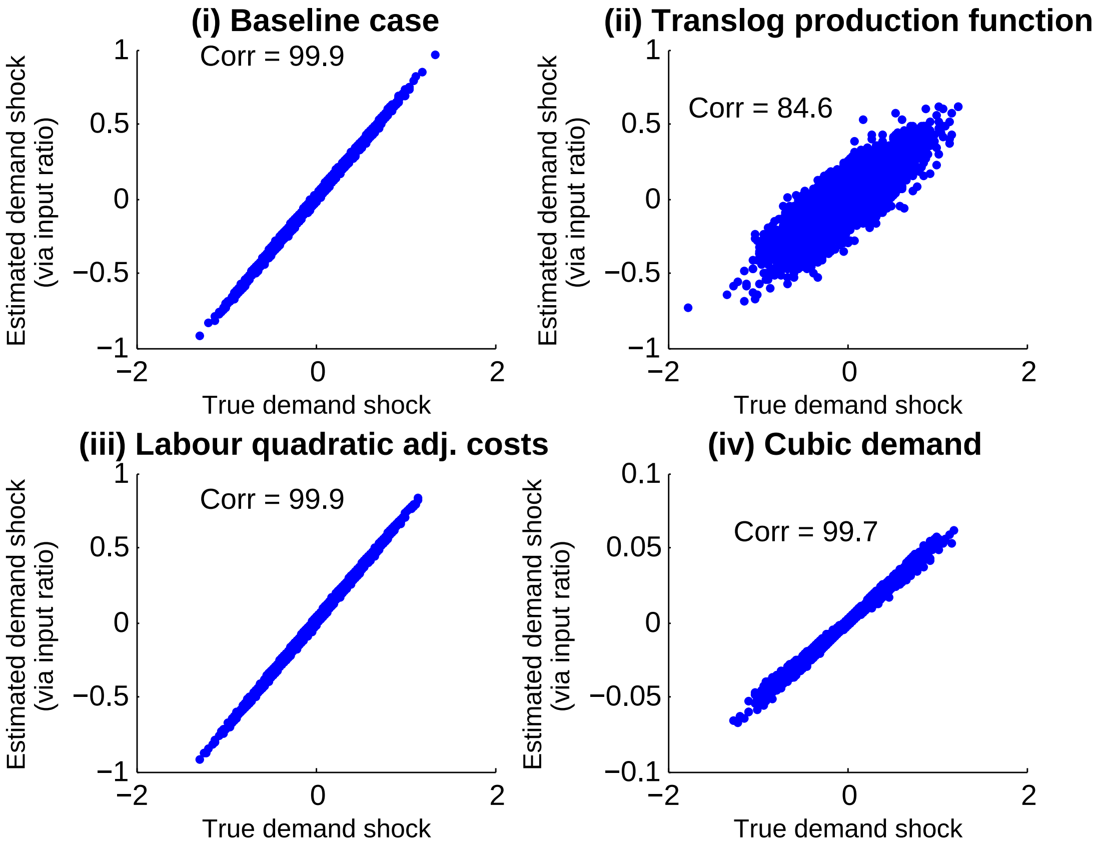
Industry	Non-metal Minerals	Basic Metal	Fabricat. Metal	Equip.	Offic./data Precision	Electric.	Vehicles	Other Transp.	Furniture	Misc.
Dep. Var:	ln(wage bill)									
Demand residual	-0.001	0.130	0.100	0.120	0.026	0.077	0.054	0.106	0.148***	0.150
Capital	0.716***	0.711***	0.628***	0.651***	0.565***	0.566***	0.742***	0.696***	0.430***	0.610***
TFP	-0.083	-0.448	0.140	0.439**	0.663**	0.745***	0.399	0.298	1.368***	0.415
Cons.	3.806***	3.660***	5.079***	5.219***	6.805***	6.645***	3.585***	4.431***	7.806***	5.284***
Dep. Var:	ln(materials)									
Demand residual	0.401***	0.601***	0.679***	0.863***	0.155	0.503***	0.178*	0.631***	0.200**	0.759*
Capital	0.731***	0.748***	0.731***	0.830***	0.692***	0.654***	0.882***	0.758***	0.438***	0.571***
TFP	0.540*	0.263	0.034	-0.430	0.536	0.942*	0.814	-0.516	1.627***	1.291**
Cons.	4.199***	3.940***	4.218***	3.193***	4.889	6.062***	1.915	4.419***	8.219***	6.474***
N	1613	694	2191	1773	359	1647	1049	516	1140	553

Notes: OLS estimates for the regression of the wage bill (top panel) and materials (bottom panel) on estimated demand shocks (via demand function), TFP and capital stock, by industry. Year dummies included. S.e.'s clustered at the firm level. All variables in logs. * $p<0.05$; ** $p<0.01$; *** $p<0.001$

Table A.5: Input demand equations (with estimated demand residuals).

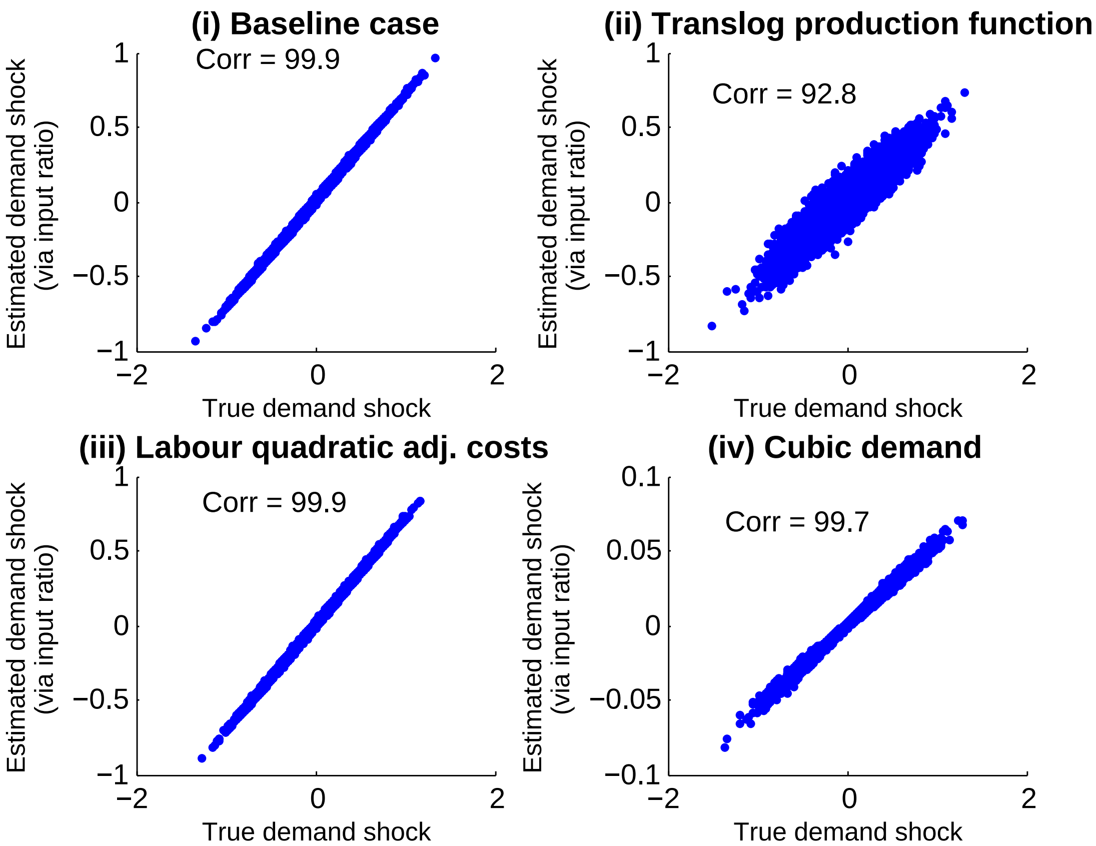
A.6 Monte Carlo results

Figure A.2: Simulated vs. estimated shocks with varying parametrizations ($\phi_z = 0.75$).



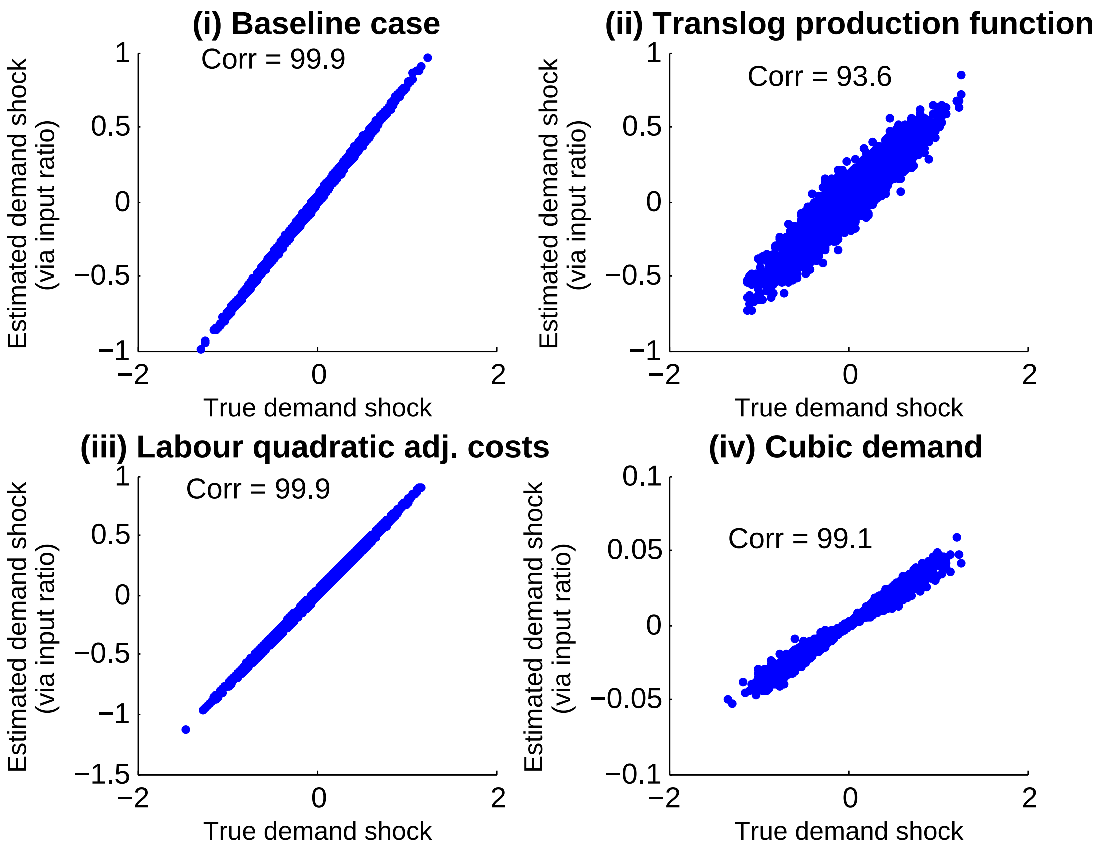
Notes: Scatterplot and correlation between simulated and estimated demand shocks using four different parametrizations ($\phi_z = 0.75$).

Figure A.3: Simulated vs. estimated shocks with varying parametrizations ($\delta = 0.66$).



Notes: Scatterplot and correlation between simulated and estimated demand shocks using four different parametrizations ($\delta = 0.66$).

Figure A.4: Simulated vs. estimated shocks with varying parametrizations ($\beta = 0.35$ and $\gamma = 0.7$).



Notes: Scatterplot and correlation between simulated and estimated demand shocks using four different parametrizations ($\beta = 0.35$ and $\gamma = 0.7$).