Confidence regions for spatial excursion sets from repeated random field observations, with an application to climate

Supplementary material

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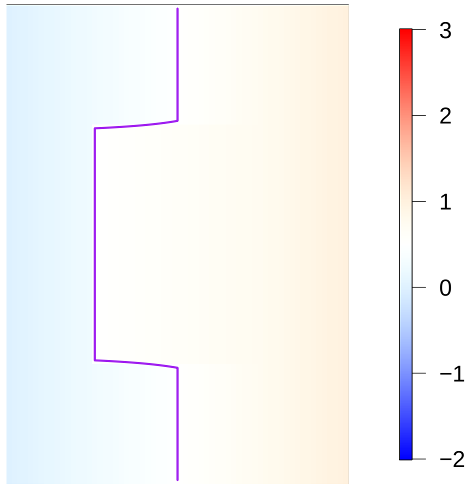


Figure 1: The discontinuous signal.

1 Simulations: Discontinuous signal

For the theoretical coverage guarantees for CoPE sets we assume that the true signal is continuous around the true contour ∂A_c. In practice, it may be difficult to distinguish a discontinuity from a large gradient if the data is given on a discrete grid.

To assess numerically whether the continuity assumption is crucial for proper coverage, we performed experiments for a signal that is discontinuous around the target level (see Figure 1). Exemplary realizations of CoPE sets for this signal are shown in Figure 2.

We repeated the experiments of Section 4 of the main article investigating empirical coverage in this setting using only i.i.d. Noise 1. The results are shown in Figure 3. Coverage quickly reaches the nominal level of 90%. This suggests that the continuity assumption is not necessary to obtain coverage at or above the nominal level.

2 Additional Data Analysis

2.1 AR coefficients in the data

To better assess the temporal correlation in the data, Figure 4 shows the spatial distribution of the coefficient of an AR(1) model fitted at each location. The maps and histograms show a range of positive and negative AR coefficients that vary smoothly over space. To simulate a situation that is similar to the data, the numerical experiments in Section 4 of the main

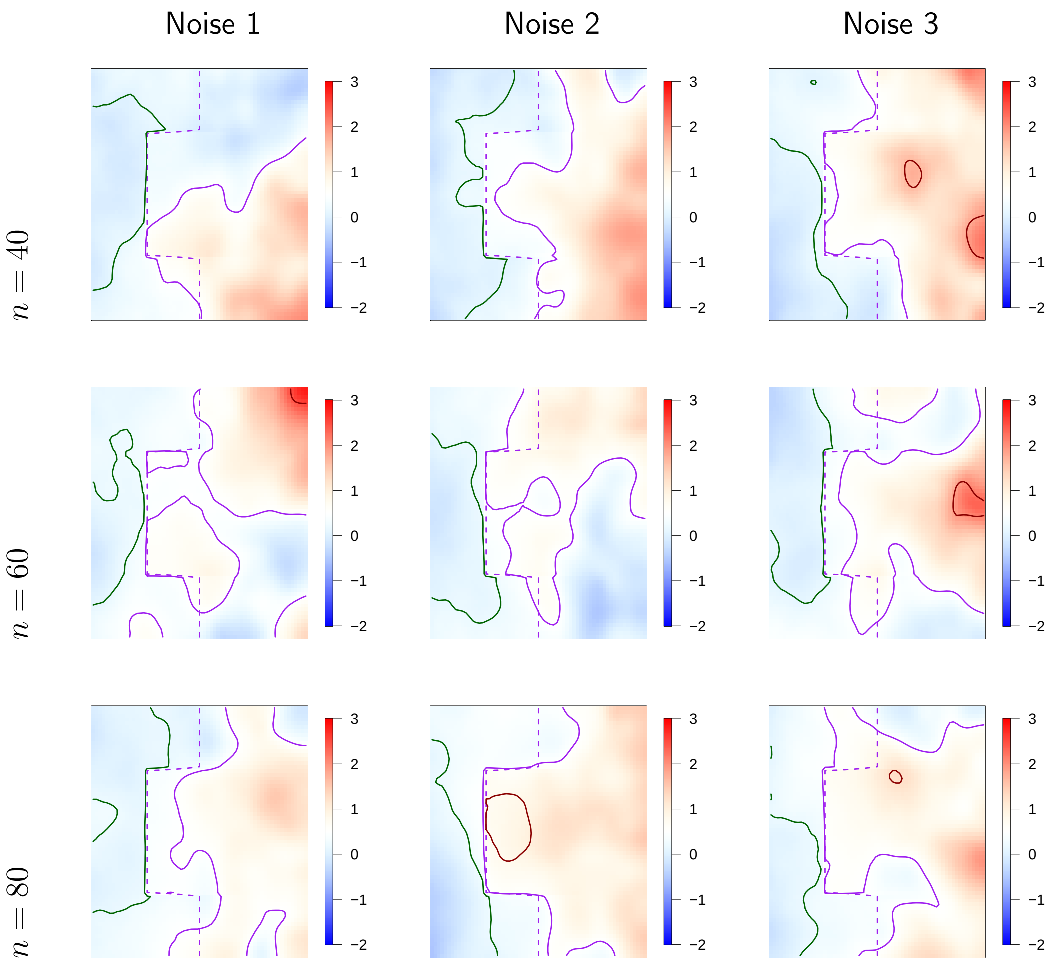


Figure 2: The output of our method for the three noise fields described above (corresponding to rows) and for sample sizes $n = 60, 120, 240$ (corresponding to columns) with the target function $\mu(\mathbf{s})$ shown in Figure 1. In all pictures we show a heat map of the estimator $\hat{\mu}_n(\mathbf{s})$, the boundary of $A_c(\mu)$ in purple as well as the boundaries of $\hat{A}_c^+$ and $\hat{A}_c^-$ in red and green, respectively.

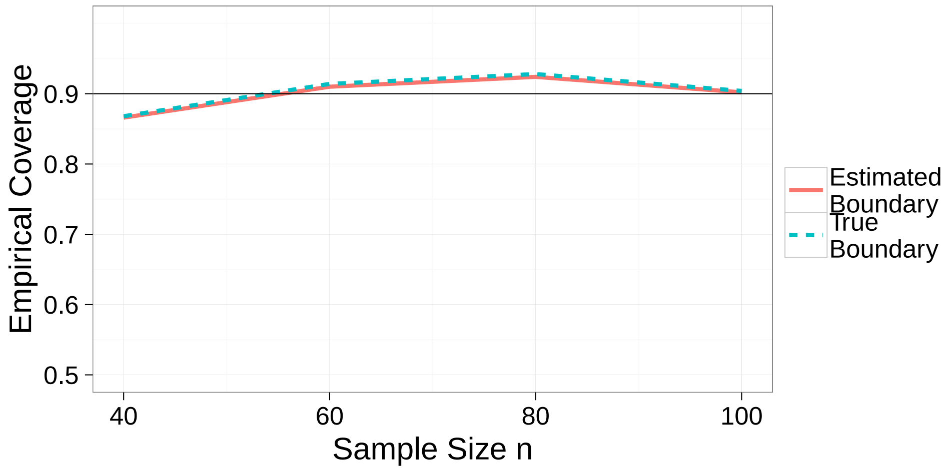


Figure 3: Empirical coverage of CoPE sets for the discontinuous signal.

article used a spatially varying AR(1) coefficient with a range and histogram similar to those here.

2.2 Choice of domain

As pointed out in the discussion, the choice of the domain S influences the CoPE sets. To demonstrate this influence we repeated the analysis of the climate data considering only the USA. The results are shown in Figure 5. The smaller domain results in slightly less conservative CoPE sets than those obtain when including the entire NARCAAP domain (Figure 1 in the main article).

2.3 Pixel-wise confidence sets via multiple testing correction

As a point of reference using a simpler method, we also computed confidence regions using pixel-wise tests for the hypotheses that the mean temperature difference $T^{(b)}(\mathbf{s}) - T^{(a)}(\mathbf{s})$ is equal to $c = 2°C$ and adjust for multiple testing with a Holm correction (Holm, 1979). In order to obtain the p-values for the individual pixels, we assume normality of the estimate. The resulting confidence regions are shown in Figure 5.

As one might expect, the regions obtained in this manner are considerably more conservative than the CoPE sets, as they do not take into account the spatial structure of the data. Naturally, they will become more conservative if the grid becomes finer since this

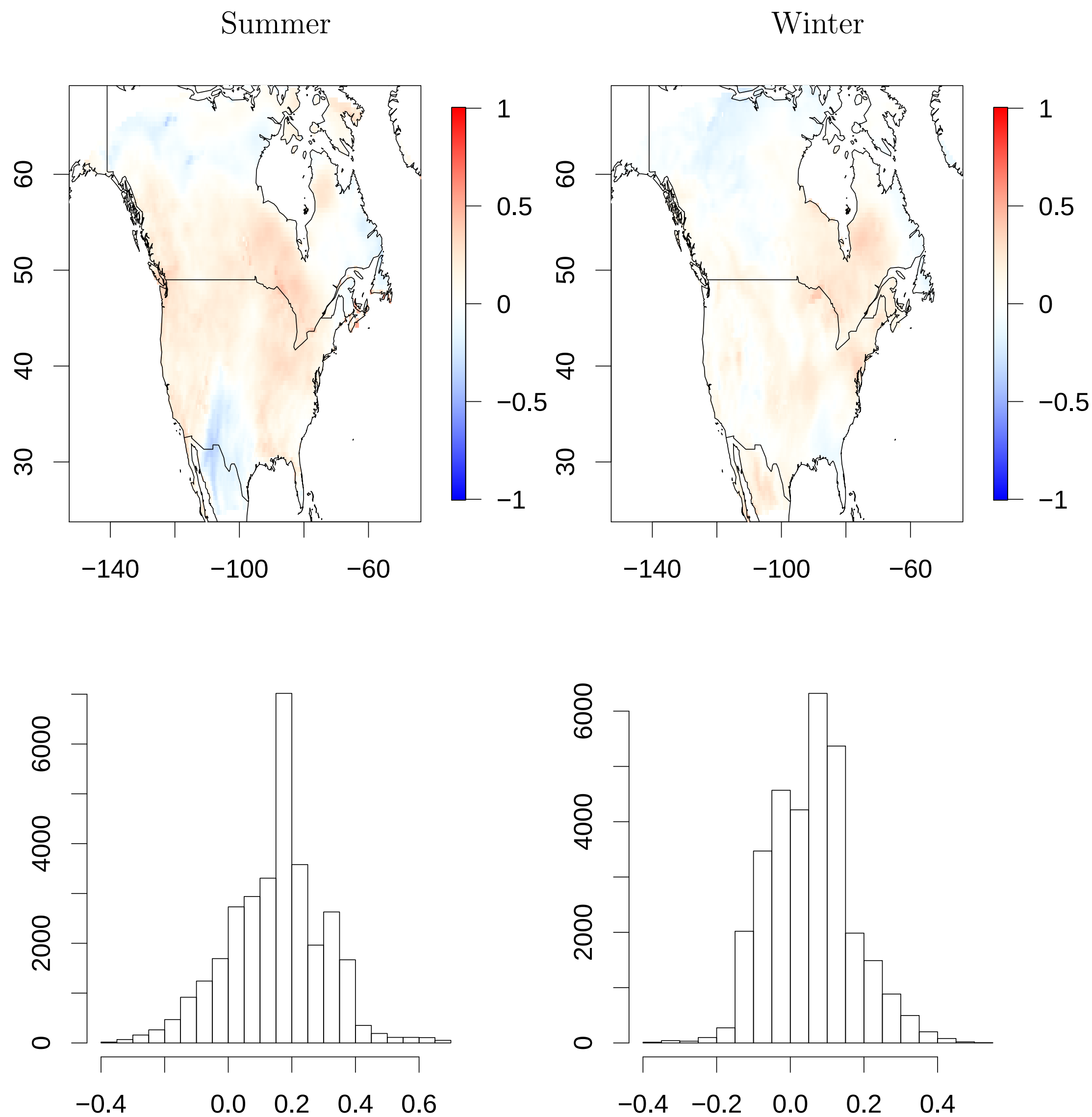
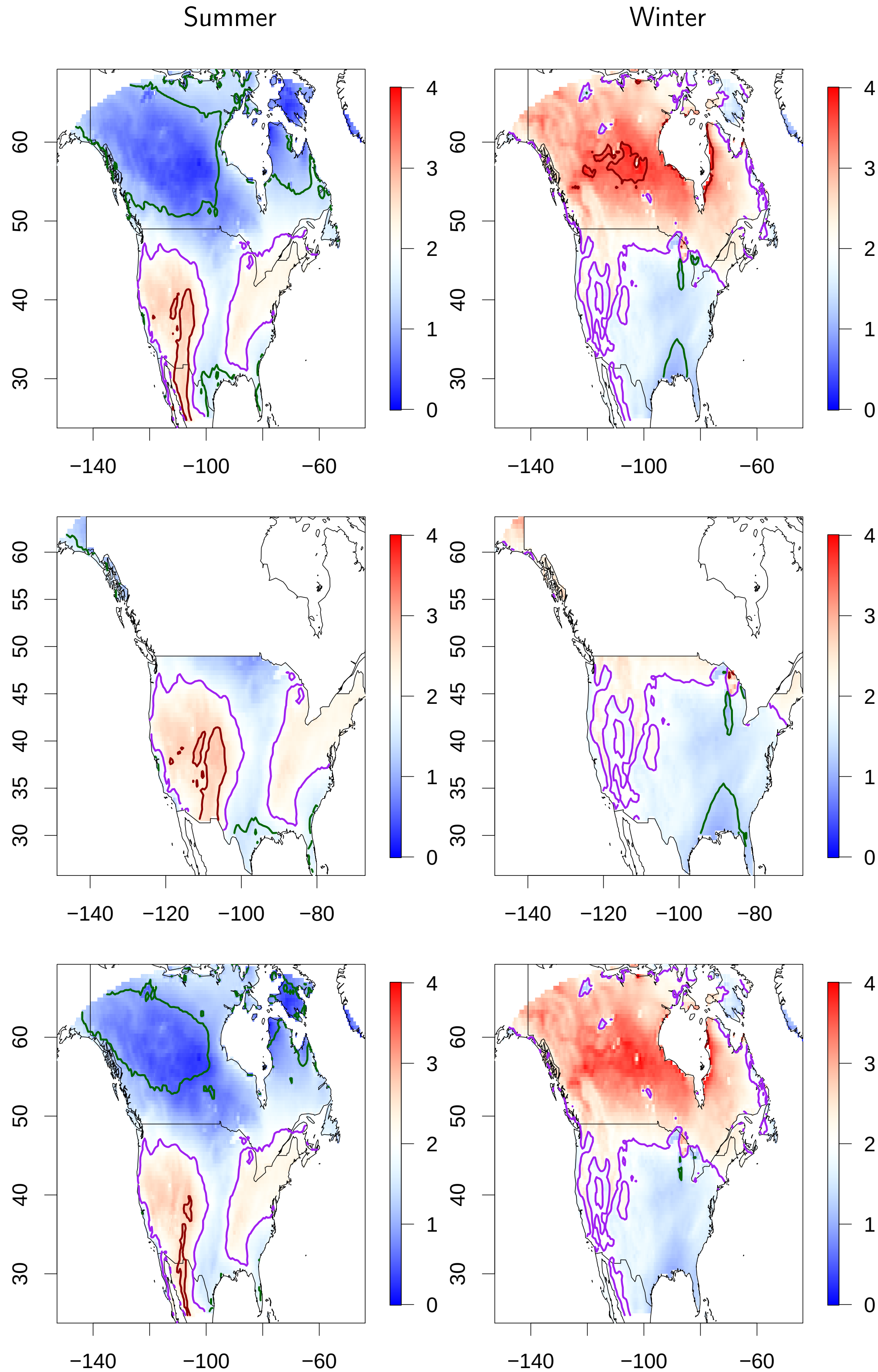


Figure 4: Estimated AR coefficients for the linear model fitted to the climate data: spatial distribution (top row) and histogram over locations (bottom row).

Figure 5: CoPE sets for entire domain (North America)(top row), US only (middle) and confidence regions for the excursion set A_c for $c = 2°C$ obtained via pixel-wise testing and Holm correction (bottom).



will increase the number of tests. The CoPE sets do not suffer from this drawback.

References

Holm, S. (1979). A Simple Sequentially Rejective Multiple Test Procedure. *Scandinavian Journal of Statistics*, 6(2):65–70.