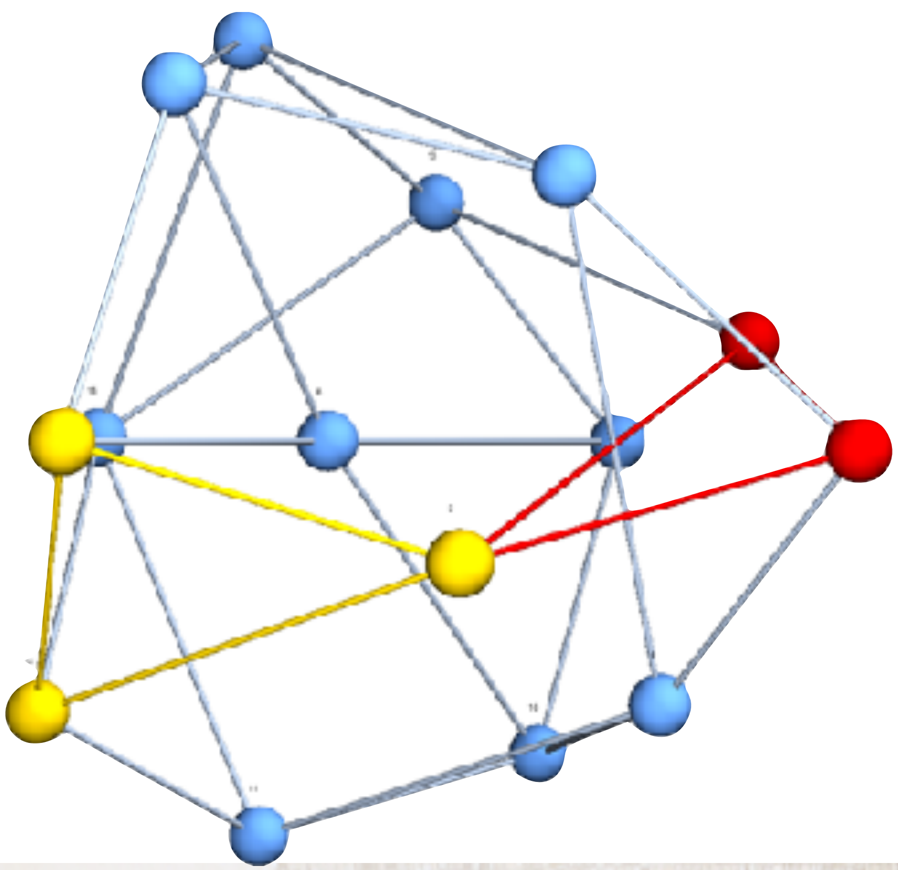
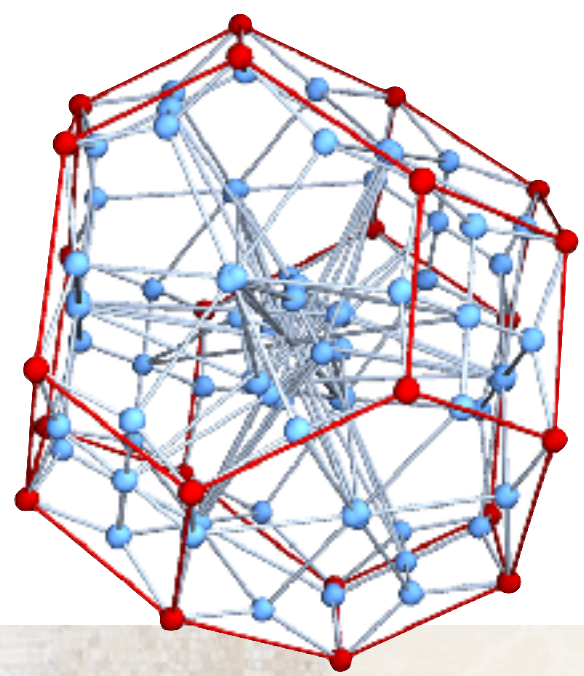
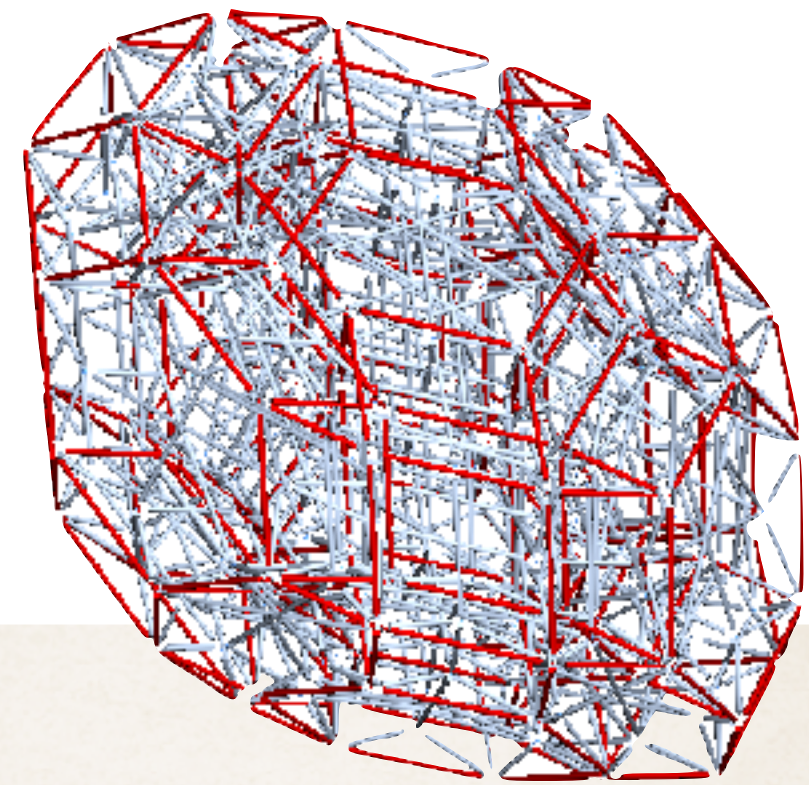
Recent progress from amplitudes

A “Big Picture” overview…







Strings 2017

June 26-30, 2017 | Tel Aviv, Israel

John Joseph M Carrasco

IPhT cea saclay

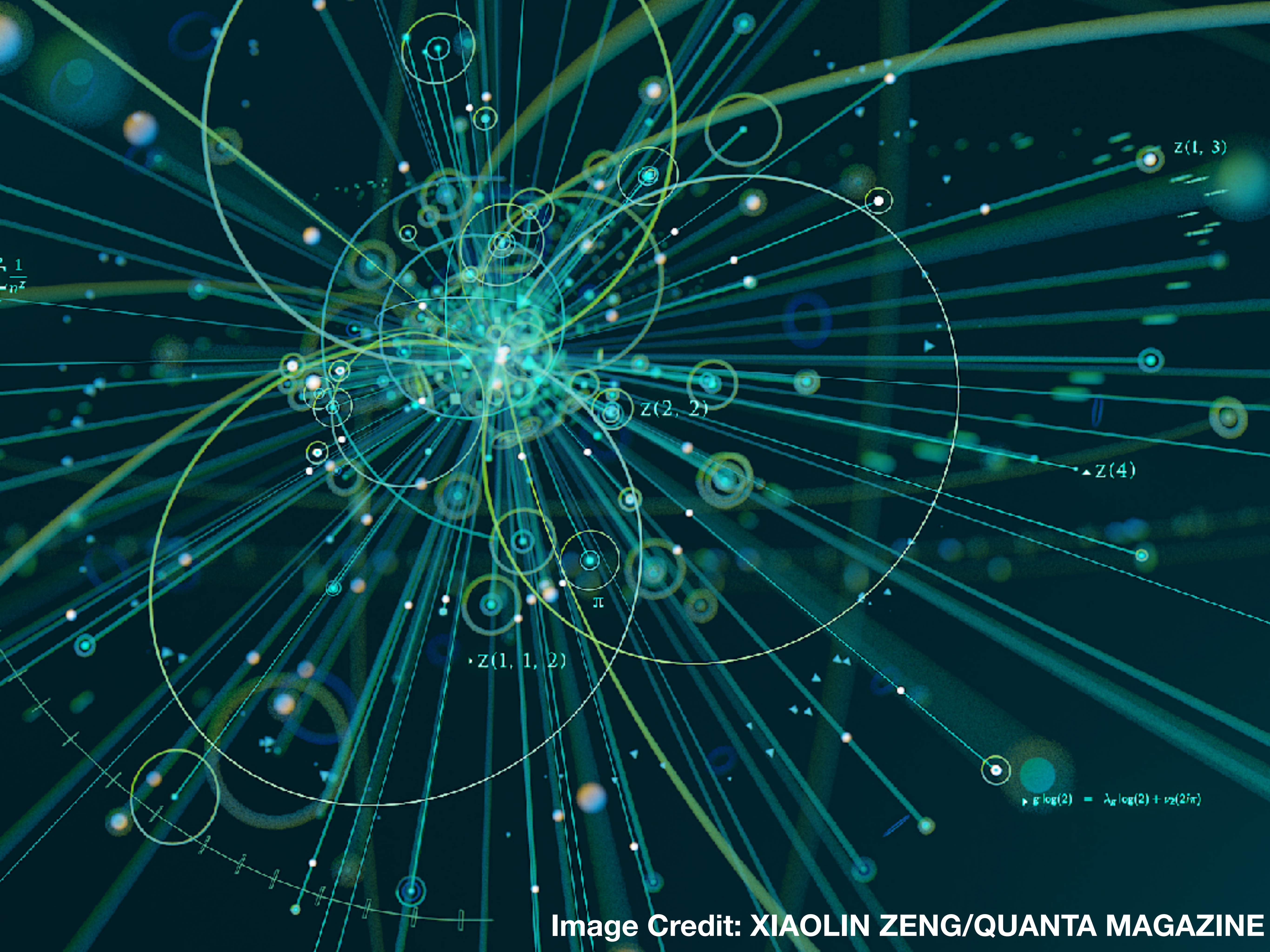


Image Credit: XIAOLIN ZENG/QUANTA MAGAZINE

Perturbative gauge theory as a string theory in twistor space

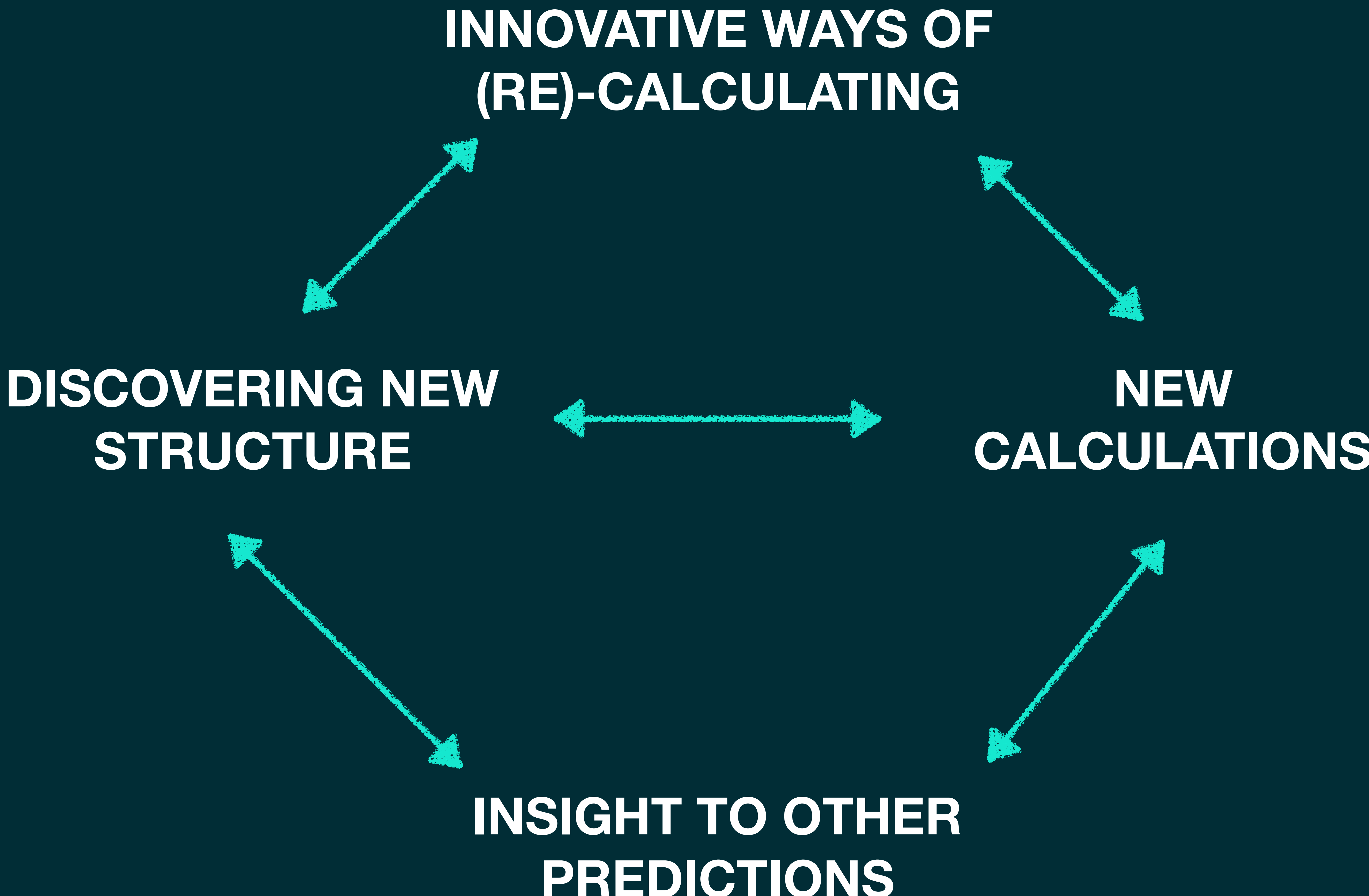
[Edward Witten](). Dec 2003.

NEW CALCULATIONS

INNOVATIVE WAYS OF (RE)-CALCULATING

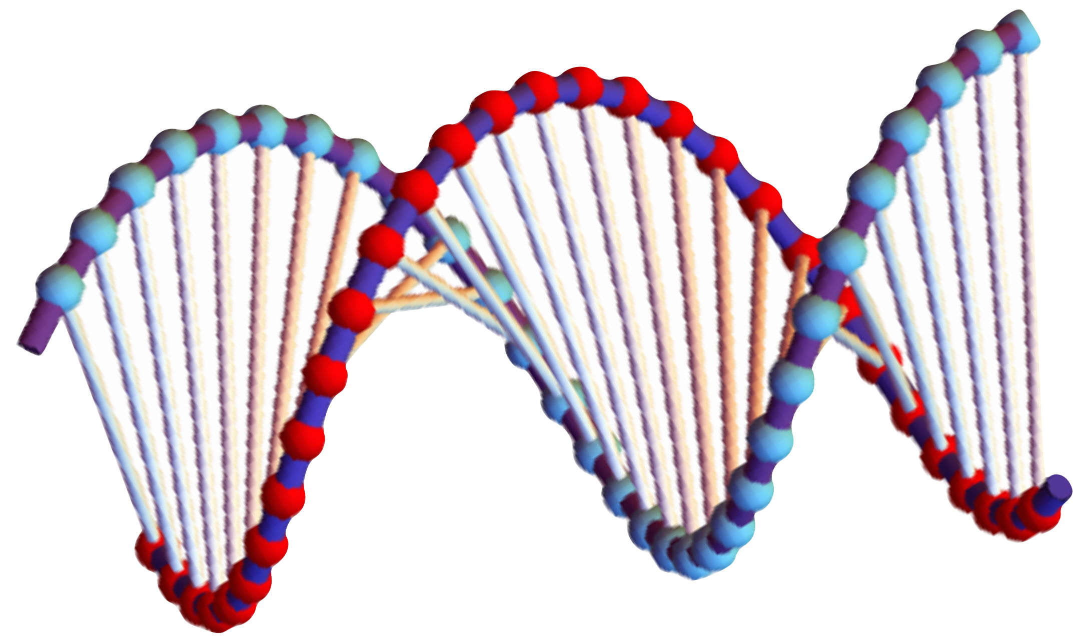
DISCOVERING NEW STRUCTURE

INSIGHT TO OTHER PREDICTIONS


INNOVATIVE WAYS OF (RE)-CALCULATING
DISCOVERING NEW STRUCTURE
NEW CALCULATIONS
INSIGHT TO OTHER PREDICTIONS

A KEY STRUCTURAL DEVELOPMENT:

Lots of theories' predictions are related to each other



they have a *Double Copy* structure
and are built out of shared ingredients:

A Relation Between Tree Amplitudes of Closed and Open Strings
H. Kawai, D.C. Lewellen, S.H.H. Tye. Sep 1985.

KLT

New Relations for Gauge-Theory Amplitudes
Z. Bern, JJMC, Henrik Johansson May 2008

BCJ

Scattering in Three Dimensions from Rational Maps
Freddy Cachazo, Song He, Ellis Ye Yuan. Jun 12, 2013.

CHY

{Abe, Abreu, Adamo, Aharony, Ahmadiniaz, Ahn, Akhoury, Akinto, Alday, Alston, Ambrosio, Anastasiou, Arkani-Hamed, Baadsgaard, Badger, Bajnok, Bargheer, Barreiro, Bastianelli, Beisert, Benincasa, Berg, Bern, Bianchi, Bissi, Bjerrum-Bohr, Bjornsson, Boels, Bonezzi, Borsten, Boucher-Veronneau, Bourjaily, Brandhuber, Brink, Broedel, Brown, Buchberger, Burger, Cachazo, Campiglia, Carballo-Rubio, Cardoso, Caron-Huot, Carrasco, Casali, Chen, Chester, Cheung, Chi, Chiodaroli, Chu, Coito, Conde Pena, Corradini, Damgaard, Davies, de la Cruz, de la Fuente, Dennen, Diaz-Cruz, Di Vecchia, Dixon, Donoghue, Drummond, Du, Duff, Duhr, Dunbar, Eden, Edison, Ellis, El-Menoufi, Elvang, Enciso, Engelund, Ettle, Farhi, Febres Cordero, Feige, Feng, Freedman, Frellesvig, Freyhult, Freytsis, Fu, Gang, Gardi, Geyer, Goddard, Goldberger, Gomez, Grassi, Green, Gromov, Gu, Gunaydin, Gurdoğan, Gurdogan, Hall, Hansen, Harland-Lang, Härtl, He, Heckman, Henn, Herrmann, Heslop, Ho, Hodges, Hoeche, Hohenegger, Hohm, Holstein, Horowitz, Horst, Huang, Huber, Hughes, Isermann, Ita, Janik, Jaquier, Jia, Jin, Johansson, Kallosh, Kampf, Kaplan, Kazakov, Keppeler, Kharel, Kiermaier, Kim, Klose, Kniehl, Kniss, Koh, Kol, Korchemsky, Korres, Kosower, Krasnov, Krauss, Kristjansen, Kuhnel, Kunszt, Lai, Larios, Larkoski, Larsen, Latini, Lee, Leoni, Li, Lipstein, Litsey, Liu, Luisoni, Luna, Luo, Lust, Ma, Mafra, Magro, Marotta, Marrani, Mason, Mastrolia, Matthes, Mauri, McGady, McLoughlin, Medina, Medrano Jimenez, Melia, Melnikov, Melville, Mezzalira, Minahan, Mirabella, Mistlberger, Mitsuka, Mizera, Mogull, Mojaza, Monteiro, Mooney, Moynihan, Murli, Murugan, Naculich, Nagy, Nampuri, Nandan, Nastase, Neill, Nekovar, Nepomechie, Nguyen, Nicholson, Nohle, Novotny, Ochirov, O'Connell, Ossola, Oxburgh, Page, Parra-Martinez, Paton, Penante, Penati, Pennington, Peraro, Perkins, Perlmutter, Plante, Planté, Plefka, Ponomarev, Porto, Postnikov, Prabhu, Primo, Ragoucy, Rao, Rej, Remmen, Ridgway, Rodina, Roehrig, Roiban, Rothstein, Sabio Vera, Santambrogio, Saotome, Schabinger, Schafer-Nameki, Schlotterer, Schnitzer, Schonherr, Schreiber, Schubert, Schwab, Schwartz, Sen, Serna Campillo, Shen, Shir, Sieg, Siegel, Siegert, Sivaramakrishnan, Sjodahl, Sjödahl, Smillie, Smirnov, Søgaard, Sommovigo, Sondergaard, Spence, Spradlin, Sprenger, Stankowicz, Staudacher, Sterman, Stewart, Stieberger, Sundborg, Sundrum, Tahir, Tan, Tarasov, Taylor, Teng, Terasoma, Thompson, Thorén, Tolotti, Torres Bobadilla, Torrielli, Tourkine, Travaglini, Trnka, Tseytlin, Tsimpis, Tye, Vaman, van Deurzen, Vanhove, Van Proeyen, Vazquez-Mozo, Vázquez-Mozo, Vergu, Verlinde, Vieira, Volin, Volovich, Wang, Wecht, Weinzierl, Weltman, Wen, Westerberg, White, Wiegandt, Wilhelm, Wise, Xie, Xu, Yamada, Yang, Yao, Young, Yuan, Zanderighi, Zeng, Zhang, Zhou, Zoccali, Zoubos}

Color-Kinematics and Double Copy Construction

Consider a Villanelle

Do Not Go Gentle Into That Good Night

Do not go gentle into that good night,
Old age should burn and rave at close of day;
Rage, rage against the dying of the light.

Though wise men at their end know dark is right,
Because their words had forked no lightning they
Do not go gentle into that good night.

Good men, the last wave by, crying how bright
Their frail deeds might have danced in a green bay,
Rage, rage against the dying of the light.

Wild men who caught and sang the sun in flight,
And learn, too late, they grieved it on its way,
Do not go gentle into that good night.

Grave men, near death, who see with blinding sight
Blind eyes could blaze like meteors and be gay,
Rage, rage against the dying of the light.

And you, my father, there on that sad height,
Curse, bless, me now with your fierce tears, I pray.
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-Dylan Thomas

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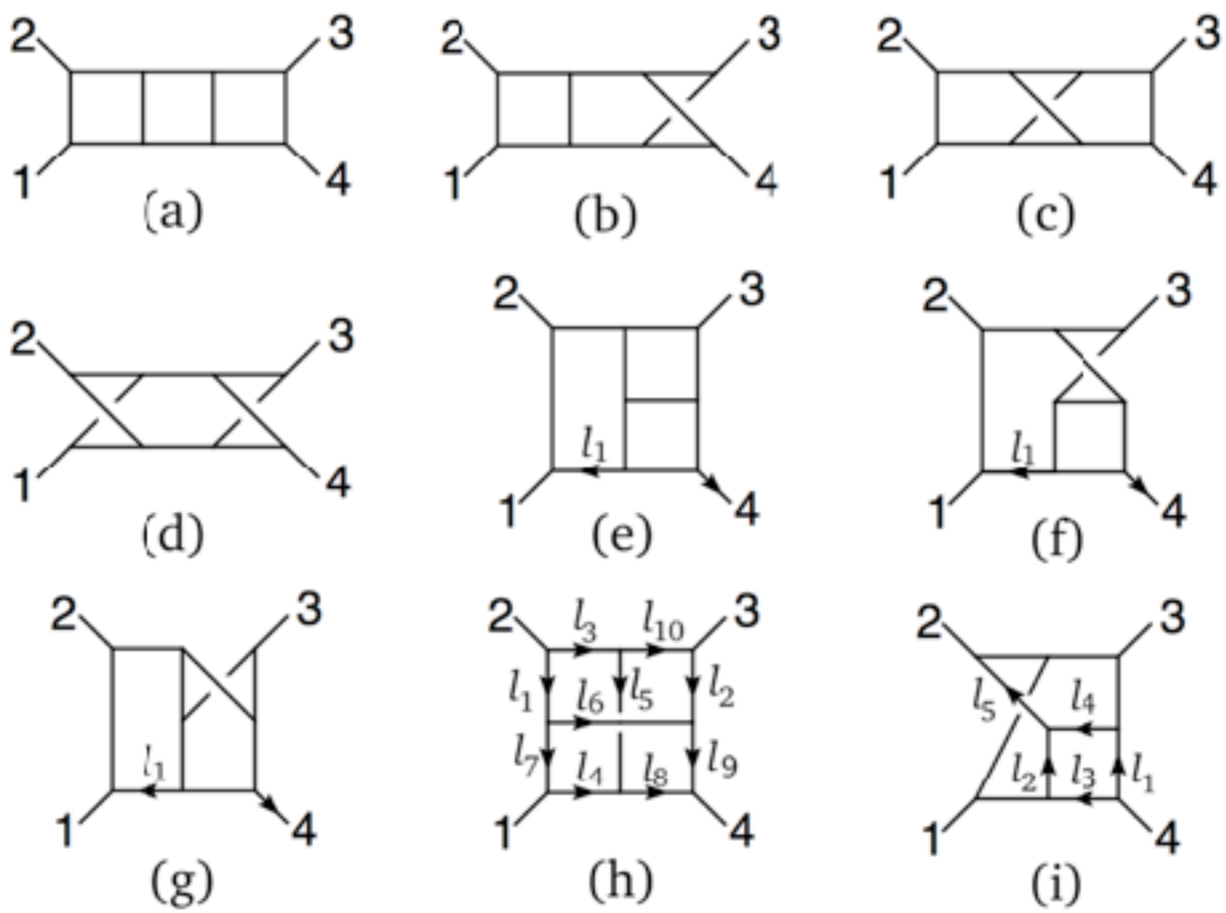
What's going on?

- Minimal information in.
- Relations propagate this information to a full solution.

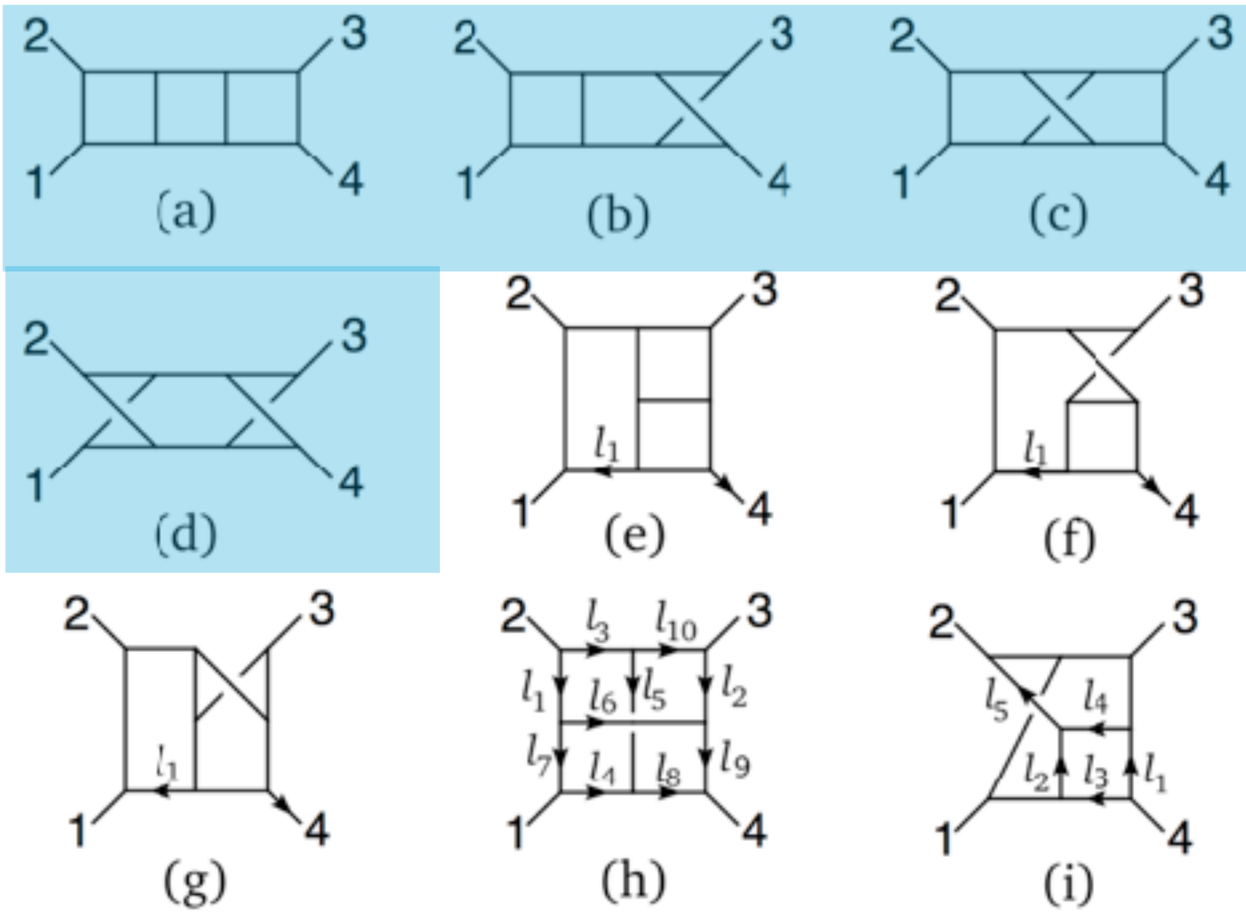
Consider an Amplitude

Bern, JJMC, Dixon, Kosower, Johansson, Roiban '07



Original solution of three-loop four-point N=4 sYM and N=8 sugra

Integral	$\mathcal{N}=4$ Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$s(l_1+l_2)^2+t(l_3+l_4)^2 - sl_5^2 - tl_6^2 - st$	$(s(l_1+l_2)^2+t(l_3+l_4)^2-st)^2 - s^2(2((l_1+l_2)^2-t)+l_5^2)l_5^2$ $-t^2(2((l_3+l_4)^2-s)+l_6^2)l_6^2 - s^2(2l_7^2l_2^2+2l_1^2l_9^2+l_2^2l_9^2+l_1^2l_7^2)$ $-t^2(2l_3^2l_8^2+2l_{10}^2l_4^2+l_8^2l_4^2+l_3^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$s(l_1+l_2)^2 - t(l_3+l_4)^2 - \frac{1}{3}(s-t)l_5^2$	$(s(l_1+l_2)^2-t(l_3+l_4)^2)^2 - (s^2(l_1+l_2)^2+t^2(l_3+l_4)^2+\frac{1}{3}stu)l_5^2$



Bern, JJMC, Dixon, Kosower, Johansson, Roiban '07

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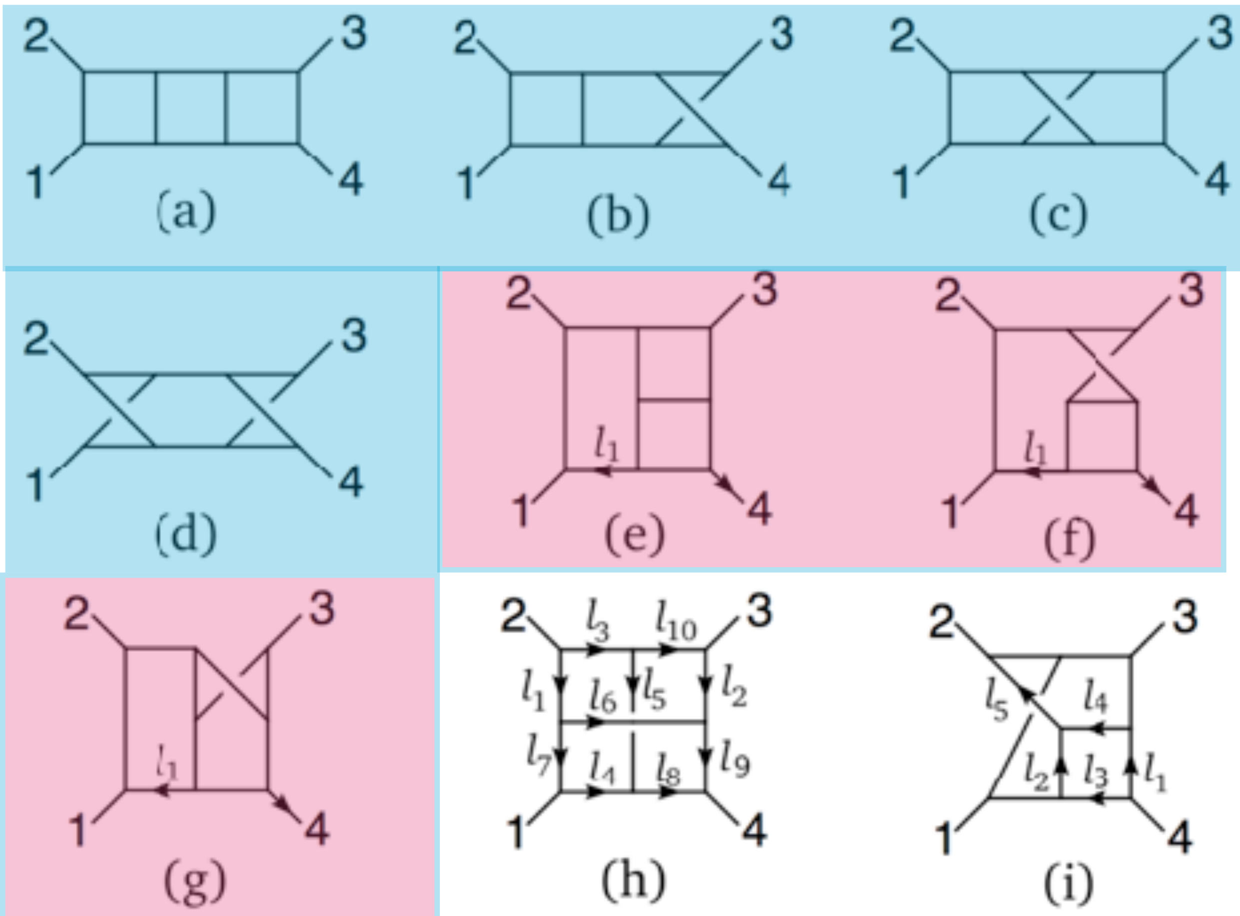

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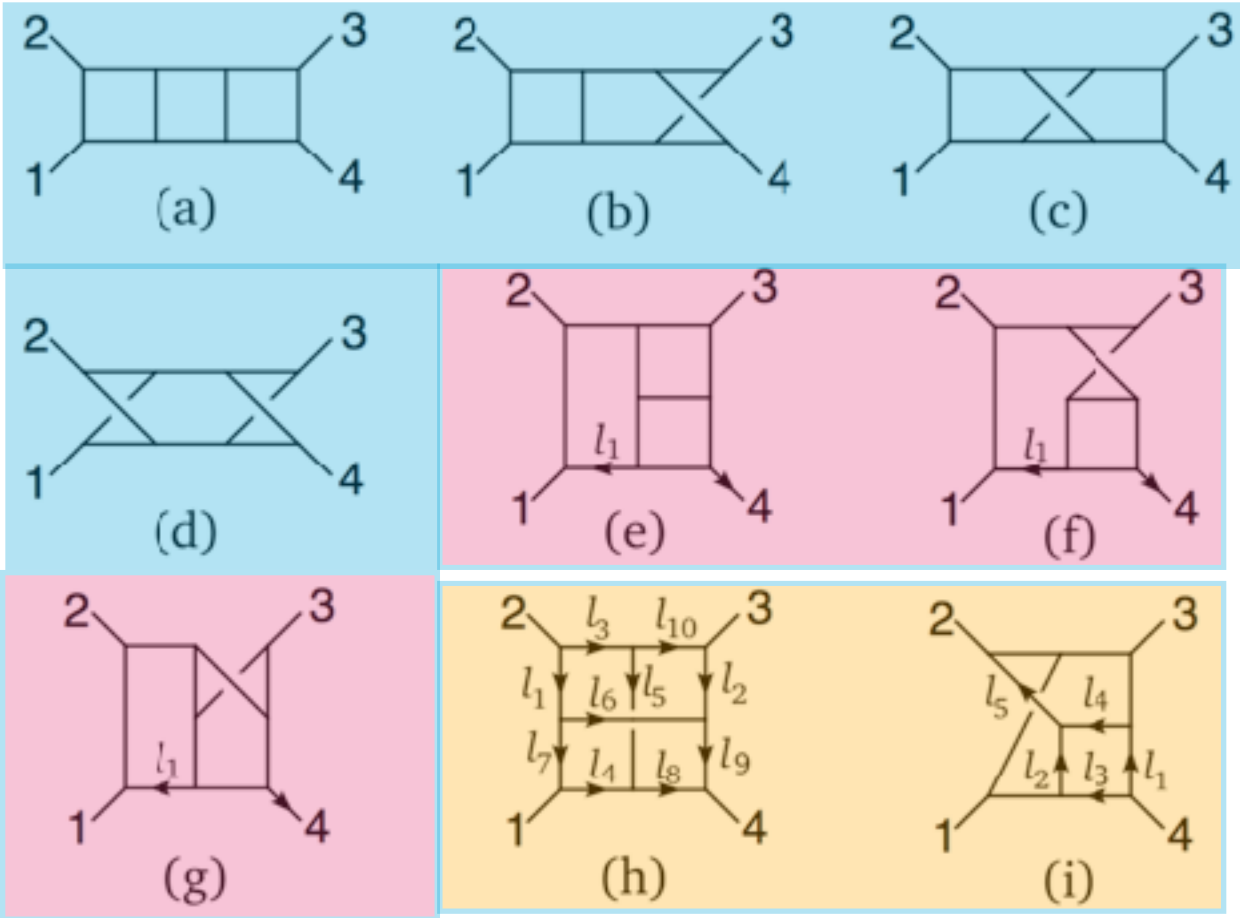

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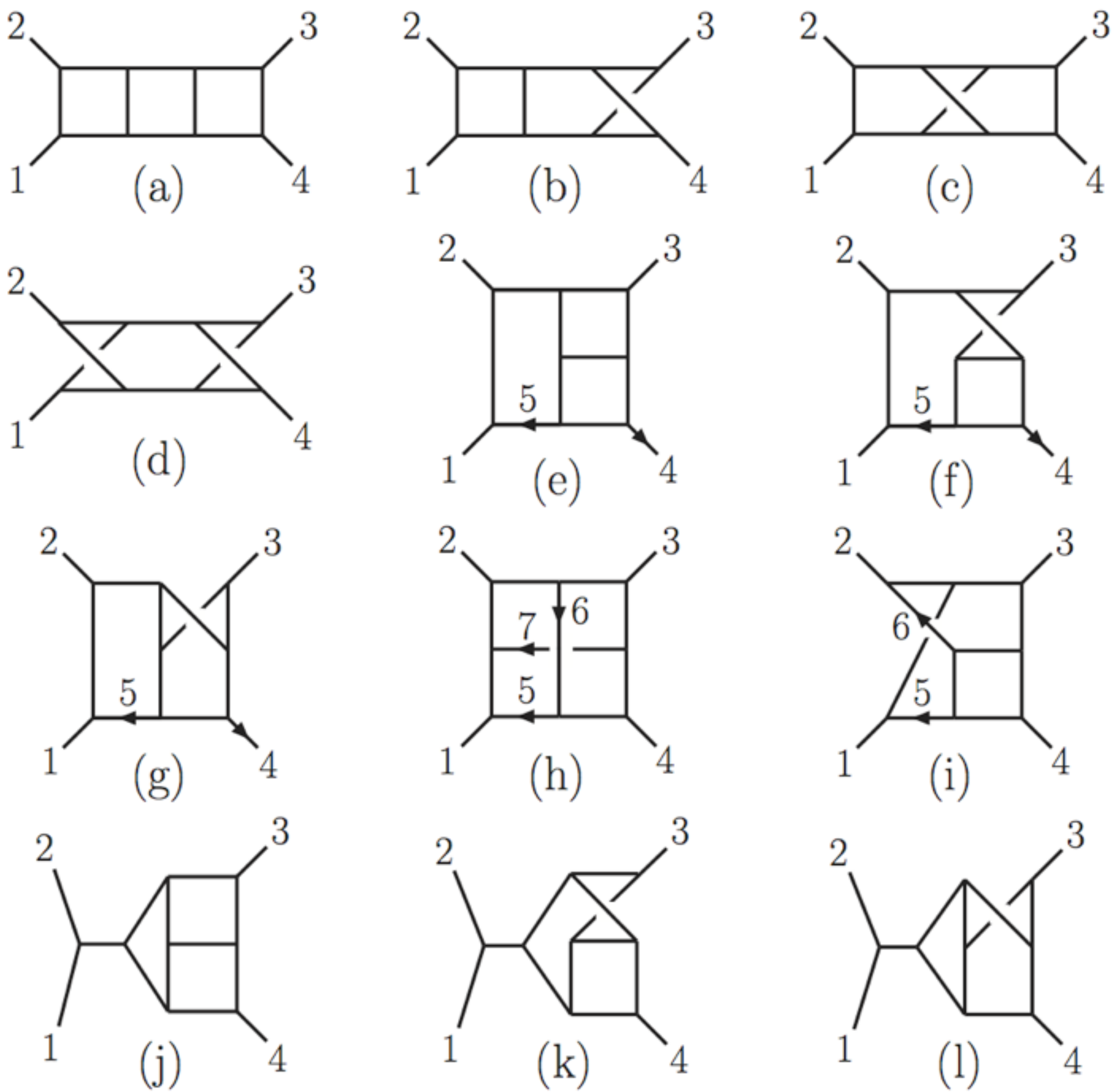
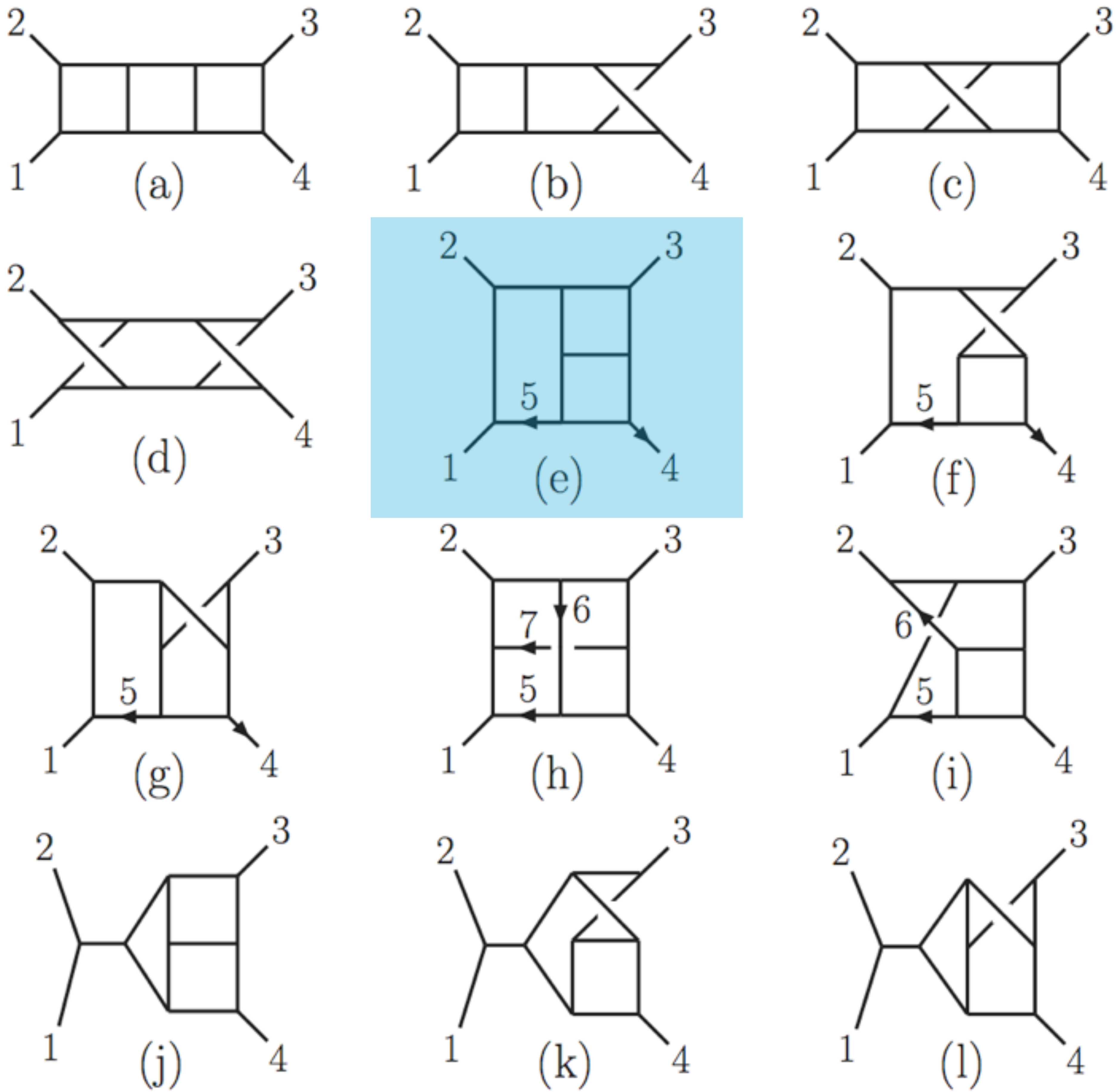
Bern, JJMC, Johansson, '10





Bern, JJMC, Johansson, '10

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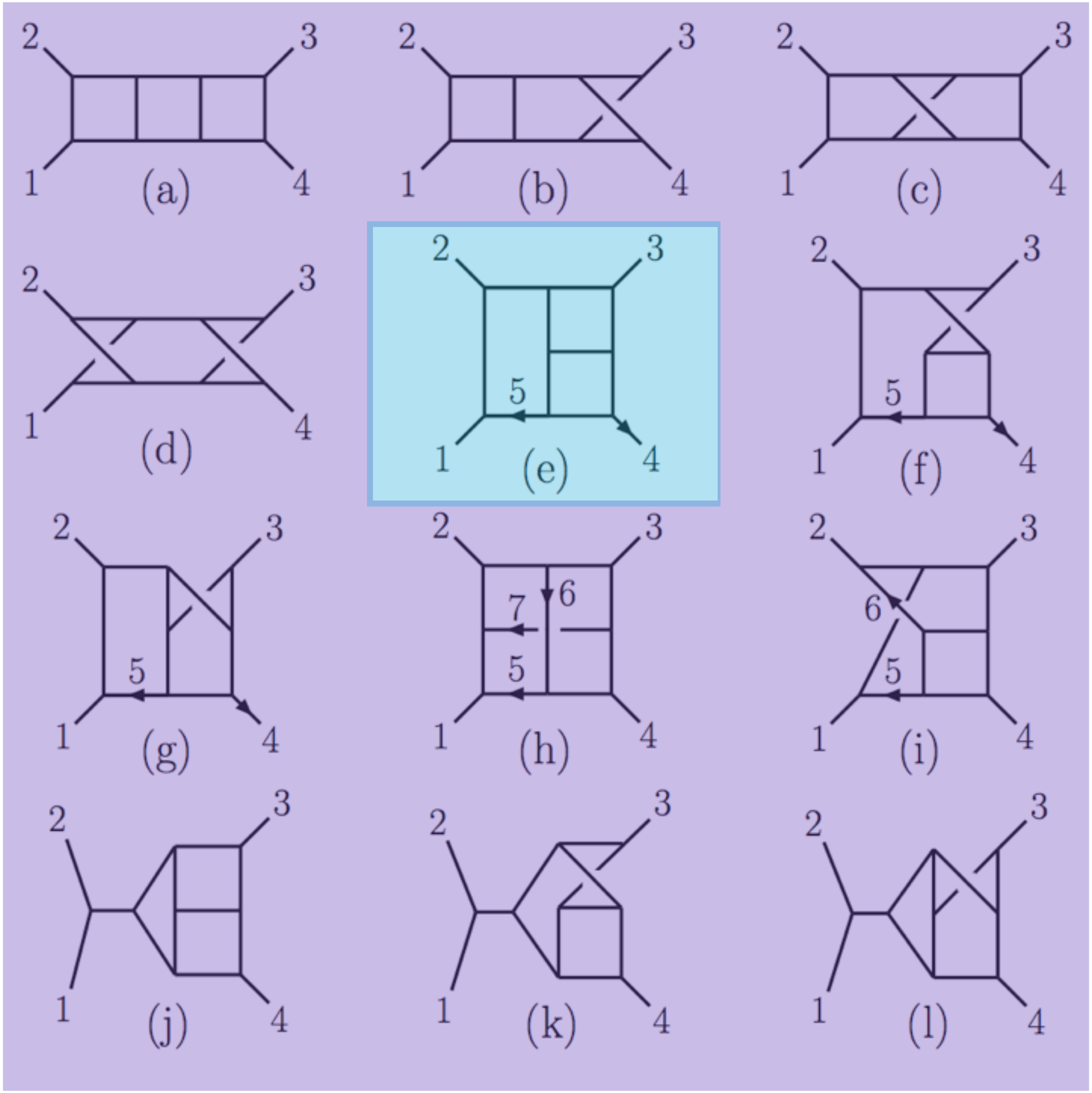
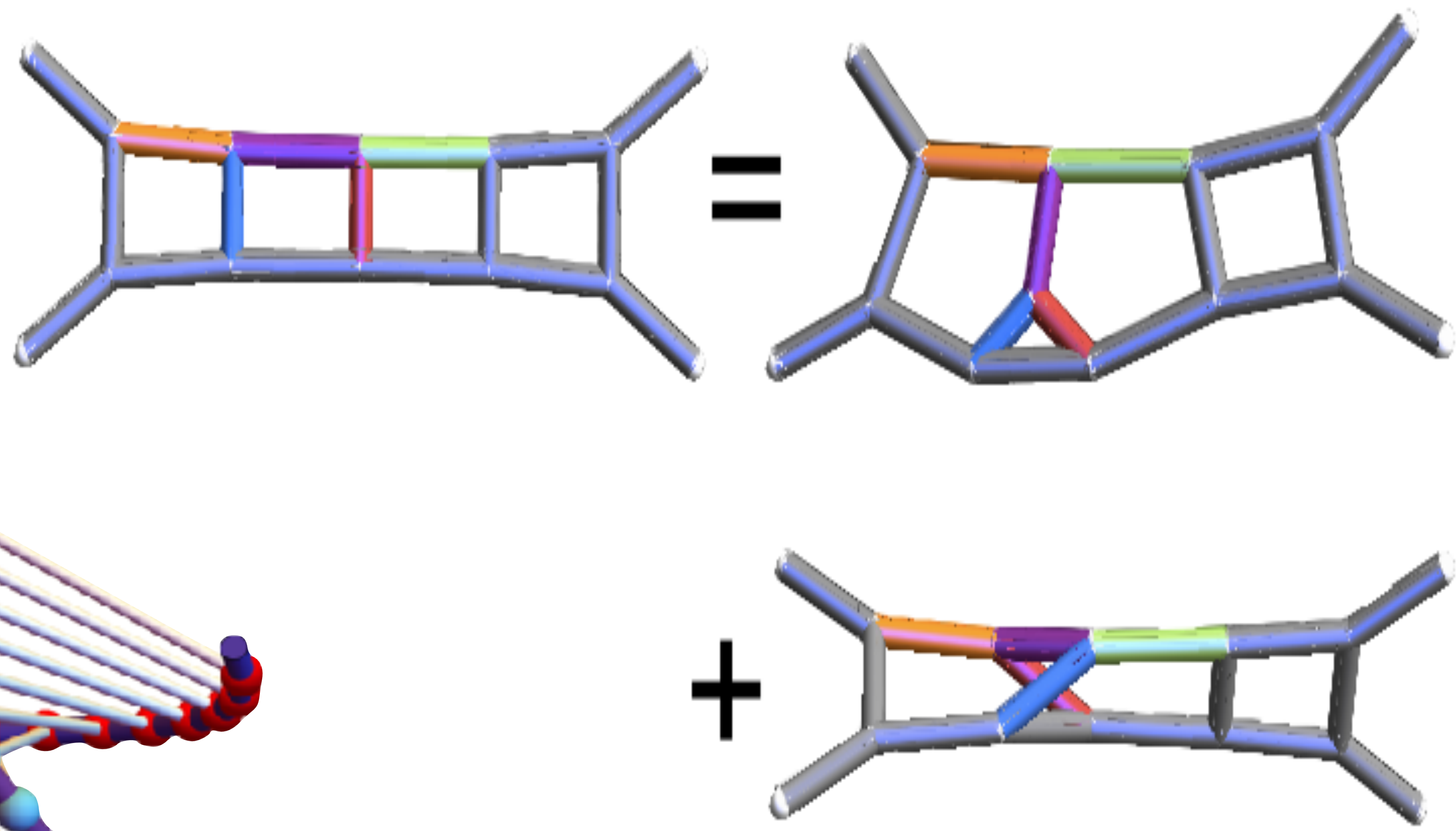
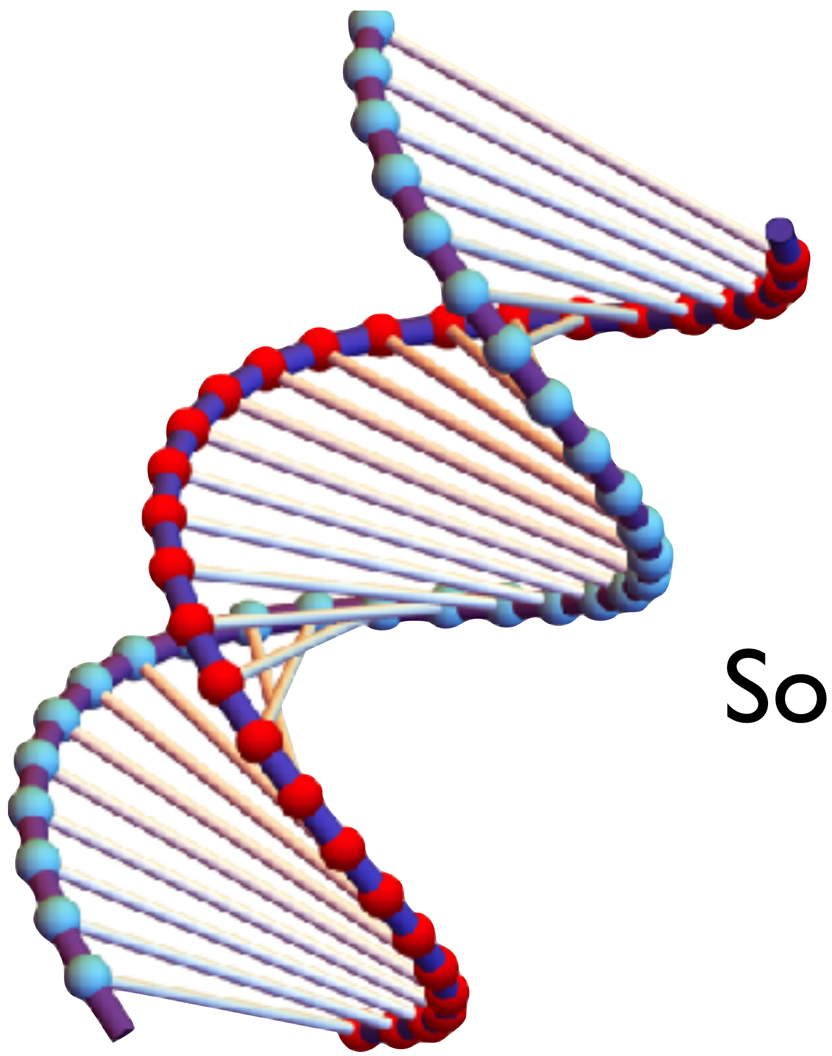


Bern, JJMC, Johansson ('08,'10)

Color and Kinematics dance together.



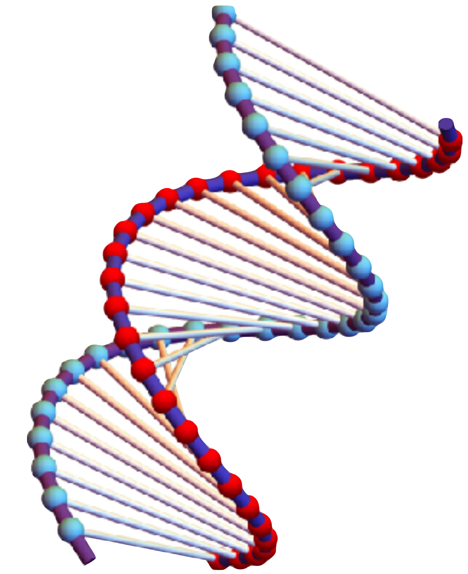


Solving Yang-Mills theories means
solving Gravity theories.

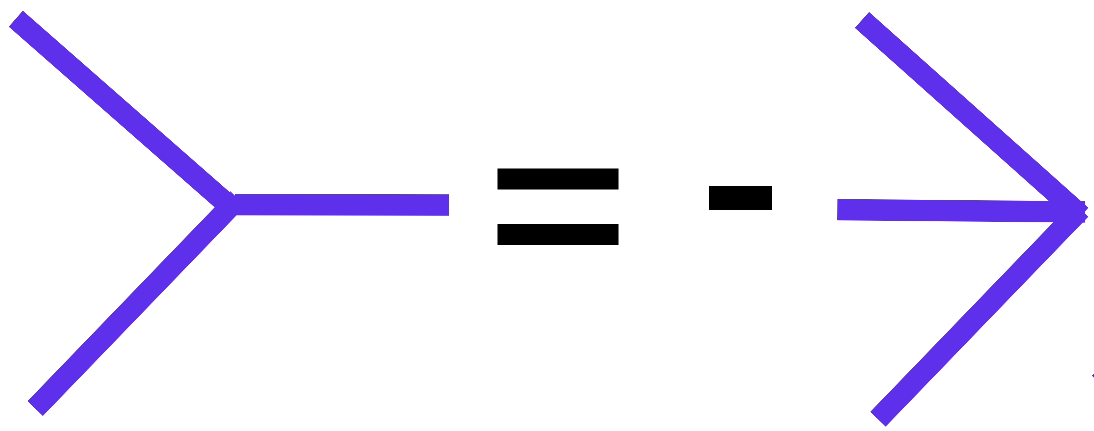
Bern, JJMC, Johansson ('08,'10)

Generic D-dimensional YM theories have a fascinating structure at tree-level

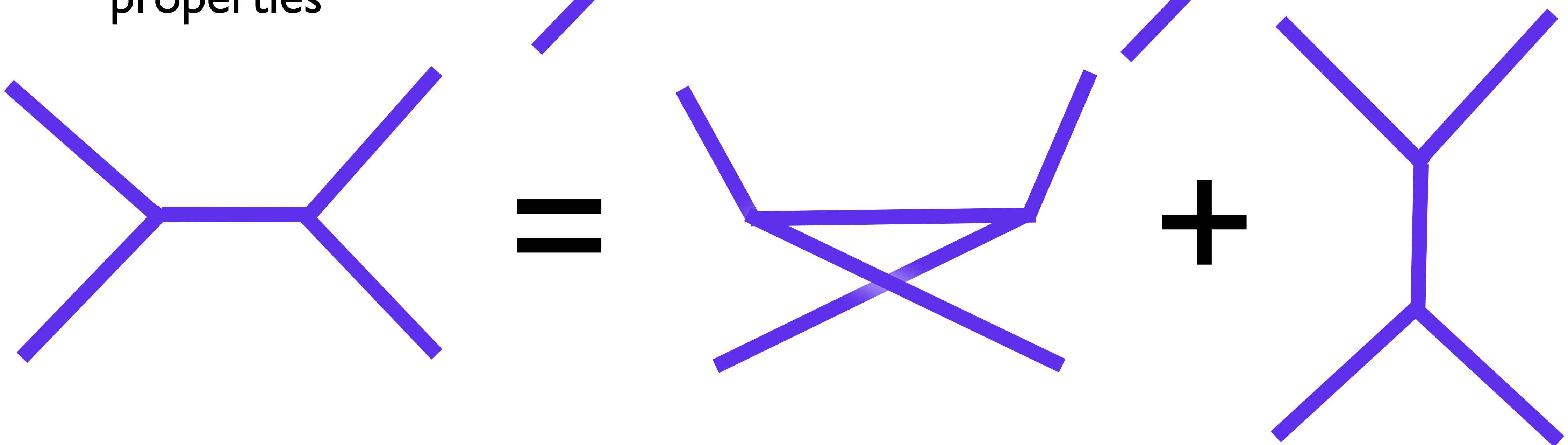
$$\mathcal{A}_m^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{c(\mathcal{G})n(\mathcal{G})}{D(\mathcal{G})}$$



Color factors and numerator factors satisfy similar lie algebra properties



Vertex Antisymmetry



Jacobi

Color-Kinematic Duality!

Bern, JJMC, Johansson ('08,'10)

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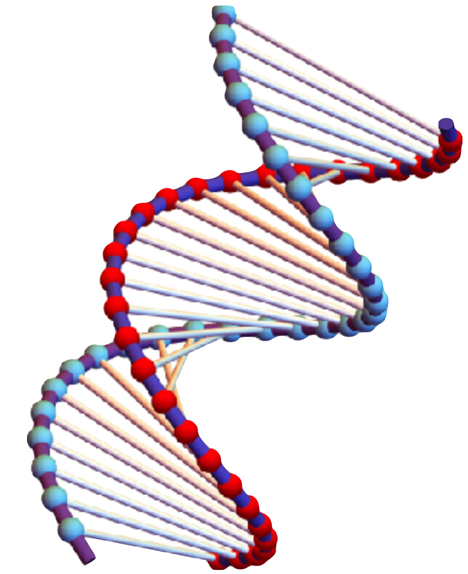
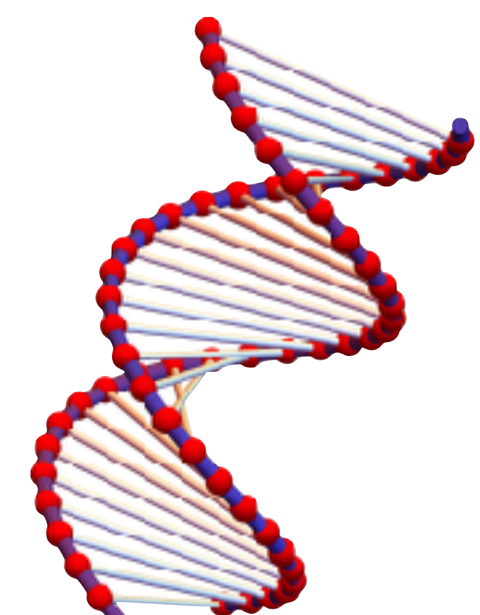
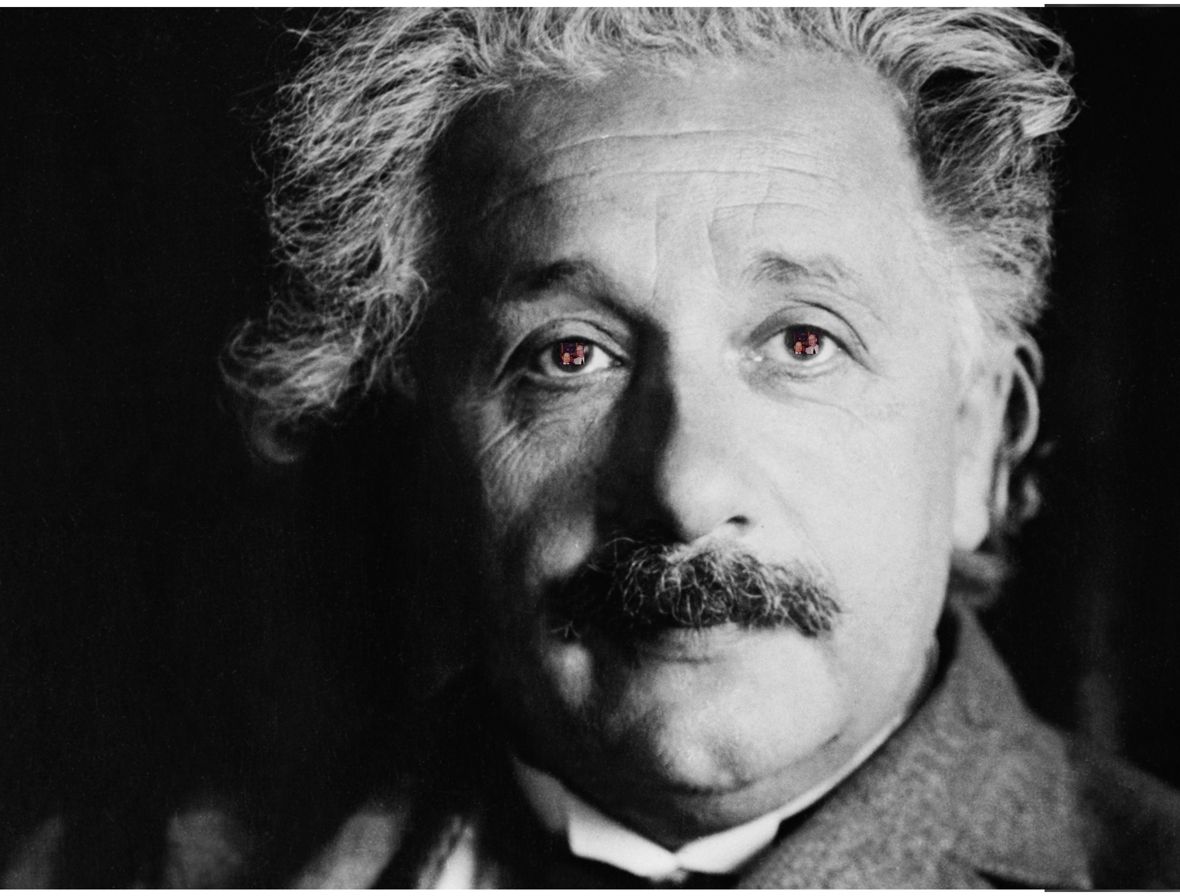


YM's Color-Kinematic duality makes manifest gravitational double copy structure:

$$-iM_n^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$



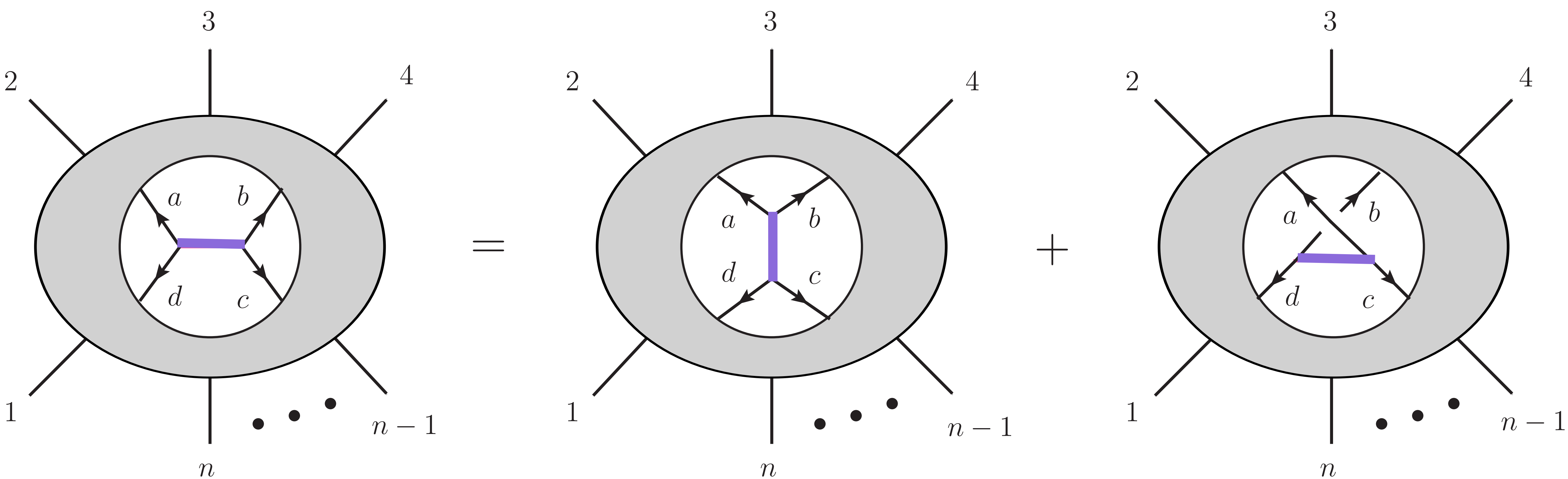



GR = YM^2

Valid multi-loop generalization?

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})c(\mathcal{G})}{D(\mathcal{G})}$$

CONJECTURE: for all graphs, can impose CK on every edge:



Consequence of unitarity: double copy structure holds.

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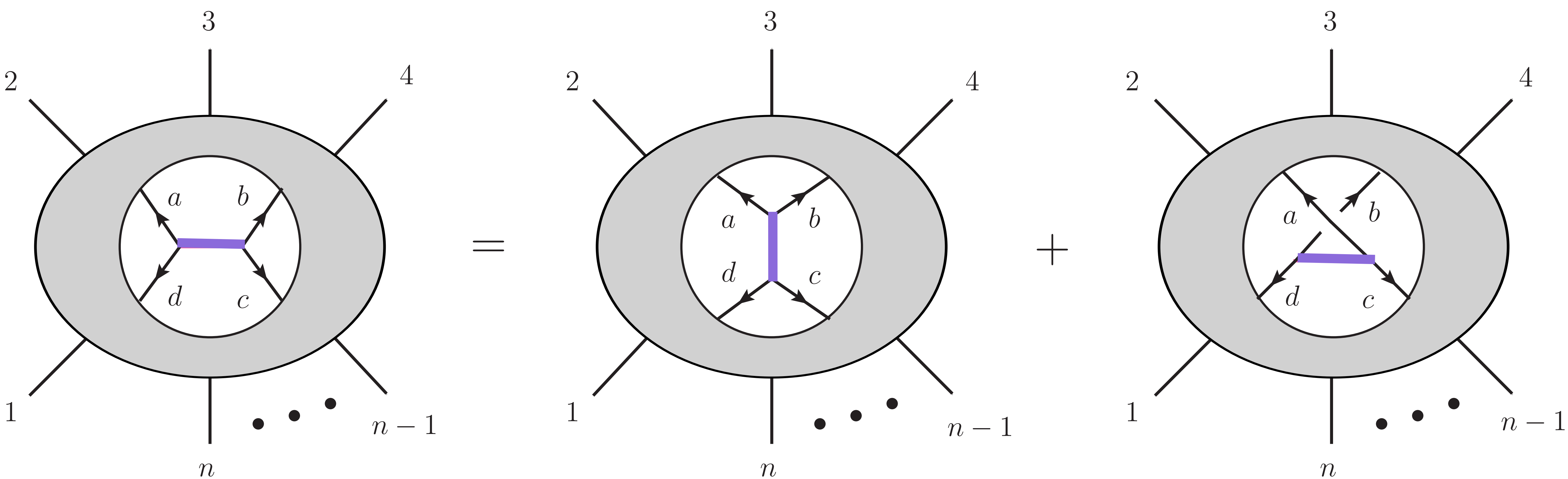


Consequence of unitarity: double copy structure holds.

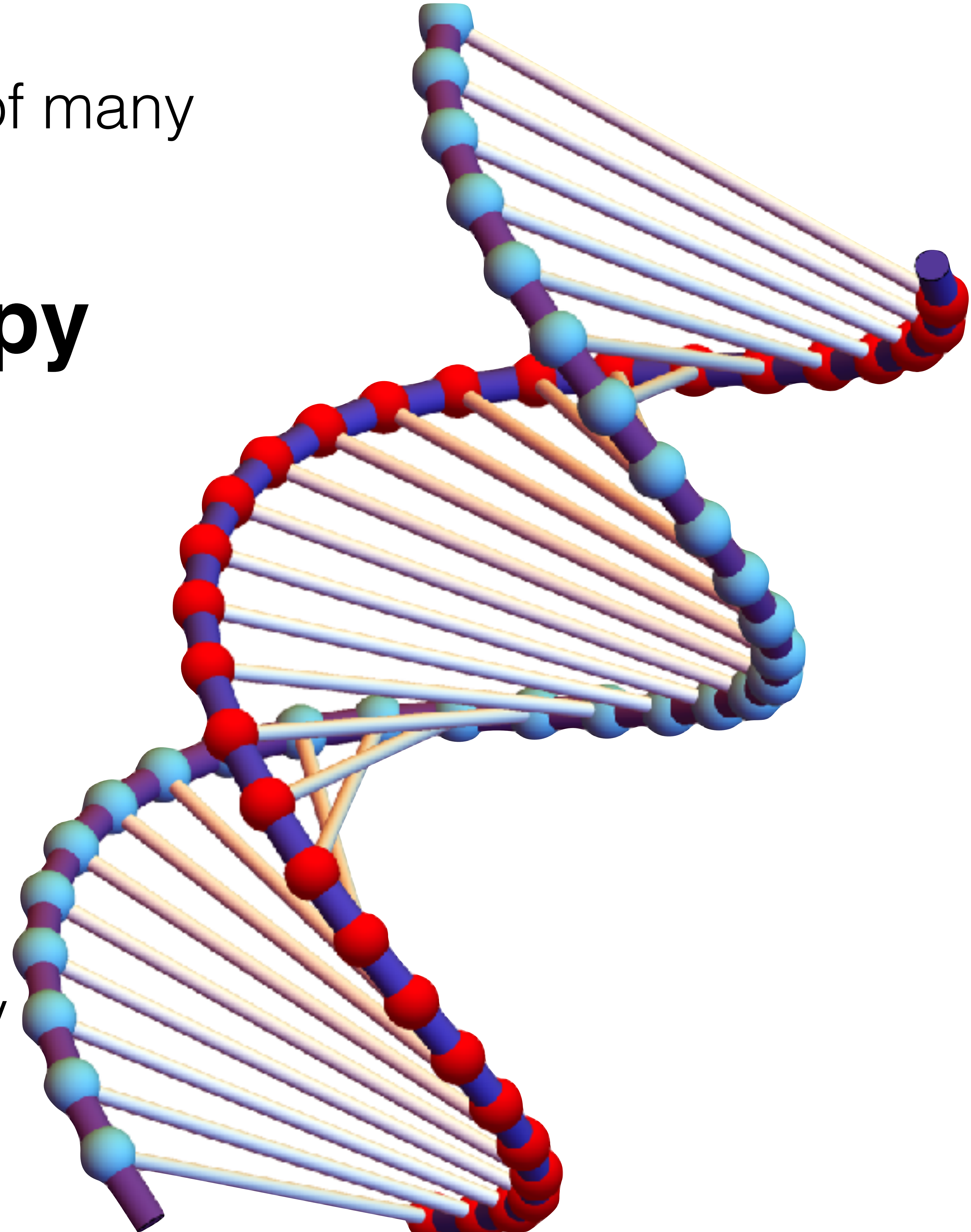
$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}^{\text{loop}} = \sum_{\mathcal{G}\in\text{cubic}} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S(\mathcal{G})} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$

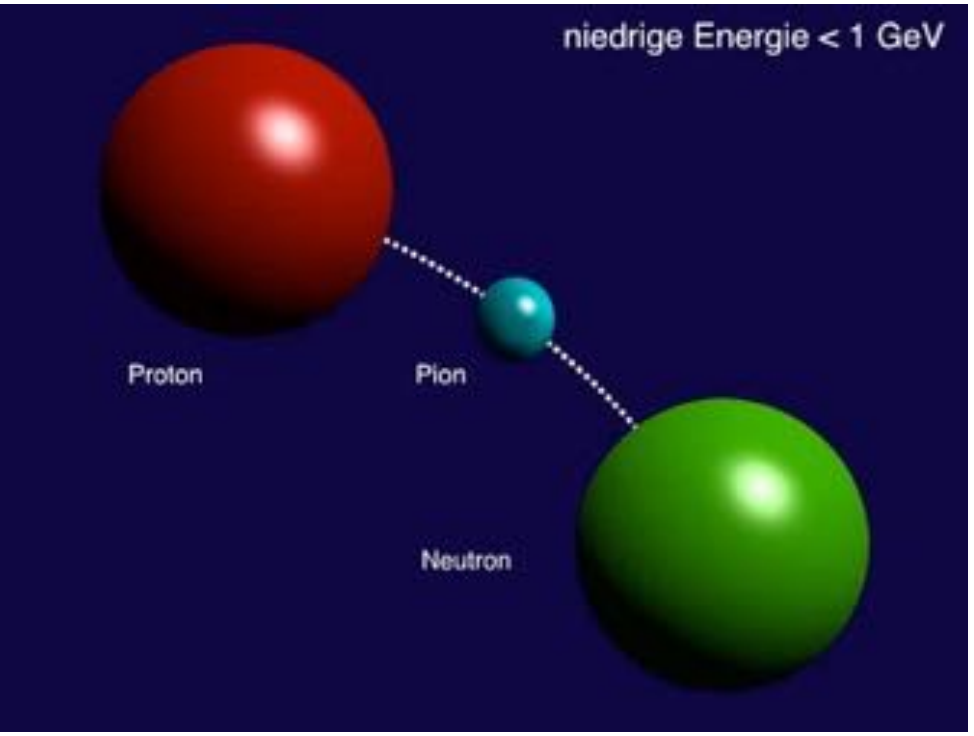
The scattering amplitudes of many relativistic theories admit a:

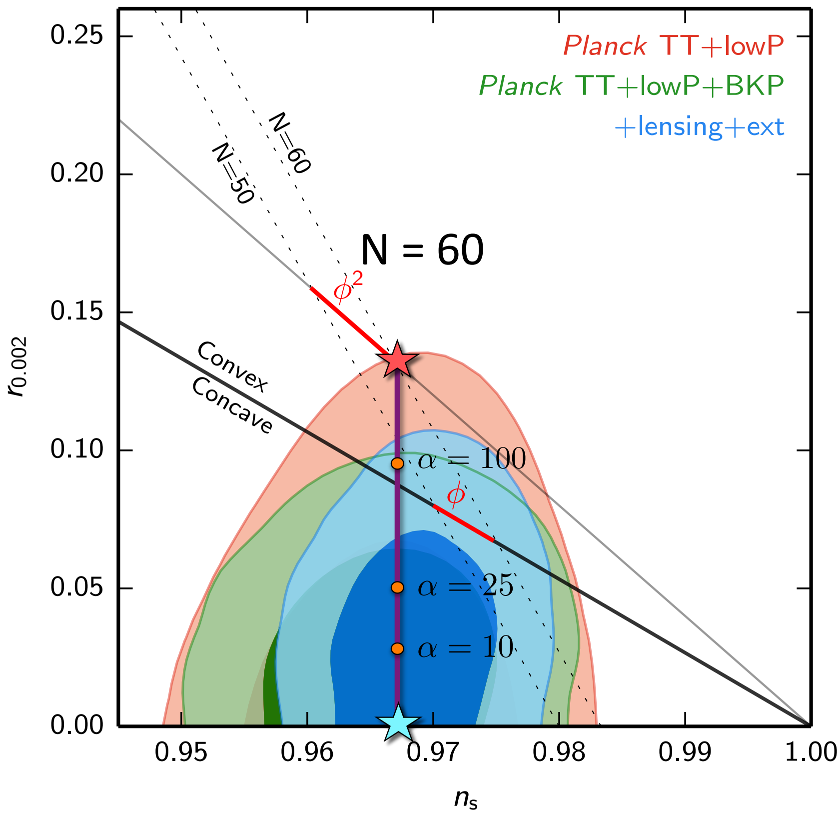
Double-copy
Numerator
Algebra

This points to previously hidden structure in many theories.

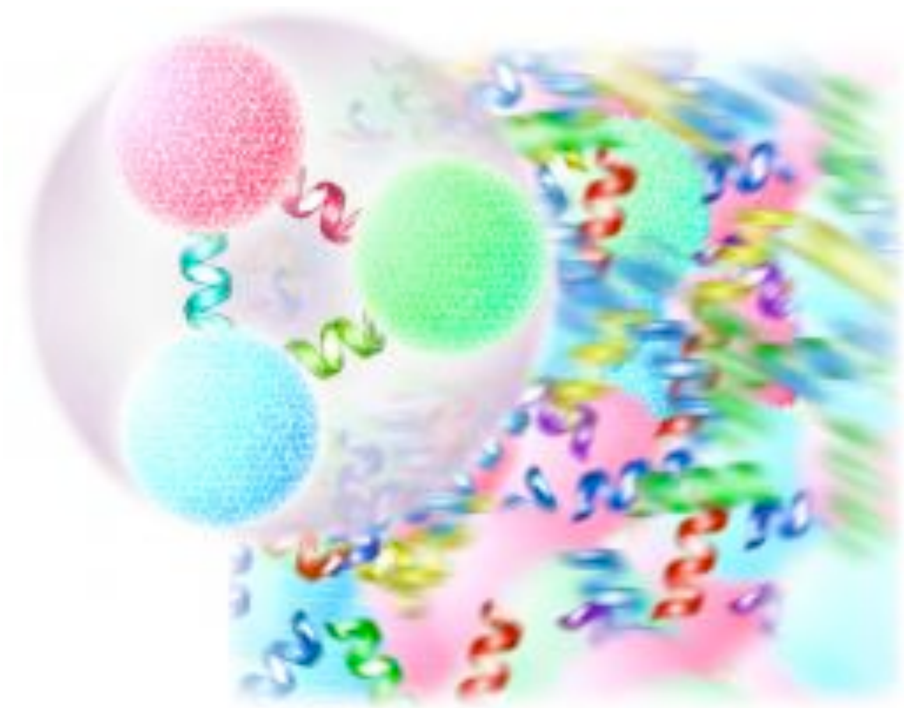
Structure yet to be generally understood at the level of the action.

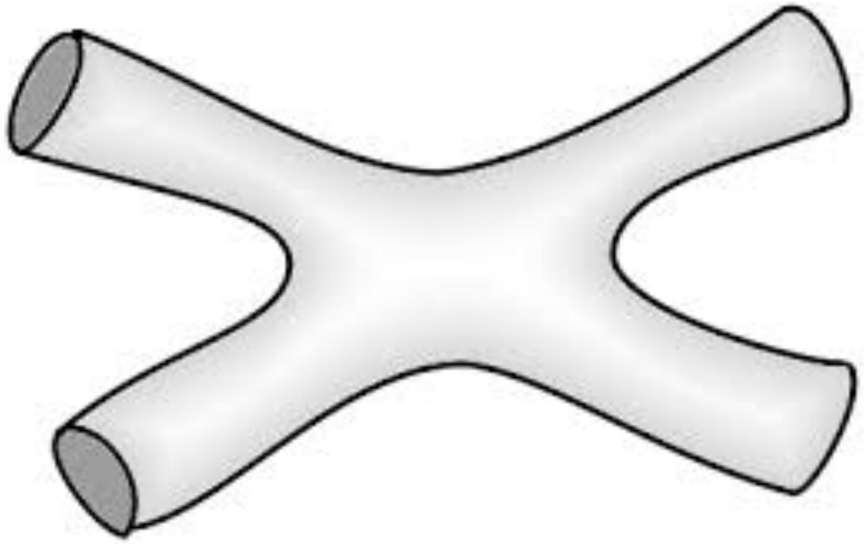






Many theories amplitudes are double copy!





Key Point: **MANY Theories are Double Copies**

Bi-Adjoint Scalar:	color	$\otimes$	color
Bern, de Freitas, Wong ('99); Bern, Dennen, Huang; Du, Feng, Fu; Bjerrum-Bohr, Damgaard, Monteiro, O'Connell			
(S)YM (…(S)QCD…):	color	$\otimes$	spin-1
BCJ ('08) Bjerrum-Bohr, Damgaard, Vanhove; Steiberger; Feng et al; Mafra, Schlotterer, ('08-'11); Johansson, Ochirov			
(S)Gr (…(S)Einstein-YM…):	spin-1	$\otimes$	spin-1
KLT('86); BCJ ('08); Chiodaroli, Gunaydin, Johansson, Roiban; Johansson, Ochirov; Johansson, Kälin, Mogull			
NLSM / Chiral Lagrangian:	"color"	$\otimes$	even-spin-0
Chen, Du '13 Cachazo, He, Yuan '14 Cheung, Shen '16			
(S)Born-Infeld:	spin-1	$\otimes$	even-spin-0
Cachazo, He, Yuan '14			
Special Galileon:	even-spin-0	$\otimes$	even-spin-0
Cachazo, He, Yuan '14 Cheung, Shen '16			
Open String:	α'	$\otimes$	spin-1
Broedel, Schlotterer, Stieberger			
Closed String:	spin-1	$\otimes$	α' corrected spin-1
Broedel, Schlotterer, Stieberger;			
Z-theory:	α'	$\otimes$	"color"
Broedel, Schlotterer, Stieberger; JJMC, Mafra, Schlotterer			

Key Point: **MANY Theories are Double Copies**

Ingredients:

α'

color

spin 0,1/2,1

For all these theories:

(S)YM
(...(S)QCD...)

Bi-Adjoint Scalar

(S)Gr
(...(S)Einstein-YM...)

NLSM

(S)Born-Infeld

Special Galileon

Z-theory

Open String

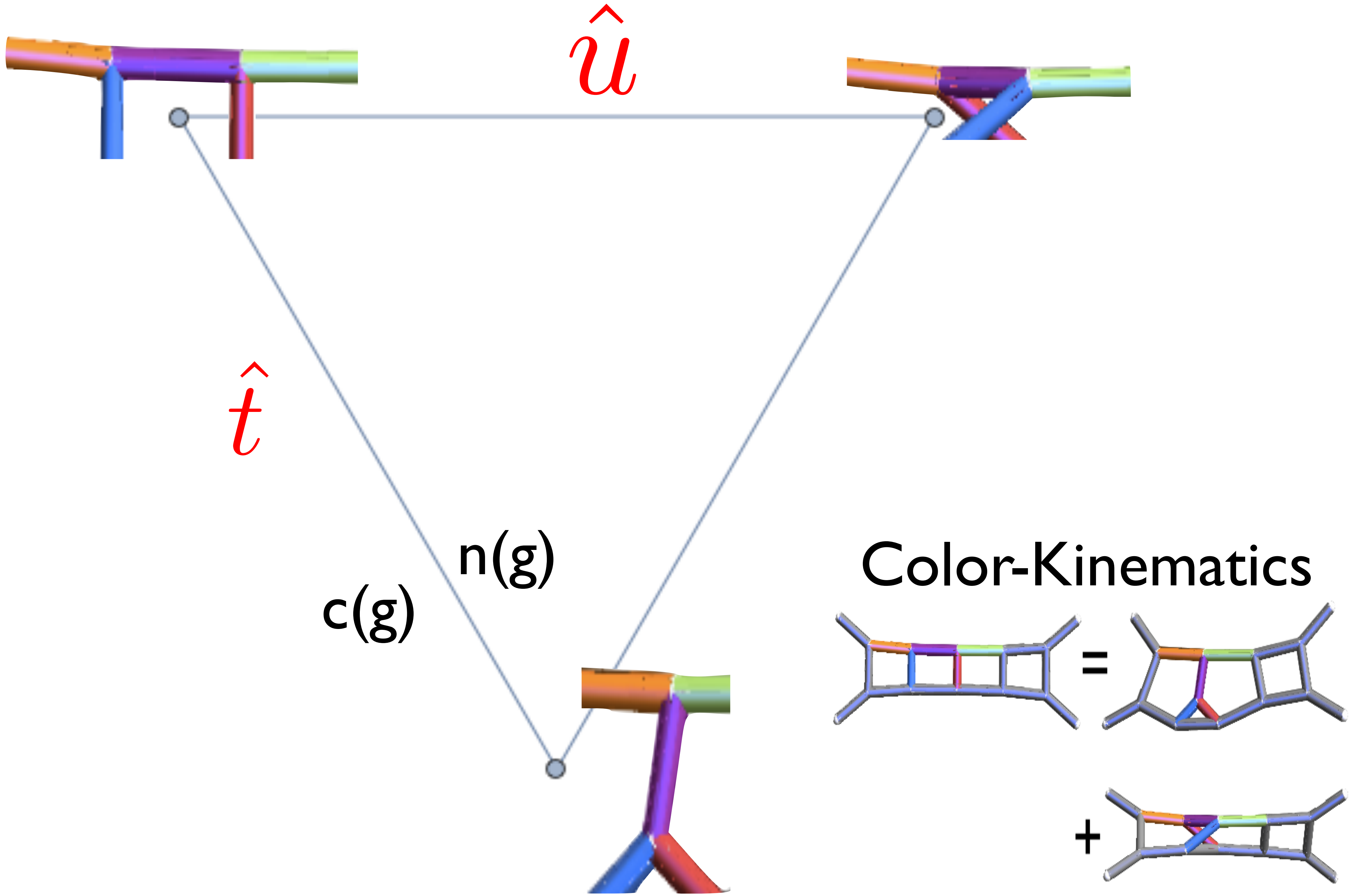
Closed String

a geometric guide to color-kinematics

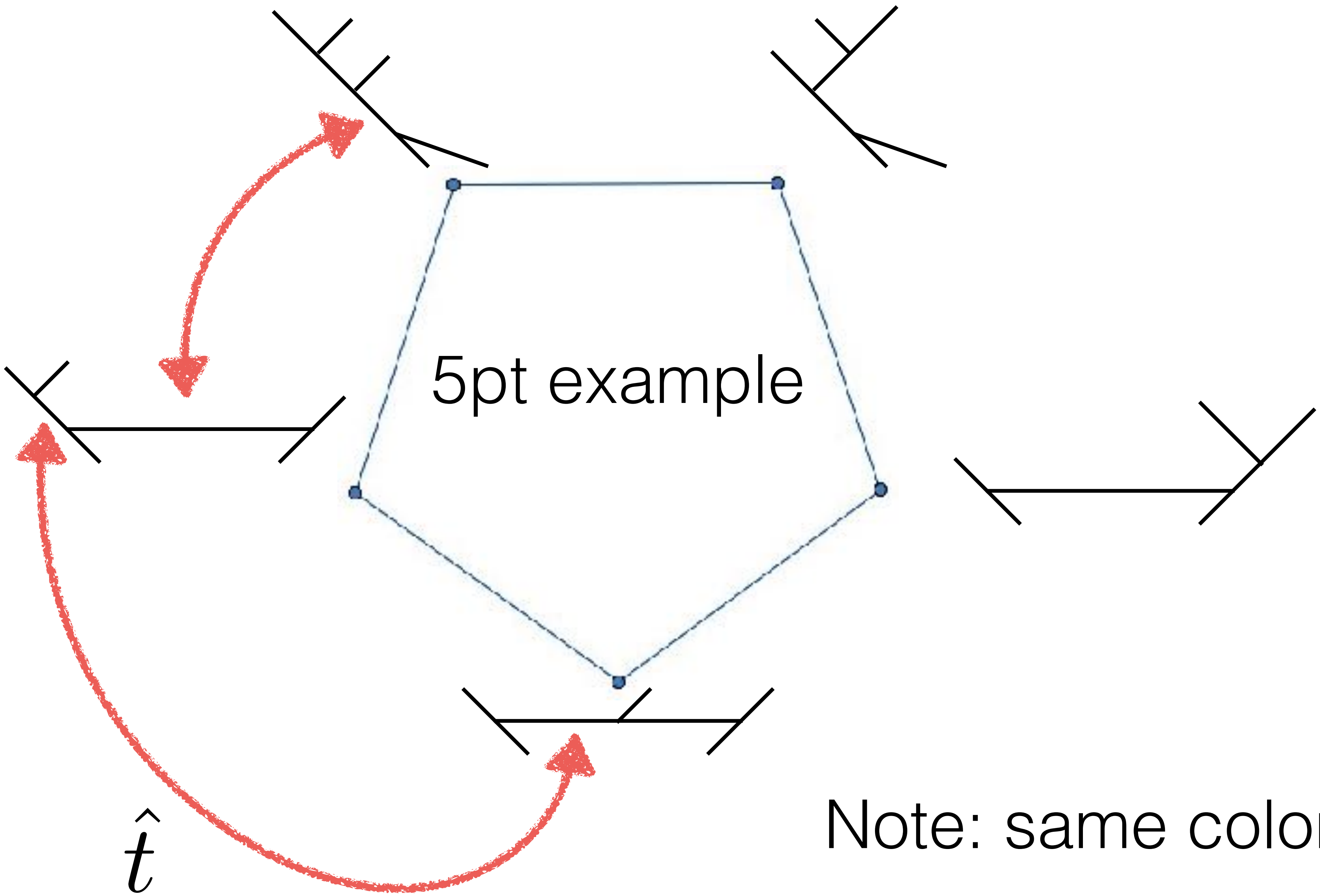
Physics = Geometry

(the best polytopes are graphs of graphs!)

Convenient language: graphs of graphs



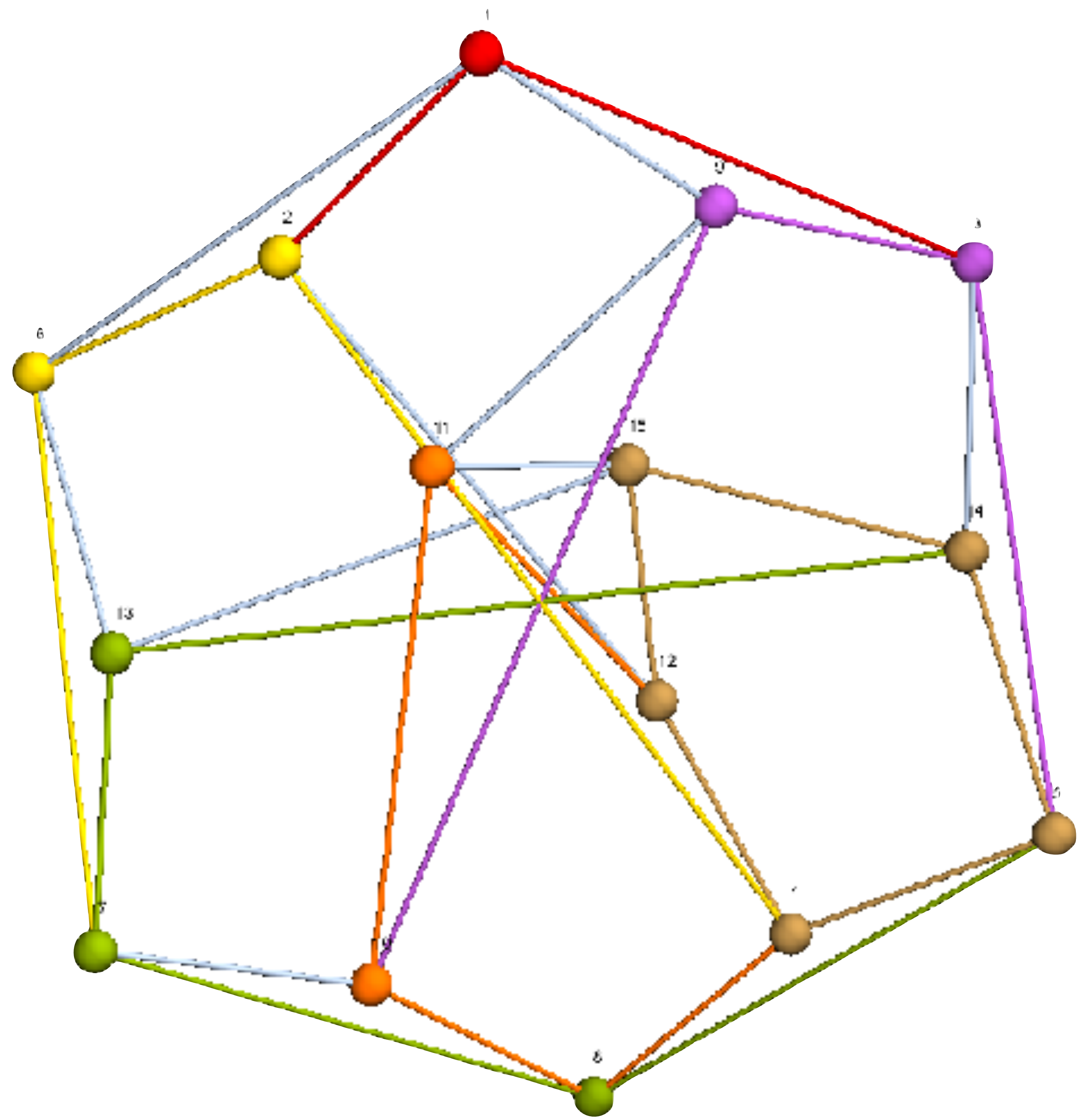
Graphs contributing to an ordered tree (color-stripped), generate the 1-skeleton of **Stasheff polytopes** joined only by $\hat{t}$



Note: same color-order!

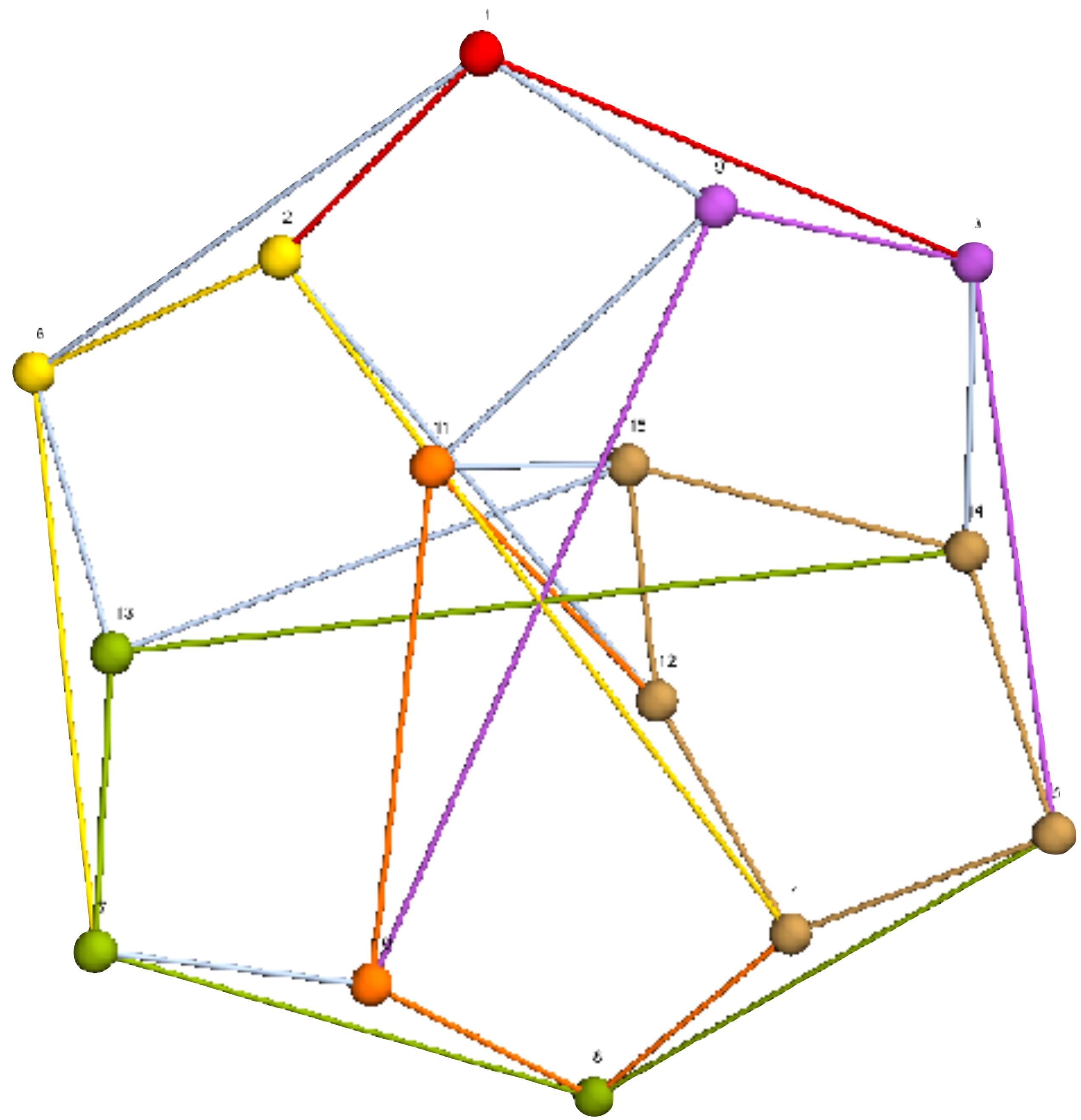
(these polytopes are also called **associahedra**)

You might think you need (m-2)! of these color-ordered amplitudes to capture everything because this is what is required to touch every vertex at least once:



JJMC

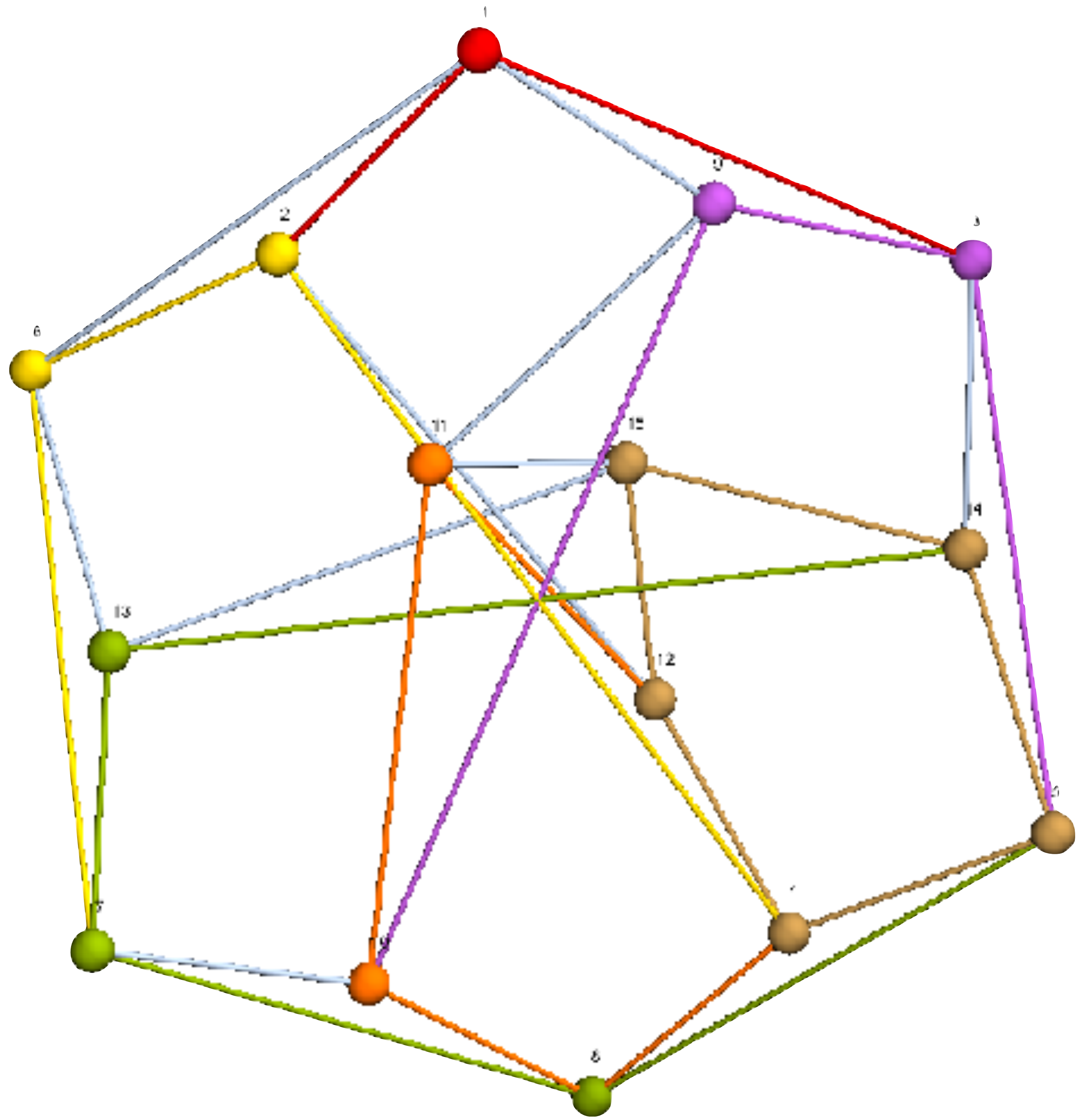
You might think you need (m-2)! of these color-ordered amplitudes to capture everything because this is what is required to touch every vertex at least once:



In fact, such a choice is the KK-basis, proven sufficient by Del Duca, Dixon, and Maltoni

JJMC

But notice, because of color-kinematics, only (m-2)! nodes are needed to specify both the color factors and numerator factors of everyone



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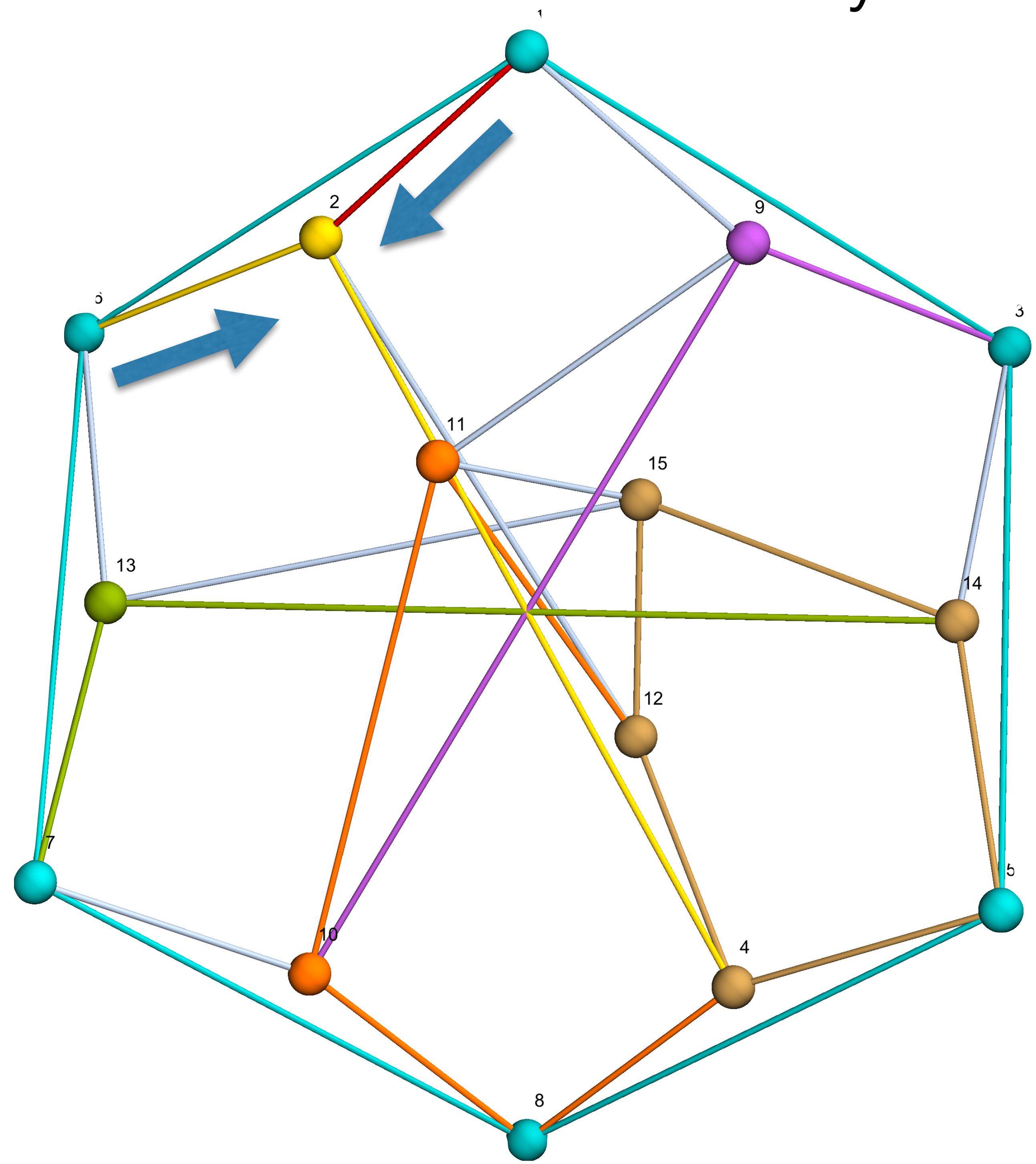

JJMC

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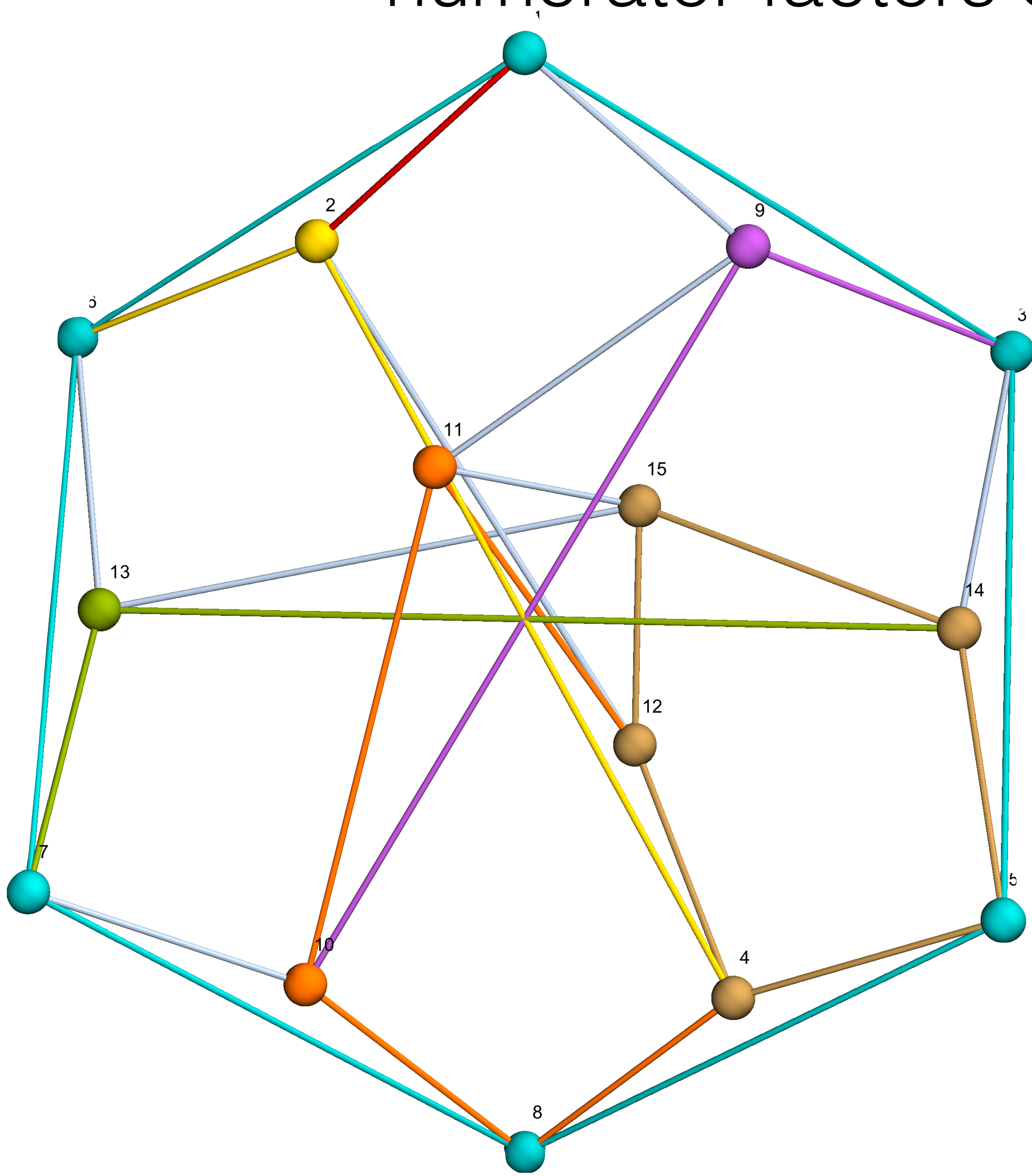
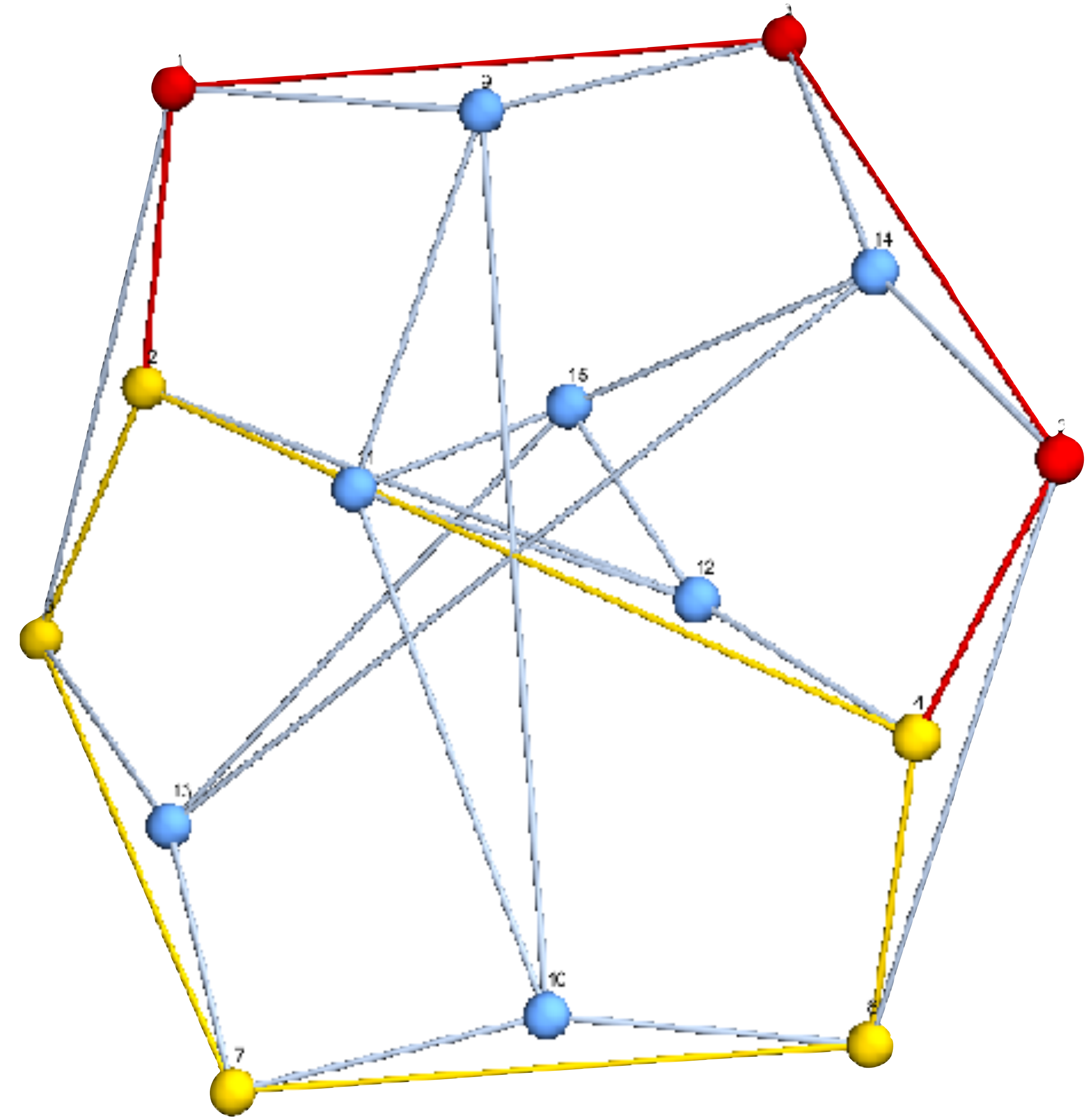
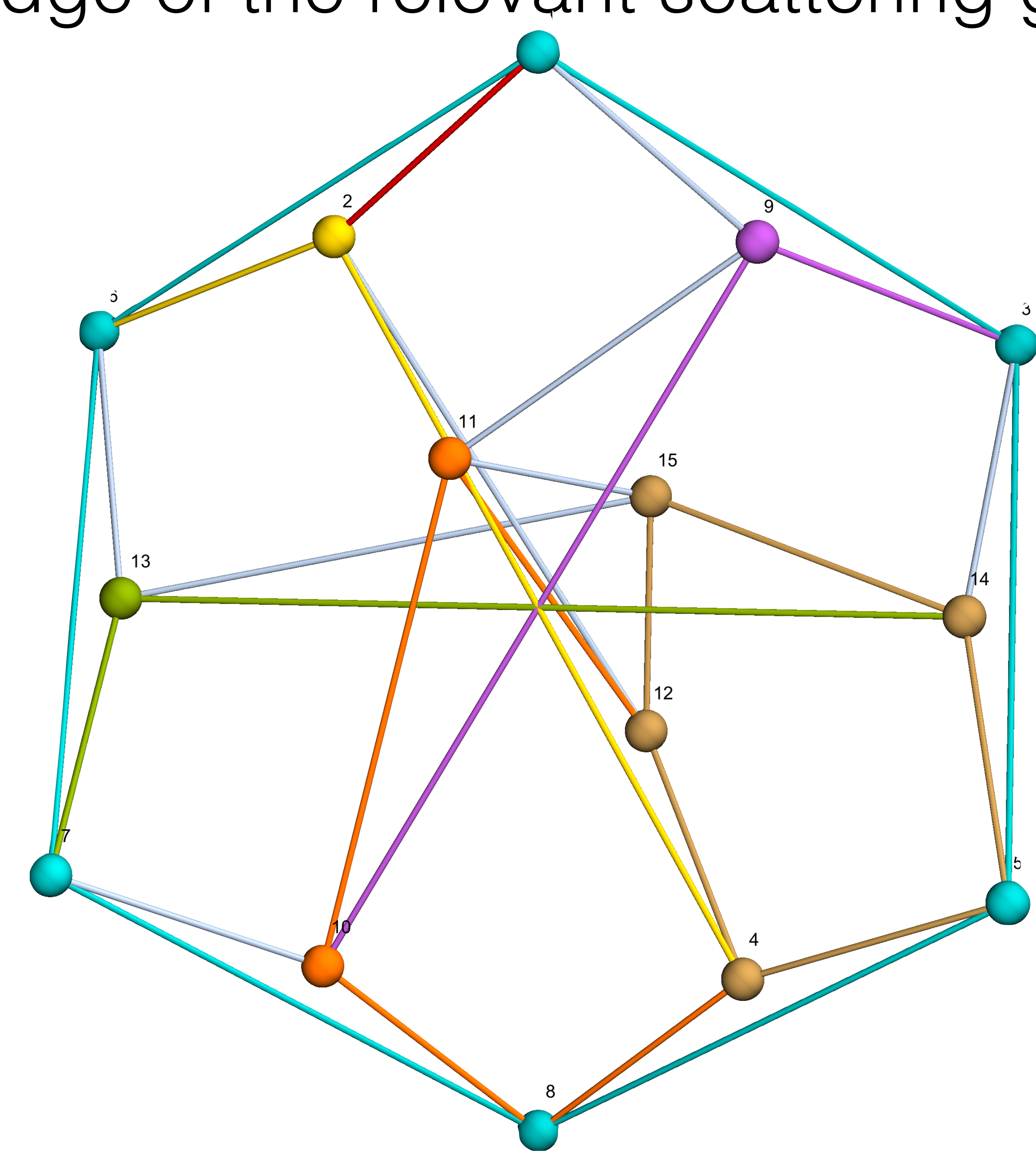


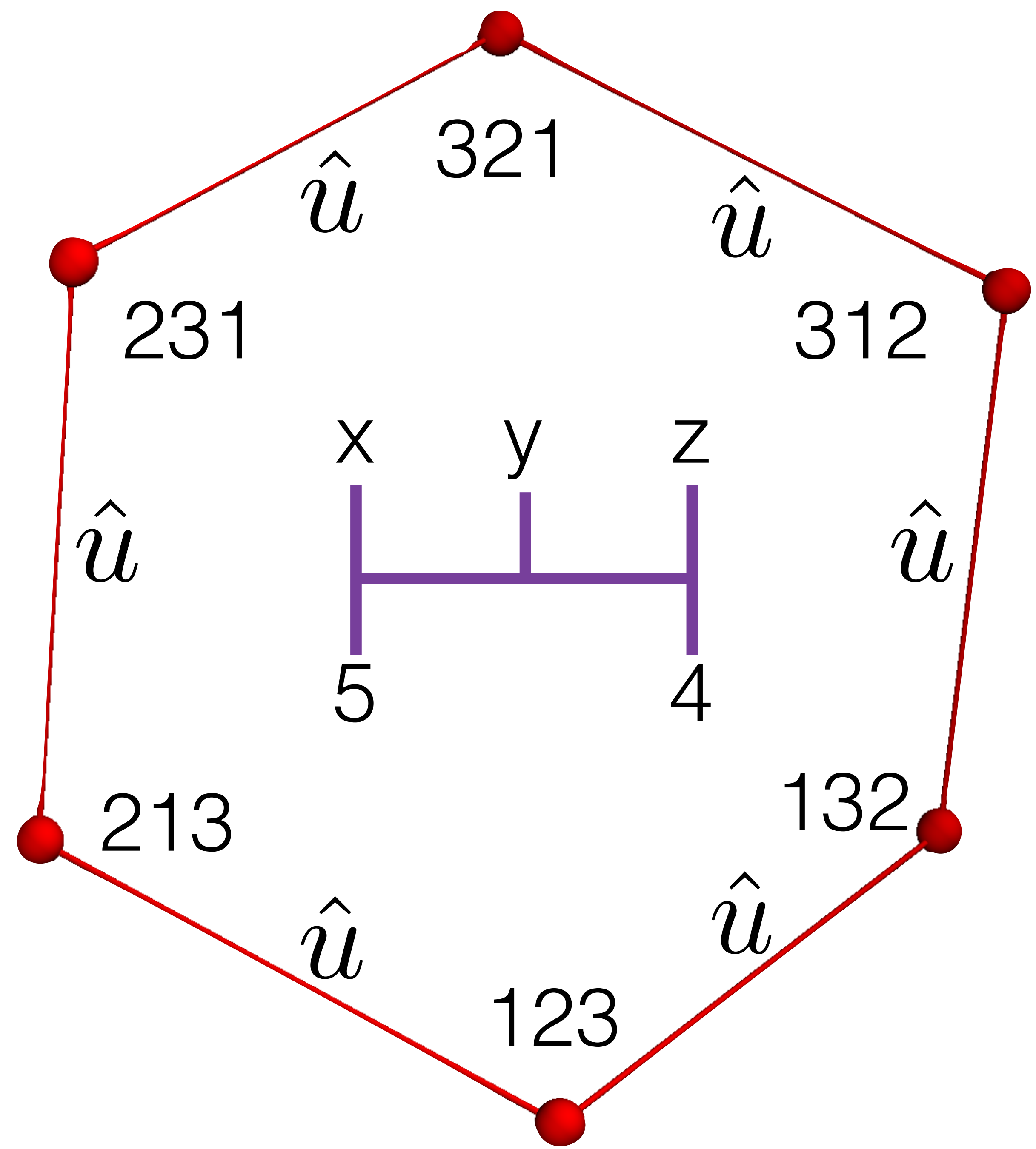


This reduces the set of necessary color-ordered amplitudes (associahedra) to (m-3)! : “BCJ” relations

JJMC

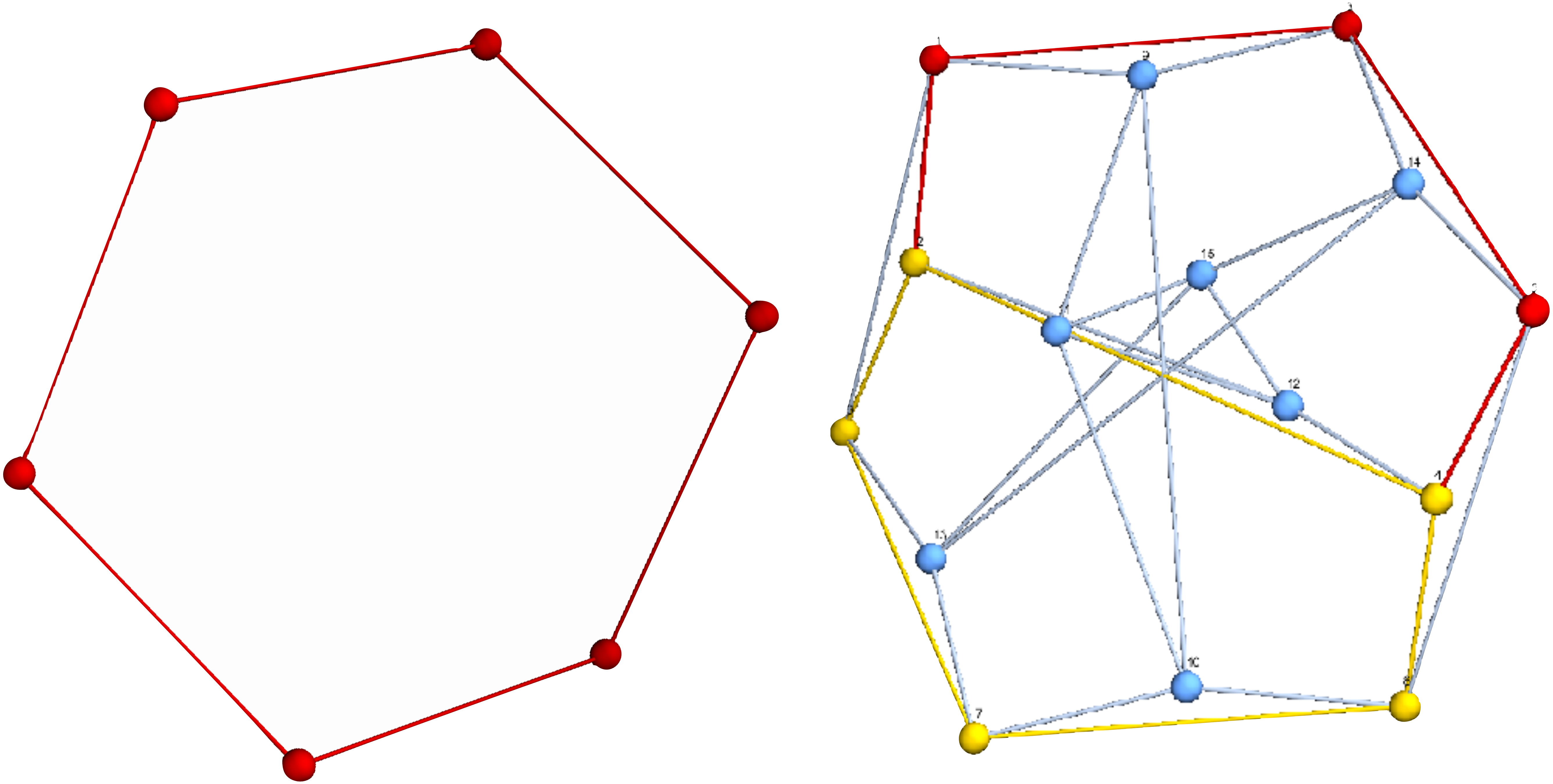
At every multiplicity the masters can be chosen to form the 1-skeleton of a polytope related by $\hat{u}$ on every internal edge of the relevant scattering graphs





(these polytopes are called **permutahedra**)

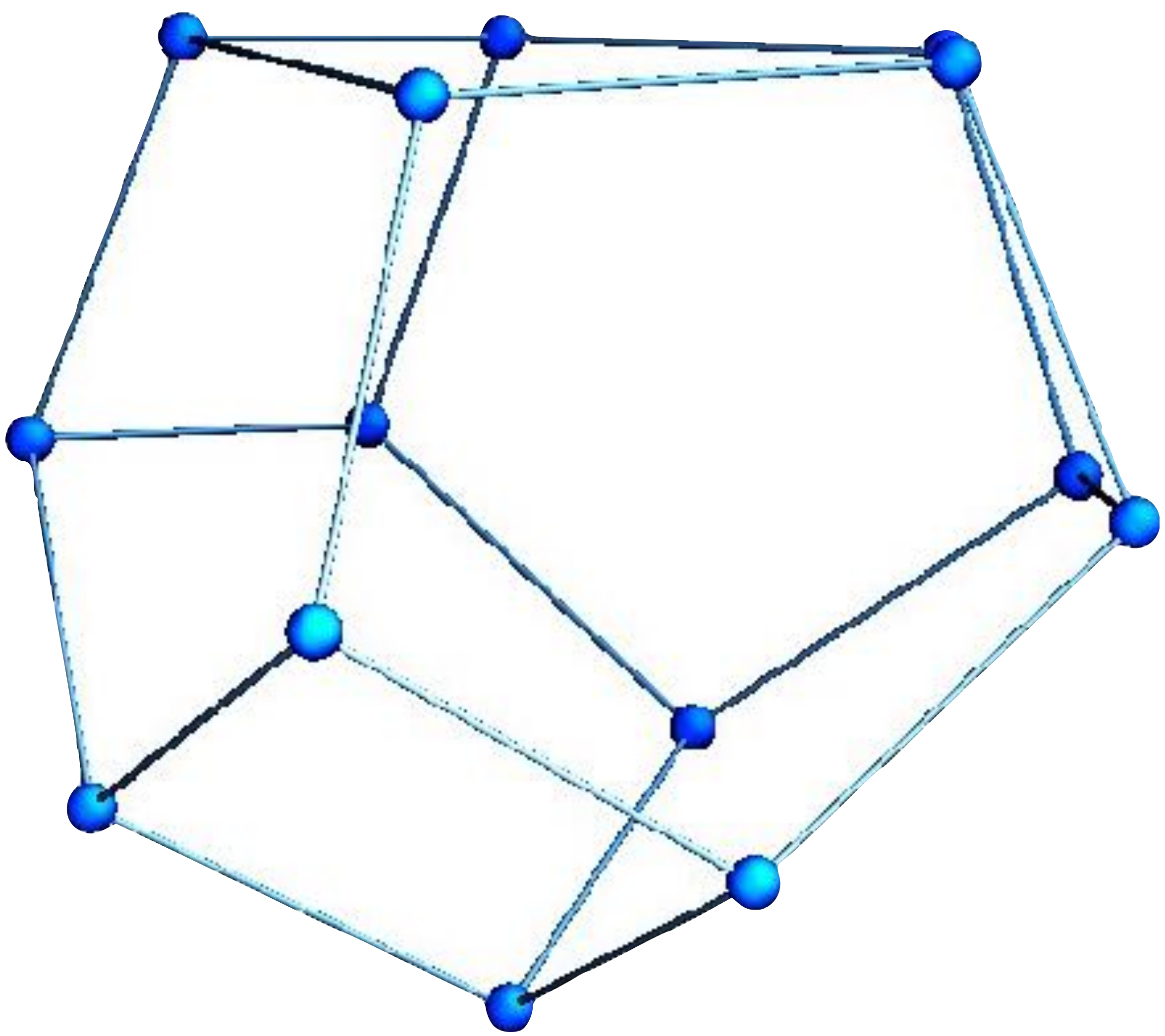
Can linearly solve for the (m-2)! numerators of the masters in terms of the (m-3)! “BCJ” independent color-ordered amplitudes. In fact you get (m-3)! numerators in terms of the color-ordered amplitudes and (m-3)(m-3)! free functions.



(generalized gauge freedom)

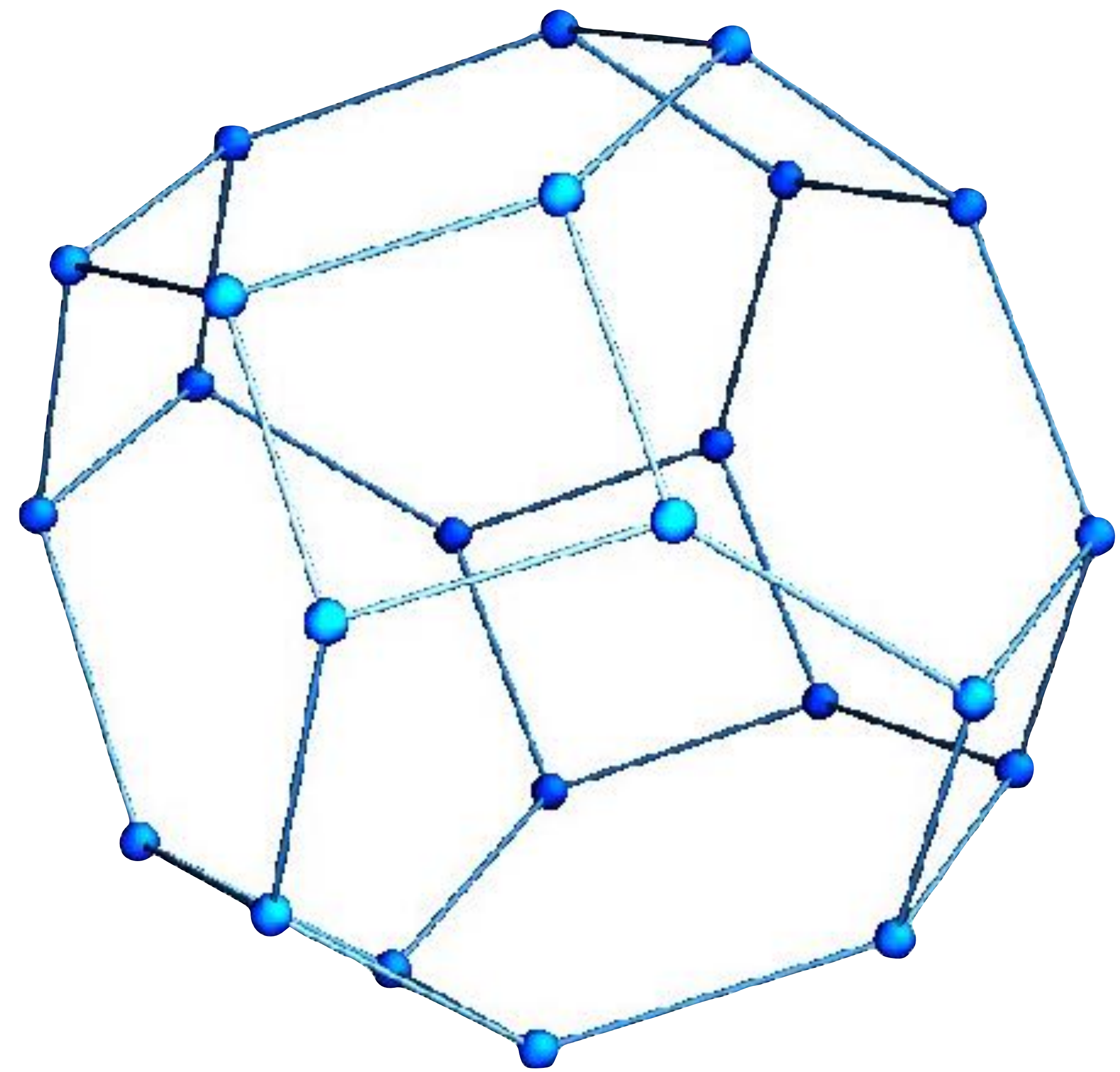
Building blocks at 6-points:

color-ordered amplitude



associahedron

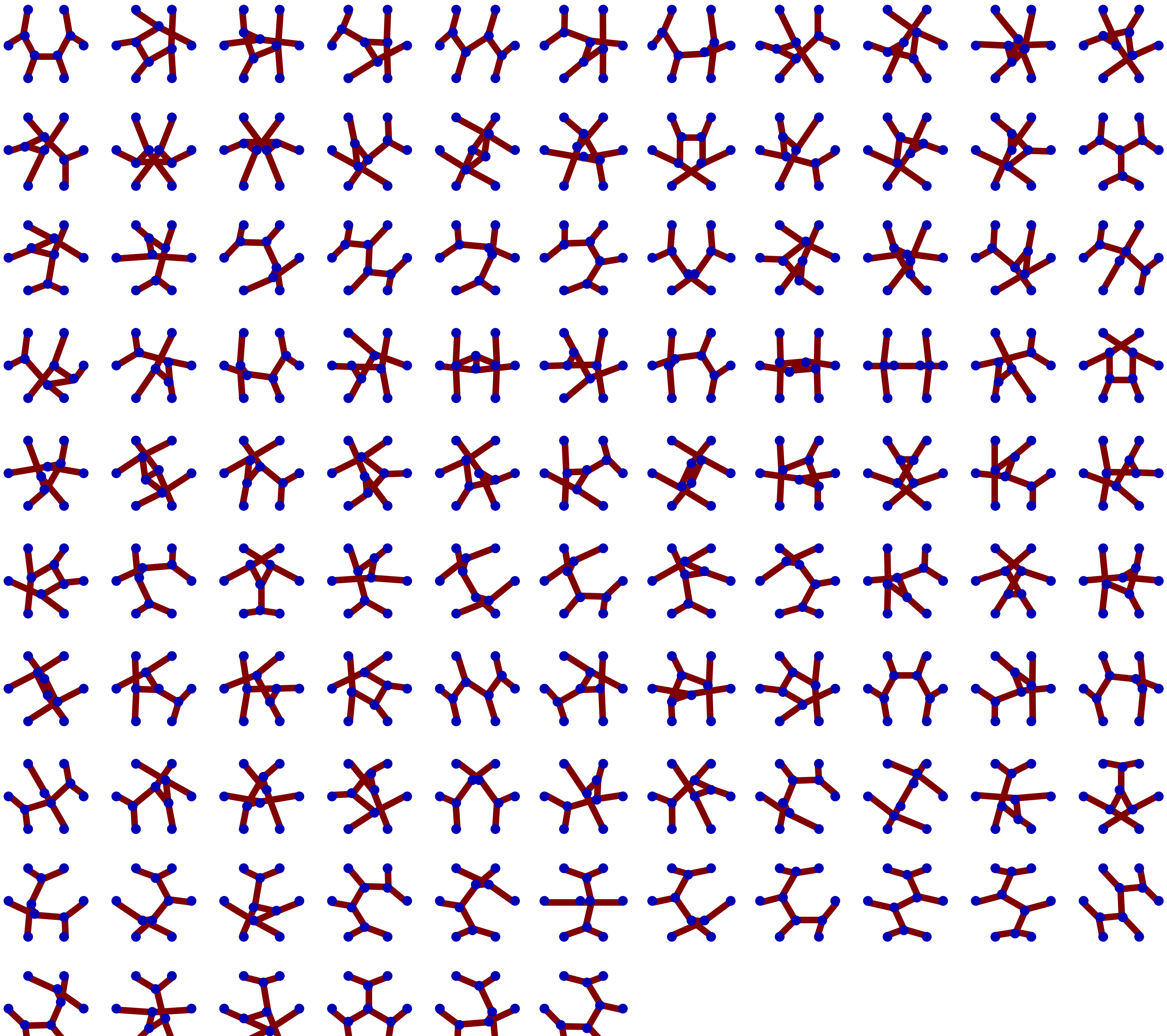
set of masters

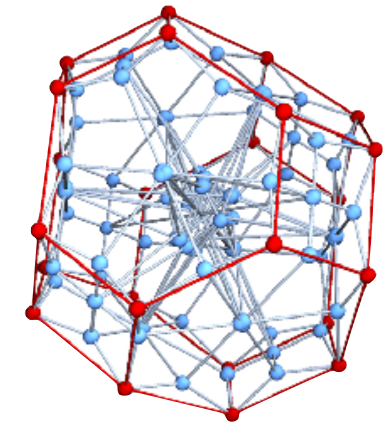


permutahedron

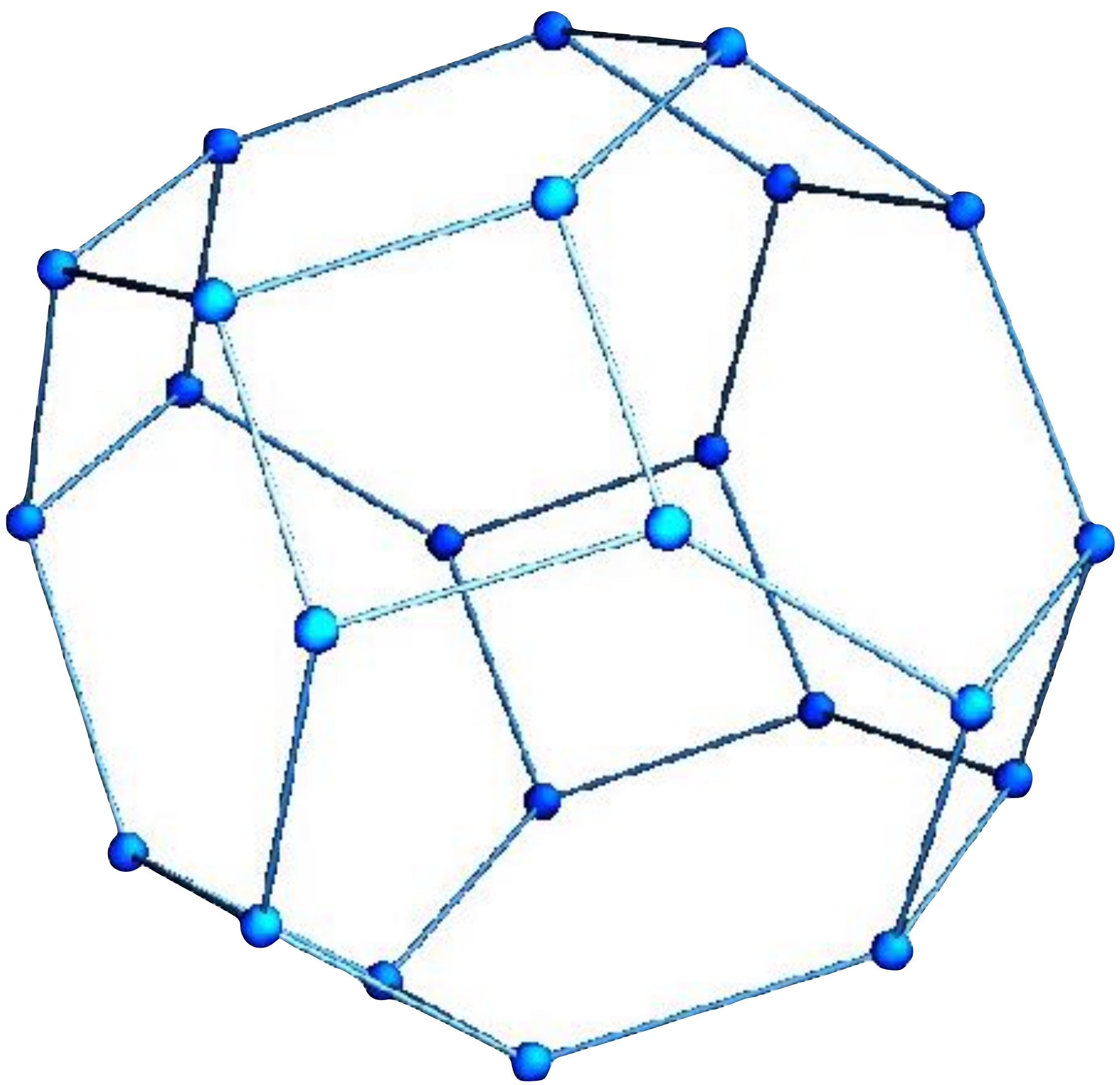
JJMC

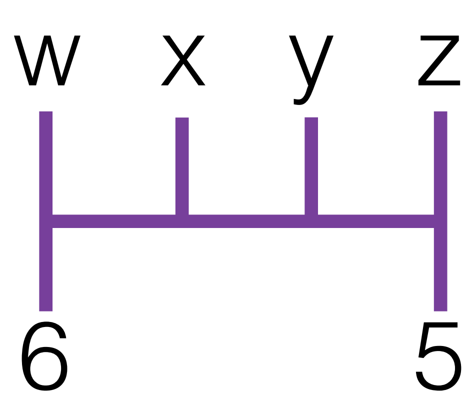
105
cubic graphs at 6 pt



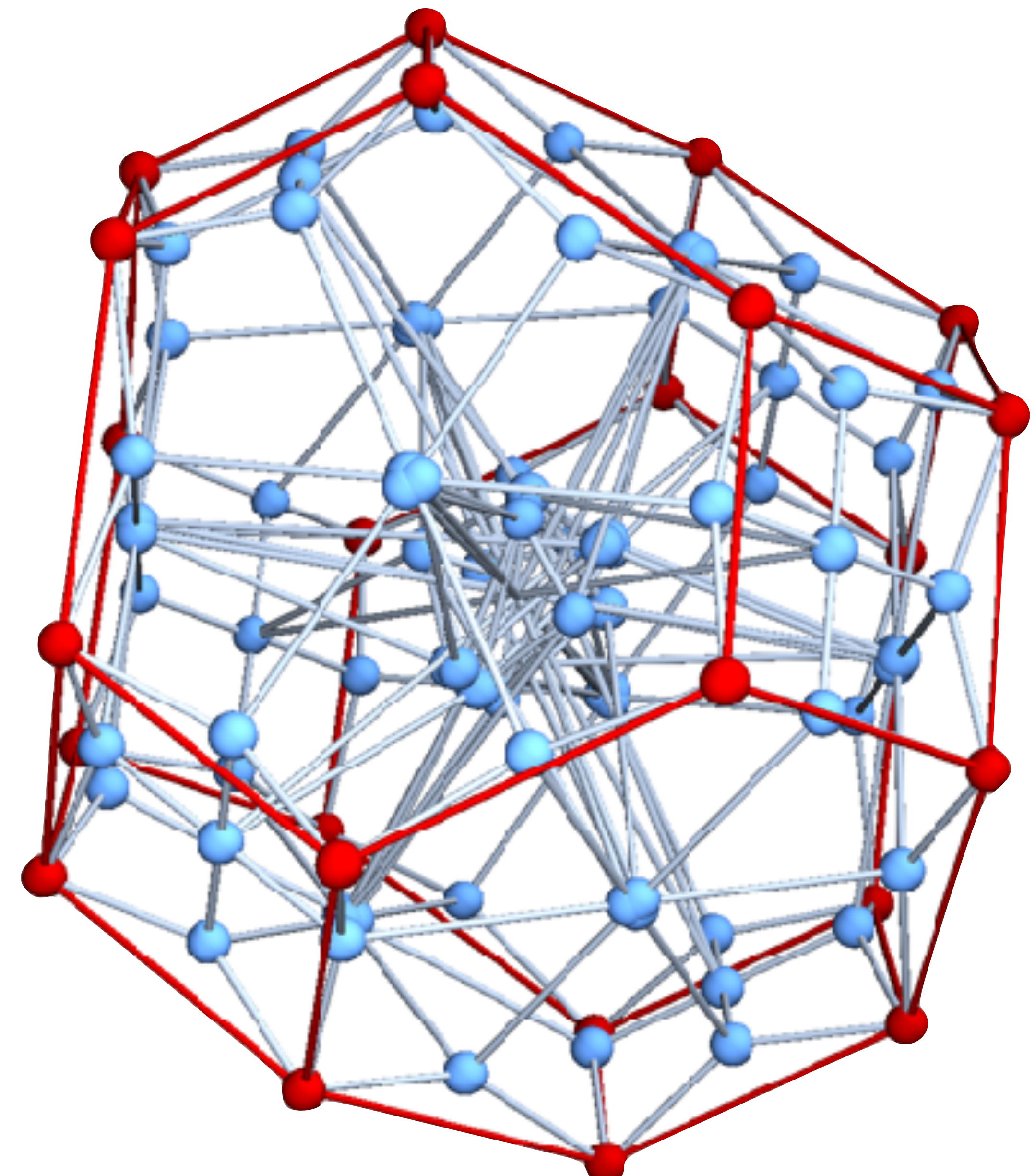


set of masters

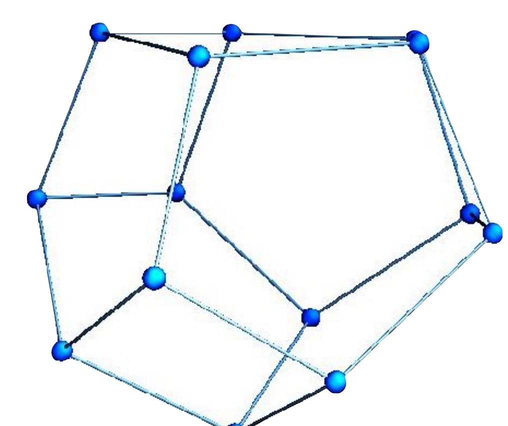




full amplitude

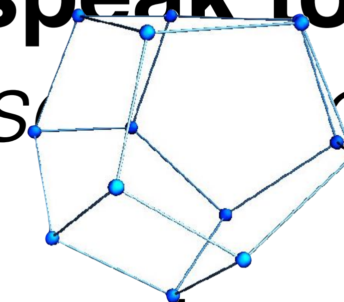


masters fixed by 6

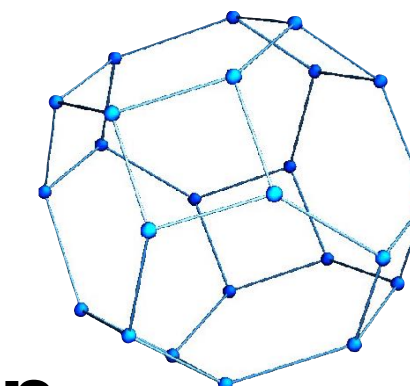


TREE-LEVEL SUMMARY

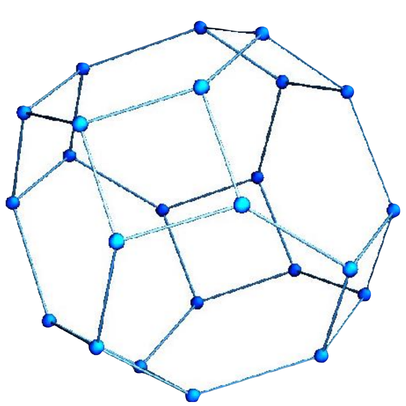
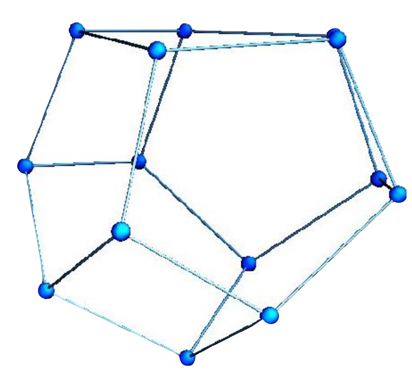
1. **Gauge invariant building blocks that speak to the theory:** color-ordered amplitudes, *associahedra*



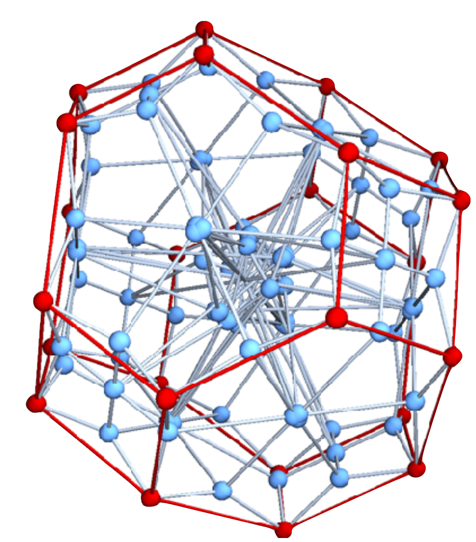
2. **CK means only need to specify the boundary data**: the master graphs, given by the relevant *permutahedron*



3. **Can solve for the *full amplitude efficiently* in terms of the (n-3)! independent *associahedra***

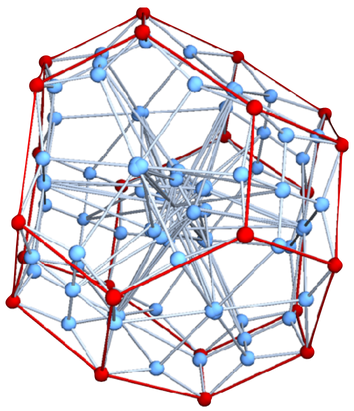
 = f()

(linear)



physics <—> geometry

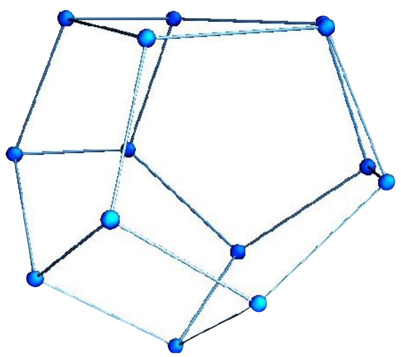
Full YM:



color $\otimes$ spin-1

$$\mathcal{A}_m^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{c(\mathcal{G})\,n(\mathcal{G})}{D(\mathcal{G})}$$

color-stripped YM

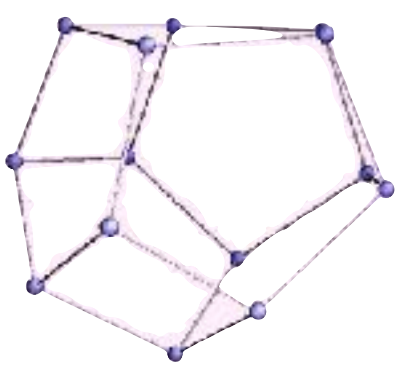


$$\mathbf{A}_{\mathbf{m}}^{\text{tree}}(\rho) = \sum_{\mathcal{G}\in\rho} \frac{\mathbf{n}(\mathcal{G})}{\mathbf{D}(\mathcal{G})}$$

(*same* as kinematic-stripped gravity

$$-iM_n^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$)

kinematic-stripped YM



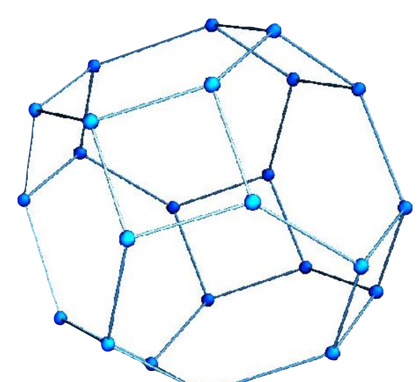
$$\mathbf{C}_{\mathbf{m}}^{\text{tree}}(\rho) = \sum_{\mathcal{G}\in\rho} \frac{\mathbf{c}(\mathcal{G})}{\mathbf{D}(\mathcal{G})}$$

(*same* as color-stripped Bi-Adjoint Scalar

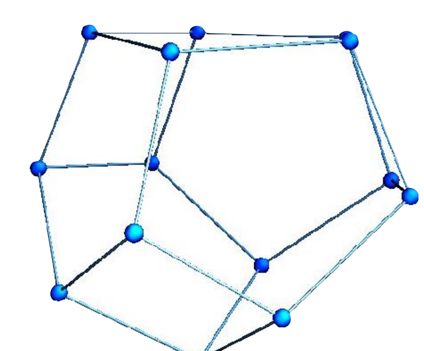
$$\mathcal{C}_{\mathbf{m}}^{\text{tree}}(\rho) = \sum_{\mathcal{G}\in\text{cubic}} \frac{\mathbf{c}(\mathcal{G})\tilde{\mathbf{c}}(\mathcal{G})}{\mathbf{D}(\mathcal{G})}$$)

Can (pseudo) invert:

$$\mathbf{n}(\mathcal{G}) = \sum_{\rho} \mathbf{D}(\mathcal{G}|\rho)\,\mathbf{A}(\rho)$$



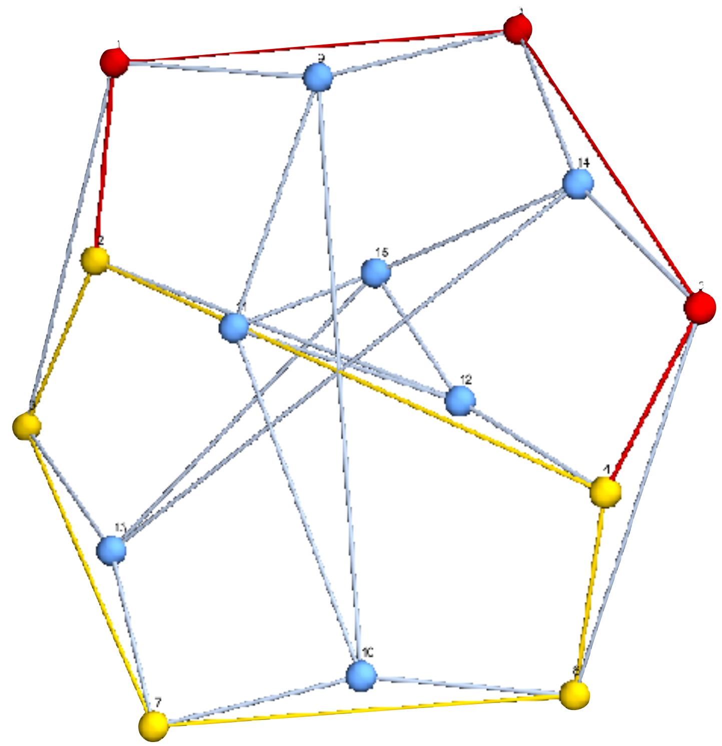
= f$_{\text{(linear)}}$(



)

Can only (pseudo) invert iff A(1,2,σ) aren't independent

$$\mathbf{n}(\mathcal{G}) = \sum_{\rho} \mathbf{D}(\mathcal{G}|\rho)\mathbf{A}(\rho)$$



This means additional relations giving (n-3)! BCJ relations:

$$A(1,2,\sigma) = \sum_{\sigma,\rho} f_{\sigma,\rho} A(1,2,\rho,n)$$

If assume A's proportional to gen. Park-Taylor factors can derive the scattering equations.

$$E_a := \sum_{\substack{b=1 \\ b\neq a}}^{n} \frac{s_{a\,b}}{\sigma_a - \sigma_b} = 0, \quad \forall a \in \{1,2,\ldots,n\}.$$

Foundation of the powerful and elegant CHY formalism.

High energy strings: Gross, Mende

4D connected prescription twistor strings:

Witten ; Roiban, Spradlin, Volovich

D-dimensions YM+Grav+….

Cachazo, He, Yuan

(See Yvonne's talk)

Bern, JJMC, Johansson (2008)

color-kinematics $\longrightarrow$ KLT-type relations

$$\mathcal{M}_m^{\text{tree}} = \sum_{\mathcal{G}\in\text{cubic}} \frac{n(\mathcal{G})\tilde{n}(\mathcal{G})}{D(\mathcal{G})}$$

$$= \sum_{\mathbf{g}\in\text{cubic},\rho,\tau} \frac{(\mathbf{D}(\mathbf{g},\rho)\mathbf{A}(\rho))(\mathbf{D}(\mathbf{g},\tau)\tilde{\mathbf{A}}(\tau))}{\mathbf{D}(\mathbf{g})}$$

$$= \sum_{\rho,\tau} \mathbf{A}(\rho)\left(\sum_{\mathbf{g}\in\text{cubic}} \frac{\mathbf{D}(\mathbf{g},\rho)\mathbf{D}(\mathbf{g},\tau)}{\mathbf{D}(\mathbf{g})}\right)\tilde{\mathbf{A}}(\tau))$$

$$= \sum_{\rho,\tau} \mathbf{A}(\rho)\mathbf{S_0}(\rho|\tau)\tilde{\mathbf{A}}(\tau)$$

Field theory KLT-type matrix / momentum kernel

Bern, Dixon, Perelstein, Rozowsky (1999)

Bjerrum-Bohr, Damgaard, Feng, Sondergaard (2010)

Bjerrum-Bohr, Damgaard,Sondergaard,Vanhove (2011)

KLT-type relations $\longrightarrow$ color-kinematics

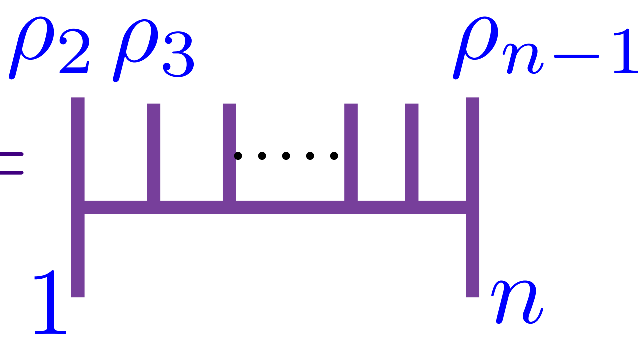
$$\mathcal{A}^{\text{tree}}_{\mathbf{m}}(\rho) = \sum_{\mathcal{G}\in\text{cubic}} \frac{\mathbf{n}(\mathcal{G})\mathbf{c}(\mathcal{G})}{\mathbf{D}(\mathcal{G})}$$

$$= \sum_{\rho,\tau} \mathbf{A}(\rho)\mathbf{S_0}(\rho|\tau)\mathbf{C}(\tau)$$

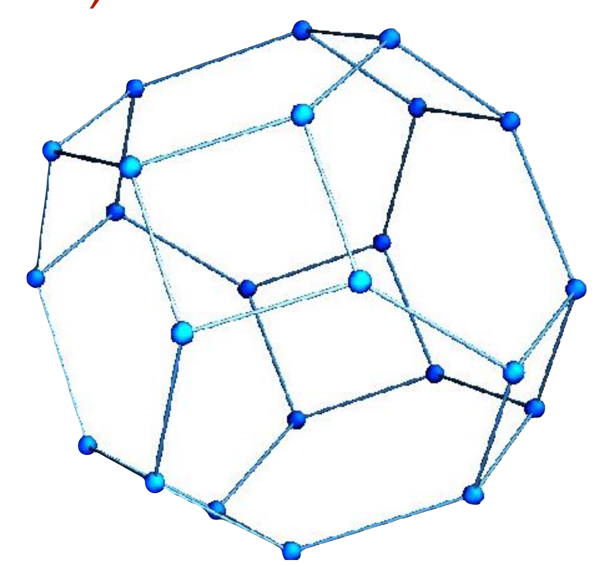
$$= \sum_{\rho} A(\rho)c(\rho)$$

Del Duca, Dixon, Maltoni (1999)

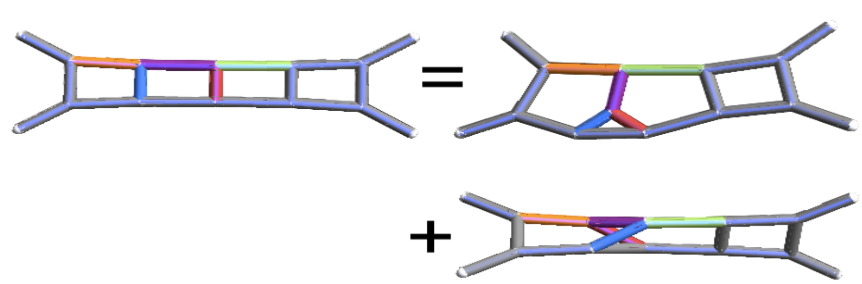
$c(\rho) =$



color weights of permutahedron:



relies only on color-Jacobi satisfaction



$$c(\rho) = \sum_{\tau} S_0(\rho|\tau)C(\tau)$$

$$D(g(\rho)|\tau) = S_0(\rho|\tau)$$

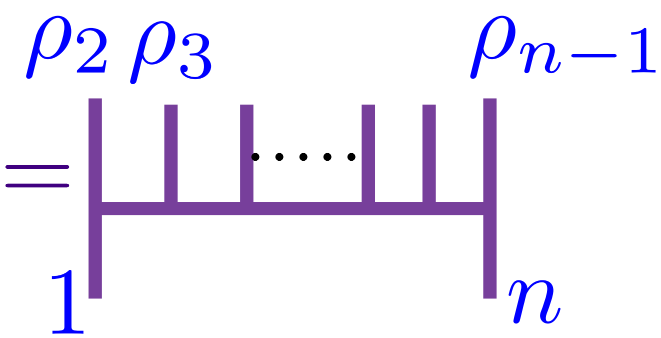
KLT-type relations $\longrightarrow$ color-kinematics

$$\mathcal{M}^{\text{tree}}_{\mathbf{m}}(\rho) = \sum_{\mathcal{G}\in\text{cubic}} \frac{\mathbf{n}(\mathcal{G})\tilde{\mathbf{n}}(\mathcal{G})}{\mathbf{D}(\mathcal{G})}$$

$$= \sum_{\rho,\tau} \mathbf{A}(\rho)\mathbf{S_0}(\rho|\tau)\tilde{\mathbf{A}}(\tau)$$

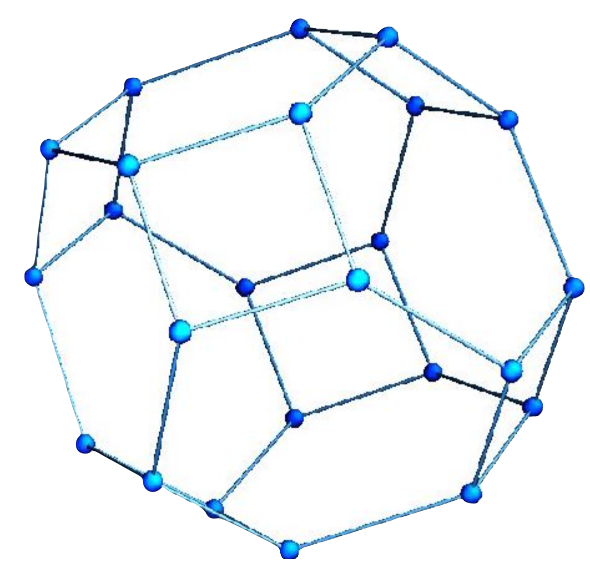
DDM basis for Gravity!

$$= \sum_{\rho} A(\rho)\tilde{n}(\rho) \qquad \tilde{n}(\rho) =$$



kinematic weights of permutahedron:

relies only on kinematic-Jacobi satisfaction



Closed form (non-local) color-dual numerators:

$$\tilde{n}(\rho) = \sum_{\tau} S_0(\rho|\tau)\tilde{A}(\tau)$$

Kiermier; Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove (2010)

Can generalize c/k numerators to off-shell multi-loop:

By introducing ansatze. **BCJ; BCDJR; CJ; Bern, Davies, Dennen, Huang, Nohle; Johansson, Ochirov; Mogull, O'Connell; Johanson, Kälin, Mogull; . . .**

Yang **(FIRST 5-loop N=8 SG Calc: Form Factor!!!!)**

By introducing massive over-redundancy in graphs: **JJMC**

By exploiting BRST invariance of pure-spinor superstrings: **Mafra, Schlotterer**

By recycling forward limits & CHY formalism: **He, Schlotterer, Zhang**

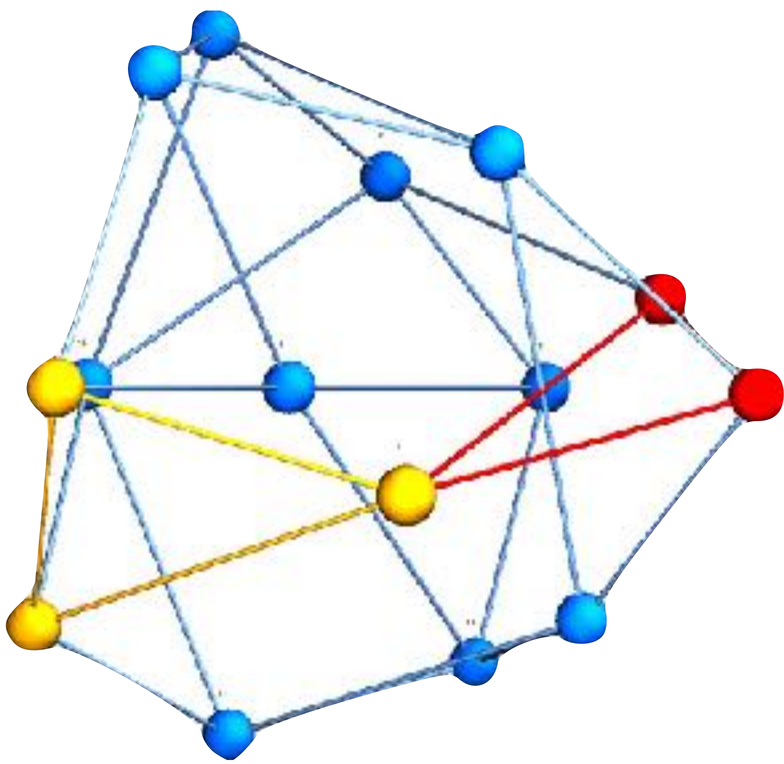
Can generalize BCJ amp relns at loops: **Vanhove, Tourkine; Hohenegger, Stieberger; He, Schlotterer; Boels, Isermann**

Can take CHY tree-rep to loop integrand via ambitwistor string:

Adamo, Casali, Skinner; Geyer, Monteiro, Mason, Tourkine; He, Yuan; Baadsgaard, Bjerrum-Bohr, Bourjaily, Damgaard, Feng

But let's say you don't want to do any of that.

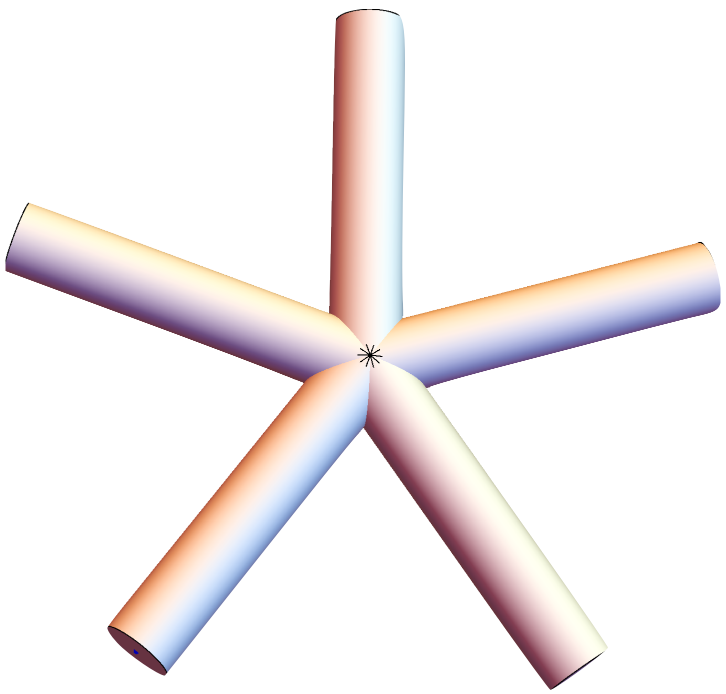
Given a generic (non color-dual) representation for a **gauge amplitude**, and all you want is the related **gravity amplitudes**.



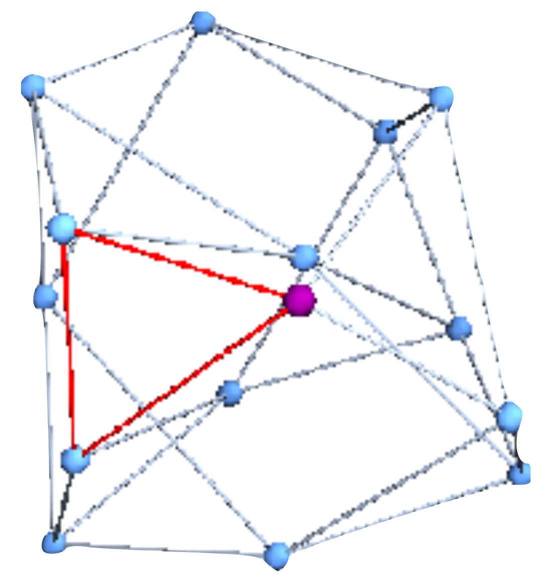
Is there a simple path forward?

YES.

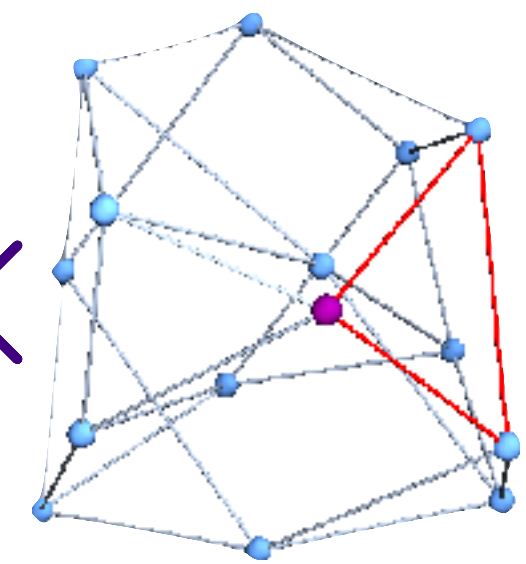
The idea is natural: take all non-vanishing kinematic-Jacobi combinations (the triangles), double-copy them with each other, use this information to ***define*** *off-shell* contact graphs in the double-copy theory.



$= \sum$



$\times$



$$= -\frac{1}{6}\sum_i \frac{J_{i,1}J'_{i,2} + J_{i,2}J'_{i,1}}{d^{(1)}_{i,1}\, d^{(1)}_{i,2}}$$

Bern, JJMC, Chen, Johansson, Roiban (2017)

How does this come together for a full integrand?

Unitarity

$$= \mathcal{U}_c \circ \sum_{g} \frac{n \circ g}{d \circ g}$$

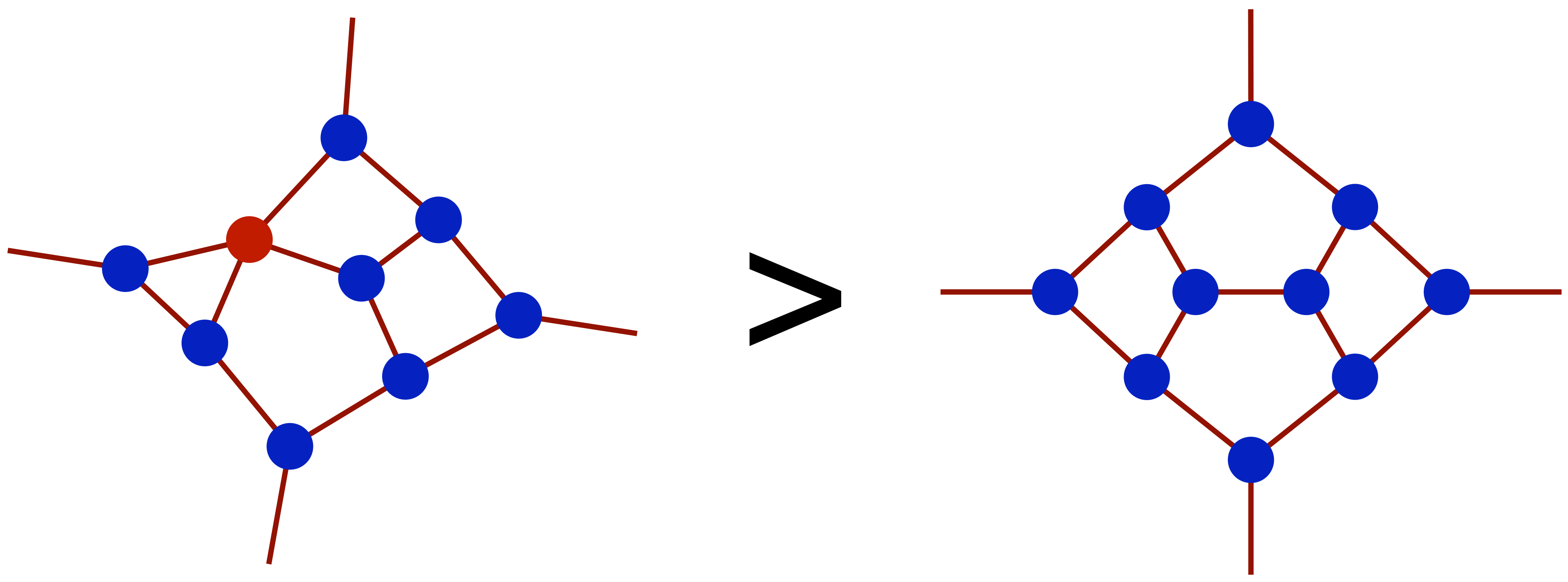
$$\forall\ \mathcal{U}_c \in \text{unitarity cuts}$$

Bern, Dixon, Dunbar, and Kosower ('94,'95)

Bern, Dixon, and Kosower ('96)

Britto, Cachazo, and Feng ('04)

SPANNING CUTS



leads to notion of a **Minimal Spanning Set**

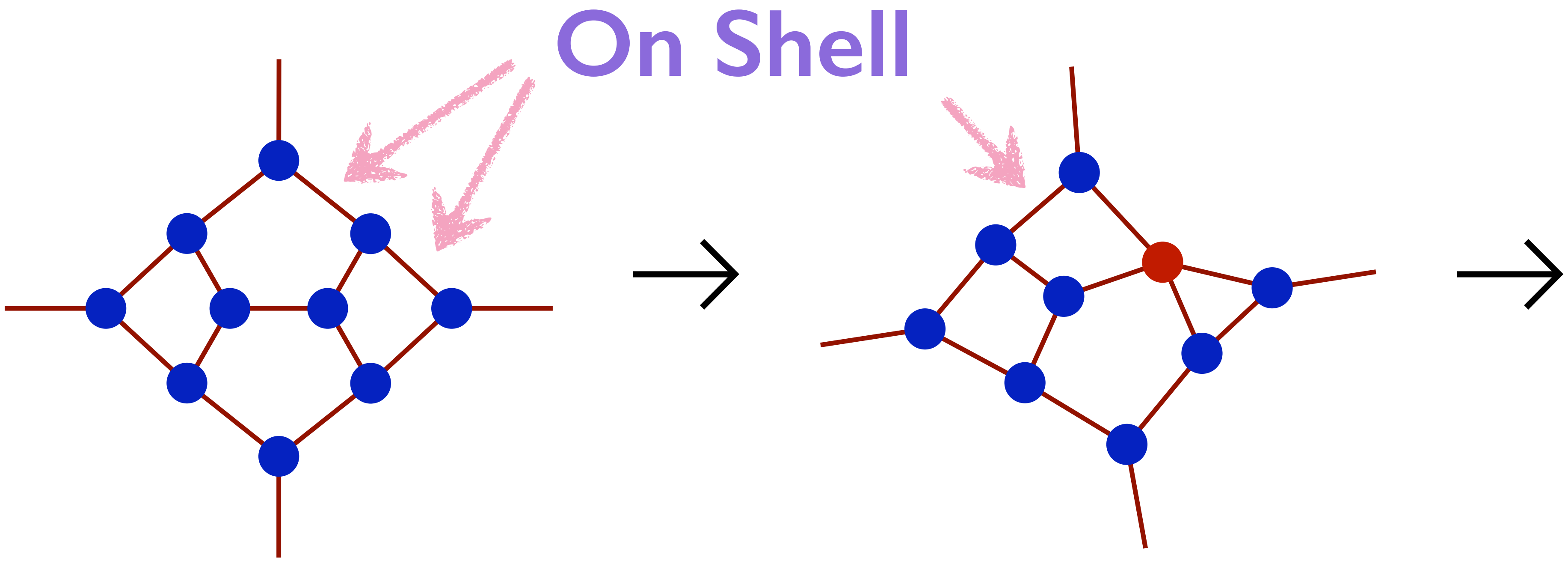
EASY VERIFICATION

EASY VERIFICATION

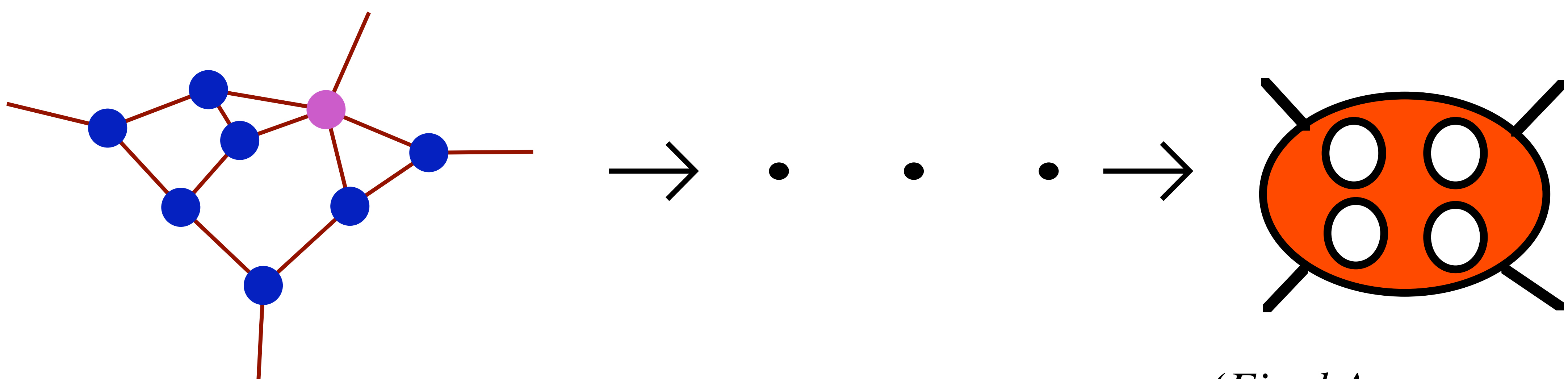
EASY VERIFICATION $\longrightarrow$ NATURAL CONSTRUCTION

METHOD OF MAXIMAL CUTS

Bern, JJMC, Kosower, Johansson (`07)



$$(\forall \text{ exposed propagators } p^2 = 0\ )$$

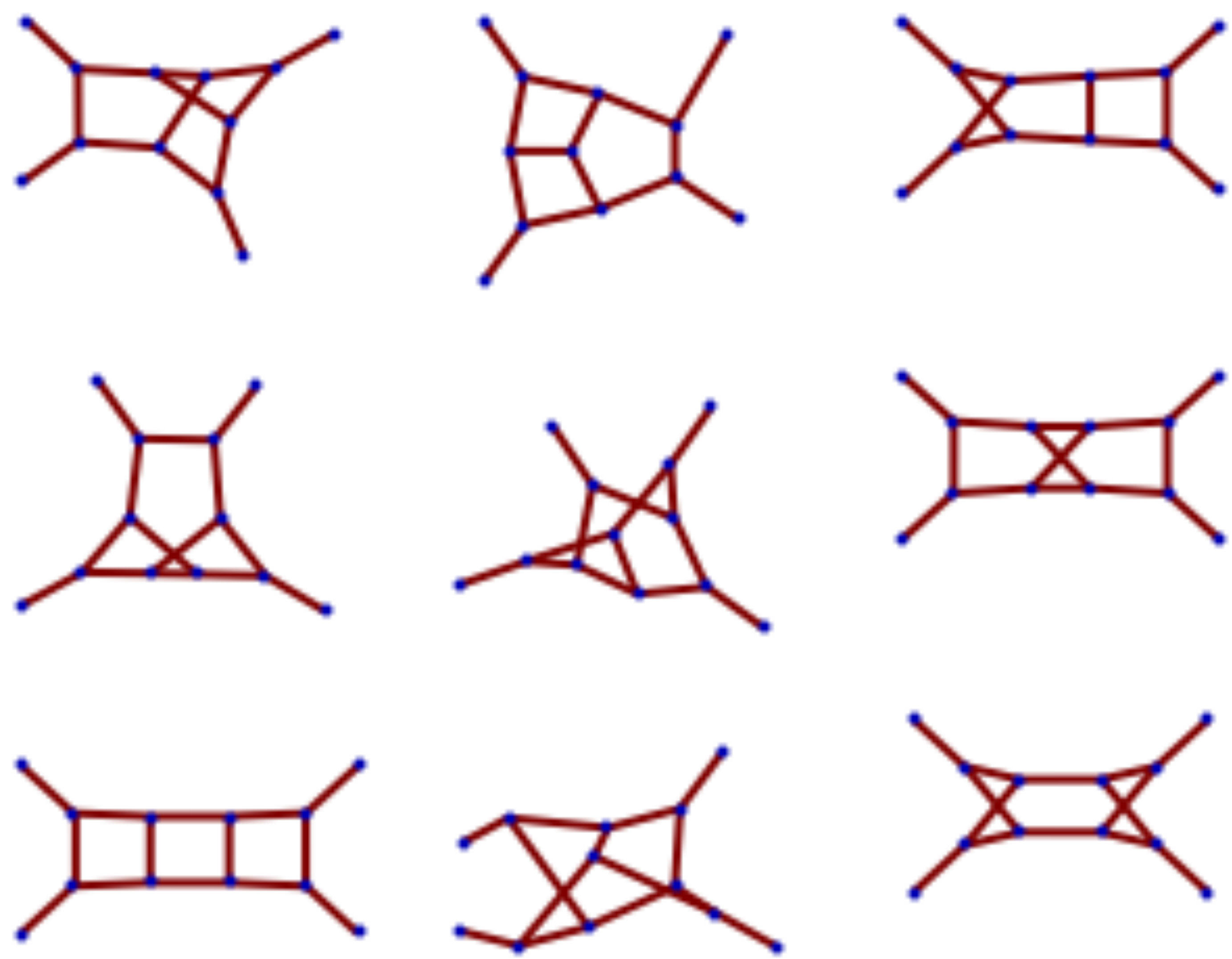


(*Final Answer*, *no cut conditions !*)

Full 3-loop Example

3-loop cubic graphs

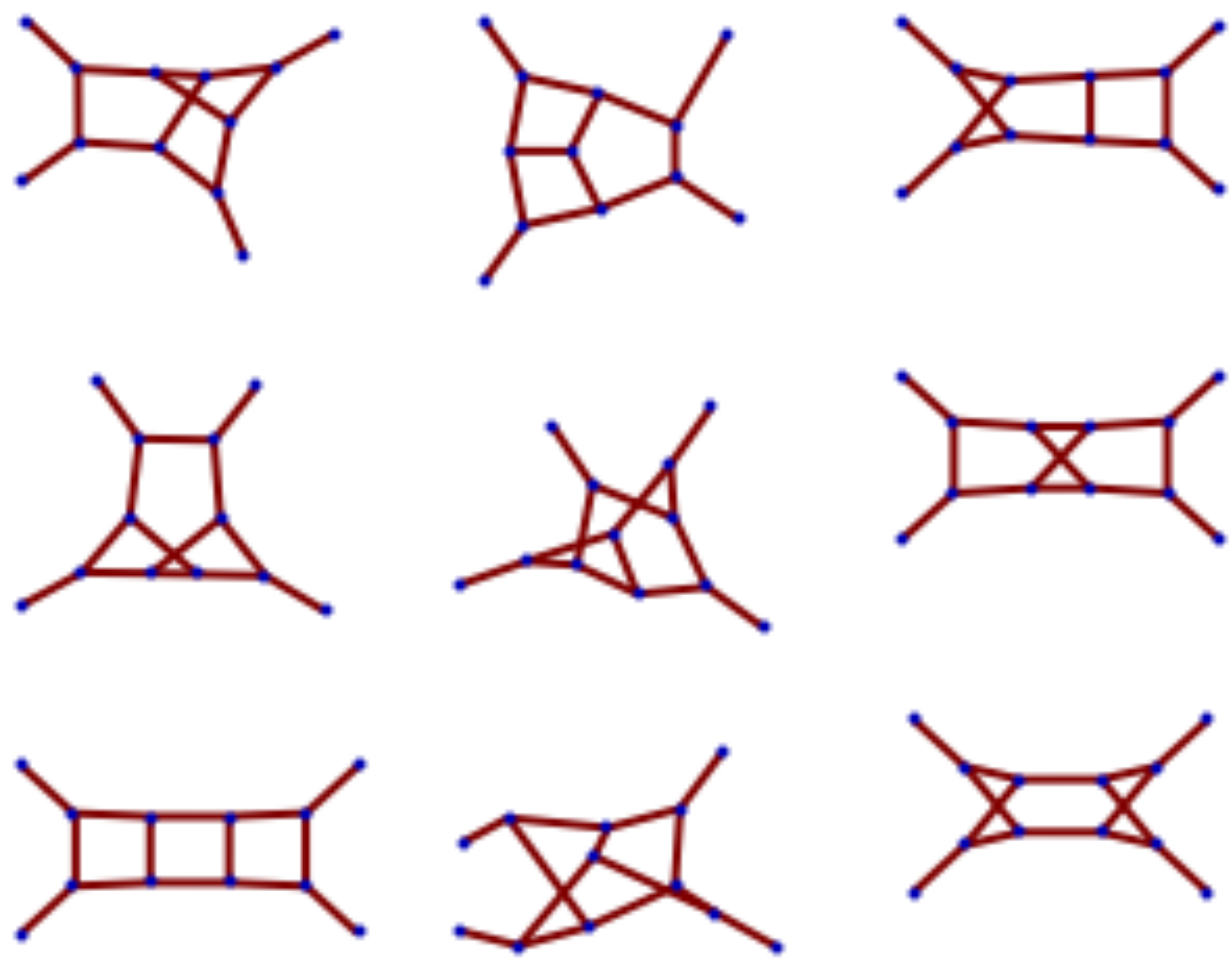
Graph	$\mathcal{N}=4$ sYM numerators.
(a)-(d)	s^2
(e)-(g)	$s(p_5^2+\tau_{45})$
(h)	$s(\tau_{26}+\tau_{36})-t(\tau_{17}+\tau_{27})+st$
(i)	$s(p_5^2+\tau_{45})-t(p_5^2+\tau_{56}+p_6^2)-(s-t)p_6^2/3$



Bern, JJMC, Chen, Johansson, Roiban

ASSIGN square of 3-loop cubic graphs to N=8 SG

Graph	$\mathcal{N}=8$ SG cubic numerators.
(a)-(d)	$\left[s^2\right]^2$
(e)-(g)	$\left[s(p_5^2+\tau_{45})\right]^2$
(h)	$\left[s(\tau_{26}+\tau_{36})-t(\tau_{17}+\tau_{27})+st\right]^2$
(i)	$\left[s(p_5^2+\tau_{45})-t(p_5^2+\tau_{56}+p_6^2)-(s-t)p_6^2/3\right]^2$



Bern, JJMC, Chen, Johansson, Roiban

ASSIGN square of 3-loop cubic graphs to N=8 SG

Graph	$\mathcal{N}=8$ SG cubic numerators.
(a)-(d)	$\left[s^2\right]^2$
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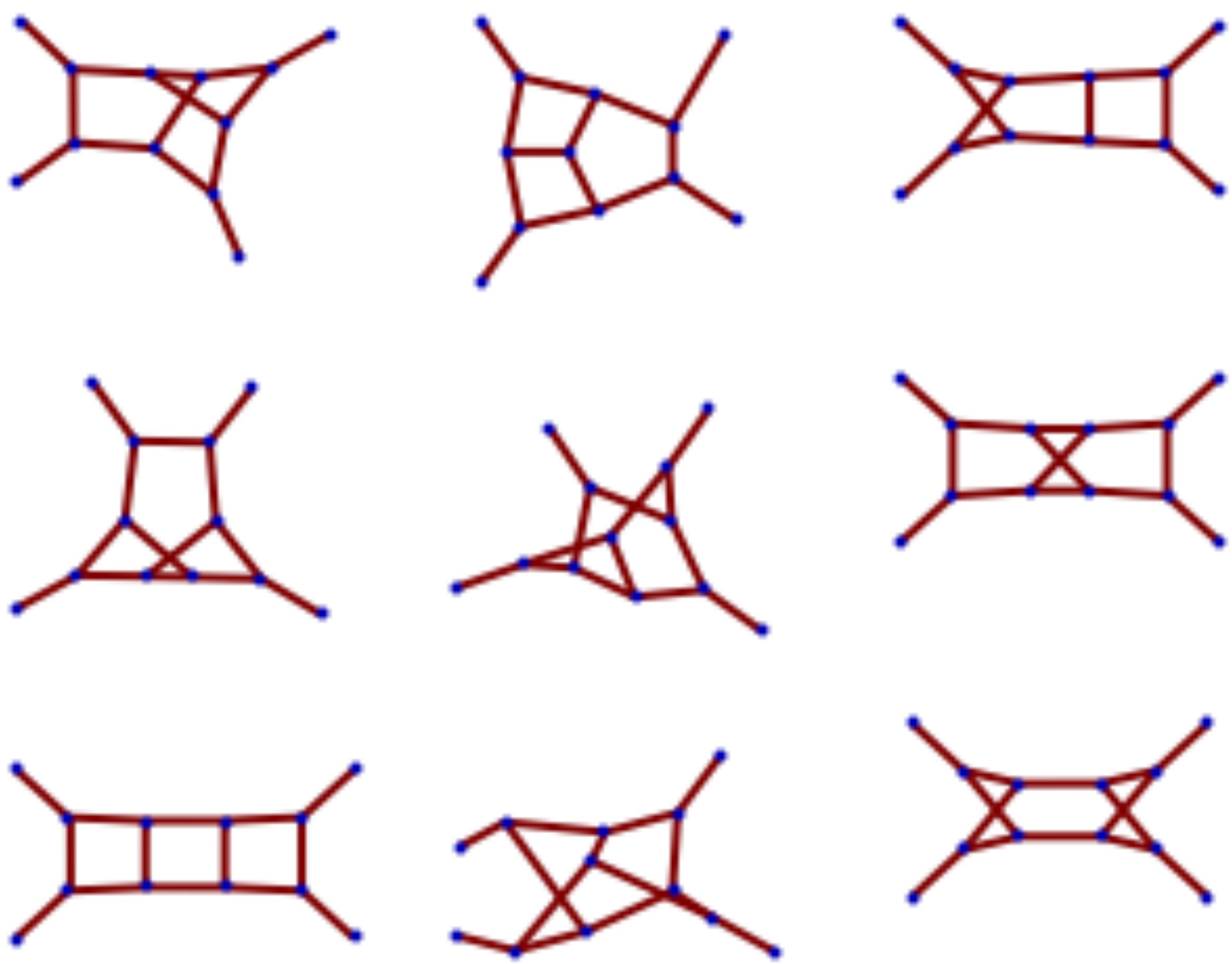
(i)	$\left[s(p_5^2+\tau_{45})-t(p_5^2+\tau_{56}+p_6^2)-(s-t)p_6^2/3\right]^2$

This is just the starting point.

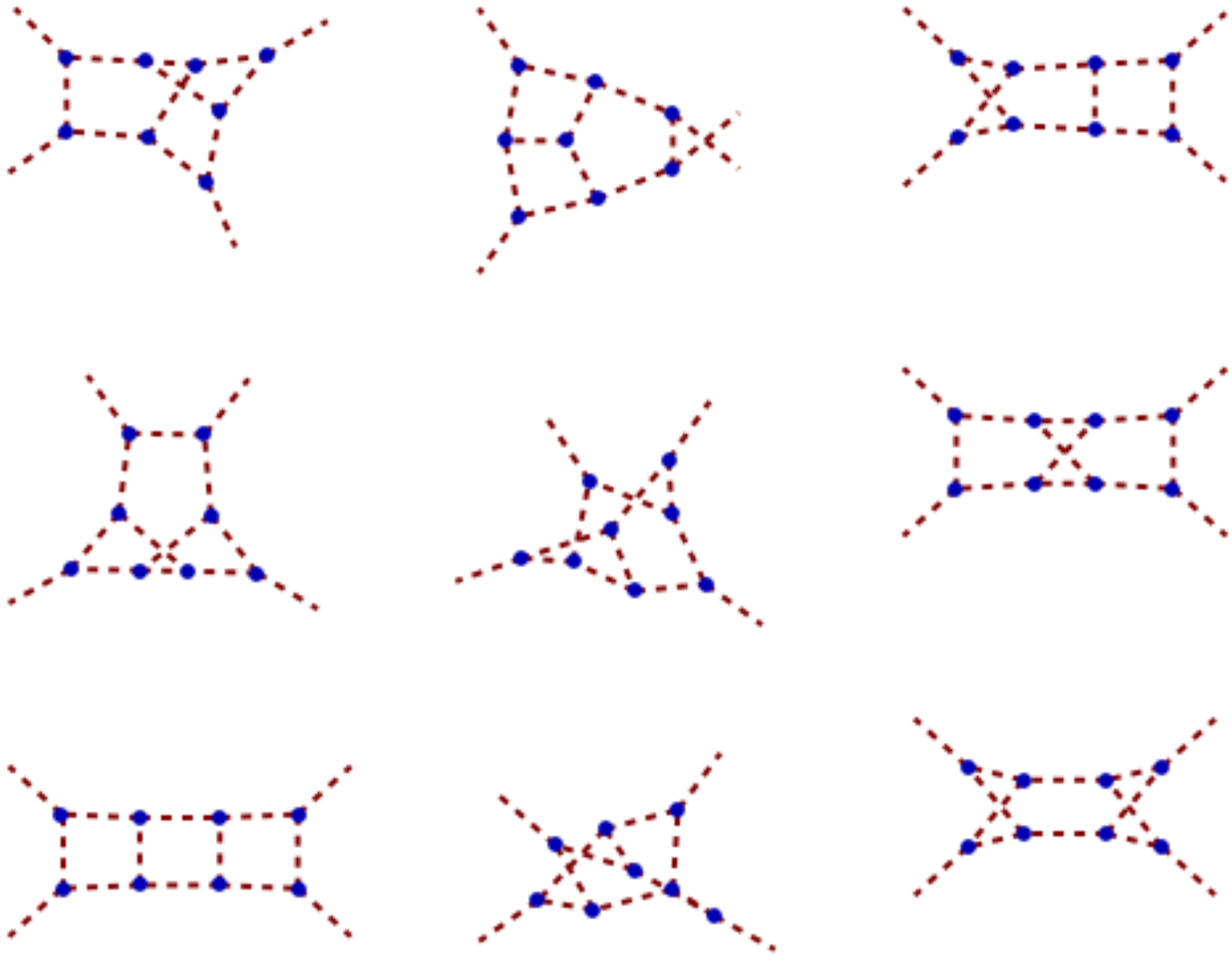


Bern, JJMC, Chen, Johansson, Roiban

Those cubic grav dressings

automatically satisfies all of these cuts

N^0 cut

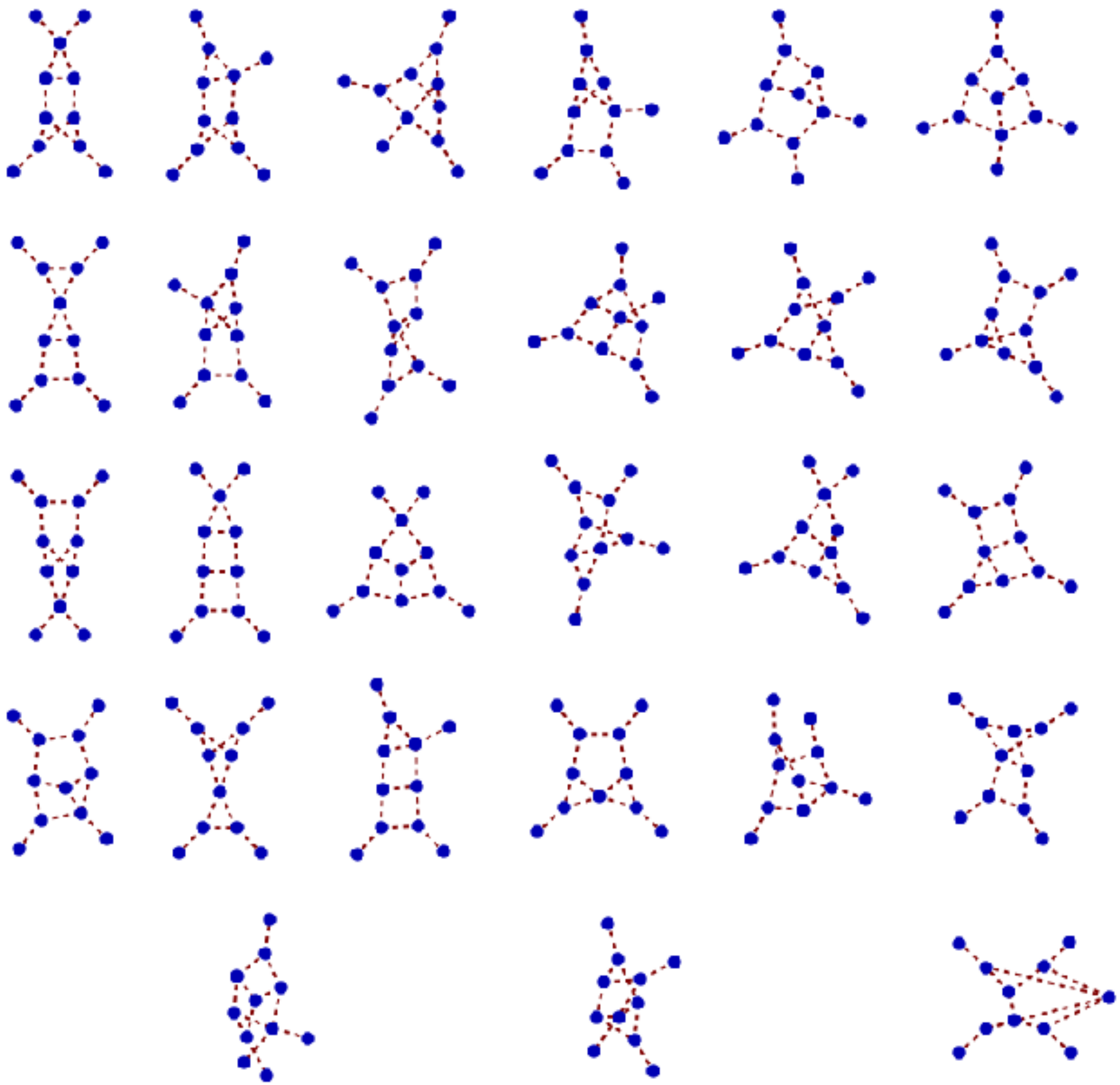


Bern, JJMC, Chen, Johansson, Roiban

Those cubic grav dressings

automatically satisfies all of these cuts too

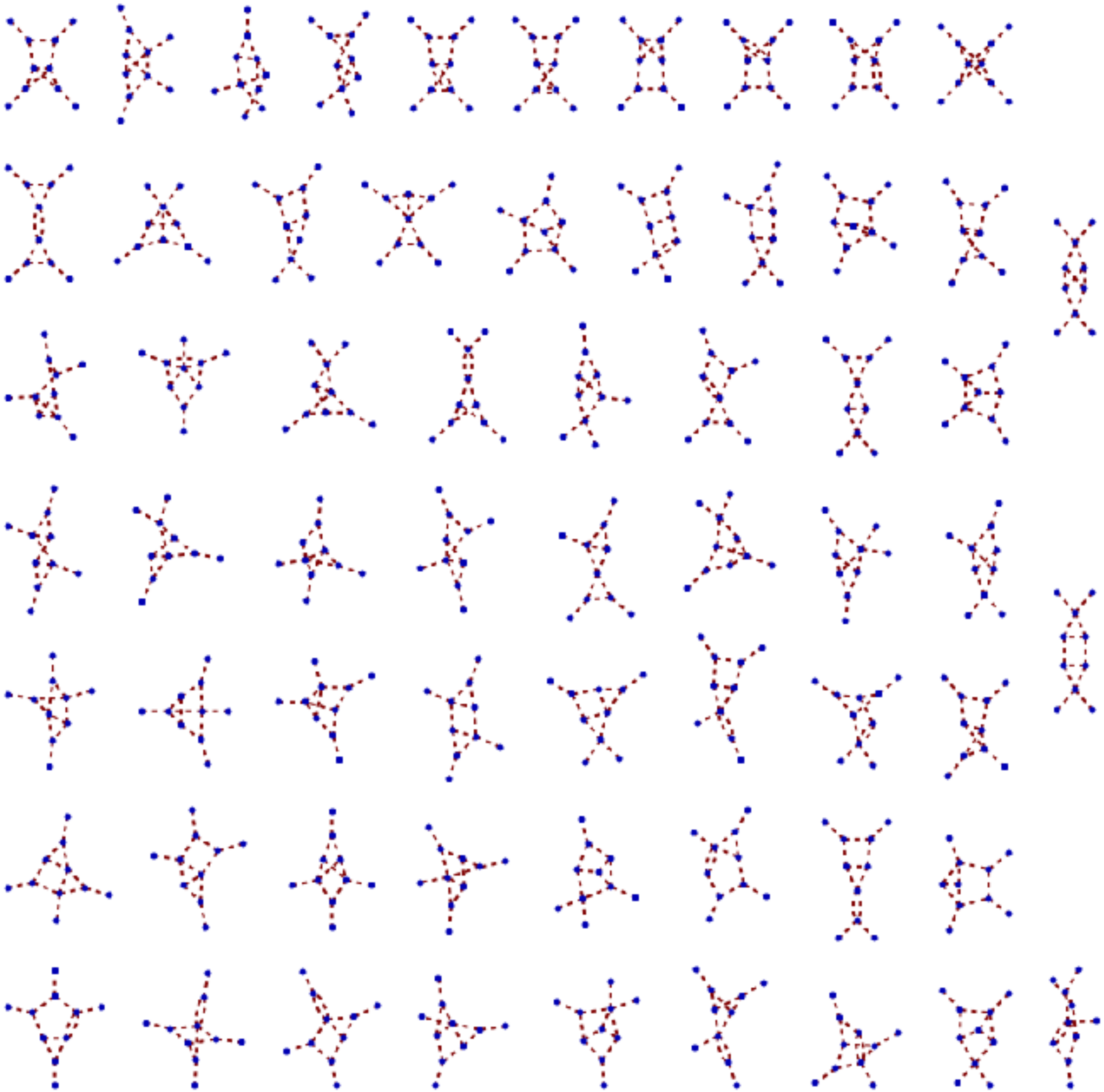
N^1 cut



Bern, JJMC, Chen, Johansson, Roiban

Those cubic grav dressings

satisfy most of these cuts!

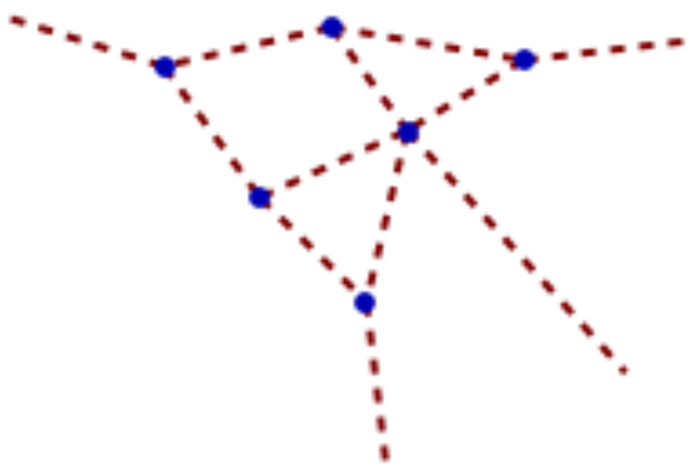


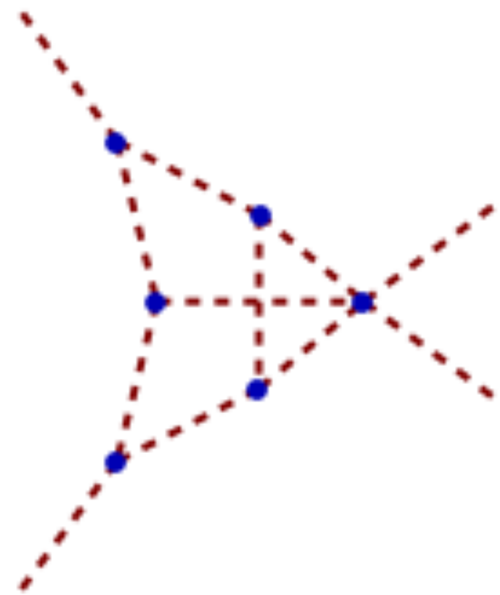
N^2 cut

Bern, JJMC, Chen, Johansson, Roiban

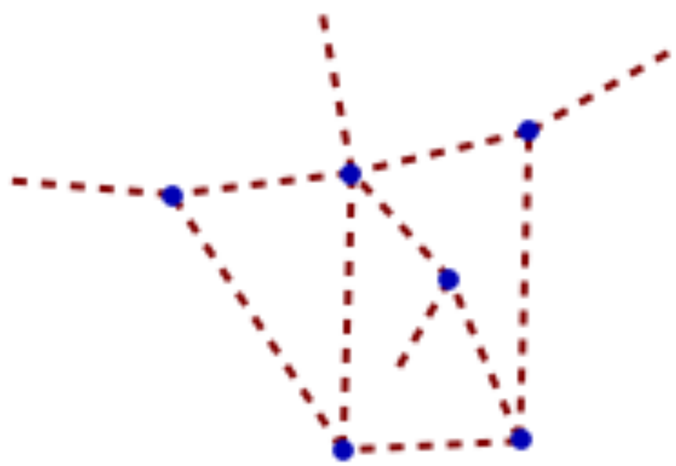
N^2 cut

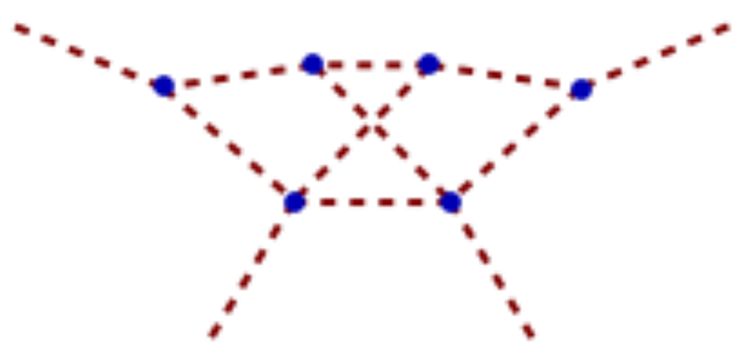
Only 4 non-vanishing cuts





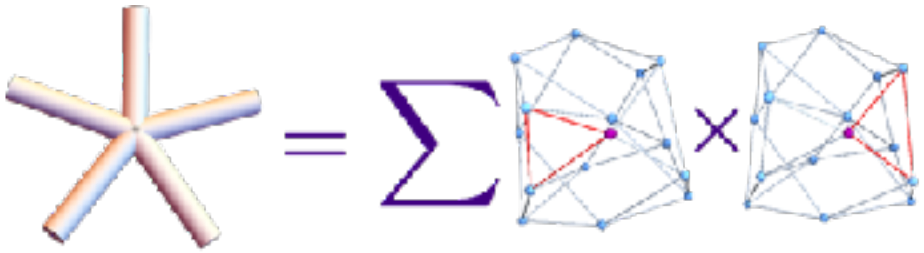
Need to add 4 "contact" contributions



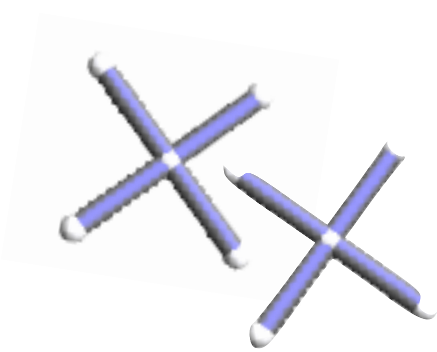


Bern, JJMC, Chen, Johansson, Roiban

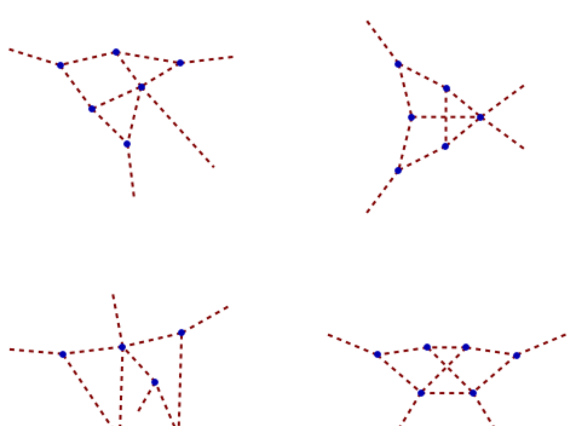
Need to add 4 contacts



$$= -\frac{1}{6}\sum_i \frac{J_{i,1}J'_{i,2}+J_{i,2}J'_{i,1}}{d^{(1)}_{i,1}d^{(1)}_{i,2}}$$

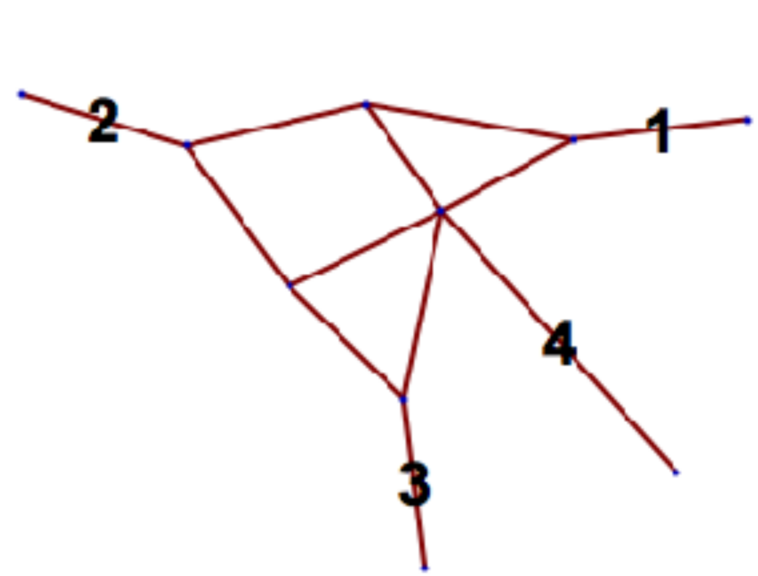


$$= -\frac{1}{9}\sum_i \frac{J_{i,1}J'_{i,2}+J_{i,2}J'_{i,1}}{d^{(1)}_{i,1}d^{(1)}_{i,2}}$$

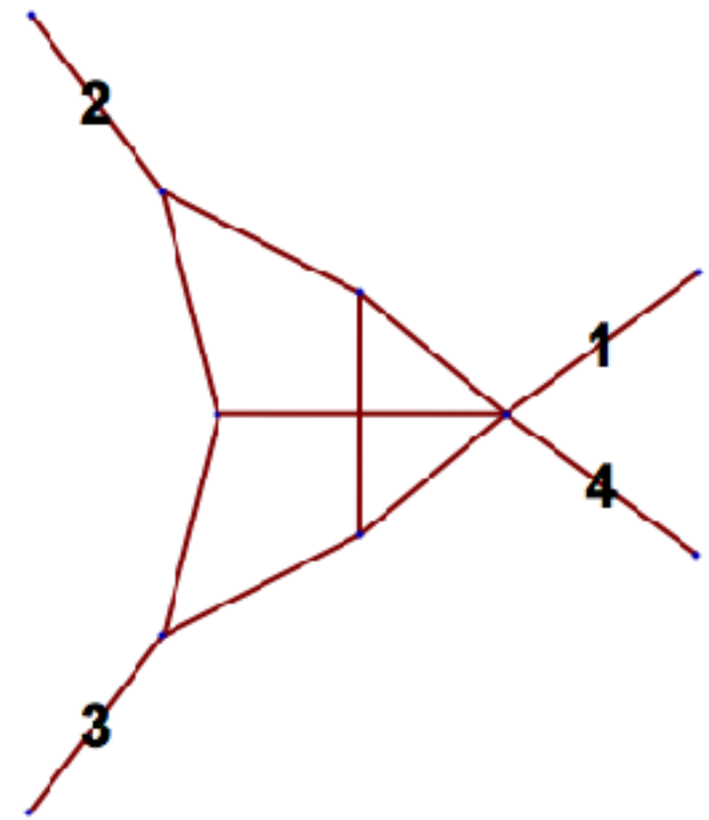


….but you just write them down

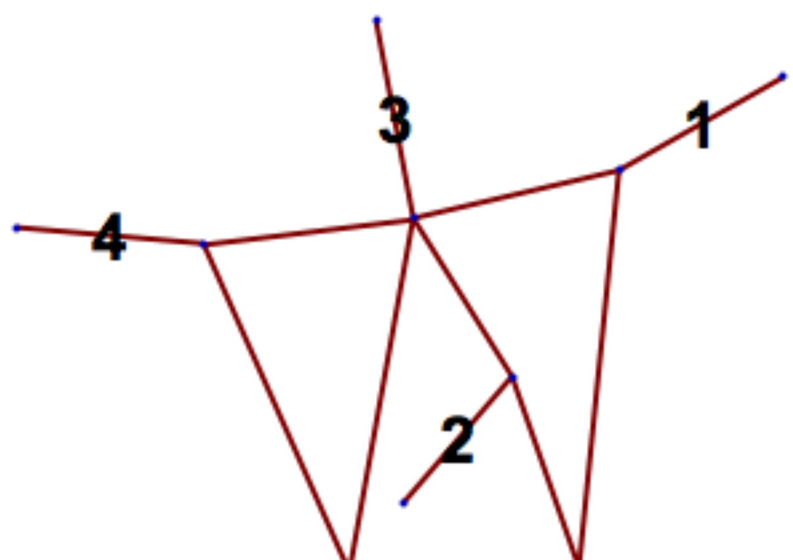
$-\frac{1}{9}(s-t)^2$



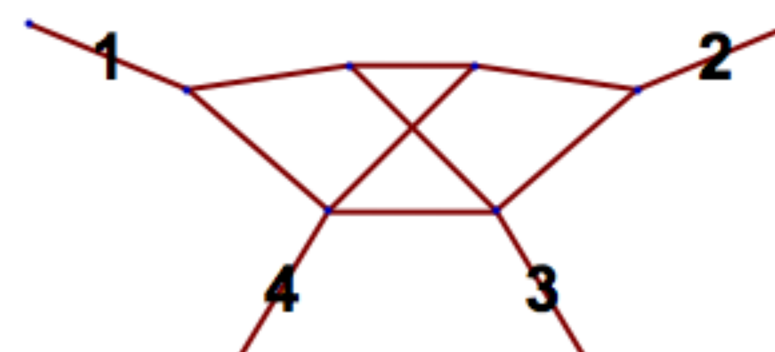
$-2t^2$



$-2s^2$



$-2t^2$

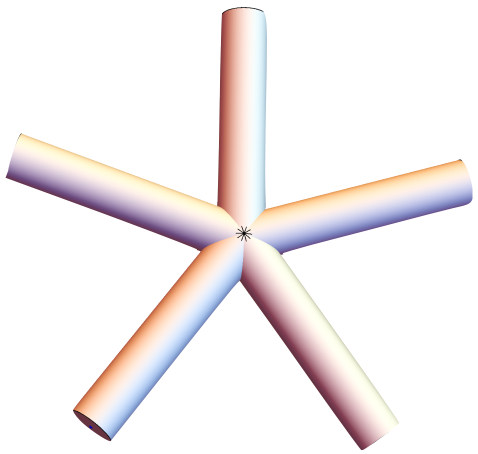


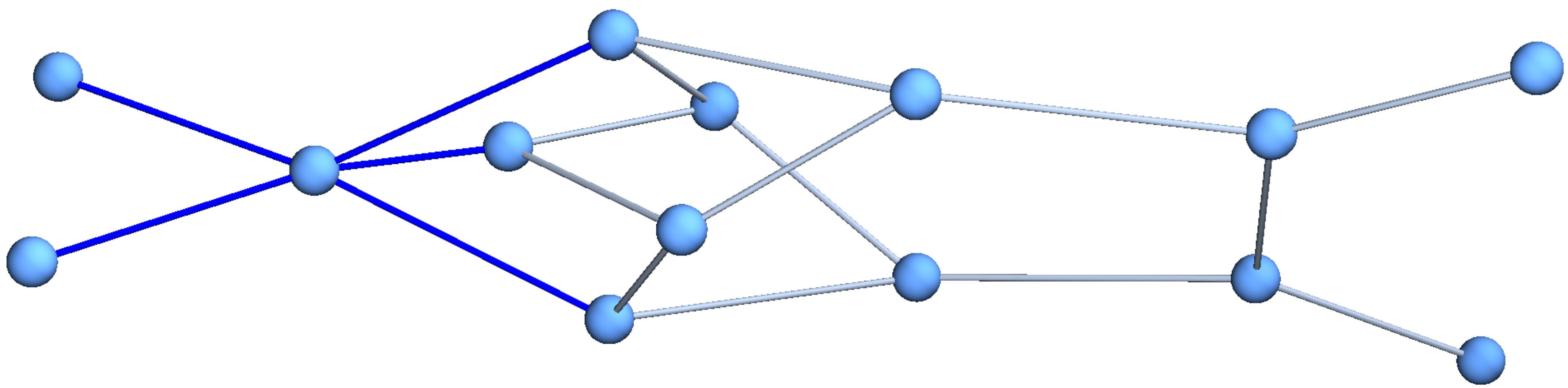
Bern, JJMC, Chen, Johansson, Roiban

Some more examples

Some 5-loop examples

5-loops, a potential N2 contact

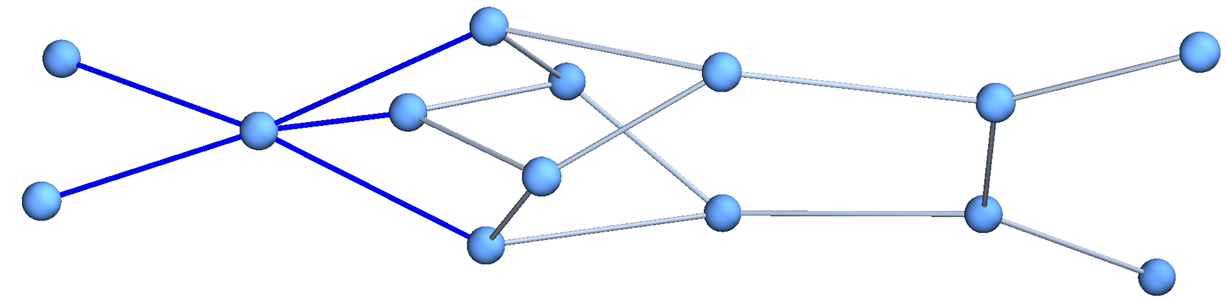




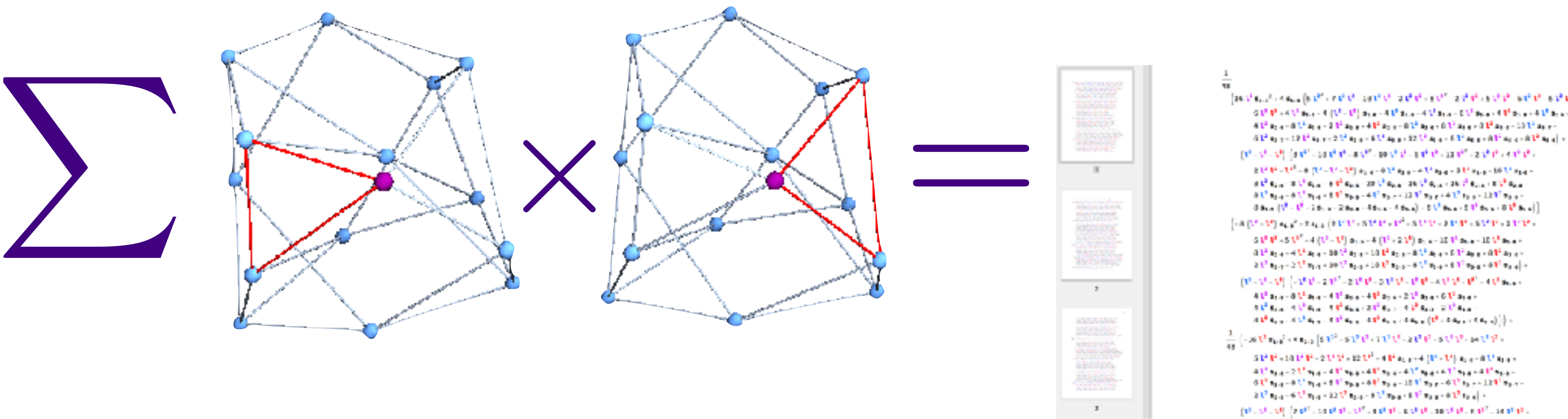
This is serious.

(5-loops is definitely not a joke)

5-loops, potential N2 contact



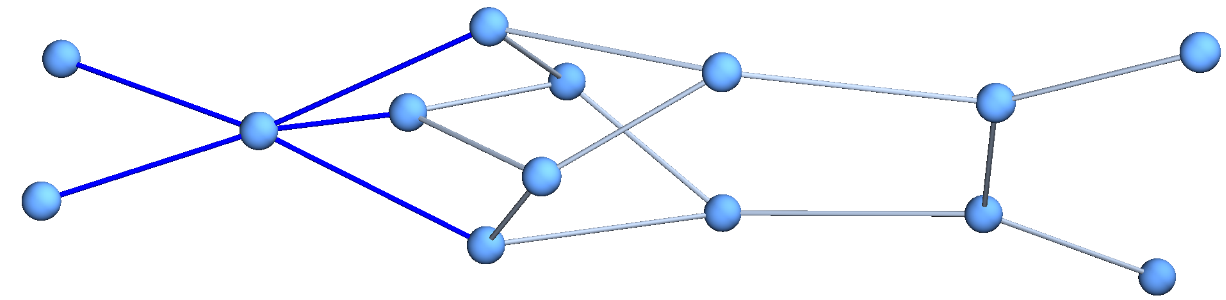
Contact / Missing Information you can just write down:

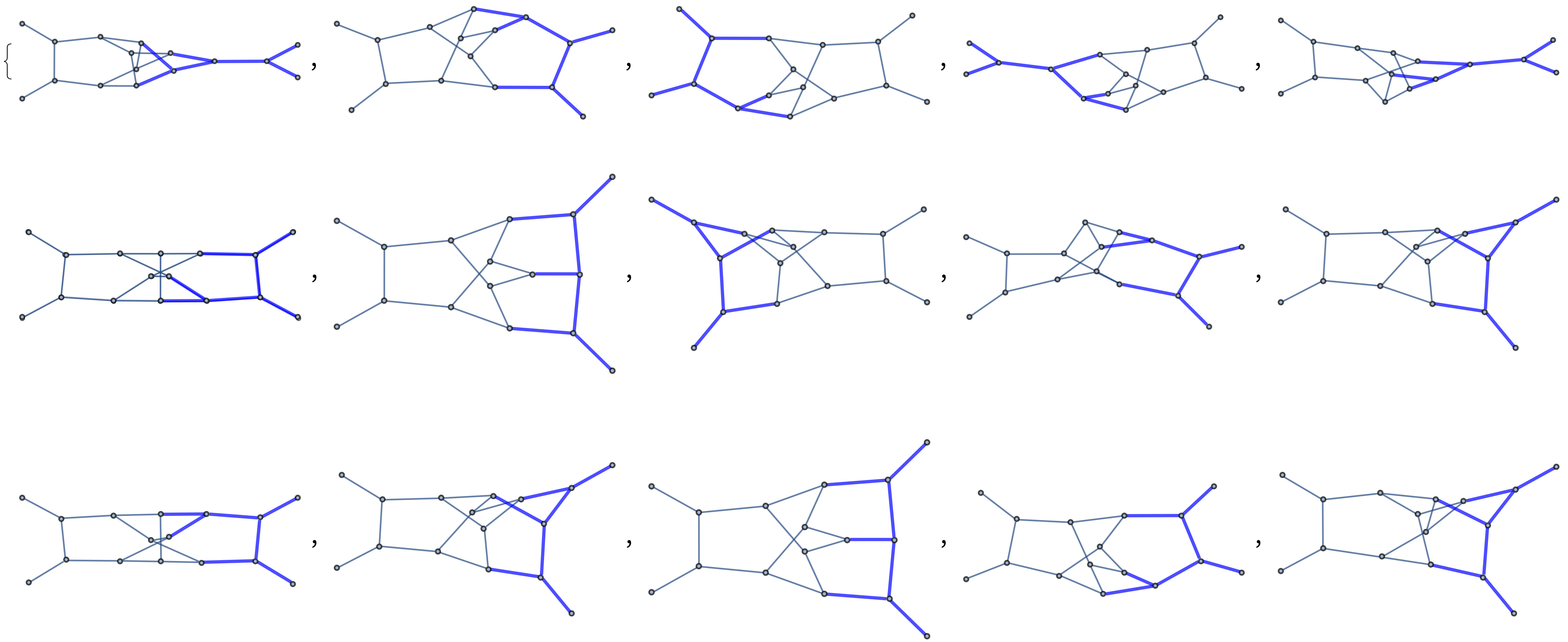


8 pages, local

5-loops isn't for the faint of heart.

5-loops, potential N2 contact

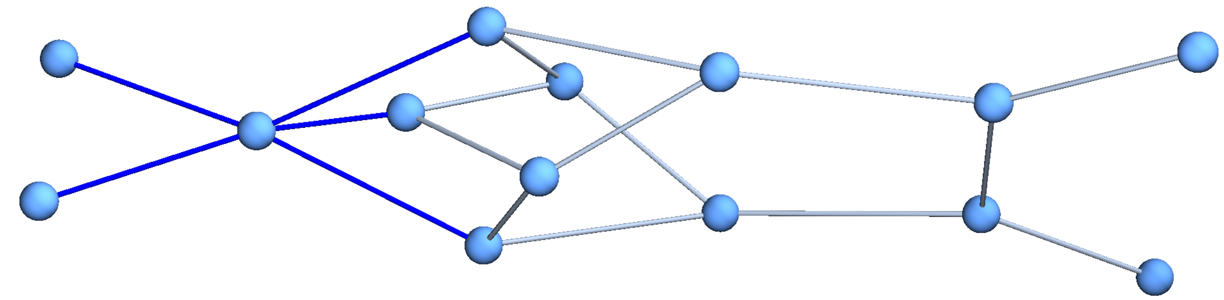




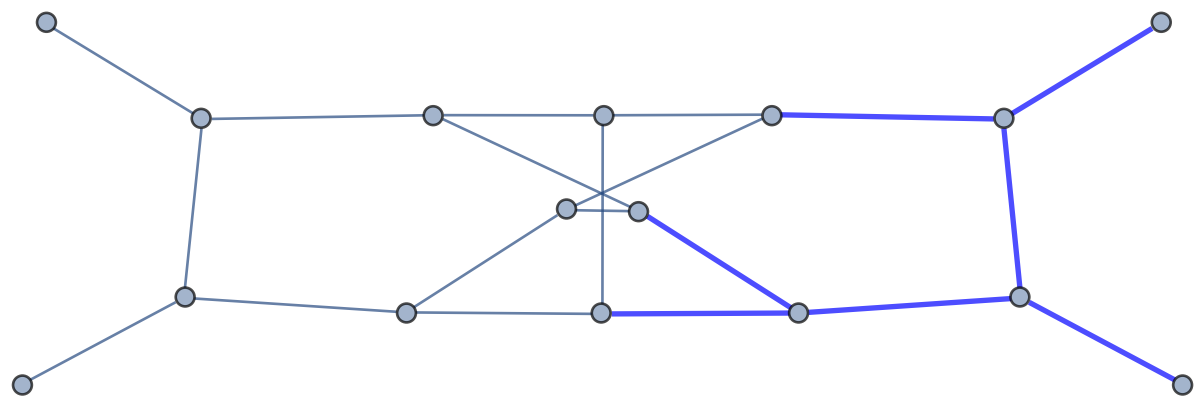
Note: very non-planar, far from the ladder graphs.

(cubic graphs)

5-loops, potential N2 contact

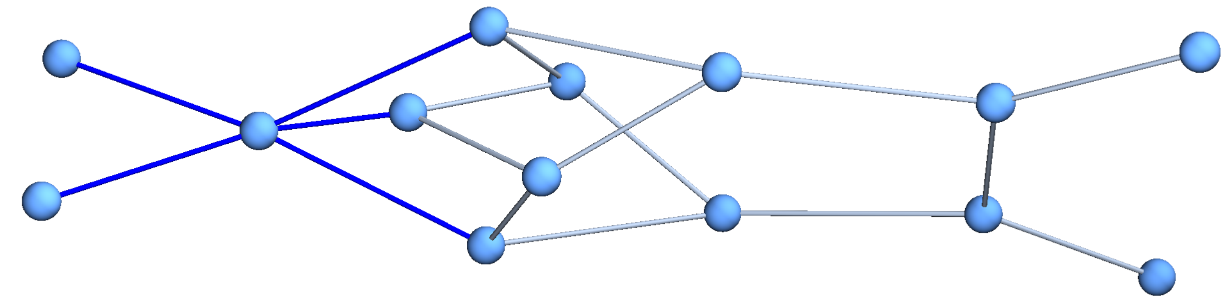


sYM Numerator

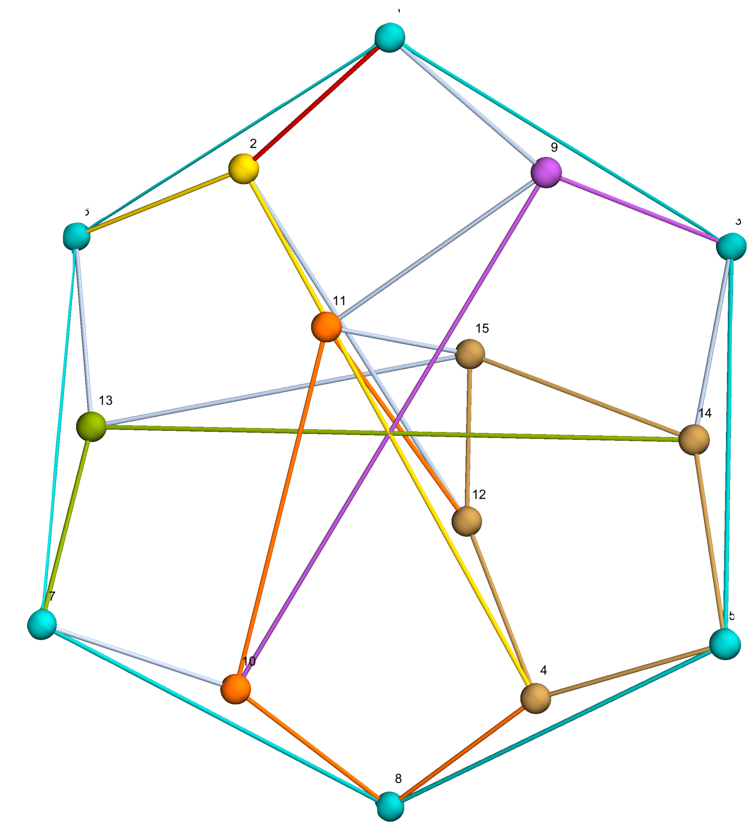


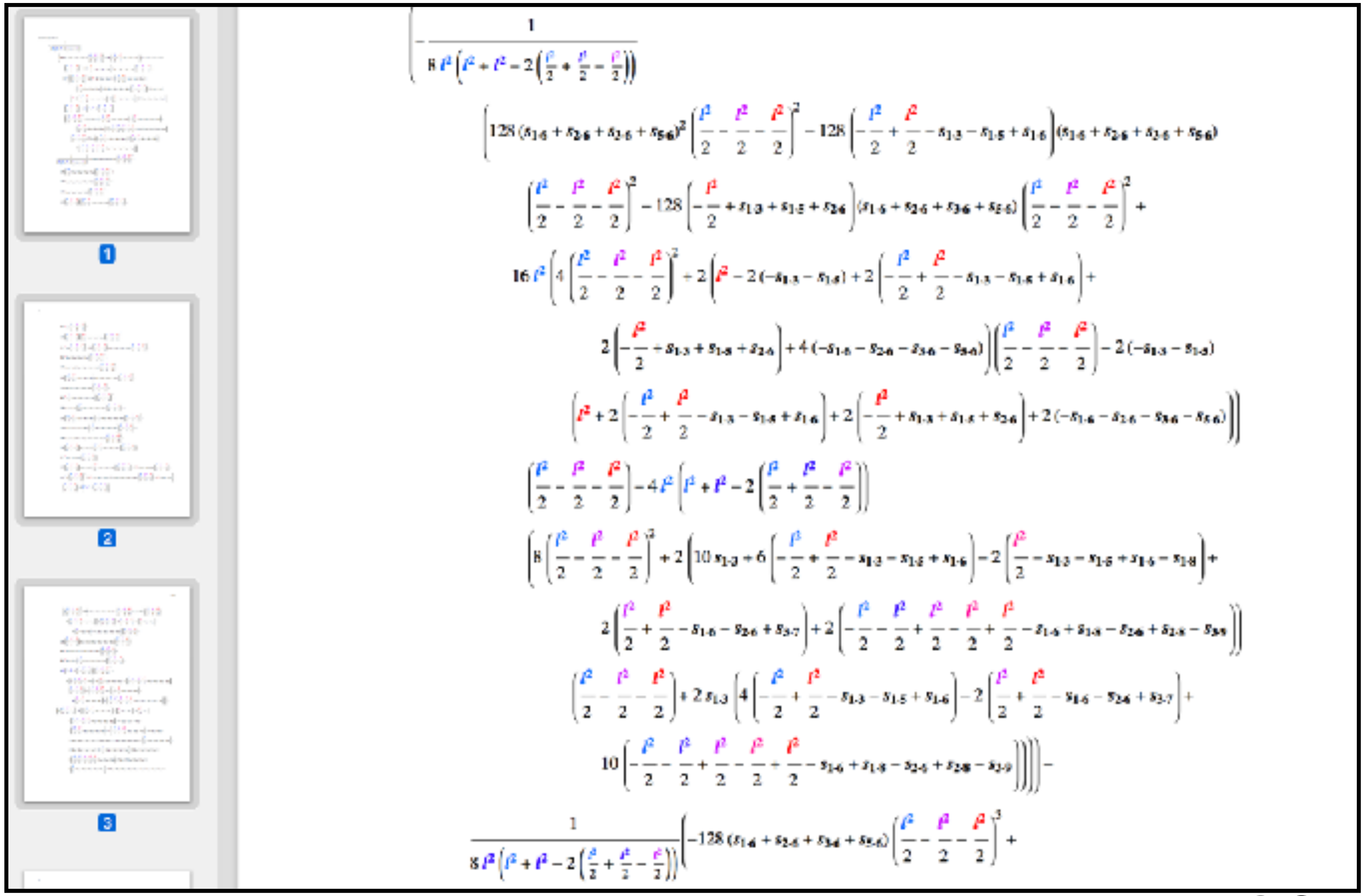
$$
\begin{aligned}
&\frac{1}{4}\Big(-8\,(l^2+l^2-l^2+l^2-l^2)\,s_{1\cdot3}\\
&\quad({l^2}^2+l^2\,l^2-l^2\,l^2-l^2\,l^2+2\,s_{3\cdot6}\,l^2-2\,s_{3\cdot8}\,l^2+2\,s_{5\cdot6}\,l^2-2\,s_{5\cdot8}\,l^2+{l^2}^2-l^2\,l^2+l^2\,l^2-l^2\,l^2+2\,l^2\,s_{1\cdot6}-2\,l^2\,s_{1\cdot8}+2\,l^2\,s_{2\cdot6}-\\
&\qquad 2\,l^2\,s_{2\cdot8}-2\,l^2\,s_{3\cdot6}-2\,l^2\,s_{3\cdot6}-2\,l^2\,s_{3\cdot7}+2\,l^2\,s_{3\cdot8}+2\,l^2\,s_{3\cdot8}+2\,l^2\,s_{3\cdot9}-2\,l^2\,s_{5\cdot6}-2\,l^2\,s_{5\cdot6}+2\,l^2\,s_{5\cdot8}+2\,l^2\,s_{5\cdot8})-\\
&\quad(l^2-l^2-l^2)\,(9\,{l^2}^3+13\,l^2\,{l^2}^2-19\,l^2\,{l^2}^2+9\,l^2\,{l^2}^2-19\,l^2\,{l^2}^2-16\,s_{2\cdot6}\,{l^2}^2-8\,s_{3\cdot6}\,{l^2}^2-8\,s_{3\cdot8}\,{l^2}^2-8\,s_{5\cdot6}\,{l^2}^2-8\,s_{5\cdot8}\,{l^2}^2+\\
&\qquad 8\,s_{6\cdot8}\,{l^2}^2+2\,{l^2}^2\,l^2+11\,{l^2}^2\,l^2+11\,{l^2}^2\,l^2-16\,s_{3\cdot6}^2\,l^2-16\,s_{3\cdot8}^2\,l^2-16\,s_{5\cdot6}^2\,l^2-16\,s_{5\cdot8}^2\,l^2-20\,l^2\,l^2\,l^2+2\,l^2\,l^2\,l^2-10\,l^2\,l^2\,l^2-\\
&\qquad 20\,l^2\,l^2\,l^2+22\,l^2\,l^2\,l^2-10\,l^2\,l^2\,l^2+4\,l^2\,s_{1\cdot8}\,l^2-16\,l^2\,s_{2\cdot6}\,l^2+32\,l^2\,s_{2\cdot6}\,l^2-16\,l^2\,s_{2\cdot6}\,l^2+32\,l^2\,s_{2\cdot6}\,l^2+4\,l^2\,s_{2\cdot8}\,l^2+\\
&\qquad 8\,l^2\,s_{3\cdot6}\,l^2+8\,l^2\,s_{3\cdot6}\,l^2+16\,s_{1\cdot8}\,s_{3\cdot6}\,l^2+16\,s_{2\cdot8}\,s_{3\cdot6}\,l^2-16\,l^2\,s_{3\cdot7}\,l^2-8\,l^2\,s_{3\cdot8}\,l^2+24\,l^2\,s_{3\cdot8}\,l^2-8\,l^2\,s_{3\cdot8}\,l^2+24\,l^2\,s_{3\cdot8}\,l^2-\\
&\qquad 16\,s_{1\cdot8}\,s_{3\cdot8}\,l^2-16\,s_{2\cdot8}\,s_{3\cdot8}\,l^2+32\,s_{3\cdot6}\,s_{3\cdot8}\,l^2-8\,l^2\,s_{3\cdot9}\,l^2+8\,l^2\,s_{5\cdot6}\,l^2+8\,l^2\,s_{5\cdot6}\,l^2+16\,s_{1\cdot8}\,s_{5\cdot6}\,l^2+16\,s_{2\cdot8}\,s_{5\cdot6}\,l^2-32\,s_{3\cdot6}\,s_{5\cdot6}\,l^2+\\
&\qquad 32\,s_{3\cdot8}\,s_{5\cdot6}\,l^2-8\,l^2\,s_{5\cdot8}\,l^2+24\,l^2\,s_{5\cdot8}\,l^2-8\,l^2\,s_{5\cdot8}\,l^2+24\,l^2\,s_{5\cdot8}\,l^2-16\,s_{1\cdot8}\,s_{5\cdot8}\,l^2-16\,s_{2\cdot8}\,s_{5\cdot8}\,l^2+32\,s_{3\cdot6}\,s_{5\cdot8}\,l^2-32\,s_{3\cdot8}\,s_{5\cdot8}\,l^2+\\
&\qquad 32\,s_{5\cdot6}\,s_{5\cdot8}\,l^2-16\,l^2\,s_{6\cdot8}\,l^2-16\,l^2\,s_{6\cdot8}\,l^2-2\,{l^2}^3-{l^2}^3-{l^2}^3+7\,l^2\,{l^2}^2-2\,l^2\,{l^2}^2+7\,l^2\,{l^2}^2-3\,l^2\,{l^2}^2+l^2\,{l^2}^2+16\,l^2\,s_{3\cdot6}^2+\\
&\qquad 16\,l^2\,s_{3\cdot6}^2+16\,l^2\,s_{3\cdot8}^2+16\,l^2\,s_{3\cdot8}^2+16\,l^2\,s_{5\cdot6}^2+16\,l^2\,s_{5\cdot6}^2+16\,l^2\,s_{5\cdot8}^2+16\,l^2\,s_{5\cdot8}^2-4\,{l^2}^2\,l^2-4\,{l^2}^2\,l^2+{l^2}^2\,l^2-4\,l^2\,l^2\,l^2-\\
&\qquad 4\,{l^2}^2\,l^2-3\,{l^2}^2\,l^2+14\,l^2\,l^2\,l^2-4\,l^2\,l^2\,l^2+2\,l^2\,l^2\,l^2-16\,(l^2-l^2-l^2)\,(l^2+l^2-l^2+l^2-l^2)\,s_{1\cdot6}+4\,{l^2}^2\,s_{1\cdot8}-4\,l^2\,l^2\,s_{1\cdot8}+\\
&\qquad 4\,l^2\,l^2\,s_{1\cdot8}-4\,l^2\,l^2\,s_{1\cdot8}-16\,{l^2}^2\,s_{2\cdot6}-16\,{l^2}^2\,s_{2\cdot6}+16\,l^2\,l^2\,s_{2\cdot6}+16\,l^2\,l^2\,s_{2\cdot6}+16\,l^2\,l^2\,s_{2\cdot6}-32\,l^2\,l^2\,s_{2\cdot6}+16\,l^2\,l^2\,s_{2\cdot6}+\\
&\qquad 4\,{l^2}^2\,s_{2\cdot8}-4\,l^2\,l^2\,s_{2\cdot8}+4\,l^2\,l^2\,s_{2\cdot8}-4\,l^2\,l^2\,s_{2\cdot8}-16\,l^2\,s_{1\cdot8}\,s_{3\cdot6}-16\,l^2\,s_{1\cdot8}\,s_{3\cdot6}-16\,l^2\,s_{2\cdot8}\,s_{3\cdot6}-16\,l^2\,s_{2\cdot8}\,s_{3\cdot6}-16\,{l^2}^2\,s_{3\cdot7}+\\
&\qquad 16\,l^2\,l^2\,s_{3\cdot7}-16\,l^2\,l^2\,s_{3\cdot7}+16\,l^2\,l^2\,s_{3\cdot7}-16\,{l^2}^2\,s_{3\cdot8}-16\,{l^2}^2\,s_{3\cdot8}+8\,l^2\,l^2\,s_{3\cdot8}+8\,l^2\,l^2\,s_{3\cdot8}+8\,l^2\,l^2\,s_{3\cdot8}-32\,l^2\,l^2\,s_{3\cdot8}+\\
&\qquad 8\,l^2\,l^2\,s_{3\cdot8}+16\,l^2\,s_{1\cdot8}\,s_{3\cdot8}+16\,l^2\,s_{1\cdot8}\,s_{3\cdot8}+16\,l^2\,s_{2\cdot8}\,s_{3\cdot8}+16\,l^2\,s_{2\cdot8}\,s_{3\cdot8}-32\,l^2\,s_{3\cdot6}\,s_{3\cdot8}-32\,l^2\,s_{3\cdot6}\,s_{3\cdot8}-8\,{l^2}^2\,s_{3\cdot9}+8\,l^2\,l^2\,s_{3\cdot9}-\\
&\qquad 8\,l^2\,l^2\,s_{3\cdot9}+8\,l^2\,l^2\,s_{3\cdot9}-16\,l^2\,s_{1\cdot8}\,s_{5\cdot6}-16\,l^2\,s_{1\cdot8}\,s_{5\cdot6}-16\,l^2\,s_{2\cdot8}\,s_{5\cdot6}-16\,l^2\,s_{2\cdot8}\,s_{5\cdot6}+32\,l^2\,s_{3\cdot6}\,s_{5\cdot6}+32\,l^2\,s_{3\cdot6}\,s_{5\cdot6}-\\
&\qquad 32\,l^2\,s_{3\cdot8}\,s_{5\cdot6}-32\,l^2\,s_{3\cdot8}\,s_{5\cdot6}+8\,(l^2+l^2-l^2+l^2-l^2)\,s_{1\cdot5}\,(l^2+2\,s_{3\cdot6}-2\,s_{3\cdot8}+2\,s_{5\cdot6}-2\,s_{5\cdot8})-16\,{l^2}^2\,s_{5\cdot8}-16\,{l^2}^2\,s_{5\cdot8}+\\
&\qquad 8\,l^2\,l^2\,s_{5\cdot8}+8\,l^2\,l^2\,s_{5\cdot8}+8\,l^2\,l^2\,s_{5\cdot8}-32\,l^2\,l^2\,s_{5\cdot8}+8\,l^2\,l^2\,s_{5\cdot8}+16\,l^2\,s_{1\cdot8}\,s_{5\cdot8}+16\,l^2\,s_{1\cdot8}\,s_{5\cdot8}+16\,l^2\,s_{2\cdot8}\,s_{5\cdot8}+16\,l^2\,s_{2\cdot8}\,s_{5\cdot8}-\\
&\qquad 32\,l^2\,s_{3\cdot6}\,s_{5\cdot8}-32\,l^2\,s_{3\cdot6}\,s_{5\cdot8}+32\,l^2\,s_{3\cdot8}\,s_{5\cdot8}+32\,l^2\,s_{3\cdot8}\,s_{5\cdot8}-32\,l^2\,s_{5\cdot6}\,s_{5\cdot8}-32\,l^2\,s_{5\cdot6}\,s_{5\cdot8}+8\,{l^2}^2\,s_{6\cdot8}+8\,{l^2}^2\,s_{6\cdot8}+16\,l^2\,l^2\,s_{6\cdot8})\Big)
\end{aligned}
$$

5-loops, potential N2 contact



$$\mathrm{truth}|_{\mathrm{cut}} = \sum_{g \in \mathrm{cut}} \frac{\mathring{n}_g^{\,2}}{d_g}$$



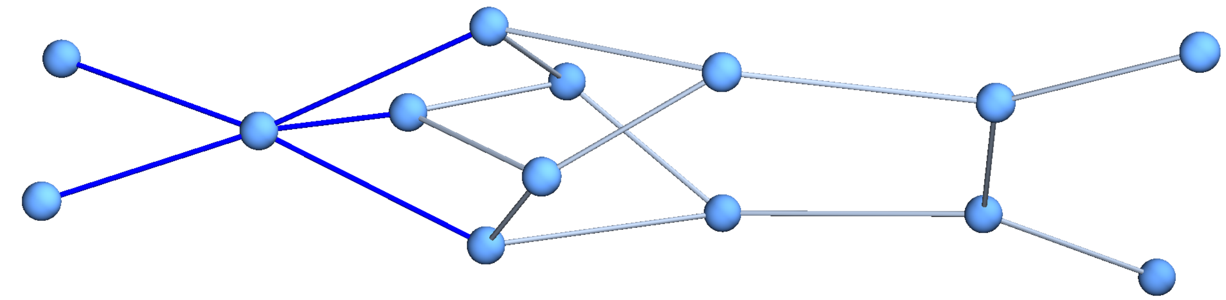


KLT→non-local c/k numerators

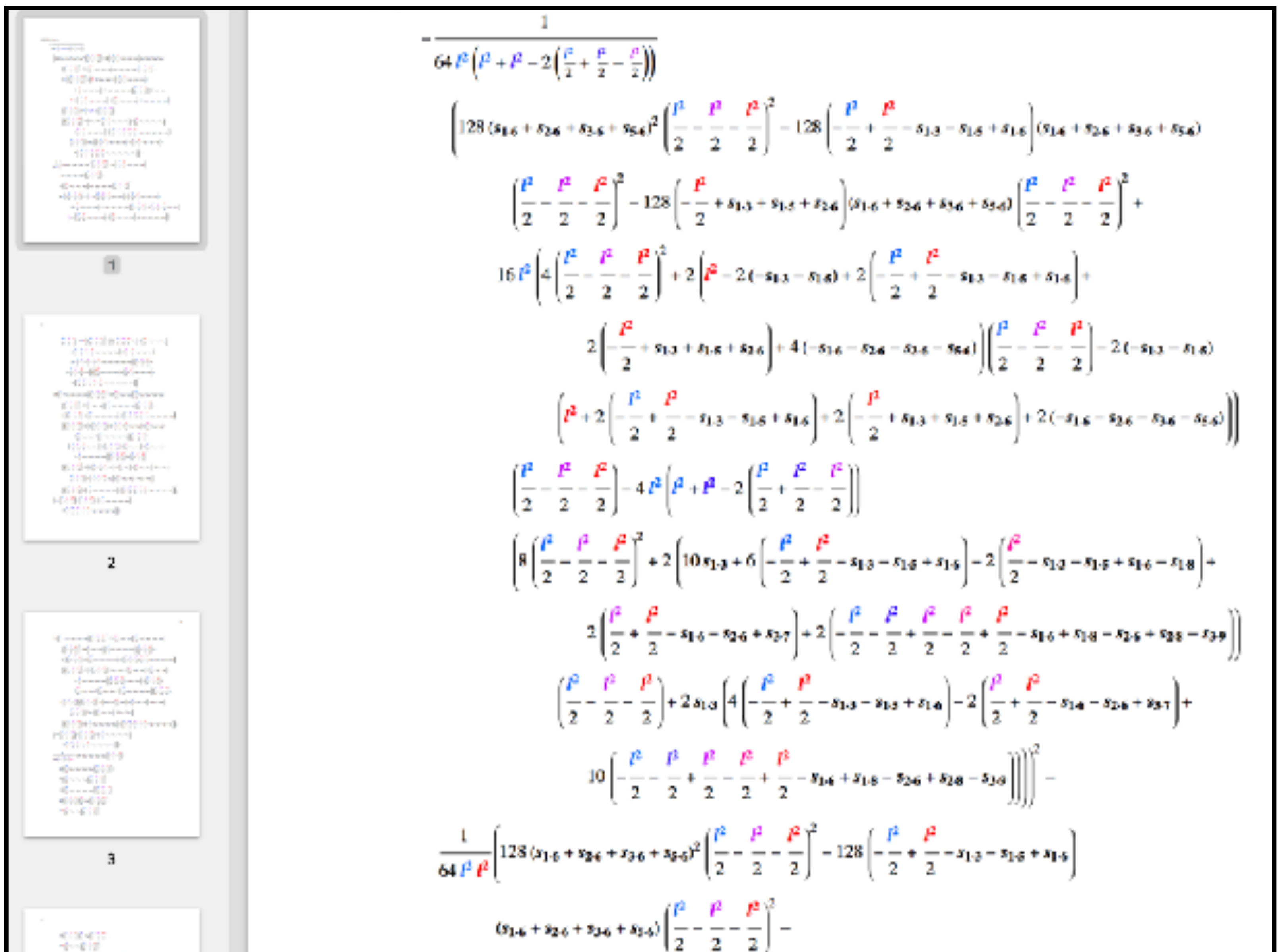
…26 pages, non-local

(truth)

5-loops, potential N2 contact



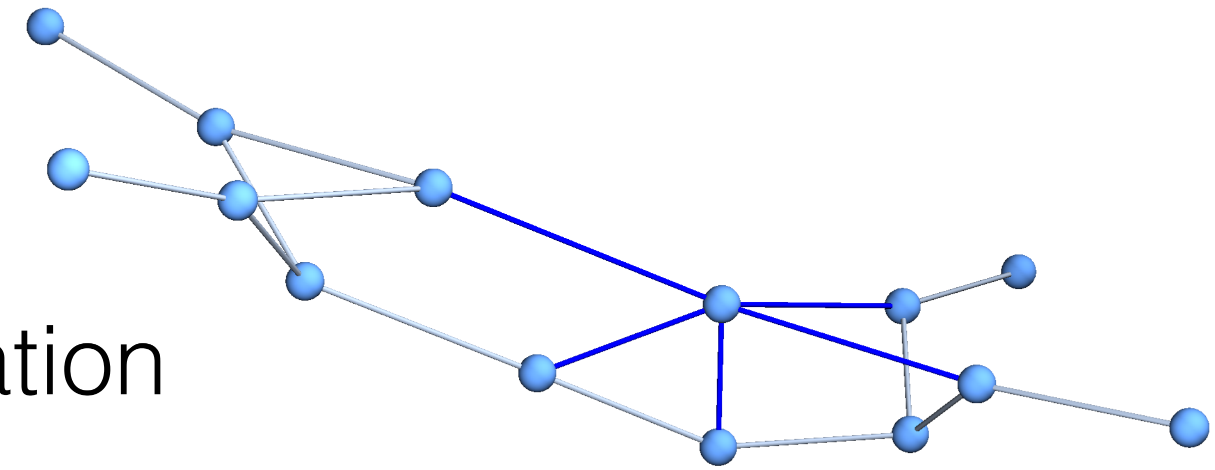
$$-\sum_{\mathbf{g}\in\mathrm{cut}} \frac{\mathbf{n_g}^2}{\mathbf{d_g}}$$



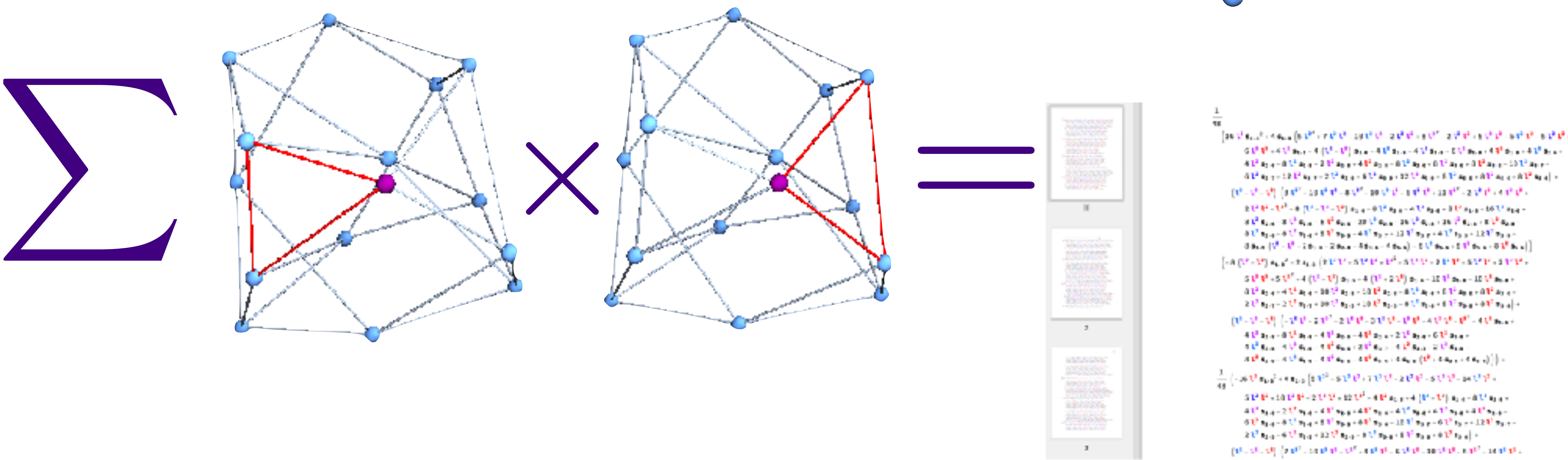
…26 pages

(double copy of cubic sYM)

5-loops, potential N2 contact

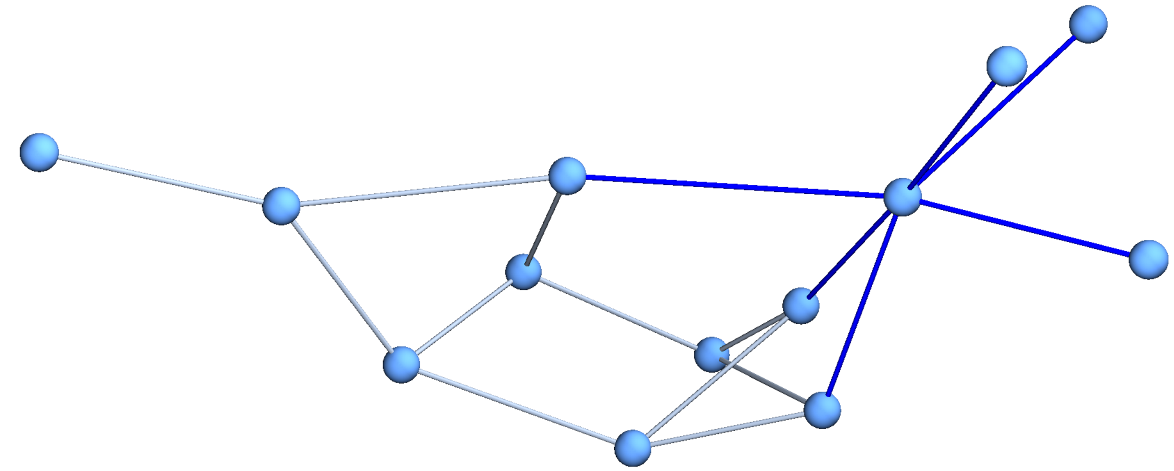


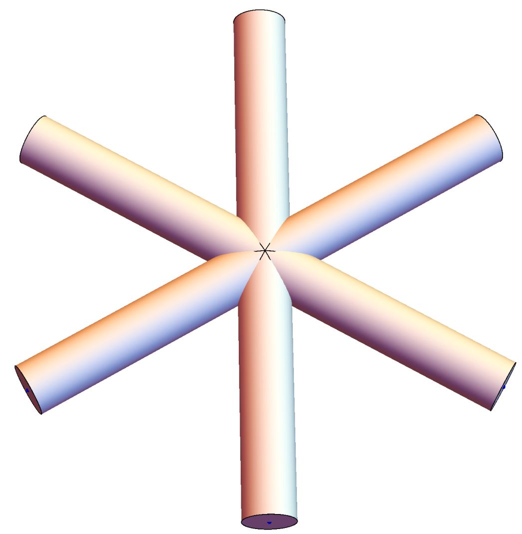
Contact / Missing Information



you just write it down!

5-loops, potential N3 contact

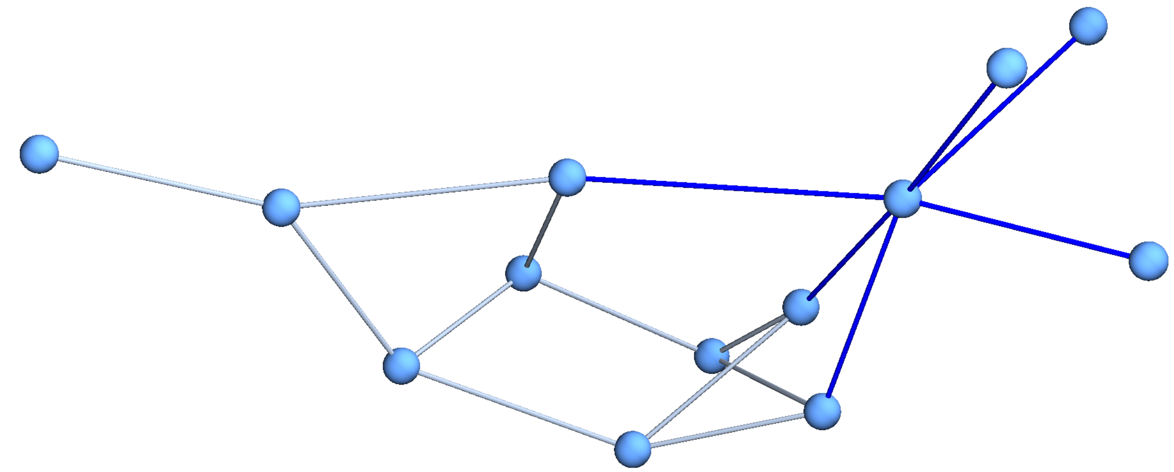




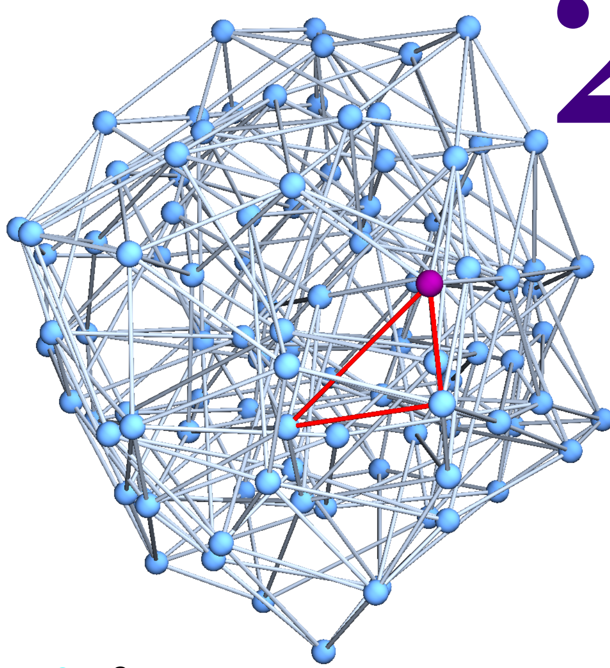
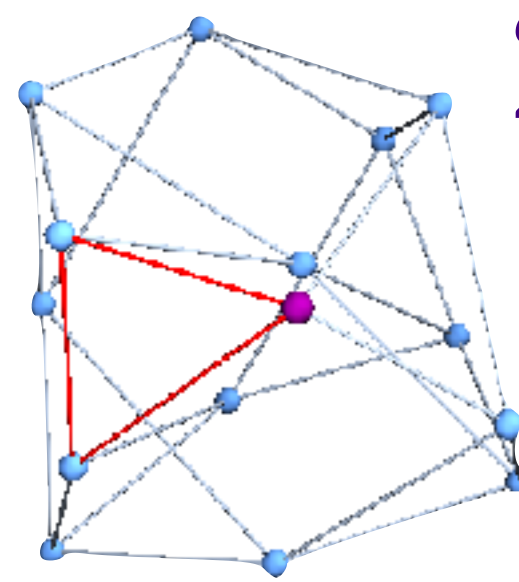
$$N^3\text{–contact} = \text{off shell}\left[\left(\text{truth} - \sum_{\mathrm{g}\in\mathrm{cut}} \frac{\mathrm{n_g}^2}{\mathrm{d_g}} - \sum_{\mathrm{g}\in\mathrm{N^2contacts}} \frac{\mathrm{N_g}}{\mathrm{d_g}}\right)\Bigg|_{\mathrm{cut}}\right]$$

$$= \text{off shell}\left[\left(\sum_{g\in\mathrm{cut}} \frac{\mathring{n}_g{}^2}{d_g} - \sum_{g\in\mathrm{cut}} \frac{n_g{}^2}{d_g} - \sum_{g\in N^2\mathrm{contacts}} \frac{N_g}{d_g}\right)\Bigg|_{\mathrm{cut}}\right]$$

5-loops, potential N3 contact

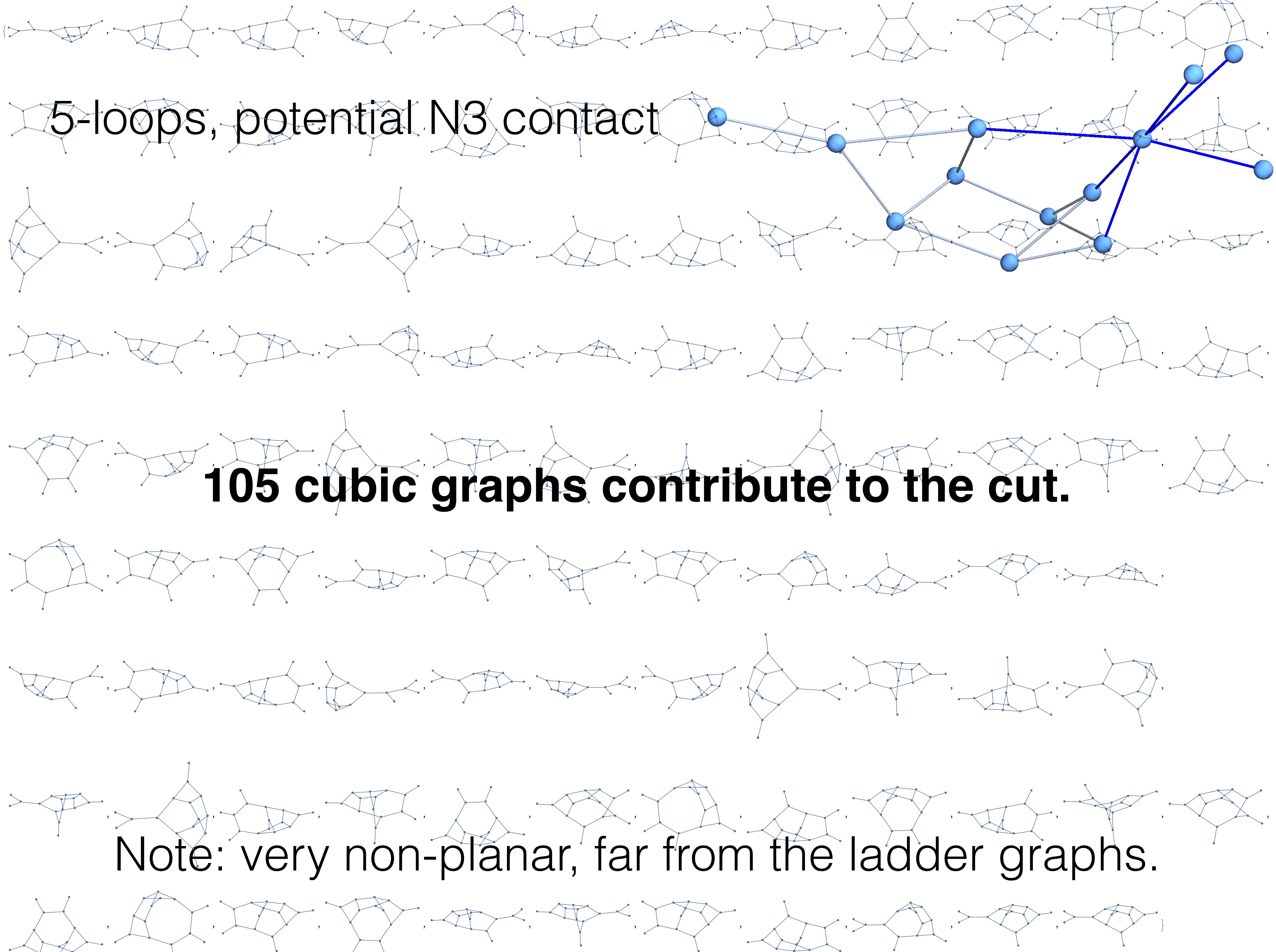


Contact / Missing Information you just write down:

$$\sum$$  $$^{2} \; - $$  $$^{2} \; =$$

$$
\begin{aligned}
&-\left(\ell^2-\ell^2-\ell^2+\ell^2\right)^2\\
&\Big(4\,{\ell^2}^2\,\ell^2-10\,\ell^2\,{\ell^2}^2+4\,{\ell^2}^3+{\ell^2}^2\,\ell^2+4\,\ell^2\,\ell^2\,\ell^2-5\,{\ell^2}^2\,\ell^2+2\,\ell^2\,{\ell^2}^2+{\ell^2}^3+{\ell^2}^2\,\ell^2+4\,\ell^2\,\ell^2\,\ell^2-5\,{\ell^2}^2\,\ell^2+\\
&\quad 4\,\ell^2\,\ell^2\,\ell^2+3\,{\ell^2}^2\,\ell^2+2\,\ell^2\,{\ell^2}^2+3\,\ell^2\,{\ell^2}^2+{\ell^2}^3+{\ell^2}^2\left(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)+{\ell^2}^2\left(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)-\\
&\quad 2\left({\ell^2}^2-{\ell^2}^2+3\,\ell^2\left(\ell^2+\ell^2\right)-2\,\ell^2\left(\ell^2+\ell^2\right)+2\left(\ell^2+\ell^2\right)^2\right)\ell^2+\\
&\quad \left(4\,\ell^2-4\,\ell^2+5\left(\ell^2+\ell^2\right)\right){\ell^2}^2-2\,{\ell^2}^3+\ell^2\Big(7\,{\ell^2}^2-2\,\ell^2\,\ell^2-2\,{\ell^2}^2-2\,\ell^2\,\ell^2-4\,\ell^2\,\ell^2-2\,{\ell^2}^2+\\
&\qquad 2\,\ell^2\left(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)-2\,\ell^2\left(3\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)-2\left(\ell^2-3\left(\ell^2+\ell^2\right)\right)\ell^2-4\,{\ell^2}^2\Big)-\\
&\quad \ell^2\left(-7\,{\ell^2}^2+2\,\ell^2\left(3\,\ell^2+\ell^2+\ell^2-2\,\ell^2\right)+2\left(\ell^2+\ell^2-2\,\ell^2\right)\left(\ell^2+\ell^2-\ell^2\right)+2\,\ell^2\left(\ell^2+\ell^2+\ell^2\right)\right)\Big)
\end{aligned}
$$

5-loops, potential N3 contact



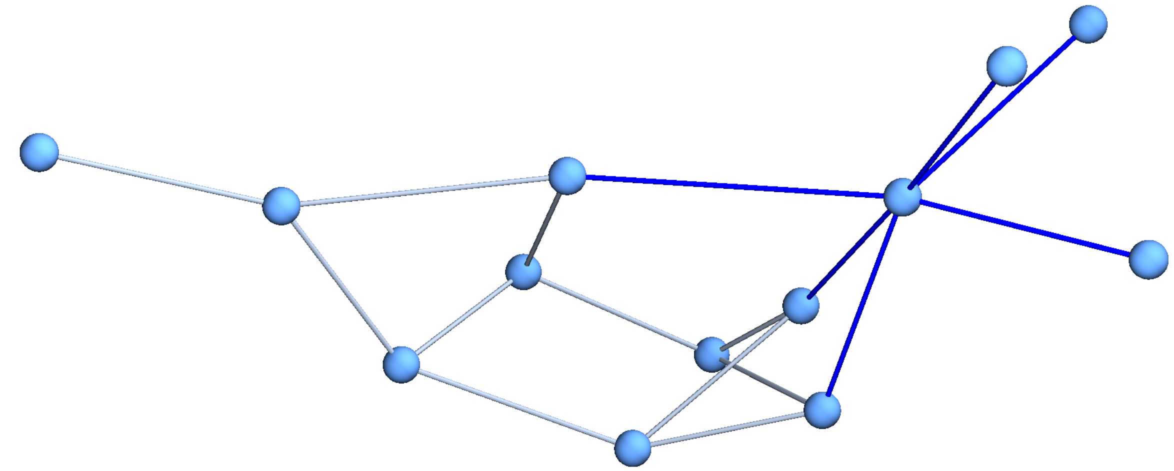
105 cubic graphs contribute to the cut.

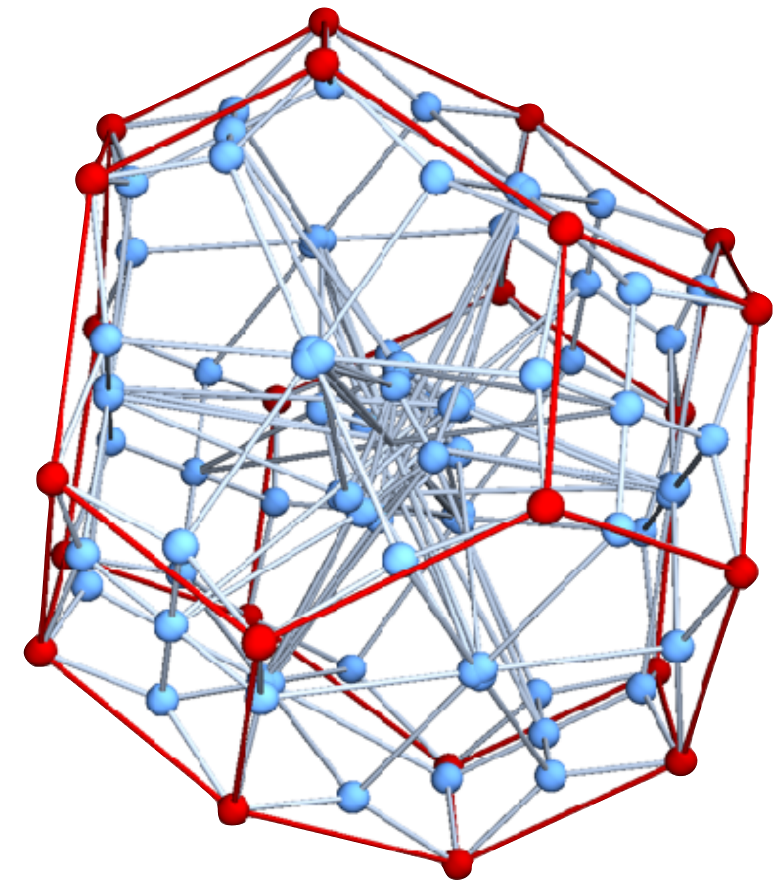
Note: very non-planar, far from the ladder graphs.

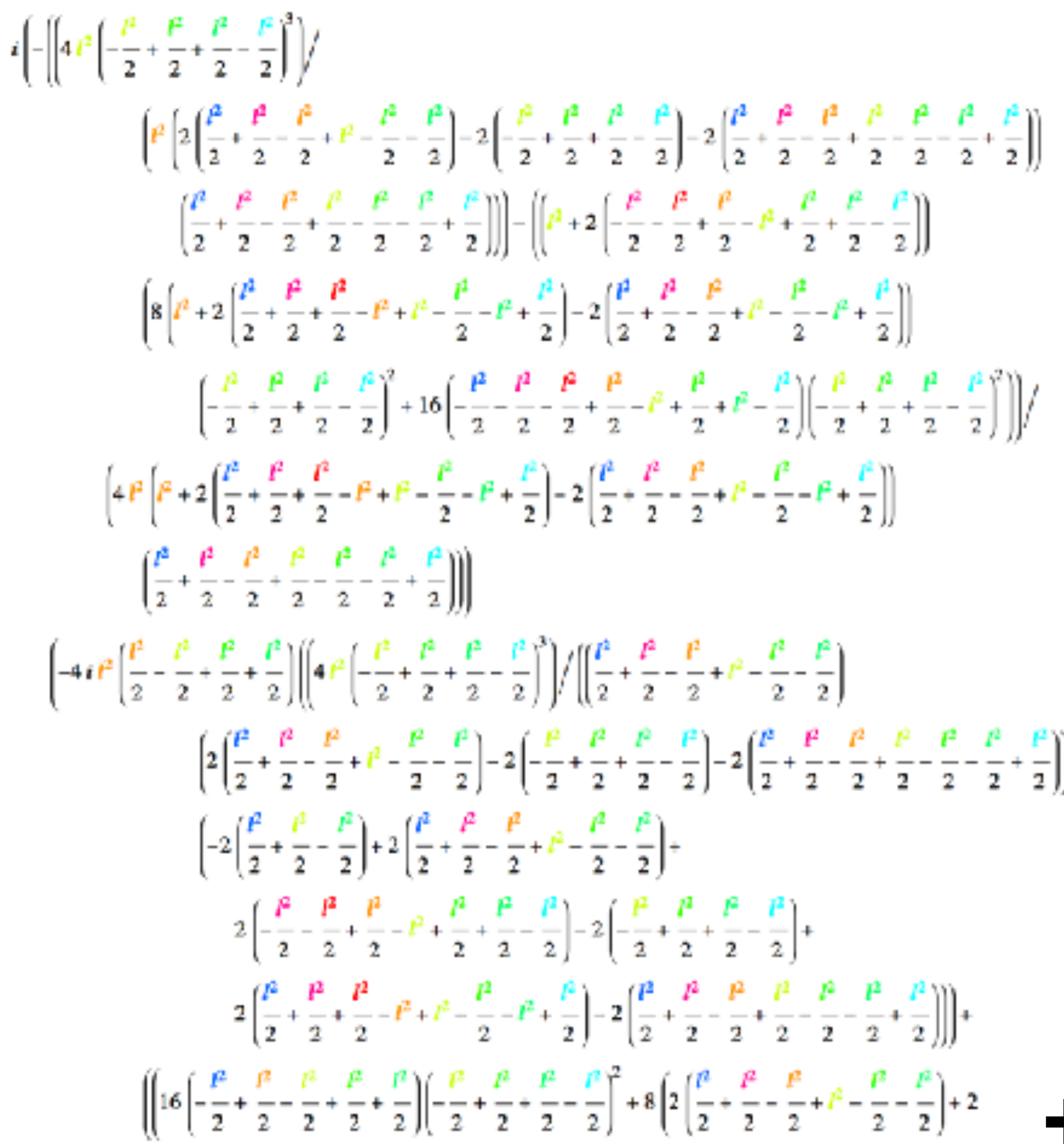
(cubic graphs)

5-loops, potential N3 contact

$$\mathrm{truth}|_{\mathrm{cut}} = \sum_{g \in \mathrm{cut}} \frac{\mathring{n}_g{}^2}{d_g}$$







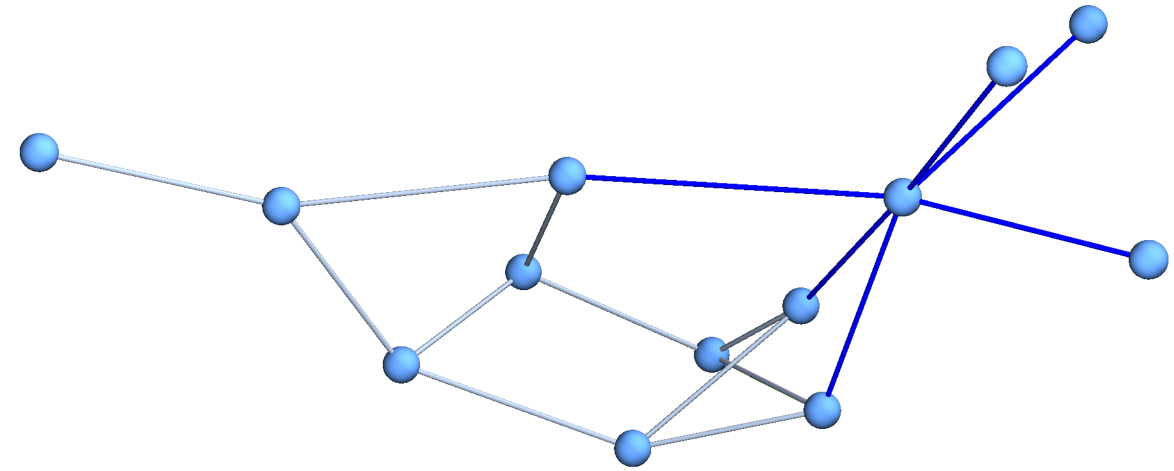
(truth)

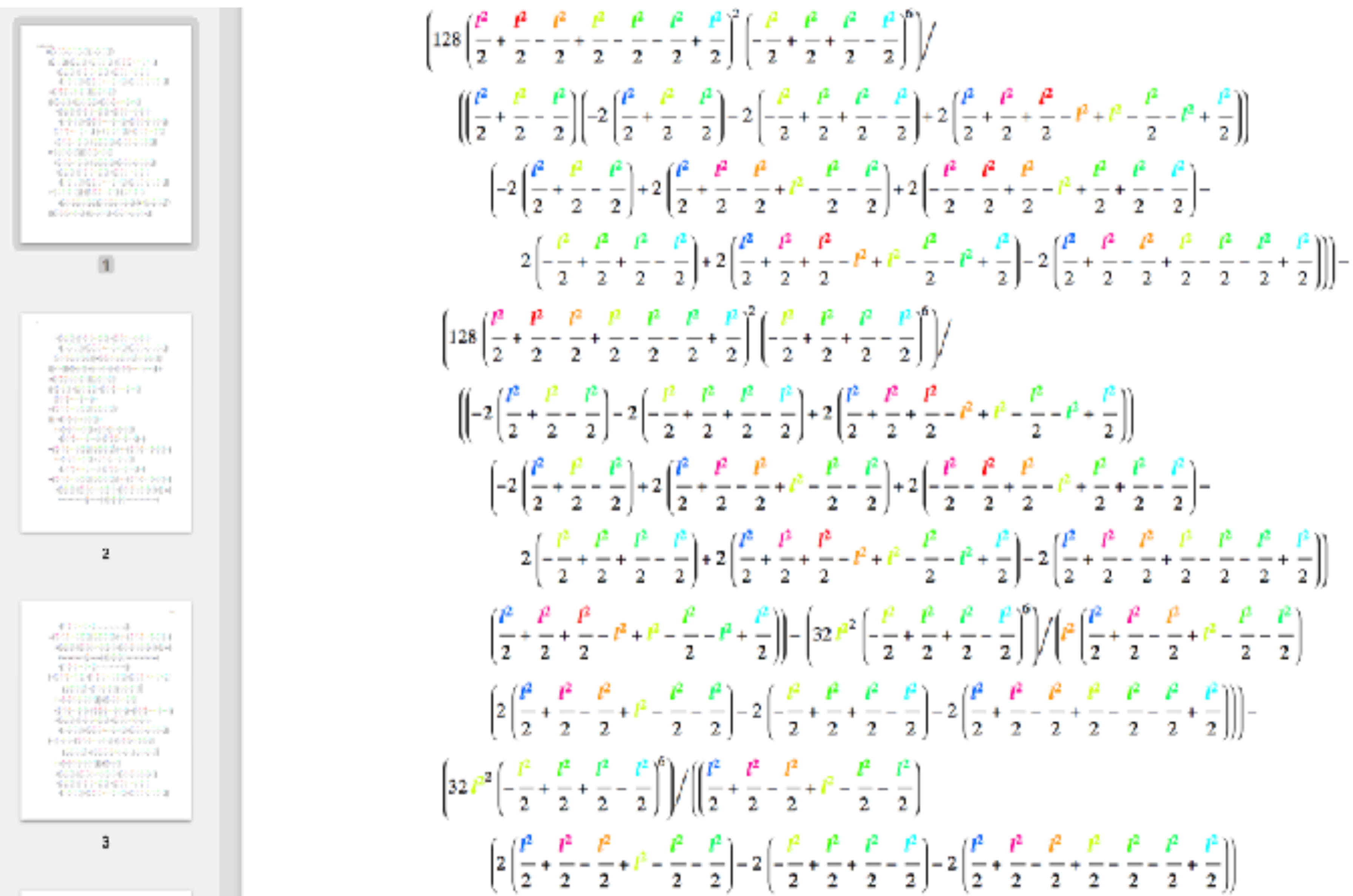
KLT→non-local c/k numerators

+30 pages, non-local

5-loops, potential N3 contact

$$-\sum_{\mathbf{g}\in\mathrm{cut}} \frac{{\mathbf{n_g}}^{2}}{\mathbf{d_g}}$$



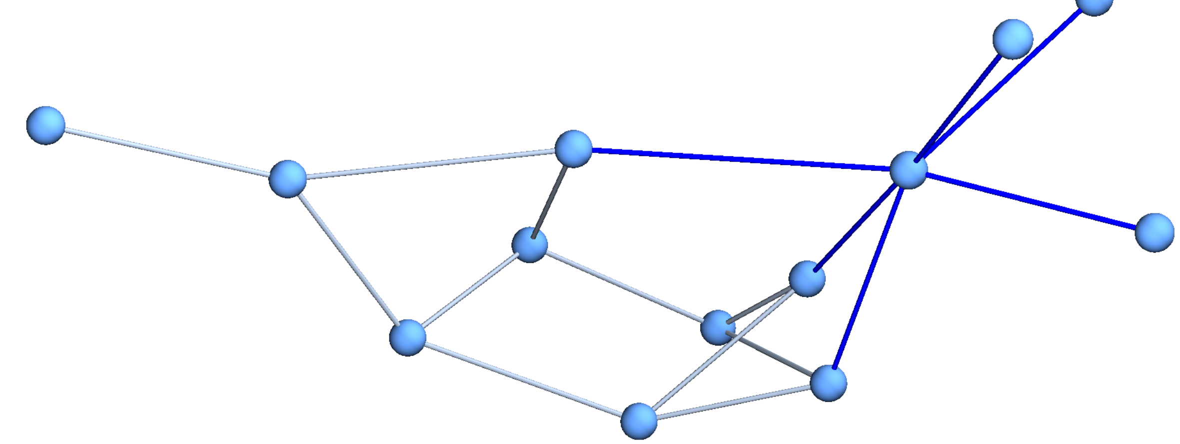


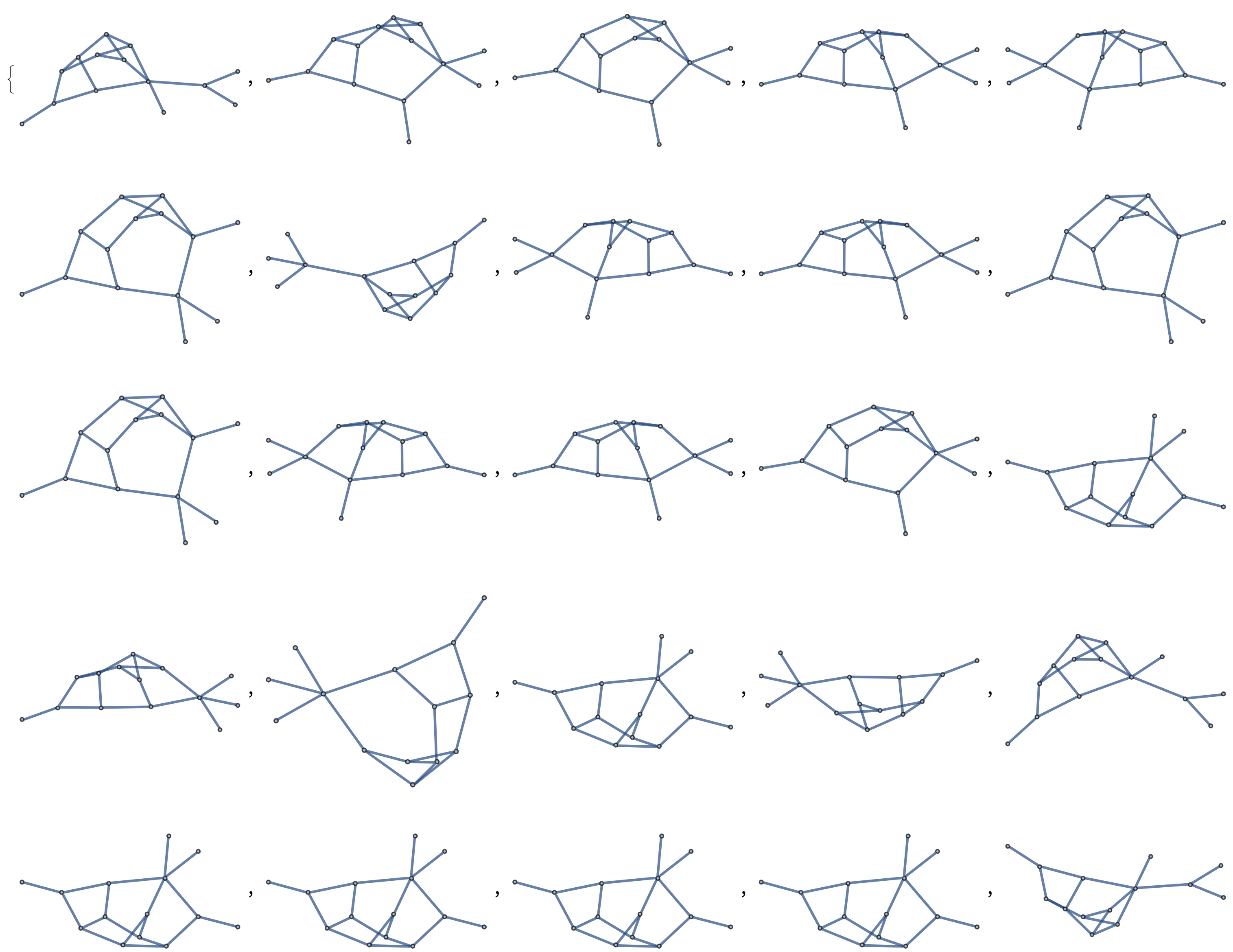
…35 pages

(double copy of cubic YM)

5-loops, potential N3 contact

$$\sum_{g \in N^2 \text{contacts}} \frac{N_g}{d_g}$$

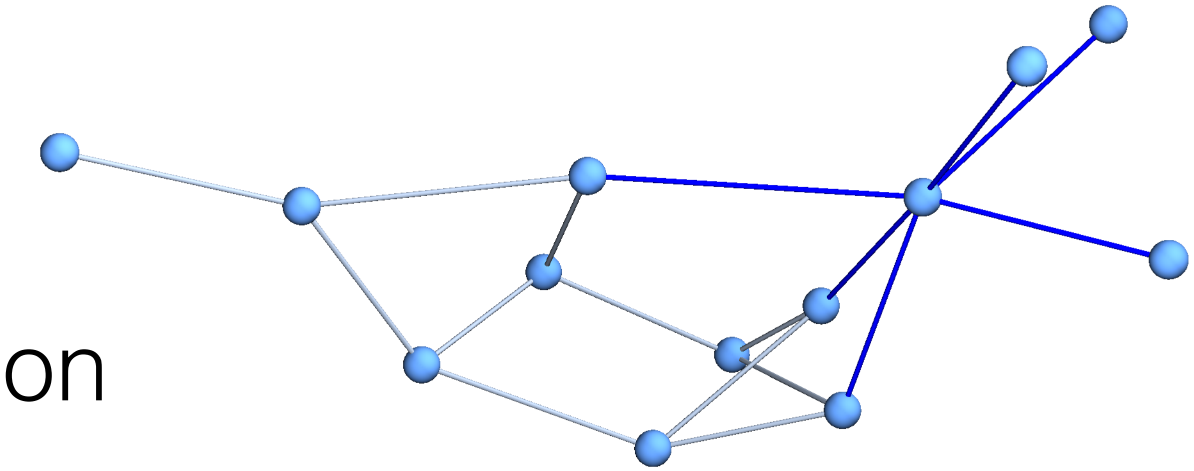




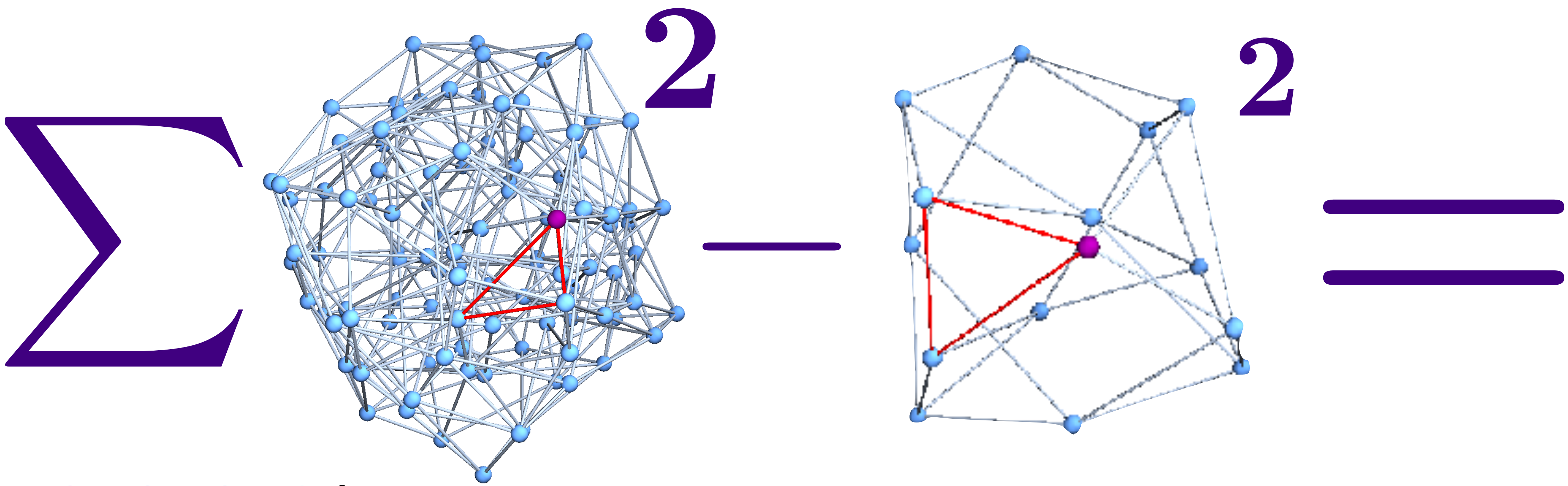
25 N2 contacts

(Necessary N2 contacts)

5-loops, potential N3 contact



Contact / Missing Information



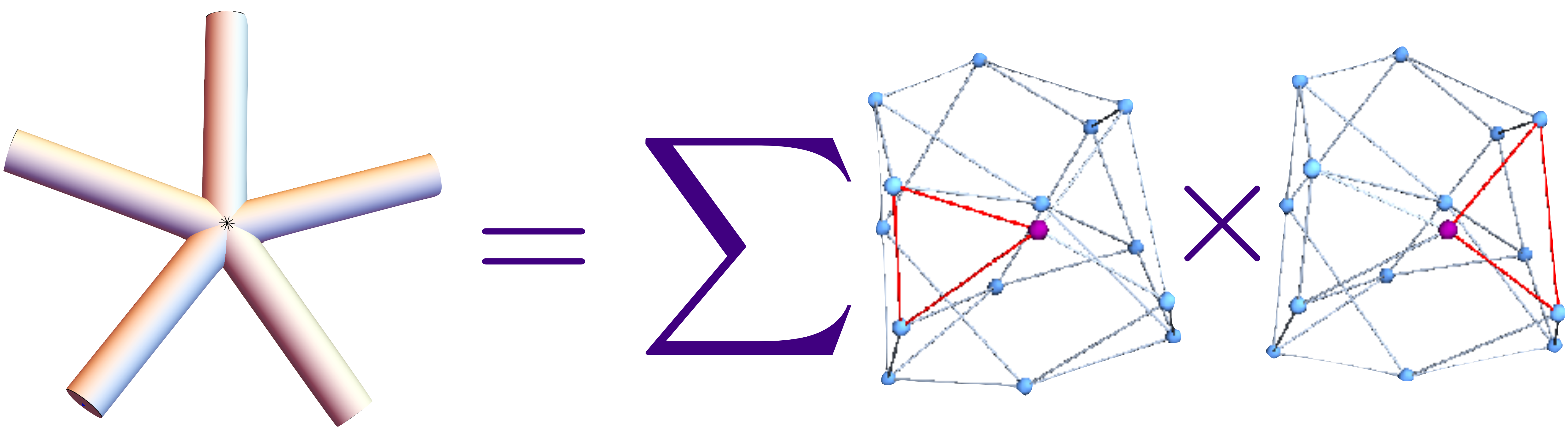
$$
\begin{aligned}
&-(\ell^2-\ell^2-\ell^2+\ell^2)^2\\
&\quad(4\,{\ell^2}^2\,\ell^2-10\,\ell^2\,{\ell^2}^2+4\,{\ell^2}^3+{\ell^2}^2\,\ell^2+4\,\ell^2\,\ell^2\,\ell^2-5\,{\ell^2}^2\,\ell^2+2\,\ell^2\,{\ell^2}^2+{\ell^2}^3+{\ell^2}^2\,\ell^2+4\,\ell^2\,\ell^2\,\ell^2-5\,{\ell^2}^2\,\ell^2+\\
&\quad\quad 4\,\ell^2\,\ell^2\,\ell^2+3\,{\ell^2}^2\,\ell^2+2\,\ell^2\,{\ell^2}^2+3\,\ell^2\,{\ell^2}^2+{\ell^2}^3+{\ell^2}^2\,(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2)+{\ell^2}^2\,(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2)-\\
&\quad\quad 2\,({\ell^2}^2-{\ell^2}^2+3\,\ell^2\,(\ell^2+\ell^2)-2\,\ell^2\,(\ell^2+\ell^2)+2\,(\ell^2+\ell^2)^2)\,\ell^2+\\
&\quad\quad\,(4\,\ell^2-4\,\ell^2+5\,(\ell^2+\ell^2))\,{\ell^2}^2-2\,{\ell^2}^3+\ell^2\,(7\,{\ell^2}^2-2\,\ell^2\,\ell^2-2\,{\ell^2}^2-2\,\ell^2\,\ell^2-4\,\ell^2\,\ell^2-2\,{\ell^2}^2+\\
&\quad\quad\quad 2\,\ell^2\,(2\,\ell^2+\ell^2+\ell^2-2\,\ell^2)-2\,\ell^2\,(3\,\ell^2+\ell^2+\ell^2-2\,\ell^2)-2\,(\ell^2-3\,(\ell^2+\ell^2))\,\ell^2-4\,{\ell^2}^2)-\\
&\quad\quad \ell^2\,(-7\,{\ell^2}^2+2\,\ell^2\,(3\,\ell^2+\ell^2+\ell^2-2\,\ell^2)+2\,(\ell^2+\ell^2-2\,\ell^2)\,(\ell^2+\ell^2-\ell^2)+2\,\ell^2\,(\ell^2+\ell^2+\ell^2)))
\end{aligned}
$$

Summary

c/k + gen. gauge transforms $\longrightarrow$

can directly **double-copy** **non-c/k representations**
resulting in add'l ***local*** higher-point contact terms

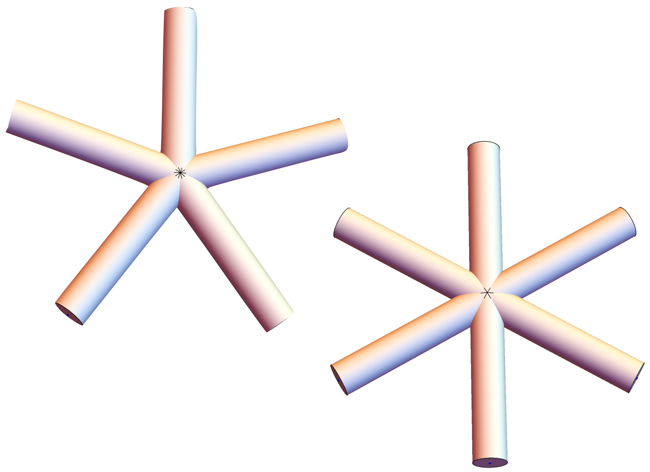
(something you can figure out more or less from tree-level considerations)



Gen. Double Copy Summary

- **Control through 5-pt => all N^2 cuts**
- **Control through 6-pt => all N^3 cuts**

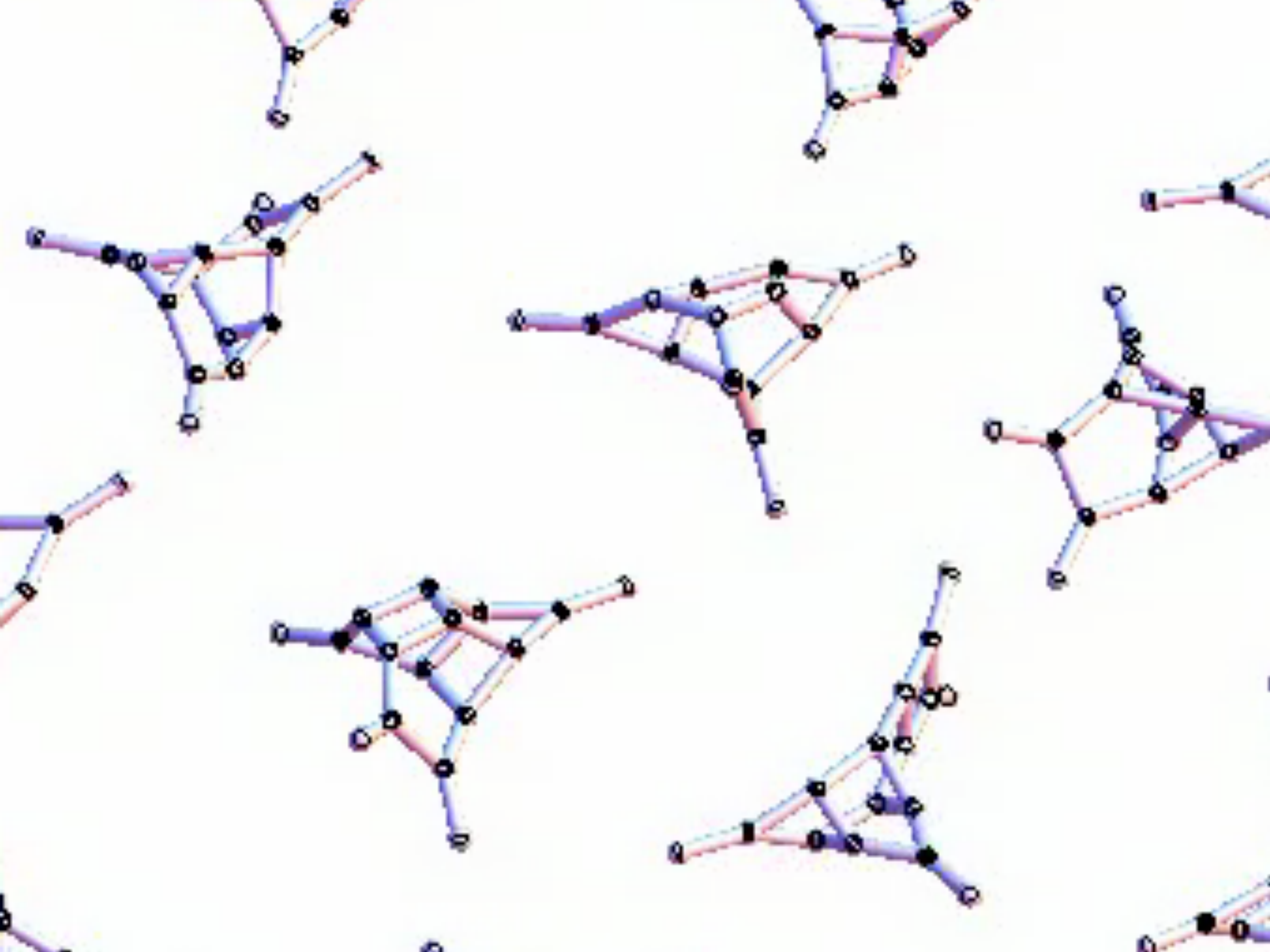
… and so on



Multiplicity and loop-order independent!

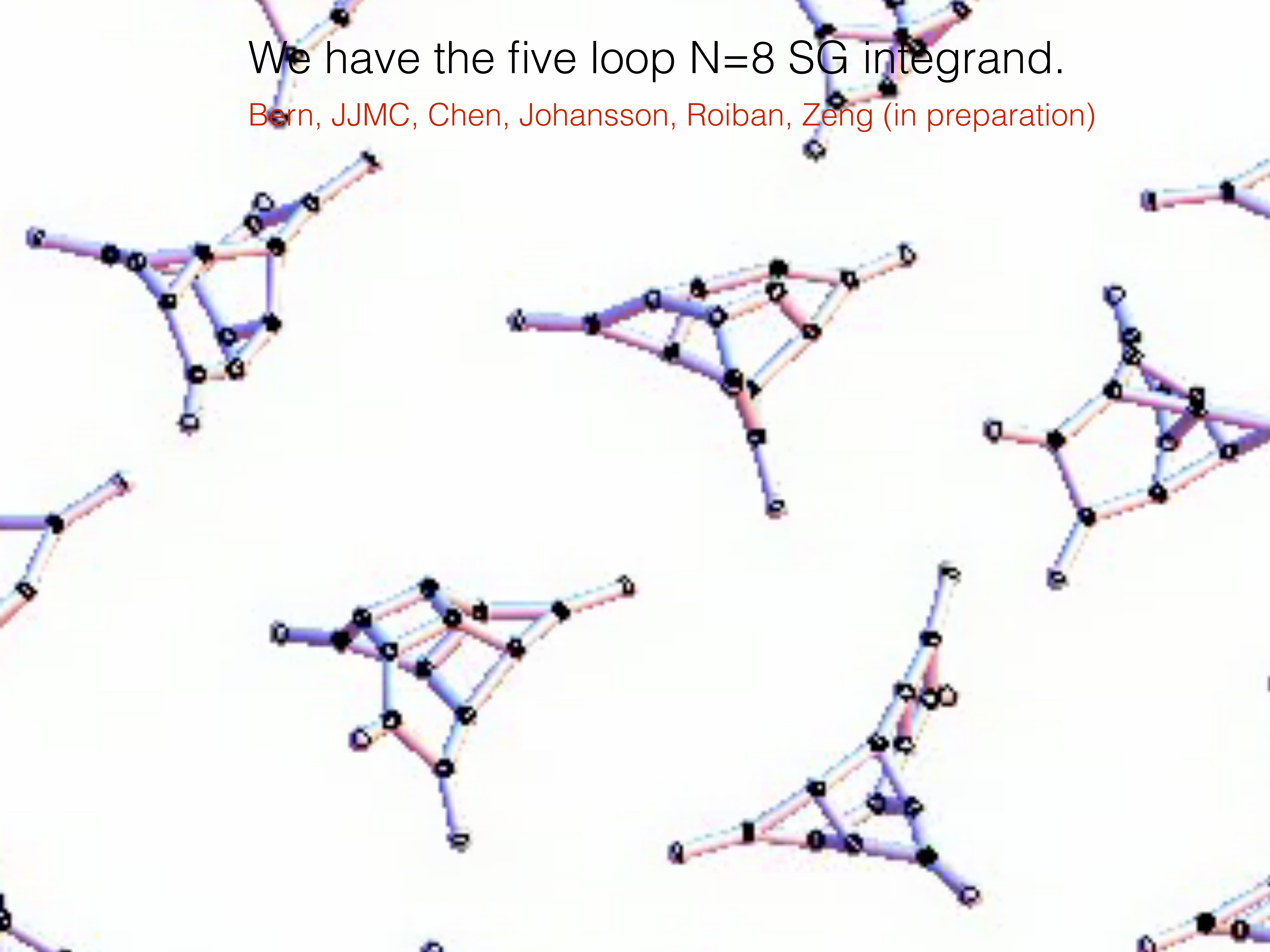
works for any double-copy theory b/c of single-copy properties (sYM/NLSM/Z-theory/…)

provides a *simple* path forward for tough to crack multi loop double-copy constructions…



We have the five loop N=8 SG integrand.

Bern, JJMC, Chen, Johansson, Roiban, Zeng (in preparation)



Integrand has passed many non-trivial tests

✓ N^7 cuts verified in *independent* checks— no missing data

✓ D=22/5 top-level UV-finite **(expected by everyone. anything else would've likely meant glitch in the calculation)**

Stay tuned for behavior in D=24/5

(may have news by Radu's talk next week at String Theory and QG, Ascona, Switzerland)

Reminder:
D=24/5 at 5-loops is the first potential critical dimension challenging N=8 SG having the same perturbative UV behavior as N=4 sYM

$$D_c^{\mathcal{N}=4\ \mathrm{SYM}}(L) = 4 + 6/L \qquad D_c^{\mathcal{N}=8\ \mathrm{SG}}(5) = ???$$

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Playful Construction

the game of Scattering Amplitudes

QFT, NR-QM, String Theory, Classical Physics…

IN

free states

(no interactions)

$$[\mathrm{S-matrix}]$$

(all the interactions!)

OUT

free states

(no interactions)

the game of Scattering Amplitudes

QFT, NR-QM, String Theory, Classical Physics…

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Symmetries, Scaling, Geometry

Playful Construction Using Double-Copy as a Principle

$$U = V \otimes W$$

1) Take theories that exhibit Double-Copy (e.g. both factors obey same algebra), strip one "factor" replace with something else that obeys the same algebra.

cf. all the E-YM work of **Chiodaroli, Gunaydin, Johansson,Roiban**

2) Start with generic ansatze, constrain engineering weight, impose algebra.

cf. explorations by **Boels; Trnka, Arkani-Hamed, Rodina; Rodoina**

Example of playful construction

Open String:

Broedel, Schlotterer, Stieberger (2013)

$$\alpha' \otimes \text{spin-1}$$

Chan-Paton Stripped open string

$$\mathrm{OS}(P(1,\ldots,n)) = Z_P \otimes A$$

Doubly-ordered Z-functions: obey monodromy relations on P

But obey field theory (n-3)! relations on it's field theory KLT with Yang-Mills A.

$$Z_P(q_1,q_2,\ldots,q_n) \equiv \alpha'^{\,n-3} \int_{-\infty\le z_{P(1)}\le z_{P(2)}\le\ldots\le z_{P(n)}\le\infty} \frac{\mathrm{dz}_1\ \mathrm{dz}_2\ \cdots\ \mathrm{dz}_n}{\mathrm{vol}(SL(2,\mathbb{R}))} \frac{\prod_{i<j}^{n} |z_{ij}|^{\alpha' s_{ij}}}{z_{q_1q_2} z_{q_2q_3}\ldots z_{q_{n-1}q_n} z_{q_nq_1}}\,.$$

Take seriously Z-functions as encoding predictions for some (effective) field theory.

JJMC, Mafra, Schlotterer (2016)

Replace sYM in OS with a color-stripped bi-adjoint Scalar

$$\mathrm{OS}(P(1,\ldots,n)) = Z_P \otimes A$$

$$\mathbf{Z}(P(1,\ldots,n)) = Z_P \otimes C$$

Dressing with Chan-Paton factors renders something that has the possibility of being interpreted as **doubly-colored** **field-theory scattering amplitudes**: we call it **Z theory**.

Color-Stripped (Chan-Paton dressed) tree-level Z-amplitude:

$$\mathcal{Z}\left(\tau(1,2,\ldots,n)\right) \equiv \sum_{P\in S_{n-1}} \mathrm{Tr}(t^1 t^{P(2)} \cdots t^{P(n)}) Z_{1,P}(\tau(1,2,\ldots,n))$$

Color Stripped (or Color-Ordered) tree-level Z-amplitude

$$\mathcal{Z}\left(\tau(1,2,\ldots,n)\right) \equiv \sum_{P\in S_{n-1}} \mathrm{Tr}(t^1 t^{P(2)} \cdots t^{P(n)}) Z_{1,P}(\tau(1,2,\ldots,n))$$

Now look at: $\mathcal{Z} \otimes C$

"Low energy limit" -> bi-adjoint scalar: $\displaystyle\sum_g \frac{\tilde{c}(g)c(g)}{D(g)}$

Higher order in α': $\displaystyle\sum_g \frac{z(g)c(g)}{D(g)}$

both CP-weights and kinematics conspire in z(g) to obey algebraic identities.

Color Stripped (or Color-Ordered) tree-level Z-amplitude

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Play with CP factors. Abelian CP generators means no-longer a bi-colored scalar.

$$\mathcal{Z}_\times \otimes C = \sum_g \frac{z_\times(g)c(g)}{D(g)}$$

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Low energy limit: $\lim_{\alpha'\to 0} \mathcal{Z}_\times \otimes C \to \mathrm{NLSM}$

JJMC, Mafra, Schlotterer (2016)

$$\mathcal{L}_{\mathrm{NLSM}} = \frac{1}{2}\mathrm{Tr}\left\{\partial_\mu\varphi \frac{1}{1-\varphi^2}\partial^\mu\varphi \frac{1}{1-\varphi^2}\right\}$$

Color Stripped (or Color-Ordered) tree-level Z-amplitude

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Abelian Z: $\displaystyle \lim_{\alpha'\to 0} \mathcal{Z}_\times \otimes C \to \mathrm{NLSM}$

JJMC, Mafra, Schlotterer (2016)

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(Cayley Parameterization)

Completely different story for the same prediction.
Chen, Du '13 showed obeyed (n-3)! relns. **Cheung,Shen '16** found an action that directly gives the color-dual kinematic story.

$$\mathcal{L}_{\mathrm{NLSM}} = Z^{a\mu}\Box X^a_\mu + \frac{1}{2}Y^a\Box Y^a - f^{abc}\left(Z^{a\mu}Z^{b\nu}X^c_{\mu\nu} + Z^{a\mu}(Y^b\overleftrightarrow{\partial}_\mu Y^c)\right)$$

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Somehow abelianization is encoding a story related to SSB

Color Stripped (or Color-Ordered) tree-level Z-amplitude

$$\mathcal{Z}\left(\tau(1,2,\ldots,n)\right) \equiv \sum_{P\in S_{n-1}} \mathrm{Tr}(t^1 t^{P(2)} \cdots t^{P(n)}) Z_{1,P}(\tau(1,2,\ldots,n))$$

Abelian Z: $$\lim_{\alpha'\to 0} \mathcal{Z}_\times \otimes C \to \mathrm{NLSM}$$

JJMC, Mafra, Schlotterer (2016)

Let's look at it's other copy, back to the superstring:

Abelian Open Superstring: $$\left[\left(\lim_{\alpha'\to 0} \mathcal{Z}_\times\right) \otimes A\right] \to [\mathrm{NLSM} \otimes A]$$

He, Liu, Wu '16; Cachazo, Cha, Mizera '16 told us:

$$[\mathrm{NLSM} \otimes A] = \mathrm{SDBIVA}$$

For maximal sYM, 16 linearly realized, 16 nonlinearly realized,

Bergshoeff, Coomans, Kallosh, Shahbazi Van Proeyen '13

$$U = V \otimes W$$

Order by order in higher derivatives can play all these constructive games (and more!) using ansatze with the correct ingredients.

Open question as to what theories can be understood as nontrivial double copies and what their dual-stories are.

The amplitudes can still be interesting even if crazy from some perspectives.

Clearly lots of fun games yet to be played — very much an open field.

Classical Solutions

Do classical solutions double-copy?

(See also work of Saotome & Akhoury and combinations of Anastasiou, Borsten, Duff, Hughes, Nagy)

Monteiro, O'Connell, and White, along with increasing list of collaborators are amassing evidence that the answer is **yes**, at least for a certain class of solutions.

Monteiro, O'Connell, White '14
Luna, Monteiro, O'Connell, White '15
Luna, Monteiro, Nicholson, O'Connell, White '16

for general perturbative solutions:

Goldberger, Ridgeway '16
Luna, Monteiro, Nicholson, Ochirov, O'Connell, Westerberg, White '16
Goldberger, Prabhu, Thompson '17

scattering on sandwich plane-waves:

Adamo, Casali, Mason, Nekovar '17

Monteiro, O'Connell, and White

3-pt Scattering Amplitude

$$\frac{\mathbf{c(g)n(g)}}{\mathbf{d(g)}} \xrightarrow{\text{Double Copy}} \frac{\mathbf{n(g)n(g)}}{\mathbf{d(g)}}$$

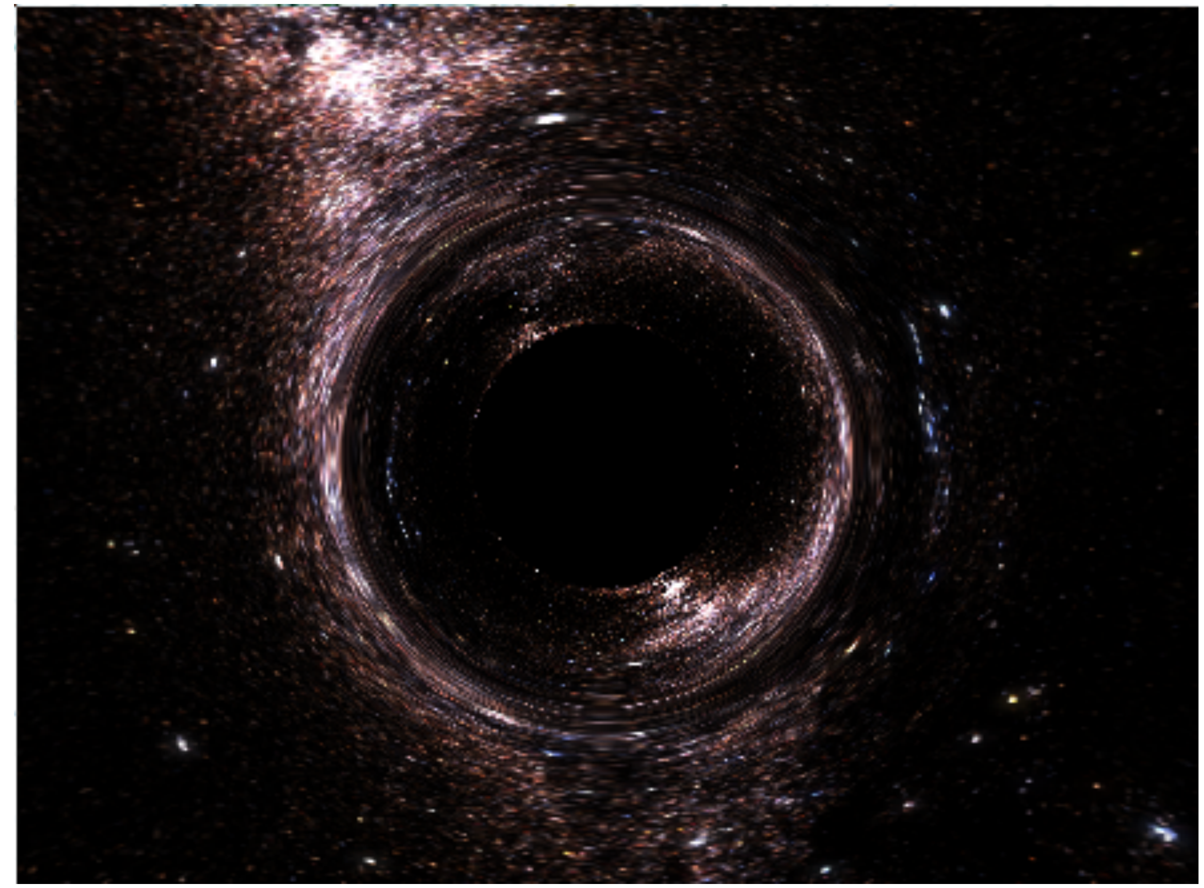
Classical Solutions (in a special class called Kerr-Schild)

$$\mathbf{A^a_m u} = \mathbf{c^a k}_\nu \phi \xrightarrow{\text{Double Copy}} \mathbf{g}_{\mu\nu} - \eta_{\mu\nu} = \mathbf{k}_\mu \mathbf{k}_\nu \phi$$

Schwarzschild

$$\mathbf{g}_{\mu\nu} - \eta_{\mu\nu} = \frac{\mathbf{2GM}}{\mathbf{r}}\mathbf{k}_\mu \mathbf{k}_\nu$$

$$\mathbf{k}_\mu = \{\mathbf{1}, \hat{\mathbf{r}}\}$$

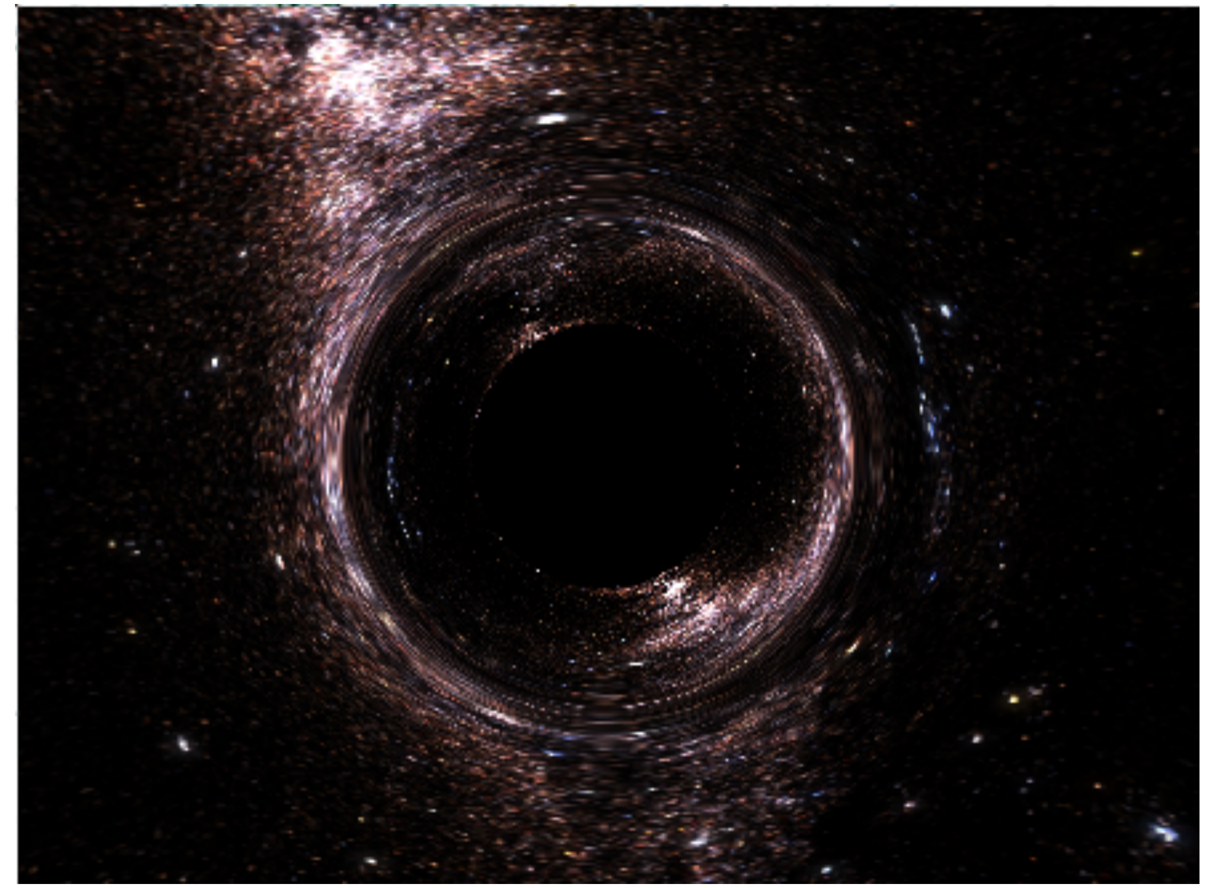


Monteiro, O'Connell, and White

Schwarzschild

$$\mathbf{g}_{\mu\nu} - \eta_{\mu\nu} = \frac{\mathbf{2GM}}{\mathbf{r}}\mathbf{k}_{\mu}\mathbf{k}_{\nu}$$

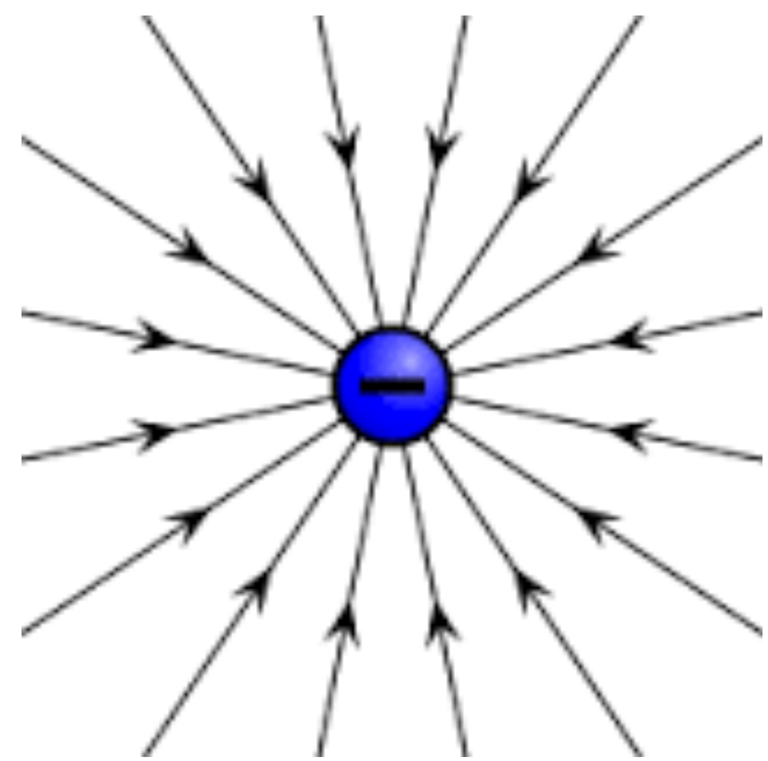
$$\mathbf{k}_{\mu} = \{\mathbf{1}, \hat{\mathbf{r}}\}$$



The double copy of

$$\mathbf{A}_{\mu} = \frac{\mathbf{2GM}}{\mathbf{r}}\mathbf{k}_{\mu}$$

abelianized point charge



Monteiro, O'Connell, and White

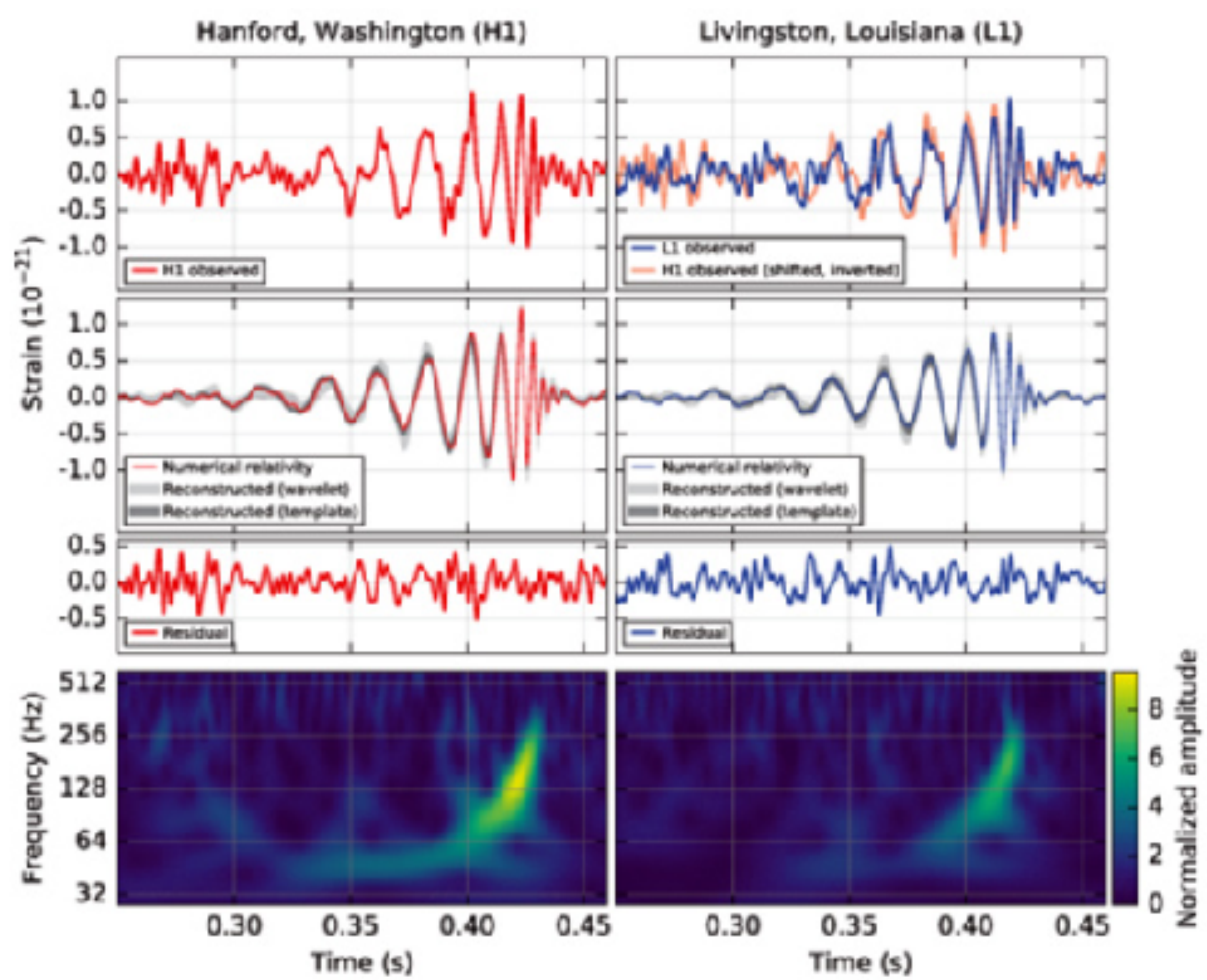
Classical gravity is a Double Copy?

Remind you of some of the double-copy positives:

+ Constrained solutions => can exploit for technical simplicity in prediction

+ Web of relationships between theories

Open question: how far can this go?



$\exists$ Tons of exciting stuff I haven't even had a chance to begin to talk about….

Beautiful body of work going into Solving and Understanding Properties of Scattering Eqns

…; Dolan, Goddard; Lam, Yao; Bjerrum-Bohr, Bourjaily, Damgaard, Feng; Du, Teng, Wu; Nandan, Pleua, Wormsbecher; He, Liu, Wu; …

Renewed interest in soft/colinear limits and EFT:

…; Cachazo, Strominger; Cheung, Kampf, Novotny, Shen, Trnka; Nandan, Plefka, Wormsbecher; Nandan, Pleua, Wormsbecher; He, Liu, Wu; Broedel, de Leeuw, Plefka , Rosso; Bern, Davies, Nohle ; Bern,Davies , Di Vecchia, Nohle; Golden, Spradlin; Di Vecchia, Marotta, Mojaza; Duo, Luo; Kallosh; Kallosh, Karlsson, Murli; Nandan, Plefka, Schlotterer, Wen;Klose, McLoughlin, Nandan, Plefka, Travaglini; Broedel, de Leeuw, Plefka, Rosso …

Universality in string interactions:

…; Huang, Schlotterer; Chepelev, Tseytlin; Caron-Huot,Komargodski, Sever, Zhiboedev; …

Non-planar on-shell diagrams:

…; Arkani-Hamed, Bourjaily, Cachazo, Postnikov, Trnka ; Herrmann, Trnka, Bourjaily ; Heslop, Lipstein ; Franco, Galloni, Penante, Wen ; Benincasa, Gordo; …

$\exists$ Tons of exciting stuff I haven’t even had a chance to begin to talk about….

Physical Understanding of Integrated Multiloop Gauge Amplitudes

…; Dixon, Hippel, McLeod, Trnka Caron-Huot; Dixon, McLeod, von Hippel; [Combinations of: {Drummond, Gloden, Goncharov,Papathanasiou,Parker,Paulos,Spradlin,Scherlis,Vergu, Volovich}],Del Duca, Duhr, Smirnov; Caron-Huot; Dixon, Drummond, Henn Dixon, Drummond, Henn; Caron-Huot, He; Dixon, Drummond, von Hippel, Pennington; …

Physical Aspects of Infinities in Gravity:

…; Bern, Edison, Kosower, Parra-Martinez; Bern, Chi, Dixon, Edison; Bern, Cheung, Chi, Davies, Dixon, Nohle; …

UV Properties & Anomalies in Lower SUSY SG:

…; Bern, Enciso, Parra-Martinez, Zeng; Bern, Davies, Nohle; Freedman, Kallosh, Murli, Van Proeyen,Yamada; Bern, Davies, Dennen; Kallosh; Bern, Davies, Dennen, AV. Smirnov, VA Smirnov; Bern, Davies, Tristan Dennen; Bern, Davies, Dennen, Y.T. Huang; JJMC, Kallosh, Tseytlyn, Roiban; …

Recent Integration innovations:

…; Bosma, Sogaard, Zhang; Gluza, Jelinski, Kosower; Georgoudis), Larsen, Zhang; Kosower; Mastrolia, Peraro, Primo; Remidd, Tancredi; Gehrmann, Henn, Presti; Baadsgaard, Bjerrum-Bohr, Bourjaily, Damgaard; Henn; Johansson Kosower, Larsen, Søgaard; Eden, VA Smirnov; Feng, Chang, Chen, Gu, Zhang; AV Smirnov; von Manteuffel, Schabinger; Caron-Huot, Henn; Johansson, Kosower, Larsen; Pierpaolo Mastrolia, Mirabella, Ossola, Peraro ; …

$\exists$ Tons of exciting stuff I haven't even had a chance to begin to talk about….

Integrability and Amplitudes and Correlation Functions

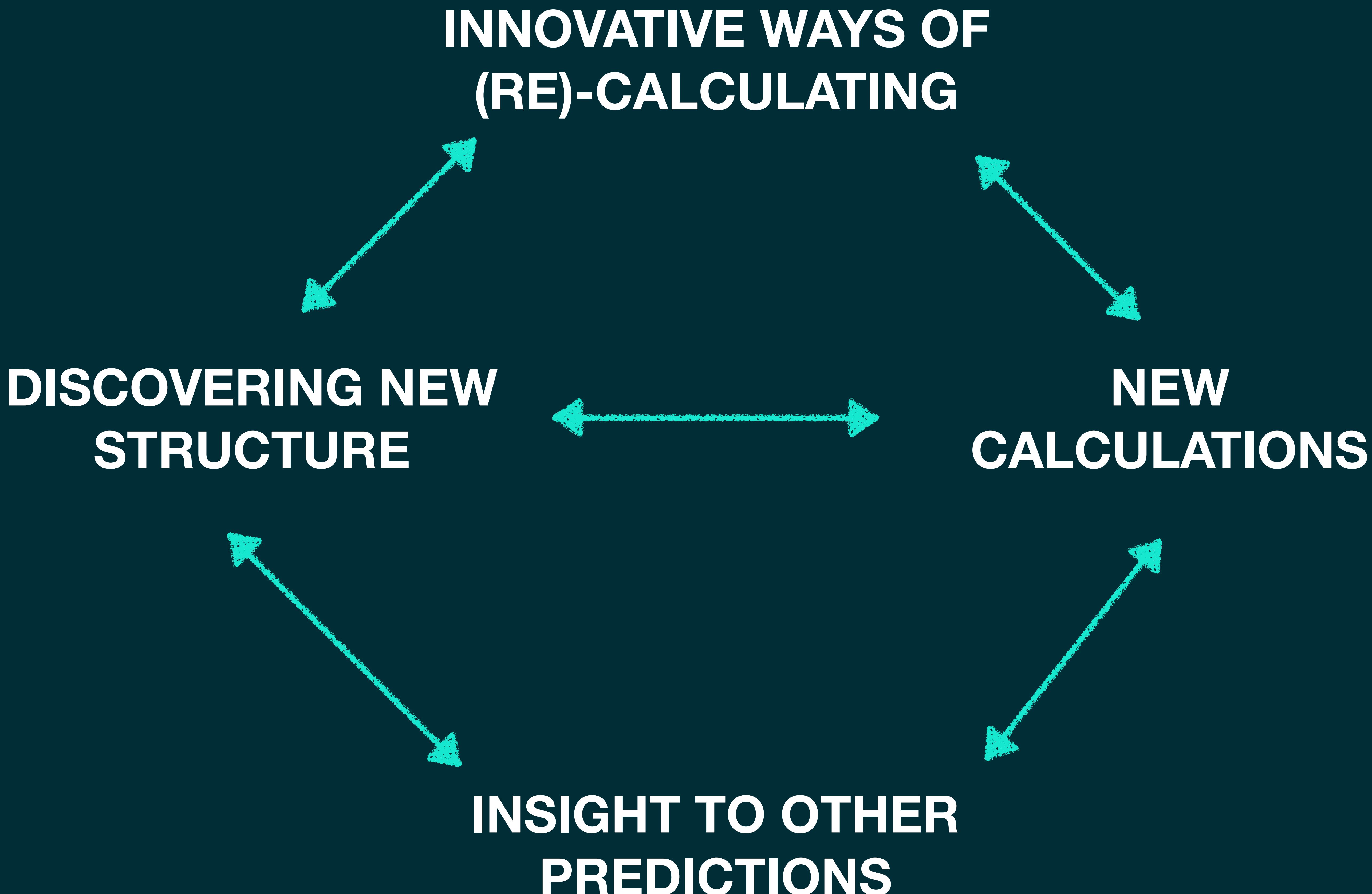
…; Gromov, Kazakov, Korchemsky, Negro, Sizov; Beisert, Garus, Rosso; Brandhuber, Hughes, Panerai, Spence, Travaglini; Aprile, Drummond, Heslop; Brandhuber, Kostacinska, Penante , Travaglini, Young; Korchemsky; Eden, Heslop, Mason; Bork, Onishchenko; Brandhuber, Hughes, Spence, Travaglini; Eden, Sfondrini ; Chicherin, Heslop, Korchemsky, Bourjaily, Heslop, Tran; Eden, Paul; Koster, Mitev , Staudacher; Sokatchev. Alday, Korchemsky;Beisert, Müller, Plefka, Vergu; Koster, Mitev, Staudacher, Wilhelm; Müller, Münkler, Plefka, Pollok, Zarembo; Kanning, Ko, Staudacher; Ferro, Łukowsk, Meneghelli, Plefka, Staudacher; …

MZV, Polylogs, …

…; Broedel, Matthes, Richter, Schlotterer; Henn, AV Smirnov, VA Smirnov; D'Hoker; Broedel; Green, Gurdogan , Vanhove; Broedel, Sprenger, Orjuela; Puhlfürst, Stieberger; D'Hoker, Green, Vanhove; Broedel, Mafra, Matthes , Schlotterer; Broedel, Schlotterer, Stieberger, Terasoma; Brown; …

Amplituhedron…

…; Arkani-Hamed, Thomas, Trnka; [[Ferro, Lukowski, Orta, Parisi]] ; Enciso; Dennen, Prlina, Spradlin, Stanojevic, Volovich; Ferro, Łukowski, Staudacher…


INNOVATIVE WAYS OF (RE)-CALCULATING
DISCOVERING NEW STRUCTURE
NEW CALCULATIONS
INSIGHT TO OTHER PREDICTIONS