New Approach to the theory of Gravity: Global Relativity

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Recently, it has been shown that general theory of relativity, in fact, can be independently derived without using Einstein's general relativity equations. In the approach, the two most used solutions - the Schwarzschild and Reissner Nordstrom solutions - have been derived explicitly. Here, an easy-to-understand self-contained introduction to the material is edited.

The approach, as a matter of fact, is not an alternative to the theory, but another way demonstrating a different understanding of the theory of gravitation. Yet, gravity through the basic ideas of the approach is very comprehensible.

INTRODUCTION

In general relativity (GR), the curvature of spacetime is directly related to the stress-energy tensor. This provides a rigorous geometric description for the gravitational phenomena. Yet, the theory is geometrically very complicated. Different attempts for alternatives have been made [1–3], yet the consistency with special relativity (SR) is confused, even with the theory of general relativity itself. In this review [4], we investigate deriving general relativity using only SR principles.

The main result of the approach is a new formula relating the spacetime interval with the stress-energy tensor, which self-sufficiently reproduces the geodesics found through GR such as the Schwarzschild and Reissner-Nordstrom metrics [7].

The results simplify the theory of gravitation very much, since the curvature in use is that of lines (not surfaces). Moreover, new insights into the quantitative description of quantum gravity are provided; the approach we present here fundamentally based on the notion of local coordinates, which can lead to new key-concepts about gravitational waves.

RIEMANN GEOMETRY

Riemann manifold is the global space on which Einstein equations solutions are represented. Each point, say p, in it corresponds to the center, say $O_l(p)$, of a local frame. Each local frame has its own local basis with respect to the gravity center. For a global observer at the gravity center, say O, this basis is the coordinates basis. The location in the global space is defined by the curved coordinates, say $\{x^\mu\}$, whereas in the local frames the flat coordinates, say $\{X^\mu\}$, are used instead. At a point M, the coordinates basis can be defined using the partial derivative of the global position with respect to the curved coordinates as $e_\mu = \partial_\mu \mathbf{OM}$. The coordinates basis is tangential to the lines of curved coordinates.

Einstein realized that local frames correspond to the case of SR, whereas the general motion in the global space (which corresponds to a continuous jumping between infinite Minkowski spaces) corresponds to the general case of the theory, from which the title general relativity. The basic idea of the theory is that, for a local observer in free-fall (moving along a given geodesic), the space with respect to him is Minkowskian. This called Einstein equivalence principle (EEP). Einstein equations are the constraints that define the geodesics, as such, define EEP.

The result of the theory is that: *energy distorts spacetime, and test particles located in spacetime follow distorted paths.*

GLOBAL RELATIVITY

SR is the local description of spacetime. For global observers, local frames (associated with test particles) change at each new point on the geodesic. This leads to the general case of the theory: *global relativity.*

In a local frame $O_l(p)$, the local observer measures the infinitesimal interval as:

$$ds^2 = g_{\mu\nu}(X) dX^\mu dX^\nu \tag{1}$$

where $g_{\mu\nu}(X)$ is the Minkowskian metric. With respect to the global observer (at the gravity center), each infinitesimal element in the coordinates is split as:

$$dx^\mu = f^\mu_\nu(x) dX^\nu \tag{2}$$

where $f^\mu_\nu(x) = \frac{\partial x^\mu}{\partial X^\nu}$. In the global space, the coordinates are curved, therefore the relations between the coordinates are not linear. Moreover, each coordinate is parametrized with the parameter of the embedded curve in the manifold of spacetime (geodesic) as $x^\mu(\tau)$. Furthermore, each coordinate can (generally) construct three planes, e.g. for x^1, we have $\{(x^1, x^2), (x^1, x^3), (x^1, x^4)\}$, therefore, each coordinate can generally construct three curves by eliminating the parameter τ between the couples, e.g. for $x^1(\tau)$, we have

$\{(x^1(\tau), x^2(\tau)), (x^1(\tau), x^3(\tau)), (x^1(\tau), x^4(\tau))\}$ Each couple corresponds to a curve. We use the polar coordinates for each couple, with the choice that the coordinate that construct the planes plays the role of the rho-coordinate, e.g. for $\{(x^1, x^2), (x^1, x^3), (x^1, x^4)\}$, the coordinate x^1 plays the role of the rho-coordinate. Those planes are used to parametrize the general form of the curve in the global space, hence they determine the explicit form of the curvature.

Since the coordinates X^μ are flat, we write

$$dX^i = \omega^i dT^i, i = \overline{1,3}. \tag{3}$$

That is, the theory is described with three constants, therefore

$$dx^\mu = f^\mu_\nu(x) dX^\nu = \left(g^\mu_\nu(X) + g^\mu_4(X)\omega_\rho \delta^\rho_\nu\right) dX^\nu \tag{4}$$

where $\omega_\rho = \frac{1}{\omega^\rho}, \omega_1 = 1$ (Note that $\{x^\nu\}$ locally reduce to $\{X^\nu\}$ and the relations between the coordinates $\{X^\nu\}$ are linear).

An important note to mention is that the differential elements used here are not infinitesimal in the mathematical sense, but in the context of EEP. That is, they correspond physically to sufficiently small regions of space.

Clearly, the quantities $\{f^\mu_4(x)\}$ correspond to curvatures of curved lines in 2d spaces, as such, they correspond to accelerations, therefore by multiplying those with the differential element of the time coordinate $d\tau$ we get the elements of velocity in the same/opposite direction of the μ-axis.

The quatity $\frac{ds}{d\tau}$ is invariant, as such, the square $g_{\mu\nu}(X)\frac{X^\mu}{d\tau}\frac{dX^\nu}{d\tau} = M$. Since the components of this four-vector are the velocity, thus by adopting constants for those components, we get a geometric stress-energy tensor. The physical interpretation of that tensor is simple: it represents locally the four-vector of impulsion-energy (caused by gravity) of the test particle under study.

Now, let us explore the physics as seen by the global observer. To proceed in that, we just apply SR principles. Considering the fact that the first SR postulate corresponds to the conservation of momentum, and the second corresponds to the constancy of the speed of light, one can get a generalized geometric stress-energy tensor, say $G_{\mu\nu}$, in which the equations of gravity lie in, via the equations

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu. \tag{5}$$

It is clear that $G_{\mu\nu}$ reduces to $M_{\mu\nu}$ locally. Considering the general case by adding the stress-energy tensor of ordinary matter gives the final form of the gravity equation:

$$ds^2 = \hat{G}_{\mu\nu} dx^\mu dx^\nu \tag{6}$$

where $\hat{G}_{\mu\nu}$ corresponds to the matter-geometry stress-energy tensor as it is measured by the global observer.

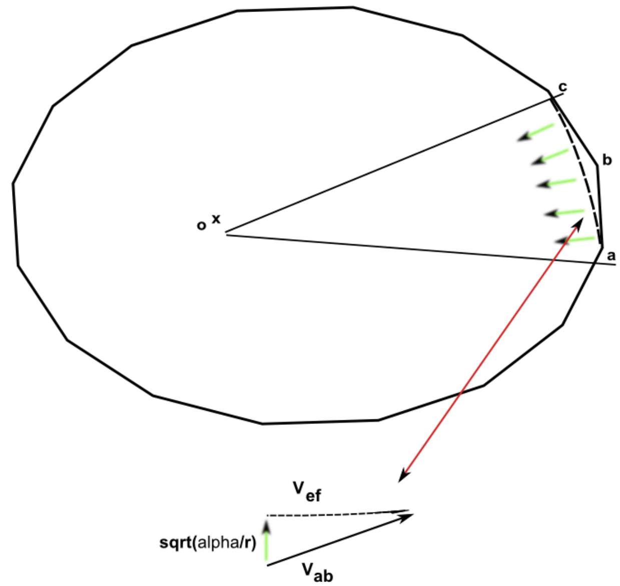

FIG. 1. Local and effective velocity as seen locally and globally

Evidently, the last equation defines the metric of space-time, together with the geodesic equations (note that the geodesics equations are closely to the local coordinate [5]) the curved paths can be determined explicitly.

THE SPHERICAL SYMMETRIC CASE

For a static spherical gravitational field, the parametrization of the geodesics must depend only on the radial dimension r. The corresponding orbits are the circular ones. Applying Einstein equivalence principle to the motion along those orbits together with the equations of geodesics will leads finally to the Spherical Symmetric metric of the manifold.

We start by describing the local velocity of our test particle v. Knowing that the velocity vector is tangent to the orbit and the change in this vector is proportional to $-\alpha/r$ (it comes from the derivation of the unit vector tangent to the curve), one is led to consider the effective local velocity (see Fig. 1), say v_{ef}, of the test particle. Using the theorem of Pythagoras, one get

$$v_{ef}^2 = v^2 - \frac{\alpha}{r}(\delta T)^2 \tag{7}$$

On the other hand, the local velocity is the derivative of the local position with respect to the proper time, that is

$$v = \frac{\delta R}{\delta T} \tag{8}$$

Which gives

$$(v_{ef}\delta T)^2 = (\delta R)^2 - \frac{\alpha}{r}(\delta T)^2 \tag{9}$$

On the other hand, the relation between the local position and local time is linear (local frames are flat)

$$v\delta T = \delta R \tag{10}$$

thus

$$(v_{ef}\delta T)^2 = (\delta R)^2 \left(1 - \frac{\beta}{r}\right) \tag{11}$$

where $\alpha/v = \beta$. Because v_{ef} corresponds to the effective coordinates, which are the curved ones, one write

$$v_{ef} = \frac{\delta r}{\delta t}. \tag{12}$$

This gives

$$(\delta r)^2 = (\delta R)^2 \left(1 - \frac{\beta}{r}\right) \tag{13}$$

which gives the space sector of Schwarzschild's metric

$$(\delta R)^2 = \frac{(\delta r)^2}{1 - \frac{\beta}{r}} \tag{14}$$

The time sector can be found easily from the space one. There are two methods to find that: (a) using spacetime diagram as in special relativity; or (b) using the constancy of the speed of light postulate. For simplicity, we follow the second choice.

Knowing that $\delta R \succ \delta r$ and the fact that the starting point (i..e the point a in Fig. 1) of the segment corresponding to δR and the final one (i..e the point c in Fig. 1) is the same locally and globally in the manifold, one can realize the only solution for time dilation that can lead to a mathematical consistency is that defined with the formula

$$(\delta T)^2 = (\delta t)^2 \left(1 - \frac{\beta}{r}\right) \tag{15}$$

The interval of the local spacetime has the form

$$(\delta S)^2 = -(\delta T)^2 + (\delta R)^2 \tag{16}$$

By substituting the above results, we get

$$(\delta S)^2 = -(\delta t)^2 \left(1 - \frac{\beta}{r}\right) + (\delta R)^2 + \frac{(\delta r)^2}{1 - \frac{\beta}{r}}, \tag{17}$$

The last equation is exactly Schwarzschild's interval of the spherical symmetric spacetime.

Note that the local intervals δt and δr correspond the Einsteinien sufficiently small regions of space mentioned in the equivalence principle, that is globally those small intervals look mathematically infinitesimal, thus we write

$$\delta s \approx ds, \tag{18}$$

$$\delta r \approx dr, \tag{19}$$

$$\delta t \approx dt \tag{20}$$

The above metric becomes

$$(dS)^2 = -(dt)^2 \left(1 - \frac{\beta}{r}\right) + (dR)^2 + \frac{(dr)^2}{1 - \frac{\beta}{r}} \tag{21}$$

This completes the derivation of the spherical symmetric spacetime interval.

REISSNER-NORDSTROM SOLUTION

In the case of gravitational field of a charged, non-rotating, spherically symmetric gravity source, the metric is extracted by taking into account the Hamiltonian terms $T_{\varphi\varphi}$ or $T_{\theta\theta}$ of the electric charge q cosmological constant Λ. Their Hamiltonian terms are given by [6] $T_{\varphi\varphi} = -r^2\Lambda$, $T_{\varphi\varphi} = \frac{\chi}{r^2}$ (for Λ and q respectively) where χ is a constant.

The geometry-matter stress-energy tensor $\hat{G}_{\mu\nu}$ can be written in its eexplicit form as

$$\hat{G}_{\mu\nu} = G_{\mu\nu} + T_{\mu\nu} \tag{22}$$

The first term (i.e. $G_{\mu\nu}$) corresponds to the contributions of the Minkowski local frame and the induced spacetime; the second one (i.e. $T_{\mu\nu}$) corresponds to the usual matter stress-energy tensor. This analysis leads to the formula

$$(dx^r)^2 = (1 - \frac{1}{r} + \frac{\chi}{r^2} - \Lambda r^2)(dX^r)^2 = K(r)(dX^r)^2 \tag{23}$$

For the element of time, we proceed as above, evidently this gives

$$(dx^t)^2 = \frac{1}{K(r)}(dX^t)^2 \tag{24}$$

By replacing those results in Eq.(16), we get the solution of Reissner-Nordstrom

$$ds^2 = -(dX^t)^2 + (dX^r)^2 + (rd\theta)^2 + (r\sin(\theta)d\varphi)^2 = -K(r)(dx^t)^2 + \frac{1}{K(r)}(dx^r)^2 + (rd\theta)^2 + (r\sin(\theta)d\varphi)^2 \tag{25}$$

REISSNER-NORDSTROM SOLUTION AS A SCHWARZSCHILD-LIKE SPACETIME

Here, we show that the Solution of Reissner-Nordstrom, in fact, is just a Schwarzschild-like one in different coordinates. For simplicity, let us suppose:
(a) $\alpha = 0$.
(b) $r = r_0$ where r_0 is a constant.
(c) the radial dimension r is replaced with a $U(1)$-variable, say $\acute{\varphi}$.
The Schwarzschild solution with the new coordinates $(\acute{\varphi}, t)$ can be written as

$$(dS)^2 = -(dt)^2 \left(1 - \frac{\beta}{\acute{\varphi}}\right) + (d\acute{\varphi})^2 \frac{1}{1 - \frac{\beta}{\acute{\varphi}}} + \acute{\varphi}^2 d\varphi^2 \quad (26)$$

When passing to Schwarzschild's coordinates (r, t), the constant of the geodesic β becomes depending on the factor $\frac{1}{r}$, i.e. $\beta = \frac{v}{r}$, where $v = \frac{dr}{dT}$. Moreover, the curvature is negative. Working with $\acute{\varphi} = \frac{r}{r_0}$, one can get

$$(dS)^2 = -(dt)^2 \left(1 + \frac{\frac{v}{r}}{\frac{r}{r_0}}\right) + (d\frac{r}{r_0})^2 \frac{1}{1 + \frac{\frac{v}{r}}{\frac{r}{r_0}}} + (\frac{r}{r_0})^2 (d\varphi)^2 \quad (27)$$

Denoting $\frac{r}{r_0}$ by $\acute{r}$ gives

$$(dS)^2 = -(dt)^2 \left(1 + \frac{vr_0}{\acute{r}^2}\right) + (d\acute{r})^2 \frac{1}{1 + \frac{vr_0}{\acute{r}^2}} + \acute{r}^2 (d\varphi)^2 \quad (28)$$

The last equation is exactly Reissner-Nordstrom's spacetime interval. This proves that Reissner-Nordstrom's spacetime is just a Schwarzschild-like one in different coordinates.

CONCLUSION

Einstein equivalence principle is the geometric realization of special theory of relativity principles.

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